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A Toolkit for the Visualization of Tensor Fields in Biomedical Finite Element Models

Burkhard Claus Wünsche

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University of Auckland
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This thesis was assessed and approved by

Dr. Richard Lobb
Director of the Graphics Group, Department of Computer Science,
University of Auckland, Auckland, New Zealand

Dr. David Kenwright
Senior Research Scientist, Industrial Research Limited, Auckland, New Zealand

Prof. Dr. Christopher R. Johnson
Director, School of Computing
Director, Scientific Computing and Imaging Institute (SCI)
Director, Center for Bioelectric Field Modeling, Simulation, and Visualization (NIH
NCRR)
Director, Engineering Scholars Program (ESP)
Co-Director, Advanced Visualization Technology Center (AVTC)
Distinguished Professor of Computer Science
Research Professor of Bioengineering
Adjunct Professor of Physics
Faculty Member, Computational Engineering and Science (CES) Program
Co-Founder, Visual Influence Inc.

Abstract

Medical imaging is an essential tool for improving the diagnoses, understanding and treatment of a large variety of diseases. Over the last century technology has advanced from the discovery of x-rays to a variety of 3D imaging tools such as magnetic resonance imaging, computed tomography, positron emission tomography and ultrasonography.

As a consequence the size and complexity of medical data sets has increased tremendously making it ever more difficult to understand, analyze, compare and communicate this data. Visualization is an attempt to simplify these tasks according to the motto “An image says more than a thousand words”.

This thesis introduces a toolkit for visualizing biomedical data sets with a particular emphasis on second-order tensors, which are mathematically described by matrices and can be used to express complex tissue properties such as material deformation and water diffusion. The toolkit has a modular design which facilitates the comparison and exploration of multiple data sets. A novel field data structure allows the interactive creation of new measures and boolean filters are introduced as a universal visualization tool. Various new visualization methods are presented including new colour mapping techniques, ellipsoid-based textures and a line integral convolution texture for visualizing tensor fields.

To motivate the design and to assist in the use of the toolkit, guidelines for creating effective visualizations are derived by using perceptual concepts from cognitive science. A new classification for visual attributes according to representational accuracy, perceptual dimension and spatial requirements is presented and the results are used to derive values for the information content and information density of each attribute. A review and a classification of visualization icons completes the theoretical background.

The thesis concludes with two case studies. In the first case study the toolkit is used to visualize the strain tensor field in a healthy and a diseased human left ventricle. New insight into the cardiac mechanics is obtained by applying and modifying techniques traditionally used in solid mechanics and computational fluid dynamics.

The second case study explores ways to obtain in vivo information of the brain anatomy by visualizing and systematically exploring Diffusion Tensor Imaging (DTI) data. Three new techniques for the visualization of DTI data are presented: Barycentric colour maps allow an integrated view of different types of diffusion anisotropy.

Ellipsoid-based textures and Anisotropy Modulated Line Integral Convolution create images segmented by tissue type and incorporating a texture representing the 3D orientation of nerve fibers. The effectiveness of the exploration approach and the new visualization techniques are demonstrated by identifying various anatomical structures and features from a diffusion tensor data set of a healthy brain.

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