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DEVELOPMENT OF AN AEROELASTIC SIMULATION
FOR THE ANALYSIS OF VERTICAL-AXIS WIND TURBINES

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Abstract

Like their horizontal-axis counterparts, as the blades of vertical-axis wind turbines increase in size they typically become relatively more flexible so a better understanding of their aeroelastic behaviour is required. This research addresses the challenges of large, flexible, vertical-axis wind turbines using methods previously unavailable, or impractical due to limited computational resources.

A weakly-coupled aeroelastic simulation was developed to investigate the dynamic behaviour of vertical-axis wind turbines. The aeroelastic simulation comprised a free vortex wake model to represent the aerodynamics, and a multibody systems model to represent the structural dynamics, with the models coupled together via an interface. A modified version of the Beddoes-Leishman dynamic stall model was used to account for unsteady aerodynamic effects on the blades. The modal characteristics of the structural model were extracted using the Eigensystem Realisation Algorithm.

The aerodynamic aspects of the simulation were validated against a wide range of experimental data. The structural aspects of the simulation were verified against an industry-standard multi-body systems package, and validated against a combination of analytical, numerical and experimental data.

An extensive set of tests were conducted on the aerodynamic and structural models demonstrating that quite computationally undemanding methods for the computational parameters, including the time-step sizes and levels of blade discretisation, were capable of producing results very similar to those predicted using much more computationally demanding values.

The aeroelastic simulation was used to conduct a number of numerical experiments designed to aid an investigation into how the aeroelastic behaviour of a large-scale baseline turbine configuration changed in response to various parameters.

It was shown that the asymmetric loading along the blades, caused by a combination of the gravitational loads and the vertical variation in height of the wind speed, results in a periodic vertical rocking motion of the blades. This rocking motion occurred at a frequency of twice per revolution. It enabled new insight into the dynamic motion of VAWT blades. It was also found that the periodic rocking motion could be reduced by increasing the stiffness of the blades and reducing the linear density of the blades.
Acknowledgements

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Chapter 1

Introduction

Engineering is a great profession. There is the satisfaction of watching a figment of the imagination emerge through the aid of science to a plan on paper. Then it moves to realization in stone or metal or energy. Then it brings jobs and homes to men or women. Then it elevates the standards of living and adds to the comforts of life. That is the engineer’s high privilege.

Herbert Hoover - The Profession of Engineering

1.1 Introduction

The purpose of this chapter is to set the scene for the remainder of this thesis. A brief overview of the problem investigated is provided to better facilitate an understanding of the specific details of the research as described in the following chapters. A summary of the approach used and of some of the contributions made by this research to the body of knowledge are also given, followed finally by a description of the structure of this thesis.

1.2 The Problem

How do variations in structural and aerodynamic parameters effect the aeroelastic behaviour of vertical-axis wind turbines?

Like their horizontal-axis wind turbine (HAWT) counterparts, as the blades of vertical-axis wind turbines (VAWTs) increase in size they typically become relatively more flexible so a better understanding of their aeroelastic behaviour is required. The aerodynamic environment within which a VAWT blade operates is a particularly hostile one. The direction of aerodynamic
load alternates from one side of the blade to the other twice per revolution and the blade passes through not only the wake of the tower but also through its own wake and the wakes left by any other blades. For a VAWT with flexible blades, therefore, there is a potential for unusual aeroelastic behaviour. Despite this, the aeroelastic analysis of VAWTs has received only very limited attention in the past (a fact discussed in more detail in section 2.6). There have been some worthwhile contributions towards the analysis of the aerodynamics (discussed in section 2.4) and towards the analysis of the structural dynamics (discussed in section 2.5), but few contributions combining the two. The work described by this thesis aimed to address, at least in part, this deficiency in the literature.

1.3 Summary of Approach

The initial intention of the work described in this thesis was that the emphasis would be on the case studies themselves, i.e. the actual investigations into the aeroelastic behaviour of VAWTs. When the decision was made, however, to implement an aeroelastic simulation, the emphasis of the work changed to the development of that simulation. The development of such a complex simulation was, in hindsight, very ambitious to attempt within the scope of a single PhD and absolutely impossible to develop, verify, validate, and then apply to the wide range of investigations initially hoped for. The bulk of this thesis, then, is dedicated primarily to the development of the model, rather than to the case studies to which the model was applied.

During the course of this work, three independent simulations were implemented; a two-dimensional aerodynamic simulation, a three-dimensional aerodynamic simulation, and a three-dimensional structural simulation. The three-dimensional aerodynamic simulation and the structural simulation were then coupled together via an interface that was also developed as part of this work.

The aerodynamics were modelled using a combination of methods. The aerodynamics of the blades were modelled using a free vortex wake approach, whereby the wakes of the blades were represented as an arrangement of vortices (vortex points in the two-dimensional case and vortex filaments in the three-dimensional case). The strengths of the vortices in the wake were calculated based on the circulations around sections along the blades, and how the circulations changed with time. The circulations around the blade sections were, in turn, calculated based on the static aerofoil load characteristics which were adjusted using a semi-empirical dynamic stall model (specifically, a version of the Beddoes-Leishman dynamic stall model) to account for various unsteady effects involved. The aerodynamics of the tower were modelled using a significantly simpler method. The influence of the tower on the overall flow field was approximated as a potential flow field around a cylinder together with a velocity deficit model behind the tower, the impact of which was based on the drag coefficient of the tower. Whereas the
approach used to model the blades was an unsteady method, the tower model is best described as a quasi-steady method.

The structural dynamics were modelled using a multibody systems approach, whereby the flexible members of the turbine such as the blades were represented by series of interconnected rigid bodies. By arranging the rigid bodies into groups connected together with a specific combination of joints and springs with appropriately calculated strengths, the dynamic behaviour of the continuous system was approximated well.

As the structural and aerodynamic models required different levels of discretisation and time-step sizes, a mapping between the two models was required to account for this. An interface between the three-dimensional aerodynamic simulation and the structural dynamics simulation was developed allowing the aerodynamic model to pull position and motion information from the structural model, and the structural model to pull the aerodynamic loads from the aerodynamic model.

Each of the simulations was thoroughly tested and validated against available data from the literature to confirm the capabilities, and limitations, of the simulations.

The three-dimensional aerodynamic simulation, and the combination of the three-dimensional aerodynamic simulation and the structural simulation were used to conduct a range of numerical experiments to investigate how the aeroelastic behaviour of a large-scale baseline turbine configuration changed in response to changing configuration parameters.

1.4 Summary of Contributions

- The free vortex wake models described in this thesis represents a significantly more advanced approach than previous vortex models developed to model VAWTs. Although related to, and drawing inspiration from, other vortex models, the specific implementations described herein are unique. Extensive validations of the aerodynamic simulations were conducted to confirm their suitability.

- The dynamic stall model described herein, although based on the Beddoes-Leishman dynamic stall model, pulls together contributions from a number of sources and adapts the method to the vortex wake model implemented to model the aerodynamics, resulting in a unique implementation.

- Previous attempts to model the structural dynamics of VAWTs have been very limited compared to the method described in this thesis. Modelling the structural dynamics of flexible VAWT blades using a multibody system approach, in particular, has not previously been attempted.

- The superelement approach used to group and connect rigid bodies in such a manner
as to approximate the dynamics of a continuous straight beam-like flexible member was applied for the first time to curved members. The suitability of the approach was validated against data from the literature.

- A method, originally intended for modelling the motion of constrained multibody systems, was adapted and used to identify and perturb the degrees of freedom of the multibody system, allowing a system identification method to be used to linearise the system such that the modal properties could be extracted. The system identification itself, although applied to various other applications, including HAWTs, had not previously been applied to VAWTs.

- An extensive range of tests was conducted to establish the relationship between the predicted results and various computational parameters, such as the levels of discretisation of the blades, the time-step sizes, the length of the wake modelled etc.

- The aerodynamic and structural models, both of which are independently more advanced than previous similar models used to analyse VAWTs, were coupled together using an interface developed specifically for this purpose.

- The combined aero-structural simulation was used to conduct a number of numerical experiments into the aeroelastic behaviour of a baseline large scale turbine configuration, helping to develop a better understanding into the relationship between various configuration parameters, structural properties, and the resulting behaviour of VAWT blades.

1.5 Structure of Thesis

- Chapter 2 (Historical development of vertical-axis wind turbines) provides an introduction to the general field of wind energy and an overview of VAWTs and how they work. This chapter also discusses several methods considered for modelling the aerodynamics and the structural dynamics, with particular emphasis given to the approaches actually selected for the work described in this thesis. Finally, this chapter introduces the field of aeroelasticity and provides a summary of previous research conducted in relation to VAWTs.

- Chapter 3 (Baseline turbine configuration) describes the properties and configuration of the VAWT used as a baseline for the case studies for which results are presented in this thesis. The description of the baseline configuration is provided prior to the chapters describing the technical details of the simulations so that the reader has an understanding of how the simulations are ultimately applied while reading those sections.

- Chapter 4 (Aerodynamic modelling) describes the modelling approach implemented to simulate the unsteady aerodynamics of VAWTs, and the results from a thorough set of validation cases confirming the suitability of the aerodynamic models, and their imple-
Chapter 1: Introduction

- Chapter 5 (Structural modelling) describes the modelling approach implemented to simulate the structural dynamics of VAWTs and the results of both verification and validation cases confirming the suitability of the aerodynamic model and its implementation.

- Chapter 6 (Aero-structural interface) describes the method implemented to couple together the three-dimensional aerodynamic and structural dynamics models.

- Chapter 8 (Case studies) provides the results from a range of numerical simulations conducted to investigate the changes to the aeroelastic behaviour of the baseline VAWT blades in response to changes to various configuration parameters.

- Chapter 9 (Conclusions and recommendations) describes the conclusions that may be drawn based on the research described by this thesis and recommendations for future work that may build on and improve this work.

- Finally, several appendices are included as a reference for the reader. The appendices contain a variety of material that supplements the main body of the thesis but is not critical to an understanding of the research described.
Chapter 2

Historical development of vertical-axis wind turbines

If I have seen further, it is by standing on the shoulders of giants.

Isaac Newton

2.1 Introduction

The primary intention of this chapter is to provide some insight into the development of wind energy technology over the years, and in particular into the development of VAWTs. This is to provide a good understanding of the background leading to the research described in this thesis, and where it fits into the existing body of knowledge. The intention of this chapter is not, however, to act as an exhaustive reference on all aspects of VAWTs.

The chapter begins with a short overview of the historical development of wind energy (section 2.2) from the earliest uses of windmills to grind grain through to the rapid development of modern wind turbines for electricity generation in the second half of the 20th century, and finally to the current state of the technology. This is followed by an explanation about VAWTs (section 2.3), and in particular lift-driven curved-bladed VAWTs, showing how they work and also giving a summary of the general advantages and disadvantages of them in comparison to the far more common modern HAWT designs.

Finally, as it is critically important in any research to have a good understanding of the strengths and weaknesses of work already done in the field, and to use this as a foundation upon which to build new knowledge, a summary of aerodynamic modelling (section 2.4), structural modelling (section 2.5) and aeroelastic modelling (section 2.6) is presented. Although the emphasis of
these sections is on methods directly related to the approach described in this thesis, popular alternative approaches are also discussed where appropriate.

2.2 Historical wind energy development

2.2.1 Harnessing the wind

Mankind has been harnessing the energy in the wind for millennia to propel sailing ships [10] and for at least a millennium to drive windmills to grind grain. The earliest texts referring to windmills date back to around 10th century Persia [11], though they were likely to have been in use for some time before then. A drawing, dated around AD 1300, shows a windmill with cloth sails oriented around a vertical axis [11]. The windmill illustrated in figure 2.1 is a representation of a design in use in Afghanistan until at least the middle of the 20th century, and believed to be reminiscent of the vertical-axis windmills in use in ancient Persia [1, 11]. The windmills consisted of either cloth or bundles of reeds oriented about the vertical shaft which sits atop the millstones. Walls are constructed forcing the flow past one side of the axis only and driving the windmill about its axis. The only major difference between the vertical-axis windmills used in Afghanistan in the 20th century and that of AD 1300 drawing is that the millstones in the original appeared above the windmill rather than below it. As noted by Shepard [11], it is unknown how early on this switch was made.

There is also general acceptance that the Chinese have been using vertical-axis windmills for at least a few hundred years [1, 11], though whether they pre-date those in use in Persia is unclear, with the earliest references dating back to the 13th century [11].

The date of the first appearance of horizontal axis windmills is uncertain, though it was almost certainly prior to the 13th century and by the 14th century they were common throughout Northwest Europe [11]. Although much more complex than the vertical-axis windmills of Persia,
the horizontal axis windmills of Europe would have been a lot more efficient. The horizontal axis windmills remained the dominant source of mechanical power for several centuries, until the advent of the steam engine [1, 12].

Towards the end of the 19th century the first efforts to produce electricity from the wind began, starting with Charles F. Brush in the United States in 1888 [1, 11] who successfully built a 17m diameter, 12kW wind turbine. Brush’s design shared more in common with the windmills common on American farms at the time, having a high solidity rotor consisting of 144 blades, than it shared in common with modern HAWTs. Not long after this, in Denmark, Poul LaCour designed several electricity generating wind turbines ranging from 5 to 25 kW based on which several hundred turbines were eventually built. LaCour’s designs, which made use of an early scientific understanding of aerodynamics (including wind tunnel testing), would be more recognisable today as wind turbines (through certainly still based on the traditional windmills), with lower solidity, leading edge camber and low drag blades [11].

For most of the 20th century wind turbine development showed steady progress, thanks largely to improvements in the understanding of aerodynamics paralleling the development of aircraft. Of particular note is the development of lower solidity, higher speed turbines with two or three aerodynamic wing-like blades easily recognisable as the direct ancestors of today’s modern HAWTs. Most importantly, at least in relation to the work described in this thesis, was the invention and subsequent patenting of the lift driven VAWT by the Frenchman George Darrieus in 1931 [13]. Darrieus’ patent described a two or three-bladed rotor with the tops and bottoms of the blades attached to a central rotating tower. The patent covered both straight-bladed rotors and the curved-bladed rotors which are the focus of this thesis. The principle behind the Darrieus rotor is that by shaping the blades into a Troposkien-shape\(^1\) the bending stresses within the blades can be minimised. In reality, under the influence of gravitational and aerodynamic loading, all VAWT blades will experience variable bending moments, even Troposkien shaped blades. The Darrieus rotor showed no significant progress for several decades until it was reinvented in Canada in the late 1960s [14].

2.2.2 The rise of the modern vertical-axis wind turbine

The curved-bladed VAWT was reinvented in late the 1960s by researchers at the National Research Council of Canada (NRC) who, on discovery of the earlier patent by Darrieus, named the configuration in his honour. South and Rangi of the NRC conducted a variety of wind tunnel experiments on VAWTs through the late 1960s and into the 1970s [15, 16], but wider interest in VAWTs (and in wind energy in general) accelerated due to the Arab Oil Embargo of 1973 [1, 11, 12, 17], which spurred governments in a number of countries to seek out alterna-

\(^1\)A Troposkien is the shape that a perfectly flexible uniform cable, rotating about an axis that passes through the fixed ends, will take under the influence of centrifugal forces only.
Chapter 2: Historical development of vertical-axis wind turbines

(a) Curved-bladed Darrieus type VAWT with rotating tower

(b) Straight-bladed Musgrove type (H-type) VAWT

(c) Straight-bladed Musgrove type (H-type) VAWT with reefing mechanism

Figure 2.2: Simplified illustration of (a) the curved-bladed Darrieus-type VAWT with a rotating tower, (b) the straight-bladed Musgrove type (H-type) VAWT and (c) the straight-bladed Musgrove type VAWT with reefing mechanism.

tatives to imported oil. Funded by the US Department of Energy (DOE) researchers at Sandia National Laboratories, working together with their Canadian counterparts, developed an extensive theoretical and experimental research programme which focused on the curved-bladed Darrieus-type turbines (figure 2.2a) until the 1980s [14]. In the UK, led primarily by Musgrove at the University of Reading, research on the straight-bladed (H-type) rotor (figure 2.2b) was favoured [1, 12, 14].

The Sandia research programme, over the course of a number of years, constructed and tested the aerodynamic characteristics of a range of curved-bladed Darrieus-type VAWTs in both wind tunnel and field tests and also conducted a range of experimental modal tests to try and understand the structural properties. The VAWTs investigated covered a wide range of sizes from relatively small 2m [18–21] and 5m [22–24] diameter turbines, and extensive investigations of the more moderately sized 17m turbine [25–27]. In 1987 installation was completed on a dedicated research curved-bladed Darrieus type VAWT test bed with a 34m diameter and rated to 626 kW which was used for a wide range of detailed experiments [28–34]. The Sandia 34m turbine was a very sophisticated VAWT with varying chord length and aerofoil type along the span, a variable speed generator, numerous measurement devices and computer control. Based on the knowledge gained at the NRC and Sandia, in 1986 the largest VAWT every constructed was installed in Canada at Cap-Chat, Quebec. Named the Éole, the turbine had a 64m diameter, a height of almost 110m and was designed with a capacity of 4MW [7] (or 3.6MW [14] or 3.8MW [35] or 4.2MW [36], depending on the source). Based on concerns about blade fatigue, however, the Éole was limited to a maximum power output of about 2MW to ensure its survival for at least five years of operation [14]. The Éole eventually ceased operation in 1993 when the bottom bearing failed [35]. While no longer in operation, the Éole remains in place as a tourist attraction.

Several interesting prototypes were also built in the UK including a 135kW 25m straight-
Chapter 2: Historical development of vertical-axis wind turbines

bladed turbine, funded by the UK Department of Energy and built by Vertical-Axis Wind Turbines Limited, that had a reefing mechanism which changed the angle of inclination of the blades to the vertical (figure 2.2c) in different wind speeds to control the loads [12, 14, 17]. The knowledge gathered from this prototype led to the construction, beginning in 1989, of the largest straight-bladed VAWT. The turbine, rated to 500kW, had a radius of 38m and a height of 45m but omitted the reefing mechanism of the smaller prototype as it was deemed unnecessarily complicated [17]. It was assembled in 1990 but operated for less than seven months until a manufacturing defect led to one of the blades breaking [17].

Although the US, Canada and the UK were the dominant forces behind VAWT research during the 1970s and 1980s, each with their dedicated research programmes, a number researchers of other nations were also involved in more isolated development. Japan [37, 38] and New Zealand [39, 40], for example, both had researchers working on VAWT projects during this period.

The Sandia 17m turbine became the basis for the only widespread commercially successful VAWT design with variants commercialised by several companies in Canada\textsuperscript{2} and the US\textsuperscript{3}, including a 19m diameter version with a height to diameter ratio of 1.31:1 [41]. The commercial use of VAWTs reached a peak in the mid to late 1980s with over 500 utility-scale VAWTs in operation in California at one time [42].

2.2.3 The decline of the modern vertical-axis wind turbine

While considerable effort was expended throughout the 1970s and 1980s to investigate VAWTs, interest in the three-bladed HAWT design that is now a common sight was also gaining popularity. In Denmark in particular, where a lack of natural resources made the effects of the Arab Oil Embargo have even more impact than in many other countries, the commitment to alternatives was very strong.

From the late 1980s onwards, interest in VAWTs and wind energy in general in North America started to wane and emphasis gradually shifted towards HAWTs instead. The last utility-scale VAWTs in California were installed in 1986 [42], although FloWind Corp. continued to investigate potential upgrades to its existing VAWTs into the early 1990s [41]. By 1995 HAWTs represented 95% of the wind generation capacity of the state [42]. In the last quarter of 1995 there were a total of 509 utility-scale VAWTs in operation in California [42] but by 1999, according to the California Energy Commission, although some were still intact in the Altamont pass region [43] none were reported to be still in operation [44].

In the UK too, research into VAWTs faded and ultimately stopped completely with the research programme terminated in 1992 [1]. In Denmark, although they did investigate some VAWTs in

\textsuperscript{2}Indal Technologies Inc., Lavalin Inc. and Adecon Inc.[14]
\textsuperscript{3}FloWind Corp. and Vawtpower [14]
the years following the Arab Oil Embargo, their focus was, and continues to be, largely on the HAWT configurations. The three-bladed upwind turbines now familiar around the world can be referred to as Danish-style turbines thanks to that country’s commitment to wind energy, and to that design in particular.

2.2.4 Recent interest in the vertical-axis wind turbine

Since the mid-1990s there has been only limited interest in VAWTs compared with HAWTs. From time to time, however, examples of VAWT research emerge, mostly from the academic sector but occasionally also from the commercial sector. Recent review articles discussing VAWTs demonstrate their continuing, though sporadic, interest as an alternative to the widespread HAWT [35, 45]. Most of the recent efforts have focused on small scale turbines such as the work of Islam et al. [46, 47] to investigate and develop improved aerofoils for small capacity straight-bladed VAWTs. Mertens et al. [48] also looked at smaller straight-bladed turbines, and investigated their suitability for roof-mounted operation (for use in a built-up area, for example). Additional, more recent, investigations on roof mounted turbines have also been pursued by Ferreira et al. [49] and by Sharpe and Proven [50]. As noted by Sharman [51], however, convincing evidence of the economic viability of small turbines in a built-up environment still needs to be established.

Another relatively popular area of VAWT research in recent years is, once again focused on small turbines, the development of self-starting VAWTs, such as investigated by Pawsey [52] and Dominy et al. [53], for example. The challenging aerodynamics of VAWTs also continued to make them interesting subjects for examining different aerodynamic modelling methods well after their commercial use began to decline, as evidenced by the work of Ponta and Jacovkis [54], by Zhang [55], by Howell et al. [56] and in this thesis. Those same challenging aerodynamics also made for interesting experimental research. Fujisawa and Shibuya [57] used particle image velocimetry to observe dynamic stall on VAWT blades. In the commercial sector, there has been some interest (although, at the time of this writing still unsuccessful) in massive multi-megawatt scale VAWTs [36]. Finally, although not strictly wind turbine research, there is still some good potential in the use of vertical-axis turbines (and cross-flow turbines) for the closely related field of river or tidal energy extraction [58–61].

2.3 Vertical-axis wind turbines

2.3.1 How they work

Figure 2.3 illustrates a top down view of a two-dimensional horizontal slice through a VAWT with a radius $R$, rotating at an angular velocity $\Omega$, presented with a blade located in each of the four quadrants of rotation. The actual lengths of the various flow velocities and load directions
are indicative only and not meant to represent actual measurements or calculations. In reality, as explained in more detail in chapter 4, the velocity of the flow over an aerofoil ($V_{\text{rel}}$) is a combination of its motion, the onset wind flow, and the effect of the wake. For simplicity figure 2.3 shows only a free-stream velocity ($V_\infty$) and a velocity due to the rotation of the blades about the central axis ($R\Omega$). This is sufficient to illustrate how a VAWT generates power without adding too much visual clutter to the figure. With the additional effects of arbitrary blade motion and the influence of the wake included, the direction and magnitude of the loads change slightly, but the general concept remains the same.

The relative flow velocity over each blade is a combination of the free-stream flow and the flow due to the rotation of the blades about the central axis. Except for cases in which the blade is moving either directly towards or away from the free-stream flow, the relative velocity over the blade is at an angle of attack to the foil and, depending on the aerodynamic characteristics of the aerofoil, results in a lift force ($L$) that acts forward of the normal to the blade. Converting the lift and drag loads into forces normal to the blade ($F_N$) and tangential to the blade ($F_T$) shows that, provided the lift to drag ratio and/or the angle of attack is sufficiently high, the tangential force acts towards the leading edge of the blade pulling the blade forward and so generating torque about the central axis.

As illustrated in figure 2.3, this effect works both upstream and downstream of the central axis and regardless of whether the blade is moving generally towards the free-stream flow or away from it. There will be a small region when the blade is moving almost directly towards the free-stream flow and when the blade is moving almost directly away from the free-stream flow that will have tangential forces towards the trailing edge of the blade. As the blade travels over the upstream side of the turbine the lift force acts towards the inside of the arc of rotation, but on the downstream side of the turbine the lift force acts away from the inside of the arc of rotation.
Chapter 2: Historical development of vertical-axis wind turbines

rotation. This means that there must be two times per revolution where the lift force is zero as the lift shifts from one side to the other, and thus the only aerodynamic force acting on the blade is drag.

2.3.2 Comparison with horizontal-axis wind turbines

The relative merits of VAWTs versus HAWTs is an ongoing debate that is unlikely to be resolved in the foreseeable future, so is left to other authors to address. The intention of this section, and indeed of this thesis, is not to enter seriously into the debate of whether VAWTs or HAWTs are “better”. The intention of this section is, however, to provide a brief summary of some of the key differences between the two configurations and some of the typical arguments made both for and against each type. The basic reality is that for every argument in favour of VAWTs there is an equally compelling argument against them, none of which actually matters in terms of the scope of this thesis. This thesis describes a tool for the analysis of VAWTs with results presented from a comprehensive set of case studies to demonstrate the use the tool. Whether or not the VAWT configuration itself is suitable in a given situation does not effect the validity of the tool described herein.

The following descriptions of some of the relative advantages of VAWTs versus HAWTs are, unless otherwise noted, related primarily to the standard curved-bladed Darrieus-type VAWT with a rotating tower, and to the common Danish-style three-bladed upwind HAWT. For each advantage described, there are either already alternative designs in the other configuration that will counter the advantage, or there could be alternatives developed in the future. The number of possible configurations already developed today is too large for every possibility to be discussed, without even taking into account possible future designs.

Advantages of vertical-axis wind turbines

- **No yaw mechanism:** HAWTs must be oriented to point towards the onset wind flow for successful operation. While some downwind wind turbines are capable of self orientation, the more common upwind HAWTs require either some sort of wind vane type mechanism in the case of small turbines, or for large scale turbines, a mechanically driven yawing mechanism with an associated control system. The yawing of such massive rotating structures can also generate large gyroscopic loads that must be accounted for in their design and construction. VAWTs, with their ability to accept wind flow from any direction, do not require any such yawing mechanism and do not suffer the same gyroscopic load issues as HAWTs.

- **Drive chain and generator located near ground level:** The generator and drive chain of a VAWT is located much closer to the ground than those of a HAWT. The generator and drive chain of a HAWT is positioned at the top of the tower in the nacelle, which not only
makes maintenance more challenging, but also requires large cranes to lift the nacelle into place during assembly. The tower of a VAWT does not need to support the weight of the generator so may be lighter, and as the generator itself does not need to be positioned at the top of the tower, its size and weight are not important so larger, more efficient and less expensive generators may be possible.

- **Smaller tower moment:** The wind pushing on the rotor of a HAWT produces a large horizontal load at the top of the tower to which it is attached. This horizontal load corresponds to a large bending moment in the tower. This may require a more substantial, and hence more expensive, tower than a VAWT.

- **Simpler blade design:** While there may be some aerodynamic benefits to using blades on VAWTs with camber, taper or twist, they typically don’t require such complications. One of the strengths of VAWTs over HAWTs is the very simple blade construction possible with a symmetric blade of uniform chord and no geometric twist. Such simple construction should keep the costs of manufacturing VAWT blades well below those of the generally much more complex HAWT blades.

### Advantages of horizontal-axis wind turbines

- **Height above ground:** Wind speed tends to increase and turbulence tends to decrease with height, so for a given tower height a HAWT has access to faster and smoother winds than a VAWT would for the same tower height. This is particularly true of the curved-bladed VAWTs which generate most of their energy from the region of the blades closer to the equator (where the radius is at a maximum), which is located only about halfway up the tower. Straight-bladed VAWTs can also benefit from the faster wind speeds with height, as their horizontal member can be attached at the top of the tower, but the large bending moments in the blades and the drag from the horizontal member counter this benefit somewhat.

- **Tower shadow:** Each blade of a HAWT only encounters the effect of the tower once per revolution; either the tower dam effect for upwind turbines, or the tower shadow effect for downwind turbines. Each blade of a VAWT on the other hand experiences both the tower dam effect on the upstream side (though this is not significant for most of the blade span) and the tower shadow effect on the downstream side. VAWT blades must also contend with their own wakes, released on the upstream side and passed through on the downstream side of the turbine. This results in a very complex, and quite aerodynamically hostile environment.

- **Active blade pitch control:** One of the great strengths of HAWTs is the ability to have active blade pitch control. This has numerous advantages. The blades can be pitched to control the loads on the blades and hence the amount of power being generated. By
careful design of a sophisticated control system, the blades can be pitched to optimise the 
energy extraction such that the turbine can maintain a high level of efficiency over a wide 
range of wind speeds. The blades can be pitched such that the wind turbine can self-start 
in low winds, which is something a curve-bladed VAWT cannot generally do. The blades 
can also be feathered in very high winds to shed energy and protect the turbine from 
damage. A curved-bladed VAWT cannot accomplish such a function and so the turbine 
components need to be designed for higher loads to survive such events.

- **Blade length:** Although VAWT blades are generally a simpler design than HAWT blades, 
so may require less complicated manufacturing techniques, for a given swept area the 
VAWT blades need to be considerably longer than their HAWT counterparts. Careful 
cost-benefit analysis would need to be done to determine whether the simpler manufactur-
ing techniques required by VAWT blades are sufficient to overcome the additional expense 
due to the additional material that may be required.

- **Access to main bearing:** Although the location of the drive train and generator nearer 
to the ground is listed as an advantage for VAWTs during assembly, in the event that the 
main bearing needs to be replaced the location of the drive train and generator becomes 
a disadvantage. Depending upon how the turbine is designed, the entire tower and rotor 
assembly may need to be removed to access the main bearing.

- **Cyclic loading:** HAWTs do experience some cyclic loading as the blades rotate from the 
lower wind speed region towards the bottom of their rotation to the higher wind speed 
region at top of their rotation, and also due to the gravitational loads that change direction 
relative to the blades as they rotate. VAWTs do not experience the cyclic loading due 
to gravity, but can experience severe cyclic loading due to the change in direction of the 
relative flow velocity. By the very nature of the way VAWTs generate energy (as explained 
in section 2.3.1) the direction of the aerodynamic lift force switches direction twice per 
revolution (from upstream to downstream, then again from downstream to upstream). 
This cyclic aerodynamic loading may fatigue VAWT blades faster than a correspondingly 
sized HAWT in the same wind conditions.

- **Investment in research:** Perhaps one of the greatest advantages of HAWTs over VAWTs, 
and one that should not be underestimated, is the considerable time and money that has 
gone into HAWT development around the world. While VAWTs did receive a lot of 
interest in North America and the UK for a number of years throughout the 1970s and 
1980s, the effort dedicated to the development of VAWTs is trivial compared to that 
of HAWTs, both before and after this period. HAWTs have received considerably more 
attention in recent years than VAWTs ever did, both in academic research and commercial 
applications. This has allowed the HAWT to evolve into a very efficient and reliable means 
of energy generation. Whether VAWTs are inherently better or worse than HAWTs may,
in fact, not really matter unless the gap in knowledge between the two configurations can be closed. The work described herein helps to close this gap.

2.4 Aerodynamic modelling

There have been a number of very good review articles covering aerodynamic modelling methods, predominately of HAWTs and rotorcraft, but also with good relevance to VAWTs. Some of the review articles specifically discuss VAWTs [45, 62–65] while others highlight issues generally applicable to any type of wind turbine [66]. The majority, however, focus on HAWTs [2, 67–70] although the techniques they discuss are also very relevant to VAWT investigations. Still others, while intended as reviews of rotorcraft aerodynamics, share so much in common with wind turbines, albeit with additional challenges caused by the high Mach numbers and flight dynamics involved, that they also provide considerable insight into the applicable aerodynamic techniques [71, 72]. The reviews of Hansen and Butterfield [68], Snel [69, 70] and Leishman [2] in particular are very thorough and, although they are all now a number of years old, they provide an excellent base of knowledge beyond the brief overview provided in this section. Also, the text by Paraschivoiu [14] describes in some detail the various aerodynamic methods, and the momentum methods in particular, as they relate specifically to VAWTs.

The aerodynamics of VAWTs have been modelled using a wide variety of different techniques. Momentum models, and Blade Element Momentum (BEM) models in particular, are easily the most widely used. The approach used for the work described in this thesis is a vortex wake model which, although not as commonly used as the momentum methods, would certainly not be described as very rare either. A number of other methods have been used to model the aerodynamics of VAWTs too, though these are much less common. These include Computational Fluid Dynamics (CFD) and Cascade models, for example. Each of these modelling approaches is discussed briefly in this section.

This section also provides an overview of the phenomenon of dynamic stall, which is very important to accurately estimate the aerodynamic loads, the effects of which are generally not represented directly by the various aerodynamic models and must be accounted for separately. The exception is CFD models which, in principle at least, may be capable of capturing the effects of dynamic stall without the inclusion of a separate model.

2.4.1 Momentum models

Despite their limitations, momentum models remain very popular for both HAWT and VAWT analysis. Their speed and simplicity, combined with their ability to predict overall performance with reasonable accuracy, make them very appealing. Their ability to accurately predict instantaneous loads at specific locations on the blades is, on the other hand, quite limited compared
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with some of the other approaches.

The momentum models used to predict the aerodynamics of wind turbines are all fundamentally based on theory originally developed for the analysis of propellers, towards the end of the 19th century, by Rankine and extended by Froude. The theory uses the laws of the conservation of mass, momentum and energy on a control volume around the rotor (i.e. a streamtube) to develop a relationship between the time-averaged aerodynamic loads of the rotor and the change in flow velocity through the rotor. Betz was the first to apply the principle to wind turbines in the early part of the 20th century. These early models were relatively simple, making the assumptions that the the rotor was a uniformly loaded disc, that the flow was steady, and that the flow was in an axial direction only. Betz determined that, given these assumptions, the maximum power coefficient\(^4\) \((C_P)\) of a wind turbine is \(C_{P_{\text{max}}}=16/27 \approx 59\%\). This maximum is referred to as the Betz limit. Some researchers, such as van Kuik [73] for example, have shown that the maximum theoretical efficiency of rotors may, in fact, be higher than this. To date, however, no actual wind turbine of any sort (either horizontal-axis or vertical-axis) has come close to the Betz limit. Glauert extended the momentum theory, applying it to multiple concentric annuli instead of a single streamtube. This provides a much better understanding of the loading characteristics along the span of the blades. Although there have been many improvements and refinements to the method over the years, Glauert’s work remains the basic foundation for a lot turbine analysis done today.

The first application of momentum theory to VAWTs was Templin’s single streamtube model [74] which was used to analyse curved-bladed Darrieus-type turbines. The method assumes an average loading over the whole turbine, combining both upstream and downstream, so is only capable of very broad predictions about the overall performance. Templin’s simple single streamtube model was soon followed by Strickland’s multiple streamtube model [75]. Strickland divided the turbine into multiple streamtubes both horizontally and vertically. Applying BEM theory to each of these streamtubes separately provided a much better understanding of the average loads on the VAWT at different locations perpendicular to the direction of wind flow. The method was still somewhat lacking, however, in that like the single streamtube model, it made no distinction between the upstream and downstream sides of the turbine so could not provide a detailed understanding of instantaneous blade loads, but only the overall performance (albeit, more accurately than the single streamtube model could). The next major advance in the application of BEM methods to VAWTs was made by Paraschivoiu with the development of the Double-Multiple Streamtube (DMST) model [76]. Like the multiple streamtube model, the DMST also divides the flow through the VAWT into a number of horizontal and vertical streamtubes, but unlike the multiple streamtube model, in the DMST model a distinction is

\(^4\)The power coefficient \((C_P)\) is the amount of energy extracted by a wind turbine as a proportion of the total energy available in the wind passing through the same swept area.
made between the upstream and downstream sides of the turbine. Paraschivoiu and Delclaux [77] soon improved on the DMST model with refinements that allowed better azimuthal variation and Paraschivoiu et al. [78] included a variety of modifications to account for a range of secondary effects such as the effect of a rotating tower and the presence of struts and spoilers. Paraschivoiu [79] demonstrated that, with the inclusion of a dynamic stall model, the DMST model could produce reasonable predictions of the variation of blade loads with azimuthal position. Although momentum-based models such as the DMST have been used together with simulations of turbulent wind fields [80] they are, at their core, still quasi-steady models. Although sophisticated versions such the DMST do appear to generate reasonable instantaneous predictions, they do so despite their known poor assumptions and the limitations of the physical principles upon which they are based, and as such, confidence in their use for anything beyond estimating average performance is not really justified.

2.4.2 Vortex wake models

Vortex wake models represent a significant step forward in their potential ability, compared to the BEM methods, to capture the instantaneous loads of more complex and unsteady cases, but also a big increase in computational cost. In vortex wake models, the vorticity in the wake is modelled and used to determine the velocity field when and where needed. For VAWTs (and for HAWTs and rotorcraft), because the flow may be considered inviscid everywhere except for a thin layer trailing behind each blade, the vorticity can be represented as a thin sheet (or lines, or blobs, or points) of vorticity. The strength of vorticity in the wake is determined from the distribution of circulation along the span of each blade at the time of separation (the details of which are provided in section 4.3).

Vortex wake models can be broadly described as either fixed-wake and free-wake models. In fixed-wake models, a number of assumptions about the structure of the wake are made which make the models considerably less computationally demanding, at the expense of physical accuracy. In free-wake models, the wake is allowed to develop over time under its own influence and the influence of the structure, which results in much higher computational costs.

The use of vortex wake models is common for rotorcraft research [81–87] and although they have sometimes been used for HAWT research [88–91] they remain less popular than the BEM methods [92]. Vortex wake models have been applied to the analysis of VAWTs for almost as long as momentum models. Wilson applied a simple (by today’s standards) two-dimensional fixed-wake vortex model to the analysis of a Giromill\(^5\) as early as the late 1970s [93]. Wilson and Walker then developed a more advanced fixed-wake model capable of modelling curved-bladed Darrieus-type turbines that used a combination of vortex and momentum theory [94].

\(^{5}\)A Giromill is a straight-bladed VAWT with articulated blades that pitch to, in theory, allow maximal energy extraction.
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The computational expense of these fixed-wake model was comparable to the pure momentum models and showed a reasonable capability to predict the overall performance of a turbine, but it is unclear whether the model offered much of an advantage over the pure momentum models. Coton et al. [95] developed a fixed-wake model for VAWT analysis with modifications to more accurately predict the tangential loads in unsteady conditions. Coton et al. demonstrated that their prescribed wake model might produce results comparable to a free-wake model (for the case tested), without the high computational burden.

Alongside the work being done on fixed-wake models at the time, Strickland et al. [96, 97] developed both two-dimensional and three-dimensional free-wake vortex models capable of modelling curved-bladed Darrieus-type VAWTs for analysis. A number of experimental tests were also performed and the numerical model compared favourably to the measurements [98, 99]. The vortex wake model lacked the advanced dynamic stall modelling capabilities described in this thesis (section 4.8), and the experimental tests were essentially two-dimensional only (straight-blades in a towing tank), but nonetheless, the comparison demonstrated the potential capabilities of free-wake vortex models for VAWTs. Modifications to the model were made later by Brownlee [100] in an effort to reduce the computational cost by assuming that the velocity of the wake vortices could be calculated at their creation time. Wilson et al. [101] also developed a two-dimensional free-vortex model that uses conformal mapping techniques to map the (uninstalled) aerofoil into a rotating cylinder for analysis and showed good agreement with analytical predictions of unsteady motion. Vandenberghhe and Dick [102] developed a free-wake vortex model for the analysis of VAWTs which, although only two-dimensional, did include a simple dynamic stall model (specifically, a modified Boeing-Vertol dynamic stall model) to account for some of the unsteady effects. Vandenberghhe and Dick’s model differed from the other examples described here in that they used a vortex-in-cell approach. The vortices in the wake move in a Lagrangian manner, as in the other models, but the velocity induced by the wake is determined using a Eulerian grid. The vorticity is redistributed to the predefined grid nodes at each time-step. A hybrid Lagrangian-Eulerian method such as this can reduce the computational costs significantly while still retaining theoretically better unsteady modelling capabilities than the momentum methods. The main danger in this approach is in how the vorticity is redistributed from the vortices to the grid nodes. If not implemented correctly, it may lead to either numerical instabilities, or artificially introduced numerical diffusion.

The inherently unsteady nature of free-wake vortex models makes them theoretically much better suited to estimating instantaneous unsteady loads, but this must be weighed up against the fact that the computational cost of doing so is considerably greater than that of the momentum based models. On balance, because the unsteady aerodynamics were considered likely to have an effect on the aeroelastic behaviour of VAWTs, a free-wake vortex model was selected for use in the work described in this thesis. The details of the model implemented are described in
section 4.3.

### 2.4.3 Other aerodynamic models

In addition to the momentum and vortex wake models discussed in the previous sections, several other methods have been used to model the aerodynamics of VAWTs, although they are generally much less common. These include cascade models, which are based on techniques typically applied to turbomachinery analysis. Modified versions of the cascade model developed by Hirsch and Mandal [103] for VAWT analysis were applied to an examination of dynamic stall and flow curvature effects by Mandal and Burton [104] and recently in an investigation on aerofoil design for small VAWTs by Islam et al. [47].

Navier-Stokes based computational fluid dynamics (CFD) modelling undoubtedly offers great potential for capturing an accurate representation of the aerodynamics. The greatest difficulty in modelling the aerodynamics of a structure as aerodynamically complex as a wind turbine using such methods, is however, the very high computational costs involved. Tchon and Paraschivoiu [105] used a two-dimensional Reynolds-averaged Navier-Stokes CFD model to simulate the flow around an aerofoil undergoing VAWT motion. Tchon and Paraschivoiu did not attempt to model the flow around the whole turbine, but rather modelled a single stationary aerofoil exposed to an flow that varied in direction in a manner approximating the variation in relative flow direction expected as a VAWT blade rotates. Allet et al. [106, 107] used a similar approach, but with an emphasis on using CFD to model the aerofoil undergoing VAWT motion as it experiences dynamic stall. Very recently, Howell et al. [56] used both two and three-dimensional CFD models to model a small VAWT and compared the results with wind tunnel tests. While such CFD work is interesting and may lead to some good insights, the very high computational cost of these CFD models limits their widespread use for the analysis of VAWTs.

One clever use of CFD modelling to examine complete (two-dimensional) VAWTs, while keeping the computational manageable, was developed by Ponta and Jacovkis [54]. Ponta and Jacovkis combined a CFD model of the flow immediately around an aerofoil with a free-wake model further away. The approach balances the potential accuracy of CFD in close proximity to the blades with the lower computational demands of vortex wake models at larger distances. The approach showed promising agreement against some experimental measurements, but whether it can be extended to a full three-dimensional turbine has yet to be proven.

### 2.4.4 Dynamic stall

Dynamic stall is a very interesting and very complex phenomenon that has a significant effect on blade loads. Dynamic stall can effect any aerofoil moving dynamically with respect to the
onset flow, and can be experienced by the blades of HAWTs and rotorcraft. With blades that are effectively pitching continuously with respect to the direction of flow, the prediction of the effect of dynamic stall on blade loads is particularly important for VAWTs, and as such the subject has occasionally received specific examination [57, 108, 109]. It is discussed briefly here because of its importance.

The dynamic aerodynamic loads measured by Hoffman et al. [3] in a wind tunnel for two sequential oscillations of a NACA 4415 aerofoil pitching sinusoidally are shown together with the measured steady state loads in figure 2.4. Additional details of the experimental set-up are given in section 4.10.2, where the results are used to validate the aerodynamic simulation described in this thesis. Figure 2.4 also illustrates the flow physics at different stages of the aerofoil experiencing dynamic stall. At stage 1, as the aerofoil pitches upwards, flow separation is delayed beyond the angle of attack at which steady state stall would occur. At stage 2, leading-edge separation begins and a vortex starts to form near the leading-edge that induces additional lift. Between stages 2 and 3, the leading-edge vortex separates and starts to travel over the upper surface of the aerofoil towards the trailing edge. The vortex continues to induce additional lift as it moves and also shifts the centre of pressure towards the trailing edge. Between stages 3 and 4, the flow fully separates (the aerofoil stalls) resulting in a sharp drop in lift. At stage 5, when the angle of attack and the rate of change of the angle of attack are low enough, the flow starts to reattach from the leading-edge first, and then towards the trailing-edge.
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While CFD models may, in theory, be capable of accounting for dynamic stall automatically, momentum and vortex wake models require either modification or separate dynamic stall models to account for such effects. A good overview of the general approach taken by several of the most common dynamic stall models is provided by Leishman [2] and by Johnson [110]. Variants of the Gormont Boeing-Vertol model were particularly popular for VAWT analysis throughout the 1980s and into the 1990s [79, 80, 102, 111]. Since that time, however, developments in HAWT and rotorcraft research have led to advances in the understanding of dynamic stall and semi-empirical models based on physical phenomenon such as the ONERA and Beddoes-Leishman models are now in frequent use for HAWT and rotorcraft blade modelling. The Beddoes-Leishman model was selected for this work because of its basis on actual physical phenomenon and its general applicability to different aerofoils. Coton et al. [95, 112] also used the Beddoes-Leishman model in VAWT analysis, but the version described in section 4.8 of this thesis contains a number of modifications improving on those earlier versions. As demonstrated by the validation cases presented in section 4.10, the Beddoes-Leishman dynamic stall model together with a free vortex wake model is very capable of capturing the unsteady aerodynamics experienced by VAWT blades.

2.5 Structural modelling

Modelling the structural dynamics of wind turbines, both VAWTs and HAWTs, has generally received a lot less attention than the aerodynamics. This was not unreasonable in the early days of wind turbine development as the structures tended to be be relatively rigid so structural dynamics was not a major concern. As wind turbines have increased in size and become relatively more flexible, however, the importance of understanding the structural dynamics has been recognised. HAWTs in particular have, and continue to see more and more attention paid to their structural dynamics. The structural dynamics of VAWTs, although receiving some detailed analysis through the 1980s and into the 1990s has, in the period since, lagged behind the research being done on the structural dynamics of HAWTS, reflecting the lower levels of interest in VAWTs overall.

Like modelling the aerodynamics, modelling the structural dynamics of VAWTs shares a lot in common with that of HAWTs and rotorcraft blades so the techniques applicable to one system are generally applicable to the others. Two of the most common techniques used to model the structural dynamics of wind turbines (and rotorcraft) are finite element models and multibody systems (MBS) models. Each of these techniques are discussed briefly in turn, followed by a brief overview of some of the previous experimental testing conducted to investigate the structural properties of VAWTs.

There have been some alternatives to finite element and MBS methods used to model VAWTs,
such as the analytical approach taken by Rosen and Abramovich [113, 114]. Methods such as this are, although interesting, far less common and quite limited in application so are not discussed in further detail here.

2.5.1 Finite element modelling

In finite element methods, the flexible continuous structure is broken up into a number (usually a very large number) of discrete geometrically simple elements. The physical properties and behaviour of the individual elements are modelled at nodes within each element. Together with any applied loads or boundary conditions, a set of algebraic equations is found approximating the whole system. Finite elements models are typically of quite a high order, and as such can be computationally very expensive. As noted by Molenaar [115], although finite element modelling offers a good approach to the examination of the loads on static structures and those undergoing small dynamic motions, the approach is less well suited to structures undergoing general (and possibly large) gross dynamic motions.

The use of finite element methods has become increasingly common as computational power has increased, although their application is certainly not new. Finite element models of VAWTs were developed at Sandia National Laboratories using MSC/NASTRAN and compared with experimental measurements of a 2m curved-bladed Darrieus-type turbine by Carne et al. [20] in the early 1980s. The finite element model was formed in terms of a coordinate system rotating with the turbine and as such centrifugal and Coriolis effects were not automatically accounted for. The model also required the assumption of small motions within the rotating reference frame [116]. Carne et al. included centrifugal and Coriolis effects by modifying the stiffness and damping matrices and the force vector. This early model did not, however, include any aeroelastic effects. Modifications by Popelka [117] and Lobitz and Ashwill [118], on the other hand, did include some aeroelastic effects. Finite element modelling was used to evaluate several conceptual designs for the Sandia 34m VAWT [119]. Once the turbine was constructed, the models were validated and tuned based on actual measurements [28, 30, 31, 34]. The finite element methods used for the analysis of VAWTs mentioned so far were all effectively limited to the frequency domain. Dohrmann and Veers [120] remedied this by including time dependent terms into the finite element matrices allowing varying speed operation, such as during start-up, to be modelled.

Veilleux and Tinawi [7, 121] also developed a finite element model for VAWT analysis and used their model to investigate the influence that variation in the stiffness of the guy cables may have had on the dynamic behaviour of the 64m Éole turbine.

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6NASTRAN is a general purpose finite element code developed at NASA in the late 1960s. MSC/NASTRAN is the commercial version of NASTRAN developed by MSC (previously called MacNeal-Schwendler Corporation).
2.5.2 Multibody systems modelling

A MBS approach involves dividing the structure up into a collection of rigid bodies which are connected together with a variety of different types of joints. Unlike the finite element methods, in which deformation may be modelled by the behaviour within each element, a multibody system consisting of rigid bodies permits deformation only at the joints between the bodies. By careful arrangement of the rigid bodies, selection of appropriate joint types, and calculation of suitable springs and dampers at the joints, the dynamics of flexible members such as wind turbine blades can be modelled accurately with relatively few degrees of freedom [115, 122, 123].

For an examination of the dynamics of an entire complex structure such as a VAWT, accurate results may be possible with much lower computational demands using a MBS method than would otherwise be possible using a finite element method. For this reason, a MBS approach was used for the work described herein, the details of which are provided in chapter 5.

The use of MBS modelling to investigate the structural dynamics of VAWTs is, thus far, very limited. Lobitz and Sullivan [124] created VAWTDYN, which can be described as a very simple MBS model in which the blades and tower are modelled as individual rigid bodies attached via ball joints. VAWTDYN is also notable as an early, albeit very simple, attempt at modelling the aeroelastic behaviour of VAWTs with its inclusion of an aerodynamic model too. Biswas et al. [125] later adopted the structural modelling approach of VAWTDYN but replaced the earlier aerodynamic model with their own newer (but still simple) model [126]. Pawsey [52] also adopted a MBS approach to simulate a VAWT. Pawsey used the general purpose structural modelling package Pro/ENGINEER Mechanica to assemble a model of a VAWT with straight pitching-blades. Pawsey’s interest was in the pitching mechanism rather than the elasticity of the blades, so the model was still quite simple and assumed perfectly rigid blades.

The work described in this thesis is considerably more advanced than these previous attempts to model VAWTs using MBS approaches, and is based on the method described by Molenaar [115, 127] for modelling flexible HAWT blades using a MBS approach.

2.5.3 Experimental testing

While not technically a “modelling” method, it is worth commenting on some of the physical experimentation done to investigate the structural dynamics of VAWTs, or more specifically, the extraction of modal properties of such. The three methods used by researchers at Sandia National Laboratories to excite the modes and frequencies of interest of VAWTs were impact [21], step-relaxation [20, 21, 30, 128], and natural excitation (i.e. the wind) [30, 34, 128, 129].

By suddenly releasing a cable under tension connected between a blade and the tower, the step-
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2.6 Aeroelastic modelling

According to Collar [5], and also to Hodges and Pierce [6], the term “Aeroelasticity” was first coined by Roxbee Cox and Pugsley in the 1930s. Figure 2.5, is a visual representation showing aeroelasticity as the interaction between the aerodynamic forces, the elastic forces, and the inertial forces. The concept of the triangle of aeroelasticity is attributed to Collar [4] but has been used and adapted by a number of researchers such as Bisplinghoff [130], Dowell [131] and Hodges and Pierce [6]. This representation provides a clear and simple way to understand how the different types of forces interact together. Aerodynamic and elastic forces combine (without consideration of the inertial forces) to give us static aeroelasticity (e.g. divergence). The elastic and inertial forces, in the absence of the aerodynamics, gives us the structural dynamics. Determination of the natural frequencies and mode shapes of the structure in the absence of any aerodynamic flows falls into this category. The term “flight mechanics”, as used by Hodges and Pierce [6], is retained here to describe the interaction between the aerodynamic and inertial forces. Although the term refers to aircraft flight, the term is still appropriate here, as wind turbine blades do effectively “fly” through the air, albeit with some part of their structure constrained to the ground. Dynamic aeroelasticity describes the interaction between all three types of forces and is the subject of the work described in this thesis.

The field of aeroelasticity owes much of its current body of knowledge to the work done with respect to aeronautical development from the period of World War II onwards. As aircraft capabilities increased, so too did the potential for aeroelastic problems, and the ability to
accurately predict such events became more important. Collar describes the recognition of aeroelasticity and its development into an established part of aircraft design over the first half of the 20th century, and any reader with an interest in the historical development of aeroelasticity as a field of research is encouraged to read Collar’s excellent review article “The First Fifty Years of Aeroelasticity” [5].

Beyond the commonalities that wind turbines share with fixed wing aircraft, one important distinction that must be considered when examining wind turbines (or rotorcraft) is that rotating beams (and blades) have higher natural frequencies than equivalent non-rotating beams, due to the additional centrifugal stiffening [132]. The amount of the centrifugal stiffening is a function of both the distribution of mass, and the rotational speed. Traditional techniques for aeroelastic analysis of aircraft wings do not, therefore, always apply to the rotating blades of turbines and rotorcraft.

The aeroelastic behaviour of HAWTs has received a good level of attention for quite some time. Good overviews of some of the possible aeroelastic issues faced by HAWTs and the approaches used to investigate them are provided by Rasmussen et al. [133], Riziotis et al. [134] and Hansen [135].

The aeroelastic behaviour of VAWTs, in contrast, has received only limited attention. Some of the work mentioned in section 2.5, describing the structural modelling methods applied to VAWTs, also includes some aeroelastic investigations. The work by Lobitz and Sullivan [124], Biswas et al. [125] and Pawsey [52] were similar to the work described in this thesis, in that they focused on the overall aeroelastic behaviour of the turbine rather than on specific instabilities, such as done by Popelka [117] and Lobitz and Ashwill [118], for example. Popelka developed a theoretical prediction for the onset of flutter instability for VAWTs (assuming a very simple aerodynamic model), and showed that an operating speed several times that of the typical operating speed would be required for flutter to be initiated [117]. Lobitz and Ashwill investigated the onset of flutter, also concluding that flutter would require rotational speeds at least two to three times higher than the typical operating speeds. Lobitz and Ashwill also investigated the onset of divergence, but concluded that this was even less likely than flutter because of the torsional stiffness due to the attachment of the blades to the tower at both ends [118]. Malcolm [136] used an aeroelastic model combining the DMST aerodynamic model with the NASTRAN finite element to investigate the dynamic response of a curved-bladed Darrieus-style VAWT to turbulent wind. An interesting finding from Malcolm’s work is that that atmospheric turbulence may have a significant effect on the fatigue life of VAWT blades beyond that expected, due to the cyclic loading due to the rotation alone. Atmospheric turbulence is not considered as part of the work described in this thesis, but may make a worthy addition for research in the future.
Chapter 3

Baseline turbine configuration

The beginning of knowledge is the discovery of something we do not understand.

Frank Herbert

3.1 Introduction

This chapter describes the turbine configuration upon which the case studies presented in chapter 8 are based. The details of the baseline turbine configuration used for the investigations are presented here, rather than in the chapter containing the results, so that the reader has a general impression of the turbine configuration investigated, prior to reading the sections describing the methods used to model the aerodynamics (chapter 4), and the structural dynamics (chapter 5). Understanding the ultimate application of the simulations at the early stage in the thesis should facilitate the ease with which the descriptions of the models may be read.

The general configuration of the baseline turbine, such as the dimensions, the baseline rate of rotation, and the geometry of the blades is described in section 3.2. This is followed by a description of the aerodynamic properties of the turbine in section 3.3, including a description of the wind field within which the turbine operates, the baseline tip speed ratio, and the aerodynamic characteristics of the blades. Finally, a description of the structural properties of the blades is provided in section 3.4.

The configuration of the baseline turbine investigated is based in general on the Éole 64m turbine (referred to for the remainder of this chapter as simply the Éole), with a number of properties modified to isolate the features of most interest (i.e. the blades only) or to reflect more modern materials (e.g. blades constructed using a fibreglass composite instead of the original steel). It must be pointed out at this stage that, in practice, many of the parameters
Chapter 3: Baseline turbine configuration

Figure 3.1: Illustration of the Éole 64m turbine (adapted from Veilleux and Tinawi [7]) and the overall dimensions of the baseline turbine.

defined in this section would actually be tailored based on a detailed design procedure for a given wind turbine at a given site. The design of an optimal turbine configuration is well beyond the scope of this work, so it should not be assumed that the precise combination of configuration parameters described would in fact exist on a real turbine at a real site. It should also not be assumed, however, that the precise combination of configuration parameters described would not, in fact, exist on a real turbine at a real site. It is simply important to recognise that the quantifiable determination of the design one way or another, is not considered within the scope of the work described by this thesis. It should be stressed, however, that every effort has been made to select the individual configuration parameters to be representative of a realistic wind turbine and achievable in practice, if not necessarily optimal in combination.

3.2 General configuration

3.2.1 Overall dimensions

The overall dimensions of the baseline turbine are shown in Fig. 3.1 alongside an illustration of the Éole adapted from Veilleux and Tinawi [7]. Like the Éole, the baseline turbine is a two-bladed Darrieus-type configuration. The overall dimensions of the baseline turbine are left unchanged from those of the actual Éole with a blade diameter at the equator of 64m, a height at the base of the blades of 14m, and a height at the top of the blades of 110m. This gives a blade radius to height ratio of 1:3, in contrast to the Sandia 17m turbine (against which much of the aerodynamic model presented in this thesis is validated), which has a blade radius to height ratio of 1:2. The chord length of the blades of the baseline turbine is set at 2.4m along their full length, matching the blades of the Éole. The diameter of the tower of the baseline turbine is 5m, also matching the diameter of the Éole tower. For the purposes of modelling the aerodynamic effects of the tower, the representation of the tower is simplified in the baseline turbine in comparison to the actual Éole. The tower has been extended above the height of the blades to avoid numerical issues in the aerodynamic simulation, due to the
Chapter 3: Baseline turbine configuration

Figure 3.2: Non-dimensional shape an ideal Troposkien, the blades of the Sandia 17m turbine, the Éole 64m turbine and a 6th order polynomial approximation for the baseline turbine configuration.

ends of the blades coinciding with the end of the region of influence of the tower\(^1\). As shown in Fig. 3.1, the large generator housing resides below the blades of the Éole. As a compromise between the additional complexity of attempting to model a large generator housing, and the over-simplification of omitting any structure completely, the tower is simply extended all the way to the ground. Details concerning the aerodynamic influence of the tower are described in section 4.5.

3.2.2 Rate of rotation

The Éole was capable of operation at various rotational speeds up to 16.3 rpm, although in practice, because of concerns about fatigue [14], the rate of rotation was not allowed to exceed 13.25 rpm, and on a day-to-day operational basis it ran at rotational speeds of 10 rpm and 11.35 rpm to ensure survival for the short five year planned operational life of the turbine.

The typical rotational speed of the baseline turbine has been selected to be 13 rpm. This choice of rotational speed was selected as it falls approximately mid-way between the Éole’s design rate of 16.3 rpm, and the lower operational rate of 10 rpm. This allows some scope for investigating changes to the rate of rotation, both increases and decreases, while still remaining within a range that can be achieved in practice for a turbine of this size.
3.2.3 Blade geometry

The curved blades of the Éole (and the Sandia 17m turbine) are not shaped into an ideal Troposkien. The shape of the blades of the baseline turbine is based on the blades of the actual physical turbines, rather than the idealised Troposkien shape. To ease implementation and modification during the various investigations, the initial shape of the blades was approximated by a 6th-order polynomial curve determined by a least-squares best fit with the average of the non-dimensional Éole’s blades, and the non-dimensional Sandia 17m turbines blades. The non-dimensional blade shapes given by an idealised Troposkien, the Sandia 17m turbine, the Éole and the 6th-order polynomial used to approximate the blade shape of the baseline turbine are presented in Fig. 3.2 where it can be seen that they are very similar. The non-dimensional shape of the ideal Troposkien and the Sandia 17m turbine are taken from Paraschivoiu [14] and the non-dimensional shape of the Éole was determined by digitising a scale diagram given in Veilleux and Tinawi [7]. The 6th-order polynomial shown in Fig. 3.2 (determined using Matlab’s polyfit function) is given in Eq. 3.1 where $\bar{r}$ is the non-dimensional radius ($r/R$), $\bar{h}$ is the non-dimensional height ($h/H$), $A=38.4530$, $B=-115.3378$, $C=131.2284$, $D=-70.2522$, $E=13.4989$, $F=2.4098$ and $G=0.0056$.

$$\bar{r} = A\bar{h}^6 + B\bar{h}^5 + C\bar{h}^4 + D\bar{h}^3 + E\bar{h}^2 + F\bar{h} + G$$ (3.1)

3.3 Aerodynamic properties

3.3.1 Wind field

The onset wind field encountered by the baseline turbine is modelled using a logarithmic velocity profile described in section 4.4 and given in Eq. 4.26. For the purposes of the work described by this thesis, the baseline turbine is assumed to be in an area classed as Terrain Category 2 according to the Australia/New Zealand standard AS/NZ1170.2:2002 [137]. Terrain Category 2 refers to open terrain and grassland with few obstructions (fairly typical terrain for a wind farm location) and has a roughness length of $z_0=0.02$ m. Note that although the simulation described by this thesis uses a steady onset wind field, the only part of the modelling methods used that makes the assumption of a steady wind field is the tower wake model (described in section 4.5). All other aerodynamic and structural modelling requires no such assumption.

1As described in section 4.5, the tower model acts as if on a two-dimensional plane at any given height. As such, there is an abrupt change in the numerical model, that would not be realistic in practice, at the top of the tower where its influence stops suddenly. Simply raising the height of the tower slightly above that of the blades avoids this problem and ensures that the blades and the wake continue to be affected by the tower in a consistent manner.

2Some typographic errors in the ideal Troposkien shape in the original source were corrected before presentation here.
transient onset wind field could be implemented in the future with minimal changes required to the existing simulation by researchers with access to a suitable transient wind field model.

### 3.3.2 Tip speed ratio

The relationship between the power coefficient ($C_P$) and the tip speed ratio (TSR) for the baseline turbine with a perfectly rigid structure (i.e. aerodynamic modelling only) is illustrated in Fig. 3.3(a). The maximum $C_P$ occurs at a TSR of about 5, which for the baseline turbine corresponds to a wind speed (at 10m reference height) of about 6.8 m/s corresponding to a wind speed at the equator of about 8.8 m/s. This does not, however, correspond with the maximum loads experienced by the wind turbine. As the wind speed increases (TSR decreases for a given rotational rate), the power available in the wind increases and although the extraction of the energy becomes less efficient, it continues to become more effective as seen in Fig. 3.3(b).

For the baseline turbine, a TSR of 3.0 was selected, which corresponds to a wind speed (at 10m reference height) of about 11.4 m/s and a wind speed at the equator of about 14.7 m/s. This velocity is likely to be above average for most sites, but not so infrequent that the behaviour of the turbine in these conditions may be disregarded. At a TSR of 3.0, the blade loads are reasonably high and some dynamic stall is experienced, which was anticipated to be an important factor in determining the aeroelastic behaviour. Like the choice of the rotational rate, this baseline value allows some scope to adjust the TSR up and down during investigations, while remaining within the scope of achievable and practical values. At very low TSRs (i.e. very high wind speeds for a given rotational rate) a real turbine will, in all likelihood given current technology, need to be shut down to protect the structure from the extreme loads. Although Fig. 3.3(c) shows velocities over 30 m/s at a TSR of 1.0, it is very unlikely a real turbine would be allowed to operate in such conditions. Such high wind velocities are rarely experienced so designing the structural properties of the turbine to withstand operation in such conditions is unlikely to be cost effective, although it must still withstand high wind speeds in the shut down mode. As the TSR increases (wind speeds decrease for a given rotational
rate), the loads on the blades decrease so although the turbine may keep operating (provided it is generating power), from an aeroelastic perspective it is not particularly interesting. It is in the low TSR/high blade load region that an understanding of the aeroelastic behaviour was anticipated to be most useful. A comparison of the relative effect of TSR on the aeroelastic behaviour of the blades is presented in section 8.4.1.

3.3.3 Aerodynamic coefficients

The Éole had a NACA 0018 aerofoil section along the full length of the blades. The aerodynamic properties of the same aerofoil section have also been used as a basis for the baseline turbine. At the very high Reynolds numbers involved for such a large scale turbine there are very limited examples of experimental data available for the NACA 0018 aerofoil up to high angles of attack, so a combination of sources (and some subjective interpretation) were used to estimate the aerodynamic coefficient curves for the baseline turbine. The procedure consisted of starting with an estimate of the average Reynolds number experienced by the blades of the baseline turbine. With a maximum radius of 32m, the average radius of the baseline turbine’s blades is 22.18m and the rate of rotation is, as specified above, 13 rpm (1.3614 rad/s). The chord length is 2.4m and assuming a kinematic viscosity of the air equal to that defined by the International Standard Atmosphere [138] of $1.46 \times 10^{-5}$ m²/s, the average Reynolds number is estimated according to Eq. 3.2.

$$Re_{avg} = \frac{|V_{rel}|_{avg} c}{\nu_{\infty}} = \frac{R_{avg} \Omega c}{\nu_{\infty}} = \frac{(22.18)(1.3614)(2.4)}{1.46 \times 10^{-5}} \approx 5.0 \times 10^6$$  (3.2)

The aerodynamic coefficient data of the NACA 0018 aerofoil up to angles of attack slightly above stall ($-25^\circ \leq \alpha \leq 25^\circ$) were taken from Sheldahl and Klimas [9], who estimated the aerodynamic coefficients for the NACA 0018 over a range of Reynolds numbers up to $5 \times 10^6$ through a combination of experimental measurements (on related aerofoils such as the NACA 0012 and NACA 0015) and numerically synthesised data. At high angles of attack Sheldahl and Klimas made the assumption that all aerofoils perform identically, regardless of shape or Reynolds number. While this approach appears suitable for some of the other aerofoils they investigated, particularly at lower Reynolds numbers, the shape of the lift coefficient curve estimated by Sheldahl and Klimas for the NACA 0018 at high Reynolds numbers does not seem plausible. The coefficients at low to moderate angles of attack do not transition smoothly to the coefficients estimated at high angles of attack. As such, an alternative approach was adopted by the author to estimate the aerodynamic properties of the baseline turbine at the higher angles of attack. The coefficient data from Sheldahl and Klimas up to just beyond stall ($-25^\circ \leq \alpha \leq 25^\circ$) was inserted into the AirfoilPrep preprocessor spreadsheet developed at the National Wind Technology Center [139]. The AirfoilPrep spreadsheet includes functionality to extend aerodynamic coefficient data from a limited range of angles of attack to a full $\pm 180^\circ$. 

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For the aerodynamic coefficients of the baseline turbine, the option to use the Viterna method for the drag coefficient was selected instead of the flat plate theory for drag. A maximum drag coefficient of 1.8 was used, matching the Sheldahl and Klimas data. Although very useful, the AirfoilPrep spreadsheet appears to be designed for cambered aerofoils rather than the symmetric ones of interest here, so to avoid any issues, only the data produced by the spreadsheet over positive angles of attack was used. These coefficients were extended separately to negative angles of attack using the symmetrical relationship of the data (i.e. identical values for the drag coefficients at negative angles of attack and reversing the signs of the data for the lift and moment coefficients).

It is important to note that for the case of the moment coefficient, even less data are available than for the lift and drag coefficients of the NACA 0018 (and most other aerofoils too) at high angles of attack, at high Reynolds numbers. In Sheldahl and Klimas [9], moment coefficient data up to a Reynolds number of only 680000 are provided, and those which are provided show considerable variation. Consequently, only a very crude (and highly smoothed) approximation of the moment coefficient could be made, and was assumed to be independent of Reynolds number.

The lift, drag and moment coefficient curves were then approximated using a series of curved and straight line segments (described in appendix A). The data were smoothed using this approach to mitigate the sharp changes that can sometimes be present in experimentally measured aerodynamic coefficient data. While in principle it is possible that some of the sharp changes observed in the aerodynamic coefficient data could be real, it is far more likely that the sharp changes are as a result of experimental error, so should be omitted where possible. Although discussed in more detail in appendix A, it may be pointed out here that interpolation between different Reynolds numbers was, in fact, done using the variables defining the approximate parametric curves. Direct interpolation of lift coefficient data yielded satisfactory results, but direct interpolation of drag coefficient data was less appropriate. Interpolation using the parametric variables produced much more consistent results.

Finally, the aerodynamic coefficient data were passed through a procedure to convert lift and drag coefficients into the normal and tangential coefficients required for the simulation method used, and a number of parameters related to the modelling of dynamic stall (such as effective separation points) were extracted from the data. Details on this procedure are also described in appendix A.

The smoothed lift, drag and moment coefficient data for the NACA 0018 at a Reynolds number of $5.0 \times 10^6$ are presented in Fig. 3.4. It should also be noted that the general procedure described in this section (and in appendix A) to obtain the aerodynamic coefficient data was repeated for every type of aerofoil at every average Reynolds number for which results are
Figure 3.4: Lift, drag and moment coefficient data used for baseline turbine configuration

presented in this thesis unless otherwise noted. The only exception to this is the validation of the 2D aerodynamic model against dynamically pitching aerofoils presented in section 4.10.2, for which the actual aerodynamic coefficient data measured experimentally were used directly without smoothing. The decision was made to use these data directly, without smoothing, because the static data were measured in the same wind tunnel, with the same models by the same group that made the dynamic measurements, so had a higher level of reliability than in the general case.

3.4 Structural properties

The structural properties of the blades are the main difference between the original Éole and the baseline turbine described herein. The blades of the Éole were fabricated from steel and approached 50 tonnes each. Rather than use blades such as those of the Éole for the baseline turbine, it was felt more appropriate to assume that a modern VAWT of the scale investigated here would exploit the advances made in blade design for use on large scale HAWTs. The properties of the blades of the baseline turbine were based on those of the LMH64-5 blade, which was designed by LM-Glasfiber Holland for a 6MW HAWT. The properties of the LMH64-5 blade were taken from Lindenburg [140].

The properties were estimated by interpolation of the data at the radius at which the chord length of the LMH64-5 blade was equal to the desired chord length of the baseline turbine’s blades (i.e. 2.4 m). Although the aerofoil cross section of the LMH64-5 blade is cambered, rather than being symmetrical like the typical blades of VAWTs, conveniently the thickness of the aerofoil section at a spanwise location where the chord length was 2.4m, matched the
Chapter 3: Baseline turbine configuration

Figure 3.5: Terminology used to define the sectional properties and the terminology used to define the locations of the sectional properties.

thickness (18% thickness) of the NACA 0018 used on the Éole, so the structural properties are similar. Differences in the inertial and flexural properties due to the camber were assumed to be sufficiently negligible that they could be ignored. Differences in the locations of sectional properties such as centre of mass, due to the difference in aerofoil shape, were assumed to be negligible in the horizontal direction, but in the vertical direction were adjusted to lie on the chordline to reflect the symmetrical nature of the NACA 0018.

It is important to note that several of the structural properties of the blades were defined, and are implemented in the simulation described in this thesis, but were not necessarily required for the case studies presented in chapter 8. The modelling methods described in this thesis are capable of modelling significantly more complex turbines and scenarios than required by the limited set of case studies presented herein. For completeness, however, all structural properties that may be specified in the simulation, whether used in the case studies or not, were defined.

3.4.1 Sectional locations

The sectional locations that may be specified and the terms used to define them are illustrated in Fig. 3.5. The horizontal locations of each property (subscript $h$) are defined along the chordline, relative to the leading edge of the aerofoil. The vertical locations of each property (subscript $v$) are defined perpendicular to the chordline, with positive values towards the outside of the arc of rotation of the turbine. The baseline values for each parameter are given in table 3.1, with all values given as a proportion of the chord length.

The elastic centre is the location of the neutral axis i.e. the line through the blade through which no change in length occurs during bending. The shear centre is the line through the blade through which an applied load will not result in twisting. From a practical point-of-view, as is discussed in more detail in section 5.3, the joints used to model bending motions are attached at the elastic centre, and the joints used to model torsional motion are attached at the shear centre.
Table 3.1: Locations of sectional properties of the blades of the baseline turbine.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Baseline Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre of Mass (Horizontal Location)</td>
<td>(cm_h)</td>
<td>0.47</td>
</tr>
<tr>
<td>Centre of Mass (Vertical Location)</td>
<td>(cm_v)</td>
<td>0</td>
</tr>
<tr>
<td>Elastic Centre (Horizontal Location)</td>
<td>(ec_h)</td>
<td>0.42</td>
</tr>
<tr>
<td>Elastic Centre (Vertical Location)</td>
<td>(ec_v)</td>
<td>0</td>
</tr>
<tr>
<td>Shear Centre (Horizontal Location)</td>
<td>(sc_h)</td>
<td>0.33</td>
</tr>
<tr>
<td>Shear Centre (Vertical Location)</td>
<td>(sc_v)</td>
<td>0</td>
</tr>
<tr>
<td>Mounting Point (Horizontal Location)</td>
<td>(mp_h)</td>
<td>0.50</td>
</tr>
<tr>
<td>Aerodynamic Centre (Horizontal Location)</td>
<td>(ac_h)</td>
<td>0.25</td>
</tr>
<tr>
<td>Geometric Angle of Attack</td>
<td>(\alpha_g)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.2: Physical properties of the blades of the baseline turbine.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Baseline Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord length [m]</td>
<td>(c)</td>
<td>2.4</td>
</tr>
<tr>
<td>Linear density [kg/m]</td>
<td>(\sigma)</td>
<td>95</td>
</tr>
<tr>
<td>Flatwise mass moment of inertia per unit length [kg.m]</td>
<td>(I_{flat})</td>
<td>2.5</td>
</tr>
<tr>
<td>Edgewise mass moment of inertia per unit length [kg.m]</td>
<td>(I_{edge})</td>
<td>37</td>
</tr>
<tr>
<td>Flatwise flexural rigidity [N.m²]</td>
<td>(EI_{flat})</td>
<td>4.4E+07</td>
</tr>
<tr>
<td>Edgewise flexural rigidity [N.m²]</td>
<td>(EI_{edge})</td>
<td>4.3E+08</td>
</tr>
<tr>
<td>Torsional rigidity [N.m²]</td>
<td>(GJ)</td>
<td>7.8E+06</td>
</tr>
</tbody>
</table>

Note that vertical distances are not defined for either the mounting point or the aerodynamic centre. Both values are assumed to always reside on the chordline, and in the case of the aerodynamic centre, it was fixed at the quarter-chord point for all cases presented in this thesis. For a symmetrical aerofoil, the centre of mass, elastic centre and shear centre will, in reality, also always reside on the chordline, but these were implemented as variable parameters to allow investigations into their influences on the behaviour of the turbine in future work.

3.4.2 Sectional physical properties

The physical properties of the blades of the baseline turbine are given in table 3.2. The physical properties were, as mentioned previously, estimated by interpolation of the properties of the LMH64-5 blade. As the purpose of this exercise was to simply obtain plausible physical properties for the baseline measurements, rather than to have accurate properties of an actual existing or proposed VAWT design, any significant figures beyond the first two have no real relevance so have been omitted. Nonetheless, it is believed that the values given in tables 3.1 and 3.2 are realistic and reasonable for a 2010 VAWT.
Chapter 4

Aerodynamic modelling

*Big whirls have little whirls, which feed on their velocity.*
*Little whirls have smaller whirls, and so on to viscosity.*

L F Richardson

4.1 Introduction

This chapter describes the approach used to model the aerodynamics of VAWTs. The chapter is introduced with an overview of the modelling methods used and a summary of the notation used throughout the rest of this chapter. Following this is a description of vortex wake modelling method itself and specific details about the processes performed during each simulated aerodynamic time-step. Finally, several sets of results are presented validating the aerodynamic simulation against experimental measurements available from the literature.

In the process of implementing the aerodynamic simulation, two aerodynamic models were actually developed which are henceforth referred to as Aeolus2D and Aeolus3D. The two-dimensional simulation Aeolus2D was developed first allowing much of the vortex wake method’s implementation to be verified (and validated against several sets of experimental data) without the additional theoretical and computational complexity of a three-dimensional environment. By developing the two-dimensional simulation first, many of the challenges in implementation could be tested and corrected more quickly than would have been possible by implementing the three-dimensional simulation in isolation. Although the emphasis of this chapter is on the three-dimensional Aeolus3D, where significant differences between the two and three-dimensional models exist, they are identified.
4.2 Overview of the aerodynamic simulation

4.2.1 Aerodynamic components

The aerodynamic model of the VAWT can, conceptually, be broken down into a number of sub-components. The free-stream wind-field is independent of the VAWT contained within it and was modelled as such, the details of which are described in section 4.4. The VAWT itself is comprised of two main parts, the blades and the tower. The blades are lifting bodies represented using bound vortex filaments running along the blade sections at the quarter-chord location. Each blade is discretised into a number of aerodynamic sections, each of which is modelled using a single bound vortex filament using a method described in section 4.3. Although the strength of individual vortices within the wake are determined at their time of release from the blades of the VAWT, once released the wake can be thought of as a separate entity acting both on the VAWT and on itself. The development of the structure of the wake over time is described in section 4.6. Finally, the tower is a non-lifting body modelled using a combination of potential flow theory and a velocity deficit model, described in section 4.5.

4.2.2 Coordinate systems and terminology

Within the VAWT literature, there does not exist a standard, single, uniform approach to describing the aerodynamic characteristics or behaviour of VAWTs. It is very important therefore to define the specific coordinate systems and terminology used throughout the rest of this chapter to avoid any ambiguity.
Azimuth and global coordinate system

Figure 4.1 shows a top down view of a two-Bladed VAWT rotating at an angular velocity of $\Omega$, to illustrate the azimuthal position of a blade and the orientation of the global coordinate system in relation to the direction of the (average) oncoming wind field. For all cases presented in this thesis the magnitude of the wind field was allowed to vary with height, but the direction of the flow was uniform throughout the wind field.

An azimuthal position ($\theta$) of $\theta = 0^\circ$ was defined as the location where the motion of the blade is parallel to, and in the opposite direction to, the average wind direction. The azimuth increases throughout a revolution with the motion of the blade such that at $\theta = 90^\circ$ the blade is directly upstream of the tower and so on.

The axes of the global coordinate system are always attached to the centre of the tower and ground level. The global x-axis ($x_0$) is aligned parallel to average wind direction, pointing directly downstream towards an azimuthal position of $\theta = 270^\circ$. The global y-axis ($y_0$) is perpendicular to the average wind direction pointing towards an azimuthal position of $\theta = 0^\circ$ and the global z-axis ($z_0$) points vertically upwards.

Flow velocity over blade section

The relative velocity ($V_{\text{rel}}$) at any point at any time in the fluid domain is determined from the Helmholtz decomposition theory which specifies that a flow field can be described by the superposition of a rotational flow field and an irrotational flow field. In this particular case, the rotational flow field is that due to the induced velocity from the bound ($V_{\text{bound}}$) and wake vortices ($V_{\text{wake}}$) (i.e. the lifting surfaces and the wake from the lifting surfaces) and the irrotational flow field is the free-stream wind field including the influence of non-lifting surfaces such as the tower ($V_{\text{wind,tower}}$) and, in the case of the velocity over a blade section, the velocity due to the motion of the blade section itself ($V_{\text{foil}}$). The total relative flow velocity over a blade
Chapter 4: Aerodynamic modelling

section is therefore given by Eq. 4.1.

\[
V_{\text{rel}} = V_{\text{bound}} + V_{\text{wake}} + V_{\text{wind,tower}} + V_{\text{foil}}
\]  

(4.1)

For the purposes of determining the loads experienced by the blade section (and the circulation of the bound vortex), only the components of the velocity in the plane of aerofoil’s cross-section are of interest. The spanwise flow is assumed not to have a significant influence on the blade loads (which are based on two-dimensional measurements). In practice, high levels of spanwise flow (caused either by a pressure gradient along the span or by sweep angle) may change the dynamic stalling behaviour of the aerofoils. Du and Selig [141] investigated the effect of rotation on the boundary layer of a HAWT blade, concluding that stall may be delayed due to the radial flow inside the boundary layer. Leishman [2] discusses this, along with several other three-dimensional effects in relation to HAWTs, indicating also that such effects may act to delay the onset of dynamic stall. Leishman also highlights, however, the many challenges and uncertainties in modelling such effects and illustrates the concept of “Ockham’s Hill”, that increasing model complexity does not always lead to increasing model accuracy. Significant spanwise flow is not expected on a VAWT, except perhaps in close proximity to the tower where the blade loads are quite small anyway. As such, modifications to the dynamic stall characteristics to account for spanwise flow is considered outside the scope of this work, although it may represent an interesting area for future investigation.

The sectional flow velocity \((V_{\text{sec}})\) over the blade section is determined, therefore, by projection of the relative flow velocity onto the plane of the blade cross-section as illustrated in Fig. 4.3. There are a variety of methods available for projecting a three-dimensional vector onto a plane. The method used to determine the flow velocity over a section in Aeolus2D and Aeolus3D is not actually the most efficient if only the velocity is required, but as the magnitudes of the components of velocity along the chordline and normal to the blade are required to calculate the angle of attack anyway (in Eq. 4.3 below), the approach described here works well. The flow
velocity over the blade section is simply the sum of the component of relative velocity along the chordline (i.e. along the blade section’s local z-axis ($\hat{z}$)) and the component of relative velocity normal to the blade (i.e. along the blade section’s local y-axis ($\hat{y}$)) as given by Eq. 4.2.

$$V_{sec} = (V_{rel} \cdot \hat{z})\hat{z} + (V_{rel} \cdot \hat{y})\hat{y} \quad (4.2)$$

Angle of attack

One of the most important parameters required in the aerodynamic simulation is the effective angle of attack ($\alpha_E$) at each blade section, which is defined as the angle between the velocity vector of the flow over the section ($V_{sec}$, defined below in the section 4.2.2) and the chordline. Complicating this, however, is the fact that the direction of the velocity vector is not, in fact, constant along the aerofoil’s chordline as illustrated in Fig. 4.4. Note that the influence of the wake has been omitted from Fig. 4.4 simply to avoid the illustration becoming too cluttered, though it too will vary along the chordline of the blade. In the case of a VAWT, the variation in velocity along the chordline is due primarily to the blade rotation about the tower and is sometimes referred to as flow curvature. It should be noted, however, that any moving aerofoil undergoing pitching motion relative to a flow field will have such a variation in flow velocity along its chordline.

Migliore et al. [142] developed a method that maps the aerofoil in a curved flow field (due to the rotation of the VAWT) to an equivalent “virtual” aerofoil in a rectilinear flow field. Essentially, the method modifies the lift coefficient curve in such a way as to approximate the effect that the introduction of camber would have on the curve. Migliore also demonstrated that the strength of the flow curvature effect is related to the chord to radius ratio of the VAWT (i.e. the solidity). The greater the ratio, the greater the effect. Cardona [143] adopted Migliore’s method and applied it, along with a dynamic stall model, to Strickland’s [96, 97] vortex wake model. Cardona demonstrated some potential improvements to the predicted tangential loads.
using this approach via a comparison against experimental data, but the results were certainly not decisive. In some cases the predictions were actually worse. Unfortunately, Cardona did not compare results that included the dynamic stall model only, so it is not possible to conclusively attribute the improvements to the flow curvature model. A similar approach was also developed by Mandal and Burton [104] for use in their cascade model by adjusting the lift coefficient curve to approximate the effect of flow curvature.

An alternative approach, rather than adjusting the lift coefficient curve directly, is to select a location along the chordline as representative of the flow over the whole section. With a changing velocity along the chordline of the aerofoil, however, the question then becomes: Where should such a single value for velocity and angle of attack be calculated?

Bramwell et al. [144] demonstrates how a component of the lift due to the bound circulation is proportional to the downwash at the three-quarter-chord point, so recommended that this point be used to determine the velocity (and hence angle of attack) over a section. This approach matches earlier work of Fung [145] demonstrating the apparent proportionality between lift and velocity at the three-quarter-chord point. In both Fung and Bramwell et al., however, the assumption is made that the onset flow is constant in magnitude and direction, the flow over the aerofoil remains fully attached, the motion of the aerofoil is harmonic and the rotation is about mid-chord, so the applicability of this to arbitrary motion in a continuously changing flow is less conclusive.

Strickland [96, 97] adopted an interesting approach, in which (based on work by Milne-Thomson [146]) the velocity over the blade section (and hence the angle of attack) was determined at mid-chord for the tangential loads and at three-quarter-chord for the normal loads. Strickland did not, however, actually calculate the velocity directly at both locations, instead the velocity due to the wake at the attachment point was calculated and the velocity at the other points approximated based on this. In effect, this means that the influence of the motion of the blade on the angle of attack at different points along the chordline was approximated, but differences in the velocity induced by the wake along the chordline were not.

It should also be noted that this issue concerning an appropriate location at which to evaluate the velocity over the section and the angle of attack is not limited to VAWTs. Garrel [90], for example, describes a vortex wake method for use with HAWTs in which the velocity over the each section is determined at the quarter-chord point. Bjorck [147], however, describes an implementation of the Beddoes-Leishman dynamic stall model for use with HAWTs, using the velocity at the three-quarter-chord location for its calculations. The PHATAS program developed at ECN for modelling HAWTs takes yet a different approach in which, according to Lindenburg [148], users can select whether the angle of attack is determined at the quarter-chord or the three-quarter-chord location. If the angle of attack at the three-quarter-chord location
is selected, however, the velocity at the quarter-chord location is still used to determine the aerodynamic loads.

Without a conclusive argument one way or the other about where the velocity and angle of attack should be determined, a conservative middle of the road approach has been adopted in Aeolus2D and Aeolus3D with the velocity over the section and the angle of attack determined at mid-chord. At mid-chord, the velocity and angle of attack will typically fall somewhere between its quarter-chord value (location of the bound vortex) and its three-quarter-chord value (analytically shown to be more accurate under some specific and tightly controlled conditions).

With the location specified at which the velocity over the section and the angle of attack are to be determined, the angle of attack is easily calculated by breaking down the relative flow velocity over the section into the magnitude of its component normal to the blade (i.e. parallel to the blades local y-axis) and its component parallel to the chordline (i.e. parallel to the blades local z-axis) as shown by Eq. 4.3.

\[
\alpha_E = \tan^{-1} \left( \frac{V_{rel} \cdot \hat{y}}{V_{rel} \cdot \hat{z}} \right)
\]  

(4.3)

**Direction of components of force**

The aerofoil coefficient curve data used to represent the aerodynamic characteristics of each blade section are traditionally specified as the lift \((C_L)\), drag \((C_D)\) and moment \((C_M)\) coefficients about the quarter-chord point of the aerofoil. For modelling a VAWT, however, the normal and tangential loads are more convenient on two counts. Firstly, the normal and tangential directions are conceptually more meaningful than the continuously changing lift and drag directions. With the normal and tangential directions referenced with respect to the orientation of the blades rather than the direction of the flow over the blades, like the lift and drag are, interpreting the loading is quite straightforward. Additionally, the Beddoes-Leishman (BL)
model used to predict unsteady effects on the aerodynamic coefficients during stall (described below in section 4.8) is formulated in terms of the normal and tangential load coefficients, rather than the lift and drag coefficients, so if for no other reason it is convenient to work exclusively with the normal and tangential components wherever possible, as the loads would need to be converted to this form for use by the BL model anyway.

The directions of the various components of force on a blade section are illustrated in Fig. 4.5. Note the size of the blade section relative to the arc of rotation has been exaggerated for visual clarity. Such a ratio between the two would really only exist very close to the tower, if at all. The direction of drag ($\hat{D}$) is defined as being parallel to the direction of the flow over the section, positive in the same direction as that of the flow, and the direction of lift ($\hat{L}$) is perpendicular to the direction of the flow over the section. The tangential direction ($\hat{T}$) is defined as parallel to the chordline of the aerofoil, positive towards the leading-edge of the aerofoil (i.e. this could also be referred to as the direction of the thrust force) and the normal direction ($\hat{N}$) is defined as perpendicular to the chordline of the aerofoil (and perpendicular to the spanline of the blade), positive towards the inside of the arc of rotation of the VAWT. The moment ($\hat{M}$) acts around the spanline of the turbine blade, positive defined from bottom to top of the blade (out of the page in Fig. 4.5) according to the right hand rule.

Given these definitions for the directions of the force components, the lift and drag force coefficients are converted to normal and tangential force coefficients according to Eqs. 4.4 and 4.5.

$$C_N = C_L \cos(\alpha_E) + C_D \sin(\alpha_E) \quad (4.4)$$

$$C_T = C_L \sin(\alpha_E) - C_D \cos(\alpha_E) \quad (4.5)$$

And, given the normal and tangential coefficients, the corresponding conversion back into the lift and drag coefficients is given by Eqs. 4.6 and 4.7.

$$C_L = C_N \cos(\alpha_E) + C_T \sin(\alpha_E) \quad (4.6)$$

$$C_D = C_N \sin(\alpha_E) - C_T \cos(\alpha_E) \quad (4.7)$$

### 4.2.3 Actions taken during a single time-step

Figure 4.6 presents a flow chart outlining (at a high level) the actions taken during a single time-step of the aerodynamic simulation. A brief summary of the actions is given here together with references to the locations of more detailed descriptions within the thesis.

**Pull structural state from aero-structural interface:** This step is explained in chap-
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Figure 4.6: Flowchart of Aerodynamic Simulation Step

ter 6 along with a detailed explanation of the interface allowing communication between the aerodynamic and structural simulations. In essence, however, the aerodynamic simulation pulls necessary information about the current structural state via the aero-structural interface. Specifically, the aerodynamic simulation requires the current location and velocities of all aerodynamic elements. The interface itself performs the mapping required between the structural simulation and the aerodynamic simulation (which do not necessarily have the same level of discretisation). In the case of perfectly rigid blades (such as for an investigation of aerodynamic behaviour only) the structural state is still pulled from the aero-structural interface. However in this case the interface calculates the location and velocities of all aerodynamic elements based at the given time on a prescribed motion rather than from the structural simulation. From the point of view of the aerodynamic simulation there is no difference between rigid and flexible blades.

**Update wake state:** This step involves updating the wake from its state at the previous time-step to its current state, a process described in detail in section 4.6. The key part of this step involves convecting all vortices in the wake from their previous locations to their new locations. This is the most computationally expensive part of the aerodynamic simulation.

**Update circulation of bound and near wake blade vortices:** During this step the current strength of the bound vortices (those attached to the blades) and the near wake vortices (those newly released from the blades) are determined. The circulation of the bound and near wake vortices are functions not only of the motion of the blades, the wind field and the wake state, but also of each other. Due to their interdependence on each other, the determination of the
circulations of the bound and near wake vortices is performed using an iterative approach. Although much less computationally expensive than updating the wake state, this step is the most complex from a purely technical point of view, involving a number of sub-steps described in detail in section 4.7.

**Update loads and push to aero-structural interface:** This fairly simple step involves calculating the actual aerodynamic loads (based on the load coefficients) and pushing the results to the Aero-Structural interface for use by the structural simulation. The details of this step are explained in chapter 6.

**Output aerodynamic data to files:** All required aerodynamic data are written to file. This includes not only load data required for analysis, but also includes sufficient information to fully describe the aerodynamic state of the system allowing further simulations to be performed with alternative parameters, or for graphics to be generated for visualisation purposes.

**Check for convergence:** If the current time-step represents the completion of a full revolution of the VAWT, a check is made to see whether the aerodynamic simulation has reached convergence. This check is described in section 4.9.

### 4.3 Vortex wake modelling

The approach used herein to capture the aerodynamic behaviour of the VAWT is a free vortex wake method. As discussed in section 2.4, the free vortex wake methods offer a good balance between the simple quasi-steady blade element momentum methods frequently used for wind turbine analysis, and the more computationally expensive Navier-Stokes based full CFD methods.

This section describes the vortex wake method used to model the blades and the wake. Vortex wake methods themselves have been a long established approach to modelling the aerodynamics of lifting bodies and the term covers a wide range of approaches. The particular subtype of vortex wake method implemented in Aeolus3D is a vortex filament method\(^1\), which can also be called a vortex line or vortex lattice method. The approach exploits the assumption that the fluid flow field is incompressible everywhere\(^2\), and inviscid\(^3\) everywhere except in a thin layer of fluid in the wake of the lifting bodies. The effect of this viscous "sheet" on the fluid

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\(^1\)Technically, the vortex wake method in Aeolus2D is also a vortex filament method except that the length of each vortex filament is infinite.

\(^2\)For the baseline turbine, the blade velocity at the equator (where the velocity is highest) due to rotation only is approximately 43.6 m/s. So, even at the very low TSR of 2.0, for example, the flow velocity over the blade at the equator has a maximum of approximately 65.3 m/s (when the blade is moving directly into the wind). Velocities below about 100 m/s (i.e. approximately Mach 0.3 at room temperature) can generally be considered incompressible.

\(^3\)The local Reynolds number is high (i.e. inertial forces much greater than viscous forces) except in the boundary layer and subsequent wake of the blades, so viscous effects can be ignored everywhere else.
flow field is approximated by an arrangement of discrete vortex filaments, the strengths of which are determined from the spatial and temporal loading variation of the lifting bodies from which the wake emanated. Many texts provide a detailed explanation of vortex wake methods so only an overview of the particular approach implemented in Aeolus2D and Aeolus3D is provided here. Katz and Plotkin [149] in particular proved to be an excellent resource during the implementation of the vortex wake method. Additionally, Leishmann [150], Houghton & Carpenter [151, 152] and Moran [153] give additional insight into the method.

The basic concept behind the vortex filament method is that the lifting bodies (i.e. the wind turbine blades) and the wake from these lifting bodies can be represented using a collection of discrete vortex filaments (which will henceforth be called simply vortices). Conceptually there are several types of vortices involved in the aerodynamic simulation, bound vortices, shed vortices and trailing vortices. Shed and trailing vortices are further divided conceptually into near wake and far wake vortices. It is important to note, however, that these distinctions are purely conceptual, and are based on how the circulation and motion of each vortex is determined. From a practical perspective, the behaviour of each vortex and in particular the influence of the vortex on the fluid flow field is treated almost identically.

Figure 4.7 shows a region of a three-dimensional vortex wake emanating from section $i$ of a blade at time-step $j$. The direction of the circulations ($\Gamma$) of the vortices has been shown arbitrarily for illustrative purposes. In reality, the circulation can act in either direction about a vortex depending on the loading conditions on the blade at the time of vortex creation and the strength of previously released vortices. An explanation of how the circulation of the bound vortices ($\Gamma^B$), the circulation of the shed vortices ($\Gamma^S$) and the circulation of the trailing vortices ($\Gamma^T$)
is calculated is provided in section 4.3.1. The near-wake vortices consist of the shed vortices newly released into the wake aligned parallel to the trailing edge of the blade, and the trailing vortices aligned along the chord of the blade connecting the ends of the bound vortices to the ends of the near-wake shed vortices. These vortices in the near-wake are distinguished from the far-wake vortices (which encompasses all other wake vortices, both shed and trailing) because, although their influence on the flow field is calculated the same way as the vortices in the far wake, their motion and circulation is calculated differently from the vortices of the far-wake.

In Fig. 4.7 the term “vortex markers” is used to indicate the end points of a vortex. This terminology is presented here because, as explained in section 4.6, to determine the motion of the far-wake vortices the velocity at the ends of each vortex is calculated. As the end of each vortex can be connected to up to three other vortices, from the point of view of implementation it is the velocity of the vortex markers that is calculated at each time-step, with the relationships between connected vortices tracked and maintained over time as the wake develops.

Shed vortices are released parallel to, and a distance behind, the trailing edge of the blade at each time-step during which the circulation of the associated bound vortex has changed. According to Katz and Plotkin [149] a shed vortex should generally be positioned between 20 and 30% of the distance from the current location of the trailing edge to the location of the trailing edge at the previous time-step. In reality, the wake behind an aerofoil forms continuously at the trailing edge. In a discretised model such as this, however, the vortex is assumed to have been released at some time between the previous time-step and the current time-step and must be placed at a specific location. Rather than placing the shed vortex at the mid-point between the current location of the trailing edge and the previous location of the trailing edge, according to Katz and Plotkin, this will underestimate the induced velocity compared to the real a continuous wake. Placing the shed vortex closer to the current location of the trailing edge will aid in correcting this error. For the purposes of this research, the new shed vortices were placed at a distance 25% of the way from the current location of the trailing edge to the location of the trailing edge at the previous time-step.

### 4.3.1 Vortex circulation

**Bound vortices**

Bound vortices are attached to the quarter-chord of the blade and move with the blade (hence the name “Bound” vortex). The strength of each bound vortex is governed by the Kutta-Joukowsky theorem, shown in Eq. 4.8 expressed in terms of the two-dimensional sectional lift coefficient ($C_L$), the chord length ($c$) and the velocity of the flow over the aerofoil section in a plane parallel to the chord and perpendicular to the span of the blade ($V_{sec}$). Developed independently by Kutta in 1902 and Joukowsky in 1906, this equation provides a basis for
calculating the circulation of a bound vortex ($\Gamma^B$). The circulation of the bound vortices are then used to determine the circulation of the shed and trailing wake vortices ($\Gamma^S, \Gamma^T$) in the near-wake.

$$\Gamma^B = \frac{1}{2} C_L |V_{sec}| c$$

(4.8)

The Kutta-Joukowsky equation is reformatted into normal ($C_N$) and tangential ($C_T$) load coefficients in Eq. 4.9, which matches the reference system used in this simulation.

$$\Gamma^B = \frac{1}{2} (C_N \cos (\alpha_E) + C_T \sin (\alpha_E)) |V_{sec}| c$$

(4.9)

**Shed vortices**

The relationship between the circulation of the bound vortex of section $i$ at time-step $j$ and the circulation of the near-wake shed vortex from that section is given by the Kelvin condition (Eq. 4.10) which requires that the circulation around any closed fluid system ($\Gamma^C$) remains constant over time.

$$\frac{d \Gamma^C_{i,j}}{dt} = \frac{d \Gamma^B_{i,j}}{dt} + \frac{d \Gamma^S_{i,j}}{dt} = 0$$

(4.10)

The fluid system in this case encompasses the blade section (i.e. its bound vortex) and the wake emanating from the section. As all vortices in the far-wake have constant circulation which was determined at their time of creation, only the circulation of the latest near-wake vortex released, needs to appear in this equation. Note also that the circulation of the trailing vortices do not appear directly in this equation. In the three-dimensional case (there are no trailing vortices in a two-dimensional model), they do appear indirectly, however, in determining the circulation of the bound vortices through their influence on the sectional flow velocity.

The Kelvin condition can be reformatted into a discrete form (Eqs. 4.11 - 4.13), where $\Delta t$ is the time-step size, and provides the method for calculating the circulation of a newly created wake vortex based on the current strength of the bound vortex and the history of the previous wake vortices released into the wake.

$$\frac{D \Gamma^C_{i,j}}{D \Delta t} = \frac{\Gamma^B_{i,j} + \Gamma^S_{i,j}}{\Delta t} = 0$$

(4.11)

$$\Gamma^S_{i,j} = - \left[ \Gamma^B_{i,j} - \Gamma^B_{i,j-1} \right]$$

(4.12)

$$\Gamma^S_{i,j} = - \left[ \Gamma^B_{i,j} + \sum_{k=1}^{j-1} \Gamma^S_{i,k} \right]$$

(4.13)

As the shed vortices are a function of the change in bound circulation over time, they appear in
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the implementation of both Aeolus2D and Aeolus3D. Clearly, however, in the two-dimensional case of Aeolus2D their implementation is much simpler, being represented by points in two-dimensional space, rather than as a filament in three-dimensional space.

**Trailing vortices**

Helmholtz’s second theorem requires that a vortex must not end in the fluid. All vortices must either terminate at a boundary (such as a solid wall), extend to infinity or form a closed loop. Helmholtz also requires that the strength (circulation) of a vortex remains constant along its length. In order to satisfy Helmholtz’s theorems, Prandtl determined that a trailing vortex must emanate from the blade (or wing, in his case) at each point where the strength of the bound vortex changes to allow for the varying circulation along the span. In practice this process is continuous along the span, but in a discretised computer model this occurs at the intersection between each discrete bound vortex. The strength of a trailing vortex (in the near-wake) is therefore the difference between the strengths of the bound vortices from which it emanates. The strength \( \Gamma_{T_{i,j}} \), for example, of a trailing vortex emanating between sections \( i \) and \( i - 1 \) at time-step \( j \) is given by Eq. 4.14.

\[
\Gamma_{T_{i,j}} = \Gamma_{B_{i-1,j}} - \Gamma_{B_{i,j}}
\]  

(4.14)

Because the trailing vortices are a direct result of the variation of circulation along a span, they clearly will not occur in a two-dimensional model. Trailing vortices therefore appear in the implementation of Aeolus3D but not Aeolus2D.

At the ends of the blades, where they attach to the tower, it is unclear whether a trailing vortex should in fact be released or not. On the one hand, the final section of the blade has a bound vortex with a finite circulation and the adjoining tower effectively has zero circulation, so to adhere to Helmholtz’s and Prandtl’s theorems the difference in circulation should be released as a trailing vortex, i.e. a trailing vortex with a strength equal to the bound circulation of the final section of blade. On the other hand, Helmholtz’s theorems allow for a vortex to terminate at a solid boundary so the bound vortex could simply stop at the tower and no trailing vortex released. For the purposes of this work, a trailing vortex with a strength equal to the final segment of the bound vortex is released into the wake. Although in some applications, such as modelling an aircraft, this decision may have significant implications, in the case of a curved bladed VAWT it does not. As the radius decreases as the blade approaches the tower, the relative velocity over each blade section decreases significantly. Hence the blade loads decrease significantly, and thus the bound circulation decreases significantly. This means that the strength of the trailing vortex released at this point is quite low. In the case of an aircraft, the section of wing close to the fuselage is travelling at the same velocity as the wing tip, and often has a larger chord and greater loads so accurate modelling of the interface between the
Figure 4.8: Illustration of relative flow velocity for a local tip speed ratio below 1.0.

wing and the fuselage is much more important than modelling the interface between a VAWT blade and the tower.

**Flow off the leading edge**

One area where the vortex wake model described breaks down is when the flow is no longer travelling from the blade’s leading edge to its trailing edge. This occurs when the local TSR at a blade section is below 1.0. The typical conditions under which this takes place are as the blade is moving away from the oncoming wind (in the vicinity of an azimuthal position of 180°) at blade sections close to the tower. The azimuthal range over which this can occur increases as the TSR of the VAWT decreases.

In such conditions there will exist a pair of blade elements somewhere along the blade, as illustrated in Fig. 4.8, where the element closer to the equator (i.e. with a greater radius and hence greater local TSR) will have the typical leading edge to trailing edge flow direction, and the element closer to the tower will have a flow in the opposite direction. There is no clear way to handle such a situation in the standard vortex wake methods, which were developed primarily for modelling aircraft and in which such a situation was not anticipated (complete flow reversal over the wing of an aircraft would in all likelihood be catastrophic). In theory, the location of the bound vortex could be moved from the quarter-chord to the three-quarter-chord position and the shed vortices then released from the leading edge instead of the trailing edge, but it is unclear in such conditions what should happen to the wake as this switch is made, particularly to the trailing vortices.

To handle this situation, as the new location of each near-wake shed vortex is determined at each time-step (explained in more detail below in section 4.7) a check is made as to whether the local flow direction is from leading to trailing edge or vice versa. If the flow direction is from leading to trailing edge, the vortices are released as normal. If the flow direction is from trailing to leading edge, no vortex is released at this time-step and the determination of the
loads on this blade element is based on the static aerofoil coefficients. When the flow returns to the typical leading to trailing edge direction, the wake restarts from scratch.

This approach accounts (in a logical manner, at least) for the fact that the flow reversal represents a complete stop and then a restart, effectively requiring a new start-up wake vortex. Additionally, the conditions under which such flow reversal occurs are also conditions in which there are very low blade loads, so good accuracy is not critical. Not only is the angle of attack approaching zero in such conditions, but the flow velocity must pass through zero (hence zero aerodynamic loading) in order to change direction over the blade. Furthermore, as the main interest of this work is the aeroelastic behaviour of the blades, loads closer to the equator (where this phenomenon is unlikely to be experienced) are expected to have a much larger influence on the aeroelastic behaviour than loads closer to the tower where the blades are physically attached.

### 4.3.2 Velocity induced by a vortex

**Biot-Savart Law**

The velocity induced by a vortex is determined by the Biot-Savart law, which for a general three-dimensional vortex filament with constant circulation $\Gamma$, such as illustrated in Fig. 4.9, states that the induced velocity $dV_{\text{ind}}$ at a point $P$ by an infinitesimal segment $dl$ of the vortex filament is given by Eq. 4.15.

$$dV_{\text{ind}} = \frac{\Gamma}{4\pi} \frac{dl \times r}{|r|^3}$$  \hspace{1cm} (4.15)

Integrating along the length of the vortex filament gives the total induced velocity $V_{\text{ind}}$ at point $P$ by the whole vortex filament gives

$$V_{\text{ind}} = \frac{\Gamma}{4\pi} \int \frac{dl \times r}{|r|^3}.$$  \hspace{1cm} (4.16)
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Figure 4.10: Notation used for determining the velocity induced by a straight vortex filament in vector form.

The application of the Biot-Savart law to a straight, finite length, vortex filament from point $A$ to point $B$ with circulation $\Gamma$ (as required in this work) is based on the approach of Bertin and Smith as cited in Mason [154] and in Melin [155]. Note, the notation used (illustrated in Fig. 4.10) has been modified slightly from the original. The angles, for example, are identified using the symbol $\beta$, rather than $\theta$ as used by Bertin and Smith as $\theta$ is used in this thesis to indicate the azimuthal position of the VAWT blades. Also, the point of interest at which the induced velocity is calculated is designated $P$ here for consistency with the general case described above, instead of the $C$ used by Bertin and Smith.

Evaluating the Biot-Savart integral of Eq. 4.16 along the straight, finite length, vortex filament gives Eq. 4.17 where $\hat{e}$ is the unit direction vector of the induced velocity.

$$V_{\text{ind}} = \frac{\Gamma}{4\pi r_p} (\cos \beta_1 - \cos \beta_2) \hat{e} \quad (4.17)$$

In the specific case of a two-dimensional vortex (i.e. an infinite vortex) $\beta_1$ is $0^\circ$ and $\beta_2$ is $180^\circ$ so $\cos \beta_1 = 1$ and $\cos \beta_2 = -1$. In this case, the velocity induced by a vortex in two-dimensions therefore reduces to Eq. 4.18.

$$V_{2D,\text{ind}} = \frac{\Gamma}{2\pi r_p} \hat{e} \quad (4.18)$$

Note that as $r_p$ approaches zero, the induced velocity becomes infinite. This deviation from physical reality is addressed with the introduction of a viscous core model, as explained below.

The derivation of Eq. 4.17 (and Eq. 4.18) from Eq. 4.16 is not provided here as this is a standard result available in a similar form in many aerodynamics texts. Converting this into vector notation, as done by Bertin and Smith, however, is less common so is explained in more detail here.

As shown in Fig. 4.10, $r_0$ is a vector aligned along the vortex filament from point $A$ to point $B$, $r_1$ is a vector from point $A$ to point $P$ and $r_2$ is a vector from point $B$ to point $P$. The angles $\beta_1$ and $\beta_2$ can be defined using the dot product of the associated vectors giving Eqs. 4.19 and...
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Potential Vortex (Biot-Savart with no viscous core model)
Rankine Vortex
Lamb-Oseen Vortex
Vatistas Vortex (with n=2)

Figure 4.11: Induced velocity predicted by several vortex models (adapted from Bhagwat and Leishman [8]).

4.20.

\[
\cos \beta_1 = \frac{\mathbf{r}_0 \cdot \mathbf{r}_1}{|\mathbf{r}_0||\mathbf{r}_1|} \\
\cos \beta_2 = \frac{\mathbf{r}_0 \cdot \mathbf{r}_2}{|\mathbf{r}_0||\mathbf{r}_2|}
\] (4.19) (4.20)

The distance \( r_p \) of point \( P \) from the vortex filament is given by Eq. 4.21.

\[
\mathbf{r}_p = \frac{|\mathbf{r}_1 \times \mathbf{r}_2|}{|\mathbf{r}_0|}
\] (4.21)

Finally, the direction \( \mathbf{\hat{e}} \) of the induced velocity is given by Eq. 4.22.

\[
\mathbf{\hat{e}} = \frac{\mathbf{r}_1 \times \mathbf{r}_2}{|\mathbf{r}_1 \times \mathbf{r}_2|}
\] (4.22)

Bertin and Smith substituted Eqs. 4.19-4.22 into Eq. 4.17 and used vector identity rules to simplify it. For the purposes of this work, however, it was more convenient to leave it in the form shown in Eq. 4.23.

\[
\mathbf{V}_{\text{ind}} = \frac{\Gamma}{4\pi r_p} \left( \frac{\mathbf{r}_0 \cdot \mathbf{r}_1}{|\mathbf{r}_0||\mathbf{r}_1|} - \frac{\mathbf{r}_0 \cdot \mathbf{r}_2}{|\mathbf{r}_0||\mathbf{r}_2|} \right) \frac{\mathbf{r}_1 \times \mathbf{r}_2}{|\mathbf{r}_1 \times \mathbf{r}_2|}
\] (4.23)

Leaving the equation in this form allowed a viscous core model to be introduced more easily than in the simplified Bertin and Smith equation.
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Viscous core model

As mentioned above, as the distance $r_p$ between point $P$ and the vortex approaches zero the induced velocity approaches infinity according to the Biot-Savart law. This is a direct consequence of assuming potential flow at all points. In reality, however, in close proximity to vortices, viscous effects are encountered that reduce the velocity to zero at the centre of the vortices. To accurately approximate this behaviour a viscous core model is introduced to account for the discrepancy between the Biot-Savart law, which works very well at large distances, and the physical reality close to the vortex where viscous effects cannot be ignored. A good summary of several different viscous core models is available in Bhagwat and Leishman [8] and in Leishman [150].

Figure 4.11 (which was based on a figure in Bhagwat and Leishman [8]) presents a comparison of the velocity induced close to a vortex (with a viscous core radius $r_c$) using four different vortex models. Note that as the distance away from the viscous core increases, the induced velocity predicted by the different models converge. At very large distances, the effect of the viscous core becomes negligible, with all models matching the original potential flow version of the Biot-Savart law.

The Rankine vortex model simply assumes solid body rotation within the viscous core and the standard potential flow outside the viscous core. The main problem with this model, as pointed out by Bhagwat and Leishman, is that the Rankine vortex model is discontinuous at the boundary of the vortex core. A much better, and with a much stronger basis in physical reality, is the Lamb-Oseen model which is in fact a solution to the one-dimensional laminar Navier-Stokes equations. The Vatistas viscous core model (Eq. 4.24), which is the viscous core model implemented in Aeolus2D and Aeolus3D, involves a simple algebraic equation that, for $n = 2$, closely approximates the more complex Lamb-Oseen viscous core model.

$$V_{2D,\text{ind}} = \frac{\Gamma}{2\pi} \left( \frac{r_p}{(r_c^{2n} + r_p^{2n})^{1/n}} \right) \hat{e}$$

(4.24)

Bhagwat and Leishman [8] describe various extensions to the viscous core models to account for growth of the viscous core over time and the effect of turbulence generation, but such refinements have not been implemented in Aeolus2D or Aeolus3D at this stage. Introducing the viscous effects of the Vatistas viscous core model into the three-dimensional potential Biot-Savart equation (Eq. 4.23) results in Eq. 4.25, which is the equation implemented in Aeolus3D to calculate the velocity induced by a single vortex filament, including the viscous core effects.

$$V_{\text{ind}} = \frac{\Gamma}{4\pi} \frac{r_p}{\sqrt{(r_c^4 + r_p^4)}} \left( \frac{r_0 \cdot r_1}{|r_0||r_1|} - \frac{r_0 \cdot r_2}{|r_0||r_2|} \right) \frac{r_1 \times r_2}{|r_1 \times r_2|}$$

(4.25)
Bhagwat and Leishman state that measurements of the trailing vortices from the tips of helicopter rotors indicated an initial core radius \( r_c \) equal to approximately 5-10% of the chord length of the blades. Although Aeolus2D includes only shed vortices, and Aeolus3D includes both trailing and shed vortices, the core radius of all wake vortices was set to 7.5% of the chord length of the blades (i.e. the mid-point of the measured range). The shed vortices are a discrete representation of a continuous region of highly sheared flow in the wake of the aerofoils. In practice, the thickness of this wake will change over time as the angle of attack of the aerofoil changes, so the core radius of the wake vortices would ideally also change. In the absence of a more detailed model to accurately represent this effect, however, the core radius of the shed vortices was assumed to be similar to that of the trailing vortices measured by Bhagwat and Leishman. To check the sensitivity of the loads on a VAWT to the size of the core radius, a set of numerical experiments were performed with viscous core radii ranging from 1% of the chord to 20% of the chord. The results of these tests are presented in section 7.3.3. The results showed that in practice, for a VAWT, the precise size of the viscous core is not critical.

### 4.4 The wind field

In practice, the velocity of an onset wind flow in open terrain varies in both space and time. Although Aeolus2D and Aeolus3D have been implemented to accept wind flow data that varies in space and time, at this stage only a limited wind field model has been included in both simulations. The wind field model currently implemented in Aeolus2D assumes steady uniform flow and the wind field model currently implemented in Aeolus3D assumes steady flow that varies with height only.

The mean velocity tends to be lower close to the ground, and increases with height (up to a height beyond that of any currently practical wind turbine designs), as illustrated in Fig. 4.12. In addition to an unrealistic (but useful for comparative purposes) uniform wind field, a logarithmic and a power law velocity profile are both used in Aeolus3D to account for the
variation in mean wind speed with height.

4.4.1 Logarithmic velocity profile

The logarithmic velocity profile, given by Eq. 4.26, is the preferred method in this work, with the power law profile included purely to allow consistency in several of the validation cases presented in section 4.10.

\[ V_{\text{wind}}(z) = V_{z_{\text{ref}}} \frac{\ln(z/z_0)}{\ln(z_{\text{ref}}/z_0)} \] (4.26)

\( V_{z_{\text{ref}}} \) is a reference velocity at height \( z_{\text{ref}} \) and \( z_0 \) is a roughness length governing the shape of the velocity profile. In general a reference height of 10m is quite common, but in Aeolus3D a reference height equal to the height of the equator of the wind turbine is more useful. The reference velocity \( V_{z_{\text{ref}}} \) is then determined based on the desired TSR at a given rotational speed. The roughness length \( z_0 \) is related to the average height of obstructions upstream from the point of interest, although it is much smaller in magnitude. The roughness length provides a convenient way of specifying the velocity profile shape over a variety of different terrains, which is independent of the mean wind speed. The Australia/New Zealand standard AS/NZ1170.2:2002 [137], for example, provides guidelines on the roughness length for several different terrain categories. Terrain category 1, corresponding to smooth and very open terrain such as snow or sandy deserts, has a roughness length of 0.002m. Terrain category 2 (the category used as the baseline for the case studies presented in this thesis), corresponds to open terrain with few obstructions such as grassland, has a roughness length of 0.02m. Terrain category 3, corresponding to more closely spaced obstructions such as suburban housing, has a roughness length of 0.2m. Finally, terrain category 4, corresponding to densely packed tall obstructions such as an urban environment, has a roughness length of 2.0m.

4.4.2 Power law velocity profile

As mentioned, in addition to the logarithmic velocity profile, a power law profile as given by Eq. 4.27 (where \( a \) defines the shape of the velocity profile) has been included in Aeolus3D.

\[ V_{\text{wind}}(z) = V_{z_{\text{ref}}} \left( \frac{z}{z_{\text{ref}}} \right)^a \] (4.27)

It is important to note that the only results presented in this thesis that use the power law profile rather than the logarithmic profile are those presented for validation against the measurements made on the Sandia 17m turbine. The power law profile was used for consistency between the experimental measurements (for which the power law velocity profile of the site is given) and the simulated results. Although the difference between the logarithmic profile with an appropriately selected roughness length and the power law profile would be negligible from a
practical point of view, the implementation of the power law into Aeolus3D was sufficiently simple to justify its inclusion.

4.5 The tower

The aerodynamic effects of the tower are accounted for using a far simpler approach than the aerodynamic effects of the blades. Most importantly, the effects of the tower are, unlike the blades, modelled assuming steady flow. Dynamic effects, such as vortex shedding, are not included in Aeolus3D at this stage. Where the blades get in close proximity to the tower this omission of the dynamic effects could lead to some differences in load prediction. As the blades approach the tower, however, the actual loads on the blades reduce significantly (because the radius, and hence relative velocity, decreases significantly) so accuracy becomes less critical. At greater distances, such as at the equator, dynamic shedding from the tower will have much less impact, particularly in comparison to the influence of the blade wakes through which the blades must pass on the downstream side of the turbine.

The aerodynamic effects of the tower are modelled in two parts, each part accounting for a different effect. Firstly, a simple potential flow model of a doublet superimposed on a uniform flow field approximates the flow in close proximity to the tower, ensuring that the wake vortices behave appropriately as they travel downstream.

In addition to the potential flow model of the tower a velocity deficit model (which is based on the drag coefficient of the tower) is included to ensure that downstream of the tower the wind velocity is reduced appropriately due to the wake of the tower. As mentioned above, dynamic effects that may occur in reality as a result of the tower, such as vortex shedding (depending on the wind velocity), are not currently included in either Aeolus2D or Aeolus3D. At the high Reynolds numbers of interest here, and based on the fact that the sections of the blade closer to the equator (which are much further from the tower) are of most importance in the work described by this thesis, the wake from the tower can be approximated by this time-averaged model. Note that even if a structurally dynamic tower is used (which the structural model allows, though it is not used in the case studies presented herein), it has been assumed that the aerodynamic model of the tower remains stationary. As it is assumed that the wake can be represented by its time-averaged influence, it is also appropriate that the tower itself is represented by its average location.

The influence of the tower acts in the horizontal plane only, i.e. the tower models do not involve a vertical component of velocity. This is a very reasonable assumption as the tower is tall and slender.
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4.5.1 Potential flow model around the tower

The flow around the tower is modelled as a doublet in a uniform horizontal flow. The approach here is based on the approach for modelling the flow around a 2D cylinder in Houghton and Carpenter [152], although the approach itself is quite standard and most aerodynamics texts on the subject present an equivalent method. The only changes made to the method from that of Houghton and Carpenter is that the cylindrical coordinate system used has been converted to a Cartesian coordinate system to match that of Aeolus2D and Aeolus3D.

Figure 4.13 illustrates the notations used to model the influence of the tower on the wind field. Note that although all simulations presented in this thesis define the wind field to be aligned along the $x$-axis of the global coordinate system ($x_0$), for the sake of generality the influence of the tower as presented in this section has been defined with respect to an arbitrary wind direction.

Given the horizontal components ($V_{\text{wind},x}$ and $V_{\text{wind},y}$) of the free stream wind velocity ($V_{\text{wind}}$), the magnitude of the horizontal component of the free stream velocity ($|V_{\text{wind},xy}|$) is determined according to Eq. 4.28.

$$|V_{\text{wind},xy}| = \sqrt{V_{\text{wind},x}^2 + V_{\text{wind},y}^2}$$  (4.28)

The direction ($\phi$) of the flow to the $x$-axis is calculated using Eq. 4.29.

$$\phi = \tan^{-1}\left(\frac{V_{\text{wind},y}}{V_{\text{wind},x}}\right)$$  (4.29)

The velocity at an arbitrary point ($x,y$) in the global coordinate system can therefore be transformed to a coordinate system aligned with the direction of the free stream flow using Eqs.
4.30 and 4.31, where \((\bar{x}, \bar{y})\) is the point in the new coordinate system.

\[
\begin{align*}
\bar{x} &= \cos(\phi)x + \sin(\phi)y \\
\bar{y} &= -\sin(\phi)x + \cos(\phi)y
\end{align*}
\]  

(4.30)  

(4.31)

The components of velocity in the new coordinate system including the effect of the potential flow model \((\bar{V}_{\text{wind},\text{pot},x}, \bar{V}_{\text{wind},\text{pot},y})\) are then calculated according to Eqs. 4.32 and 4.33, where \(r\) is the radius of the tower.

\[
\begin{align*}
\bar{V}_{\text{wind},\text{pot},x} &= |V_{\text{wind},xy}| \left( \frac{2r^2\bar{y}^2}{(\bar{x}^2 + \bar{y}^2)^2} - \frac{r^2}{\bar{x}^2 + \bar{y}^2} + 1 \right) \\
\bar{V}_{\text{wind},\text{pot},y} &= -|V_{\text{wind},xy}| \frac{2r^2\bar{y}\bar{x}}{(\bar{x}^2 + \bar{y}^2)^2}
\end{align*}
\]  

(4.32)  

(4.33)

Equations 4.32 and 4.33 were derived by superposition of the stream functions of a uniform flow field and a doublet using the procedure explained in Houghton and Carpenter [152] (and in many other aerodynamics texts). In comparison to the influence of the wake of the tower, as discussed in the section below, or the wakes from the blades themselves, the model described in this section to account for the flow around the tower has very little effect in practice on the overall aerodynamics, and could in fact have been omitted from Aeolus2D and Aeolus3D entirely without compromising the results. The computational cost of its inclusion, however, is trivial so it has been implemented for thoroughness, and also because its effect was not known a priori.

### 4.5.2 Velocity deficit downstream of the tower

In addition to the potential flow model described in the previous section, a velocity deficit model is also included to account for the flow retardation in the wake of the tower. The velocity deficit model used in Aeolus2D and Aeolus3D is taken directly from the model implemented
in AeroDyn for use with downwind HAWTs, as described in the AeroDyn theory manual [156] and based on the work of Powles [157]. Note that, although the method is almost identical to that of AeroDyn, the equations are presented in a slightly modified form. In the AeroDyn theory manual, it is stated that distances are normalised with respect to the tower radius. As presented here, however, the tower radius is explicitly included in the equations for consistency with the potential flow model described in the previous section. It should also be noted that differences exist between versions of the AeroDyn theory manual. The January 2005 version [158], for example, specifies that the velocity deficit of the wake model should be applied to the components of velocity in both the $x$ and $y$ directions, but the December 2005 version [156] specifically states that the velocity deficit model is only applied to the component of velocity in the $x$ direction. For the purposes of this work, it was assumed that the more recent theory manual is the more accurate.

One important difference between the method used in AeroDyn and that described here is that in the AeroDyn model, at all points within the wake, the velocity deficit model is used instead of a potential flow model of the tower. This could, however, cause physically implausible flow. In Aeolus2D and Aeolus3D the velocity deficit model is used in addition to the potential flow model of the tower. If the wake is modelled using the velocity deficit model only, rather than in addition to the potential flow model, then at points along the boundary of the wake it is possible for the flow to be faster inside the wake than outside the wake. The actual differences to the predicted loads are unlikely to be significant, but this difference from the original AeroDyn model needs to be pointed out, as it does result in some difference in how the equations of the model are applied.

The width of the influence of the wake of the tower is assumed to expand downstream at a rate proportional to the square root of the distance from the centreline of the tower [157] (i.e. the distance from the origin of the global coordinate system). A velocity deficit ($\bar{v}_{wake,x}$) is applied to the flow velocity (including the effect of the potential flow model described in the section above) for all points downstream of the tower and within the area of influence of the wake (i.e. where $x > 0$ and $|\bar{y}| \leq r\sqrt{d}$) according to Eq. 4.34.

$$\bar{v}_{wake,x} = \frac{C_D}{\sqrt{d}} \cos^2 \left( \frac{\pi}{2} \frac{\bar{y}}{r\sqrt{d}} \right), \text{ for } x > 0 \text{ and } |\bar{y}| \leq r\sqrt{d}$$  \hspace{1cm} (4.34)

Where the distance from the centreline of the tower ($d$) is defined in Eq. 4.35.

$$d = \frac{\sqrt{\bar{x}^2 + \bar{y}^2}}{r}$$  \hspace{1cm} (4.35)

The velocity deficit can then be used to adjust the component of velocity, determined using the
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potential flow model, in the \( \bar{x} \) direction \( (\bar{V}_{\text{wind, pot, } x}) \) as shown in Eq. 4.36.

\[
\bar{V}_{\text{wind, wake, } x} = (1 - \bar{v}_{\text{wake, } x})\bar{V}_{\text{wind, pot, } x} 
\] (4.36)

The final velocity components are then simply transformed back into the original global coordinate system according to Eqs. 4.37 and 4.38.

\[
V_{\text{wind, tower, } x} = \cos(\phi)\bar{V}_{\text{wind, wake, } x} - \sin(\phi)\bar{V}_{\text{wind, pot, } y} 
\] (4.37)

\[
V_{\text{wind, tower, } y} = \sin(\phi)\bar{V}_{\text{wind, wake, } x} + \cos(\phi)\bar{V}_{\text{wind, pot, } y} 
\] (4.38)

An example of the velocity flow field due to both the potential flow model and the velocity deficit model of the wake is illustrated in Fig. 4.14, for a flow aligned with the \( x \)-axis of the global coordinate system and a tower drag coefficient of \( C_D = 1 \).

This tower wake model appears qualitatively accurate, expanding in width and reducing in influence with distance downstream, but it is certainly open to significant improvement beyond the scope of this work. For the purposes of the work in this thesis, the wake model of the tower is adequate, in that it gives an indication of the general effect of the tower wake, but quantitative conclusions based on the tower wake model should be made with caution. Of particular note, significant care should be taken if the drag coefficient \( (C_D) \) of the tower is greater than one. This will predict a velocity deficit value greater than one for some locations which means that the flow velocity becomes negative (i.e. the flow is back towards the tower rather than away from it). Specifically, for locations directly downstream of the tower \( (\bar{y} = 0) \) the flow will be negative for any distance less than the square of the drag coefficient (i.e. \( \bar{v}_{\text{wake, } x} > 1 \) when \( d < C_D^2 \)). While such flow reversals are theoretically and practically possible in the wake of an object, in terms of the vortex wake model (particularly the three-dimensional model) this could potentially cause numerical problems if a vortex wake marker were to become trapped in such an eddy. One end of the vortex would remain close to the tower while the other end would continue downstream and the vortex would continue to stretch to physically impossible lengths. This could be overcome with a model to estimate when a vortex filament is stretched to its breaking point, but the rarity of such an event does not warrant its inclusion in Aeolus3D. Rather, an awareness of the risks of using high tower drag coefficients was sufficient to mitigate such a possibility in this research.

4.6 Updating the wake state

This section describes the process of updating the state of the wake. Tracking the development of the wake over time represents, as mentioned above, the most computationally expensive part of the simulation. However, the use of a freely developing wake is also what gives the vortex
wake methods an advantage over the simpler methods such as the blade element momentum based methods. The freely developing wake requires no prior assumptions about the structure of the wake, such as whether the flow itself or the loads on the VAWT are symmetric in nature, either parallel to the oncoming wind field, or between the upstream and downstream sides of the wind turbine.

Although very computationally demanding, the process of updating the wake is actually quite straightforward, and consists of the following three main steps.

1. Calculate the velocity of all vortex markers in the wake (section 4.6.1).
2. Move all vortex markers to their new location (section 4.6.2).
3. Trim off excess wake vortices to limit computational demands (section 4.6.3).

### 4.6.1 Determining velocity of vortex markers

**Full self-interacting wake**

At the end of each time-step (or beginning of the next time-step), the velocity of all wake markers is determined by summing, as per Helmholtz decomposition theory, the velocity due to the rotational \( \mathbf{V}_{\text{bound}}, \mathbf{V}_{\text{wake}} \) and the irrotational \( \mathbf{V}_{\text{wind,tower}} \) flow fields.

The velocity due to the bound vortices is the sum of the induced velocity from each bound vortex (Eq. 4.39) and the induced velocity due to the wake vortices is the sum of the induced velocity from each wake vortex (4.40).

\[
\mathbf{V}_{\text{bound}} = \sum_{i=1}^{N_B} \mathbf{V}_{\text{ind},i} \tag{4.39}
\]

\[
\mathbf{V}_{\text{wake}} = \sum_{j=1}^{N_W} \mathbf{V}_{\text{ind},j} \tag{4.40}
\]

\( i \) is the index of the bound wake vortex, \( N_B \) is the total number of bound vortices, \( j \) is the index of the wake vortex and \( N_W \) is the total number of wake vortices. The induced velocity of each vortex was shown in section 4.3.2 to be calculated by Eq. 4.25. It is these two equations (4.39 and 4.40) which lead to the \( O(N^2) \) complexity of the aerodynamic simulation. The velocity of every vortex marker is a function of every vortex in the system (other than the, up to four, to which it is attached). The \( N_W \) parameter in particular is key to the total computational demands which can increase with every time-step, unlike \( N_B \) which remains constant for any given simulation.
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Partial self-interacting wake

As an alternative to the fully self-interacting wake described above in which the velocity of every vortex marker is calculated using the induced velocity of every vortex, several alternative approaches were investigated in an attempt to reduce the computational cost of updating the wake state. The approaches looked at were, in general, based on the concept of allowing the wake to self-interact over only a limited distance downstream. Vortex markers beyond this limited distance would then be moved using various other methods to calculate the speed. The alternative approaches all have a self-interacting wake to a set distance downstream, then beyond that point:

- Vortex markers move at the free stream wind speed.
- Vortex markers move at the average velocity of the self-interacting region of the wake.
- Vortex markers move at the average velocity of the vortex markers in the self-interacting region of the wake, with a tube of set diameter parallel to the flow direction centred at each vortex marker in question (in principle, to allow for variation in velocity with height and parallel to the direction of flow.
- Vortex markers continue to move downstream at the velocity they were travelling downstream when they crossed beyond the self-interacting wake region.

These represent only a summary of the variations investigated. In most cases, although greatly reducing computational time, the results rarely matched those predicted by the full self-interacting wake. The most promising approach was the final one listed above, in which the vortex markers continue to travel downstream at the velocity they were travelling downstream when they passed beyond the self-interacting wake length. Unfortunately, however, the results were somewhat inconsistent. Rather than having a short self-interacting wake followed by a longer non-self-interacting wake, one could simply use a full self-interacting wake with a length somewhere between the two. There is a length in-between that will have the same computational cost but slightly different results, and no obvious indication regarding which is the more accurate.

No self-interacting wake

In addition to the partial self-interacting wake alternatives mentioned above, the idea was investigated of having, initially, no self-interacting wake (i.e. a prescribed wake shape) until some predefined conditions were met, then allowing either the partial self-interacting wake or the full self-interacting wake to start. The condition for switching from the prescribed wake to the self-interacting wake was based on either the torque generated by the turbine reaching equilibrium, or by the wake fully extending to its pre-set limit.
The principle behind this approach was that a prescribed wake could be formed very rapidly as it has an almost trivial computational cost compared with the computational cost of a self-interacting wake. If the prescribed wake was close enough to the self-interacting wake, then the whole simulation may be able to reach equilibrium faster than if the self-interacting wake had to form right from the start.

In practice, however, there was no clear advantage to this approach. For some turbine configurations the simulation reached equilibrium faster, but for other configurations the total computational time was, in fact, longer. Although there may be some combination of prescribed wake type, self-interacting wake length and conditions for switching between the two, there was no obvious pattern uncovered during this research that indicates what such a combination might be. In the end, such efforts to accelerate the simulations were unsuccessful (or more precisely, unpredictable in their success) using these methods though there may be merit in re-examining these and other concepts in the future.

4.6.2 Updating location of vortex markers

Time-stepping scheme

With the velocity of all vortex markers calculated, they are moved to their new locations using the simple explicit Euler method. While the Euler method can (and probably will) lead to increasing error over time, this is not of great concern here. As wake vortices travel downstream away from the VAWT their precise locations become much less important. As was seen above in section 4.3.2, the velocity induced by a vortex filament of constant strength, and assuming its length remains approximately constant\(^4\), at a significant distance from the point of interest (i.e. well away from the viscous effects of the vortex core) is inversely proportional to the distance. Therefore, as errors accumulate as the vortices travel downstream, the error in their positions become less important with respect to their effect on the flow over the VAWT's blades. As an area of future improvement to the simulation, the adoption of a higher order method may be worth investigation but within the scope of this work it is unlikely to offer any advantages. Care is definitely required for any stability analysis as the explicit Euler method can be unstable (though not necessarily so in the cases presented in this thesis).

Correction of misplaced wake vortices

Because the wake evolves at discrete time-steps rather than in a continuous manner it is sometimes possible (in the absence of any correction) for wake vortex markers close to the tower to end up inside the tower at the following time-step. If a vortex marker crosses the boundary of the tower it can become stuck inside the tower, unable to escape. To counteract this issue, as the

\(^4\)The length of the vortices will not remain constant, but it is helpful to make such an assumption here to highlight the strong relationship between induced velocity and distance.
new location of each wake vortex marker is determined a check is made to see whether or not it is inside or outside of the tower. If the vortex marker is located within the tower it is simply pushed horizontally in a radial direction from the centreline of the tower to a distance of 1.01 times the radius (i.e. a distance of 1% of the radius beyond the wall of the tower). This is a very minor and simple fix that had no measurable effect on the loads, but was necessary to account for a purely numerical imperfection resulting from the discretisation of the system.

4.6.3 Trimming excess wake vortices

With the vortices moved to their new locations, any vortex beyond a pre-set wake length is removed from the simulation. This is a necessary step due to finite computational power. Ideally, the wake would be allowed to convect without bounds, but as the influence a vortex particle has on the flow velocity experienced by the wind turbine diminishes with distance, at long wake lengths, although still having an influence on the flow velocity at the wind turbine, the magnitude of such influence becomes so small that the high computational cost cannot be justified. The computational cost of a vortex in close proximity to the wind turbine is identical to the computational cost of a vortex a long way away, so the elimination of the furthest vortices allows additional vortices to be modelled closer to the wind turbine without affecting the total computational cost.

The trimming of the vortices was done in several stages. First, both ends of all vortices in the wake are checked and if both ends are beyond the pre-set wake length then the vortex between them is removed from the simulation (but the vortex markers are left, for now). Next, if one end of a vortex is beyond the pre-set wake length, but the other end is is still within the pre-set wake length, the vortex is split so that only the proportion of the vortex within the pre-set wake length continues to induce a velocity on anything else. Finally, each vortex marker is checked to see whether there are still any vortices attached to it. If a vortex marker no longer has any vortices attached (all previously attached vortices have now exceeded the pre-set wake length and been eliminated from the simulation) then it too is eliminated from the simulation.

In addition to the pre-set wake length limitation, Aeolus2D and Aeolus3D have both been implemented to allow a maximum total number of wake vortices. If the number of wake vortices exceeds this maximum then excess vortices, starting with the furthest wake vortices are eliminated, with the distance defined by the upwind end of the vortex. In practice, controlling the computational costs by limiting the wake length rather than a set maximum number of vortices appeared to be the best approach. At low TSRs, the vortices are transported downstream rapidly so much longer wakes could, in fact, have been used without significant increases to the computational demands. At high TSRs, the vortices take longer to travel downstream so for the same wake length, the computational costs are much greater due to the number of vortices at any given time. Unfortunately, on investigation, higher TSRs also required longer wake
lengths than lower TSRs for the predicted loads to converge so there is actually no advantage in allowing longer wakes at lower TSRs. Details are provided in chapter 7 on how an appropriate balance between wake length, vortex release rate and accuracy was found.

From the point of view of implementation, it should be noted that details of the far-wake are all stored dynamically with memory being allocated and freed on the fly with each time-step to ensure that the overall memory demands of the simulation remains manageable. With new vortices being released into the wake at every time-step, memory demands could easily become excessive without proper management.

4.7 Updating bound and near-wake vortices

As mentioned above, updating bound and near wake vortices was an iterative process (or more specifically, the determination of the circulation of the bound and near wake vortices is an iterative process). The circulation of the near-wake shed and trailing vortices were shown, in section 4.3.1 (Eqs. 4.13 and 4.14), to be functions of the circulation of the bound vortices. The circulation of the bound vortices, however, were shown to be a function of the velocity across the section (Eq. 4.8) which includes the velocity induced by all wake vortices, including those of the near wake. An outline of the process of updating the bound and near-wake vortices is illustrated in Fig. 4.15, in which one can see that several stages of the process may be repeated until the solution converges. Each step of this process is explained below.

**Updating the location of the vortex markers:** The first step in the process is to move the vortex markers of the bound vortices and the near-wake vortices to their new locations at the
current time-step. Unlike the locations of the vortex markers of the far-wake, the locations of
the bound vortices and the near-wake vortices are governed only by the motions of the blades.
The bound vortex markers are placed along the blade at the quarter-chord location and the
near-wake vortex markers are placed a distance of 25% from the current location of the trailing
edge to the location of the trailing edge at the previous time-step as explained above in section
4.3.

**Calculating the independent velocity:** The calculation of the velocity over each blade
section is divided into two separate components, referred to here as the independent velocity and
the dependent velocity. The independent velocity result from those contributions to the velocity
that are not influenced by the bound or near wake circulation. This means the independent
components only need be calculated once outside the iterative solver. The dependent velocity
components, however, are those that need to be recalculated repeatedly with each iterative
step. The independent velocity at each section therefore consists of the the velocity induced by
the far-wake vortices, the velocity due to the motion of the blades, and the velocity due to the
free-stream wind field.

**Making an initial estimate of the circulation of bound vortices:** An initial estimate
of the circulation of the bound vortices is required to initiate the iterative process. Using an
initial circulation of the bound vortices equal to the circulation at the previous time-step was
found to be very stable and typically found the new circulations in, at most, a few iterations.
Using an initial value of zero for the circulations of all bound vortices was also very stable
and typically solved in a few iterations too, but showed no obvious advantage. Attempting to
predict an initial value of the circulation of the bound vortices using the previous value and the
rate of change estimated by a backwards differencing method worked quickly in most cases, but
could be numerically unstable in others, with the iterative solver failing to reach convergence.
Because of this danger, the possibly slightly slower, but definitely much more reliable approach
of using the circulation from the previous time-step was adopted. No quantitative analysis
was done to determine whether there was a speed advantage to using the circulation from the
previous time-step versus setting the initial value of the circulation of all bound vortices to zero.
As the computational bottleneck in the aerodynamic simulation is determining the motion of
the wake, even an order of magnitude increase in computational performance in the iterative
solution would be unlikely to make a noticeable difference to the total computational cost.
Reliability is much more important here, and both methods performed well in this regard. In
the case of a single blade section, a one-dimensional root finding algorithm was used (instead of
the multi-dimensional root finding algorithm used for multiple blade sections) that required a
bracketed range of initial estimates that the solution is expected to remain within rather than
a single estimate. In these cases, the initial range was set to the value of the circulation at the
previous time-step plus and minus a value several orders of magnitude higher than the highest
expected circulation in the system (based on chord length, velocity and peak lift coefficient). This is certainly not the most efficient approach and probably requires an extra iterative step or two (or more) to solve, but as this method is only used when there is a single blade section being simulated, the total computational costs were so light that this was of no concern.

Calculating the circulation of the near-wake vortices: Using the current estimate of the bound vortices, the circulations of the near-wake shed and trailing vortices are calculated using Eqs. 4.13 and 4.14, as explained in section 4.3.1.

Calculating the dependent velocity: As mentioned above, the dependent velocity over each blade section consists of the components of velocity that change as a function of the circulation of the bound and near-wake vortices so must be recalculated with each iterative step, (i.e. the velocity induced by the bound vortices, the near-wake shed vortices and the near-wake trailing vortices). The dependent velocity is added to the independent velocity previously calculated giving the relative velocity over each blade section \( \mathbf{V}_{\text{rel}} \), which is projected onto the plane of the cross-section of each blade section giving the sectional velocity over the blade \( \mathbf{V}_{\text{sec}} \) as described above in section 4.2.2.

Determining the direction of the flow over the sections: The direction of the flow over each section is determined to confirm whether the flow separates off the trailing-edge or the leading-edge of the aerofoil (for reasons explained above in section 4.3.1).

Calculating the blade load coefficients: The aerodynamic lift, drag and moment coefficients across each section of the blade is calculated, including any dynamic stall effects. Because of the complexity of the process of calculating the blade load coefficients, this is explained in full in section 4.8.

Checking for convergence: The iterative process for determining the circulations of the bound and near wake vortices is considered to have been solved when the results of a governing equation is lower than a predefined absolute tolerance. A tolerance of \( 1 \times 10^{-3} \) is used in Aeolus2D and Aeolus3D for this process. While a lower tolerance is tempting, it is unnecessary here. With the many assumptions and approximations required by the aerodynamic simulation, a very low tolerance here would simply be more precise than accurate and as such would add little value. The governing equation in Aeolus2D and Aeolus3D is a rearrangement of the Kutta-Joukowsky equation (Eq. 4.9) and the introduction of the tolerance is given by Eq. 4.41.

\[
\frac{1}{2} (C_N \cos (\alpha_E) + C_T \sin (\alpha_E)) |\mathbf{V}_{\text{sec}}| c - \Gamma^B \leq \text{TOLERANCE} \tag{4.41}
\]

If Eq. 4.41 holds true for all blade sections in the system, then convergence is considered to have been reached. If Eq. 4.41 does not hold true for all blade sections, the iterative process must continue with an improved estimate of the circulations of the bound vortices.
Making an improved estimate of the circulation of bound vortices: If the solution has not been found, an improved estimate of the circulation of the bound vortices is made such that the iterative process of finding a solution can continue. The determination of the value of the improved estimate is managed automatically in Aeolus2D and Aeolus3D by two different iterative methods from the GNU Scientific Library (GSL) [159]. If only a single blade section is being simulated (such as an individual, harmonically pitching aerofoil) the solution is iteratively found using the one dimensional Brent-Dekker root finding algorithm implemented in GSL\(^5\), which is a bracketed algorithm requiring an initial interval within which the solution lies. According to Galassi et al. [159] the Brent-Dekker method is "a fast algorithm which is still robust". When simulating more than one blade section, the Brent-Dekker method is insufficient and a multi-dimensional root finding algorithm is required. Aeolus2D and Aeolus3D utilise a hybrid multi-dimensional root finding algorithm implemented in GSL\(^6\), when solving simulations with more than one blade section. The GSL version of the multi-dimensional root finding algorithm is based on the MINPACK implementation of Powell’s Hybrid method (i.e. HYBRJ).

4.8 Calculating the blade load coefficients

An accurate model of the loads on the blades themselves is one of the most important parts of the aerodynamic simulation, particularly the phenomenon of dynamic stall. Whereas a fairly simple model of the wind field itself (section 4.4) and the influence of the tower on that wind field (section 4.5) is adequate, such a simple approach towards modelling the loads on the blades themselves would not be sufficient. The loads on the blades are inherently unsteady and non-linear. Every effort to capture the transient and complex nature of the loads accurately must be made. If the intention were only to model averaged or quasi-steady loads, then a simple blade element momentum method would have resulted in vastly lower computational cost and required a lot less time to implement. Such an approach would not, however, have given the level of insight desired into the transient behaviour of the turbine’s blades.

The basic modelling approach used in this research to capture the dynamic stall behaviour is, as mentioned previously, the BL dynamic stall model [160]. Some minor modifications are made to the original BL mode, taken from a number of researchers, particularly Pierce [161] and Minnema [162], upon which much of the dynamic stall model described herein is based. The model was further adapted slightly for use with the vortex modelling approach used herein, rather than the blade element momentum based approach for which the BL dynamic stall model was originally developed and traditionally applied.

Only a summary of the dynamic stall model used in Aeolus2D and Aeolus3D is included herein,

\(^5\)The one-dimensional GSL algorithm used is called *gsl_root_fsolver_brent*.

\(^6\)The multi-dimensional GSL algorithm used is called *gsl_multiroot_fsolver_hybrids*. 

with an emphasis on differences from the original model as described by Beddoes and Leishman. For a more detailed description of the model the reader is directed to Leishman and Beddoes [160], Leishman [150], Gupta and Leishman [163], Pierce [161] and Minnema [162].

The BL model divides various effects into attached flow and separated flow models with the separated flow effects broken down further into trailing-edge separation, leading-edge separation and the vortex shedding. At a high level, the approach followed here is the same as the original BL model. There are, however, a number of minor differences at the lower levels of each sub-model. The attached flow model, in particular, differs significantly from the approach proposed by Beddoes and Leishman. The approach used in the original BL model is not ideal for modelling the blades of VAWTs, where the wake shed off blades during the upstream pass may have a strong influence on the blade loads during the downstream pass. The separated flow models, on the other hand, remain very similar in nature to the original BL model, with only a number of minor modifications made to the actual implementation to make the model more suitable for application to wind turbine blades than the original helicopter blades for which the BL model was originally intended. The modifications made to the separated flow models come from a variety of sources and these modifications are described in detail in the following sections.

It should be noted that although for a given periodic motion, the unsteady aerodynamic model described here produces periodic load curves, in reality the loads appear much more chaotic. McAlister, Carr and McCroskey [164], for example, stated that the ensemble average of 50 or more cycles of data was required to represent the aerodynamic loads during their experimental work. A qualitative observation of any of the OSU test data (e.g. Hoffmann et al. [3] and Ramsay et al. [165]), can similarly be seen to have a high degree of variation from one cycle to the next, some of which can be seen in section 4.10.2 where it is used to validate Aeolus2D. This is particularly apparent at the top of a pitch cycle when flow separation has taken place. The variation in measured loads at a given angle of attack can, in fact, be quite large.

The model described here should therefore be thought of as an attempt to approximate the “average” unsteady loads. The reality of dynamic stall is very complex, with many uncertainties still unresolved. Given this qualifier however, the model described here predicts the general nature of the unsteady loads both in and out of stall very well, and appears sufficiently accurate qualitatively for the purposes of this research, where the emphasis is on comparisons between different turbine configurations and parameters and not on actual absolute loads.

4.8.1 Attached flow

The attached flow comprises circulatory and non-circulatory effects. The circulatory effects on the attached flow are related to the downwash from the wake changing the effective angle of
attack of the aerofoil\textsuperscript{7}. The non-circulatory effects include compressibility and apparent mass effects. The approach adopted in the original BL model was to determine the attached flow in terms of indicial aerodynamic responses, but a different method has been adopted in Aeolus2D and Aeolus3D.

The circulatory effects are accounted for directly in a free vortex wake model. The actual structure of the wake is known at all times so the downwash can be calculated at any location and any point in time. The circulatory components of the normal attached flow load ($C_N^C$) is calculated, as shown in Eq. 4.42, from the slope of the normal load coefficient curve ($C_{na}$) and the effective angle of attack ($\alpha_E$).

\[ C_N^C = C_{na} \alpha_E \]  

(4.42)

The BL indicial approach is based on piston theory which accounts for compressibility. While this is very important for rotorcraft applications (for which the method was originally developed and applied), it is less important for wind turbine modelling where the Mach numbers are sufficiently low (i.e. $M < 0.3$) that the flow can be considered incompressible. A consequence of this assumption is that any pressure changes due to non-circulatory effects can be assumed to propagate at infinite velocity through the fluid domain. As such, for wind turbine applications the non-circulatory loads can be determined by the instantaneous motion of the aerofoil only, which in the absence of any compressibility effects are simply the apparent mass loads.

The apparent mass loads are estimated by examining the loads required to accelerate a volume of air surrounding the aerofoil. The mass of the air being accelerated is typically calculated by assuming that the volume of air being accelerated is a cylindrical tube of air with a diameter equal to the chord of the aerofoil section. Adapted from Bisplinghoff [130], the non-circulatory normal apparent mass loads due to the acceleration of the air mass can be calculated as shown in Eq. 4.43.

\[ C_{NC} = \frac{\pi c}{2} \left[ \frac{\ddot{\alpha}}{|V_{sec}|} + \frac{\ddot{h}}{|V_{sec}|^2} - \frac{ca\ddot{\alpha}}{2|V_{sec}|^2} \right] \]  

(4.43)

$\ddot{h}$ is the acceleration of the aerofoil in the plunging direction, which is assumed to be negligible in comparison to the pitching rate for the cases presented in this thesis, and $a$ is the distance (in semi-chords) from the mid-point of the aerofoil to the pitching axis. In the case of a VAWT, the angular acceleration term is much smaller than the velocity term, due to the square of the relative wind speed appearing in the denominator of the acceleration term. Hansen \textit{et al.} [166]

\textsuperscript{7}One may also consider the flow curvature effects discussed in section 4.2.2 as circulatory effects, but these are not addressed by the dynamic stall model, which was not originally developed with VAWTs in mind, so are not described in this section.
make a similar assumption, pointing out that at moderate reduced frequencies the acceleration
terms are an order of magnitude lower. Given this, the apparent mass loads can be simplified
to Eq. 4.44.

\[ C_N^{NC} \approx \frac{\pi c \ddot{\alpha}}{2|V_{sec}|} \]  

(4.44)

The rate of change of the angle of attack (\( \dot{\alpha} \)) at time-step \( k \) is estimated numerically in Aeolus2D
and Aeolus3D using the backwards difference theorem according to Eq. 4.45.

\[ \dot{\alpha}_k \approx \frac{\alpha_E^k - \alpha_E^{k-1}}{\Delta t} \]  

(4.45)

Similar to the apparent mass normal load estimation, the pitching moment may have a non-
circulatory component which is calculated in a similar fashion. Again, from Bisplinghoff [130],
the non-circulatory pitching moment coefficient about the mid-chord is given by Eq. 4.46.

\[ C_{NC}^{M_{1/2}} = \frac{\pi}{2} \left[ \frac{ca\ddot{h}}{2|V_{sec}|^2} - \frac{c^2}{4|V_{sec}|^2} \left( \frac{1}{8} + a^2 \right) \ddot{\alpha} \right] \]  

(4.46)

So, the pitching moment about the quarter-chord point is given by Eq. 4.47.

\[ C_{NC}^{M_{1/4}} = C_{NC}^{M_{1/2}} - \frac{C_N^{NC}}{4} \]  

(4.47)

As before, making the assumption that the accelerations are negligible compared to the ve-
ocities, the non-circulatory pitching moment about the quarter-chord point reduces to Eq. 4.48

\[ C_{NC}^{M_{1/4}} \approx -\frac{\pi c \ddot{\alpha}}{8|V_{sec}|} \]  

(4.48)

Note that this method is certainly not an ideal estimation of the apparent mass effects. The
approach is, in reality, formulated for an aerofoil undergoing harmonic movement in a uniform
constant flow field. Clearly, this is not the case for a VAWT, which can experience arbitrary motion
in a continuously changing onset flow. It does, however, provide a general approximation of
the apparent mass effects. In practice, the apparent mass effects are not particularly significant
and at the low to moderate TSRs of interest in this work the loads due to dynamic stall effects
dominate over the secondary effects such as the apparent mass loads [14]. In hindsight, the
apparent mass effects could have been omitted from the simulation entirely without impacting
the results, but this was not realised until late into the implementation of the models (and after
some validation had already been completed) so the effects were retained for completeness.
4.8.2 Separated flow

The approach used to model separated flow in this work remains largely unchanged from the original model developed by Beddoes and Leishman, who state “The most critical aspect of modeling dynamic stall is to define the conditions under which leading edge separation occurs.” [160]. The approach employed in the BL model is to define a critical normal load coefficient which is assumed to correspond to the pressure at which leading edge separation will occur.

The aerofoil will experience a lag in the leading edge pressure in response to a changing angle of attack. As per the original BL model, the normal load coefficient including the lag \(C'_{N_k}\) is approximated at time-step \(k\) by a first-order, deficit function \(D_{N_k}\), given by Eq. 4.49.

\[
C'_{N_k} = C_{N_k}^C - D_{N_k}^p
\]

where:

\[
D_{N_k}^p = D_{N_{k-1}}^p \exp \left( -\frac{\Delta s}{T_p} \right) + \left( C_{N_k}^C - C_{N_{k-1}}^C \right) \exp \left( -\frac{\Delta s}{2T_p} \right)
\]

\(T_p\) is a time constant (measured in semi-chords) related to the aerofoil and \(\Delta s\) is the time-step size made dimensionless with respect to the semi-chord and the velocity, as shown in Eq. 4.50. Making the time-step size dimensionless assists in ensuring applicability of the methods across all chord length and velocity scales.

\[
\Delta s = \frac{2|V_{sec}|}{c} \Delta t
\]

Leading edge separation is assumed to occur when \(C'_{N}\) exceeds the critical normal load coefficient. Trailing edge separation is modelled using an approach based on Kirchoff theory [167], which relates the post-stall normal and tangential loads on a flat plate to the separation point \(f\). The original BL model uses a single value for the separation point at any given angle of attack and represents the relationship between angle of attack and separation point with curve fitting techniques. It should be stressed, however, that the separation point described here is not a real physical separation point. Instead, the separation point is simply a representative point that on an idealised flat plate would produce the equivalent load coefficients.

For this research, the approach used by Pierce [161] was adopted instead of the original BL approach. Instead of a single relationship between angle of attack and separation point, separate normal and tangential separation points were used based on the normal and tangential load coefficients \((f_N\) and \(f_T\) respectively). A look-up table, with simple linear interpolation between points, was used instead of a curve fitting approach for the relationship between the angle of attack and the separation points. Finally, the Kirchoff equations were modified to keep track of the sign of the separation point. The original BL model assumes only positive values for the separation point, but as the separation point does not actually represent a real physical
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separation point, negative values are possible and should be accounted for. The relationship between the separation points and the load coefficients, using modified versions of Kirchoff’s equations are therefore as specified by Eqs. 4.51 and 4.52, where $\alpha_0$ is the zero lift angle of attack.\(^8\)

\[
C_N = C_{N\alpha}(\alpha - \alpha_0) \left(1 + \frac{\sqrt{|f_N| \text{sgn}(f_N)}}{2}\right)^2 \tag{4.51}
\]

\[
C_T = C_{N\alpha}\alpha \sin(\alpha - \alpha_0) \sqrt{|f_T| \text{sgn}(f_T)} \tag{4.52}
\]

Rearrangement of these equations in terms of the separation points provides a relationship between the angle of attack and the separation points based on static measurements of the load coefficients. These equations are used to pre-process the load coefficient data into the look-up tables relating the angles of attack to the separation points. The normal separation point is calculated as shown in Eq. 4.53.

\[
f_N = z_N^2 \text{sgn}(z_N) \tag{4.53}
\]

where: \(z_N = 2 \sqrt{\frac{C_N}{C_{N\alpha} \sin(\alpha - \alpha_0)}} - 1\)

The tangential separation point is calculated as shown in Eq. 4.54.

\[
f_T = z_T^2 \text{sgn}(z_T) \tag{4.54}
\]

where: \(z_T = \frac{C_T}{C_{N\alpha} \alpha \sin(\alpha - \alpha_0)}\)

Using the normal load coefficient including the leading edge pressure lag (\(C'_{Nk}\)), an equivalent angle of attack $\alpha'_k$ can be calculated that includes the pressure lag effect, as shown in Eq. 4.55.

\[
\alpha'_k = \frac{C'_{Nk}}{C_{N\alpha}} \tag{4.55}
\]

The equivalent angle of attack $\alpha'_k$ is used to find equivalent separation points for both the normal and tangential loads (Eqs. 4.56 and 4.57 respectively).

\[
f_{Nk}' = f_N(\alpha'_k) \tag{4.56}
\]

\[
f_{Tk}' = f_T(\alpha'_k) \tag{4.57}
\]

Another first-order deficit function ($D^f_N$) is applied to the separation points to account for un-

---

\(^8\)More precisely, in Aeolus2D and Aeolus3D, $\alpha_0$ is the angle of attack at which the normal load coefficient is zero but for symmetric aerofoils such as those used on VAWTs this will be the same as the zero lift angle of attack.
steady boundary-layer effects. The normal separation point with unsteady boundary-layer effects included is given by Eq. 4.58 and the tangential separation point with unsteady boundary-layer effects included is given by Eq. 4.59.

\[
f''_N = f'_{N_k} - D'_{N_k}
\]

where:

\[
D'_{N_k} = D'_{N_{k-1}} \exp \left(-\frac{\Delta s}{T_f}\right) + \left(f'_{N_k} - f'_{N_{k-1}}\right) \exp \left(-\frac{\Delta s}{2T_f}\right)
\]

\[
f''_T = f'_{T_k} - D'_{T_k}
\]

where:

\[
D'_{T_k} = D'_{T_{k-1}} \exp \left(-\frac{\Delta s}{T_f}\right) + \left(f'_{T_k} - f'_{T_{k-1}}\right) \exp \left(-\frac{\Delta s}{2T_f}\right)
\]

\(T_f\) is a time constant related to the aerofoil. These separation points, which account for the lag in the leading edge pressure due to changes in angle of attack, and the lag due to unsteady boundary-layer effects are used in the modified Kirchoff equations, together with the effective angle of attack \(\alpha_E\) to determine the normal \(C''_N\) and tangential \(C''_T\) load coefficients which also accounts for these effects, as given by Eqs. 4.60 and 4.61 respectively.

\[
C''_N = C_N(\alpha_{E_k} - \alpha_0) \left(1 + \sqrt{\left|f''_{N_k}\right| \text{sgn}(f''_{N_k})}\right)^2
\]

\[
C''_T = C_N \alpha_{E_k} \sin(\alpha_{E_k} - \alpha_0) \sqrt{\left|f''_{T_k}\right| \text{sgn}(f''_{T_k})}
\]

Not included in the original BL model are the effects that the unsteady boundary layer has on the moment coefficient. To include these effects, modifications suggested by Minnema [162] are included in Aeolus2D and Aeolus3D. Minnema models the effect on the moment coefficient in a similar manner to the normal and tangential load coefficients, but rather than applying the deficit function to a separation point, a deficit function is applied to the equivalent angle of attack \(\alpha'\). \(\alpha''_k\), as given by Eq. 4.62, is therefore the equivalent angle of attack at time-step \(k\) including the unsteady boundary layer effects.

\[
\alpha''_k = \alpha'_k - D_{\alpha_k}
\]

where:

\[
D_{\alpha_k} = D_{\alpha_{k-1}} \exp \left(-\frac{\Delta s}{T_\alpha}\right) + \left(\alpha'_{k} - \alpha'_{k-1}\right) \exp \left(-\frac{\Delta s}{2T_\alpha}\right)
\]

The moment coefficient, including the unsteady boundary layer effects, is then estimated as the static moment coefficient at an angle of attack equal to \(\alpha''\), i.e.

\[
C''_{M_k} = C_{M\text{static}}(\alpha''_k)
\]
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Vortex build-up

During dynamic stall, a vortex builds up on the low pressure side of the aerofoil near the leading edge. McAlister et al. [164] determined, from a series of wind tunnel experiments investigating the dynamic stall of a NACA 0012 aerofoil, that the strong lift forces experienced by an aerofoil undergoing dynamic pitching beyond angles of attack at which stall would normally (i.e. in static conditions) occur are caused primarily by the presence of the vortex over the aerofoil and its subsequent shedding. The results of McAlister et al. also demonstrated that the influence of the vortex increases as the pitch frequency increases.

In the BL model the vortex can continue to build in strength until the load reaches the critical normal coefficient, at which time the vortex separates and travels over the aerofoil and off the trailing edge. Prior to separation the strength of the vortex can increase or decrease depending on the motion of the aerofoil. According to the BL model, the vortex lift ($C_{vk}$) is defined as the difference between the linearised unsteady circulatory lift and the unsteady non-linear lift, and is calculated as shown in Eq. 4.64.

$$C_{vk} = C_{C_Nk}^C (1 - K_{nk})$$

(4.64)

where:

$$K_{nk} = \left( \frac{1 + \sqrt{|f''_{Nk}| \text{sgn}(f''_{Nk})}}{2} \right)^2$$

The effect of the vortex on the normal load coefficient is allowed to build or decay in time according to Eq. 4.65.

$$C_{V_Nk} = C_{V_Nk-1} \exp \left( -\frac{\Delta s}{T_v} \right) + (C_{vk} - C_{vk-1}) \exp \left( -\frac{\Delta s}{2T_v} \right)$$

(4.65)

When the vortex has separated (when the critical normal load coefficient is reached) the build-up or decay of the vortex is assumed to continue as usual until the vortex has passed the trailing edge of the aerofoil. The position of the vortex on the aerofoil is tracked with the non-dimensional travel time ($\tau_v$) since the vortex has separated from the leading edge and starts travelling over the aerofoil towards the trailing edge. The non-dimensional travel time is is zero at separation (i.e. $\tau_v = 0$) and passes the trailing edge when the non-dimensional time equals the time constant $T_{vl}$ (i.e. $\tau_v = T_v$). When the vortex has passed the trailing edge ($\tau_v > T_v$), the vortex continues to influence the loads on the aerofoil, but no longer builds in strength and is assumed to decay more rapidly. This more rapid decay is accomplished by temporarily doubling the time constant $T_v$. Some qualitative tests conducted by selecting different values of $T_v$, at reduced frequencies and variations in angle of attack of interest to VAWT modelling, appeared to indicate that the results are not very sensitive to this parameter. The method has been implemented in Aeolus2D and Aeolus3D with the time constant (and the temporary
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modification to it) as defined for consistency with other implementations of the BL model.

As the vortex travels over the aerofoil, the centre of pressure also moves, which according to the original BL model will have an effect on the moment coefficient. The location of the centre of pressure due to the movement of the vortex is modelled empirically by Beddoes and Leishman using Eq. 4.66, and is used herein.

\[ x_{cp_v} = 0.20 \left[ 1 - \cos \left( \frac{\pi \tau_v}{T_{vl}} \right) \right] \]  

(4.66)

The moment coefficient about the quarter-chord is therefore given by Eq. 4.67.

\[ C_{M_k}^{V} = -x_{cp_v} C_{Nk}^{V} \]  

(4.67)

The vortex may, in theory, also have an effect on the tangential load coefficient, although this is less clearly defined. Beddoes and Leishman [160] do not specify that it should be included in the calculation of the tangential load coefficient, and nor does it appear to be included by Gupta and Leishman [163], but Pierce [161] for example, concluded it should be included and did so. Pierce modelled the effect on the tangential component in the same manner as the normal component but made it proportional to the distance the vortex has travelled along the aerofoil such that it has maximum effect when the vortex is at the leading edge, but no effect when the vortex is at the trailing edge.

In practice, the inclusion of the effect the vortex has on the moment coefficient, in particular, and on the tangential coefficient, to a lesser extent, was found to be quite dependent on the time-step size in Aeolus2D and Aeolus3D. It appears that the issue lies in the fact that the system takes discrete steps that are, by computational necessity, quite large compared to the chord length. As such, the transition between the vortex separating and the vortex passing beyond the trailing edge is quite sudden. At most practical time-step sizes the vortex is likely to go from separating away from the leading edge, to beyond the trailing edge within a single time-step. As the normal coefficient is not directly dependent on the location of the vortex (only indirectly dependent through the rate of build-up and decay of the vortex) it appears to be largely independent of time-step size. Because of this independence, in the results presented in this thesis, the leading edge build-up only contributes towards the normal load coefficient and not the tangential load coefficient or the moment coefficient. As demonstrated by the validation cases provided below in section 4.10, this method produces good results.

In the work described in this paper, the flow is assumed to reattach when either the low pressure side of the aerofoil switches from one side to the other, or the aerofoil changes from a state of moving away from stall, to a state of moving towards stall. The determination of whether the aerofoil is moving towards stall or away from stall is based on whether \( C_{N}^\prime \) is increasing or
decreasing, respectively. $C_N'$ is used as an indicator of whether the aerofoil is moving towards stall or away from stall as it appears to be a better indicator than parameters such as the angle of attack, which tends to be more sensitive to small changes in flow direction.

### 4.8.3 Total loads

The total load coefficients are determined by summing the various contributions, as shown by Eqs. 4.68, 4.69 and 4.70.

\[
C_N = C_N'' + C_N^{NC} + C_N^V + C_{D_0} \sin \alpha_E 
\]

\[
C_T = C_T'' - C_{D_0} \cos \alpha_E 
\]

\[
C_M = C_M'' + C_M^{NC} 
\]

(4.68)

(4.69)

(4.70)

Note that the minimum drag coefficient $C_{D_0}$ is added back on here. This component of drag is due to viscous forces only (friction on the surface of the aerofoil) and is assumed to be unaffected by dynamic stall\(^9\). As such, it is subtracted off the static aerofoil coefficient data prior to use in the dynamic stall model and so must be added back on at the end. The process of preparing the aerofoil coefficient data for use in the simulation is explained in more detail in appendix A.2.

### 4.8.4 Time constants

The equations in the preceding section included several time constants. Ideally these time constants would be tailored to each case (e.g. aerofoil shape, Reynolds number etc.) based on dynamic measurements. The time constants do not vary significantly with aerofoil type, however [160, 163], so in the absence of experimental measurements (as is the case with most of the aerofoils used in this thesis) a standard set of values can be applied to all aerofoils. The value of $T_\alpha$ presented in table 4.1 matches that of Minnema [162], and the values of $T_p$, $T_f$, $T_v$ and $T_{vl}$ match that of Minnema [162], Pierce [161] and Gupta and Leishman [163].

### 4.9 Determination of convergence

A fairly simple method was used using the average torque of the turbine over one revolution to determine the convergence of the aerodynamic model at steady state. If the average torque predicted by the aerodynamic model was within 1% of the average torque predicted on the previous revolution, the aerodynamics were assumed to have converged to a level suitable for the intended purposes of the work described by this thesis. The convergence was tested once per revolution on one of the blades. Note, in practice that the convergence could have been

\(^9\)While this is not strictly true, when dynamic stall effects are significant the minimum drag is considerably lower in magnitude than the total load coefficients so such an approximating is perfectly adequate.
tested each time any blade passed the same point due to the periodic nature of the loads, but for simplicity of implementation this was not done. In a worst case scenario this means the simulation may have run for up to 1 revolution more than actually required to reach equilibrium\textsuperscript{10}.

### 4.10 Validation

#### 4.10.1 Overview

The purpose of this section is to demonstrate the validity of the aerodynamic simulations (both Aeolus2D and Aeolus3D) by simulating and comparing against a variety of experimentally measured cases. As pointed out previously, all computational modelling requires a series of compromises and approximations to be made. As shown by the validations presented below, however, with careful consideration about such compromises and approximations it is possible to achieve acceptable results across a wide range of cases. It should be remembered when examining the comparisons presented in this section, that the purpose of the simulations implemented as part of this work is not to precisely model any single existing turbine. The purpose of the simulations is to capture the key behaviour of VAWTs in general with sufficient accuracy and, just as importantly, with sufficient consistency to allow comparisons to be made between similar turbines with varying parameters and characteristics. The modelling of a single specific turbine would have involved a slightly different approach and an effort to “tune” the simulations to the specific design. No such effort has been employed here. The simulations must be capable of producing reasonable approximation across a range of turbine designs that are not known a priori. Having stated that, both models perform admirably under a wide range of conditions with just a few limitations which are discussed below. The determination of suitable computational parameters (time-step size, wake length, discretisation) is presented in chapter 7.

\textsuperscript{10}This worst case scenario occurs for an infinite number of blades. In practice it runs for an additional \((1-1/(\text{Number of Blades}))\) revolutions, i.e. an extra \(1/2\) revolution for two-bladed turbines, \(2/3\) revolution for three-bladed turbines etc.
In this section, the selection of the computational parameters used for each validation case are not explained, they are simply stated here without justification. They are, of course, based on the work described in chapter 7.

4.10.2 Dynamically pitching aerofoil loads (2D)

This section uses the two dimensional aerodynamic model (Aeolus2D) to simulate a variety of wind tunnel tests conducted at the Ohio State University (OSU) on a dynamically pitching aerofoil [3, 165]. The purpose of this validation is to confirm that the model accurately predicts unsteady loads, and in particular the unsteady loads during dynamic stall (something that occurs regularly on VAWTs at low local TSRs) of a moving aerofoil. The dynamically pitching aerofoil is a somewhat simpler situation than that experienced by a VAWT blade in that the free stream velocity remains constant and the blade never crosses over its own wake. Modelling the dynamically pitching aerofoil allows the validity of the methods used to model the unsteady loads to be examined without these additional complications. An important parameter to consider when dealing with dynamically pitching aerofoils is the reduced frequency, defined in Eq. 4.71, where $V_\infty$ is the free stream flow velocity.

$$k = \frac{\omega c}{2V_\infty}$$  (4.71)

In the case of a pitching aerofoil experiencing a constant oncoming wind speed, as in the case of the OSU wind tunnel experiments, the reduced frequency has a single value. In the case of a VAWT, however, where velocity is continually changing with azimuthal position, the implication of the reduced frequency is less clear. For the purposes of this validation, however, we may assume that the average velocity is the velocity due to the motion of the blades only (i.e. $V_\infty \approx \omega R$) enabling an approximation of the reduced frequency of the VAWT to be calculated as shown in Eq. 4.72.

$$k \approx \frac{\omega c}{2\omega R} = \frac{c}{2R}$$  (4.72)

As an example of the typical reduced frequency of a VAWT, the Sandia 17m turbine has an average radius of approximately 5.65m (and a maximum radius of 8.36m), and chord length of 0.612m, which gives a reduced frequency of $k \approx 0.054$ according to Eq. 4.72. This is just an indication of the average value, however, as the actual velocity varies continually during each rotation and the radius changes along the blade.

Description of setup

A variety of different aerofoil shapes were tested at OSU. As the emphasis of the work done at OSU was for HAWTs, none of the aerofoils tested was a symmetrical aerofoil like those
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typically found on VAWTs. The closest match that was tested at OSU was the NACA 4415 which although cambered has the closest characteristics to aerofoil profiles such as the NACA 0015 frequently used in VAWT designs. In addition to the results for the NACA 4415 presented in this section, the S809 aerofoil was also simulated for comparison against the experimental measurements of Ramsay et al. [165] to ensure the results were consistent for more than just a single aerofoil. The success of the S809 aerofoil comparisons were very similar to those of the NACA 4415 aerofoil presented here so have not been included herein.

A fully, detailed, description of the experimental set-up is available in Hoffmann et al. [3]. To summarise, a NACA 4415 aerofoil section with a chord of 0.457m and a span filling the wind tunnel (approximating 2D conditions) was forced to oscillate in a sinusoidal motion about its quarter-chord at a range of angles of attack and wind speeds. The intended frequencies of oscillation were 0.6, 1.2 and 1.8Hz with a magnitude of oscillation of ±5.5° and ±10° about mean angles of attack of 8°, 14° and 20°. Reynolds numbers of 0.75, 1, 1.25 and 1.5 million were aimed for. A subset of the experimental set-ups, equivalent to a low reduced frequency (\(k \approx 0.018\)), a medium reduced frequency (\(k \approx 0.054\)) and a high reduced frequency (\(k \approx 0.11\)) was selected for comparison, providing a good range of parameters against which to validate the simulation. Note that Hoffmann et al. performed measurements of both “clean” aerofoils, and of aerofoils with artificially added leading edge grit roughness. Only comparisons with the “clean” aerofoil measurements have been made as part of this work.

Rather than simulating the intended parameters of the experiments, an examination of the data revealed small variations in wind speed, frequency of oscillation, mean angle of attack and angle of attack amplitude. The actual average wind speeds and frequencies of oscillation reported were simulated instead of the intended values. The amplitude of the angle of attack range was established by examining the actual time history measurements and estimating the approximate maximum and minimum angles of attack (ignoring clear outliers) and setting the mean value to the average of the maximum and minimum values. This is not a precise replication of the experimental conditions, but does allow for a more realistic comparison to be made between the experimental measurements and the simulated results than could have been achieved by simulating the intended experimental parameters. The intended experimental parameters for each case simulated are summarised in table 4.2 with the actual parameters used in the simulation in square brackets.

All simulations of the dynamically pitching aerofoil were run at a rate of 50 time-steps per oscillation. No restrictions were placed on the wake length or number of vortices present at any given time. The aerofoil was pitched 5 times to ensure periodic equilibrium was reached, and the results of the final oscillation are presented.
Table 4.2: Intended parameters of dynamically pitching aerofoil experiments and actual parameters used in simulated comparison.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Amplitude (deg)</th>
<th>Mean Angle (deg)</th>
<th>Frequency (Hz)</th>
<th>Reynolds No. (x10^6)</th>
<th>Reduced Freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>±5.5 ±4.90</td>
<td>8 [7.70]</td>
<td>0.6 [0.60]</td>
<td>1.50 [1.51]</td>
<td>0.0180 [0.0179]</td>
</tr>
<tr>
<td>2</td>
<td>±5.5 ±4.75</td>
<td>14 [14.15]</td>
<td>0.6 [0.60]</td>
<td>1.50 [1.50]</td>
<td>0.0180 [0.0180]</td>
</tr>
<tr>
<td>3</td>
<td>±5.5 ±4.75</td>
<td>20 [19.55]</td>
<td>0.6 [0.60]</td>
<td>1.50 [1.51]</td>
<td>0.0180 [0.0179]</td>
</tr>
<tr>
<td>4</td>
<td>±5.5 ±5.25</td>
<td>8 [7.75]</td>
<td>1.2 [1.18]</td>
<td>1.00 [1.01]</td>
<td>0.0539 [0.0525]</td>
</tr>
<tr>
<td>5</td>
<td>±5.5 ±5.30</td>
<td>14 [14.10]</td>
<td>1.2 [1.22]</td>
<td>1.00 [1.01]</td>
<td>0.0539 [0.0543]</td>
</tr>
<tr>
<td>6</td>
<td>±5.5 ±5.05</td>
<td>20 [19.65]</td>
<td>1.2 [1.22]</td>
<td>1.00 [1.01]</td>
<td>0.0539 [0.0543]</td>
</tr>
<tr>
<td>7</td>
<td>±5.5 ±5.40</td>
<td>8 [7.80]</td>
<td>1.8 [1.85]</td>
<td>0.75 [0.75]</td>
<td>0.1079 [0.1109]</td>
</tr>
<tr>
<td>8</td>
<td>±5.5 ±5.35</td>
<td>14 [14.05]</td>
<td>1.8 [1.85]</td>
<td>0.75 [0.75]</td>
<td>0.1079 [0.1109]</td>
</tr>
<tr>
<td>9</td>
<td>±5.5 ±5.30</td>
<td>20 [19.20]</td>
<td>1.8 [1.85]</td>
<td>0.75 [0.76]</td>
<td>0.1079 [0.1094]</td>
</tr>
<tr>
<td>10</td>
<td>±10.0 ±10.20</td>
<td>8 [7.20]</td>
<td>0.6 [0.62]</td>
<td>1.50 [1.50]</td>
<td>0.0180 [0.0186]</td>
</tr>
<tr>
<td>11</td>
<td>±10.0 ±10.10</td>
<td>14 [13.90]</td>
<td>0.6 [0.60]</td>
<td>1.50 [1.49]</td>
<td>0.0180 [0.0181]</td>
</tr>
<tr>
<td>12</td>
<td>±10.0 ±10.05</td>
<td>20 [18.55]</td>
<td>0.6 [0.58]</td>
<td>1.50 [1.48]</td>
<td>0.0180 [0.0176]</td>
</tr>
<tr>
<td>13</td>
<td>±10.0 ±10.70</td>
<td>8 [7.20]</td>
<td>1.2 [1.18]</td>
<td>1.00 [1.01]</td>
<td>0.0539 [0.0525]</td>
</tr>
<tr>
<td>14</td>
<td>±10.0 ±10.60</td>
<td>14 [13.90]</td>
<td>1.2 [1.18]</td>
<td>1.00 [1.01]</td>
<td>0.0539 [0.0525]</td>
</tr>
<tr>
<td>15</td>
<td>±10.0 ±10.55</td>
<td>20 [18.55]</td>
<td>1.2 [1.18]</td>
<td>1.00 [0.97]</td>
<td>0.0539 [0.0547]</td>
</tr>
<tr>
<td>16</td>
<td>±10.0 ±11.00</td>
<td>8 [7.40]</td>
<td>1.8 [1.85]</td>
<td>0.75 [0.75]</td>
<td>0.1079 [0.1109]</td>
</tr>
<tr>
<td>17</td>
<td>±10.0 ±10.85</td>
<td>14 [14.15]</td>
<td>1.8 [1.79]</td>
<td>0.75 [0.74]</td>
<td>0.1079 [0.1087]</td>
</tr>
<tr>
<td>18</td>
<td>±10.0 ±10.80</td>
<td>20 [18.70]</td>
<td>1.8 [1.85]</td>
<td>0.75 [0.72]</td>
<td>0.1079 [0.1155]</td>
</tr>
</tbody>
</table>

Normal load coefficients

A comparison between the experimentally measured normal load coefficient and that predicted by Aeolus2D is presented in Fig. 4.16 for cases 1 through 9 (amplitude of oscillation ≈ ±5.5°) and Fig. 4.17 for cases 10 through 18 (amplitude of oscillation ≈ ±10°). The individual plots within each figure are arranged such that the columns represent increasing mean angles of attack from left to right and the rows represent increasing reduced frequencies from top to bottom. In all cases, the prediction of the normal load coefficient by Aeolus2D is excellent. The only cases which may be described as merely very good are cases 16 through 18, shown in Fig. 4.17. These cases, representing a large amplitude of oscillation at a high reduced frequency, show a slight over-estimation of the normal load coefficient on the lower side of the hysteresis loop (as the nose of the aerofoil pitches downwards). The over estimation is present at all mean angles of attack, but is most noticeable at the highest mean angle of attack (i.e. Case 18). Overall however, the comparison between Aeolus2D and the experimental measurements is excellent.

Tangential load coefficients

A comparison between experimentally measured tangential load coefficients and those predicted by Aeolus2D is presented in Fig. 4.18 for cases 1 through 9 (amplitude of oscillation ≈ ±5.5°) and Fig. 4.19 for cases 10 through 18 (amplitude of oscillation ≈ ±10°). Although not
as impressive as the prediction of the normal load coefficients shown above, the comparison between Aeolus2D and the experimental measurements is still very good overall. In general, Aeolus2D appears to slightly underestimate the tangential load coefficient on the upper side of the hysteresis loop (as the nose of the aerofoil pitches upwards). At large amplitudes of oscillation at high mean angles of attack (Cases 12, 15 and 18), and also at moderate mean angles of attack at a high reduced frequency (Case 17), Aeolus2D appears to slightly overestimate the tangential load coefficient on the lower side of the hysteresis loop (as the nose of the aerofoil pitches downwards). The poorest comparison between the experimental measurements and Aeolus2D is clearly at a high mean angle of attack, at a high reduced frequency and a low amplitude of oscillation (i.e. Case 9). Aeolus2D does a reasonable job of predicting the
Figure 4.17: Normal load coefficients of a NACA 4415 aerofoil dynamically pitching through a geometric angle of attack of approximately $\alpha_g \pm 10^\circ$ at various mean geometric angles of attack (increasing left to right) and reduced frequencies (increasing top to bottom).

tangential load coefficient as the nose is pitching upwards, but as the nose pitches downwards the normal tangential coefficient is significantly underestimated. The dynamic stall model used to predict the loads at these high angles of attack is dependent on the static aerodynamic characteristics. In the instance of Case 9, the static measurements of the tangential load coefficient versus angle of attack can be seen to drop very sharply just prior to the maximum angle of attack being reached during the dynamic pitching. The sharp drop at this point has a strong impact on the loads predicted by the dynamic stall model. For the larger amplitude of oscillation (e.g. Case 18), the maximum angle reached while pitching is several degrees higher than the sharp drop and the predicted tangential loads do not appear to be strongly affected by it. Also, at lower reduced frequencies (e.g. Cases 3 and 6), the higher Reynolds numbers
involved (approximately 1.5 and 1.0 million respectively, versus 0.75 million of case 9) show the sharp drop-off in the static tangential load coefficient just above the maximum angle of attack reached during pitching and as such are not as strongly affected by it. The tangential loads predicted by Aeolus2D were, as mentioned, very good overall. However, the discrepancy in accuracy observed in the case of a pitching aerofoil for which the maximum angle of attack coincided closely with a sharp change in the static tangential load coefficient should be taken into consideration when interpreting the results of VAWTs operating at very low TSRs.
Figure 4.19: Tangential load coefficients of a NACA 4415 aerofoil dynamically pitching through a geometric angle of attack of approximately $\alpha_g \pm 10^\circ$ at various mean geometric angles of attack (increasing left to right) and reduced frequencies (increasing top to bottom).

**Moment coefficients**

A comparison between the experimentally measured moment coefficient and that predicted by Aeolus2D is presented in Fig. 4.20 for cases 1 through 9 (amplitude of oscillation $\approx \pm 5.5^\circ$) and Fig. 4.21 for cases 10 through 18 (amplitude of oscillation $\approx \pm 10^\circ$). From the results presented it is clear that, although certainly not bad, the ability to accurately predict the moment coefficient of the dynamically pitching aerofoil by Aeolus2D is weaker than its ability to predict the normal and tangential load coefficients. In general Aeolus2D appears to both underestimate the moment coefficient on the upper side of the hysteresis loop (as the nose of the aerofoil pitches upwards) and overestimate the moment coefficient on the lower side of the hysteresis loop (as the nose of the aerofoil pitches downwards). At low reduced frequencies...
the moment coefficient of the dynamically pitching aerofoil tracks very closely to the moment coefficient of the static aerofoil so unsurprisingly, in such cases (Cases 1,2,3 and 10,11,12) the predictions by Aeolus2D compare very well. Also, the predictions are reasonable, albeit conservative, at low mean angles of attack (Cases 1,4,7 and 10,13,16). It is at moderate to high mean angles of attack and moderate to high reduced frequencies (Cases 5,6,8,9 and 14,15,17,18) that all the dynamic effects do not appear to be fully accounted for by Aeolus2D. Having stated that, when a full understanding of all the dynamic effects are unknown, a slightly conservative estimate of the effects is preferable to an overestimation of their impact. If the effects are overestimated there is a risk that aeroelastic behaviour or instabilities may be predicted that would not, in fact, occur in reality. With a more conservative estimate of the effects, any unusual aeroelastic behaviour predicted can be interpreted with more confidence as likely to being real.

4.10.3 One-bladed turbine blade loads (2D)

Building on the validation of Aeolus2D against a dynamically pitching aerofoil in the previous section, this section presents a comparison between experimentally measured results of a one-bladed VAWT in a water towing tank and the loads predicted by Aeolus2D simulating the same case. The normal ($F_N$) and tangential ($F_T$) loads experimentally measured by Graham [168] were (and are, in this section) presented in a non-dimensional form ($F_N^*$ and $F_T^*$) according to Eqs. 4.73 and 4.74.

\[
F_N^* = \frac{|F_N|}{(1/2)\rho V_\infty^2 c} = C_N \left( \frac{|V_{sec}|}{V_\infty} \right)^2 \quad (4.73)
\]

\[
F_T^* = \frac{|F_T|}{(1/2)\rho V_\infty^2 c} = C_T \left( \frac{|V_{sec}|}{V_\infty} \right)^2 \quad (4.74)
\]

The loads predicted by Aeolus2D are also transformed, in this section, into the same non-dimensional representation for consistency with the original experimental measurements.

Description of setup

A detailed explanation of the experimental set-up is given by Graham [168], and a brief summary is provided in section 7.3.1 (where the configuration is identified as VAWT A) in relation to the determination of the computational parameters required, the key properties of which are repeated in table 4.3.

It should be pointed out that the measurements from Graham’s experiments were also presented in Oler et al. [108], however the attachment point\textsuperscript{11} is identified there as 0.5, not 0.467. As

\textsuperscript{11}The attachment point is the location along the chord of the blade where it is attached to the rotating support arm, given as the proportion of the chord as measured from the leading-edge of the blade.
Figure 4.20: Moment coefficients of a NACA 4415 aerofoil dynamically pitching through a geometric angle of attack of approximately $\alpha_g \pm 5.5^\circ$ at various mean geometric angles of attack (increasing left to right) and reduced frequencies (increasing top to bottom).

Table 4.3: Key properties of the one-bladed wind turbine configuration used to validate Aeolus2D.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blades</td>
<td>1</td>
</tr>
<tr>
<td>Radius</td>
<td>0.62m</td>
</tr>
<tr>
<td>Chord</td>
<td>0.1524m</td>
</tr>
<tr>
<td>Rate of Rotation</td>
<td>0.7484 rad/s</td>
</tr>
<tr>
<td>Aerofoil</td>
<td>NACA 0015</td>
</tr>
<tr>
<td>Attachment Point</td>
<td>0.467</td>
</tr>
<tr>
<td>Medium</td>
<td>Water</td>
</tr>
</tbody>
</table>
Figure 4.21: Moment coefficients of a NACA 4415 aerofoil dynamically pitching through a geometric angle of attack of approximately $\alpha_g \pm 10^\circ$ at various mean geometric angles of attack (increasing left to right) and reduced frequencies (increasing top to bottom).

The original measurements were made by Graham [168], the configuration described there was assumed to be the correct one. It is unlikely that the small difference will have a measurable influence on the results, but the difference needed to be highlighted nonetheless. The VAWT had an average blade Reynolds number of approximately 67000. The aerodynamic lift and drag coefficients of the aerofoil at a Reynolds number of 67000 were approximated, for use in Aeolus2D, by linear interpolation between the lift and drag coefficient data (with the curves first parameterised prior to interpolation, see appendix A.1 for more details) given in the appendix of Graham [168] at Reynolds numbers of 40000 and 30000. The VAWT was towed at velocities of 0.183 m/s, 0.091 m/s and 0.061 m/s giving TSRs of 2.5, 5.1 and 7.6 respectively. Measurements were made by Graham using both strain gauges (at the top of the blade, at
Figure 4.22: Comparison of non-dimensional normal force pressure measurements and strain gauge measurements with Aeolus2D predictions for repeated revolutions of the one-bladed VAWT.

mid-chord) and pressure taps (located at chord positions of 0.017, 0.042, 0.1, 0.36 and 0.81, approximately 300mm below the surface of the water). The experimental measurements were corrected for centrifugal forces (i.e. the centrifugal forces were estimated and subtracted from the measurements) and finite aspect ratio effects, the details of which are given in Graham [168]. Because of the limited number of pressure taps, the pressure measurements may be quite prone to error.

The simulations were run at a rate of 72 time-steps per revolution with no restrictions placed on the length of the wake or the number of vortices present at any given time.
Comparison of several sequential revolutions

The non-dimensional normal and tangential blade loads for seven revolutions of the turbine are presented together with the the available experimental pressure and strain gauge measurements in Figs. 4.22 and 4.23. Experimental measurements for the first revolution at any given TSR were not provided and the total number of revolutions measured varied depending on TSR, presumably due to limitations in the length of the water towing tank being used (i.e. fewer revolutions would be possible at low TSRs as the experimental rig must be towed more quickly), but possibly because at very low TSRs periodic equilibrium will be reached much more quickly so additional revolutions are less necessary. Note, Aeolus2D effectively assumes infinite acceleration is possible (i.e. the turbine goes from rest to its required rotational velocity
instantly). Although the experimental rig will accelerate to its operational speed quickly, it will have inertia to overcome so if data from the first revolution were available differences would be expected during that initial revolution. The differences would be expected to disappear very quickly, however, and by the second revolution this difference should be immeasurably small.

The normal pressure measurements, strain gauge measurements and predictions from Aeolus2D over several subsequent revolutions are presented in Fig. 4.22 which illustrates several important factors. Firstly, it is clear that at the lower TSR, periodic equilibrium is reached very quickly and at the higher TSRs, additional revolutions are required to reach equilibrium. Arguably, at a TSR of 7.6 (Fig. 4.22(c)), even after seven revolutions the system may not quite have reached periodic equilibrium. At a TSR of 2.5, on the other hand (Fig. 4.22(a)), the difference between the second and seventh revolutions is quite minor. Secondly, the strain gauge measurements show much less variation from one revolution to the next than the pressure measurements. This was also commented on by both Graham [168] and Oler et al. [108]. The variation does not appear to be very significant at a TSR of 2.5, but with data over much fewer revolutions available, definitive conclusions about this would be premature. The final factor of note in Fig. 4.22 is that the comparison between Aeolus2D and the experimental measurements is excellent at a TSR of 2.5, very good at a TSR of 5.1 and only qualitatively good (e.g. the peaks are in the right places) at a TSR of 7.6. This is not completely clear-cut, however, as is discussed below. A closer examination of an individual revolution shows that the accuracy of the measurements at a TSR of 7.6 is debatable.

The tangential pressure measurements, strain gauge measurements and predictions from Aeolus2D over several subsequent revolutions are presented in Fig. 4.23. Like the normal loads, it is clear from Fig. 4.23 that the pressure measurements are quite variable from one revolution to the next, although even more so than the normal loads. In fact, the tangential pressure measurements appear quite erratic, particularly at a TSR of 7.6 (Fig. 4.23(c)). This should not be completely surprising, however, as estimating tangential loads on an aerofoil by pressure tap measurements is notoriously challenging. Considerably more pressure tap locations would be required to obtain very accurate results. Interestingly, at a TSR of 7.6 in particular (but also at the other TSRs to a lesser degree), the strain gauge measurements also show more variation from one revolution to the next. The variation is much lower than that of the pressure measurements, but should be noted in the detailed examination of a single revolution made below.
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Figure 4.24: Comparison of non-dimensional normal and tangential force pressure measurements, strain gauge measurements, and VDART2 predictions with Aeolus2D predictions for the 2nd revolution of the one-bladed VAWT at a tip speed ratio of 2.5.

Comparison of single revolution

Figure 4.24 presents the experimentally measured and computationally predicted results over the second revolution at a TSR of 2.5. In addition to the pressure and strain gauge measurements, the predictions of the vortex wake model VDART2 (developed by Strickland et al. [96, 97]) are included for comparison with the predictions of Aeolus2D. It is clear from this figure that the comparison between the normal loads predicted by Aeolus2D and the experimental measurements are excellent (as are the predictions of VDART2). At such a low TSR, the variation in angle of attack is large and dynamic stall effects are present. The accuracy of these predictions confirm the capabilities of the dynamic stall model used in Aeolus2D to capture these effects.

On the upstream side of the turbine (azimuthal positions of 0° to 180°), Aeolus2D shows a considerably better comparison with the tangential experimental measurements than VDART2. The maximum magnitude of the tangential load coefficient is predicted very well by Aeolus2D and grossly over estimated by VDART2. On the downstream side of the turbine, both com-

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12This was probably due to the fact that the pressure measurements were point measurements, whereas the strain gauge measurements integrated the pressure over area.
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Figure 4.25: Comparison of non-dimensional normal and tangential force pressure measurements, strain gauge measurements, and VDART2 predictions with Aeolus2D predictions for the 4th revolution of the one-bladed VAWT at a tip speed ratio of 5.1.

Computational models compare well to the experimental measurements. Assuming that the strain gauge measurements are more accurate than the pressure measurements, VDART2 is arguably slightly more accurate over the 180° to 270° azimuthal range and Aeolus2D more accurate over the 270° to 360° range. Allowing for experimental measurement error, however, both computational models do very well.

Figure 4.25 presents the experimentally measured and computationally predicted results over the fourth revolution at a TSR of 5.1. Again, the normal loads predicted by Aeolus2D (and VDART2) compare very well with the experimental measurements. The normal loads predicted by Aeolus2D may appear to be a slight underestimation, but considering experimental measurement error, the difference is too small to draw such a conclusion decisively. The tangential loads also compare well, particularly over the downstream side (again, assuming the strain gauge measurements are more reliable than the pressure measurements). Interestingly, at this TSR, VDART2 also does a better job (than it did at a TSR of 2.5) of predicting the tangential loads over the upstream side of the turbine. At this TSR, dynamic stall effects are likely to be much lower (if present at all). This may be an indication that the dynamic stall model used in VDART2 lacks the capabilities of the BL dynamic stall model implemented in Aeolus2D at predicting the influence of dynamic stall effects on the tangential loads.
The final set of comparisons in this section are the predicted and measured results of the sixth revolution of the turbine at a TSR of 7.6, as presented in Fig. 4.26. At first glance, Aeolus2D appears to under-predict the normal loads at all azimuthal positions. With more consideration, however, this is not as clear cut. Examining the strain gauge measurements, the normal load remains positive at all azimuthal positions throughout the full revolution. This would mean that the flow over the blade remains on the same side at all times and the load is always directed towards the inside of the arc of rotation. This should be impossible as rotation in a non-stationary flow field would require the direction of normal loading on the aerofoil alternate on the upstream and downstream sides of the turbine. Most likely, the strain gauge measurements are in error and should, in reality, be shifted towards the predictions of Aeolus2D. It should be remembered, also, that at such a high TSR the magnitude of the angle of attack at any point is very small so even minor errors in measurement could lead to quite large relative differences. Less easy to explain is how VDART2 produces predictions so close to the strain gauge measurements. For the same reasons, however, these too are likely to be in error but an explanation of why the loads remain positive throughout a full revolution was not addressed by either Graham [168] or Oler et al. [108]. It is tempting to point out how close the pressure tap measurements are to the Aeolus2D predictions, in particular the quite
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Figure 4.27: Overall performance of both the Sandia 17m VAWT at rotational rates of (a) 42.2 rpm and (b) 50.6 rpm and the Éole 64m VAWT at rotational rates of (c) 10.00 rpm and (d) 11.35 rpm.

flat relationship between load and azimuthal position on the downstream side. As already discussed, however, the considerable variation possible with the pressure tap measurements from revolution to revolution means that this cannot reliably be attributed to anything other than coincidence. The tangential loads are quite well predicted by Aeolus2D, comparing well with the strain gauge measurements. The variation of tangential load with azimuthal position is, perhaps, predicted to be a little too flat by Aeolus2D on the downstream side of the turbine. But, in the vicinity of about 0° to 40° and 330° to 360° (i.e. when the aerofoil is almost moving straight into the flow) Aeolus2D does a much better job than VDART2, which over-predicts the tangential loads to the extent that the magnitude is similar but the sign is wrong.

4.10.4 Overall performance (3D)

For the purposes of the work described by this thesis the actual overall power production is not, in fact, of particular importance. However, for the sake of completeness this section presents comparisons between the overall performance as predicted by Aeolus3D and the experimentally measured performances of four different VAWT cases. The cases used for comparison are the Sandia 17m turbine at two different rotational rates (42.2 rpm [14] and 50.6 rpm [169]) and
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the 64m Éole at two different rotational rates (10 rpm and 11.35 rpm [14]). The results are presented in Fig. 4.27. The performance coefficient, as defined in Eq. 4.75 is used in place of the power coefficient (given by Eq. 4.76) because, as pointed out by Worstell [25], the power coefficient’s sensitivity to wind speed (which can be very challenging to measure accurately in field tests) makes it less suitable for comparison than the performance coefficient.

\[
K_P = \frac{T \omega}{(1/2) \rho A(R \omega)^3} \quad (4.75)
\]

\[
C_P = \frac{T \omega}{(1/2) \rho A V_\infty^3} \quad (4.76)
\]

The performance coefficient is sensitive to the radius of the VAWT and the angular velocity, and these terms can generally be measured with more accuracy than the wind speed. The performance coefficient is plotted against the advance ratio (the inverse of the TSR) for consistency with the convention used by Sandia, though TSR (or indeed, wind speed) would have sufficed for the comparisons shown in this section.

In this validation case (and the validation cases presented in the following sections), the simulation was run at a rate of 36 time-steps per revolution until equilibrium was reached, and then a final additional revolution was run at a rate of 72 time-steps per revolution from which the results are based. The wake length was restricted to three turbine diameters downstream of the turbine (downstream of the furthest point the blade passes, i.e. 3.5 turbine diameters downstream of the tower). Each turbine blade was divided into nine equal length aerodynamic elements along their span. The tower drag coefficient was assumed to be \( C_{D_{\text{tower}}} = 0.5 \).

An examination of the predicted performance of the Sandia 17m turbine (Fig. 4.27(a) and (b)) shows that for these cases at high advance ratios (low TSRs) Aeolus3D appears to slightly under-predict the overall performance, and at very low advance ratios (very high TSRs) appears to slightly over-predict the overall performance. At moderate advance ratios (moderate TSRs) the predictions made by Aeolus3D compare very well with the measured results. At the high advance ratios, where the performance is under-predicted by Aeolus3D, the magnitude of the performance coefficient is large, and as a proportion the predictions are (although certainly not perfect) are acceptable. At very low advance ratios, where Aeolus3D over-predicts the performance the magnitudes of the performance coefficient is very small. Although as a ratio the predictions made by Aeolus3D are very poor at these very low advance ratios (particularly at 50.6 rpm) the actual coefficient magnitude is so small that this is not significant for the purposes of the work described in this thesis.

It must be noted that when interpreting the comparison between the performance predicted by Aeolus3D and the actual measurements made of the Éole 64m turbine (Fig. 4.27(c) and (d)) that several approximations were required that reduce the accuracy of the model. Unlike
the Sandia 17m turbine, the Éole 64m turbine has two support struts between the blades and the tower. The drag on these struts will act to reduce the torque (and hence the performance coefficient) from that of a strut-free structure. These struts are not modelled by Aeolus3D, so all else being equal it is expected that Aeolus3D should predict higher performance coefficients than the actual turbine will experience. Additionally, unlike the Sandia 17m turbine where properties of the typical boundary layer wind profile were readily available (e.g. Worstell [25]), assumptions were required regarding the wind field experienced by the Éole 64m turbine for use in Aeolus3D. The wind field was assumed to have a logarithmic profile with a roughness length of 0.02m (i.e. open terrain and grassland with few obstructions). Bearing in mind the expected over estimation by Aeolus3D of the overall performance due to a lack of the strut drag, the accuracy of the predictions at 10 rpm for the Éole 64m turbine appears similar to the accuracy of the predictions of the Sandia 17m turbine at 50.6 rpm, i.e. slight under-prediction at high advance ratios, reasonable predictions at moderate advance ratios and an over-prediction at low advance ratios. The comparison between the performance coefficients predicted by Aeolus3D and the measured results for the Éole 64m turbine at 11.35 rpm are the poorest of the four cases presented here. Even allowing for the unaccounted drag of the struts, the predicted values appear to be too high at all advance ratios. Although the magnitude appears consistently over estimated for this case, the shape of the predicted performance coefficient curve tracks quite well with the measured values which is an important consideration. The variation in performance as advance ratio changes (i.e. as the wind speed changes) appears to be predicted well, albeit not the correct magnitude.

On balance, despite a less than excellent prediction of the performance of the Éole 64m turbine at 11.35 rpm, and some limitations at very low and high advance ratios (very high and low TSRs) for the other cases, in general the overall performance is well predicted using Aeolus3D.

4.10.5 Two-bladed turbine torque (3D)

Of more interest (for the focus of this thesis) than the overall performance presented in the previous section, is the variation in loading as the turbine rotates. Akins et al. [169] conducted a series of tests on the Sandia 17m turbine, at 50.6 rpm, measuring the total torque as it varied with azimuthal position at a variety of TSRs.

Figure 4.28 presents the total turbine torque as predicted by Aeolus3D and the total turbine torque experimentally measured by Akins et al. versus azimuthal position at low (TSR=2.87), moderate (TSR=4.61) and high (TSR=6.40) TSRs. Note, that only one half of a revolution is shown as the total torque of a two-bladed turbine has a period of 180°, i.e. the total turbine torque at any given azimuthal position $\Theta$ is equal to the total turbine torque at azimuthal position $\Theta + 180^\circ$.
At a TSR of 2.87 (Fig. 4.28(a)) the predictions from Aeolus3D are generally very good compared with the experimental measurements. From 0° to about 45° the predictions from Aeolus3D are slightly less than the measurements but considering the challenges in modelling the dynamic stall experienced by all parts of the blades on both the upstream and downstream sides of the turbine at this TSR, the predictions are still very good. At all other azimuthal positions, the predictions are excellent at this TSR.

At a TSR of 4.61 (Fig. 4.28(b)) the mid-sections of the blades are only just stalling on the upstream side of the turbine, but the flow over the mid-sections remains attached on the downstream side of the turbine. The prediction of the torque by Aeolus3D between 0° and 45° compares better with the measurements than at a TSR of 2.87, but from about 80° to about 130° the predictions by Aeolus3D are slightly higher than the measurements. Overall, the predictions at this TSR are very good.
At a TSR of 6.40 (Fig. 4.28(c)) the flow over most of the span of the blades remains fully attached throughout each revolution. Only the parts of the blades closer to the tower continue to experience stall at this TSR. From 0° to about 70° the predictions of Aeolus3D compare very well with the measurements, but are over-predicted for the remaining azimuthal positions. At this TSR, secondary effects such as drag from support struts or the aerodynamic effect of a rotating tower are more significant than dynamic stall effects [14] and these are not fully accounted for by Aeolus3D. Additionally, as the flow remains fully attached at most sections of the blades, the slope of the aerofoil lift coefficient curve, and the minimum drag coefficient become much more important factors. Very slight changes to these parameters have a more significant impact on the predicted torque than at lower TSRs, where the blade sections spend a much greater proportion of their time in stall. Finally, the magnitude of the torque at this TSR is also much less than at the lower TSRs, and the accuracy of the measurements themselves become more debatable, though this is unlikely to account for all of the differences between the predictions and the measurements.

Importantly, for the purposes of the work described by this thesis, the accuracy of the predictions at high TSRs is of less concern than at low TSRs as the actual loads are so much lower. Any aeroelastic deflections or instabilities are more likely to cause concern at the lower TSRs where very high and variable loads are encountered.

### 4.10.6 Two-bladed turbine equatorial loads (3D)

The validation presented in previous section was the total turbine torque, which is an averaged value for the whole turbine. In this section, the normal and tangential coefficients at the equator of one of the two blades is compared with a series of measurements made by Akins [170]. Akins measured the surface pressures along the chord at the equator (or more precisely, 0.25 of a chord below the equator) of one of the blades of the Sandia 17m turbine at 38.7 rpm at a variety of TSRs. A detailed description of the experimental setup is available in Akins [170], with only an overview provided here. Initially 14 pressure transducers were mounted along both the inner and outer surfaces of the blade but as several transducers failed during the experiment, the presented measurements were based on the measurements from 9 transducers along the inner surface and 11 along the outer surface. Normal and tangential force coefficients ($C_N$ & $C_T$) were estimated via integration of the pressure coefficient ($C_P$) curves along the chord of the blade, which were estimated via cubic spline interpolation of the transducer measurements.

The normal load coefficients predicted by Aeolus3D compared with the experimental measurements for TSRs ranging from 2.20 to 4.60 are presented in Fig. 4.29. At all TSRs, the comparison between the normal coefficients predicted by Aeolus3D and those measured experi-
mentally are very good on the upstream side of the turbine (azimuthal positions of 0° to 180° ). At TSRs of 3.70 and 4.60 the comparison is excellent at all azimuthal positions on both the upstream and downstream sides of the turbine. At a TSR of 2.20, at an azimuthal position of around 220° the experimental measurements show a sudden drop in negative normal load (a drop in suction), which recovers before reaching an azimuthal position of 270° . Akins notes that this may be related to where the blade crosses the wake left by earlier blade passes. The sudden drop in negative normal load is not predicted by Aeolus3D, in fact Aeolus3D predicts an increase in negative normal load in the vicinity of these azimuthal positions. Although an interesting effect, a further examination into the cause of this drop in normal load coefficient is beyond the scope of the work described by this thesis. At the only slightly higher TSR of 2.33, this sudden change in normal load coefficient diminishes dramatically, and at a TSR of 2.49 it is barely observable.
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With the caveat that Aeolus3D predicts normal coefficients lower than those experimentally measured between azimuthal positions of about 200° and 250° at the lower TSRs, the ability of Aeolus3D to accurately predict the normal load coefficients at the equator of the blade is clearly very good overall.

The tangential load coefficients predicted by Aeolus3D compared with the experimental measurements are presented in Fig. 4.30. It should be pointed out at this stage, that the estimation of the experimental tangential load coefficients by integration of the pressures measured by the pressure transducers is very sensitive to error. The tangential load coefficients can be almost an order of magnitude less than the normal load coefficients, which in itself poses challenges in obtaining accurate measurements, but additionally the greatest contribution to the determination of the tangential load coefficient is the single pressure transducer at the leading edge of the blade. With such dependence on a single pressure transducer, any errors in the measurement
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by the transducer will have much greater effect on the tangential load coefficient than an error from a single pressure transducer on the normal load coefficient.

At TSRs of 3.70 and 4.60, the shape of the tangential coefficient curve is predicted very well by Aeolus3D, however the magnitude of the predicted tangential loads are generally less than the experimentally measured loads. The experimental approach of using pressure transducers does not account for skin friction drag, so the experimental measurements of the tangential loads are expected to be slightly higher than the real loads.

At the lower TSRs presented (2.20 through 2.87) a comparison between the magnitudes of the tangential load coefficients predicted by Aeolus3D and the tangential load coefficients experimentally measured appears, at first, to be quite poor. On closer examination however, from an azimuthal position of 0° to about 60° - 70° the prediction by Aeolus3D tracks closely with the experimental measurements. Also, from about 310° to 360° the results also compare very well. The qualitative shapes of the curves are also similar. Aeolus3D predicts two peaks on the upstream side of the turbine and two peaks on the downstream side of the turbine with the first peak on each side being larger than the second peak. The experimental measurements show this same behaviour. The location and magnitude of the first peak on the upstream side of the turbine and the location of the first peak on the downstream side of the turbine is very well predicted by Aeolus3D. Although the magnitude of the first peak on the downstream side of the turbine is overestimated by Aeolus3D, the prediction of the magnitude of the second peak is much better. The main difference between the tangential loads predicted by Aeolus3D and the experimental measurements is that Aeolus3D predicts that between the peaks, the tangential load coefficient drops to about zero, whereas the experimental measurements only drop this low at an azimuthal position of 0° / 360° as the blade points directly into the wind. Whether the differences are due to measurements errors, or due to some effect not being modelled in Aeolus3D would be an interesting investigation, but is beyond the scope of the work described by this thesis. Even in the vicinity of an azimuthal position of 180° where the blade is pointing directly downstream, the fact that the measured tangential loads remain quite high could indicate that the problem lies with the experimental measurements rather than a genuine effect not being captured in Aeolus3D. As the blade passes from the upstream side of the turbine to the downstream side of the turbine, the direction of the lift forces move from one side of the blade to the other (move from inner surface to outer surface). At this point, the only contribution towards the tangential load coefficient is the drag force. At all TSRs above one, the drag force opposes the direction of the motion of the blade. It is technically possible that there is some dynamic effect that has not previously been observed, and hence is not modelled in Aeolus3D, that allows the tangential load to remain high in this situation but it is more likely that the experimental measurements are not correctly detecting the drop in tangential load here.
4.10.7 Conclusion

The validations presented in this section demonstrate that both Aeolus2D and Aeolus3D are capable of modelling the aerodynamic behaviour of a VAWT, including the important and challenging dynamic stall effects. In general, the predictions of normal loads are very good across a wide range of angle of attack variations at various reduced frequencies (in the case of the dynamically pitching aerofoil), and across a wide range of TSRs (in the case of VAWTs). The tangential loads are also generally well predicted, although not with the same level of accuracy as the normal loads. The difference between the predictions of Aeolus3D and the tangential loads measured at the equator of the Sandia 17m VAWT at low TSRs is more than desired but, as discussed above, could be due to a variety of factors. The general characteristics of the moment of the dynamically pitching aerofoil are qualitatively captured, but the ability to accurately predict the moment certainly appears to be weaker than the ability to predict normal and tangential loads. On the positive side, however, the predictions of dynamic effects were on the conservative side, tending to predict moments closer to the static measurements which is favourable to over-predicting these effects.
Chapter 5

Structural modelling

5.1 Introduction

This chapter describes the approach used to model the structural dynamics of VAWTs. The chapter is introduced with an overview of the modelling methods used and a summary of some of the terminology used throughout the rest of this chapter. Following this is a description of the general approach used to model the elastic behaviour of the structural members in the simulation. Details of the processes and calculations performed during each simulated structural time-step are described, specifically the modelling of the motion of the rigid bodies themselves, and the details of the method for modelling the joint constraints. This is followed by a description of a method for extracting the modal characteristics from the structural simulation, which is needed for validation. Finally, several sets of results are presented verifying the structural simulation against a commercial package, and validating the structural simulation against experimental measurements and theoretical models available from the literature.

The three-dimensional simulation with the structural aspects included will continue to be referred to as Aeolus3D. Aeolus3D was implemented to allow either a fully integrated simulation (which is discussed in chapter 6), a purely aerodynamic simulation (i.e. prescribed motion) or a purely structural simulation (i.e. operation within a vacuum). Whereas all examples presented in chapter 4 used the aerodynamic aspects of the simulation in isolation from the structural aspects of the simulation, in this chapter all examples presented use the structural aspects of the simulation in isolation from the aerodynamic aspects. No structural simulation capabilities
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5.2 Overview of the structural simulation

5.2.1 Structural components

The structural model of the VAWT can, like the aerodynamic model, be broken down into a number of sub-components. The physical members of the structural model consist of a number of rigid bodies (section 5.4) arranged in series in groupings of four bodies each. The rigid bodies are connected together with joints constraining the relative motion between them to certain degrees of freedom (section 5.5). Springs and dampers are used at each joint to model the elasticity of the members (section 5.3). The motor/generator model (henceforth referred to as just the generator) is implemented as part of the joint constraint model to ensure a predefined angular velocity is maintained (section 5.5.3). Additionally, a method was developed to perturb the structural system allowing the transient behaviour to be measured, and a linearised model of the system to be automatically produced for modal analysis (section 5.6).

5.2.2 Coordinate systems and terminology

Directions of motion

There are a variety of terms used in aeroelastic research to describe the directions of motion of blades and wings. The terms are not always used consistently, however, so for the purposes of the work described here, the most important ones, as illustrated in Fig. 5.1, are defined. Flatwise, edgewise and torsional are all motions relative to the orientation of blade sections themselves. Flatwise motion is an in-plane motion of the blades, i.e. bending about an axis parallel to the blade’s chordline. Edgewise motion is an out-of-plane motion of the blades, i.e. bending about an axis normal to the blade. Torsional motion, also called twist, is a rotation.

Figure 5.1: Direction of the motion of a blade section terminology

were added to Aeolus2D and as such are not discussed further in this chapter.
about an axis along the blade spanline. There are also two terms (*flapwise* and *lead-lag*) used to describe the direction of motion relative to the fluid flow rather than to the orientation of the blade section. These terms are not usually necessary for VAWT analysis as the constantly changing direction of fluid flow makes them less conceptually useful. They are defined here, however, to avoid any confusion with the blade section centric terms. Flapwise motion is a motion in a direction perpendicular to the direction of fluid flow and lead-lag motion is a motion in a direction parallel to the direction of fluid flow.

**Directions of components of force**

The directions of the components of force on the blades in the structural simulation are identical to those of the aerodynamic simulation. The local $y$-axis is normal to the blade, positive in a direction towards the inside of the arc of rotation of the VAWT. The local $z$-axis is parallel to the chordline of the blade, positive from leading to trailing edge. The local $x$-axis of a blade rigid body is oriented along the span of the blade, orthogonal to the $y$ and $z$-axes, positive in a direction from the base to the top of the VAWT.

The main difference between the aerodynamic and structural simulations is that the local coordinate system of each rigid body is centred at the centre of mass of the rigid body. Keeping the local coordinate system centred at the centre of mass results in quite straightforward equations of motion. The initialisation of the system can in some cases be slightly more complex, as parameters such as the mass moment of inertia and the locations of the elastic and shear centres must be determined relative to the centre of mass, instead of somewhere conceptually simpler like the mid-chord or trailing edge, but once initialised the actual simulation becomes simpler.

### 5.2.3 Initialising the simulation

The simulation is initialised with the physical configuration and characteristics describing the system along with any relevant initial conditions, such as the initial velocities of each body. This initialisation includes the determination of how the system is discretised and the appropriate masses of each rigid body and the mass moments of inertia in each body’s local coordinate system. Parameters such as the mass and mass moments of inertia are assumed to remain constant for the duration of the simulation. This is a very reasonable assumption for solid beam-like structures such as the blades and tower of a wind turbine in which very little warping of the cross-sections are expected. The flexural and torsional rigidities of members, and the amount of damping is assigned so that suitable spring and damping coefficients at each joint can be calculated. The locations of the elastic and shear centres are also defined so that the correct locations of the joints relative to the centres of mass may be calculated.
5.2.4 Actions taken during a single time-step

Figure 5.2 presents a flow chart outlining (at a very high level) the actions taken during a single time-step of the structural simulation. A brief summary of the actions is given here together with references to the locations of more detailed descriptions within this thesis. Note that Fig. 5.2 is shown as if each process is only performed once during each time-step, but in practice, because a higher-order solver and an adaptive time-step is used, several of the processes (e.g. updating the elastic and the constraint loads) are performed several times during each time-step. As this is due to the specific solver being used and not the actual processes shown in figure 5.2, this looping has been omitted for simplicity here.

**Reset the loads on the bodies:** The loads applied to each rigid body are reset at the start of each time-step back to either zero or to the gravitational loads only (if gravity is included). If gravitational loads are included in the simulation, they are assumed to remain constant with respect to time.

**Pull aerodynamic loads from aero-structural interface:** This step is explained in detail in chapter 6. In essence, the structural simulation pulls the current aerodynamic loads, as calculated by the aerodynamic simulation, from the aero-structural interface.

**Update non-aerodynamic external loads:** Aeolus3D was implemented to allow arbitrary external loads to be applied. The main intention of this functionality was to allow loads due to
guy cables to be applied to the system. After investigating various methods for modelling the guy cables, such functionality was omitted from the simulations presented in this thesis. The capability to allow arbitrary external loads, however, was retained in Aeolus3D. The method of modelling the guy cables range from the very simple (e.g. Carne et al. [20]) to the more advanced (e.g. Veilleux and Tinawi [7, 121] and Reuter [171]).

Update elastic loads: The internal elastic loads due to deformation and the rate of change of deformation of the elastic members are calculated based on the superelement method described in section 5.3.

Update constraint loads: This is the most complex process involved in each structural time-step (except for the linearisation process, which is arguably more complex but considered separately as not every simulation involves linearisation). Based on the current state of the structural system and all loads applied to the rigid bodies (both internal and external), the loads required to maintain the predefined joint constraints (without doing work on the system) are calculated using the Lagrangian multiplier approach, as described in section 5.5.

Update the state of each rigid body: The rate of change of the state of each rigid body is determined (based on the current states of the rigid bodies and the loads being applied) and is used to estimate the new states of the rigid bodies at the end of the time-step. This process is described in section 5.4.

Push structural state data to aero-structural interface: Each rigid body is updated with its new state information and the results are pushed to the aero-structural interface for use by the aerodynamic simulation. The details of this step are explained in chapter 6.

Output structural data to files: All required structural data are written to file. This includes not only load data required for analysis, but also includes sufficient information to fully describe the structural state of the system allowing further simulations to be performed with alternative parameters, or for graphics to be generated for visualisation purposes.

Check for convergence: If the current time-step represents the completion of a full revolution of the VAWT, a check is made to see whether the structural simulation has reached convergence. This check is described in section 5.7.

5.3 Elasticity

To accurately predict the dynamic behaviour of any structure, the model must accurately approximate the equivalent dynamic behaviour. The approach used in Aeolus3D to ensure

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[1] Note that the convergence of the system to a periodic equilibrium condition is not guaranteed. Some configurations could exhibit instabilities that would prevent the system converging to an equilibrium condition.
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Undeformed Superelement

Universal joints to model in-plane and out-of-plane (flatwise and edgewise) bending motion

Revolute (twist) joint to model twisting motion

Ends may be attached to other bodies/superelements, the "world", or left free

Deformed Superelement

Figure 5.3: Illustration of an undeformed superelement and a deformed (exaggerated) superelement.

This prediction does that is to arrange the rigid bodies of the structure using the superelement method described by Molenaar [115]. The superelement method approximates the dynamic behaviour of beam-like structures by arranging the rigid bodies of the system into groups of four, as illustrated in Fig. 5.3, and connecting them together with suitable joint models, each axis of which has a spring and damper. Each superelement contains two universal joints, located at the elastic axis between bodies, used to model the bending motion in the in-plane (flatwise) and out-of-plane (edgewise) directions and a revolute joint located at the shear axis between bodies used to model the torsional motion. A cylindrical joint (allowing rotation about an axis and translation along the axis) could have been used instead of the revolute joint to simulate extensional motion. For the structures of interest in VAWT analysis, however, the stiffness is so much greater (and movement, so much less) in the extensional direction than in the bending and torsional directions that it was considered unnecessary for this work. A very similar method to the superelement approach of Molenaar, though slightly more elaborate, was also described and used by Zhao et al. [122] for modelling the dynamics of HAWTs. The main difference between the two is simply that Molenaar’s work is based on Euler-Bernoulli beam theory (which is appropriate for the blades and towers of wind turbines which are generally very long relative to their thickness), whereas the work of Zhao et al. is based on Timoshenko beam theory. The approach employed in Aeolus3D is modelled on Molenaar’s earlier, and simpler, method but the work of Zhao et al. further confirms the validity of the general concept.

It is important to note that the superelement method does require the assumption of small deflections within each superelement. In the general case, the relative rotation between two bodies connected with a universal joint (or any joint allowing rotation about more than a single axis) is governed by the order of rotation about each axis of the joint. For small angles of...
rotation, however, the relative motion is effectively independent of the order of rotation\textsuperscript{2}. Evidence of this is provided in appendix B. For highly flexible structures, additional superelements may sometimes be required to ensure that the angles of deflection remain small within each superelement.

The ends of the superelements may be rigidly connected to an adjoining superelement to build up an entire beam-like structure. The bodies of two adjoining superelements could in fact be treated as a single rigid body but in Aeolus3D this approach was not taken. The bodies are modelled separately and connected together with joint constraint equations removing all relative degrees of freedom between them. This approach does increase the computational cost slightly, but as the end bodies of the superelements of a curved system have relative rotations between them, the use of joint constraints to connect them together greatly simplifies the initialisation of the models. The calculation of the more complex inertia tensor of the arbitrarily combined bodies is not required using this approach, which is a significant advantage with respect to the implementation of the simulation. Furthermore, as discussed in relation to the aerodynamic simulations (chapter 4), simulating the development of the free wake of the turbine has the highest computational cost of any of the processes involved in the simulation. As such, the computational improvement to the structural model would need to be significant (which it would not be) to make any noticeable difference to the overall simulation time.

An illustration of how a complete VAWT may be modelled using superelements is presented in Fig. 5.4. In this example, each blade is modelled using four superelements, and the central tower is modelled using two superelements. The VAWT illustrated in Fig. 5.4 is a “classic”

\textsuperscript{2}“Effectively” independent because although not truly independent of order of rotation, the differences become negligible as the angles get smaller. For an angle of rotation of 5\degree about each axes, the difference is less than 1\% for all elements in the resulting rotation matrices.
configuration in that the blades are fixed to the tower which rotates with the blades. Aeolus3D is not limited, however, to this configuration of VAWT. The tower can be omitted completely (allowing the motion of the blades to be studied in isolation from the tower, as done for the case studies described in chapter 8) or with the top and bottom of the blades either constrained to each other or allowed to revolve freely about the central axis (such as via a bearing). Aeolus3D also allows a non-rotating tower to be included with the top (or bottom) of the blades to be attached via a bearing to the tower. Additionally, despite the fact that all examples presented in this thesis are for Darrieus-style curved-bladed VAWTs, this is not a limitation of Aeolus3D. Most general beam-like structures could be modelled using this approach including Musgrove-style straight-bladed VAWTs, HAWTs, or even rotorcraft. In the case of HAWTs and rotorcraft, however, some additional work would be required for Aeolus3D to be useful in practice (actively controllable blade pitching would need to be implemented, for example) but all of the required building blocks are in place.

5.3.1 Joint stiffnesses

Molenaar demonstrated that if the first and last bodies in the superelement have a length of \( l \), as given by Eq. 5.1 where \( L_{se} \) is the total length of superelement, then no spring is required between the first and last bodies of the superelement to accurately model the elastic behaviour of an equivalent beam.

\[
l = \frac{1}{2} \left( 1 - \frac{1}{\sqrt{3}} \right) L_{se} \approx (0.2113) L_{se} \tag{5.1}
\]

With the first and last bodies of the superelement defined as having a length of \( l \) then the spring stiffnesses of the in-plane (flatwise) and out-of-plane (edgewise) bending directions are, based on Euler-Bernoulli beam theory, given by Eqs. 5.2 and 5.3 respectively.

\[
k_{flat} = \frac{2EI_{in}}{L_{se}} \tag{5.2}
\]
\[
k_{edge} = \frac{2EI_{out}}{L_{se}} \tag{5.3}
\]

\( EI_{in} \) and \( EI_{out} \) are the flexural rigidities of the member in the in-plane and out-of-plane directions respectively (or the flatwise and edgewise flexural rigidities in the case of blade superelements). Holierhoek [123] demonstrated that for tapered members some improvements to the method may be possible by calculating the spring stiffnesses using flexural rigidities averaged over half a superelement rather than the full superelement. This is, however, more relevant to HAWTs than VAWTs as VAWT blades are rarely tapered significantly (and none of the VAWTs examined in this thesis are tapered) but could be worthy of future investigation should an analysis of a VAWT with highly tapered blades be required. It should be noted, however,
that Holierhoek’s work is based on a very limited number of superelements per blade. As the curved blades of a VAWT require additional superelements just to model the curvature sufficiently, it is debatable whether Holierhoek’s modified method would still offer a measurable difference from Molenaar’s original method, and further work would be required to confirm this.

The spring stiffness of the torsional joint between the middle two bodies of the superelement is given by Eq. 5.4, where $GJ$ is the torsional rigidity of the member.

$$k_{tor} = \frac{GJ}{L_{se}} \quad (5.4)$$

### 5.3.2 Joint damping

Each axis of each joint is also modelled with a viscous damper to approximate the small amount of structural damping that is present in all real structures. In practice, the accurate estimation of actual damping is incredibly complex and not possible within the scope of this work. However, the amount of actual damping in the structure of a VAWT is expected to be very small so a precise measurement is less critical than for some other structures. The dampers used in Aeolus3D are standard viscous torsional dampers with a damping coefficient assigned to each axis of each joint, and so apply a counter load proportional to the rate of change of the joint angles. The inclusion of damping in the system is, in fact, as much for numerical stability as for physical accuracy. Although there are other methods for modelling the damping within a structure (e.g. hysteretic damping) which may in fact be more accurate, the low level of damping in a VAWT makes the inclusion of more complex methods unjustified.

In general, the damping coefficient was estimated according to Rayleigh damping ($c = \eta_1 m + \eta_2 k$, where $c$ is the damping coefficient, $m$ is the mass, $k$ is the spring stiffness, and $\eta_1$ and $\eta_2$ are the Rayleigh damping parameters defining the relationship between the mass, stiffness, and damping), with $\eta_1 = 0$. So the damping at each axis of each joint is assumed to be directly proportional to the stiffness at the joint axis. This is not, however, an intrinsic part of Aeolus3D and the damping coefficient could be set to any value. This approach means that the damping will appear to increase for higher modes in comparison to lower modes. Although not really physically realistic, as the amount of damping cannot be based on actual measurements of actual structures, it is impossible to know the true effects of damping, so this widely used approximation is adequate for the purposes of this work. It should be noted, however, that if any structures with high levels of damping were modelled using Aeolus3D, a more advanced method of modelling the damping might be required.
5.4 Rigid body motion

As the joint constraints are determined using the Lagrangian multiplier approach (section 5.5), the equations of motion of each rigid body can be treated independently of each other. This separation of the motion of the rigid bodies from the joint constraints allows for a very general method which requires no prior knowledge of the motion of the system. New, and very different, system configurations can be assembled quickly without any consideration of the equations of motion.

The motion of each of the independent rigid bodies of the system may be solved using any standard rigid body simulation approach. The method implemented in Aeolus3D was based largely on an excellent set of notes presented at SIGGRAPH 2001 by Baraff [172], but very similar approaches are described by others, e.g. Coutinho [173].

The general approach of the rigid body simulation is to determine a state vector \( \mathbf{X} \) and its derivative with respect to time \( \dot{\mathbf{X}} \) which contain sufficient information to fully describe the condition of any arbitrary rigid body at any instant in time. The state vector used by Aeolus3D to describe each rigid body is given by Eq. 5.5. Note that, unless otherwise stated, all kinematic variables are measured in the global coordinate system.

\[
\mathbf{X} = \begin{pmatrix} \mathbf{x} \\ \mathbf{q} \\ \mathbf{P} \\ \mathbf{L} \end{pmatrix}
\] (5.5)

In Eq. 5.5, \( \mathbf{x} \) is the position vector of the centre of mass, \( \mathbf{q} \) is the orientation of the rigid body represented as a unit quaternion \(^{3}\), \( \mathbf{P} \) is the linear momentum vector and \( \mathbf{L} \) is the angular momentum vector. Although in many engineering disciplines orientation is frequently represented by a rotation matrix, in the field of computer graphics and animation the benefits of using quaternions instead have been realised for some time. In any type of computer simulation, numerical drift is inevitable. It is an inherent limitation of numerical modelling and cannot be avoided completely, only managed. One method for reducing the numerical drift is by careful selection of state variables. By using quaternions instead of rotation matrices, there are only four parameters required to identify the orientation of each rigid body instead of the nine required in a \( 3 \times 3 \) rotation matrix. In addition to the reduced memory demands (which is not really a significant concern compared to the memory demands of the aerodynamic free wake), the reduced number of parameters reduces the amount of numerical drift that will occur. Perhaps their most useful feature is, however, that because the quaternions used to represent orientation

\(^{3}\)Quaternions are members of a four-dimensional number system represented by one real dimension and three imaginary dimensions. A unit quaternion can be used to represent a rotation by an angle equal to the real dimension about a vector represented by the imaginary dimensions.
are unit length quaternions, they may be easily renormalised to unit length in the event that any of the quaternions is found to have deviated from unit length and hence no longer represent a pure rotation. At certain stages of the simulation, rotation matrices are still needed by Aeolus3D (such as converting the inertia tensor between local and global coordinate systems, or generating graphics for visualisation). When rotation matrices are actually needed by the simulation, they are assembled as needed from the quaternions (see appendix C for details).

The state vector given above (Eq. 5.5) is differentiated with respect to time to give the rate of change of the state vector of a rigid body, as shown in Eq. 5.6.

\[
\dot{X} = \begin{pmatrix} \dot{x} \\ \dot{q} \\ \dot{P} \\ \dot{L} \end{pmatrix} = \begin{pmatrix} v \\ (1/2)\omega q \\ F \\ T \end{pmatrix} = \begin{pmatrix} P/m \\ (1/2)I^{-1}Lq \\ F \\ T \end{pmatrix}
\]

\(v\) is the velocity vector of the centre of mass of the rigid body and is determined from the linear momentum and the mass \((m)\) of the rigid body \((v = P/m)\). \(\omega\) is the angular velocity vector of the rigid body about its centre of mass and is determined from the angular momentum and the \(3 \times 3\) mass moment of inertia tensor \((I)\) of the rigid body \((\omega = I^{-1}L)\). \(F\) is the vector sum of all forces acting on the rigid body (including the calculated constraint forces) and \(T\) is the vector sum of all the torques acting on the body (also including the calculated constraint torques).

By assuming that the rigid bodies maintain their mass and shape throughout the simulation, the inertia tensor of each rigid body in its local coordinate system \((I_{\text{body}})\) and its inverse \((I_{\text{body}}^{-1})\) can be determined in advance. The inverse of the local inertia tensor can then be used at any instant in time, together with the orientation of the rigid body, to determine the inverse of the inertia tensor in the global coordinate system as the simulation progresses. This transformation between coordinate systems is shown in Eqs. 5.7 and 5.8.

\[
I = RI_{\text{body}}R^T \\
I^{-1} = RI_{\text{body}}^{-1}R^T
\]

With all elements of the state vector and its rate of change with respect to time defined, any suitable numerical ordinary differential equation (ODE) solver can be used in a time marching scheme to simulate the motion of all the rigid bodies in the system. All of the Aeolus3D results presented in this thesis used the Runge-Kutta-Fehlberg method (RKF45) with an adaptive time-step to solve the equations of motion of the system. Unless otherwise stated, the RKF45 method was run with a relative error of \(1 \times 10^{-4}\) and an absolute error of \(1 \times 10^{-6}\), which are
commonly used values\textsuperscript{4}.

\section*{5.5 Joint constraints}

The Lagrangian multiplier approach involves the determination of the applied loads required to maintain the defined constraints. The constraint loads are determined at each time-step by calculating the loads required to maintain the joint constraints without adding to or removing energy from the system (i.e. no work is done on the system by the joint constraints, according to d’Alembert’s principle \cite{174}).

The particular implementation used for Aeolus3D was, again, inspired by a set of notes presented at SIGGRAPH 2001, in this case by Witkin \cite{175}. The form of the joint constraint equation presented here differs slightly from Witkin’s to accommodate the inertia tensor of the rigid bodies (Witkin’s formulation was for three-dimensional particles with mass but no inertia). The system constraint equation (Eq. 5.9), which is a system of linear algebraic equations, is solved to find the vector of Lagrangian multipliers ($\lambda$). The constraint forces ($F_c$) required to ensure that the joint constraints are maintained during the time marching simulation are then determined according to Eq. 5.10.

\begin{align*}
JM^{-1}J^T\lambda &= -\dot{J}\dot{z} - JM^{-1}F_{nc} + JM^{-1}\dot{M}\dot{z} - 2\alpha J\dot{z} - \beta^2 C_c + c \\
F_c &= J^T\lambda
\end{align*}

\begin{align*}
5.9
\end{align*}

\begin{align*}
F_c &= J^T\lambda
\end{align*}

\begin{align*}
5.10
\end{align*}

$J$ is the Jacobian constraint matrix defining the degrees of freedom removed from the system, $M$ is the system mass matrix defining the masses and inertia tensors of all the rigid bodies in the system, $z$ is the vector of generalised coordinates of the system, $F_{nc}$ is the vector sum of all non-constraint forces (i.e. all applied loads except the constraint loads), $C_c$ is the vector of constraints\textsuperscript{5}, $c$ is the desired value of the constraint\textsuperscript{6} and $\alpha$ and $\beta$ are Baumgarte parameters which are involved in the Baumgarte stabilisation method \cite{176} used by Aeolus3D to mitigate the numerical drift that could otherwise take place.

It is worth noting that the matrix $JM^{-1}J^T$ in Eq. 5.9 is typically sparse for a VAWT (chains of bodies connected together in series) so computational gains can be made by utilising a method of solving systems of linear equations specifically tailored for sparse matrix systems. In the case of Aeolus3D, the general purpose library, SuperLU, developed by Demmel \textit{et al.} \cite{177} for the

\textsuperscript{4}The default relative and absolute tolerances used by the ordinary differential equation solver in Matlab are $1 \times 10^{-3}$ and $1 \times 10^{-6}$ respectively.

\textsuperscript{5}The Jacobian constraint matrix $J$ is derived from the vector of the constraints $C_c$ and the vector of generalised coordinates $z$, i.e. $J = \partial C_c / \partial z$.

\textsuperscript{6}In Aeolus3D, every row of $c$ is zero (i.e. zero relative acceleration between bodies in the degree of freedom removed by the constraint equation) with the exception of those constraint rows corresponding to prescribed velocity joints. In such cases, the corresponding rows of $c$ hold the value of the acceleration required during the current time-step to maintain constant velocity.
solution of large, sparse, non-symmetric systems of linear equations was used.

5.5.1 Generalised coordinates

A brief explanation of the vector of generalised coordinates $z$ is required so as to avoid confusion with the components of the state vector defined in section 5.4. Each body has six degrees of freedom, three translational and three rotational. The important difference from the state vector is that quaternions are not used here to describe the orientations. Instead, a vector of three rotational coordinates ($\Theta$) is used. The vector of rotational coordinates does not appear directly in the constraint equation so its actual definition is not critical. $\Theta$ can simply be thought of as the integral with respect to time of the angular velocity (which does appear directly in the constraint equation). The vector of generalised coordinates is shown in Eq. 5.11 and its differential with respect to time is given in Eq. 5.12 where $n$ is the number of rigid bodies in the system.

$$
z = \begin{bmatrix} x_1 & \Theta_1 & x_2 & \Theta_2 & \ldots & x_n & \Theta_n \end{bmatrix}^T \quad (5.11)$$

$$
\dot{z} = \begin{bmatrix} v_1 & \omega_1 & v_2 & \omega_2 & \ldots & v_n & \omega_n \end{bmatrix}^T \quad (5.12)
$$

5.5.2 Derivation of system constraint equation

If $C_c(z)$ is a set of functions, in terms of the vector of generalised coordinates $z$, that define the spatial constraints (either position or orientation). Differentiating once gives a set of velocity constraint equations as shown in Eq. 5.13 where the term $\partial C_c / \partial z$ is known as the Jacobian matrix of $C_c$ and is typically denoted as $J$.

$$
\dot{C}_c = \frac{\partial C_c}{\partial z} \dot{z} = J \dot{z} \quad (5.13)
$$

Differentiating once more produces the acceleration level constraints according to Eq. 5.14.

$$
\ddot{C}_c = \dot{J} \dot{z} + J \ddot{z} \quad (5.14)
$$

Newton’s second law of motion is shown in Eq. 5.15, and is re-arranged to be in terms of $\ddot{z}$.

$$
\sum F = \frac{d}{dt} (M \ddot{z}) = F_c + F_{nc} = M \ddot{z} + \dot{M} \ddot{z} \quad (5.15)
$$

$$
\ddot{z} = M^{-1} \left( F_c + F_{nc} - \dot{M} \ddot{z} \right)
$$
Substitution of $\ddot{z}$ back into Eq. 5.14 gives an expression for $\ddot{C}_c$ in Eq. 5.16.

$$\ddot{C}_c = \dot{J}\dot{z} + JM^{-1}\left(F_c + F_{nz} - \ddot{M}z\right)$$

(5.16)

$$= \dot{J}\dot{z} + JM^{-1}F_c + JM^{-1}F_{nc} - JM^{-1}\ddot{M}z$$

The Lagrangian multipliers ($\lambda$) are introduced into the equation by replacing the constraint forces $F_c$ with $J^T\lambda$ according to the principle of virtual work\footnote{Requires the constraint forces to do no work on the system. This principle is explained in more detail by Witkin [175]} giving Eq. 5.17.

$$\ddot{C}_c = \dot{J}\dot{z} + JM^{-1}J^T\lambda + JM^{-1}F_{nc} - JM^{-1}\ddot{M}z$$

(5.17)

Eq. 5.17 is the acceleration constraint equation in terms of Lagrangian multipliers. Although correct, without a stabilisation method numerical drift may, over time, violate the defined joint constraints and cause the simulation to behave inaccurately. The method used in Aeolus3D to counteract this problem is the Baumgarte stabilisation method [176]. Baumgarte stabilisation is a very simple, yet very effective method of numerical stabilisation. Rather than using just the acceleration constraint equation, a linear combination of the positional, velocity and acceleration constraints are used. Using $c$ to signify the magnitude of the acceleration that the particular constraint should try to maintain over the current time-step, the acceleration constraint equations without any stabilisation would simply be $\ddot{C} = c$. With the Baumgarte stabilisation, this equation is replaced with a combination of the velocity and spatial constraints, as given in Eq. 5.18.

$$\ddot{C} + 2\alpha\dot{C} + \beta^2C = c$$

(5.18)

According to Witkin [175] the precise values of the parameters controlling numerical drift are not critical. The values of the Baumgarte parameters need to be large enough that they provide sufficient stabilisation, but not so large that they make the system of equations unnecessarily stiff, hence making numerical integration more computationally demanding than necessary. Aeolus3D uses the approach recommended by Burgermeister et al. [178], that the Baumgarte parameters should both be greater than zero and less than $1/\Delta t$, where $\Delta t$ is the time-step size of the simulation. Both of the Baumgarte parameters in Aeolus3D were defined using the midpoint of this recommendation as shown in Eq. 5.19. Some qualitative experimentation was conducted by varying the Baumgarte parameters up and down from this value and the simulations appeared to be fairly insensitive to the precise magnitude.

$$\alpha = \beta = \frac{1}{2\Delta t}$$

(5.19)
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Substituting Eqs. 5.13 and 5.17 into 5.18 and rearranging it so that only terms involving the Lagrange multipliers are on the left hand side and all other terms are on the right hand side gives Eq. 5.9 which, as explained, is the complete constraint equation with Baumgarte stabilisation.

5.5.3 Assembling and solving the constraint equations

The assembly and subsequent solving of the constraint equation using full matrix multiplication and dense linear algebra techniques is unnecessarily computationally expensive. The construction of the linear algebra problem is greatly accelerated by using block matrix multiplication to ensure that only those parts of the final matrices with non-zero blocks are considered. Additionally, the blocks within the Jacobian and mass matrices are themselves often fairly sparse so further gains can be made by performing block matrix multiplication within the blocks themselves at the lowest level. Because the structure of the mass matrix is always the same, and the structure of the Jacobian and the derivative of the Jacobian matrices are defined for each type of joint, the location of the zeroes within the blocks to be multiplied is known in advance. By only multiplying through the parts of the matrices that are known not to be guaranteed to be zeros, the complete Jacobian matrix, derivative of the Jacobian matrix, and the inverse mass matrix, need never actually be assembled in full to solve the constraint equations. It is also important to note that as the $\mathbf{J M}^{-1} \mathbf{J}^T$ term on the left hand side of the constraint equation is a square symmetric matrix, only the upper or lower half of the matrix needs to be included in the block matrix multiplication process, further reducing computational demands.

The mass matrices

The mass matrix of the total system ($\mathbf{M}$) is a block diagonal matrix with the diagonal elements being the mass matrices of each individual rigid body in the system ($\mathbf{M}_1 \ldots \mathbf{M}_n$), where $n$ is the number of rigid bodies in the system and as the mass matrix is a diagonal matrix, its inverse is found by simply inverting each element along its diagonal. The mass matrix of the system and its inverse are shown in Eq. 5.20.

\[
\mathbf{M} = \begin{bmatrix}
\mathbf{M}_1 & & \\
& \mathbf{M}_2 & \\
& & \ddots \\
& & & \mathbf{M}_n
\end{bmatrix}, \quad 
\mathbf{M}^{-1} = \begin{bmatrix}
\mathbf{M}_1^{-1} & & \\
& \mathbf{M}_2^{-1} & \\
& & \ddots \\
& & & \mathbf{M}_n^{-1}
\end{bmatrix} \quad (5.20)
\]
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The mass matrix for an individual body \((M_i)\) and its inverse are \(6 \times 6\) matrices defined according to Eq. 5.21.

\[
M_i = \begin{bmatrix}
m & m & m \\
m & m & m \\
m & m & m \\
\end{bmatrix}, \quad M_i^{-1} = \begin{bmatrix}
m^{-1} & m^{-1} & m^{-1} \\
m^{-1} & m^{-1} & m^{-1} \\
m^{-1} & m^{-1} & m^{-1} \\
\end{bmatrix}
\]

(5.21)

As shown previously in Eqs. 5.7 and 5.8, the inertia tensor and its inverse in the global coordinate system are calculated from the inertia tensor of the body in its local coordinate system, and the rotation matrix describing its orientation. By storing the mass \((m)\), its inverse \((m^{-1})\), the mass moment of inertia tensor in local coordinates \((I_{body})\) and its inverse \((I_{body}^{-1})\) during the initialisation of the simulation, no matrix inversion actually needs to be performed during the running of the simulation to find the inverse of the full mass matrix of the whole system. Only simple (and fast) matrix multiplication of the individual \(3 \times 3\) rotation matrices and inertia tensors is required.

Determining the rate of change of the mass matrix is slightly more complex than determining the inverse of the mass matrix. Like the inverse, the rate of change of the mass matrix is determined by calculating the rate of change of each element of the diagonal. The rate of change of the mass matrix of the system \((\dot{M})\) and the rate of change of the mass matrix of an individual body \((\dot{M}_i)\) are shown in Eq. 5.22.

\[
\dot{M} = \begin{bmatrix}
\dot{M}_1 \\
\dot{M}_2 \\
\vdots \\
\dot{M}_n \\
\end{bmatrix}, \quad \dot{M}_i = \begin{bmatrix}
\dot{m} \\
\dot{m} \\
\vdots \\
\dot{m} \\
\end{bmatrix}
\]

(5.22)

As the rigid bodies in the system maintain their physical properties, such as size, shape and density, the rate of change of the mass will be zero (i.e. \(\dot{m} = 0\)). Only changes in orientation will impact upon the rate of change of the mass matrix. So, the inertia tensor in the global coordinate system (defined in Eq. 5.7) can be differentiated with respect to time as shown in Eq. 5.23.

\[
\dot{I} = \frac{d}{dt}(RI_{body}R^T) = \dot{R}I_{body}R^T + RI_{body}\dot{R}^T
\]

(5.23)
The rate of change of the rotation matrix is defined as $\dot{R} = \tilde{\omega}R$ (shown in appendix C), where $\tilde{\omega}$ is the angular velocity of the body in skew symmetric matrix form. The transpose of the rate of change of the rotation matrix can therefore be calculated as shown in Eq. 5.24.

$$
\dot{R}^T = (\tilde{\omega}R)^T
= R^T \tilde{\omega}^T
= -R^T \tilde{\omega}
$$

Substituting these back into Eq. 5.23 gives the rate of change of the mass moment of inertia of a rigid body in the global coordinate system as shown in Eq. 5.25.

$$
\dot{\mathbf{I}} = \tilde{\omega} \mathbf{R}_{\text{body}} \mathbf{R}^T - \mathbf{R}_{\text{body}} \mathbf{R}^T \tilde{\omega}
= \tilde{\omega} \mathbf{I} - \mathbf{I} \tilde{\omega}
$$

The Jacobian constraint matrices

Like the mass matrix of the whole system described above, the Jacobian constraint matrix for the whole system ($\mathbf{J}$) and its derivative with respect to time ($\dot{\mathbf{J}}$) is also assembled in a modular manner. Where the elements of the mass matrix of the system were related to individual rigid bodies, the elements of the Jacobian constraint matrix of the system are related to the individual joints in the system. Each row of the constraint matrix of the system represents a joint between either a single rigid body and the global coordinate system (such as between the base of the wind turbine and the ground) or between two rigid bodies. As such, each row of the constraint matrix of the system will have at least one block element (i.e. a joint involving a single rigid body) and no more than two block elements (i.e. a joint involving two rigid bodies). As the size of the system increases, so too does the sparsity of the constraint matrix.

Each block element in the constraint matrix of the system is itself a matrix relating the constraint equations for the particular joint to the body or bodies to which it is attached. Each of these sub-matrices has six columns corresponding to the six degrees of freedom of a free rigid body (three translational and three rotational) and a number of rows equal to the number of degrees of freedom removed from the system by that particular joint. As such, the constraint matrix for the whole system has a number of columns equal to six times the number of rigid bodies in the system and a number of rows equal to the number of degrees of freedom removed from the entire system. Or, put another way, the matrices have a number of rows equal to six times the number of bodies in the system less the number of degrees of freedom in the system.

The rate of change of the constraint matrix with respect to time ($\dot{\mathbf{J}}$) has the same dimensions as the constraint matrix and each block element of the matrix represents the rate of change of the corresponding element of the constraint matrix.
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The Jacobian constraint matrices of each joint type can be derived from the velocity constraint equation (Eq. 5.13). The velocity constraint equation for a single joint between two bodies can be expanded out and rewritten as shown in Eq. 5.26, where the superscripts $i$ and $j$ are used to indicate the terms associated with body $i$ and $j$ respectively and the subscripts “lin” and “ang” refer to the linear (i.e. translational) and angular (i.e. rotational) constraints respectively.

$$\dot{C}_c = J\dot{z}$$

$$= J^i\dot{z}^i + J^j\dot{z}^j$$

$$= \begin{bmatrix} J^i_{\text{lin}} & J^i_{\text{ang}} \end{bmatrix} \begin{bmatrix} \nu^i \\ \omega^i \end{bmatrix} + \begin{bmatrix} J^j_{\text{lin}} & J^j_{\text{ang}} \end{bmatrix} \begin{bmatrix} \nu^j \\ \omega^j \end{bmatrix}$$

$$= J^i_{\text{lin}} \nu^i + J^i_{\text{ang}} \omega^i + J^j_{\text{lin}} \nu^j + J^j_{\text{ang}} \omega^j$$

The procedure to determine the Jacobian constraint matrices for each joint type, therefore, is to develop a constraint equation in terms of the velocities of the bodies. By rearranging the constraint equation of the joint into a form matching that of Eq. 5.26, the Jacobian constraint matrices can be identified.

Revolute joint

The procedure to derive the Jacobian constraint matrices for a revolute joint is presented here. A revolute joint, as depicted in Fig. 5.5, constrains the relative motion between two bodies in all three translational degrees of freedom, and in two of the three rotational degrees of freedom. The location of the revolute joint is defined by two position vectors, $\mathbf{r}^i$ and $\mathbf{r}^j$, indicating the location of the joint relative to the centre of mass of body $i$ and body $j$ respectively. In practice, because the location of the joint with respect to the bodies remains constant, the vectors are limited to any location along the axis of rotation of the revolute joint.

---

The vectors $\mathbf{r}^i$ and $\mathbf{r}^j$ can, in fact, point to any location along the axis of rotation of the revolute joint.
defined during initialisation in the local coordinate systems of each body and transformed into the global coordinate system at each time-step based on the current orientation of the bodies. Two vectors \((s_1 \text{ and } s_2)\) are defined which are orthogonal to the axis of rotation of the revolute joint. These vectors indicate the axes about which angular motion is not permissible. Vectors \(s_1\) and \(s_2\) are also stored in terms of the local coordinate system of one of the bodies and transformed into the global coordinate system at each time-step.

It is worth noting that although the revolute joint illustrated in Fig. 5.5 is aligned with the local \(x\)-axes of the bodies (and vectors \(s_1\) and \(s_2\) aligned with the local \(y\)-axis and \(z\)-axis of body \(i\)), such as is the case for the revolute joint in the middle of each superelement, this is not necessary to the procedure described here. The Jacobian constraint matrices formed are for a general revolute joint about any arbitrary axis.

To ensure that the translational position of the joint is maintained, the position of the joint in the global coordinate system, as referenced by body \(i\) and body \(j\), must remain the same. The translational position constraint equation can therefore be expressed as Eq. 5.27.

\[
x^i + r^i = x^j + r^j \tag{5.27}
\]

The translational position constraint is differentiated with respect to time to determine the translational velocity constraint equation as shown in Eq. 5.28.

\[
\frac{d}{dt} (x^i + r^i) = \frac{d}{dt} (x^j + r^j) \\
v^i + \omega^i \times r^i = v^j + \omega^j \times r^j \\
v^i - \tilde{r}^i \omega^i = v^j - \tilde{r}^j \omega^j
\tag{5.28}
\]

Rarrangement of Eq. 5.28 to match the form of Eq. 5.26 gives Eq. 5.29.

\[
v^i - \tilde{r}^i \omega^i - v^j + \tilde{r}^j \omega^j = 0 \tag{5.29}
\]

Comparing Eq. 5.29 with Eq. 5.26 reveals that the three rows of each Jacobian matrices corresponding to the translational constraints as shown in Eqs. 5.30 through 5.33, where \(I^3_{\text{ID}}\) is a \(3 \times 3\) identity matrix.

\[
J^i_{\text{lin}} = I^3_{\text{ID}} \tag{5.30}
\]
\[
J^i_{\text{ang}} = \tilde{r}^i \tag{5.31}
\]
\[
J^j_{\text{lin}} = -I^3_{\text{ID}} \tag{5.32}
\]
\[
J^j_{\text{ang}} = \tilde{r}^j \tag{5.33}
\]
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The two rotational constraints require that the component of the angular velocity of body $i$ about the constrained axes $s_1$ and $s_2$ is equal to the components of angular velocity of body $j$ about the same axes. The rotational velocity constraint (for axis $s_1$) is, therefore, shown in Eq. 5.34.

$$s_1 \cdot \omega^i = s_1 \cdot \omega^j$$  \hspace{1cm} (5.34)

As before, the velocity constraint equation is rearranged into a form matching 5.26, as shown in Eq. 5.35.

$$s_1 \cdot \omega^i - s_1 \cdot \omega^j = 0$$  \hspace{1cm} (5.35)

Comparison between Eqs. 5.35 and 5.26, reveal the Jacobian matrices for the rotational constraint about axes $s_1$ as shown by Eqs. 5.36 through 5.39.

$$J^i_{\text{lin}} = 0$$  \hspace{1cm} (5.36)

$$J^i_{\text{ang}} = s_1$$  \hspace{1cm} (5.37)

$$J^j_{\text{lin}} = 0$$  \hspace{1cm} (5.38)

$$J^j_{\text{ang}} = -s_1$$  \hspace{1cm} (5.39)

The rotational constraint about axis $s_2$ is formed in exactly the same way as that of axis $s_1$, so the complete Jacobian matrices $J^i$ and $J^j$ for the revolute joint are assembled as shown in Eqs. 5.40 and 5.41.

$$J^i = \begin{bmatrix} I_3 & -\ddot{r}^i \\ 0 & s_1 \\ 0 & s_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -r^i_z & r^i_y \\ 0 & 1 & 0 & r^i_z & 0 & -r^i_y \\ 0 & 0 & 1 & -r^i_y & r^i_x & 0 \\ 0 & 0 & 0 & s_{1,x} & s_{1,y} & s_{1,z} \\ 0 & 0 & 0 & s_{2,x} & s_{2,y} & s_{2,z} \end{bmatrix}$$  \hspace{1cm} (5.40)

$$J^j = \begin{bmatrix} -I_3 & \ddot{r}^j \\ 0 & -s_1 \\ 0 & -s_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & r^j_z & -r^j_y \\ 0 & -1 & 0 & -r^j_z & 0 & r^j_y \\ 0 & 0 & -1 & r^j_y & -r^j_x & 0 \\ 0 & 0 & 0 & -s_{1,x} & -s_{1,y} & -s_{1,z} \\ 0 & 0 & 0 & -s_{2,x} & -s_{2,y} & -s_{2,z} \end{bmatrix}$$  \hspace{1cm} (5.41)

The rates of change of the Jacobian matrices ($\dot{J}^i$ and $\dot{J}^j$) are found by differentiating each part
of the Jacobian matrices as shown in Eqs. 5.42 and 5.43.

\[
\dot{J}^i = \begin{bmatrix}
\dot{I}^3_{ID} & -\dot{\tilde{r}}^i \\
0 & \dot{s}_1 \\
0 & \dot{s}_2
\end{bmatrix}
\] (5.42)

\[
\dot{J}^j = \begin{bmatrix}
-\dot{I}^3_{ID} & \dot{r}^j \\
0 & -\dot{s}_1 \\
0 & -\dot{s}_2
\end{bmatrix}
\] (5.43)

The rate of change of an identity matrix is zero, i.e. \( \dot{I}^3_{ID} = 0^3 \) where \( 0^3 \) is a \( 3 \times 3 \) matrix in which all elements are zero. Given that all of the vectors involved in the revolute joint are attached to one or other of the bodies, and that they do not shift with respect to the local coordinate system of that body, the rate of change of a vector is found using the angular velocity of the body to which the vector is attached. So, assuming that the constraint axes \( s_1 \) and \( s_2 \) are attached to body \( i \), the rates of change of each vector involved are shown by Eqs. 5.44 through 5.47.

\[
\dot{r}^i = \omega^i \times r^i
\] (5.44)

\[
\dot{r}^j = \omega^j \times r^j
\] (5.45)

\[
\dot{s}_1 = \omega^i \times s_1
\] (5.46)

\[
\dot{s}_2 = \omega^i \times s_2
\] (5.47)

**Universal joint**

The Jacobian constraint matrices of the universal joint are derived in similar way to those of the revolute joint. The first three rows of each Jacobian constraint matrix of the universal joint, corresponding to the translational constraints, are in fact identical to those of the revolute joint. The rotational constraint of the universal joint is, however, slightly more complex than that of the revolute joint. In the case of the revolute joint, the rotational constraint axes remained fixed to one or other of the bodies, but for the universal joint there is a single rotational constraint axis which moves as the relative orientation between the two bodies changes. As shown in Fig. 5.5, the axes of rotation of the universal joint are designated as \( a^i \) and \( a^j \), where \( a^i \) maintains a fixed orientation with respect to body \( i \) and \( a^j \) maintains a fixed orientation with respect to body \( j \). With rotational motion permitted only about axes \( a^i \) and \( a^j \), the axis about which rotational motion is constrained (s) must be orthogonal to both rotational axes as shown in Eq. 5.48.

\[
s = a^i \times a^j
\] (5.48)
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The rows of the Jacobian constraint matrices are now derived exactly as for the revolute joint (Eqs. 5.34 through 5.39), resulting in the Jacobian matrices $\mathbf{J}^i$ and $\mathbf{J}^j$ for the universal joint as shown in Eqs. 5.49 and 5.50.

$$
\mathbf{J}^i = \begin{bmatrix}
\mathbf{I}_{\text{ID}}^3 & -\mathbf{\tilde{r}}^i \\
0 & \mathbf{s}
\end{bmatrix} \quad (5.49)
$$

$$
\mathbf{J}^j = \begin{bmatrix}
-\mathbf{I}_{\text{ID}}^3 & \mathbf{\tilde{r}}^j \\
0 & -\mathbf{s}
\end{bmatrix} \quad (5.50)
$$

The rates of change of $\mathbf{I}_{\text{ID}}^3$, $\mathbf{\tilde{r}}^i$ and $\mathbf{\tilde{r}}^j$ are as shown for the revolute joint, so only the procedure to determine the rate of change of $\mathbf{s}$ is required. With $\mathbf{a}^i$ fixed to body $i$ (so rotating at the angular velocity of body $i$) and $\mathbf{a}^j$ fixed to body $j$ (so rotating at the angular velocity of body $j$), the rate of change of the constrained axis is determined using the product rule for the cross product of two vectors as shown in Eq. 5.51.

$$
\dot{s} = \frac{d}{dt} (\mathbf{a}^i \times \mathbf{a}^j) \quad (5.51)
$$

$$
= \mathbf{a}^i \times \dot{\mathbf{a}}^j + \dot{\mathbf{a}}^i \times \mathbf{a}^j
$$

$$
= \mathbf{a}^i \times (\mathbf{\omega}^j \times \mathbf{a}^j) + (\mathbf{\omega}^i \times \mathbf{a}^i) \times \mathbf{a}^j
$$

Generator joint

The actual behaviour of a real generator will have an affect on the dynamic behaviour of the rest of the wind turbine structure. Rather than selecting an actual generator design and attempting to model its dynamic behaviour, however, an effort was made to minimise the influence of the generator on the system by including the generator directly into the joint constraint mechanism. Integrating the generator into the constraint equation of the system has the advantage that the required loads that need to be applied to the generator to achieve or maintain the desired velocity automatically take into account the mass moment of inertia of the complete structure at every point in time. The approach described here is similar to that described by Smith [179] for simulating motors. The main difference here is that Smith’s approach is based on velocity level constraints, whereas the constraints in Aeolus3D are acceleration constraints.

A generator joint can maintain a constant relative velocity between two bodies or a constant angular velocity of a single body about any point in space. All uses of generator joints in this thesis are of the latter variety so this section describes that type only.

The first five rows of the Jacobian matrix (and its rate of change) are identical to that of body $i$ of the revolute joint. The revolute joint is extended to a generator joint with the inclusion of a sixth row to the Jacobian matrix representing the axis of rotation ($\mathbf{s}_3$) which is orthogonal to both constraint axes (i.e. $\mathbf{s}_3 = \mathbf{s}_1 \times \mathbf{s}_2$).
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The component of angular velocity of the body about the axis of rotation must be equal to the target velocity of the generator joint as shown in Eq. 5.52.

\[ s_3 \cdot \omega = \omega_{\text{target}} \]  \hspace{1cm} (5.52)

The Jacobian constraint matrix for the generator joint is, therefore, that shown in Eq. 5.53.

\[ J = \begin{bmatrix} I^3_{\text{ID}} & -\tilde{r} \\ 0 & s_1 \\ 0 & s_2 \\ 0 & s_3 \end{bmatrix} \]  \hspace{1cm} (5.53)

The rate of change of the Jacobian constraint matrix is determined using exactly the same procedure as described for the elements of the revolute joint so is not repeated here.

Recall that in the full constraint equation (Eq. 5.9) the right hand side of the equation included the term \( c \). For every other constraint row in the system the value of the element of \( c \) corresponding to each constraint is zero, indicating that there should be no relative acceleration between the two bodies in the direction of the degree of freedom restricted by the constraint row. For the row corresponding to the axis of rotation of the generator, however, the element of \( c \) takes the value of the required acceleration (i.e. \( \dot{\omega}_{\text{constraint}} \)). If the target velocity of the generator at any given time-step is set, then the target acceleration about the generator axis is approximated as shown in Eq. 5.54.

\[ \dot{\omega}_{\text{constraint}} = \frac{\omega_{\text{target}} - \omega_{\text{current}}}{\Delta t} \]  \hspace{1cm} (5.54)

Other joints

In addition to the revolute joint, the universal joint, and the generator joint described here, several other joints types were implemented in Aeolus3D as well. The rigid joint, for example, removes all relative degrees of freedom between two bodies or completely locks the position and orientation of one body regardless of the loads applied. Individual rotational or translational constraints about any arbitrary axes are also implemented which enable any arbitrary joint type to be assembled using a combination of the constraints. These individual joints were particularly useful for assembling rarely used joint types, such as were sometimes needed at the top of each blade. If, for example, a blade that allowed purely flatwise motion (a rigid joint at the centre of each superelement and with the universal joints replaced with revolute joints aligned with the chordline of the blade) and no structural tower is used, then the top of the blade cannot be connected using a rigid joint. A rigid joint in such cases would over-constrain the system and the set of linear equations forming the system constraint equation would not
be linearly independent of each other. The individual arbitrary joint constraints allow only the degrees of freedom necessary to be constrained so that the set of system constraint equations remain linearly independent. The derivation of additional joints is not included in this thesis as the general approach for all joint constraints follows the same basic procedure as that described for the revolute and universal joints. The derivation of the Jacobian constraint matrices (but not necessarily the rates of change of the matrices) for several different joint types is presented by Shabana [174, 180] and by Smith [179], and for a wide variety of joint types by Erleben [181] in particular.

5.6 Determination of modal properties

The Lagrangian multiplier approach used to implement the multibody system simulation provides a number of benefits with respect to rapid assembly of new models for simulation. No prior knowledge of the equations of motion of the system are necessary using this method as the forces required to maintain the appropriate joint constraints are determined at each time-step as the simulation progresses. It turned out, however, that the benefits of this approach were countered somewhat when attempting to extract the modal characteristics of the system. Without explicit equations of motion defining the whole system, the modal characteristics cannot be determined directly.

The method ultimately employed in this work to determine the modal characteristics of the system is called the Eigensystem Realisation Algorithm (ERA), developed by Juang [182]. The ERA is a system realisation method which allows the full non-linear dynamic multibody system to be approximated as a state-space model, linearised about the equilibrium points of the system. The ERA, although originally developed for electrical circuit systems, has since been applied successfully to numerous physical systems, including HAWTs (Meng et al. [183]), aeroelasticity of aircraft (Farhat et al. [184]), aeroelasticity of bridges (Zhang et al. [185]) and ship motion (Suleiman [186]) for example. The application of ERA requires perturbing the system and measuring the transient responses, much like a physical experiment. To reduce the subjective nature of applying impulsive loads to the system, a coordinate partitioning method described by Shabana [174, 180] was employed to determine the degrees of freedom of the system and the relationships between the generalised coordinates when perturbed. Each independent coordinate could then be perturbed in turn, mitigating the risk of failing to excite one or several of the modes of vibration.
5.6.1 State-space model

The modal properties of the non-linear model are determined by identifying a discrete periodic linear state-space model, as shown in Eq. 5.55, about the equilibrium point of the system.

\[
x(k + k_T + 1) = A_{ss}x(k + k_T) + B_{ss}u(k + k_T) \\
y(k + k_T) = C_{ss}X(k + k_T) + D_{ss}u(k + k_T)
\]

(5.55)

\(k\) is the current sample, \(k_T\) is the number of samples per period, \(x\) is the state vector, \(u\) is the input vector and \(y\) is the output vector. \(A_{ss}, B_{ss}, C_{ss}\) and \(D_{ss}\) are the state matrix, input matrix, output matrix and direct transmission matrix, respectively. This section describes how these matrices are determined. The procedure used by Aeolus3D to determine the modal characteristics of the system can be summarised with the following steps.

1. Determine the equilibrium point of the system (section 5.6.2).
2. Determine the independent coordinates of the system and the relationship between the independent and dependent coordinates (section 5.6.3).
3. Perturb each independent coordinate of the system in turn and measure the transient response (section 5.6.4).
4. Assemble Hankel matrices from the transient responses (5.6.5).
5. Use the ERA to determine a state-space realisation of the system (5.6.6).
6. Perform an eigenanalysis on the resulting state-space realisation (all modal analyses of the state-space models were performed using MATLAB).

5.6.2 Determination of the equilibrium point

The first step towards an analysis of the dynamic characteristics of the system is to determine the equilibrium point. The system is initially integrated over time until the model has converged to its periodic (or steady state) equilibrium point as described in section 5.7. When measuring the transient responses to perturbations, the responses are normalised with respect to this equilibrium point. The measurements used by the ERA method, which in the case of Aeolus3D are the angles and angular velocities of each joint of each superelement, are measurements relative to the equilibrium condition rather than absolute measurements. It is important to note, however, that the ERA method itself is independent of the type of measurements being used. The angles of the joints of the superelements were merely a convenient and physically meaningful set of measurements for the cases of interest in this work.

Starting from the system state of interest (e.g. the azimuthal position in the case of a VAWT) the simulation was rerun for the number of time-steps required to assemble the Hankel matri-
ces (section 5.6.5) at a time-step size equal to that used when the system is perturbed (which is typically much smaller than that used normally for the structural simulation). The transient measurements of this equilibrium condition are recorded for subtraction off the transient measurements of each set of perturbations.

5.6.3 Determination of independent coordinates

The approach used in Aeolus3D to determine the independent coordinates of the system and their relationship to the dependent coordinates is based on an approach described by Shabana [174, 180] which makes use of the Jacobian constraint matrix of the system.

Shabana’s method involves partitioning the vector of generalised coordinates \( \mathbf{z} \) into a set of independent coordinates \( \mathbf{z}_i \) and dependent coordinates \( \mathbf{z}_d \) (coordinates that are dependent on the independent coordinates), as shown in Eq. 5.56.

\[
\mathbf{z} = \begin{bmatrix} \mathbf{z}_d \\ \mathbf{z}_i \end{bmatrix}
\]  

(5.56)

There are \( n_c \) dependent coordinates and \( n - n_c \) independent coordinates, where \( n \) is the total number of generalised coordinates in the system (i.e. \( 6 \times N \), where \( N \) is the number of rigid bodies in the system) and \( n_c \) is the number of degrees of freedom removed from the system by joint constraints (i.e. the number of rows in the Jacobian constraint matrix). A virtual change in the system coordinates, as shown by Eq. 5.57, can be used to determine a relationship between the independent and dependent coordinate of the system.

\[
\mathbf{J} \delta \mathbf{z} = 0
\]  

(5.57)

A matrix decomposition with full column pivoting can be performed on the Jacobian constraint matrix to determine the independent Jacobian constraint matrix \( \mathbf{J}_i \) and the dependent Jacobian constraint matrix \( \mathbf{J}_d \) as shown in Eq. 5.58.

\[
\begin{bmatrix} \mathbf{J}_d \\ \mathbf{J}_i \end{bmatrix} \begin{bmatrix} \delta \mathbf{z}_d \\ \delta \mathbf{z}_i \end{bmatrix} = 0
\]  

\( \rightarrow \)  

(5.58)

By using a decomposition method with full pivoting the coordinates are automatically rearranged such that the first \( n_c \) elements represent the dependent coordinates and the remainder are the independent coordinates. Aeolus3D uses the QR decomposition with full column pivoting functionality of the GNU scientific library [159] for this operation. \( \mathbf{J}_d \) is a square, upper triangular, non-singular matrix with dimensions of \( n_c \times n_c \). \( \mathbf{J}_i \) is a rectangular matrix with dimensions of \( n_c \times (n - n_c) \).

Eq. 5.58 can be rearranged to give the perturbations of the dependent coordinates in terms of
perturbations of the independent coordinates, as shown in Eq. 5.59.

\[ \delta z_d = (-J_d^{-1} J_i) \delta z_i \]  

(5.59)

Using Eqs. 5.56 and 5.59, the virtual change to all the generalised coordinates of the system can be determined as shown in Eq. 5.60.

\[ \delta z = \left[ -J_d^{-1} J_i \right] \delta z_i = B_i \delta z_i \]  

(5.60)

\( B_i \) is the transformation matrix that maps the virtual changes in the independent coordinates to the virtual changes in all the generalised coordinates. \( I_{ID} \) is an identity matrix with dimensions of \( n_c \times n_c \). Shabana showed that the same approach, as described above, can be applied to find the relationship between the velocities of the independent coordinates and the velocities of all the generalised coordinates, resulting in Eq. 5.61.

\[ \dot{z} = B_i \dot{z}_i \]  

(5.61)

### 5.6.4 Perturbing the system

Perturbing a system was not the original purpose of Shabana’s method for identifying the independent coordinates of the system. The intent of Shabana’s method was actually related to deriving the equations of motion of systems of rigid bodies. As part of the work described by this thesis, however, the method was identified as being suitable for the alternative purpose of perturbing a system of rigid bodies and was successfully used as such. Rather than applying impulsive loads to the system to excite the various modes as is performed in physical modal testing, the approach used in Aeolus3D is to perturb each of the velocities of the independent coordinates one at a time and to measure the transient response. The coordinates themselves could also have been perturbed from their equilibrium points to produce a similar transient response. This, however, would have required a great deal more care to ensure that the joint constraints between the rigid bodies were not violated by the perturbation. Some investigation into this approach was conducted, and acceptable results were obtained by perturbing orientations using a linearised form of a rotation matrix. The results showed no obvious advantage, however, over the much simpler (and with no risk of constraint violations) velocity perturbation approach.

### 5.6.5 Assembly of Hankel matrices

These normalised measurements are assembled into the Markov parameters used to construct the Hankel matrices required for the ERA method. The Markov parameters are the pulse
response matrices, which in the general case are constructed as shown in Eq. 5.62.

\[
Y_k = \begin{bmatrix}
 y_{1,1}(k) & y_{1,2}(k) & \cdots & y_{1,q}(k) \\
 y_{2,1}(k) & y_{2,2}(k) & \cdots & y_{2,q}(k) \\
 \vdots & \vdots & \ddots & \vdots \\
 y_{p,1}(k) & y_{p,2}(k) & \cdots & y_{p,q}(k)
\end{bmatrix}
\]  (5.62)

\(p\) and \(q\) are the total number of responses being measured and the independent coordinates and velocities being perturbed respectively. \(k\) is the index of the particular time-step and \(y\) is the measured response, after subtraction of the equilibrium point.

Hankel matrices are formed using the Markov parameters as shown in Eq. 5.63.

\[
H(k-1) = \begin{bmatrix}
 Y_k & Y_{k+1} & \cdots & Y_{k+r}
\end{bmatrix}
\]  (5.63)

\(r\) can in theory be any arbitrary value greater than or equal to the number of modes of interest in the system, provided that the system response measurements are reduced using a reduced order system identification method such as the proper orthogonal identification method (POD) used by Meng et al. [183] to study HAWTs, for example. For the work presented in this thesis, however, the number of degrees of freedom are small enough that no system reduction method is necessary. \(r\) is set equal to the number of degrees of freedom \((q\) in this case\) of the system such that every mode of vibration of the system can be captured. As explained in the following section, to apply the ERA method, the first two Hankel matrices are required, as shown in Eqs. 5.64 and 5.65.

\[
H(0) = \begin{bmatrix}
 Y_1 & Y_2 & \cdots & Y_q
\end{bmatrix}
\]  (5.64)

\[
H(1) = \begin{bmatrix}
 Y_2 & Y_3 & \cdots & Y_{q+1}
\end{bmatrix}
\]  (5.65)

### 5.6.6 The Eigensystem Realisation Algorithm

As discussed previously, the particular system identification method used in Aeolus3D is the Eigensystem Realisation Algorithm (ERA) developed by Juang and Pappa [182] and described in detail in Juang [187]. Only a summary of of the final equations used to determine the
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state-space matrices in Eq. 5.55 is included here and are shown as Eqs. 5.66 through 5.69.

\[
A_{ss} = \left[ U_n \Sigma_n^{1/2} \right] H(1) \left[ \Sigma_n^{1/2} V_n^T \right] \dagger
\]  
\[ (5.66) \]

\[
B_{ss} = \left[ \Sigma_n^{1/2} V_n^T \right]_{1st\ column}
\]  
\[ (5.67) \]

\[
C_{ss} = \left[ U_n \Sigma_n^{1/2} \right]
\]  
\[ (5.68) \]

\[
D_{ss} = Y_0
\]  
\[ (5.69) \]

U_n, \Sigma_n and V_n are the terms, with all minor trivial modes eliminated, resulting from the single value decomposition of the Hankel matrix H(0) and \( \dagger \) indicates the pseudoinverse of the associated matrix.\(^9\)

5.7 Determination of convergence

To determine whether the system has converged to its periodic equilibrium state, the same general procedure as that described for the determination of the convergence of the aerodynamic simulation (section 4.9) in that the check for convergence is carried out once per revolution. Determining the convergence of the structural system was somewhat more complex, however, than the aerodynamic system. Rather than using a broader, averaged value such as in the case of the aerodynamic system (i.e. the aerodynamic torque), in the case of the structural system the state of each rigid body of each blade is evaluated to determine whether it is the same (i.e. within acceptable tolerances) as at the same azimuthal position during the previous revolution. Three of the states used to determine convergence match those of the state vector used to calculate the equations of motion (Eq. 5.5), the location of the centre of mass (\( x \)), the linear momentum (\( P \)) and the angular momentum (\( L \)). The unit quaternion representing the orientation (\( q \)) does not lend itself easily to a direct test for convergence. Instead, a relative quaternion (\( q_{rel} \)) mapping of the orientation at the previous revolution (\( q_{prev} \)) to the orientation at the current revolution (\( q_{curr} \)) is determined, as shown in Eq. 5.70, and are converted to Euler angles which are used to determine convergence. When all of the Euler angles are below the predefined tolerance, the rigid body can be assumed to be the same orientation as it was at the same azimuth at the previous revolution.

\[
q_{rel} = q'_{prev} q_{curr}
\]  
\[ (5.70) \]

The position of each rigid body of each blade at the azimuth being evaluated is considered the same two revolutions in a row when the value of each component of the position vector is within

\(^9\)The pseudoinverse of a matrix, also known as the Moore-Penrose inverse or the generalised inverse, is a matrix that has some, but not necessarily all, of the properties of the inverse matrix. Unlike a standard inverse matrix, however, the pseudoinverse is not limited to square and non-singular matrices.
a absolute tolerance equal to 1% of the chord length. The orientation is considered to be the same two revolutions in a row when the Euler angles determined from $q_{\text{rel}}$ are all within an absolute tolerance of $0.5^\circ$. The linear and angular momentum are considered to be the same two revolutions in a row when the values of their components are within 1% of their values at the previous revolution down to an minimum absolute tolerance of $1 \times 10^{-2}$.

The check for convergence was performed when one of the blades passed an azimuthal position of $\theta = 45^\circ$. Initially, the check for convergence was attempted as one of the blades passed an azimuthal position of $\theta = 0^\circ$, but it proved quite difficult to reliably detect whether the system had converged at this point. As the blades pass the $\theta = 0^\circ$ and the normal blade loads pass through zero as they transition from one side of the blade to the other, the convergence of the linear momentum in particular was very difficult to assess accurately. At an azimuthal position of $\theta = 45^\circ$, however, the normal blade loads are consistently pointed in the same direction and have a moderate magnitude between their minimum and maximum values. In these conditions, assessing convergence was found to be much more reliable and consistent.

5.8 Verification and validation

There are two important aspects to confirming the appropriateness of the structural modelling method implemented in Aeolus3D, verification and validation. Verification is required to ensure that the analysis method itself will provide accurate results and that the simulation has been correctly implemented in the software. Validation is required to demonstrate that the modelling approach, and in particular the superelement method applied to curved systems, accurately estimates the dynamic characteristics of particular cases. It is also useful in validation not only to confirm the appropriateness of the methods for the primary cases of interest (i.e. VAWT blades) but also to establish the limitations of the methods, beyond which they become invalid.
Table 5.1: Properties of joints of model used for verification of structural simulation.

<table>
<thead>
<tr>
<th>Joint axis</th>
<th>k</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal joint 1 (in-plane)</td>
<td>10000</td>
<td>1500</td>
</tr>
<tr>
<td>Universal joint 1 (out-of-plane)</td>
<td>20000</td>
<td>2000</td>
</tr>
<tr>
<td>Revolute joint</td>
<td>3000</td>
<td>150</td>
</tr>
<tr>
<td>Universal joint 2 (in-plane)</td>
<td>1500</td>
<td>50</td>
</tr>
<tr>
<td>Universal joint 2 (out-of-plane)</td>
<td>2000</td>
<td>100</td>
</tr>
</tbody>
</table>

5.8.1 Verification of the structural simulation

To verify that all parts of the structural simulation (except the linearisation method) were working as intended, an abstract system, as illustrated in Fig. 5.6, was developed, which although simple challenges the many aspects of the structural simulation. The purpose of the model was to verify the equations of motion of the individual rigid bodies, the application of a “gravitational” load offset from both the initial plane of motion and the axis of rotation \( g = [-5.0 \ -5.0 \ -5.0] \text{ms}^{-2} \) ensuring both centrifugal and gyroscopic loads must be correctly accounted for, the joint constraints between the bodies (some of which are offset from the centreline of the bodies and offset from each other), the springs and dampers at each joint axis and the constant velocity generator joint \((\Omega = 2\text{rad}\cdot\text{s}^{-1} \text{about the global } z\text{-axis})\). The system described here requires all of these features to function correctly. To further challenge the simulation, some of the joints were offset from the centreline of the bodies and from each other and the spring and damper values assigned to each joint axis were all different. Also, the dimensions, masses and mass moments of inertia of each body were made unique from each other.

The centre of mass of each body is located at the mid-point of that body. Also, the centre of mass of each body initially lies on the global \( x \)-axis. The masses (in kg) and and mass moments of inertia (in kg·m²) of the bodies are as follows:

\[
\begin{align*}
    m_1 &= 100 & m_2 &= 200 & m_3 &= 150 & m_4 &= 300 \\
    I_1 &= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} & I_2 &= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{bmatrix} & I_3 &= \begin{bmatrix} 15 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix} & I_4 &= \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 10 \end{bmatrix}
\end{align*}
\]

Each joint is initially located on the global \( x - y \) plane with the axes of the universal joints parallel to the global \( y \) and \( z \)-axes and the axis of the revolute joint parallel to the global \( x \)-axis. Each joint axis in the system (other than the generator) has a linear torsional spring and viscous torsional damper assigned with the spring \((k \text{ in N·m·rad}^{-1})\) and damping \((c \text{ in N·m·s·rad}^{-1})\) coefficients as shown in table 5.1 (where Universal joint 1 is closest to the generator):

The system was assembled and simulated in Aeolus3D and in MATLAB’s SimMechanics package for comparison. Where possible, matching solver settings were used. Both simulations were run using variable time-step solvers at a maximum permissible step-size of 0.05s, with relative
and absolute tolerances of $1 \times 10^{-6}$. One difference between the two simulations, however, was the numerical solver used. The RKF45 method used in Aeolus3D was not built into SimMechanics, so the Dormand-Prince fourth/fifth order method (ode45) was used instead. Both methods are of a similar order, however, so a comparison of the results was expected to be valid.

The angle of displacement about each joint axis was recorded during ten minutes of simulated time, with the results from the first and last five seconds presented in Fig. 5.7. Even after ten minutes of simulated time (at least 12000 times-steps), there is no discernible difference between the angles predicted by Aeolus3D and the angles predicted by SimMechanics. It is clear from these results that the structural simulation functionality of Aeolus3D operates as intended.

### 5.8.2 Verification of modal properties

The ERA method, as applied by Aeolus3D, was verified against a linearisation in MATLAB by analysing identically configured models simulated with both Aeolus3D and the MATLAB SimMechanics toolbox. Although the ERA method itself has been successfully applied a number of times, it was important to verify the Aeolus3D implementation against an established linearisation implementation.

**Single superelement at various rotation rates**

The verification presented here to prove that the modal properties are being correctly calculated is that of a straight uniform beam rotating at various angular velocities about one end, as illustrated in Fig. 5.8. The selected properties of the beam are taken from a verification study conducted by Ortiz and Bir [188] and also described by Bir [189] of the linearisation capability.
Chapter 5: Structural modelling

Figure 5.8: Pictorial representation of a single superelement approximating a straight rotating beam with uniform cross-sectional properties and of two superelements approximating a bent, closed (attached at both ends), rotating beam with uniform cross-sectional properties.

The properties of the beam are as follows.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of beam</td>
<td>$L$</td>
<td>31.62 m</td>
</tr>
<tr>
<td>In-plane flexural rigidity</td>
<td>$EI_{\text{in}}$</td>
<td>$1.0 \times 10^8$ N m$^2$</td>
</tr>
<tr>
<td>Out-of-plane flexural rigidity</td>
<td>$EI_{\text{out}}$</td>
<td>$1.0 \times 10^9$ N m$^2$</td>
</tr>
<tr>
<td>Torsional rigidity</td>
<td>$GJ$</td>
<td>$1.0 \times 10^5$ N m$^2$</td>
</tr>
<tr>
<td>Linear density of beam</td>
<td>$\mu$</td>
<td>100 kg m$^{-3}$</td>
</tr>
<tr>
<td>In-plane radius of gyration</td>
<td>$R_{g,\text{in}}$</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Out-of-plane radius of gyration</td>
<td>$R_{g,\text{out}}$</td>
<td>0.001 m</td>
</tr>
</tbody>
</table>

The damping coefficients at each joint (not included in the work of Ortiz and Bir) were set at 2% of the spring stiffness at the same joint i.e. Rayleigh’s proportional damping, $c = \eta_1 m + \eta_2 k$ with $\eta_1 = 0$ and $\eta_2 = 0.02$. The beam was modelled using a single superelement in both Aeolus3D and also in SimMechanics for comparison. The modal properties were extracted from the SimMechanics model using MATLAB’s \textit{linmod} function. The gravitational acceleration in this case was set to $g = 0$ m s$^{-2}$. The predicted natural frequencies and damping ratios of the two in-plane bending, two out-of-plane bending and the torsional model of vibration are shown in Fig. 5.9. It is clear from these results that there is very good agreement between the Aeolus3D and SimMechanics predictions across a broad range of rotational rates.
Figure 5.9: Comparison between the natural frequencies and damping ratios predicted by Aeolus3D and SimMechanics for a straight rotating beam modelled using a single superelement.

Figure 5.10: Comparison between the natural frequencies of the flatwise modes of vibration at various azimuthal positions predicted by Aeolus3D and SimMechanics for a straight beam modelled using a single superelement rotating at 2 rad s\(^{-1}\), with gravity acting across the plane of rotation.

**Single superelement at various azimuthal positions**

The previous example was subjected to no external loading so the results were independent of azimuthal position. In the next case, however, the beam is rotated at an angular velocity of 2 rad s\(^{-1}\) and a gravitational acceleration of \(g = 9.81\) m s\(^{-2}\) is applied in a direction across the plane of rotation of the blade, as shown in Fig. 5.8. This introduces a slight azimuthal dependency to the predicted modal characteristics, which are shown in Fig. 5.10. Only the natural frequencies of the first and second out-of-plane bending modes of vibration are shown. Changes to the in-plane bending natural frequencies, the torsional natural frequencies and all damping ratios are negligible in this case. The results again show good agreement between the
Aeolus3D and the SimMechanics predictions across all azimuthal positions. Small differences can be seen in the prediction of the second mode of vibration at azimuthal positions of 90° and 180°, but as a percentage error of only 0.05%, this is a very trivial difference and may simply reflect the different numerical models or numerical rounding used in each simulation.

Two superelements in bent closed-loop configuration

Both of the previous cases investigated straight beams only. As the primary purpose of the work described in this thesis is to investigate curved beam-like structures such as VAWT blades, it was important to verify that Aeolus3D is not limited to such simple cases. A highly simplified VAWT blade configuration was modelled by bending the beam used in the previous cases through 90° at its mid-point and fixing both ends to a rotating axis, as illustrated in Fig. 5.8. Each straight segment of the bent beam was modelled with a single superelement. Figure 5.11 shows the natural frequencies and damping ratios of the modes of vibration. Like the straight beam case, there is very good agreement between the Aeolus3D and SimMechanics predictions across a wide range of rotational rates.

5.8.3 Validation of modal properties

Although the validation of the superelement approach for the modelling of straight beams was conducted through the course of this work for both stationary and rotating beams, the results do not need to be included here. Validation of the superelement method for straight uniform-beams (for modelling HAWT blades) has already been presented by Molenaar [115] and for straight tapered-beams by Holierhoek [123].

Like their horizontal axis counterparts, VAWTs are constructed with long flexible beam-like blades, which typically become relatively more flexible as the scale of the wind turbine increases.
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With curved blades physically attached at both ends, however, it does not automatically follow that a method suitable for modelling HAWTs would be suitable for modelling VAWTs. The validation that the dynamic characteristics of curved-beams are modelled correctly using this method was an important step to confirm the suitability of the method. The purpose of this section is to demonstrate the validity of the superelement approach for curved beam-like structures.

Three different cases of varying shapes and slenderness ratios, as illustrated in Fig. 5.12, with properties shown in table 5.2, were examined and compared with results available in the literature. The slenderness ratio is defined as \( S = \sqrt{AR^2/I} \) where \( A \) is the cross-sectional area, \( R \) is the radius of the arc and \( I \) is the second moment of area. In each case the number of superelements used to approximate the arcs was varied to evaluate the rate of convergence with respect to the level of discretisation of the curves. All three cases are semi-circular arcs with an opening angle of 180°.

### Table 5.2: Physical properties of the curved beams used for validation.

<table>
<thead>
<tr>
<th>Parameter (units)</th>
<th>Symbol</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of arc (m)</td>
<td>( R )</td>
<td>0.5642</td>
<td>0.1128</td>
<td>0.305</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>( E )</td>
<td>210</td>
<td>210</td>
<td>68.13</td>
</tr>
<tr>
<td>Shear modulus (GPa)</td>
<td>( G )</td>
<td>81</td>
<td>81</td>
<td>25.61</td>
</tr>
<tr>
<td>Density (kg m(^{-3}))</td>
<td>( \rho )</td>
<td>7850</td>
<td>7850</td>
<td>2882</td>
</tr>
<tr>
<td>Diameter of beam (mm)</td>
<td>( d )</td>
<td>22.57</td>
<td>22.57</td>
<td>-</td>
</tr>
<tr>
<td>Width of beam (mm)</td>
<td>( w )</td>
<td>-</td>
<td>-</td>
<td>18.90</td>
</tr>
<tr>
<td>Height of beam (mm)</td>
<td>( h )</td>
<td>-</td>
<td>-</td>
<td>6.20</td>
</tr>
<tr>
<td>In-plane slenderness ratio (-)</td>
<td>( S_{in} )</td>
<td>100</td>
<td>20</td>
<td>170</td>
</tr>
<tr>
<td>Out-of-plane slenderness ratio (-)</td>
<td>( S_{out} )</td>
<td>100</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>
Case 1

In case 1, a semi-circular arc with a circular cross-section, fixed at both ends, was examined with the results presented in Fig. 5.13. The results were compared with the natural frequencies predicted by Yildirim’s TMARC program [190] for this case, which also showed very good agreement with Irie et al. [191, 192]. The comparison shows very good agreement between the natural frequencies predicted by the Aeolus3D approach and the results obtained from the literature. From these results it appears that Aeolus3D requires that the number of superelements be at least two more than the number of in-plane modes of interest to achieve very good results for this particular case (e.g. to predict the fourth in-plane mode, six superelements are required).

Case 2

In case 2, a semi-circular arc with cross-sectional and material properties identical to the beam in case 1 and again fixed at both ends was examined. Only the radius of the arc was shortened, leading to a reduced slenderness ratio. The results were again compared, as shown in Fig. 5.14, with the natural frequencies predicted by Yildirim [190], who achieved very good agreement with Irie et al. [191, 192]. Aeolus3D again shows very good agreement with results in the literature when the number of superelements exceeds the number of modes of interest by at least two. Although Aeolus3D shows very good agreement with this thicker beam, it is expected that if the slenderness ratio is further reduced the accuracy of the predictions will eventually deteriorate. The approach used in Aeolus3D to determine the spring coefficients is based on Euler-Bernoulli, so is strictly only applicable to thin beams. For modelling thick beams using superelements an alternative approach based on Timoshenko beam theory, for example, such
Case 3

In case 3, a semi-circular arc with a rectangular cross-section, fixed at one end and free at the other, was examined with the results presented in Fig. 5.15. The results show very good agreement again with Yildirim [190].
Table 5.3: Comparison between the natural frequencies of the first in-plane and first out-of-plane modes predicted by Aeolus3D, Yildirim (TMARC) and Tabarrok of a curved beam with a rectangular cross-section, an in-plane slenderness ratio of 170 and an out-of-plane slenderess ratio of 20, fixed at one end and free at the other.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{\text{in}}$</th>
<th>$\omega_{\text{out}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aeolus3D (seven superelements)</td>
<td>41.74</td>
<td>56.12</td>
</tr>
<tr>
<td>Yildirim [190]</td>
<td>40.71</td>
<td>54.80</td>
</tr>
<tr>
<td>Tabarrok (finite element) [193]</td>
<td>44.3</td>
<td>59.6</td>
</tr>
<tr>
<td>Tabarrok (experimental) [193]</td>
<td>41</td>
<td>56</td>
</tr>
</tbody>
</table>

The first in-plane and first out-of-plane modes, predicted using both theoretical and experimental methods, were also presented by Tabarrok and Sinclair [193] and also show good agreement as shown in table 5.3 with seven superelements in Aeolus3D (selected because Tabarrok’s finite element results were calculated using a model with seven finite elements).

5.8.4 Conclusion

The verifications and validations presented in this section demonstrate that Aeolus3D is capable of modelling the structural dynamics of a VAWT. The multibody system simulation as implemented in Aeolus3D shows excellent agreement with the established industrial package SimMechanics confirming that the motion of the rigid bodies, the joint constraints, the springs and dampers at the joints and the constant velocity generator joint are all implemented correctly. Furthermore, the novel method presented of using the Jacobian constraint matrix of the system to determine the independent coordinates and to perform the subsequent perturbation of each of the independent velocities, which subsequently provides the necessary measurements of for the ERA method of system realisation, has been validated using several test cases over a wide range of rotation rates. Finally, the suitability of the superelement approach of multibody system modelling, originally developed for modelling straight beam-like structures, has been validated for the first time as an appropriate method for also modelling curved beams. The results obtained for both fixed-fixed and fixed-free beams clearly demonstrate that the method may be used with confidence.
6.1 Introduction

This chapter describes the approach used to interface the structural and aerodynamic simulations, which are described in isolation from each other in chapters 4 and 5 respectively. The chapter is introduced with an overview of the purpose of the interface, some of the challenges faced in coupling the aerodynamic and structural models together, and the general approach taken in Aeolus3D. Following this is an explanation of how information about the structural state of the system is passed to the aerodynamic model, and then information about how the aerodynamic loads are passed from the aerodynamic model to the structural model.

Coupling structural and aerodynamic models together can be very challenging. As pointed out by Rasmussen [133], care must be taken to ensure that any numerical instability is not misinterpreted as a physical instability. This is of particular concern in Aeolus3D, considering that its primary purpose is to investigate the dynamic behaviour of VAWTs.

6.2 Overview of the aero-structural interface

Figure 6.1 illustrates the specific data required to pass between the aerodynamic and structural simulations of Aeolus3D via the aero-structural interface. The details on how the data required are passed from the structural model to the aerodynamic model are described in section 6.3, and from the aerodynamic model to the structural model in section 6.4.

In general, the approach used to couple together the fluid components of a simulation and the
Chapter 6: Aero-structural interface

Figure 6.1: Conceptual illustration of aero-structural interface and data transferred between the aerodynamic and structural simulations in Aeolus3D.

structural components of a simulation can be broadly described as either weakly coupled or strongly coupled. A strongly coupled simulation requires combining the equations describing the fluid motion together with the equations describing the structural motion. For simple cases this may be a good approach, as combining the equations of both the aerodynamic and structural models ensures that all physical elements are accounted for at once and can be solved simultaneously at every time-step. To simulate more complex or arbitrary structures such as VAWTs, however, combining the equations describing the fluid and structural elements becomes very difficult. In Aeolus3D, the structural and aerodynamic elements were developed as two separate simulations (as described in chapters 4 and 5) and a weakly coupled method was used to combine the two together into a fully aeroelastic simulation. In a weakly coupled simulation, the equations representing the fluid and the equations representing the structural elements of the simulations remain separate. The separate simulations pass data to and from each other and can either be solved iteratively at each time-step, or in a time-marching approach (as implemented in Aeolus3D), where the output from one model is used as the input to the other, but they are not solved together in an iterative manner.

There are, of course, an infinite variety of ways in which the aerodynamic and structural simulations may be coupled together. The general approach employed in Aeolus3D errs on the conservative side by using weighted averages of the parameters to be transferred between simulations, with the weighting based on the relative overlap between the elements of each simulation. It is probable that a more sophisticated interface might provide greater accuracy, but until sufficient and reliable experimental aeroelastic measurements of a VAWT are readily available, the present more conservative approach is not just adequate, but also it is more appropriate than a more complex, but unprovable, coupling method.

6.2.1 Temporal coupling

The aerodynamic and structural simulations typically require different time-step sizes from each other, with the structural simulation generally requiring a much smaller time-step size than the aerodynamic simulation. This becomes especially true as the stiffnesses of the structural
members of the system are increased. The stiffer the members, the smaller the structural time-steps required. Ideally, the time-step size of the aerodynamic simulation would match the time-step size of the structural simulation. As discussed in chapter 4, however, the computational demands of the aerodynamic simulation escalate rapidly as the time-step size is decreased, so the resulting combined simulation would be computationally impractical.

### 6.2.2 Geometric coupling

The two separate simulations may require not only different time-step sizes, but also different geometric discretisations. The case studies presented in chapter 8, as it turned out, were run with the number of aerodynamic elements matching the number of structural superelements. It was not known in advance, however, what the suitable level of discretisation for both the aerodynamic and structural models would be (the suitability of which is demonstrated in chapter 7). Even given matching numbers of aerodynamic elements and structural superelements, as each superelement consists of four rigid bodies, the interface must transfer the data between models of different geometric discretisation. The main advantage of restricting the number of aerodynamic elements to match the number of structural superelements, would have been a simpler implementation of the mapping procedures, but the general method would have been the same. The method implemented in Aeolus3D allows completely arbitrary and different numbers of aerodynamic elements and structural superelements.

The side view of a VAWT blade modelled using three structural superelements and four aerodynamic elements is illustrated on the left hand side of Fig. 6.2. Realistically, using only three structural superelements as illustrated here is not likely to produce satisfactory results, but is useful to highlight the challenge in mapping the two different simulations together. It is clear in Fig. 6.2 that when there are a different number of aerodynamic elements and structural superelements making up a curved blade, the rigid bodies of the structural simulation do not align directly with the elements of the aerodynamic simulation. The first step in mapping the
two models together was to “unwrap” each blade model, laying out the individual elements flat, as shown on the right hand side of Fig. 6.2, with their lengths normalised so that the two may be compared directly with each other. The actual length of each discretised blade is clearly quite different for so few elements, but as the number of elements in each model increases their actual lengths will become similar (and the length of each discretised blade will approach the length of the actual VAWT blade being modelled).

### 6.3 Structural to aerodynamic coupling

The structural data required by the aerodynamic simulation is pulled, via the interface, at the start of each aerodynamic time-step. The data required by the aerodynamic simulation are the current locations of the leading edges of the blades at the end of each aerodynamic element, the current locations and velocities of the trailing edges of the blades at the end of each aerodynamic element, and the current velocities of the mid-chord points of the blades (which, as noted in section 4.2.2 is where the angles of attack are determined in Aeolus3D).

Mapping the data from the structural to the aerodynamic simulation was done using two main methods. One method maps the structural data to the ends of the aerodynamic elements. This is the method used to map the positions of the leading and trailing edges, and the velocities of the trailing edges. The other method, which is needed for the velocities at the angle of attack locations, maps the structural data to the middle of each aerodynamic element’s span.

#### 6.3.1 Location of leading and trailing edges

A simple illustration of the method used to map the structural data to the end of an aerodynamic element is shown in Fig. 6.3. The approach taken in Aeolus3D is to assume that the locations of the leading and trailing edges of the end of each aerodynamic element can be described as a weighted average of their locations relative to the centres of gravity of each of the rigid
bodies that are overlapped by the adjacent aerodynamic elements up to their mid-span. The weighting of each body’s contribution to the averages is based on the length of overlap between each body as a proportion of the total overlap of all the bodies. The locations of the leading and trailing edges are determined using the weighted average of several rigid bodies rather than relative to a single rigid body because although the edges of the rigid bodies are collocated with each other during initialisation, as the simulation progresses and the blades are allowed to deform, the edges of the rigid bodies will no longer be collocated. Using a weighted average between the rigid bodies eliminates the necessity to make an assumption about which single rigid body best reflects the location of the end of an aerodynamic element. Using more than just the rigid bodies immediately either side of the end of the aerodynamic element to determine the locations of the leading and trailing edges is an attempt to, at least partially, account for the deformation of all rigid bodies, despite the fact that the aerodynamics of each element is ultimately represented by a single bound vortex overlapping several rigid bodies at a time.

Like the situation with the different time-step sizes required by the aerodynamic and structural simulations mentioned in section 6.2.1, ideally the number of aerodynamic elements used to model the blades would be at least as many as (and preferably exceed) the number rigid bodies used to model the blades. This would allow every small deformation of the blades to be captured by both the structural and aerodynamic models, instead of needing to approximate the deformation of the rigid bodies with the weighted average approach described here. Just like the restriction on the practical limit for the time-step size of the aerodynamic model, the computational cost also grows rapidly with an increase in additional aerodynamic elements. It is, unfortunately, still not practical at this stage to model the blades with as many aerodynamic elements as desired.

The locations of the mid-span of the two adjacent aerodynamic elements of the unwrapped and normalised aerodynamic blade are projected onto the unwrapped and normalised structural blade so that the rigid bodies overlapped by the relevant halves of the aerodynamic elements can be identified (designated as bodies \(i, j, p\) and \(q\) in this example) and the length of overlap of each measured \((l_i, l_j, l_p\) and \(l_q)\). Two of the rigid bodies in this example \((j\) and \(p\) are completely overlapped and the other two rigid bodies \((i\) and \(q\) are partially overlapped. The location of the centre of gravity of each rigid body along the span of the structural blade is projected onto aerodynamic blade. The vectors from the centres of gravity of each rigid body to the leading \((r_{le}^i, r_{le}^j, r_{le}^p\) and \(r_{le}^q)\) and trailing \((r_{te}^i, r_{te}^j, r_{te}^p\) and \(r_{te}^q\) edges of the aerodynamic element are determined. Note that although the vectors in Fig. 6.3 are shown as two-dimensional vectors on the unwrapped model they are, in fact, determined based on the three-dimensional model, stored in terms of the local coordinate system of each body, and converted to the global coordinate system each time they are required. They are shown as two-dimensional vectors on the unwrapped model for visual clarity only.
6.3.2 Velocity of the trailing edge

The velocity of the trailing edge at the end of each aerodynamic element is needed to check whether the direction of the fluid flow is in fact off the trailing edge as described in section 4.3.1. The velocity at the trailing edge is calculated using the same weighted average approach as used to determine the position, except that the velocity at that position relative to each rigid body is used instead of just the position itself. The velocity at the trailing edge relative to body $i$, for example, is calculated as shown in Eq. 6.3.

$$ v_{le}^i = v^i + \omega^i \times r_{le}^i $$

The weighted average of the velocity at the trailing edge ($v_{te}$) is, therefore, calculated as shown in Eq. 6.4.

$$ v_{te} = \frac{l^i v_{le}^i + l^j v_{le}^j + l^p v_{le}^p + l^q v_{le}^q}{L} $$

6.3.3 Velocity of the angle of attack location

The velocity of each aerodynamic element at its angle of attack location is required to determine the angle of attack, and also the magnitude of the aerodynamic loads experienced by the blades.
A simple illustration of the interface used to map the velocities determined by the structural simulation to an aerodynamic element is shown in Fig. 6.4. In this interface the overlap is based, not on the two halves of adjoining aerodynamic elements, but based on each whole individual aerodynamic element.

Each end of the aerodynamic element is projected onto the structural blade to, once again, identify the rigid bodies being overlapped, and the length of the overlap of each rigid body. Like the previous example, in this example there are two rigid bodies \((j \text{ and } p)\) that are completely overlapped and two rigid bodies \((i \text{ and } q)\) are partially overlapped. The chordwise location at which the angle of attack is to be calculated (i.e. mid-chord in Aeolus3D) is projected onto the structural blade and the velocity at mid-span of the overlap of each rigid body is calculated \((v^i, v^j, v^p \text{ and } v^q)\). The weighted average of the velocity over the whole aerodynamic element \((v)\) is shown in Eq. 6.5.

\[
v = \frac{l^i v^i + l^j v^j + l^p v^p + l^q v^q}{L}
\]

**6.4 Aerodynamic to structural coupling**

The aerodynamic data required by the aerodynamic simulation is pulled from the aerodynamic simulation, via the interface, at the start of each structural time-step that immediately follows an aerodynamic time-step. For each subsequent structural time-step, until another aerodynamic time-step is taken, the loads normal and tangential to the blade sections are assumed to remain constant. Ideally, the loads would be allowed to change continuously as the blade bends and twists between aerodynamic time-steps to account for the changing flow velocity and angle of attack. Some consideration was given to various methods that would allow this, such as fixing the structure of the wake between major aerodynamic time-steps, but allowing the velocity and angle of attack to be recalculated at each structural time-step. Such approaches might have been beneficial, but without adequate experimental measurements against which to compare the various approaches, the introduction of more complex methods could not be justified. In addition, for small time-steps the changes to the angle of attack are small.

A simple illustration of the interface used to map the aerodynamic loads to the rigid bodies is shown in Fig. 6.5. In Aeolus3D, the loads are calculated at the mid-span and are assumed to be constant across the span of each aerodynamic element. The proportion of the load to be applied to each overlapped rigid body is estimated from the length of the overlap of each rigid body as a proportion of the length of the whole aerodynamic element. The normal force
applied to body “i” of the example shown in Fig. 6.5, for example, is defined in Eq. 6.6.

\[
F_i^N = F_N \left( \frac{l_i}{l_i + l_j + l_p + l_r} \right) \tag{6.6}
\]

The loads are applied to the rigid bodies at locations corresponding to the middle of each overlapped region in a spanwise direction, and to the quarter-chord position in a chordwise direction.

### 6.5 Determination of convergence

The process for the determination of convergence of the aerodynamic simulation is described in section 4.9, and the process for the determination of convergence of the structural simulation is described in section 5.7. The determination of the combined aeroelastic simulation simply requires that the criteria for convergence of both the structural and aerodynamic simulations are met. Convergence of a VAWT is checked as one of the blades passes an azimuthal position of 45°.

### 6.6 Verification and validation

#### 6.6.1 Verification

A variety of testing was done to verify that the data were being transferred between simulations as intended. In the case of the transfer of data from the structural simulation to the aerodynamic simulation, this was done by inspecting and checking by hand each stage of the process for several elements at various azimuthal positions. In the case of the transfer of data from the aerodynamic simulation to the structural simulation, a qualitative test was also possible by plotting the distribution of loads (per unit length) along the blades as calculated by the aerodynamic simulation, and then as applied to the structural simulation. Also, calculating the
total torque at each azimuthal position via the structural model and via the aerodynamic model yielded very similar results, with some slight numerical differences that could be attributed to the different radii of the rigid bodies compared to the radii of the aerodynamic elements. Overall, it was possible to be confident that the interface between the aerodynamic and structural models performs as described in this chapter.

### 6.6.2 Validation

Although, as demonstrated in sections 4.10 and 5.8, the aerodynamic and structural simulations can be used with confidence in isolation from each other, the combination of the aerodynamic and structural models could not be validated against experimental measurements. Every effort has been made to minimise the risk that the combined model is invalid by carefully validating each component independently and using the clear and simple coupling scheme described in this chapter. The physical measurements required to fully validate this combined model are not currently available as far as the author is aware, and the process of actually acquiring such data experimentally for the purposes of this work was out of scope of the present PhD research.

The mere acquisition of reliable experimental aeroelastic data is, in fact, quite a complex undertaking in its own right, aside from even considering the appropriate interpretation of such data. Of particular note is the considerable complexity involved in accurately measuring the wind field experienced by the turbine - and yet, an accurate knowledge of the conditions within the wind field is vital to the interpretation of any aeroelastic behaviour. The author suggests that should efforts be pursued in the future to gather experimental data related to the aeroelastic behaviour of VAWTs, those efforts should first focus on wind tunnel tests where the conditions of the airflow can be carefully controlled and, perhaps more importantly, accurately measured. In order to obtain reliable experimental data regarding the structural behaviour of the turbine, the use of lightweight strain gauges should prove suitable for flapwise and edgewise bending moments at various locations along the blades. Lightweight accelerometers may also be a worthy consideration, but a great deal of care would need to be taken to ensure that the mass of the accelerometers themselves are either negligible compared to the mass of the turbine components to which they are attached, or that their effect can be accurately removed from the results during post-processing of the data. Otherwise, the very presence of the accelerometers themselves may artificially skew the results. The use of pressure taps within the blades could provide some very useful data, especially with regards to measuring the aerodynamic pitching moment experienced by the blades and give greater insight into the dynamic stall experienced by VAWT blades. As discussed previously, however, care must be taken when determining the instantaneous loads based on pressure tap measurements - especially with respect to tangential loads. The author also suggests that any future experimental work may benefit from an investigation into the possible use of optical systems for measuring blade motion. While there would, undoubtedly,
be some considerable challenges in the use of such a system - the potential for a completely non-invasive system of measurement is sufficiently tempting that its use in aeroelastic experimental work should not be overlooked.

In addition to the challenges of measuring loads, displacements, and flow conditions, there is also the issue of how modal properties can be extracted. Typically, to extract the modal properties of a structure, the structure is excited in some way and the response measured. With a non-rotating structure, one may need to choose a suitable excitation load and location, but for a rotating structure such as a wind turbine there is the added complexity of how to apply the excitation loads while the structure is in motion. Assuming the application of suitable excitation loads and accurate measurements of the response, the aeroelastic experimentalist is still faced with the non-trivial task of interpreting the measurements. As the loads are continually changing as the blades rotate, so too are the effective stiffness and damping - which means that the modal properties are also a function of the azimuthal position and must be measured and interpreted at multiple orientations while the turbine rotates. Clearly, such an experimental investigation would represent a very worthwhile, but very challenging piece of work.

One area where a considerable amount of time and effort was dedicated was the extraction of modal properties from the combined aero-structural model. It was hoped during much of the PhD programme, that by extracting the modal properties from the combined model, the influence that various parameters had on the stability of VAWTs could be investigated. Without complete validation, however, the author considers that such analysis could not be conducted with complete confidence. The method presented in section 5.6 can be used with confidence (as demonstrated by the verification and validation conducted and presented in section 5.8) to extract the modal characteristics of the structural system in isolation from the aerodynamic model, but further work, and validation in particular, is required before the method can be used with confidence for the fully combined model. Additionally, unlike purely structural systems which were found to be very tolerant of linearisation parameters (time-step sizes, perturbation strengths etc.), with the aerodynamics included the results were far more sensitive to linearisation parameters. Again, without validation it is not possible to know whether this is a limitation with the coupling method, or with the linearisation method applied to the combined models. A number of assumptions were needed to apply the linearisation method to the combined model, particularly with regard to how the free wake should be modelled during linearisation. On the other hand, there are no other methods available to accurately investigate the aeroelastic stability of VAWTs before they are built. The details of the attempted implementation are not included here due to lack of space, but may represent a significant area for future research should experimental validation data become available.
Chapter 7

Determination of required computational parameters

*Aristotle maintained that women have fewer teeth than men; although he was twice married, it never occurred to him to verify this statement by examining his wives’ mouths.*

Bertrand Russell

7.1 Introduction

In an ideal world, with unlimited computing power, this chapter would not be necessary. Arbitrarily small time-steps with any level of discretisation and no restriction on the length of the wake could be modelled without hesitation. In reality, however, all computational modelling requires a compromise to be made between a desire for very detailed simulations, and a practical restriction on the available computational resources. The purpose of this chapter is to describe the computational parameters that were selected for both the aerodynamic and structural simulations and to demonstrate the suitability of those selections. The general process was to test representative models with a range of each computational parameter and examine the rate of convergence of either the loads (in the case of the aerodynamic simulations) or the natural frequencies (in the case of the structural simulation) in response to that parameter. The chapter starts with a section describing the key features of the aerodynamic and structural simulations determining the overall computational cost and the relationship between those features and the computational cost (section 7.2). The determination of the required computational parameters of the aerodynamic simulation are then presented in two parts. The determination of the key computational parameters of the two-dimensional simulation (i.e. Aeolus2D) are presented in section 7.3, and the determination of the key computational parameters of the
three-dimensional simulation (i.e. Aeolus3D) are presented in section 7.4. The determination of the key computational parameter of the structural simulation (there is only a single relevant parameter for the structural simulation) is presented in section 7.6.

7.2 Overview of computational costs

7.2.1 Computational cost of aerodynamic simulations

The computational cost of the aerodynamic simulation is strongly governed by the number of vortices being simulated at any given time, as illustrated in Fig. 7.1. The computational cost per time-step is shown as a function of the number of vortices being modelled for the Sandia 17m turbine for two very different TSRs and combinations of computational parameters. The times indicated on these plots are approximate values only, based on the system time rather than the precise process time. It is clear from these plots, however, that the required computational time per time-step does increase parabolically with the number of vortices (confirming that the aerodynamic simulation is an \( O(N^2) \) operation). In order to keep the computational cost to manageable levels, it was important to determine (and restrict) the number of vortices being simulated during any given time-step. The number of vortices being simulated is essentially a function of the time-step size, the wake length, the number of blades, the TSR, and (in the case of the three-dimensional model) the number of aerodynamic sections per blade. The number of blades and the TSR are fixed parameters for a particular case and as such cannot be modified to restrict computational cost. The other parameters listed, however, can all be adjusted in an effort to balance the computational cost against the accuracy.

It should be noted that the values ultimately selected were not an optimised set, but were based on a subjective evaluation of a generally acceptable set of values across a wide range of turbine configurations. At low TSRs, for example, a much higher level of blade discretisation and smaller time-step sizes could be used than at high TSRs. This is because at high TSRs the vortices released into the wake remain closer to the turbine for longer (as they convect downstream much more slowly relative to the rate of vortex release than at low TSRs) and the turbine requires many more revolutions to reach equilibrium. The combination of the proximity of many more vortices and the longer simulation runs compound with each other resulting in simulations taking much longer than required for the same turbine configuration at low TSRs. Although it is tempting to allow different computational parameters at different TSRs (and turbine configurations), selecting a single set of computational parameters that is applied to all cases reduces the risk that apparent differences are, in fact, due to differing computational parameters rather than genuine differences. As such, a compromise was made in an effort to find acceptable values for the computational parameters across a wide range of cases.
Chapter 7: Determination of required computational parameters

![Graph showing computational time required](image)

**Figure 7.1:** Relationship between number of vortices being simulated and the computational cost of the three-dimensional aerodynamic simulation per time-step.

### 7.2.2 Computational cost of structural simulations

The computational cost of the structural simulation is strongly governed by the number of superelements per blade, as illustrated in Fig. 7.2. The computational cost, normalised against the computational cost with 9 superelements, is shown as a function of the number of superelements per blade for the baseline turbine configuration with both one and two blades.

From Fig. 7.2 it is clear that the computational cost increases exponentially with the number of superelements per blade. Interestingly, however, the computational cost does not increase exponentially with the total number of superelements in the system as illustrated by the fact that the computational cost of the one and two-bladed turbines maintain an almost constant difference regardless of the number of superelements per blade. This is because the computational demand of modelling the bodies and their interconnections forms only a small part of the total computational cost of the structural simulation. Additional bodies (such as due to an additional blade) do not, in of themselves, add significantly to the total cost. The primary reason that the computational cost increases with additional superelements per blade is that as additional superelements are added to the same blade, they must become shorter. As demonstrated in section 5.3.1, the stiffness of each joint is inversely proportional to the length of each superelement. It is this increased stiffening of the system as additional superelements are added to each blade that increases the computational demands of the whole simulation.
7.3 Aerodynamic parameters - Aeolus2D

7.3.1 Description of turbine configurations

Two different turbine configurations were used to investigate the effects that the computational parameters have on the results of Aeolus2D, a one-bladed turbine (VAWT A) with quite a large chord to radius ratio \((c/R \approx 0.25)\), and a two-bladed turbine (VAWT B) with a lower chord to radius ratio \((c/R \approx 0.073)\). The one-bladed turbine is a configuration used in a set of water towing tank tests conducted by Graham [168], against which Aeolus2D was validated (section 4.10.3). The two-bladed turbine is included as it is more representative of the typical proportions (and number of blades) of the turbines of interest in the work described in this thesis. The properties of the two-bladed turbine are based on the Sandia 17m turbine [170]. The key properties of both turbines are listed in table 7.1.

7.3.2 Vortex release rate (time-step size)

This section describes how the loads and wake shapes of the two-dimensional turbines are affected by the rate at which vortices are released into the wake, i.e. the time-step size. The vortex release rate is defined by the number of vortices released per revolution of the turbine. A wide range of release rates were tested other than the 18, 36, 72 and 180 per revolution.
Figure 7.3: The variation of normal, tangential and moment load coefficients with azimuth over the seventh revolution of the one-bladed turbine at TSRs of 2.5 and 7.6 for a range of vortex release rates.

Load coefficients of the one-bladed turbine (VAWT A)

Figure 7.3 presents the normal ($C_N$), tangential ($C_T$) and moment ($C_M$) load coefficients predicted for VAWT A, at a TSR of 2.5 and a TSR of 7.6 over a single revolution of the blade for vortex release rates of 18, 36, 72 and 180 steps per revolution. In each case the results presented are for the seventh revolution of the turbine, allowing a sufficient number of vortices to be released into the wake to ensure that their presence affect the blade loads. At a TSR of 7.6, the turbine was unlikely to have reached equilibrium by this stage, but this is unimportant here as the purpose of these tests was to establish the relationship between the computational

presented here, but the results were consistent with those presented.
parameters and the results, not the actual equilibrium loads of the turbines.

It is clear from Fig. 7.3 that at a TSR of 2.5, the predicted loads are not very dependent on the number of steps taken per revolution. There is little difference in the values of the coefficients at any azimuthal position. At a TSR of 7.6, there is more variation in the predicted results. At 18 steps per revolution, two distinct spikes appear at azimuths of 200° and 300° that do not occur at the other rates. With as few as 36 steps per revolution, however, the differences in the predicted $C_N$ and $C_T$ with increasing vortex release rates becomes quite minor, despite the very large difference in computational cost between 36 and 180 steps per revolution. $C_M$ appears quite chaotic on the downstream side (180° - 360°) at a TSR of 7.6. This can be partially attributed to the very low values of $C_M$ at high TSRs (low angles of attack, and no stall) but not entirely. The fact that the $C_M$ curve is much smoother over the upstream side of the turbine implies that $C_M$ is probably a lot more sensitive to the proximity of wake vortices than $C_N$ and $C_T$. This would need to be considered when attempting to interpret any results involving $C_M$ at high TSRs.

**Load coefficients of the two-bladed turbine (VAWT B)**

Figure 7.4 presents the normal, tangential and moment load coefficients predicted for VAWT B, at a TSR of 2.87 and a TSR of 6.40 over a single revolution of a blade for vortex release rates of 18, 36, 72 and 180 steps per revolution. It should be noted that the TSRs selected for testing were not arbitrary, which is why the TSRs tested with VAWT A and VAWT B are not the same. The TSRs were selected to match TSRs for which some experimental measurements were available (although in the case of VAWT B, only for the three-dimensional turbine) and against which the simulations were validated.

Figure 7.4 shows similar results to those found for VAWT A (Fig. 7.3) with just some minor differences. $C_N$ at both TSRs appears to be very tolerant of the number of steps per revolution, and like VAWT A, $C_M$ of VAWT B at high TSR is quite chaotic on the downstream side of the turbine, though arguably less chaotic than VAWT A. It is possible that the presence of the additional blade, and hence more evenly spread wake vortices, tends to smooth out the impact on $C_M$. There is, however, insufficient information here to confirm this hypothesis. $C_T$ shows some variation at certain azimuthal angles at 18 steps per revolution (this is particularly noticeable at an azimuth of 140° at a TSR of 2.87 and 180° at a TSR of 6.4), but by 36 steps per revolution these differences are gone.

**Wake shape of the two-bladed turbine (VAWT B)**

Less critical than the load coefficients for the purposes of the work described by this thesis, though interesting none-the-less, is how the shape of the wake differs as the steps per revolution changes. The wake shape (i.e. the location of the wake vortices) from one of the two blades of
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VAWT B after four revolutions is illustrated in Fig. 7.5 at TSRs of 2.87 and 6.40. The general shape of the wake is basically the same for the various numbers of steps per revolution, but as the wake travels downstream the differences become more apparent. As the wake vortices move further downstream, their influence on the blades reduces so the differences become less important the further downstream they travel. In close proximity to the blades, where the vortices have a stronger influence on the blades, the shape is very similar for all numbers of steps per revolution.
Figure 7.5: The wake shape after four revolutions of the two-bladed turbine at TSRs of 2.87 and 6.40 for a range of vortex release rates.

**Load coefficients of the two-bladed turbine with tower (VAWT B)**

To determine whether the presence of a tower influences the required number of time steps per revolution, VAWT B was simulated with two different configurations of tower. One tower had a diameter of 0.5 m and a drag coefficient of 0.5, and the other tower had a diameter of 2.0 m and a drag coefficient of 1.0 (in other words, one tower with a weak effect and one with a strong effect). The results at a TSR of 4.61 are presented in Fig. 7.6. Tests were also conducted at TSRs of 2.87 and 6.40 but the results were consistent with those shown here so were omitted for the sake of brevity. Furthermore, only the results for the downstream side have been shown as the presence of the tower had a negligible impact on the upstream side. It is apparent from Fig. 7.6 that although the inclusion of a tower may have a significant impact on the predicted loads, producing a potentially sharp change in the loads as the blade passes through the tower shadow, the relationship between the number of steps per revolution and the predicted loads is not strongly affected by the tower. At 18 steps per revolution, there are several azimuthal positions that deviate from the results obtained at the higher numbers of steps per revolution.
Figure 7.6: The variation of normal and tangential load coefficients with azimuth over the seventh revolution of the two-bladed turbine at a TSR of 4.61 for a range of vortex release rates with (a) & (b) No Tower, (c) & (d) a tower with a diameter of 0.5 m and a drag coefficient of 0.5 and (e) & (f) a tower with a diameter of 2.0 m and a drag coefficient of 1.0.

(e.g. at 200°, 220° and in the case of the tower with a diameter of 2.0 m, at 320°). At 36 steps per revolution, there is a slight difference at an azimuthal position of 310° that is not evident at the higher numbers of steps per revolution, but in general 36 steps per revolution appears again to be sufficient to capture the key characteristics of the variation in loads.

7.3.3 Vortex core radius

It was specified in section 4.3.2 that the radius of the vortex core was set to 7.5% of the chord length based on the fact that this value falls in the middle of the range recommended by Bhagwat and Leishman for trailing vortices from helicopter rotors [8]. To determine how
sensitive the predicted loads are to the radius of the vortex core, a set of simulations were run with various vortex core radii at 72 steps per revolution. In addition to the results presented in Fig. 7.7 of VAWT B at TSRs of 2.87 and 6.40, a number of tests were also run using an isolated dynamically pitching aerofoil. For the pitching aerofoil, the results appeared to be independent of vortex core radius so have not been included here. Only the results for the downstream side of the turbine are presented, as the size of the vortex core had no effect on any of the results on the upstream side. From Fig. 7.7, it is clear that at the higher TSR of 6.40 the results are unaffected by the size of the vortex cores. At the lower TSR of 2.87, however, the size of the vortex core can be seen to influence the predicted loads from an azimuth of about 235° through to about 280°. Fortunately, it is only at the smallest vortex core size tested of 1% of the chord
length that the difference become significant. Although the predicted results are different at 5% and 10% of chord length, the difference is acceptably small that the selection of a vortex core size at the centre of this range could be used with confidence.

7.4 Aerodynamic parameters - Aeolus3D

In the two-dimensional cases described in the previous section, two different turbine configurations were used. For the three-dimensional cases described in this section, only the Sandia 17m turbine was used. The reason for using the one-bladed turbine configuration in the previous section was because experimental data for that configuration were available and were used for validation of Aeolus2D (section 4.10.3). The Sandia 17m turbine configuration was used in the current section for the same reason. A variety of experimental measurements from the Sandia 17m turbine were available and used for validation of Aeolus3D (sections 4.10.4, 4.10.5 and 4.10.6). The baseline turbine configuration described in chapter 3 and used in the case studies presented in chapter 8 has a similar chord to radius ratio as the Sandia 17m turbine (0.075 versus 0.073), so the general effect of the aerodynamic computational parameters for one configuration could be assumed to hold true for the other.

The properties of the Sandia 17m turbine used in this section are the same as given for the two-dimensional two-bladed turbine listed as VAWT B in table 7.1. Additionally, the height at the base of the blades is 4.88 m, the height at the top of the blades is 21.88 m and the turbine has a swept area of 187 m$^2$. For the purposes of the test cases included in this section, the wind field was assumed to be uniform in both space and time. For the two-dimensional cases described in the previous section, the average Reynolds number (and hence the aerodynamic characteristics) of the blades was estimated based on the velocity due to rotation only, and not on the oncoming wind field. The Reynolds number as the blade moves into the wind represents a maximum and the Reynolds number as the blade moves away from the wind represents a minimum. The actual average Reynolds number cannot be calculated a priori, but the Reynolds number based on the velocity due to the rotation only represents a good approximation. In the three-dimensional case, rather than estimating the Reynolds number experienced at the equator, a more representative approach is to use the Reynolds number (also based on the velocity due to rotation only) experienced at the average blade radius. In the case of the Sandia 17m turbine, the average radius is approximately 5.65 m, which combined with the low air density (because of high altitude) of about $\rho = 1.0$ kg/m$^3$ experienced at the Sandia 17m test site [25], gave an average Reynolds number (at 50.6 rpm) of about 1 million (using the radius at the equator would have put the Reynolds number estimate closer to 1.5 million). The aerodynamic load coefficients of the NACA 0015 aerofoil at a Reynolds number of 1 million, as used in Aeolus3D to produce the results presented in this section, were taken from Sheldahl and Klimas [9] and processed using the approach described in section 3.3.3.
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Figure 7.8: The variation of normal, tangential and moment load coefficients at the equator with azimuth over the third revolution of the two-bladed Sandia 17m turbine, with 21 aerodynamic blade sections per blade, at TSRs of 2.87 and 6.40 for a range of vortex release rates.

7.4.1 Vortex release rate (time-step size)

Variation with azimuth

As in section 7.3.2, in which the effects of the vortex release rate on the results of the two-dimensional model were investigated, a similar investigation is presented here for the three-dimensional model. Figure 7.8 shows the normal, tangential and moment load coefficients predicted at the equator of the Sandia 17m turbine rotating at 50.6 rpm, at a TSR of 2.87 and a TSR of 6.40 over the third revolution of the blade for vortex release rates of 18, 36, 72 and 180 steps per revolution. Each blade was modelled with 21 aerodynamic elements per blade,
which was the maximum number of sections tested, in an attempt to isolate the influence of the number of steps per revolution from the number of sections per blade. Like the two-dimensional cases, the results from the other rates tested were consistent with those presented here. In each case, the results presented are for the third revolution of the turbine. As discussed in section 7.3.2 for the two-dimensional cases, reaching equilibrium was not necessary for these tests as the purpose here was to establish the relationship between the computational parameters and the results, not the actual periodic equilibrium loads of the turbines.

From Fig. 7.8 it appears that at TSRs of both 2.87 and 6.40, the predicted loads at the equator are not strongly dependent on the number of steps taken per revolution. At a TSR of 2.87, there are small, but noticeable, spikes in $C_N$ and $C_T$ at 36 steps per revolution at an azimuth of 340° that do not appear at the other vortex release rates. This slight numerical aberration would have negligible impact on the aeroelastic behaviour of the turbine, however, so is of trivial concern. $C_M$ shows the same generally chaotic nature on the downstream side at a TSR of 6.40, as the two-dimensional one-bladed turbine at a TSR of 7.6 (Fig. 7.3) which likewise, needs to be considered when interpreting any results involving $C_M$ at high TSRs.

**Variation along the blade span**

In addition to establishing the relationship between the loads at the equator and the vortex release rate, as this VAWT is actually three-dimensional, it was also important to check the relationship between the vortex release rate and the loads along the span of the blade. Figure 7.9 presents the normal and tangential forces per unit length ($F_N$ and $F_T$ respectively) and the spanwise torque per unit length ($T_S$) shown along the span of the blade ($x/L$), normalised against their maximum values, at 80° azimuthal spacings, at a TSR of 2.87. The loads presented were normalised against their absolute maximum magnitudes as shown in Eqs. 7.1 through 7.3.

$$F_N^* = \frac{F_N}{\max(|F_{N_{\text{min}}}|, |F_{N_{\text{max}}}|)} \quad (7.1)$$

$$F_T^* = \frac{F_T}{\max(|F_{T_{\text{min}}}|, |F_{T_{\text{max}}}|)} \quad (7.2)$$

$$T_S^* = \frac{T_S}{\max(|T_{S_{\text{min}}}|, |T_{S_{\text{max}}}|)} \quad (7.3)$$

The variation of load along the blade span with vortex release rate is even less at a TSR of 6.40 than at a TSR of 2.87, so their inclusion would be of no additional value here. From Fig. 7.9 it is apparent that the greatest variation with vortex release rate for all load types occurs closest to the equator, with almost no variation towards the ends of the blades. This confirms that the equatorial load coefficients shown in Fig. 7.8 actually represent a worst case scenario, so determining the minimum vortex release rate required at the equator ensures that the minimum vortex release rate is met at all other points along the span.
7.4.2 Blade discretisation (number of sections per blade)

Variation with azimuth

This section describes how the loads of three-dimensional turbines are affected by the number of aerodynamic sections per blade, i.e. the number of bound vortices along each blade. A wide range of levels of discretisation were tested other than the 5, 9, 13 and 21 per blade presented here. The other levels of discretisation tested gave trends that were consistent with those presented here.
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Figure 7.10: The variation of normal, tangential and moment load coefficients with azimuth at the equator over the third revolution of the two-bladed Sandia 17m turbine, modelled with 180 steps per revolution, at TSRs of 2.87 and 6.40 for a range of discretisation levels.

Figure 7.10 shows the normal, tangential and moment load coefficients predicted at the equator of the Sandia 17m turbine rotating at 50.6 rpm, at TSRs of 2.87 and 6.40 over the third revolution of the blade with 5, 9, 13 and 21 aerodynamic sections per blade. The turbine was simulated at a rate of 180 steps per revolution, which was the maximum rate tested, in an attempt to isolate the effect of the number of aerodynamic sections per blade from the number of steps per revolution. $C_N$ is largely unaffected by the number of vortices per blade with little separating the predictions over the very wide range of 5 to 21 sections per blade. $C_T$ at a TSR of 6.40, also shows little variation with number of aerodynamic elements, but at a TSR of 2.87 there is a noticeable difference between the predictions with 5 aerodynamic elements per blade and the higher levels of discretisation. This is most apparent at each of the peaks.
and troughs (particularly on the downstream side) with the peaks being overestimated and the troughs being underestimated with 5 sections per blade. However, there is little difference in $C_T$ predicted with 9, 13 and 21 sections per blade, however. $C_M$ at a TSR of 2.87, is not particularly affected by the number of sections per blade, but at a TSR of 6.40, there is again a noticeable difference at 5 sections per blade. As with previous cases, however, $C_M$ at a TSR of 6.40 is quite chaotic and the magnitude very small, so differences due to the number of sections per blade are less important than the overall behaviour for this parameter.
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Variation along blade span

Figure 7.11 presents the normal and tangential forces per unit length and the spanwise torque per unit length shown along the span of the blade, normalised against their maximum values, at 80° azimuthal spacings, for a TSR of 2.87. Again, the results for a TSR of 6.40 are omitted as there is even less variation than at a TSR of 2.87, along the span of the blade from number of sections per blade. In general, the variation due to the number of sections per blade is greatest at the equator, although there are some exceptions. At an azimuth of 0°, there are clear differences in \( F_N \) and \( F_T \) above and below the equator with 5 sections per blade. Overall, however, the results are consistent with those predicted at the equator, there being some variation with 5 sections per blade, but little difference from 9 sections per blade and more.

7.4.3 Wake length

Variation with azimuth

This section describes how the loads of three-dimensional turbines are affected by the length of the free vortex wake modelled. The wake lengths tested are given in terms of number of rotor diameters downstream of the turbine, measured from the furthest point reached by the rotor blade on the downstream side (e.g. 3 diameters is the same as saying 3.5 turbine diameters downstream from the tower). Figure 7.8 shows the normal, tangential and moment load coefficients predicted at the equator of the modelled Sandia 17m turbine rotating at 50.6 rpm, at a TSR of 2.87 and a TSR of 6.40 over the third revolution of the blade for wake lengths of 1, 2, 3, 4 and 5 diameters downstream. At a TSR of 2.87, the predicted results for all load types are almost indistinguishable from each other regardless of the wake length. This is because at low TSRs, the vortices are quickly transported downstream away from the turbine and as the influence of the vortices decreases with distance, there is little difference between a wake 1 turbine diameter in length and a wake 5 turbine diameters in length. At the higher TSR of 6.40, however, the wake length has a more noticeable impact, particularly on the downstream side. At higher TSRs, the vortices in the wake are not transported away from the turbine as quickly so tend to bunch up close to the turbine and continue to have a significant influence on the blade loads. For \( C_N \) and \( C_T \), there is only a slight variation in results on the downstream side for wake lengths of 2 turbine diameters and longer. There is no noticeable variation on the upstream side at any wake length. \( C_M \) at a TSR of 6.40, like the previous cases, remains quite chaotic so is of limited value. Interestingly, however, although still chaotic, \( C_M \) at a TSR of 6.40 does appear to be less affected by the wake length than by the vortex release rate or by the number of aerodynamic sections per blade.
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Figure 7.12: The variation of normal, tangential and moment load coefficients with azimuth over the third revolution of the two-bladed Sandia 17m turbine, at the equator, at TSRs of 2.87 and 6.40 for a range of wake lengths.

Variation along blade span

Figure 7.13 presents the normal and tangential forces per unit length and the spanwise torque per unit length shown along the span of the blade, normalised against their maximum values, at 80° azimuthal spacings, at a TSR of 6.40. The results at a TSR of 2.87 are omitted in this case because, contrary to the previous cases, there is less variation at a TSR of 2.87 than at a TSR of 6.40. The variations with wake length, although still minor, are once again greatest close to the equator.
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Figure 7.13: The variation of normalised normal force, tangential force and spanwise torque along the blade span with azimuth over the third revolution of the two-bladed Sandia 17m turbine, at 80° azimuthal spacings, at a TSR of 6.40 for a range of wake lengths.

7.5 Attempts to improve the aerodynamic simulations

This section describes two of the investigations conducted to reduce the computational cost of the simulations without significantly affecting their accuracy.

7.5.1 Partially prescribed wake

Because the most significant contribution towards the total computational cost is updating the state of the self-interacting free wake, it was thought that modelling the free wake for
Figure 7.14: The variation of normal and tangential load coefficients with azimuth over the third revolution of the modelled two-bladed Sandia 17m turbine, at the equator, at a TSR of 6.40 for different wake types.

A shorter distance downstream, followed by a prescribed wake for a further distance would reduce the total computational cost without impacting too severely on the accuracy. Figure 7.14 presents the normal and tangential load coefficients of the blade predicted at the equator over the downstream side of the turbine (the upstream side did not appear to be affected so is not included here) at a TSR of 6.40 for three different types of wake. The wakes include a fully self-interacting wake extending 2 diameters downstream (henceforth referred to as wake F2), a fully self-interacting wake extending 3 diameters downstream (henceforth referred to as wake F3), and a self-interacting wake extending 1 diameter downstream that continues as a prescribed wake for an additional 2 diameters downstream (henceforth referred to as wake PF). A wide range of other combinations of free and prescribed wake lengths were tested with results exhibiting similar trends to those presented here. Wake F2 was selected for inclusion as the computational cost was similar to that of wake PF. For wake PF to offer an advantage it therefore needed to produce results closer to those of wake F3, than wake F2 did. At some azimuths the loads predicted with wake PF are, in fact, closer to the results predicted with wake F3 than the results predicted with wake F2 ($C_N$ at an azimuth of 200°, for example) but in general the results with wake F2 more closely resemble the results with wake F3 than the results with wake PF ($C_N$ and $C_T$ at azimuths of 210° and 220°, for example), or show little difference.

### 7.5.2 Increased vortex release rate for final revolution

Figure 7.15 presents the normal and tangential load coefficients of the blade predicted at the equator over the downstream side of the turbine at TSRs of 2.87 and 6.40 with 36 steps per revolution, 72 steps per revolution, and 36 steps per revolution till convergence is reached followed by 72 steps per revolution for a single revolution. Although at an azimuth of 210° the
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Figure 7.15: The variation of normal and tangential load coefficients with azimuth over the third revolution of the modelled two-bladed Sandia 17m turbine, at the equator, at TSRs of 2.87 and 6.40 for 36 steps per revolution, 72 steps per revolution and 36 steps per revolution till convergence is reached, followed by 72 steps per revolution for a single revolution.

36 steps per revolution simulation predicts load coefficients that are closer to the 72 steps per revolution results than the combined approach, this is the exception. At all other azimuths, the combined approach very closely resembles the results of the far more computationally demanding 72 steps per revolution for every revolution until convergence.

7.6 Structural parameters

The number of superelements per blade is the only computational parameter influencing the structural simulation. It was determined by evaluating the relationship between the number of superelements per blade and the predicted natural frequencies. The first five flatwise modes for a baseline turbine configuration blade that can bend in a flatwise direction only (illustrated in Fig. 7.16) were selected for this evaluation. In practice, for the cases of interest in the work described by this thesis, the first one or two modes only would actually have been sufficient, but because the structural simulation is so much less demanding than the aerodynamic simulation, a greater level of accuracy could be used without increasing the overall simulation.
Figure 7.16: Illustration of the first five flatwise modeshapes of the baseline turbine blade with bending in the flatwise direction only.

Table 7.2: Natural frequencies of the first five flatwise modes with 9 and 15 superelements.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\omega_n$ (9 elements)</th>
<th>$\omega_n$ (15 elements)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0898</td>
<td>3.0833</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>6.4570</td>
<td>6.4101</td>
<td>0.73</td>
</tr>
<tr>
<td>3</td>
<td>10.619</td>
<td>10.544</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
<td>15.557</td>
<td>15.454</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>21.526</td>
<td>21.464</td>
<td>0.29</td>
</tr>
</tbody>
</table>

time of the combined model significantly. Furthermore, the additional superelements required to model higher modes of vibration was advantageous in that it was better able to model the curvature and instantaneous shape of the blades themselves. Only the convergence of the flatwise modes for blades with flatwise bending only are presented here, as such configurations were the emphasis of most of the case studies presented in the thesis. Blades with flatwise, edgewise and torsional motion were also examined, although the results are not included here. Convergence of the modes that are dominated by edgewise motion requires a similar number of superelements to the flatwise modes of the flatwise blade. Convergence of the modes that are dominated by torsional motion, however, typically require additional superelements for a given number of modes. This was expected, as each superelement consists of two joints that allow flatwise bending, two joints that allow edgewise bending, and only one joint that allows torsional motion. For any given number of superelements then, there are twice as many bending joints (in each direction) as torsional joints so additional superelements were expected in order to obtain the same level of modelling capability for the torsional modes.

Figure 7.17 presents the natural frequencies of the first five modes of the baseline turbine blade versus number of superelements, and the percentage difference between the natural frequencies at each number of superelements compared with the natural frequency predicted with 15 su-
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Figure 7.17: Natural frequencies of the first five flatwise modes of the baseline turbine versus number of superelements and the percentage difference between the natural frequencies at each number of superelements compared with the natural frequency predicted with 15 superelements.

7.7 Conclusions

7.7.1 Aeolus2D

Although it appeared that 36 steps per revolution was a sufficient number to capture the varying loads on the blades of the two-dimensional turbine, the relatively low computational cost of the two-dimensional model (compared with the three-dimensional model) meant that 72 steps per revolution could be used instead without significant concern about the computational burden and so it was decided to use 72 steps in the simulations.

A vortex core radius of 7.5% of chord length was selected because it is in the middle of the 5% to 10% of chord length range recommended by Bhagwat and Leishman for trailing vortices and was shown here to be reasonable for VAWT applications because of the lack of sensitivity of the loads to vortex core radius over that range.
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7.7.2 Aeolus3D

Aerodynamics

Like the two-dimensional simulations, 36 steps per revolution appeared to be a sufficient number to capture the aerodynamic loads of the turbine. Unlike the two-dimensional model, however, the additional computational cost of increasing the vortex release rate as high as 72 steps per revolution could not be justified. As a compromise, the three-dimensional aerodynamic model was run at 36 steps per revolution until convergence was reached, then at 72 steps per revolution for a single, final revolution.

Using 9 aerodynamic sections per blade was found to be sufficient to produce results very similar to those predicted with more aerodynamic sections per blade.

At low TSRs there was little dependence on wake length so a very short wake would, in fact, be adequate. At high TSRs, however, a longer wake length was required for the results to converge. Although a wake length of 2 turbine diameters appeared sufficient, even at the higher TSR of 6.40, for the turbine configuration tested, a wake length of 3 turbine diameters was actually selected for the validations presented in chapter 4 and the case studies presented in chapter 8. A longer wake length than appeared necessary was selected to reduce the risk that the different case studies would require a longer wake length to reach convergence than the configuration used to test the effect of wake length in this chapter. At the baseline turbine’s TSR of 3.0, the turbine would typically only require an additional one or two revolutions to reach convergence with a wake length of 3 diameters versus a wake length of 2 diameters. At higher TSRs, the turbines would typically require several more revolutions to reach convergence but there were sufficiently few such cases examined that the additional computational burden was acceptable.

Structural dynamics

Using 9 superelements per blade was found to be sufficient to predict the natural frequencies of the first five flatwise modes of vibration to within 1% of the natural frequencies predicted using the far more computationally demanding 15 superelements per blade.
Chapter 8

Case studies

However beautiful the strategy,
you should occasionally look at the results.

Winston Churchill

8.1 Introduction

This chapter describes a range of simulations that were run to examine the effect on the blade loads and deflections of a large VAWT in response to changing various parameters. The results from the simulations are presented in a variety of different ways in order to provide insight into the observed effects. The results of the simulations are divided into two main sections. Firstly, the behaviour of the baseline turbine configuration itself is examined (section 8.3). Secondly, various parameters of the baseline turbine configuration are modified and the effects investigated (section 8.4).

Although the simulations presented in this thesis are capable of modelling a wide range of complex structures, because of the considerable complexity and time required to implement the simulations, investigations which make full use of simulation’s capabilities were not a practical objective within the timespan of this research. As such, the case studies presented in this chapter were restricted to only an important limited set of the full functionality. The focus of the case studies presented are blades with flatwise bending only, attached to a perfectly rigid tower. The results from some simulations of perfectly rigid blades, and blades with flatwise and edgewise bending are included to contrast them with the blades with flatwise bending only, but blades with twisting motion are not included. Using the blade parameters for the baseline turbine configuration defined in chapter 3 proved problematic when trying to reach periodic equilibrium for blades with twisting motion included. This is discussed briefly in section 8.3.5.
As the purpose of the work described by this thesis, and this chapter in particular, is neither to design a new turbine nor to predict precisely the behaviour of an existing turbine, the actual magnitudes of the loads and deflections are not included. The purpose of the cases presented in this chapter is to understand the general aeroelastic behaviour of representative large-scale VAWT blades and understand the effect of changing various parameters so the loads and deflections are (unless otherwise stated) normalised against the maximum magnitude of the same parameter for the baseline turbine configuration to ease comparison.

It is often the case in previous VAWT research that only very limited information is presented, which can be a problem. Overall parameters such as the power coefficient or the torque make for easy comparison, but in themselves are not particularly informative when examining aeroelastic behaviour. Load coefficients at the equator are another popular choice and are slightly more useful, but once again they only tell a small part of the story. Presenting the massive volume of data generated by each simulation (or set of simulations) in a meaningful and informative way was, in itself, a significant challenge. When deciding which information to present and how it should be presented, the aim was to give clear visual insight into the nature of how the loads and deflections vary along the span of the blade and at different azimuthal positions. It was also decided that, in order to compare the effect of changing different properties, a limited set of quantitative parameters would be useful. The absolute maximum loads and deflections are an obvious choice as the ultimate required strength of the blades is directly affected by such parameters. The other parameters chosen for analysis are the maximum peak-to-peak loads and deflections, as these cyclic parameters have an impact on the potential lifespan of the blades due to fatigue.

For consistency, all loads and deflections presented in this chapter are extracted from the structural simulation, including the aerodynamic loads after they have been pulled from the aerodynamic simulation via the interface. The effective angle of attack is the only parameter extracted from the aerodynamic simulation as this term is not passed to the structural simulation.

### 8.2 Terminology and conventions

This section defines a number of terms and conventions used throughout the rest of the chapter. The terminology generally simplifies expressions or phrases that are used frequently to assist in the flow of reading. The conventions described generally address how the information is presented. Presenting three-dimensional transient loads and deflections of a moving structure on a static two-dimensional page is quite challenging, but the approach defined should assist in visualising the information in a consistent way. The terms and conventions used are:

- The baseline turbine configuration is referred to as simply the **baseline**.
• A blade that does not permit any bending or twisting is referred to as a **rigid blade**. Simulation of a rigid blade is equivalent to an aerodynamic-only simulation.

• A blade that permits bending in the flatwise direction but does not permit bending in the edgewise direction or twisting about its span is referred to as a **flatwise blade**.

• A blade that permits bending in both the flatwise and edgewise directions but does not permit twisting about its span is referred to as a **flatwise-edgewise blade**.

• Unless otherwise stated, loads and deflections are normalised against the maximum predicted load or deflection of the equivalent baseline blades.

• Unless otherwise stated, loads are per unit length along the blade span.

• Flatwise blade deflections are shown exaggerated from their mean shape to assist in visual clarity. The actual magnitudes of deflection are very small for all but the most extreme cases, so differences would not be apparent if plotted without exaggeration. For plots showing deflections for more than one case, the deflections of each case are exaggerated by the same factor allowing a comparison to be made.

• Spanwise locations along a blade are measured from base to top, normalised against the blade’s total length and referred to as **span**, e.g. the base of the blade is 0.0 span, the mid-point of the blade is 0.50 span and the top of the blade is 1.0 span\(^1\).

• Although all results in this chapter were determined after periodic equilibrium conditions were reached so should be the same regardless of the blade being examined, the results as presented generally reflect the average of both blades to reduce the risk of numerical discrepancies from one of the blades artificially distorting the results. The exception to this is the comparison between different numbers of blades, in which case the results are from a single blade instead of from an average.

• All edgewise forces, edgewise deflections and spanwise torques are presented as viewed end-on to the plane of the blade, as if seen from the outside of the turbine looking towards the inside with the direction of motion of the blade being from left to right on the page.

• Flatwise forces and flatwise deflections are presented on a side-on view of the plane of the blade as if viewed from behind the blade with the direction of motion of the blade being into the surface of the page.

### 8.3 Baseline configuration

This section presents an examination of the behaviour of the baseline, as described in chapter 3. A good understanding of the characteristics of the baseline is important so that any effect

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\(^1\)The spanwise location is measured along the actual blade itself, as if the curved blade was unwrapped and laid out flat. So 0.25 span, for example, is not the same as 25% of the distance from base to top of the blade if measured vertically along the tower.
due to modification of parameters can be recognised. This section includes an examination of the baseline with rigid blades (section 8.3.1), with flatwise blades (section 8.3.2) and with flatwise-edgewise blades (section 8.3.3). A comparison of the baseline rigid, flatwise and flatwise-edgewise blades is presented in section 8.3.4, and a brief discussion regarding the inclusion of twisting motion is given in section 8.3.5.

### 8.3.1 Rigid blades

In investigating the aeroelastic behaviour of flexible blades it is useful, and appropriate, to firstly investigate the characteristics of the baseline with perfectly rigid blades. This section presents some of the results of this investigation, describing the general loading characteristics experienced by the rigid blades.

**Relative flatwise forces along span of blade**

The total flatwise loads experienced by the blades are due to the aerodynamic forces, the gravitational forces and the inertial forces. The total flatwise loads and the relative contributions from each type of load are presented in Fig. 8.1, at azimuthal positions of $90^\circ$ (directly upstream) and $270^\circ$ (directly downstream). The lengths of the gravitational loads have been exaggerated by a factor of 5 for visual clarity as their actual magnitudes are significantly smaller than the aerodynamic and inertial loads.

In the case of the rigid blades, the gravitational and inertial loads are constant at all azimuthal positions (in contrast to flexible blades, for which only the gravitational loads are constant) and only the aerodynamic loads vary with azimuthal position. On the upstream side, the flatwise
Figure 8.2: Aerodynamic flatwise forces per unit length along blade span at 30° azimuthal spacings around a complete 360° revolution for the baseline rigid blade.

Aerodynamic loads act towards the centre of the turbine, opposing the centrifugal forces, and on the downstream side the flatwise aerodynamic loads act away from the centre of the turbine adding to the centrifugal forces.\(^2\)

To further illustrate the variation in loads along the blade span with azimuthal position, the aerodynamic forces at 30° azimuthal spacings are presented in Fig. 8.2, with the total forces presented in Fig. 8.3. As expected, the magnitude of the aerodynamic loads are greatest towards mid-span of the blade where the larger radius results in higher relative velocities and loads. The magnitude of the aerodynamic loads tails off towards the ends of the blades where the relative velocities are significantly lower. The magnitude of the aerodynamic loads on the upper half of the blade are slightly larger than the loads on the lower half of the blade due to the logarithmic shape of the wind field’s velocity profile. From Fig. 8.2 it can also be seen that, overall, the magnitudes of the aerodynamic forces on the downstream side of the turbine are lower than those on the upstream side. This is because as the wind flows through the turbine the velocity decreases, so less energy is available for extraction on the downstream side of the turbine.

Examining the total loads in Fig. 8.3 it is clear that there exists a considerable difference between the total flatwise loads experienced by the turbine blades on the upstream side from those experienced on the downstream side of the turbine. As noted previously, the aerodynamic loads oppose the centrifugal loads on the upstream side but add together on the downstream side. At azimuthal positions of 60° and 90° in Fig. 8.3 it is apparent that the aerodynamic loads act towards the centre of the turbine, opposing the centrifugal forces, and on the downstream side the flatwise aerodynamic loads act away from the centre of the turbine adding to the centrifugal forces.\(^2\)

\(^2\)In reality, depending on the aerofoil section and VAWT configuration, the switch between the direction the aerodynamic loads act may not occur precisely at 0° and 180°. The aerodynamic loads are, however, sufficiently small in this vicinity that the upstream/downstream generalisation is suitable and convenient.
loads are of a similar order of magnitude to the centrifugal loads resulting in very low net loads (relative to the magnitudes on the downstream side of the turbine).

Note also the variation in the vertical component from the loads with azimuthal position and with location along the blade span. On the upstream side, the aerodynamic loads which act towards the centre of the turbine add to the gravitational loads on the upper half of the blade and act against the gravitational loads on the lower half of the blade. On the downstream side of the wind turbine, the effect is the other way around with the aerodynamic loads opposing the gravitational loads on the upper half of the blades and adding together on the lower half.

As shown in section 8.3.2, when the blades are allowed to flex in the flatwise direction, this differential vertical loading causes the blades to rock vertically as they rotate, which in turn produces Coriolis loads acting in the edgewise direction.

**Relative edgewise forces along span of blade**

The relative total edgewise forces at 30° azimuthal spacings are presented in Fig. 8.4. Note that only the horizontal component of the forces are displayed. The vertical components of the forces were included in the figures illustrating the flatwise forces so can be omitted here as their inclusion would make interpretation of the edgewise forces more difficult. Unlike the flatwise loads presented previously, the aerodynamic loads are not presented separately in this case as there is very little contribution to the total loads from anything other than the aerodynamics. There is a very small edgewise component of the centrifugal loads because of the offset between the mounting location of the blades and the centre of mass but its magnitude is so small, compared to the aerodynamic loads, that it may be considered irrelevant and certainly would not be visible on these plots.
Unlike the flatwise aerodynamic loads, which alternate in direction on the upstream and downstream sides of the turbine, the edgewise aerodynamic loads act forwards of the blade at all azimuthal positions (or have magnitudes so low as to be approximately zero, such as in the vicinity of 0° and 180°) thus pulling the blade forwards and producing turbine torque. Thus these edgewise forces are those that produce power, and it can be seen that most of the power contribution comes from the angle ranges 60° - 120° and 240° - 300°.

From Fig. 8.4 it can be seen that the maximum edgewise loads are experienced on the upstream side, but that the loads are more uniform over a wide azimuthal range on the downstream side of the turbine due to the slowing down of the wind as it passes through the turbine. The wind slows down more in the centre of the turbine than the outer sides, which is why the loads are more even across a range of azimuthal positions on the downstream side than the upstream side.

**Relative spanwise torsion along span of blade**

The relative total spanwise torsion about the shear centre at 30° azimuthal spacings are presented in Fig. 8.5. The relative magnitudes are presented here with positive values to the right of the blades indicating positive spanwise torsion along the blade in an upwards direction (i.e. a positive torsion is one attempting to rotate the leading edge of the blade towards the centre of the VAWT and a negative torsion is one attempting to rotate the leading edge of the blade away from the centre of the VAWT).

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3Note that this is the case at this particular TSR, but is not the case at all TSRs. At some TSRs, particularly at very high or very low values, the edgewise loads will point backwards, reducing or even negating the torque produced by the turbine.
Not surprisingly, the greatest torsion occurs towards the middle of the blade where the loads are greatest. Of more interest is the fact that the torsion is considerably higher on the upstream side than the downstream side of the turbine. This is because the aerodynamic centre is located forwards of the shear centre and the centre of mass is located behind the shear centre. On the upstream side, where the aerodynamic forces act towards the centre of the turbine and the centrifugal forces act away from the centre of the turbine, they both contribute to a positive torsion. On the downstream side of the turbine, where both the aerodynamic and centrifugal forces act away from the centre of the turbine, only the centrifugal forces contribute to a positive torsion. The aerodynamic forces produce a negative torsion about the shear centre, opposing the torsion from the centrifugal forces.

Also of note from Fig. 8.5 is that the magnitude of torsion is skewed slightly towards the lower half of the blade. Because of the curvature of the blade, the gravitational force on the lower half of the blade adds to the torsion, but on the upper half of the blade opposes it. Although only a minor contribution, the difference is sufficient to produce this noticeable variation between the upper and lower halves of the blade.

Load type contributions at selected spanwise locations

Figures 8.1 through 8.5 provide a good understanding of the overall blade loading at various azimuthal positions, but it is also helpful to examine in more detail the azimuthal variation in loads at specific locations along the blades. Figure 8.6 shows the normal and tangential forces, and the spanwise torsion at locations of 0.25, 0.50 and 0.75 span. Figure 8.7 presents a breakdown of the relative contribution from each load type to the normal force, tangential force and spanwise torsion at spanwise locations of 0.25, 0.50 and 0.75 blade span. The loads
at 0.25, 0.50 and 0.75 span are determined by linear interpolation between the loads at the centres of mass of the rigid bodies either side of each spanwise location. \( F_N^*, F_T^* \) and \( T_S^* \) are the loads per unit length along the blade span, normalised against their maximum magnitudes, as defined in section 7.4.1.

Note that the small spike that appears at an azimuth of about 310° to 320° in some of the data presented in this chapter is most likely numerical in nature, and related to the discretisation of the blades and the time-step size, and triggered as the blade passes through the wake. As demonstrated in chapter 7, with a much finer blade discretisation or a much smaller time-step size, this small spike is not observed. This indicates that it is most likely not a real physical effect, but a numerical one (although it is impossible to say so definitively). The magnitude of the spike is small enough, over a short enough time-span, and barely observable except at the equator, so it is not expected to influence the general behaviour of the turbine.

In Fig. 8.6 it appears that for \( F_N^* \) and \( T_S^* \) there is a fairly uniform difference between the loads predicted at 0.25 and 0.75 span. An examination of Fig. 8.7 reveals that although there is a small difference between the two due to the aerodynamic loads (which are different because of the logarithmic velocity profile), the major difference is due to the gravitational loads which have an opposite effect on \( F_N \) and \( T_S \) between the upper and lower halves of the blade.

For the baseline configuration, the maximum magnitude of the aerodynamic forces on the upstream side are very similar to the magnitude of the inertial loads at all three spanwise locations. As the aerodynamic and inertial loads oppose each other on the upstream side, the magnitude of \( F_N^* \) is quite similar at all three spanwise locations near an azimuthal position of 90°. On the downstream side, the magnitudes of the aerodynamic loads at 0.25 and 0.75 span are noticeably less than on the upstream side. In contrast, at 0.50 span the magnitudes of the aerodynamic loads on the downstream side are less than on the upstream side, but not as significantly reduced as at 0.25 and 0.75 span. This may be due to the greater radius at 0.50 span keeping the blade further away from the influence of the wake of the tower and from the
Chapter 8: Case studies

Figure 8.7: Contributions of different force types to the total loads on a baseline rigid blade predicted at 0.25, 0.50 and 0.75 span.

blades’ passage on the upstream side. The result of this is that on the downstream side, where the aerodynamic and inertial loads act in the same direction, the magnitude of $F^*_{N}$ becomes much greater at 0.50 span due to the larger aerodynamic forces relative to the inertial forces compared with the 0.25 and 0.75 span values. For the rigid blade case, the aerodynamic loads are the only significant contribution to $F^*_{T}$. It is also noticeable that, like $F^*_{N}$, the aerodynamic loads at 0.25 and 0.75 span are more uniform over a wider range of azimuthal positions on the downstream side than at 0.50 span.

Angle of attack at selected spanwise locations

Figure 8.8 illustrates the effective angles of attack predicted at spanwise locations of 0.25, 0.50 and 0.75 span. This figure demonstrates one of the reasons why the TSR of 3.0 selected for the baseline was a good choice for an investigation of the aeroelastic behaviour. At 0.50 span the angle of attack exceeds the static stall angle of attack by a small amount on the upstream side, but remains within the fully attached region on the downstream side. At 0.75 span, the blade
experiences slightly deeper stall on the upstream side, and only just exceeds the static stall angle of attack on the downstream side. At 0.25 span, the blade experiences quite deep stall on the upstream side, and also comfortably exceeds the static stall angle of attack on the downstream side of the turbine. For the baseline configuration, at a TSR of 3.0, the blade will experience different combinations of stalled and attached flow over much of its power generating spanwise region. At higher TSRs, more of the blade will experience fully attached flow throughout its full revolution. Such conditions may be important for fatigue analysis as the turbine will ideally spend most of its time operating closer to the TSR at which its maximum power coefficient occurs, but for the purpose of understanding the aeroelastic behaviour in more aerodynamically challenging conditions, higher TSRs are less informative.

8.3.2 Flatwise blades

Following the analysis of the loads experienced by the rigid blades, the baseline was simulated with blades capable of flatwise bending only. While there are a lot of similarities between a blade with flatwise bending and a perfectly rigid blade, the bending blade adds an interesting element, in that any bending motion will produce edgewise Coriolis forces, which are not present on a perfectly rigid blade. This section presents some of the results of this investigation, describing the general loading characteristics and deflections experienced by the VAWT with blades that bend in a flatwise direction only.

The aerodynamic pitching moment

Because of the limited reliable experimental data available for the aerodynamic pitching moment for aerofoils at high Reynolds numbers, and the weaker dynamic stall modelling capabilities of the pitching moment compared with the normal and tangential forces, it is important to understand the effect that the aerodynamic moment actually has on the behaviour being examined. Figure 8.9 presents the normal and tangential forces and the spanwise torque at 0.50 span of a flatwise blade with and without the aerodynamic moment included. The loads in both cases are normalised against the maximum loads predicted for the case with the aerodynamic moment included. The offset between the aerodynamic centre and the shear centre is still taken into
account in both cases, as is the moment due to the apparent mass. In the “no aerodynamic moment” case, the static moment coefficient data used by the dynamic stall model is simply assumed to be zero at all angles of attack. Similar comparisons were also conducted for the rigid and flatwise-edgewise blades, and in both cases the differences were very similar to the flatwise blade shown here. Other locations along the span also showed the same general behaviour, but at reduced magnitudes. It is clear from Fig. 8.9 that the aerodynamic pitching moment has no effect on $F_N^*$ or $F_T^*$ (and it also has no effect on the deflections of the flatwise or flatwise-edgewise blades). Only the spanwise torque changes, due to the inclusion of the aerodynamic pitching moment, becoming more irregular as the angle of attack changes with azimuth. For simulations that include twisting motion, the inclusion of the aerodynamic pitching moment may be important, but for the flatwise and flatwise-edgewise blades examined in this chapter the inclusion of the pitching moment adds no significant value, only uncertainty. The spanwise torque is of only limited interest for non-twisting blades, so the remainder of the simulations presented in this chapter were run without the aerodynamic moment included.

**Deflections**

The flatwise blade deflections are compared with the equilibrium blade shape in the absence of any aerodynamic loads at 30° azimuthal spacings in Fig. 8.10. It is clear from this figure that there is a flatwise rocking motion of the blades when the aerodynamic loads are included. The blades rock downwards on the upstream side and upwards in the vicinity of the upstream-to-downstream, and downstream-to-upstream transitions.

The flatwise forces on the bending blades are sufficiently similar to the perfectly rigid blades that an examination of Figs. 8.2 and 8.3 is appropriate here without needing to include the actual flatwise forces from the bending blades. There is, in reality, a very small difference in both aerodynamic and inertial forces from the rigid blades (due predominately to the changing radii as the blade deflects) but the difference is so small that it is barely visible when plotted. The other important loads that are not present for the perfectly rigid blade are the elastic loads
resulting from the blade being deformed from its original undeformed shape$. The elastic loads are not shown here as their relative strength can be inferred directly from the blade shape. The larger the change in the curvature of the blade from its undeformed shape, the greater the elastic loads attempting to restore the blade to its original shape. This information, together with the flatwise forces on the rigid blades, can be used to understand why the blade moves the way that it does.

As the blade passes an azimuth of $0^\circ$ the blade is in a deflected shape above its mean, which is actually closer to the undeformed shape so the elastic loads are quite low. The aerodynamic loads in this vicinity are also very low but as the blade moves upstream, the aerodynamic loads build up and oppose the centrifugal loads. With the loads pulling the blade outwards decreasing, the gravitational loads pulling the blade downwards cause the shape to droop. As the blades approach an azimuth of $90^\circ$ the aerodynamic loads have started to decrease and eventually the elastic and centrifugal loads are sufficiently strong compared to the aerodynamic loads, that they begin to dominate again and pull the blade back out and upwards. As the blade moves to the downstream side, the momentum of its motion carries it beyond its mean shape but not far, as the combination of the aerodynamic and centrifugal loads now working in the same direction pull the blades tighter and outwards. Beyond an azimuth of about $270^\circ$ the blade starts to move into the wind and the aerodynamic loads continue to increase. Due to the logarithmic velocity profile of the wind, the aerodynamic loads on the top half of the blade are

$^4$Note that the undeformed blade shape of the baseline configuration is defined as symmetrical but in the presence of gravity the blade shape become asymmetric, so elastic restoring loads will be present even in its mean shape.
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Relative edgewise forces along span of blade

As the blades rock up and down in a flatwise direction, the changing radii along the blade result in edgewise inertial forces due to the Coriolis effect. The inertial edgewise forces are presented in Fig. 8.11 and the total edgewise forces are presented in Fig. 8.12. As the radius at a part of the blade decreases, an inertial edgewise forward force on the blade is produced and as the radius at part of the blade increases, an inertial edgewise backwards force on the blade is produced. The presence of the inertial edgewise forces changes the distribution of the total edgewise forces by making the loading on the upper and lower halves of the blade slightly less balanced. This imbalance can be seen by looking at $F_T^*$ at 0.25, 0.50 and 0.75 span as shown in Fig. 8.13. At 0.50 span, there is little deflection of the blade, so the inertial loads remain very small compared to the aerodynamic loads. At 0.25 and 0.75 span, the blade experiences a lot more motion so the inertial loads (i.e. the Coriolis forces) are much greater. The inertial loads at 0.25 and 0.75 are also approximately opposite in sign to each other because of the rocking motion of the blade, as a decreasing radius on one half corresponds to an increasing radius on the other so the Coriolis forces act in opposite directions.

8.3.3 Flatwise-edgewise blades

The edgewise blade deflections are compared with the equilibrium blade shape in the absence of any aerodynamic loads at 30° azimuthal spacings in Fig. 8.14. Like the flatwise case, the deflections from the mean shape have been exaggerated. The magnitudes of deflections in the
flatwise direction are slightly different from those of the flatwise-only blade, but the general behaviour is sufficiently similar that the inclusion of figures showing the flatwise bending of the flatwise-edgewise blades are unnecessary here. It can been seen in Fig. 8.14 that there is a distinctive periodic edgewise rocking motion with the blades moving front-to-back twice per revolution.

The inertial edgewise forces are presented in Fig. 8.15, and the total edgewise forces presented in Fig. 8.16. It is clear that these are quite different from the edgewise forces for the flatwise blade. As the blade rocks backwards and forwards in an edgewise direction, the inertial loads also act backwards and forwards of the blade. The inertial forces are dominated here by the centrifugal forces, as the blade bends in an edgewise direction the centrifugal force no longer
acts in the plane of the blades’ undeformed shape, but fore or aft of it. Also, as the blade bends edgewise, the centre of rotation and the angular velocity are no longer constant along the full length of the blade, further changing the direction and magnitude of the centrifugal forces. Unlike the flatwise blade, the magnitude of the edgewise inertial loads on the flatwise-edgewise blade are much larger compared to the aerodynamic forces, as can be seen by the total edgewise forces in Fig. 8.16.

As the blade moves over the upstream side, the edgewise forces pull the blade forward, but
as it approaches the transition from the upstream to downstream side, the edgewise forces drop sharply and the blade starts to spring back due to the elastic restoring loads. The blade overshoots and starts to bend backwards, but a combination of the elastic restoring loads and the edgewise forces, which are once again building in forward strength, pull the blade forward again. As the blade approaches the downstream to upstream transition again, the edgewise forces drop sharply once more, and the cycle repeats. This forwards pull, then release on both the upstream and downstream sides, explains the twice per revolution edgewise rocking motion observed in figure 8.14.

### 8.3.4 Comparison of rigid, flatwise and flatwise-edgewise blades

Figure 8.17 presents comparisons of the loads predicted at spanwise locations of 0.25, 0.50 and 0.75 span of the rigid blade, the flatwise blade, and the flatwise-edgewise blade. The loads are all normalised against the absolute maximum magnitude of the loads on the rigid blade. Except for some slight variation for \( F_T^* \) at 0.25 and 0.75 span, the loads on the rigid and flatwise blades are very similar. With the exception of some variation at 0.50 span, \( F_N^* \) and \( T_S^* \) are also reasonably similar for the flatwise-edgewise blade. The major difference is that \( F_T^* \) of the flatwise-edgewise blade has considerably more variation than the rigid and flatwise blades. Despite the large difference in the variation of \( F_T^* \) over a revolution, the power coefficient is essentially the same for all three blade types \( (C_{P_{\text{rigid}}} = 0.299, C_{P_{\text{flat}}} = 0.298, C_{P_{\text{flat-edge}}} = 0.300) \). This highlights why the averaged properties, such as the power coefficient, can be insufficient to give a true understanding of the aeroelastic behaviour.
Figure 8.17: Comparison between the total loads per unit length along blade span predicted at spanwise locations of 0.25, 0.50 and 0.75 blade span for the rigid blade, the flatwise blade, and the flatwise-edgewise blade, normalised against the absolute maximum magnitude on the loads on the rigid blade.

8.3.5 Flatwise-edgewise-torsional blades

As mentioned in the introduction to this chapter, problems were encountered in attempting to reach periodic equilibrium when the baseline turbine was simulated with twisting motion included. To resolve this issue, there are a number of possible explanations that would need to be investigated. Perhaps the blade design, which is based on a HAWT blade, does in fact lead to a real physical instability when used on a VAWT and the motion is not periodic. Increasing the torsional stiffness and/or damping sufficiently did allow the system to reach periodic equilibrium. This was not, however, an adequate solution to the issue as the problem may not actually be a real physical instability but an as yet unidentified numerical instability occurring when blade twisting is permitted. Or, perhaps, the convergence criteria selected to evaluate whether periodic equilibrium has been reached are simply stricter than necessary. Because of the indeterminate amount of additional work required to establish the true cause of the issues with the full flatwise-edgewise-torsional blades, it had to be ruled out of scope of
8.4 Modified configurations

This section presents a range of results from investigations into the effects on the aeroelastic behaviour of modifying various parameters of the baseline. The parameters modified were the TSR (section 8.4.1), the amount of blade damping (section 8.4.2), the wind field’s velocity profile with and without gravity included (section 8.4.3), the rate of rotation (section 8.4.4), the blade stiffness (section 8.4.5), the linear density of the blade (section 8.4.6), the mass moment of inertia of the blade (section 8.4.7), the number of blades (section 8.4.8), and the cross-sectional location of the centre of mass and the elastic centre section (section 8.4.9).

8.4.1 Tip Speed Ratio

This section describes a brief examination of how the flatwise blades are affected by different TSRs. The TSR was modified by holding the rate of rotation constant and varying the wind speed. Figure 8.18 presents the predicted power coefficients for the rigid and flatwise blades at a range of TSRs. The difference in power coefficient between the baseline rigid and the flatwise blades at all TSRs tested is barely discernible. This was important to confirm as it reinforces the claim made previously that averaged parameters such as the power coefficient are not always very informative on their own when trying to understand aeroelastic behaviour.

Figure 8.19 presents the flatwise deflections of the flatwise blade at TSRs of 2.0, 3.0 (the baseline) and 6.0. While the magnitudes of the deflections are very different, the general behaviour is the same at all TSRs, with the rocking motion following the same pattern in each case. As expected, the higher the TSR, the smaller the deflections experienced. This is explained by looking at the loads presented in Fig. 8.20. At a TSR of 6.0, because the aerodynamic loads are very small compared to the inertial loads, there is little variation with azimuth compared with at TSRs of 2.0 and 3.0 and hence there are very small deflections seen in Fig. 8.19. At a TSR of 2.0, the blade is passing in and out of deep stall with large variations
in angle of attack so larger cyclic variations in aerodynamic loads, and hence larger deflections, are experienced. In addition to the greater variation of the aerodynamic loads, the variation of inertial loads are also greater at the lower TSR due to the larger changes in radii.

8.4.2 Blade damping

This section describes a brief examination of how the flatwise blades are affected by different amounts of damping. For the baseline blade, a small amount of damping (Using the Rayleigh damping model, as described in section 5.3.2, with damping equal to 2% of stiffness) was introduced to approximate the low level of damping expected in the blades and also to reduce the risk of numerical instabilities due to the discrete time-step sizes used. The purpose of the simulations presented here is to determine the effect that additional damping would have on the behaviour of the blades. The flatwise deflections of the flatwise blade at damping levels of 2%, 10% and 20% of stiffness are presented in Fig. 8.21. The magnitudes of the deflections are similar across this wide range of damping. The higher levels of damping have marginally smaller maximum deflections, and the motion is slowed down slightly, with the more highly damped blades lagging slightly behind the lower damped blades, but even this difference is quite minor.

\( F_N \) and \( T_S \) remain almost identical regardless of the damping. Only \( F_T \) shows a noticeable variation, as presented in Fig. 8.22, although this too is not major. At 0.50 span, where there is very little blade motion, the loads are essentially unaffected by the damping. At 0.25 and
0.75 span, where the motion is more significant, the loads are slightly different for the various levels of damping. With higher levels of damping the flatwise movement of the blade slows down so there is a reduction in the Coriolis forces. The effect of this is a slightly more gradual change of $F_T$ at higher levels of damping. The aerodynamic loads also change slightly, but this is less significant than the change in inertial loads.

### 8.4.3 Wind field velocity profile

The intention of the cases presented in this section was to examine the effect that the vertical velocity profile of the wind had on the behaviour of the flatwise blades. The behaviour of the VAWT with flatwise blades was tested both with and without gravity to isolate the effect of the wind profile. The flatwise deflections of the flatwise blade in a wind field with a uniform wind flow and logarithmic velocity profiles with roughness lengths of 0.02m (the baseline) and 2.0m are presented with gravity included in Fig. 8.23, and without gravity included in Fig. 8.24. These figures illustrate that the magnitude of the rocking motion of the flatwise blade
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Figure 8.21: Flatwise blade deflections for a flatwise blade at damping levels of 2%, 10% and 20% of stiffness at 30° azimuthal spacings around a complete 360° revolution.

Figure 8.22: Total tangential loads on a flatwise blade at 0.25, 0.50 and 0.75 blade span with damping levels of 2%, 10% and 20% of stiffness.

is influenced by both the presence of gravity and by the roughness length of a logarithmic velocity profile. An examination of Fig. 8.23 reveals that the magnitude of the flatwise motion is greater for a roughness length of 2.0m compared to a roughness length of 0.02m, which in turn experiences a greater magnitude of motion than for uniform flow. The interesting feature to note here is that the rocking motion is present, and still quite significant, for the uniform flow. This is due to the gravity force, as evidenced by the absence of the rocking motion for the uniform flow when gravity is omitted, presented in Fig. 8.24. In the absence of gravity, only the blades exposed to a logarithmic velocity profile continue to exhibit the rocking motion,
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Figure 8.23: Flatwise blade deflections for a flatwise blade (with gravity included) in a wind field with a uniform wind flow, and logarithmic velocity profiles with roughness lengths of 0.02m and 2.0m at 30° azimuthal spacings around a complete 360° revolution.

Figure 8.24: Flatwise blade deflections for a flatwise blade (without gravity included) in a wind field with a uniform wind flow, and logarithmic velocity profiles with roughness lengths of 0.02m and 2.0m at 30° azimuthal spacings around a complete 360° revolution.
Figure 8.25: The difference between the normal force at 0.75 span and the normal force at 0.25 span, with and without gravity included, in a wind field with a uniform wind flow, and logarithmic velocity profiles with roughness lengths of 0.02m and 2.0m.

although it is noticeably smaller than the cases which include gravity. Under the influence of gravity, the blades sag downwards so even in the presence of a uniform wind flow, the loading is no longer symmetrical between the upper and lower halves of the blade and a rocking motion is initiated. Likewise, when gravity is omitted but a logarithmic wind field is used, although the geometric shape of the blades is symmetrical, the varying wind speed with height results in asymmetric loading which, once again, initiates a rocking motion.

To illustrate this asymmetry, the difference between $F_N^*$ predicted at 0.75 span and at 0.25 span in a uniform flow and in logarithmic velocity profiles with roughness lengths of 0.02m and 2.0m, both with and without gravity included, are presented in Fig. 8.25. In the absence of gravity, only the logarithmic velocity profiles show asymmetric loading, both of which vary with azimuth, leading to the vertical rocking motion. The uniform flow with no gravity, for which no vertical blade rocking motion was predicted, is zero at all azimuths, confirming that the normal loads on the upper and lower halves of the blade remain symmetrical for this case. With gravity included, however, all three wind fields including the uniform flow produce an asymmetry in loading that changes with azimuth. With gravity included, the average difference between the upper and lower halves of the blade becomes more positive. On the upper half of the blade the gravity force acts towards the inside of the blade, but on the lower half of the blade the gravity force acts towards the outside, causing a constant difference between the two halves.

Figure 8.26 presents the variation of absolute and peak-to-peak maxima of $F_N^*$, $F_T^*$ and $w_f^*$ with roughness length (where $w_f^*$ is the magnitude of flatwise deflection from the initial blade shape normalised against the maximum flatwise deflection of the baseline flatwise blade). The interesting features to note in Fig. 8.26 are the small range of magnitudes of the absolute maximum $F_T^*$ (within about 1% of the baseline) and the absolute and peak-to-peak maximum $F_N^*$ (within about 2% of the baseline) over the wide range of roughness lengths tested. Also, the peak-to-peak maximum $F_T^*$ and the absolute and peak-to-peak maximum $w_f^*$ show steadily increasing magnitudes with roughness length, the rate of which decrease with higher roughness lengths. As shown by Fig. 8.25, the larger roughness length creates a greater variation with
azimuth between the upper and lower halves of the turbine, which leads to greater rocking motion. This is why both the absolute and peak-to-peak maximum $w_f^*$ increase with roughness length. The greater rocking motion leads to higher variations in tangential Coriolis forces. The absolute maximum $F_T^*$ occurs close to mid-span (actually, just above mid-span) where the rocking motion has little effect, so there are minimal Coriolis forces and hence the absolute maximum value of $F_T^*$ remains largely unaffected by roughness length. The maximum motion occurs higher on the blade (approximately 0.65 span) where the absolute maximum $F_T^*$ is less. This means that as the motion increases, the region of the blade experiencing the greater motion also experiences greater variation in Coriolis forces, which increases the peak-to-peak $F_T^*$, but does not effect the absolute maximum closer to mid-span.

### 8.4.4 Rate of rotation

The intention of the cases presented in this section was to examine the effect that the rate of rotation of the turbine had on the behaviour of the flatwise blades. A direct comparison is made difficult because, in order to maintain the same TSR at different rates of rotation, the wind speed must also adjusted by the same proportion. In reality, the aerodynamic characteristics of the aerofoil would also change in response to the changing Reynolds Number but this has been kept constant in these cases to minimise the number of parameters being changed at the same time. The centrifugal forces increase with the square of the angular velocity, but an increase in the angular velocity results in a proportional increase in the velocity of the blade (at a given radius) and an increase in the wind speed (to maintain the TSR). As the aerodynamic loads increase with the square of flow velocity over the blade, the investigation described in this section is best approached as a examination of the relationship between the different load types as the rate of rotation increases rather than a direct comparison of the magnitudes.

Figure 8.27 presents the flatwise deflections for a flatwise blade, at a TSR of 3.0, at rates of
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Figure 8.27: Flatwise blade deflections for a flatwise blade at a TSR of 3.0, at rotation speeds of 8 rpm, 13 rpm and 18 rpm, at 30° azimuthal spacings around a complete 360° revolution.

Figure 8.28: Variation with rate of rotation of absolute and peak-to-peak maximum normal force, tangential force and flatwise displacement of a flatwise blade from initial blade shape.

rotation of 8 rpm, 13 rpm (the baseline) and 18 rpm. The interesting feature to note here is that although the deflected blade shape is similar at all three rates of rotation from about 270° to 90° (i.e. as the blade is travelling towards the wind), the shape at 8 rpm in particular is quite different from about 90° to 270° (i.e. as the blade is travelling away from the wind). Interestingly, the azimuth at which the maximum deflection occurs shifts as the rate of rotation changes. At 8 rpm the maximum flatwise deflection occurs at an azimuth of 90°, at 13 rpm it occurs at 110° and at 18 rpm it occurs at 120°. At 8 rpm, as the blade rocks down on the upstream side, the blade deflects almost as much as at 13 rpm and 18 rpm, but as the blade
rocks up on the downstream side, the blade does not deflect much beyond the blade shape with no aerodynamic loads. A consequence of this behaviour can be observed in the absolute and peak-to-peak maximum $w_f^*$ presented in Fig. 8.28. Although the absolute maximum does not vary greatly with the rate of rotation, the peak-to-peak maximum increases significantly with rate of rotation up to about 14 rpm, and then continues to increase with rate of rotation, but at a slower rate. It is also interesting to note that the absolute maximum is reached at a rate of rotation of about 13 or 14 rpm and reduces gradually at both lower and higher rates of rotation.

The relative contributions from the aerodynamic, gravitational and inertial loads to the total normal force, at a TSR of 3.0, at 0.75 span, at rates of rotation of 8 rpm, 13 rpm and 18 rpm are presented in Fig. 8.29. Unlike most of the other cases in this chapter, the loads at each rate of rotation are normalised against their own maxima rather than the baseline, so that a comparison of the relative contributions can be made. The tangential forces showed just the expected variations due to the motion of the blade so are not included here. The loads at 0.75 span only are shown in Fig. 8.29 as a representative location. The loads at 0.50 span show very little variation with rate of rotation, and the loads at 0.25 span are similar to those at 0.75 span, except that the maximum magnitude of the aerodynamic loads are slightly lower and the sign of the gravitational loads are reversed.

The proportion of the aerodynamic and inertial loads to the total loads remains very similar at the various rates of rotation. The inertial loads show some very minor variations, which are expected, due to the changing radii of the blades. The main effect of the different rates of rotation is in the relative magnitude of the gravitational loads. The gravitational loads themselves remain constant, but at low rates of rotation the aerodynamic and centrifugal forces
are lower so the gravitational loads become more significant in relation to them. At higher rates of rotation, the aerodynamic and centrifugal forces dominate and the relative contribution from the gravitational forces becomes quite small. In essence, at a low rate of rotation, the blade sags lower because the ratio between the centrifugal forces pulling the blades out and the gravitational forces pulling the blades down is less, which is why its maximum deflection from the initial shape is similar to the more highly loaded blades at higher rates of rotation. At the low rates of rotation the lower aerodynamic and centrifugal forces are insufficient to lift the blades far in the other direction compared with the higher rates of rotation, which is why the peak-to-peak deflection is much lower.

### 8.4.5 Blade stiffness

The intention of the cases presented in this section was to examine the effect that the flatwise flexural rigidity had on the behaviour of the flatwise blades. The flatwise deflections of the flatwise blade with flexural rigidities of $2.0 \times 10^7$ N.m$^2$, $4.4 \times 10^7$ N.m$^2$ and $7.0 \times 10^7$ N.m$^2$ at $30^\circ$ azimuthal spacings around a complete $360^\circ$ revolution.
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Figure 8.31: Variation with flexural rigidity of absolute and peak-to-peak maximum normal force, tangential force and flatwise displacement of a flatwise blade from the initial blade shape.

an azimuth of 125° for a flexural rigidity of $2.0 \times 10^7$ N.m², an azimuth of 110° for a flexural rigidity of $4.4 \times 10^7$ N.m², and an azimuth of 95° for a flexural rigidity of $7.0 \times 10^7$ N.m².

The variation of the absolute and peak-to-peak maxima of $F_N^*$, $F_T^*$ and $w_f^*$ with flatwise flexural rigidity are presented in Fig. 8.31. There is a steep drop in absolute and peak-to-peak maximum $F_N^*$ as flexural rigidity increases at very low flexural rigidities, then a gentle and fairly insignificant decrease over most of the range of flexural rigidities tested. $F_T^*$ shows more variation with flexural rigidity, also decreasing with increased flexural rigidity, because the greater motion at lower flexural rigidities results in greater Coriolis forces acting tangentially. $w_f^*$ is significantly affected by the flexural rigidity, decreasing as the flexural rigidity is increased in a non-linear way. The higher the flexural rigidity, the lower the rate at which $w_f^*$ changes with respect to flexural rigidity.

The power coefficient does not change significantly with flexural rigidity ($C_{P,2.0\times10^7\text{ N.m}^2} = 0.297$, $C_{P,4.4\times10^7\text{ N.m}^2} = 0.298$, $C_{P,7.0\times10^7\text{ N.m}^2} = 0.299$) so there appears to be an advantage in using stiffer blades. The stiffer blades have similar power coefficients but reduced deflections and loads. There are, however, diminishing returns as the flexural rigidity is increased. Increasing the flexural rigidity from $1.0 \times 10^7$ N.m² to $2.0 \times 10^7$ N.m², for example, has a much greater effect than increasing the flexural rigidity from $9.0 \times 10^7$ N.m² to $10.0 \times 10^7$ N.m². As shown in the following section, however, there appears also to be an advantage by using lighter blades, so how the additional stiffness is achieved would be quite important. If the linear density (mass per unit length along blade) is increased along with the stiffness, the advantages may not be seen.
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8.4.6 Blade linear density

The intention of the cases presented in this section was to examine the effect that the linear density of the blades has on the behaviour of the flatwise blades. The flatwise deflections of the flatwise blade with linear densities of 30 kg/m, 95 kg/m (the baseline) and 160 kg/m are presented in Fig. 8.32. The general behaviour is similar at all three linear densities, with the maximum deflection being reached at a similar azimuth in each case. The maximum magnitude of the deflection appears to increase with linear density, which is confirmed by examining the absolute and peak-to-peak maximum $w_f^*$ shown in Fig. 8.33.

The peak-to-peak maximum $w_f^*$ appears to increase almost linearly with linear density, but the rate of increase of the absolute maximum diminishes as the linear density increases. The absolute maxima of both $F_N^*$ and $F_T^*$, as shown in Fig. 8.33, increase linearly with linear density. The inertial loads are proportional to mass, so if aerodynamic and gravitational loads remain essentially the same, as they appear to here, then the linear increase of the absolute maximum loads with linear density is as expected. The peak-to-peak maximum $F_N^*$ does not vary with linear density because the centrifugal forces always act away from the axis of rotation, so the magnitude of $F_N^*$ is shifted by approximately the same amount (allowing for some variation due to the changing radii) at all azimuths, as shown in Fig. 8.34.

The power coefficient does not change significantly with linear density ($C_{P_{30 \text{ kg/m}}} = 0.299$, $C_{P_{95 \text{ kg/m}}} = 0.298$, $C_{P_{160 \text{ kg/m}}} = 0.297$) so there appears to be an advantage in using lighter
The absolute maximum $F^*_N$, the absolute and peak-to-peak maximum $F^*_T$ and the absolute and peak-to-peak maximum $w^*_f$ are all reduced with lighter blades. As mentioned in the previous section, however, consideration would need to be given in practice to the relationship between the stiffness and the linear density of the blades. Ideally, to minimise the loads and deflections, the blades should be both stiff and light.

### 8.4.7 Blade inertia

To investigate the possibility that the magnitude of the mass moment of inertia may influence the motion of the blades, possibly delaying the motion and shifting the azimuth at which maximum values occur, a range of simulations using flatwise blades were run with flatwise mass moments of inertia per unit length ranging from 1.60 to 3.60 kg.m (the baseline is 2.5 kg.m) in increments of 0.1 kg.m. It was found that the loads and deflections at all mass moments of inertia tested were indistinguishable from each other.
A similar set of simulations were also run using flatwise-edgewise blades with edgewise mass moments of inertia per unit length ranging from 10 to 60 kg.m (the baseline is 37 kg.m) in increments of 5 kg.m. Like the flatwise blades, it was found that the behaviour of the VAWT was unaffected by the mass moment of inertia.

Over the range of values tested, the mass moments of inertia about either axis do not appear to have any effect on the behaviour of the blades.

8.4.8 Number of blades

The intention of the cases presented in this section was to examine the effect that the number of blades of the turbine has on the behaviour of the flatwise blades. Testing the number of blades was not, in fact, a trivial exercise. The results presented in this section should be considered as an indication of the trend in behaviour only, because a number of assumptions were required that introduced some uncertainty. Although the actual magnitudes may not be accurate, the assumptions made were such that the qualitative change in behaviour could be examined with confidence.

In order to compare the behaviour of turbines with different numbers of blades at the same TSR, the solidity of the turbines needed to be kept constant. This meant changing the chord length to compensate for the changing number of blades. The aerodynamic characteristics of the aerofoil were not changed in response to the changing Reynolds number due to the different chord lengths. In practice, for shorter chord lengths, the Reynolds number will be less and stall will occur at a slightly lower angle of attack and have a slightly sharper drop in lift, but the effects were not expected to be significant compared to the changes in the physical properties. Initially, several simulations with different numbers of blades were run with all the physical properties kept constant, but it was soon apparent that this was not particularly meaningful. Changing the chord length has a direct effect on the magnitude of the aerodynamic loads, but if all other physical properties are kept the same then larger blades simply deflect a lot more than smaller blades. This does not take into account the fact that geometrically similar larger blades, of the same design, will have higher mass and stiffness. In an effort to account for the changing physical properties with chord length, the same approach as that described in chapter 3 was repeated for chord lengths suitable for one and three-bladed turbines. The properties of the LMH64-5 HAWT blade were, once again, used to approximate the properties of an equivalent VAWT blade. For the one-bladed turbine, the chord length required to maintain the solidity is greater than the LMH64-5 blade at any point along its span. As such, the properties of the one-bladed turbine blade were estimated by linear extrapolation of the LMH64-5 blade properties from tip to root, beyond the region where the blade has the thickness of 18% matching the NACA 0018 blade of the baseline. This is where the greatest uncertainty was introduced as the properties had to be extrapolated from a chord length of about 3m to 4.8m, which is quite a lot.
Table 8.1: Physical properties of blades for investigations of turbines with different numbers of blades, but the same solidity.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>1-blade</th>
<th>2-blades</th>
<th>3-blades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord length [m]</td>
<td>$c$</td>
<td>4.8</td>
<td>2.4</td>
<td>1.6</td>
</tr>
<tr>
<td>Linear density [kg/m]</td>
<td>$\sigma$</td>
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<td>95</td>
<td>49</td>
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<td>Flatwise mass moment of inertia [kg.m]</td>
<td>$I_{\text{flat}}$</td>
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<td>0.57</td>
</tr>
<tr>
<td>Edgewise mass moment of inertia [kg.m]</td>
<td>$I_{\text{edge}}$</td>
<td>540</td>
<td>37</td>
<td>7</td>
</tr>
<tr>
<td>Flatwise flexural rigidity [N.m$^2$]</td>
<td>$EI_{\text{flat}}$</td>
<td>3.6E+08</td>
<td>4.4E+07</td>
<td>1.00E+07</td>
</tr>
<tr>
<td>Edgewise flexural rigidity [N.m$^2$]</td>
<td>$EI_{\text{edge}}$</td>
<td>4.5E+09</td>
<td>4.3E+08</td>
<td>1.00E+08</td>
</tr>
<tr>
<td>Torsional rigidity [N.m$^2$]</td>
<td>$GJ$</td>
<td>5.3E+07</td>
<td>7.8E+06</td>
<td>2.70E+06</td>
</tr>
</tbody>
</table>

when assuming linear variation. The extrapolated properties should be sufficiently accurate, however, to give an indication of the general behaviour of the one-bladed turbine compared with the two and three-bladed turbines. In reality, even if some theoretical advantage was uncovered, a one-bladed VAWT is unlikely to be considered because of the difficulty the rotating radial load would cause to a real tower. The perfectly rigid tower simulated here does not interact with the blades at all, so this was not an issue, but on a real turbine the cyclic loading on the tower caused by a single blade would be problematic.

The properties of the blades of the one-bladed, two-bladed (baseline) and three-bladed turbines are presented in table 8.1. The edgewise flexural rigidity and the torsional rigidity do not actually have any effect on the flatwise blade, but they are also included in the table to show how they would vary at the different chord lengths. The locations of the sectional properties as a proportion of chord length were kept the same as those of the baseline (table 3.1).

The flatwise deflections of the flatwise blade for one-bladed, two-bladed and three-bladed turbines are presented in Fig. 8.35 and the absolute and peak-to-peak maximum $F_N^*$ and $F_T^*$ are presented in Fig. 8.36, together with the azimuthal variation of the power coefficient of the whole turbine. As anticipated, by keeping the solidity constant, the average power coefficient remains similar for all three cases ($C_{P_{NB=1}} = 0.313$, $C_{P_{NB=2}} = 0.298$, $C_{P_{NB=3}} = 0.291$). The instantaneous blade loads and power coefficient of the turbine are, however, quite different for the three cases. The loads are much greater with fewer blades because, not only are the aerodynamic loads larger due to the longer chord length, but the higher mass per unit length leads to greater inertial and gravitational loads. Despite the much higher loads, however, the blade also deflects a lot less (as seen in Fig. 8.32) with fewer blades, due to the associated increased stiffness. While the actual loads are relatively small for the three-bladed turbine, the deflections are very large because of the much lower blade stiffness.

Using additional blades certainly appears advantageous in some respects. The much smoother delivery of overall torque to the generator is very beneficial, but the large deflections would need to be addressed with additional blade stiffness. A far more detailed analysis would be required than the brief one conducted here in order to weigh up the relative advantages and
disadvantages of using three blades instead of two. The additional cost of manufacture may prove too much to be justified, but it is worthy of consideration.

### 8.4.9 Centre of mass and elastic centre

The intention of the cases presented in this section was to examine the effect on the behaviour of the flatwise-edgewise blades by moving the centre of mass and the elastic centre to different locations along the chord line. The aerodynamic centre, the mounting point and the shear centre were all kept at 0.25, 0.50 and 0.33 of chord respectively. The power coefficient and percentage difference in absolute and peak-to-peak maximum normal and tangential loads and
Table 8.2: Power coefficient and percentage differences in absolute maximum (abs) and peak-to-peak maximum (p-p) loads and deflections from baseline for flatwise-edgewise blades with various horizontal locations of the centre of mass (CoM) and elastic centre (EC).

<table>
<thead>
<tr>
<th>Case</th>
<th>CoM</th>
<th>EC</th>
<th>(C_P)</th>
<th>(F_N) (abs)</th>
<th>(F_N) (p-p)</th>
<th>(F_T) (abs)</th>
<th>(F_T) (p-p)</th>
<th>(w_f) (abs)</th>
<th>(w_f) (p-p)</th>
<th>(w_e) (abs)</th>
<th>(w_e) (p-p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>0.10</td>
<td>0.302</td>
<td>+1.0</td>
<td>+1.1</td>
<td>+6.7</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0.0</td>
<td>+5.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>0.42</td>
<td>0.301</td>
<td>+0.7</td>
<td>+1.0</td>
<td>+6.1</td>
<td>-0.4</td>
<td>-0.1</td>
<td>-0.3</td>
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<td>-0.9</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>0.25</td>
<td>0.301</td>
<td>+0.7</td>
<td>+1.0</td>
<td>+3.6</td>
<td>-0.6</td>
<td>-0.1</td>
<td>0.0</td>
<td>+2.6</td>
<td>-1.7</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.42</td>
<td>0.301</td>
<td>+0.2</td>
<td>+0.1</td>
<td>+4.1</td>
<td>+0.1</td>
<td>-0.3</td>
<td>+0.3</td>
<td>+1.5</td>
<td>-0.4</td>
</tr>
<tr>
<td>5</td>
<td>0.33</td>
<td>0.42</td>
<td>0.301</td>
<td>+0.0</td>
<td>-0.2</td>
<td>+2.3</td>
<td>+0.0</td>
<td>-0.2</td>
<td>+0.1</td>
<td>+0.7</td>
<td>-0.4</td>
</tr>
<tr>
<td>6</td>
<td>0.47</td>
<td>0.10</td>
<td>0.301</td>
<td>+0.9</td>
<td>+1.2</td>
<td>+0.6</td>
<td>-0.8</td>
<td>-0.5</td>
<td>-0.3</td>
<td>+3.1</td>
<td>-2.4</td>
</tr>
<tr>
<td>7</td>
<td>0.47</td>
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<td>0.301</td>
<td>+0.2</td>
<td>+0.2</td>
<td>+0.5</td>
<td>-0.1</td>
<td>-0.4</td>
<td>+0.1</td>
<td>+1.5</td>
<td>-1.2</td>
</tr>
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<td>0.47</td>
<td>0.33</td>
<td>0.300</td>
<td>+0.1</td>
<td>+0.1</td>
<td>+0.2</td>
<td>-0.1</td>
<td>-0.2</td>
<td>-0.1</td>
<td>+0.7</td>
<td>-0.7</td>
</tr>
<tr>
<td>9</td>
<td>0.47</td>
<td>0.50</td>
<td>0.299</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.4</td>
<td>-0.1</td>
<td>+0.2</td>
<td>-0.2</td>
<td>-0.8</td>
<td>+0.4</td>
</tr>
<tr>
<td>10</td>
<td>0.47</td>
<td>0.75</td>
<td>0.300</td>
<td>-0.1</td>
<td>+0.2</td>
<td>-1.6</td>
<td>-0.5</td>
<td>0.9</td>
<td>-0.3</td>
<td>-3.0</td>
<td>+2.0</td>
</tr>
<tr>
<td>11</td>
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<td>0.302</td>
<td>+0.0</td>
<td>0.0</td>
<td>-0.6</td>
<td>-0.1</td>
<td>0.0</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>12</td>
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<td>0.50</td>
<td>0.300</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-1.0</td>
<td>-0.2</td>
<td>+0.2</td>
<td>-0.2</td>
<td>-1.0</td>
<td>+0.5</td>
</tr>
<tr>
<td>13</td>
<td>0.75</td>
<td>0.42</td>
<td>0.301</td>
<td>+0.3</td>
<td>+0.6</td>
<td>-5.5</td>
<td>-0.8</td>
<td>+0.3</td>
<td>-0.6</td>
<td>-2.3</td>
<td>-0.2</td>
</tr>
<tr>
<td>14</td>
<td>0.75</td>
<td>0.75</td>
<td>0.300</td>
<td>-0.2</td>
<td>+0.1</td>
<td>-6.6</td>
<td>-0.8</td>
<td>+1.2</td>
<td>-0.8</td>
<td>-5.2</td>
<td>+2.1</td>
</tr>
</tbody>
</table>

the flatwise and edgewise deflections \(w_e\) from the baseline flatwise-edgewise blades for each of the cases tested are presented in Table 8.2. For these cases, the locations of the centre of mass and elastic centre have minimal effect on \(F_N\) and \(w_f\). As a general observation, it appears that moving the centre of mass towards the leading edge (cases 1-5) progressively increases the absolute maximum \(F_T\), but has hardly any effect on the peak-to-peak \(F_T\). Likewise, moving the centre of mass towards the trailing edge (cases 13 and 14) decreases the absolute maximum \(F_T\). Moving the centre of mass has a similar, although less significant effect on \(w_e\). Moving the elastic centre towards the leading edge (cases 1, 3 and 6-8) also appears to progressively increase the absolute maximum \(w_e\) but decreases the peak-to-peak \(w_e\). Moving the elastic centre towards the trailing edge (cases 10 and 14) decreases the absolute maximum \(w_e\) but increases the peak-to-peak \(w_e\).

Compared with the other modifications to the parameters discussed in this chapter, even the most extreme relocation of the centre of mass and elastic centre still only produced fairly minor changes to the loads and deflections, and trivial changes to the power coefficient. There will, almost certainly, be combinations of the locations of sectional properties that will result in instabilities in some situations, but none of the cases tested here had any difficulty reaching periodic equilibrium. Additional combinations of the placement of the centre of mass and elastic centre, and of the shear centre and mounting point, may lead to some interesting insights into the aeroelastic stability of the turbine blades in different conditions, but the twisting motion of the blade would, in all likelihood, need to be included in the simulation to obtain truly useful results.
Chapter 9

Conclusions and recommendations

Reasoning draws a conclusion, but does not make the conclusion certain, unless the mind discovers it by the path of experience.

Roger Bacon

9.1 Introduction

This chapter describes the conclusions that were drawn from the research described in this thesis, and recommendations for future work emanating from it. Many conclusions about the individual aspects of the modelling approach, the implementation of the simulations, and the findings of the case studies are discussed in their relevant sections of this thesis. The primary purpose of this chapter is not to simply restate those conclusions and findings, but to pull together the key conclusions for interpretation and to provide some understanding and insight from them.

The work described by this thesis consists of two quite distinct parts, the development and validation of the simulations themselves, and the subsequent use of the simulations to perform a variety of case studies. The conclusions and the recommendations for future work, as they relate to either the simulations or the case studies are, therefore, discussed separately.

9.2 Conclusions

9.2.1 The simulations

The simulations of the two-dimensional aerodynamics, the three-dimensional aerodynamics, and the structural dynamics were all implemented independently of each other, allowing each
to be used in isolation to investigate different aspects of VAWT behaviour if desired.

Although both computationally more demanding and a lot more time-consuming to implement than the quasi-steady blade element momentum methods typically used for VAWT analysis, the use of the vortex wake methods were justified for this research because, unlike the blade element momentum methods, the vortex wake methods are capable of providing the time dependent results required.

The comparison between predictions from the two-dimensional aerodynamic simulation and experimental measurements of a dynamically pitching aerofoil confirmed that the modified version of the Beddoes-Leishman model described in this thesis did an excellent job at capturing the normal loads during dynamic stall, a very good job at capturing the tangential loads during dynamic stall, and an adequate job at capturing the general characteristics of the pitching moment during dynamic stall. The prediction of the pitching moment was, however, weaker than the ability to predict the normal and tangential loads. This was, in itself, an important discovery as it is always important to understand not only where simulations are particularly strong, but also where their results should be interpreted with more caution.

A very thorough analysis was conducted into all three simulations to determine the relationship between a wide range of computational parameters, such as time-step sizes, levels of discretisation and the length of the wake being modelled, to establish acceptable levels of computational parameters balanced against the computational demands of the simulations. The analyses demonstrated that quite computationally inexpensive values for the various computational parameters were sufficient to predict results very similar to those predicted with much more computationally expensive values for the various computational parameters.

Predictions from both the two and three-dimensional aerodynamic simulations were compared with a wide range of experimental measurements of VAWTs ranging from the loads on a simple (effectively two-dimensional) straight one-bladed VAWT in a water towing-tank up to loads at the equator of an actual three-dimensional rotating VAWT. The comparisons thoroughly tested the capabilities of both the two and three-dimensional simulations in a variety of challenging conditions. Overall, the results of the simulations compared very favourably with the experimental measurements. Where differences were observed, they have been explained in section 4.10. The simulated results compared particularly well with the experimental measurements at the low to moderate TSRs of interest in this work, and which formed the basis of the cases studies examined.

Predictions from the structural dynamics simulation were verified against the commercial multi-body systems package SimMechanics with excellent results. The modal properties of curved beams predicted with the structural dynamics simulation were compared with finite element
predictions, analytical predictions and experimental measurements, all of which demonstrated both the capability of the structural simulation to accurately model the dynamic characteristics of the system and the capability of the linearisation method used to extract the modal properties. Although the linearisation method should, in principle, also work on the combined aeroelastic system, a lack of suitable experimental data against which to validate the approach limited its use in this research to the structural model only.

9.2.2 The case studies

Although the focus of the work described in this thesis was the development of the simulations themselves, the combined three-dimensional aerodynamic and structural dynamic simulations were used to conduct a range of numerical experiments. The purpose of these numerical experiments was to investigate how the aeroelastic behaviour of a baseline VAWT configuration changed in response to changes in various configuration parameters. The emphasis of the case studies described in this thesis focused on blades that allowed bending in only the more flexible flatwise direction. Where appropriate, some investigations were also conducted on blades that allowed bending in both a flatwise and edgewise direction.

The case studies demonstrated the twice-per-revolution periodic vertical rocking motion of the blades that occurs due to the changes in the variation in loading along the blade span as the blades rotate. The rocking motion occurs because of the asymmetry of the loads on the upper and lower halves of the blades, and specifically the fact that the relationship between the loads on the upper and lower halves of the blades changes with azimuth. The asymmetry in loading is caused by a combination of gravity and the variation of wind speed with height, both of which lead to an asymmetry in loading between the upper and lower halves of the blades that varies with azimuth.

It was also shown that the vertical rocking motion of the blades in the flatwise direction introduced edgewise Coriolis forces. With edgewise bending allowed, however, these edgewise Coriolis loads became quite small relative to the edgewise centrifugal forces. With edgewise bending included, an edgewise rocking motion occurred with a frequency of two per revolution.

It was discovered that as the stiffness of the blades was increased, the absolute maximum and peak-to-peak maximum loads and deflections decreased. Although the effect of blade stiffness on the normal loads were minor, the effect on the tangential loads and the flatwise deflections were significant. With increased blade stiffness, the magnitude of the vertical rocking motion was restricted, so the flatwise deflections were lessened, and with reduced motion the edgewise Coriolis forces were likewise reduced.

It was also discovered that as the linear density of the blades increased, the absolute maximum normal load, and the absolute maximum and the peak-to-peak maximum tangential loads and
flatwise deflections increased. The peak-to-peak normal loads were essentially unaffected by the linear density of the blades. The absolute maximum normal and tangential loads increased linearly with linear density because of the increased centrifugal loading. The peak-to-peak maximum tangential load increased as a result of both the increased centrifugal loads and the increased rocking motion experienced by the blades with greater linear density.

How the aeroelastic behaviour of the VAWT was affected by different numbers of blades was examined by making some assumptions about how the blade properties would be designed differently with different chord lengths (to maintain the turbine solidity). While certainly not conclusive, due to the assumptions required, this investigation suggested that despite the expected aerodynamic advantages of using additional blades, there may be some aeroelastic disadvantages. This is because although the loads reduce with additional numbers of blades, the deflections may tend to be much greater due to the reduced stiffness.

On the basis of this research it is recommended that VAWT blades are designed with a high stiffness and low linear density. As noted in chapter 8, however, the relationship between the blade stiffness and the linear density can make achieving both quite challenging. This research also indicates that it might be possible to further reduce the rocking motion of the blades by designing the undeformed shape to be asymmetric, such that under the influence of gravity and the logarithmic velocity profile of the wind, the loading over the upper and lower halves of the blades become more even. It is unlikely that the rocking motion can be eliminated completely, except in the ideal case of uniform wind flow, but with careful design it may be significantly reduced.

9.3 Recommendations

9.3.1 The simulations

One interesting piece of work would be to assume that there are no aeroelastic concerns with VAWTs and given that assumption, determine under which circumstances VAWTs would actually be preferred over HAWTs. There are many qualitative advantages and disadvantages discussed in the literature (some of which are discussed in section 2.3.2), but few quantitative analyses. This would be a very difficult piece of work, but determining first the conditions under which VAWTs may offer a real viable alternative to the very successful HAWTs would lead to further focused research on the specific configurations with the most potential.

The aerodynamic simulations, specifically the motion of the free wake vortices, were implemented using a simple explicit Euler method. While not critical for this research, as discussed in section 4.6.2, an improved numerical integration method may allow the structure of the wake to be more accurately modelled as it evolves over time. A more accurate wake model (or more
specifically, a wake model that remains accurate further downstream of the turbine) would open up opportunities for some interesting turbine interaction investigations to be conducted. Understanding how the wakes of multiple turbines affect each other is important for determining suitable turbine spacings for a wind farm, which is an active area of wind turbine research.

Some additional work, and in particular some additional validation, would be useful to make better use of the linearisation capabilities of the combined aeroelastic model. Although there are no known theoretical reasons why the linearisation method should not work for the combined aeroelastic model, without validation it cannot be used with total confidence. Such validation would probably require physical testing of an aeroelastic VAWT model, which would be quite a demanding piece of work.

Validation of the structural simulation for rotating curved beam-like systems, like the blades of a VAWT, would give additional confidence in its use. Between the verification of the structural simulation against a commercial package, the validation of rotating straight-beams and the validation of non-rotating curved-beams, there is no reason to believe that the structural simulation is invalid for a rotating curved beam-like system. However, suitable data against which to perform this validation could not be found despite extensive searching of the literature.

While some research is available in the literature on the effect of spanwise flow on HAWTs, none could be found related to VAWTs. Although this is only likely to have a very minor effect on the behaviour of the blades (most likely causing stall to delay close to the blade roots), this could be an interesting area for future investigation and make a useful contribution to the accuracy of the three-dimensional aerodynamic simulation.

Including the tower into the structural dynamics is another logical step. Modelling the tower itself is already within the capabilities of the existing simulation, but a suitable general-purpose guy-cable model was not found in the literature. It is for this reason that the tower was omitted from the case studies described within this thesis.

9.3.2 The case studies

The scope for future case studies is, in fact, limitless. The purpose of this section is, therefore, not to provide an exhaustive list of every possible additional case study of value but to highlight some of the case studies that were initially envisioned during the development of the simulations, or revealed as potentially valuable cases during the case studies that were conducted, but could not be included as part of this research because of either technical difficulties or, more often, because of time constraints.

A different baseline turbine configuration would, of course, provide a rich source of potential new case studies. Wind turbines with quite different configurations from the baseline turbine
configuration selected for investigation in this work might result in very different aeroelastic behaviour.

One clear area for additional research is to perform more investigations into the aeroelastic behaviour of blades with degrees of freedom beyond flatwise bending. Although some studies were conducted that also included edgewise bending, for a fuller understanding of the aeroelastic behaviour it would be necessary to conduct investigations that also include torsional motion.

Another area for additional work could be to conduct investigations into how the aeroelastic behaviour changes in response to different aerodynamic parameters. As part of the research described by this thesis, the influence of the TSR and the shape of the wind field velocity profile were investigated, but the simulations were developed to allow a much wider range of changes to the aerodynamic parameters. How changes to the aerodynamic characteristics of the aerofoils affected their response would be an interesting area for future research. Such work was not performed as part of the research described by this thesis predominantly because of the very large scale of the baseline VAWT configuration examined, and hence the very high blade Reynolds numbers involved. In an investigation of a smaller VAWT (with correspondingly lower Reynolds numbers), the aerodynamic behaviour of the aerofoils used may have a more significant influence than at very high Reynolds numbers for which stall tends to be a more gradual phenomenon than at low Reynolds numbers.

Finally, a more detailed investigation into the advantages and disadvantages of using more than two blades is warranted given the results presented in section 8.4.8. If a larger number of blades can be made stiff, without excessive additional mass or cost, the aerodynamic benefits may justify their use.
References


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Appendix A

Aerofoil data preparation

A.1 Parameterisation of lift and drag curves

As mentioned in section 3.3.3, rather than using available experimental aerofoil data directly, it was first translated into a series of straight and curved line segments defined with a minimal set of parameters\(^1\). This eliminated any sharp discontinuities in the data and reduced the risk of experimental error at particular angles of attack being falsely introduced into the simulation. This approach also made interpolation between data at different Reynolds numbers much easier and more consistent. While not really an issue for the lift curve, the region of the drag curve near stall where there is a sudden sharp rise in drag did not appear to interpolate well between Reynolds numbers\(^2\). Simple interpolation based on the actual drag coefficient resulted in a softening of the severity of stall than what it was likely to experience in reality. Interpolation of the parameters of the parameterisation, however, produced a much more plausible set of data.

Figure A.1 illustrates an example of the parameterisation of the experimentally measured aerodynamic data of the NACA 0015 aerofoil at a Reynolds number of \(1 \times 10^6\).

The lift curve consists of a straight line over the linear (attached flow) region of the curve up until the aerofoil starts to stall (the slope of the line is equal to \(C_{N\alpha}\)). The rest of the lift curve consists of a series of cubic spline curves connected together. The parameters of the cubic splines were defined manually rather than using any form of regression analysis so that obvious outliers or discontinuities could be ignored.

The drag curve consists of a cubic spline at low angles of attack (as seen in figure A.1(b)) until the aerofoil starts to stall. At this point, a straight line is used over the region where the drag increases sharply. Post stall, as the lift starts to recover, the rate of increase of drag decreases.

\(^1\)With the exception of the OSU data, which as discussed, was used directly.
\(^2\)This was certainly the case for the NACA 00xx and NACA 44xx series aerofoils.
A.1. Parameterisation of lift and drag curves

![Graph showing lift, drag, and moment coefficients](image)

**Figure A.1:** Parameterised and measured (from Sheldahl and Klimas [9]) aerodynamic data of a NACA 0015 aerofoil at a Reynolds number of $1 \times 10^6$ (a) from $0^\circ$ to $180^\circ$ and (b) close-up of the drag and moment curves from $0^\circ$ to $20^\circ$.

and the remainder of the drag curve consists of a series of cubic splines.

It should be noted that the measured moment coefficient data in figure A.1 is at a Reynolds number of 680000, not $1 \times 10^6$. Sheldahl and Klimas [9] did not provide (or presumably, test) the moment coefficients at higher Reynolds numbers. The moment coefficient data were, in fact, quite erratic compared to the lift and drag data. This is particularly apparent in figure A.1(a) in the vicinity of an angle of attack of about $90^\circ$.

The moment coefficient is approximately zero up until some point (still with the attached flow region) where it increases gradually (represented with a straight line). At a point close to stall (just before stall in this example) the moment drops sharply (again, represented with a straight line). The remainder of the moment coefficient curve is approximated with cubic splines. The parameterisation of the moment curves is less reliable than the parameterisation of the lift and drag curves, but considering the considerable uncertainty in the experimentally measured moment coefficients, the parameterisation still represents an improvement over the use of the raw data in terms of consistency.

This parameterisation was done for all aerofoils examined with the exception of the validations against the OSU dynamically pitching aerofoil experiments. In these cases, the static aerodynamic coefficients for the aerofoils were measured using the same physical aerofoil models and in the same wind tunnel as the dynamic tests so could be used with more confidence for
A.2 Preparing aerofoil data files for input

The lift, drag and moment coefficient curves do not, on their own, provide sufficient information about the aerofoil for use in Aeolus2D and Aeolus3D. The BL model in particular requires a variety of additional information describing the aerofoils. The information required on each aerofoil includes $C_N$, $C_T$, $C_M$, $f_N$ and $f_T$ at angles of attack between at least $-90^\circ$ and $+90^\circ$ (but preferably from $-180^\circ$ to $+180^\circ$). Furthermore, in addition to the time constants for the BL dynamic stall model (which are kept the same for all aerofoils in this work), $\alpha_0$, $C_{D_0}$, $C_{N_0}$, $C_{N_{max}}$ and $C_{N_{min}}$ are also required where $C_{N_{max}}$ and $C_{N_{min}}$ are the critical values for the normal load coefficient discussed in section 4.8.2 at positive and negative angles of attack respectively. The additional information required (except the time constants), was all derived directly from the static lift, drag and moment curves.

The aerofoil data for use in Aeolus2D and Aeolus3D were prepared semi-automatically by a MATLAB script based on the lift, drag and moment coefficient data at the Reynolds number of interest. In reality, where unsteady experimental data are available, the parameters for a given aerofoil should be "tuned" to provide better results. Such an approach is not attempted here however as unsteady experimental results are only available for a limited selection of aerofoil shapes. In an effort to ensure consistency, the process was standardised and applied equally to all aerofoils examined. The processing of all data was treated identically, whether taken directly from wind tunnel experiments or from a parametric approximation.

1. First the minimum drag coefficient $C_{D_0}$ was determined and subtracted off the drag coefficient comparison than most aerofoil data.
A.2. Preparing aerofoil data files for input

Figure A.3: Effective separation points calculated from parameterised (i.e. smoothed) aerodynamic data of a NACA 0015 aerofoil at a Reynolds number of $1 \times 10^6$

Coefficient curve (then was added back on the dynamic stall calculations) and the lift and drag were converted to normal and tangential load coefficients according (i.e. equations A.1 and A.2).

\begin{align*}
C_N &= C_L \cos(\alpha) + (C_D - C_{D_0}) \sin(\alpha) \tag{A.1} \\
C_T &= C_L \sin(\alpha) - (C_D - C_{D_0}) \cos(\alpha) \tag{A.2}
\end{align*}

2. A linear least-squares method was used to estimate the slope of $C_N$ over the linear attached flow region and the angle of attack at which $C_N$ is zero (i.e. $C_{N_0}$ and $\alpha_0$ respectively).

3. Determine angle of attack at which the normal coefficient is at a maximum during stall (but before post stall recovery) and use this to determine what the normal coefficient would be at this angle of attack if stall did not occur (i.e. $C_{N_{\text{max}}}$) using the slope of the normal coefficient curve estimated previously and extrapolating out to this angle of attack.

4. Repeat the previous step at negative angles of attack, finding the angle of attack at which the normal coefficient is at a minimum during stall to determine $C_{N_{\text{min}}}$. 

5. Calculate $f_N$ and $f_T$ at all angles of attack using equations 4.53 and 4.54 respectively.

Figure A.3 illustrates the effective separation points calculated using the parameterised data of a NACA 0015 aerofoil at a Reynolds number of $1 \times 10^6$. $f_N$ and $f_T$ typically have values between zero (fully separated) and one (fully attached) but as they are, in fact, effective separation points (according to Kirchoff’s theory), not actual physical separation points they can (and often do) have values greater than one at low angles of attack when the flow is fully attached (and equally, can have values less than one even if the flow is fully attached in reality).
Appendix B

Linearisation of rotation matrices

This appendix is provided to demonstrate that for small angles of rotation about the $x$, $y$ and $z$ axes, the order of rotation is not critical. The rotation matrices defining rotations about the $x$-axis, $y$-axis and $z$-axis are given by equations B.1, B.2 and B.3 respectively.

\[
R_x = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\phi_x) & -\sin(\phi_x) \\
0 & \sin(\phi_x) & \cos(\phi_x)
\end{bmatrix}
\] (B.1)

\[
R_y = \begin{bmatrix}
\cos(\phi_y) & 0 & \sin(\phi_y) \\
0 & 1 & 0 \\
-\sin(\phi_y) & 0 & \cos(\phi_y)
\end{bmatrix}
\] (B.2)

\[
R_z = \begin{bmatrix}
\cos(\phi_z) & -\sin(\phi_z) & 0 \\
\sin(\phi_z) & \cos(\phi_z) & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (B.3)

$\phi_x$, $\phi_y$ and $\phi_z$ are the angles of rotation about axes $x$-axis, $y$-axis and $z$-axis respectively. To ensure that the remainder of the equations in this section fit onto the page, the cos and sin terms are simplified to $\cos(\phi_x) \Rightarrow c_x$, $\sin(\phi_x) \Rightarrow s_x$, $\cos(\phi_y) \Rightarrow c_y$ etc. Multiplying the three

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matrices in each possible order gives equations B.4 through B.9.

\[
\begin{align*}
\mathbf{R}_{z} \mathbf{R}_{y} \mathbf{R}_{z} &= 
\begin{bmatrix}
    c_z c_y & -s_z c_y & s_y \\
    c_z s_y s_x + s_z c_x & -s_z s_y s_x + c_z c_x & -c_y s_x \\
    -c_z s_y c_x + s_z s_x & s_z s_y c_x + c_z s_x & c_y c_x
\end{bmatrix} \\ 
\mathbf{R}_{z} \mathbf{R}_{x} \mathbf{R}_{y} &= 
\begin{bmatrix}
    c_y c_z - s_x s_y s_z & -c_x s_z & c_x s_y + s_x s_z c_y \\
    c_y s_z + s_x s_y c_z & c_x c_z & s_z s_y - s_x c_z c_y \\
    -c_x s_y & s_x & c_x c_y
\end{bmatrix} \\ 
\mathbf{R}_{y} \mathbf{R}_{z} \mathbf{R}_{x} &= 
\begin{bmatrix}
    c_y c_z & -c_y s_x s_y + s_x s_y c_y & c_x s_y c_z \\
    c_y s_x s_y + s_x c_y & c_x c_z & s_x s_y s_z - s_x c_y c_z \\
    -s_z & c_x s_y & c_x c_y
\end{bmatrix} \\ 
\mathbf{R}_{z} \mathbf{R}_{y} \mathbf{R}_{x} &= 
\begin{bmatrix}
    c_y c_z & c_y s_x s_y + s_x c_y c_z & s_x s_y c_z \\
    c_y s_x s_y + s_x s_y c_z & c_x c_z & s_x s_y s_z + s_x s_y c_z \\
    -c_y s_x & s_x & c_x c_y
\end{bmatrix} \\ 
\mathbf{R}_{y} \mathbf{R}_{x} \mathbf{R}_{z} &= 
\begin{bmatrix}
    c_y c_z & -c_y s_x s_y + s_x s_y c_z & c_x s_y c_z \\
    c_y s_x s_y + s_x c_y c_z & c_x c_z & s_x s_y s_z - s_x c_y c_z \\
    -c_y s_x & s_x & c_x c_y
\end{bmatrix}
\end{align*}
\]

The rotation matrices can be linearised by assuming that \( s \approx \phi, c \approx 1 \) and that multiplication of any two or more small angles is approximately zero (e.g. \( s_x s_y \approx \phi_x \phi_y \approx 0 \)). Given these assumptions, equations B.4 through B.9 all reduce to equation B.10.

\[
\mathbf{R}_{\text{lin}} = 
\begin{bmatrix}
    1 & -\phi_z & \phi_y \\
    \phi_z & 1 & -\phi_x \\
    -\phi_y & \phi_x & 1
\end{bmatrix}
\]
Appendix C

Useful mathematical operations

Throughout the work described by this thesis, a wide range of mathematical theory, particularly related to the three-dimensional geometry was needed. Some were basic knowledge for any engineering graduate (e.g. dot products and cross products of vectors), some were less basic (e.g. rotation of vectors using quaternions). A collection of some of the most useful and less common mathematical operations and relationships is provided here.

C.1 Vector operations

Skew symmetric matrix form of vector:

$$\tilde{a} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$  \hspace{1cm} (C.1)

Multiplication of a skew symmetric matrix and a vector:

$$\tilde{a} b = a \times b$$  \hspace{1cm} (C.2)

Differentiation of a vector rotating at an angular velocity $\omega$:

$$\frac{da}{dt} = \omega \times a$$  \hspace{1cm} (C.3)

Differentiation of the dot product of two vectors:

$$\frac{d}{dt}(a \cdot b) = a \cdot \frac{db}{dt} + b \cdot \frac{da}{dt}$$  \hspace{1cm} (C.4)
Differentiation of the cross product of two vectors:
\[
\frac{d}{dt}(a \times b) = a \times \frac{db}{dt} + \frac{da}{dt} \times b \tag{C.5}
\]

C.2 Quaternion Operations

General form of a quaternion expressed as a scalar and a vector:
\[
q = s + v_x i + v_y j + v_z k = [s, \ b] \tag{C.6}
\]

Creation of a quaternion from a rotation angle \( \phi \) about unit vector \( \hat{a} \):
\[
[s, \ b] = \left[ \cos(\phi/2), \ \hat{a} \sin(\phi/2) \right] \tag{C.7}
\]

Conjugation of a quaternion:
\[
q' = [s, -b] \tag{C.8}
\]

Length of a quaternion:
\[
|q| = \sqrt{s^2 + v_x^2 + v_y^2 + v_z^2} \tag{C.9}
\]

Normalisation of a quaternion:
\[
\hat{q} = \frac{q}{|q|} = \left[ \frac{s}{|q|}, \frac{b}{|q|} \right] \tag{C.10}
\]

Multiplication of quaternions:
\[
q_1 q_2 = [s_1, v_1] [s_2, v_2] = \left[ s_1 s_2 - v_1 \cdot v_2, s_1 v_2 + s_2 v_1 + v_1 \times v_2 \right] \tag{C.11}
\]

Multiplication of a quaternion and a vector:
\[
aq = [0, a] [s, v] \tag{C.12}
\]

Rotation of a vector:
\[
a = q \hat{a} q' \tag{C.13}
\]
C.3 Matrix Operations

General form of a rotation matrix:

\[ \mathbf{R} = \begin{bmatrix} r_1x & r_2x & r_3x \\ r_1y & r_2y & r_3y \\ r_1z & r_2z & r_3z \end{bmatrix} = [ \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 ] \quad (C.14) \]

Transpose of a matrix product:

\[ (\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T \quad (C.15) \]

Differentiation of a rotation matrix:

\[ \frac{d\mathbf{R}}{dt} = \begin{bmatrix} \frac{d\mathbf{r}_1}{dt} & \frac{d\mathbf{r}_2}{dt} & \frac{d\mathbf{r}_3}{dt} \end{bmatrix} \quad (C.16) \]

\[ = [ \omega \times \mathbf{r}_1 \quad \mathbf{r}_2 \quad \omega \times \mathbf{r}_3 ] \]

\[ = [ \mathbf{\omega r}_1 \quad \mathbf{\omega r}_2 \quad \mathbf{\omega r}_3 ] \]

\[ = \mathbf{\dot{\omega}R} \]

Creation of a rotation matrix from a quaternion:

\[ \mathbf{R} = \begin{bmatrix} 1 - 2v_y^2 - 2v_z^2 & 2v_xv_y - 2sv_z & 2v_xv_z + 2sv_y \\ 2v_xv_y + 2sv_z & 1 - 2v_x^2 - 2v_z^2 & 2v_yv_z - 2sv_x \\ 2v_xv_z - 2sv_y & 2v_yv_z + 2sv_x & 1 - 2v_x^2 - 2v_y^2 \end{bmatrix} \quad (C.17) \]