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The fractal modelling of turbulent surface-layer winds

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A dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy, The University of Auckland, 1999
Mt Cook, the highest mountain in the Southern Alps of the South Island of New Zealand, and setting for some of this research.

'But why some say the moon? Why choose this as our goal? ... and they may well ask, why climb the highest mountain? Why, 35 years ago, fly the Atlantic? ...

We choose to go to the moon, we choose to go to the moon in this decade and do the other things, not because they are easy, but because they are hard ...'

United States President
John Fitzgerald Kennedy
September 12, 1962
Abstract

Multiscaling analysis and cascade simulation techniques, which form part of the more general field of fractals, are introduced as a method for characterising and simulating surface-layer winds, particularly for time scales associated with the energy-containing range. This type of analysis consists of determining the power-law parameter of the spectrum of the data, and the scaling of the statistical moments. These techniques were applied to determine how the statistics depended on the duration (or scale) of the fluctuations in wind speed, the atmospheric conditions, and the topography of the site. It was found that the parameterisations produced using multiscaling analysis characterised differences in the statistics for each of these cases. Furthermore, the fractal cascade simulation techniques used provided simple methods for reproducing these statistics. This analysis is followed by an investigation into the robustness of some of these results. In particular, the data is examined for the existence of self-similar distributions of the cascade weighting factor, W. Such self-similar analysis allows the direct simulation of the data via a cascade. Cascade models have the virtue of being able to reproduce statistical properties such as intermittency, and in particular, the nesting of intermittency from different wavenumber bands in the same region of space. The existence of these properties in both the experimental and simulated data is investigated, with consideration given to the consequence of the results for simulation techniques. One notable discovery is the failure of these methods to reproduce the bias in the distribution of the gradients in the wind velocity field. This result has important implications for all workers dealing with simulation of geophysical data by fractal cascades. Finally, a brief numerical experiment is carried out to both demonstrate how this bias may be exploited to construct a model, and to test some of the analysis techniques presented on non-cascade based data. While not a particularly convincing simulator of turbulence, the model nevertheless displays some interesting turbulence-like characteristics.
Preface

This thesis is an account of nearly four years' work toward the degree of Doctor of Philosophy in physics. As such, it contains much work which has either been published during this period or is due to be published in international refereed journals. The work is presented in an order which I feel appropriate, rather than as a collection of papers. Certain chapters contain large portions taken more-or-less directly from the papers, particularly chapter 2, based extensively on Lauren et al [1998b], and chapter 3, based on Lauren et al [1998a]. Additionally, chapter 4 contains various results which appeared in different contexts in Revell et al [1996], a paper for which I was a co-author, and Lauren et al [1998b]. At the time of writing, a fourth paper was being planned dealing more directly with the issue of the bias in the distribution of the gradients of the velocity field and other multiscaling geophysical signals, and its implications for both the structure function and the spectrum of such data.

Michael K. Lauren

November 14, 1998
Acknowledgements

The work for this thesis was largely conducted as a stand-alone project, which presented a slightly double-edged sword, in that while I had considerable independence in my pursuits, they lacked the financial backing and infrastructure of a large-scale project. For this reason I am grateful for the support offered by several groups, particularly the “fractal” group within the Atmospheric Physics Group of the Department of Physics, our departmental technicians, as well as those from the faculty of engineering who were always so cheerful when lending me their expensive equipment, and the group from NIWA with whom I was lucky enough to be involved in a joint publication. In particular, I would like to thank Geoffrey Austin for his excellent supervision, and for rescuing me from a tight spot when other supervisory options disappeared; Alan Seed for his constant support during the experimental set up; Merab Menabde, who provided the theoretical platform on which the work in this thesis is based, and who also played an excellent support role in my supervision; Daniel Harris, who was always happy to answer questions and offer advice on multiscaling analysis techniques, and who also provided some of the computer code used in the analysis; Brian Watson and Darryl Visser, who managed to keep a sense of humour when I brought back their hot-wire anemometer in pieces; Mike Revell and Don Purnell for getting me involved in their project, particularly Mike, with whom I made an expedition to Mt Cook to gather data; and finally Janetta Mackay, who proof read this work.
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1. Introduction

1.1 Origins and context to the work

This thesis principally concerns itself with the use of a relatively new type of mathematical analysis for geophysical data — multiscaling analysis. This kind of analysis goes hand-in-hand with so-called fractal cascade models, which are a practical and powerful tool for simulating various natural signals. Consequently, the kind of analysis conducted here not only has the possibility of characterising experimental data, but also of producing simulated data with indistinguishable statistical properties from nature.

The study of multiscaling, or more generally multifractal, models within the atmospheric group at the University of Auckland began with the arrival of Geoffrey Austin and Daniel Harris from McGill University in Canada, leading to work on the multiscaling analysis of rain fields. They were soon joined in their efforts by Alan Seed, with much expertise on rainfall from his work in South Africa, and Merab Menabde, whose mathematical knowledge, acquired in the former Soviet Union, gave the group a sharp theoretical edge.

Rainfall proved to be a successful choice of study for the group. Although the cascade models employed were developed originally to describe turbulent fields, turbulence is already extensively reported on in the literature. Additionally, much of the thrust in the atmospheric group had been in the area of describing rain processes, and these endeavours had provided a wealth of data to work with.

However, the use of cascade models to describe rain fields has one serious flaw: that there is no apparent justification for doing so. The formation of rain is reasonably well understood, and though the processes are often strongly non-linear, there is no particularly obvious reason why rain fields should behave in a cascade-like way. On the other hand, cascade models for turbulence can be phenomenologically justified. It would seem reasonable then to suppose that turbulence might play a role in causing rain to exhibit cascade-like behaviour.
To this end, the work of the group logically must return to the issue of turbulence.

Turbulence, of course, has been studied by many highly regarded researchers, including the likes of Reynolds, Richardson, Kolmogorov, Taylor, Batchelor, Novikov, Steward, Obukhov, Monin, Yaglom and Mandelbrot, to name some, so that the likelihood of making an advance these workers have not is small. However, there appears to be some room to extend upon their work, particularly in the application of cascade models to the simulation of surface winds.

In this regard, one of the objectives of this thesis is to look beyond the areas which these workers have studied. In practical terms, this means looking beyond the inertial range to the energy-containing range.

Workers have tended to concentrate, for example, on the dissipation field, largely due to the important role given it by Kolmogorov’s revised similarity hypothesis (see later sections for details). However, for most engineering-type considerations, the simulation of the wind field itself is of more interest.

For all practical purposes, it is only necessary to use these models as a statistical method, ignoring their relationship with the physical processes. This certainly applies to rain data, for which neither a theory equivalent to Kolmogorov’s exists, nor is it even intuitively clear that such a phenomenological cascade model is appropriate. Given the increasingly widespread use of such models to simulate geophysical data, it seems reasonable to try this approach with the energy-containing range for turbulence, and much of this thesis proceeds on this basis.

Numerical models which try to simulate turbulent wind signals based on closure schemes for the Navier-Stokes equation, often for engineering-type purposes, usually fail to reproduce the scaling properties of experimentally measured signals, and are not helped by the limitations placed upon them by grid scale. Cascade models, by contrast, can accurately reproduce such signals without being computationally expensive, so are potentially a powerful modelling tool, particularly if they can be made to reproduce, say, fluctuations in wind signals on time scales which might be useful for engineering purposes.

Initial experimentation conducted for this thesis cleared some misconceptions and revealed a few surprises. For example, most of the workers spoken to in this
country generally believed that turbulent spectra possessed only a $-5/3$ spectral power law, in keeping with Kolmogorov's theory, yet the first investigations with a cup anemometer revealed power laws of much flatter slopes of around 1. As it turned out, this observation in itself was no revelation in terms of previous workers' studies. Because the cup anemometer collects data with a reasonably similar temporal resolution to the rain gauges used in the study of rain fields, it soon became clear it could be used to investigate the possibility of applying multiscaling analysis to turbulence on these time scales, which were generally much larger than inertial-range scales. This in turn might lead to a better understanding of the meteorological processes occurring on these larger scales, and eventually an understanding of the origin of the multiscaling character of rainfall (although, this is beyond the scope of this thesis). Being able to use a cup anemometer to study the energy-containing range made life a lot easier, because the finer-resolution scales characteristic of the inertial range tend to have to be measured using a fast-response (and therefore invariably more expensive and fragile) device, such as a hot-wire anemometer. Experiments with a hot-wire anemometer showed that the inertial range $-5/3$ spectrum broke down for temporal scales larger than about 1s in the surface layer at the site used, while the limit to which tests showed the cup anemometer could resolve spectra accurately was about 5s. Thus the cup anemometer could measure fluctuations in the wind signal for the majority of the energy-containing range.

The work contained in this thesis largely concentrates on two practical themes, firstly, whether or not there is a range of scales for which inertial-range statistics do not apply yet nevertheless is suitable for description by "cascade" statistics, and secondly, how surface winds can be characterised and simulated using these descriptions. Before describing the aims of this thesis in more detail, a brief account of the theory is given.

1.2 An introduction to the necessary theory

1.2.1 Looking for order in chaos

In spite of the fact that all of us have to contend with surface-layers winds virtually every day, there are still many grey areas in our understanding of the
turbulent fluctuations they convey. Needless to say, turbulence appears to be unpredictable in nature.

A great deal of work has been done this century in an effort to describe turbulence and much progress has been made, thanks largely to the development of such fields as non-linear dynamics, fractal geometry, and the study of complex systems.

In the study of the dynamics of turbulence, it is necessary to think about what sort of processes are important to the description. For example, turbulence-producing fluid shears generated in narrow pipes, or even over surfaces such as aircraft wings, are clearly dependent on the frictional drag of the fluid. The nature of friction — a subject which is only slowly becoming better understood (as is evident from articles such as Krim [1996] on the mysteries of friction) — is such that it might be supposed that the quirks of the small-scale frictional processes play a role in the unpredictability and non-linearity of the dynamics in these situations. At the other end of the scale, cyclone systems are of such a large scale that it is likely that small-scale frictional effects are unimportant, in which case, the non-linearity of the system comes from elsewhere.

Thus, in the study of turbulence, the scale being dealt with is crucial.

One of the most significant advances in the understanding of turbulence has been Kolmogorov's hypotheses (Kolmogorov [1941], [1962]). These suppose that for some range of scales, turbulent motion becomes essentially independent of the boundary conditions of the system. This range, known as the inertial range, was conceived as a transitional range between the input of energy into the system due to large-scale forcing (such as eddy-shedding from obstacles in the flow and thermal convection) and molecular processes which act over small scales to dissipate turbulent kinetic energy.

This picture of an energy "cascade," in which large-scale eddies shed energy to eddies of increasingly smaller scale until the dissipation range is reached, led to cascade models for turbulence [e.g. Novikov and Steward 1964]. In such a scheme, it is envisaged that the cascade process becomes the dominating feature of the dynamics, so that the boundary conditions imposed on the largest-scale of the
eddies (which introduce energy into the flow) are "forgotten" about by the smaller-scale eddies.

These models allowed turbulence to be characterised in terms of fractal geometry, a subject popularised by works such as Mandelbrot [1983]. Fractal descriptions may be used for a vast array of natural systems, with cascades just one type of fractal model. While the definition of a fractal remains somewhat unclear [see Saucier 1991], generally speaking, a fractal is a discontinuous function which displays structure on all scales, and usually exhibits at least qualitative self-similarity — that is, when examined on increasingly smaller scales, the fractal resembles miniature copies of itself.

Fractals, therefore, are mathematical abstractions which no natural system exactly mimics, simply because such systems usually possess terminal scales at which self-similarity breaks down (e.g. the dissipation scale in inertial-range turbulence).

While fractals are one convenient method for describing such systems, advances have also been made in the understanding of non-linear systems. Lorenz [1963], for example, invented a system of equations to describe convective turbulence. As it turns out, the phase-space trajectories of such systems of equations lie on a fractal set. Any given trajectory never intersects with another, and two neighbouring trajectories diverge exponentially quickly [see Tsonis and Elsner 1989 for more detail]. Yet, the trajectories seem to be drawn towards a "strange attractor," and these unusual objects usually require fractal geometry to describe them. Thus, fractals are not only useful tools for simulating turbulent signals, they are also necessary to describe them.

1.2.2 Kolmogorov's similarity hypothesis

In a 1941 paper Kolmogorov proposed two similarity hypotheses regarding the form of the velocity correlation function in isotropic turbulence of high Reynolds number.

The first hypothesis was that locally isotropic turbulence distributions are uniquely determined by the quantities $\nu$, the molecular viscosity, and $\bar{\varepsilon}$, the average energy dispersion per unit volume. Here, locally isotropic turbulence is a reference to an assumed property of small-scale turbulence, introduced by Taylor
and modified by Kolmogorov, which is that at these scales the turbulence is sufficiently decoupled from large-scale turbulence (usually anisotropic), that large-scale forcing due to the boundary conditions of the system do not affect the small-scale dynamics. Kolmogorov's hypothesis implies the existence of small-scale turbulent processes for which the boundary conditions are unimportant, so that this type of turbulence has essentially a universal structure.

Kolmogorov used a co-ordinate transformation, constructing units of time and distance from the quantities \( \nu \) and \( \overline{\epsilon} \) (with units of \((\text{length})^2 \cdot \text{(time)}^{-1} \) and \( l^2 \cdot t^{-3} \) respectively), to produce the well-known Kolmogorov microscales, defined as:

\[
\eta = \frac{\nu^{3/4}}{\overline{\epsilon}^{1/4}}, \quad \tau_0 = \left( \frac{\nu}{\overline{\epsilon}} \right)^{1/2}, \quad u_0 = \left( \frac{\nu}{\overline{\epsilon}} \right)^{1/4}
\]

where \( \eta \) is the "inner" or "dissipative" scale of the turbulence, and has units of length, \( \tau \) units of time, and \( u_0 \) units of velocity.

Kolmogorov's second hypothesis supposes that the velocity structure functions, \( < |\Delta u(l)|^h > = < |u(x_0+l) - u(x_0)|^h > \), are independent of \( \nu \) when \( l >> \eta \), and depend only on \( l \) and \( \overline{\epsilon} \). Note that in the context of the structure function, \( l \) is the spatial separation between two points. Dimensional arguments then imply:

\[
< |\Delta u|^q > = C(\overline{\epsilon} l)^{\nu/3}
\]

where the angled brackets represent an ensemble average, and \( C \) is a constant.

When \( q = 2 \), the Fourier transform of equation 1.1, by virtue of the Wiener-Khinchine theorem, implies the form of the energy spectrum is such that:

\[
E(k) = C_{K} \overline{\epsilon}^{2/3} k^{-5/3}
\]

where \( C_{K} \) is the Kolmogorov constant, which has a value of approximately 0.55. Once again, it is assumed that equation 1.2 is a universal property of locally isotropic turbulence. There is a great deal of empirical evidence which supports equation 1.2 (e.g. Grant, Stewart and Molliet [1962], Obukhov [1961]). However, equation 1.1 does not appear to hold experimentally for \( q > 2 \), and instead has the form \( < |\Delta u|^q > \propto \zeta^q \), where \( \zeta \) is a non-linear function of \( q \) (Anselmet [1984]).

The range of wavelengths which obey equation 1.2 can conveniently be defined to be the inertial range. Phenomenologically speaking, it is imagined that this range acts as an energy cascade, as discussed above, which terminates at a
dissipation scale, when the eddy sizes are of a scale of the order of \( \eta \). This “energy cascade” process has been modelled phenomenologically by various workers who suppose that eddies of each scale \( l \), where \( \eta < l < \Lambda \), shed their energy to daughter eddies of some scale \( l/n \), where \( n \) is usually an integer for the purpose of cascade models, and \( \Lambda \) is the “outer scale” of the scaling region of the spectrum.

Such an eddy energy cascade picture borrows heavily from Richardson’s earlier hierarchical model of turbulence. However, Kolmogorov’s theory quantitatively captured the concept of a range of velocity distributions which depend only on the cascade process itself, rather than the mean flow and boundary conditions, and correctly predicted the form of the spatial correlations and their scaling properties, at least for \( q = 2 \).

Note that Kolmogorov’s 1941 paper, however, does not explicitly require a cascade process. The cascade models themselves grew out of the consequences of a number of objections to Kolmogorov’s 1941 theory, notably from Landau [Landau and Lifshitz 1959] as to the universality of the constant \( C_K \), who pointed out that the law in equation 1.2 is not invariant with respect to the composition of the statistical ensembles, since the left-hand side is an average, while the right-hand side has a power of 2/3 of an average.

Objections such as Landau’s lead Kolmogorov [Kolmogorov 1962] and Obukhov [1962] to modify the 1941 theory in a way which led to a more explicit phenomenology involving an energy cascade based on \( \varepsilon \), which plays the role of a measure of the energy flux between scales. The 1962 theory replaced the quantity \( \bar{\varepsilon} \) with a local spatial average \( \varepsilon_v \), interpreted as the local energy flux, such that:

\[
\varepsilon_v = \int \varepsilon (x+y,t) \, d^3y
\]

and equation 1.1 is replaced by the Kolmogorov-Obukhov law:

\[
<|\Delta u|> \propto (\varepsilon_v l)^{1/3}
\]  

Because the scaling properties of \( \varepsilon_v \) are not known, it is not clear from equation 1.3 what form the energy spectrum should take, without the use of further assumptions. Obukhov imagined an ensemble of flows within the volume such that \( \varepsilon_v \) has a fixed value (Obukhov [1962]). The 1941 theory is then assumed to hold for this ensemble.
However, the revised theory raises further questions, particularly because it is not clear why $\varepsilon_V$ should have such an important role. Though cascade models assume that the energy flux (taken to be the dissipation rate in some volume of space) associated with each eddy is shed to daughter eddies, that contain some fraction of the energy flux of the original, the physical mechanism responsible for this shedding of energy flux is not known. Nor is it clear that this splitting of dynamical quantities into daughter eddies actually occurs, or is even possible (Kraichnan 1974). Even the use of $\varepsilon_V$ as a local energy flux is somewhat controversial (Saucier 1991). Since the dynamical quantity responsible for the flow of energy from larger-scale fluctuations to smaller scales is the spectral flux, which is not, in general, equal to $\varepsilon_V$ (Kraichnan 1974). Nevertheless, this interpretation is currently used by many workers (e.g. Anselmet [1984], Schmitt et al [1993], Meneveau and Sreenivasan [1990, 1987a, 1987b], Novikov [1990]).

In this thesis, not much concern is paid to these issues, as it is the intention here to address the practical issues of modelling surface winds using cascades. Rather, use is made of the cascade models which evolved as a direct consequence of the 1962 theory, with the principal aim to determine their usefulness in characterising and reproducing turbulent signals in the surface layer.

1.2.3 Overview of fractal and multiscaling statistics

To give a full account of fractals would require a small novel. In fact, this is what Benoit Mandelbrot [1983] has done, and it is perhaps he who has the best claim to popularising these mathematical abstractions, as well as coining the term "fractal". Even so, the field of fractals remains in its infancy, particularly in terms of its applications to dynamical systems, with many aspects still controversial.

Here, the discussion is confined to just those aspects of fractals and their related statistical properties which will be used to analyse data in the later chapters. As mentioned, even the definition of a fractal varies from author to author. To steer clear of such debates, definitions are given just to the extent that they are necessary for the analysis. For this work, a curve is defined to be a fractal if it possesses a fractal dimension. This kind of dimension is effectively a measure of the degree to which a curve "meanders" between two points in space. Perhaps the best illustration of this concept is given by the work of Richardson (discussed...
in Mandelbrot [1993]). Richardson noted that measurements of Britain’s west coast and the Spanish-Portuguese land frontier depended heavily on the scale of the map used. It is not hard to see why. As the resolution of the map increases, more and more bays and outcrops appear, which add to the length of the coastline.

Now suppose we measure the coastline with a yard stick of length $a$. If the yardstick is used $N$ times, the total length of the coast is $s = Na$. Mandelbrot then gives a convenient definition of the fractal dimension:

$$D = \lim_{a \to 0} \frac{\log N}{\log \left( \frac{1}{a} \right)}$$

If $D$ is a non-integer, then the coastline is a fractal. If this is so, then the length of the coastline goes as:

$$s = \left( \frac{1}{a} \right)^{D-1}$$

that is, $s$ depends on the scale, or resolution, $a$, at which it is measured, and that dependence is a power law. In the case of the wind signals studied in the present work, it is the dependence of the statistical properties of the signal — such as the value of its statistical moments — on the resolution which is of interest.

One simple example of a mathematical abstraction with a fractal dimension is the Koch curve. This is constructed by starting with a line segment, removing the middle third and replacing it with an equilateral triangle, as in figure 1.1. This is repeated for each of the four new line segments. After several iterations, the result looks like figure 1.2, and so on. The analogy with the coastline is clear, as the curve is examined on increasing smaller scales, ever smaller outcrops become visible, adding to its length. Using equation 1.4, the fractal dimension for the Koch curve is approximately 1.26.

Generally speaking, fractals scale. This means that the curve or signal examined at increasingly small scales appears to be a miniature version of the large scale. This is clearly the case for the Koch curve, but it is also true of less-regular, unpredictable systems, such as inertial-range turbulence.
Figure 1-1: Construction of the Koch fractal.

Figure 1-2: A Koch “island”.

Figure 1-3: The signal at the bottom is a “zoom” in on the first quarter of the signal at the top. Note that the zooming reveals ever finer-scale features.

Figure 1.3 shows an inertial-range turbulence wind velocity signal for two different scales to demonstrate the point. Zooming in on the signal reveals ever more fine-scale structure, just as is the case for a coastline. The scale dependence of inertial-range turbulence is embodied in equation 1.1, and the power-law spectrum that results consequently.

Such power-law spectra are a direct manifestation of scale invariance, although, as will be discussed in the later chapters, real geophysical data does not possess indefinite scaling ranges. Also, since this kind of spectrum has no characteristic time scale, the data may be statistically self similar.

The existence of such scaling is a feature of complex systems, such as weather, whose dynamics are chaotic but governed by the existence of strange attractors (Tsonis and Elsner [1989]). The behaviour of the dynamical equations of these systems in state space is such that the trajectory describing their evolution never repeats itself (since it is non-periodic), and never crosses itself. If a strange attractor exists, the trajectory is confined to a finite region of the state space, but
must be of infinite length. In that case, the attractor is not a topological manifold, but a fractal set. Consequently, dynamical quantities in such system are also describable in terms of fractal indices. The spectral power-law slope, and the slope of the scaling of the moments, are two such indices.

More generally, it should be expected that other complex systems besides inertial-range turbulence should exhibit such behaviour. This is certainly the case for rain distributions, for which no equivalent phenomenological model to that of Kolmogorov's exists, yet cascade models have been successfully used to characterise their statistics [Schertzer and Lovejoy 1987, Lovejoy and Schertzer 1995, Over and Gupta 1994, Gupta and Waymire 1993]. Additionally, in the case of rain, at least one study [Harris et al 1996] suggests that the model parameters necessary to describe the statistics are not universal.

Even for inertial-range turbulence, it is not clear that Kolmogorov's theory must be correct, particularly the 1941 theory. As mentioned above, equation 1.1 does not hold for moments of higher-order than the second. This behaviour introduces the concept of multiscaling statistics. The 1962 theory does not describe the behaviour of the spectrum either unless \( \langle \varepsilon V^3 \rangle \) is known. As it turns out, experimental evidence shows that \( \varepsilon V \) is extremely inhomogeneously distributed in space (see Batchelor and Townsend [1949], Kuo and Corrsin [1971]). The distribution of \( \varepsilon \) has been attempted to be modelled phenomenologically using various fractal cascade models.

These models were necessary in order to reproduce other important properties of inertial-range turbulence, those of intermittency — that is, that the largest fluctuations in the small-scale structure of turbulent signals tend to be clustered in small spatial regions, while most of the rest of the space is relatively empty of strong fluctuations — and "multiscaling" statistical moments (see Davis et al [1994]).

For example, Yaglom [1966] developed a cascade model based on \( \varepsilon V \), which considers an interval, \( \Lambda \), over which the local energy flux \( \varepsilon V (\Lambda) \) is uniformly distributed.
Figure 4: An example of a cascade model (from Davis et al [1994]).
Figure 5: An illustration of how intermittency in data causes the statistical moments to scale. In (a)-(c), the data \( \varepsilon(l;x) \) is averaged locally over every other neighbouring pair of points, then globally averaged. Clearly the global average in each case will be the same. On the other hand, (d)-(f) show the same procedure, except prior to averaging, \( \varepsilon(l;x) \) is taken to the power of \( \pi \) (i.e. we are exploring the scaling of the \( \pi \)th-order moment). Clearly, this moment is scale dependent. Plotting \( \langle \varepsilon(l;x) \rangle \) as a function of \( l \) and \( \pi \) on a log-log graph produces a straight-line, the slope of which gives \( K(\pi) \). (From Davis et al [1994]).
On the first step of the cascade, the interval is split in half (if the branching number is two), and each half is assigned a new value for energy flux, $\varepsilon_V(\Lambda/2) = W \varepsilon_V(\Lambda)$, where $W$ is a weighting factor taken from some random distribution. The process is then repeated on each of the new intervals, where the $W_i$ for each step are statistically independent and identically distributed.

Thus:

$$\varepsilon_V(\Lambda/2^n) = W_1 W_2 \ldots W_n \varepsilon_V(\Lambda)$$  \hspace{1cm} (1.6)

with the constraint that $< \varepsilon_V(\Lambda/2^n) > = \varepsilon_V(\Lambda)$, which implies $< W > = 1$.

Equation (1.6) leads to:

$$\left< \varepsilon_V(\Lambda/2^n)^q \right> = \left< W^q \right> \varepsilon_V(\Lambda)^q = \left( \frac{l}{\Lambda} \right)^{-K(q)} \varepsilon_V(\Lambda)^q,$$

$$K(q) = \log_2 \left< W^q \right>, \quad l = \Lambda/2^n$$  \hspace{1cm} (1.7)

When $K(q)$ is a linear function of $q$, equation 1.7 has a similar form to Kolmogorov's original suggestion, formulated in equation 1.1. However, intermittency results in statistics for which $K(q)$ is a non-linear function. The field $\varepsilon_V$ is then said to possess multiscaling statistics. Thus $K(q)$ is a key indicator of the degree of intermittency of the field. The co-dimensional parameter $C_1$ is popularly used as an intermittency parameter (see Davis et al [1994]), and is defined as the first derivative of $K(q)$ evaluated at $q = 1$. Generally speaking, the greater the degree of curvature of the $K(q)$ function, the more intermittent the field.

Figure 1.4 shows an example of a cascade generation process. A lognormal multiplicative cascade is shown (i.e. $W$ has a lognormal distribution), with a branching number of three (i.e. each interval is split into three for each iteration). Figure 1.5 demonstrates how multiscaling statistics arise from such intermittent data (both taken from Davis et al [1994]).

1.2.4 Link between multiscaling statistics and the fractal dimension

The multiscaling description in equation 1.7 can be formulated in a different way in terms of the concept of a fractal dimension introduced in equation 1.4 (see Menabde [1997]), by use of the concept of a “multifractal” [Parisi and Frisch 1985, and Halsey et al 1986].
Take an interval \( \Lambda \) partitioned into pieces of size \( l_0 \) and define a scaling exponent \( \alpha_j \) such that:

\[
\varepsilon_j \sim l_0^{\alpha_j}
\]

where \( \varepsilon_j \) is the average dissipation on the \( j \)-th interval, \( l_0 \). Assuming that the set of intervals with the same \( \alpha_j \) has the fractal dimension \( f(\alpha_j) \), and so is "multifractal", we get:

\[
\left\langle \varepsilon_i(\lambda)^q \right\rangle \sim \sum_j l_0^{q\alpha_j} l_0^{-f(\alpha_j)} \sim \int d\alpha l_0^{q\alpha - f(\alpha)}
\]

Since \( t_0 \) is very small, the integral can be approximately evaluated by the steepest descent method [Halsey et al 1986], yielding:

\[
\left\langle \varepsilon_i(\lambda)^q \right\rangle \sim t_0^{-K(q)}
\]

where \( K(q) = f(\alpha(q)) - \alpha(q)q \) and \( \alpha(q) \) is defined by the external condition:

\[
d / d\alpha(q\alpha - f(\alpha)) = 0
\]

That is to say, by adopting this hierarchy of fractal indices (multifractal model), we produce the same form for the statistics as obtained using equations 1.6 and 1.7.

1.2.5 Characteristics of atmospheric surface-layer spectra

Spectra of boundary-layer wind velocity usually consist of three parts: the energy-containing range, the inertial range and the dissipation range (see Kaimal and Finnigan [1994]). For the purposes of this thesis, the scales were mostly discussed using units of time or frequency. Although more usually scales in turbulence are measured in terms of wavenumber, \( k \), the use of units of time is the more natural way of thinking about the resolution capabilities of the instruments used here. This is important when considering which part of the spectrum is being examined by which instrument. However, the time scales discussed can easily be converted into wavenumber if desired by use of Taylor's hypothesis, such that:

\[
k = 2\pi f / U
\]

where \( f \) is frequency and \( U \) is the mean wind speed. Mean wind speeds for most of the data used were around 6 to 7ms\(^{-1}\). The typical turbulent intensity range (standard deviation/mean) for the hot-wire anemometer data was 0.35.
Figure 1-6: This plot of spectra of the $x$ component of the wind velocity, taken from Kaimal et al. 1972, shows the behaviour of the spectra with varying values of $z/L$.

Figure 1-7: Kaimal's proposed scheme to describe spectra including unstable conditions, taken from Kaimal [1978]. Note the power-law in the middle portion is flatter than the $-5/3$ typical of the inertial range. Here, $f$ on the $x$ axis is actually normalised frequency, equivalent to $n$ used in figure 1.6.
The spectral properties of these ranges were generally well studied and characterised in a comprehensive field experiment nearly 30 years ago. Kaimal et al [1972], for example, presented an extensive survey of wind characteristics using the framework of Monin-Obukhov similarity theory based on dimensionless quantities to construct generalised curves for spectra in a variety of atmospheric stability conditions.

Monin-Obukhov similarity theory still forms the foundation of understanding of the properties of the surface layer (Kaimal and Finnigan [1994], Garrett [1992], Panofsky and Dutton [1984]). The theory supposes that the statistical characterisation of turbulent wind velocity fluctuations, such as the spectra, in this layer are universal functions of $z/L$ when normalised by the appropriate powers of $u^*$ (the "friction" velocity, defined as $(\tau/\rho)^{1/2}$, where $\tau$ is the Reynolds stress and $\rho$ the density, see Panofsky and Dutton [1984]) and $L$, where $L$ is the Monin-Obukhov length (effectively a measure of the degree of atmospheric stability), and $z$ is the height.

The experimental results presented by Kaimal et al [1972] show that the spectra of horizontal wind velocity fluctuations produce a family of curves which, with the appropriate normalisation, converge to a $-5/3$ power law in the inertial range, but spread out into separate functions depending on the value of $z/L$ at the low-frequency end of the spectrum in neutral or stable conditions, as shown in figure 1.6 for the $x$ component of the velocity.

Note from the figure the existence of an excluded region (shown with dark shading) separating the stable and unstable spectra. For unstable conditions, the progression of the spectral form with decreasing $z/L$ breaks down so that Monin-Obukhov scaling does not appear to hold, and the spectra are contained within the lightly shaded region in a non-systematic way. This suggests that some length scale other than $z$ controls the low-frequency behaviour of the spectrum. For unstable conditions, for which convective mixing becomes important, it appears that the spectra become functions of $z_i$, the height of the lowest temperature inversion [Kaimal 1978, Højstrup 1981, and Højstrup 1982].
Kaimal [1978] suggested that the unstable spectra of the $u_z$ component could be generalised if different portions of the spectra were expressed in different coordinates.

In unstable conditions, he suggested splitting the spectra into a high-frequency inertial range, which obeys Monin-Obukhov similarity, a low-frequency energy-containing range, and a transitional range between the two which takes the form of a spectral power law.

Figure 1.7 shows a schematic of this suggested spectral scheme. Kaimal effectively interpolates between the high and low-frequency regions, by assuming a power-law to describe this "middle" part of the spectrum, such that:

\[ f_n E(f_n) = 0.3 f_n^{-2/3} \quad l \leq 2z \]
\[ f_n E(f_n) = 0.48 (2 f_n)^{-p} \quad 2z \leq l \leq 0.67 z_i \]

where $f_n$ is the normalised frequency, defined as $f_n = f z / U$, $U$ is the mean wind speed, $f$ is frequency, and the quantity $f_n E(f_n)$ is the "frequency-weighted" spectrum. The value of $p$ is given by:

\[ p = \frac{\ln 0.44 A}{\ln 0.33 B} \]

where:

\[ A \approx \frac{(12 + 0.5 z_i / L)^{2/3}}{1 + 0.75 z_i / L} \]

and

\[ B = \frac{z_i}{z} \]

However, this description of the "middle" portion of the spectrum oddly does not appear in some recent texts (e.g. Kaimal and Finnigan [1994], Garrett [1992], Panofsky and Dutton [1984]), and later works tend to concentrate on using more than one dimensional wavenumber descriptions, e.g. Kristensen et al [1989], Mann [1994], and Peltier [1996]. Such a variable power-law region lends itself well to a multiscaling study, particularly since it provides the value for one of the parameters, the power-law slope.

Besides Kaimal's suggestion for an energy-containing range power law, there are other examples of workers reporting low-wavenumber spectral power-law
behaviour. Theoretical work as early as 1953 by Tchen predicted a $-1$ power law for low wavenumbers for a neutral-atmospheric surface layer based on simplifications of the Fourier-transformed Navier-Stokes equations.

However, until the early 1990s, the existence of this power law had been questioned by several workers (see Raupach et al [1991] and Antonia and Raupach [1993]), despite evidence for its existence being obtained first by Klebanoff [1954] using a flow over a flat plate with zero pressure gradient, and later by Perry and Abell [1975] and subsequent papers (e.g. Perry et al [1986]).

More recent papers on longitudinal low-wavenumber wind spectra provide evidence of a $-1$ power law in the atmospheric surface layer for neutral conditions, and a $-5/3$ power law for convective conditions (Kader and Yaglom [1991], Katul et al [1995], Katul et al [1996], and Katul and Chu [1998]). This description makes use of Townsend's [1961] hypothesis that turbulent eddy motion in the "inner" region of a boundary layer consist of an "active" part which produces shear stress and an "inactive" irrotational part determined by the turbulence in the outer region of the boundary layer. According to this scheme, for large-scale eddy motions much larger than the height of the anemometer, $z$, the spectra should scale with the outer region. In that case, the spectrum is independent of $z$, and goes as:

$$E(k) = C_u u'^2 k^{-1}$$

(see Katul et al [1995]) where $C_u$ is a similarity constant, with a typical value of 1.

1.3 Equipment and experimental set-up

The principal site used for data collection was the University of Auckland's Ardmore field station, situated in the gently rolling countryside south of Auckland city. This site is relatively flat, but features such as tree-line wind breaks and low hills within a few kilometres could have potentially affected the spectrum.

The site is next to a small airport (used mostly by single-engined aircraft) and is often used by local farmers for grazing livestock. The plain where the station sits extends approximately 5km in the direction from which the prevailing wind blows (roughly east). Beyond this, the landscape becomes residential for another 2km, then becomes an estuary for the Manukau Harbour. In the directions perpendicular to the prevailing winds, there are low hills 4km to the north and
2km to the south, which run parallel to the direction of the wind for approximately 4km. In the direction of the wind there are scattered structures typical of farmland, the most significant of which are tree-line wind breaks 3m to 4m in height a few hundred meters ahead of the site. Although these structures undoubtedly produce lee-effects, it seems unlikely they would have much of an effect on the low-wavenumber end of the spectrum, for which scales of between 5m and 5000m are considered.

Cup anemometers were mounted on a 10m tower and measured wind speed during most of 1996 and early 1997. Since the cup anemometers could not resolve the three-dimensional components of the winds, the analysis of the data obtained from these instruments was actually for the wind speed, rather than the longitudinal component. If it is assumed that the longitudinal component dominates the fluctuations in wind speed for low wavenumbers, then the recorded signal can be treated as a close representation of the x-component of the velocity. Comparison of the results from the cup and a hot-wire anemometer (see chapter 2) suggested the above assumption was reasonable. In any case, it may do just as well to characterise and simulate low-wavenumber wind speed rather than a velocity component for most practical applications. The term "velocity field" in the following chapters refers to the x-component of the velocity, and where this refers to cup anemometer data, the above assumption has been made.

The cup anemometers were relatively cheap and easy to operate, as opposed to more expensive hot-wire or sonic anemometers, and their durability made them suitable for longer-term studies, such as the one conducted here. Owing to their light construction, the cup anemometers had relatively fast response rates by comparison to typical long-term field-deployed cup anemometers. Testing of the anemometers suggested that they were capable of producing reasonably accurate wind spectra for periods longer than a few seconds (as will be seen in the next chapter). The cup anemometer made two voltage pulses per rotation, and for the particular set-up used the number of pulses per 10 second interval was counted and digitally stored on the hard drive of a personal computer. The relationship between the frequency of the pulses and wind velocity had been determined in a wind tunnel, so that the pulse-count data could easily be converted into wind speed with 10s resolution.
Three cup anemometers measured wind speed in the surface layer, generally considered to be the lowest 10 per cent of the atmospheric boundary layer (Panofsky and Dutton [1984]), at heights of 10m, 5m, and 1.5m, so that information on wind shear was available.

In order to study the behaviour of the finer-scale fluctuations in the wind signal and to test the capability of the cup anemometer to measure the spectrum accurately, a hot-wire anemometer was used. For the hot-wire anemometer experiments, the output was recorded with a sampling frequency of 1000Hz on a PC via an analogue to digital card.

With the length of time the cup anemometers were in the field, each was able to collect several million data points. However, because the wind conditions were often relatively calm, the amount of useful data available was much less, roughly in the high hundreds of thousands of data points.

The hot-wire anemometers collected data rapidly, and did not need to be in the field for long periods to build large data sets. Several million useful data points were obtained with this instrument.

1.4 Aims of the work presented and content

Although there remain a large number of unresolved theoretical issues in regards to turbulence and cascade models, this thesis largely takes a practical and pragmatic approach. Practical, in that the experiments and analysis conducted were designed solely to seek the multiscaling characteristics of wind signals for the surface layer, describe them and simulate them; pragmatic in that these models are simply employed for their ability to do this, with little regard for theoretical concerns.

At the same time, the significance of the work conducted here should not be understated. It is apparent from the literature that little effort has been made to apply such models beyond the inertial-range dissipation field, and similarly little has been done on the practical uses of such models for the modelling of surface-layer winds. Yet potentially these models represent powerful tools, at least for engineering-type purposes. In this regard, another point made in this thesis is that such multifractal models do describe wind statistics much more accurately than
most conventional numerical simulations based on fluid dynamics, such as that described in chapter 4.

In going beyond the inertial range, the study presented examines a range of wind fluctuations which are much more energetic than the inertial range, and for this reason perhaps more important for practical applications. From a more esoteric point of view, the description of this low-wavenumber, energy-containing range using such cascades is significant because it is on these scales that numerical models based on known dynamical equations are generally used, whereas inertial-range behaviour is typically modelled stochastically. Because examination of larger-scale fluctuations allows meteorological processes to be taken into account, which can be modelled by deterministic numerical models, it is conceivable that some physical insight as to the link between cascade models and the dynamical equations might be gained, if cascade-like scaling behaviour exists for these scales. While this is beyond the scope of this thesis, the work presented lays the foundations for such a study.

Chapter 2 largely concerns itself with the question of what are the multiscaling statistics of the energy-containing spectral range. This was no easy task, requiring data to be collected over many months. It will be seen that not only can such signals be approximately described by multiscaling statistics, but the spectral slopes of the signals varied depending on the conditions on the day. This is an extremely important result in the context of the work presented here and does not appear to have been reported before. It raises the possibility that the parameters of such multiscaling models may be able to be associated with quantitative meteorological parameters, such as atmospheric stability. It is also important because there seems to be a lack of examples in the literature of geophysical power laws that depend on well-defined parameters such as, say, the Richardson number (which describes the degree of stability).

The third chapter examines a variation on the method of multiscaling analysis, that of self-similarity analysis. This type of analysis, developed by Novikov, is potentially a more rigorous way of analysing the wind signals than multiscaling analysis, as well as providing a robust method for simulating such signals. Issues arising from simulations using cascades are addressed in this chapter.
Having seen in chapter 2 that the multiscaling statistics of surface layer winds (for the energy-containing range) depend on atmospheric conditions, the investigation is extended to examine the influence of topography on the statistics. The case study presented in chapter 4 reveals the statistical properties of wind in the lee of a mountain range which includes Mt Cook, New Zealand’s highest peak. The work conducted at this site contributed to a paper dealing with the modelling of such a wind flow, using a large-eddy simulation (this paper is included in the appendix for the reader’s convenience). This provided an interesting opportunity to compare the output of such a model with a cascade model.

In chapter 5 the nature of surface-wind intermittency, particularly for the energy-containing range, is examined in more detail. This leads to a brief examination of an alternative method for simulating power-law signals. The principal reason for doing this is to demonstrate that the properties of the energy-containing range are not merely incidentally able to be simulated by cascades, but that cascades are the most reasonable way of reproducing both the power law spectra and the intermittent behaviour of the observed signal.

Chapter 6 is an attempt to formulate a turbulence model which uses the same sort of dimensional arguments as Kolmogorov’s 1941 theory, but is not cascade-based. To do this, use is made of the experimental observations in chapters 2 and 3. While the algorithm presented does not involve a cascade, it nevertheless reproduces, to an extent anyway, some of the multiscaling characteristics of turbulence for a limited range. As will be clear from the experimental analysis in the following chapters, real turbulence is not a true fractal, since it has definite upper and lower limits for which scaling holds. This is an important theme in this thesis, since the analysis strives to match the experimental data to multiscaling models. While it is not clear that the model in chapter 6 has any real merit as a turbulence model, it does appear to be quite an interesting system in itself.
2. Characterisation and simulation of the multiscaling properties of horizontal surface-layer winds

2.1 Introduction

This chapter is the basis for the main thrust of this thesis — to demonstrate that multiscaling analysis is a useful tool for characterising and simulating the low-wavenumber statistical properties of surface-layer winds. For the low-wavenumber statistics, it is neither clear that the use of a cascade model is phenomenologically valid, nor that the model parameters should be universal. Workers who model turbulence using cascade models appear to have ignored the low-wavenumber range, presumably for these reasons. However, just as other studies have done for rain, the authors contend that such models may be treated simply as a statistical method, which determines the appropriate scaling indices.

There are two aspects to the experimental study: a) hot-wire anemometer data is used to demonstrate the existence of two power law and multiscaling regimes. b) cup anemometer data is used to show how such techniques might be applied for long-term field studies where it is not feasible to deploy and supervise more expensive and fragile anemometer types for long periods. In the case of the cup anemometer data, the slow-response rate meant only the low-wavenumber end of the spectrum was able to be examined.

As discussed in chapter one, the inertial range is understood in terms of a theory originated by Kolmogorov [1941], which implies that the spectrum of velocity fluctuations in the inertial-range goes as the power law:

\[ E(k) \propto k^{-5/3} \]

where \( k \) is the wavenumber.
The independence from boundary conditions suggests a universal nature to the dynamics. The cascade models used to simulate inertial-range turbulence signals should thus be expected to have constant values for their parameters. To this end, various workers have attempted to find both sufficiently parameterised models and the value of those parameters, which then presumably completely describe the statistical properties of such turbulence (e.g. Schmitt et al [1992]).

These models have latterly been used to simulate the statistical properties of other geophysical fields, particularly rain [Lovejoy 1981, Schertzer and Lovejoy 1987, Lovejoy and Schertzer 1995, Over and Gupta 1994, Gupta and Waymire 1993], for which it is not clear that the phenomenological cascade picture is valid. In the same vein, many of these workers have attempted to parameterise rain fields by assuming that such universal constants can be found to completely describe rain statistics. However, there is no theory of the kind Kolmogorov used for inertial-range turbulence that suggests that rain processes should have a universal nature independent of boundary conditions, or even that rain processes are always the same. Indeed, recent work by Harris et al [1996] suggests that the multiscaling parameters necessary to describe rain field statistics can depend strongly on the topography of the site where the data is collected.

Rain data collected by Harris et al showed power-law spectra for time scales above 15 seconds up to periods of the order of an hour, obeying power laws which varied from $f^{-0.92}$ to $f^{-1.54}$ depending on the site of the rain gauge, where $f$ is frequency. The time scales over which these rain fields were studied correspond well with the energy-containing range in turbulence. This led our group to speculate that turbulent wind behaviour for similar temporal scales might also be describable in terms of multiscaling statistics with variable parameters, which may indeed be related to the same processes determining the nature of the rain statistics.

Multiscaling models at the very least represent a convenient way of simulating signals from this range (which is interesting in itself), so that it seems reasonable to set aside theoretical concerns (at least temporarily) and determine if these methods can reveal new information about the nature of such turbulence and the sort of dynamical processes which occur on these scales. It is particularly interesting to ask if the non-universal behaviour of the power laws described in
Harris et al's rain work might also exist for low-wavenumber turbulence, remembering that for inertial-range turbulence the parameters are generally taken to be universal.

If wind fluctuations on these time scales could be described in multiscaling terms, then fractal cascades can be used to simulate them. Although numerical simulations based on fluid dynamical equations are capable of reasonably simulating surface layer winds (e.g. Revell et al [1996]), they are expensive in terms of computational time, are limited by their grid scale, and generally speaking are unable to reproduce multiscaling statistics, due to their limitations in reproducing extremely intermittent behaviour.

Also, if inertial-range turbulence indeed has a universal nature, which is already well studied, it is less interesting to measure than non-universal turbulence, because in the latter case, provided such turbulent signals can be modelled using multiscaling cascades, it may be asked how such factors as atmospheric stability might affect the parameters of these models.

From an esoteric point of view, the energy-containing range is interesting to examine because tangible dynamical and meteorological processes occur on these scales, whereas the inertial range is generally described by stochastic models. Thus there is potential to gain a physical insight into how the cascade-like features of the wind signals arise. This is particularly interesting in view of the fact that no one has been able to demonstrate that such cascade models represent solutions of the Navier-Stokes equations, leaving cascade models firmly in the phenomenological category.

Seeking a multiscaling description of the statistical properties of wind on these scales also has merit for practical reasons. Since fluctuations of these periods fall into the energy-containing range, these fluctuations are energetically more significant than inertial-range turbulence. Consequently, it must be expected that wind fluctuations of these time scales are more important in terms of some applications, like the modelling of output of wind turbines, rather than the shorter time scale inertial range.

In the following sections, the spectra of horizontal surface wind fluctuations for the region of the spectrum immediately above the inertial range are examined for power-law behaviour. It was typically found that this part of the spectrum was
described by a power law with a slope usually steeper than $-1$, depending on the conditions, but less than $-5/3$. This range is taken to be approximately the range of fluctuations from 1s up to $10^3$ s ($k \approx 1.2 \text{ m}^{-1}$ to $1.2 \times 10^3 \text{ m}^{-1}$).

The scaling of the moments of the data was examined, assuming that this low-frequency power-law region was also multiscaling. Studying the scaling of the moments and the $K(q)$ function (described in chapter 1) has the additional benefit of quantifying the degree of intermittency of wind fluctuations from this part of the spectrum, something which the results of Kaimal et al [1972 and 1978] do not.

This is followed by an examination of the relationship between atmospheric conditions and the spectral power-law slope, with the results obtained from the data collected here compared with the theoretical function suggested by Kaimal [1978].

Finally, a bounded fractal cascade is used to demonstrate how it is possible to simulate such signals once the multiscaling parameters have been determined.

**2.2 Multiscaling analysis and scaling breaks**

**2.2.1 Multiscaling parameters**

The relevant parameters for multiscaling analysis are the slope of the power-law spectra, which, following the notation widely used in the literature, is denoted by $\beta$, so that:

$$E(f) \propto f^{-\beta}$$

and the scaling exponents $K(q)$ for the $q$th-order statistical moments of the field, where frequency, $f$, is dealt with rather than wavenumber, $k$, since the experimental data is presented in terms of temporal units, as discussed in chapter 1.

To determine $\beta$, a standard fast-Fourier transform (as found in Press et al [1992]) was used to produce horizontal wind spectra. The power-law exponent $\beta$ was determined by fitting a straight line to the spectra on a log-log scale using a linear regression.

Recall that a field $\epsilon$ is multiscaling if the $q$th-order moments behave such that:
\(< \varepsilon, q > \propto \left( \frac{t}{T_0} \right)^{-K(q)} \), \ t \ll T_0 \)

and \(K(q)\) is a non-linear function. Here, \(\varepsilon\) denotes that the variable was measured with a finite temporal resolution, \(t\) (note that when \(t\) is larger than the resolution of the instrument, it is effectively a temporal average of \(\varepsilon\) over that time period), for a time series of readings of \(\varepsilon\), where the angled brackets represent an ensemble average over some time of observation, \(T_0\).

The codimensional parameter \(C_1\) is popularly used as a measure of intermittency (see Davis et al [1994]), and is defined as the first derivative of \(K(q)\) evaluated at \(q = 1\). Generally speaking, the greater the curvature of \(K(q)\), the more intermittent the field.

Unfortunately, \(K(q)\) does not exist for fields with \(\beta > 1\) [Menabde et al 1997], which is the case for most of the wind velocity data used here. \(K(q)\) is instead found for a field \(\varepsilon\) derived from the raw data field, \(u\), where \(\varepsilon\) is found by taking the square of the difference between the points in the field \(u\).

If the interval between points is small enough, then the gradients approximate the derivative of the velocity field at that point, so that \(\varepsilon\) has a simple interpretation as energy dissipation.

However, for low wavenumbers, the intervals between points are large, so this interpretation is not valid, and \(\varepsilon\) is simply an intermediate field to which the analysis is applied.

The function \(K(q)\) is found using the following method, borrowed from Harris et al [1996]. The scaling of the \(q\)th-order moment is determined by:

1. Renormalising the data so that \(\bar{\varepsilon} = 1\).
2. Computing the \(q\)th-order moment for the data set \(\varepsilon\).
3. Degrading the resolution by half by averaging pairs of data points.
4. Repeating steps (2) and (3).
Figure 2-1: Fast-Fourier transform of the wind signal, showing the break from approximate Kolmogorov scaling, with $\beta = 1.87$, into a flatter low-wavenumber scaling range, where $\beta = 1.12$ for this case. Note that the break between the power laws seems quite marked, and in this case occurs at a frequency a little under 1Hz.

Figure 2-2: The inertial-range spectrum, shown here, for our data tended to a spectral slope of $\beta = 1.7$ with increasing averaging of ensembles of spectra.
Figure 2-3: Comparison of the hot-wire anemometer data spectrum with the cup anemometer spectrum.

The power-law exponent is found by fitting a straight line to the log-log plot of the $q$th-order moment versus resolution. This is then repeated for every desired $q$. However, this process appeared to produce an anomalously steep slope for the first three or four points. This was clearly an artifact of the above process, because it appeared regardless of the scales examined.

2.2.2 Scaling breaks and scaling of the energy-containing range

Fast-Fourier transforms of data obtained with the hot-wire anemometer were fitted with power laws for two ranges of frequencies.

These were the usual inertial range, for which a slope of approximately $-5/3$ could be expected, and a second range of periods from 1 s up to the order of an hour, which obeyed an approximate spectral power law with a slope usually much flatter than $-5/3$. This is demonstrated in figure 2.1, which shows the spectrum of wind velocity fluctuations from hot-wire anemometer data collected at the Ardmore site, where the spectrum was produced from an ensemble average of eight separate spectra made using 16,384 data points. For this data set, the spectral slope was found to be around $\beta = 1.87$ for the higher-frequency inertial range part
of the spectrum, greater than the \(-5/3\) expected by Kolmogorov's theory. For reference, a \(-5/3\) line is included on the plot. Other observations of inertial-range spectra [e.g. Saucier 1991] put \(\beta\) for the inertial range at around \(-1.7\).

Heavier averaging of the spectra using a larger number of data points gave a similar value to Saucier's work for \(\beta\) for the data used here, as figure 2.2 shows, for which \(\beta = 1.74\). This would appear to agree with observations by Szilagyi et al. [1996] that the inertial-range spectral slope varies around \(-5/3\) depending on local intermittency and tends to a value close to \(-5/3\) only with increasing averaging.

The lower-frequency portion of the spectrum in figure 2.1 also appeared to follow a power law, but with a shallower slope of \(\beta = 1.12\). It is supposed here that this constitutes a low-wavenumber scaling regime, which, the results of Kaimal [1978] suggest, is a function of stability and inversion height.

Figure 2.3 shows the low-wavenumber end of the spectrum obtained simultaneously by hot-wire and cup anemometers. This figure is presented to establish that the cup anemometer obtained spectra for which the power-law slope was similar to the hot-wire data, and was hence able to be used to estimate this parameter for this part of the spectrum. Both spectra have a slope of about \(\beta = 1.12\), clearly flatter than expected for inertial-range turbulence. Such agreement between the two instruments was observed on the other occasions for which the hot-anemometer was used. The largest value for the spectral slope observed for the low-wavenumber range by the hot wire was 1.35.

The hot-wire spectrum was an average of three 512-point spectra from data sets 2048 points in length, with the resolution reduced to 10Hz. The cup anemometer data was an average of three 128-point spectra, obtained from consecutively recorded data sets of 512 points for which the sampling period was 10s, so that the data was recorded over a period of just over four hours. The point at which the spectrum changes from the inertial range to the energy-containing range can be derived analytically by matching the inertial-range power law and the power-law scheme suggested by Kaimal (as described in chapter 1).
This gives a transition frequency between the two of:

\[ f_n = (2^{-p} 1.6)^{1/(p-2/3)} \]

where \( f_n \) is the normalised frequency introduced in chapter 1, and \( p \) is the spectral slope for the "frequency-weighted" spectrum used in the description of Kaimal's scheme discussed in the introductory chapter, which is related to \( \beta \) by

\[ p = \beta - 1 \]

Alternatively, in terms of the form of the low-wavenumber portion of the spectrum suggested by Kader and Yaglom [1991] and Katul et al [1995, 1996, 1998], matching of power laws (assuming that for the surface layer \( \varepsilon = u_*^3 / K_v z \), where \( K_v \) is the von Karman constant, which has a typical value of 1.02) gives the transition wavenumber between the two regimes as:

\[ k = \frac{1}{K_v z} \left( \frac{C_K}{C_u} \right)^{3/2} \]

Studies by the latter group of workers suggest that this transition point is independent of surface roughness and is a function of dynamics rather than lower boundary-layer conditions.

From figures 2.1 and 2.3, the transition from the \(-5/3\) slope to the flatter power law appears to occur over a short range, in agreement with observations in Kader and Yaglom and Katul et al. For \( z = 3 \)m, the Kader and Yaglom and Katul et al models suggest a transition frequency of 1Hz, and the Kaimal model a value of 0.5Hz.

Comparison with figure 1 suggests that the latter value is a better match for the transition frequency. The observed transition frequency varied little between hot-wire anemometer runs.

2.2.3 \( K(q) \) and the degree of intermittency

The scaling of the statistical moments of data obtained with the hot-wire anemometer displayed two distinct regions, as was the case for the power laws observed from the FFTs of the data.
Figure 2.4: The scaling of the second-order moments shows a marked break between the inertial range and energy-containing range, occurring at a similar frequency to that observed for the spectrum shown in figure 2.1.

Figure 2.5: The $K(q)$ functions for both the inertial (top) and energy-containing (bottom) ranges. The dashed curve is a theoretical function suggested by Schmitt et al [1992].
Band-passed filtered signals from inertial and post-inertial ranges

Figure 2.6: Band-passed filtered signals for ranges of frequencies lying in the inertial and energy-containing ranges. This qualitatively shows that inertial-range fluctuations are much more intermittent than energy-containing range fluctuations, in agreement with the more quantitative multiscaling analysis that was also carried out.

The slope of the scaling became flatter for periods longer than about 1s. This can be seen in figure 2.4, which shows the scaling of the second-order moment. Two straight lines (representing scaling ranges) were fitted to the plot on the same basis as had been done for the spectrum — that is, it was assumed that there existed an inertial scaling range for periods less than a second, and an energy-containing scaling range for periods longer than this and up to about $10^3$s. For this realisation, the scaling of the second-order moment for the inertial range was characterised by a power exponent of about $K(2) = 0.40$, whereas for the low-wavenumber region, the exponent was only $0.065$.

The $K(q)$ functions obtained by assuming the existence of these two separate multiscaling regions are shown in figure 2.5, with the two curves in each case showing extreme fits to the data used.
Also included is the $K(q)$ function suggested by Schmitt et al [1992]:

$$K(q) = C_1 \frac{q^\alpha - q}{(\alpha - 1)}$$

with $\alpha = 1.4$ and $C_1 = 0.25$, where $C_1$ is the intermittency parameter discussed earlier.

From the data obtained it was estimated that $C_l$ had a value of around 0.15 for inertial-range turbulence, but $C_l$ ranged from just 0.015 to 0.038 for the energy-containing range. This makes inertial-range turbulence much more intermittent than larger-scale turbulence (signals that are more strongly intermittent tend to have their strongest gusts clustered together in space). This is also qualitatively illustrated in figure 2.6. This shows the signal band-pass filtered for a range of frequencies from 350Hz to 450Hz, compared with a band from 0.035Hz to 0.045Hz (or for fluctuations of periods 22s to 29s). The high-frequencies, belonging to the inertial range, exhibit localised intermittent bursts, in agreement with observations by Batchelor and Townsend [1949]. The lower-wavenumber, energy-containing band of frequencies appears much less intermittent.

The scaling for the third-order moments for data collected with the cup anemometer during two eight-hour periods of differing atmospheric conditions are shown in figure 2.7. Note from this figure that there was some difficulty in fitting straight lines to the cup anemometer data, as the slopes were not steep, which tended to exaggerate the scatter from a straight-line fit. Additionally, the anomalously steep slope that occurs for the first three points, as mentioned in introduction, is clearly evident in the figure, which adds to the difficulty in determining which points to use.

However, higher-resolution data collected by the hot-wire anemometer produced plots for which a straight-line could be much more convincingly fitted, as can be seen from figure 2.4. As a result, the range for which straight lines were fitted was chosen on the basis of what the hot-wire data had revealed. As will be seen later, the scaling of the moments of the simulated signal behaved in the same way as those of the experimental data, which is all that is required for such a method to be useful.
The answer to the more esoteric question of whether or not these non-inertial range signals are truly multiscaling is unclear from the data, but more light will be thrown on this by the investigations in chapter 3.

The necessity of such an arbitrary scheme demonstrates a problem with this type of analysis: the method loses some of its objectivity in the process of deciding which data points should be used when fitting the straight line. Fortunately, this problem can be at least partially addressed by examining the self-similar properties of the signal, as is done in the next chapter.

The plot at the top of figure 2.7 shows wind which appears to be more intermittent than that in the figure below. $K(3)$ for the top case was estimated to be 0.11, and 0.14 for the bottom, where the first three points have been skipped in order to avoid the artifact problem described earlier. It was found that typically the degree of intermittency varied little, regardless of the atmospheric conditions and the value of the spectral power-law slope, $\beta$.

Some data appeared to suggest that periods of slightly higher intermittency often coincided with periods of heavy rain and storm fronts, as was the case for the more intermittent of the two signals in figure 2.7, so that perhaps the strongest gusts were nested within showers of rain. However, this was based on a very qualitative analysis of the data, while examination of rain data recorded at the
Ardmore site did not reveal a quantitative link, although the amount of rain data available was much more limited than that for the wind study.

**2.3 Dependence of parameters on atmospheric conditions**

**2.3.1 Choice of period and construction of spectra**

During the course of a day, atmospheric conditions cannot be expected to remain constant. The transition from day-time heating to night-time cooling, the arrival of storm fronts and sea breezes all affect conditions on these time scales. For this reason, care was needed to construct spectra from data obtained in relatively constant conditions.

The spectrum for any given period was constructed by averaging spectra of subsets of the data obtained during a period of roughly constant atmospheric conditions. Typically, periods of about eight hours were divided into 512-point data subsets, representing periods of about an hour-and-a-half, to create 128-point spectra, which were averaged together.

![Construction of wind spectrum for period 25-5-96 10am-6:30pm](image)

*Figure 2-8:* Construction of the spectra for a given period involved averaging of several spectra obtained during that period, to find a spectral slope representative of the entire period.
Figure 2-9: Surface-layer wind signals obtained in different conditions appeared both qualitatively and quantitatively different. Above, the bottom signal (b) is qualitatively more “spikey” (has more large fluctuations between points separated by small distances) than the top signal (a), indicating the existence of a quantitatively higher proportion of high-frequency components than for the top signal.

Figure 2-10: Spectra for the signals shown in figure 2.9.
Figure 2.8 illustrates this procedure. Here, six individual spectra are shown, constructed from data sets obtained consecutively over a period of about eight-and-a-half hours. Although the power-law slopes of each of the spectra obtained in the relatively constant conditions shown in figure 2.8 vary slightly, this variation was much less than the variation between groups of averaged spectra from different days of differing conditions, suggesting that the conditions have a strong effect on the spectral slope. By comparison, averaged spectra obtained on days of different conditions varied from $\beta = 0.81$ to $\beta = 1.35$.

Examination of the low-wavenumber spectra revealed that the winds were qualitatively and quantitatively different in different conditions. This is illustrated in figures 2.9 and 2.10. Here wind velocity and spectra from two consecutively recorded periods, each lasting about eight hours, are shown. A marked contrast is seen in the appearance of the velocity signal and spectral slope.

Noticeably, figure 2.9 (b), with a spectral slope of $\beta = 0.96$ (see figure 2.10(b)), exhibits large-amplitude, short-period fluctuations by comparison with figure 2.9 (a), indicating the presence of relatively higher frequency components, and hence a flatter spectrum.

Figure 2.9 (a), on the other hand, exhibits large-amplitude fluctuations which occur over longer time periods, hence it has more low-frequency components, and a steeper spectral slope with $\beta = 1.32$.

2.3.2 Dependence of spectral slope on conditions

From the data, it was found that the parameter $\beta$ did appear able to be organised into a relationship based on atmospheric conditions. The Richardson number $Ri$ was used as the non-dimensional parameter to describe atmospheric conditions, defined as:

$$Ri = \frac{g}{T} \frac{\partial T / \partial z}{(\partial U / \partial z)^2}$$

where $g$ is the local gravitational acceleration, $T$ is temperature, which is a function of height $z$, and $U$ is the mean wind speed, also a function of $z$. Although it is more typical in the literature for the Monin-Obukhov length $L$ to be used to describe conditions, the estimation of the value of this parameter required high-resolution instruments not available for this experiment.
Fortunately, $L$ has been shown by the Businger-Dyer-Pandolfo empirical result (see Panofsky and Dutton [1984]) to be easily related to $R_i$, such that, for unstable conditions $z/L \approx R_i$ where $z$ is height, and for stable conditions, $z/L = R_i / (1 - 5R_i)$.

The wind shear was estimated by comparing the wind speed as measured by the 10m and 5m mounted anemometers, to obtain a quantity $\Delta U/\Delta z$.

Since $\Delta T$ is tiny for a spatial separation of 5m, and so difficult both to distinguish in a noisy signal and to use to gauge the conditions prevailing in the boundary layer, balloon soundings from a meteorological station near to the Whenuapai Air Force base (just north of Auckland) were used to determine the temperature gradient up to a height of the order of 1000m. Although the experimental site at Ardmore was south of Auckland, the Whenuapai station was used on the basis that the synoptic-scale conditions were likely to be reasonably similar at both locations.

The temperature gradient was near to constant over such heights, so it was assumed that the $\Delta T / \Delta z$ obtained was valid at the height of the anemometers. Most of the measurements of temperature gradients were at midday, so conditions were usually likely to be unstable.

Figure 2.11 shows a plot of $\beta$ as a function of $R_i$. Here, the circles indicate those data points for which the height of the lowest inversion was estimated at around 1000-2000m. The solid line shows Kaimal’s theoretical function for the spectral slope parameter, assuming the height of the lowest inversion, $z_l$, is 1000m, and using the unstable case of the Businger-Dyer-Pandolfo result. The dashed line is the same, this time using the stable case.

The plot shows some organised features: The data plausibly follows Kaimal’s function for $R_i > 0.7$, but $\beta$ splits into two groups, with either very high values of around 1.3 or very low values of less than 1, for most of the points with $R_i$ below this value.
Figure 2.11: Plot of spectral slope versus Richardson number. The value of $\beta$ appears to increase with increasing Richardson number for values of $R_i > 0.7$, in keeping with the solid line, representing Kaimal's [Kaimal 1978] theoretical function for the spectral slope, assuming that the height of the lowest inversion, $z_i$, was 1000m. However, this function does not explain the split in $\beta$ values for $R_i < 0.7$. Circles represent just those data points collected on days when the height of the lowest inversion was around the 1000m to 2000m range.

For an arbitrarily long field produced by a cascade with multiscaling parameters, Harris et al [1998b] suggest that to estimate $\beta_0$ to within ±0.1 with a 95 % confidence level requires a ensemble of the order of $10^4$ data points, where $\beta_0$ is the parameter which determines the limiting value of the spectral slope of the cascade output, as opposed to $\beta$, the spectral slope for that particular ensemble. If it is to be assumed that the observed signals can be simulated using such a cascade, then the estimate for $\beta_0$ from the experimental data must have an associated uncertainty (because the experimental data sets themselves must be treated as though they are sub-ensembles of some larger set for which there is a limiting value of $\beta_0$). The experimental data sets used in figure 2.11 each had between 3000 and 10,000 data points, suggesting a typical uncertainty of ±0.10
for $\beta_0$. Bearing this in mind, figure 2.11 includes error bars representing an uncertainty range of $\pm$ 10%, close to the 95% confidence level for the value of $\beta_0$ for most of the points plotted.

### 2.4 Simulation of surface winds

A bounded-cascade model suggested by Menabde et al [1997a] was used to produce simulated surface wind signals. This model was used not because it was necessarily the most appropriate for this kind of simulation, but because it allowed the direct simulation of the wind. Cascade models which are not bound are only capable of producing signals with $\beta < 1$, and are typically used for the simulation of the energy dissipation field, rather than the velocity field. Bounded cascades, on the other hand, can produce fields with $\beta > 1$ because the probability distribution for the weighting factors, $W$, is such that it depends on the scale.

The cascade model starts by considering an initial homogeneous distribution of the signal, $u_x$, over some interval $T$. On the first step of the cascade, the interval is divided into halves, and each new interval assigned a new value, $u_x = W_k u_x$. On the $k^{th}$ step of the cascade, $W_k$, takes on the value

$$W_{1k} = 1 + (W_{10} - 1)2^{-(k-1)H}$$

with a probability of $p_1$, and with a probability of $p_2$ the value

$$W_{2k} = 1 + (W_{20} - 1)2^{-(k-1)H}$$

with the constraints that $p_1 + p_2 = 1$, and $p_1 W_{10} + p_2 W_{20} = 1$.

This leaves three free parameters, $p_1$, $W_{10}$, and $H$. It can be shown that the spectral slope $\beta$ tends to $1+2H$ (see Menabde [1997]) for a large number of simulated points.

Since the numerical studies of cascade models by Harris et al [1998b] suggested that of the order of $10^4$ data points were required to estimate $\beta_0$ with an uncertainty of $\pm 0.1$ with 95% confidence, it follows that the same number of points must be generated by the cascade model for its spectral slope to converge to $\beta_0$ to within the same degree of certainty (i.e. roughly 13 iterations).
Figure 2-12: Simulated wind data with $\beta = 1.31$ (top) and $\beta = 1.0$.

Figure 2-13: Spectra and third-order moments for the simulated data shown in figure 2.12.
Figure 2.14: The correlation functions for the real (solid line) and simulated (dashed) wind signals for the low-wavenumber range.

Qualitatively speaking, tests with the model used here suggested that the output possessed a spectral slope close to $\beta_0$ after just 11 iterations.

Figure 2.12 shows two simulated fields, made in three steps. A raw field was constructed using this model with parameters of $H = 0.01$, $H = 0.13$, $W_{10} = 0.9$ and $p_1 = 0.35$ for both cases.

Note that although for strongly intermittent fields, the free parameters can be found analytically (see Appendix A, from Menabde et al), this is not the case here, and these values were chosen heuristically by finding the best-approximation to the desired degree of moment scaling.

This field was then reduced in resolution (a process know as “dressing”, see Schertzer and Lovejoy [1987]) by one fifth using averaging. (Note that this process increased $\beta$, so that the values for $\beta$ for the field obtained were not exactly $1 + 2H$.) Finally, the field was scaled by an appropriate value to produce some desired mean wind speed.

Figure 2.13 shows the spectra and third-order moments for the simulated data. As a further comparison of the statistics of the real and the simulated field, figure 2.14 shows the correlation functions $c(l) = \langle u_s(x+l) u_s(x) \rangle / \langle u_s(x) u_s(x) \rangle$ for each.
Note that since the degree of intermittency of the wind signals tended to be small and varied little between realisations, for most practical purposes only a change in the parameter $H$ was required to be able to simulate a wide range of conditions.

2.5 Conclusions

Examination of the $x$-component of surface-layer winds using a hot-wire anemometer showed two spectral power law and multiscaling regimes — a low-wavenumber regime, corresponding to length scales of about 5m to 5000m, and a high-wavenumber inertial range. For longer time scales, the flatter low-wavenumber power law appeared to break down, limiting this scaling range to between two and three decades — a relatively short range of scaling. Nevertheless, this appears to be comparable with the scaling range obtained by Harris et al [1996] for rain data collected with similar resolution to the cup anemometer. The principal difficulty in testing the applicability of this method to turbulent fluctuations on time scales longer than those typical of the inertial range was the limited number of data points able to be obtained for any given period of windiness. Given that strong winds were only likely to last at the site used for periods of at the most 8 hours, and that the general conditions were also likely to change over similar periods, the number of points obtainable with the cup anemometer (at 10s resolution) was limited to around 3000 points for any given period of constant wind. This also limited the ability to examine the behaviour of the spectrum for periods longer than this.

Although the hot-wire anemometer used was capable of recording data at a much higher rate than the cup anemometer, because the energy-containing range started for periods longer than 1s, the number of data points obtained could be increased by an order of magnitude with this device. The hot-wire anemometer, however, was too fragile to leave in the field for extended periods of time.

In the case of the data collected at Ardmore, the scaling of the moments for low wavenumbers was characterised by gentle slopes, so that the scaling was not so convincing as for the inertial range. Nevertheless, simulated data which exhibited similar behaviour for the spectrum and the scaling of moments was able to be constructed for the low-wavenumber range using a bounded-random cascade
model. The bounded-cascade model used was able to simulate the wind signal directly, rather than the more usual energy dissipation field, which appears to be an original result. This is significant because it is more usual for turbulence (particularly for the inertial-range) to model an intermediate field, such as dissipation. The fact that wind signals from the energy-containing region of the spectrum can be modelled directly suggests that the processes which occur within these two spectral ranges (i.e. the inertial and energy-containing ranges) are different.

This method of characterising the data had the benefit of describing the degree of intermittency of the wind for this part of the spectrum, something which the results of Kaimal et al [1972 and 1978] and Kader and Yaglom [1991] and Katul et al [1995, 1996, 1998] do not.

From the cup-anemometer data, the spectral slope parameter $\beta$ for the low-wavenumber end of the spectrum showed a dependence on the atmospheric conditions, characterised in this study by the Richardson number. This dependence appeared to follow the theoretical suggestion of Kaimal [1978] for the spectral slope, except for the values of $Ri < 0.6$. It was possible that the split into high and low $\beta$ branches for $Ri$ smaller than this represented periods of either instability or stability which were not well characterised by the estimates of $Ri$.

The spectral slope parameter in this study took on a range of values between 0.8 and 1.35 at the farmland site, whereas Kader and Yaglom and Katul et al reported slopes of either 1 for neutral conditions or 5/3 for convective conditions. While the present results support the existence of a $-1$ power law, they also suggest a more-complex character to the behaviour of the low-wavenumber end of the spectrum for this site, more in keeping with Kaimal's [1978] suggestion. This is supported by the fact that the value of $\beta$ for spectra obtained on the same day varied little, whereas much larger variations in $\beta$ were seen for spectra obtained on different days (and different atmospheric conditions). Note too that the hot-wire anemometer data revealed values for $\beta$ ranging from 1.12 to 1.35. The overwhelming majority of the data points for which $\beta \leq 1.1$ were cases where an inversion layer existed below 2000m, suggesting a dampening on convective motion. This agrees with the results of Kader and Yaglom and Katul et al, which
suggest that the non-convective case produces a lower value (around \(-1\)) for the spectral slope.

It is possible that a mechanism for producing a sharp jump from low values for \(\beta\), of beneath 1, to the higher values operates in conditions of strong wind shear (and therefore small \(Ri\)). Qualitative observations of conditions suggested \(\beta\) obtained values of around 1.3 in conditions where there was little cloud and bright sunlight, whereas the lower values were at times of cloud cover. This suggests convection was responsible for the switch in behaviour, and that for strong winds, the difference between mechanically dominated and convectively dominated turbulence in the surface layer is quite distinctive in terms of its effect upon the spectral slope.

However, there is an alternative view, in the light of observations on the effect of intermittency on the inertial range spectrum by Szilagyi et al. [1995]. That work found that the slope of the energy spectrum is steeper than \(-5/3\) during well-developed intermittent events, but flatter than \(-5/3\) in less-active regions. This tends to imply that the inertial range might not be as universal as might otherwise be supposed, and the slope of the inertial-range spectrum might indeed depend on some spatially local condition, as the spectral slope of the energy-containing range seems to, though it would be on a much smaller scale. It must be noted that the energy-containing range spectral slope might just as easily be averaged using all the spectra collected to produce some seemingly universal slope of around \(-1.2\). However, the evidence appeared to be very clear that the spectral slope parameter depended on the conditions on the day, since sub ensembles taken from data collected on the same day had values for the spectral slope which were reasonably consistent.

The change from the \(-5/3\) inertial-range power law to the less-steep power laws of the energy-containing range appeared to be quite marked, supporting observations by other workers (Kader and Yaglom [1991] and Katul et al [1995]) and the generality of using two slopes to simulate the spectrum, or more generally, two scaling regimes to simulate the multiscaling statistics. The transition frequency between the two spectral slopes also appeared to be more in keeping with the Kaimal scheme than the Kader and Yaglom and Katul et al studies.
One point to note is that the site (typical of flat farmland) was less uniform than that used in the Kader and Yaglom and Katul et al studies. Thus day-to-day variations in the direction of the prevailing wind may have meant changes in the surface roughness, which may in turn have affected the value of $\beta$. However, it should be clear from figure 2.11 that there appears to be a systematic dependence of $\beta$ upon $Ri$, so that changes in surface roughness do not appear to have had a strong effect at this location.

There seems to be a lack of examples in the literature of geophysical power laws that depend on well-defined parameters such as the Richardson number. Indeed, it may be possible that multiscaling fields exist in two categories: those which appear to be universal in nature, such as inertial-range turbulence, and those which do not, such as low-wavenumber range turbulence and rain fields. The former, then, might fit more neatly as an example of the so-called self-organised criticality phenomena described by Bak et al [1988], where the small scale workings of the dynamical system determine the behaviour regardless of the initial conditions, whereas the latter might represent some other kind of process, for which large-scale conditions are also important.
3. Self-similarity analysis and simulation of surface-layer winds

3.1 Introduction

3.1.1 Overview

Having used multiscaling statistics to determine the characteristics of the energy-containing range, further investigation of this range and simulation techniques suitable for it was conducted. Rather than use multiscaling statistics, which reveal the statistical nature of the data but do not allow direct simulation, the analysis was extended by looking for self-similarity in the distribution of the cascade weighting factors, \( W \). This method, as will be seen in this chapter, appears to be a more robust way of characterising these signals, and allows the direct simulation of the field from the results.

The intention of this chapter is again largely to explore the practicalities of analysing and simulating surface-layer winds on various scales using multiplicative cascades. The previous chapter demonstrated that bounded multiplicative cascades combined with multiscaling analysis could be a practical analysis tool for surface-layer wind studies, and it was shown that not only did inertial-range turbulence (the dynamics of which are widely believed to behave as a cascade, owing to the ideas of Richardson and Kolmogorov) demonstrate multiscaling properties, but so did the energy-containing range for the region of the spectrum immediately above the inertial range. However, in the case of the energy-containing range, the multiscaling properties existed for the velocity signal, as opposed to the more usually studied dissipation signal. While this range was able to be simulated using a bounded multiplicative random cascade, it was noted that not only was the scaling range of the experimental data relatively short, multiscaling analysis left it unclear whether the signals were truly multiscaling, or simply possessed spurious multiscaling characteristics.
A particular difficulty with multiscaling analysis (which analyses the behaviour of the moments of the data as the resolution of the signal changes) appears to be the choice of scaling range. This arises because often the so-called scaling ranges for multifractal geophysical signals (such as wind and rain) appear to be somewhat arbitrarily chosen in the literature, particularly since most such data usually does not obey a true straight-line power law (or if it does, only for one or two decades), but rather shows some curvature (e.g. Pedrizetti et al [1996]), which often appears to make the decision as to where to fit a straight line subjective. Davis et al [1996] discuss the limitations of estimating multifractal parameters from short scaling ranges, while Hamburger et al [1996] have shown that apparent fractal behaviour observed experimentally over a limit range may often have its origin in processes governed by uniformly random distributions. Relying on such scaling gives rise to the possibility of “spurious” as opposed to “real” multiscaling parameters.

Alternatively, seeking self-similar distributions for the weighting factor, $W$, is perhaps a much more robust method for determining the multiscaling parameters of the signal. It will be seen that this method is potentially capable of differentiating the effects of noise and scaling breaks from the “genuinely” multiscaling properties of the signal. By doing so, additional evidence was obtained for the multiscaling character of the energy-containing range velocity signal discussed in chapter 2.

3.1.2 Method and analysis

Recapping on the origin of cascade methods, these models were developed for turbulence following the work of Kolmogorov [1941, 1962], principally as a means of modelling the distribution of the dissipation field, $\varepsilon$, in space (e.g. Yaglom [1966]). The motivation for this approach stemmed directly from aspects of Kolmogorov’s suggestion [Kolmogorov 1941] for the velocity structure function for $\eta < l < \Lambda$:

$$\left\langle |u(x + l) - u(x)|^\kappa \right\rangle \propto (\varepsilon l)^{\kappa/3}$$

(3.1)

The quantity $\bar{\varepsilon}$ was somewhat ill-defined as a average local energy dissipation, because experimental observations of the dissipation field revealed it
to be extremely inhomogeneously distributed, so that $\varepsilon$ fluctuates strongly in space. The revised Kolmogorov-Obukhov law replaces $\bar{\varepsilon}$ with the locally defined quantity, $\varepsilon_V$, which is the average of $\varepsilon$ over a sphere of volume $V$. This led workers to suggest models for describing the distribution of $\varepsilon_V$ in space.

Cascade models of the type suggested by Yaglom [1966] phenomenologically describe turbulent dynamics by assuming that eddies of size $l$ shed their share of energy dissipation to daughter eddies of size $l/2$ with some random weighting $W$. When such models are iterated over several scales, they produce an artificial $\varepsilon$ field, whose spectrum obeys a power-law, such that:

$$E(f) \propto f^{-\beta}$$

(3.2)

where $\beta < 1$ (see Monin and Yaglom [1975]).

Although many definitions exist for fractals (see Saucier [1991]), their use often goes hand-in-hand with the concept of self-similarity, that is, qualitatively speaking, that the field when examined on increasingly small scales resembles miniature versions of its large-scale structure.

The theory of self-similar random fields was developed by Novikov [1966, 1969, 1994] for use in describing the dissipation field. This quantitatively incorporates self-similarity by assuming that the probability distribution of the random weightings $W$ is the same for each scale $l/2^{(n-1)}$. Menabde [1997b] discusses how such a self-similar field can be formulated to incorporate multiscaling behaviour.

For our analysis, consider a one-dimensional non-negative scalar field $\varepsilon (r)$ and its spatial average

$$\bar{\varepsilon}_S(r) = \frac{1}{l} \int_S \varepsilon (r) \, dr$$

where $S$ is a line segment of length $l$, centred at some point $r$. Following Novikov, breakdown coefficients, $a_{\text{scale}}, a_{\text{outer scale}}$, are defined:

$$a_{l:\Lambda} = \frac{\varepsilon_l(r)}{\varepsilon_{\Lambda}(r')} \quad a_{l:R} = a_{l:R} a_{r:R}$$

with $l < r < \Lambda$ and $l \subset S_r \subset S_\Lambda$. These coefficients are defined to be self-similar if they are random variables that belong to a distribution which is independent of the
ratio \( \varepsilon / \Lambda \). The coefficients can be easily related to the so-called weighting factors used in cascade models.

Such models typically generate signals by redistributing the value of a field \( \varepsilon_n \) for some interval, (which, for a branching number of 2, is of size \( l_n = \Lambda 2^{-n} \)) between two sub-intervals of size \( \Lambda 2^{-(n+1)} \), such that the new intervals have a value:

\[
\varepsilon_{n+1} = W \varepsilon_n
\]

It follows that this process gives rise to multiscaling statistics via equations 1.6 and 1.7 in chapter 1.

Since cascades of the type described above generate fields for which \( \beta \leq 1 \), in order for a field to have \( \beta > 1 \), the cascade process needs to cause the field to be smoother for smaller-scale fluctuations than larger ones. This can be achieved using bounded cascades (Cahalan [1990], Marshak et al [1994], Menabde et al [1997a], also see the previous chapter). However, such cascades deliberately alter the probability distribution of \( W \) with decreasing scale, so are not self-similar by our definition. Consequently, in order to find self-similarity, fields with \( \beta \leq 1 \) are required.

Although for inertial-range turbulence \( \beta = 5/3 \), a field \( \varepsilon \) with \( \beta \leq 1 \) can be derived from the velocity field by taking the square of the gradients, \((\partial u / \partial x)^2\), using finite differences to approximate the derivative. The resultant field, \( \varepsilon \), is proportional to the energy dissipation field, and must itself possess a power-law spectrum to be of use for self-similarity analysis.

Likewise, standard multiplicative cascades of the type described in the introduction cannot be used to directly simulate inertial-range velocity signals. This difficulty is overcome by modelling the square-of-the-gradient field and power-law filtering it to increase \( \beta \) to an appropriate value, as Menabde et al [1997b] do for rain fields.

By contrast, self-similarity analysis can be applied directly to the velocity field for the energy-containing range in some cases. The previous chapter showed that such velocity signals could be modelled directly using cascades, although in that case a bounded cascade was used to produce simulated data. For the particular
data examined in this part of the analysis, \( \beta \) was less than 1, allowing a self-similar distribution for \( W \) for the low-wavenumber velocity field to be sought directly. This type of analysis for the energy-containing range does not appear to have been explored by previous workers.

The probability density functions \( p(W, l_n/l_{n-1}) \) were found by taking the ratio of the value of individual points in the field, \( \varepsilon_i \), with the average of that point and its next neighbour, \( (\varepsilon_i + \varepsilon_{i+1})/2 \). Note that the distributions associated with this ratio depended on whether \( 2 \varepsilon_i/(\varepsilon_i + \varepsilon_{i+1}) \) or \( 2 \varepsilon_{i+1}/(\varepsilon_i + \varepsilon_{i+1}) \) was used, as has been reported by Pedrizzetti et al [1996]. The first one of these ratios was chosen and used to find the distribution for the \( Ws \) for the various fields examined here.

Pedrizzetti et al in a wind-tunnel experiment found that the probability distribution for the dissipation field was only self-similar for scales sufficiently larger than the Kolmogorov internal scale \( \eta \). Specifically, they found the distribution tended to a unique statistic for a range \( 700 \eta < l_n < l_{n-1} < 2 \Lambda \), where in this case \( \Lambda \) is determined by the dimensions of the wind tunnel.

Given their estimate that for the wind tunnel used \( \Lambda/\eta = 12,000 \), this implied that the range for which self-similarity was found to hold was for spatial scales of about 1m to 60m. For reference, the wind tunnel was 175m long and tapered down from an area of 22m x 32m at one end to 22m x 20m at the other.

Note that the conventions discussed in chapter 1 and used in the previous chapter apply here. References to the “velocity field” in fact denote the \( x \)-component of the velocity field, and therefore these are scalar fields.

### 3.2 Experimental results

#### 3.2.1 The inertial-range velocity field

Inertial-range statistics were determined entirely from the hot-wire anemometer data, as with the previous chapter. Surface-layer wind data was collected with a sampling frequency of 1000Hz with this instrument.
Figure 3-1: The spectrum of the velocity field, constructed from 12,288 points. The spectral slope, obtained using a linear regression fit, was $\beta = 1.63$.

Figure 3-2: The probability distributions $p(W, l_n / l_{n+1})$ for the velocity field, for $N-5$ to $N$, where $N$ denotes the scale corresponding to the signal's highest resolution. Here, the dashed line is the distribution with the signal at 1000Hz, the points are with the signal resolution reduced to 500Hz by averaging, the dotted line is 250Hz, the dash-dotted line 125Hz, the solid line 62.5Hz and the pluses 31.25Hz. Note the distribution depends (becomes narrower with finer resolution) on $l_n / l_{n+1}$, which allows $\beta > 1$. The field is clearly not self-similar for this reason.
A 128-point spectrum of the velocity field is shown in figure 3.1. This spectrum is in fact an ensemble average of spectra constructed from several subsets of the data, and in all, 12,288 data points were used. The slope of the spectrum was estimated using a linear regression, which gave $\beta = 1.63$. However, this value was quite variable and as with the previous chapter depended on the amount of averaging done and the particular realisation.

Figure 3.2 shows the probability distribution of $W$ as a function of $l_n/l_{n-1}$ for the velocity signal. This was made by finding the probability density $p(W, l_n/l_{n-1})$ for $l_{N-5}$ to $l_N$, where $l_N$ is the highest resolution of the signal, and reducing the resolution for each step $l_{N-1...5}$ by half by averaging pairs of data points.

From the figure, it is clear that $p(W, l_n/l_{n-1}) \neq p(W, l_{n-1}/l_{n-2}) \neq p(W, l_{n-2}/l_{n-3})$ etc., so that the velocity field is not self-similar, as must be expected, since for this signal the spectrum showed that $\beta > 1$. Note too that the field was smoothest at the smallest scales (i.e. the $W$ distribution was narrower for the higher resolutions), which is a requirement for the signal to have $\beta > 1$.

3.2.2 $p(W, l_n/l_{n-1})$ distribution for the dissipation field

Here the methods of Pedrizetti et al [1996] and Menabde [1997b] were followed, by analysing the square of gradients of the experimentally obtained signal. For turbulence, the field obtained, $\varepsilon$, is proportional to energy dissipation.

The spectrum of $\varepsilon$ is shown in figure 3.3, for which the spectral slope parameter was $\beta = 0.62$, in agreement with observations by other investigators, who found $\beta = 0.5$ to 0.7 for the dissipation field (Saucier [1991], Monin and Yaglom [1975]).

The probability density $p(W, l_n/l_{n-1})$ for $l_N$ to $l_{N-5}$ is shown in figure 3.4. Here, it is notable that the distribution changes significantly for each of $l_N$, $l_{N-1}$, and $l_{N-2}$, and begins to converge to a single distribution for scales $l_{N-3}$ and coarser. The distributions for the $l_N$, $l_{N-1}$, and $l_{N-2}$ ranges do not obey a self-similar distribution, due to the influence of noise.
so as to be proportional to energy dissipation. The spectral slope is $\beta = 0.62$.

This will be discussed further in 3.3.1. Using the $l_{N-3}$ and coarser scales, for which the statistical distribution appeared to become independent of $l_n/l_{n-1}$, a mean distribution referred to as $p(W_{\text{dissipation}})$ was obtained and used in the cascade simulations in the later sections.

### 3.2.3 Distribution for the energy-containing range

The previous chapter found that the multiscaling properties of surface winds changed from those of the inertial range for time scales from 1s to $10^3$s for the data examined (or, about 5m to 5000m in terms of horizontal length scales, by virtue of Taylor's frozen turbulence hypothesis).

This region of the energy-containing range had different and distinct values for the spectral slope and $K(q)$ parameters from the inertial range, and it was shown that wind signals for these scales appeared to be able to be modelled using bounded cascades.

For the part of the study examining the energy-containing range in this chapter, cup anemometer data was used, with a sampling period again of 10s. For the particular set of data examined, $\beta$ was less than 1, allowing a self-similar distribution for $W$ for the velocity field to be sought directly. The spectral slope for this data was $\beta = 0.88$, as shown in figure 3.5.
Figure 3-4: The probability distributions for \( p(W, l_n / l_{n-1}) \) for the field generated by taking the square of the gradients for the velocity field, for \( N \) to \( N-5 \). Here, \( l_n/l_{n-1} \) gives an almost uniform distribution of \( W \), but gradually converges to a single distribution for \( l_{N-1}/l_{N-1} \) and larger scales (see figure 2 for key).

Figure 3-5: The spectrum for the energy-containing range velocity signal, which had a spectral slope of \( \beta = 0.89 \).
Figure 3-6: The probability density for $p(W, l_n / l_{n-1})$ for $N-3$ to $N$ for the energy-containing range signal (finest resolution is 0.1Hz).

The probability density $p(W, l_n / l_{n-1})$ for this field is shown in figure 3.6, for $l_{N-3}$ to $l_N$.

Here, a difficulty arose owing to the relatively slow rate at which data was able to be accrued with the instrument. Because the previous chapter found that the statistical characteristics for the energy-containing range varied depending on the conditions (which changed usually over the course of a period of eight to 12 hours), the number of data points was limited to just 3000 to 4000 for any such period.

The data set used here was chosen to be representative of days for which the spectral slope parameter, $\beta$, had a value of around 0.9, which made a total of some 70,000 data points.

For $l_{N-3}$ to $l_N$, it was found that the distributions $p(W, l_n / l_{n-1})$ were approximately the same, but the distributions for larger scales become too poorly defined due to the limited number of data points available.
Nevertheless, the existence of at least some range of self-similarity supports the contention that the energy-containing range possesses multiscaling properties (by virtue of equation 1.7).

In the next section, a simulated velocity field for this range is constructed by assuming that the scales larger than the ones shown in figure 3.6 have the same distribution as $l_{N-3}$ to $l_N$, which will be referred to as $p(W_{\text{energy}})$.

### 3.3 Simulations and their multiscaling statistics

#### 3.3.1 The dissipation field

Using the probability density function $p(W_{\text{dissipation}})$ obtained in the previous section from the inertial-range dissipation field, a simple cascade of branching number two was used to generate a simulated field.
Figure 3.8: The spectrum of the simulated dissipation signal, with $\beta = 0.65$.

This was done by dividing a homogeneous distribution of the dissipation field $\varepsilon_0$ over an interval $L$ into two halves and then assigning each half values of $\varepsilon_1 = W\varepsilon_0$ and $\varepsilon_2 = 2\varepsilon_1/W - \varepsilon_1$ respectively, where $W$ is a random variable chosen from the probability density $p(W_{\text{dissipation}})$. This process was then reiterated on the new intervals.

Rather than attempt to find an analytical function to describe the distribution, the histogram of the experimental distribution was used to generate each $W$. This process involved taking random fractions of the area under the probability density curve and finding the corresponding $W$ value.

This method was able to produce the same $W$ distributions for the simulated data as were observed in the experimental data (as shall be seen). The dissipation field generated using this procedure is shown in figure 3.7. For comparison, an experimentally recorded dissipation signal is also shown below it.

Figure 3.8 shows the spectra of the simulated field, for which the spectral slope was $\beta = 0.65$.

Figure 3.9 compares the scaling of the second-order moments of the simulated and experimental data and figure 3.10 the same for the third-order moments. These two figures illustrate the point made in the introduction, that the scaling of such geophysical signals is not always good.
The scaling of the second-order moments of the simulated (crosses) and experimental (circles) dissipation signals.

The scaling of the third-order moments of the simulated (crosses) and experimental (circles) dissipation signals.
Figure 3-11: The probability distributions for $p(W, l_n / l_{n+1})$ for the simulated signal with "noise" added. Note the similarity to figure 4 (see figure 2 for key).

Clearly, to find a straight-line fit to the plots of the moments of the experimental data (circles), some subjective decision must be made as to which points to include. At a quick glance, there appears to be at least three "scaling" regions.

However, for the simulated data, constructed as described above, the moment scaling was near perfect, as must be expected from equation 1.7. Since this simulated field was constructed using the self-similar distribution for $W$ obtained from the experimental data, it seems reasonable to take only those experimental points which approximately match the scaling of the simulated data to be the range for which multiscaling holds. This procedure is presented as a simple and practical method for objectively choosing the range of scaling.

Recall also that the experimental field did not display a self-similar distribution until it had been averaged several times. Consequently, the first three or four points at the high-frequency end in figures 3.9 and 3.10 must be regarded with suspicion.
Although it is not clear what the cause of this break from the scaling for these points is, it is most likely a consequence of high-frequency noise in the raw signal which filtering failed to eliminate.

This assertion was supported by the fact that the spectra of the data often flattened at the extreme high-frequency end, a characteristic of the presence of "white" noise. Note that excessive filtering is undesirable, as it smoothes some of the very structure being investigated.

The effect of such noise can be shown by taking a simulated velocity field — the construction of which is described below — and adding a uniformly random field to it to simulate noise. While the dissipation field of the simulated velocity field was self-similar, when noise was added it lacked self-similarity for its finest scales, as can be seen in figure 3.11, which bears a strong resemblance to figure 3.4. The points in figures 3.9 and 3.10 which correspond to these scales diverge from the scaling displayed by the simulated signal. Thus, by retrieving the self-similar distributions, the effect on the scaling of the moments of this noise can be detected and taken into account in determining the multiscaling statistics.

At the low-frequency end of the plots in figures 3.10 and 3.11, the experimental points flatten out relative to the slope in the middle part of the plot.
Figure 3-13: The distributions $p(W, l_n / l_{n-1})$ for simulated and experimental dissipation fields.

Figure 3-14: Simulated (top) and experimental inertial range velocity data.
Figure 3.15: Probability density of normalised simulated (dashed) and experimental inertial-range velocity gradients.

This corresponds with the start of the region for which the inertial-range ends and the energy-containing range begins. Comparison with the scaling of the simulated data clearly identifies these points as not obeying inertial-range scaling. Figure 3.12 shows the $K(q)$ functions for both the simulated (solid line) and experimental (dot-dashed line).

The match between experimental and simulated $K(q)$ is obviously good, owing to the method used to select the scaling range for the experimental data. The intermittency parameter, $C_1$ (taken to be the gradient of $K(q)$ for $q = 1$), was approximately 0.11 for both cases.

Figure 3.13 shows $p(W_n/l_n/l_{n-1})$ obtained from the experimental and simulated fields, demonstrating that the cascade process used here produces the same probability distribution as the experimental signal.

Because inertial-range turbulent velocity signals have $\beta > 1$, standard multiplicative cascades of the type described above cannot be used to simulate these signals directly. However, the dissipation field (which had $\beta < 1$) may be used to obtained a velocity signal by finding the real square root of each point,
then using a power-law filter to increase $\beta$ to an appropriate value, as Menabde (1997a) has done with rain fields, for example.

A simulated velocity field was produced using the method described above. The resultant field is shown in figure 3.14, along with an experimentally obtained velocity signal.

While the experimental and simulated signals are both qualitatively and quantitatively a good match, a subtle difference not characterised by the analysis method presented here was discovered. This was found by examining the probability density of the velocity gradients $\Delta u_i = u_{i+1} - u_i$ for both the experimental and simulated data. The distributions of 8192 such data points from each case are shown in figure 3.15, for which the experimental data shows a clear bias towards negative gradients. The skew of the distribution of the experimental data, calculated using the standard definition:

$$\text{skew} = \frac{1}{N} \sum_{j=1}^{N} \left( \frac{x_j - \bar{x}}{\sigma} \right)^3$$

where $\sigma$ is the standard deviation, was 0.28 from a sample of greater than $10^6$ experimental data points. In terms of the weightings of the signs of the gradients, it was found that for this data set the gradients were negative with a roughly 51 per cent probability and positive with a 47.3 per cent probability (the remaining gradients having a value of zero), whereas for the simulated data the split appeared to be roughly 50-50.

At first sight, one may believe this bias to be the result of non-stationary data, i.e. the result of a "trend" in the wind speed. However, these ratios were reasonably consistent, even for small ensembles of only a few thousand points, and appeared in all the data collected. This suggests that the bias owes its origin to a consistent property of the turbulence, more specifically, that the average magnitude of the positive gradients must be larger than that for negative gradients.

This suggests that the average of the velocity difference $\langle |u_t(x + l) - u_t(x)| \rangle$ depends on the ratio of each case in the ensemble.
Simulated velocity field for post-inertial range turbulence

Velocity field data for post-inertial range turbulence

Figure 3-16: Simulated (top) and experimental velocity signals for the energy-containing range.

Spectra for simulated and experimental data, post-inertial range

Figure 3-17: Spectra of experimental (top) and simulated data for the energy-containing range, with spectral slopes of $\beta = 0.89$ and $\beta = 0.82$ respectively.
Scaling of the moments of the simulated and experimental post--inertial velocity fields

**Figure 3-18:** The scaling of the second-order moments of the simulated (crosses) and experimental (circles) energy-containing range velocity signals.

Scaling of the moments of the simulated and experimental post--inertial velocity fields

**Figure 3-19:** The scaling of the third-order moments of the simulated (crosses) and experimental (circles) energy-containing range velocity signals.
If it does, then this might be expected to affect the structure function, and consequently the spectral slope, perhaps providing an explanation for why different ensembles of turbulence data collected on different days sometimes obey Kolmogorov's $-5/3$ law and sometimes do not, without any apparent reason. In chapter 6 some possibilities which arise from this observation are explored in the form of a turbulence model.

### 3.3.2 Simulating the energy-containing range of the velocity field

Using the probability density function $p(W_{\text{energy}})$ obtained from the cup anemometer data, a cascade was used to generate a simulated velocity field for the low-wavenumber, energy-containing range. The resultant field is shown in figure 3.16, along with the experimental data used to generate $p(W_{\text{energy}})$. Figure 3.17 shows the spectra of both the simulated (slope of $\beta = 0.82$) and experimental (slope of $\beta = 0.88$) signals. Figures 3.18 and 3.19 show the scaling of the second and third-order moments of the experimental and simulated velocity data. Note that the slopes for the simulated signal (crosses) here generally appear to be steeper than the experimental data. However, the first six or so points (starting from the high-resolution end) of the experimental data (circles) obey a reasonably similar slope to the simulated data.
Figure 3.21: The distributions $p(W, l_n / l_n)$ for simulated and experimental energy-containing range fields.

Figure 3.22: Probability density of normalised simulated (dashed) and experimental energy-containing range velocity gradients.
By restricting ourselves to this limited range, an energy-containing scaling regime is chosen which covers no more than three orders of magnitude (say, time scales of $1s$ to $10^3s$), in agreement with observations in the previous chapter. Using just these points, the $K(q)$ functions were constructed, and shown in figure 3.20. Here, $\beta$ was less than one, so that unlike the previous chapter, the $K(q)$ of the velocity field itself was examined rather than of the square gradients.

Note that while the experimental and simulated $K(q)$s appeared to be less of a match than was the case for the dissipation signal of inertial range turbulence, the overwhelming conclusion was, as in chapter 2, that both experimental and simulated signals were extremely weakly intermittent, so that the difference between the two was largely inconsequential. The intermittency parameter for the experimental signal was estimated at $C_1 = 0.0041$ and $C_1 = 0.0064$ for the simulated signal.

Figure 3.21 compares $p(W, l_n / l_{n-1})$ for the simulated field with that of the experimental field.
For this range, as with the inertial range, the probability density of the gradients of the experimental and simulated fields differed. This is shown in figure 3.22, using an ensemble of 8192 points. The skew parameter was calculated at 0.31 for the $5 \times 10^4$ data points used, with the sign of the velocity gradients negative with a roughly 50.5 per cent probability and positive with a 45.9 per cent probability. This result also appeared to hold for sub-ensembles of the data set.

On an aside, it should be noted that though the cascade model used here failed to reproduce this property for the turbulence signals examined, this is also likely to be true for other types of geophysical signals to which cascade models have been applied.

To test this, the probability density of the gradients of rain data collected at the Ardmore site using the same rain gauge instrument as described in Menadbe [1997a], Harris et al [1996], and Harris [1998a,b] was examined. Figure 3.23 shows a plot of the density of gradients using an ensemble of 8192 data points, where zeros have been skipped (since the data set included periods when no rain was recorded). Note that there was considerable quantisation in the recorded values. The figure also shows a bias to negative gradients. While this is hardly an extensive survey of rain data, it must be remembered that a cascade model used to simulate this number of data points would not produce such a bias, as figures 3.15 and 3.21 demonstrate.

### 3.4 Conclusions

Analysis of inertial-range turbulence in the surface layer of the atmospheric boundary layer demonstrated that the break-down coefficients for the dissipation signal, equivalent to the weighting factor used for cascades, $W$, was a random variable which possessed a single probability distribution for a certain range of scales, as Pedrizetti et al [1996] observed in wind tunnel experiments. Self-similarity appeared to hold for the data set used for spatial scales down to the order of 10cm, with finer scales unable to be resolved with the instrument used. This suggests that self-similarity might occur on finer scales in the atmospheric surface layer than in the type of wind-tunnel simulation conducted by Pedrizetti et al. The upper scale at which the self-similar distribution for the dissipation field broke down could not be determined from the data used here. However, since the
x-component of the velocity signal obtained from the cup-anemometer at a sampling frequency of 0.1Hz, or in spatial terms, 50m, demonstrated a self-similar distribution for the velocity field, the self-similar distribution for the inertial-range dissipation field must have broken down at smaller scales than these.

The simulation of the dissipation signal was achieved using a simple cascade. The scaling of the moments of the experimental data appeared to follow a curve (rather than a straight-line fit characteristic of power laws), whereas the simulated moments scaled well. The "scaling" range was chosen to be just that range on the plot of the scaling of the moments for which the experimental points matched the simulated points, on the basis that such a simulated field constructed from a self-similar cascade (where the self-similar distribution was obtained from the experimental data) must be multiscaling. This is presented as a method for eliminating the effects of noise and scaling breaks from the estimation of multiscaling parameters. The existence of these self-similar distributions is more convincing evidence of the multiscaling character of both than simply relying on multiscaling analysis, which appears to be susceptible to spurious scaling, a particularly worrying property given that the choice of the "range of scaling" in most such studies appears to be somewhat arbitrary, as any particular wind (rain or other such "multifractal" geophysical signal) data set will not necessary scale perfectly, even after heavy averaging of data ensembles.

In the case of the low-wavenumber energy-containing velocity signal, it was found that such a field could be simulated directly with a self-similar cascade. Analysis of this signal revealed that self-similarity approximately applied, although there is no theory of the type proposed by Kolmogorov to explain its existence. Such an analysis for this range does not appear to have been presented previously, and this is a key result of this chapter. The existence of at least some range for which self-similarity applies is strong evidence that the energy-containing range is multiscaling. A simulated field was able to be produced which was weakly intermittent, in agreement with the experimentally obtained signal, though the $K(q)$ function was somewhat of a poorer match than had been the case for the simulated and experimental dissipation fields. It might be that the degree of intermittency was so small that errors in its estimation were much more significant. At any rate, the difference in the level of intermittency between
simulated and experimental signals was likely to be inconsequential for practical or engineering purposes.

In the process of constructing a simulated velocity signal for both the inertial and energy-containing ranges, it was noted that the experimental signal seemed to possess a bias towards negative gradients. For the inertial range, the distribution of the gradients had a skew of 0.28, while for the energy-containing range the skew was 0.31. This skew has implications for the form of the structure function should an ensemble of data contain unusually high proportions of either positive or negative gradients. The skew was not able to be characterised or simulated by the multiscaling and self-similar analysis methods discussed here, and raises the question of whether this property has been addressed before. The origin of this skew in the probability distribution of the gradients is curious, and is perhaps a property of the interaction between the turbulent components of the flow and the mean flow. It also raises the question of whether other types of geophysical data which have been characterised and simulated using multiscaling or self-similar analysis — a brief look at some rain data, for instance, revealed similar results — possess this curious property, and are consequently not able to be simulated in all aspects by cascade models.
4. Statistics of wind in the lee of Mt Cook and a comparison between fractal and dynamical models

4.1 Introduction

This chapter deals with a case study of surface winds collected at Mt Cook in the South Island of New Zealand, using the same analysis methods presented in chapter 2.

Experimental work was conducted at this location primarily to provide experimental support for a three-dimensional compressible fluid-dynamical model developed by workers at New Zealand’s National Institute of Water and Atmospheric Research (NIWA) which simulated surface wind gusts caused by large eddies blown down a mountain valley in neutrally stratified conditions. The resulting paper [Revell et al 1996] is provided in appendix 2 for readers’ convenience.

While this was the primary reason for obtaining the data, it also enabled a comparison to be made of the multiscaling statistical properties of the wind signal in the lee of the mountain range with those obtained at the Ardmore site. As will be seen, the difference in the statistical properties proved significant and provided insight to an important question, that of the influence of the topography on the statistics. Not only was the spectrum (which was the primary method used to compare the experimental data with the Revell et al fluid dynamical model) unlike those obtained at Ardmore, but the multiscaling statistics also gave important insight into the differences in the degree of intermittency of the wind signals at the different sites.

4.1.1 The site and results for the fluid-dynamical model

Data was collected at the Tasman aerodrome at Mt Cook, the highest mountain of the Southern Alps. The anemometer was the same type used to collect
the data for the previous chapters. The site was in a steep-walled glacial valley, downwind of the main divide of the Southern Alps. The divide was about 2km in height, with peaks of 3km. The valley down which the wind blew had walls 1 to 2km in height, and a width of only about 3km at a position 4km upstream and 1km above the anemometer. The floor of the valley was remarkably flat. A map of the topography is contained in appendix 2.

The fluid dynamical model developed by NIWA had the advantage that it was designed to conserve total energy so as to remain stable without the need for any artificially high level of diffusion. This meant that the diffusion in the model could be set up solely to represent the physics of mixing by sub-grid scale processes (most models of the RAMS type, for instance, require a higher level of dissipation than dictated by the physics just to remain numerically stable). This is particularly advantageous when the viscosity must be small so as to simulate flow separation or turbulence, i.e. in high Reynolds number flows.

The model was used to simulate wind at the Mt Cook site assuming neutrally stratified conditions. The work in Revell et al principally set out to explore the behaviour for this model for a range of parameters in order to determine the requirements for accurately simulating surface wind gusts in the glacial valley. This was done by comparing the model output with the experimental data recorded by the anemometer at the site.

It was found that in order to approach the experimentally observed behaviour of the wind (described in the next sections), the model had to be three-dimensional, with horizontal resolution better than 250m and with Reynolds-stress eddy-viscosity of less than 5m^2.s^{-1}. The three-dimensional simulations conducted by Revell et al were computed by assuming the wind was blown down a 10km-wide channel. The results with the model in three dimensions produced eddies which were not so big in size and amplitude as those produced by two dimensional models, so that the results gave a more realistic “gusting” effect. The reverse flow produced by the three-dimensional model was also reduced from that of the two dimensional case, so that there was only reverse flow at the steepest part of the lee slope.
While the details of the workings of this model and its results are not central to the aims of this thesis, in order to give the reader an appreciation for the type of output the model produced and its capabilities, figures 4.1 and 4.2 show a cross-section of the simulated fluid field for the two and three dimensional simulations respectively.

4.2 Experimental results

4.2.1 Comparison of spectrum with fluid dynamic model

The experiment was conducted over a period of just two days, so that, unfortunately, the amount of data obtained was small by comparison with the Ardmore data set.

During the period of the experiment, there were two types of prevailing conditions: a pre-frontal wind phase and the arrival of the front. The front, which arrived from the east, blew from the north-west over the mountain range and down the Tasman glacial valley.
Figure 4.2: Cross-section of flow for three-dimensional simulation using the Revell et al. model. Here the reverse flow is confined to the steepest part of the lee slope of the mountain.

For both phases the spectrum obtained was almost identical. Thus, a composite spectrum was able to be constructed from all suitable available data, which was about 5000 data points (note that the wind did not blow strongly during the entire time of the experiment). The spectrum is shown in figure 4.3.

Note that there appears to be a spectral peak/scaling break between periods of approximately 4 to 8 minutes. This appears to be evidence for a source of eddies with periods greater than 4 min. At higher frequencies than this the spectrum appears to follow a steep power law. Using a linear regression fit (solid line), the spectral slope was $\beta = 1.5$, considerably steeper than the slopes observed at the Ardmore site.

This appears to agree with the results of previous workers (e.g. Dutton et al. [1980], Panofsky and Dutton [1984] pp 200), which found relatively more low-frequency energy at sites surrounded by hills. The dashed line in figure 4.3 shows a $-5/3$ power law for comparison.

The observation of the spectral peak was supported by data obtained from the permanently stationed Munro anemometer at the aerodrome. The instrument recorded data on a chart, which was carefully digitised using a pen swinging on an arm with the same radius as the original recorder.
Figure 4-3: The spectrum from data recorded at Mt Cook. The dashed line is a $-5/3$ power law, and the solid line is a linear-regression fit, for which the slope is $-1.5$.

The spectrum obtained from this data is shown in figure 4.4. Note that the limitations of digitising the chart in this way produced an artefact for periods below 2 minutes, which should be ignored. For the data obtained during the period of the experiment, the mean wind velocity was 9.5 ms$^{-1}$, so that by Taylor’s hypothesis, the spectral peaks observed represented wind structures of 2280m to 4560m in size for 4 to 8 minute periods.

In the context of the Revell et al simulations, the spatial scale of these eddies is compatible with eddies in a turbulent wake which have their source at the flow separation occurring at the ridge crests. Phenomenologically speaking, this envisages that the spectrum has the form it does because eddies are introduced into the flow at this large scale, and transfer energy to smaller scales via an inertial range, presumably explaining the near $-5/3$ slope of the higher-frequency end of the spectrum.

Having obtained this spectral form, much of the rest of Revell et al deals with the requirements for simulating gusts which match the observed behaviour. The model is reasonably successful at doing this. Figure 4.5 shows the spectrum generated from the output of the model.
The spectrum of the output from the Revell et al model shows a spectral peak (here at about a period of 8 minutes) and a power law tailing off from this. Note that the power-law obtained from this model data was much steeper (with a spectral slope parameter approaching 3) than experimentally observed, and that the model could only reproduce the spectrum for time periods longer than a couple of minutes. Such limitations due to grid size are a difficulty with this kind of model when the detailed statistical properties of the kind discussed in the preceding two chapters are important. In the case of the simulation discussed here, the horizontal mesh spacing was 250m, while the lowest layer thickness was 20m, with layers above that thicker by nominally 15 per cent per layer up to a maximum thickness of 350m.

4.2.2 Comparison with multiscaling model and degree of intermittency

Of principal concern for multiscaling analysis is the spectral slope parameter and the scaling of the moments. The spectral slope of the data, at 1.5, is considerably less than that of the inertial range, especially given that measurements of the inertial-range spectral slope in the earlier chapters indicated it was usually steeper than the \(-5/3\) value suggested by Kolmogorov's theory.
Figure 4.5: The spectrum made from the Revell et al. model output. The dashed lines show $-5/3$ and $-3$ power laws.

It thus seems reasonable that what this slope represents is not a low-frequency inertial range, as perhaps the Revell et al. interpretation might have it, but rather part of the energy-containing range (which is, by definition, the scales at which energy is injected into the turbulent flow).

This seems likely as it is hard to imagine that the eddies just smaller than those of the scale at which the "forcing" due to boundary conditions occurs would have evolved enough to have "forgotten" those boundary conditions, as is envisaged to be the case for an inertial range.

Whether this spectral region is an inertial range or not is largely unimportant from a pragmatic point of view.

For the purposes of constructing cascade simulations, the relevant point is that the wind generated in the lee of the mountain range had a significantly higher value for $\beta$ than that observed over flat terrain.

In order to determine the degree of intermittency exhibited by the wind signal obtained at Mt. Cook, the $K(q)$ function was found for this data and compared with the energy-containing and inertial-range wind data obtained at the Ardmore site.
Scaling of third-order moment

Figure 4.6: The scaling of the moments for inertial-range turbulence (circles), and for the energy-containing range at the Mt Cook site (pluses), and the Ardmore site (crosses).

The scaling of the third-order moment for each of these cases is shown in figure 4.6, where, as with chapter 2, the first three points of the plot have been ignored (and are not shown here). Here, the ‘+’s represent the third-order moments of the Mt Cook data (sampling frequency 0.1Hz), the ‘o’ s are those of the inertial range (1000Hz), and the ‘x’ s those of the Ardmore data (0.1Hz). Figure 4.7 shows the $K(q)$ function for each case, where the solid line is the function for the inertial range data, the dot-dashed line that of the Mt Cook data, and the dashed line that of the Ardmore data. The corresponding intermittency parameter, $C_1$, for each case was: Mt Cook 0.0792, Ardmore 0.0328, and inertial range 0.0758. It must be stressed that each of these estimates is for a single realisation. Note from the previous chapters that the intermittency parameter for inertial-range wind varied from values of $C_1 = 0.106$ to 0.157 for various ensembles of 16,384 points. This suggests that while for the realisations examined, the Mt Cook data was more intermittent than the inertial-range data, the value for the Mt Cook intermittency parameter was more typical of the low-end of the range of intermittency for the inertial range.
K(q) for various wind signals

Figure 4-7: The K(q) functions for inertial-range turbulence (solid line), and for the energy-containing range at the Mt Cook site (dot dashed), and the Ardmore site (dashed).

Unfortunately, the Mt Cook data set was limited to this single realisation, so that it was not possible to obtain a broader picture of intermittency at this location. Nevertheless, it was clear that the winds at this site were much more intermittent than winds of the same time scales at the Ardmore site.

4.3 Simulations

Using the same bounded-cascade model described in chapter 2, a simulation of the wind at Mt Cook was able to be produced. Once again, the parameters for this model, $H$, $W$ and $p$, were not able to be determined analytically using the method suggested by Menabde et al [1996] (see appendix 1), and were found by heuristic methods, comparing the multiscaling properties of the simulated cascade with the experimental data, as was the case in chapter 2. Here, $W_{10} = 0.9$ was again used, with $H = 0.25$, and $p_1 = 0.65$. In order to produce the spectral peak observed in the experimental data, the signal was constructed by producing sets of simulated data just 32 points in length, corresponding to a period of about 5 minutes. These were then joined to make a continuous signal of some arbitrary length.
Figure 4-8: Experimental (top) and simulated (bottom) signals for the wind data obtained at Mt Cook, using a bounded cascade.

Figure 4-9: Simulated wind signal produced by the Revell et al model.
Figure 4.8 shows the simulated signal and the experimental signal. Figure 4.10 shows the spectra of the experimental and simulated signals.

For comparison, figure 4.9 shows the signal resulting from Revell et al.'s numerical model. It is clear from this figure that the output is too continuous to possess multifractal indices (recalling from chapter 1 that such indices are effectively a measure of how much a signal "meanders" across space), and does not have structure on all scales.

Thus, the cascade model reproduces the features of the experimental signal, as well as its statistical properties, more convincingly than Revell et al.'s model, although this may be partly explained by the fact that the grid scale for the fluid dynamical model was too coarse to model the fine-scale structure. Note that the peaks and troughs in this figure were much more pronounced than for the experimental data. Such unrealistically dramatic gusts raise questions about this model's suitability for all practical situations. Even so, it should be pointed out that Revell et al.'s model gives a three-dimensional flow field for the entire valley, while the cascade model represents simulated data obtained from a single instrument at a single point.
Scaling of third-order moment for simulated and experimental signals

Figure 4.11: The moment scaling for experimental and simulated signals for the Mt Cook site.

However, combining the multiscaling and fluid dynamical model might represent a method of reproducing both the low and high frequency statistics. For example, rather than adding small sets of cascade output to simulate the spectral peak, the same effect could be achieved by superimposing the cascade output on the Revell et al output. This might be done by averaging the Revell et al output over intervals of a few minutes (thus losing the unrealistic fine-scale structure while retaining the large-scale information) and adding sets of the appropriate length cascade model output to it (thus replacing the fine-scale structure).

Returning to the cascade simulation, figure 4.11 shows the scaling of the third-order moment of the experimental and simulated signals, and figure 4.12 the $K(q)$ functions. Here, it was found that the bounded cascade did not reproduce the $K(q)$ function well for values of $q$ less than 1, though the match with the experimental data for $q > 1$ was quite good. Theoretically this means that the probability distribution of the weighting factor, $W$, must be different for the experimental data than for that used in the bounded cascade. While different distributions could be tried to reproduce the behaviour of the experimental data to some arbitrary degree, in this case the available data was limited, so that the uncertainty associated with the presented $K(q)$ function was high (see Harris et al [1998b]). This being the case, it would seem pointless trying to reproduce it too closely.
It seems reasonable enough that the cascade simulation here has reproduced the gross characteristics of the experimental data.

4.4 Conclusions

It was found that a surface wind signal collected in the lee of the Mt Cook mountain range was able to be characterised and simulated using multiscaling models. The resolution of the signal obtained was the same as for the energy-containing range at Ardmore. It seems reasonable that since eddies shedding from the ridge of the Mt Cook range are injecting turbulent energy into the flow at a characteristic temporal scale of 4 to 8 minutes, that the portion of the spectrum characterised by the data collected for this study is the energy-containing range, rather than some extended inertial range.

This is important because the statistical character of this energy-containing range was quite different from those of the signals recorded at the Ardmore site. This difference was almost certainly attributable to the topography of the site. The data obtained at Mt Cook suggests that wind in the lee of a mountain range possesses both a steeper spectral slope (around 1.5 for the Mt Cook site, compared with 0.8 to 1.35 for the Ardmore site), and a higher degree of intermittency ($C_1 = 0.0792$ for Mt Cook and 0.0328 for Ardmore). For the Mt Cook energy-containing
range, the scaling of the moments was not only more convincing, it also lent weight to the generality of using multiscaling analysis for the low-wavenumber part of the spectrum, by demonstrating that wind signals for these wavenumbers can show significant intermittency, making $C_1$ a useful model parameter.

In order to reproduce the spectrum of the experimentally measured signal more closely, the number of steps used in the cascade was kept small enough so as to cause the spectrum to flatten for time scales larger than those of the largest eddies expected to be shed from the mountain ridge (in fact, a series of short signals of length equivalent to this time scale were generated using cascades and jointed to produce a signal of some desired length).

Since the model presented in the Revell et al paper was also capable of producing this spectral peak, it is clear that the two methods might be used in some complementary way. Indeed, it is possible to imagine adding a “fractal fur” to the signal generated by the Revell et al model in order to take into account the fine-scale statistics.

It must be remembered that while the cascade models clearly produced a more realistic simulation of the wind for a single location in space, the numerical model used by Revell et al was capable of producing a three-dimensional flow field through the entire space. Thus, the appropriateness of each model depends largely on the application. Where detailed knowledge of the statistical properties of the fine-scale structure of the wind is required, multiscaling analysis and cascade models appear a better option. Note that “fine-scale” structure here refers to the smallest scales that fluid dynamical models typically run to (i.e. gusts lasting a few seconds), but will usually be large enough to be significant to an observer on the ground. But the statistics of such a model at these scales are likely to be suspect, because scales finer than the model resolution are ignored. These statistics are likely to be relevant to applications such as wind turbines, and where an understanding of pressure gradient variations are important (i.e. pressure equilibrium in skyscrapers to keep moisture out).
5. Nested intermittency and its significance for simulation techniques

5.1 Introduction

In the previous chapters, surface layer winds were simulated using cascades, which are designed to display scaling behaviour for both the spectrum and moments. Unfortunately, as already noted, the justification for using such cascades is only phenomenological, and little real insight into the dynamics responsible for such signals can be gained. This is particularly so in the case of the energy-containing range, because the time scales of the wind-strength fluctuations in this range correspond to spatial scales of the order of hundreds of metres for the largest eddies (assuming an eddy-cascade phenomenological picture), while the depth of the surface layer is no more than a few tens of metres.

This raises the possibility of an alternative phenomenological argument for the processes occurring on energy-containing range time scales. If large eddies are supposed to be shedding daughter eddies close to the ground, they are likely to be elongated due to the boundary. In that case, does a cascade of elongated eddies as opposed to fully developed eddies occur? Or, is such an eddy cascade process confined to the fully developed three-dimensional eddies higher up. An alternative phenomenology might then have such eddies generated above the surface layer and carried down to the surface layer by chance, interrupting the fully developed three-dimensional cascade process. In the latter case, it might be imagined that these eddies provide random input to the surface-layer wind signals, which, if an eddy of size $l_n$ has an associated mean amplitude $A_n$ and decay time $t_n$, is on different characteristic time scales depending on the size of the eddy. Such a model resembles some biological models which generate power-law behaviour by summing random input of varying characterised periods. In this way, the eddies might still originate from a universal cascade process, but their statistics at ground level may be strongly dependent on the prevailing atmospheric conditions (such as
wind shear and temperature gradient), which may affect the likelihood of transport of such eddies to the surface layer.

In order to try to gain some more insight into the dynamical processes which might be responsible for generating the power-law features of the energy-containing range, the signal was examined to determine if it possessed an intermittent structure characteristic of a cascade process. The intermittency of turbulent signals (the property that the strongest fluctuations in the signal tend to be clustered in the same regions of space), first noted by Batchelor and Townsend [1949] and others (e.g. Sandborn [1959], Kuo and Corrsin [1971]), has played a significant part in the characterisation and modelling of turbulence since its discovery. Some workers (e.g. Frisch, Sulem and Nelkin [1978]) have assumed that small-scale fluctuations are nested within larger-scale fluctuations, occupying smaller regions of space at smaller scales. Some insight into this process was gained from the experimental results of Saucier [1991]. In that work, high-resolution, hot-film anemometer data was digitally band-pass filtered for bands around wavenumbers $k_0, 2k_0, \ldots, 2^N k_0$. When compared, the strongest turbulent fluctuations in each band appeared to occupy the same region of space. Clearly a cascade process, owing to the way in which it is constructed — with the distribution of the largest scales directly affecting the smallest — must be expected to behave in this way. In this chapter, the energy-containing range is examined for the existence of this characteristic.

The properties of the energy-containing range are also compared with the output of a biologically motivated, non-cascade model which generates signals which exhibit $E(f) \propto f^{-\beta}$ behaviour. The model, suggested by Hausdorff and Peng [1996], simulates such signals by adding unrelated random inputs of differing time scales. Such a model may be adapted to represent the non-cascade phenomenological suggestion for the energy-containing range described above. Since each frequency band is independent of the others, one would not expect to find strong fluctuations in the Hausdorff and Peng signal represented in all bands simultaneously. On the other hand, one would for a cascade-generated signal.
Figure 5-1: Raw and band-passed hot-wire anemometer data (inertial range).

Figure 5-2: Band-passed hot-wire anemometer data for a range of frequencies.
Given that not all signals that exhibit power-law behaviour necessarily need be produced by processes with similar dynamics, a lack of correlation between fluctuations in different wavenumber bands may provide a method for distinguishing different types of power-law producing dynamics.

Of particular interest to the central theme of this thesis, is the point made by Hausdorff and Peng that the power-law scaling observed in biological systems need not reflect any "special" phenomena. Extending this argument to the energy-containing range for turbulence, it is interesting to ask if there is any "special" need for a cascade model to describe this range. Indeed, a model such as that described by Hausdorff and Peng might provide a better description for surface-layer turbulent fluctuations of scales larger than those of the inertial range, given the difficulty in justifying a cascade process on large scales in this layer. The work in this chapter investigates the appropriateness of such an "unspecial" model for describing this kind of turbulent behaviour.

5.2 Results

5.2.1 Digital filtering

Data was obtained with a sampling frequency of 1000Hz from the hot-wire anemometer, and 0.1Hz from the cup anemometer. These data sets were low-pass filtered to eliminate contributions from frequencies higher than the sampling rate of the instrument used to collect them.

The signals were band-passed filtered for different frequency bands in order to examine the correspondence between intermittent bursts on each scale, using the same method described by Saucier [1991]. Filtering for this procedure was done using a third-order Butterworth digital filter with the Matlab signal-processing package.

Figure 5.1 shows 10,000 hot-wire anemometer data points (and so inertial-range data) and the same data band-pass filtered for frequencies from 160Hz to 320Hz. The figure demonstrates the intermittent nature of turbulence, with strong fluctuations confined locally in space. Note that these bands can be described in terms of spatial scales by use of Taylor's hypothesis, by taking the mean wind speed to be 6m/s.
Figure 5.3: Band-pass filtered cup anemometer data for a range of frequencies.

Figure 5.2 shows the sequence of band-pass frequencies (from the bottom) for 160Hz to 320Hz, 80Hz to 160Hz, 40Hz to 80Hz, 20Hz to 40Hz, 10Hz to 20Hz, and 5Hz to 10Hz, so the frequency range for each band is halved for each successive step. Although it may only be judged qualitatively, this figure reveals that strong fluctuations occur in the same portion of the signal for different frequency bands, as reported by Saucier.

Though it must be remembered that a sharp discontinuity in the raw signal will contribute to all frequency components (and thereby cause strong fluctuations in the same portion of the signal for all bands), there are two things to note from figure 5.2. First, the regions of strong fluctuations are reasonably broad, and thus not likely to be associated with a single discontinuity in the signal. Secondly, the strong fluctuations themselves seem to be clustered together, a property also suggestive of a cascade-like process.

Note from figure 5.1 that these intermittent bursts appear to coincide with strong fluctuations in the non-filtered signal (that is, the signal changes markedly in a very short interval).
The cup anemometer data, recording wind-speed fluctuations on time scales typical of the energy-containing range, was filtered in the same way, and the results shown in figure 5.3. Here, 10,000 points were again used, with the frequency bands being 0.016Hz to 0.032Hz, 0.008Hz to 0.016Hz, 0.004Hz to 0.008Hz, 0.002Hz to 0.004Hz, 0.001Hz to 0.002Hz, and 0.0005Hz to 0.001Hz. From this figure, again qualitatively speaking, the signals appear not only much less intermittent (in agreement with observations in chapter 2), but there appears to be only slight correlation between the fluctuations on large scales with those on small scales. Additionally, there is no strong indication that the fluctuations on the smaller scales are nested in smaller regions of space than the fluctuations on the larger scales.

5.2.2 The Hausdorff and Peng model as a turbulence simulator

The model suggested by Hausdorff and Peng [1996] to describe biological systems considers the sum of multiple random signals, with each signal having its own decay constant, \( \tau_n \), and, in this way, simulating the input into the system of unrelated random processes of different time scales, a situation typical of some biological systems. Such a model might be adopted to describe surface-layer winds if it is imagined that fluctuations in such winds occur as eddies of different time scales randomly drift from the atmospheric boundary layer into the surface layer, where they decay at some mean characteristic rate related to the size of the eddy.

The Hausdorff and Peng model sums a series of functions \( x_n(t) \) of discrete time steps \( t \). For \( t_1 \), \( x_n(t_1) = R \), where \( R \) is a normally distributed random amplitude. For each \( t_i \), \( R \) takes on a new value with a probability of \( 1/\tau_n \). Consequently, the correlation function for each \( x_n \) goes as \( \exp(-t/\tau_n) \). The sum of the components \( x_i \)

\[
y(x) = \sum_n A_n x_n \tag{5.1}
\]

where the \( A_i \) determine the relative importance of each input, can be shown to tend to power-law behaviour (see Hausdorff and Peng [1996]).
Figure 5-4: A simulated wind signal using the Hausdorff and Peng model, compared with experimentally obtained wind data.

Figure 5-5: Spectra of simulated and experimental energy-containing range signals.
Using a non-cascade based phenomenological interpretation for energy-containing range turbulence, such a model might be adapted using spatial scales instead of temporal, where various eddies of spatial scales $L2^{n-1}$ for $n = 1...N$ possess mean characteristic decay constants of $\lambda_n \equiv \tau_n$. Then the decay constants for each $x_i$ is given by:

$$\lambda_n = \frac{\pi 2^n}{L}$$  \hspace{1cm} (5.2)

Figure 5.4 shows a 10,000-point signal generated using equations 5.1 and 5.2, with $A_n = 1$ and $N = 20$. This is compared with an experimentally obtained wind signal for the energy-containing range. Figure 5.5 shows the spectrum of these signals, and figure 5.6 the $K(q)$ functions.

The spectral slope for the simulated signal — using the same method as the previous chapters to produce a 128-point spectrum from 3072 points — yielded $\beta = 0.80$, compared with $\beta = 0.94$ for the experimental data shown, although in principle the model could produce a value closer to the experimental signal by choosing the $A_n$ differently.
The $K(q)$ function produced from the simulated and experimental signals indicated that both signals were only weakly intermittent, although, as was the case with the simulations in the previous chapter, the $K(q)$ functions were not a particularly close match. Significantly, the extent to which the Hausdorff and Peng model reproduced the scaling of the moments of the experimental data was no worse than for cascade models for the Ardmore wind signals.

Figure 5.7 shows the Hausdorff and Peng simulated signal band-passed filtered for various frequencies, as in the previous sections. Clearly, the bands show little intermittency or correlation between intermittent bursts on different scales. Additionally, although it is only a subjective assessment, it would appear that the band-passed signals for the energy-containing range collected with the cup anemometer in figure 5.3 showed at least some correlation between various scales, while the Hausdorff and Peng model shows none. This is in contrast with signals which were simulated with the cascade models described in chapter 3.

Figure 5.8 shows band-passed filtered signals for inertial-range data simulated with a cascade model (as in chapter 3), and figure 5.9 for cascade-simulated energy-containing range data.
Velocity data simulated using a cascade

Figure 5-8: Simulated inertial-range band-pass filtered data.

Energy-containing range data simulated with cascade

Figure 5-9: Simulated energy-containing range band-pass filtered data.
While once again only a subjective assessment, it would appear that both these figures illustrate correlation between frequency bands.

Figure 5.10 shows the distribution for the cascade weighting factor $W$, as described in chapter 3, for the Hausdorff and Peng model signal. While it is clear, by comparison with the distribution obtained in chapter 3 for the energy-containing range (shown with 'x's in the figure), that this distribution is narrower and has a higher peak than the experimental wind data, this simulation nevertheless appears to be at least as self-similar as the experimental data, even though it was not generated by a cascade. The shape of this self-similar distribution could be modified by altering the decay constants $\lambda_n$ by introducing a new parameter $B$, so that:

$$\lambda_n = B \frac{\pi 2^n}{L}$$

Putting $B = 1.83$ produced a distribution of the same height as the experimental data, as figure 5.11 shows, but still of the wrong shape.

Thus the Hausdorff and Peng model fails to reproduce the properties of the experimental signal in this respect as well, despite the moments scaling similarly to the real data.
Figure 5-11: The probability density distribution for the cascade weighting factor $W$ for the Hausdorff and Peng model signal, this time with $B = 1.83$. The 'x's again show the distribution for the energy-containing range wind velocity signal.

Figure 5-12: The output from the Hausdorff and Peng model with decay constants altered to make the signal more intermittent.
Figure 5.13: The scaling of the third-order moment of the signal shown in figure 5.10 clearly does not obey a straight-line scaling law.

Figure 5.14: The distribution for $W$ for the signal shown in figure 5.12 does not possess a self-similar distribution either.
In the case of simulating data which is more strongly intermittent, such as that collected in the surface layer at Mt Cook, or inertial-range turbulent signals, the Hausdorff and Peng model appears to be inadequate. While attempts may be made to make the output of the Hausdorff and Peng model more intermittent by altering the parameters $A$ and $B$, it was found that the output, while qualitatively appearing to be more intermittent, as illustrated by the case shown in figure 5.12, failed to produce multiscaling moments, as can be seen from figure 5.13, and the signal was not self similar, as shown for figure 5.14.

Note that this result is not surprising, since the model was intended to simulate spectral power-law scaling, not multiscaling moments, which are a trait of cascade models.

5.3 Conclusions

Analysis of the energy-containing range wind signals obtained at the Ardmore site by band-pass filtering demonstrated that these signals showed an apparent correlation between large-scale and small-scale fluctuations, as has been reported by Saucier [1991] for the inertial range, though the degree of the correlation was much less significant. Also, there did not appear to be the nesting of small-scale fluctuations within ever-smaller regions of space (or at least, it was not qualitatively obvious that this was occurring). In principle, the existence of multiscaling moments and self-similar distributions is quantitative evidence for this sort of nesting, since the existence of such correlations between bands can occur as a consequence of cascade processes. Indeed, simulations of both the inertial and energy-containing ranges using cascade models possessed this trait.

Using the Hausdorff and Peng model (for which an alternative phenomenology was imagined to describe the existence of a power-law for the energy-containing range), it was possible to generate simulated energy-containing range winds with spectral slope and multiscaling parameters similar to the real data. However, the correlation between fluctuations in different frequency bands was absent, and while the data appeared to be self-similar, it had a different distribution to the experimental data. Since the model possessed insufficient parameters to produce an arbitrary distribution, it seems a self-similar cascade approach as described in chapter 3 might be a more suitable way of characterising
and simulating the statistical properties of the experimentally observed signal. In terms of the alternate phenomenological interpretations discussed in the introduction, the results presented here suggested that simply adding random functions of differing time scales (representing different-sized eddies randomly drifting into the surface layer) is insufficient to reproduce all the properties of the experimental signal. The Hausdorff and Peng model, originally intended to describe biological systems, is also unsuitable for simulating turbulence of more strongly intermittent varieties.

The existence of a self-similar distribution for the output of the Hausdorff and Peng model presented here might simply serve to demonstrate the versatility of self-similar cascade models, in that these models are capable of reproducing the statistics produced by the Hausdorff and Peng model, while the reverse is not generally true.

The frequency band analysis conducted in this chapter might, then, represent a test which may be applied to various types of signals that possess power-law scaling, to determine the existence of a cascade process, as opposed to some other power-law generating process, such as that envisaged by Hausdorff and Peng.

Yet at the same time, it must be remembered that the band-pass filter method is subjective, so that it may be left unclear whether a cascade process is genuinely at work when a signal is only weakly intermittent. It would also seem that nested intermittency is only readily apparent in strongly intermittent signals.

While this leaves room for some doubt as to whether the energy-containing range behaviour observed in the previous chapters is necessarily the result of a cascade process, there are some good reasons to suppose that it is. Central to this is that the Hausdorff and Peng model is not suitable for producing more intermittent signals such as those observed at Mt Cook. This is because the model is not designed to produce multiscaling moments. Though it appears to reproduce the scaling of the moments for the Ardmore wind signals as well as the cascade models do, this is most likely a consequent of both the model output and the experimental signal having little intermittency. However, given that the Mt Cook energy-containing range was more intermittent, with multiscaling moments characteristic of a cascade-type process, it does not seem far-fetched to suppose that the Ardmore wind signals were produced by similar dynamics. It is also much
more elegant from a modelling point of view to use the same models for both the Ardmore and Mt Cook sites and assume that the difference in the statistical character of signals from each site is a property of conditions (e.g. wind shear and stability) at the site, with the possibility that such conditions may be directly related to the parameters of the models (as indeed appeared to be the case for the energy-containing range wind signals at Ardmore described in chapter 2).

In summary, the existence of correlations between the frequency bands and the shape of the self-similar distribution (different from a Hausdorff and Peng style model) for the energy-containing range suggests that it is "special" enough to justify the use of cascade models.
6. A suggestion for a turbulence model

6.1 Introduction

6.1.1 Overview

The analysis of inertial and energy-containing range turbulence signals conducted in the previous chapters determined that these could be described in terms of multiscaling analysis and possessed self-similarity. As such, these properties were able to be reproduced by the use of randomly weighted multiplicative cascades. While this sort of cascade model is an effective method for reproducing such signals, it is not clear what its use says about the physical processes which occur to produce such signals. Though such models may be justifiable phenomenologically — at least for the inertial range, thanks to the ideas of Richardson and Kolmogorov — they become increasingly dubious when applied to the energy-containing range, and even more so for other such multifractal geophysical signals, such as rain.

It is also clear from the analysis in the preceding chapters that turbulent signals are far from ideal multifractals. These signals showed multiscaling behaviour for limited ranges only, and, as was seen, as well as has been reported by other workers, the scaling of the moments does not necessarily obey an obvious straight-line fit. Conversely, cascades are mathematical abstractions which are perfectly multifractal and scale endlessly. While it is true that these models can be set up to produce outer and inner limits to the scaling ranges, these are by their very nature artificial.

Additionally, the modelling of the dissipation range as opposed to the velocity signal, in the case of the inertial range at least, presents the problem of how to then obtain a velocity signal.

Probably the most interesting deficiency of the cascade models discussed in this thesis is that described in chapter 3, their failure to reproduce the experimentally observed property of less positively signed gradients in the velocity
signal than negatively signed gradients. This chapter is a slight departure from the central theme of this thesis and primarily serves to present a speculative examination of a non-cascade model based on this experimental observation. This is done by using similar dimensional arguments to Kolmogorov’s original scaling formulation with the aim of generating signals with similar statistical properties to turbulence using a finite step model of the form:

\[ u_i = f(u_j, u_k, t_n) \]  

(6.1)

where \( u_i \) is some dynamical quantity, say, the \( i \)th component of the velocity field, \( u_j \) and \( u_k \) are variables correlated with \( u_i \) (the \( j \)th and \( k \)th velocity components, say), and \( t_n \) is a series of discreet points in time. While the model to be presented is essentially just another phenomenological model, for which there is no particular connection to the well-known Navier-Stokes equations for a viscous fluid:

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}
\]

(6.2)

it is suggested that, provided the statistical properties of the output of the model match the experimentally observed properties of turbulence, they provide some insight to the form of the actual solution, much as it is implicit that cascade models offer insight into the nature of turbulent dynamics because they reproduce them so well (note that the summation convention for equation 6.2 is that repeated indices in any term indicate a summation over all three values of the index).

6.1.2 Model construction

The aim here is to model the evolution of the fluid as it flows past a point in space with some mean speed \( \bar{U} \), so that the output simulates an instrument measuring wind speed at a fixed location. Consider the \( x \) component of the velocity at that point and express the model in a form:

\[ U_x(t_0 + t) = U_x(t_0) + f(t) \]  

(6.3)

Also introduce two extra variables correlated with \( U_x \), say \( U_y \) and \( U_z \), the \( y \) and \( z \) components of the velocity. If the turbulence is evolving much more slowly than the rate at which the mean wind speed carries it past the reference point \( x_0 \), then \( U_x(t_0 + t) \) and \( U_x(t_0) \) are effectively measures of the velocity field at two separate points in the reference frame of the mean fluid velocity, and as such, by Taylor’s
hypothesis, temporal scales may be converted into spatial scales with $\bar{U}$ the proportionality constant.

If so, then equation 6.3 may equivalently have the form:

$$U_x(x_0 + l) - U_x(x_0) = \Delta U_x = f(l)$$  \hspace{1cm} (6.4)

The model is then constructed by guessing the form of $f$ by using dimensional arguments for the same quantities used in Kolmogorov's 1941 formulation:

$$\left( |U_x(x_0 + l_0) - U_x(x_0)|^p \right) \propto (\varepsilon l_0)^{p/2}$$  \hspace{1cm} (6.5)

that is, as a first assumption:

$$f = f(l, \varepsilon)$$  \hspace{1cm} (6.6)

However, recall the result from chapter 3 that the distribution of the velocity gradients was biased towards the negative side, from which it follows that if there is to be a mean value for the velocity of the flow, then on average the negative gradients must be of smaller magnitude than the positive gradients. Clearly, however, equation 6.6 takes no account of the sign of the gradient, since $\varepsilon$ is proportional to the square of the gradient. In order to incorporate the sign,

$$f = f \left( \nu^{1/2}, \left( \frac{\partial U_i}{\partial x_j} \right)^A, l^{D(A)} \right)$$  \hspace{1cm} (6.7)

is tried instead, where we introduce $A$ as a new parameter. The most obvious method for choosing the value of $A$ is to use dimensional arguments, following Kolmogorov's example. $D$ is introduced as a function of $A$, so that equation 6.7 does not necessarily give a single possible value for $A$ from these dimensional requirements. Note that since equation 6.4 must be able to be negative and that for the purposes of this model $l$ and $\nu$ can effectively be treated as constants, then the range of possible values for the power is restricted. Additionally, taking into account the mentioned experimental observation, $f$ must be an asymmetrical function about the origin. Here we note that if the value for $A$ is restricted to ranges $0.5 < A < 1.5, 2.5 < A < 3.5, A \neq 1, 2$, etc., then the real part of $(\partial U_i / \partial x_j)^A$ is asymmetrical. For example, if $A = 2/3$, the real part behaves as shown in figure 6.1, satisfying the requirement for an asymmetrical function, while maintaining the dimensional requirements if $D = 1/3$. 

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This leads to the rather ugly function:

\[ \Delta U_x \propto v^{1/3} \text{Real}\left(\frac{\partial U_x}{\partial y}\right)^{2/3} l^{1/3} \]  

(6.8)

Unfortunately, \( \partial U_x / \partial y \) is an unknown. Having already abandoned the use of \( \varepsilon \), so that a postulate is effectively being made as to the dependence of \( \Delta U_x \), this quantity might just as well be replaced by any quantity with the same units.

Thus, it is proposed — principally for the sake of expediency — to use a quantity with the same units, \((U_l - U_y)/l\), which both provides a closure scheme for the model and maintains dimensional requirements, so:

\[ \Delta U_x \propto v^{1/3} \text{Real}\left(\frac{(U_x - U_y)}{l}\right)^{2/3} l^{1/3} \]  

(6.9)

By itself, equation 6.9 is unlikely to produce suitable output, since the model needs to behave like an oscillator, so that the value of \( U_x \) tends neither to infinity nor zero. At least a second term is required to prevent equation 6.9 from taking either of these paths. Note that with the appropriate powers of \( v, l, (U_x - U_y)/l_0 \) other dimensionally correct terms of this form can be constructed.

As a guide to choosing such terms, it is noted that if \( A > 0 \), then the term \( f \) grows larger as the difference between \( U_x \) and \( U_y \) grows, but if \( A < 0 \), the opposite
is true. This has the effect of rotating the velocity vector $U$ in opposite directions depending on $U_x - U_y$. If these two cases can be incorporated into terms which act in opposite ways on the direction of $U$, then an equation may be sought in which these terms act to balance each other, with the dynamical interpretation that one is a rotational term and the other anti-rotational. This is the principle upon which the present model is based.

Below are two sets of equations, obtained largely by trial and error, for which neither the rotational nor anti-rotational term becomes large enough for the output to diverge from a chaotic attractor. These equations are used to generate the signals examined in the next sections:

$$U_x((n+1)\ t_0) = U_x(nt_0) + t_0 \left(-C_1 U_y \left(1 + \text{Real}\left[(U_x - U_y)^A\right]\right) + B \text{ Real}\left[(U_x - U_y)^{A-1}\right]\right)$$

$$U_y((n+1)\ t_0) = U_y(nt_0) + t_0 \left(-C_2 U_x \left(1 + \text{Real}\left[(U_y - U_x)^A\right]\right) + B \text{ Real}\left[(U_y - U_x)^{A-1}\right]\right)$$

(6.10)

for a two-dimensional model with components $U_x$ and $U_y$. Note from equation 6.9 and the use of Taylor's hypothesis that the functions $C$ and $B$ must be functions of $l, \bar{U}$, and $v$. Using three components the following form is suggested:

$$U_x((n+1)\ t_0) = U_x(nt_0) + t_0 \left(-C_1 U_z \left(1 + \text{Real}\left[(U_x - U_z)^A\right]\right) + B \text{ Real}\left[(U_x - U_z)^{A-1}\right]\right)$$

$$U_y((n+1)\ t_0) = U_y(nt_0) + t_0 \left(-C_2 U_x \left(1 + \text{Real}\left[(U_y - U_x)^A\right]\right) + B \text{ Real}\left[(U_y - U_x)^{A-1}\right]\right)$$

$$U_z((n+1)\ t_0) = U_z(nt_0) + t_0 \left(-C_3 U_y \left(1 + \text{Real}\left[(U_z - U_y)^A\right]\right) + B \text{ Real}\left[(U_z - U_y)^{A-1}\right]\right)$$

(6.11)

### 6.2 Results

#### 6.2.1 Scaling of the ‘velocity’ field, $U_x$

If the model presented above is to be useful it must produce scaling behaviour. However, it is difficult to justify why the model should scale appropriately. In order for scaling to occur, the structure function of the output must behave such that:

$$\left\langle \left| \Delta U_x^p \right|^2 \right\rangle \sim l^{\xi(p)}$$

(6.12)
A field which behaves this way is said to be multiaffine [Benzi et al 1993], a property which is closely related to multiscaling behaviour [Vainshtein et al 1994, Menabde et al 1997a].

If $\zeta$ is a linear function of $p$ then equation 6.12 has the same form as Kolmogorov’s 1941 hypothesis as given in equation 6.5, and if it is a non-linear function of $p$, the structure function is multiscaling, as is the case for real turbulence.

Numerical experiments were carried out on equations 6.11 to ascertain if this was the case. Figure 6.2 compares the scaling of the second-order structure functions for the three-dimensional model output (obtained with parameter vales of $A = 2/3$, $B = 1$, $C_1 = 1$, $C_2 = 0.8$, and $C_3 = 0.8$, with initial values $U_x = 1$, $U_y = 1.6$, $U_z = 2$ and step size $t_0 = 0.002$) with experimentally obtained inertial-range turbulence data. In both cases, the structure functions scaled with a power-law exponent of $\alpha = 0.8$, implying a spectral slope of about $-1.8$. Thus Kolmogorov’s 1941 hypothesis, in the form of equation 6.5, holds for both the model output and the experimental signal for $p = 2$. 

Figure 6-2: Scaling of the second-order structure function for the model output (crosses) and the experimental inertial-range velocity signal.
This suggests that the quantity $(U_x - U_y)/l_0$ for the model might play an equivalent role to that of $\varepsilon$ in Kolmogorov's formulation, so that, it might be speculated that for some range of scales:

$$\left\langle |\Delta U_x|^2 \right\rangle = \left\langle \nu^{1/3} \text{Re}\left( \frac{(U_x - U_y)^{2/3}}{l} \right) \right\rangle l^{2/3}$$  \hspace{1cm} (6.13)

is the form of the behaviour of the second-order structure function, which in turn implies that the parameter $A$ in the model determines the spectral slope. Of course, equation 6.13 would be extremely difficult to prove analytically, and is arrived at by nothing more than a similar dimensional argument to Kolmogorov's.

Nevertheless, the behaviour of the structure function shown in figure 6.2 suggests a power-law behaviour of the type in equation 6.12. The behaviour of the function $\zeta$ for the model output not surprisingly depended on the model parameters (more detail of this dependence is given in the following sections). Figure 6.3 shows $\zeta(p)$ using the parameters given above, and also for $A = 0.64$, $B = 0.5$ and a step size of $t_0 = 0.001$. Included in the plot is the behaviour for the experimental data. None of the data obeys the "monoscaling" suggested by Kolmogorov's 1941 theory and equation 6.5.
Figure 6-4: The model output with $A = 0.6$ and $B = 0.5$ (top), $A = 2/3$ and $B = 1$ (middle), and an experimentally obtained turbulence signal. Note the spikes in the middle signal for points around $n = 13,000$.

Figure 6-5: The spectra for the signals shown in figure 6.4, in the same order.
Figure 6-6: Output of the model using $B = 0$ and $A = 2/3$.

Note however that only the second model case (shown with asterisks in the figure) exhibits a behaviour similar to the experimental data, whereas for the first case $\zeta$ becomes a linear function of $p$ for values $p > 3$, i.e., it is no longer multiscaling.

The different behaviour of the two cases can partially be understood by examining the output signals, shown in figure 6.4 for $n = 5000$ to 15000. The model output has a similar appearance to the experimental data, but it is noticeable that for the model data there are “spikes” in the signal, particularly for the middle case, for several points around $n = 13,000$.

These spikes are not seen in the experimental signal, and occur when $U_i - U_j$ becomes small, making the second term in equation 6.11 large. Unfortunately, the presence of the spikes dominates the scaling as the order of the exponent $p$ becomes large. Note that the signal with the most spikes in figure 6.4 is the one for which multiscaling behaviour for the structure function broke down for the higher-order values of $p$ in figure 6.3. It will be seen that this breakdown is even more pronounced for the square of the gradients field in the following sections, since the amplitude of the spikes are squared to start with.
Figure 6-7: Output of the model using $B = 0.14$ and $A = 2/3$.

Figure 6-8: Output of the model using $B = 0.15$ and $A = 2/3$. 
6.2.2 Parameter dependence

Having speculated that the parameter $A$ controls the spectral slope of the output, numerical experiments were conducted to try to quantify this. Setting $B = 1$ and $C_i = 1$ to effectively eliminate these parameters, and using a step size of $t_0 = 0.01$ and the same initial values as before, the model produced power-law spectra output for a range of values for $A$ of about 0.5 to 0.7.

<table>
<thead>
<tr>
<th>$B$</th>
<th>$\beta$</th>
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<tbody>
<tr>
<td>0.27</td>
<td>1.70</td>
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<tr>
<td>0.50</td>
<td>1.64</td>
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<tr>
<td>1</td>
<td>1.56</td>
</tr>
<tr>
<td>2</td>
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<td>0.60</td>
</tr>
<tr>
<td>20</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table 6-1: The dependence of spectral slope on the parameter $B$.

The results give $\beta = 1.46$ for $A = 0.6$, $\beta = 1.67$ for $A = 0.65$, and $\beta = 1.88$ for $A = 0.7$. The statistical properties of the model did not appear to depend strongly on the initial values when these values were changed by small amounts (e.g. using $U_x = 2$, $U_y = 1$, and $U_z = 1$ as initial values, say, produced similar results).

Using a value of $A = 2/3$, the output’s dependence on $B$ was examined. Figure 6.6 shows the output for $B = 0$. The output is at first transient, then settles into periodic behaviour. Increasing $B$ up to a value of 0.14 produces output with similar results (figure 6.7 shows the signal with $B = 0.14$).

However, above this value the output changes suddenly, becoming unpredictable and possessing a power-law spectra. Figure 6.8 shows the output of the signal for $B = 0.15$, which possesses a spectral slope of $\beta = 1.7$ for points $n = 2000$ to 4048. As $B$ increases, the spectral slope decreases, tending to a value of $\beta = 0.6$. Table 6.1 summarises some values, using $A = 0.7$, where the spectral slope is determined for the same range of $n$ as before.
Figure 6-9: Comparison of the output of the two-component model with experimentally recorded inertial-range wind data.

Figure 6-10: The spectrum of the model output shown in figure 6.9.
Since $B$ determines the relative importance of the two terms in equations 6.10 and 6.11, it seems reasonable that for small $B$ the first term dominates in determining the spectral slope, while for larger values of $B$ the second term dominates, i.e. the slope appeared to tend to either $A + 1$ or $A$ depending on $B$.

### 6.2.3 Reducing the dimensions of the model

The two-dimensional model given by equation 6.10 was also examined. It has the obvious advantage of having less complexity than 6.11. Figure 6.9 shows the signal produced with this algorithm, using $A = 0.75$, $B = 1$, $C_1 = 1$ and $C_2 = 0.8$, with initial values $U_x = 1$, $U_y = 1.6$, and the step size set at $t_0 = 0.02$, along with experimentally recorded data.

Clearly the model output has some of the visual characteristics of the experimental signal, but is easily distinguishable from the experimental data. Figure 6.10 shows the spectrum of the simulated signal, for which the spectral slope was $\beta = 1.4$. 
6.2.4 Multiscaling behaviour from the model

As discussed in section 6.2.1, the "spikes" which appeared in the model output tended to dominate the scaling of the structure function.

Likewise, it is reasonable to suppose that taking the square gradients of the model output to find the "dissipation" field for the model will serve to exaggerate the effects of these spikes further. Figure 6.11 shows the square gradient field of the model's output using the second case discussed in section 6.2.1 (i.e., with $A = 0.64$ and $B = 0.5$). Here, both fields have been normalised to have the same mean, so that clearly the model output produces a field with much larger extreme values than the experimental data. Figure 6.12 shows the spectrum of the model signal shown in figure 6.11.

This spectrum was virtually flat, i.e. $\beta = 0$, implying that the field does not scale, although the spectrum of the field generated by taking the absolute value of the gradients, which is used by some workers to obtain a field for which to apply multiscaling analysis (as in Harris et al [1996]), had a spectral slope of $\beta = 0.25$. 
Figure 6.13: The scaling of the second-order moment of the model output shown in figure 6.11 shows what appears to be a steep, short scaling range (due to a few extreme values in the data). For scales larger than this range, there appears to be no scaling.

Figure 6.14: $K(q)$ function for the model output shown in figure 6.11.
This spectrum is also shown in the figure, along with a spectrum of the experimental dissipation field, which, as discussed in chapter 3, has a spectral slope of 0.6 to 0.7.

In contrast to the scaling behaviour of the structure function, the model did not produce convincing multiscaling behaviour for the square gradients field. Figure 6.13 shows the scaling for the second-order moment for the output shown in figure 6.11. While there appears to be a short, steeply sloping scaling range, this is due to the presence of the extreme values in the field. Evidence that a few data points dominate the scaling can be seen by examining the $K(q)$ function for the output, shown in figure 6.14. It shows that the function becomes essentially linear for $q$ bigger than 1.5, as would be expected in such a case. Examination of the scaling of the moments of the absolute gradients field showed little to suggest multiscaling behaviour.

Despite that lack of scaling of the square gradient field of the output, the model could still produce a field which displayed multiscaling behaviour by adjusting the parameters to produce output with $\beta < 1$, and examining $U'$ directly.
Figure 6.16: The scaling of the second-order moment for the model output to the fifth power (crosses) and the experimental dissipation field.

Figure 6.15 shows the model output using parameters of $A = 0.7$, $B = 30$, $C_1 = 1$, $C_2 = 0.8$, and $C_3 = 0.8$, with initial values $U_x = 1$, $U_y = 1.6$, $U_z = 2$ and step size $t_0 = 0.01$. The figure shows a plot for $n = 1000$ to 6000, where the minimum value of the output for these points has been subtracted from the field so that the new minimum value is zero. Note that the output suffers from the "spikes" discussed in the earlier sections. The largest of these were removed by simply taking them out of the data (five or so points in this case), and the resulting field squared. The result is also shown in figure 6.15. The spikes were removed purely for cosmetic reasons (i.e. to make a more easily interpretable plot), because as higher powers of the output field are examined, they dominate the plot of the field, so that the rest of the signal becomes too small to see. While it should not be forgotten that the spikes are actually part of the model output, experimentation showed that their presence was not the key determiner of the statistical properties of the output. Figure 6.15 also includes a plot of this output to the fifth power.

The reason for including the plots of the field squared and to the fifth power is to demonstrate how the field can be made to appear more like the dissipation field for turbulence data.
Figure 6-17: The $K(q)$ functions for the model output and the experimental dissipation field (dashed line).

Figure 6-18: The spectra for the model output (top) and the experimental dissipation field.
Figure 6-19: The distributions of the weighting factor, $W$, of the model output for $I_{N,3}$ to $I_N$. Here, the dashed line is the distribution with the finest resolution. The order of increasing coarseness is: points, dotted line, dash-dotted line, solid line and pluses.

Figure 6-20: The distributions of $W$ for the model output for $I_{N,3}$ to $I_{N,4}$.
In the case of the fifth power, the result looks almost indistinguishable from the dissipation field shown in figure 6.11. Of course, this is not a particularly useful result unless the statistical properties of the two cases are alike. If the field is scaling, taking the appropriate power of the model output can produce scaling of a desired slope. Figure 6.16 shows scaling of the second-order moment for the model output to the fourth power, using point \( n = 11,000 \) to 50,000, compared with the scaling of the same moment for experimental data using an ensemble with the same number of points. This shows a good match between the scaling of the experimental data and the model output. Figure 6.17 shows the \( K(q) \) functions for both cases. Once again, the match between the model and the experimental data is good, suggesting that the scaling behaviour of both cases is similar. The spectra for both cases in shown in figure 6.18. Here, both have a spectral slope of about 0.35.

The model signal was then examined for self-similar behaviour, using the methods discussed in chapter 3.

Figure 6.19 shows the \( W \) distributions for the model signal for points \( n = 11,000 \) to 200,000 for \( \ell_{N,4} \) to \( \ell_N \), where \( N \) is the smallest scale.
While the distributions shown are not particularly self-similar, for scales larger than $l_{N,3}$, a rough self-similarity does appear. Figure 6.20 shows the distributions for $l_{N,3}$ to $l_{N,6}$.

The distributions for $l_{N,4}$ to $l_{N,6}$ were used to construct an average distribution, which is shown in figure 6.21, along with the self-similar distribution for the experimental dissipation field.

Despite the close agreement achieved between the appearance of the $K(q)$ functions for the model and experimental data, this self-similar distribution demonstrates that the statistical properties of the two signals were different. On an aside, note that this difference in the agreement of these two methods supports the argument in chapter 3 that self-similarity analysis (provided self-similarity exists) is a more rigorous method for establishing the statistical character of scaling data than multiscaling analysis.

While the model provides output with reasonably similar appearance and statistical properties to the experimental dissipation field, power-law filtering of the signal to make $\beta > 1$ (as was done in chapter 3) failed to produce a velocity signal for which the square gradients field showed convincing scaling of the moments.
Rather, the behaviour was similar to that of the model output which directly produced a field with $\beta > 1$, although the square gradients did have a scaling spectrum with $\beta = 0.7$. This lack of ability to generate a velocity signal with the appropriate statistical behaviour suggests a superficiality to the multiscaling behaviour observed for the model dissipation field.

Finally, the distribution of the gradients was examined, since the main point of this model was to exploit the asymmetry in this distribution observed in the experimental data.

Figure 6.22 shows the distribution, using the top case of the data shown in figure 6.4. While the distribution is biased towards negative gradients, as was observed for the experimental data in chapter 3, the model distribution is much narrower than the experimental one, as can be seen by comparison with figure 3.15.

6.3 Conclusions

Here an attempt was made to construct a model using the experimental observation from chapter 3 that the gradients of the turbulent velocity field are asymmetrically distributed. Using a simple assumption as to the form of function $U_x (x + l) - U_x(x)$, such that this function was assumed to be proportional to a power, denoted $A$, of the quantity $U_x - U_y / l$, the value of $A$ is immediately limited to non-integer values if the function is to be asymmetrical. The model was then constructed using two such terms, with the output at least superficially reproducing some aspects of turbulent behaviour. Thus the model was successful in achieving one of its goals, to show that using an asymmetrical function can produce at least some kinds of scaling behaviour. In particular, the scaling of the structure functions of the experimental and model data behaved in a similar way for some cases.

Since the model has more of the form of a dynamical equation than a cascade model, it is interesting to speculate whether a similar model, based much more on dynamics rather than simple dimensional arguments, might be constructed using such an asymmetrical function. However, what the present model says about turbulent dynamics is unclear, if indeed it contributes anything to the understanding of turbulence. While the model output, made with parameters
chosen to produce a field with $\beta > 1$, had a clear physical interpretation as a velocity signal and produced similar spectral and multiaffine behaviour to the experimental data, multiscaling analysis of the square gradient (dissipation) field found the model an inadequate representation of turbulence. On the other hand, the model with parameters chosen to produce output with $\beta < 1$ appeared to display multiscaling behaviour after reasonably trivial manipulation, but could not be used to produce a velocity field which matched the statistical behaviour of the experimental data. It was also unclear what the particular field examined for this latter case, which was taken to the fourth power to produce the appropriate slope for the scaling of the moments, represented in terms of a physical interpretation.

Nevertheless, there are at least two points of view which might be taken to justify the usefulness of research on this model.

Firstly, it might be that the model is somehow incomplete, and that some modification produces much better results. This would be particularly worth investigating if certain points could be established experimentally. It is implicit in the construction of the model using dimensional arguments that the quantity $U_i - U_j / l$ serves as a substitute for the gradient, $\partial U_i / \partial x_j$, in order for the model to have closure. If experimental data was available for all three velocity components, it would be worth investigating whether there exists some range of scales $l$ or values $U_i - U_j$ for which $U_i - U_j / l \approx \partial U_i / \partial x_j$ is a reasonable approximation. For example, it could be that this approximation is good when the difference between $U_i$ and $U_j$ is not too small. If the model equations were altered to eliminate this case, it may also eliminate the "spikey" appearance of the output and improve the match between the statistical behaviour of the model and experimental data.

Secondly, while this model is inadequate for producing the statistical properties of turbulence, the output is nonetheless interesting in terms of its own behaviour, and might be a useful model for other types of signals, whether geophysical or otherwise (such as financial market data, for instance). However, this goes beyond the scope of this thesis.

The behaviour of the model depended on two principal parameters. The power parameter $A$ determined the spectral slope of the model output, with the spectral slope tending to $A + 1$. The proportionally constant $B$ also affected the value of the
output's spectral slope. The results suggested that the effect of the parameter on the spectral slope was due to the increased weighting it could give either of the power law terms. When the $A > 0$ term was weighted relatively heavily, the output tended to a spectral slope of greater than 1, while when the $A < 0$ term was large, the spectral slope tended to a value less than one. This balancing of the terms which generate two different power laws might provide some explanation as to the mechanism which causes the spectral slope of the experimental data to vary between realisations for no apparent reason. Note too that since $B$ must be a function of $I$, $\bar{U}$, and $v$, it seems reasonable that this parameter is related to the Reynolds number.

Although cascade models, for comparison, are capable of producing indefinite scaling ranges, real turbulence does not scale indefinitely. It is worth bearing in mind that whether or not the model presented possesses spurious multiscaling moments (in that, the scaling is too limited to be considered multiscaling), that natural signals tend to produce only limited scaling. Of course, the experimental data produced longer and more convincing scaling than the model output.

It is also worth noting that while the model produced data with apparently quite convincing $K(q)$ functions, power-law filtering this data to increase $\beta$ to greater than one did not produce a velocity field with any better multiscaling properties than using the model to produce output with $\beta > 1$ directly. Yet producing a simulated dissipation field with a cascade model and doing the same thing to that, as was done in chapter 3, did produce a velocity field with this behaviour. This suggests there is some characteristic of the two fields which is not well differentiated by multiscaling analysis. How this may be characterised, short of going through the procedure of power-law filtering the data in this way, is unclear. Note too that the $\zeta$ function which characterises multiaffine behaviour was non-linear for both real and model data. This is interesting because it is often assumed that this function is non-linear because the signal is intermittent, and by implication has multiscaling moments. Yet this does not appear to be the case for the model output.

Finally, the model was able to reproduce at least one property of the experimental data that cascade models do not: the gradients of the output signals
were biased in that they had more negative signs than positive, as might be expected given the way the model was constructed. However, the usefulness of this was limited by the failure of the model to reproduce other key statistical properties of turbulence.
7. Conclusions

Development of Western Science is based on two great achievements; the invention of the formal logical system (in Euclidean geometry) by the Greek philosophers, and the discovery of the possibility to find out causal relationship by systematic experiment (Renaissance). In my opinion one has not to be astonished that the Chinese sages have not made these steps. The astonishing thing is that these discoveries were made at all.

Albert Einstein, 1953

7.1 Overview

The primary objective of this thesis was to investigate if multiscaling models could be used to characterise and simulate surface layers winds. The results demonstrate that these models can usefully do so, within limitations. It is clear from the investigations that the range over which either the inertial range or energy-containing range could be simulated was limited.

The results obtained found a spectral slope of around $-1$ for low wavenumbers and $-5/3$ for high wavenumbers. These results appear to be consistent with, and indeed support, observations by other workers (e.g. Katul et al). As is detailed in the manuscript, the results obtained from the cup anemometer were consistent with those obtained by the much faster response hot-wire anemometer. Of course, no instrument perfectly reproduces reality, and it is implicit in all experimental work that what is often being dealt with is a signal which could have occurred.

It must be pointed out that in the terms of using the present study to describe the universal structure of turbulence, the site (not located on a barren expansive plain) and the choice of instrument (cup anemometers have a slow response) are not ideal in terms of previous studies, and the style of presentation (multiscaling analysis, as opposed to more familiar similarity analysis) is a departure from more
traditional methods. However, this study was not conducted within such a traditional context. The goal of this work was to demonstrate the versatility of "fractal" analysis to show how it may be applied in practical situations, such as the study of low-wavenumber turbulence at a particular site for some engineering purpose. While the analysis could have been confined to the hot-wire anemometer data, part of the aim was to demonstrate that not only could these multiscaling techniques be used with very-high resolution data obtained under close supervision, as is typically the case with such studies in the literature, but also in situations where it is undesirable to leave more-expensive and fragile anemometer types in the field for long periods.

The fact that once such wind signals are obtained they can then be simulated by a fractal cascade makes these methods a potentially powerful tool for such engineering-style studies.

From a more esoteric point of view, it appears from the results of previous workers and the publishability of the results presented here (e.g. Revell et al [1996], Lauren et al [1998a, 1998b]), that the work may also be significant in terms of furthering the understanding of the structure of turbulence. Since the manuscript does not mislead the reader as to the methods used to obtain the results, the reader is free to make any conclusion he/she likes about their usefulness in this regard.

It must also be pointed out that the use of the multiscaling methods for the energy-containing range lacks the theoretical basis that such models have when applied to the inertial range. However, the results presented here suggest that these methods are useful enough as a statistical method alone that they should be persevered with. Rather than worry too much about such theoretical concerns at this point, it is perhaps better to think that the results presented may in future help lead to some theoretical justification for their use.

The methods are an excellent way of characterising statistical features such as intermittency. They reveal separate regimes of intermittency for low and high wavenumbers. To be able to characterise the degree of intermittency of wind velocity at given locations may be of particular importance in engineering-type applications, in addition to more esoterical questions, such as to what degree wind is intermittent for various wavenumber ranges. The utility of these methods is
illustrated using data obtained in the lee of a mountain range, from which the low-wavenumber turbulence revealed a much more intermittent character than was seen above the relatively flat Ardmore field station. This supports the generality of using cascade models for the energy-containing range, because cascades are ideal for high-intermittency signals — such as those observed at Mt Cook — whereas it is not obvious that cascades are necessarily the best way of characterising the weakly intermittent signals observed at the Ardmore site.

It should be stressed that the characterisations obtained by applying these methods are new results in that they do not appear to have been previously applied to low-wavenumber turbulence, as is the application of a cascade model to directly simulate low-wavenumber wind velocity. Part of the aim in producing this work and the journal papers which it spawned was to present these kinds of analytical methods to the main-stream boundary layer community. This was particularly encouraged by the number of inquiries made by workers following presentation to the Australasian meteorological community of these results at conferences held in New Zealand and overseas at EGU and AGU meetings.

7.2 The use of multiscaling models to simulate surface winds

Chapter 2 contains an original investigation of the properties of surface-layer winds on time scales from fractions of a second up to the order of $10^3$ seconds. The originality of the analysis here has its roots in the fact that the experimental workers studying surface winds in the 70s and 80s were not doing so from a "fractal" perspective.

In that chapter, the main tool for examining the surface-layer winds was multiscaling analysis. It was found that the spectra of the winds could be broken into separate regimes — an inertial and an energy-containing range — according to the parameters given by this analysis method. Fluctuations in wind speed belonging to the energy-containing range were usually described by a spectral slope parameter of between 0.8 and 1.35, as opposed to the 5/3 typically assumed for the inertial range. This result supports the generality of using two scaling ranges to describe surface-layer wind spectra, as suggested by Kader and Yaglom [1991], Katul et al [1995], Katul et al [1996], Katul and Chu [1998], who studied surface-layer winds in the context of similarity theories. Support of these workers'
results is reasonably important given that the existence of a low-wavenumber range power law had been questioned by several workers relatively recently (e.g. Raupach et al [1991] and Antonia and Raupach [1993]).

The work here extends the results of these previous workers by using separate scaling ranges for the moments of the data as well, allowing the intermittency to be characterised for the energy-containing range. This range was found to be much less intermittent than the inertial-range.

The investigation differs from other workers who have used multifractal-type analysis to describe turbulence (particularly Schertzer and Lovejoy), in that rather than attempting to find a single set of “universal” parameters to describe the multiscaling properties of atmospheric turbulence, the behaviour of the turbulence characterised in this study was far from universal, depending on both the topography and atmospheric conditions (in this regard, unlike inertial-range turbulence, it appears that large-scale environmental factors, or boundary conditions, affect the value of the parameters). This has analogies to the multiscaling properties of rainfall, the parameters for which were shown by Harris et al [1996] to be dependent on terrain, while other workers had appeared to claim that rain should be describable by a single set of parameters, presumably simply on the basis that if inertial-range turbulence was universal, and cascade models were developed to describe this type of turbulence, then the cascade models used to describe rain distributions should also have universal parameters.

To demonstrate that the energy-containing range for turbulence signals was not universal, multiscaling analysis was conducted on data sets obtained in differing atmospheric conditions at a single site over a period of a year-and-a-half. The spectral parameter was characterised in terms of atmospheric conditions, rather than simply as an average over the entire period (which tends to obscure temporally local influences) for which data was recorded. The result of this was the discovery that the spectral slope parameter for the energy-containing range appeared to be a function of atmospheric conditions. This does not appear to have been reported previously in these terms, although work by Kaimal [1978] suggests the existence of such behaviour (but the Kader and Yaglom, Katul et al papers do not). This seems to be a result of some esoteric interest as well as practical, as it may provide a place to begin the understanding of how the spectral slope
parameter may be related to physical processes, beyond mere dimensional arguments.

The dependence of the spectral slope parameter on atmospheric conditions was compared with theoretical behaviour suggested by Kaimal. Our results suggested that Kaimal's function was not a particularly good fit in conditions with a low Richardson number, although this may have been because of the way the Richardson number was calculated in the study (see chapter 2).

In regards to the characterisation of the intermittency of the energy-containing range, it must be remembered that intermittency is a property more usually associated with small-scale turbulence. Little has been said in the literature about the degree of intermittency for larger scales, particularly in a multiscaling context. The results presented here confirmed that signals from this range were only weakly intermittent. While this was the case for the energy-containing range signals collected at Ardmore, it was not so for the data (collected with the same temporal resolution) from the Mt Cook experiment. Thus intermittency is an important parameter for characterising the energy-containing range as well as for the inertial range.

The great advantage of characterising such signals using multiscaling analysis as opposed to similarity theory is that simulated signals which possess the same scaling statistics as the experimental signals can be produced with the use of cascade models. In chapter 2, the multiscaling properties of the energy-containing range were simulated using a bounded cascade technique developed by Merab Menabde. Although it is clear that such a simulation technique should be applicable to any signal for which multiscaling parameters can be determined, the actual application and recognition of its applicability to energy-containing range turbulent velocity signals in this chapter appears to be original.

7.3 The importance of self-similarity analysis

Chapter 3 was essentially an extension of the study in chapter 2 by exploring the concept of self-similarity and its application to multiscaling analysis. Once again, the originality of the analysis is largely in the examination of the energy-containing range statistics. Given that the self-similarity method suggested by Novikov was developed specifically with small-scale intermittency in mind, it is
hardly surprising that this method has not been applied to the energy-containing range. It was found that this method was able to be applied directly to the velocity field for energy-containing range (the work of Novikov and Menabde deals principally with the square of the gradients field, which for turbulence, is proportional to dissipation). Finding such self-similar behaviour allowed a cascade model to be used to simulate the data directly from the distribution of the weighting factors. As with the previous chapter, this does not appear to have been previously done for the energy-containing range.

An additional result was that it was found that the probability densities of the gradients of the velocity fields for both the inertial and energy-containing range cases were skewed, being more likely to be positive than negative. This does not appear to have been reported before. Neither the multiscaling nor self-similarity analysis methods discussed here characterise this feature, so that it is potentially a blind-spot for these methods.

Generally speaking, the results of chapter 3 suggest that the use of self-similar analysis is a more solid foundation from which to study the multiscaling properties of surface-layer wind signals, since the existence of such distributions imply multiscaling behaviour, by virtue of equation 1.7. While in principle the two methods are equivalent, since \( K(q) \) depends on the distribution for \( W \), multiscaling analysis by itself appears to be susceptible to spurious scaling, a fact illustrated both in chapter 3 and chapter 6, a particularly worrying property given that the choice of the “range of scaling” in most such studies appears to be somewhat arbitrary, as any particular wind (rain or other such multifractal geophysical signal) data set will not necessarily scale perfectly, even after heavy averaging. Using simulated data based on the self-similar distribution obtained from the experimental data, a practical and objective method for choosing the scaling range was presented. This appears to circumvent some of the difficulties described with multiscaling analysis in chapter 2.

7.4 The influence of topography on statistical parameters

In order to put into context the results obtained at the Ardmore site, which was relatively flat, the effects topographical features on the wind statistics were examined. This was done to the extreme, by choosing a site downwind of New
Zealand's highest mountain, Mt Cook. A fluid dynamical model study conducted by Revel et al [1996] at this site suggested that it was reasonable to expect eddies with a largest characteristic period of 4 min to be shed off the mountain range, as had been observed in experimental data obtained for this thesis, which formed part of the Revel et al paper. The wind signal obtained at this location possessed a much steeper spectral power-law slope than the Ardmore data, with $\beta = 1.5$. Multiscaling analysis revealed the wind there was much more intermittent than at Ardmore, with the intermittency parameter, $C_1$, being about 0.08 for Mt Cook, compared with 0.03 for Ardmore. This result demonstrates that multiscaling techniques can usefully be used for analysing the energy-containing range, since intermittency is well characterised by these methods, and the observed variability of the degree of intermittency between these sites opens possibilities for further studies with the aim of relating energy-containing range intermittency to topography.

A cascade simulation of the Mt Cook wind was made, with the resultant field reproducing reasonably closely the experimental statistics. This was done by using cascades to create a series of signals of length equivalent to a period of about 4 min, thus producing separately generated subsets of the signal, and connecting them to make a single record of arbitrary length. In this way, the “scaling break” observed in the spectrum (or, the point where the spectrum flattened off), was able to be reproduced in the simulated data. In a similar way, once the cascade parameters are known, output from a model such as that suggested by Revell et al might be used to provide an initial value for each of the cascade subsets, thus using the cascade model to simulate the high-resolution details of the signal, essentially adding a fractal “fur”.

The applicability of the multiscaling analysis to both the Ardmore and Mt Cook cases demonstrates the potential of the method. It is tempting to think that just as the parameters for the wind statistics were able to be related to the atmospheric conditions in chapter 2, that some classification scheme might also be possible for lee effects resulting from topography. Once again, such a study has the potential to establish a link between fluid dynamical and fractal modelling methods.
7.5 The importance of nesting of intermittency

While chapters 2, 3 and 4 largely treated surface-layer wind signals in terms of cascades, chapter 5 examined an alternative simulation method in the hope of determining something about the dynamical processes. This was done by questioning whether the experimental data contained the traits that might be expected of a cascade process.

The fact that the energy-containing range data was not particularly intermittent at the Ardmore site suggests that the lower-wavenumber range signal for this case might be just as well simulated by a non-cascade method, raising the question of whether the fluctuations in the energy-containing range are the result of a cascade process at all. Since the fluctuations correspond to spatial scales from 5m to 5000m, it seems unlikely that this range could represent a genuinely fully developed three-dimensional eddy cascade process.

Study of a model suggested by Hausdorff and Peng for biological systems, which was capable of producing power-law signals, showed that their method was capable of producing simulations with multiscaling statistics close to those of the energy-containing range turbulence over flat terrain (i.e. at the Ardmore site). This was most likely because both the experimental signal and the model output were only weakly intermittent, in which case, even if the experimental signal is multiscaling, it does not possess strong multiscaling characteristics, such as strong intermittency.

This ties up with the more esoteric question of whether or not the energy-containing range is multiscaling, or whether the application of the methods used have simply produced spurious multiscaling parameters. Although from a practical point of view, this question does not matter provided the model being used reproduces the appropriate statistics, it is important in terms of understanding the processes. One possible alternative to the cascade dynamical picture for energy-containing range surface-layer dynamics which might fit a description based on a Hausdorff and Peng-like model is to imagine that the fluctuations in the surface winds are produced somewhere above the surface layer, with some mechanism bringing the gusts down to the surface. In this alternative phenomenology, the surface-layer wind signals might be described as the combination of random events occurring on differing time scales.
In order to investigate this possibility, some small extensions were made on Saucier's [1991] observation that intermittent bursts in different wavenumber bands appear to be nested in the same spatial regions of the signal. Results from this chapter confirmed this observation for the inertial-range. Energy-containing range signals were examined in the same way. The result was, not surprisingly, that the energy-containing range signal was not particularly intermittent, but, more significantly, there appeared to be some correlation between wavenumber bands, as was the case for the inertial range. Intuitively, it must be expected that this behaviour should exist if a cascade-like dynamical process is occurring, since the large-scale distribution of the signal directly affects the smaller scales.

It must also be noted that the results of chapter 4 suggest that surface winds in the lee of orographic influences are much more intermittent than those observed at the Ardmore site. As such, the wind collected at Mt Cook had much more definite features associated with multiscaling behaviour, and so were not suitable for description with the Hausdorff and Peng model. It seems entirely reasonable that the sort of processes which occur in the surface layer at Mt Cook are similar to those which occur at the Ardmore site, the differences in their character being perhaps determined by some quantifiable parameter, such as wind shear. In this case, it is much more elegant to imagine that both situations should be modelled by the same phenomenology (e.g. cascade models), since they are from the same region of the spectrum.

7.6 Bias in the gradient distribution and the model suggestion

The observation of the bias in the distribution of the gradients, far from being merely an interesting aside, is perhaps one of the most significant results in this work. There are a number of reasons for this. The bias suggests that the ensemble average of the velocity difference \( \langle |u(x + l) - u(x)| \rangle \) will depend on the proportion of negative and positive values of \( u(x + l) - u(x) \) in the ensemble. This is because if there are more negative gradients than positive in a flow with a mean velocity, then the negative gradients must be on average smaller than the positive. This has interesting consequences for the structure function, as it raises the question of what form this function should take for various ratios of negative and positive gradients in the ensemble. Since this ratio might be expected to affect
the structure function, and consequently the spectrum, it perhaps provides an explanation as to why different ensembles of turbulence data have quite variable spectral slope parameters.

This bias is also interesting because the standard multiscaling analysis techniques and cascade models used widely in the literature take no account of it. This insensitivity to this property has serious ramifications for all such previously published works.

Chapter 6 was essentially an aside to examine a possible model based on this behaviour, and explore the application of the methods presented in the earlier chapters to a non-cascade based signal. The model possessed two principal parameters. The power parameter $A$ seemed to determine the spectral slope of the model output, while the proportionally constant $B$ also affected the value of the output's spectral slope, by determining which of the two power terms dominated. From the physical interpretation of the model variables, $B$ was a function of $l, \bar{U},$ and $\nu,$ so that it was a Reynolds-number-like parameter. Numerical experiments suggested that $B$ could change the behaviour of the system from quasi-periodic to chaotic behaviour.

The "velocity" output produced by the model displayed good scaling for the structure functions, but only limited scaling ranges for the "dissipation" fields. Although cascade models, for comparison, are capable of producing indefinite scaling ranges, real turbulence does not scale indefinitely. It is open to debate whether the model presented here possessed spurious scaling, or whether in fact the model reproduced nature by possessing only limited scaling.

The model was able to reproduced the bias in the velocity gradients, which is what it was intended to do.

The investigations of the model output also raised some further questions as to the reliability of multiscaling analysis. This was particularly so for the case of the model output which appeared to behave in a multiscaling way, but which was not able to be manipulated to produce a "velocity" field with both $\beta > 1$ and a square gradient (or dissipation) field with multiscaling characteristics.
7.7 Discussion and thoughts for further work

Broadly speaking, this work appears to have been successful in achieving its aim of extending the use of cascade models beyond inertial-range turbulence. In particular, it would appear that bounded cascades are a powerful and simple tool for producing simulated wind signals of energy-containing range time scales. This may hold many benefits for engineering-style purposes, such as designing wind turbines, or indeed any structure for which wind variability on these time scales is important. To make use of these results, an effort should be made to apply them to such practical situations. It would be nice to think that such work might find funding in a joint engineering/commercial context. A systematic study of the effect of topography on the multiscaling characteristics of the energy-containing range is one of the principal areas where the work presented could be extended.

The self-similarity analysis, on the other hand, was a little less useful from a practical point of view for at least two reasons. Firstly, a larger number of data points is required to accurately characterise the self-similar distribution, and secondly, it is more computationally expensive to use the self-similar distribution to generate the simulated signal, because the probability density function for the method used in chapter 3 was not analytical.

It must be noted that some of the questions raised by the results in this thesis were not able to be fully answered by the data obtained, in particular, the multiscaling characteristics and self-similar distributions for the signals of the energy-containing range were difficult to determine and justify. In this respect, it must be stressed that the cascade-model approach presented is well capable of producing simulated fields with statistical characteristics which are extremely close to those of the experimentally observed data, which is in itself an important result and achievement. What is less clear is whether these models are necessary to do this, as would be the case if the experimental data is genuinely multiscaling, or if the methods are simply a nice statistical trick for reproducing the properties of wind on energy-containing range scales.

The major reason for the remaining doubt is the quality of the data. Although the cup anemometer was adequate for the purposes of this work, it was far from ideal, and certainly would have been insufficient without the support of data from the hot-wire anemometer. Broadly speaking, the hot-wire anemometer was the
principal instrument used to determine the multiscaling characteristics, while the cup anemometer provided information as to how these properties changed in varying conditions. As discussed in the relevant sections, the hot-wire anemometer was not-suited to long periods in the field, and was essentially an instrument designed to be used in an indoors lab (indeed, the hot wire was unfortunately broken several times in the course of the field work). It could not be deployed in adverse conditions, such as rain, during which time winds were often strongest. Data collection was thus a balance of high resolution but short-term deployment, and low resolution but long-term deployment — a trade off which is often faced in field work. Nevertheless, there are more expensive field instruments suitable for such deployment (in particular, sonic anemometers). With the strong impression that as much was wrung out of the data as was possible, it is recommended that further studies into the nature of the energy-containing range should be done using such higher-resolution instruments. While saying this, the analysis presented here does lay the ground work for use of cup anemometers for multiscaling analysis of the energy-containing range for practical, rather than esoteric, purposes. That is, there is no reason why these methods should not be applied in field studies immediately, provided that the purpose of these studies is to characterise the wind rather than increase the understanding of the processes that cause those characteristics. Additionally, it would be useful to compare the results presented with an independent data set — either collected by a separate group of workers, or having been recorded on a different type of instrument to those used in this study. This would allow a greater confidence in some of the finer details of the results reported.
Appendix A: Determination of parameters for Menabde's bounded cascade model

Consider a wind or other geophysical field \( u \) sampled with some finite time resolution \( t = 2^n T \), or spatial resolution \( l = 2^{-n} L \), so as to produce a series of discrete values \( u_i \), which we wish to simulate using a cascade model. The second order moment is:

\[
M_n = \langle u_i^2 \rangle = \langle W_1^2 \rangle \langle W_2^2 \rangle \ldots \langle W_n^2 \rangle \langle Z_{N-n}^2 \rangle
\]

where:

\[
Z_{N-n} = 2^{(n-N)} \sum_{j=1}^{N-n} W_{N+n+1} \ldots W_N
\]

and \( N \to \infty \). Here, \( W \) is the weighting factor discussed earlier.

The ratio of the second-order moments for the cascade field produced after \( n \) steps and \( n+1 \) step is then:

\[
F_n = M_{n+1} / M_n = \langle W_{n+1}^2 \rangle \langle Z_{N-n-1}^2 \rangle / \langle Z_{N-n}^2 \rangle
\]

When \( (N-n) \gg 1 \), the ratio \( \langle Z_{N-n-1}^2 \rangle / \langle Z_{N-n}^2 \rangle \), is very close to unity.

Recall that for this bounded cascade model, \( W \) is given with probability \( p_1 \) by

\[
W_1 = 1 + (W_{10} - 1)2^{-(n-1)H}
\]

and with probability \( p_2 \)

\[
W_2 = 1 + (W_{20} - 1)2^{-(n-1)H}
\]

with the constraints that \( p_1 + p_2 = 1 \) and \( p_1 W_1 + p_2 W_2 = 1 \).

Taking the logarithm of both sides of (1) then gives:

\[
\log_2(F_n - 1) = \log_2(p_1 (W_{10} - 1)^2 + p_2 (W_{20} - 1)^2) - 2nH
\]
Plotting the function $\log_2(F_n - 1)$ against $n$ and finding the slope gives the parameter $H$, and

$$\log_2(p_1(W_{10} - 1)^2 + p_2(W_{20} - 1)^2)$$

(2)

from the intersection with the ordinate. Although (2) contains two unknowns, one can be fixed, because in order to be able to generate a high degree of intermittency, the value of one of the initial weighting factors, say $W_{10}$, needs to be very small, say $= 0$. Then (2) takes the form:

$$\log_2(p_1(W_{10} - 1)^2 + p_2(W_{20} - 1)^2) = \log_2(p_1 / (1 - p_1))$$

(3)

from which $p_1$ can easily be found. Menabde reports that simulations are reasonably insensitive to the actual value of $W_{10}$, so we may take $W_{10} = 0.1$. 
Appendix B: Requirements for Large-eddy Simulation of Surface Wind Gusts in a Mountain Valley


The following pages contain a copy of the above paper, printed in 'Boundary-Layer Meteorology', volume 80, pages 333-353.

The purpose of reproducing this publication here is to allow the reader to gain further insight into the workings of the fluid dynamical model described in chapter 4 that are beyond the main themes of this thesis.
1. Introduction

During moderate to strong north-westerlies, strong wind gusts with periods greater than 2 min are observed in the lee of the Mt Cook ridge, at Tasman aerodrome in the Southern Alps of New Zealand. These gusts are associated with large eddies which, marked by the glacial dust they lift, can be seen as vigorous overturning at the foot of the lee slope and surface wind surges propagating for several kilometres down the valley. These wind gusts accompany the passage of fronts with a northwest-southeast orientation and are significant for the risk of wind damage to structures or forests. In other locations, updrafts and downdrafts associated with eddies on scales of a few km could affect the distribution of rain, and possibly its transport across the main dividing range to hydro-electric power storage catchments. Although the air is stable ahead of the front, the high moisture content at the front itself, together
with intense uplift associated with flow across the main divide, frequently saturate the air, producing a low-level layer which is near moist neutral.

Discussions with the local pilots indicate that predictable lee wave structures do exist down to a kilometre or so as the fronts approach. Pilots use them to derive lift and ensure a smooth ride for passengers taking scenic flights over the surrounding mountains. Just before the arrival of one such front, a short flight through the first lee wave downstream of the main divide, at a height of about 2 km, revealed a very smooth but narrow updraft 100 m wide. As the main divide was approached the smooth flow terminated abruptly and extremely turbulent air was encountered – possible evidence for eddies due to flow separation at the upstream ridge crest. Unfortunately with the arrival of the front and less stable air, all light aircraft ceased flying due to poor visibility and an uncomfortable level of turbulence. However, at this time the surface wind features could still be seen in the glacial dust.

A problem for modellers of flow over hilly terrain is that quite ordinary phenomena such as flow separation, which may produce a turbulent wake in the lee of a hill for example, occur at Reynolds numbers above $10^3$, which is beyond the reach of many mesoscale models. One reason for this limitation is that many numerical methods in use require a minimum level of diffusion to ensure computational stability, thereby achieving effective eddy-stress Reynolds numbers of only a few hundred. Reynolds-average models use a turbulence closure scheme to compute a steady mean flow, thus precluding any ability to describe the statistics of the turbulence itself. As examples we note the simulation of non-linear flow over a bell-shaped mountain by Saito and Ikawa (1991), using a conventional second order, turbulent kinetic energy based closure scheme following Klemp and Wilhelmson (1978) and Deardorff (1980), showing accelerated but smooth flow in the lee, and the simulations of flow over an isolated hill by Wood and Mason (1993).

Scinocca and Peltier (1989) list many other studies of downslope windstorms over mountainous terrain in stably stratified conditions. As they point out, most of these studies have investigated the factors affecting the wave breaking mechanism by which the storm initially develops. Although observations of these downslope windstorms, e.g. that of 11 January 1972, described in Klemp and Lilly (1975), reveal intense surface gustiness, only the papers by Scinocca and Peltier (1989) and Clark and Farley (1984) address this aspect of the resulting flow. They show the two-dimensional flow can be unstable due to Kelvin Helmholtz (KH) instability and suggest this as a possible explanation for the unsteadiness of the surface wind. However, the above studies are for stably stratified conditions and none generates flow separation. When conditions closer to neutral occur at Tasman aerodrome, it thus seems likely that flow separation at ridge crests and downstream propagation of eddies are a source of wind fluctuations, but with the limited observations available we cannot rule out the possibility of KH instability.

In contrast to the Reynolds average models, large eddy simulations (LES) obtain unsteady solutions to the Navier Stokes equations and employ a turbulence closure to account only for stresses due to eddies on scales too small to be resolved by the
model mesh. A theoretical justification for this strategy may require the existence of a gap in the eddy spectrum, coincident with the limit of model resolution. In practice there is no such gap, and turbulence closure models assume similarity relations, e.g. for an inertial subrange, to deduce the effect of unresolved eddies. Various deterministic and stochastic sub-grid models for eddy simulation have been evaluated over flat ground by Mason and Thompson (1992) and Mason and Brown (1994).

The purpose of this paper is to assess some of the requirements for simulating surface wind gusts in neutrally stratified flow over mountainous terrain, by exploring parameter space with LES of flows over the Mt Cook region for a range of resolutions, eddy viscosities, stabilities and flow speeds and comparing the model generated wind statistics with those observed at Tasman aerodrome.

2. Wind Data in the Mt Cook Region

Anyone who has hiked in hilly country will be familiar with the gusty nature of the wind flow there, with periods of intense wind interrupted by equally dramatic lulls. The frequency of these gusts appears to be related to the scale of the valley and the prevailing wind speed. Applying the Taylor hypothesis that these gusts are associated with eddies of length scale $L$ advected with a mean speed of $V$, this implies a period of $L/V$. In New Zealand there are very few long term records of wind statistics in these mountainous regions where the most intense rainfalls and wind gusts are produced. One of the few is at Tasman aerodrome at Mt Cook, located about five kilometres downstream (to the east) of the main divide at an altitude of 665 m. Near here the main divide is about two kilometres high and the peaks are about three kilometres high. A contour map of the topography indicating the location of this station is shown in Figure 1. The anemometer is sited in a steep-walled glacial valley. Typical heights of the valley walls are 1–2 km. For some 5 km upstream of the anemometer, the width of the valley is only about 3–5 km. At the foot of the main divide the valley floor is obstructed by glacial debris and ice falls at scales below 50 m. Within a 3 km radius of the anemometer the valley floor is remarkably flat with roughness elements of less than 1 m. A more detailed map of the local terrain is shown in Figure 2.

An example chart of the wind record at this Mt Cook station made with a 10 m Munro anemometer is shown in Figure 3 for 21 June 1993. This is typical of a severe (but not extreme) wind event in this area, with local pilots suggesting on average five such events occurring in a given year. Initially we attempted to digitise a seven hour record from this chart, but limitations of reading the chart record prevented us from estimating a corresponding frequency spectrum for periods below 2 min.

In order to overcome this problem we installed a digital recording, light-weight cup anemometer and wind vane from Auckland University at the Tasman aerodrome site for the period 18–20 November, 1995. During this time a frontal system moved
Figure 1. Contour map of orographic heights in the Mt Cook area, showing the cross-section, Tasman aerodrome and places referred to in the text. Shading changes every 500 m beginning at 1000 m.

over the Southern Alps bringing strong northwest winds, of similar intensity to the 21 June 1993 event, along the Hooker valley. The anemometer was calibrated in a wind tunnel, and was capable of accurately resolving wind fluctuations down to periods of 2 s. The raw data were extracted at 10 s intervals, and a standard spectral analysis of these data using 100 frequency bins is shown in Figure 4.
Figure 2. Contour map of orographic heights in the 3D channel. Contours every 200 m.

The spectrum (Figure 4) is evidence of a source of eddies with periods greater than 4 min. Using the Taylor hypothesis and a mean wind speed of 15 m s$^{-1}$, implies an eddy length scale of at least 3 km. We suggest that this is compatible with a model in which eddies in a turbulent wake are associated with flow separation at the ridge crests, and large eddies are shed at periods of 4 min or more.

Reference lines with $-5/3$ and $-3$ slopes are plotted on Figure 4. A $-5/3$ power law is characteristic of an "inertial sub-range" for three-dimensional turbulence in which neither the value of viscosity of the fluid, nor the eddy source mechanism, are significant (see e.g. Tennekes and Lumley, 1972), whereas a $-3$ power law might be expected of an "inertial subrange" for two-dimensional turbulence (Kraichnan, 1967).
3. Flow Computations

3.1. Channel Geometry

The placement of the vertical section used for two-dimensional simulations and for display of results is marked by a NW-SE line on the map of the experimental area shown in Figure 1. Height of the terrain and location of the grid cells in this two-dimensional section is shown in Figure 5. In the central area the horizontal mesh spacing $\delta x = 250$ m, the lowest layer thickness $\delta z = 20$ m, and layers above
that are thicker by nominally 15% per layer up to a maximum thickness of 350 m. A grid-smoothing operation has modified the nominal layer thicknesses.

3.2. ENVIRONMENTAL DATA

Environmental data to initialise the model simulations were pieced together from a number of sources. The 1000 m wind at Hokitika (marked in Figure 1) was approximately 15 m s\(^{-1}\), so our control simulation was started with an initial wind
\( U = 15 \text{ m s}^{-1} \) everywhere. The resulting mean wind is close to this initial value at upstream locations such as Hokitika, but is modified by the topography in the region of interest. The only tephigram available was at Invercargill (also marked in Figure 1) and this is shown in Figure 6. Consistent with a moist north-westerly flow ahead of a front approaching from the west the lowest 3 km is approximately moist neutral. To keep the analysis simple, and because we do not include moisture in our simulations, we have approximated this with a dry adiabatic layer below 3 km and a moderately stable layer above this.

3.3. MODEL DESCRIPTION

A Lagrangian version of the model for fully-compressible fluid described by Purnell et al. (1995) was used for the simulations of valley flow. In this Lagrangian version
the computational mesh moved with the flow, but was re-mapped to a standard mesh at frequent intervals by interpolation of the flow variables. The interpolation conserved momentum, mass, and internal- and gravitational-energy, using a Euler-backward flux algorithm. For the hydraulic jump test in Purnell and Revell (1993), the model gives the same results for short time steps as the energy-bounded method described there, but a slightly smoother solution for long timesteps. This is the behaviour expected of a first-order accurate, high-frequency damping scheme such as this one.

In simulations of homogeneous turbulence, Bardina et al. (1983) found that the resolved motions depended very little upon the choice of parametrisation scheme for subgrid-scale dissipation. They found that a constant eddy viscosity was as satisfactory as the variable eddy viscosity generated by a variety of turbulence closure schemes, provided only that the eddy-viscosity constant was equal to the mean value of the eddy-viscosity function in a turbulence closure scheme.

For the case of turbulent flow over flat ground, Mason and Brown (1994) have investigated the same question of sensitivity to choice of parametrisation scheme for subgrid-scale dissipation. They found that the choice is significant only near the surface, where only a "back-scatter" scheme gave a good approximation to the correct logarithmic profile of the mean velocity.

We are here interested only in large eddies that are resolved by a coarse grid, and the gross influence of these large eddies on surface wind gusts, rather than any fine details of flow near the ground. Transport of small eddies will tend to cause short-period fluctuations in wind, and short-period fluctuations will be damped by the time-integration scheme used here.

In our simulations a standard Smagorinsky (1963) first-order turbulence closure was used, modified to match a logarithmic "law of the wall" at the lower boundary. The stress tensor $F$ at a position $\mathbf{x}$ was computed from the velocities $\mathbf{v}$ by

$$ F_{ij} = l^2 |S| S_{ij} $$

$$ |S| = \left( \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} S_{ij}^2 \right)^{1/2} $$

$$ S_{ij} = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \sum_{i=1}^{3} \frac{\partial v_i}{\partial x_i} $$

$$ l^2(x) = c^2 d^2(x) r^2(z)/(r^2(z) + c^2 d^2(x)) $$

$$ r(z) = k(z - z_s + z_0) $$

where $k = 0.4$ is the von Karman constant, $z - z_s$ is the height above the surface, $z_0 = 0$, $c$ is a dimensionless constant and $d(x)$ is the largest of the three distances...
between the three pairs of cell face centres of a cell with centre \( x \). In the experiments this meant that \( d(x) \) was independent of \( x \) except where the mesh was stretched approaching the upstream and downstream domain walls. Hence, in the region of interest, the mixing length \( l(x) \) varied only vertically close to the surface.

Given that we are interested in spatial scales of only a few kilometres and time scales of several minutes, for simplicity we have neglected Coriolis effects.

3.4. COMPARISON OF METHODS OVER AGNESI HILL

This method was compared with the Regional Atmospheric Modelling System (RAMS) model (Pielke et al., 1992) for the case of flow over a "witch of Agnesi" hill with a height of 2 km and a half width of 2 km, quite similar to the main Divide shown in Figure 5. The RAMS model was set up with Smagorinski (1963) horizontal diffusion, using the RAMS recommended minimum effective horizontal eddy viscosity of \( K = 250 \text{ m}^2 \text{s}^{-1} \) in the area of interest where the horizontal grid spacing is 250 m, and with a Mellor–Yamada (1982) vertical mixing rate dependent on a time-varying turbulent kinetic energy.

Figure 7 shows a solution at 60 min for flow over this ideal hill as computed by RAMS. Figure 8 shows the solution to the same problem as computed by the test model on the same grid, but using \( K = 5 \text{ m}^2 \text{s}^{-1} \). The main features clearly match, but the test model has a more explicit representation of turbulent flow near the ground. To compare the models at similar eddy-viscosity constants, an attempt was made to run RAMS using similar minimum eddy-viscosity constants, but this was not possible because of computational instability.

3.5. SIMULATED SURFACE WIND

For each experiment, the model was run for one hour to allow the model state to adapt to the boundary conditions and approach a stochastic equilibrium. A time series of the lowest-level wind in the model, corresponding to a height of \(~10 \text{ m}\) above the ground, was sampled for the subsequent two hours, averaged over consecutive 15 s intervals, and a sub-sampled series of these averages constructed. This sub-sampling procedure is a filter which will remove a small amount of power from the upper frequency bands near the Nyquist period of 30 s, but at 1-min periods this discrepancy in the power amounts to only 20\%, which is not enough to alter the picture significantly. The sub-sampled series at 15 s intervals was processed in the same way as the observed data shown in Figure 3, to produce a spectrum of kinetic energy as a function of period.

3.6. TWO-DIMENSIONAL MODEL OF THE ALPS

Two-dimensional simulations in a cross-section across the Southern Alps of New Zealand in near-neutral conditions were computed for a range of values of eddy viscosity, to find a range which, though corresponding to Reynolds numbers much
Figure 7. Streamlines after one hour for flow over a "witch of Agnesi" hill simulated using the RAMS model. Background flow is 15 m s$^{-1}$, with neutral conditions below 3 km and a constant lapse rate of 6.7 K km$^{-1}$ above this. Abscissa axis is horizontal position.

les than the real atmosphere, nevertheless produce large eddies with sizes of the order of 1 km or more.

The solid line in Figure 9 shows the spectrum for a case which will be used as a basis for comparison. In this case the horizontal resolution is 250 m, the upstream wind is 15 m s$^{-1}$, the eddy-viscosity parameter $K = 5$ m$^2$ s$^{-1}$, and the temperature profile has a neutral dry-adiabatic lapse rate up to a height of 3 km and a more stable lapse rate of 6.7 K km$^{-1}$ above that. Since the model has no moisture in it, the temperature profile corresponds to neutral conditions below the mountain tops. Compared to the observed spectrum shown in Figure 4 the power is generally shifted toward longer periods.

3.7. THREE-DIMENSIONAL EDDY SIMULATIONS OVER THE ALPS

Three-dimensional simulations were computed in a channel 10 km wide, spanned by 20 grid cells, aligned with the section drawn on the map of Figure 1. A two dimensional ridge was still employed in these computations, thus the only change
to the two-dimensional simulations was to allow an extra degree of freedom in the direction parallel to the ridge. A further simulation was performed with the actual three dimensional terrain heights shown in Figure 2. The wind spectrum generated from this run will also be discussed.

4. Results

4.1. Varied Eddy Viscosity

Altering only the the eddy-viscosity parameter in the reference run used to generate the solid line in Figure 9, by a reduction factor of 10 to $K = 0.5 \, \text{m}^2 \, \text{s}^{-1}$, has not significantly changed the spectrum shown by the dashed line in Figure 9. A snapshot of the flow streamlines (Figure 10) and another 4 min later (Figure 11) show an eddy forming and breaking away from the peak, with reversed flow at the surface extending about 10 km downstream. This is not what is observed. Note, however, that over this 4-min interval the large eddies have been displaced by about half a wavelength, corresponding to a period of about 8 min.
Period (minutes)

Figure 9. Two decades of the kinetic energy density spectrum (units as in Figure 4) for the control simulation (solid line) with horizontal mesh spacing $\delta x = 250$ m, $K = 5$ m$^2$ s$^{-1}$, $U = 15$ m s$^{-1}$, lowest layer thickness $\delta z = 20$ m and a lapse rate above 3 km of 6.7 K km$^{-1}$. Low viscosity simulation (dashed line), as above except $K = 0.5$ m$^2$ s$^{-1}$, High viscosity simulation (dotted line), as above except $K = 50$ m$^2$ s$^{-1}$. Course resolution simulation (dash dotted line) as above except $\delta x = 1000$ m. Doubled wind simulation (dot double-dashed line), as above except $U = 30$ m s$^{-1}$. Increased stability simulation (dash double-dotted line), as above except lapse rate 5.0 K km$^{-1}$ above 3 km. The dotted lines show the slopes of $-5/3$ and $-3$ power laws.

Figure 10. A snapshot of the streamlines for the two-dimensional, low-viscosity flow in Figure 9.

Altering only the eddy-viscosity parameter in the reference run (solid line in Figure 9), to increase it by a factor 10 to $K = 50$ m$^2$ s$^{-1}$, has generated the dotted spectrum in Figure 9. This differs from the reference spectrum by a shift toward
longer periods, taking the eddies further away in period from what is observed in Figure 4.

4.2. VARYING RESOLUTION

Altering only the horizontal resolution in the reference run used to generate the solid line in Figure 9, from 250 m to a coarser 1000 m grid, generated the dash-dotted spectrum in Figure 9. This spectrum is significantly shifted towards long periods, much like the high-viscosity ($K = 50 \text{ m}^2 \text{s}^{-1}$) simulation, displacing the eddies further away in period from what is observed in Figure 4.

4.3. INCREASED WIND SPEED

The far-field upstream wind speed of the reference two-dimensional model run was doubled in this experiment. Since eddies in the flow would be transported by the model at twice the previous rate, it was expected that this would increase the predicted frequencies, and bring the spectrum closer to the observed one. Figure 12 and the dot double-dashed line in Figure 9 show the two-dimensional model wind and corresponding kinetic energy spectrum at Mt Cook Airfield, for this doubled wind. The spectrum has been shifted to the right, but not quite as far as a doubling of frequencies. Perhaps the tendency of the Euler-backward time-integration scheme to damp high frequency modes has counteracted the effect of faster transport by reducing the amplitude of the smaller eddies.
4.4. INCREASED STATIC STABILITY

The dash double-dotted spectrum in Figure 9 was produced by changing the temperature profile above a height of 3 km, from the reference lapse rate of 6.7 K km\(^{-1}\) (used to generate the solid line in Figure 9) to a more stable lapse rate of 5 K km\(^{-1}\). This does not significantly change the spectrum, except perhaps to inhibit the largest eddies by the effect of a firmer lid on the turbulence below.

4.5. THREE-DIMENSIONAL TURBULENCE

A simulation with an extra degree of freedom, allowing the possibility of three-dimensional turbulence, but still using a two-dimensional mountain was made with a horizontal resolution of 500 m. The resulting spectrum for a point corresponding to the Tasman aerodrome is shown by the dashed line in Figure 13. A snapshot of the flow streamlines (Figure 14), and another 4 min later (Figure 15), in the centre of the channel show an eddy forming and breaking away from the peak, with reversed flow confined to a zone on the lee slope. The downstream eddies perturb the flow, but are generally not big enough to cause reversed flow further downstream. Over this 4-min interval the large fluctuations associated with shedding of eddies near the ridge crest have been displaced by about half a cycle, corresponding to a period of about 8 min.

The model results also indicate the eddies have a three-dimensional "cat’s paw" like structure — larger scale versions of the features commonly observed over water in the lee of small headlands. This is illustrated in Figures 16 and 17 which show two horizontal sections, four minutes apart, of horizontal wind vectors at 30 m and vertical velocity contours at 60 m above the surface. These eddies are noticeably stronger at the foot of the mountain and appear to dissipate as they propagate down the valley. This effect can be seen by comparing the dot dashed wind spectrum in Figure 13, corresponding to a point at the foot of the lee slope, with the dashed spectrum at Tasman aerodrome. The dashed spectrum is not unlike the coarser two-dimensional spectrum shown by the dotted line in Figure 9 suggesting that

![Figure 12. Time series of the lowest level, model-generated winds at the Tasman aerodrome point for the $U = 30$ m s\(^{-1}\) (doubled) wind speed run.](image)
Figure 13. As in Figure 9, except $\delta x = 500$ m, and there are 20 points 500 m apart across the channel. Full stress formulation with three-dimensional terrain at the Tasman aerodrome point (solid line) and at the foot of the lee slope (dotted line). Full stress formulation with two-dimensional terrain at the Tasman aerodrome point (dashed line) and at the foot of the lee slope (dot dashed line). Simplified stress formulation with two-dimensional terrain at the Tasman aerodrome point (dot double dashed line) and at the foot of the lee slope (dash double dotted line).

Figure 14. A snapshot of the streamlines along the centre of the channel for the three-dimensional flow with full stress formulations over two-dimensional terrain in Figure 18.

500 m resolution is too coarse to allow the higher frequency eddies to develop or persist downstream.
The insensitivity of the wind spectra to the precise form of the sub-grid scale model for Reynolds stress can be seen by comparing the dot double-dashed and dash double-dotted spectra in Figure 13, which correspond to points at the Tasman aerodrome and foot of the lee slope respectively, to a three-dimensional simulation with a simplified model of stress. This did not include any explicit stresses due to horizontal shear, and a constant value of eddy viscosity $K$ was used at all altitudes to infer a rough approximation of the stress force vector $\mathbf{f}$ on each cell due to vertical shear in the velocity $\mathbf{v}$:

$$
\mathbf{f} = K d_z^{-2} \Delta_z^+ \Delta_z (\mathbf{v} - (n_z \cdot \mathbf{v}) n_z).
$$

Here $z$ is a label for the upwards co-ordinate direction shown in Figure 5, $n_z$ is a normal vector pointing in this upwards direction, $d_z$ is the distance in this direction between cell faces, and $\Delta_z^+ \Delta_z$ is the usual second-difference across cells in this upwards direction. We suggest that this insensitivity to the details of stress formulation is because boundary-layer separation is forced in any case by the pressure gradients induced by steep terrain.

A final simulation was performed with a complete formulation of stress and the actual three-dimensional terrain heights shown in Figure 2. The model generated wind spectra corresponding to Tasman aerodrome and a point at the foot of the lee slope are shown by the solid and dotted lines respectively in Figure 13. The insensitivity of the wind spectra to the complete terrain detail is some justification for our approximation of the Hooker valley as a two-dimensional channel. It is further evidence that the large eddies are produced by separation at the ridge crests and not by edge effects around headlands.

Figure 15. As in Figure 14, but 4 min later.
5. Summary and Discussion

Evidence of large eddies being shed from the upstream ridge and propagating down the Hooker valley to Tasman aerodrome has been found from high resolution anemometer data collected in this region. A fully-compressible, fluid dynamical model has been used to compute flow over a cross section through the Hooker valley down to the head of Lake Pukaki. A range of model parameters was explored to assess some of the requirements for simulating surface wind gusts in mountainous terrain in New Zealand.

The three most crucial requirements for the simulation of surface wind gusts at Mt Cook appeared to be the need for three-dimensional rather than two-dimensional
eddy dissipation, the need for sufficient resolution to model the eddies of interest, and the need for a model of sub-grid dissipation which influenced the resolved motions in a way similar to the effect of an inertial subrange. In our model we required the sub-grid dissipation to be sufficiently weak to allow significant amplitude of the smaller eddies. We have not attempted to model some of the major features of the mean flow at the surface.

In the two-dimensional simulations, the eddies were too big in size and in amplitude, and at the surface this was associated with reversed flow extending much too far downstream. Three-dimensional simulations, in contrast, produced large-scale “cat’s paws” with a realistic gusting at the surface and reversed flow only at the steep part of the lee slope.

The spectra of large eddies simulated in steep terrain were not very sensitive to the details of the eddy-stress model. We suggest that this was because boundary
layer separation near ridge crests was forced in any case by terrain-induced pressure gradients.

The resolution required for a particular application will depend on how much of the wind spectrum is needed. We found resolutions coarser than 250 m tended to cut the energy spectra off below periods of four minutes. Fortunately the larger eddies account for most of the energy, so that fairly coarse models such as have been explored may be adequate for applications where wind energy is the primary consideration. For example, an assessment of risk of wind damage to structures or forests could fall into this category. In other locations, updrafts and downdrafts associated with eddies on scales of a few kilometres might affect the distribution of rain, and possibly its transport across the main Dividing range to hydro-electric power storage catchments.

Acknowledgements

We thank John Kidson for providing spectral analysis code, and Roger Ridley for assistance with running the RAMS model. We are grateful to the staff at Tasman aerodrome, Mount Cook for their cooperation and enthusiasm. In particular we thank Tony Delaney for help with setting up the instruments, Phil Galloway for sharing his extensive knowledge of wind structures in the area and Ross Anderson for teaching us a great deal about turbulence in a 15 min flight. This work was supported by the New Zealand foundation for research, science and technology.

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