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Abstract

This paper incorporates partial consumer participation in a model of competition between telecommunications networks with two-way interconnection. It is shown, in contrast to the results of similar models with full participation, that the firms' equilibrium profits depend on the level of a reciprocal access charge under two-part retail pricing. Under some simplifying assumptions, it is shown that firms prefer the access charge be set equal to the marginal cost of termination, which coincides with the social optimum. Without these additional assumptions the model is analytically complex and simulation results are presented that suggest firms prefer the access charge to be less than marginal cost, while the socially optimal access charge may be above or below cost depending on the differentiation of the firms.

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1 Introduction

With the advent of mobile telephony and deregulation of the local call market,[1] competition in situations of two-way interconnection has become the focus of much research. In order to provide ubiquitous service, competing local or mobile telephony suppliers must interconnect and agree to terminate calls originating on each other's network. To cover the costs of termination, firms typically pay each other an access charge. While it is generally illegal for firms to collude over retail prices, access charges are frequently set cooperatively. Beginning with the seminal papers of Armstrong (1998), Laffont, Rey and Tirole, hereafter LRT, (1998a), and Carter and Wright (1999), researchers have therefore sought to determine whether the access charge can be used by the firms as an instrument of collusion, and what, if any, regulations should be imposed on interconnection.

From this literature has emerged what could be called the 'standard' model of competition between telecommunications firms under two-way interconnection. The details of this model will be reviewed in the next section, but one of the main results of all these papers is that, under linear retail prices, the access charge is an instrument of collusion. The basic intuition for this result is that when the access charge is above cost, if a firm lowers its retail price, its subscribers will make more calls which triggers an access deficit at the margin.[2] If the access charge is above marginal cost, the incentive to lower the retail price is reduced and retail competition between the firms is softened. However, under two-part pricing, firms' equilibrium profits are neutral with respect to the access charge. The basic intuition for this result stems from the fact that firms can build market share by lowering

[1]For example, as provided for by the United States Telecommunications Act of 1996.

[2]A lower price will also attract more subscribers in competition with the other firm, however this does not affect the net outflow of calls as long as subscribers make calls according to a balanced calling pattern – see the next section.

their rentals while keeping usage prices constant, and hence not incurring an access deficit.

One of the assumptions of the standard model is that there is full consumer participation, that is, whatever the prices, all consumers choose to participate in the market. This is a realistic approximation for competition in local fixed-line telecommunications markets and a useful simplifying assumption, however it does not fit well with in emerging markets such as cellular telephony. In this paper, the extent to which the results of the standard model under the more realistic assumption of two-part pricing are affected by partial consumer participation is examined.

When there is partial participation, a firm lowering its prices will gain customers through two channels. First, some customers who already participate in the market will switch from the other firm(s) ('business stealing'). Second, some consumers who did not participate in the market will choose to enter the market ('market expansion'). The model with full participation captures the former effect but not the latter. To the extent that the market expansion effect is important, as will probably be the case when the participation rate is significantly below 100%, the strategic interaction between the firms will change and the profit neutrality result may no longer hold. Furthermore, the socially optimal access charge may no longer be equal to marginal cost.

Introducing partial participation creates industry-wide network effects in the model. Consumers get utility from making calls, and this utility will be proportional to the number of other consumers that can be called, i.e., the market participation rate. As will be shown, the presence of network effects, coupled with the endogenous determination of the participation rate, makes the model difficult to solve analytically. In order to gain some insights into firms' behavior under partial participation, some additional simplifying assumptions need to be imposed. It is easily shown that when the participation rate is exogenous, firms remain indifferent over the level of the access charge. However, when the participation rate is endogenous but there are no network effects, firms prefer that the access charge be set equal to marginal cost. This result is also obtained in a fulfilled-expectations equilibrium.

Finally, numerical simulations are used to analyze the full model. It is found that the profit-maximizing access charge is below cost, for a wide range of parameter values. As will be explained, the presence of partial participation by consumers causes the firms to compete even more fiercely than with full participation. In this case, below cost access charges soften competition by reducing the incentive to sign up new customers. On the other hand, the socially optimal access charge is typically above cost. This is because firms ignore some of the externalities that increased participation create. Above-cost access charges mean lower rentals and increased participation and welfare. Only when the competition between the firms is very weak does 'taxing' the firms through a below-cost access charge become socially desirable.

The format of the rest of this paper is as follows. The next section reviews the standard model of two-way interconnection and discusses the reasons for the profit-neutrality result. Section three then discusses how to incorporate partial consumer participation in this model analyses this case under the simplifying assumptions given above, and presents the numerical results. Section four concludes.

2 Profit neutrality under full participation

Let us briefly review the basic structure of the standard model and the profit neutrality result. The same basic structure and notation will be used in the model in the next section. The standard model first appears in its basic form in LRT (1997) and was further developed, almost simultaneously, in three subsequent papers: Armstrong (1998), LRT (1998a), and Carter and

Wright (1999).

The essential features of the standard model are as follows. There are two horizontally differentiated and symmetric firms that supply telecommunications services, labelled firm 1 and firm 2. The networks of the two firms are transparently interconnected, so that a consumer who subscribes to one network can call any other consumer on either network. Both networks have full coverage, which means that both are available to all consumers, and there is full participation in the market by consumers. Firms face a constant marginal cost, c, per unit of origination and termination[3] and a fixed cost, f, per customer. Each firm charges a per-unit price for making calls, p_i, and a fixed subscription fee, r_i, to each customer. Firms are assumed not to price discriminate between calls that terminate on- and off-net.[4] The assumptions of interconnection, full participation and no price discrimination together mean that there are no network effects. For calls that originate on one network and terminate on the other, the originating network pays a reciprocal access (or termination) charge, a, per unit of termination to the terminating network. The access charge is negotiated or set by a regulator first, and then firms compete in retail prices with the access charge remaining constant. Firms set prices simultaneously.

Consumers get utility from making calls but not from receiving calls.[5] Consumers choose how many minutes of calls to make, q, to maximize their

[3]The total marginal cost of a call is therefore $2c$. In LRT (1998a, b), marginal cost is broken down into the marginal costs at the originating and terminating ends, c_0, and the marginal cost in between, c_1, so that the total marginal cost of a call is $2c_0 + c_1$. This difference does not significantly affect the results of the model and for simplicity of notation it is not used here.

[4]The implications of termination-based price discrimination are analysed by LRT (1998b). Such price discrimination is outside the scope of this paper.

[5]Note, however, that since all calls are willingly received and the length of calls is determined by the caller, an implicit assumption is that consumers do not get disutility from receiving calls.

net utility $u(q) - p_i q$ where $u(q)$ is the consumers' variable gross surplus function and is homogeneous across consumers. As usual it is assumed that $u'(q) > 0$ and $u''(q) < 0$. This yields a downward-sloping demand function $q(p_i)$ which gives the number of minutes of calls that every consumer wishes to make at the per-minute price p_i. Consumers are assumed to make calls according to a balanced calling pattern. This means that consumers make calls in a random fashion, and the probability of any consumer calling another consumer who subscribes to a particular network is equal to that network's market share.

Let $s_i \in [0,1]$ be the market share of firm i. Given the above specification of the model, and since $s_1 + s_2 = 1$, firm i's profit function can be written as

$$\pi_i = s_i \left[(p_i - 2c)\, q(p_i)\right] + s_i s_j (a - c) \left[q(p_j) - q(p_i)\right] + s_i (r_i - f),$$

for $i \neq j = 1, 2$. We can see that firm i's profit can be decomposed into the profit it would make if all calls terminated on-net, plus an access revenue or deficit, plus the profit from subscriptions. Let us call the sum of the first two terms firm i's 'profit from calls' and the last term its 'profit from subscriptions'. Let $w_i = v(p_i) - r_i$ be the net utility offered by firm i to consumers, where $v(p_i)$ is consumers' variable net surplus from making calls. There is a linear relationship between w_i and r_i and it is analytically convenient to imagine firms competing over p and w rather than p and r.

Following LRT (1997, 1998a) and Armstrong (1998), the Hotelling model with linear transportation costs is used to determine market shares. Under the Hotelling model in its simplest form, there is a continuum of consumers uniformly distributed on $[0,1]$, and firms 1 and 2 are located at 0 and 1 respectively. A consumer located at $x \in [0,1]$ who subscribes to firm i located at $\alpha_i \in \{0,1\}$ has net utility $v_0 + w_i - t\,|x - \alpha_i|$ where v_0 is the consumer's fixed surplus from being connected to either network (consumers are homogeneous with respect to v_0), and t is the Hotelling 'transportation cost'. To ensure full participation, it is necessary to assume that v_0 is sufficiently large so that all consumers always have positive net utility from participating in the market. Under this specification, firm i's market share is $s_i = \frac{1}{2} + \gamma (w_i - w_j)$, for $i \neq j = 1, 2$, where $\gamma = 1/2t$ is a measure of the differentiation of the firms.

Firms choose p_i and w_i to maximize profits. The first-order condition for p_i yields $p_i = 2c + s_j (a - c)$, reflecting a standard result of two-part pricing with known demands, that usage prices are set equal to marginal cost. The actual marginal cost of a call is $2c$, however both firms *perceive* the marginal cost to be $2c + s_j (a - c)$.[6] A non-zero access charge affects each firm's perceived marginal cost and therefore the retail prices that it sets, even if the flow of calls is balanced and the networks do not actually pay any net access fees to each other. This is termed the *raise-each-other's-cost effect* by Laffont and Tirole (2000).

The first-order condition for w_i gives

$$\begin{aligned} w_i &= v(p_i) - f - \frac{s_i}{\gamma} + (p_i - 2c)\, q(p_i) \\ &\quad + (s_j - s_i)(a - c)\left[q(p_j) - q(p_i)\right]. \end{aligned}$$

In a symmetric equilibrium,[7] $s_i^* = s_j^* = \frac{1}{2}$, $p_i^* = p_j^* = p^* = 2c + \frac{1}{2}(a - c)$, and hence the equilibrium rental is $r_i^* = r_j^* = r^* = v(p^*) - w^*$, which gives

$$r^* = f - \tfrac{1}{2}(a - c)\, q(p^*) + \frac{1}{2\gamma}.$$

The rental in a symmetric equilibrium is thus equal to the marginal cost of an additional subscriber, $f - \frac{1}{2}(a - c)\, q(p^*)$ plus the Hotelling markup,

[6] Observe that for firm i, the marginal cost of an on-net call is $2c$ and the marginal cost of an off-net call is $c + a = 2c + (a - c)$. Under the balanced calling pattern assumption, the fraction s_j of calls originating on network i are off-net calls. Hence network i's perceived marginal cost of calls is $2c + s_j (a - c)$.

[7] LRT (1998a, proposition 7) have shown, as with linear pricing, that this symmetric equilibrium exists and is unique, as long as the access markup is not too large and the degree of substitutability between the firms is not too great.

$1/2\gamma$. To see why $f - \frac{1}{2}(a-c)\,q(p^*)$ is the marginal cost of an additional customer, note that since $p_i = 2c + s_j(a-c)$, a firm's profits from calls is $\pi_i^C = s_i s_j (a-c)\,q(p_j)$, so that firms make a profit from calls when $a > c$. In a symmetric equilibrium, each customer generates a profit from calls of $\frac{1}{2}(a-c)\,q(p^*)$. Competition for market share causes the firms to reduce their rentals by the amount of profit from calls that each customer generates.

Profit in a symmetric equilibrium is $\pi^* = \frac{1}{4\gamma}$, which is a constant. The intuition for this is that if the access fee increases, to maintain market share both firms must reduce their rentals, which lowers the gain from attracting a new customer. However, the increase in the access fee raises the incentive to attract a new customer since the profit from calls is now hither. Under full participation these two effects exactly cancel out and hence the firms are indifferent over the level of the reciprocal access charge. In the case of two-part tariffs, the socially optimal access charge is $a = c$ and hence the firms could agree on the socially optimal access charge, however there is no special reason to expect that they will do so.

3 Partial consumer participation

The model outlined above utilizes the assumption of full consumer participation. That is, whatever the equilibrium prices, all consumers choose to participate in the market. This is a particularly strong assumption if the model was to be applied to cellular telephony, for example. In that industry, consumer participation rates are typically closer to 50% than 100%.[8] In this section, a model of partial consumer participation is developed and grafted on to the standard model and its implications are discussed.

The Hotelling model is maintained for the determination of market shares. For simplicity it is assumed that consumers' subscription and par-

[8]The *OECD Communications Outlook* 1999 gives the OECD average participation rate in cellular telephony as about 28%.

ticipation decisions are independent. This implies that the number of subscribers to firm i is given by $n_i = s_i \rho$, where s_i is firm i's Hotelling market share and $\rho \in [0,1]$ is the market participation rate. Aside from simplifying the analysis, this assumption can be justified by realizing that if consumers' participation decisions depend on their horizontal locations, there will be some consumers on the margin of participating that the firms are not competing over. On the other hand, consumers who strictly prefer to participate but are indifferent between the firms will be competed over relatively more fiercely. In specification used here, firms are equally competitive over all types of marginal consumer. This issue is discussed more fully in Schiff and Wright (2001).

In general, the participation rate will be a function of all the prices, i.e., $\rho = h(p_1, p_2, r_1, r_2)$ where ρ is decreasing in all arguments, and both firms' prices have symmetric effects on ρ. For some of the propositions and for the numerical simulation results, a specific model of ρ is needed. Such a model that maintains independence of the consumers' participation and subscription decisions is outlined in the next subsection.

3.1 A model of partial participation

Assume that consumers are differentiated in two dimensions: their horizontal preference between the firms, represented by their location $x \in [0,1]$ and their valuation of participating in the market, represented by v_0. Consumers know their realization of v_0 and observe the deals offered by the firms (w_1 and w_2), however consumers must conduct some evaluation of the firms before knowing their preference between the firms. In order to know their horizontal location, consumers must spend time and effort to evaluate the exact packages offered by the firms. This evaluation causes consumers to incur a disutility of $k > 0$. Consumers then calculate the probability that they will join each firm if they do participate, and make their participation decision based on the expected utility of participating.

Each consumer's valuation is assumed to be drawn from a distribution with density function $g(v_0)$ defined over $(-\infty,\infty)$. A consumer's location and valuation are independent. Regardless of their value of v_0, the consumer located at $\widehat{x}=\frac{1}{2}+\gamma(w_1-w_2)$ is indifferent between subscribing to either firm. A consumer's expected utility from participating in the market is therefore

$$E(u)=v_0-k+\widehat{x}\left[w_1-\tfrac{1}{2}t\widehat{x}\right]+(1-\widehat{x})\left[w_2-\tfrac{1}{2}t(1-\widehat{x})\right].$$

Consumers will participate if $E(u)\geq 0$, which implies that $v_0\geq v_H$ where

$$v_H=k+\tfrac{1}{4}t-\tfrac{1}{2}(w_1+w_2)-\tfrac{1}{4t}(w_1-w_2)^2. \tag{1}$$

If a consumer gets a realization of v_0 that is sufficiently high, they will choose to participate in the market and will evaluate which of firm 1 or firm 2 they prefer, incurring the evaluation cost k in the process. After evaluation, their horizontal location (x) is realized and the consumer chooses which firm to purchase from. It is possible that some consumers whose v_0 is not sufficiently high could get a realization of x for which they would have negative *ex post* utility from joining either firm and would instead prefer to drop out of the market. It is assumed that k is sufficiently large so as to avoid this possibility; that is, all consumers who chose to participate in the market before knowing their x will still prefer to do so once their x is realized.[9]

Consumers participate in the market if $v_0\geq v_H$ and purchase from firm 1 if their realization of x is less than $\widehat{x}$ and purchase firm 2 if it is greater than $\widehat{x}$. Therefore, the number of consumers who purchase from firm i is

$$n_i=\left[\tfrac{1}{2}+\gamma(w_i-w_j)\right]\int_{v_H}^{\infty}g(v_0)\,dv_0. \tag{2}$$

[9]This requires that (i) $v_0+w_1-tx\geq 0$ for all $v_0\in[v_H,\infty)$ and all $x\in[0,\widehat{x}]$, and (ii) $v_H+w_2-t(1-x)\geq 0$ for all $v_0\in[v_H,\infty)$ and all $x\in[\widehat{x},1]$. Both of these conditions imply that $k\geq\frac{1}{4}t+\frac{1}{4t}(w_1-w_2)^2$.

The market participation rate is given by $\rho = n_1 + n_2 = \int_{v_H}^{\infty} g(v_0)\, dv_0$. Finally, consumer surplus is given by

$$CS = n_1 w_1 + n_2 w_2 - T_1 - T_2 + \int_{v_H}^{\infty} v_0 g(v_0)\, dv_0,$$

where T_i is the total disutility of subscribers to network i from not being able to subscribe to their most preferred network (the 'transportation cost'). For any such $v_0 \geq v_H$, the total 'transportation cost' for subscribers to network 1 is $\int_0^{\widehat{x}} tx dx = \frac{1}{2}t\widehat{x}^2$, hence $T_1 = \int_{v_H}^{\infty} \frac{1}{2}t\widehat{x}^2 g(v_0)\, dv_0 = \frac{1}{2}t\widehat{x}n_1$. Similarly, $T_2 = \frac{1}{2}t(1-\widehat{x})\, n_2$ and hence

$$CS = n_1 \left[w_1 - \tfrac{1}{2}t\widehat{x}\right] + n_2 \left[w_2 - \tfrac{1}{2}t(1-\widehat{x})\right] + \int_{v_H}^{\infty} v_0 g(v_0)\, dv_0. \tag{3}$$

3.2 Partial participation in the standard model

Now let us consider the implications for the standard model of partial participation, using the model developed above. First note that subscribers to firm i now make only $\rho q(p_i)$ calls and hence $w_i = \rho v(p_i) - r_i$. Since ρ is a function of w_1 and w_2, there is no longer a linear relationship between r_i and w_i and we cannot replace r_i with w_i in firm i's profit function as was done to simplify the analysis under full participation. With this in mind, firm i's profit function is given by

$$\begin{aligned}
\pi_i &= n_i s_i (p_i - 2c)\, \rho q(p_i) + n_i s_j (p_i - a - c)\, \rho q(p_i) \\
&\quad + n_j s_i (a - c)\, \rho q(p_j) + n_i (r_i - f)\,.
\end{aligned}$$

The first term is profit from calls that originate and terminate on firm i's own network. This term is derived as follows. Firm i has n_i subscribers, each of which make $\rho q(p_i)$ minutes of calls. Under the balanced calling pattern assumption, the probability of any of these calls terminating on firm i's own network is s_i. The quantity of such calls is therefore $n_i s_i \rho q(p_i)$. Similarly, the second term is profit from calls that originate on firm i's network and terminate on firm j's network, the third term is profit from calls that originate on firm j's network and terminate on firm i's network, and the fourth term is profit from subscriptions. Using the facts that $s_i = n_i/\rho$ and $s_i + s_j = 1$, firm i's profit function becomes

$$\begin{aligned} \pi_i &= \rho^2 \left[s_i \left(p_i - 2c \right) q\left(p_i \right) + s_i s_j \left(a - c \right) \left(q\left(p_j \right) - q\left(p_i \right) \right) \right] \\ &\quad + \rho s_i \left(r_i - f \right). \end{aligned} \tag{4}$$

The analysis of this model somewhat complex, due to the presence of an endogenous participation rate and network externalities. In order to gain some insights into firms' behavior under partial participation, let us consider the results of the model under some additional simplifying assumptions. Note that in all cases, it is straightforward to show that the first-order condition for p_i again gives $p_i = 2c + s_j (a - c)$, i.e., usage prices equal to perceived marginal cost.

3.2.1 Exogenous participation rate

Suppose that ρ is entirely determined by factors that are outside the firms' control. To some extent this is true in the cellular telephony market, for example, where changes in participation may be due to social trends rather than prices. In this case it is straigthtforward to show the following proposition.

Proposition 1 *If the participation rate is exogenous, firms are indifferent over the access charge and the socially optimal access charge is equal to marginal cost.*

Proof. Note that in this case the linear relationship between w_i and r_i is restored. Substituting w_i for r_i in (4), the first-order condition for w_i is

$$\begin{aligned} \frac{\partial \pi_i}{\partial w_i} &= \rho^2 \left(\gamma \left(p_i - 2c \right) q\left(p_i \right) + \gamma \left(s_j - s_i \right) \left(a - c \right) \left[q\left(p_j \right) - q\left(p_i \right) \right] \right) \\ &\quad + \gamma \rho \left[\rho v\left(p_i \right) - w_i - f \right] - \rho s_i \\ &= 0. \end{aligned}$$

In a symmetric equilibrium this implies that $w^* = \rho v(p^*) - f + \frac{1}{2}\rho(a-c)q(p^*) - \frac{1}{2\gamma}$. Profit in a symmetric equilibrium is therefore given by $\pi^* = \frac{\rho}{4\gamma}$, which is independent of a. From (3), consumer surplus in a symmetric equilibrium is $CS^* = \rho\left(w^* - \frac{1}{4}t\right) + X$ where X is a constant representing the utility from participating in the market. Substituting in w^*, this becomes

$$CS^* = \rho\left[\rho v(p^*) - f + \tfrac{1}{2}\rho(a-c)q(p^*) - \tfrac{1}{2\gamma} - \tfrac{1}{4}t\right].$$

Noting that $v'(p^*) = -q(p^*)$, this gives

$$\frac{dCS^*}{da} = \tfrac{1}{4}\rho^2(a-c)q'(p^*).$$

Clearly, $a = c$ satisfies the first-order condition for maximizing consumer surplus and hence welfare, since profits are constant. Furthermore,

$$\left.\frac{d^2CS^*}{da^2}\right|_{a=c} = \frac{1}{2}\rho^2 q'(p^*) < 0.$$

■

To interpret this result, let us make the comparison with the full participation case. With an exogenous participation rate, the equilibrium rental is again equal to the marginal cost of adding a customer plus the Hotelling markup. The difference compared to full participation is that the call revenue generated by an additional consumer is multiplied by ρ, since in equilibrium each consumer makes $\rho q(p^*)$ minutes of calls. The Hotelling markup per customer, $\frac{1}{2\gamma}$, is independent of the participation rate because when the participation rate is exogenous it does not affect the intensity of competition for the consumers who do participate in the market. The equilibrium profit level is equal to the full participation profit level multiplied by the participation rate. This suggests that when the participation rate is endogenous, the firms will take account of how the access charge affects the participation rate through usage prices and rentals. To investigate this, consider the next simplifying assumption.

3.2.2 Calls are perfect substitutes

Industry-wide network externalities arise in the model because consumers get more utility from making calls the more consumers participate in the market. Let us assume instead that calls to all consumers are perfect substitutes and hence all subscribers to firm i make $q(p_i)$ units of calls regardless of the participation rate. This implies that $w_i = v(p_i) - r_i$ and there is again a linear relationship between w_i and r_i. Using the assumption of a balanced calling pattern, firm i's profit function in terms of p and w is given by

$$\begin{aligned}\pi_i &= \rho\left[s_i(p_i - 2c)\,q(p_i) + s_i s_j(a-c)\left(q(p_j) - q(p_i)\right)\right] \\ &\quad +\rho s_i\left[v(p_i) - w_i - f\right], \end{aligned} \tag{5}$$

which is equal to ρ multiplied by firm i's profit under full participation. In this case we have

Proposition 2 *If calls are perfect substitutes, firms' equilibrium profits, consumer surplus, and total welfare are maximized when the access charge is set equal to marginal cost.*

Proof. Using the model of participation given above, first note that

$$\frac{\partial\rho}{\partial w_i} = \left[\tfrac{1}{2} + \gamma(w_i - w_j)\right] g(v_H) = s_i g(v_H).$$

Therefore the first-order condition for w_i is

$$\begin{aligned}\frac{\partial \pi_i}{\partial w_i} &= \rho\left[\gamma(p_i - 2c)\,q(p_i) + \gamma(s_j - s_i)(a-c)\left(q(p_j) - q(p_i)\right)\right] \\ &\quad +\rho\left[\gamma\left(v(p_i) - w_i - f\right) - s_i\right] \\ &\quad +s_i g(v_H)\left[s_i(p_i - 2c)\,q(p_i) + s_i s_j(a-c)\left(q(p_j) - q(p_i)\right)\right] \\ &\quad +s_i^2 g(v_H)\left[v(p_i) - w_i - f\right] \\ &= 0.\end{aligned}$$

In a symmetric equilibrium, $s_1^* = s_2^* = \frac{1}{2}$, $p_1^* = p_2^* = p^* = 2c + \frac{1}{2}(a-c)$, $w_1^* = w_2^* = w^*$, and $\rho^* = \int_{v_H^*}^{\infty} g(v_0)\, dv_0$ where $v_H^* = k + \frac{1}{4}t - w^*$. This gives

$$w^* = v(p^*) + \tfrac{1}{2}(a-c)\, q(p^*) - f - \frac{\frac{1}{2}\rho^*}{\gamma\rho^* + \frac{1}{4}g\left(v_H^*\right)},$$

and hence equilibrium profit is

$$\pi^* = \frac{\frac{1}{4}(\rho^*)^2}{\gamma\rho^* + \frac{1}{4}g\left(v_H^*\right)}.$$

Unlike the full participation case, equilibrium profits depend on a since both the numerator and denominator depend on w^*, which depends on a. Thus,

$$\frac{\partial \pi^*}{\partial a} = \frac{\frac{1}{2}\rho^*\left[\gamma\rho^* + \frac{1}{8}g(v_H^*)\right]\frac{\partial \rho^*}{\partial a} - \frac{1}{4}(\rho^*)^2\left[\gamma\frac{\partial \rho^*}{\partial a} + \frac{1}{4}g'(v_H^*)\frac{\partial v_H^*}{\partial a}\right]}{\left[\gamma\rho^* + \frac{1}{8}g\left(v_H^*\right)\right]^2}.$$

Furthermore, from (3), in a symmetric equilibrium $CS^* = \rho^*\left(w^* - \frac{1}{4}t\right) + \int_{v_H^*}^{\infty} v_0 g(v_0)\, dv_0$ and hence

$$\frac{\partial CS^*}{\partial a} = \left[w^* - \tfrac{1}{4}t\right]\frac{\partial \rho^*}{\partial a} + \rho^*\frac{\partial w^*}{\partial a} - v_H^* g(v_H^*)\frac{\partial v_H^*}{\partial a}.$$

Note that $\rho^* = \int_{v_H^*}^{\infty} g(v_0)\, dv_0$, where $v_H^* = k + \frac{1}{4}t - w^*$. Since w^* is a function of ρ^*, this is only an implicit expression for ρ^*. Totally differentiating with respect to a gives

$$\frac{\partial \rho^*}{\partial a} = \frac{\frac{\partial w^*}{\partial a} g\left(v_H^*\right)}{1 - \frac{\partial w^*}{\partial a^*} g\left(v_H^*\right)}.$$

In addition,

$$\frac{\partial v_H^*}{\partial a} = \frac{-\frac{\partial w^*}{\partial a}}{1 - \frac{\partial w^*}{\partial v_H^*}}.$$

Differentiating w^* with respect to a while holding ρ^* (or, equivalently, v_H^*) constant gives

$$\frac{\partial w^*}{\partial a} = \frac{1}{2}v'(p^*) + \frac{1}{2}q(p^*) + \frac{1}{4}(a-c)\, q'(p^*).$$

Since $v'(p^*) = -q(p^*)$ this is zero when $a = c$ and hence $a = c$ satisfies the first-order condition for maximizing equilibrium profit and consumer surplus. ■

To interpret this result, let us compare it with the full participation case. First, observe that the equilibrium rental is again equal to the marginal cost of an additional customer, plus the Hotelling markup. In particular,

$$r^* = f - \tfrac{1}{2}(a-c)\,q(p^*) + \frac{\tfrac{1}{2}\rho^*}{\gamma\rho^* + \tfrac{1}{4}g\left(v_H^*\right)}. \tag{6}$$

The perfect substitutability of calls means that the marginal cost of an additional consumer is independent of the participation rate, since all consumers make $q(p^*)$ minutes of calls in equilibrium. However, the Hotelling markup per customer now depends on the participation rate.[10] That is, the firms' ability to price rentals above cost, and hence the intensity of competition, depends on the participation rate. As under full participation, equilibrium profit is equal to the Hotelling markup multiplied by the number of customers of each firm, which in this case is $\frac{1}{2}\rho^*$. Since the participation rate is endogenous and depends on w^*, equilibrium profits are no longer independent of the access charge. In fact, the Hotelling profit is maximized when the access charge is equal to marginal cost.

3.2.3 Fulfilled expectations equilibrium

A standard device in the literature on network externalities (for example, Katz and Shapiro, 1985) is to use the concept of a fulfilled-expectations Nash equilibrium. To use this in the present model, the game is modified as follows. First, the firms negotiate the access charge or it is set by a regulator. Consumers then form some expectation of the market participation rate, ρ^e, without knowing the access charge. Firms maximize profits by choosing prices while taking the expected participation rate as given. Finally

[10] To see that the last term in (6) is indeed the Hotelling markup, consider the standard Hotelling model with unit demands. In this model, firm i's profit is $\pi_i = n_i(p_i - c)$ where c is the constant marginal cost. The first-order condition for p_i is $\partial\pi_i/\partial p_i = n_i + \partial n_i/\partial p_i\,(p_i - c) = 0$ which gives $p_i = c - \frac{n_i}{\partial n_i/\partial p_i}$. The Hotelling markup is therefore $-\frac{n_i}{\partial n_i/\partial p_i}$. Note that $n_i = s_i\rho$ and $\partial n_i/\partial p_i = -s_i^2 g(v_H) - \gamma\rho$. In a symmetric equilibrium, the Hotelling markup is $\frac{1}{2}\rho\big/\left[\gamma\rho + \frac{1}{4}g(v_H)\right]$.

consumers observe the prices and make their subscription and participation decisions. In equilibrium, consumers' expectations are correct, so $\rho^e = \rho^*$.

In this case, firm i's profit function is

$$\begin{aligned}\pi_i &= (\rho^e)^2 \left[s_i (p_i - 2c) q(p_i) + s_i s_j (a - c)(q(p_j) - q(p_i))\right] \\ &\quad + \rho^e s_i (r_i - f).\end{aligned}$$

It turns out that the firms' preferred access charge and the socially optimal access charge are the same as in the previous subsection where there were no network externalities. In particular, we have,

Proposition 3 *In a fulfilled-expectations equilibrium, firms' equilibrium profits, consumer surplus, and total welfare are maximized if the access charge is set equal to marginal cost.*

Proof. First note that $w_i = \rho^e v(p_i) - r_i$, hence there is a linear relationship between w_i and r_i, so we can imagine firms competing over w and p. The first-order condition for w_i gives

$$\begin{aligned}w_i &= \rho^e (p_i - 2c) q(p_i) + \rho^e (s_j - s_i)(a - c)(q(p_j) - q(p_i)) \\ &\quad + \rho^e v(p_i) - f - \frac{s_i}{\gamma}.\end{aligned}$$

In a symmetric fulfilled expectations equilibrium, $\rho^e = \rho^*$, $s_i = s_j = \frac{1}{2}$ and $p_i = p_j = p^* = 2c + \frac{1}{2}(a - c)$ and hence

$$w^* = \rho^* v(p^*) - f + \frac{1}{2}\rho^* (a - c) q(p^*) - \frac{1}{2\gamma}.$$

Profit in a symmetric fulfilled-expectations equilibrium is

$$\pi^* = \frac{1}{2}(\rho^*)^2 (p^* - 2c) q(p^*) + \frac{1}{2}\rho^* (\rho^* v(p^*) - w^* - f)$$

and substituting in for p^* and w^* gives

$$\pi^* = \frac{\rho^*}{4\gamma}.$$

Therefore, in the first stage of the game, firms choose a to maximize the equilibrium participation rate. Note that

$$\rho^* = \int_{v_H^*}^{\infty} g(v_0)\, dv_0$$

where $v_H^* = k+\frac{1}{4}t-w^* = k+\frac{1}{4}t+\frac{1}{2\sigma}-\rho^* v(p^*)-\frac{1}{2}\rho^*(a-c)\,q(p^*)$. Therefore,

$$\frac{\partial \rho^*}{\partial a} = \frac{\frac{1}{4}\rho^*(a-c)\,q'(p^*)\,g(v_H^*)}{1-\left[v(p^*)+\frac{1}{2}(a-c)\,q(p^*)\right]g\left(v_H^*\right)}$$

and hence $a = c$ satisfies the first-order condition for maximizing equilibrium profits. Consumer surplus in a symmetric equilibrium is $CS^* = \rho^*\left(w^*-\frac{1}{4}t\right)+\int_{v_H^*}^{\infty} v_0 g(v_0)\, dv_0$ and hence

$$\frac{\partial CS^*}{\partial a} = \left(w^*-\frac{1}{4}t\right)\frac{\partial \rho^*}{\partial a}+\rho^*\frac{\partial w^*}{\partial a}-v_H^* g(v_H^*)\frac{\partial v_H^*}{\partial a}.$$

From above, $\frac{\partial \rho^*}{\partial a} = 0$ when $a = c$. Furthermore,

$$\frac{\partial w^*}{\partial a} = \frac{\frac{1}{4}\rho^*(a-c)\,q'(p^*)}{1-\left[v(p^*)+\frac{1}{2}(a-c)\,q(p^*)\right]\frac{\partial \rho^*}{\partial w^*}},$$

and

$$\frac{\partial v_H^*}{\partial a} = \frac{-\frac{1}{4}(a-c)\,q'(p^*)}{1+\left[v(p^*)+\frac{1}{2}(a-c)\,q(p^*)\right]\frac{\partial \rho^*}{dv_H^*}},$$

therefore $a = c$ satisfies the first-order condition for maximizing consumer surplus and welfare. ■

3.2.4 Returning to the full model

Let us now consider the full model with endogenous participation and network effects. A firm's profit function is given by (4). The first-order condition for p_i gives $p_i = 2c + s_j(a-c)$, as in the full participation case. The first-order condition for r_i is

$$\begin{aligned}\frac{\partial \pi_i}{\partial r_i} &= \rho^2 \left[\frac{\partial s_i}{\partial r_i}(p_i - 2c)\, q(p_i) + \left(s_i \frac{\partial s_j}{\partial r_i} + s_j \frac{\partial s_i}{\partial r_i}\right)(a-c)\left(q(p_j) - q(p_i)\right)\right] \\ &\quad + \rho \frac{\partial s_i}{\partial r_i}(r_i - f) + \rho s_i \\ &\quad + 2\rho \frac{\partial \rho}{\partial r_i}\left[s_i (p_i - 2c)\, q(p_i) + s_i s_j (a-c)\left(q(p_j) - q(p_i)\right)\right] \\ &\quad + \frac{\partial \rho}{\partial r_i} s_i (r_i - f) \\ &= 0.\end{aligned}$$

In a symmetric equilibrium, $r_1^* = r_2^* = r^*$, $p_1^* = p_2^* = p^* = 2c + \frac{1}{2}(a-c)$ and the first-order condition for the rental becomes[11]

$$\begin{aligned}&\tfrac{1}{2}(\rho^*)^2 (a-c)\, q(p^*) \left.\frac{\partial s_i}{\partial r_i}\right|_{r_i = r_j = r^*} + \rho^* (r^* - f) \left.\frac{\partial s_i}{\partial r_i}\right|_{r_i = r_j = r^*} + \tfrac{1}{2}\rho^* \\ &+ \tfrac{1}{2}\rho^* (a-c)\, q(p^*) \left.\frac{\partial \rho}{\partial r_i}\right|_{r_i = r_j = r^*} + \tfrac{1}{2}(r^* - f) \left.\frac{\partial \rho}{\partial r_i}\right|_{r_i = r_j = r^*} \\ =\;& 0\end{aligned}$$

Note that firm i's market share is given by $s_i = \frac{1}{2} + \gamma\left[\rho\left(v(p_i) - v(p_j)\right) - r_i + r_j\right]$ and hence

$$\frac{\partial s_i}{\partial r_i} = \gamma \left[\frac{\partial \rho}{\partial r_i}\left(v(p_i) - v(p_j)\right) - 1\right],$$

so that in a symmetric equilibrium,

$$\left.\frac{\partial s_i}{\partial r_i}\right|_{r_i = r_j = r^*} = -\gamma,$$

which is the same as under full participation. This is because starting from a symmetric equilibrium, the effect of a change in firm i's rental on the participation rate and hence the indirect utility from subscribing to either

[11] To save on notation, $r_i = r_j = r^*$ is used to denote a derivative that is evaluated at the symmetric equilibrium point. Note also that $p_i = p_j = p^*$ at this point.

firm is equal. Therefore, the equilibrium rental is implicitly defined by

$$
\begin{aligned}
r^* &= f - \frac{1}{2}\rho^* (a-c)\, q(p^*) \left[1 + \frac{\frac{1}{2} \left.\frac{\partial \rho}{\partial r_i}\right|_{r_i=r_j=r^*}}{\frac{1}{2} \left.\frac{\partial \rho}{\partial r_i}\right|_{r_i=r_j=r^*} - 2t\rho^*} \right] \\
&\quad - \frac{\frac{1}{2}\rho^*}{\frac{1}{2} \left.\frac{\partial \rho}{\partial r_i}\right|_{r_i=r_j=r^*} - 2t\rho^*}.
\end{aligned}
\tag{7}
$$

Let $\delta = \left.\frac{\partial \rho}{\partial r_i}\right|_{r_i=r_j=r^*} < 0$ denote the rate at which the participation rate declines as firm i raises its rental, starting from a symmetric equilibrium. The solution for r^* in the partial participation case is analogous to the solution in the full participation case. In particular, the rental in a symmetric equilibrium is equal to the marginal cost of adding a new customer, plus the Hotelling markup. Recall that under full participation, the marginal cost of an additional customer is $f - \frac{1}{2}(a-c)\, q(p^*)$. That is, with full participation, an additional customer costs the firm f, but gains the firm additional profit from calls of $\frac{1}{2}(a-c)\, q(p^*)$. Now, with partial participation, an additional customer makes only $\rho^* q(p^*)$ calls. In addition, the profit from calls gained is multiplied by a term reflecting the fact that if a firm lowers its rental, the participation rate increases which causes all consumers to make more calls. To explain this term, note that a reduction in r_i causes firm i to gain customers through two channels: (i) increased participation, $-\frac{1}{2}\delta$, and (ii) customers who switch from firm j, $\gamma\rho^*$. The former causes all consumers to make more calls while the latter does not. Therefore the change in the quantity of calls made by all consumers that is due to an increase in the participation rate following a reduction in a firm's rental is $\frac{1}{2}\delta \big/ \left(\frac{1}{2}\delta - \gamma\rho^*\right)$. The last term in (7) is the Hotelling markup per customer, which is now endogenous.

Profits in a symmetric equilibrium are given by

$$
\pi^* = -\tfrac{1}{4}(\rho^*)^2 (a-c)\, q(p^*) \left[\frac{\frac{1}{2}\delta}{\frac{1}{2}\delta - 2t\rho^*} \right] - \frac{\frac{1}{4}(\rho^*)^2}{\frac{1}{2}\delta - 2t\rho^*}.
$$

To find the profit-maximizing access charge, we would like to know the sign of $d\pi^*/da$ evaluated at $a = c$. Since δ is endogenous and depends on the distribution of v_0, this derivative is very complex. Instead, let us turn to a numerical simulation to investigate how equilibrium profits and welfare depend on the access charge.

Numerical simulation results The simulation program was written in *Matlab* version 12 and is available in electronic form from the author. In order to obtain numerical results, a specific distribution for v_0 must be chosen. A normal distribution with mean μ and standard deviation σ has been used, and the simulation programs also permit the use of a uniform distribution. The latter does not qualitatively affect the results but is somewhat more difficult to work with since one must ensure that v_H falls within the range specified for v_0. Only results from the normal distribution will be reported here. Second, an exact demand function for usage must be specified. LRT (1998a, b) use a constant elasticity demand function of the form $q(p) = p^{-\eta}$, where η is the elasticity of demand, to derive their analytical results and it will also be used here. In addition, a linear demand function of the form $q(p) = (\alpha - p)/2\beta$ has been tested and it does not qualitatively affect the results. Only results from the constant elasticity demand will be reported here.

The purpose of the simulation program is to investigate the access charges that maximize firms' equilibrium profits, consumer surplus, and social welfare. To do this, for any given set of parameter values the program takes a range of access charges and attempts to find, for each, the Nash equilibrium of the firms' game. From these prices, the firms' profits, consumer surplus, and total welfare can be calculated.

The most fundamental algorithm of the simulation program is to attempt to find a Nash equilibrium $\{(p_1^*, r_1^*), (p_2^*, r_2^*)\}$ of the firms' pricing game for a given access charge. The user supplies numerical values for the model's set of parameters: $\{a, c, f, k, t, \eta, \mu, \sigma\}$ and the equilibrium-finding algorithm then proceeds as follows. First, recall that under two-part pricing, usage prices are set equal to perceived marginal cost, i.e., $p_i = 2c + s_j (a - c)$, so the program only specifically searches for an equilibrium in (r_1, r_2) space. To do this, the program starts with an initial guess of r_2, calculates firm 1's best response, then calculates firm 2's best response, and so on. This iterative procedure continues until the changes in r_1 and r_2 between iterations falls below some specified tolerance level. The equilibrium rentals should be in the region of the per-customer fixed cost, f, hence the initial guess for r_2 is f, and the tolerance for finding the equilibrium is $f/1000$.

Evaluating one firm's best response for a given rental of the other firm is complicated by the presence of network effects and involves solving a fixed point problem. Recall that the number of subscribers to firm i is given by $n_i = s_i \rho$ where $s_i = \frac{1}{2} + \gamma (w_i - w_j)$ and $\rho = \int_{v_H}^{\infty} g(v_0)\, dv_0$ with $v_H = k + \frac{1}{4}t - \frac{1}{2}(w_1 + w_2) - \frac{1}{4t}(w_1 - w_2)^2$ and $w_i = \rho v(p_i) - r_i$. For a given r_1 and r_2, the number of subscribers to firm i is therefore solved numerically as follows. Starting from an initial guess of $n_1 = n_2 = 0.5$ and hence $\rho = 1$, p_1 and p_2 are determined and w_1 and w_2 are calculated. These numbers are then substituted into the above equations to calculate n_1 and n_2, which are then used to re-calculate w_1 and w_2, which gives new values for n_1 and n_2, and so on. This process continues until the change in n_1 and n_2 between iterations is below a certain tolerance. Once n_1 and n_2 have been calculated in this manner, it is straightfoward to calculate the firm's profit level associated with the rentals.

Using the above procedure to solve for n_1, n_2 and the usage prices, figure 1 shows a typical plot of a firm's profit against its rental, for a given rental of the other firm. The profit function is concave in the firm's own rental and otherwise well behaved. The shape of the profit function can be explained as follows. When r_i is low compared to r_j, firm i captures all of the consumers on the Hotelling interval ($s_i = 1$), given the participation rate, thus profits are increasing in r_i. When r_i is close to r_j, both firms have positive market share and the slope of the profit function decreases rapidly and eventually starts to fall. Finally, when r_i is very high compared to r_j (greater than $r_j + t$), firm i's market share is zero, and firm i's profit is equal to zero.

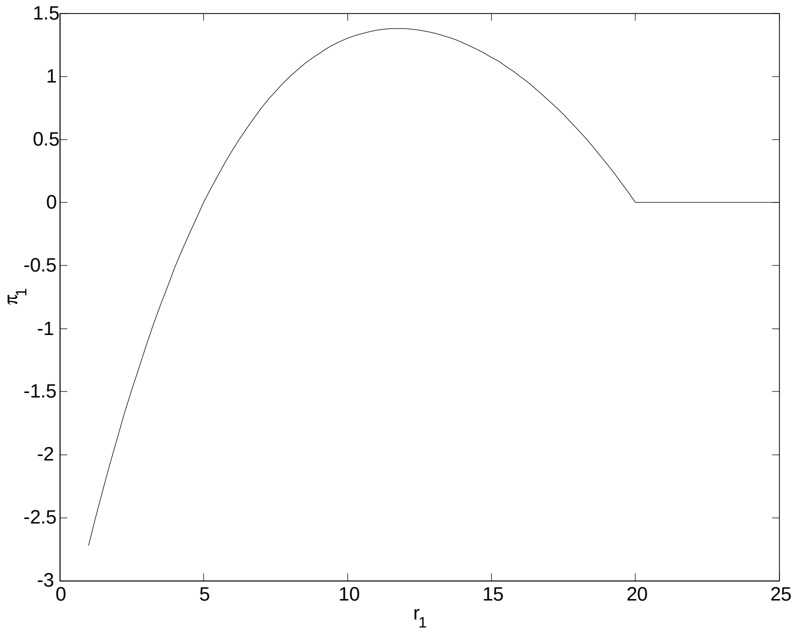

Figure 1: Firm i's profit as a function of its rental, for a given rental of the other firm. Parameter values: $r_2 = 10$, $a = 0.05$, $c = 0.05$, $t = 10$, $k = 100$, $f = 5$, $\eta = 1.5$, $\mu = 110$, $\sigma = 15$.

As discussed above, the program searches for an equilibrium by iterating along the best-responses of the firms with respect to their rentals. Figure 2 shows a typical plot of these pseudo best response curves.[12] The solid line is firm 1's best response and the dashed line is firm 2's. We can see

[12] A firm's strategic variables are p_i and r_i. Hence these are not proper best response curves because p_1 and p_2 are changing along the curves.

that the curves are generally upward-sloping, however they briefly become downward sloping. This is an interesting observation as prices are usually strategic complements. Furthermore, when firm j's rental increases beyond a certain level, firm i's best response is to undercut and take the whole market, hence firm i's best response becomes linear for high values of r_j. Figure 2 also seems to indicate that an equilibrium, if it exists, is unique and symmetric. Extensive testing with different parameter values has produced only graphs that look like figure 2.

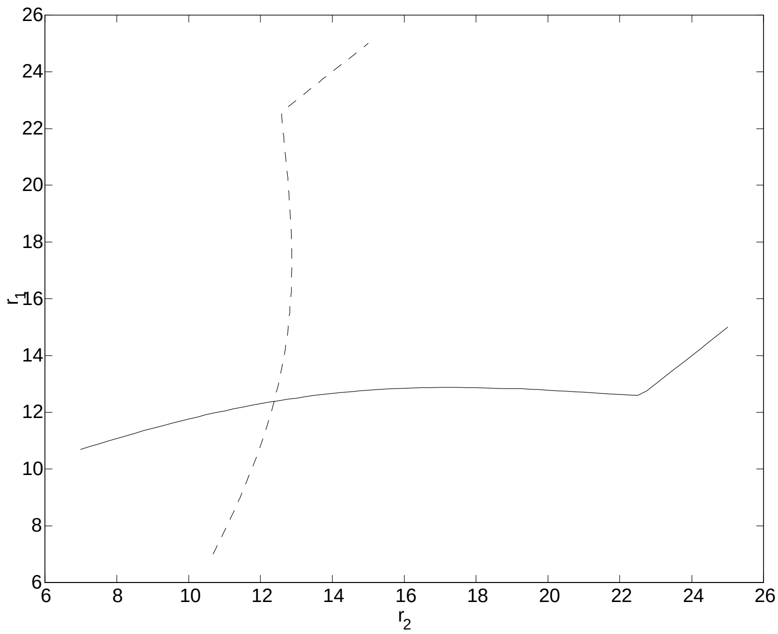

Figure 2: Pseudo best responses for firm 1 and firm 2. Firm 1's best response is the solid line and firm 2's is the dashed line. Parameter values: $a = 0.05$, $c = 0.05$, $t = 10$, $k = 100$, $f = 5$, $\eta = 1.5$, $\mu = 110$, $\sigma = 15$.

Let us now turn to the results of the simulations with regard to the question of the profit-maximizing and socially optimal access charges. First, a set of benchmark parameter values were chosen that, in a symmetric equilibrium, give a participation rate of around 40%, and that allow a reasonable

c	t	k	f	η	μ	σ
0.05	10	100	5	1.5	110	15

Table 1: Benchmark parameter values.

range of unilateral deviations in each parameter from the benchmark without violating any constraints of the model in equilibrium.[13] These parameter values are given in table 1. For the benchmark parameters, the profit maximizing access charge is -0.047 and the welfare maximizing access charge is 0.073. The characteristics of the equilibria corresponding to these two access charges are given in table 2.

The profit maximizing access charge for these parameter values is below marginal cost. As in the full participation case, the access charge can be used to manipulate equilibrium per-minute prices and rentals. Compared to full participation, competition in rentals is even more fierce because there are new customers outside the market to be competed for, as well as existing customers. Setting a below-cost access charge makes the competition less fierce since additional consumers are less attractive. On the other hand, the welfare maximizing access charge is above cost. Higher access charges cause firms to cut rentals which increases participation and thus welfare through two channels. First, the extra consumers who participate get intrinsic benefits from simply participating. Second, higher participation means that there are more people who can be called by subscribers to either network. On the other hand, higher access charges raise per-minute prices which will reduce consumer surplus. The specifications of the utility function and distribution of benefits used in the simulations mean that the first effect usually

[13]Specifically, recall the constraint that $k \geq \frac{1}{4}t + \frac{1}{4t}(w_1 - w_2)^2$. In a symmetric equilibrium this implies that $k \geq \frac{1}{4}t$. However, a higher value of k typically needs to be chosen, since the algorithm for finding the equilibrium iterates along the best-responses of the firms which implies that $w_1 \neq w_2$. Another constraint is that the elasticity of demand, η, must be strictly greater than one in order for the indirect utility from calls to be positive.

Profit maximising: $a = -0.047$					
p^*	$q(p^*)$	r^*	ρ^*	CS^*	π^*
0.051	85.3	15.77	0.37	40.68	1.71
Welfare maximising: $a = 0.073$					
p^*	$q(p^*)$	r^*	ρ^*	CS^*	π^*
0.11	26.8	12.20	0.45	49.66	1.64

Table 2: Characteristics of the equilibria corresponding profit maximising and welfare maximising access charges for the benchmark parameter values.

dominates, so welfare is increased by having an above-cost access charge.

As mentioned above, to investigate how these access charges change with the parameter values, some parameters were varied unilaterally from their benchmarks. The results of these experiments are represented in the figures on the following pages. In all the figures, the solid line represents the profit-maximizing access charge and the dashed line is the socially optimal access charge. The top half of each plot shows the profit-maximizing access charge on the left-hand vertical axis and the socially optimal access charge on the right-hand vertical axis. The bottom half shows the corresponding equilibrium participation rates, both of which are plotted on the left-hand vertical axis.

First, figure 3 shows the results from changing the marginal cost, c. The profit maximizing access charge is decreasing in marginal cost, while the socially optimal access charge is increasing. In both cases the equilibrium participation rate decreases.

Figure 4 shows the simulation results for different levels of the per-customer fixed cost, f. The opposite behavior to changes in marginal cost is observed. The profit maximizing access charge increases while the welfare maximizing access charge decreases. However, the changes in both are

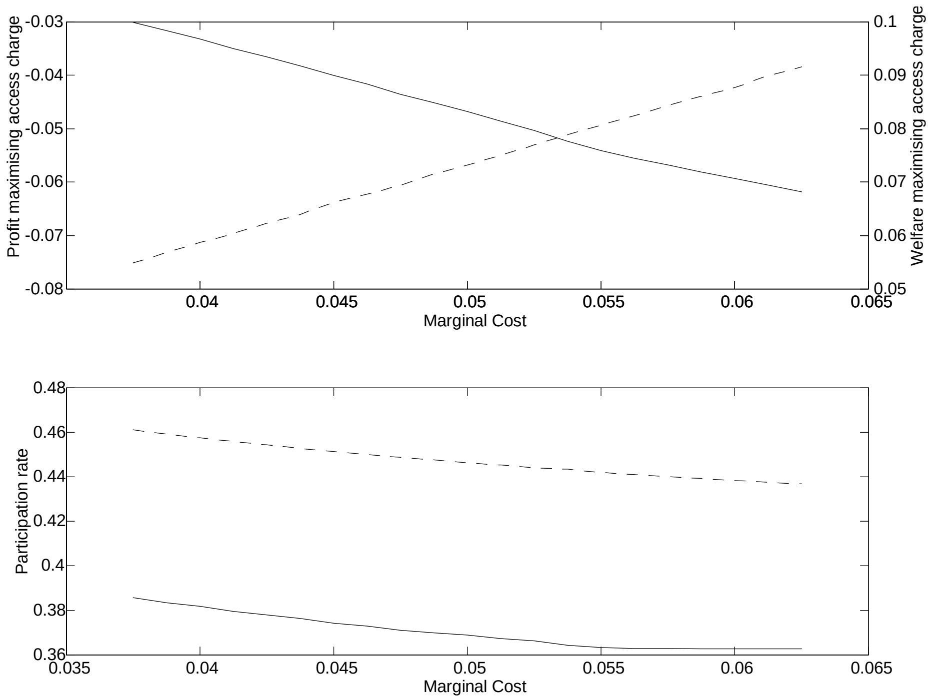

Figure 3: Profit maximising (solid line, left-hand scale) and welfare maximising (dashed line, right-hand scale) access charges and the associated participation rates for different marginal costs.

very small. The apparently nonlinearity of the welfare maximizing access charge is due to numerical errors induced by the range of variation being approximately only 4%.

Figure 5 shows the effects of changing the elasticity of demand. For elasticities close to one, the profit maximizing access charge remains around -0.05. This is an artificially imposed limit. Recall that the equilibrium usage price is given by $p^* = 2c + \frac{1}{2}(a - c)$. In order for p^* to remain positive, we must have $a > -3c$. However, in the equilibrium solving process described above we must allow for the possibility that in the process of finding the equilibrium, one firm may have a market share close to one, and hence it is required that $a > -c$. As the elasticity increases, the profit maximiz-

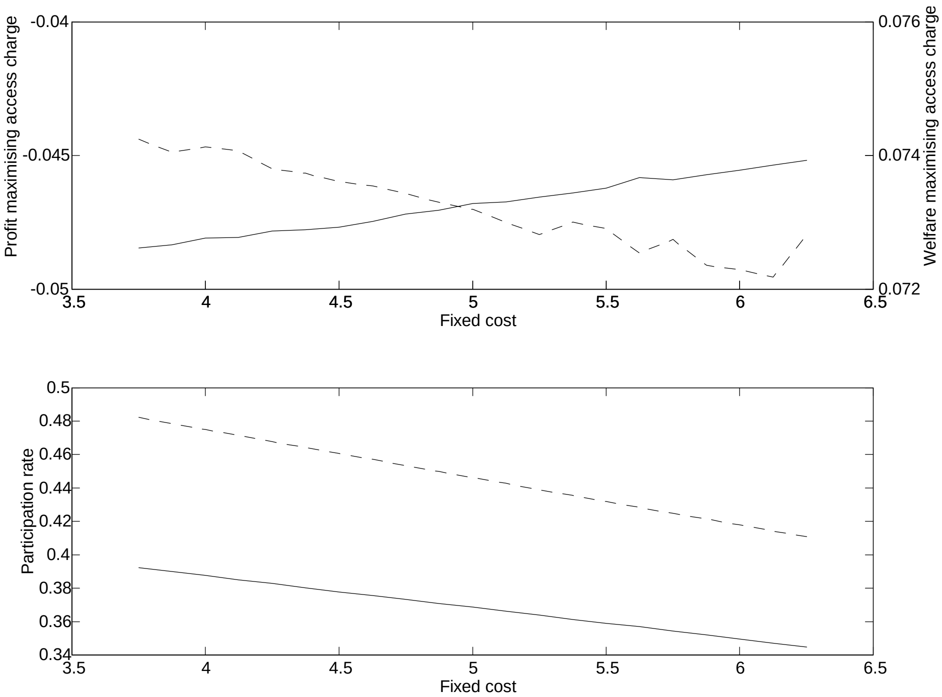

Figure 4: Profit maximising (solid line, left-hand scale) and welfare maximising (dashed line, right-hand scale) access charges and the associated participation rates for different fixed costs.

ing access charge increases towards marginal cost. The welfare maximizing access charge decreases, but remains above cost.

Figure 6 shows the effect of changing the Hotelling 'transportation cost parameter', t. As t increases the firms' products become more differentiated and the firms are less competitive. When t is low and firms are very competitive, the profit-maximizing access charge is low. As firms become less competitive, the profit maximizing access charge increases asymptotically towards marginal cost. This is because when the firms are highly differentiated there is less need for a below-cost access charge to soften retail competition through the channels discussed earlier. On the other hand, the socially optimal access charge decreases and eventually goes below cost

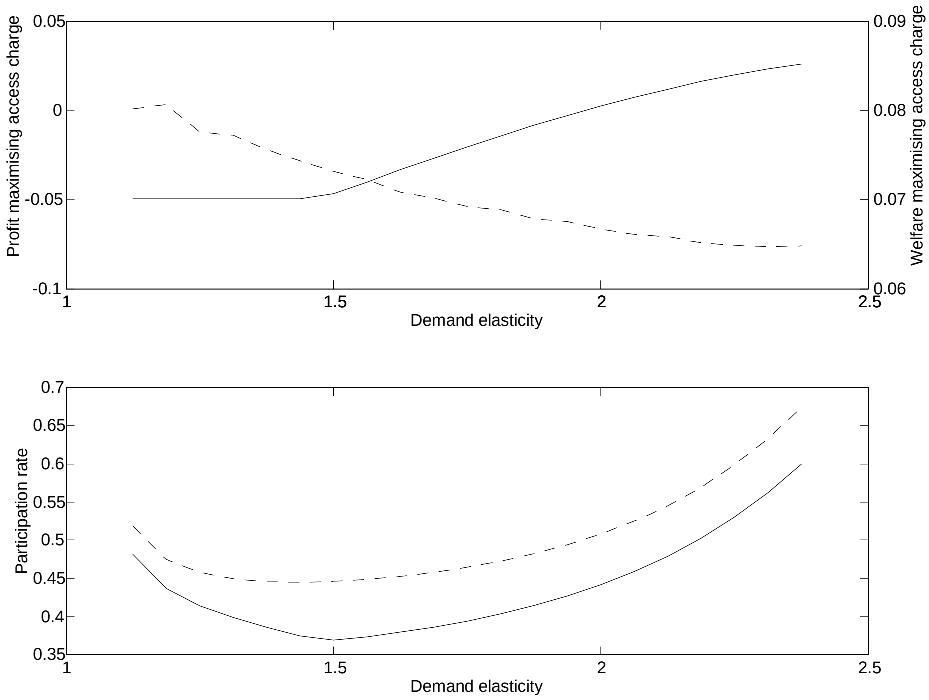

Figure 5: Profit maximising (solid line, left-hand scale) and welfare maximising (dashed line, right-hand scale) access charges and the associated participation rates for different demand elasticities (constant elasticity demand).

when firms are highly differentiated. This is because when firms are highly differentiated the Hotelling markups are a large source of welfare loss. To offset this, the firms need to be 'taxed' through a below-cost access charge.

Finally, figure 7 shows the effects of changing the mean of v_0, which is distributed according to a normal distribution. The main effect of changing the mean of v_0 is to change the equilibrium participation rate. When the mean of v_0 is low, the equilibrium participation rate is low and the profit-maximizing access charge is close to, but less than, marginal cost. As the mean of v_0 increases, the profit-maximizing access charge decreases, towards the limit discussed above. This is because, in the limit when the partici-

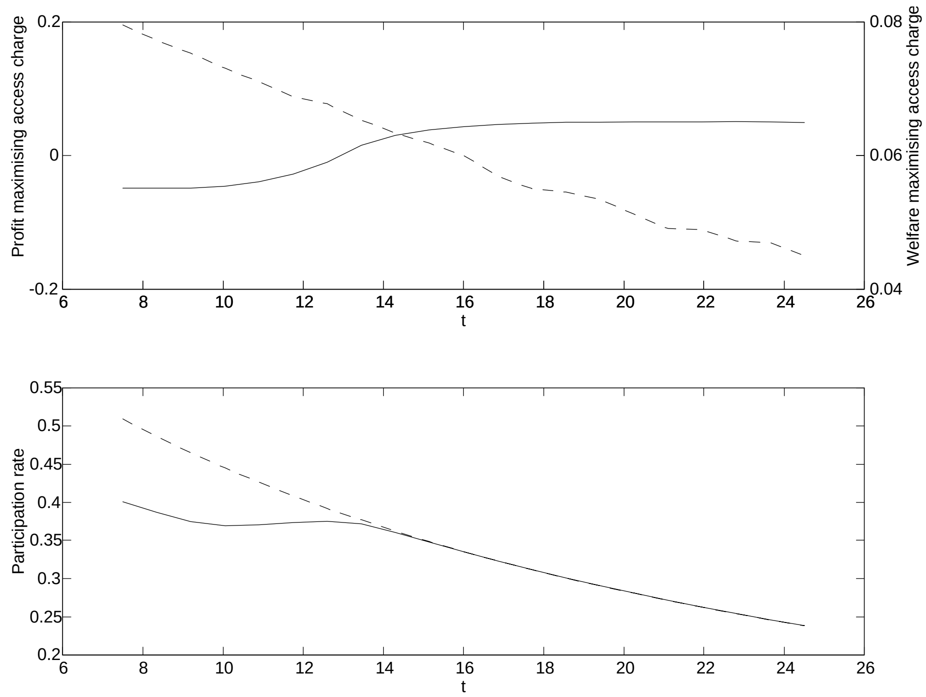

Figure 6: Profit maximising (solid line, left-hand scale) and welfare maximising (dashed line, right-hand scale) access charges and the associated participation rates for different values of the 'transportatation cost' parameter..

pation rate goes to one, equilibrium profits are independent of the access charge. The socially optimal access charge is initially above cost and rises as the mean of v_0 rises. At some point it starts to fall and the socially optimal access charge converges to marginal cost as the participation rate goes to one. This reflects the result that under full participation the socially optimal access charge is equal to marginal cost.

4 Conclusion

This paper has analyzed the implications of partial consumer participation on the standard model of competition in telecommunications markets where

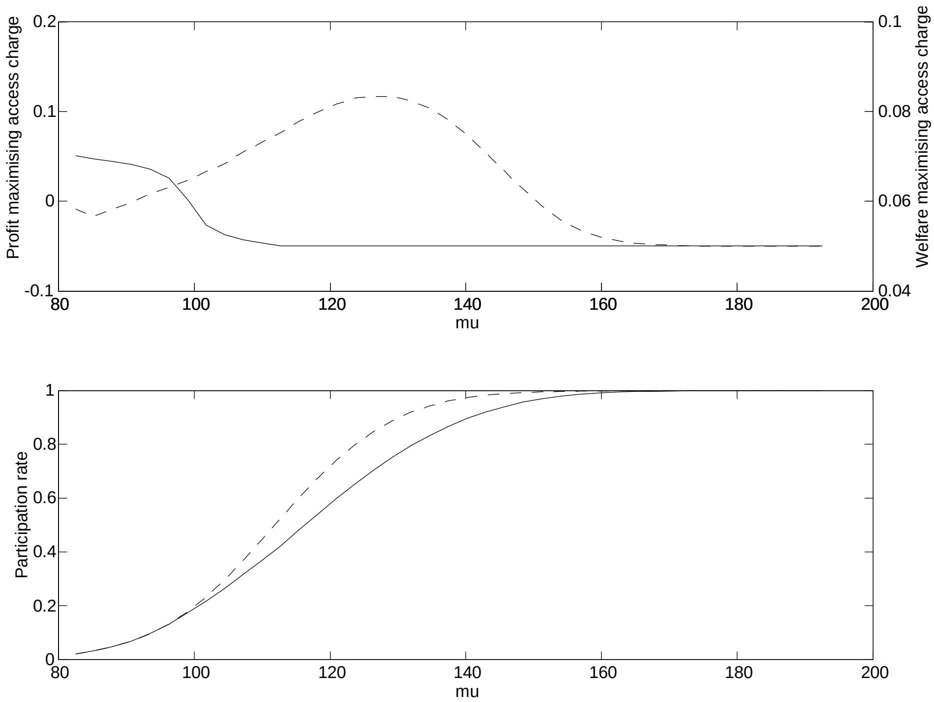

Figure 7: Profit maximising (solid line, left-hand scale) and welfare maximising (dashed line, right-hand scale) access charges and the associated participation rates for different values of the mean of the distribution of v_0 (normal distribution).

there is two-way interconnection. Partial participation introduces industry-wide network effects and means that firms must take account of the market expansion effects, as well as business stealing effects, of their pricing strategies.

It was first shown that an endogenous participation rate is crucial for the non-neutrality of the access charge. If the participation rate is fixed, the firms remain indifferent over the level of the access charge. On the other hand, it was shown that, under two-part retail pricing, in a fulfilled-expectations equilibrium, or if the network effects are ignored, firms prefer that the access charge is set equal to the marginal cost of termination. These results are in stark contrast with the full participation case where firms are indifferent over the level of the access charge and suggest that partial participation by consumers plays an important role in the determination of access prices by the firms.

In some sense, the fulfilled-expectations model provides a good approximation to the results of the unsimplified model, which was analyzed with the aid of a numerical simulation. In the fulfilled-expectations model, when firms are setting the access charge, they take account of the fact that changes in their prices will have a direct effect on the market participation rate. The unsimplified model includes this effect, plus an additional 'second-round' effect whereby the change in the participation rate will induce other consumers to enter or leave the market. This second-round effect makes the model difficult to solve analytically. However, the numerical results reveal that it leads firms to prefer that the access charge is set below the marginal cost of termination. This is not a general claim, but appears to hold for all the sets of parameter values that have been investigated. The reason seems to be that with partial participation the firms reduce equilibrium rentals by even more than the additional profit from calls generated by an above-cost access charge. This is because the firms know that when they cut their rentals the participation rate will be increased which will increase the quantity of calls made by all consumers. Competition in rentals is thus more intense than the full participation case, and a below-cost access charge is necessary to offset this. The simulation results also show that the socially optimal access charge may be above or below cost depending on the parameters of the model. It is typically above cost but converges to cost when the equilibrium participation rate is high. It may be below cost when the firms are highly differentiated.

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