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We consider a model of vertical quality differentiation. We show that in Cournot (quality setting) competition firm's profit is increasing in its own quality and decreasing in its rival's quality. This differs from the results for Bertrand (price setting) competition and conforms to some previously made assumptions concerning profit functions in a setting of vertical quality differentiation. However, even in this case, when an initial stage in which firms make as costly investment in quality is added, an asymmetric equilibrium results. This follows from the fact that in both types of competition, it is possible to improve profit by moving away (either by choosing higher or lower quality) from rival's quality. This paper is the same as manuscript dated 1988 of the same name.

Key words: vertical quality differentiation, Cournot and Bertrand competition, product innovation

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1 Introduction

We consider a model of two firms competing in a market for a good with vertically differentiated quality. (That is, given two different qualities one is unambiguously better than the other.) If the firms engage in Cournot (quantity setting) competition, a firm's equilibrium profit increases when its quality increases and decreases when its rival's quality increases. This is in contrast to the apparently paradoxical fact that in Bertrand competition a firm's profit is, in some of the quality space decreasing in its own quality and increasing in its rival's quality. In some sense the Cournot model seems to conform more closely to some intuition as to how equilibrium profit depends on quality.

Significant differences between models with vertical and horizontal quality differentiation have been demonstrated by Shaked and Sutton[1983]. In their model the two types of product quality differentiation may have different implications for the possible number of firms in the market. Gabszewicz and Thisse[1979] have examined a model of vertical quality differentiation and how profit depends on the quality of the firm's own product as well as that rival's when two firms engage in Bertrand (price setting) competition. Shaked and Sutton[1982] extended this analysis by adding two stages prior to the Bertrand subgame: a first stage where entry decisions are made and a willingness to pay for better quality among the consumers is bounded in a certain way, only two firms will enter the market in the perfect equilibrium. In the second stage, the two firms choose qualities that are distant from each other. This polarization of quality is a result similar to results obtained for models with horizontal quality differentiation (D'Aspremont, Gabszewicz and Thisse[1979], Neven[1985], Novshek[1980]). The result in the vertical quality model follows from an aspect of the profit function which is similar to the profit function of the horizontal quality models. As in the horizontal quality models, when the quality of the two firms approach each other (either an improvement of lower quality firm or worsening of the higher quality firm), profit for each firm approaches zero. This occurs because when the qualities are very close, the two products are almost homogeneous and the equilibrium is close to the Bertrand equilibrium with homogeneous products in which case each firm has zero profit. This occurs in both vertical and horizontal quality models.

In this paper we will see that the profit function is qualitatively different when firms engage in Cournot (quantity setting) competition. Cournot competition with vertical quality has been considered by Moorthy[5] when the quality and quantity choices are made simultaneously. Here, we will consider a two–stage game where quality choices are made and observed, and then quantity choices are made. By restricting ourselves to more explicit functional forms similar to those in D'Aspremont, Gabszewicz, and Thisse[1979] or Shaked and Sutton[1982,83], we are able to explicitly examine the profit functions as well as the equilibrium quality choices.

The monotonicity of Cournot profit function is significant in interpreting the R&D and innovation literature. The difference between cost reducing and product enhancing innovations was explicitly noted by Arrow[1962], but in the subsequent literature the difference or similarity of the two types of innovation has largely been neglected, although there seems to be consensus that both cost reduction and quality improvement increase profits and thus it was largely a matter of choice which type of innovation to consider.[1] However the results about vertical quality (Gabszewicz and two innovations have qualitatively different effects upon profits and known results about cost reducing innovation (Arrow[1962], Kamien and Tauman[1986], Katz and Shapiro[1985], Reinganum[1981], to name a few) have limited applicability to the study of product innovation. With what we show here about Cournot competition, we know that quality improving innovation in fact will have some of the same effects upon the profit of the producer as cost reduction. The results about cost reducing innovation may be relevant for the case of quality improving innovations as well.

We consider a model corresponding to the last two stages of Shaked and Sutton[1982]. There are two firms that are going to sell a product differentiated by vertical quality. In the first stage, firms make their quality choices, incurring a cost for choosing higher quality. The quality choices become common knowledge at the end of this stage. In the second stage firms sell to consumers that differ in their willingness to pay. Our primary interest is to consider a case where this subgame is a Cournot game. We also analyze the game where the subgame is a Bertrand (sub)game and compare the perfect equilibria for the two games.

The perfect equilibrium is characterized by solving the model backwards. Given two qualities, we find the Cournot and Bertrand equilibrium strategies for each subgame as function of the qualities. The payoff or profit of the second stage as function of the two qualities is defined by these equilibrium strategies. We find that this function is non–monotonic in the Bertrand subgame case, while in the Cournot subgame, the profit is monotonically increasing in firm's own quality and decreasing in the quality of the rival.

Next we explore the implication this Cournot profit function has for the quality choice. In the Bertrand game it is clearly not the case that there is a symmetric equilibrium because by choosing quality below that of the rival, a firm can increase profit while simultaneously investing less in quality. However in the Cournot case, profit is increasing in firm's own quality, independent of the rival's quality, raising at least the possibility of a symmetric equilibrium. However this is not the case. A discontinuity in the firm's marginal profit function at the point at which the two qualities are equal implies that it is never a best response for a firm to choose the same quality as its rival. Thus in spite of some qualitative differences in the equilibrium profit function the equilibrium quality choice displays the same features as in models of horizontal quality or Bertrand competition. That is, the two firms will never choose the same quality (Hotelling[1928], Gabszewicz and Thisse[1979], Osborne and Pitchik[1987]).

The main purpose of this paper is to examine the Cournot model and to compare the equilibrium of that model with the equilibrium of the Bertrand model. Our analysis of the Bertrand model confirms the results of earlier work (Gabszewicz and Thisse[1979], Shaked and Sutton[1982]) although the consumers are modeled slightly differently here.

The full model is presented in the next section. The equilibrium strategies for both the Cournot and Bertrand subgames are characterized in Section 3. The profits in equilibrium as functions of qualities is derived there also. In Section 4, we characterize the equilibrium quality choices for the Cournot and Bertrand games using those profit functions. Section 5 includes the concluding remarks.

2 The Model

There are two, initially identical, firms which we will call firm 1 and firm 2. The firms compete in a two stage game. In the first stage the firms simultaneously undertake investment which determines the quality of the product to be sold in the second stage. The quality of firm i is represented by the quality index $q_i \in [0,\infty)$. We assume that the relationship between investment and quality is deterministic and strictly increasing. Thus we may think of the firm as choosing its quality q and deterministically incurring the cost C(q). At the end of the first stage the quality choice becomes common knowledge.

In the second stage, with their qualities determined, the firms sell to a heterogeneous set of consumers. We assume that the quality differences are vertical, that is, that consumers agree on which of any two qualities is better, though they may differ in their willingness to pay for the higher quality. Consumers who are willing to pay more for the good are assumed to also be more willing to pay for an increase in quality. More specifically, we assume that the consumers are uniformly distributed over the interval [0,1]. For consumer $t \in [0,1]$ we let $r(t,q)$ be his reservation price for one unit of the good of quality q. When more than one quality is available consumer t chooses the quality which maximizes $r(t,q) - p_q$, where p_q is the price of a unit of the good with quality q, as long as this maximum is positive. We assume that the reservation price has the particular functional form $r(t,q) = \beta(q)t$, where $\beta(\cdot)$ is a differentiable fucntion, $\beta(q) > 0$ and $\beta'(q) > 0$ for all $q \in [0,\infty)$.

We consider two different games, one in which the firms' strategies in the second stage are to choose quantities and one in which their strategies are to set prices. In the case that the firms' strategies are to choose quantities we call the subgame which follows the firms' quality choices the Cournot subgame, and in the case that the strategies are to choose prices we call the subgame the Bertrand subgame. The entire two stage game will be named according to the form of the second stage. Thus a strategy for a firm will be a pair consisting of a quality choice and a function which gives the quantity or price choice in the second stage as a function of the quality choice of the other firm.

Because it significantly simplifies the analysis we assume that the marginal cost of production is zero. It is possible to get qualitatively similar results if the marginal cost is constant and independent of quality.

In the next section we calculate the equilibrium for the Cournot subgame and the Bertrand subgame and characterize the equilibrium as a function of the qualities chosen in the first stage. In Section 4 we use these results to find the subgame perfect equilibrium of the entire game.

3 Cournot and Bertrand Competition – Second Stage

In this section we examine the subgames that follow the firms' choice of qualities, that is, we take the qualities as given. It will be convenient to index the qualities not by the particular firm which chose that quality but by the levels of the quality. Thus we consider two qualities q_L and q_H with $q_H \geq q_L$. Also we let $\beta_i = \beta(q_i)$ and $v_i(t,p_i) = r(t,q_i) - p_i$ for $i = L,H$, where p_i is the price of the product of quality q_i. Let x_i be the quantity demanded of the product of quality q_i. Restating the consumers' behavior given in Section 2 we have: consumer t will purchase the product of quality q_i if $v_i(t,p_i) > 0$ and $v_i(t,p_i) > v_j(t,p_j)$ for $j \neq i$. The weak versions of these inequalities will affect only a zero measure of consumers (actually one for each inequality) and so we need not be concerned with them.

Now, let us define $t_i(p_i)$ by $v_i(t_i(p_i),p_i) = 0$, or, equivalently $\beta_i t_i(p_i) = p_i$. For the case in which $1 \geq t_H(p_H) > t_L(p_L) \geq 0$, we also define $\hat{t}(p_H,p_L)$ by $v_H(\hat{t}(p_H,p_L),p_H) = v_L(\hat{t}(p_H,p_L),p_L)$. Thus $\hat{t}(p_H,p_L)$ is the point at which the lines $v_H(t,p_H)$ and $v_L(t,p_L)$ intersect. This is shown in Figure 1. All consumers to the right of $\hat{t}(p_H,p_L)$ will purchase q_H while those between $t_L(p_L)$ and $\hat{t}(p_H,p_L)$ will purchase q_L. Because of our assumption that the consumers are uniformly distributed with total mass 1 the quantities demanded, x_L and x_H, are given by the lengths of the intervals $[t_L(p_L),\hat{t}(p_H,p_L)]$ and $[\hat{t}(p_H,p_L),1]$ respectively. This allows us to characterize the demand functions $x_H(p_H,p_L)$ and $x_L(p_H,p_L)$ and the inverse demand functions $p_H(x_H,x_L)$ and $p_L(x_H,x_L)$. We do this in the following two lemmas.

LEMMA 1:

$x_H(p_H,p_L)$

$$= \text{Min}\{\, 1-p_H/\beta_H \,,\, 1-(p_H-p_L)/(\beta_H-\beta_L) \,\} \qquad \text{for } p_H < \text{Min}\{\, \beta_H+p_L-\beta_L, \beta_H \,\}$$

$$= 0 \qquad \text{for } p_H \geq \text{Min}\{\, \beta_H+p_L-\beta_L \,,\, \beta_H \,\}$$

$x_L(p_H,p_L)$

$= \text{Min}\{\ 1-p_L/\beta_L,\ (p_H-p_L)/(\beta_H-\beta_L)-p_L/\beta_L\ \}$ for $p_L < \text{Min}\{\ \beta_L p_H/\beta_H,\ \beta_L\ \}$

$= 0$ for $p_L \geq \text{Min}\{\ p_H\beta_L/\beta_H\ ,\ \beta_L\ \}$.

Proof: First we calculate x_H. For $p_L \geq \beta_L$ no consumer values q_L non–negatively. So as long as the price of q_H is low enough, namely $p_H < \beta_H$, consumers on $[t_H(p_H),1]$ all buy q_H and the demand is $1 - t_H(p_H)$. If $p_H \geq \beta_H$, no consumer values q_H non–negatively and demand is zero.

For $p_L < \beta_L$, consumers on $[t_L(p_L),1]$ value q_L non–negatively. If $\frac{p_H}{\beta_H} < \frac{p_L}{\beta_L}$, $[t_L(p_L),1] \subset [t_H(p_H),1]$ but those on $[t_L(p_L),1]$ value q_H higher. Thus all consumers on $[t_H(p_H),1]$ buy q_H. If $\frac{p_H}{\beta_H} \geq \frac{p_L}{\beta_L}$ but $p_H < \beta_H - \beta_L + p_L$, then consumers on $[t_L(p_L),1]$ value at least one of the products non–negatively. Those on $[\hat{t}(p_H,p_L),1]$ value q_H more, thus demand is $1 - \hat{t}(p_H,p_L)$. Demand is zero when the price is too high, $p_H \geq \beta_H - \beta_L + p_L$.

Now we calculate x_L. For $p_H \geq \beta_H$, the price of p_H is too high and if $p_L < \beta_L$, all consumers on $[t_L(p_L),1]$ will buy q_L. When the price of q_L is also too high, i.e., $p_L \geq \beta_L$, then the demand for q_L is zero.

For the case $p_H < \beta_H$, if $\beta_L > \beta_H - p_H$, then it is possible to charge price p_L such that $p_L < p_H - \beta_H + \beta_L$. Although consumers on $[t_H(p_H),1]$ value q_H non–negatively, $[t_H(p_H),1] \subset [t_L(p_L)\ 1]$ and those on $[t_H(p_H),1]$ value q_L more. Thus consumers on $[t_L(p_L),1]$ all buy q_L and demand is $1 - t_L(p_L)$. If p_H is such that there can be no p_L to satisfy this condition, then demand takes only one of the next two forms. In case $\frac{\beta_L p_H}{\beta_H} > p_L \geq p_H - \beta_H + \beta_L$, those on $[t_L(p_L),1]$ value one of the two products non–negatively but those on $[t_L(p_L),\ \hat{t}(p_H,p_L)]$ value q_L more. Thus demand is $\hat{t}(p_H,p_L) - t_L(p_L)$. If p_L is so high that $\frac{\beta_L p_H}{\beta_H} \leq p_L$, then even those that value q_L non–negatively value q_H more. □

We can characterize $p_H(x_H,x_L)$ and $p_L(x_H,x_L)$ as below.

LEMMA 2: $p_H(x_H,x_L) = (\beta_H-\beta_L)(1-x_H) + [\beta_L(1-x_H-x_L)]^+$ and $p_L(x_H,x_L) = [\beta_L(1-x_H-x_L)]^+$.[2]

Proof. Consider the case $x_H + x_L \leq 1$, $x_H > 0$ and $x_L > 0$. By solving the two relations $1 - x_H = \hat{t}(p_H,p_L)$ and $1 - x_H - x_L = t_L(p_L)$ for p_H and p_L, we have $p_H = \beta_H(1-x_H) - \beta_L x_L$ and $p_L = \beta_L(1 - x_H - x_L)$. This is the value of the expressions in the Lemma for this case. When $x_H + x_L > 1$, price of q_L must drop to zero. The price of q_H is determined by $1 - x_H = \hat{t}(p_H,0)$ which is equal to $p_H = (\beta_H - \beta_L)(1 - x_H)$. In the cases $x_H = 0$ or $x_L = 0$, there is actually indeterminacy of prices. $p_H(0,x_L)$ may be any p_H such that $p_H \geq \beta_H-\beta_L + p_L(0,x_L)$ and $p_L(x_H,0)$ may be any price such that $p_L \geq \beta_L - \beta_H + p_H(x_H,0)$. □

A. Equilibrium of the Cournot Subgame

When firms set quantities as strategies, the market clearing price is determined according to the functions given in Lemma 2. Profit of each firm as functions of quantities is given by the following.

$$
\begin{aligned}
p_H(x_H,x_L)\,x_H &= \tilde{\pi}^{H1}(x_H,x_L) \equiv x_H\,(\beta_H - \beta_H x_H - \beta_L x_L) && x_H + x_L \leq 1.\\
&= \tilde{\pi}^{H2}(x_H,x_L) \equiv x_H\,(\beta_H - \beta_L)(1 - x_H) && x_H + x_L > 1.\\
p_L(x_H,x_L)\,x_L &= \tilde{\pi}^{L}(x_H,x_L) \equiv \beta_L\, x_L\,(1 - x_H - x_L) && x_H + x_L \leq 1.
\end{aligned}
$$

Profit of the lower quality firm when it produces x_L and $x_L+x_H > 1$ is zero since price will be zero. The maximum of $\tilde{\pi}^L(x_H,x_L)$ will occur at $x_L = (1-x_H)/2 < 1-x_H$ for any x_H. The maximum value will always be positive. Thus the reaction function $x_L{}^c(x_H) = \frac{1-x_H}{2}$. Given the quantity for the high quality firm, the low quality firm will sell to exactly half of the consumers that do not buy from the high quality firm.

The maximum of $\tilde{\pi}^{H2}(x_H,x_L)$ occurs at $x_H = 1/2$ for any x_L and the maximized value is $(\beta_H-\beta_L)/4$. Maximum of $\tilde{\pi}^{H1}(x_H,x_L)$ occurs at $x_H = (\beta_H-\beta_L x_L)/2\beta_H$ and the maximized value is $(\beta_H-\beta_L x_L)^2/(4\beta_H)$ which is decreasing in x_L for $x_L \leq 1$. Define $\tilde{x}_L$ as the solution (which is less than 1) of $(\beta_H-\beta_L x_L)^2/(4\beta_H) = (\beta_H-\beta_L)/4$. Thus high quality firm's maximum is on the $\tilde{\pi}^{H1}(x_H,x_L)$ portion for $x_L < \tilde{x}_L$ and on the $\tilde{\pi}^{H2}(x_H,x_L)$ portion for $x_L > \tilde{x}_L$. When $x_L = \tilde{x}_L$, there are two maximizers. Thus the reaction function for the high quality firm is

$$x_H{}^c(x_L) = \frac{\beta_H-\beta_L x_L}{2\beta_H} \qquad x_L \leq \tilde{x}_L,$$
$$= 1/2 \qquad x_L \geq \tilde{x}_L.$$

When the quantity of the low quality firm is large ($x_L > \tilde{x}_L$), the high quality firm will dump $x_H = 1/2$ on the market and drive the price of q_L to zero. The intersection of the two reaction functions will occur on the $\frac{\beta_H-\beta_L x_L}{2\beta_H}$ portion of $x_H{}^c(\cdot)$ because $x_L{}^c(\cdot)$ is under the $x_H = 1/2$ line (Figure 2). The Cournot equilibrium quantities, (x_H^c, x_L^c) and the corresponding prices are,

$$x_H^c = \frac{2\beta_H - \beta_L}{4\beta_H - \beta_L}, \qquad p_H^c = \frac{\beta_H (2\beta_H - \beta_L)}{4\beta_H - \beta_L},$$
$$x_L^c = \frac{\beta_H}{4\beta_H - \beta_L}, \qquad p_L^c = \frac{\beta_L\beta_H}{4\beta_H - \beta_L}.$$

The firm with the higher quality will always sell more and charge a higher price.

LEMMA 3: The Cournot equilibrium profit functions are,

$$\pi^{cH}(q_H,q_L) = \beta_H \left[\frac{2\beta_H - \beta_L}{4\beta_H - \beta_L} \right]^2 \text{ and } \pi^{cL}(q_H,q_L) = \beta_L \left[\frac{\beta_H}{4\beta_H - \beta_L} \right]^2 .$$

Remember that β_i is a function of q_i. Marginal profits are easily calculated using the notation $\beta_i' \equiv \beta'(q_i) > 0$. Let $\pi^{ci}{}_j(q_H,q_L) \equiv \frac{\partial}{\partial q_j}\pi^{ci}(q_H,q_L)$, i,j = H,L and higher derivatives are defined similarly.

LEMMA 4:

$$\pi^{cH}{}_H(q_H,q_L) = \frac{\beta_H' \, (2\beta_H - \beta_L)}{(4\beta_H - \beta_L)^3}(8\beta_H^2 - 2\beta_H\beta_L + \beta_L^2) > 0,$$

$$\pi^{cH}{}_L(q_H,q_L) = \frac{-4\beta_H{}^2\beta_L' \, (2\beta_H - \beta_L)}{(4\beta_H - \beta_L)^3} < 0,$$

$$\pi^{cL}{}_H(q_H,q_L) = \frac{-2\beta_H'\beta_L{}^2\beta_H}{(4\beta_H - \beta_L)^3} < 0,$$

$$\pi^{cL}{}_L(q_H,q_L) = \frac{\beta_L'\beta_H{}^2 \, (4\beta_H + \beta_L)}{(4\beta_H - \beta_L)^3} > 0.$$

Profit is increasing in the firm's own quality and decreasing in the quality of the rival. The cross derivatives are given in the following lemma. Let $\beta_i'' \equiv \beta''(q_i)$.

LEMMA 5:

$$\pi^{cH}{}_{HL}(q_H,q_L) = \pi^{cH}{}_{LH}(q_H,q_L) = \frac{8\beta_H'\beta_L'\beta_H\beta_L \, (\beta_H - \beta_L)}{(4\beta_H - \beta_L)^4} > 0,$$

$$\pi^{cH}{}_{LL}(q_H,q_L) = \frac{-4\beta_H{}^2\beta_L''(2\beta_H - \beta_L)}{(4\beta_H - \beta_L)^3} - \frac{8\beta_H{}^2\beta_L'{}^2(\beta_H - \beta_L)}{(4\beta_H - \beta_L)^4},$$

$$\pi^{cH}{}_{HH}(q_H,q_L) = \frac{\beta_H''(2\beta_H - \beta_L)(8\beta_H{}^2 - 2\beta_H\beta_L + \beta_L{}^2)}{(4\beta_H - \beta_L)^3} - \frac{8\beta_L{}^2\beta_H'{}^2(\beta_H + \beta_L)}{(4\beta_H - \beta_L)^4},$$

$$\pi^{cL}{}_{HL}(q_H,q_L) = \pi^{cL}{}_{LH}(q_H,q_L) = \frac{-2\beta_H'\beta_L'\beta_H\beta_L\,(8\beta_H + \beta_L)}{(4\beta_H - \beta_L)^4} < 0,$$

$$\pi^{cL}{}_{LL}(q_H,q_L) = \frac{\beta_H{}^2(4\beta_H + \beta_L)\beta_L''}{(4\beta_H - \beta_L)^3} + \frac{\beta_L'{}^2(16\beta_H + 2\beta_L)\beta_H{}^2}{(4\beta_H - \beta_L)^4},$$

$$\pi^{cL}{}_{HH}(q_H,q_L) = \frac{-2\beta_L{}^2\beta_H\beta_H''}{(4\beta_H - \beta_L)^3} + \frac{2\beta_L{}^2\beta_H'{}^2(8\beta_H + \beta_L)}{(4\beta_H - \beta_L)^3}.$$

We can sign $\pi^{cL}{}_{HH}(q_H,q_L)$ and $\pi^{cH}{}_{HH}(q_H,q_L)$ if $\beta''(q) \leq 0$. We may sign all the cross and second derivatives if $\beta''(q) = 0$. The "horizontal effect" appears at the margin as summarized in the following theorem.

THEOREM 1: In the equilibrium of the Cournot subgame,

(i) The profit of a firm is increasing in its own quality and decreasing in that of its rival.

(ii) If $\beta(q)$ is linear in q, then the marginal profit with respect to its own quality increases (decreases) as qualities become closer (more distant). The marginal loss with respect to the rival's quality increases (decreases) as the qualities become closer (more distant).

B. Equilibrium of the Bertrand Subgame

Using Lemma 1, we obtain the profit functions $\tilde{\pi}^i(p_h,p_L)$ i = H,L in prices. For convenience, let us define two forms of profit functions for each quality, corresponding to the different segments of the demand functions.

$$\tilde{\pi}^{H1}(p_H,p_L) = p_H\left(1-\frac{p_H}{\beta_H}\right),$$
$$\tilde{\pi}^{H2}(p_H,p_L) = p_H\left(1-\frac{p_H - p_L}{\beta_H - p_H}\right),$$
$$\tilde{\pi}^{L1}(p_H,p_L) = p_L\left(1-\frac{p_L}{\beta_L}\right),$$
$$\tilde{\pi}^{L2}(p_H,p_L) = p_L\left(\frac{p_H - p_L}{\beta_H - p_H}-\frac{p_H}{\beta_H}\right).$$

Using these, we obtain

$$\begin{aligned}
\tilde{\pi}^{H}(p_H,p_L) &= \tilde{\pi}^{H1}(p_H,p_L) && p_H \le \mathrm{Min}\{\, \beta_H,\, \beta_H p_L/\beta_L \,\} \\
&= \tilde{\pi}^{H2}(p_H,p_L) && \mathrm{Min}\{\, \beta_H, \beta_H p_L/\beta_L \,\} < p_H < \mathrm{Min}\{\, \beta_H,\, \beta_H - \beta_L + p_L \,\} \\
&= 0 && p_H \ge \mathrm{Min}\{\, \beta_H,\, \beta_H - \beta_L + p_L \,\}.
\end{aligned}$$

$$\begin{aligned}
\tilde{\pi}^{L}(p_H,p_L) &= \tilde{\pi}^{L1}(p_H,p_L) && p_L \le \mathrm{Min}\{\, \beta_L,\, \beta_L + p_H - \beta_H \,\} \\
&= \tilde{\pi}^{L2}(p_H,p_L) && \mathrm{Min}\{\, \beta_L,\, \beta_L + p_H - \beta_H \,\} < p_L < \mathrm{Min}\{\, \beta_L,\, \beta_L p_H/\beta_H \,\} \\
&= 0 && p_L \ge \mathrm{Min}\{\, \beta_L,\, \beta_L p_H/\beta_H \,\}.
\end{aligned}$$

The reaction functions $p_H^b(p_L)$ and $p_L^b(p_H)$ for the extreme cases are obvious: for $p_L \geq \beta_L$, $p_H^b(p_L) = \arg\max \tilde{\pi}^{H1}(p_H,p_L) = \frac{\beta_H}{2}$, and for $p_H \geq \beta_H$, $p_L^b(p_H) = \arg\max \tilde{\pi}^{L1}(p_H,p_L) = \frac{\beta_L}{2}$. For cases where the rival's price is too high for any consumers to buy, a firm will set its price independent of that of the rival. These are actually monopoly prices.

For $p_L < \beta_L$, the profit function is a combination of $\tilde{\pi}^{H1}(p_H,p_L)$, $\tilde{\pi}^{H2}(p_H,p_L)$ and 0. The reaction function is summarized as,

$$p_H^b(p_L) = \begin{cases} \dfrac{\beta_H + p_L - \beta_L}{2} & \dfrac{p_L\beta_H}{\beta_L} < \dfrac{\beta_H + p_L - \beta_L}{2}, \\ \dfrac{p_L\beta_H}{\beta_L} & \dfrac{\beta_H + p_L - \beta_L}{2} \leq \dfrac{p_L\beta_H}{\beta_L} < \dfrac{\beta_H}{2}, \\ \dfrac{\beta_H}{2} & \dfrac{\beta_H}{2} \leq \dfrac{p_L\beta_H}{\beta_L} < \beta_H. \end{cases}$$

The reaction function of the low quality firm can be found in a similar fashion.

$$p_L^b(p_H) = \begin{cases} \dfrac{\beta_L p_H}{2\beta_H} & p_H - \beta_H + \beta_L < \dfrac{\beta_L p_H}{2\beta_H}, \\ p_H - \beta_H + \beta_L & \dfrac{\beta_L p_H}{2\beta_H} \leq p_H - \beta_H + \beta_L < \dfrac{\beta_L}{2}, \\ \dfrac{\beta_L}{2} & \dfrac{\beta_L}{2} \leq p_H - \beta_H + \beta_L < \beta_L. \end{cases}$$

Both reaction functions are increasing. However there always is an intersection between the first portions of the reaction functions (Figure 3). Thus the Bertrand equilibrium prices are given as solutions to the system of equations,

$$p_H = \frac{\beta_H + p_L - \beta_L}{2} \quad \text{and} \quad p_L = \frac{\beta_L p_H}{2\beta_H}.$$

The Bertrand equilibrium prices (p_H^b, p_L^b) and the corresponding quantities are,

$$x_H^b = \frac{2\beta_H}{4\beta_H - \beta_L}, \qquad p_H^b = \frac{2\beta_H\,(\beta_H - \beta_L)}{4\beta_H - \beta_L},$$
$$x_L^b = \frac{\beta_H}{4\beta_H - \beta_L}, \qquad p_L^b = \frac{\beta_L\,(\beta_H - \beta_L)}{4\beta_H - \beta_L}.$$

Again, the firm with better quality sells more at higher price. But for the same quality configuration, prices are higher in Cournot than they are in Bertrand and more of the higher quality is sold in Bertrand. Sale of the low quality good is the same in both subgames.

LEMMA 6: The Bertrand equilibrium profits are,

$$\pi^{bH}(q_H, q_L) = \frac{4\beta_H^2\,(\beta_H - \beta_L)}{(4\beta_H - \beta_L)^2} \quad \text{and} \quad \pi^{bL}(q_H, q_L) = \frac{\beta_H\beta_L\,(\beta_H - \beta_L)}{(4\beta_H - \beta_L)^2}.$$

LEMMA 7: The derivatives of the profit functions are

$$\pi^{bH}{}_H(q_H, q_L) = \frac{4\beta_H'\beta_H}{(4\beta_H - \beta_L)^3}\left[4\beta_H^2 - 3\beta_H\beta_L + 2\beta_L^2\right] > 0,$$
$$\pi^{bH}{}_L(q_H, q_L) = \frac{-4\beta_H^2\beta_L'\,(2\beta_H + \beta_L)}{(4\beta_H - \beta_L)^3} < 0,$$
$$\pi^{bL}{}_H(q_H, q_L) = \frac{\beta_H'\beta_L^2\,(2\beta_H + \beta_L)}{(4\beta_H - \beta_L)^3} > 0,$$
$$\pi^{bL}{}_L(q_H, q_L) = \frac{\beta_H^2\beta_L'\,(4\beta_H - 7\beta_L)}{(4\beta_H - \beta_L)^3}\begin{bmatrix} < \\ = \\ > \end{bmatrix} 0 \iff \frac{4}{7}\beta_H \begin{bmatrix} < \\ = \\ > \end{bmatrix} \beta_L.$$

For completeness and use in the later section, we list the higher derivatives. They follow from simple differentiation.

LEMMA 8:

$$\pi^{bH}_{HH}(q_H,q_L) = \frac{-8(\beta_H')^2\beta_L^{\,2}}{(4\beta_H - \beta_L)^4}(5\beta_H + \beta_L) + \frac{4\beta_H''\beta_H}{(4\beta_H - \beta_L)^3}\left[4\beta_H^{\,2} - 3\beta_H\beta_L + 2\beta_L^{\,2}\right] < 0,$$

$$\pi^{bH}_{HL}(q_H,q_L) = \pi^{bH}_{LH}(q_H,q_L) = \frac{8\beta_H'\beta_L'\beta_H\beta_L(5\beta_H + \beta_L)}{(4\beta_H - \beta_L)^4} > 0,$$

$$\pi^{bL}_{LL}(q_H,q_L) = \frac{-2\beta_H^2(\beta_L')^2}{(4\beta_H - \beta_L)^4}(8\beta_H + 7\beta_L) + \frac{\beta_L''\beta_H^{\,2}(4\beta_H - 7\beta_L)}{(4\beta_H - \beta_L)^3} < 0,$$

$$\pi^{bL}_{LH}(q_H,q_L) = \pi^{bL}_{HL}(q_H,q_L) = \frac{2\beta_H'\beta_L'\beta_H\beta_L(8\beta_H + 7\beta_L)}{(4\beta_H - \beta_L)^4} > 0.$$

We can summarize the results in the following theorem. It coincides with results in [4,10].

THEOREM 2: In the equilibrium of the Bertrand subgame,

(i) Prices and profits tend to zero as the two qualities become closer.

(ii) The high quality firm's profit is increasing in its own quality and decreasing in that of the rival.

(iii) An improvement in the rival's quality is profitable for the low quality firm. An improvement of its own quality is profitable if the two qualities are far apart ($4\beta_H > 7\beta_L$) and unprofitable if two qualities are very similar ($4\beta_H < 7\beta_L$).

We see that the improvement of its own quality is not always profitable for the firm. When the two qualities are close ($4\beta_H < \beta_L$), it is more profitable if the two qualities become further apart. This implies that for the low quality firm, the profit increases when its own quality gets worse. It is always profitable for the high quality firm if the quality becomes further apart. However, if the qualities are very different ($4\beta_H > \beta_L$), the low quality firm is able to exploit greater local monopoly power among the consumers with low evaluation and the profit becomes an increasing function of its own quality in that range.

Compared to the Cournot profits (Theorem 1), the most striking feature of the Bertrand profits (Theorem 2) is the non–monotonicity. Note also that for the same configuration of qualities, both low and high quality firms have larger profits in Cournot than in Bertrand markets. Consumers are better off with Bertrand competition than with Cournot since more consumers buy the high quality product at lower price and just as many consumers buy low quality with Bertrand than with Cournot also at a lower price. The efficiency advantage of Bertrand competition over Cournot competition is a familiar one from other models of product differentiation[12].

4 Quality Investment Decision – First Stage

Anticipating the profit functions of Theorem 1 or 2 in the second stage, firms make their quality choice in the first stage. We assume the two firms incur the same cost for choosing a quality q, given by $C(q) = kq^2$ where k is a positive constant, thus $C'(q) > 0$ and $C''(q) > 0$. The payoff of a firm from the second stage is the profit derived in the previous section. The total payoff from the whole game is the payoff from the second stage less the cost of the quality investment. Firm i maximizes $\pi^c(q_i,q_j) - C(q_i)$ in the Cournot game and $\pi^b(q_i,q_j) - C(q_i)$ in the Bertrand game. Using the notation of preceding sections,

$$\pi^{c}(q_1,q_2) = \pi^{cL}(q_2,q_1) \qquad q_1 < q_2,$$
$$= \lim_{q_1 \to q_2} \pi^{cL}(q_1,q_2) \qquad q_1 = q_2,$$
$$= \pi^{cH}(q_1,q_2) \qquad q_1 > q_2.$$
$$q_1^{\,c}(q_2) \equiv \arg\max_{q_1} \pi^{c}(q_1,q_2) - C(q_1).$$

$$\pi^{b}(q_1,q_2) = \pi^{bL}(q_2,q_1) \qquad q_1 < q_2,$$
$$= \lim_{q_1 \to q_2} \pi^{bL}(q_1,q_2) \qquad q_1 = q_2,$$
$$= \pi^{bH}(q_1,q_2) \qquad q_1 > q_2.$$
$$q_1^{\,b}(q_2) \equiv \arg\max_{q_1} \pi^{b}(q_1,q_2) - C(q_1).$$

Profits for the case $q_1 = q_2$ are equal to the profit functions of Cournot and Bertrand competition with homogeneous products and zero marginal cost. We assume $\beta(q) = q$, so $\beta_H = q_H$ and $\beta_L = q_L$. Since the firms are identical, we construct here only the reaction functions $q_1^{\,c}(q_2)$ and $q_1^{\,b}(q_2)$ for firm 1. The reaction functions of firm 2, $q_2^{\,i}(q_1)$, i=c,b, will have identical forms.

Properties of the reaction function in the Cournot case are summarized in the following two Lemmas.

LEMMA 9: (1) $q_1^{\,c}(0) = 1/(8k)$.

(2) $q_1^{\,c}(q_2) > q_2$ for $q_2 \leq 1/(18k)$.

(3) $q_1^{\,c}(q_2) < 1/(9k) \leq q_2$ $q_2 \geq 1/(9k)$.

(4) $q_1^{\,c}(q_2) \to 0$ as $q_2 \to \infty$.

(5) $q_1^c(q_2)$ is strictly increasing for $q_1^c(q_2) > q_2$ and
$q_2^c(q_1)$ is strictly decreasing for $q_1^c(q_2) < q_2$.

Proof: (1) $q_1^c(0) = \arg\max_q q/4 - kq^2 = 1/(8k)$.
(2) For any q_2, $\pi^{cH}(q_2,q_2) = \pi^{cL}(q_2,q_2) = q_2/9$. From monotonicity, $\pi^{cL}(q_2,q_1) < q_1/9$ for all $q_1 < q_2$, and $\pi^{cH}(q_1,q_2) > q_1/9$ for all $q_1 > q_2$. Note that $1/(18k) = \arg\max_q q/9 - C(q)$. Thus for any $q_1 < q_2 \le 1/(18k)$, we have the following relations: $\max_{0\le q_1\le q_2} \pi^{cL}(q_2,q_1) - C(q_1) <$ $\max_{0\le q_1\le q_2} q_1/9 - C(q_1) \le \max_{q_2\le q_1\le\infty} q_1/9 - C(q_1) \le \max_{q_2\le q_1\le\infty} \pi^{cH}(q_1,q_2) - C(q_1)$. Thus $\max_{0\le q_1\le\infty} \pi^c(q_1,q_2) - C(q_1) = \max_{0\le q_1\le q_2} \pi^{cH}(q_1,q_2) - C(q_1)$, which implies $q_1^c(q_2) > q_2$ for $q_2 \le 1/(18k)$.
(3) Since $\pi^{cL}(1/(9k),(1/9k)) = \pi^{cH}(1/(9k),(1/9k)) = 1/(9k)$ and $\pi^{cH}_{HH}(q_1,q_2) < 0$, we have $\pi^{cH}(q_1,q_2) < C(q_1)$ for all $q_1 > q_2 \ge 1/(9k)$. Thus $q_1^c(q_2) = \arg\max_q \pi^{cL}(q_2,q) - C(q)$. Since $C(q) \lesseqgtr q/9$ for all $q \lesseqgtr 1/(9k)$, $\pi^{cL}(q_2,q) - C(q) < 0$ for all $q_1 > 1/(9k)$. Thus $q_1^c(q_2) < 1/(9k)$ for $q_2 \ge 1/(18k)$.
(4) Follows from the fact that $\pi^{cH}(q_1,q_2) \to 0$ as $q_2 \to 0$.
(5) The reaction function satisfies the First Order Condition of maximization, thus $\frac{dq_1}{dq_2} =$

$$\left. \frac{-\ \pi^{cH}_{HL}(q_1,q_2)}{\pi^{cH}_{HH}(q_1,q_2) - k} \right|_{q_1 = q_1^c(q_2)} > 0$$

from Lemma 5. Similarly for the case $q_1^c(q_2) < q_2$. □

If the rival's quality is low, firm 1 chooses to be the higher quality firm. If firm 2's quality is very high, firm 1 chooses the lower end of the market. Profit function for the portion above the quality of the rival is concave while cost function is convex. Although profit is increasing in firm's own quality, it becomes more profitable to be the lower quality firm when the rival's quality is very high. Thus we have (4).

From (2) and (3), we know that $q_1^c(q_2)$ will cross the 45 degree line in the interval [1/(18k),1/(9k)] if at all. But we have the following Lemma.

LEMMA 10: For any q, $\pi^{cH}{}_H(q,q) = 7\,\beta'(q)/27 > \pi^{cL}{}_L(q,q) = 5\,\beta'(q)/27$.

Proof: Follows from formula given in Lemma 4.

This implies that for *any* differentiable cost function C(q), q will never be the best response to q. The right derivative of the profit function at the quality level equal to that of the rival is given by $\pi^{cH}{}_H(q,q)$ while the left derivative is equal to $\pi^{cL}{}_L(q,q)$. According to the Lemma, if the First Order Condition of profit maximization holds for the right derivative, marginal condition will be negative with respect to the left derivative. If the FOC holds for the left derivative, marginal condition will be positive with respect to the right derivative. Thus profit can be increased by choosing quality either slightly lower or slightly higher than that of the rival. The reaction functions will not cross the 45 degree line and there is a discontinuity somewhere between 1/(18k) and 1/(9k). There is no symmetric equilibrium in which $q_1 = q_2$. From Lemma 8, we also know that the reaction functions will intersect at least twice, once in the $q_1 < q_2$ region and once in the $q_1 > q_2$ region (Figure 4). We summarize the preceding discussion in the following theorem.

THEOREM 3: (i) There are at least two asymmetric equilibria in the Cournot game, one in which $q_1 > 1/(8k)$ and $1/(18k) > q_2$ and one in which $q_2 > 1/(8k)$ and $1/(18k) > q_1$.
(ii) There is no symmetric equilibrium where the two firms chose the same quality.

The following two Lemmas characterize the reaction functions for the Bertrand game.

LEMMA 11: (1) ${q_1}^b(0) = 1/(8k)$.

(2) ${q_1}^b(q_2) > q_2$ and increasing in the neighborhood of $q_2 = 0$.

(3) ${q_1}^b(q_2) < q_2$ and increasing for $q_2 \geq 1/(12k)$.

(4) ${q_1}^b(q_2) \to 1/(32k)$ as $q_2 \to \infty$.

Proof: (1) ${q_1}^b(0) = \arg\max_q q/4 - kq^2 = 1/(8k)$.

(2) In the neighborhood of $q_2 = 0$ where q_2 is very small, $\max_q \pi^b(q_1,q_2) - C(q) = \max_{q \geq q_2} \pi^{bH}(q,q_2) - C(q)$. ${q_1}^b(q_2)$ satisfies the First Order Condition. By totally differentiating the FOC,

$$\frac{dq_1}{dq_2} = -\left.\frac{\pi^{bH}_{HL}(q_1,q_2)}{\pi^{bH}_{HH}(q_1,q_2) - k}\right|_{q_1={q_1}^b(q_2)} > 0 \text{ from Lemma 8.}$$

(3) It can be shown that at $q_2 = 1/12k$, $\pi^{bH}(q_1,q_2) = C(q_1)$. For larger q_2, $\pi^{bH}(q_1,q_2) - C(q_1) < 0$. Thus $\max_q \pi^b(q_1,q_2) - C(q) = \max_{q \leq q_2} \pi^{bL}(q2,q) - C(q)$. ${q_1}^b(q_2)$ satisfies the First Order Condition. By totally differentiating the FOC, $\frac{dq_1}{dq_2} = -\left.\frac{\pi^{bL}_{LH}(q_2,q_1)}{\pi^{bL}_{LL}(q_2,q_1) - k}\right|_{q_1={q_1}^b(q_2)} > 0$ from Lemma 8.

(4) As q_2 gets large, since $q_2 > 1/(12k)$, $\max_q \pi^b(q_1,q_2) - C(q) = \max_{q \leq q_2} \pi^{bL}(q2,q) - C(q)$. As $q_2 \to \infty$, $\pi^{bL}(q_2,q_1) \to q_1/16$. $\arg\max (q_1/16 - k{q_1}^2) = 1/(32k)$. □

The important difference between Cournot and Bertrand is property (3). In the Cournot game, the reaction function of firm 1 was decreasing for high q_2's. Since with Bertrand competition, increase of higher quality (for some range) increases profit of the low quality firm, the low quality firm invests more as the rival's quality increases. In the Bertrand case, it is clear from the profit function that there is a discontinuity in the reaction function. Quality level q will never be the best response to q since revenue is zero at $q_1 = q_2$. Lemma 11 is not sufficient to ensure that the reaction curves will intersect. However, the following is true.

LEMMA 12: $q_1^b(q_2) > q_2$ at $q_2 = 1/(32k)$.

Proof. We show the existence of $M > 0$ and $Q > q_2$ such that

$$\pi^{bH}(Q, q_2 = 1/(32k)) - C(Q) > M > \pi^{bL}(q_2 = 1/(32k), q_1) - C(q_1) \ \ \forall q_1 < q_2.$$

$\pi^{bL}(q_2, q_1)$ takes maximum for $q_1 < q_2$ in q_1 at $q_1 = 4q_2/7$ and the maximum value is $q_2/24$. At $q_2 = 1/(32k)$, $\pi^{bL}(q_2 = 1/(32k), q_1) - C(q_1) < \pi^{bL}(q_2 = 1/(32k), q_1 = 4q_2/7) = q_2/24$ for all $q_1 < q_2$. We can choose $M = 1/(32 \times 24 \times k)$. To find Q, we let $q_1 = \alpha q_2$ and show that there is $\alpha > 1$ such that $\pi^{bH}(q_1 = \alpha q_2, q_2 = 1/(32k)) - C(q_1) > M$. In fact, $\pi^{bH}(\alpha/(32k), 1/(32k)) - C(\alpha/(32k)) - M = \frac{1}{8k}\{\frac{\alpha^2(\alpha-1)}{(4\alpha-1)^2} - \frac{3\alpha^2+4}{32\times 12}\}$. When $\alpha = 3$, the value is $3161/(11^2 \times 32 \times 12)$. So let $Q = 3 \times 1/(32k)$.□

The reaction curves must intersect at least once in the region $q_1 > 1/(8k)$ and $q_2 < 1/(32k)$ and at least once in the region $q_1 < 1/(32k)$ and $q_2 > 1/(8k)$ (Figure 5). The fact that in the Bertrand case the firms will never chose the same quality follows from the fact that profits for both the high quality firm and the low quality firm will go to zero as the two qualities approaches each other.

THEOREM 4: (i) There are at least two asymmetric equilibria in the Bertrand game with $q_1 > 1/(8k)$ and $q_2 < 1/(32k)$ or $q_1 < 1/(32k)$ and $q_2 > 1/(8k)$.
(ii) There never will be a symmetric equilibrium.

The upper bound of lower quality is lower than that of the Cournot game. Remember that the Bertrand profit function for the lower quality firm had a maximum point. Because of the increasing cost, firm chooses quality below this maximum point. Of course this result is consistent with results in [9].

5 Conclusion

In this paper we analyzed a model of two firms that sell products differentiated by vertical quality. The game evolves in two stages: in the first stage firms simultaneously choose their respective quality; in the second stage, firms sell to consumers who differ in their willingness to pay for better quality. Since we have vertical quality differentiation, all consumers agree on what is better. Our major interest was the perfect equilibrium of the game in which the second stage is a quantity setting game, i.e., a Cournot game. For comparison, we also considered a game in which this subgame is Bertrand, or price setting for comparison. We verified that the results for the Bertrand subgame coincided with results previously reported in slightly different models.

The second stage payoff or the Cournot profit induced by perfect equilibrium strategies as function of qualities differs from the Bertrand subgame profits. In the Cournot subgame case, profit of a firm is greater when firm's own quality is higher and less when quality of the rival is higher. This was found to be true for any pair of qualities. This is in contrast with the Bertrand subgame case where for some pair of qualities, both firms may benefit from improvement of the higher quality and both may lose profit when the lower quality improves. On can say that the Cournot profit function reflects the vertical aspect of quality differentiation, that is, since all the consumers agree on what is better, firm always benefits from improving its own quality. On the other hand, Bertrand subgame profits has property common with the horizontal quality models: as qualities become very close, the competition becomes more like a homogeneous quality Bertrand competition.

Despite the fact that profit is increasing for a firm in its own quality independent of the quality level of the rival, a firm will still avoid choosing the same quality as that of the rival for any differentiable cost of quality function. This is because marginal profit is very sensitive to the quality level of the rival in the Cournot subgame case as well. Marginal profit will discontinuously increase at the level of the rival. Thus a firm will always find it profitable to either increase its quality above or decrease it below the level of the rival. Even though the second stage profits as functions of quality choices differ in the Cournot and Bertrand subgames, the quality choices are similar in that firms never choose the same quality level.

Footnotes

[1]For an example of how two innovations yield different reuslts, see Kamien and Tauman[1986,88].

[2]$[x]^+ = \text{Max}\{ x, 0 \}$.

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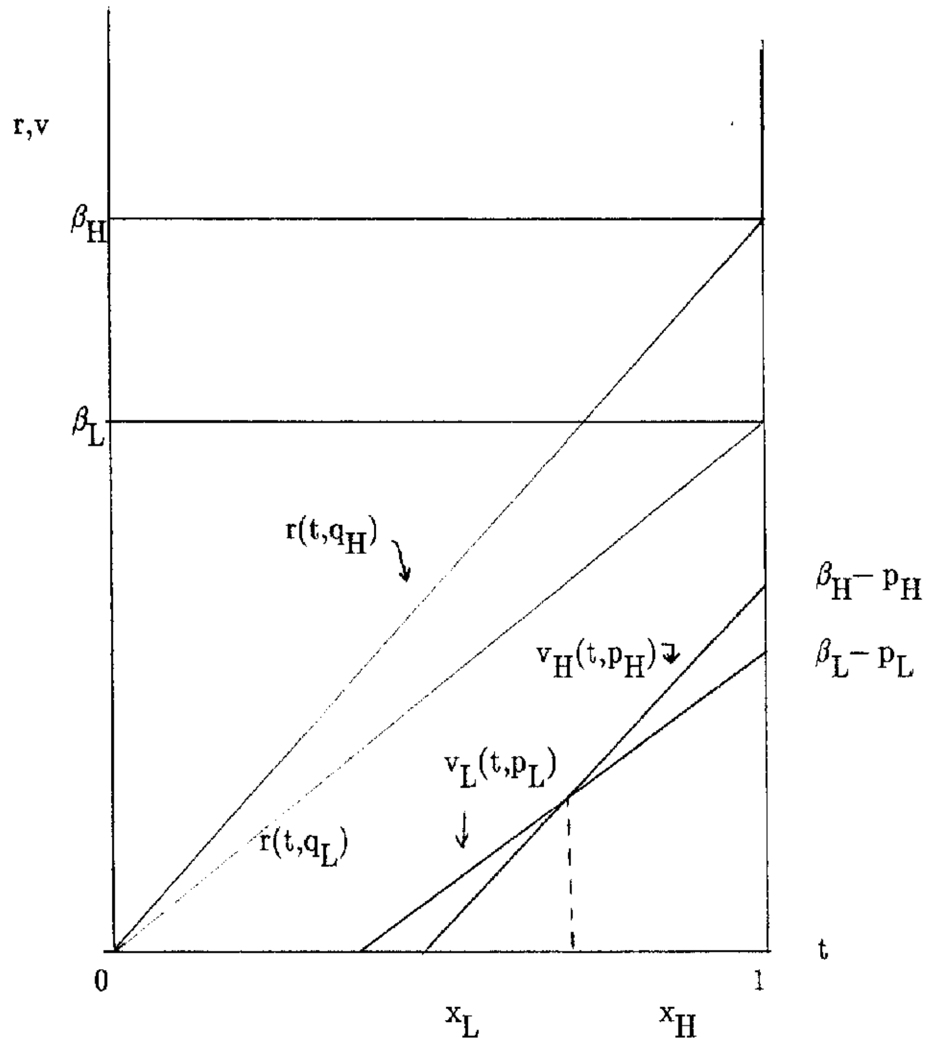
Figure 1

Model of the Consumers

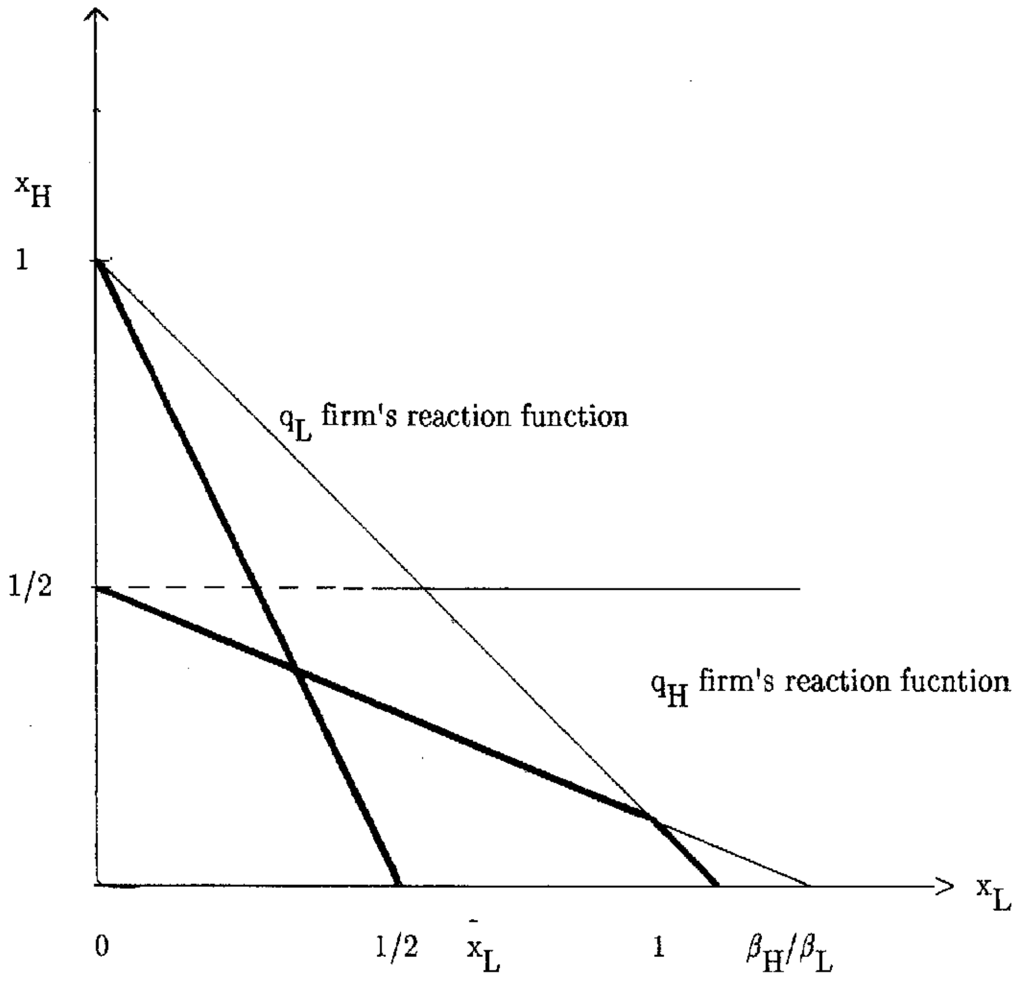


$$v_H(t,p_H) = r(t,q_H) - p_H$$

$$v_L(t,p_L) = r(t,q_L) - p_L$$

Figure 2

Cournot Subgame Reaction Functions



q_L firm's reaction fucntion $x_L(x_H) = (1 - x_H)/2$

q_H firm's reaction fucntion $x_H(x_L) = \text{Max}\left[\frac{\beta_H - \beta_L x_L}{2\beta_H}, 1 - x_L\right]$

$\tilde{x}_L$ satisfies $\tilde{\pi}^H(x_H^c(\tilde{x}_L),\tilde{x}_L) = (\beta_H - \beta_L)/4$.

$(\beta_H - \beta_L)/4$ is profit of q_H firm when $p_L = 0$ and $x_H = 1/2$.

Figrue 4

Cournot Game First Stage Reaction Functions

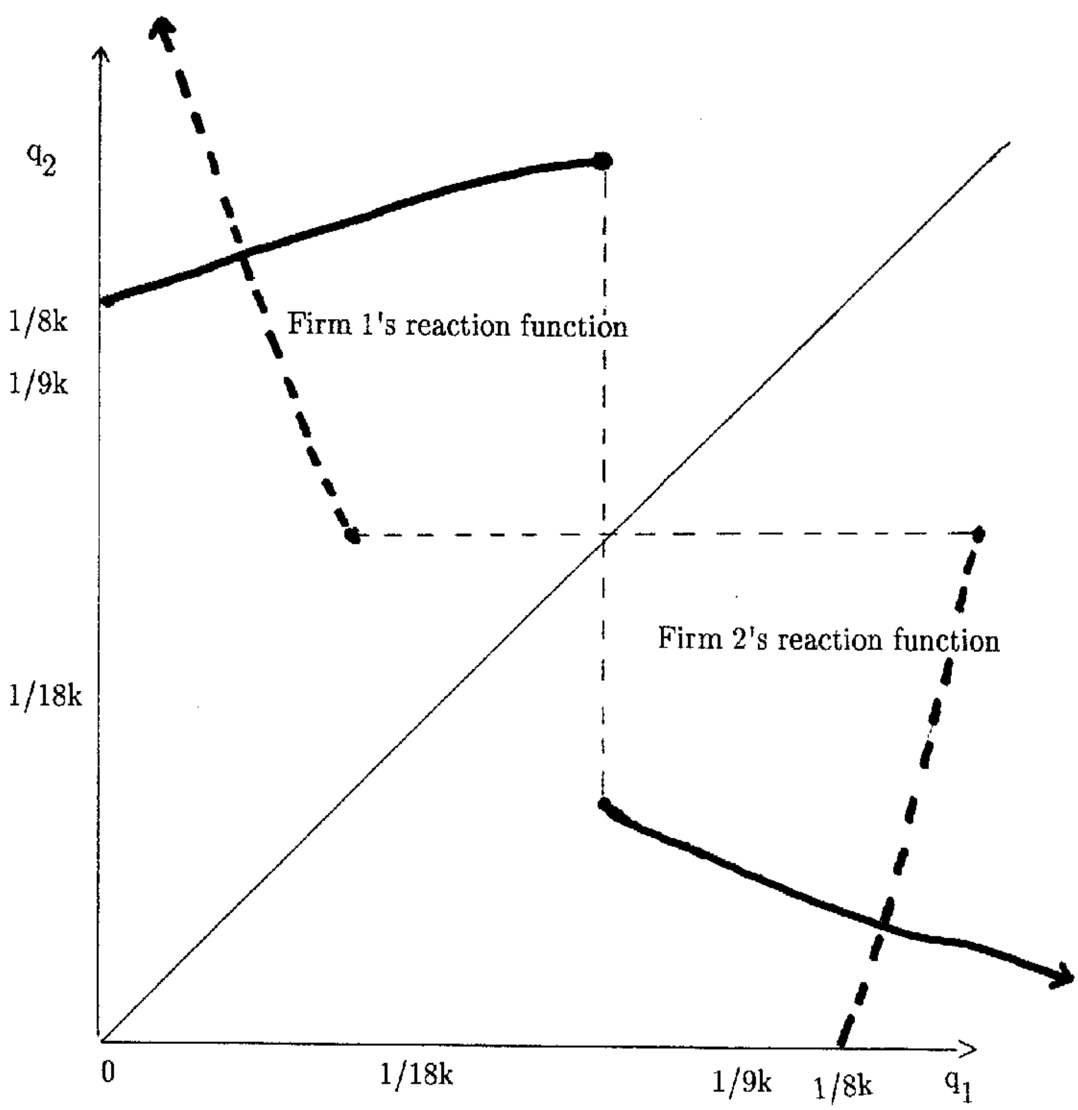


Figure 5

Bertrand Game First Stage Reaction Functions