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Seismic Assessment of Timber Floor Diaphragms in Unreinforced Masonry Buildings

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

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Abstract

An integrated experimental and analytical study investigating the in-plane performance of timber floor diaphragms in URM buildings is presented. The research presented in this thesis was conducted with the primary aim of developing a revised procedure for the detailed seismic assessment of timber floor diaphragms in URM buildings.

The deformation mechanics of timber floor diaphragms are presented and were used to develop an analytical model that can be used to predict diaphragm nonlinear performance. The analytical model was used to update the current idealisation of diaphragm behaviour in current seismic assessment documents.

A three-phase experimental testing program is presented. Nail connection testing results indicated that connections in diaphragms contained within existing URM buildings have substantially lower initial stiffness, strength, and displacement capacity than do new connections. Small-scale diaphragm testing results proved that friction resistance between straight-edge or tongue and groove floorboards is negligible. Full-scale diaphragm testing results confirmed the highly flexible and orthotropic behaviour of timber floor diaphragms, and that a small stairwell penetration or the presence of discontinuous joists having a reliable mechanical connection do not significantly influence diaphragm performance.

A finite element (FE) modelling method for timber floor diaphragms using the structural analysis software SAP2000 was appropriately validated using analytical modelling results and experimental data. Salvaged nail connection load-slip test data was used to program a diaphragm FE model to establish the representative performance of diaphragms in ~100 year old URM buildings in New Zealand. A comprehensive parametric analysis of diaphragm behaviour was subsequently undertaken.

Finally, a revised assessment procedure for timber floor diaphragms was developed, which incorporates representative performance parameters, and provides suitable provisions to account for diaphragm orthotropic behaviour and for variations in key diaphragm configuration characteristics.
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Notation

**Roman**

- $A$ Beam cross-sectional area
- $A_{d(gross)}$ Diaphragm gross area
- $A_{d(net)}$ Diaphragm net area
- $A_e$ Area underneath diaphragm force-displacement backbone curve
- $A_i$ Arbitrary coefficient for diaphragm deformation analysis of $i^{th}$ section
- $A_k$ Arbitrary coefficient for diaphragm deformation analysis of $k^{th}$ section
- $b_p$ Diaphragm penetration width
- $b_0$ Parameter for diaphragm deformation analysis
- $b_1$ Parameter for diaphragm deformation analysis
- $b_2$ Parameter for diaphragm deformation analysis
- $b_3$ Parameter for diaphragm deformation analysis
- $B$ Diaphragm width
- $B_i$ Arbitrary coefficient for diaphragm deformation analysis of $i^{th}$ section
- $B_k$ Arbitrary coefficient for diaphragm deformation analysis of $k^{th}$ section
- $c_E$ Modification factor for timber elastic modulus
- $c_{tf}$ Modification factor for floorboard thickness
- $c_{tj}$ Modification factor for joist thickness
- $c_{df}$ Modification factor for floorboard width
- $c_{dj}$ Modification factor for joist depth
- $c_{L(para)}$ Modification factor for joist spacing, for parallel-to-joist loading
- $c_{L(perp)}$ Modification factor for joist spacing, for perpendicular-to-joist loading
- $c_{p(para)}$ Modification factor for penetrations, for parallel-to-joist loading
- $c_{p(perp)}$ Modification factor for penetrations, for perpendicular-to-joist loading
- $c_{\ell(para)}$ Modification factor for joist spacing, for parallel-to-joist loading
<table>
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<tr>
<th>Notation</th>
<th>Description</th>
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<td>$c_{l(perp)}$</td>
<td>Modification factor for joist spacing, for perpendicular-to-joist loading</td>
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<td>$C_i$</td>
<td>Arbitrary coefficient for diaphragm deformation analysis of $i^{th}$ section</td>
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<td>$C_k$</td>
<td>Arbitrary coefficient for diaphragm deformation analysis of $k^{th}$ section</td>
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<td>Upper and lower bound values for 95% confidence interval</td>
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<td>Joist depth</td>
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<tr>
<td>$d_p$</td>
<td>Width of URM wall pocket</td>
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<td>$e$</td>
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<td>Timber elastic modulus</td>
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<td>$f_1$</td>
<td>Diaphragm fundamental frequency</td>
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<td>$F_{cr}$</td>
<td>Diaphragm force corresponding to critical midspan displacement</td>
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<td>Total diaphragm seismic load</td>
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<td>$F_{d,max}$</td>
<td>Maximum diaphragm load</td>
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<td>$F_e$</td>
<td>Total diaphragm design earthquake load for $\mu = 1.0$</td>
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<td>$F_{n,max}$</td>
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<td>$F_{n,ult}$</td>
<td>Nail connection strength load</td>
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<td>Nail connection yield load</td>
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<td>Peak diaphragm force</td>
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<td>$F_{res}$</td>
<td>Diaphragm force resistance</td>
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<tr>
<td>$F_u$</td>
<td>Weight tributary to the diaphragm under 1.0g acceleration</td>
</tr>
<tr>
<td>$F_y$</td>
<td>Diaphragm yield load</td>
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<td>$F_{y1}$</td>
<td>Effective yield force of Pivot model for positive displacements</td>
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<td>$F_{y2}$</td>
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<tr>
<td>$F_0$</td>
<td>Initial force of Foschi (1974) nail connection load-slip characteristic</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity</td>
</tr>
<tr>
<td>$g_e$</td>
<td>Initial opening of Gap element in SAP2000</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus</td>
</tr>
</tbody>
</table>
Notation

\( G_d \) Diaphragm shear stiffness
\( G_{d(mean)} \) Standard diaphragm shear stiffness
\( \tilde{G}_d \) Normalised diaphragm shear stiffness
\( I \) Second moment of inertia
\( I_f \) Second moment of inertia of floorboard
\( I_j \) Second moment of inertia of joist
\( k_n \) Linear-elastic nail connection stiffness
\( \bar{k} \) Generalised stiffness of a structure
\( K_a \) Beam laminar slip
\( K_e \) Diaphragm effective stiffness
\( K_d \) Diaphragm stiffness
\( K_{d1} \) Diaphragm initial stiffness
\( K_{d2} \) Diaphragm secondary stiffness
\( K_g \) Stiffness of gap element in SAP2000
\( K_{rot} \) Link element rotational stiffness in SAP2000
\( K_a \) Nail connection initial stiffness
\( K_\beta \) Nail connection secondary stiffness
\( K_0 \) Initial stiffness of Foschi (1974) nail connection load-slip characteristic
\( K_1 \) Secondary stiffness of Foschi (1974) nail connection load-slip characteristic
\( l_p \) Diaphragm penetration length
\( L \) Diaphragm span
\( \ell \) Joist spacing
\( m \) Mass
\( \bar{m} \) Mass per unit distance
\( \vec{m} \) Generalised mass of a structure
\( M_k \) Restoring moment generated by \( k^{th} \) nail connection couple
\( M_n \) Restoring moment generated by nail connection couple
\( n_b \) Number of floorboards
\( n_c \) Number of nail connection couples
\( n_j \) Number of joists
### Notation

<table>
<thead>
<tr>
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<th>Description</th>
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<tr>
<td>( N )</td>
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<td>Beam point load</td>
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<tr>
<td>( P_n )</td>
<td>Nail connection load</td>
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<td>( P_{n_{\text{max}}} )</td>
<td>Nail connection maximum load</td>
</tr>
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<td>( Q_n )</td>
<td>Nominal nail connection capacity</td>
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<td>Diaphragm shear strength per lineal metre width</td>
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<td>( s )</td>
<td>Nail couple spacing</td>
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<tr>
<td>( t_w )</td>
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<td>Diaphragm fundamental period</td>
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<tr>
<td>( u )</td>
<td>Assumed displacement of a structure</td>
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<tr>
<td>( v )</td>
<td>Diaphragm in-plane deformation</td>
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<td>( v_i )</td>
<td>Diaphragm in-plane deformation within ( i^{th} ) section</td>
</tr>
<tr>
<td>( v_s )</td>
<td>Shear force per lineal metre width</td>
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<td>Diaphragm yield shear force per lineal metre width</td>
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<td>( V )</td>
<td>Total shear force</td>
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<td>( w )</td>
<td>Uniformly distributed load per floorboard</td>
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<td>( w_E )</td>
<td>Parabolic load distribution as defined by ASCE 41-06 (2007)</td>
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<tr>
<td>( W )</td>
<td>Total uniformly distributed load on diaphragm</td>
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<td>( \bar{x} )</td>
<td>Population mean</td>
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<tr>
<td>( z )</td>
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<tr>
<td>( %\text{NBS} )</td>
<td>Percentage new building standard</td>
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### Greek

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>( \alpha_c )</td>
<td>Modification factor for diaphragm configuration</td>
</tr>
</tbody>
</table>
Notation

\( \alpha_g \quad \) Modification factor for diaphragm geometry
\( \alpha_{jp} \quad \) Joist rotation within URM wall pocket
\( \alpha_n \quad \) Initial angle of nail connection load-slip curve
\( \alpha_p \quad \) Modification factor for diaphragm penetrations
\( \alpha_1 \quad \) Parameter controlling the unloading gradient of Pivot model for positive displacements
\( \alpha_2 \quad \) Parameter controlling the unloading gradient of Pivot model for negative displacements
\( \beta \quad \) Parameter for diaphragm deformation analysis
\( \beta_d \quad \) Parameter for Granholm (1961) diaphragm deformation analysis
\( \beta_n \quad \) Secondary angle of nail connection load-slip curve
\( \beta_1 \quad \) Parameter controlling hysteretic pinching of Pivot model for positive displacements
\( \beta_2 \quad \) Parameter controlling hysteretic pinching of Pivot model for negative displacements
\( \gamma \quad \) Uniform spacing between nail connections
\( \delta_n \quad \) Nail slip
\( \delta_{n,\text{max}} \quad \) Maximum nail slip
\( \delta_{n,\text{ult}} \quad \) Ultimate nail slip
\( \delta_{n,y} \quad \) Yield nail slip
\( \delta_{gf} \quad \) Deflected geometry of a fixed-ended flexural beam
\( \delta_{pf} \quad \) Deflected geometry of a pin-ended flexural beam
\( \delta_e \quad \) Deflected geometry of a shear beam
\( \delta_{y1} \quad \) Effective yield displacement of Pivot model for positive displacements
\( \delta_{y2} \quad \) Effective yield displacement of Pivot model for negative displacements
\( \Delta \quad \) Displacement
\( \Delta_{cr} \quad \) Diaphragm critical midspan displacement
\( \Delta_d \quad \) Diaphragm midspan displacement
\( \Delta_{\text{demand}} \quad \) Diaphragm demand midspan displacement for a load of \( F_e \)
\( \Delta_{\text{failure}} \quad \) Diaphragm midspan displacement at failure
\( \Delta_{gf} \quad \) Midspan displacement of fixed-ended flexural beam
### Notation

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<th>Symbol</th>
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<td>Diaphragm midspan displacement limit for serviceability associated with the failure of the stapled sheet metal blocking system</td>
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<tr>
<td>$\Delta_{L2}$</td>
<td>Diaphragm midspan displacement limit for serviceability associated with buckling of plywood overlay sheets</td>
</tr>
<tr>
<td>$\Delta_{\text{max}}$</td>
<td>Maximum midspan diaphragm displacement</td>
</tr>
<tr>
<td>$\Delta_{\text{peak}}$</td>
<td>Peak diaphragm midspan displacement</td>
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<tr>
<td>$\Delta_{pf}$</td>
<td>Midspan displacement of pin-ended flexural beam</td>
</tr>
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<td>Midspan displacement of shear beam</td>
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<td>Ultimate diaphragm midspan displacement</td>
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<tr>
<td>$\theta_{k}$</td>
<td>Rotation angle of $k^{th}$ nail connection couple</td>
</tr>
<tr>
<td>$\theta_{\text{max}}$</td>
<td>Maximum nail connection couple rotation angle</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Rotation of steel side frame during parallel-to-joist diaphragm testing</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>Rotation of steel side frame during parallel-to-joist diaphragm testing</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Form factor to account for shear stress distribution</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Diaphragm area ratio</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Diaphragm ductility</td>
</tr>
<tr>
<td>$\mu_{\text{demand}}$</td>
<td>Diaphragm demand ductility</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>Nail connection ductility</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>Modification factor for Granholm (1961) diaphragm deformation analysis</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Shape function</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Diaphragm overstrength ratio</td>
</tr>
</tbody>
</table>
Unreinforced masonry (URM) buildings can generally be described as buildings that are constructed from individual units of brick or stone, bound together with a layer of mortar without the use of tensile reinforcement. Widespread construction of URM buildings occurred throughout New Zealand during the late 19th and early 20th centuries until the 1931 Hawke’s Bay earthquake completely destroyed the cities of Napier and Hastings (Dowrick 1998), and graphically illustrated that URM construction provided insufficient strength to resist lateral earthquake forces. The popularity of URM rapidly declined in New Zealand following the Hawke’s Bay earthquake and was formally precluded for use in any new construction after the introduction of the NZS 1900 building bylaw in 1965 (NZSI 1965).

The poor earthquake performance of URM buildings has been widely recognised for decades (Drysdale et al. 1999; Megget 2006; Paulay and Priestley 1992). The recent 22 February 2011 Christchurch earthquake in New Zealand that severely damaged the URM building stock in Christchurch and killed more than 150 people is testament to their brittle nature and inability to dissipate hysteretic energy (see Figure 1-1). Although many URM buildings have been demolished as a result of past earthquakes or to make way for new construction, this class of structure remains prevalent throughout New Zealand, with many of these buildings left unstrengthened. Conflicting imperatives currently exists regarding
the preservation of URM buildings, which, despite posing a serious life-safety hazard, form a significant percentage of New Zealand’s building stock and represent the predominant architectural heritage of the nation (Goodwin 2009), so must be retained for commercial as well as cultural heritage reasons. The decision to strengthen or demolish a URM building is typically a function of cost and cultural significance. The accurate assessment of URM building seismic performance is fundamental in this decision process, because it firstly determines whether strengthening is required, and if so, provides an estimate of retrofitting requirements.

While URM building perimeter walls are constructed of rigid clay bricks and mortar, the floor diaphragms are almost always comprised of comparatively light timber framing. Timber floor diaphragms in URM buildings have routinely demonstrated significant influence on the seismic performance of the complete URM structure due to their flexible nature and often inadequate connection to the URM perimeter walls (Bruneau 1994a). The accurate prediction of diaphragm in-plane strength and stiffness is therefore a crucial component within the framework of URM building seismic assessments.
1.1 Diaphragm Characterisation

A number of URM buildings throughout New Zealand have been surveyed during the course of this study. Relevant characteristics of the observed timber floor diaphragms are provided hereafter, to ensure that this research was suitably scoped for representative diaphragm configurations.

1.1.1 Diaphragm construction details

The general configuration of timber floor diaphragms in New Zealand URM buildings is depicted in Figure 1-2. Timber floorboards are orientated perpendicular to timber joists and typically connected with two nails at each intersection. Timber cross-bracing is generally provided at intermittent locations along the joist lengths to mitigate out-of-plane buckling. Diaphragm blocking and chord elements are almost never present.

From discussion with engineering practitioners and current URM building owners, it is understood that timber diaphragms in New Zealand were constructed with native
hardwoods, such as Rimu, Kauri, and Matai. For the investigation of diaphragm in-plane performance, the most important timber material property is modulus of elasticity, $E$, which dictates timber stiffness. The following values are published for the relevant hardwoods (NZ Wood 2011a): $E_{\text{Rimu}} = 9.7$ GPa, $E_{\text{Kauri}} = 13.0$ GPa, and $E_{\text{Matai}} = 8.0$ GPa. Many nails were recovered from the inspected floorboard-to-joist nail connections, and appeared to be consistent with turn-of-the 20th century wire-drawn nails, as described by Isaacs (2009).

Floorboard profiles were observed to be either straight-edge or tongue and groove (T&G), with neither profile appearing to be more prevalent. Floorboard length did not exceed approximately 6.0 m, so arrangements of discontinuous floorboards were observable, although no consistent patterns could be identified. The abutments of discontinuous floorboards were almost always located over joist members. Rectangular joist and cross-bracing framing members of various sizes were observed. The typical arrangement of the described timber framing members is illustrated in Figure 1-3. From the performed diaphragm inspections and through correspondence with eminent New Zealand structural engineers (Oliver 2010b), a set of realistic ranges for relevant diaphragm configuration parameters were established, which are provided in Table 1-1. Accordingly this research focussed on diaphragm configurations within these ranges.

<table>
<thead>
<tr>
<th>Floorboard thickness, $t_f$</th>
<th>Floorboard width, $d_f$</th>
<th>Joist thickness, $t_j$</th>
<th>Joist depth, $d_j$</th>
<th>X-bracing thickness, $t_b$</th>
<th>X-bracing depth, $d_b$</th>
<th>Joist spacing, $\ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>15 – 30</td>
<td>100 – 150</td>
<td>40 – 65</td>
<td>150 – 400</td>
<td>40 – 65</td>
<td>40 – 130</td>
<td>300 – 650</td>
</tr>
</tbody>
</table>

Diaphragm joists were observed to be either simply supported on a brick ledge resulting from the URM walls reducing in width at each storey height, or pocketed into the URM walls to a depth equal to one brick width, as illustrated in Figure 1-4. Pocketed joists were either loosely seated within the URM wall pockets (with variable lateral clearance), or tightly fitted using mortar or grout. This research was concerned with diaphragm configurations comprising pocketed joists only.
Figure 1-3: Underside of diaphragm showing joists, floorboards, and cross-bracing

(a) Joist seated on ledge of URM wall  (b) Joists pocketed into URM wall

Figure 1-4: Joist seating details

Maximum joist length was observed to be approximately 6.0 m. For small URM buildings where the perimeter walls were close enough, the joists typically spanned continuously between these elements. For most URM buildings having dimensions that exceed approximately 6.0 m, joists were typically lapped or butted, either with or without some kind of mechanical connection, over intermediate steel or timber cross-beams that are supported on columns. Two examples of intermediate support conditions for discontinuous joists are provided in Figure 1-5. Both diaphragm configurations with and without discontinuous joists were addressed in this research.
1.1.2 Diaphragm geometry

Diaphragm geometry is generally dictated by the footprint of the URM building that the diaphragm is contained within. In a companion study, Russell (2010) compiled a comprehensive inventory of the typical geometries and corresponding dimensions of New Zealand’s URM building stock. From the typology study, Russell identified four basic URM building footprints that exist throughout the country: (1) square or rectangular, (2) acute intersection, (3) obtuse intersection, and (4) right-angle intersection. The identified geometries are illustrated in Figure 1-6. Although a number of URM buildings corresponding to geometries (2), (3), and (4) were observed, Russell determined that the predominant type of URM buildings in New Zealand are one- to three-storeys in height and have a rectangular or square geometry. Accordingly, this research focussed specifically on rectangular (four-sided) timber floor diaphragms, which represent the majority of diaphragm configurations currently in existence throughout New Zealand. Diaphragms associated with URM building geometries (2), (3), and (4), or geometries with even greater complexity, are outside the scope of this research.

Based upon the research of Russell (2010), it has been determined that rectangular one- to three-storey URM buildings in New Zealand will generally have dimensions of between
1.1 Diaphragm Characterisation

6.0 m × 4.0 m and 24.0 m × 20.0 m, although many dimension combinations are possible between these limiting values.

Floor diaphragms almost always contain a penetration to enable floor access for building requirements such as stairwells or lift shafts. Diaphragm penetrations are known to reduce diaphragm stiffness and strength (ASCE 2007; NZSEE 2006), so the effect of a range of penetration sizes was investigated in this research. From the inspection of existing diaphragms and from communication with New Zealand structural engineers (Oliver 2010b), it has been determined that penetration size can vary from a small single case stairwell measuring approximately 1.0 m × 3.0 m, to more complicated stairwells (such as those with mid-height landings) or lift shafts measuring up to approximately 4.0 m × 6.0 m.

Figure 1-6: Typical URM building geometries in New Zealand
[reproduced from Russell (2010)]
1.1.3 *Standard* diaphragm

It is evident from Sections 1.1.1 and 1.1.2 that many different diaphragm configurations exist throughout New Zealand, all of which cannot be considered individually. In order to establish the performance of a representative diaphragm within the scope of this research, a *standard* diaphragm configuration was established for use in numerous analyses throughout this thesis. The *standard* diaphragm is intended to represent a typical rectangular diaphragm contained within a one- to three-storey URM building, which Russell (2010) has determined to comprise approximately 85% of New Zealand’s URM building stock.

Based upon the diaphragm construction details and geometries outlined above, a set of mean parameter values have been established for the *standard* diaphragm, as outlined in Table 1-2 and illustrated in Figure 1-7. The *standard* diaphragm joists are continuously spanning along the 8.0 m dimension. It is acknowledged that joists exceeding approximately 6.0 m in length in existing URM buildings must be discontinuous and supported at intermediate span locations. However for the purpose of establishing fundamental diaphragm performance, this practical limitation has been ignored, as the influence of discontinuous joists is investigated separately during this research. In addition, the *standard* diaphragm does not contain a penetration, because this configuration feature is also specifically investigated during this research.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm dimensions (m)</td>
<td>12.0 × 8.0</td>
<td>Joist spacing, ℓ (mm)</td>
<td>400</td>
</tr>
<tr>
<td>Floorboard thickness, $t_f$ (mm)</td>
<td>25</td>
<td>Cross-bracing thickness, $t_b$</td>
<td>50</td>
</tr>
<tr>
<td>Floorboard width, $d_f$ (mm)</td>
<td>135</td>
<td>Cross-bracing depth, $d_b$</td>
<td>100</td>
</tr>
<tr>
<td>Joist thickness, $t_j$ (mm)</td>
<td>50</td>
<td>Cross-bracing spacing (m)</td>
<td>2.0</td>
</tr>
<tr>
<td>Joist depth, $d_j$ (mm)</td>
<td>300</td>
<td>Timber elastic modulus, $E$ (GPa)</td>
<td>10.0</td>
</tr>
</tbody>
</table>
1.2 Research Motivation and Objectives

While extensive research has been conducted on plywood-sheathed diaphragms, straight-sheathed timber floor diaphragms in URM buildings have received very little research attention (ABK 1981; Baldessari 2010; Brignola 2009; Peralta 2003). As a result, several aspects regarding the evaluation of heritage diaphragm\(^1\) seismic performance have remained poorly understood by structural engineers. Of particular importance is the consideration of diaphragm orthotropic behaviour, which has been largely ignored in

\(^1\)Straight-sheathed timber floor diaphragms in URM buildings are referred to as ‘heritage diaphragms’ many times throughout this thesis to avoid confusion with other common diaphragm configurations, such as plywood-sheathed diaphragms.
published research which has focussed almost exclusively on parallel-to-joist performance. The influence of several diaphragm configuration features on the performance of heritage diaphragms has also yet to be suitably investigated, such as variations in diaphragm construction details, the presence of discontinuous joists, and diaphragm penetrations. Furthermore, heritage diaphragm research to date has been restricted to the testing and analysis of newly constructed diaphragm components; the effects of deterioration and outdated construction materials on the performance of ~100 year old diaphragms are yet to be quantified.

The lack of experimental data and suitable analysis pertaining to heritage diaphragms has been reflected in current seismic assessment documents ASCE 41-06 (2007) and NZSEE (2006). Structural engineers have communicated that these guidelines are insufficiently prescriptive and difficult to logically follow, and have expressed concern with the published performance parameters which appear to be inconsistent between the international documents. Consequently, significant motivation exists to consolidate the current assessment procedures into one harmonised guideline, and to provide heritage diaphragm performance parameters that are truly representative of the timber floor diaphragms in New Zealand URM buildings.

In response to the above, the principal objective of this research is to develop a revised procedure to assess the seismic performance of timber floor diaphragms in URM buildings. Within the scope of the principal objective, and to address current research limitations, the major objectives of this study were as follows:

- Deepen the understanding of heritage diaphragm deformation mechanics.
- Establish representative performance parameters for ~100 year old timber floor diaphragms in heritage New Zealand URM buildings, such as stiffness, strength and displacement capacity.
- Characterise the above performance parameters for both principal loading directions (parallel-to-joist and perpendicular-to-joist), in order to appropriately establish orthotropic behaviour.
1.3 Thesis Outline

To achieve the research objectives stated above, an integrated experimental and analytical program investigating the in-plane performance of timber floor diaphragms in URM buildings was undertaken. The basic concept of the research program involved using an analytical model and experimental data to calibrate and validate a finite element modelling method for straight-sheathed timber floor diaphragms. Diaphragm finite element models were subsequently used to establish representative diaphragm performance and to undertake a comprehensive parametric analysis to investigate the influence of key configuration parameters on diaphragm performance. Using the finite element analysis results, the principal objective of this research was achieved by developing a revised detailed seismic assessment procedure for timber floor diaphragms. The research program described above is separated into chapters, each describing a distinct component of the research agenda. The contents of this thesis are organised as follows:

**Literature Review:** A comprehensive literature review of published research pertaining to timber floor diaphragms in URM buildings is presented in Chapter 2. The significance of diaphragm performance on the overall seismic response of URM buildings is firstly established. Past research regarding the lateral load-slip performance of nailed timber
connections is then presented, followed by a detailed account of the experimental and modelling research that has been undertaken on timber floor diaphragms.

Mechanics of Diaphragm Deformations: The deformation mechanics of straight-sheathed timber diaphragms is described in detail in Chapter 3. A new analytical model is developed that mathematically describes diaphragm deformations when loaded in either principal loading direction. Application of the model for both linear-elastic and nonlinear nail connection load-slip behaviour is subsequently demonstrated, followed by a validation of model response to changes in selected diaphragm configuration parameters.

Idealisation of Diaphragm Behaviour for Assessment: Using the analytical model developed in Chapter 3, an evaluation of diaphragm behaviour idealisation for the assessment of diaphragm seismic performance is presented in Chapter 4. The equations for diaphragm fundamental period determination and for diaphragm displacement determination that are currently published in assessment documents ASCE 41-06 (2006) and NZSEE (2006) are critiqued. A revised idealisation of diaphragm behaviour is subsequently derived, and used to propose updated fundamental period and displacement equations for timber floor diaphragms.

Nail Connection Testing: A summary of lateral tests performed on nail connections to establish their hysteretic behaviour is presented in Chapter 5. The load-slip test results of new connections constructed with new pine timber and new wire-drawn nails, and of salvaged connections extracted from the timber floor diaphragms of two New Zealand heritage URM buildings are presented. The load-slip test data is used to determine averaged backbone curves for each connection type, which are used to compare new and salvaged connection performance, and also for implementation into the diaphragm finite element models described in Chapters 8 and 9.

Small-Scale Diaphragm Testing: A series of small-scale diaphragm tests is presented in Chapter 6. The magnitude of inter-floorboard friction is determined from racking tests of small-scale diaphragm assemblages. The performance of newly constructed small-scale
diaphragms and small diaphragm sections extracted from a New Zealand heritage URM building are discussed.

**Full-Scale Diaphragm Testing:** Chapter 7 presents the in-plane cyclic response of four full-scale as-built timber diaphragms, and of four full-scale diaphragms retrofitted with a plywood sheet overlay and stapled sheet metal blocking system. The diaphragms were tested either parallel-to-joist or perpendicular-to-joist. Numerous aspects of diaphragm response are analysed, with a specific focus on determining orthotropic behaviour and quantifying the influence of a small stairwell penetration and of discontinuous joists having a lapped two-bolt connection.

**Finite Element Modelling:** A comprehensive procedure to model straight-sheathed timber diaphragms in the structural analysis software SAP2000 (CSI 2004) is presented in Chapter 8. Topics discussed include the suitable treatment of timber framing members, nail connections, and diaphragm boundary conditions. Using the analytical model developed in Chapter 3, and the test data generated from the experimental program presented in Chapters 5, 6, and 7, the proposed modelling method is appropriately validated for both monotonic and cyclic response.

**Parametric Analysis of Diaphragm Behaviour:** The representative performance of existing timber floor diaphragms in heritage URM buildings is established in the first section of Chapter 9. The salvaged nail connection load-slip data generated in Chapter 5 is used to program a diaphragm finite element model to ensure that analysis data reflects the deteriorated performance of ~100 year old timber diaphragms. In the second section of Chapter 9, the details of a comprehensive parametric analysis of diaphragm behaviour are provided. Modification factors are developed to suitably adjust diaphragm performance for possible variations in diaphragm geometry, penetration size, and diaphragm configuration.

**Seismic Assessment of Timber Floor Diaphragms:** Chapter 10 presents a revised procedure to assess the seismic performance of timber floor diaphragms in URM buildings. Three assessment examples are provided to demonstrate suitable application of the procedure.
**Summary and Conclusions:** Chapter 11 summarises the main conclusions of the thesis and provides recommendations for further research.
Chapter 2

LITERATURE REVIEW

The purpose of the literature review presented in this chapter was to establish the current state-of-the-art pertaining to timber floor diaphragm testing and analysis, and to subsequently identify specific knowledge gaps that were ultimately targeted in the research objectives of this study.

2.1 Diaphragm Influence

The response of unreinforced masonry (URM) buildings to earthquakes differs substantially from comparable modern-day construction materials such as reinforced masonry, reinforced concrete or steel. The incompatibility of rigid and brittle perimeter walls with light, highly flexible timber diaphragms results in the inability of URM buildings to effectively transmit lateral loads, rendering them extremely vulnerable to earthquakes. The unique behaviour of URM was captured in the ‘special procedure’ developed by the Agbabian Barnes Kariotis (ABK 1984) research collaboration in the early 1980’s. While this procedure is by no means the definitive analysis method for URM, it has enjoyed widespread acceptance and has been shown to be a sound assessment practice (Bruneau 1994a). Furthermore, the ABK procedure serves as a rational and
representative approach for URM construction, and highlights the important role of timber diaphragms on the overall seismic response of URM buildings.

To illustrate the basis of the ABK method, take Figure 2-1, which shows a URM structure separated into three primary components: (1) end walls (parallel to the direction of excitation), (2) head walls (perpendicular to the direction of excitation), and (3) horizontal diaphragms. When a URM building is subjected to earthquake loads, the ABK method assumes that the head walls provide zero strength, and that the comparatively ‘infinitely’ stiff end walls resist all earthquake excitation. Earthquake ground motion is therefore transmitted unmodified to the ends of each floor diaphragm. These diaphragms, in turn, load the head walls which are excited in their out-of-plane direction (Bruneau 1994a). Based on this idealised behaviour, the in-plane performance of the timber diaphragms plays a critical role in the complete URM building seismic response.

2.1.1 Common URM failures

Poorly performing URM structures has been extensively documented in published earthquake reconnaissance reports (see for instance Adham 1985; Cross and Jones 1991; Deppe 1988; Ingham and Griffith 2011; Kariotis 1984). Consistent damage observations

![Figure 2-1: Idealised response of URM buildings to earthquakes](taken from (Bruneau 1994a))
illustrated in these reports have led to two primary URM building failure mechanisms directly attributed to the influence of diaphragm configuration and performance (Bruneau 1994b):

- Out-of-plane failure
- Corner ‘punching’ failure

It is noteworthy that damage to the diaphragm itself is rarely observed. This is likely attributed to ductility of these systems but possibly due to the tendency of earthquake reconnaissance teams to report predominantly on the exterior of the building. Whatever the reason, unless the diaphragm decouples from the perimeter walls, damage to the diaphragm generally does not have life-threatening consequences and the diaphragm usually remains serviceable after an earthquake (Bruneau 1994b).

Out-of-plane wall failure is considered the most hazardous URM failure mode to life-safety due to falling debris from parapets and unrestrained walls, and because it potentially compromises the gravity-load carrying capacity of the structure, rendering it susceptible to catastrophic collapse (Bruneau 1994a). The distinct influence that floor diaphragms have on out-of-plane building response is governed by the presence of anchors to the perimeter walls. Often insufficient or no anchorage is provided, in which case the head walls behave as cantilevers over the entire height of the building because they are not adequately tied into the structure (Bruneau 1994b). As a result, large flexural demand is placed on the base of the walls and an out-of-plane collapse is likely to occur. When sufficient floor-to-wall anchorage is provided, the highly flexible nature of the floor diaphragms and subsequent inability to effectively transfer lateral loads to the end walls results in large mid-span deformations and flexural demand on the head walls. The in-plane performance of the diaphragm therefore becomes critical to the out-of-plane response of the URM building. Figure 2-2 clearly illustrates the impact of floor diaphragms on out-of-plane response.

Floor diaphragms in URM buildings are typically considered as deep beams spanning between perimeter walls. The flexible in-plane rotation of these diaphragms combined with the inability to transfer shear forces into the end walls often results in damage to the corner of the building (see Figure 2-3). Corner or ‘punching’ failure is especially common when
long narrow URM structures are excited in their short direction and large diaphragm excitations cannot be transmitted over such small lengths of URM wall (Bruneau 1994b).

### 2.1.2 Experimental research

Consistent damage observations from past earthquakes have spurred numerous laboratory studies focussed on experimentally determining the seismic performance of URM

![Figure 2-2: Typical out-of-plane wall failure, 1989 Loma Prieta earthquake](www.nisee.org)

![Figure 2-3: Typical corner ‘punching’ failure, 1989 Loma Prieta earthquake](www.nisee.org)
structures with flexible timber diaphragms. Not only have these tests provided much needed data on the system-level behaviour of URM buildings, but also many of the behaviours observed were notably linked to the performance of the diaphragms. Perhaps one of the first experimental studies on system-level URM building seismic response was undertaken by Tomazevic et al. (1993) who dynamically tested four, ¼ scale simplified two-storey URM building models. To investigate the influence of diaphragms and adequate floor-to-wall coupling on URM behaviour, each model was constructed with identical masonry but a different diaphragm configuration. Model A comprised a flexible wooden diaphragm with intentionally poor quality anchorage to the out-of-plane URM walls to simulate the behaviour of commonly observed URM structures with insufficient floor-to-wall coupling. Model B was constructed with a comparatively rigid reinforced concrete floor slab to compare the effect of flexible vs. rigid diaphragms. To provide direct comparison to Modal A, Model C was constructed identically, but steel-tie anchors were used to effectively connect the diaphragm to the out-of-plane walls. Model D simulated an alternative construction practice, comprising a timber floor diaphragm at mid-height and a ‘brick vault’ roof diaphragm. Figure 2-4 shows the general elevation of the URM models. Two distinct performances were observed during testing. Total collapse of the second storey occurred in Model A, while damage was primarily focussed at the first floor for Models B, C and D. It was concluded from these observations that adequate floor-to-wall anchorage must be provided to avoid full-height out-of-plane wall collapse, and that the structural characteristics of the diaphragms are critical for the seismic performance of URM structures.

Costley and Abrams (1995) performed dynamic testing of two 1/3 scale, two-storey URM structures constructed from cut pavers with a number of window and doorway openings. At each storey height a flexible diaphragm was built from steel bars that were firmly tied to the exterior walls. Although these diaphragms did not mirror typical URM construction practice, this configuration was considered necessary to resist the considerable vertical loads applied whilst remaining sufficiently flexible to simulate light timber construction. The structures were placed individually on a shake table and subjected to 1985 Nahanni earthquake excitations. The primary observed failure mechanism was the stable rocking of the in-plane piers. As illustrated by the crack pattern in Figure 2-5, this rocking mechanism
effectively isolated the second storey from significant displacement, remaining almost fixed in space while the first floor rocked beneath it. It was noted that the flexible diaphragm systems amplified in-plane wall displacements prior to cracking and provided negligible coupling between the in-plane walls. It was also shown that differential displacement of the in-plane walls was accompanied by the in-plane shear rotation of the diaphragms.

To further investigate flexible floor and rigid URM wall interaction, Paquette and Bruneau (2003) tested a full scale, one-storey URM building pseudo-dynamically using hydraulic actuators attached at roof height. The structure had dimensions of 4.089 m long by 5.690 m wide and 2.724 m high, and was built with standard modular clay bricks. The roof-height diaphragm comprised timber joists with diagonally laid floorboards and a straight-sheathing overlay. To ensure effective floor-to-wall coupling, joist anchors were installed in accordance with the Uniform Code for Building Conservation (ICBO 1997). A doorway and window opening were present in both in-plane shear walls, and an expansion joint was incorporated between one of the out-of-plane walls to investigate the effect of corner
2.1 Diaphragm Influence

Figure 2-5: Wall crack patterns [taken from Costley and Abrams (1995)]

continuity. Observations and test data confirmed that stable pier rocking and sliding occurred without significant global strength reduction. Although not mentioned by the authors, this suggests that the rocking and sliding response of the in-plane piers limited the force developed in the diaphragms, thus enabling them to remain elastic throughout testing (Moon 2004).

In 1997, the Mid-America Earthquake Centre (MAE Centre) was established to address the seismic vulnerability of URM buildings across the Mid-Western United States. This research collaboration comprised several studies investigating URM building component-level and system-level behaviours. Perhaps of most relevance was the study by Simsir (2004) who investigated the influence of diaphragm flexibility on the dynamic response of out-of-plane URM walls. In this study three idealized models representing the essential features of low-rise URM buildings were constructed and dynamically tested on a shake table. ‘Essential’ features included two out-of-plane and two in-plane freestanding walls with a steel floor (or roof) beam installed to simulate a typical timber diaphragm (see Figure 2-6). Because this research was focussed primarily on bearing walls responding out-of-plane, the applied floor mass was supported solely by the out-of-plane walls, while in-plane walls acted as shear walls by resisting the lateral inertial forces exerted by the floor
mass. As outlined in Table 7-1, each model was constructed with a flexible or stiff diaphragm, and varying levels of out-of-plane wall gravity load.

<table>
<thead>
<tr>
<th>Test specimen</th>
<th>Diaphragm beam</th>
<th>Floor mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Stiff</td>
<td>28.9 kN (6.5 kips)</td>
</tr>
<tr>
<td>S2</td>
<td>Flexible</td>
<td>28.9 kN (6.5 kips)</td>
</tr>
<tr>
<td>S3</td>
<td>Flexible</td>
<td>15.6 kN (3.5 kips)</td>
</tr>
</tbody>
</table>

Experimental results showed that diaphragm flexibility amplified mid-span displacement and acceleration with respect to the in-plane wall tops by as much as 7 and 2, respectively, when compared to equivalent tests with a rigid diaphragm. As a consequence, it was concluded that flexible diaphragms can reduce the force demand on out-of-plane walls (at the expense of amplifying out-of-plane wall response), while a stiffer diaphragm can increase these forces and potentially compromise out-of-plane wall stability. Moreover, experimental observations showed that if adequate anchorage is provided, stable rocking of the out-of-plane walls can be generated with a flexible diaphragm. The implications of
these findings are significant as it shows that overall response of the out-of-plane walls is governed by the flexibility and aspect ratio of the diaphragm, and in addition, though it is not mentioned by the author, questions the typical assumption that timber diaphragms in historic URM buildings require mandatory stiffening.

As a capstone to the MAE-Centre projects, Yi et al. (2006) tested a full-scale, two-storey URM building with flexible timber diaphragms before and after the application of several retrofits to investigate system-level URM building response. The structure measured 7.32 m wide by 7.32 m long and 7.14 m high, and was intended to emulate a typical fire station with numerous openings in the URM walls. The building was loaded pseudo-statically at increasing levels of displacement amplitude using hydraulic actuators attached at first and second storey heights (see Figure 2-7). While typical rocking and sliding failure mechanisms of the in-plane walls were observed, a considerable increase in lateral capacity was measured due to the participation of the out-plane-walls to in-plane response (labelled ‘flange effects’), and also overturning moment (illustrated by the wall crack patterns in Figure 2-8). The unexpected influence of these additional mechanisms resulted in maximum wall strength up to 46% larger than that predicted by FEMA 356 (ATC 2000) (now ASCE 41-06). The authors do not discuss the direct influence of the diaphragms but a vastly different response would be expected if the diaphragm stiffness and configuration were altered.

![Figure 2-7: URM test structure](taken from Yi et al. (2006))
Complimentary to the numerous laboratory studies conducted on the seismic performance of URM buildings with flexible diaphragms, a computational analysis of three different structures was presented by Tena-Colunga and Abrams (1996). While two of the buildings are of limited relevance, one of the modelled buildings was a two-storey firehouse with URM walls and timber floor and roof diaphragms. This model represented an actual URM building located in the Mid-Western United States. Diaphragm configuration and boundary conditions were implemented into the model with best practice, including sufficient floor-to-wall steel anchors that were present in the building. Comparative analyses were conducted on the computed seismic response of the structure with flexible diaphragms against a counterpart structure with rigid diaphragms. It was concluded that as diaphragm flexibility increases, diaphragm and shear wall accelerations can increase. Conversely, as diaphragm flexibility decreases, torsional effects can be reduced considerably. It was also noted that when assessing flexible diaphragm systems, design criteria based on rigid diaphragms are not necessarily conservative.
Grubbs et al. (2007) developed a prototype two-storey URM building in SAP2000 to investigate the effect that diaphragm retrofitting, as designed using FEMA (ATC 2000), has on the overall response of URM structures. Analysis results showed that the behaviour of the URM building differed dramatically after diaphragm strengthening. Before retrofitting, the out-of-plane walls appeared to arch away from the building as a cantilever pivoting about the base (see Figure 2-9a). However, when the diaphragm was retrofitted the out-of-plane walls tended to behave as two-span beams supported at floor and roof heights (see Figure 2-9b). Retrofitting the diaphragm was shown to significantly reduce diaphragm and out-of-plane wall displacements, at the expense of higher out-of-plane and in-plane wall stresses, as well as higher base shear values. Grubbs et al. recommend a complete URM building analysis when assessing diaphragm performance as stiffening the diaphragm could have undesirable effects on other components of the structure.

Giongo et al. (2011) also investigated the influence of diaphragm retrofitting on URM building seismic performance using an equivalent-frame model of a two-storey Italian-style URM building developed in SAP2000. The diaphragms were modelled as nonlinear shell elements that were assigned as-built and retrofitted shear stiffness ($G_d$) values determined from experimental testing (Baldeessari 2010). Analysis results indicated that base shear was unaffected by the type of diaphragm retrofit, with all retrofits improving base shear by approximately 20%. This result suggests that diaphragm stiffness only requires retrofitting above a certain threshold in order to adequately improve URM building base shear resistance.

Figure 2-9: Prototype URM building FE model [taken from Grubbs et al. (2007)]
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2.2 Nailed Timber Connections

As a construction material, timber offers little ductility. Timber systems therefore rely primarily on their nailed connections to provide ductility and energy dissipation when subjected to earthquakes. Subsequently, the representative characterisation of individual nailed connections is critical in order to accurately predict the response of complete timber systems. While not all investigations have been focussed specifically on earthquake response, extensive research has been conducted on the monotonic and cyclic performance of isolated nailed timber connections, and serves as an important basis for the current study.

2.2.1 Experimental testing

2.2.1.1 Monotonic loading

The load-slip profile of nailed connections is highly dependent on numerous connection parameters that can be grouped into three primary categories: (1) materials and dimensions of joint components, (2) joint configuration and (3) loading conditions. Monotonic testing has served primarily to characterise the influence of these parameters on joint strength and stiffness (Antonides 1979; Antonides et al. 1980; Ehlbeck 1976).

Several investigations have shown that the moisture content of timber significantly influences connection performance (Albert and Johnson 1967; Leach 1964; Mack 1962). By testing specimens with either dry or green timber, Mack (1962) concluded that the strength of joints constructed with dry timber was on average 30% higher than of joints fabricated with green timber. Only a small difference in joint stiffness was observed, with green timber joints being slightly lower than joints with dry timber.

Leach (1964) reported that the fibre saturation point governs the influence of timber moisture content and found that above the fibre saturation point, moisture content had a negligible effect on the strength and stiffness of nailed connections. However provisions
were made for a 25% reduction in the strength of green timber joints when compared to connections fabricated with dry timber.

Mohammad and Smith (1994; 1996) studied the effects of multi-phased moisture conditioning on the stiffness of oriented strand board (OSB)-to-spruce nailed timber connections. For kiln-dried timber it was found that repeated moisture cycling after initial conditioning resulted in a 20-30% decrease in joint stiffness when compared to joints only subjected to initial conditioning. For specimens fabricated with green timber, no statistically significant changes to joint stiffness were observed as a result of moisture cycling, after the initial moisture conditioning. While not mentioned by the authors, these findings indicate that timber connections subjected to numerous wetting and drying cycles during service could behave considerably different than their expected design performance.

The response of nailed connections to lateral loading is typically a combination of nail yielding and localised timber crushing (Johansen 1949). It is logical then that numerous studies have shown the specific gravity of timber to significantly affect the strength and stiffness of nailed connections (Brock 1957; Mack 1960b; Scholten 1965; Stern et al. 1973). Brock (1957) found that strength increased linearly with timber specific gravity. Scholten (1965) later extended Brock’s findings and developed a curvilinear relationship that predicted maximum connection strength with respect to nail and timber material properties:

$$p_{max} = C_k(SG)^{5/4}$$  \hspace{1cm} (2-1)

where $p_{max}$ is maximum lateral strength, $C_k$ is a constant depending on the size and type of nail, and $SG$ is the specific gravity of the timber members.

The interface characteristics of timber members have a marked effect on joint stiffness (Antonides et al. 1980; Malhotra and Thomas 1982; Thomas and Malhotra 1985). Antonides et al. (1980) found that joint stiffness decreased with increasing interface gap and that gaps produced from wetting and drying cycles had the same quantitative effects on joint stiffness as those fabricated with metal shims. Using experimental results, Antonides et al. established the following equation to predict joint stiffness:
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\[ \log K = C_a - C_b g \]  \hspace{1cm} (2-2)

where \( K \) is joint stiffness (kN/mm), \( g \) is interlayer gap size (mm), and \( C_a \) and \( C_b \) are experimentally determined constants.

Malhotra and Thomas (1982) completed an extensive study on the influence of interface friction and gap size. To determine the lateral force-deformation response of joints with and without friction, nailed connections with interface gaps of 0 mm (no gap), 0.58 mm, 1.19 mm and 1.61 mm were monotonically loaded until ultimate load. It was found that the strength and stiffness of joints tested with friction were higher than those that were frictionless. This effect was more pronounced at proportional limit slip than at ultimate slip (5 mm), indicating that interface friction contributes more significantly to the initial stiffness of nailed connections. The effect of interface gap was observed to increase with increasing gap size. In an attempt to quantify frictional forces, a unique set of tests were performed to determine the coefficient of friction between members and the normal force generated from nailing, which were found to be 0.437 and 163 N, respectively.

In an extension of their 1982 study, Thomas and Malhotra (1985) tested numerous joints fabricated with multiple nails. The objectives of this investigation were to determine whether the product of individual nail capacity and number of nails could predict the strength of multiple-nailed joints, and to also determine whether interface friction influenced these results. Joints fabricated with 2-8 nails, and interface characteristics similar to Malhotra and Thomas (1982) were tested and compared against equivalent joints with single nails. Experimental results indicated that for joints with multiple nails, individual nail contribution is less than that carried by a corresponding joint with a single fastener. It was concluded that for joints with multiple nails, their capacity is up to 10% less than the value of individual nail capacity multiplied by the number of nails. Interface friction was observed to increase linearly with number of nails, in joints with up to three nails. For joints with three to eight nails, interface friction was on average a constant 230 N, which, for practical purposes, suggests that friction has an insignificant effect on multi-nailed joints.
2.2.1.2 Reversed cyclic loading

One of the earliest published studies on cyclic performance was Mack (1960a), who investigated the fatigue characteristics of nailed connections by performing repetitive load testing. Single-nail joints were cycled either 1000 or 10,000 times between various displacement amplitudes up to a maximum of 1.52 mm (0.06 in). At the conclusion of cyclic loading, each joint was loaded monotonically until failure. It was found that ultimate joint capacity was largely unaffected by the number of loading cycles, while residual joint-slip increased with increasing cycles and displacement amplitude.

Wilkinson (1976) subjected nailed and bolted connections to pseudo-static and vibration loads to compare their performances under each loading condition. Nailed connections were tested under various combinations of applied weights (87.1 N to 595.5 N), frequencies (5 Hz to 20 Hz), and accelerations (0.2g to 0.8g). Experimental results indicated that joints were considerably stiffer under dynamic loads than for corresponding joints tested pseudo-statically. It was concluded that a 33% increase in earthquake and wind loads was probably conservative.

Single-nail joints constructed of either plywood or gypsum board sheathing were subjected to four fully-reversed cycles at five increasing levels of load by Atherton et al. (1980). Timber specific gravity, interface friction and sheathing thickness were found to have minimal effect on energy absorption and slip modulus, compared with the effect of load-level. Figure 2-10 provides examples of the observed influences of specific gravity, sheathing thickness and interface friction. It was also observed that joint friction increased joint damping and stiffness, particularly at low loads. While this observation was made for newly constructed joints, the authors recommended that no design provisions be made to account for joint friction because timber used in construction is conventionally prepared at moisture contents that allow shrinkage during service, therefore reducing any initial friction component. Future researchers were also cautioned that newly constructed timber systems may not be representative of actual in-service conditions.
To supplement research on the composite response of wooden floors to vertical loading, Gromala (1985) tested nail connections fabricated with several commonly used sheathing materials, including plywood, flakewood, hardboard and gypsum board. Analytical procedures at the time typically required a ‘slip modulus’ to account for the lateral resistance of nailed sheathing in the combined flexure of wooden floors when loaded vertically. Results from testing showed that the selection of linearised slip moduli was extremely sensitive to connection parameters and subjective in nature. It was suggested that the characterisation of nonlinear load-slip relationships for various nailed connections would be more beneficial.

In an apparent extension of Wilkinson (1976), Soltis and Mtenga (1985) sought to determine whether static load design criteria applies to dynamically loaded timber structures. To achieve this objective, single-nail joints were tested either monotonically until failure, or cycled forty times at several combinations of frequency level and displacement amplitude. Comparison of test results showed that at low displacement levels, a higher cycling frequency increased lateral joint resistance, while at higher displacement levels the opposite was true. Strength and stiffness degradation per cycle was
observed to increase in magnitude as displacement amplitude increased. It was also shown that ultimate joint strength after forty cycles was slightly lower than static tests loaded to the same displacement level. It was concluded that deformation level and number of cycles determines whether or not there is a significant difference between statically and dynamically loaded joints.

As a contribution to the design of timber structures for wind and earthquake loads in New Zealand, Dowrick (1986) performed cyclic tests on a number of timber assemblages to establish their hysteretic response. Testing included moment resisting joints, shear walls clad with various materials and several connection configurations. Force-displacement curves plotted from nailed-connection tests showed changing hysteretic shape at increasing levels of displacement. As illustrated in Figure 2-11, a number of commonly observed hysteretic characteristics were demonstrated: (1) highly nonlinear response with no distinct yield point, (2) progressive loss of strength and stiffness with increasing number of cycles, (3) heavily pinched hysteretic loops.

Chou and Polensek (1987) reported that two sources of damping are present within timber structures: (1) damping within the timber itself, and (2) damping due to slip or frictional forces between members. Test results from various plywood-to-wood connection configurations provided several noteworthy observations:

![Figure 2-11: Hysteretic response of nailed plywood-to-timber joint](taken from Dowrick (1986))
At small loads, damping between members dominates response. Damping ratio was less for specimens fabricated with an interface gap than those tested without. For connections without an interface gap, damping ratio decreased with increasing load. Due to shrinkage, connections constructed of green timber and allowed to dry before testing performed synonymous to connections fabricated with an interface gap.

Polensek and Bastendorff (1987) extended the study of Chou and Polensek (1987) by cyclically testing a variety of connection configurations. With respect to a given nail size and load level, it was found that timber species had the greatest effect on damping in comparison to grain orientation. This suggests that a large variability in damping ratio can exist between nailed connections constructed with different timber species.

Dean et al. (1989) explained that the highly pinched hysteresis loops of nailed timber connections is caused by slackness developed during cyclic loading. When the nail shank is in direct contact with the timber, its resistance to lateral deformation is generated through combined nail yielding and localised timber crushing. Figure 2-12 illustrates the cavity formed behind the nail when a joint is loaded laterally. However when this load is reversed, nail bending provides the sole resistance to lateral deformation and subsequently a significant drop in joint strength and stiffness is observed (see Figure 2-11). This reduction in performance exists until previous deformation levels are exceeded, at which time joint strength and stiffness once again increase as new timber is crushed.

As part of an extensive study investigating the lateral behaviour of timber shear walls, Dolan and Madsen (1992) tested several nail connections fabricated with either plywood or waferboard sheathing. For each of the monotonic and cyclic tests performed, force-displacement response was fitted to the exponential functions proposed by Dolan and Foschi (1991), and used to compare the influence of grain orientation and loading rate. Test results indicated that grain orientation had a minimal effect on joint performance; apart from post-yield stiffness, which was shown to be higher for joints loaded
2.2 Nailed Timber Connections

(a) Localised wood cavity under monotonic loading
(b) Localised wood cavity under reversed cyclic loading

Figure 2-12: Nail yielding and localised timber crushing [taken from Ni (1997)]

perpendicular to grain. The premise that hysteretic response is fully enclosed within the monotonic force-displacement curve was confirmed. Comparison of slow (10 mm/min) and rapid (300 mm/min) tests indicated that load rate has little effect on connection performance. Though not mentioned by the authors, this suggests that pseudo-static test data may be reliable for dynamic loading of timber systems. Dolan and Madsen concluded that nail properties govern the load-displacement response of sheathing connections with common nails. Sheathing type does have an influence the load-slip relationship but primarily near the ultimate load capacity of the connection.

The effects of cyclic loading on timber joint performance was investigated by Dolan et al. (1994), who compared monotonic and cyclic test results for a range of nailed and bolted connection configurations. With applied cyclic loads reaching over 2.0 times typical design values, it was shown that prior cyclic loading of nailed joints has no adverse effect on ultimate capacity and ductility.

Ni and Chui (1994) further studied the effects of loading regime on the hysteretic behaviour of nailed joints. Twenty groups of specimens were subjected to both monotonic and reversed-cyclic loads. Comparison of joints with and without interface gaps showed that interface friction has negligible effect for low-density members. Overall, a higher ultimate load was obtained under reversed cyclic loading than for monotonic loading,
though a degradation of strength was observed with increasing number of cycles. Ni and Chui also reported that low cycle fatigue of the nail could occur under reversed cyclic loading.

2.2.2 Empirical modelling

Numerous efforts have been made to describe the hysteretic response of nailed timber connections using empirical equations. Models of this form are based solely on experimental observation and are generally developed by fitting mathematical functions to force-displacement curves generated from physical testing. An early study conducted by Mack (1966) found that the monotonic load-displacement relationship of nailed connections could be described as:

\[
P = k_\lambda (C_a \delta + C_b) \left(1 - e^{C_c \delta}\right)^{C_d}
\]

(2-3)

where \(P\) is load, \(k_\lambda\) is the product of parameter functions, \(\delta\) is joint slip, and \(C_a, C_b, C_c\) and \(C_d\) are constants.

McLain (1975) proposed a two-coefficient logarithmic load-slip model:

\[
P = A \log(1 + B \delta)
\]

(2-4)

where \(P\) is load, \(\delta\) is joint slip, and \(A\) and \(B\) are coefficients. Using the results of forty nailed connection tests, McLain was able to show that coefficient \(A\) was strongly correlated to the specific gravity of the timber members, while coefficient \(B\) could be predicted by rearranging Equation 2-4 and selecting known coordinates from test results. The application of McLain’s equation was extended by Stone (1980), who calculated correction factors for coefficients \(A\) and \(B\) to account for side member thickness, nail diameter and interface gap. Further provisions were made to Equation 2-4 for timber moisture content and specific gravity (SaRibeiro and SaRibeiro 1991), and a range of commonly available sheathing products (Pellicane 1985).
When subjected to reversed-cyclic loads, nailed timber joints display a unique hysteretic trace (Figure 2-11). Developed empirical models have generally comprised two parts: a virgin envelope curve (which connects all the peak points of the hysteresis loops), and hysteresis loops (a closed loop under reversed cyclic loads) (Ni 1997). Kivell et al. (1981) adapted Takeda’s idealised reinforced concrete model (1970) to predict the behaviour of moment-resisting nailed joints. As illustrated in Figure 2-13, symmetric bilinear paths were used for the backbone curves while trilinear paths were used to describe the hysteretic loops during unloading and reloading phases. Locations ABCD on the trilinear trace were determined by assuming a cubic function between maximum positive and maximum negative displacements. Polensek and Laursen (1984) later modified Kivell’s model for plywood-to-wood connections. The bilinear backbone curve was replaced with a trilinear idealisation while governing points on the hysteretic loops were determined by statistically fitting test data. Both of these models successfully captured some pinching behaviour but failed to account for strength and stiffness degradation.

Stewart (1987) developed a similar model to Kivell et al. for sheathing-to-lumber nailed connections. This model employed a trilinear backbone trace that included strength degradation. Idealised hysteretic pinching and stiffness degradation were also accounted for using sets of force-history rules.

Instead of using a multi-linear idealisation, Dolan (1989) described the envelope curve using the exponential function proposed by Foschi (1974). Each hysteresis loop was divided into four segments and described by an individual exponential equation with four boundary conditions (Figure 2-14). Strength was assumed to go to zero if joint-slip exceeded a pre-described maximum value. Load intercept $P_0$ and stiffness at zero slip $K_d$ were assumed to remain constant.

A different approach was offered by Foliente (1995), who applied the so-called Bouc-Wen-Barbi-Noori (BWBN) hysteresis model to timber connections. Hysteretic force in this model was related to joint-slip by a first order nonlinear differential equation. While the model was considered applicable to many joint configurations, it failed to adequately capture the heavy pinching characteristic of timber connections.
2.2.3 Theoretical modelling

Unlike an empirical approach, theoretical models are derived from fundamental mechanical behaviour so that the functions governing response have real physical meaning. Early analytical efforts assumed the fastener to act like a beam on an elastic foundation, with resistance being proportional to deformation (Kuenzi 1955; Noren 1961;
Wilkinson 1971; Wilkinson 1972). This technique successfully predicted connection response until the proportional limit; meaning that it is capable of predicting initial stiffness, but not maximum strength, ductility or residual deformation because true nailed connection behaviour is nonlinear.

Larsen (1973) proposed that the stress-strain relationships of the fastener steel and timber embedment were perfectly elasto-plastic. This enabled the prediction of performance values such as bearing capacity, yield strength and ultimate strength but not the complete load-slip response. Foschi (1974) offered a solution to this limitation in the form of a small-displacement finite element model for single shear nailed connections. Like Larsen (1973), fastener behaviour was considered to be perfectly elasto-plastic, however a nonlinear expression was developed for timber load-embedment (see Figure 2-15). Using the principles of virtual work the entire force-displacement response could be generated, from which initial stiffness and ultimate load could be determined.

Numerous improvements have been made to the finite element analysis performed by Foschi. Malhotra and Thomas (1982) included the effect of interface friction when timber members were in contact. This was achieved by proposing that friction force was equal to the coefficient of friction multiplied by the normal force generated from fastener tensioning. Hunt and Bryant (1990) accounted for different fastener end conditions by including a rotational spring, provided by the nail head. This was particularly relevant for

![Figure 2-15: Mechanical basis of Foschi model [taken from Foschi (1974)]](image)

(a) Assumed elastic perfectly-plastic stress-strain relationship of fastener

(b) Load-embedment relationship for timber

- 37 -
connections fabricated with thin steel side plates, where the nail head is restricted from rotating freely. Koponen (1991) modified Foschi’s model for double shear wood-to-wood and steel-to-wood joints with dowel type fasteners. Adjustments for the foundation modulus were also made to cater for initial slip caused by bolt-hole clearances.

Smith (1983) and Erki (1991) both developed finite element models to predict the load-slip behaviour of two and three member non-symmetric connections. Smith targeted single circular shaped dowels with no rotational end restraints, and provided provisions for small and large displacements, and also interface friction. Instead of focussing on individual fastener configurations, Erki included key characteristics of fastener behaviour for nail, glulam rivet, bolt, dowel and lag screws. Interface friction characteristics, withdrawal effects, and combined fastener bending and axial tension were also taken into consideration for joint-slip.

More recently, theoretical modelling has focussed on the response of nailed timber connections to reversed-cyclic loading. Ni et al. (1993) proposed a simplified model based on the nail shank acting as a continuous beam bearing on a nonlinear Winkler foundation. The moment-curvature relationship of the fastener was taken as rigid perfectly-plastic, while the load-embedment relationship proposed by Dolan (1989) was adopted. Cruz (1993) offered improvements to this model by proposing a bi-linear moment-rotation relationship for the fastener, and by fitting Dolan’s model to test data. The model provided a reasonable qualitative representation of hysteretic response but failed to capture typical pinching characteristics due to the neglect of fastener elastic deformation.

A more sophisticated finite element model that included large displacement theory was developed by Chui and Ni (1997), and Chui et al. (1998). The model was comprised of three primary elements: (1) a three-node plane beam element to represent the fastener, (2) a spring element for embedment, and (3) a linkage element for the friction force between the fastener shank and wood embedment. The embedment springs were defined in two parts: backbone response, which was defined by Foschi’s (1974) relationship that was modified by the authors to account for strength degradation, and hysteresis response, which was defined using the method proposed by Dolan (1989). The model was advantageous as it
2.3 Timber Floor Diaphragms

This section summarises published literature regarding experimental testing and analytical modelling of the in-plane performance of timber floor diaphragms.
2.3.1 Experimental testing

Timber floor diaphragms typically found in historic URM buildings have received very little research attention. The Agbadian Barnes Kariotis joint venture (ABK 1981) was perhaps the first significant investigation on the seismic performance of such diaphragms, as part of an extensive study tasked with developing a methodology for mitigation of seismic hazards in existing URM buildings in California. Diaphragm testing consisted of intermingled pseudo-static and dynamic loading of fourteen different diaphragm specimens (see Table 2-2). It is noteworthy that only configuration ‘E’ has particular relevance to unretrofitted diaphragms in New Zealand URM buildings, and even this configuration includes ‘built-up roofing’ which is not applicable to floors. The total number of specimens tested by ABK comprised a number of modifications or retrofits to already tested specimens in order to save time and construction costs, and to develop a direct comparison between performances. All diaphragms measured 18.3 m by 6.1 m (aspect ratio 3:1) and were constructed from new Douglas-fir timber. Applied retrofits included plywood, steel decking and concrete slab overlays. As shown in Figure 2-16, pseudo-static testing was performed using hydraulic actuators attached at each end of the diaphragm specimens and by providing reaction points installed at 1/3-locations. Figure 2-17 illustrates the unique test setup used to replicate dynamic earthquake excitation of the diaphragm. Fifteen 1-tonne weights on low friction rollers were attached to the out-of-plane sides of the diaphragm to simulate wall inertia as the diaphragm was dynamically loaded at its ends.

Tested diaphragms demonstrated highly nonlinear hysteretic stiffness characteristics, particularly during dynamic excitation above an effective peak acceleration of 0.1g. Built-up roofing was reported to add considerable stiffness to the diaphragm, as long as it remained attached during loading. For the most part, diaphragms exhibited little damage and remained serviceable after testing. At the conclusion of the testing program, the authors recommended that reverse-cyclic testing should be performed to provide complete hysteretic behaviour. They also recommended the benefits of dynamic testing and investigating the effect of diaphragm aspect ratio.
Table 2-2: Diaphragm specimen description [reproduced from ABK (1981)]

<table>
<thead>
<tr>
<th>Diaphragm identification</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>20-ga steel decking, unfilled, unchorded, button-punched seams at 457 mm c/c</td>
</tr>
<tr>
<td>R</td>
<td>20-ga steel decking, unfilled, chorded, button-punched seams at 152 mm c/c</td>
</tr>
<tr>
<td>C</td>
<td>13 mm plywood, unblocked, unchorded, built-up roofing</td>
</tr>
<tr>
<td>D</td>
<td>13 mm plywood, unblocked, chorded, built-up roofing, roofing retrofit nailing</td>
</tr>
<tr>
<td>B</td>
<td>13 mm plywood, unblocked, chorded</td>
</tr>
<tr>
<td>E</td>
<td>25 mm x 152 mm straight sheathing, unchorded, built-up roofing</td>
</tr>
<tr>
<td>E₁</td>
<td>25 mm x 152 mm straight sheathing, unchorded, built-up roofing, roofing retrofit nailing</td>
</tr>
<tr>
<td>H</td>
<td>25 mm x 152 mm straight sheathing, 8 mm plywood overlay, chorded</td>
</tr>
<tr>
<td>I</td>
<td>25 mm x 152 mm diagonal sheathing, unchorded, built-up roofing</td>
</tr>
<tr>
<td>I₁</td>
<td>25 mm x 152 mm diagonal sheathing, unchorded, built-up roofing, roofing retrofit nailing</td>
</tr>
<tr>
<td>K</td>
<td>25 mm x 152 mm diagonal sheathing, 25 mm x 152 mm straight sheathing overlay, chorded</td>
</tr>
<tr>
<td>N</td>
<td>13 mm plywood, blocked, chorded</td>
</tr>
<tr>
<td>P</td>
<td>19 mm plywood, 19 mm plywood overlay, blocked, chorded</td>
</tr>
<tr>
<td>S</td>
<td>20-ga steel decking, 63.5 mm concrete fill, chorded, button-punched seams 457 mm c/c</td>
</tr>
</tbody>
</table>

The seismic performance of timber floor diaphragms was later investigated by Peralta (2003) who tested three specimens with varying construction details typical of pre-1950’s URM buildings in the Mid-Western United States. Three floor specimens measuring 7.32 m by 3.66 m (aspect ratio 2:1) were constructed from new Southern Pine timber and tested in their unretrofitted states. Each diaphragm was then retrofitted using common techniques and retested to determine the effectiveness of the retrofit. The first unretrofitted specimen, labelled MAE-1, comprised tongue and groove (T&G) floorboards and was loaded in the direction orthogonal to the joists. Unretrofitted diaphragms MAE-2 and MAE-3 were both constructed with straight-edge floorboards and loaded parallel to the joists with the only difference being that MAE-3 contained a corner penetration to simulate a stairwell opening. Table 2-3 summarises the tested diaphragms including a description of the retrofit techniques employed. Pseudo-static loads were provided by a hydraulic actuator and introduced into the diaphragm using a large H-shaped steel frame that was bolted through the floorboards alongside the joists at one third diaphragm width locations (see Figure 2-18).
Figure 2-16: Pseudo-static test schematic [taken from ABK (1981)]

Figure 2-17: Dynamic test schematic [taken from ABK (1981)]
2.3 Timber Floor Diaphragms

Table 2-3: Diaphragm specimen description [reproduced from Peralta (2003)]

<table>
<thead>
<tr>
<th>Diaphragm identification</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE-1</td>
<td>25 mm x 100 mm T&amp;G sheathing, star anchors</td>
</tr>
<tr>
<td>MAE-1A</td>
<td>MAE-1 plus enhanced bolted connections and perimeter steel strapping</td>
</tr>
<tr>
<td>MAE-1B</td>
<td>MAE-1A plus steel truss</td>
</tr>
<tr>
<td>MAE-2</td>
<td>25 mm x 150 mm straight-sheathing, bolted connections, unchorded</td>
</tr>
<tr>
<td>MAE-2A</td>
<td>MAE-2 plus steel truss</td>
</tr>
<tr>
<td>MAE-2B</td>
<td>MAE-2 plus 9.5 mm unblocked plywood overlay</td>
</tr>
<tr>
<td>MAE-2C</td>
<td>MAE-2 plus 9.5 mm blocked plywood overlay</td>
</tr>
<tr>
<td>MAE-3</td>
<td>25 mm x 150 mm straight-sheathing, bolted connections, unchorded, corner opening</td>
</tr>
<tr>
<td>MAE-3A</td>
<td>MAE-3 plus 9.5 mm unblocked plywood overlay</td>
</tr>
<tr>
<td>MAE-3B</td>
<td>MAE-3 plus 9.5 mm blocked plywood overlay, steel strap at opening</td>
</tr>
</tbody>
</table>

Figure 2-18: Photograph of diaphragm test setup [taken from Peralta (2003)]

Test observations from Peralta (2003) confirmed that the unstrengthened diaphragms had high flexibility by exhibiting large deformations at relatively low loads. This was particularly true for diaphragm MAE-1, which resisted around 6 times less force than diaphragms MAE-2 and MAE-3, at corresponding displacement levels. All diaphragms displayed little damage, with the mid-span deflection reaching the 2% drift limit (ATC
2000) without significant yielding occurring. Experimental results indicated that the four rehabilitation methods accomplished the objectives of increasing in-plane lateral shear strength and stiffness. The steel truss retrofit was found to provide the largest increase, followed by the plywood overlays. A comparison between experimental results and predictions using FEMA 273 and FEMA 356 (now ASCE 41-06) was also reported. Using guidelines from FEMA 273, a bilinear representation of the backbone hysteretic response curves was developed to establish yield strength, yield deformation and pre- and post-yield stiffness values. These values were compared to those predicted by the equations provided in the FEMA 273 and FEMA 356 assessment guides. It was concluded that the backbone curves developed from FEMA 273 tended to over-predict stiffness and significantly under-predict yield displacement, while FEMA 356 conservatively predicted strength, stiffness and deformability values. Dynamic testing of timber diaphragms with varying aspect ratios and consideration of the effects of aging and decay on their performance were recommended for future research.

Corradi et al. (2006) reported on the shear behaviour of unretrofitted and retrofitted European-style floor diaphragms typically found in historical URM buildings in Italy. These types of diaphragms employ considerably different configurations than those common to the United States or New Zealand. Specimens were constructed using joists with 180 mm by 180 mm cross-section, and either 600 mm long timber floor planks or floor bricks. For the purposes of the current study, only test results from plank-diaphragms are discussed. All specimens measured 3.0 m x 3.0 m and were pseudo-statically tested using the setup illustrated in Figure 2-19. This loading configuration was designed to isolate shear response by racking the diaphragm. For each test specimen the measured force-displacement response was used to calculate the shear stiffness; defined as the secant of the backbone curve at 1/3 maximum load:

\[
k_{1/3} = \frac{F_{1/3} - F_i}{\gamma_{1/3}d}
\]  

\[
F = kx \equiv k(\gamma d)
\]
2.3 Timber Floor Diaphragms

where \( k \) is shear stiffness, \( F \) is applied force, \( \gamma \) is angular strain, and \( d \) is the length of the edge perpendicular to the direction of the applied shear force. Test results showed that unretrofitted diaphragms offered extremely low shear stiffness and strength, and retrofit techniques were shown to increase the shear stiffness, \( k \), by as much as 20 times.

Baldessari (2010) and Piazza et al. (2008a; 2008b) also investigated the behaviour of Italian-style diaphragms, with a focus on diaphragm stiffness and the effectiveness of retrofit techniques. Small (1 m x 2 m) and large (4 m x 5 m) specimens were pseudo-statically loaded in the direction of the joists before and after retrofitting (Figure 2-20a). At the expense of physical representation, the authors avoided realistic boundary conditions due to the inherent difficulty of characterising them during FE analysis. Instead, the end joists were fixed to steel beams that pivoted about a single central hinge, hence generating a simple and analysis-friendly boundary condition. An equivalent shear modulus was calculated for each test and used to compare performance. Figure 2-20b illustrates that size-effects were considerable and that the adopted retrofitting techniques increased diaphragm stiffness by as much as one hundred times. For ease of construction and improved seismic energy dissipation, the authors recommend the diagonal floorboard overlay and plywood overlay diaphragm retrofit systems.

**Figure 2-19: Test setup [taken from Corradi et al. (2006)]**
In a joint research study between the University of Genoa in Italy, and the University of Canterbury in New Zealand, Brignola (2009) experimentally evaluated the in-plane stiffness of timber diaphragms for a proposed procedure for the performance-based retrofit procedure of URM buildings. Brignola pseudo-statically tested five medium-scale as-built diaphragms in the direction parallel-to-joists, before retrofitting each with a plywood overlay and retesting using the same loading protocol. Four of the as-built diaphragms represented typical New Zealand construction (see Chapter 1), while the other represented typical Italian-style construction with 200 mm x 100 mm joists. All diaphragms measured 4 m perpendicular to the applied loading (span) and 3 m parallel to the applied loading (width). The as-built diaphragms were either chorded or unchorded, while the plywood retrofits were either left unblocked or blocked with stapled sheet metal straps.

Test results from Brignola (2009) demonstrated that the addition of a plywood overlay significantly improves diaphragm performance, with shear strength increasing by two to three times, and initial stiffness increasing by up to seven times. The test results suggest that as-built diaphragm shear strength is not significantly influenced by the presence of perimeter chords, although initial stiffness generally increased by 150%. Both shear strength and initial stiffness were considerably improved by perimeter chording for the plywood-retrofitted diaphragms. In comparison to assessment procedures, diaphragm shear
2.3 Timber Floor Diaphragms

stiffness was found to be considerably under-predicted using the methodology offered by NZSEE (2006) and over-predicted using the guidelines of ASCE 41-06 (2007).

Though it offers less relevance to the current study, extensive testing has also been performed on plywood-sheathed floor diaphragms. A brief summary of literature relating to this field of research is reported hereafter. The Engineered Wood Association, formerly known as the American Plywood Association (APA) has performed extensive diaphragm testing since the 1950’s. Countryman (1952; 1955) conducted a series of tests that provided the design framework for the use of plywood as an efficient shear-resistant system which was later updated by Tissell (1967) due to changes in plywood manufacturing. More recently, Tissell and Elliot (1997) tested eleven diaphragm configuration to resist high shear loads sometimes required for buildings in high seismic regions. It was concluded from this research that increasing the number of fasteners, which prevents the timber from splitting, and adding a second layer of plywood in the areas of high shear, are the best methods for obtaining higher diaphragm shear strength. These methods are particularly useful around openings in the diaphragm by offsetting the weakening effect that they have.

The strength and stiffness of timber diaphragms was investigated by Johnson (1956) at the Forest Products Laboratory at Oregon State University. Sixteen full-scale lumber sheathed and four plywood sheathed diaphragms were pseudo-statically deformed to various levels while comparing differing characteristics, including timber sheathing humidity, nailing pattern, type of boundary members, bridging, blocking, connections, openings and aspect ratios. Each diaphragm was monotonically loaded at fifth points on the transverse chord member while displacement readings were taken at midspan on the non-loaded chord member and end posts. From test observations and data analysis, it was concluded that the straight-sheathed timber diaphragms were inadequate in resisting lateral loads. It was also concluded that the use of additional chord members on one of the diaphragms increased the strength by 30% while using dry timber instead of green timber resulted in a strength increase of about 40%. The use of blocking was found to significantly reduce the deflection of plywood sheathed diaphragms.
Zagajeski et al. (1984) further investigated the in-plane response of plywood-sheathed floor diaphragms. Several full-scale diaphragms measuring 4.90 m by 7.32 m and 4.90 m by 4.90 m were constructed and subjected to dynamic, pseudo-static monotonic and pseudo-static cyclic loads. Different diaphragm configurations were tested and compared to determine the effect of various parameters, including: blocking, diaphragm openings, plywood thickness, corner stiffeners and nail size. Applied load, lateral shear displacement, interpanel slip, relative slip between the diaphragm specimen and steel loading frame, and support displacements were all measured during testing. An equivalent viscous damping ratio of 15% to 20% was determined from dynamic testing. It was found that blocking significantly improved diaphragm behaviour, while thicker plywood panels were shown to have a negative effect on nail slip due to the reduction in nail penetration depth. A significant conclusion of this study was that in-plane response was controlled by the nail slip characteristics of joints between adjacent plywood panels and between panels and boundary elements.

Filiatrault et al. (2002) explored the influence of in-situ conditions on the in-plane flexibility of plywood-sheathed diaphragms. Quasi-static testing was performed on a full-scale wood floor diaphragm in a two-storey wood frame house measuring 4.9 m by 6.1 m. Chord members were fixed to the longer edges while the shorter sides were blocked. A total of fourteen different configurations were tested. The initial eight tests were performed with only the ground floor constructed, during which the influence of nail schedule, panel edge blocking and addition of sub-floor adhesive were compared. The upper storey of the house was completed for the final six tests that were performed to compare the effects of modifications to the building, while the diaphragm configuration was kept constant. Applied load and displacement response were measured during testing, and the global, shear and flexural stiffness of each diaphragm configuration were subsequently calculated. Experimental results indicated that the nailed diaphragms exhibited significant shear hysteretic behaviour even at low levels of deformation and that shear deformations contribute up to approximately 75% of this response. It was believed that this response was caused by friction between wood members, as opposed to inelastic response of the nails. It was concluded that blocking at unsupported plywood edges significantly increases shear rigidity, while flexural rigidity is largely dependent on the chord members.
Significant damage to wood frame residential buildings observed after the 1994 Northridge earthquake motivated focussed research to develop reliable and economical methods of improving the seismic performance of these buildings. Subsequently, the Consortium of Universities for Research in Earthquake Engineering (CUREE) and the California Institute of Technology collaborated on a project that involved testing plywood panel diaphragms to investigate the factors contributing to stiffness (CUREE 2002). A number of conclusions and recommendations were made as follows:

- The negative effect that unbalanced nailing patterns have on shear wall capacity is directly applicable to diaphragms. Nail patterns should be symmetric.
- The use of adhesives between joists and plywood sheathing significantly increases the shear stiffness.
- Stiffness is greatly increased when the edges of the diaphragm are chorded.
- The addition of blocking increases the shear stiffness of the diaphragm and is an effective method for reducing deflections.
- Openings in the diaphragm have a significant effect on the overall stiffness and local shear stiffness.

2.3.2 Modelling

Few numerical models have been developed for timber diaphragms typical of those found in URM buildings. One of the early efforts was the lumped parameter model developed by Ewing et al. (1980) as part of the ABK (1981) joint research venture on timber diaphragms. The model smeared nonlinear response by idealising the diaphragm as a deep shear beam, divided into several shear segments with a lumped mass at each interface. Each segment contained a nonlinear spring characterised by a second-order backbone curve and multi-linear hysteretic trace, and also a viscous damper (see Figure 2-21). Calibration of the spring force-displacement relationship using experimental data was suggested. The analytical model was subjected to dynamic earthquake records, and a range of representative parameters were used for plywood-sheathed and diagonal-sheathed diaphragm configurations.
Further analysis of diaphragms in URM buildings did not come until Peralta (2003), who as part of an investigation into the seismic performance of rehabilitated floor diaphragms developed FE models using the structural analysis software ABAQUS (2003). Diaphragms with typical straight-edge and T&G sheathing (as-built) were modelled, as well as diaphragms with plywood overlays (rehabilitated). The models generally consisted of linear 2D shell elements (flooring) connected to linear frame elements (joists) using nonlinear spring pairs orientated in each principal direction (nailed connections). In an attempt to improve computational efficiency, each floorboard was assumed to span the entire width of the diaphragm, when in fact they were staggered in their arrangement, and was modelled using only one shell element. While analysis may have been less cumbersome, this approach introduced unnecessary error into the performance prediction of the as-built specimens. Spring elements can only be placed at nodal locations in ABAQUS, meaning that nail couple spacing at each floorboard end was increased by approximately 38 mm (~40%). This increase provided a larger moment couple and subsequently over-predicted resistance to board rotation during diaphragm loading. The load-slip relationship of the nonlinear springs was characterised in two parts: (1) monotonic response, using the empirical equations developed by McLain (1975), and (2) hysteretic trace, described by adapting the three-parameter model developed by Park et al. (1987) originally designed for reinforced concrete. Representative values of parameters $A$ and $B$ for McLain’s equation were selected, while the parameters controlling hysteretic stiffness, strength degradation and pinching behaviour were calibrated by comparing FE
2.3 Timber Floor Diaphragms

model predictions against experimental results. All frictional forces within the diaphragm were ignored.

FE models developed for unretrofitted diaphragm specimens MAE-1, MAE-2 and MAE-3 (see Table 2-3) were subjected to displacement amplitudes equivalent to the performed experiments, and the response for each was compared. It was shown that for diaphragm MAE-1, which was loaded in the direction perpendicular to the joists, the FE model grossly under predicted diaphragm strength and stiffness, and failed to capture a representative force-displacement profile (see Figure 2-22a). This was primarily attributed to the inability to accurately model the anchorage of the joists. For diaphragms MAE-2 and MAE-3, which were loaded in the direction parallel to the joists, the FE model offered a more comparable performance prediction when compared to corresponding experimental results (see Figure 2-22b).

Brignola (2009) developed FE models of tested timber diaphragm configurations in structural analysis software ANSYS (1998) in order to evaluate diaphragm effective elastic stiffness, \( G_d \). Similar to Peralta (2003), the FE models were developed on the assumption that timber framing members (floorboards and joists) remain elastic during diaphragm deformation, and that no mechanical interaction occurs between these members. The nail connections were modelled as elastic beam elements that were assigned fictitious

![Diaphragm MAE-1](a) Diaphragm MAE-1 ![Diaphragm MAE-2](b) Diaphragm MAE-2

Figure 2-22: FE model comparison [taken from Peralta (2003)]
material properties to capture representative effective nail connection stiffness ($k_n$). The developed models were shown to capably predict diaphragm effective elastic stiffness, but by design were unable to capture the nonlinear force-displacement response of timber diaphragms.

A simplified numerical modelling technique for as-built and retrofitted timber diaphragms was proposed by Baldessari et al. (2010; 2009) using the structural analysis program SAP2000 (CSI 2004). Instead of modelling every floorboard and nail connection individually, a one-dimensional grid system was developed between the joist members, which were modelled as elastic frame elements. The reticular grid system was formed using nonlinear spring elements orientated perpendicular and at 45° angles to the joist elements, as shown in Figure 2-23. The nonlinear springs were assigned force-displacement curves which were calibrated from diaphragm experimental results in order to match overall diaphragm behaviour. The simplified form of the model enabled the analysis of different retrofitting techniques, which would have been difficult to model explicitly. The model was shown to capably capture experimental results for both as-built diaphragm configurations, and for a range of retrofitted diaphragm configurations. Analysis was only performed in the direction parallel-to-joists.

Figure 2-23: 1-D diaphragm FE model [taken from Baldessari (2010)]
2.3 Timber Floor Diaphragms

To the best of the author’s knowledge, the above represents the only published analytical models developed specifically for diaphragms in URM buildings. While these types of diaphragms have received little analytical attention, plywood floor and plywood wall diaphragms have been modelled extensively at various levels of complexity. The simplest shear wall models take their form as single degree of freedom (SDOF) systems (Foliente 1995; Stewart 1987). SDOF models are developed by calibrating the relationship between applied force and displacement at the top of the shear wall using experimental data. Because the equations of motion are straightforward, the advantage of this approach is that the developed models may easily be employed in dynamic analysis. The drawback, however, is that the model is not ubiquitous as it must be calibrated to the specific diaphragm configuration and material properties used in each construction. It is also noteworthy that while the idealised racking behaviour employed for SDOF systems is appropriate for shear walls, it is not so relevant for horizontal diaphragms.

A number of numerical models have been developed based on the premise that nonlinear global diaphragm deformation is attributed to the nonlinear load-slip characteristics of the sheathing-to-framing connectors (Dinehart and Shenton Ii 2000; Filiatrault 1990; Folz and Filiatrault 2001; Gupta and Kuo 1985; Gupta and Kuo 1987). In this case, the timber framing is assumed to remain rigid, and all sheathing panels are assumed to undergo equivalent rotation and translation. Generally the global force-displacement relationship for these models is developed by equating internal strain energy with the potential energy generated from virtually applied displacements. This modelling approach is considered computationally efficient, though only overall wall response can be predicted.

Sophisticated FE models that are capable of capturing intercomponent response and force distribution within the diaphragm have also been developed (Dolan 1989; Dolan and Foschi 1991; Falk and Itani 1989; Foschi 1977; Itani and Cheung 1984; Judd and Fonseca 2005). For this class of model, joists and other framing are represented by linear beam elements. Sheathing is modelled using linear plane-stress (shell) elements, and sheathing-to-framing and framing-to-framing connections are generally provided by nonlinear spring pairs that are characterised by representative load-slip relationships of nailed timber connections. Interestingly, despite increased complexity, these models have been shown to
provide similar accuracy to simpler numerical models when predicting global shear wall response (Folz and Filiatrault 2001).

2.4 Summary

An extensive literature review was provided to establish the current state-of-the-art relating to the in-plane performance of timber floor diaphragms in URM buildings. A review of published earthquake reconnaissance reports and system-level URM research has illustrated that floor diaphragms have a significant influence on the overall seismic performance of URM structures, yet there appears to be a considerable lack of data associated with the behaviour of such diaphragms.

Nailed connections have been comprehensively studied through experimental testing and analytical modelling. Research has shown that the load-slip relationship of these joints is primarily controlled by joint configuration and material properties. Numerous empirical and theoretical models have successfully captured the highly nonlinear behaviour of nailed timber connections, including undefined yield point, strength and stiffness degradation and hysteretic pinching. There remains a need to quantify the performance of typical nailed connections in New Zealand URM buildings, as the effects of deterioration and out-dated construction materials are largely unknown.

Despite extensive testing on plywood-sheathed diaphragms, square-sheathed and T&G-sheathed diaphragms have received little research attention. While limited testing has demonstrated that straight-sheathed diaphragms are highly nonlinear, flexible, and remain largely serviceable after loading, it is evident that diaphragm scale-effects and orthotropic behaviour require further investigation. Friction mechanisms between diaphragm components are yet to be established, as well as the influence that sheathing type has on these mechanisms and overall diaphragm performance.
Analytical modelling of timber diaphragms has generally been based on the premise that diaphragm nonlinearity is fully attributable to the nonlinear load-slip behaviour of its primary nailed connections. Numerous shear wall and plywood-sheathed models of varying complexity have largely shown this notion to be accurate, although this has yet to be confirmed for straight-sheathed diaphragms. A validated model that accurately captures the behaviour of representative diaphragms is required for comprehensive parametric analyses, and to provide engineering practitioners with the necessary modelling guidelines for independent URM building time-history analyses.

Using the specific knowledge gaps identified in this literature review, an integrated experimental and modelling program was initiated to improve the current understanding of the in-plane seismic response of timber floor diaphragms representative of those found in historic New Zealand URM buildings.
Chapter 3

MECHANICS OF DIAPHRAGM DEFORMATIONS

The lateral deformation of timber floor diaphragms in URM buildings is difficult to describe analytically due to the complex interaction of framing deformations and intermittent nail couple rotations (Dean et al. 1982). Previous models have either overly simplified diaphragm behaviour by assuming rigid framing rotations, or idealised diaphragm response indirectly as the partial laminar interaction of a multilayer beam-type structure. Due largely to the unpopularity of straight-sheathed diaphragms for new construction, further development of appropriate analysis methods has not taken place and subsequently diaphragm deformation mechanics has remained ambiguous.

A new generalised analytical model for straight-sheathed timber floor diaphragms is presented. The model appropriately captures framing flexural deformations and corresponding nail couple resistance, and is capable of predicting diaphragm displacement at any span location. The model is used to describe the mechanics of diaphragm deformations and in Chapter 4 is used to inform the appropriate idealisation of diaphragm behaviour for fundamental period determination and for the assessment of seismic performance.
3.1 Previous Analysis Methods

The analysis of square-sheathed timber floor diaphragm deformations has commonly been simplified to an equivalent shear wall analysis, for which rigid framing rotations are assumed. While this assumption is appropriate for point loaded shear walls with only base fixity, it overly simplifies the behaviour of floor diaphragms which are supported at both ends and which have seismic loads distributed across their span, therefore generating flexural bending of framing members. Figures 3-1a and 3-2a depict the assumed rigid-rotation deflections for diaphragms loaded perpendicular-to-joists and for diaphragms loaded parallel-to-joists, respectively. From inspection of Figures 3-1b and 3-2b, it can be seen that a resisting moment is generated at each floorboard-to-joist connection equal to 

\[ M_n = F_n s. \]

If the total number of nail couples within half the diaphragm span is equal to \( n_j \) multiplied by the number of floorboards \( n_b \), then from moment equilibrium, it can be shown that,

\[ V \frac{L}{2} = n_j n_b M_n \] (3-1)

Substituting the expression for \( M_n \), and acknowledging that \( n_b = \frac{L}{2d_f} \) and \( n_j \approx \frac{B}{\ell} \) for loading perpendicular-to-joists, and that \( n_b = \frac{B}{d_f} \) and \( n_j \approx \frac{L}{2\ell} \) for loading parallel-to-joists, Equation 3-1 can be rearranged for an expression of \( F_n \) that is applicable to either principal direction of loading:

\[ F_n = \frac{d_f \ell V}{s B} \] (3-2)

From inspection of Figures 3-1b and 3-2b it can be shown that the rotation angle of the diaphragm, \( \theta \), is related to individual nail-slip, \( \delta_n \), by Equation 3-3:

\[ \theta = \frac{2\delta_n}{s} \] (3-3)

If a linear-elastic nail-slip characteristic with a stiffness coefficient \( k_n \) is assumed, then \( \delta_n = \frac{F_n}{k_n} \). Substituting Equation 3-2 into this expression for \( \delta_n \), and substituting the
3.1 Previous Analysis Methods

product into Equation 3-3, gives,

$$\theta = \frac{2d_f \ell V}{k_n s^2 B}$$  \hfill (3-4)

Finally, recognising that $\Delta_d = \frac{L}{2} \theta$, and that $V = WL/2$ from Figures 3-1a and 3-2a, diaphragm midspan displacement can be determined from,

$$\Delta_d = \frac{d_f \ell W L^2}{k_n s^2 2B}$$  \hfill (3-5)

Additional information regarding the above methodology is presented in Dean et al. (1982).

A more rigorous deformation analysis was offered by Granholm (1961) for straight-sheathed diaphragms loaded parallel-to-joists, which was modified by Walford (1978) for integration into an early version of the New Zealand timber design standard, NZS 3603:1980. Granholm’s model considers the diaphragm to be a nail-laminated beam.

![Figure 3-1: Simplified diaphragm analysis, loading perpendicular-to-joists](image)
Figure 3-2: Simplified diaphragm analysis, loading parallel-to-joists

that resists lateral loading through interlaminar rigidity, which is provided by the joists to which the floorboards are nailed to, as well as any fasteners provided between the floorboards (Dean et al. 1982). Based on this analogy, diaphragm midspan displacement under uniformly distributed loading conditions is determined by adjusting the flexural deformation of a simply supported beam using a modification factor, $\phi_1$:

$$\Delta_d = \frac{5WL^3}{384EI} \phi_1$$

(3-6)

where

$$\phi_1 = \frac{512}{5\pi^5} \sum_{n=1}^{\infty} \left(\frac{-1}{n^4}\right) \frac{(\beta_a n)^2}{n^5 \left(1 - \tanh \beta_a n\right)}$$

(3-7)

and

$$\beta_a = \left(\frac{\pi B}{2L}\right) \sqrt{\frac{E}{K_a d_f}}$$

(3-8)
where \( K_a \) is the beam laminar slip parameter which is evaluated from nail connection stiffness. The variables \( B \), \( L \), \( d_f \) and \( W \) in Equations 3-6 to 3-8 are consistent with those illustrated in Figures 3-1 and 3-2, while \( E \) and \( I \) are conventional sectional properties.

The analytical models described above are reasonable idealisations of diaphragm response that have previously been adopted to predict diaphragm deformations in design and assessment standards such as NZS 3603:1981 and NZSEE (2006), but which fail to represent the physical behaviour of diaphragms subjected to lateral loading. The proposed analytical model developed in this chapter attempts to mathematically describe the true mechanics of square-sheathed timber floor diaphragm deformations in both principal loading directions.

### 3.2 Mechanics of Diaphragm Deformations

The deformation of straight-sheathed timber floor diaphragms in each principal loading direction is presented. By making appropriate assumptions, a differential equation is developed that mathematically describes diaphragm deformation at any span location.

#### 3.2.1 Diaphragm deformations parallel-to-joists

Figure 3-3a shows a timber floor diaphragm with a span of \( L \) and depth of \( B \) deforming under a uniformly distributed load, \( W \), that is applied parallel to its joists and that represents the out-of-plane wall loads imposed onto a diaphragm during an earthquake\(^1\). The out-of-plane wall loads are transmitted into the diaphragm through the ends of the joists and into the floorboards through the nail connections. As illustrated in Figure 3-3a, the transmitted forces cause relative joist displacement and flexural bending of the joists.

\(^1\)Out-of-plane wall loads are conventionally distributed parabolically across the diaphragm span to account for increased stiffness and load sharing within proximity of the in-plane masonry walls (ASCE 2007). However, for the purposes of this analysis, a uniformly distributed load is an appropriate simplification.
floorboards, which, in turn, induces nail couple rotation at each floorboard-to-joist connection. Figure 3-3b depicts the imposed nail-slip caused by floorboard rotation and demonstrates that nail couple rotation, $\theta$, is equal to floorboard rotation.

The induced nail couple rotations generate small restoring moments at each floorboard-to-joist connection which attempt to resist diaphragm deformation. From Figure 3-3b, the magnitude of individual restoring moments is obtained by,

$$M_k = sF_n$$  \hspace{1cm} (3-9)

where $M_k$ is the restoring moment at the $k^{th}$ nail connection, $s$ is nail couple spacing, and $F_n$ is the force resisted by each nail, which is directly proportional to nail couple rotation and the load-slip characteristics of the nail connections themselves. The treatment of $F_n$ for linear-elastic and nonlinear diaphragm deformations is detailed in Sections 3.3 and 3.4, respectively.

If the imposed wall loads are assumed to be evenly shared by the floorboards, then for any realistic diaphragm configuration with many joists, each floorboard can be assumed to be uniformly loaded with $w$, defined as,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{diaphragm_deformation}
\caption{Diaphragm deformation parallel-to-joists}
\end{figure}
3.2 Mechanics of Diaphragm Deformations

\[ w = \frac{W}{n_b} \quad (3-10) \]

where \( n_b \) is the number of rows of floorboards, equal to \( n_b = B/d_f \), where \( d_f \) is floorboard width. Given that the experimental results presented in Chapter 6 indicate that inter-board friction is negligible for square-sheathed timber diaphragms, this assumption of even load sharing between floorboards is considered appropriate, and total diaphragm deformation can subsequently be determined from the superposition of individual floorboard response.

Figure 3-4 depicts a generalised floorboard section subjected to a uniformly distributed load \( w \) with point-moments, \( M_k \), spaced at \( \ell \) (joist spacing), representing the floorboard-to-joint nail connections. The first point-moment, \( M_1 \), is located at centre-width of the end joist, which is assumed to occur at \( x = 0 \) with any floorboard length before this point considered negligible. If the floorboard is assumed to be simply supported and continuous over the full diaphragm span, \( L \), then by considering internal moments at a distance \( x \) along the floorboard, it can be shown that for \( 0 \leq x \leq \frac{L}{2} \),

\[ M_x = \frac{wx}{2} (L - x) - \sum_{k=1}^{i} M_k \bigg|_{x = \xi_k} \quad k = 1, 2, 3 \ldots i \quad (3-11) \]

where,

\[ \xi_k = (k - 1)\ell \quad (3-12) \]

\[ i = \left\lceil \frac{x}{\ell} \right\rceil \quad (3-13) \]

Fundamental beam theory states that flexural deformations can be related to internal bending moments by Equation 3-14,

\[ \frac{d^2v}{dx^2} = -\frac{M}{EI} \quad (3-14) \]
therefore, by substituting Equation 3-11 into Equation 3-14, it can be shown that,

$$\frac{d^2v_i}{dx^2} = \frac{wx}{2EI}(x - L) + \frac{1}{EI} \sum_{k=1}^{i} M_k \bigg|_{x = \xi_k}$$

which represents the governing differential equation describing diaphragm deformations for $0 \leq x \leq L/2$ when loaded parallel-to-joists. Given that each floorboard is effectively pinned at its ends, is symmetrically loaded, and for realistic timber sections is uniformly stiff, then Equation 3-15 is subject to the boundary conditions $v(0) = 0$ and $v'(L/2) = 0$ (i.e. the slope at $x = \frac{L}{2}$ must be zero).

### 3.2.2 Diaphragm deformations perpendicular-to-joists

Figure 3-5a shows a timber floor diaphragm with a span of $L$ and depth of $B$ deforming under a uniformly distributed load, $W$, that is applied perpendicular to its joists and that represents the out-of-plane wall loads imposed onto a diaphragm during an earthquake. As Figure 3-5a illustrates, the imposed wall loads cause the weak-axis flexural bending of joists and the relative displacement of floorboards, which are assumed to be rigid and only capable of transferring shear forces. Figure 3-5b depicts the nail-slip resulting from the relative floorboard rigidity and induced joist curvature at a nail connection location. Unlike nail couples loaded parallel-to-joists, Figure 3-5b demonstrates that for perpendicular-to-joist loading, individual nail-slip is dissimilar due to changing joist curvature.
3.2 Mechanics of Diaphragm Deformations

Therefore, the restoring moment generated by each floorboard-to-joist connection is strictly defined by,

$$M_k = \frac{s}{2}(F_{n1} + F_{n2})$$  \hspace{1cm} (3-16)

However, given that the change of curvature over a realistic nail couple spacing is small, the resisting forces $F_{n1}$ and $F_{n2}$ can be considered equal, and defined by the average of the two values. Individual restoring moments can therefore be adequately determined using Equation 3-9, as previously shown.

The transmission of perpendicular-to-joist wall loads into typical as-built diaphragm configurations is often troublesome due to the absence of a reliable load-path. The automatic assumption of even load-sharing between joists can therefore not be made with true physical relevance. However, given that the construction of a subdiaphragm\(^2\) or similarly reliable load path into the diaphragm is mandatory for any URM building seismic retrofit, the applied uniformly distributed load, $W$, can be assumed to be evenly shared between the joists for the purpose of this analysis. Diaphragm deformations can therefore
be evaluated through the consideration of an individual joist subjected to a uniformly distributed load, \( w \), defined as,

\[
w = \frac{W}{(n_j - 1)}
\]

(3-17)

where \( n_j \) is the number of joists, equal to \( n_j = B / \ell + 1 \).

Figure 3-6 depicts a generalised joist section subjected to a uniformly distributed load \( w \) with point-moments, \( M_k \), spaced at \( d_f \) (floorboard width), representing the floorboard-to-joist nail connections. The first point-moment, \( M_1 \), is located at \( x = \xi \), equal to half the width of a floorboard plus any additional length of joist to the effective support location. If the joist is assumed to be simply supported and continuous over the full diaphragm span, \( L \), then by considering internal moments at a distance \( x \) along the joist, it can be shown that for \( 0 \leq x \leq \frac{L}{2} \)

\[
M_x = \frac{wx}{2} (L - x) - \sum_{k=1}^{i} M_k \bigg|_{x = \xi_k} \quad k = 1, 2, 3 \ldots i
\]

(3-18)

where,

\[
\xi_k = \xi + (k - 1)d_f
\]

(3-19)

\[
i = \left\lfloor \frac{x}{d_f} \right\rfloor
\]

(3-20)

Considering that floorboards are typically flush against the perimeter URM walls, and that half the width of a floorboard is typically negligible compared with diaphragm span, the

\[\text{The subdiaphragm methodology is a design method whereby the main diaphragm is broken up into a number of smaller subdiaphragms at the diaphragm perimeter, which are designed to resist the amplified out-of-plane wall anchorage forces and span between diaphragm cross-ties to effectively transmit wall loads into the diaphragm (Oliver 2010a).}\]
value of $\xi$ in Equation 3-19 can be considered zero for the purposes of this analysis. If $\xi = 0$, Equations 3-18 to 3-20 are identical to Equations 3-11 to 3-13 for parallel-to-joist loading, except that joist spacing, $\ell$, is replaced with floorboard width, $d_f$. For the simplicity of analysis, the joists are assumed to be pinned at their ends, which, considering the typical over sizing of the URM wall pockets (see Chapter 1) is physically representative of in-situ conditions, at least until large diaphragm deformations are reached. Given symmetrical loading conditions and uniform member stiffness, Equation 3-18 is also subject to the boundary conditions $u(0) = 0$ and $u'(L/2) = 0$, and can be solved in exactly the same manner as Equation 3-11 for parallel-to-joist loading.

### 3.3 Linear-Elastic Deformations

As previously shown by Equation 3-9, the restoring moments, $M_k$, in Equations 3-11 and 3-18 may be defined in terms of individual nail resistance, $F_n$, which is directly proportional to nail couple rotation, $\theta$, and the load-slip characteristics of the nail connections themselves. If a linear-elastic nail-slip relationship with a stiffness coefficient of $k_n$ is assumed, then using basic mechanics from Figure 3-3b, the restoring moment can be calculated as,

$$M_k = \frac{s^2 k_n \theta_k}{2}$$  \hspace{1cm} (3-21)
where \( \frac{s}{2} \theta_k = \delta_n \) (see Equation 3-3), and \( \delta_n \) is individual nail-slip. Recognising that,

\[
\theta_k = \frac{dv}{dx} \left( \xi_k \right) = v' \bigg|_{x = \xi_k} \quad (3-22)
\]

Equation 3-15 may be re-written in general terms as,

\[
v''_i(x) = \frac{1}{EI} \left( \frac{wx^2}{2} - \frac{wLx}{2} + \beta \sum_{k=1}^{i} v' \bigg|_{x = \xi_k} \right) \quad k = 1, 2, 3 \ldots i \quad (3-23)
\]

where,

\[
\beta = \frac{k_n s^2}{2} \quad (3-24)
\]

and in general terms,

\[
\xi_k = (k - 1) \gamma \quad (3-25)
\]

\[
i = \left\lfloor \frac{x}{\gamma} \right\rfloor \quad (3-26)
\]

where \( \gamma \) is the uniform spacing between nail connections. Equation 3-23 can subsequently be solved for linear-elastic diaphragm deformation at any location between \( x = 0 \) and \( x = L/2 \), which, given symmetry about diaphragm midspan, can be used to determine linear-elastic diaphragm deformation at any span location.

### 3.3.1 General solution

For a generalised distribution of nail connections evenly spaced at \( \gamma \) over a diaphragm span of \( L \), there are two distinct distribution possibilities: (1) an odd number of nail connections, and (2) an even number of nail connections, as depicted in Figure 3-7. For an odd number of nail connections, it is shown that a nail connection occurs at exactly
midspan, whereas for an even number of nail connections, it is shown that midspan is located half-way between the two central nail connections. For both possibilities, the generalised solution to Equation 3-23 may be developed using the same methodology, where the distinction between the two cases influences only the simplification of the final equations. The generalised solution process is reported hereafter, followed by the reduced equations for each distribution case.

Figure 3-7 illustrates that there are \( m \) sections between the nail connections within each half of the diaphragm. Due to the restoring moment that occurs at each floorboard-to-joist nail connection, the solution to Equation 3-23 is discontinuous and must be considered separately for each of the individual sections. For any \( i^{th} \) section (defined by the ceiling function in Equation 3-26), the general solution to Equation 3-23 can, by inspection, be expressed as a quadratic polynomial of the form,

\[
v_i(x) = \frac{wx^4}{24EI} - \frac{wLx^3}{12EI} + A_ix^2 + B_ix + C_i
\]  

\subequations \begin{align}  
\text{(a) Odd number} \\
\|
\begin{array}{ccccccc}
0 & M_1 & M_2 & M_3 & M_4 & M_i & M_{i+1} & M_m \\
\gamma & \gamma & & & & & & \\
\end{array} \\
\end{align} 

\subequations \begin{align}  
\text{(b) Even number} \\
\|
\begin{array}{ccccccc}
0 & M_1 & M_2 & M_3 & M_4 & M_i & M_{i+1} & M_m \\
\gamma & \gamma & & & & & & \\
\end{array} \\
\end{align} 

Figure 3-7: Generalised distribution of nail connections
for which the derivatives are,

\[ v_i'(x) = \frac{wx^2}{6EI} - \frac{wLx^2}{4EI} + 2A_i x + B_i \]  
(3-28)

\[ v_i''(x) = \frac{wx^2}{2EI} - \frac{wLx}{2EI} + 2A_i \]  
(3-29)

where \( A_i, B_i, \) and \( C_i \) are arbitrary coefficients that are unique to the \( i^{th} \) section. If the total number of sections within half of the diaphragm span is \( m \), and there are three arbitrary coefficients per section, then the total number of unknown coefficients for any given solution is \( 3m \), where,

\[ m = \left\lfloor \frac{n_c}{2} \right\rfloor \]  
(3-30)

and \( n_c \) is the number of nail connections within the full diaphragm span, \( L \). To evaluate the \( 3m \) unknown coefficients, consider firstly the \( m \) differential equations defined by Equation 3-23 for \( i = 1, 2, 3 \ldots m \). If Equation 3-23 is substituted into the trial solution in Equation 3-27, it can be shown that,

\[ 2A_i = \frac{\beta}{EI} \sum_{k=1}^{i} \left( \frac{w}{6EI} \left( (k - 1)\gamma \right)^3 - \frac{wL}{4EI} \left( (k - 1)\gamma \right)^2 \right) + 2A_k (k - 1)\gamma + B_k \]  
\[ i = 1, 2 \ldots m \]  
\[ k = 1, 2 \ldots i \]  
(3-31)

which can be rewritten as,

\[ \frac{2EI}{\beta} A_i = \frac{w\gamma^3}{6EI} \sum_{k=1}^{i} (k - 1)^3 - \frac{wLy^2}{4EI} \sum_{k=1}^{i} (k - 1)^2 \]  
\[ i = 1, 2 \ldots m \]  
\[ k = 1, 2 \ldots i \]  
(3-32)
Using the expressions,

\[
\sum_{k=1}^{i} (k - 1)^2 = \frac{1}{6} i(2i - 1)(i - 1) \quad (3-33)
\]

\[
\sum_{k=1}^{i} (k - 1)^3 = \frac{1}{4} i^2(i - 1)^2 \quad (3-34)
\]

Equation 3-32 can be rearranged and simplified to,

\[
\left(\frac{2EI}{\beta} - 2(i - 1)\right)A_i - 2\gamma \sum_{k=1}^{i-1} A_k(k - 1) - \sum_{k=1}^{i} B_k = \frac{wy^2}{24EI} i(i - 1) \left(\gamma i(i - 1) - L(2i - 1)\right) \quad (3-35)
\]

The second manner in which the unknown coefficients are evaluated is the \(2(m - 1)\) consistency conditions at the junction of each section,

\[
v_{i-1}(i - 1)\gamma = v_i((i - 1)\gamma) \quad i = 2, 3 \ldots m \quad (3-36)
\]

\[
v'_{i-1}(i - 1)\gamma = v'_i((i - 1)\gamma) \quad i = 2, 3 \ldots m \quad (3-37)
\]

which, when substituted into the trial solution in Equation 3-27, gives,

\[
A_{i-1}(i - 1)^2\gamma^2 + B_{i-1}(i - 1)\gamma + C_{i-1} - A_i(i - 1)^2\gamma^2 - B_i(i - 1)\gamma - C_i = 0 \quad i = 2, 3 \ldots m \quad (3-38)
\]

and

\[
2A_{i-1}(i - 1)\gamma + B_{i-1} - 2A_i(i - 1)\gamma - B_i = 0 \quad i = 2, 3 \ldots m \quad (3-39)
\]
Finally, the two remaining unknown coefficients are evaluated using the two known boundary conditions,

\[ v_1(0) = 0 \]  
(3-40)

\[ v_m'(L/2) = 0 \]  
(3-41)

which, when substituted into the trial solution in Equation 3-27, show that,

\[ C_1 = 0 \]  
(3-42)

and that,

\[ A_m L + B_m = \frac{wL^3}{24EI} \]  
(3-43)

Using Equations 3-35, 3-38, 3-39, 3-42, and 3-43, a system of equations can be constructed which can be readily solved for \( A_i, B_i, \) and \( C_i \) using any suitable mathematical software. The determined coefficients can then be substituted back into Equation 3-27 for an expression of linear-elastic diaphragm displacement within any \( i^{th} \) section. The following sections demonstrate how such a system of equations may be developed for the two distribution cases: (1) an odd number of nail connections, and (2) an even number of nail connections.

### 3.3.2 Generalised system of equations for an odd number of nail connections

Figure 3-7a illustrates that for an odd number of nail connections,

\[ \gamma = \frac{L}{2m} \]  
(3-44)

Substituting the above expression for \( \gamma \) into Equation 3-35 and factorising, gives,
\[
\left( \frac{2EIm}{\beta L} - (i - 1) \right) A_i L - L \sum_{k=1}^{i-1} A_k (k - 1) - m \sum_{k=1}^{i} B_k = \frac{wL^3}{96Em} (i - 1) \left( \frac{i^2(i - 1)}{2m} - i(2i - 1) \right)
\]

which, if divided through by \( \frac{wL^3}{EI} \) simplifies to,

\[
\left( \frac{2m}{\beta} - (i - 1) \right) \bar{A}_i - \sum_{k=1}^{i-1} \bar{A}_k (k - 1) - m \sum_{k=1}^{i} \bar{B}_k = \frac{(i - 1)}{96m} \left( \frac{i^2(i - 1)}{2m} - i(2i - 1) \right)
\]

where,

\[
\bar{A}_i = \frac{A_i}{\left( \frac{wL^2}{EI} \right)}
\]

\[
\bar{B}_i = \frac{B_i}{\left( \frac{wL^3}{EI} \right)}
\]

\[
\bar{C}_i = \frac{C_i}{\left( \frac{wL^4}{EI} \right)}
\]

\[
\bar{\beta} = \frac{\beta L}{EI}
\]

Equations 3-38, 3-39, 3-42, and 3-43 can be similarly simplified by substituting in Equation 3-44 and normalising the coefficients with respect to Equations 3-47 to 3-50, giving,
Equations 3-46 and 3-51 to 3-54 represent the generalised system of equations for an odd number of joists, that can be solved simultaneously for the unknown coefficients $\bar{A}_i$, $\bar{B}_i$, and $\bar{C}_i$. For mathematical convenience, the system of equations may be expressed in matrix form as,

$$GX = H \quad (3-55)$$

where, $G$ is a $3m \times 3m$ matrix containing the coefficients of $\bar{A}_i$, $\bar{B}_i$, and $\bar{C}_i$, $X$ is a $3m$ column vector containing $\bar{A}_i$, $\bar{B}_i$, and $\bar{C}_i$, and $H$ is a $3m$ column vector containing the corresponding solutions to Equations 3-46 and 3-51 to 3-54. The unknown coefficients $\bar{A}_i$, $\bar{B}_i$, and $\bar{C}_i$ in $X$, may subsequently be determined by left matrix division,

$$X = G\backslash H \quad (3-56)$$

which, for clarity, is approximately equivalent to,

$$X = G^{-1}H \quad (3-57)$$

The generalised system of equations for an odd number of nail connections is presented in matrix form in Equation 3-58.
\[
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 1 & 1 & 0 \\
\frac{2m}{\beta} & -m & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
-\frac{m}{\beta} & -m & 0 & \frac{2m}{\beta} & -1 & -m & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & -m & 0 & -1 & -m & 0 & \ldots & \frac{2m}{\beta} & -(i-1) & -m & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & -m & 0 & -1 & -m & 0 & \ldots & -(k-1) & -m & 0 & \ldots & \frac{2m}{\beta} & -(m-1) & -m & 0 \\
\frac{1}{4m^2} & \frac{1}{2m} & 1 & \frac{1}{4m^2} & -\frac{1}{2m} & -1 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{m} & 1 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & (i-1)^2 & \frac{1}{4m^2} & -(i-1) & 2 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & -(m-1)^2 & \frac{4m^2}{2m} & -(m-1) & -1 \\
\frac{1}{m} & 1 & 0 & -\frac{1}{m} & -1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2}{m} & 1 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & (i-1) & \frac{2m}{m} & -1 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & -(m-1) & \frac{m}{m} & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\hat{A}_1 \\
\hat{B}_1 \\
\hat{C}_1 \\
\hat{A}_2 \\
\hat{B}_2 \\
\hat{C}_2 \\
\vdots \\
\hat{A}_m \\
\hat{B}_m \\
\hat{C}_m
\end{bmatrix} =
\begin{bmatrix}
0 \\
\frac{1}{24} \\
0 \\
\frac{5}{192} \\
\vdots \\
\frac{(i-1)(k^2(i-1) - i(2i-1))}{96m^2} \\
\frac{(m-1)(m-1)(m-1) - (2m-1)}{96m^2} \\
\dot{\hat{A}}_1 \\
\dot{\hat{B}}_1 \\
\dot{\hat{C}}_1 \\
\vdots \\
\dot{\hat{A}}_m \\
\dot{\hat{B}}_m \\
\dot{\hat{C}}_m
\end{bmatrix}
\tag{3.58}
\]
3.3.3 Generalised system of equations for an even number of nail connections

Figure 3-7b illustrates that for an even number of nail connections,

\[ \gamma = \frac{L}{2m - 1} \]  \hspace{1cm} (3-59)

which, by substituting into Equations 3-35, 3-38, 3-39, 3-42, and 3-43, and factorising in the same manner as were the corresponding equations in Section 3.3.2, the generalised system of equations for an even number of nail connections can be expressed as,

\[ \left( \frac{2(m - \frac{1}{2})}{\bar{\beta}} - (i - 1) \right) \ddot{A}_i - \sum_{k=1}^{i-1} \ddot{A}_k(k - 1) = 0 \quad i = 1, 2 \ldots m \]  \hspace{1cm} (3-60)

\[ \left( \frac{i - 1}{4} \right) \dddot{A}_{i-1} + \frac{(i - 1)}{2} \dddot{B}_{i-1} + \dddot{C}_{i-1} - \frac{(i - 1)^2}{4} \dddot{A}_i - \frac{(i - 1)}{2} \dddot{B}_i + \dddot{C}_i = 0 \quad i = 2, 3 \ldots m \]  \hspace{1cm} (3-61)

\[ \frac{(i - 1)}{(m - \frac{1}{2})^2} \dddot{A}_{i-1} + \frac{(i - 1)}{(m - \frac{1}{2})} \dddot{B}_{i-1} - \dddot{C}_{i-1} = 0 \]  \hspace{1cm} (3-62)

\[ \dddot{C}_1 = 0 \]  \hspace{1cm} (3-63)

\[ \dddot{A}_m + \dddot{B}_m = \frac{1}{24} \]  \hspace{1cm} (3-64)
Using the same matrix convention described in Equation 3-55, the generalised system of equations shown in Equations 3-60 to 3-64 can be expressed in matrix form, as presented in Equation 3-66.

### 3.4 Nonlinear Deformations

The assumption of linear-elastic nail connection behaviour was shown in Section 3.3 to be a convenient idealisation for diaphragm deformation analysis, as it enables the unknown coefficients $\tilde{A}_i, B_i$, and $\tilde{C}_i$ in Equation 3-27 to be isolated and readily solved for using a system of equations. While this simplified method of solution is mathematically desirable, the determined linear-elastic deformations fail to capture the nonlinear behaviour of timber floor diaphragms, and are consequently limited to elastic analysis applications. To capture such diaphragm nonlinearity, the highly nonlinear behaviour of the primary nail connections must be incorporated into the deformation analysis. The development of a differential equation governing nonlinear diaphragm deformations and the corresponding solution methodology are presented in this section.

For nonlinear diaphragm deformation analysis, the linear-elastic relationship $F_n = k_n \delta_n$ used in Section 3.3 was replaced with the expression presented in Equation 3-65 relating nail connection force, $F_n$, to nail connection lateral slip, $\delta_n$. This exponential load-slip function was developed by Foschi (1974) and is widely recognised to representatively describe nonlinear nail connection behaviour, having been successfully implemented into several analytical studies of timber structures with dowel connections (Dolan 1989; Dolan and Madsen 1992; Judd and Fonseca 2002; Loo 2010).

$$F_n = (F_0 + K_1 \delta_n) \left(1 - e^{\left(-\frac{K_0 \delta_n}{F_0}\right)}\right) \tag{3-65}$$

where $F_0$ is initial force, $K_0$ is initial stiffness, and $K_1$ is secondary stiffness, as illustrated in Figure 3-8.
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\[
\begin{bmatrix}
0 & 1 - \frac{(\xi - w)}{(1 - u)} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
0 & 0 & 0 & \cdots & 1 - \frac{(\xi - w)}{(1 - t)} & \cdots & 0 & 0 & 0 & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 1 - \frac{(\xi - w)}{(1 - t)} & \cdots & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 1 & \frac{\partial}{\partial \xi} & \frac{(\xi - w)}{(1 - t)} & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
\end{bmatrix}
\]

\[
\left(1 - mu \right) \begin{bmatrix}
0 & 1 - \frac{(\xi - w)}{(1 - u)} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
0 & 0 & 0 & \cdots & 1 - \frac{(\xi - w)}{(1 - t)} & \cdots & 0 & 0 & 0 & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 1 - \frac{(\xi - w)}{(1 - t)} & \cdots & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
\end{bmatrix}
\frac{1}{1 - \xi} \begin{bmatrix}
0 & \frac{(\xi - w)}{(1 - u)} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
0 & 0 & 0 & \cdots & 1 - \frac{(\xi - w)}{(1 - t)} & \cdots & 0 & 0 & 0 & 0 & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 1 & \frac{\partial}{\partial \xi} & \frac{(\xi - w)}{(1 - t)} & 0 & 0
\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots
\\
\end{bmatrix}
\]
Nail connection nonlinearity is incorporated into the deformation analysis by substituting Equation 3-65 into the expression for restoring moment generated at each floorboard-to-joist connection, $M_k$, given in Equation 3-9, and substituting these combined equations into the governing differential equation presented in Equation 3-15. Rewriting Equation 3-15 with an arbitrary nail connection spacing of $\gamma$ gives a generalised expression relating internal bending moments to diaphragm deformation,

$$\frac{d^2 v_i}{dx^2} = \frac{wx}{2EI} (x - L)$$

$$+ \frac{1}{EI} \sum_{k=1}^{i} s(F_0 + K_1 \delta_n) \left(1 - e^{(-\frac{K_0 \delta_n}{F_0})}\right) x = \xi_k$$

where,

$$\xi_k = (k - 1)\gamma$$

$$i = \left\lceil \frac{x}{\gamma} \right\rceil$$

---

Figure 3-8: Illustration of nail connection load-slip curve developed by Foschi (1974)
Recognising that \( \frac{s}{2} \theta_k = \delta_n \), and that,

\[
\theta_k = \frac{dv}{dx}(\xi_k) = v' \bigg|_{x = \xi_k} \quad (3-70)
\]

Equation 3-67 may be rewritten in general terms as,

\[
v''(x) = \frac{wx^2}{2EI} - \frac{wLx}{2EI} + b_0 \sum_{k=1}^{i} \left( b_1 + b_2 v''(\xi_k) \right) \left( 1 - e^{-b_3 v''(\xi_k)} \right) \quad k = 1, 2, 3 \ldots i \quad (3-71)
\]

where,

\[
b_0 = \frac{s}{EI} \quad (3-72)
\]

\[
b_1 = F_0 \quad (3-73)
\]

\[
b_2 = \frac{K_1 s}{2} \quad (3-74)
\]

\[
b_3 = \frac{K_0 s}{2F_0} \quad (3-75)
\]

Equation 3-71 represents the governing differential equation for nonlinear diaphragm deformation, which can subsequently be solved for nonlinear deformations at any location between \( x = 0 \) and \( x = L/2 \), which, given symmetry about diaphragm midspan, can be used to determine nonlinear diaphragm deformation at any span location.

### 3.4.1 General solution

Despite the increased complexity in the treatment of \( M_k \), Equation 3-71 remains a linear differential equation, and is therefore identical in form to the differential equation
3.4 Nonlinear Deformations

describing linear-elastic diaphragm deformations, presented in Equation 3-23. Consequently, the solution to Equation 3-71 for nonlinear diaphragm deformations and its corresponding derivatives are also described by the polynomial expressions given in Equations 3-27 to 3-29. As for the linear-elastic analysis in Section 3.3, the two distinct nail connection distribution cases are illustrated in Figure 3-7, for which there are \( m \) mathematically discontinuous sections within each half of the diaphragm, as described by Equation 3-30. With three unknown coefficients within each section \( i \) (\( A_i, B_i, \) and \( C_i \)), there are a total of \( 3m \) unknown coefficients to be determined for any given diaphragm. The unknown coefficients are once again evaluated by considering (1) the \( m \) differential equations defined by Equation 3-71 for \( i = 1, 2, 3 \ldots m \), (2) the \( 2(m - 1) \) consistency conditions at the junction of each section, as described by Equations 3-36 and 3-37, and (3) the two known boundary conditions presented in Equations 3-40 and 3-41.

Considering first the governing differential equation, when Equation 3-71 is substituted into the solution in Equation 3-27, it can be shown that,

\[
\frac{2A_i}{b_0} = i b_1 + \sum_{k=1}^{i} \left( -b_1 e^{-b_3 v'(\xi_k)} + \frac{b_2 \gamma y^3}{6EI} (k - 1)^3 ight) - \frac{b_2 \gamma y^2}{4EI} (k - 1)^2 + 2b_2 \gamma A_k (k - 1) + b_2 B_k \\
- \frac{b_2 \gamma y^3}{6EI} (k - 1)^3 e^{-b_3 v'(\xi_k)} \\
+ \frac{b_2 \gamma y^2}{4EI} (k - 1)^2 e^{-b_3 v'(\xi_k)} \\
- 2b_2 \gamma A_k (k - 1)e^{-b_3 v'(\xi_k)} - b_2 B_k e^{-b_3 v'(\xi_k)}
\]

where,

\[
v'(\xi_k) = \frac{(k - 1)^3 y^3}{6EI} - \frac{(k - 1)^2 y^2}{4EI} + 2(k - 1) \gamma A_k + B_k
\]

Using the expressions,
\[ \sum_{k=1}^{i} (k - 1)^2 = \frac{1}{6} i(2i - 1)(i - 1) \quad (3-78) \]
\[ \sum_{k=1}^{i} (k - 1)^3 = \frac{1}{4} i^2 (i - 1)^2 \quad (3-79) \]

Equation 3-76 can be rearranged and simplified to,
\[
\frac{2A_i}{b_0} + \sum_{k=1}^{i} \left( b_1 e^{-b_3 v'(\xi_k)} - 2b_2 y A_k (k - 1) - b_2 B_k \right) + \frac{b_2 w y^3}{6EI} (k - 1)^3 e^{-b_3 v'(\xi_k)} \\
- \frac{b_2 w L y^2}{4EI} (k - 1)^2 e^{-b_3 v'(\xi_k)} - i b_1 \quad \text{for } i = 1, 2 \ldots m \\
+ 2b_2 y A_k (k - 1) e^{-b_3 v'(\xi_k)} + b_2 B_k e^{-b_3 v'(\xi_k)} - i b_1 \\
- \frac{b_2 w y^2}{24EI} (i - 1) \left( i^2 (i - 1) y - i(2i - 1)L \right) = 0 \quad (3-80) 
\]

The equations developed from the \( 2(m - 1) \) consistency conditions and two known boundary conditions for nonlinear deformation analysis are identical to those developed in Section 3.3 for the linear-elastic analysis. Therefore using Equations 3-80, 3-38, 3-39, 3-42, and 3-43, a system of equations may once again be constructed to solve for the unknown coefficients \( A_i, B_i, \) and \( C_i \) for any \( i^{th} \) section between \( x = 0 \) and \( x = L/2 \).

However, inspecting Equation 3-80, it is evident that the unknown coefficients \( A_k \) and \( B_k \) cannot be individually isolated and consequently the system of equations must be solved using a suitable numerical procedure. The following sections demonstrate how the above system of equations is developed for the two nail connection distribution cases: (1) an odd number of nail connections, and (2) an even number of nail connections, and how these systems of equations may be solved using the Newton-Raphson method.
3.4.2 Generalised system of equations for an odd number of nail connections

It was previously illustrated in Figure 3-7a that for an odd number of nail connections, nail connection spacing $\gamma$ can be related to diaphragm span $L$ by Equation 3-44. Substituting this expression for $\gamma$ into Equation 3-80, and multiplying through by $\frac{mEI}{wL^3}$ and appropriately factorising, gives,

$$0 = \frac{2m\bar{A}_i}{b_0L} + \sum_{k=1}^{i} \left( \frac{mEIb_1}{wL^3} e^{-b_3v'(\xi_k)} - b_2\bar{A}_k(k-1) - mb_2\bar{B}_k ight)$$

$$+ \frac{b_2}{48m^2}(k-1)^3e^{-b_3v'(\xi_k)} - \frac{b_2}{16m}(k-1)^2e^{-b_3v'(\xi_k)}$$

$$+ b_2\bar{A}_k(k-1)e^{-b_3v'(\xi_k)} + mb_2\bar{B}_k e^{-b_3v'(\xi_k)}$$

$$- \frac{imb_1EI}{wL^3} - \frac{b_2(i-1)}{96m} \left( \frac{i^2(i-1)}{2m} - i(2i-1) \right)$$

where,

$$v'(\xi_k) = \frac{wL^3}{EI} \left( \frac{(k-1)^3}{48m^3} - \frac{(k-1)^2}{16m^2} + \frac{(k-1)}{m}\bar{A}_k + \bar{B}_k \right)$$

and where $\bar{A}, \bar{B},$ and $\bar{C}$ are defined by Equations 3-47, 3-48, and 3-49, respectively.

As previously demonstrated, the equations developed from the $2(m-1)$ consistency conditions and two known boundary conditions for nonlinear deformation analysis are identical to those developed in Section 3.3 for the linear-elastic analysis, and subsequently, Equations 3-51 to 3-54 also apply to the nonlinear analysis. Equations 3-82 and 3-51 to 3-55 therefore represent the generalised system of equations for an odd number of nail connections, which can be solved numerically using the Newton-Raphson method to determine the values of the unknown coefficients $\bar{A}_i, \bar{B}_i,$ and $\bar{C}_i$. 
To perform the Newton-Raphson method, the nonlinear system of equations are firstly expressed as a $3m$ column vector of the form,

$$
R(u) = \begin{bmatrix}
    R_1(u) \\
    R_2(u) \\
    \vdots \\
    R_n(u)
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
$$

(3-83)

where $u$ is a $3m$ column vector containing the unknown coefficients $\vec{A}_i, \vec{B}_i, \text{ and } \vec{C}_i$, and is of the form,

$$
u = \begin{bmatrix}
u_1 \\
u_2 \\
\vdots \\
u_n
\end{bmatrix}
$$

(3-84)

The unknown coefficients contained in $u$ are then iteratively updated using the following algorithm,

$$
u_i = \nu_{i-1} + \Delta \nu_i
$$

(3-85)

where,

$$
\Delta \nu_i = -[K_T(u_{i-1})]^{-1}R(u_{i-1})
$$

(3-86)

and where $K_T$ is the tangent matrix, defined as,

$$
K_T = \frac{\partial R}{\partial \nu} = \begin{bmatrix}
    \frac{\partial R_1}{\partial u_1} & \frac{\partial R_1}{\partial u_2} & \cdots & \frac{\partial R_1}{\partial u_n} \\
    \frac{\partial R_2}{\partial u_1} & \frac{\partial R_2}{\partial u_2} & \cdots & \frac{\partial R_2}{\partial u_n} \\
    \vdots & \vdots & \ddots & \vdots \\
    \frac{\partial R_n}{\partial u_1} & \frac{\partial R_n}{\partial u_2} & \cdots & \frac{\partial R_n}{\partial u_n}
\end{bmatrix}
$$

(3-87)
To commence the Newton-Raphson algorithm, the values of $\bar{A}_i, \bar{B}_i$, and $\bar{C}_i$ determined from linear-elastic analysis can be used for the initial $u^{i-1}$. The algorithm described by Equations 3-85 and 3-86 is then repeated until the desired tolerance is reached (the magnitude of $\Delta u^i$), at which point, the coefficients contained in $u^i$ adequately satisfy the system of equations and can be substituted into Equation 3-27 for an expression of nonlinear diaphragm deformation.

For an odd number of nail connections, the matrices required for the Newton-Raphson method may be expressed in generalised form as presented in Equations 3-88 to 3-99 on the following pages.
\[ \begin{bmatrix} u_w \\ w_g \\ w_y \\ \vdots \\ w_{y_k} \end{bmatrix} = \mathbf{n} \begin{bmatrix} 68 \cdot 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \]
3.4 Nonlinear Deformations

\[
\mathbf{K}_r = \begin{bmatrix}
\frac{\partial R_1}{\partial u} & \frac{\partial R_2}{\partial u} & \cdots & \frac{\partial R_{i}}{\partial u} & \cdots & \frac{\partial R_{m}}{\partial u} & \frac{\partial R_{m+1}}{\partial u} & \cdots & \frac{\partial R_{2m+1}}{\partial u} & \frac{\partial R_{2m+2}}{\partial u} & \cdots & \frac{\partial R_{2m+m}}{\partial u} \\
\frac{\partial R_1}{\partial A_i} & \frac{\partial R_2}{\partial A_i} & \cdots & \frac{\partial R_{i}}{\partial A_i} & \cdots & \frac{\partial R_{m}}{\partial A_i} & \frac{\partial R_{m+1}}{\partial A_i} & \cdots & \frac{\partial R_{2m+1}}{\partial A_i} & \frac{\partial R_{2m+2}}{\partial A_i} & \cdots & \frac{\partial R_{2m+m}}{\partial A_i} \\
\frac{\partial R_1}{\partial B_i} & \frac{\partial R_2}{\partial B_i} & \cdots & \frac{\partial R_{i}}{\partial B_i} & \cdots & \frac{\partial R_{m}}{\partial B_i} & \frac{\partial R_{m+1}}{\partial B_i} & \cdots & \frac{\partial R_{2m+1}}{\partial B_i} & \frac{\partial R_{2m+2}}{\partial B_i} & \cdots & \frac{\partial R_{2m+m}}{\partial B_i} \\
\end{bmatrix}
\]

(3-90)

where \( \frac{\partial R_k}{\partial u} \) to \( \frac{\partial R_{2m+m}}{\partial u} \) are the partial derivatives of Equation 3-81 for \( i = 1, 2 \ldots m \), where for \( k = i \),

\[
\frac{\partial R_{2+i}}{\partial A_i} = \frac{2m}{b_0 L} - b_2(i-1) + \left( -b_1b_3(i-1) - \frac{b_2b_3(i-1)^4wL^3}{48m^3EI} + \frac{b_2b_3(i-1)^3wL^3}{16m^2EI} + b_2(i-1) - \frac{b_2b_3(i-1)^2wL^3}{mEI} \right) \frac{A_i}{E} + \frac{b_2b_3(i-1)^2wL^3}{48m^3EI} \tilde{A}_i + \frac{b_2b_3(i-1)^2wL^3}{16mEI} \tilde{A}_i \tag{3-91}
\]

\[
\frac{\partial R_{2+i}}{\partial B_i} = -mb_2 + \left( -mb_1b_3 - \frac{b_2b_3(i-1)^3wL^3}{48m^3EI} + \frac{b_2b_3(i-1)^2wL^3}{16mEI} \right) + \frac{b_2b_3(k-1)wL^3}{EI} \frac{A_i}{E} - \frac{mb_2b_3wL^3}{EI} \tilde{A}_i \tag{3-92}
\]

- 87 -
\[ \frac{\partial R_{2+i}}{\partial C_k} = 0 \]  

(3-93)

for \( k < i \),

\[ \frac{\partial R_{2+i}}{\partial A_k} = -b_2 (k - 1) + \left( -b_1 b_3 (k - 1) - \frac{b_2 b_3 (k - 1)^4 wL^3}{48m^3EI} \right) + \frac{b_2 b_3 (k - 1)^3 wL^3}{16m^2 EI} + b_2 (k - 1) - \frac{b_2 b_3 (k - 1)^2 wL^3}{mEI} \]

\[ - \frac{b_2 b_3 (k - 1) wL^3}{EI} \left( \frac{b_2 b_3}{EI} (k-1)^2 \frac{1}{16m^2} \frac{1}{m} \right) \left( \frac{1}{E} \right) \left( \frac{1}{L} \right) \left( \frac{1}{k} \right) \left( \frac{1}{A_k + B_k} \right) \]

(3-94)

\[ \frac{\partial R_{2+i}}{\partial B_k} = -mb_2 + \left( -mb_1 b_3 - \frac{b_2 b_3 (k - 1)^3 wL^3}{48m^2 EI} \right) + \frac{b_2 b_3 (k - 1)^2 wL^3}{16mEI} \]

\[ + mb_2 - \frac{b_2 b_3 (k - 1) wL^3}{EI} \left( \frac{b_2 b_3}{EI} (k-1)^2 \frac{1}{16m^2} \frac{1}{m} \right) \left( \frac{1}{E} \right) \left( \frac{1}{L} \right) \left( \frac{1}{k} \right) \left( \frac{1}{A_k + B_k} \right) \]

(3-95)

\[ \frac{\partial R_{2+i}}{\partial C_k} = 0 \]  

(3-96)

and for \( k > i \),

\[ \frac{\partial R_{2+i}}{\partial A_k} = 0 \]  

(3-97)

\[ \frac{\partial R_{2+i}}{\partial B_k} = -mb_2 + \left( -mb_1 b_3 - \frac{b_2 b_3 (k - 1)^3 wL^3}{48m^2 EI} \right) \]

\[ + \frac{b_2 b_3 (k - 1)^2 wL^3}{16mEI} + mb_z - \frac{b_2 b_3 (k - 1) wL^3}{EI} \left( \frac{b_2 b_3}{EI} (k-1)^2 \frac{1}{16m^2} \frac{1}{m} \right) \left( \frac{1}{E} \right) \left( \frac{1}{L} \right) \left( \frac{1}{k} \right) \left( \frac{1}{A_k + B_k} \right) \]

(3-98)
3.4 Nonlinear Deformations

\[ \frac{\partial R_{2+i}}{\partial C_k} = 0 \quad \text{(3-99)} \]

### 3.4.3 Generalised system of equations for an even number of nail connections

It was previously illustrated in Figure 3-7b that for an even number of nail connections, the uniform nail connection spacing \( \gamma \) can be related to diaphragm span \( L \) by Equation 3-59. Substituting this expression for \( \gamma \) into Equation 3-80, and multiplying through by \( \frac{mEI}{wL^3} \) and appropriately factorising, gives,

\[
0 = \frac{2\left(m - \frac{1}{2}\right) \bar{A}_i}{b_0 L} + \sum_{k=1}^{i} \left( \frac{m - \frac{1}{2}}{wL^3} \right) e^{-b_2 v'(\xi_k)} - b_2 \bar{A}_k (k - 1) \\
- \left( m - \frac{1}{2} \right) b_2 \bar{B}_k + \frac{b_2}{48 \left( m - \frac{1}{2} \right)} (k - 1)^3 e^{-b_2 v'(\xi_k)} \\
- \frac{b_2}{16 \left( m - \frac{1}{2} \right)} (k - 1)^2 e^{-b_2 v'(\xi_k)} + b_2 \bar{A}_k (k - 1) e^{-b_2 v'(\xi_k)} \\
i = 1, 2 ... m \\
k = 1, 2 ... i \\
(3-100)
\]

\[
+ \left( m - \frac{1}{2} \right) b_2 \bar{B}_k e^{-b_2 v'(\xi_k)} \\
- \left( m - \frac{1}{2} \right) b_2 E I \\
\frac{wL^3}{wL^3} \\
- \left( m - \frac{1}{2} \right) \left( \frac{i^2 (i - 1)}{2 \left( m - \frac{1}{2} \right)} - i(2i - 1) \right) \\
\]

where,

\[
v'(\xi_k) = \frac{wL^2}{EI} \left( \frac{(k - 1)^3}{48 \left( m - \frac{1}{2} \right)^3} - \frac{(k - 1)^2}{16 \left( m - \frac{1}{2} \right)^2} + \frac{(k - 1)}{\left( m - \frac{1}{2} \right)} \bar{A}_k + \bar{B}_k \right) \quad \text{(3-101)}
\]
and where $\bar{A}, \bar{B},$ and $\bar{C}$ are defined by Equations 3-47, 3-48, and 3-49, respectively.

As previously demonstrated in Section 3.4.2, the consistency conditions and boundary conditions are consistent with the linear-elastic analysis presented in Section 3.3. Equation 3-100, and Equations 3-51 to 3-54 with $m$ replaced with $\left(m - \frac{1}{2}\right)$, therefore represent the generalised system of equations for an even number of nail connections. Following the methodology outlined in Section 3.4.2, the system of equations can be expressed in matrix form and solved numerically using the Newton-Raphson method to determine the unknown coefficients $\bar{A}_i, \bar{B}_i,$ and $\bar{C}_i$. The matrices required to perform the Newton-Raphson numerical algorithm are presented in Equations 3-100 to 3-102 on the following pages.

It should be noted that all equations presented in this section are identical to the equations presented in Section 3.4.2 for an odd number of nail connections, except that $m$ is replaced with $\left(m - \frac{1}{2}\right)$ with respect to Equation 3-59.
\[
\begin{aligned}
R_1 & \rightarrow \\
R_2 & \rightarrow \\
R_3 & \rightarrow \\
R_4 & \rightarrow \\
R_5 & \rightarrow \\
R_6 & \rightarrow \\
\end{aligned}
\]

\[
\begin{aligned}
R(u) &= \begin{bmatrix}
\tilde{A}_1 \\
\vdots \\
\tilde{A}_n \\
\tilde{B}_1 \\
\vdots \\
\tilde{B}_n \\
\tilde{C}_1 \\
\vdots \\
\tilde{C}_n
\end{bmatrix} \\
&= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \quad (3-103), \\
\begin{bmatrix}
u_1 \\
\vdots \\
u_n
\end{bmatrix} = \begin{bmatrix}
\tilde{A}_1 \\
\vdots \\
\tilde{A}_n \\
\tilde{B}_1 \\
\vdots \\
\tilde{B}_n \\
\tilde{C}_1 \\
\vdots \\
\tilde{C}_n
\end{bmatrix} \quad (3-102)
\end{aligned}
\]

3.4 Nonlinear Deformations
where $\frac{\partial R_3}{\partial u}$ to $\frac{\partial R_{2+m}}{\partial u}$ are the partial derivatives of Equation 3-100 for $i = 1, 2 \ldots m$, where for $k = i,$

\[
\frac{\partial R_{z+i}}{\partial A_k} = \frac{2 \left( m - \frac{1}{2} \right)}{b_0 L} - b_2 (i - 1) + \left( -b_1 b_3 (i - 1) - \frac{b_2 b_3 (i - 1)^4 w L^3}{48 \left( m - \frac{1}{2} \right)^3 E I} \right. \\
+ \frac{b_2 b_3 (i - 1)^3 w L^3}{16 \left( m - \frac{1}{2} \right)^2 E I} + b_2 (i - 1) - \frac{b_2 b_3 (i - 1)^2 w L^3}{(m - \frac{1}{2}) E I} \bar{A}_i \\
\left. - \frac{b_2 b_3 (i - 1) w L^3}{EI} \left( \frac{(i-1)^3}{48(m-\frac{1}{2})^2} + \frac{(i-1)^2}{16(m-\frac{1}{2})} + \frac{(i-1)^2}{(m-\frac{1}{2})} \bar{A}_i + \bar{B}_i \right) \right) \\
\]

(3-105)
\[
\frac{\partial R_{2+i}}{\partial B_k} = -\left(m - \frac{1}{2}\right)b_2 + \left(-\left(m - \frac{1}{2}\right)b_1b_3 - \frac{b_2b_3(i - 1)^3wL^3}{48\left(m - \frac{1}{2}\right)^2EI}\right) + \frac{b_2b_3(i - 1)^2wL^3}{16\left(m - \frac{1}{2}\right)EI} + \left(m - \frac{1}{2}\right)b_2 - \frac{b_2b_3(i - 1)wL^3}{EI}\tilde{A}_i
\]

\[
- \left(m - \frac{1}{2}\right)b_2b_3WL^3 \bigg(\frac{\partial}{\partial B_i}\bigg) \frac{-b_2WL^3}{EI} \left(\frac{(i-1)^3}{48\left(m - \frac{1}{2}\right)^2} + \frac{(i-1)^2}{16\left(m - \frac{1}{2}\right)} + \frac{(i-1)}{\left(m - \frac{1}{2}\right)^2}\right) + \frac{b_2b_3(i - 1)^3wL^3}{16\left(m - \frac{1}{2}\right)^2 EI} + \frac{b_2b_3(i - 1)^2wL^3}{\left(m - \frac{1}{2}\right)EI}\tilde{A}_i + B_i
\]

\[
\frac{\partial R_{2+i}}{\partial C_k} = 0
\]

for \(k < i\),

\[
\frac{\partial R_{2+i}}{\partial \tilde{A}_k} = -\frac{b_2(k - 1)}{\left(m - \frac{1}{2}\right)^2} + \left(-b_1b_3(k - 1) - \frac{b_2b_3(k - 1)^4wL^3}{48\left(m - \frac{1}{2}\right)^3 EI}\right) + \frac{b_2b_3(k - 1)^3wL^3}{16\left(m - \frac{1}{2}\right)^2 EI} + b_2(k - 1) - \frac{b_2b_3(k - 1)^2wL^3}{\left(m - \frac{1}{2}\right)EI}\tilde{A}_k
\]

\[
- \frac{b_2b_3(k - 1)wL^3}{EI} \bigg(\frac{\partial}{\partial B_k}\bigg) \frac{-b_2WL^3}{EI} \left(\frac{(k-1)^3}{48\left(m - \frac{1}{2}\right)^2} + \frac{(k-1)^2}{16\left(m - \frac{1}{2}\right)} + \frac{(k-1)}{\left(m - \frac{1}{2}\right)^2}\right) + \frac{b_2b_3(k - 1)^3wL^3}{16\left(m - \frac{1}{2}\right)^2 EI} + \frac{b_2b_3(k - 1)^2wL^3}{\left(m - \frac{1}{2}\right)EI}\tilde{A}_k + B_k
\]
\[ \frac{\partial R_{2+i}}{\partial B_k} = -\left( m - \frac{1}{2} \right) b_2 + \left( -\left( m - \frac{1}{2} \right) b_1 b_3 - \frac{b_2 b_3 (k - 1)^3 w L^3}{48 \left( \frac{1}{2} \right)^2 E I} \right) + \frac{b_2 b_3 (k - 1)^2 w L^3}{16 \left( m - \frac{1}{2} \right) E I} + \left( m - \frac{1}{2} \right) b_2 - \frac{b_2 b_3 (k - 1) w L^3}{E I} \bar{A}_k \]

\[ -\left( m - \frac{1}{2} \right) b_2 b_3 w L^3 \right) e^{\frac{-b_2 w L^3 E I}{48 \left( \frac{1}{2} \right)^3 E I} + \frac{(k-1)^2}{16 \left( \frac{1}{2} \right)^2 E I} + \frac{(k-1) A_k}{E I} B_k} \]

\[ \frac{\partial R_{2+i}}{\partial C_k} = 0 \quad (3-109) \]

and for \( k > i \),

\[ \frac{\partial R_{2+i}}{\partial A_k} = 0 \quad (3-110) \]

\[ \frac{\partial R_{2+i}}{\partial B_k} = -\left( m - \frac{1}{2} \right) b_2 + \left( -\left( m - \frac{1}{2} \right) b_1 b_3 - \frac{b_2 b_3 (k - 1)^3 w L^3}{48 \left( \frac{1}{2} \right)^2 E I} \right) + \frac{b_2 b_3 (k - 1)^2 w L^3}{16 \left( m - \frac{1}{2} \right) E I} + \left( m - \frac{1}{2} \right) b_2 - \frac{b_2 b_3 (k - 1) w L^3}{E I} \bar{A}_k \]

\[ -\left( m - \frac{1}{2} \right) b_2 b_3 w L^3 \right) e^{\frac{-b_2 w L^3 E I}{48 \left( \frac{1}{2} \right)^3 E I} + \frac{(k-1)^2}{16 \left( \frac{1}{2} \right)^2 E I} + \frac{(k-1) A_k}{E I} B_k} \]

\[ \frac{\partial R_{2+i}}{\partial C_k} = 0 \quad (3-113) \]
3.5 MATLAB programming

The mechanical models developed in Sections 3.2 to 3.4 were implemented into MATLAB (The Mathworks 2011) for the automated calculation of: (1) diaphragm displacement profile, and (2) diaphragm force-displacement response. The code for (1) and (2) was developed for both linear-elastic and nonlinear diaphragm deformation analysis, making a total of four individual analysis programs, which are provided in Appendix A with appropriate commentary for reference. Each set of code is completely generalised, enabling the user to specify any rectangular diaphragm geometric configuration (assuming that the floorboards are fastened orthogonal to the joists) subjected to any uniformly distributed load that is applied in either principal direction (parallel-to-joist or perpendicular-to-joist).

For linear-elastic analysis, the user is required to provide an effective nail connection stiffness. For nonlinear analysis, the user is required to provide realistic values for the three characteristic parameters $K_0$, $K_1$, and $F_0$, that are required for the nonlinear load-slip function developed by Foschi (1974).

For diaphragm displacement profile, the user is required to specify a uniformly distributed load and a span location between $0 \leq x \leq L/2$, for which the MATLAB code calculates and displays both the full span displacement profile of the diaphragm and the displacement of the diaphragm at the specified span location.

For force-displacement response, the user is required to specify a vector of uniformly distributed load values and a span location between $0 \leq x \leq L/2$, for which the MATLAB code calculates and displays the force-displacement response of the diaphragm at the specified location for the given loading range.
3.6 Discussion

Using the MATLAB code described in Section 3.5, the mechanical model developed in Sections 3.2 to 3.4 is demonstrated to appropriately capture diaphragm behaviour. While numerous parameters influence diaphragm behaviour, for the purpose of illustrating the effective response of the developed mechanical model when altering the input parameters, nail couple spacing \(s\), nail connection spacing \(y\) and nail connection performance parameters \(k_n, K_0, K_1,\) and \(F_0\) were selected for discussion. Consequently, the following parameters were assigned constant values:

- Diaphragm span, \(L = 10.4\) m
- Diaphragm width, \(B = 5.5\) m
- Floorboard thickness, \(t_f = 25\) mm
- Floorboard width, \(d_f = 135\) mm
- Joist thickness, \(t_j = 50\) mm
- Joist depth, \(d_j = 300\) mm
- Timber elastic modulus, \(E = 8\) GPa
- Applied load, \(W = 5\) kN/m

Consider firstly the theoretical limits of nail connection stiffness \(k_n\) using the linear-elastic analysis model. For the lower limit \(k_n \rightarrow 0\) the nail connections provide zero resistance to diaphragm deformation and the model should respond identically to a simply supported beam with an equivalent configuration. It is illustrated in Figure 3-9a that for \(k_n \rightarrow 0\), the displacement profile generated by the analytical model exactly matches the displacement function of a simply supported beam, as described in the following equation:

\[
v(x) = \frac{wx}{24EI} \left( L^3 - 2Lx^2 + x^3 \right)
\]  

For the upper limit \(k_n \rightarrow \infty\), the nail connections theoretically provide infinite resistance to member rotation at each floorboard-to-joist nail connection, meaning that the slope at these locations must be zero. It is shown in Figure 3-9b that for \(k_n \rightarrow \infty\), the analytical model appropriately captures the predicted zero-slope behaviour at the nail connection locations, and accordingly, a dramatic reduction in displacement magnitude (in comparison to the lower limit analysis \(k_n \rightarrow 0\)) is observable.
3.6 Discussion

(a) For $k_n \rightarrow 0$

(b) For $k_n \rightarrow \infty$

**Figure 3-9: Analytical displacement profile for theoretical limits of $k_n$**

The response of the analytical model to changes in nail connection performance, nail connection spacing, and nail couple spacing are illustrated in Figures 3-10 to 3-12, respectively. For each figure, the plots were generated using three data sets labelled *low*, *middle* and *high*, corresponding to their influence on the model (refer to Table 3-1). For Figure 3-10, the middle data set of $K_0$, $K_1$, and $F_0$ was established from experimental values determined by Dolan and Madsen (1992), from which the high and low data sets were calculated by simply doubling or halving the middle data set values, respectively. For Figures 3-11 and 3-12, the middle data sets of $\gamma$ and $s$ were based upon typical diaphragm configurations parameters (see Chapter 1). The general effect that $\gamma$ and $s$ have on diaphragm deformations was demonstrated by appropriately modifying these values (see Table 3-1).

It is shown in Figure 3-10 that the analytical model appropriately responds to variations in nail connection performance, by exhibiting lower stiffness and greater displacements for
decreasing values of $K_0$, $K_1$, and $F_0$. Reduced nail connection performance translates into lower restoring moments at each floorboard-to-joist connection, and in turn, reduced resistance to diaphragm deformations from applied loading.

For loading parallel-to-joist, reduced nail connection spacing corresponds to an increase in the number of floorboard-to-joist nail connections, which logically provides greater resistance to lateral loading, and therefore greater diaphragm strength and diaphragm stiffness. Accordingly, the analytical model is shown to correctly respond to decreasing nail connection spacing by exhibiting greater stiffness and lower displacements, as illustrated in Figure 3-11.

Nail couple spacing directly influences the magnitude of the restoring moment at each floorboard-to-joist nail connection. The analytical model should therefore appropriately capture reduced diaphragm stiffness and increased diaphragm displacements for decreasing values of nail couple spacing. From the displacement profile and force-displacement plots in Figure 3-12, the analytical model is proven to correctly respond to changes in nail couple spacing, based on the above logic.

<table>
<thead>
<tr>
<th>Parameter group</th>
<th>Parameter</th>
<th>Set label</th>
<th>Low</th>
<th>Middle</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nail connection performance</td>
<td>$K_0$ (N/mm)</td>
<td>591</td>
<td>1182</td>
<td>2364</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_1$ (N/mm)</td>
<td>25</td>
<td>50</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_0$ (N)</td>
<td>460</td>
<td>920</td>
<td>1840</td>
<td></td>
</tr>
<tr>
<td>Nail connection spacing</td>
<td>$\gamma$ (mm)</td>
<td>800</td>
<td>400</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Nail couple spacing</td>
<td>$s$ (mm)</td>
<td>75</td>
<td>95</td>
<td>115</td>
<td></td>
</tr>
</tbody>
</table>
3.6 Discussion

Figure 3-10: Analytical model response to varying nail connection parameters, $K_0$, $K_1$, and $F_0$

Figure 3-11: Analytical model response to varying nail connection spacing, $\gamma$

Figure 3-12: Analytical model response to varying nail couple spacing, $s$
3.7 Conclusions

The in-plane deformation mechanics of straight-sheathed timber floor diaphragms was presented. By appropriately idealising diaphragm behaviour, an analytical model was developed that mathematically describes diaphragm deformations when loaded in either principal loading direction (parallel-to-joists and perpendicular-to-joists). Application of the model for both linear-elastic and nonlinear nail connection load-slip behaviour was presented. The model is restricted to symmetrical diaphragm configurations with laterally applied uniformly distributed loads, and to the theoretical nail connection load-slip characteristic developed by Foschi (1974).

The developed model demonstrated logical responses to changing diaphragm parameters \( (s, \gamma, K_0, K_1, \text{ and } F_0) \), and was shown to appropriately capture the theoretical limits of nail connection stiffness, \( k_n \to 0 \) and \( k_n \to \infty \).

The analytical model is used in Chapter 4 to inform the appropriate characterisation of diaphragm behaviour for fundamental period determination and for the assessment of seismic performance.
IDEALISATION OF DIAPHRAGM BEHAVIOUR FOR ASSESSMENT

Diaphragm fundamental period and diaphragm strength and stiffness are critical parameters for URM building seismic assessment, as they significantly influence design loads and predicted wall displacements. To simplify the estimation of these parameters, in-plane diaphragm behaviour is typically idealised in common assessment guides (ASCE 2007; NZSEE 2006) to derive convenient expressions for parameter calculation. ASCE (2007) and NZSEE (2006) stipulate that diaphragm fundamental period be calculated as,

\[ T = \sqrt{3.07\Delta_d} \]  

(4-1)

where \( \Delta_d \) is maximum diaphragm deformation due to a lateral load of 1.0g. ASCE (2007) recommends that diaphragm strength and stiffness be evaluated through use of default shear strength \( R_d \) and shear stiffness \( G_d \) parameters, from which diaphragm midspan displacement at yield is determined from,

\[ \Delta_y = \frac{v_y L}{2G_d} \]  

(4-2)
where $v_y$ is shear force per unit width of diaphragm at yield, and $L$ is diaphragm span. However neither assessment document clarifies the sources of these equations, and consequently the basis of the assessment procedures remain unclear.

In this chapter, the diaphragm behaviour idealisations used to derive Equations 4-1 and 4-2 are determined and evaluated against predicted diaphragm response using the analytical model developed in Chapter 3. Wherever necessary, the methodology for diaphragm fundamental period assessment and for the assessment of diaphragm seismic performance is appropriately modified.

### 4.1 Current Assessment Equations

**4.1.1 Fundamental period assessment**

To determine the origin of Equation 4-1, consider the deflected geometry of a fixed-ended flexural beam sagging under its own self-weight,

\[
\delta_{ff}(x) = \frac{m g x^2}{24EI} (L - x)^2 \tag{4-3}
\]

where $x$ is distance from the support, $m$ is mass per unit distance, $E$ is elastic modulus, and $I$ is second-moment of area. From Equation 4-3, the maximum displacement can be determined as,

\[
\Delta_{ff} = \frac{m g L^4}{384EI} \tag{4-4}
\]

As outlined in Chopra (2007), a structure comprising distributed mass and elasticity can be approximated as a generalised single degree-of-freedom system in order to estimate its fundamental period. The basis of this method is to lump the mass and stiffness of the true infinite degree-of-freedom structure into a single degree-of-freedom, and to restrict the
displacements of this system to an assumed shape function $\psi(x)$ that approximates the fundamental mode of vibration. From this shape function, expressions for generalised mass $\tilde{m}$ and generalised stiffness $\tilde{k}$ can be derived, which can then be introduced to the generalised period equation to obtain a specific expression for fundamental period.

The shape function is defined by,

$$\psi(x) = \frac{u(x,t)}{z(t)}$$

(4-5)

where $u(x,t)$ is the assumed displacement of the structure, and $z(t)$ is the generalised displacement. Assuming $u(x,t)$ to be the displacement function described in Equation 4-3 and taking $z(t)$ to be midspan (maximum) displacement, the shape function for a fixed-ended flexural beam can be considered to be,

$$\psi(x) = \frac{16x^2}{L^4} (L - x)^2$$

(4-6)

The generalised mass and generalised stiffness may subsequently be determined by substituting Equation 4-6 into Equations 4-7 and 4-8, respectively,

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx$$

(4-7)

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx$$

(4-8)

giving,

$$\tilde{m} = \frac{128}{315} \bar{m}L$$

(4-9)

$$\tilde{k} = \frac{1024 EI}{L^3}$$

(4-10)
Substituting Equations 4-9 and 4-10 into the generalised period equation,

\[ T = 2\pi \sqrt{\frac{m}{k}} \]  

(4-11)

gives,

\[ T = 2\pi \sqrt{\frac{1}{504 EI} \frac{\bar{m} L}{\bar{m}}} \]  

(4-12)

which, by comparison with Equation 4-4, may be re-expressed in terms of maximum displacement as,

\[ T = \sqrt{3.07\Delta_{ff}} \]  

(4-13)

Equation 4-13 is identical to Equation 4-1, demonstrating that the assessment of diaphragm fundamental period in ASCE (2007) and NZSEE (2006) most probably derives from a fixed-ended flexural beam idealisation of diaphragm behaviour.

### 4.1.2 Displacement assessment

To evaluate the origin of Equation 4-2, consider a rectangular shear beam subjected to an applied point-load at midspan. For this idealisation, the deflected geometry is given by,

\[ \delta_x(x) = \frac{\kappa P x}{2 G B t} \]  

(4-14)

where, \( \kappa \) is form factor (which accounts for the distribution of shear stress over the rectangular section), \( P \) is applied point load, \( G \) is shear modulus, \( B \) is beam depth, and \( t \) is beam thickness. Replacing \( P \) with \( 2V \) in Equation 4-14 and recognising that \( V = v_s B \), the maximum displacement can be expressed as,
Using Equation 4-15, it is postulated that the authors of ASCE (2007) assumed $\kappa \approx 1$ and considered $G_d = Gt$ in the derivation of Equation 4-2, which, upon substitution, makes the two equations identical. In the absence of a formally published derivation of Equation 4-2, the presented derivation is considered a logical demonstration that diaphragm behaviour was idealised in ASCE (2007) as a point-loaded shear beam for the purposes of diaphragm deformation assessment.

4.2 Idealisation of Diaphragm Behaviour

The analysis presented in Section 4.1 highlighted two important concerns: (1) there is inconsistency between the idealisations of diaphragm behaviour for fundamental period determination and for seismic performance assessment in current assessment guides ASCE (2007) and NZSEE (2006), with a different type of beam analogy having been adopted for each specific application, and (2) neither idealisation is supported by documented analysis verifying that it suitably captures diaphragm behaviour.

To demonstrate which diaphragm idealisation is most appropriate for assessment purposes, true diaphragm behaviour was examined against: (1) a fixed-ended flexural beam, (2) a pin-ended flexural beam, and (3) a shear beam. Using the analytical model developed in Chapter 3, the displacement profile and the relationship between midspan displacement and span length of a standard timber floor diaphragm were compared against the three proposed beam analogies in order to judge which idealisation most adequately captures diaphragm behaviour. Following the selection of an appropriate beam analogy, a parametric analysis was performed to ensure that the selected idealised behaviour was consistently suitable for all realistic variations in diaphragm configuration parameters.

A standard diaphragm configuration was established from the characterisation survey presented in Chapter 1 as a representative example of timber floor construction in historic
New Zealand URM buildings. The parameters selected in Section 4.2.1 for standard diaphragm analysis are listed in Table 4-1 along with the corresponding realistic upper and lower bound values that were used for parametric analysis in Section 4.2.2. The nail connection parameters $K_0$, $K_1$, and $F_0$ required for the analytical model were taken from Dolan and Madsen (1992) for the standard diaphragm configuration (see Table 4-1). Although realistic variations of these parameters are unknown, upper and lower bound values were included in the parametric analysis by doubling or halving their value, respectively, in order to assess the sensitivity of diaphragm behaviour to changes in these parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm span, $L$ (m)</td>
<td>10 12 24</td>
</tr>
<tr>
<td>Diaphragm width, $B$ (m)</td>
<td>6 8 16</td>
</tr>
<tr>
<td>Floorboard section (mm)</td>
<td>100 × 15 135 × 25 150 × 30</td>
</tr>
<tr>
<td>Joist section (mm)</td>
<td>150 × 40 300 × 50 400 × 65</td>
</tr>
<tr>
<td>Timber elastic modulus, $E$ (GPa)</td>
<td>4.0 10.0 15.0</td>
</tr>
<tr>
<td>Nail couple spacing, $s$ (mm)</td>
<td>Varies but taken as 2/3 × floorboard width</td>
</tr>
<tr>
<td>Joist spacing (c/c), $\ell$ (mm)</td>
<td>300 400 650</td>
</tr>
<tr>
<td>$K_0$ (N/mm)</td>
<td>591 1182 2364</td>
</tr>
<tr>
<td>$K_1$ (N/mm)</td>
<td>25 50 100</td>
</tr>
<tr>
<td>$F_0$ (N)</td>
<td>460 920 1840</td>
</tr>
</tbody>
</table>

### 4.2.1 Selection of diaphragm idealisation

Consider the displacement geometries of the three proposed beam idealisations for an applied uniformly distributed load $w$:

1. Fixed-ended flexural beam,

$$
\delta_\theta(x) = \frac{wx^2}{24EI} (L - x)^2
$$

(4-16)
(2) Pin-ended flexural beam,
\[
\delta_{pf}(x) = \frac{wx}{24EI} \left( L^3 - 2Lx^2 + x^3 \right) \tag{4-17}
\]

(3) Shear beam,
\[
\delta_s(x) = \frac{wk}{GA} \left( L \left( \frac{x}{2} - \frac{x^2}{2} \right) \right) \tag{4-18}
\]

The displacement profiles described by Equations 4-16 to 4-18 are plotted against the displacement profile of the standard diaphragm in Figure 4-1 for both principal loading directions. To make an appropriate comparison, displacements were normalised to the maximum displacement of the standard diaphragm and span location was normalised to diaphragm span.

![Figure 4-1: Displacement profile comparison](image-url)
Figure 4-1 illustrates that diaphragm displacements trace between the displacement profile of a pin-ended flexural beam and the displacement profile of a shear beam, although slightly closer to a pin-ended flexural beam. A fixed-ended flexural beam is shown to be an inappropriate characterisation of diaphragm displacement profile.

Consider next the relationship between diaphragm midspan displacement and diaphragm span,

\[ \Delta \propto L^n \]  

(4-19)

For the three proposed beam idealisations, this relationship is defined by the following midspan displacement equations,

\[ \Delta_{ff} = \frac{wL^4}{384EI} \]  

(4-20)

\[ \Delta_{pf} = \frac{5wL^4}{384EI} \]  

(4-21)

\[ \Delta_s = \frac{wkL^2}{8GA} \]  

(4-22)

Using Equations 4-20 to 4-22, the relationship described by Equation 4-19 is plotted in Figure 4-2 for a range of span lengths for the proposed beam idealisations and for the standard diaphragm configuration. To generate Figure 4-2, midspan displacements were determined for span lengths up to 48 m, and for comparative purposes, were normalised to the midspan displacement determined for the maximum applied span length (48 m). Figure 4-2 clearly demonstrates that for both principal loading directions, the proportionality of diaphragm displacement to diaphragm span is most closely approximated by a shear beam idealisation.

To further illustrate the above observation, the values of \( n \) in Equation 4-19 for the adopted range of span lengths may be evaluated using the logarithmic properties,
4.2 Idealisation of Diaphragm Behaviour

\[ n = \log_{L} \Delta = \frac{\log_{\Delta} \Delta}{\log_{L} L} \quad (4-23) \]

Applying Equation 4-23 to the standard diaphragm configuration, the relationships between \( n \) and diaphragm span shown in Figure 4-3 are generated for each principal loading direction.

(a) Loading parallel-to-joist

(b) Loading perpendicular-to-joist

**Figure 4-2: Relationship between \( \Delta \) and \( L \)**

(a) Loading parallel-to-joist

(b) Loading perpendicular-to-joist

**Figure 4-3: Relationship between \( n \) and \( L \)**
In Figure 4-3, the value of $n$ is shown to descend rapidly from $n \approx 4$ for $L \to 0$ to $n \approx 2$ for $L \to \infty$, further demonstrating that even for small span lengths, the proportionality of diaphragm midspan displacement to diaphragm span is closely approximated by a shear beam idealisation ($\Delta \propto L^2$). It is important to recognise that the observations made from Figures 4-2 and 4-3 are not only a product of increasing diaphragm span, but an increase in the number of nail connections. However given that changes in diaphragm span will always be accompanied by corresponding changes in the number of nail connections, the relationships between $\Delta$, $n$, and $L$ shown in Figures 4-2 and 4-3 will always hold true.

From the analysis results presented above, it is evident that diaphragm behaviour is most appropriately idealised by a shear beam analogy. Although the displacement profile of the standard diaphragm configuration was shown to be captured slightly better by the idealised pin-ended flexural beam, overwhelming evidence was presented which demonstrated diaphragm behaviour to be almost identical to that of an idealised shear beam, with respect to the relationship of midspan displacement and diaphragm span. Given that the proportionality of $\Delta$ and $L$ is more important for diaphragm assessment purposes than is displacement profile, the shear beam analogy is undoubtedly the most suitable idealisation of diaphragm behaviour.

### 4.2.2 Sensitivity of diaphragm behaviour

A sensitivity analysis was performed to ensure that the proposed shear beam analogy remains an appropriate idealisation of diaphragm behaviour for realistic variations in parameter values. The analysis was performed by programming the analytical model with the upper and lower bound values of an individual parameter set (see Table 4-1), while keeping all other parameters equal to the standard diaphragm configuration. For each parameter set, plots of the relationship between $\Delta$ and $L$ and the relationship between $n$ and $L$ were generated for comparison (see Figures 4-4 to 4-8). It should be noted that the upper and lower bound values for parameters $K_0$, $K_1$, and $F_0$ were analysed simultaneously under the label variations in nail connection characteristic. It should also be noted that all parametric analyses were performed for loading parallel-to-joists except for variations in joist size, which was performed for loading perpendicular-to-joists.
4.2 Idealisation of Diaphragm Behaviour

Inspecting Figures 4-4 to 4-8 it is apparent that little variation in diaphragm behaviour occurs with realistic variations in diaphragm parameter values. Consequently, the sensitivity analysis has adequately demonstrated that a shear beam analogy is an appropriate idealisation of diaphragm behaviour for any realistic diaphragm configuration.
Chapter 4: Idealisation of Diaphragm Behaviour for Assessment

(a) Relationship between $\Delta$ and $L$

(b) Relationship between $n$ and $L$

Figure 4-6: Diaphragm behaviour for variations in timber elastic modulus ($E$)

(a) Relationship between $\Delta$ and $L$

(b) Relationship between $n$ and $L$

Figure 4-7: Diaphragm behaviour for variations in joist spacing ($\ell$)
4.3 Updating assessment equations

The analyses presented in Sections 4.1 and 4.2 have indicated that the current procedure for diaphragm fundamental period assessment in ASCE (2007) and NZSEE (2006) is incorrectly based upon a fixed-ended flexural beam idealisation, which was shown to poorly capture diaphragm behaviour. Although a shear beam idealisation is correctly assumed for seismic performance assessment in ASCE(2007), the applicability of a centrally applied point-load is questionable for realistic earthquake loading applications, and must be reviewed. Accordingly, the assessment procedure for timber floor diaphragms in ASCE (2007) and NZSEE (2006) is updated and harmonised hereafter, to reflect the appropriate idealisation of diaphragm behaviour for suitable earthquake loading conditions.

4.3.1 Fundamental period assessment

Following the methodology presented in Section 4.1.1, the fundamental period of an idealised shear beam may be evaluated for the correct seismic loading conditions.
ASCE (2007) recommends that the distribution of horizontal inertial forces on a diaphragm be given by,

\[ w_E = \frac{1.5F_d}{L} \left[ 1 - \left( \frac{2x}{L} - 1 \right)^2 \right] \]  
(4-24)

where \( F_d \) is total diaphragm seismic load, \( L \) is diaphragm span, and \( x \) is distance from the side of the diaphragm. It can be shown that the deflected geometry of a rectangular shear beam subjected to the parabolic load distribution given in Equation 4-24 is,

\[ \delta_s = \frac{6F_d}{5GA} \left( \frac{x}{2} - \frac{x^3}{L^2} + \frac{x^4}{2L^3} \right) \]  
(4-25)

from which the displacement at midspan can be determined as,

\[ \Delta_s = \frac{3F_dL}{16GA} \]  
(4-26)

Replacing \( F_d \) with \( mg \), Equations 4-25 and 4-26 become, respectively,

\[ \delta_s = \frac{6mg}{5GA} \left( \frac{x}{2} - \frac{x^3}{L^2} + \frac{x^4}{2L^3} \right) \]  
(4-27)

\[ \Delta_s = \frac{3mgL}{16GA} \]  
(4-28)

where \( m \) is the total seismic mass attributed to the diaphragm and \( g \) is gravity. As presented in Section 4.1.1, the fundamental period of a structure can be approximated using a generalised single degree-of-freedom system, comprising a generalised mass \((\bar{m})\), a generalised stiffness \((\bar{k})\), and an assumed shape function \((\psi(x))\) which approximates the fundamental mode of vibration. With reference to Equation 4-5, assuming \( u(x,t) \) to be the displacement function described in Equation 4-27, and taking \( z(t) \) to be the midspan
displacement given in Equation 4-28, the shape function for a rectangular shear beam can be considered to be,

\[
\psi(x) = \frac{32}{5L} \left( x - \frac{x^3}{L^2} + \frac{x^4}{2L^3} \right) \quad (4-29)
\]

Substituting this shape function into Equations 4-7 and 4-8, the generalised mass and generalised stiffness of the generalised single degree-of-freedom system can be expressed as,

\[
\tilde{m} = \frac{3968}{7875} m \quad (4-30)
\]

\[
\tilde{k} = \frac{4352GA}{875\kappa L} \quad (4-31)
\]

Introducing Equations 4-30 and 4-31 into the generalised period equation given in Equation 4-11, the fundamental period of rectangular shear beam can be calculated as,

\[
T = \sqrt{\frac{3.9994}{3.9994}} \frac{mkL}{GA} \quad (4-32)
\]

which, if compared to the expression for midspan displacement in Equation 4-28, can be re-expressed as,

\[
T = \sqrt{2.61\Delta_s} \quad (4-33)
\]

Finally, if \( A = Bt \), and \( G_d = Gt \), then by substituting Equation 4-26 into Equation 4-32, fundamental period can also be expressed as,
\[ T = 0.7 \sqrt{\frac{F_d L}{G_d B}} \]  (4-34)

From the analysis presented above, it is recommended that the equation for fundamental period determination in ASCE (2007) and NZSEE (2006) be updated with either Equation 4-33 or Equation 4-34 (which is a more explicit expression of Equation 4-33).

### 4.3.2 Seismic performance assessment

The fundamental period analysis presented in Section 4.3.1 has provided sufficient evidence for the appropriate assessment of diaphragm seismic performance. Based on the parabolic load distribution recommended by ASCE (2007) (see Equation 4-24) and the proposed shear beam idealisation, the midspan displacement of timber floor diaphragms can be evaluated using Equation 4-26. If midspan displacement is considered at yield, and \( F_y \) is replaced with \( v_y B \), then Equation 4-26 becomes,

\[ \Delta_y = \frac{3v_y L}{16G_d} \]  (4-35)

which is analogous to the equation for \( \Delta_y \) published in ASCE (2007) except for the appropriate modification of the coefficient value. Consequently, it is recommended that the current equation for \( \Delta_y \) given in Equation 4-2 be updated with Equation 4-35 in the relevant assessment documents. Furthermore, based on the proposed shear beam analogy, it is recommended that diaphragm seismic strength be evaluated using default shear strength values, \( R_d \), which may be substituted for \( v_y \) in Equation 4-35 to determine diaphragm yield displacement.
4.4 Conclusions

It is concluded that diaphragm behaviour is most suitably captured by a shear beam idealisation for the detailed assessment of fundamental period and seismic performance. Based on this conclusion, the current procedure for diaphragm fundamental period determination in ASCE (2007) and NZSEE (2006) was shown to be incorrectly derived from a fixed-ended flexural beam idealisation. Although the assessment of diaphragm seismic performance is correctly formulated in ASCE (2007) based upon a shear beam idealisation, the adopted point-load conditions were concluded to be an inappropriate reflection of realistic earthquake loads.

From the analysis presented in this chapter, it is recommended that diaphragm fundamental period be evaluated using,

\[ T = \sqrt{2.61 \Delta_d} \]  \hspace{1cm} (4-36)

or, more explicitly,

\[ T = 0.7 \sqrt{\frac{V_d L}{G_d B}} \]  \hspace{1cm} (4-37)

It is recommended that diaphragm yield displacement be determined as,

\[ \Delta_y = \frac{3v_y L}{16G_d} \]  \hspace{1cm} (4-38)

It is recommended that current assessment guides ASCE (2007) and NZSEE (2006) be updated to reflect the recommendations determined in this chapter.
Chapter 5

NAIL CONNECTION TESTING

Nail-fastened timber structures depend heavily on the load-slip characteristics of their primary nail connections (Dolan and Madsen 1992; Filiatrault and Folz 2002; Foschi 1977; Foschi and Bonac 1977). Quantifying the performance of floorboard-to-joist nail connections when subjected to lateral loading is therefore essential for the accurate seismic assessment of timber floor diaphragms.

This chapter summarises lateral tests performed on nail connections to establish their hysteretic behaviour, for implementation into finite element (FE) models developed in SAP2000 (CSI 2004) which are presented in Chapter 8. Testing was performed on new connections constructed with pine timber and wire drawn nails, and on salvaged connections extracted from the timber diaphragms of two New Zealand heritage URM buildings. New connections were built with the same materials as used for the sub-component diaphragms described in Chapter 6, and the full-scale diaphragms described in Chapter 7, from which the customised load-slip response data was used to calibrate and validate sub-component and full-scale diaphragm FE models that are presented in Chapter 8. The results obtained from testing of salvaged connections were used to calibrate the finalised FE model to ensure that the seismic assessment procedures developed in Chapter 10 were representative of existing timber diaphragms in heritage URM buildings.
Due to an opportunity to study abroad, nail connection testing was performed in two parts. Connections constructed with sub-component diaphragm materials were tested at Drexel University in Philadelphia, USA, while connections constructed with full-scale diaphragm materials and connections salvaged from heritage URM buildings were tested at the University of Auckland in New Zealand. This chapter describes the nail connections tested and the testing details employed at each institution. For each connection type, an averaged backbone load-slip relationship was developed from the measured hysteretic responses, and was further characterised to compare new and salvaged connection performance. In addition, a discussion of the effects of connection deterioration on performance is provided.

5.1 New Nail Connections

As outlined above, nail connections constructed with sub-component diaphragm materials were tested in the USA and nail connections constructed with full-scale diaphragm materials were tested in New Zealand, and for simplicity are hereafter reported as New-USA and New-NZ, respectively.

5.1.1 Construction of New-USA connections

A total of six New-USA test units were constructed using 19 mm x 89 mm Eastern White Pine floorboards and 38 mm x 140 mm Hem-Fir joists which were nailed together using 64 mm long bright common wire nails with a shank diameter of 3.33 mm. The White Pine timber had an average density of 390 kN/m$^3$ (specific gravity (s.g.) of 0.36) and average moisture content of 7.1%, while the Hem-Fir timber had an average density of 448 kN/m$^3$ (s.g. of 0.42) and average moisture content of 7.3%.

The overall configuration of the nail connection test units is depicted in Figure 5-1, along with nominal dimensions. The test units comprised two sheathing members nailed to one joist member with blocking in between, and were constructed to fit the custom designed test rig described in Section 5.3. While the performance of a single nail connection was of
interest, the two-connection test units created a concentric loading condition which prevented overturning moments and the need to provide a restoring force. Individual nail connection performance was deduced through data post-processing by halving the applied load and averaging the relative displacements between the joist and floorboards. Figure 5-1 also shows details of holes that were drilled through the top and bottom sections of the test units for bolts to be inserted and securely fastened to the test rig.

Figure 5-1: New-USA test unit details
5.1.2 Construction of New-NZ connections

A total of twelve New-NZ test units were constructed using 18 mm x 135 mm Radiata Pine floorboards and 45 mm x 290 mm Radiata Pine joists which were nailed together using 75 mm x 3.15 mm bright power-driven nails. The floorboard timber had an average density of 487 kN/m³ (s.g. of 0.43) and average moisture content of 12.7%, while the joist timber had an average density of 517 kN/m³ (s.g. of 0.46) and average moisture content of 13.0%.

As depicted in Figure 5-2, New-NZ test units were comprised of two floorboard-to-joist nail connection assemblages, each constructed with a 400 mm length floorboard nailed to a 315 mm length joist with two nails spaced at 95 mm. The two assemblages were then fastened together along the floorboard faces using high strength wood glue to produce a symmetric, four-nail test unit that was compatible with the custom-built test rig described in Section 5.4. This configuration was necessary for stability and to create a concentric loading condition which prevented overturning moments and the need to provide a restoring force. Individual nail connection performance was deduced through data post-processing by dividing the applied load by four, and averaging the two displacement readings taken during testing.

5.2 Salvaged Nail Connections

During the course of this study, two opportunities arose to extract timber floor sections from historic New Zealand URM buildings. The first extraction opportunity was a two-storey URM building located in Parnell, Auckland that was constructed during the 1890’s for a local business and had served many commercial purposes since. In February 2010 the building was demolished by a professional contractor to make way for new construction, during which a number of floor sections were extracted and gifted to this study. All floors of the Parnell building were built with Kauri joists and Kauri T&G floorboards that were generally fastened together with two nails at each floorboard-to-joist interface. Joist
sections were found to have an average profile of 52 mm x 222 mm, and the floorboard profile was on average 20 mm x 135 mm. The nails appeared consistent with turn-of-the 20\textsuperscript{th} century wire-drawn nails, as described by Isaacs (2009), and were found to have an average length of 64 mm and an average shank diameter of 2.95 mm. A set of example nails removed from the salvaged connections are shown in Figure 5-3. Before connection testing was performed, the Kauri floorboard timber was found to have an average density of 546 kN/m\textsuperscript{3} (s.g. of 0.49) and average moisture content of 11.2\%, while the Kauri joist timber was found to have an average density of 537 kN/m\textsuperscript{3} (s.g. of 0.48) and average moisture content of 11.6\%. No material testing was performed on the salvaged wire-drawn nails.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_2.png}
\caption{New-NZ test unit details}
\end{figure}
Chapter 5: Nail Connection Testing

The second extraction opportunity was the three-storey ‘T Adair Building’ located in the central business district of Gisborne, named after its original owner Mr. Thomas Adair. The URM building was constructed in 1914 for commercial purposes and has remained in-service for numerous local businesses since this time. It is registered as a Category B building in the Gisborne District Plan, meaning that it has historical significance within the area. On July 1\textsuperscript{st}, 2009 the T Adair building was damaged by a fire that destroyed the roof and many sections of the upper floor but which left much of the first floor undamaged. Because of its heritage categorisation, demolition of the building was prohibited and the building was instead commissioned for extensive strengthening and refurbishment works to restore its integrity. During this process the remaining floors were slated for removal and authorisation was granted to extract sections for nail connection testing.

The floors of the T Adair building were built with Rimu joists and Rimu T&G floorboards that were generally fastened together with two nails at each floorboard-to-joist interface. Joists sections were found to have an average profile of 50 mm x 300 mm, while floorboards were found to have an average profile of 24 mm x 100 mm. The nails from the salvaged connections appeared consistent with turn-of-the 20\textsuperscript{th} century wire-drawn nails, as described by Isaacs (2009), and were found to have an average length of 63 mm and an average shank diameter of 2.95 mm (see Figure 5-3). Before connection testing was performed, the Rimu floorboard timber was found to have an average density of 534 kN/m\textsuperscript{3} (s.g. of 0.47) and average moisture content of 13.9%, while the Rimu joist...
timber was found to have an average density of 600 kN/m³ (s.g. of 0.53) and average moisture content of 14.1%. Again, no material testing was performed on the salvaged wire-drawn nails.

5.2.1 Floor section extraction

The extraction of timber floor sections required diligence to ensure that nail connection integrity was maintained. Fortunately, the Parnell building was demolished by professional contractors who had specialist equipment onsite to facilitate the careful extraction of medium-to-large sized floor sections. The basic process involved removing perimeter floorboards and attaching the exposed joists to the hydraulic arm of a 20 tonne excavator using chains. The perimeter of the floor section was then chain sawed to release it from the superstructure and was lifted away and placed on a Hiab truck for transportation. The Parnell building was also fortunately located in close proximity to the University of Auckland, so the floor sections were delivered directly to the laboratory in whole sections for test preparation, as shown in Figure 5-4a.

Floor sections in the T Adair building were extracted in smaller sections than were those extracted from Parnell because no hydraulic lifting equipment was available, and the floor was required to remain in-service for the refurbishment construction that was underway at the time. As illustrated in Figures 5-4b and 5-4c, individual joist sections were isolated by chain sawing the floorboards half-way between the adjacent joists. Floorboards were subsequently removed at each end of the section and the joist was cut to detach it from the surrounding floor. Due to their small scale, the isolated floor sections could be hand lifted out and stacked inside a small cargo van that was hired for transportation. This process was repeated at approximately seven locations across the first floor diaphragm, after which the penetrations were repaired with plywood to ensure that a safe working environment was maintained for onsite construction workers.
5.2.2 Test unit preparation

Having successfully extracted sections of flooring from historic URM buildings, further preparation was required for nail connection testing. Salvaged connections were tested at the University of Auckland and were prepared to match the New-NZ test unit configuration illustrated in Figure 5-2 in order to fit the custom-built test rig described in Section 5.4.

The comparatively large floor sections from Parnell were firstly chain sawed into individual joist sections, analogous to the process illustrated in Figures 5-4b and 5-4c for the T Adair building. Alternating floorboards were then removed from the individual joist

(a) Delivery of Parnell floor sections

(b) Chain sawing of T Adair floorboards

(c) Removal of T Adair floor sections

Figure 5-4: Extraction of floor sections
sections to isolate single floorboard-to-connections and to provide sufficient clearance between floorboards for the 315 mm length of joist required for the test unit configuration (see Figure 5-5a). Following this, the joists were carefully cut between the isolated floorboards using a drop saw. The floorboard surfaces were then cleaned of unwanted debris and glued and clamped to produce the final test unit configuration, as shown in Figures 5-5b and 5-5c. A total of twelve test units were prepared from the Parnell building floor sections, and twenty two test units were prepared from the T Adair Building floor sections.

![Figure 5-5: Preparation of salvaged nail connections](image)

(a) Removal of alternating floorboards  
(b) Applying high strength wood glue  
(c) Clamping floorboards together

Figure 5-5: Preparation of salvaged nail connections
5.3 Testing at Drexel University

New-USA connection tests were performed at Drexel University in Philadelphia, USA, in accordance with ISO 16670 (2003) using a Tinius Olsen universal testing machine. Test units were fixed to custom-made upper and lower brackets using carriage bolts, as shown in Figure 5-6. The upper bracket was directly attached to a 10.7 kN Omegadyne load cell that was used to measure total applied load. Custom-made Perspex brackets were fixed to the test units using high strength epoxy resin and used to mount two ±25 mm Polaris linear potentiometers that measured the relative displacement of the left and right joints.

Each test unit was subjected to pseudo-static loads following a reversed-cyclic loading protocol of single cycle displacement to peak amplitudes of 0.375 mm, 0.75 mm, 1.125 mm and 1.5 mm, and three repeated cycles to peak displacement amplitudes of 3.0 mm, 6.0 mm, 9.0 mm and 15.0 mm, as shown in Figure 5-7. Repeated cycling was used to establish performance degradation to the same displacement amplitude. Loading was applied at an average rate of 20 mm/min. Downward loading was defined as positive and upward loading was defined as negative.

![Test set-up at Drexel University](image)
5.4 Testing at the University of Auckland

New-NZ connections and salvaged connections were tested at the University of Auckland in New Zealand, in accordance with ISO 16670 (2003) using a uniaxial Instron testing machine. The test rig shown in Figure 5-8 was custom-designed to accommodate new and salvaged connections by catering for the conventional perpendicular arrangement of floorboard-to-joist connections. The test rig was comprised of two steel angle brackets that were positioned on each side of the test unit and securely clamped to the Instron base plate using C-clamps. Using threaded rods and a steel plate, each joist was tightly post-tensioned to the Instron machine for complete restraint against movement, as shown in Figure 5-8a. A custom-made steel loading bracket that was attached to the Instron loading arm was then lowered into place over the floorboards. As illustrated in Figure 5-8b, the loading bracket comprised an upper plate with a threaded connection for the load cell, and two sets of long prongs with adjustable (upper) loading clamps located at approximately one third down their length. The loading bracket was lowered until these ‘upper’ clamps contacted the upper surface of the floorboards, at which time detachable clamps were bolted to the bottom section of the prongs. The adjustable clamps shown in Figure 5-8d were then

![Figure 5-7: Reversed-cyclic loading schedule](image-url)
carefully tightened to securely fix the floorboards to the loading bracket, ready for cyclic loading (see Figure 5-8c).

Total applied load was recorded by a 10 kN Revere load cell that was attached between the loading bracket and Instron loading arm. Displacement of the connection was measured by two ±25 mm LVDTs positioned at each end of the floorboards, as shown in Figure 5-8d.

Figure 5-8: Test set-up at the University of Auckland
Each test unit was subjected to pseudo-static loads following the identical reversed-cyclic loading protocol used for connections tested at Drexel University (see Figure 5-7). As before, loading was applied at an average rate of 20 mm/min with downward loading defined as positive and upward loading defined as negative.

5.5 Test Results

A summary of the hysteretic behaviour of new and salvaged nail connections is presented. The development of an averaged backbone curve for each connection type is outlined and further characterised using the provisions of ISO 16670 (2003) and the methodology reported by Ceccotti (1995). The characterised test results were used to compare new and salvaged connection performance and to discuss the effects of connection deterioration.

5.5.1 Observations

Before commencing each test, an inspection of the nail connection test unit was performed to appraise its condition. As would be expected, all New-USA and New-NZ connections that were purpose-built for testing appeared in visually perfect condition, with no timber degradation, nail rusting or any other signs of damage. Floorboard and joist members were tightly fastened together with the power-driven nails, meaning that the connection was effectively rigid during handling. In contrast, varying amounts of degradation of the salvaged nail connections was observable. The Rimu timber of the T Adair connections appeared to have suffered wetting and drying cycles during their lifetime, with shrinkage, warping, and some surficial cracking of the floorboards clearly evident, as shown in Figure 5-9a. Such evidence of wetting and drying was largely undetectable for the Parnell salvaged connections, and as such, the Kauri floorboards generally appeared in relatively good condition, as illustrated in Figure 5-9b. Slight gapping between the floorboard and joist members was observed for both the T Adair and Parnell connections, although this gapping was more severe in the T Adair test units. Member gapping combined with what appeared to be minor degradation and loosening of the timber immediately surrounding the
nails, caused the salvaged connections to be slightly wobbly during handling, which would have undoubtedly reduced initial strength and stiffness performance. In addition to this, all nails recovered from the T Adair and Parnell salvaged floor sections were rusted to some extent, with some localised rusting having considerably eroded the shank of several inspected nails (see Figure 5-3). Nail rusting and reduced shank diameter would ultimately further reduce connection performance. Overall the performed condition inspections demonstrated that the T Adair salvaged connections were in poorer condition than the Parnell salvaged connections.

From an observational standpoint, the performed nail connection tests were largely uneventful, due to the unavoidable obstruction-of-view caused by the timber members. As a consequence, the dominant visual observation during testing was simply the loaded floorboards being cyclically displaced relative to the fixed joists. Intermittent timber cracking sounds were often heard for both new and salvaged connections, although these sounds were almost never accompanied by a visual observation. Numerous salvaged connection nails were observed to fracture during testing (typically at displacements in excess of ±6 mm) (see Figure 5-10a), although these observations were only possible when gapping between floorboard and joist members was sufficiently large. As a consequence, the displacement at which these fractures occurred was unable to be precisely determined.

Figure 5-9: General condition of salvaged nail connections before testing
Lastly, in some cases the timber floorboards of salvaged connections split due to the forces generated within the nail connection during testing (see Figure 5-10b).

The deformation mechanics of nailed timber connections has been comprehensively documented in published research (for example Dean et al. 1989; Dolan and Madsen 1992; Ni 1997), so given unavoidable time constraints placed on the testing program, dissection of the test units to examine nail deformation within the timber members was not performed. Based on the wealth of existing knowledge regarding nail connection behaviour, it is expected that both new and salvaged nail connections generated their strength through a combination of nail yielding and localised timber crushing within the timber-to-nail bearing surface, as described in detail by Johansen (1949) and Dean et al. (1989) (see Section 2.2.1). It is acknowledged that different proportions of nail yielding and timber crushing would have occurred based on the relevant material properties and connection integrity, but in general the deformation of the tested nail connections would be representatively illustrated by Figure 2-12 (see Section 2.2.1.2).

### 5.5.2 Force-displacement response

The force-displacement responses of all nail connections are presented in Appendix B for reference. For each connection type, a representative force-displacement response was selected for commentary and for the FE modelling reported in Chapter 8, where the shape

![Fractured nails within floorboard](image1.png)  ![Floorboard splitting](image2.png)

*Figure 5-10: Examples of damage to salvaged nail connections after testing*
of the hysteretic force-displacement response was required for calibrating the hysteretic coefficients of nonlinear spring elements. Figure 5-11 shows the selected representative force-displacement responses, which illustrate the highly nonlinear behaviour and the lack of a distinct yield point that was demonstrated by all connection types. Connection strength is shown to be primarily generated during the initial loading cycle when the nail bears against uncrushed timber (Dean et al. 1989; Dowrick 1986), after which connection strength and stiffness is reduced for repeated loading cycles, leading to a considerable degree of hysteretic pinching. An initial comparison of new and salvaged connection response suggests that salvaged connections have reduced load carrying capacity and

Figure 5-11: Representative nail connection force-displacement responses with backbone curves
reduced ductility capacity, although these performance parameters will be characterised and further discussed in subsequent sections.

5.5.3 Backbone curve development

Backbone (envelope) curves were developed for each hysteretic response using a multi-linear line passing through the maximum force recorded at each displacement amplitude. Figure 5-11 illustrates that the backbone curves appropriately enclose the entire nail connection force-displacement response.

In order to prepare the backbone data for FE model implementation, it was necessary to establish a representative data set for each connection type. Given that the diaphragm FE models reported in Chapter 8 were highly sensitive to the initial response of nail connections, the representative backbone curves were developed by averaging positive-displacement backbone test data only, which comprised complete force-displacement responses from zero to +15 mm displacement. Averaging was achieved by reconfiguring individual positive-displacement backbone data to twenty eight standardised displacement amplitudes between zero and +15 mm, using linear interpolation. Once the backbone curves were standardised, the force values at each assigned displacement amplitude were averaged to obtain an overall single-direction backbone curve representing the performance of each connection type. To accommodate bi-directional nail couple displacement, the averaged positive-displacement backbone curve was mirrored about the x and y axes to produce an averaged and symmetric backbone curve for each connection type, as shown in Figure 5-12 and summarised in Tables B-1 to B-4 in Appendix B.

Nail connection performance varies considerably due to natural differences in material properties and connection configuration (such as nail penetration and driving angle). This variability is further exaggerated for connections salvaged from historic URM buildings which have undergone decades of service and variable weathering processes. To capture realistic variations in connection response, a simple statistical analysis was performed to establish upper bound and lower bound backbone curves for each connection type. Assuming that nail connection test data is normally distributed, the upper and lower
bounds were computed for a 95% confidence interval. This computation was achieved by firstly calculating the standard deviation of the standardised positive-displacement backbone force data at each displacement amplitude, in accordance with Equation 5-1. For each displacement amplitude, an upper and lower bound force value was then determined by adding or subtracting 1.96 times the relevant standard deviation to or from the relevant average force, as described in Equation 5-2.

Figure 5-12: Nail connection averaged backbone curves for FE modelling
5.5 Test Results

\[ sd = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2} \]  \hspace{1cm} (5-1)

Where \( sd \) is standard deviation of the sample, \( N \) is sample size, \( \bar{x} \) is population mean, and \( x_i \) are data set values.

\[ ci = \bar{x} \pm 1.96s \]  \hspace{1cm} (5-2)

Where \( ci \) is the upper or lower bound of the 95% confidence interval.

Again, the developed positive-displacement backbone curves were mirrored about the \( x \) and \( y \) axes to produce symmetric bi-directional upper bound and lower bound backbone curves for each connection type, as shown in Figure 5-12. It should be noted that if the lower bound forces changed sign when applying Equation 5-2, then these values were considered to be zero.

Figure 5-12 illustrates that the hysteretic response of salvaged nail connections was considerably more variable than for new nail connections. Although the pronounced configuration inconsistencies of salvaged connections (compared to laboratory-built new connections) will have undoubtedly contributed to the observed performance variations, the results suggest that connection integrity, and therefore performance, is primarily influenced by the variable processes of aging and deterioration.

5.5.4 Backbone characterisation

To compare the performance of new and salvaged connections, comparative parameters were established from the averaged backbone curves developed in Section 5.5.3. The backbone curves were characterised using the methodology outlined by Ceccotti (1995) to define initial stiffness and the effective yield point. As illustrated in Figure 5-13, initial stiffness \( (K_o) \) was calculated as the line joining the data points corresponding to 10% and 40% of maximum load \( (F_{n,max}) \), while the effective yield point \( (F_{n,y}, \delta_{n,y}) \) was determined as
the intersection of the initial stiffness line and a second tangent line ($K_\beta$) with a gradient equal to one sixth of the initial stiffness. Positioning of the second tangent line was achieved by optimising the tangency of angle $\beta_n$ with respect to the relevant backbone curve data points. The intersection of the initial stiffness line and second tangent line was subsequently determined and the effective yield point established. For New-USA connections, in which no strength degradation occurs on the averaged backbone curve shown in Figure 5-12a, $F_{n,max}$ was not necessarily reached and was defined instead as the maximum recorded load for comparative purposes.

Displacement capacity values were established in accordance with ISO 16670 (2003) for each connection type, in order to estimate the deformation capabilities of timber diaphragms using the FE models presented in Chapter 8, and to specifically establish whether premature connection failure would occur in heritage timber diaphragms subjected to earthquake loading. Displacement capacity (also referred to as ultimate displacement), $\delta_{n,ult}$, was defined as the displacement at $0.8F_{n,max}$ on the descending portion of the averaged backbone curves. When this value was not reached, $\delta_{n,ult}$ was indicated as greater than 15 mm.

Having defined an effective yield point, connection ductility was calculated using Equation 5-3. As discussed by Munoz et al. (2008), the ductility of nailed timber
connections can be described as the ratio of either: (1) displacement at maximum load to effective-yield point displacement, or (2) ultimate displacement to effective yield-point displacement. Because ultimate displacement was not reached for New-NZ and New-USA connections, ductility was defined as the ratio between displacement at maximum load, $\delta_{n,\text{max}}$, and effective yield-point displacement, $\delta_{n,y}$, for the comparative purposes of this analysis.

$$\mu_n = \frac{\delta_{n,\text{max}}}{\delta_{n,y}}$$ (5-3)

A list of all the performance parameters described above is provided in Table 5-1 for comparison.

### Table 5-1: Nail connection performance parameters

<table>
<thead>
<tr>
<th>Connection type</th>
<th>$F_{n,y}$</th>
<th>$\delta_{n,y}$</th>
<th>$F_{n,\text{max}}$</th>
<th>$\delta_{n,\text{max}}$</th>
<th>$\delta_{n,\text{ult}}$</th>
<th>$K_a$</th>
<th>$\mu_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New-USA</td>
<td>0.71</td>
<td>0.84</td>
<td>&gt;1.25</td>
<td>&gt;15</td>
<td>&gt;15</td>
<td>0.72</td>
<td>17.9</td>
</tr>
<tr>
<td>New-NZ</td>
<td>0.86</td>
<td>0.82</td>
<td>1.67</td>
<td>12</td>
<td>&gt;15</td>
<td>0.90</td>
<td>14.7</td>
</tr>
<tr>
<td>Salvaged-Parnell</td>
<td>0.95</td>
<td>2.10</td>
<td>1.33</td>
<td>9</td>
<td>12.0</td>
<td>0.44</td>
<td>4.3</td>
</tr>
<tr>
<td>Salvaged-T Adair</td>
<td>0.84</td>
<td>2.69</td>
<td>1.01</td>
<td>6</td>
<td>10.5</td>
<td>0.30</td>
<td>2.2</td>
</tr>
</tbody>
</table>

### 5.6 Discussion

The relationships between nail connection performance and constituent properties have been investigated extensively for decades (see Section 2.2.1). Using the current test results to quantify and further characterise such relationships was therefore not considered a necessary objective for this research. However, a candid discussion regarding the relative effects of timber material properties and nail dimensions on new and salvaged nail connection performance was required to draw any veritable conclusions about the effects of connection age and condition on nail connection performance.
Relevant properties of the tested nail connections are summarised in Table 5-2, as well as a connection condition ranking based on visual inspections conducted prior to testing. Correlating the parameters listed in Tables 5-1 and 5-2, it is evident that New-NZ connections were comprised of higher density timber with higher moisture content and smaller nail shank diameter, but exhibited greater strength and stiffness than New-USA connections. This observation suggests that timber density had a greater influence on connection performance than did the differences in moisture content and nail dimensions. Past research validates this claim by showing connection strength and stiffness to be directly proportional to timber density and nail shank diameter (Brock 1957; Scholten 1965), and that below the fibre saturation point (which is typically approximately 30% timber moisture content (Forest Products Laboratory 1999)), decreasing moisture content should generate higher connection strengths but not significantly influence connection stiffness. Based on this logic and the nail connection properties presented in Table 5-2, if all tested connections were in equally perfect condition, then the salvaged nail connections would have exhibited greater strength and stiffness than the new nail connections due to considerably higher timber densities. However, the $P_{\text{max}}$ and $K_{\alpha}$ values in Table 5-1 clearly show this supposition to be not true; indicating that the deterioration of connection integrity must influence connection performance.

<table>
<thead>
<tr>
<th>Connection type</th>
<th>Timber material properties</th>
<th>Nail dimensions</th>
<th>Overall condition rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Floorboard density (kN/m³)</td>
<td>Joist density (kN/m³)</td>
<td>Floorboard m/c (%)</td>
</tr>
<tr>
<td>New-USA</td>
<td>390</td>
<td>448</td>
<td>7.1</td>
</tr>
<tr>
<td>New-NZ</td>
<td>487</td>
<td>517</td>
<td>12.7</td>
</tr>
<tr>
<td>Salvaged-Parnell</td>
<td>546</td>
<td>537</td>
<td>11.2</td>
</tr>
<tr>
<td>Salvaged-T Adair</td>
<td>534</td>
<td>600</td>
<td>13.9</td>
</tr>
</tbody>
</table>

Having demonstrated that timber material properties and nail dimensions were unlikely to have been the primary reason for performance discrepancies between new and salvaged nail connections, the following discussion highlights notable performance differences and
associated physical observations that may have contributed towards these differences. An inspection of Table 5-1 shows the connection stiffness \( K_a \) of salvaged connections to be between 33% and 66% of new connection stiffness, which was likely to have been caused by a combination of nail rusting, localised timber degradation surrounding the nail shank, and gapping between the floorboard and joist members. Rusting was observed to reduce the shank diameter of several nails while the localised timber degradation was observed to loosen the nail’s contact with the surrounding timber, both of which would have reduced connection stiffness. Antonides et al. (1980) showed that inter-member gapping created by wetting and drying cycles substantially reduced connection stiffness, providing evidence of the significance of the observed gapping between salvaged connection members.

Although effective yield loads \( F_{n,y} \) were comparable, lower stiffness values resulted in considerably higher effective yield displacements \( \delta_{n,y} \) for salvaged connections. The maximum load-carrying capacities \( F_{n,\text{max}} \) of salvaged connections were shown to be lower than New-NZ connections but comparable to New-USA connections. As explained previously, without deterioration the salvaged connections would theoretically generate larger strengths due to their higher timber densities and comparable nail dimensions, but given the observed nail fractures and the reduction in nail ductility evident in Table 5-1, maximum load carrying capacity would have ultimately been reduced to values comparable to the New-USA connections.

Table 5-1 also illustrates that displacement capacity \( \delta_{n,lt} \) and ductility capacity \( \mu_{n} \) of salvaged connections were considerably reduced when compared to the corresponding values of new nail connections. This performance reduction is likely to be a direct consequence of the observed nail rusting and reduced shank diameter causing premature nail fracture during testing. This finding demonstrates that nail fracture will likely occur at lower lateral slip amplitudes for salvaged nail connections than for new nail connections with comparable configuration.

It is acknowledged that in the absence of nail material testing, it is unknown whether variations in new and historic nail manufacturing processes would have contributed to the observed performance differences highlighted above. However, given that both new and
salvaged nails were made of wire-drawn steel and were of comparable dimensions, it is theorised that the influence of nail material properties on connection performance was insignificant in comparison to the effects of connection deterioration. It is further acknowledged that disturbance of the salvaged nail connections during extraction and test unit preparation was unavoidable, which may have contributed to reduced strength and stiffness performance. It is considered however that these disturbances were within acceptable limits, and that the condition of the salvaged nail connections during testing was representative of their in-situ condition at the time of extraction.

Although connection deterioration cannot be explicitly quantified, and some ambiguities surround the exact influence of material properties, the above performance comparison demonstrates that the nail connections in heritage timber floor diaphragms have substantially lower initial stiffness and displacement capacity than do similar configurations built with new timber and new nails. The implications of these results are that overall diaphragm stiffness and displacement capacity will be lower than the values determined from newly constructed full-scale diaphragm tests. To address this issue, the diaphragm FE models presented in Chapter 8 were populated with the salvaged nail connection data to ensure that the seismic assessment procedure developed in Chapter 10 was truly representative of timber floor diaphragms in heritage URM buildings.

5.7 Conclusions

Nail connections constructed with new pine timber and wire-drawn nails, and nail connections salvaged from two New Zealand URM buildings were tested to characterise their hysteretic response for FE modelling that is reported in Chapter 8. New connection data was used to calibrate predictive models for the diaphragm tests described in Chapters 6 and 7, while salvaged nail connection data was used to ensure that the seismic assessment procedure developed in Chapter 10 was appropriate for timber floor diaphragms in heritage URM buildings.
New and salvaged connections exhibited highly nonlinear behaviour with similar hysteretic shape and no distinct yield point. Salvaged connection performance was shown to be highly variable in comparison to new connections, with considerably wider upper bound and lower bound response curves for a 95% confidence interval. Given that salvaged connection material properties are unlikely to have originally varied more significantly than new connection material properties, it is concluded that the variable process of connection deterioration during its lifetime is the primary reason for increased performance variation.

Salvaged nail connections demonstrated substantially reduced performance in comparison to new nail connections, indicating that connection integrity deteriorates during its lifetime. Connection stiffness was shown to be up to 66% less for salvaged connections, resulting in higher effective yield displacements of two to four times those determined for new nail connections. Displacement capacity was found to be substantially reduced for salvaged nail connections with maximum load-carrying capacity on average occurring at between 6 mm and 9 mm displacement, in comparison to between 12 mm and 15 mm for new connections. Reduced connection displacement capacity corresponds to reduced diaphragm displacement capacity, suggesting that diaphragm ductility needs to be adjusted to account for this reduced performance. An investigation of diaphragm ductility is presented in Chapter 9.
Chapter 6

SMALL-SCALE DIAPHRAGM TESTING

Two small-scale diaphragm testing programs are presented in this chapter: (1) *racking testing*, performed on diaphragm assemblages constructed with new timber and new nails, and (2) *sub-component testing*, performed on diaphragm sections constructed with new timber and new nails, and on diaphragm sections extracted from a New Zealand heritage URM building.

Racking tests were performed to determine the magnitude (if any) of friction resistance between common floorboard types in timber floor diaphragms subjected to lateral loads. The sub-component diaphragm tests provided experimental data concomitant to the full-scale diaphragm tests reported in Chapter 7, with which to calibrate and validate the diaphragm FE models presented in Chapter 8. If the proposed FE models are capable of capturing diaphragm behaviour for both small and large experimental configurations, then the mechanical basis of the models is further substantiated. The performance of new and salvaged sub-component diaphragms were also characterised and compared in this chapter, to determine whether these performances are consistent with those established in Chapter 5 for corresponding new and salvaged nail connections.
6.1 Racking Testing

Friction forces between floorboard edges in heritage timber floor diaphragms are not well understood, and often are ignored for the purpose of analysis but are then claimed to be a factor to explain unexpected diaphragm performance (Brignola 2009; Peralta 2003). NZSEE (2006) further convolutes the treatment of friction forces by offering a procedure for diaphragm shear strength assessment which includes an allowance for improved performance through the consideration of inter-floorboard friction resistance, yet does not provide an appropriate reference for the published resistance values. Although friction coefficients for various timber products have previously been investigated (Bejo et al. 2000; Chen et al. 2005; McKenzie and Karpovich 1968), given the apparent confusion surrounding inter-floorboard friction in timber diaphragms, motivation exists to experimentally determine representative values of inter-floorboard friction resistance for typical sheathing members.

In response to the above, a series of racking tests was performed on small-scale diaphragm assemblages, as schematically illustrated in Figure 6-1. The premise of inter-floorboard friction determination was to firstly test an assemblage with floorboards sufficiently apart so that contact would not occur (Figure 6-1a), and then test an identical assemblage with floorboards fastened tightly together (Figure 6-1b), with any difference in overall load resistance being attributed to frictional resistance.

6.1.1 Construction details

Two racking assemblages were constructed for each of the eight configurations summarised in Table 6-1. As discussed in Section 6.1.2, each assemblage pair was tested simultaneously, meaning that a total of eight racking tests were performed. The eight tests consisted of the four geometric configurations illustrated in Figure 6-2, which were constructed with either 19 mm × 140 mm White Pine straight-edge floorboards or 19 mm × 140 mm White Pine tongue and groove (T&G) floorboards, fastened to 38 mm × 140 mm Hem-Fir joists using 64 mm long bright common wire nails with a shank diameter of 3.33 mm. As shown in Figure 6-2, two nails spaced at \( s \) were used to fasten the floorboards perpendicularly to the joists at each intersection.
6.1 Racking Testing

Figure 6-1: Schematics of small-scale racking tests

Table 6-1: Configuration details of racking assemblages

<table>
<thead>
<tr>
<th>Test reference</th>
<th>Floorboard type</th>
<th>Floorboard arrangement</th>
<th>Number of joists</th>
<th>Joist spacing, ℓ (mm)</th>
<th>Nail couple spacing, s (mm)</th>
</tr>
</thead>
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<td>1015</td>
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<td>Straight-edge</td>
<td>Together</td>
<td>2</td>
<td>1015</td>
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<tr>
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<td>3</td>
<td>507</td>
<td>95</td>
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<tr>
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<td>Straight-edge</td>
<td>Together</td>
<td>3</td>
<td>507</td>
<td>95</td>
</tr>
<tr>
<td>T&amp;GA-1</td>
<td>T&amp;G</td>
<td>Apart</td>
<td>2</td>
<td>1015</td>
<td>90</td>
</tr>
<tr>
<td>T&amp;GT-1</td>
<td>T&amp;G</td>
<td>Together</td>
<td>2</td>
<td>1015</td>
<td>90</td>
</tr>
<tr>
<td>T&amp;GA-2</td>
<td>T&amp;G</td>
<td>Apart</td>
<td>3</td>
<td>507</td>
<td>90</td>
</tr>
<tr>
<td>T&amp;GT-2</td>
<td>T&amp;G</td>
<td>Together</td>
<td>3</td>
<td>507</td>
<td>90</td>
</tr>
</tbody>
</table>

The White Pine timber was found to have an average density of 390 kN/m³ (specific gravity (s.g.) of 0.36) and average moisture content of 7.1%, while the Hem-Fir timber was found to have an average density of 448 kN/m³ (s.g. of 0.42) and average moisture content of 7.3%. From visual inspection, the T&G floorboards appeared to contain a greater number of defects (knots) than the straight-edge floorboards.
6.1.2 Testing details

Racking tests were performed using the testing facilities at Drexel University in Philadelphia, USA. The typical test set-up is shown in Figure 6-3, consisting of a central steel loading beam that was mounted on four fixed-direction castors to enable frictionless uniaxial movement, and two steel side frames which were tightly bolted to the laboratory strong floor to prevent any movement from occurring. The assemblage pairs were bolted symmetrically between the loading beam and the two steel side frames to create a
concentric loading condition which prevented overturning moments from occurring and eliminated the need to provide a restoring force (as would have been the case if each assemblage was tested individually). Loading was provided by an MTS Model 244 hydraulic actuator that was bolted to the central beam and to a rigid reaction column. Total load resistance was recorded by a 244.7 kN MTS load cell that was attached to the hydraulic arm of the actuator (refer to Figure 6-3b). Given that the inside joists of the assemblages were tightly bolted to the loading beam, displacements were recorded directly from the loading beam using a Celesco Model PT101 string potentiometer (as shown in Figure 6-3b).

Each pair of assemblages were subjected to pseudo-static loads following a reversed-cyclic loading protocol of three repeated cycles to peak displacement amplitudes of 6.4 mm, 12.7 mm, 19.1 mm, 25.4 mm, 38.1 mm, 50.8 mm, 63.5 mm, 88.9 mm, and 114.3 mm, as shown in Figure 6-4. Loading was applied at an average rate of 20 mm/min. The push direction was defined as positive and the pull direction was defined as negative.

To ensure that resistance from the loading beam would not significantly influence test results, three tests were performed without assemblages fitted to the testing frame. Results from these tests indicated that less than 20 N of force was resisted by the loading beam, which was found to be negligible (< 1%) compared to the forces recorded during testing (see Section 6.1.3.2).

6.1.3 Test results

A summary of the hysteretic behaviour of the tested racking assemblages is presented. For each hysteretic response, a backbone curve was developed for the investigation of inter-floorboard friction.

6.1.3.1 Observations

From an observational standpoint, all assemblage configurations behaved similarly during testing. For each test, the applied loading caused relative joist displacement and rigid-rotation of the floorboards, as illustrated in Figure 6-5. The nails at each floorboard-to-joist
Chapter 6: Small-Scale Diaphragm Testing

(a) Photograph of set-up

(b) Schematic of set-up

Figure 6-3: Test set-up
connection were seen to displace equally and oppositely in the plane perpendicular to the applied load. Intermittent timber creaking and cracking sounds were often heard but were almost never accompanied by a visual observation. These sounds were more prominent for assemblage configurations with the floorboards fastened together, where a greater degree of timber rubbing is thought to have occurred. It is noteworthy that the centre loading beam appeared to remain on its assigned uniaxial trajectory. With no lateral movement of the loading beam, it can be assumed that the applied loads remained appropriately symmetrical during testing.

Figure 6-4: Reversed-cyclic loading schedule

![Figure 6-4: Reversed-cyclic loading schedule](image)

(a) SBA-2 @ +144.3 mm displacement  (b) SBT-2 @ +144.3 mm displacement

Figure 6-5: Examples of racking assemblages during testing
6.1.3.2 Force-displacement response

The force-displacement responses of the racking assemblages are presented in Figure 6-6. The highly nonlinear behaviour of nail connections that was described in Chapter 5 is shown to feature prominently, with no clearly defined yield point and a considerable degree of hysteretic pinching. As was expected, the assemblages with three joists were shown to resist greater loads than assemblages with two joists. In order to compare the response of assemblage configurations with floorboards apart, with the corresponding assemblage configurations with floorboards together, a backbone curve was developed for each hysteretic response using a multi-linear line passing through the maximum force recorded at each displacement amplitude. Figure 6-6 illustrates that the backbone curves appropriately enclose the entire force-displacement response of the racking assemblages.

6.1.4 Inter-floorboard friction

The backbone curve of each assemblage configuration with floorboards apart is plotted in Figure 6-7 against the backbone curve of the corresponding assemblage configuration with floorboards together. It is evident that up to displacements of approximately ±64 mm, the force-displacement responses of racking assemblages with floorboards apart were almost identical to the force-displacement responses of corresponding racking assemblages with

![Figure 6-6: Force-displacement response](image-url)
6.1 Racking Testing

Figure 6-6 continued
floorboards together. For displacements greater than approximately \( \pm 64 \) mm, a minor strength increase is generally noticeable for assemblage configurations with floorboards together, resisting on average 12\% greater load at the maximum displacement of \( \pm 114 \) mm. Straight-edge floorboards and T&G floorboards are shown in Figure 6-7 to not affect relative assemblage performance, indicating that floorboard type does not significantly influence potential inter-floorboard friction mechanisms.

The results suggest that inter-floorboard friction is generally negligible for timber floor diaphragms but may have a potential effect on diaphragm performance at large displacement amplitudes. To evaluate whether this potential inter-floorboard friction resistance could influence realistic diaphragm response, consider the floorboard rotation angle, \( \theta \), for the tested racking assemblages,

\[
\theta = \frac{\Delta}{d}
\]

(6-1)

where \( \Delta \) is displacement and \( d \) is the distance between the fixed joist and the loaded joist shown in Figure 6-3. Using Equation 6-1 it can be shown that the observed strength increase occurred from floorboard rotation angles greater than approximately 0.063 rads. If this angle is taken as the maximum floorboard rotation before which inter-floorboard friction resistance begins to influence diaphragm performance, then the midspan displacement of a realistic diaphragm configuration may be evaluated using the shear beam analogy developed in Chapter 4 for the idealisation of diaphragm behaviour. From Equation 4-25, it can be shown that diaphragm angle of rotation may be evaluated using,

\[
\frac{d\delta_s}{dx} = \frac{6F_d}{5G_dB} \left( \frac{1}{2} - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \right)
\]

(6-2)

The maximum angle of rotation for a shear beam is located at \( x = 0 \), which, substituting into Equation 6-2 gives,

\[
\theta_{max} = \frac{3F_d}{5G_dB}
\]

(6-3)
If $B$ is taken as 8 m (from the *standard* diaphragm configuration presented in Chapter 4), and $G_d$ is taken as 350 kN/m (from the current ASCE 41-06 (2007) default shear stiffness values), then by substituting $\theta = 0.063$ into Equation 6-3, the maximum $F_d$ before potential inter-floorboard friction effects would occur is found to be 252 kN. Substituting $F_d$ into the current ASCE 41-06 diaphragm displacement equation (see Equation 4-2) and taking diaphragm span $L$ to be 12 m (from the *standard* diaphragm configuration presented in Chapter 4), a midspan displacement of 270 mm is determined for the standard diaphragm configuration. This midspan displacement is well in excess of typical diaphragm displacement limits employed by engineering practitioners, such as the 150 mm
or half out-of-plane wall thickness rule-of-thumb offered by Oliver (2010a). The above analysis therefore demonstrates that the potential inter-floorboard friction resistance observed for racking displacements in excess of ±64 mm will not occur in a realistic diaphragm configuration deforming within allowable limits. Furthermore, even at midspan displacements in excess 270 mm, only a small length of the floorboards near the diaphragm sides will have rotated through the observed limiting value of 0.063 rads, meaning that the effects of inter-floorboard friction on diaphragm performance will be considerably limited.

The results and analysis presented in this section adequately demonstrate that inter-floorboard friction can be considered negligible for timber floor diaphragms with either straight-edge floorboards or T&G floorboards. It is acknowledged that larger scale testing could be warranted, to investigate the effects of floorboard length and flexural bending (as opposed to rigid rotation) on inter-floorboard friction. However given that even small amounts of floorboard shrinkage will eliminate contact between the floorboards in existing heritage timber diaphragms, the conclusion of negligible inter-floorboard friction is considered appropriate for any realistic diaphragm configuration. Consequently, it is recommended that any allowances for diaphragm strength improvement through consideration of inter-floorboard friction be removed from current assessment documents (NZSEE 2006).

As a final note, the presence of inter-floorboard friction will be inherently investigated further in Chapter 8 through the comparison of a diaphragm FE model with the experimental results presented in Chapters 6 and 7. If the model is capable of capturing diaphragm response without including inter-floorboard friction effects, then the conclusion of negligible inter-floorboard friction will be appropriately verified.
6.2 Sub-Component Testing

Sub-component testing refers to a series of in-plane pseudo-static tests performed on small-scale diaphragm sections having spans of between 2.4 m and 3.2 m, and widths of between 1.2 m and 2.2 m. Unlike the racking tests reported in Section 6.1, the sub-component diaphragms were tested with realistic boundary conditions and realistic loading conditions. Testing was performed on diaphragm sections constructed with new timber and new wire-drawn nails, and on salvaged diaphragm sections extracted from a New Zealand heritage URM building.

The primary purpose of sub-component diaphragm testing was to provide small-scale experimental data with which to validate and calibrate the diaphragm FE models reported in Chapter 8. Scale-effects are known to influence diaphragm performance (Piazza et al. 2008a), so it is imperative that the developed FE model be shown to appropriately capture both the full-scale diaphragm performances reported in Chapter 7 and the small-scale diaphragm performances presented in this section. Concomitant objectives of the sub-component testing program include: (1) investigating the influence of floorboard type (straight-edge or T&G) and floorboard arrangement (continuous or staggered) on the performance of newly constructed diaphragm sections, and (2) comparing new and salvaged sub-component performance to determine whether the effects of age and deterioration on nail connection performance established in Chapter 5 translate consistently into larger scale diaphragm sections comprising identical nail connections.

Due to an opportunity to study abroad, sub-component testing was performed in two parts. Sub-components constructed with new timber and new nails were tested at Drexel University in Philadelphia, USA, while sub-components salvaged from a heritage URM building were tested in a commercial warehouse in Auckland, New Zealand. This section describes the sub-components tested and the testing details employed at each institution. For each tested diaphragm section a backbone curve was developed from the relevant hysteretic response and was further characterised for a comparative discussion.
6.2.1 New sub-components

Two sub-component diaphragms were constructed for each of the four configurations outlined in Table 6-2 and illustrated in Figure 6-8. Test units SC-1 and SC-3 were constructed with 19 mm × 140 mm White Pine straight-edge floorboards, while test units SC-2 and SC-4 were constructed with 19 mm × 140 mm White Pine tongue and groove (T&G) floorboards. For all sub-components, the floorboards were fastened to 38 mm × 140 mm Hem-Fir joists using two 64 mm long bright common wire nails with a shank diameter of 3.33 mm spaced at \( s \). Figure 6-8 illustrates the two floorboard arrangements that were adopted. The *continuous* arrangement comprises all floorboards spanning continuously over the diaphragm (Figure 6-8a), while the *staggered* arrangement comprises repeating rows of a continuous floorboard and a floorboard that is discontinuous at midspan (Figure 6-8b).

The White Pine timber was found to have an average density of 390 kN/m\(^3\) (specific gravity (s.g.) of 0.36) and average moisture content of 7.1%, while the Hem-Fir timber was found to have an average density of 448 kN/m\(^3\) (s.g. of 0.42) and average moisture content of 7.3%. Tests to determine timber modulus of elasticity \( E \) were not performed, however from the extensive testing of American timber species, the Forest Products Laboratory

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<th>Test reference</th>
<th>Floorboard arrangement</th>
<th>Floorboard type</th>
<th>Number of joists</th>
<th>Joist spacing, ( \ell ) (mm)</th>
<th>Nail couple spacing, ( s ) (mm)</th>
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<td>406</td>
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</tbody>
</table>
6.2 Sub-Component Testing

(a) Continuous floorboard arrangement

(b) Staggered floorboard arrangement

Figure 6-8: Schematics of new sub-component test units

(1999) recommends $E$ of White Pine timber to be 8.5 GPa, and $E$ of Hem-Fir timber to be 11.0 GPa. From visual inspection, the T&G floorboards appeared to contain a greater number of defects (knots) than the straight-edge floorboards.

6.2.2 Salvaged sub-components

Details of diaphragm sections extracted from a two-storey URM building in Parnell, New Zealand, were presented in Chapter 5 for the testing of salvaged nail connections. During the extraction process, two salvaged diaphragm sections were preserved for sub-component testing, which were labelled *Parnell-large* and *Parnell-small* based on their
Chapter 6: Small-Scale Diaphragm Testing

Figure 6-9: Schematics of salvaged sub-component test units

(a) Parnell-large

(b) Parnell-small

Figure 6-9: Schematics of salvaged sub-component test units
6.2 Sub-Component Testing

relative sizes. The relevant geometry and configuration details of the two salvaged sections are illustrated in Figure 6-9, including the observed floorboard arrangement. Both sections were built with Kauri joists and Kauri T&G floorboards that were fastened together with two nails at each floorboard-to-joist interface. Joist sections were found to have an average profile of 52 mm × 222 mm, and the floorboard profile was on average 20 mm × 135 mm. The nails were found to have an average length of 64 mm and an average shank diameter of 2.95 mm. Before testing was performed, the Kauri floorboard timber was found to have an average density of 546 kN/m³ (s.g. of 0.49) and average moisture content of 11.2%, while the Kauri joist timber was found to have an average density of 537 kN/m³ (s.g. of 0.48) and average moisture content of 11.6%. Testing to determine the modulus of elasticity (E) of the Kauri timber was not performed, however published information regarding native New Zealand timber species recommends E of old-growth Kauri timber to be 13.0 GPa (NZ Wood 2011b), which is considered a representative value for the extracted diaphragm sections. Further details of the extracted sections can be found in Section 5.2.

The salvaged diaphragm sections were generally in good condition. Although it is acknowledged that some disturbance of the sections would have occurred during extraction, these disturbances were considered to be within acceptable limits and the condition of the salvaged sections was considered to be representative of their in-situ condition. As shown in Figure 6-10, the surface of the floorboards were scratched and

![Figure 6-10: General condition of salvaged diaphragm sections](image-url)

(a) Parnell-large (b) Parnell-small
gouged in places but generally insufficient to compromise the structural integrity of the diaphragm. However each salvaged section had one or two isolated locations of floorboard damage which could have potentially reduced the strength and stiffness of the diaphragm sections. It was apparent that some kind of varnish or resin had been poured over the floor sometime in the past. The glue-like product was easily seen as a thin layer on the floorboard surfaces that filled at least the upper portion of the T&G floorboard interfaces. All nail connections appeared to be intact and in the same condition as that outlined in Section 5.5.1.

6.2.3 Testing details for new sub-components

Newly constructed sub-component diaphragms SC-1 to SC-4 were tested at Drexel University in Philadelphia, USA, using a modified version of the racking assemblage test set-up described in Section 6.1.2 and illustrated in Figure 6-3. For sub-component diaphragm testing, a custom-designed loading ‘trolley’ was fabricated from steel angles and welded to the end of the existing centre loading beam. As shown in Figure 6-11, the loading trolley was fabricated with the precise height and dimensions to enable the two joists either side of midspan to be bolted directly to the frame, creating a symmetrical two-point loading system. The sub-component test units therefore spanned directly over the loading frame and were bolted to the side frames which were fixed to the laboratory strong floor to prevent any movement from occurring (see Figure 6-12). Like the existing centre loading beam, the loading trolley was placed on fixed-direction castors to enable frictionless uniaxial movement.

Loading was again provided by an MTS Model 244 hydraulic actuator that was bolted to the central beam and to a rigid reaction column. Total load resistance was recorded by a 244.7 kN MTS load cell that was attached to the hydraulic arm of the actuator (refer to Figure 6-11). Displacements were recorded directly from the midspan joist using a Celesco Model PT101 string potentiometer, as shown in Figure 6-11.
Each sub-component diaphragm was subjected to pseudo-static loads following a reversed-cyclic loading protocol identical to that which was adopted for the racking testing (see Figure 6-4), except that the 88.9 mm and 114.3 mm displacement amplitudes were removed. The push direction was defined as positive and the pull direction was defined as negative.
6.2.4 Testing details for salvaged sub-components

Salvaged sub-component diaphragms were tested in a commercial warehouse in Auckland, New Zealand, as part of the full-scale diaphragm testing program reported in Chapter 7. It follows that the salvaged sub-component diaphragm test set-up was almost identical to that used for full-scale diaphragm testing (parallel-to-joist), for which comprehensive details can be found in Section 7.2.1. The typical reconfigured test set-up for salvaged sub-components is illustrated in Figures 6-13 and 6-14. Loading was provided by a single hydraulic actuator attached to a large reaction frame and a steel truss mounted on castors, which distributed the point load into two equal simply-supported loads that were applied to the diaphragm sections at the illustrated joist locations. Reversed cyclic loading was achieved by positioning loaders on both ends of the loaded joists and post-tensioning these together using threaded rods that spanned the width of the diaphragm section (see Figure 6-14). The diaphragm sections were bolted to two steel side frames that were specially positioned for each unique salvaged sub-component span and that were tightly bolted to the warehouse concrete slab to prevent any movement from occurring. Intermediate vertical support was provided by small steel beams with timber blocking and Teflon pads positioned under alternating joists.
6.2 Sub-Component Testing

Total load resistance was recorded by a PT-LPCH 250 kN load cell that was attached to the hydraulic arm of the actuator (refer to Figure 6-14). Displacements were recorded directly from the midspan joist using a Siko Model SG10 string potentiometer, as shown in Figure 6-14. The salvaged sub-component diaphragms were subjected to pseudo-static loads following a reversed-cyclic loading protocol of three repeated cycles to peak displacement amplitudes of 5 mm, 10 mm, 15 mm, 25 mm, and 50 mm, as shown in Figure 6-15. An additional displacement amplitude of 75 mm was added for the Parnell-large test unit due to its larger span. The push direction was defined as positive and the pull direction was defined as negative.

6.2.5 Test results

A summary of the test observations and hysteretic behaviour of the tested sub-component diaphragms is presented. For each hysteretic response, a backbone curve was developed and further characterised for a comparative discussion of new and salvaged sub-component diaphragm performance.
Figure 6-14: Schematic of test set-up for salvaged sub-components
6.2.5.1 Observations

Newly constructed sub-components with the staggered floorboard configuration (test units SC-1 and SC-2) performed visually similarly during testing, regardless of floorboard type. Up to midspan displacements of approximately ±25 mm, the continuous floorboards deformed flexurally with little or no cracking occurring when subjected to the applied loading, while the discontinuous floorboards were seen to simply rotate rigidly (analogous to the racking tests described in Section 6.1.3.1). Localised cracking of the continuous floorboards was observed to occur for midspan displacements exceeding approximately ±25 mm, generally occurring in the outermost continuous floorboards first, followed by all other continuous floorboards at greater displacements. The observed floorboard cracking was generally isolated to within 200 mm of midspan location, which was likely to have been caused by the concentration of floorboard curvature in this region induced by the decoupled rotation of the discontinuous floorboards. Overall the new sub-component configurations with staggered floorboards were observed to deform in two distinct manners: (1) before cracking of the continuous floorboards occurred, deformations were primarily flexural (generated through the flexural strength of the continuous floorboards), and (2) after cracking of the continuous floorboards occurred, deformations were V-shaped due to rigid rotation of the continuous floorboards having an effective plastic hinge at midspan, and the rigid rotation of discontinuous floorboards being effectively pinned at
midspan. An example of the described deformation observations is provided in Figure 6-16a and Figure 6-16b.

Floorboard cracking was also a prominent feature for new sub-component diaphragms with the continuous floorboard configuration, although some notable differences were observed. Cracking of straight-edge floorboards (SC2-3) generally did not occur until midspan displacements exceeded approximately ±25 mm, while cracking of T&G floorboards (SC2-4) began occurring at midspan displacements as low as ±5 mm. Given that the T&G floorboards appeared to contain more defects than the straight-edge floorboards (as noted in Section 6.2.1), the observed premature cracking of the T&G floorboards was to be expected. Consequently the number and severity of cracking was observed to be greater in the sub-components constructed with T&G floorboards. For both SC2-3 and SC2-4, cracks almost always propagated from timber defects (such as knots) located between the two joists either side of midspan, once again beginning in the outermost floorboards first, followed by cracking in the other floorboards at greater displacements. The observed floorboards cracks in sub-components with the continuous floorboard configuration (SC-3 and SC-4) were also considerably more elongated than for sub-components with the staggered floorboard configuration (SC-1 and SC-2), having lengths of up to 800 mm along the span of the floorboards. It is postulated that without the presence of discontinuous floorboards that led to a concentration of flexural stresses at midspan (due to their rigid rotation), the curvature of continuous floorboards was more likely to have been distributed over greater lengths of the floorboards, therefore leading to longer crack propagation. The elongated floorboard cracks can be seen in Figures 6-16c and 6-16d.

The salvaged sub-component diaphragms labelled Parnell-large and Parnell-small both failed prematurely during testing. For both test units, issues with the loading system were observed to be the cause of these failures. During positive (pushing) loading, it was observed that the far ends of the diaphragm sections (opposite to the loading frame) were lifting considerably due to apparent out-of-plane buckling of the floorboards, as shown in Figure 6-17a. This upward lifting created sufficiently large tension forces between the floorboards and joists to cause the floorboards of Parnell-small to pull completely away from one of the side joists at -25 mm of the first -50 mm displacement amplitude cycle,
meaning that the diaphragm section no longer had restraint against lateral loading (see Figure 6-17b). Conversely, during negative (pulling) loading, the salvaged diaphragm sections were observed to sag at the same end, resulting in downward pulling of the loaded joists. At approximately -25 mm of the first -50 mm displacement cycle, this downward pulling caused one of the loaded joists of Parnell-large to pull completely away from the floorboards (see Figure 6-17c), meaning that the load path into the diaphragm section was destroyed.

The T&G floorboards of salvaged sub-component diaphragm Parnell-large were observed to deform flexurally with no splitting up to a midspan displacement of approximately
±15 mm. Between displacement amplitudes ±15 mm and ±25 mm, cracks began to propagate from the isolated section of floorboards which had been damaged during extraction (refer to Section 6.2.2), as shown in Figure 6-17d. Crack propagation worsened until the described joist failure occurred. For all displacement amplitudes up until failure, no splitting or cracking of the T&G floorboards was observed for salvaged sub-component diaphragm Parnell-small. It was apparent that the varnish or resin filling the T&G interfaces was generating composite action between the floorboards, as only a limited amount of relative floorboard slip was observed in both Parnell-large and Parnell-small sub-components for midspan displacements below approximately ±10 mm. This observed resistance to inter-floorboard slip is expected to have greatly improved diaphragm stiffness due to effective deepening of the floorboard sections.

Figure 6-17: Testing observations for salvaged sub-components
6.2 Sub-Component Testing

6.2.5.2 Force-displacement

The force-displacement responses of new sub-component diaphragms (SC-1 to SC-4) and salvaged sub-component diaphragms (Parnell-large and Parnell-small) are presented in Figures 6-18 and 6-19, respectively. The observed fracturing of new sub-component floorboards is clearly identifiable in Figure 6-18 as the reduction of stiffness and load-carrying capacity. Interestingly, up until floorboard fracture begins to occur, the new sub-component diaphragms respond with greater elastic stiffness in comparison to the highly nonlinear behaviour of the racking assemblages presented in Section 6.1.3.2. Considering the short spans of the tested sub-components, the demonstrated elastic-type behaviour indicates that the flexural strength of the floorboards potentially dominated performance. The hysteretic shape exhibited by all new sub-component configurations was comparable.

Only a small amount of hysteretic behaviour was recorded for the salvaged sub-component diaphragms before failure of the loading systems occurred. From what was recorded, Figure 6-19 shows the hysteretic behaviour of Parnell-large and Parnell-small to be comparable except for apparent displacement increases during the unloading phase of positive displacement amplitudes for the Parnell-large test unit. The cause of these displacement increases was not specifically identified during testing but it is postulated that when each positive displacement amplitude was reached, the observed out-of-plane lifting of the test unit began to settle, resulting in increased in-plane displacements. For both salvaged sub-components, considerable stiffness degradation is exhibited between midspan displacements of ±5 mm and ±10 mm in Figure 6-19. Based on the testing observations, the primary cause of this stiffness degradation was likely to have been localised failure of the glue-like bonds between the T&G interfaces, which would have only had limited resistance to inter-floorboard slip before fracturing. Once the varnish or resin between the T&G interfaces had fractured, floorboard composite action would have greatly reduced, leading to an overall reduction in sub-component stiffness. For the Parnell-large sub-component, the observed floorboard splitting which occurred between midspan displacements of ±15 mm and ±25 mm would have also contributed to the observed stiffness degradation.
(a) SC-1a

(b) SC-1b

(c) SC-2a

(d) SC-2b

(e) SC-3a

(f) SC-3b

Figure 6-18: Force-displacement response of new sub-components
For the characterisation of sub-component performance in Section 6.2.6, a backbone curve was developed for each hysteretic response using a multi-linear line passing through the maximum force recorded at each displacement amplitude. Figures 6-18 and 6-19 illustrate that the backbone curves appropriately enclose the entire force-displacement response of the sub-component diaphragms.
6.2.6 Performance characterisation

To compare the performance of differing new sub-component diaphragm configurations, and to compare the performance of new and salvaged sub-component diaphragms, maximum load and effective stiffness values were determined from the backbone curves developed in Section 6.2.5.2. Although many aspects of sub-component performance could be characterised for comparison, strength and stiffness were selected as the two primary parameters which suitably describe performance for the purposes of this comparative analysis.

Maximum load, \( F_{d,max} \), was taken as the maximum absolute value of each backbone curve presented in Figures 6-18 and 6-19. An effective stiffness, \( K_e \), was calculated for each sub-component using the methodology outlined by Ceccotti (1995) for nailed timber connections, whereby \( K_e \) is defined as the line joining data points corresponding to 10% and 40% of maximum load. \( K_e \) values were determined for both the positive and negative portions of the backbone curves, and were subsequently averaged for a final value of effective shear stiffness. Considering that no standardised procedure has been established to determine the effective stiffness of diaphragm sections such as those tested, the adopted methodology was considered appropriate for the purpose of performance comparison. The values of \( F_{d,max} \) and \( K_e \) determined for each tested sub-component are listed in Table 6-3.

Although the strength and stiffness values presented in Table 6-3 can be directly compared for new sub-component diaphragms due to their identical geometries, in order to compare the performance of new and salvaged sub-component diaphragms it is necessary to derive corresponding strength and stiffness parameters which are independent of geometry. Accordingly, shear strength per lineal metre width, \( R_d \), and effective shear stiffness, \( G_e \), values were calculated for each sub-component diaphragm. Shear strength per lineal metre was calculated by taking the relevant maximum load, halving it to find shear resistance, and dividing it by sub-component diaphragm width, as described in Equation 6-4,

\[
R_d = \frac{F_{d,max}}{2B}
\]  

(6-4)
where \( B \) is sub-component diaphragm width (being the dimensions parallel to the applied load). It was shown in Chapter 4 that diaphragm behaviour can be idealised as a rectangular shear beam with a shear stiffness of \( G_d \). Taking \( G_e = G_d \), and acknowledging that \( G_d = Gt \) where \( G \) is shear modulus and \( t \) is shear beam thickness, effective shear stiffness \( G_e \) may be evaluated for the tested sub-components by considering a rectangular shear beam with a thickness of unity (\( t = 1 \)). Consider the shear beam illustrated in Figure 6-20 subjected to two equivalent point loads. From Figure 6-20 it can be shown that shear strain \( \gamma \) is given by,

\[
\gamma = \frac{\Delta}{a}
\]  

(6-5)

and that shear stress is given by,

\[
\tau = \frac{F}{2Bt}
\]  

(6-6)

Given that \( G = \tau / \gamma \), it can shown from Equations 6-5 and 6-6 that the shear stiffness of a beam with normalised thickness (\( t = 1 \)) and subjected to two equivalent point loads is defined as,
Using Equation 6-7, \( G_e \) values were calculated for the positive and negative portions of the backbone curves developed in Section 6.2.5.2 by taking \( F = 0.4 F_{\text{max}} \) and \( \Delta = F/K_e \) using the relevant backbone curve data. For sub-components SC-1 to SC-4, \( a = 0.812 \text{ m} \) and \( B = 1.260 \text{ m} \), for sub-component Parnell-large, \( a = 1.060 \text{ m} \) and \( B = 2.160 \text{ m} \), and for sub-component Parnell-small, \( a = 0.958 \text{ m} \) and \( B = 1.890 \text{ m} \). The calculated \( G_e \) values were subsequently averaged for a final value of effective shear stiffness. The determined values of \( Q_{\text{max}} \) and \( G_e \) are presented hereafter in Table 6-3.

### 6.2.7 Discussion

Inspecting Table 6-3 it is evident that for new sub-components with straight-edge floorboards (SC-1a, SC-1b, SC-3a, and SC-3b), configurations with the continuous floorboard arrangement (SC-3a and SC-3b) exhibited higher shear strength and higher shear stiffness than corresponding configurations with the staggered floorboard arrangement (SC-1a and SC-1b). This performance distinction was expected, given the

\[
G_e = \frac{F_a}{2B\Delta} \quad (6-7)
\]

### Table 6-3: Sub-component performance parameters

<table>
<thead>
<tr>
<th>Test reference</th>
<th>( F_{d,\text{max}} ) [kN]</th>
<th>( K_e ) [kN/m]</th>
<th>( R_d ) [kN/m]</th>
<th>( G_e ) [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-1a</td>
<td>20.8</td>
<td>1036</td>
<td>8.2</td>
<td>381</td>
</tr>
<tr>
<td>SC-1b</td>
<td>26.0</td>
<td>967</td>
<td>10.3</td>
<td>352</td>
</tr>
<tr>
<td>SC-2a</td>
<td>18.1</td>
<td>906</td>
<td>7.2</td>
<td>326</td>
</tr>
<tr>
<td>SC-2b</td>
<td>27.8</td>
<td>899</td>
<td>11.0</td>
<td>334</td>
</tr>
<tr>
<td>SC-3a</td>
<td>38.6</td>
<td>1401</td>
<td>15.3</td>
<td>498</td>
</tr>
<tr>
<td>SC-3b</td>
<td>35.1</td>
<td>1313</td>
<td>13.9</td>
<td>470</td>
</tr>
<tr>
<td>SC-4a</td>
<td>22.1</td>
<td>883</td>
<td>8.8</td>
<td>327</td>
</tr>
<tr>
<td>SC-4b</td>
<td>24.0</td>
<td>992</td>
<td>9.5</td>
<td>365</td>
</tr>
<tr>
<td>Parnell-large</td>
<td>49.1</td>
<td>7541</td>
<td>11.4</td>
<td>1850</td>
</tr>
<tr>
<td>Parnell-small</td>
<td>52.2</td>
<td>6680</td>
<td>13.8</td>
<td>1693</td>
</tr>
</tbody>
</table>
greater number of continuously spanning floorboards present in sub-components SC-3a and SC-3b providing increased flexural strength and stiffness, and therefore greater load carrying capacity than corresponding sub-components SC-1a and SC-1b. However the same logic did not hold true for new sub-components with T&G floorboards (SC-2a, SC-2b, SC-4a, and SC-4), where no consistent distinction can be made between the performance values for configurations with the continuous floorboard arrangement (SC-4a and SC-4b) and configurations with the staggered floorboard arrangement (SC-2a and SC-2b). The performance of sub-components with T&G floorboards highlights the obvious impact that timber defects had on sub-component performance. Although floorboard defects were not explicitly mapped before testing, it is postulated that the continuous floorboards present in sub-component SC-2b contained fewer defects than corresponding configurations with T&G floorboards, explaining why SC-2b had greater shear strength and comparable shear stiffness than did sub-components SC-4a and SC-4b. The observations made from Table 6-3 above are further illustrated in Figures 6-21a and 6-21b for clarity, where shear force per lineal metre was determined from the backbone curve force data using Equation 6-4, and shear strain was determined from the backbone curve displacement data using Equation 6-5.

Comparing the performance parameters of sub-components SC-3a, SC-3b, SC-4a, and SC-4b in Table 6-3, it is evident that for sub-component diaphragms with the continuous floorboard arrangement, configurations with straight-edge floorboards exhibited greater shear strength and greater shear stiffness than corresponding configurations with T&G floorboards. This reduced performance of sub-components SC-4a and SC-4b is consistent with the testing observations reported in Section 6.2.5.1, which describes the T&G floorboards in sub-components SC-4a and SC-4b to crack at considerably lower midspan displacements than the straight-edge floorboards in sub-components SC-3a and SC-3b. With smaller nail couple spacing and premature floorboard cracking, the shear strength and shear stiffness of sub-components SC-4a and SC-4b would logically be lower. Based on this argument, the same performance discrepancy was expected for sub-component diaphragms with the staggered floorboard arrangement (SC-1a, SC-1b, SC-2a, and SC-2b). However inspecting Table 6-3 it is evident that no clear distinction can be made between the performances of sub-component configurations with straight-edge floorboards (SC-1a
and SC-1b) and sub-component configurations with T&G floorboards (SC-2a and SC-2b), further highlighting the variable impact of timber defects on sub-component behaviour. The above observations can be identified in the shear force per lineal metre versus shear strain backbone curves provided in Figures 6-21c and 6-21d.

The presented comparative discussion indicates that new sub-component performance was primarily influenced by floorboard arrangement and timber defects. These observations are logical for diaphragm sections with such small spans (< 3.5 m), where the flexural capacity of the timber floorboards is reached well before other failure mechanisms, such as nail

![Shear force vs shear strain graphs](image)

(a) Configurations with straight-edge floorboards
(b) Configurations with tongue and groove floorboards
(c) Configurations with continuous floorboard arrangement
(d) Configurations with staggered floorboard arrangement

Figure 6-21: Comparison of new sub-component diaphragm performance
pullout or imposed displacement limits. For any realistic diaphragm configuration (see Chapter 1), it is anticipated that the observed effects of floorboard arrangement and timber defects will not so greatly influence diaphragm performance. The dimensions of existing timber floor diaphragms will always exceed typical floorboard lengths and therefore full-scale diaphragms will always be comprised of some selected arrangement of discontinuous floorboards. The overall effect of these floorboard arrangements is unlikely to significantly affect diaphragm performance. Given the arrangement of discontinuous floorboards in realistic diaphragm configurations, timber defects are also less likely to influence diaphragm performance because timber floorboard flexural capacity would almost never be reached before typical displacement limits are reached.

Table 6-3 shows salvaged sub-component shear stiffness to be substantially higher than for all new sub-components, which is an observation that is not consistent with the results presented in Chapter 5 for New-USA and Salvaged-Parnell nail connection performance. Salvaged-Parnell nail connection stiffness was shown to be on average 61% of New-USA nail connection stiffness, demonstrating the reduced stiffness of salvaged nail connections. Considering that new sub-components (SC-1 to SC-4) were constructed with nail connections identical to New-USA connections, and that salvaged sub-components (Parnell-large and Parnell-small) were comprised of nail connections identical to Salvaged-Parnell nail connections, it was anticipated that analogous observations would be drawn from new and salvaged sub-component testing. However it is apparent that the varnish or resin product which filled a considerable portion of the T&G floorboard interfaces (see Section 6.2.2) significantly improved the stiffness of both Parnell-large and Parnell-small sub-components. The composite floorboard action observed during testing (see Section 6.2.5.1) could only have been generated through increased inter-floorboard bonding which was undoubtedly provided by the varnish or resin product. Such composite panel action in timber floor diaphragms has previously been shown to considerably increase diaphragm shear stiffness (Bott 2005). Bott showed that for a plywood sheathed diaphragm, the application of a foam adhesive to the underside of the diaphragm increased shear stiffness by up to 89%.
It is acknowledged that the higher modulus of elasticity of Kauri timber \( E = 13.0 \) GPa compared with White Pine timber \( E = 8.5 \) GPa would also have contributed to improved extracted sub-component stiffness. However based on simple logic, timber material stiffness could have only improved extracted sub-component stiffness by \( 13.0/8.5 \approx 1.5 \) times, whereas the stiffness of Parnell-large and Parnell-small sub-components was shown to be up to 5.7 times greater than new sub-component stiffness. Such significant stiffness discrepancy is therefore more likely to have been generated by the observed composite action between the extracted sub-component T&G floorboards.

The described shear stiffness discrepancy between salvaged sub-components and new sub-components is further illustrated in Figure 6-22, which plots shear stiffness per linear metre versus shear strain for salvaged sub-components and for new sub-components SC-3a, SC-3b, SC-4a, and SC-4b.

Direct comparison of new and salvaged sub-component shear strength is difficult because the failure mechanisms were different. Nonetheless, salvaged sub-component shear strength was comparable to new sub-components constructed with straight-edge floorboards in the continuous arrangement (SC-3a and SC-3b), but the shear strength was higher than for all other new sub-component diaphragms.

![Figure 6-22: Comparison of new and salvaged sub-component performance](image-url)
6.3 Conclusions

Frictional resistance between either straight-edge floorboards or T&G floorboards was found to be insignificant. Inter-floorboard friction can therefore be considered negligible for existing timber floor diaphragms having no apparent glue-like products present between the floorboards. It is recommended that any allowances for diaphragm strength improvement through consideration of inter-floorboard friction be removed from current assessment documents (NZSEE 2006).

Floorboard configuration and defects in floorboard timber affect the performance of short spanned diaphragms ($L < 3.5$ m) but are not expected to significantly influence realistic diaphragm configurations due to reduced reliance on floorboard flexural capacity. Sub-component testing results indicate that floorboard profile type (straight-edge or T&G) is unlikely to affect full-scale diaphragm performance.

Varnish, resin or other glue-like products applied to floor diaphragms for floorboard protection can measurably improve diaphragm stiffness by generating composite actions between the floorboards. Diaphragm sections extracted from a heritage URM building in Parnell, New Zealand, were shown to have shear stiffnesses ($G_s$) of up to 5.7 times greater than diaphragm sections constructed with new timber and new nails, despite the extracted sections comprising nail connections with an average stiffness proven to be 61% of the nail connections used to construct the new diaphragm sections (see Chapter 5). The varnish or resin that was observed to cover the extracted diaphragm sections and to fill the T&G interfaces was observed to resist inter-floorboard slip during testing, which was shown to significantly improve sub-component diaphragm stiffness.

Careful inspection of inter-floorboard condition is necessary for the seismic assessment of timber floor diaphragms in order to make accurate predictions of performance. If no glue-like products are observed then no inter-floorboard bonding effects should be allowed for, whereas if glue-like products are present, then the appropriate judgement of its condition, reliability, and influence on diaphragm performance should be catered for. A
comprehensive analysis of inter-floorboard gluing effects is outside the scope of this research, but further testing of heritage timber floor diaphragms is recommended to suitably quantify the influence of applied floor varnishes or any other structural additions which may influence the performance of heritage timber floor diaphragms.
Chapter 7

FULL-SCALE DIAPHRAGM TESTING

The performance of nailed timber connections and diaphragm sub-components presented in Chapters 5 and 6 cannot be reliably extrapolated to capture full-scale diaphragm response without experimental validation. In order to develop a finite element modelling method and a detailed assessment procedure for representative timber floor diaphragms in the proceeding chapters, a series of full-scale timber floor diaphragms were tested to establish key seismic performance characteristics.

Table 7-1 summarises the key aspects of the experimental program. Four diaphragms measuring 10.400 m × 5.535 m were constructed with typical as-built configurations and were tested in either the direction parallel-to-joists or perpendicular-to-joists. Free-vibration testing was performed on each diaphragm both before and after the application of reversed-cyclic pseudo-static loads. Once each series of tests was completed, the as-built diaphragm was retrofitted and re-tested following an identical testing protocol. Although diaphragm retrofitting was not a specific focus of this research, the established full-scale testing program was an ideal opportunity to verify the performance of a cost-effective and readily repeatable retrofitting technique for timber floor diaphragms. The influence of a typical diaphragm stairwell penetration and of discontinuous joists with a typical bolted connection was investigated in the parallel-to-joist and perpendicular-to-joist directions, respectively.
Due to the size of the diaphragms, the University of Auckland’s city campus laboratory could not accommodate the testing program and testing was subsequently performed at a leased commercial warehouse in Henderson, west of Auckland.

Few research studies have experimentally determined in-plane timber floor diaphragm performance (ABK 1981; Baldessari 2010; Brignola 2009; Peralta et al. 2004), and have particularly ignored the potential orthotropic nature of these structures by limiting testing to the direction parallel-to-joists. To the best of the author’s knowledge, the full-scale diaphragm testing performed in both principal directions is the first of its kind. The study presented in this chapter investigated the orthotropic behaviour of timber diaphragms as well as quantifying diaphragm performance degradation from repetitive loading, seismic retrofit performance and the effects of diaphragm configuration variations. A summary of diaphragm construction, test set-up and the testing methodology are also summarised.

**Table 7-1: Test matrix**

<table>
<thead>
<tr>
<th>Test reference</th>
<th>Loading direction</th>
<th>Dimensions</th>
<th>State</th>
<th>Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a-PARA</td>
<td>Parallel-to-joists</td>
<td>10.4 m × 5.535 m</td>
<td>As-built</td>
<td>Homogeneous</td>
</tr>
<tr>
<td>1b-PARA</td>
<td>Parallel-to-joists</td>
<td>10.4 m × 5.535 m</td>
<td>Retrofitted</td>
<td>Homogeneous</td>
</tr>
<tr>
<td>2a-PARA</td>
<td>Parallel-to-joists</td>
<td>10.4 m × 5.535 m</td>
<td>As-built</td>
<td>Corner penetration</td>
</tr>
<tr>
<td>2b-PARA</td>
<td>Parallel-to-joists</td>
<td>10.4 m × 5.535 m</td>
<td>Retrofitted</td>
<td>Corner penetration with specific retrofitting</td>
</tr>
<tr>
<td>1a-PERP</td>
<td>Perpendicular-to-joists</td>
<td>5.535 m × 10.4 m</td>
<td>As-built</td>
<td>Homogeneous</td>
</tr>
<tr>
<td>1b-PERP</td>
<td>Perpendicular-to-joists</td>
<td>5.535 m × 10.4 m</td>
<td>Retrofitted</td>
<td>Homogeneous</td>
</tr>
<tr>
<td>2a-PERP</td>
<td>Perpendicular-to-joists</td>
<td>5.535 m × 10.4 m</td>
<td>As-built</td>
<td>Discontinuous joists with bolted lapped connection</td>
</tr>
<tr>
<td>2b-PERP</td>
<td>Perpendicular-to-joists</td>
<td>5.535 m × 10.4 m</td>
<td>Retrofitted</td>
<td>Discontinuous joists with bolted lapped connection</td>
</tr>
</tbody>
</table>
7.1 Construction of Full-Scale Diaphragms

7.1.1 As-built

Four as-built diaphragms labelled 1a-PARA, 2a-PARA, 1a-PERP, and 2a-PERP were constructed for testing. Diaphragms with the suffix –PARA were tested in the direction parallel-to-joists and diaphragms with the suffix –PERP were tested in the direction perpendicular-to-joists.

The diaphragms were constructed with new timber and new nails and were assigned representative framing parameters to replicate as much as possible existing diaphragm construction, as outlined in Chapter 1. Each diaphragm measured 10.400 m × 5.535 m and comprised 135 mm × 18 mm straight-edge timber ‘clears’ floorboards fastened perpendicular to 45 mm × 290 mm structural grade MSG8 joists spaced at 400 mm centres. The joists were orientated in the 5.535 m dimension, and the floorboards were orientated in the 10.4 m dimension in an identical arrangement to remove the influence that this configuration feature had on diaphragm performance. Two 3.15 mm (diameter) × 75 mm common bright roundhead nails were power driven at approximately 95 mm spacing to fasten the floorboards at each joist location.

1a-PARA and 2a-PARA were fitted with 45 mm × 75 mm timber cross-bracing at 1/3 joist length locations to replicate typical restraint against lateral joist buckling. 1a-PARA was a homogeneous configuration that enabled a comparison to be performed for 2a-PARA which comprised a 3.2 m × 1.08 m corner penetration, to establish the effect that a typical stairwell opening has on diaphragm performance.

For 1a-PERP and 2a-PERP, the cross-bracing was replaced with full-depth blocking at the locations of load application to effectively transmit applied pseudo-static loads into the diaphragm. 1a-PERP was considered to be homogeneous with complete sheathing and continuous joists spanning between their supports. To quantify the influence that discontinuous joists have on diaphragm performance, 2a-PERP comprised discontinuous joists with a typical two-bolt lapped connection at diaphragm midspan, while all other
configuration parameters remained identical to 1a-PERP. It should be noted that for 2a-PERP, the two end joists remained continuous to provide a consistent loading surface for pseudo-static testing.

Details of the constructed diaphragms are provided in Figure 7-1 and Figure 7-2.

7.1.2 Retrofitting

After each as-built diaphragm was tested, a retrofit system was applied and the diaphragm was re-tested using an identical testing methodology. The retrofitted diaphragms were labelled 1b-PARA, 2b-PARA, 1b-PERP, and 2b-PERP corresponding to their respective as-built configurations. All retrofits consisted of 2400 mm × 1200 mm × 15 mm AS/NZS 2269:2004 structural grade plywood laid over the existing floorboards with 75 mm × 24 gauge sheet metal straps fastened to the plywood edges with ECKO SF-9215 staples at 100 mm centres, as shown in Figure 7-3. The staple wire had a rectangular cross-section of 1.24 mm × 1.00 mm and a leg length of 15 mm. Field nailing (approximately 300 mm centres) was applied to the plywood sheets at the locations of the joists to mitigate buckling of the panels during large diaphragm displacements, while nailing was provided at 100 mm centres around all diaphragm edges to effectively transfer shear forces. All nails were 3.15 mm (diameter) × 75 mm roundhead power driven nails.

Each retrofitted diaphragm was fitted with chords to resist the tension and compression forces generated during lateral deformation, as illustrated in Figure 7-4. Chord members are required at diaphragm edges and adjacent to penetrations, but are almost always absent from existing timber diaphragms in URM buildings and are necessary as part of any diaphragm retrofit.

Compression chords were constructed for 1b-PARA and 2b-PARA by nailing full-depth blocking between the joists while tension chords were comprised of 40 mm × 6 mm mild-steel flats fastened to the timber blocking with 75 mm × 10 gauge screws at 100 mm centres, as illustrated in Figure 7-5. Full-depth blocking was constructed by nailing two cut-to-length timber members together with the same sectional properties as the joists. This
7.1 Construction of Full-Scale Diaphragms

(a) 1-PARA

(b) 2-PARA

Figure 7-1: Diaphragm construction details
Figure 7-1 continued
7.1 Construction of Full-Scale Diaphragms

Figure 7-2: 2-PERP joist connection details

(a) Joist arrangement

(b) Joist connection

(c) Connection details

Figure 7-3: Plywood overlay with stapled sheet metal blocking system

(a) Plywood overlay

(b) Stapled sheet metal straps

Figure 7-3: Plywood overlay with stapled sheet metal blocking system
is a typical technique used for in-service plywood-overlay retrofits that allows the steel tension chord to be fastened adjacent to the plywood panels, producing a near-flush finished surface level.

Additional retrofitting details were required for diaphragm 2b-PARA to address increased stress concentrations generated in the area adjacent to the corner penetration. As well as providing diaphragm edge chords analogous to those included in 1b-PARA, an additional chord was required along the edge of the penetration, as illustrated in Figure 7-5. Full-depth timber blocking was extended to diaphragm midspan from the penetration in order to disperse compression forces and to provide sufficient development length for the steel tension chord that was fastened to the blocking. During retrofit construction, a section of
existing floorboards were removed to permit access for timber blocking application and were subsequently reinstated before installing the plywood panelling. The tension chord was again fabricated from 40 mm × 6 mm steel flat and was fixed to the blocking with 75 mm × 10 gauge screws at 50 mm centres. The frequency of screws was also increased to 50 mm centres up to diaphragm midspan. Sheet metal stapling was increased to 50 mm centres in the front diaphragm area from the penetration to diaphragm midspan. All other stapling within the half of the diaphragm containing the penetration was increased to 75 mm centres. Nails along the joist adjacent to the penetration were provided at 50 mm centres up until the end of the first plywood panel, from which they were provided at 100 mm centres to the opposite end of the diaphragm. Nails at 100 mm spacing were provided along the full length of the midspan diaphragm joist. To account for reduced diaphragm width connected to the side frame, the frequency of diaphragm edge nailing was increased to 80 mm centres on this side.

1b-PERP and 2b-PERP did not require the double blocking and steel flat chord elements provided in 1b-PARA and 2b-PARA, as the continuous joists at each end of the diaphragm could be utilised as chord members. Edge nailing provided at 100 mm centres was sufficient to engage the joists as combined compression and tension chords. A single layer of full-depth blocking was fastened between the joists along the sides of the diaphragm to provide a consistent line of framing for diaphragm edge nailing and effective shear transfer. During retrofit construction, three courses of floorboards were removed and reinstated once the blocking had been fitted.

Plywood panelling is a popular and cost-effective diaphragm retrofit technique. Given that existing diaphragms in URM buildings are almost always constructed of timber, the implementation of plywood and other timber members to strengthen the diaphragm is comparatively simple. The plywood sheets can be fastened either over the existing floorboards, or to the underside of the floor as a ‘ceiling’ diaphragm, depending on aesthetic requirements. The stapled sheet metal blocking system (SMBS) provides the necessary transfer of shear flow between plywood panels and eliminates the need for conventional blocking that involves nailing timber framing between joists along plywood panel boundary lines. The less invasive nature of this retrofit allows existing diaphragm
materials to be retained and promotes the preservation of architectural heritage. The purpose of the plywood overlay and SMBS was therefore to quantify the improvement in diaphragm performance using a cost-effective and repeatable retrofitting method that encourages the preservation of existing diaphragm construction.

All retrofitting details are illustrated in Figure 7-6 and a summary of the retrofit design procedure is provided in Appendix D.

### 7.2 Pseudo-Static Testing

#### 7.2.1 Test set-up for loading parallel-to-joists

The test set-up for diaphragms loaded parallel-to-joists is shown in Figure 7-7 and Figure 7-8. Loading was provided by a single hydraulic actuator connected to a large box-frame fabricated from 460 UB 75 steel sections and with two concrete slabs on top to provide rigid reaction to applied loads. A distribution frame comprising a primary truss structure and two secondary beams on castors was used to distribute the actuator point load into four equal, simply supported loads that were applied to the diaphragm at joist locations. The primary truss, secondary beams and joist loaders were connected with purpose-built hinge joints that enabled the secondary beams to rotate with the deforming diaphragm and to ensure that applied loading was free of any induced moments. Reversed cyclic loading was achieved by positioning loaders on both ends of the loaded joists and post-tensioning these together using M16 threaded rods that spanned the length of the diaphragm. Retracting the actuator would therefore transmit tension through the post-tensioned rods and apply load to the opposite end of the diaphragm, subsequently pulling it towards the reaction frame. The primary truss and secondary beams were fabricated from 200 UC 60 steel sections, and the joist loaders were fabricated from 100 UC 15 steel sections with stiffening plates welded to each end. Two large steel beams were also placed on top of the primary truss to ensure that uplift did not occur during loading.
7.2 Pseudo-Static Testing

(a) General retrofit details

(b) Additional retrofit details for 2b-PARA

Figure 7-6: Diaphragm retrofit details
Figure 7-7: Test set-up for loading parallel-to-joists
Figure 7-8: Test schematic for loading parallel-to-joists
The distributed loading mechanism was a practical replication of diaphragm earthquake loading that involves the inertial mass of the out-of-plane walls being transmitted into the diaphragm through its joists. These loads are typically idealised as a parabolic distribution across the diaphragm (ASCE 2007), as described by Equation 7-1.

\[ w_E = \frac{1.5F_d}{L} \left[ 1 - \left( \frac{2x}{L} \right)^2 \right] \]  

(7-1)

Where \( F_d \) is total diaphragm load, \( L \) is diaphragm span, and \( x \) is the distance from diaphragm midspan. The locations of the applied loads were therefore determined by calculating four equivalent tributary areas described by Equation 7-1, and by converting these into single point loads acting at their centroid locations.

To provide the necessary restraint against lateral loading, the two side-joists were fastened to inverted T-sections fabricated from 6.0 m long steel plates (see Figure 7-9a). Holes were drilled in the side joists at twelve prefabricated bolt-hole locations in the T-section web and CL8.8 M16 bolts were used to create a tight friction connection between the steel and timber to prevent any lateral slip from occurring. The T-sections were anchored to the concrete floor of the warehouse using M16 studs and high strength epoxy mortar to completely fix against movement.

150 UB 14 steel sections were bolted to the concrete slab and blocked with timber to provide vertical support at the joist ends, as shown in Figure 7-9b. This intermediate support was necessary as the floorboards spanning between the two fixed side-joists could not carry the self-weight of the diaphragm. Teflon pads were fastened to the supports at joist locations to minimise friction resistance to diaphragm displacement.

The uplifting of extracted sub-diaphragms during pseudo-static testing reported in Chapter 6 motivated the design and implementation of custom steel frames that enclosed the front ends of the full-scale diaphragm joists and prevented this undesirable response mechanism from occurring. A total of five frames were fabricated from 40 mm × 40 mm × 4 mm steel SHS and were bolted to the concrete slab to prevent all
intermediate joists from out-of-plane movement, as shown in Figure 7-9b. The front ends of the joists were extended approximately 400 mm beyond the sheathing to allow sufficient clearance for a maximum midspan displacement of ±200 mm. A minimum clearance of 10 mm was provided between the underside of the frames and the joists to allow unimpeded lateral movement. In case of uplift occurring, a Teflon pad was provided at each joist location so that if the joists came into contact with the out-of-plane restraint frames, further uplift would be prevented whilst imposing minimal resistance to lateral diaphragm displacement.

Figure 7-9b shows that the joists within the corner penetration of diaphragm 2-PARA were extended to the location of the support frames and the out-of-plane restraint frames. This was a necessary measure to reduce set-up time and costs, and the number of holes drilled into the warehouse concrete slab. The presence of the joists within the corner penetration did not influence diaphragm response.

Additional photos of the test set-up are provided in Section 7.5.

### 7.2.2 Test set-up for loading perpendicular-to-joists

The test set-up for diaphragms loaded perpendicular-to-joists is shown in Figure 7-10. Due to a considerable reduction in diaphragm span, the loading system used for diaphragms...
tested parallel-to-joists was reconfigured for two points of loading, instead of four, by removing the secondary beams and connecting the joist loaders directly to the primary truss. The locations of the applied loads were determined as the centroid in each half of the parabolic loading profile described in Equation 7-1. Reversed-cyclic loading was again achieved by post-tensioning loaders at each end of the diaphragm.

Two short URM walls were constructed to provide a realistic boundary condition for testing diaphragms perpendicular-to-joists. The walls were constructed with solid clay bricks recycled from a heritage URM building in Auckland and a mortar composition of one part cement to one part lime to six parts sand (1:1:6 mortar), which is acknowledged to be a comparatively strong mortar mix but because the testing focus was on diaphragm response, any failure of the URM wall was undesirable and was therefore mitigated by using a stronger mortar. Overall the walls were six bricks high, two bricks wide and approximately 11.5 m long. Diaphragm joists were seated in pockets that were one brick deep and approximately 49 mm wide provided at 400 mm centres along the URM walls, as shown in Figure 7-12. This detail replicated a typical joist seating condition found in many existing URM buildings and provided the necessary resistance to lateral diaphragm loading.

Each wall was constructed around sixteen 1.0 m M16 threaded rods that were anchored into the concrete slab using high strength epoxy mortar (see Figure 7-10). After wall construction was completed and the joists were placed into the pockets, each threaded rod was post-tensioned to approximately 30 kN, producing a total wall compression load of almost 500 kN. Post-tensioning was performed to generate sufficient shear strength within the wall and sufficient friction resistance between the wall and the concrete slab to prevent sliding from occurring. Steel plates and timber blocking were used to effectively distribute the post-tensioned loads into the URM walls.
7.2 Pseudo-Static Testing

Figure 7-10: Test set-up for loading perpendicular-to-joists

(a) Overall set-up

(b) Loading frame
Figure 7-11: Test schematic for loading perpendicular-to-joists
7.2 Pseudo-Static Testing

Figure 7-12: Joists pocketed into URM wall

One wall was used for all diaphragms tested perpendicular-to-joists, while the other was rebuilt for 2a-PERP and 2b-PERP to relocate the pockets for the offset joists created from lapped connections. 150 UB 14 steel sections with timber blocking were also bolted to the concrete floor of the warehouse to provide vertical support for the discontinuous joists at midspan. This detail replicated typical diaphragm support conditions where discontinuous joists are seated on intermediate timber or steel cross-beams that are supported on columns.

Additional photos of the test set-up are provided in Section 7.5.

7.2.3 Instrumentation and test procedure

The instrumentation used to capture essential diaphragm response in each principal loading direction is illustrated in Figure 7-8 and Figure 7-11. During each test, total load ‘F’ was recorded using a load cell attached to the actuator, while the diaphragm deformation profile was measured at three locations ‘DISP1’, ‘DISP2’ and ‘DISP3’ using string potentiometers. In the loading direction parallel-to-joists, additional strain ‘portal’ gauges were used to monitor the in-plane and out-of-plane displacement of the steel side frames to
ensure that these values remained negligible. These devices were labelled $S_{IP1}$ to $S_{IP2}$ and $S_{OP1}$ to $S_{OP4}$, respectively.

Each diaphragm was subjected to pseudo-static reversed-cycle loading to midspan displacement amplitudes of 2.5 mm, 5 mm, 15 mm, 25 mm, 50 mm, 75 mm, 100 mm and 150 mm, as shown in Figure 7-13. Each displacement amplitude was repeated three times to investigate the cyclic degradation of diaphragm performance. Once this loading schedule had been completed, an attempt was made to push and pull the diaphragm to the maximum stroke of the actuator, which was ±150 mm. Because it was difficult to set the actuator perfectly at the centre of its stroke, the maximum negative displacement generally exceeded the maximum positive displacement. Hydraulic pressure was supplied to the actuator with a two-way hand pump, allowing extension and retraction of the actuator piston. The push direction was defined as positive and the pull direction was defined as negative.

### 7.2.4 Test results

Diaphragm performance is presented and characterised to compare response and to establish the influence of key configuration attributes. Performance degradation is also characterised to further compare diaphragm behaviour.

![Figure 7-13: Reversed-cyclic loading schedule](image-url)
7.2.4.1 Observations

As-built diaphragms tested parallel-to-joists responded almost identically when subjected to lateral loading, despite a reduction in floor area of approximately 6% between 1a-PARA and 2a-PARA, created by the corner penetration. Both diaphragms demonstrated highly flexible characteristics with no indications of floorboard cracking or splitting, nail pullout or shear failure, or any other structural failure up to the maximum midspan displacement of almost 200 mm. This observation was particularly surprising for 2a-PARA where floorboard cracking or separation was expected to occur adjacent to the penetration due to increased stress concentrations, but no such failures were observed. As a product of their flexibility, both diaphragms exhibited no residual damage and remained completely serviceable at the conclusion of testing. The mechanism for diaphragm deformation appeared to be relative joist displacement (racking) inducing nail couple rotation and flexural bending of the floorboards. As shown in Figure 7-14, the deformation profile appeared to be parabolic and symmetric about diaphragm midspan, even for 2a-PARA that was expected to deform asymmetrically from imbalanced resistance to lateral loading caused by the corner penetration. The diaphragm deformation mechanism appeared consistent throughout the loading protocol, with only magnitude increasing with increasing midspan displacement amplitudes.

Similar to the parallel-to-joist direction, diaphragms tested perpendicular-to-joists were highly flexible, suffering no observable damage up to a maximum midspan displacement of ±150 mm and remaining completely serviceable at the conclusion of each test. The out-of-plane flexural bending of the joists in 1a-PERP and 2a-PERP appeared to engage the nail couples, but to less of an extent than when testing was performed parallel-to-joists. The majority of deformation was observed to occur between the sides of the diaphragms and the two loading locations, while only low levels of relative displacement occurred within the central section of the diaphragms, as shown in Figure 7-14. This displacement profile was likely caused by zero shear force and constant moment existing between the two point loads, and could also be the reason that almost no difference was observed between the response of 1a-PERP with continuous joists, and the response of 2a-PERP with discontinuous joists and bolted lapped connections. Overall the URM walls remained
largely undamaged throughout testing and no sliding was observed between the walls and the concrete slab. A small amount of brick crushing and spalling of some joist pockets occurred above midspan displacements of approximately ±50 mm, primarily at the diaphragm ends. This damage appeared to be caused primarily by direct bearing of the joists against the bricks, although some prying actions may have occurred where joists were forced into double bending due to excessive rotation within the pocket. Joist end supports therefore appeared to be effectively pinned up to displacement amplitudes of approximately ±50 mm.
Retrofitted diaphragm 1b-PARA responded without observable damage up to a midspan displacement of ±25 mm. However, between displacement amplitudes of ±25 mm and ±50 mm, the sheet metal blocking that was used to transfer shear flow between plywood panels began to buckle and the staples began to pull out, as shown in Figure 7-16. This damage occurred primarily in the two outer third spans of the diaphragm, with little sheet buckling and staple slip observable within the central region of the diaphragm. By the end of the first cycle to displacement amplitude ±75 mm, all sheet metal staples had completely pulled out of the plywood in the two outer-third spans of the diaphragm. Despite early failure of the sheet metal blocking system, the diaphragm remained largely serviceable up to a midspan displacement of ±100 mm. Between ±100 mm and the maximum positive and negative displacement amplitudes, significant buckling and uplifting of plywood panels occurred throughout the diaphragm as well as nail pullout or tear-through caused by heavy panel rotation and uplifting (see Figure 7-16). Maximum actuator stroke was reached at +141 mm in the push direction and the -150 mm displacement amplitude was replaced with maximum negative actuator stroke which was -163 mm.

The initial response of diaphragm 2b-PARA, that included a corner penetration and additional retrofitting provisions to address increased stress concentrations surrounding this penetration, was similar to 1b-PARA. Little damage was observed up to ±25 mm but during the ±50 mm displacement cycle, sheet metal buckling and staple pullout began to occur in the outer regions of the diaphragm outside of the loaded joists. When a midspan displacement of ±50 mm was reached, all staples within these outer regions had completely pulled out. Sheet metal buckling and staple pullout continued towards the centre of the diaphragm during the ±75 mm displacement cycle but terminated within approximately the central third of the diaphragm which was largely undamaged throughout testing, likely due to lower shear forces existing within this region. Localised buckling of plywood panels began at ±75 mm within the region of the diaphragm containing the penetration and continued to worsen throughout the remainder of testing. Figure 7-16 shows that at maximum diaphragm displacement, significant plywood panel buckling as well as nail pullout and tear-through were observed within the region of the diaphragm containing the penetration, severely compromising the serviceability of the diaphragm. Such damage was also observed in the opposite side of the diaphragm, in the region
outside of the loaded joist, but to a lesser extent. The deformation profile of 2b-PARA was observed to be influenced by the locations of applied load, with the majority of displacement occurring between the side frames and the two outer loaded joists, and comparatively small amounts of displacement occurring within the central region of the diaphragm. Maximum actuator stroke was reached at +131 mm in the push direction and was limited to -150 mm in the pull direction.

Retrofitted diaphragms 1b-PERP and 2b-PERP were observed to behave similarly. Given the plywood arrangement, panel boundary lines in the direction of loading extended the full width of the diaphragm without interruption. A primary observation was that diaphragm damage was concentrated along the two plywood boundary lines outside of the
applied load locations, and almost no damage was observed within the central region of the
diaphragm (see Figure 7-16). Specifically, between ±15 mm and ±25 mm displacement
amplitude, a significant amount of the sheet metal staples along the outside boundary lines
pulled out of the plywood, and at the ±50 mm displacement cycle the staples were
completely withdrawn and ineffective in both diaphragms. Buckling of the sheet metal
during these displacement cycles was also observed. Plywood uplift along the outside
boundary lines began from a midspan displacement of ±75 mm and continued to worsen as
diaphragm deformation was increased. The diaphragm deformed primarily between the
side walls and the applied loads, with relatively little displacement occurring between the
applied loads. Similar to the unretrofitted configurations tested perpendicular-to-joists, the
presence of discontinuous joists with bolted lapped connections did not observably affect
diaphragm performance. Again, no significant brick crushing or spalling was observed
around the joist pockets of the URM walls. Maximum actuator stroke was reached at
+135 mm in the push direction and at -135 mm in the pull direction.

Overall the plywood overlay and sheet metal blocking retrofit performed well up to
±25 mm for diaphragms tested parallel-to-joists, and to at least ±15 mm for diaphragms
tested perpendicular-to-joists, but displayed potential serviceability issues above ±75 mm
with considerable plywood panel distortion that compromised the finished floor and that
would require considerable remedial work to rectify.

Additional photos of as-built and retrofitted diaphragm testing are provided in Section 7.5.

7.2.4.2 Force-displacement response

The force-displacement response of as-built diaphragms and their corresponding retrofitted
configurations are presented in Figure 7-17, where force is total load recorded from the
load cell and displacement is midspan displacement recorded from the central string
potentiometer.
Chapter 7: Full-Scale Diaphragm Testing

Figure 7-17: Force-displacement response

(a) 1a-PARA

(b) 1b-PARA

Figure 7-17: Force-displacement response
Figure 7-17 continued

(a) 2a-PARA

(b) 2b-PARA
Figure 7-17 continued
7.2 Pseudo-Static Testing

(a) 2a-PERP

(b) 2b-PERP

Figure 7-17 continued
As-built diaphragms exhibited flexible and highly nonlinear characteristics with no clearly defined yield point. The force-displacement responses display no indication of strength degradation up to midspan displacements of almost 200 mm, confirming diaphragm flexibility and the absence of observed structural failures during testing. Only small strength losses are evident between cycles one, two and three at each displacement amplitude, indicating that as-built diaphragm performance is not significantly degraded when repeatedly loaded to the same displacement. However, due to hysteretic pinching caused by nail slip, it is evident that once a displacement amplitude has been exceeded, diaphragm strength and stiffness is considerably reduced at lesser displacements. All diaphragms appear to have responded relatively symmetrically when subjected to reversed-cyclic loading, resisting similar loads at equivalent positive and negative midspan displacements.

Retrofitted diaphragms demonstrated a considerable increase in stiffness and in strength. While force-displacement response remained primarily nonlinear, a significant difference between initial stiffness and secondary stiffness is evident, making an effective yield point more distinguishable. It is likely that the comparatively high initial stiffness is attributable to the stapled sheet metal blocking system that effectively transferred shear flow between plywood panels up to midspan displacements of between approximately ±15 mm and ±25 mm, after which the majority of staples were ineffective, causing reduced shear transfer between plywood panels and a reduction in overall diaphragm stiffness. This is particularly true for diaphragms 1b-PERP and 2b-PERP that exhibited force-displacement responses similar to elastic-perfectly plastic behaviour.

Overall, diaphragms 1b-PARA and 2b-PARA demonstrated strength integrity up to a midspan displacement of ±100 mm, after which considerable strength degradation occurred; likely to be associated with the observed plywood panel buckling and nail pullout at large displacement amplitudes. After initial response, diaphragms 1b-PERP and 2b-PERP showed inconsistent changes in maximum load between displacement amplitudes but no overall strength degradation is evident. Unlike the as-built configurations, a reduction in load carrying capacity between cycles to the same displacement amplitude was a feature of retrofitted diaphragm response. It can be observed for diaphragms 1b-
PARA and 2b-PARA that considerable degradation occurred between the first and second cycles for displacement amplitudes of ±25 mm and above, although it appears not to increase in magnitude until the maximum positive and negative midspan displacements are reached. Degradation between the first and second cycles did not significantly occur in diaphragm 1b-PERP until ±15 mm midspan displacement, but is evident as early as ±10 mm in diaphragm 2b-PERP, although the magnitude of degradation does not increase with increasing midspan displacement for both of these tests. For all retrofitted diaphragms, strength degradation was only considerable between the first and second loading cycles, and was negligible between cycles two and three.

7.2.4.3 Backbone curve development

Backbone (envelope) curves were developed for each hysteretic response using a multi-linear line passing through the maximum force recorded at each displacement amplitude for the first, second and third cycles, respectively. Plots of the backbone curves for as-built and retrofitted diaphragms with the relevant hysteretic response are provided in Appendix C. Unless specified otherwise, backbone response discussed hereafter will correspond to the first cycle.

7.2.4.4 Bilinear idealisation

For comparative purposes it is necessary to characterise diaphragm response in such a way that performance parameters such as stiffness, strength and ductility can be established. To fulfil this need, a bilinear idealisation was selected as the most appropriate characterisation of hysteretic response to capture essential diaphragm performance and to provide the necessary characteristics to define recognisable performance parameters. Due to the distinct behaviour of as-built and retrofitted diaphragms, specific methodologies were adopted to develop a bilinear idealisation that suitably matched hysteretic response.

No universally accepted method currently exists to characterise the highly nonlinear behaviour of unretrofitted timber diaphragms. A rational methodology using the principle of hysteretic energy conservation (Mahin and Bertero 1981) was adopted that enabled objective and repeatable characterisation of as-built diaphragm performance. The idealised
curves follow the form shown in Figure 7-18(a) and a summary of the methodology is provided below.

1) The area beneath the backbone curve was calculated using the ‘trapezoidal rule’.
2) The proposed bilinear curve was given the following constraints to ensure that a unique solution could be found when equating hysteretic energy:
   a. Must pass through zero load and zero displacement.
   b. Secondary stiffness, $K_{d2}$, was taken as the average gradient of the linear portion of displacement amplitudes above 50 mm.
   c. Final displacement, $F_{d,max}$, for the purposes of energy conservation was taken as the maximum displacement of the linear portion of displacement amplitudes above 50 mm.
3) Yield displacement, $\Delta_y$, was calculated using Equation 7-2.

$$\Delta_y = \frac{\text{Area}_{backbone} - \frac{1}{2} (2F_{d,max}A_{max} - K_{d2}A_{max}^2)}{\frac{1}{2} (K_{d2} - F_{d,max})}$$  (7-2)

4) The corresponding yield load, $F_y$, was calculated using Equation 7-3.

$$F_y = K_{d2} \Delta_y + F_{d,max} - K_{d2}A_{max}$$  (7-3)

5) The bilinear curve was drawn using the calculated characteristic values.

The above process was repeated for the positive and negative displacement regions separately, and subsequently averaged to produce an average bilinear curve that was symmetrical about the origin. For diaphragms 1a-PARA and 2a-PARA the linear portion of the backbone curve existed between displacement amplitudes of ±75 mm, ±100 mm and ±150 mm, while for diaphragms 1a-PERP and 2a-PERP the linear portion existed between displacement amplitudes of ±50 mm, ±75 mm and ±100 mm. It should be noted that although the ±150 mm displacement amplitudes were removed during the idealisation
process for diaphragms 1a-PERP and 2a-PERP, the bilinear curves were extrapolated to ±150 mm to present a consistent maximum displacement for comparison.

The developed bilinear curves presented in Figure 7-19 illustrate that this is an appropriate idealisation that captures essential diaphragm response. It was found that excluding the final displacement amplitude of diaphragms tested perpendicular-to-joists generated bilinear curves that slightly underestimate response at large displacement amplitudes. This was considered an acceptable conservatism given that it permitted the unbiased definition of secondary stiffness.

Retrofitted diaphragm performance was characterised using two widely accepted methods for the bilinear idealisation of timber shear wall response, due to the similarities that exist

![Figure 7-18: Bilinear idealisations of hysteretic response](image)

- (a) Idealisation for as-built diaphragms [from Peralta (2003)]
- (b) EEEP curve for retrofitted diaphragms
- (c) SEAOSC curve for retrofitted diaphragms

[(b) and (c) from Salenikovich (2000)]
between these two timber systems. The first method used ASTM standard E2126 (2010) to develop an equivalent energy elastic-plastic (EEEP) curve that idealises average backbone response with an elastic-perfectly plastic bilinear curve dissipating equivalent hysteretic energy, as shown in Figure 7-18. Average backbone response was taken as the average of the positive and negative displacement backbone curves and specific definitions of characteristic performance parameters were used to assemble the EEEP curve. Peak force, \( F_{\text{peak}} \), and the corresponding peak displacement, \( \Delta_{\text{peak}} \), were defined as the maximum load-carrying capacity of the diaphragm before consistent strength degradation occurs. Failure displacement, \( \Delta_{\text{failure}} \), also known as ultimate displacement, \( \Delta_{\text{ult}} \) (ISO 2003), was considered to occur at 0.8\( F_{\text{peak}} \) in the descending portion of the response curve if data points less than this value existed. Otherwise \( \Delta_{\text{ult}} \) was considered to be the maximum displacement reached by the structure. The elastic portion of the EEEP curve was defined by the line that passes through the origin and the point on the response curve equal to 0.4\( F_{\text{peak}} \), and has a gradient equal to initial stiffness, \( K_{d1} \). The plastic portion of the EEEP curve is horizontal (\( K_{d2} = 0 \)) and is positioned at yield force, \( F_y \), so that the area under the EEEP curve equals the area under the response curve from zero displacement to ultimate displacement, \( A_e \). Yield force was calculated using Equation 7-4:

\[
F_y = \left( \Delta_{\text{ult}} - \sqrt{\frac{\Delta_{\text{ult}}^2 - 2A_e}{K_{d1}}} \right) K_{d1}
\]  

(7-4)

The standard published by the Structural Engineers Association of Southern California (SEAOSC 1997) was used as a second method of retrofitted diaphragm performance characterisation, to compare its suitability with the ASTM method. SEAOSC (1997) specifies two characteristic points on the hysteretic response curve that are considered to effectively capture performance; the yield limit state (YLS) and the strength limit state (SLS). The YLS is defined as the last displacement amplitude at which the load-carrying capacity of the structure does not decrease by more than 5% between the first (initial) and second (stabilised) cycles. The SLS is the maximum force resisted by the structure before consistent strength degradation occurs. A bilinear curve is subsequently assembled by joining the origin, YLS and SLS locations, as illustrated in Figure 7-18. Many researchers
(Salenikovich 2000) believe that this is an unconservative idealisation because it does not require averaging and permits the occurrence of the YLS and the SLS on separate sides of the hysteretic response curve, which can be confusing and may over estimate performance, particularly if unsymmetrical behaviour develops from symmetrical excitation. Nevertheless, the SEAOSC limit states have been determined for both sides of the hysteretic response curves. For comparison purposes, minimum yield load and maximum peak load were taken as characteristic values and the theoretical line joining their absolute locations was used to determine initial and secondary stiffness.

The bilinear idealisations developed using the provisions of ASTM and SEAOSC are presented in Figure 7-19 with the relevant retrofitted diaphragm backbone response curves. The figures illustrate that these two methods generated vastly different characterisations with unique shapes and subsequently unique definitions of characteristic values. In general, the ASTM method estimated higher yield load but lower initial stiffness, and generated lower secondary stiffness and peak load when compared to the SEAOSC approach. Overall the ASTM standard is considered to more appropriately characterise retrofitted diaphragm response and will represent idealised performance hereafter. In addition, based on the intended implementation of these results to the New Zealand engineering design community, the ASTM curve appropriately captures essential performance and provides conventional design parameters.

The performance parameters defined by the bilinear idealisations developed for as-built and retrofitted diaphragms are outlined in Table 7-2 for comparison and are discussed in the discussion section.

7.2.4.5 Shear strength

For seismic assessment purposes, diaphragm strength is conventionally presented as shear strength per lineal metre width of the diaphragm, $R_d$, which removes the influence of diaphragm geometry. For each diaphragm, shear strength was calculated by taking the relevant bilinear yield load, halving it to find shear resistance, and dividing it by the width of the diaphragm, as described in Equation 7-5.
Figure 7-19: Backbone curves with bilinear idealisations
\[ R_d = \frac{F_y}{2B} \quad (7-5) \]

where \( B \) is diaphragm width, being the dimension parallel to applied load. The values calculated for tested diaphragms are presented in Table 7-2 for comparison.

**7.2.4.6 Stiffness**

Diaphragm stiffness, \( K_d \), is considered to be initial stiffness, \( K_1 \), for the determination of diaphragm period and diaphragm deformation during seismic assessments. Diaphragm stiffness is conventionally converted to shear stiffness, \( G_d \), to achieve independence from diaphragm geometry and to allow the comparison of varying configurations. It was shown in Section 6.2.6 that diaphragm shear stiffness may be evaluated using a rectangular shear beam analogy subjected to the relevant loading conditions. For full-scale diaphragms tested perpendicular-to-joists (two point loading system), Figure 6-20 and Equation 6-7 appropriately define diaphragm shear stiffness, where \( G_e = G_d \), \( F = F_y \), \( \Delta = \Delta_y \), \( a = 1.731 \text{ m} \) (from Figure 7-11), and \( B = 10.4 \text{ m} \). For full-scale diaphragms tested parallel-to-joists (four point loading system), consider the rectangular shear beam depicted in Figure 7-20, subjected to four equivalent point loads applied at the given arbitrary locations. From Figure 7-20 it can be shown that shear strains \( \gamma_1 \) and \( \gamma_2 \) are given by,
Figure 7-20: Shear beam deformation for shear stiffness evaluation

\[ \gamma_1 = \frac{\Delta_1}{a} \]  \hspace{1cm} (7-6)

\[ \gamma_2 = \frac{\Delta_2}{b} \]  \hspace{1cm} (7-7)

and that the corresponding shear stresses are defined as,

\[ \tau_1 = \frac{F}{2Bt} \]  \hspace{1cm} (7-8)

\[ \tau_2 = \frac{F}{4Bt} \]  \hspace{1cm} (7-9)

where \( t \) is shear beam thickness. Given that \( G = \tau/\gamma \), it can be shown from Equations 7-6 to 7-8 that for \( t = 1 \), shear stiffnesses can be expressed as,

\[ G_1 = \frac{Fa}{2B\Delta_1} \]  \hspace{1cm} (7-10)

\[ G_2 = \frac{Fb}{4B\Delta_2} \]  \hspace{1cm} (7-11)
Acknowledging that $\Delta_1 + \Delta_2 = \Delta_d$ (where $\Delta_d$ is overall midspan displacement), and that $G_1 = G_2 = G_d$ for uniform sectional properties, using Equations 7-10 and 7-11 it can be shown that,

$$\Delta_1 = \frac{\Delta_d}{\left(1 + \frac{b}{2a}\right)} \quad (7-12)$$

which, substituting back into Equation 7-10, gives the following expression for diaphragm shear stiffness,

$$G_1 = G_d = \frac{F \left(a + \frac{b}{2}\right)}{2B \Delta_d} \quad (7-13)$$

Taking $F = F_y$ and $\Delta_d = \Delta_y$ and knowing that $K_d = F_y/\Delta_y$, the shear stiffness of each full-scale diaphragm tested parallel-to-joists was determined using Equation 7-14 below,

$$G_d = \frac{K_d \left(a + \frac{b}{2}\right)}{2B} \quad (7-14)$$

where $a = 2.4$ m and $b = 2.0$ m (from Figure 7-8), and $B = 5.535$ m. The shear stiffness values determined for diaphragms tested parallel-to-joists and for diaphragms tested perpendicular-to-joists are presented in Table 7-2 for comparison.

### 7.2.4.7 Ductility

The idealised backbone curves in Figure 7-19 illustrate that as-built and plywood-retrofitted timber diaphragms are capable of undergoing large deformations without significant strength degradation, demonstrating the highly ductile nature of these structures. Ductility can be defined as the ratio of ultimate (elastic + plastic) displacement to elastic displacement based on the equal displacements principle of elastic-perfectly plastic behaviour (Park et al. 1987), as shown in Figure 7-21 and described by Equation 7-15. For this reason, ductility is often utilised in structural engineering to reduce
Figure 7-21: Ductility and overstrength

Ductility as described by Equation 7-15 was directly determined for retrofitted diaphragms by using the ultimate displacement and yield displacement values defined by the elastic-perfectly plastic idealisations shown in Figure 7-19. These values are presented in Table 7-2. Equation 7-15 may also be applied to as-built diaphragms, although ultimate load is significantly underestimated due to the ‘work hardening’ of post-yield deformations, as shown in Figure 7-21. This underestimation is considered to be an acceptable conservatism during the seismic assessment of timber diaphragm strength and deformation. However, the reserve capacity ignored by idealised ductile behaviour is addressed in the concept of overstrength that must be considered for the design of associated structural components such as floor-to-wall connections that are typically designed to remain elastic during response. An overstrength factor, $\Omega$, has been calculated as the ratio of ultimate load to yield load, as described by Equation 7-16.

$$\mu = \frac{\Delta_{ult}}{\Delta_y}$$  \hspace{1cm} (7-15)
7.2 Pseudo-Static Testing

\[ \Omega = \frac{F_{\text{ult}}}{F_y} \]  \hspace{1cm} (7-16)

Figure 7-17 shows that ultimate displacement was not captured during as-built diaphragm testing, meaning that ductility could not be calculated explicitly from test results. In order to gauge some level of diaphragm ductility capacity, Equation 7-15 was applied by taking the maximum recorded displacement value of each as-built test as ultimate displacement and the corresponding yield displacement determined from the bilinear idealisations shown in Figure 7-19. As outlined in Table 7-2, despite using conservative definitions, ductility capacity was found to be between 6.7 and 8.9 for as-built diaphragms, which considerably exceed the typical values published in earthquake loading standards such as AS/NZS 1170 (2002a) that recommend a maximum ductility capacity of \( \mu = 6 \). Extrapolation of the force-displacement response to estimate ultimate displacement was therefore considered unnecessary. Overstrength factors were also determined using the loads corresponding to maximum recorded displacement and the relevant yield displacement.

7.2.4.8 Energy dissipation

While diaphragm framing members effectively respond elastically, diaphragm energy dissipation is sourced from the plastic deformation and associated local timber crushing of sheathing-to-underfloor framing nail connections that are forced to deform during diaphragm loading. Hysteretic energy is calculated as the area enclosed by the force-displacement response loops, such as those shown in Figure 7-17.

The energy dissipated within the first hysteretic loop of each displacement amplitude was calculated for both as-built and retrofitted diaphragms, as shown in Figure 7-22. It can be seen that as-built diaphragms tested perpendicular-to-joist dissipated more hysteretic energy than the diaphragms tested parallel-to-joists, with this discrepancy increasing with increasing displacement amplitude. The lower aspect ratio and subsequent greater load-carrying capacity of diaphragms 1a-PERP and 2a-PERP will have contributed to this. As expected, all retrofitted diaphragms dissipated considerably higher hysteretic energy than
### Table 7-2: Diaphragm performance values

<table>
<thead>
<tr>
<th>Diaphragm</th>
<th>$F_y$ (kN)</th>
<th>$\Delta_y$ (mm)</th>
<th>$F_{ult}$ (kN)</th>
<th>$\Delta_{ult}$ (mm)</th>
<th>$K_{d1}$ (kN/m)</th>
<th>$K_{d2}$ (kN/m)</th>
<th>$R_d$ (kN/m)</th>
<th>$G_d$ (kN/m)</th>
<th>$\mu$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a-PARA</td>
<td>17.2</td>
<td>26.8</td>
<td>36.8</td>
<td>150</td>
<td>644</td>
<td>159</td>
<td>1.6</td>
<td>198</td>
<td>7.2</td>
<td>2.6</td>
</tr>
<tr>
<td>1b-PARA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>175.8</td>
<td>12.1</td>
<td>175.8</td>
<td>149.9</td>
<td>14,518</td>
<td>0</td>
<td>15.9</td>
<td>4459</td>
<td>12.4</td>
<td>1.0</td>
</tr>
<tr>
<td>(b)</td>
<td>72.4</td>
<td>5.0</td>
<td>200.9</td>
<td>99.1</td>
<td>14,598</td>
<td>1365</td>
<td>6.5</td>
<td>4484</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2a-PARA</td>
<td>17.7</td>
<td>29.4</td>
<td>35.9</td>
<td>150</td>
<td>601</td>
<td>151</td>
<td>1.6</td>
<td>185</td>
<td>6.7</td>
<td>2.5</td>
</tr>
<tr>
<td>2b-PARA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>171.9</td>
<td>12.5</td>
<td>171.9</td>
<td>127.9</td>
<td>13,768</td>
<td>0</td>
<td>15.5</td>
<td>4229</td>
<td>10.2</td>
<td>1.0</td>
</tr>
<tr>
<td>(b)</td>
<td>70.0</td>
<td>4.9</td>
<td>205.8</td>
<td>96.7</td>
<td>14,210</td>
<td>1480</td>
<td>6.3</td>
<td>4364</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1a-PERP</td>
<td>27.0</td>
<td>16.9</td>
<td>102.9</td>
<td>150</td>
<td>1605</td>
<td>569</td>
<td>1.3</td>
<td>134</td>
<td>8.8</td>
<td>4.1</td>
</tr>
<tr>
<td>1b-PERP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>204.7</td>
<td>9.1</td>
<td>204.7</td>
<td>132.6</td>
<td>22,409</td>
<td>0</td>
<td>9.8</td>
<td>1864</td>
<td>14.6</td>
<td>1.0</td>
</tr>
<tr>
<td>(b)</td>
<td>101.9</td>
<td>5.0</td>
<td>223.4</td>
<td>20.5</td>
<td>20,505</td>
<td>7799</td>
<td>4.9</td>
<td>1706</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2a-PERP</td>
<td>30.2</td>
<td>16.7</td>
<td>99.1</td>
<td>150</td>
<td>1743</td>
<td>517</td>
<td>1.5</td>
<td>145</td>
<td>8.9</td>
<td>3.5</td>
</tr>
<tr>
<td>2b-PERP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>192.6</td>
<td>6.4</td>
<td>192.6</td>
<td>132.7</td>
<td>29,960</td>
<td>0</td>
<td>9.3</td>
<td>2493</td>
<td>20.6</td>
<td>1.0</td>
</tr>
<tr>
<td>(b)</td>
<td>92.5</td>
<td>2.4</td>
<td>225.6</td>
<td>148.5</td>
<td>38,919</td>
<td>911</td>
<td>4.4</td>
<td>3238</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(a) ASTM E2126 (2010)  
(b) SEAOSC (1997)

Unlike the as-built diaphragms however, retrofitted diaphragms tested in either principal direction dissipated similar amounts of energy, despite orthogonal aspect ratios. It is believed that the early localised failure of the plywood panel boundary lines outside of the applied load locations in diaphragms 1b-PERP and 2b-PERP contributed to this observation.

#### 7.2.4.9 Side frame deformation

For the primary purpose of verifying fixed side conditions for diaphragm finite element modelling in Chapter 8, the steel side frames of diaphragms tested parallel-to-joists were...
instrumented with strain gauges to measure in-plane and out-of-plane displacements during diaphragm lateral loading. The recorded displacements from each side frame transducer are plotted against diaphragm load in Appendix C, from which the maximum absolute displacement values are listed in Table 7-3 to gauge overall side frame fixity. Maximum recorded in-plane displacements of less than 0.25 mm for as-built and retrofitted diaphragms demonstrates that movement of the side frames in this direction is negligible, and can be considered fixed. Out-of-plane displacements of the side frames were comparatively higher than in-plane displacements but still insignificant in the context of the overall diaphragm dimensions. This is evidenced by the corresponding effective rotations (θ₁ and θ₂) calculated by assuming rigid rotation between the two maximum out-of-plane displacement values at each end of the side frames. Effective rotations of less than 0.001 radians demonstrates that the side frames can also be considered fixed against out-of-plane displacements.

Movement of the URM side walls for diaphragms tested perpendicular-to-joists was not measured with digital transducers. The unreliability of the walls to remain rigid during testing meant that displacement measurements taken at discrete locations would not have been representative of the entire wall, and could pick up localised brick movements that would not be indicative. Obvious in-plane wall displacements however were monitored by
Table 7-3: Maximum side frame displacements

<table>
<thead>
<tr>
<th>Diaphragm</th>
<th>Maximum displacement (mm)</th>
<th>Effective rotation (rads)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S_{IP1}</td>
<td>S_{IP2}</td>
</tr>
<tr>
<td>1a-PARA</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>1b-PARA</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>2a-PARA</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>2b-PARA</td>
<td>0.21</td>
<td>0.13</td>
</tr>
</tbody>
</table>

drawing a reference line parallel to the ends of each wall and observing relative displacements during testing. No observable in-plane displacement of the URM side walls was confirmed during testing of all diaphragms. Given this observation, and the high level of post-tensioning force (approximately equivalent to a three-storey URM building above the diaphragm level), the URM side walls can be considered fixed against in-plane and out-of-plane displacements.

7.2.4.10 Deformation profiles

As detailed in Section 7.2.3, displacement gauges were provided at three diaphragm span locations to capture the deformation profile of each diaphragm at increasing displacement amplitudes. Deformation profiles provide additional information on diaphragm behaviour, specifically the shape and symmetry of diaphragm deformation from the given loading conditions. Figure 7-23 shows the deformation profiles of each diaphragm with respect to midspan displacement amplitudes of 10 mm, 25 mm, 50 mm, 75 mm, 100 mm, and 150 mm (or the closest possible displacements to these values) for the first loading cycle. From the results of the previous section, the displacements at each side of the diaphragms were considered to be zero. The locations of applied load are also shown in hatched lines for reference.

Figure 7-23 shows that for as-built diaphragms, the deformation profiles were generally symmetrical, and that greater relative displacement occurred between the two outside measurement locations when compared with retrofitted diaphragms. The considerably
higher stiffness of retrofitted diaphragms and comparatively low shear forces within the central diaphragm region is likely to have caused this observation. It is also apparent that localised failures have resulted in asymmetrical deformation of retrofitted diaphragms, particularly at higher displacement amplitudes.

Figure 7-23: Diaphragm deformation profiles
7.2.5 Discussion

Diaphragm behaviour mechanisms are discussed with a specific focus on performance degradation, retrofit performance, orthotropic behaviour, and the influence of diaphragm penetrations and joist connections.

7.2.5.1 Performance degradation

The degradation of diaphragm performance is difficult to characterise with a single parameter due to the complexity of hysteretic response. The highly nonlinear force-displacement responses presented in Figure 7-17 illustrate that diaphragm stiffness decreases with increasing displacement. Pinched hysteretic loops demonstrate degradation of diaphragm stiffness and energy dissipation capacity with continued cycling to the same displacement amplitude, particularly for the retrofitted diaphragms and at larger displacement amplitudes.

To establish trends in stiffness degradation with increasing displacement and continued cycling to equivalent displacement amplitudes, an effective stiffness may be calculated. Using a method originally developed for timber shear walls (Shenton III et al. 1998), effective stiffness was determined for each hysteretic loop using Equation 7-17:
\[
K_{ei} = \frac{F_{i}^{+} - F_{i}^{-}}{\Delta_{i}^{+} - \Delta_{i}^{-}}
\]  

(7-17)

Where \(F_{i}^{+}\) is the force corresponding to the maximum positive displacement \(\Delta_{i}^{+}\), and \(F_{i}^{-}\) is the force corresponding to the maximum negative displacement \(\Delta_{i}^{-}\), for loop \(i\). By using Equation 7-14 the effective stiffness values were converted to effective shear stiffnesses in order to remove the influence of diaphragm geometry and allow better comparison between diaphragm configurations. Normalised effective shear stiffness values, calculated by dividing the effective shear stiffness of the relevant loading cycle by the effective shear stiffness of the first cycle, plotted against load cycle number, and effective shear stiffness plotted against displacement amplitude for the three loading cycles separately, are provided in Appendix C. These figures show that diaphragm effective stiffness degrades considerably with increased displacement. The stiffness degradation between the first and last loading cycles of as-built diaphragms is generally shown to be less than 10%, while for retrofitted diaphragms the degradation is up to 40%.

The degradation of energy dissipation capacity can be quantified using a similar analysis to that used for stiffness degradation. Dissipated energy per cycle was normalised by dividing the energy dissipated in the relevant loading cycle by the energy dissipated in the first loading cycle and was plotted against cycle number (refer to Appendix C). Plots of dissipated energy versus displacement amplitude of the three separate loading cycles are also provided in Appendix C, from which two examples are shown in Figure 7-24. A large reduction in energy dissipation is evident between the first and second loading cycles, but is significantly less pronounced between the second and third cycles. The normalised data indicates that for as-built diaphragms, energy dissipation capacity reduces by up to 40% between the first and third loading cycles, whereas up to 60% reduction is evident for retrofitted diaphragms between the first and third loading cycles. For both as-built and retrofitted diaphragms, the magnitude of energy dissipation reduction increases with increasing displacement. These results confirm that hysteretic response is considerably pinched after the virgin loading cycle, but does not considerably worsen with continued cycling.
7.2.5.2 Retrofit performance

The plywood overlay and stapled sheet metal blocking retrofit was proven to significantly increase as-built diaphragm performance in both principal loading directions. Key performance parameters for diaphragm strength ($R_d$), shear stiffness ($G_d$) and ductility capacity ($\mu$) are relisted in Table 7-4 with a corresponding percentage increase between the as-built and retrofitted configurations. A dramatic improvement in all of these parameters is clearly evident.

<table>
<thead>
<tr>
<th>Diaphragm</th>
<th>$R_d$ kN/m (% increase)</th>
<th>$G_d$ kN/m (% increase)</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a-PARA</td>
<td>1.6 (894%)</td>
<td>198 (2152%)</td>
<td>7.2 (72%)</td>
</tr>
<tr>
<td>1b-PARA</td>
<td>15.9</td>
<td>4459</td>
<td>12.4</td>
</tr>
<tr>
<td>2a-PARA</td>
<td>1.6 (869%)</td>
<td>185 (2186%)</td>
<td>6.7 (52%)</td>
</tr>
<tr>
<td>2b-PARA</td>
<td>15.5</td>
<td>4229</td>
<td>10.2</td>
</tr>
<tr>
<td>1a-PERP</td>
<td>1.3 (654%)</td>
<td>134 (1291%)</td>
<td>8.8 (66%)</td>
</tr>
<tr>
<td>1b-PERP</td>
<td>9.8</td>
<td>1864</td>
<td>14.6</td>
</tr>
<tr>
<td>2a-PERP</td>
<td>1.5 (520%)</td>
<td>145 (1619%)</td>
<td>8.9 (131%)</td>
</tr>
<tr>
<td>2b-PERP</td>
<td>9.3</td>
<td>2493</td>
<td>20.6</td>
</tr>
</tbody>
</table>

Figure 7-24: Degradation of dissipated energy between loading cycles
The efficacy of a retrofit system is not only measured by improved stiffness and strength but also the serviceability of the diaphragm during and after earthquake loading. As described in Section 7.2.4.1, potential performance issues associated with the buckling and uplifting of sheet metal straps and plywood panels was observed during retrofitted diaphragm testing. To establish whether these serviceability issues would occur during a design earthquake, diaphragm displacement demand was determined for design elastic earthquake loading and compared against the observed midspan displacements that caused the sheet metal straps and plywood panels to begin buckling, labelled as serviceability limits $\Delta_{L1}$ and $\Delta_{L2}$, respectively. Diaphragm displacement demand was calculated using Equations 7-18 and 7-19:

$$\mu_{demand} = \frac{F_e}{F_y} \quad (7-18)$$

$$\Delta_{demand} = \mu_{demand} \times \Delta_y \quad (7-19)$$

Where $\mu_{demand}$ is ductility demand, $F_y$ is effective yield load defined by the idealised bilinear response curve outlined in Table 7-2, and $F_e$ is the design elastic earthquake load, determined from the provisions of NZS 1170.5 (2004) for a 1/500 year return period, assuming $\mu = 1.0$ and based on the same two-storey URM building that was used as the basis for retrofit design (see Appendix D). The calculated values are outlined in Table 7-5 and illustrated in Figure 7-25.

<table>
<thead>
<tr>
<th>Diaphragm</th>
<th>$F_e$</th>
<th>$F_y$</th>
<th>$\mu_{demand}$</th>
<th>$\Delta_y$</th>
<th>$\Delta_{demand}$</th>
<th>$\Delta_{L1}$</th>
<th>$\Delta_{L2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1b-PARA</td>
<td>524</td>
<td>175.8</td>
<td>2.98</td>
<td>12.1</td>
<td>36.1</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>2b-PARA</td>
<td>524</td>
<td>171.9</td>
<td>3.04</td>
<td>12.5</td>
<td>38.0</td>
<td>25</td>
<td>75</td>
</tr>
<tr>
<td>1b-PERP</td>
<td>289</td>
<td>204.7</td>
<td>1.41</td>
<td>9.1</td>
<td>12.8</td>
<td>15</td>
<td>75</td>
</tr>
<tr>
<td>2b-PERP</td>
<td>289</td>
<td>192.6</td>
<td>1.50</td>
<td>6.4</td>
<td>9.6</td>
<td>15</td>
<td>75</td>
</tr>
</tbody>
</table>
By comparing actual displacement demand with the observed serviceability limits, it can be seen that considerable sheet metal buckling and staple pullout would be expected for diaphragms loaded parallel-to-joists, with less damage expected for loading perpendicular-to-joists. The displacements necessary to cause plywood buckling and uplift are shown to be unlikely for both principal loading directions, as displacement demands are less than one third of the observed upper serviceability limits. It is acknowledged that the idealised bilinear curves slightly over predict response in the effective yield point region, meaning that displacement demands would exceed the values listed in Table 7-5. However, these increases would be small and it remains unlikely that plywood buckling would begin to occur.
Also shown in Figure 7-25 are the expected response curves generated from the retrofit design process, the details of which are provided in Appendix D. The design curves are shown to reasonably match experimental response for diaphragms 1b-PERP and 2b-PERP, but significantly under-predict stiffness and yield strength for diaphragms 1b-PARA and 2b-PARA. This discrepancy may be the result of extensive field nailing that was applied to the plywood panels to mitigate buckling but which was not explicitly accounted for during design.

While conservative design procedures are more desirable than the contrary, greater diaphragm stiffness and strength with corresponding lower ductility demand may generate unexpectedly higher URM wall accelerations (Simsir 2004) with potentially detrimental effects on overall building seismic performance. Further discussion on the influence of diaphragm stiffness on the performance-based retrofit of URM buildings is offered in Brignola (2009).

Overall the plywood overlay and SMBS is a unique retrofitting method that allows the preservation of existing diaphragm materials. It was shown to significantly increase diaphragm stiffness and strength in both principal loading directions but is likely to suffer serviceability issues associated with the buckling of sheet metal straps and pullout of stapling. Depending on the in-service use of the floor diaphragm, the reinstatement of the SMBS could be troublesome if internal partitions and heavy office furniture exist, and is therefore an important retrofit design consideration. Finally, it is recommended that the plywood panels be orientated in the direction orthogonal to that which was tested. Given the orientation of the existing floorboards beneath the overlay, the constructed overlays caused localised shear flow weaknesses that were particularly evident in diaphragms tested perpendicular-to-joists. The same retrofit with panels orientated in the orthogonal direction is likely to have further improved performance.

7.2.5.3 Orthotropic behaviour

The performance of timber floor diaphragms was shown to be distinctly different in the principal loading directions parallel-to-joists and perpendicular-to-joists. This orthotropic
behaviour was expected from the orthogonal arrangement of floorboards and joists in the as-built diaphragm configurations. Although shear strength was similar for as-built diaphragms in both loading directions, shear stiffness was shown to be up to 32% less for loading perpendicular-to-joists. This dissimilarity was further exaggerated for retrofitted diaphragms with a reduction in shear stiffness of up to 60% between the parallel- and perpendicular-to-joist loading directions. Shear strength was also reduced from approximately 16 kN/m in the direction parallel-to-joists, to approximately 9.5 kN/m in the direction perpendicular-to-joists.

The orthotropic behaviour of timber floor diaphragms was further evidenced by comparing moment-rotation response curves which include the influence of diaphragm geometry. To generate these plots, midspan moment per unit width was calculated from the given loading conditions and rotation was defined as the ratio of midspan displacement to half diaphragm span. Figure 7-26 shows the moment-rotation responses of as-built and retrofitted diaphragms separately. For both as-built and retrofitted diaphragms, testing parallel-to-joists demonstrated higher rotational stiffness and overall moment resistance than those tested perpendicular-to-joists. It is also clear that due to reduced diaphragm span, maximum rotation reached by diaphragms tested perpendicular-to-joists considerably exceeded the maximum rotation reached by diaphragms tested parallel-to-joists.

7.2.5.4 Penetrations

Comparing the force-displacement responses of diaphragms 1a-PARA and 2a-PARA in Figure 7-17, the presence of a corner penetration equal to approximately 6% of the floor area appeared to have little effect on diaphragm performance. The values listed in Table 7-2 show that shear strength is unchanged and that shear stiffness is marginally reduced from 198 kN/m to 185 kN/m. These results show that a typical single-case stairwell opening is not significantly detrimental to as-built diaphragm performance in the loading direction parallel-to-joists.

Retrofitted diaphragms 1b-PARA and 2b-PARA also responded similarly to lateral loading, as shown in Figure 7-17, although shear strength and shear stiffness were slightly
7.2 Pseudo-Static Testing

Figure 7-26: Diaphragm moment-rotation

reduced for 2b-PARA (see Table 7-2). Such reduced performance suggests that when a diaphragm is retrofitted, the potential for localised failures caused by increased stress concentrations around a penetration is higher, and highlights the importance of incorporating specific retrofitting details immediately adjacent to penetrations. Without the additional chord member, and increased stapling and nailing provided in the vicinity of the corner penetration, retrofitted diaphragm 2b-PARA may have performed more poorly than that which was tested.

The influence of penetration size and location in both principal loading directions is investigated in Chapter 9 using an FE model.

7.2.5.5 Joist connections

The lapped and bolted joist connections in diaphragms 2a-PERP and 2b-PERP were observed to suffer no damage during testing, even at maximum midspan displacements of ±150 mm. Surprisingly, using the adopted force-displacement characterisation methodologies, shear strength and shear stiffness were found to be higher for 2a-PERP than 1a-PERP. For retrofitted diaphragms shear strength slightly decreased but shear stiffness dramatically increased between 1b-PERP and 2b-PERP. These results were
unexpected as diaphragm response perpendicular-to-joists seemingly relies heavily on the out-of-plane flexural capacity of the joists, which would be expected to reduce for discontinuous joists with only a two-bolt lapped connection. However, it is possible that the diaphragm action of the floorboards combined with the two-bolt lapped joist connections was sufficient to resist the induced joist bending moments and not compromise diaphragm performance.

The above results suggest that the presence of discontinuous joists with reliable mechanical connections do not adversely affect diaphragm performance. The influence of discontinuous joists without a mechanical connection, which are also common in New Zealand timber floor diaphragms, is investigated in Chapter 9 using an FE model.

### 7.3 Free-Vibration Testing

Diaphragm fundamental horizontal period, \( T \), features extensively throughout the seismic assessment of URM buildings and specifically governs the determination of in-plane and out-of-plane wall loads that underpin any retrofit design. It is therefore an immensely influential assessment parameter that requires accurate prediction to avoid the under- or over-estimation of structural design actions.

For the primary purpose of critiquing the diaphragm period equation offered by ASCE 41-06 (2007) and NZSEE (2006), and the updated diaphragm period equation presented in Chapter 4, horizontal free-vibration tests were performed before and after pseudo-static testing to experimentally determine the fundamental period and associated mode shape of each constructed diaphragm, and to determine the sensitivity of these properties to induced damage. Results were also used to estimate the diaphragm inherent damping ratio.

Experimental modal analysis is a proven technique for obtaining natural vibration properties and has been performed extensively on civil structures (for example Brownjohn et al. 2003; Ewins 2000; Pavic et al. 2007; Ramos 2007; Soltis et al. 2002). A concise
7.3 Free-Vibration Testing

summary of the adopted testing procedure and modal analysis using a graphical interface toolbox for system identification developed in MATLAB (Beskhyroun 2011; The Mathworks 2011) is presented. A comprehensive discussion of modal testing practices, signal processing and system identification techniques is beyond the scope of this study, and the references provided throughout this section are recommended for detailed reading.

7.3.1 Testing details

Modal tests were performed by providing impact excitation with a Dytran Model 5803A instrumented hammer with a medium softness head attachment. Input force was measured using the hammer’s inbuilt load cell. Structural response was measured using Honeywell QA750 accelerometers mounted to base plates that could be levelled to ensure correct vertical alignment. All measurements were recorded at 2000 Hz with a 16-bit National Instruments DAQ and purpose-built multi-channel signal conditioner.

Figures 7-27 and 7-28 show the impact excitation and accelerometer locations adopted for modal testing in each principal direction. Generally, a single row of eleven accelerometers were provided for diaphragms tested parallel-to-joists, and two rows of five accelerometers were provided for diaphragms tested perpendicular-to-joists, although these numbers were dependant on equipment availability at the time of testing. The number of accelerometers used for diaphragms tested parallel-to-joists varied between five, seven, nine and eleven, while ten accelerometers were available for all diaphragms tested perpendicular-to-joists except for 1a-PERP after cyclic testing, where no accelerometers were available and modal testing was not performed.

As previously outlined, free-vibration tests were performed on each diaphragm configuration (see Table 7-1) before and after pseudo-static testing, meaning that a total of sixteen modal tests were performed. Each test was performed by intermittently striking the diaphragm with the hammer at the locations shown in Figure 7-28 until six consistent impacts were recorded, at which point testing was concluded. For –PARA diaphragms, excitation was provided at the centre location only, whereas for –PERP diaphragms, excitation was provided at the two full-depth blocking locations, separately, to guarantee a
reliable distribution of the impulse into the diaphragm. The selected impact locations were also appropriate for exciting the desired first mode of vibration.

### 7.3.2 Data analysis

All modal analysis was performed using a graphical interface ‘toolbox’ that was developed in MATLAB (The MathWorks 2011) for signal processing and system identification (Beskhyroun 2011). Each data set obtained from the sixteen modal tests was analysed separately using the same methodology and same analysis parameters to ensure consistency.

The data from individual excitations was firstly isolated by taking a two-second sample of the input and output signals immediately before the hammer impact was made. This data was then filtered using a low-pass Butterworth filter with a system order of 9 and cut-off frequency of 70 Hz, which appropriately removed spurious spikes and ambient noise that were not associated with the structural response of the diaphragm. Figure 7-29 provides an example of the filtered free-vibration response captured by an accelerometer. Having appropriately processed the response signals, multiple system identification techniques programmed into the toolbox were used to determine the modal parameters of the diaphragms. The following system identification techniques were employed:
7.3 Free-Vibration Testing

(a) Diaphragms tested parallel-to-joists

(b) Diaphragms tested perpendicular-to-joists

Figure 7-28: Impact excitation and accelerometer locations
• Peak picking (PP) (Bendat and Piersol 1993)
• Frequency domain decomposition (FDD) (Brincker et al. 2000)
• Enhanced frequency domain decomposition (EFDD) (Brincker et al. 2000; Jacobsen et al. 2007)
• Eigen realisation algorithm (ERA) (Juang and Pappa 1985) combined with the natural excitation technique (NeXT) (James et al. 1993)
• Stochastic subspace identification (SSI) (Overschee and Moor 1996). Two variations of the SSI algorithm, labelled SSI1 and SSI2, were used, that differ only in the way that stable poles are identified.

The peak picking, frequency domain decomposition and enhanced frequency domain decomposition techniques are frequency-domain based, while the Eigen realisation algorithm and stochastic subspace identification techniques are time-domain based. The inclusion of both frequency-based and time-based system identification techniques, which are based on completely different mathematical algorithms, is an important feature of the toolbox. Experimental modal analysis is notoriously variable as a result of random noise, testing inconsistencies and often complex structural boundary conditions, so agreement between fundamentally different identification techniques provides confidence that the identified modal properties are truly associated with the structure.

Figure 7-29: Example of free-vibration response signal
The following parameters were kept constant for the relevant identification techniques:

- PP, FDD, and EFDD: Hamming window size of 1024 (windowing not required for ERA or SSI).
- ERA: Hankel matrix size of 100.
- SSI:
  - SSI1 frequency margin: 1 Hz
  - SSI2 frequency interval: 1 Hz
  - System order: 100
  - Hankel matrix size: 100
  - Limits for stable pole selection: <1% frequency change, <100% damping change, >0.90 modal assurance criteria.

### 7.3.3 Test results

A comprehensive summary of the modal analyses are provided in Appendix C. For each impact excitation, the fundamental frequencies and respective mode shapes identified by the different identification techniques are provided, as well as a plot of modal assurance criteria (MAC) values. MAC is a correlation coefficient between the modal ordinates predicted by identification techniques, having a scalar value between 0 and 1. Well correlated modes generally have MAC > 0.9, while poorly correlated modes have MAC < 0.5 (Brownjohn et al. 2003). An example of the modal analysis output is provided in Table 7-6.

It was found that for diaphragms tested perpendicular-to-joists, almost identical results were obtained from gridline A and gridline B accelerometers (see Figure 7-28), and as such, only the results from gridline A have been reported. As previously stated, accelerometers were not available for diaphragm 1a-PERP after cyclic testing and therefore no results are presented for this configuration.

Inspection of the identified mode shapes presented in Appendix C confirms that the first horizontal mode of vibration was well excited in the constructed diaphragms, with the
exception of 2a-PERP and 2b-PERP for which mixed mode shapes were identified. Given that the central support beam provided for these configurations did not include Teflon pads, considerable midspan friction would have affected the ability of diaphragms 2a-PERP and 2b-PERP to vibrate freely in the first mode. Overall the modal analysis results demonstrate excellent consistency between identification methods and between different impact excitations with less than 5% variation in fundamental frequency and generally agreeable mode shapes. MAC values were generally above 0.9, confirming excellent correlation between identification techniques and providing evidence that the identified fundamental frequencies are true dynamic properties of the diaphragms. As explained previously, a high level of confidence can be drawn from these modal analysis results due to the excellent correlation between frequency-domain based and time-domain based system identification techniques.

Table 7-7 presents the average fundamental frequency values identified by the analysis techniques for each diaphragm configuration tested, as well the average diaphragm damping ratios estimated by the ERA and SSI techniques. Presented also is the mean absolute deviation (MAD) of these values to indicate the variation of results.
### Table 7-7: Modal analysis summary

<table>
<thead>
<tr>
<th>Test</th>
<th>Before cyclic testing</th>
<th>After cyclic testing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$</td>
<td>$MAD_{f_1}$</td>
</tr>
<tr>
<td></td>
<td>Hz</td>
<td>Hz</td>
</tr>
<tr>
<td>1a-PARA</td>
<td>23.45</td>
<td>0.05</td>
</tr>
<tr>
<td>1b-PARA</td>
<td>27.17</td>
<td>0.22</td>
</tr>
<tr>
<td>2a-PARA</td>
<td>19.29</td>
<td>0.31</td>
</tr>
<tr>
<td>2b-PARA</td>
<td>25.75</td>
<td>0.36</td>
</tr>
<tr>
<td>1a-PERP</td>
<td>11.45</td>
<td>0.27</td>
</tr>
<tr>
<td>1b-PERP</td>
<td>21.61</td>
<td>0.23</td>
</tr>
<tr>
<td>2a-PERP</td>
<td>19.61</td>
<td>0.10</td>
</tr>
<tr>
<td>2b-PERP</td>
<td>29.01</td>
<td>0.29</td>
</tr>
</tbody>
</table>

1. $MAD = \text{mean absolute deviation}$

Overall, the fundamental frequency values listed in Table 7-7 provide further evidence of the excellent correlation between individual impact excitations and different identification techniques with less than 0.4 Hz MAD for all tested diaphragms. The results show that fundamental frequency increased between all unretrofitted and retrofitted configurations, which was expected due to the considerably increased diaphragm stiffness. Except for diaphragm 2a-PARA, the opposite was true between undamaged and damaged diaphragms as a result of reduced stiffness between these condition states. It is unknown why a slight increase in average fundamental frequency occurred between the undamaged and damaged conditions of diaphragm 2a-PARA. Such comparison could not be made for diaphragm 1a-PERP due to equipment unavailability. Considerably higher fundamental frequencies were identified for 2a-PERP and 2b-PERP when compared with 1a-PERP and 2b-PERP, confirming the frictional influence of the centre support beam on diaphragm vibration behaviour. The specific influence of the joist connections in 2-PERP configurations was therefore unable to be determined. A comparison of 1-PARA and 1-PERP configurations shows that both unretrofitted and retrofitted diaphragms tested parallel-to-joists have a higher fundamental frequency than corresponding orthogonal configurations tested perpendicular-to-joists. This difference in fundamental frequency can be explained by the floorboards bending about their strong axis when the diaphragm is vibrating parallel-to-
joists, whereas when the diaphragm is vibrating perpendicular-to-joists, the joists are bending about their weak axis.

Average damping ratios estimated by the ERA and SSI identification techniques were varied and no conclusive trends could be drawn between diaphragm configurations and principal loading directions. In addition, MAD values of up to 62% of the respective damping ratio indicate that these estimation techniques have only moderate consistency. However despite this, the results provide no evidence against the 5% inherent damping that is typically assumed for dynamically responding structures, and this value is therefore recommended for timber floor diaphragms.

### 7.3.4 Discussion

#### 7.3.4.1 Comparison with fundamental period prediction

Seismic assessment codes ASCE 41-06 (2007) and NZSEE (2006) recommend that for URM buildings with single span flexible diaphragms, fundamental period be calculated using Equation 7-20:

\[
T = \sqrt{\frac{3.07 \Delta_d}{g}}
\]  

(7-20)

where \( \Delta_d \) is the maximum in-plane diaphragm displacement in metres due to a lateral load in the direction under consideration, equal to the weight tributary to the diaphragm \( F_u \). \( \Delta_d \) is generally calculated using Equation 7-21:

\[
\Delta_d = \frac{F_u}{K_d}
\]  

(7-21)

Based on the analysis presented in Chapter 4, it has been recommended that Equation 7-20 be update with the following expression,

\[
T = \sqrt{\frac{2.61 \Delta_d}{g}}
\]  

(7-22)
which appropriately captures diaphragm shear behaviour with realistic earthquake loading conditions (see Chapter 4). Both assessment equations are compared against experimental results hereafter.

Using Equation 7-21, $\Delta_d$ was calculated for each unretrofitted diaphragm configuration by taking the relevant $K_{dl}$ value from Table 7-2, and in the absence of out-of-plane URM walls, taking $F_u$ as the self-weight of the diaphragm, equal to approximately 15.4 kN. The resulting diaphragm fundamental period estimations using Equations 7-20 and 7-22 are compared against values identified from experimental modal analysis in Table 7-8. Comparison has been limited to unretrofitted configurations because Equations 7-20 and 7-22 do not apply to stiff or rigid diaphragms and the prediction of fundamental period for such diaphragms is outside the scope of this study. Period values from testing were determined by simply inverting the frequency values outlined in Table 7-7.

<table>
<thead>
<tr>
<th>Test</th>
<th>Fundamental period, $T$ (seconds)</th>
<th>Predicted</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Eq. 7-20</td>
<td>Eq. 7-22</td>
</tr>
<tr>
<td>1a-PARA</td>
<td>0.27</td>
<td>0.23</td>
<td>0.04</td>
</tr>
<tr>
<td>2a-PARA</td>
<td>0.28</td>
<td>0.24</td>
<td>0.05</td>
</tr>
<tr>
<td>1a-PERP</td>
<td>0.17</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>2a-PERP</td>
<td>0.16</td>
<td>0.13</td>
<td>0.05</td>
</tr>
</tbody>
</table>

(I) Before cyclic testing
(II) After cyclic testing

In Table 7-8 severe discrepancy is clearly evident between the predicted and experimentally determined fundamental period values. This discrepancy could be explained by the small displacement nature of modal testing using impact excitation. At very small displacements, the stiffness of straight-sheathed timber diaphragms could be significantly higher than the bilinear stiffness values ($K_{dl}$) used for the prediction of fundamental period in Equations 7-20 and 7-22. However without large-displacement free-vibration test data, this argument cannot be substantiated.
Chapter 7: Full-Scale Diaphragm Testing

The results indicate that modal properties identified from small-displacement free-vibration testing using hammer impact excitation are not indicative of the dynamic response of timber floor diaphragms under large-displacement seismic loading. Modal analysis results generated from impact testing therefore cannot be used to establish realistic diaphragm fundamental period values, and large-displacement snap-back tests are required to properly critique the current prediction equation offered in ASCE 41-06 and NZSEE, and the updated prediction equation recommended in Chapter 4.

7.3.4.2 Damping

Although small displacement free-vibration testing was potentially not suitable for fundamental period determination, the estimated inherent damping ratios are still representative, as they are a function of material properties and diaphragm configuration. The results suggest that a 5% inherent damping ratio is appropriate for timber floor diaphragms in URM buildings.

7.4 Conclusions

Pseudo-static testing of full-scale timber floor diaphragms in both principal loading directions is, to the best of the author’s knowledge, the first of its kind. The results of this chapter are used in Chapter 8 to calibrate FE models.

As-built configurations demonstrated highly nonlinear hysteretic response with no clearly defined yield point, and were extremely flexible, exhibiting no strength degradation up to drift ratios of 3.8% and 5.4% in the parallel-to-joist and perpendicular-to-joist loading directions, respectively. No structural failures were observed and all as-built diaphragms remained fully serviceable at the conclusion of testing.

The plywood overlay and SMBS retrofit dramatically improved as-built diaphragm strength and stiffness. The displacement demand for a typical 1/500 year return period earthquake demonstrated that serviceability issues associated with the buckling of sheet
metal blocking would likely occur but that the displacement levels required to cause plywood panel uplift would not be reached. If failure of the SMBS is considered to be acceptable performance, the results indicate that plywood panel overlay with SMBS is an effective retrofitting technique that can be implemented into current URM building stock whilst preserving heritage diaphragm construction. To ensure the best possible performance possible, it is recommended that the plywood overlay panels be orientated parallel-to-joists to avoid localised shear flow weaknesses.

Retrofit design procedures currently employed by New Zealand engineering practitioners were shown to under-predict retrofit performance, particularly in the direction parallel-to-joists. Simsir (2004) reports that unexpectedly higher diaphragm stiffness can have detrimental effects on the URM building performance, so further research is recommended to properly validate relevant design procedures.

The load carrying capacity of as-built diaphragms did not significantly degrade between loading cycles to the same displacement amplitude, however due to hysteretic pinching, energy dissipation decreased by up to 40% between cycles. Strength and energy dissipation degradation was more pronounced for retrofitted diaphragms.

Testing in both principal loading directions confirmed the orthotropic nature of timber diaphragms. While shear strength remained consistent for as-built diaphragms, shear stiffness in the direction perpendicular-to-joists was up to 32% less than the corresponding value in the orthogonal configuration. For retrofitted diaphragms, the difference in shear stiffness increased to 60%, and the shear strength in the direction perpendicular-to-joists was almost 50% of the shear strength parallel-to-joists.

It was concluded that a typical stairwell penetration insignificantly influences as-built diaphragm performance, having almost no effect on shear strength, and only reducing shear stiffness from 198 kN/m to 185 kN/m. The almost identical response of diaphragms 1b-PARA and 2b-PARA demonstrated that additional retrofitted details are necessary adjacent to corner penetrations to maintain desired performance.
Discontinuous joists with a midspan two-bolt lapped connection were shown to have no detrimental impact on diaphragm performance. It was concluded from these results that discontinuous joists with a reliable mechanical connection do not adversely affect diaphragm performance.

Fundamental period values determined from experimental modal analysis were found to be significantly different to predicted values using the provisions of ASCE 41-06 (2007) and NZSEE (2006), and to predicted values using the recommended period equation presented in Chapter 4. Such significant discrepancy suggests that small-displacement free-vibration testing using impact excitation is not representative of the dynamic response of timber floor diaphragms subjected to earthquake loading. The determined fundamental period values can therefore not be used to update current predictive equations. Consequently, large-displacement snap-back tests are recommended to properly critique the current period assessment provisions in ASCE 41-06 (2007) and NZSEE (2006), and the recommended period equation reported in Chapter 4.

An inherent damping ratio of 5% is recommended for as-built timber floor diaphragms in URM buildings.
7.5 Photos of Testing

(a) Reaction frame  
(b) Loading frame  
(c) Hinge connection  
(d) Joist loader with hinge connection  
(e) Post-tensioned rods between loaders  
(e) Joist loader at opposite end

Figure 7-30: Loading system
Chapter 7: Full-Scale Diaphragm Testing

(a) Inverted T-beams with portal gauge  
(b) Joist support with Teflon pad

(c) Out-of-plane joist restraints  
(d) End of joist details

(e) Side frame displacement measurement  
(f) String potentiometer measuring diaphragm displacement

Figure 7-31: Test set-up for loading parallel-to-joists
(a) Loading frame
(b) Joist loader
(c) Wall construction
(d) Post-tensioning rod
(e) Rod’s being post-tensioned
(e) Central joist support for diaphragms 2a-PARA and 2b-PARA

Figure 7-32: Test set-up for loading perpendicular-to-joists
Figure 7-33: Diaphragm testing
The analytical model developed in Chapter 3 capably captures nonlinear diaphragm behaviour but is restricted to: (1) symmetrical and rectangular diaphragm configurations with applied uniformly distributed lateral loads, (2) continuously spanning floorboards, and (3) a theoretical load-slip characteristic (Foschi 1974) governing nail connection nonlinear behaviour. To enable greater geometrical versatility and improved parameter control, a method for diaphragm finite element (FE) modelling using the structural analysis software SAP2000 (CSI 2004) was developed for use in the parametric analysis presented in Chapter 9, and for potential integration into future URM building models suitable for use to undertake nonlinear time-history analyses in subsequent research programs. SAP2000 was selected because of its comprehensive nonlinear analysis capabilities, prevalence throughout the structural engineering industry, and comparatively simple user-interface.

The objective was to formulate an FE model based on true geometrical configuration, realistic deformation mechanics, and physically representative input parameters, which can suitably capture diaphragm nonlinear behaviour without requiring optimisation to match predicted response to experimental data. A model of this form can be confidently extrapolated for parametric analysis in Chapter 9, and confidently reproduced by
researchers and engineering practitioners for independent analyses because it will not require specific calibration to confirm its accuracy.

This chapter describes the developed method for modelling heritage timber floor diaphragms in SAP2000. The method is subsequently validated through application to the analytical model developed in Chapter 3, and to the experimental test results reported in Chapters 6 and 7. Reasons for any notable discrepancies between predicted performance and experimental performance are reviewed and discussed.

8.1 Modelling of Timber Floor Diaphragms in SAP2000

Diaphragm FE models may be formulated based upon the assumptions that: (1) diaphragm nonlinearity is consigned completely to the primary nail connections, (2) all timber framing members (floorboards, joists, and cross-bracing) remain linear-elastic, and (3) no mechanical interaction occurs between the timber framing members. Based on the experimental results presented in Chapters 6 and 7, which showed that friction between diaphragm floorboards is negligible and that full-scale diaphragms remain serviceable after undergoing large lateral deformations, these assumptions are considered to be valid for FE modelling of realistic diaphragm configurations.

The proposed modelling technique is summarily illustrated in Figure 8-1. The geometrical configuration of the diaphragm is replicated by modelling the floorboards, joists, and cross-bracing at their true centreline locations using elastic frame elements. Each floorboard-to-joist nail connection is modelled using nonlinear link elements, which are connected to the joists and coupled to the floorboards using rigid frame elements at the appropriate nail couple spacing. To ease model construction, only one nail connection couple needs to be modelled at the locations of abutting discontinuous floorboards. The described modelling components are subsequently assigned representative properties to suitably characterise the diaphragm for performance analysis.
Conceptually the proposed modelling method has previously been successfully employed for timber shear wall analysis in SAP2000 (Loo 2010). Loo modelled timber framing as elastic frame elements and plywood sheathing as elastic area sections, while all shear wall nonlinearity was consigned to nonlinear link elements representing the plywood-to-framing nail connections. Through a comparative analysis Loo demonstrated excellent correlation between predicted response and experimental shear wall data. Peralta (2003) also used a similar modelling approach for evaluating heritage timber diaphragm performance using the structural analysis software ABAQUS (2003), although as outlined in Section 2.3.2, was unable to accurately capture diaphragm behaviour in both principal loading directions (parallel-to-joists and perpendicular-to-joists).
Comprehensive details for modelling the diaphragm components as well as realistic boundary conditions and other relevant configuration features are presented in the following sections, which are titled accordingly. It should be noted that the proposed methodology considers only the timber diaphragm itself – guidance on modelling other structural members such as intermediate steel cross-beams are outside the scope of this research.

### 8.1.1 Modelling of joists

Joists can be modelled as elastic frame elements with representative section and material properties. An isotropic material with appropriate weight per unit volume \( \gamma \) and modulus of elasticity \( E \) is considered adequate for diaphragm performance analysis, although additional material complexity may be considered if deemed necessary. Using the assigned joist properties, elastic frame elements can be drawn at the centreline locations of the diaphragm joists. It is recommended that joist elements be constructed with continuous moment releases between nodes spaced at a minimum distance equal to the joist spacing, \( \ell \), in order to sufficiently mesh the frame elements for deformation analysis (only nodes can be assigned analysis data in SAP2000).

As outlined in Chapter 1, when URM perimeter walls are close enough (less than approximately 6 m) joists often span continuously between these elements. For larger spans, joists are typically lapped or butted, either with or without some kind of mechanical connection, over intermediate steel or timber cross-beams supported on columns. For FE modelling purposes, continuous joists can simply be represented as single frame elements spanning the full dimension of the diaphragm, while the four discontinuous joist connections can be modelled as follows:

1. **Butted without mechanical connection** – The discontinuous joists in this arrangement are decoupled, except from transferring compressive axial loads. Consequently at each joist discontinuity, one of the joist frame elements should be released from all shear, torsion, and moment transfer using the frame releases/partial fixity control.
8.1 Modelling of Timber Floor Diaphragms in SAP2000

(2) Butted with steel strap or timber connection – From inspection of existing diaphragms and discussion with engineering practitioners, these connections are expected to provide little moment transfer but be capable of transferring some shear. Consequently at each joist discontinuity, one of the joist frame elements should be released from torsion and moment transfer using the frame releases/partial fixity control.

(3) Lapped without mechanical connection – The frame elements representing the discontinuous joists are simply lapped at their geometrical location without any connection between them. The modelled joists are therefore assumed to respond without any mechanical interaction occurring between them.

(4) Lapped with mechanical connection – As outlined in Chapter 1, this type of joist connection typically comprises two bolts suitably spaced along the lapped length of the joists, which fasten the joists together at mid-height location. To model the mechanical fasteners, representative material and frame section properties should be defined using best available knowledge. Using the adopted section, frame elements may be assigned between the lapped joists at the relevant centreline locations of fasteners. Although each bolt will invariably generate some moment fixity between the joists, it is considered appropriately conservative to model the mechanical fasteners with pin-end conditions. Consequently one end of the frame elements representing the mechanical fasteners should be released from all moment transfer using the frame releases/partial fixity control. By way of example, modelling of the two-bolt lapped joist connection incorporated into tested diaphragm configuration 2a-PERP (see Chapter 7) is illustrated in Figure 8-2.

8.1.2 Modelling of floorboards

Analogous to the joist members, floorboards can be modelled as elastic frame elements with representative section and material properties. Although T&G floorboard interfaces may be modelled if desired, an appropriate rectangular section is considered adequate for both straight-edge and T&G type floorboards. Using the defined floorboard property, elastic frame elements can be assigned at the centreline locations of the diaphragm
Figure 8-2: FE modelling of the lapped two-bolt joist connection that was used in tested diaphragm 2a-PERP

floorboards. It is recommended that the floorboards be assigned with *continuous moment releases* between every joist location, to sufficiently mesh the *frame* elements for deformation analysis (only nodes can be assigned analysis data in SAP2000), and to provide nodal locations for assigning moment releases to discontinuous floorboards, which almost always occur at joist locations. Subsequently, wherever discontinuous floorboards are abutted over joists, one of the floorboard *frame* elements should be released of all moment and torsion transfer using the *frame releases/partial fixity* control. Because only one nail connection couple is modelled at each discontinuous floorboard abutment, preservation of moment and torsion fixity in the other floorboard *frame* element is required in order to engage this nail couple. Shear transfer between the discontinuous floorboard *frame* elements is preserved to account for the shear flow that would otherwise travel between the discontinuous floorboards through the nail connections, which are fastened to a mutual joist.

### 8.1.3 Modelling of cross-bracing

As outlined in Chapter 1, timber cross-bracing was typically fitted between the joists of URM building diaphragms to mitigate out-of-plane joist buckling (see Figure 1-3). Although the cross-braces are unlikely to contribute significantly to diaphragm performance in the parallel-to-joist direction, they may provide some degree of load distribution in the perpendicular-to-joist direction, and therefore require modelling. However because the joists are modelled in a one-dimensional plane, the cross-bracing timber members cannot be explicitly modelled and thus some effective section must be considered. Given the potential unreliability of the cross-braces, a *frame section* equal to
one of the individual cross-bracing timber members is considered an appropriate representation. Suitable timber material properties should be concurrently assigned. Using the defined cross-bracing section, elastic frame elements with pinned moment releases can be assigned between the joists at the appropriate centreline locations for the cross-bracing. Little or no moment fixity is likely to be developed between the cross-bracing and the joists, so the pinned moment releases ensure that the ends are appropriately treated as pinned conditions.

### 8.1.4 Modelling of couplers

Couplers refers to the frame elements coupling the nonlinear link elements (representing the nail connections, see Section 8.1.5) to the modelled timber floorboards, as shown in Figure 8-1. Because the floorboards are modelled as frame elements with only centreline geometry, the additional couplers are required to represent the width of the timber floorboards between the nail connections.

Given that timber shear deformation between the nail connections is negligible relative to nail connection slip, the coupler frame elements can be considered comparatively rigid. Accordingly, an isotropic material property with a modulus of elasticity equal to one thousand times that of the floorboard timber should be defined for the couplers. With such a high material stiffness, the specific section property defined for the couplers will not affect the deformation analysis, but for ease of configuration, it is recommended that an identical section to the timber floorboards be assigned. Weight per unit volume should be set to zero as the mass of the floorboard timber is already accounted for in the floorboard frame elements. Using the defined coupler property, elastic frame elements with continuous moment releases (fully fixed end conditions) should be connected to the floorboard frame elements at the joist intersections, and assigned in both perpendicular directions to a distance of \( s/2 \); resulting in a total coupler length equal to the nail couple spacing, \( s \), as illustrated in Figure 8-3. As previously outlined in Section 8.1, only one nail connection couple needs to be modelled at the locations of abutting discontinuous floorboards, to ease model construction.
8.1.5 Modelling of nail connections

Diaphragm floorboard-to-joist nail connections can be modelled in SAP2000 using nonlinear two-joint link elements assigned vertically between the joists and the rigid couplers, as illustrated in Figure 8-1. Link elements have six controllable degrees-of-freedom, comprised of three translational springs ($U_1$, $U_2$, and $U_3$) and three rotational springs ($R_1$, $R_2$, and $R_3$), as illustrated in Figure 8-4, which can be assigned force-displacement data (for translational springs) and moment-rotation data (for rotational springs) to achieve the desired nonlinear behaviour. Based on the SAP2000 local axis coordinate convention shown in Figure 8-4, $U_1$ is an axial spring, $U_2$ and $U_3$ are shear springs, $R_1$ is a torsional spring, and $R_2$ and $R_3$ are pure bending springs. Of the available link element degrees-of-freedom, the lateral load-slip characteristics of nail connections are

![Figure 8-4: Schematic of SAP2000 Link element showing the six controllable degrees-of-freedom [taken from Loo (2010)]
most appropriately attributed to the mutually perpendicular shear springs $U_2$ and $U_3$. Details of assigning monotonic (backbone) and hysteretic (cyclic) nail connection data to the shear springs is presented in the subsequent sections, while the treatment of the remaining four degrees-of-freedom is discussed hereafter.

Axial spring $U_1$ should be fixed to prevent any compression or elongation of the link elements from occurring during analysis. Fixing $U_1$ is considered physically representative because compression of the nail connection timber is negligible while any nail connection withdrawal effects are already included in the hysteretic behaviour attributed to the shear springs $U_2$ and $U_3$. Pure bending springs $R_2$ and $R_3$ should also be fixed, to prevent any relative rotation of the floorboard and joist frame members from occurring during analysis. Rotation of the nail connections, and consequently the floorboard and joist members, is assumed to be negligible when timber floor diaphragms deform under lateral loading, and thus the treatment of $R_2$ and $R_3$ is considered appropriate. Torsional spring $R_1$ should be assigned a stiffness of zero to allow free torsional rotation of the link elements. $R_1$ represents the physical spinning of the nail within the timber members. Although some frictional resistance would be generated between the timber and embedded nail, this friction would be difficult to model accurately and therefore it is suitably conservative to allow the link elements to remain torsionally unrestrained during deformation analysis.

### 8.1.5.1 Monotonic response

There are several types of link properties available in SAP2000 for the characterisation of nonlinear behaviour. Of the available options, the multi-linear Takeda plasticity property is most suitable for nail connection representation as it allows the definition of a force-displacement data set, which controls nonlinear monotonic response. Using experimental backbone curve data or a discretised theoretical load-slip characteristic, representative nail connection monotonic behaviour can be assigned to the $U_2$ and $U_3$ shear springs. This assignment can be readily achieved by simply copying and pasting a tab-delimited force-displacement data set from any suitable text or spreadsheet computer software. All remaining link element parameters, including total mass and weight, factors for line, area
Chapter 8: Finite Element Modelling

and solid springs, and p-delta parameters should remain as default values, as they do not affect the deformation analysis.

Dolan and Madsen (1992) found that timber grain orientation had a minimal effect on nail connection performance (apart from post-yield stiffness), so it is recommended that link element shear springs $U_2$ and $U_3$ be assigned identical multi-linear backbone curve data. The response of the link elements in the $U_2$ and $U_3$ degrees-of-freedom are assumed to be independent and should therefore be completely uncoupled.

Definition of the link element backbone curve is completely dependent on the timber structure being analysed. If nail connection test data is available, then this data should be directly assigned to the link elements for the most accurate representation of nail connection behaviour. If test data is not available, then a suitable theoretical load-slip characteristic (Dolan and Madsen 1992; Foschi 1974; McLain 1975) can be calibrated to exhibit the desired behaviour, and subsequently discretised for definition of link element monotonic response. To ensure sufficient analysis resolution, it is recommended that discretisation follow the same set of displacement amplitudes as used for the nail connection backbone curves presented in Tables B-1 to B-4 (see Appendix B). Failing the above options, any other suitable multi-linear force-displacement response for nail connections may be assigned to the link elements if desired, using relevant design codes (such as NZS 3603:1993) or similar.

8.1.5.2 Cyclic response

The multi-linear plastic link element in SAP2000 has three idealised hysteretic models to choose from: (1) kinematic, (2) Takeda, and (3) pivot. Based on the nail connection test results presented in Chapter 5, the pivot model is recommended as the most suitable representation of nail connection hysteretic response because it enables the greatest control over the unloading gradient and hysteretic pinching. The generalised pivot model was originally developed by Dowell et al. (1998) to capture the observed strength and stiffness degradation of reinforced concrete members, but has demonstrated good compatibility with dowel-fastened timber structures (Loo 2010). Figure 8-5 illustrates that hysteretic
behaviour is characterised by directing the unloading and reloading displacement cycles towards predefined pivot points, which are defined by the effective yield forces, $F_{y1}$ and $F_{y2}$, and the empirical parameters $\alpha_1$, $\alpha_2$, $\beta_1$, and $\beta_2$. Using the experimental test results reported in Chapter 5, the appropriate definition of the effective yield points and the empirical parameters for floorboard-to-joist nail connections is presented hereafter. Comprehensive details of the pivot model development and the rules governing hysteretic response can be found in Dowell et al. (1998).

The pivot model in SAP2000 assumes that the defined monotonic force-displacement response includes an initial linear-elastic region with corresponding yield points $[F_{y1}, \delta_{y1}]$ and $[F_{y2}, \delta_{y2}]$, as depicted in Figure 8-5. These yield points are defined automatically in SAP2000 as the first positive and negative force-displacement data points of the assigned backbone curve data set. For highly nonlinear nail connections, which do not exhibit an initial linear-elastic region, the inability to explicitly define effective yield points independent of the assigned monotonic force-displacement response imposes a trade-off between initial performance accuracy and hysteretic pinching characterisation. To accurately capture initial nail connection performance, a highly refined force-displacement data set is required (displacement intervals $< \sim 0.05$ mm), meaning that the automatically
selected positive and negative yield points will have extremely small force magnitude. An issue arises with the pivot model if the yield forces are too small because the pivot points $PP_1$ and $PP_2$ (see Figure 8-5) are functions of parameters $\beta_1$ and $\beta_2$, respectively, which can only be assigned values of between zero and one, meaning that the reloading force-displacement lines will pass through the zero-displacement axis at forces much smaller than what is representative for true nail connection performance. Consequently hysteretic pinching will be grossly exaggerated, as illustrated in the example cyclic analysis output provided in Figure 8-6 for a link element assigned with the complete backbone curve recorded from nail connection test New-NZ 3 (see Chapter 5), and with $\beta_1 = \beta_2 = 1$. The implication of this pivot model limitation in SAP2000 is that initial backbone curve data points must be removed in order to establish effective positive and negative yield forces which are sufficiently large to enable the representative characterisation of hysteretic pinching. Figure 8-7 illustrates the data removal process for an arbitrary force-displacement backbone curve in the positive direction. To retain as much force-displacement resolution as possible, the effective yield forces must remain as small as possible, which is achieved by setting $\beta_1$ and $\beta_2$ to the maximum value of one. By assigning $\beta_1 = \beta_2 = 1$, the effective yield forces $F_{y1}$ and $F_{y2}$ are approximately equal to the forces at which the reloading lines intercept the zero-displacement axis in the positive and

![Figure 8-6: Example of FE model link element hysteretic response exhibiting excessive hysteretic pinching](image-url)
negative directions, respectively. Therefore, by establishing a correlation between the location at which the reloading lines intercept the zero-displacement axis and some known force-displacement value, appropriate effective yield forces may be determined. Using the nail connection experimental data presented in Appendix B, the reloading lines of the hysteretic responses were on average found to intercept the zero-displacement axis at \(0.17 \times F_{n,\text{ult}}\), where \(F_{n,\text{ult}}\) is the ultimate strength of the nail connection in the positive or negative direction. Effective yield forces based on \(\beta_1 = \beta_2 = 1\) can therefore be calculated as,

\[
F_{n,y} = 0.17F_{n,\text{ult}}
\]  

(8-1)

Corresponding yield displacement values \((\delta_{n,y})\) can subsequently be determined through linear-interpolation of the original backbone curve data set. Using the same nail connection experimental data, it was found that parameters \(\alpha_1\) and \(\alpha_2\) should be assigned a value of one thousand \((\alpha_1 = \alpha_2 = 1000)\) to suitably capture the steep gradient of the hysteretic lines during unloading.

To demonstrate the suitability of the pivot model, the methodology described above was applied to four of the tested nail connections reported in Chapter 5 (one from each nail connection type; New-USA, New-NZ, Salvaged-Parnell, and Salvaged-T Adair). Effective

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**Figure 8-7: Illustration of nail connection initial force-displacement data removal for cyclic analysis**
yield forces were calculated by applying to Equation 8-1 the ultimate positive and negative forces recorded during testing. Corresponding yield displacements were determined through linear interpolation of the original backbone curve data set. The calculated force and displacement values are listed in Table 8-1 for reference. The FE model output for each nail connection is compared against experimental data in Figure 8-8. The link elements are shown to capture nail connection hysteretic behaviour well when characterised with the recommendations presented in this section.

To summarise, it is recommended that the pivot model be used to capture nail connection hysteretic behaviour in SAP2000. For cyclic analysis, the first positive and negative force-displacement data points of the assigned backbone curve should be determined from Equation 8-1 and from linear interpolation of the original monotonic data set. Pivot model parameters controlling unloading gradient ($\alpha_1$ and $\alpha_2$) and hysteretic pinching ($\beta_1$ and $\beta_2$) should be assigned the following values; $\alpha_1 = \alpha_2 = 1000$ and $\beta_1 = \beta_2 = 1$.

<table>
<thead>
<tr>
<th>Nail connection test reference</th>
<th>$F_{y1}$</th>
<th>$\delta_{y1}$</th>
<th>$F_{y2}$</th>
<th>$\delta_{y2}$</th>
<th>$F_{n,ult}(+)$</th>
<th>$F_{n,ult}(-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New-USA 4</td>
<td>0.23</td>
<td>0.10</td>
<td>-0.21</td>
<td>-0.12</td>
<td>1.38</td>
<td>-1.22</td>
</tr>
<tr>
<td>New-NZ 12</td>
<td>0.28</td>
<td>0.07</td>
<td>-0.22</td>
<td>-0.17</td>
<td>1.62</td>
<td>-1.28</td>
</tr>
<tr>
<td>Salvaged-Parnell 8</td>
<td>0.21</td>
<td>0.31</td>
<td>-0.17</td>
<td>-0.28</td>
<td>1.23</td>
<td>-0.98</td>
</tr>
<tr>
<td>Salvaged-T Adair 21</td>
<td>0.18</td>
<td>0.35</td>
<td>-0.18</td>
<td>-0.57</td>
<td>1.03</td>
<td>-1.08</td>
</tr>
</tbody>
</table>

### 8.1.6 Boundary conditions

Having prescribed a suitable method for modelling the structural components of heritage timber diaphragms in SAP2000, the final consideration for diaphragm FE modelling is boundary condition idealisation. Based on the diaphragm characterisation survey presented in Chapter 1, diaphragm boundary conditions are primarily governed by the interaction between the joists and the URM perimeter walls. The appropriate treatment of these boundary conditions for each principal loading direction is discussed in the following sections. For the purposes of diaphragm analysis, the URM perimeter walls are assumed to
8.1 Modelling of Timber Floor Diaphragms in SAP2000

Figure 8-8: Comparison of FE model link element hysteretic response with experimental nail connection force-displacement response
remain completely rigid. Intermediate cross-beams are assumed to provide vertical joist support only, while any in-plane friction resistance is considered to be zero. Roller-type supports should therefore be provided to the joists wherever intermediate cross-beams are located.

8.1.6.1 Modelling of parallel-to-joist boundary conditions

The idealisation of diaphragm boundary conditions for parallel-to-joist analysis is relatively straightforward. The two side joists should be modelled with fixed boundary conditions to restrain these members against any translation and rotation during analysis. The fixed restraints represent the mechanical connection of the side joists to the in-plane URM perimeter walls, which would have either been provided during original diaphragm construction or as part of any basic diaphragm retrofit. Fixing the side joists can be readily achieved in SAP2000 by dividing the relevant frame elements into four equal lengths and assigning fixed restraints to the newly created joints. For diaphragm-only analysis, the out-of-plane URM walls are assumed to provide vertical joist support only and to not contribute any in-plane resistance during diaphragm analysis. Accordingly, the intermediate joists should be assigned roller-type supports at their ends to provide the necessary vertical support but allow free in-plane translation and rotation. A roller-type restraint can be modelled in SAP2000 by selecting the relevant joints and assigning a
translation-3 restraint only. As previously outlined, roller-type supports should also be assigned to the joists at any cross-beam location. An illustrative summary of the idealised boundary conditions for parallel-to-joist diaphragm analysis is provided in Figure 8-9.

**8.1.6.2 Modelling of perpendicular-to-joist boundary conditions**

Boundary conditions for perpendicular-to-joist diaphragm analysis represent joist end restraints provided by the perimeter URM walls. It was reported in Chapter 1 that joist ends were typically either simply supported on a brick ledge resulting from the URM walls reducing in width at each storey height, or pocketed into the URM walls to a depth equal to one brick width (see Figure 1-4). As previously stated, this research is concerned with diaphragm configurations comprising pocketed joists only; FE modelling of diaphragm configurations with joists that are supported on a brick ledge is outside the scope of this research.

Pocketed joist boundary conditions are inherently difficult to characterise due to variable pocket geometry, joist embedment length, and embedment condition. There are three basic joist pocket configurations to consider: (1) oversized pockets, (2) intermediate pockets, and (3) tight pockets. Oversized pockets refer to pocket widths which sufficiently exceed

![Figure 8-9: Idealised diaphragm boundary conditions for parallel-to-joist analysis](image-url)
joist thickness such that joist pinching is avoided during realistic diaphragm lateral deformations. Figure 8-10 illustrates that when clearance exists between the pocketed joist and the adjacent bricks, the joists are able to rotate freely within the URM wall pocket up until rotations of $\alpha_{jp}$ are reached, which for oversized pockets will exceed expected joist end rotations. For intermediate pockets however, pocket width is sufficiently small such that $\alpha_{jp}$ is reached during diaphragm deformation and joist pinching begins to occur due to contact with both sides of the URM wall pockets, as illustrated in Figure 8-10b. Joist pinching consequently generates rotational fixity, which must be accounted for when modelling the diaphragm boundary conditions. As pocket width reduces, the described joist pinching occurs at smaller angles of rotation until $\alpha_{jp}$ reaches zero for tight pocket configurations, which comprise no clearance between the joist and the adjacent bricks. Tight joist pockets were typically constructed by either tightly filling the recessed support with masonry, or grouting the joist inside the wall pocket. Depending on embedment condition, a degree of fixity will be generated at the ends of tightly pocketed joists immediately upon diaphragm deformation.

In order to model the correct joist end condition, $\alpha_{jp}$ must firstly be determined to establish whether joist pinching will occur. Inspecting Figure 8-10b it can be shown through simple geometry that,

$$\theta_p = \sin^{-1}\left(\frac{d_p}{\sqrt{t_j^2 + e^2}}\right)$$

(8-2)

and that,

$$\beta_p = \tan^{-1}\left(\frac{t_j}{e}\right)$$

(8-3)

where $d_p$ is the URM wall pocket width, $t_j$ is joist thickness, and $e$ is joist embedment length. Figure 8-10b also demonstrates that,
so, by substituting Equations 8-2 and 8-3 into Equation 8-4, an expression for the limiting rotation before joist pinching occurs can be formulated;
\[ \alpha_{jp} = \sin^{-1} \left( \frac{d_p}{t_j^2 + e^2} \right) - \tan^{-1} \left( \frac{t_j}{e} \right) \]  

(8-5)

The relative values of \( \alpha_{jp} \) and expected joist end rotation dictate the most appropriate idealisation of diaphragm boundary conditions for perpendicular-to-joist analysis. Oversized joist pockets are the simplest boundary condition to idealise, in which the unrestrained joist rotations can be modelled as simple pinned restraints at the intersections of the joists and the URM perimeter walls (i.e. embedment length ignored), as illustrated in Figure 8-11. Contrastingly, pocket configurations that generate rotational restraint through pinching effects are significantly more difficult to idealise. Intermediate pockets require an idealised boundary condition that sequentially changes from pinned to partially fixed when joist end rotations reach \( \alpha_{jp} \) during the deformation analysis. Tight pockets require partially fixed end restraints for all potential joist end rotations. The magnitude of partial fixity generated through joist pinching is difficult to quantify due to highly variable embedment conditions and consequently the variable contributions of joist sliding, brick movement, and localised timber, brick and mortar crushing. The accurate determination

![Idealised diaphragm boundary conditions for oversized joist pocket configurations for perpendicular-to-joist analysis](image)

**Figure 8-11:** Idealised diaphragm boundary conditions for oversized joist pocket configurations for perpendicular-to-joist analysis
of joist pinching fixity requires experimental confirmation and extensive analysis which is beyond the scope of this study. The development of a generalised procedure to accurately model embedded joist boundary conditions is therefore recommended as the subject of future research. However two potential methods for idealising sequential pinned-to-partially fixed boundary conditions for pocketed joists are presented in Section 8.2.4. The two procedures were empirically calibrated so that FE model output suitably captured the force-displacement responses and deformation profiles of tested full-scale diaphragms 1a-PERP and 2a-PERP that were reported in Chapter 7.

8.2 Model Validation

To validate the proposed modelling method presented in Section 8.1, FE models were developed in SAP2000 for comparison with the analytical model reported in Chapter 3, and the experimental test results reported in Chapters 6 and 7. A description of each developed model and the corresponding analysis output comparison is presented in the following sections, which are titled accordingly.

8.2.1 Comparison with analytical diaphragm model

Serving as a two-way validation of the analytical model developed in Chapter 3 and the proposed FE modelling method presented in Section 8.1, a comparative analysis was performed using the standard diaphragm configuration established in Chapter 1 when subjected to a uniformly distributed lateral load (UDL). For each model, a force-displacement response and a diaphragm deformation profile were generated for each principal loading direction to evaluate the mechanical basis of the models. If both models are capable of predicting similar diaphragm response given identical input parameters, then the consistency of model formulation is appropriately verified.

The key parameters of the standard diaphragm configuration are listed in Table 8-2, as well as the arbitrary UDL chosen for deformation analysis. Predicted response using the analytical model was readily achieved by programming the listed diaphragm parameters...
into the nonlinear MATLAB code described in Section 3.5 and provided in Appendix A. Using the proposed modelling guidelines detailed in Section 8.1, an FE model of the standard diaphragm was constructed in SAP2000. Based on the analytical model limitations, the FE model was developed with the following characteristics:

1. Floorboards were modelled as continuously spanning frame elements across the diaphragm. This was a necessary simplification because the analytical model was formulated based upon the assumption that the timber framing members span continuously between their end supports.

2. The theoretical nail connection load-slip characteristic developed by Foschi (1974), which the analytical model is based upon (see Chapter 3), was discretised and assigned to the FE model link elements representing the floorboard-to-joist connections. The theoretical load-slip curve was defined using $K_0 = 1182 \text{ N/mm}$, $K_1 = 50 \text{ N/mm}$, and $F_0 = 920 \text{ N}$, which are typical values established from past research (Dolan and Madsen 1992).

3. The pinned boundary conditions described in Section 8.1.6 were adopted for both parallel-to-joist analysis and perpendicular-to-joist analysis because the analytical model was formulated based upon the assumption of pinned end conditions.

4. For parallel-to-joist analysis, the UDL was applied directly to the modelled floorboards, as was the basis for analytical model formulation. The effective load applied to each floorboard was determined using Equation 3-10. For perpendicular-to-joist, the effective load ($w$) was determined using Equation 3-17 and was applied directly to the modelled joists.

A nonlinear monotonic pushover analysis was performed on the developed FE model to generate a force-displacement response and a diaphragm deformation profile in each principal loading direction. Comparison of the FE model output and the analytical model output is provided in Figures 8-12 and 8-13. It is clearly evident that having been developed with the same boundary conditions and loading conditions, the FE model and analytical model predict almost identical responses for the standard diaphragm configuration. The observed consistency provides good confidence that the diaphragm models are formulated upon representative mechanical principles. The FE modelling
8.2 Model Validation

(a) Parallel-to-joist analysis  
(b) Perpendicular-to-joist analysis

*Figure 8-12: Comparison of FE model and analytical model force-displacement output*

(a) Parallel-to-joist analysis  
(b) Perpendicular-to-joist analysis

*Figure 8-13: Comparison of FE model and analytical model displacement profile*
method is further evaluated in the subsequent sections to demonstrate that it capably captures experimentally determined diaphragm behaviour with realistic boundary conditions and loading conditions.

Table 8-2: Standard diaphragm parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm dimensions (m)</td>
<td>12 × 8</td>
</tr>
<tr>
<td>Floorboard section (mm)</td>
<td>135 × 25</td>
</tr>
<tr>
<td>Joist section (mm)</td>
<td>300 × 50</td>
</tr>
<tr>
<td>Timber elastic modulus, $E$ (MPa)</td>
<td>10</td>
</tr>
<tr>
<td>Nail couple spacing, $s$ (mm)</td>
<td>95</td>
</tr>
<tr>
<td>Joist spacing (c/c), $\ell$ (mm)</td>
<td>400</td>
</tr>
<tr>
<td>Total applied UDL, $W$ (kN/m)</td>
<td>7</td>
</tr>
</tbody>
</table>

1. Joists in direction of 8.0 m dimension

8.2.2 Comparison with racking assemblage tests

FE models of the tested racking assemblages reported in Chapter 6 were developed in SAP2000 for monotonic analysis. Instead of mimicking the actual test set-up described in Section 6.1.2, which involved symmetrically arranged test unit pairs, the racking assemblages were modelled individually to ease model construction and because the idealised boundary conditions could readily cater for the eccentrically applied loads. The models were developed using the configurations detailed in Table 6-1 and Figure 6-2, an example of which is provided in Figure 8-14, which shows a screen-capture of modelled racking assemblage SB-1. Joist and floorboard frame elements were assigned the relevant material properties and sectional properties reported in Section 6.1.1. The averaged backbone curve that was developed in Section 5.5.3 for New-USA nail connections was used to characterise the nonlinear link elements. Boundary conditions were modelled according to the test set-up described in Section 6.1.2, in which one joist was fixed against all translation and rotation, while the other was restrained against all movement except for uniaxial lateral translation in the direction of loading. The assigned
A monotonic pushover analysis was performed on each racking assemblage FE model by applying increasing displacements to the end joint of the joist which was released for uniaxial translation, as depicted in Figure 8-14. The force-displacement responses generated from the analyses are compared against the relevant experimental results in Figure 8-15. Overall the FE models predicted racking assemblage performance well, with generally good agreement between FE model output and test data. The exception was the FE model of racking assemblages T&GA-2 and T&GT-2, which was shown to slightly under-predict performance at all displacement amplitudes. Given the accuracy of the other racking assemblage models, the only logical explanation for this performance discrepancy is that the nail connections comprising test units T&GA-2 and T&GT-2 must have been on average stronger than the New-USA nail connection backbone curve that was used to characterise the nonlinear link elements. The FE models were also not capable of capturing the stiffness and strength increase exhibited by racking assemblages with floorboards together at displacements exceeding ±64 mm. As discussed in Section 6.1.4, this performance improvement was possibly generated through inter-floorboard contact mechanisms arising at large displacements, which were ignored for FE modelling purposes.
8.2.3 Comparison with sub-component diaphragm tests

FE models of the tested sub-component diaphragms reported in Chapter 6 were developed in SAP2000 for monotonic analysis. FE models of the newly constructed sub-component diaphragms (SC2-1 to SC2-4) were developed using the configuration details listed in Table 6-2 and illustrated in Figure 6-8. The joist and floorboard frame elements were assigned the relevant material properties and sectional properties reported in Section 6.2.1, while the nonlinear link elements were characterised with the averaged backbone curve.
developed in Section 5.5.3 for New-USA nail connections. Boundary conditions were modelled in accordance with the methodology presented in Section 8.1.6.1 for parallel-to-joist diaphragm analysis. To introduce loads into the modelled sub-component diaphragms, the loading frame described in Section 6.2.3 was modelled explicitly with representative steel material properties and sectional properties, and was connected to the loaded joists at bolt locations (see Figure 6-11). The modelled loading frame was suitably assigned roller supports to provide the necessary vertical support, while allowing free lateral translation. A screen-capture of the modelled sub-component diaphragm SC2-1 is provided in Figure 8-16 for illustration of the described model development.

FE models of the salvaged sub-component diaphragms (Parnell-large and Parnell-small) were developed using the configuration details illustrated in Figure 6-9. Joist and floorboard frame elements were assigned the relevant material properties and sectional properties reported in Section 6.2.2, and nonlinear link elements were characterised with the averaged backbone curve developed in Section 5.5.3 for Salvaged-Parnell nail connections, which were taken from the same heritage timber floor diaphragm that the salvaged sub-component diaphragms were extracted from. Boundary conditions were modelled in accordance with the methodology presented in Section 8.1.6.1, which was

Figure 8-16: SAP2000 model of sub-component diaphragm test SC2-1
directly applicable to the salvaged sub-component diaphragm tests. The steel loading truss described in Section 6.2.4 was modelled as a simple beam with representative sectional properties and comparatively rigid material properties. The loading frame elements were connected to the appropriate joists with pinned end restraints to represent the hinged joist loaders described in Section 6.2.4. The loading frame joints were assigned roller restraints to provide vertical support while allowing free lateral translation.

A monotonic pushover analysis was performed on each sub-component diaphragm FE model by applying increasing displacements to the modelled loading frames, as depicted in Figure 8-16. The generated force-displacement responses are plotted in Figure 8-17 against corresponding experimental data, which demonstrates mixed FE model accuracy. Generally the performance of newly constructed diaphragms was accurately predicted up until midspan displacements of ±25 mm were reached, but was over-predicted for midspan displacements exceeding ±25 mm. As shown in Figure 8-17d, the FE model of sub-component diaphragms SC2-4a and SC2-4b over-predicted response for almost all midspan displacements. The observed performance discrepancies can be attributed to floorboard flexural cracking, which cannot be explicitly captured in SAP2000 using elastic frame elements alone. Floorboard cracking was shown in Sections 6.2.6 and 6.2.7 to considerably reduce sub-component diaphragm strength and stiffness, so it is logical that the corresponding FE models developed with elastic frame elements would over-predict force-displacement response. Considering that floorboard cracking is unlikely to occur in any realistic diaphragm configuration (as evidenced by the full-scale diaphragm tests reported in Chapter 7), the development of a method to model progressive floorboard cracking in SAP2000 is outside the scope of this research. Figures 8-17e and 8-17f show that salvaged sub-component diaphragm performance was considerably under-predicted until midspan displacements of between ±25 mm and ±35 mm were reached. As discussed

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1Effective cracked section properties could be assigned to the floorboards in order to capture overall post-floorboard-cracking response if desired, however initial response would be considerably under-predicted in such a case. Floorboard cracking could also be modelled in SAP2000 using link elements or hinges assigned to predefined locations along the floorboard frame elements, although this approach would only be effective for monotonic pushover analysis.
8.2 Model Validation

(a) SC2-1 (b) SC2-2
(c) SC2-3 (d) SC2-4
(e) Parnell-Large (f) Parnell-Small

Figure 8-17: Comparison of FE model monotonic output with experimental sub-component diaphragm backbone curves
in Section 6.2.7, composite floorboard action, which was generated through the glue-like product present between the tongue and groove interfaces, was observed to significantly increase the initial stiffness of the tested salvaged sub-component diaphragms. The FE models were developed with the assumption of no mechanical interaction between the timber framing members, so are therefore unable to capture the considerably enhanced initial stiffness of the tested salvaged sub-component diaphragms. Further FE model development to include inter-floorboard gluing effects is outside the scope of this research. In addition to the recommendation of further heritage diaphragm testing outlined in Section 6.3, a method for modelling inter-floorboard gluing in SAP2000 is recommended as the subject of future research.

8.2.4 Comparison with as-built full-scale diaphragm tests

FE models of the tested as-built full-scale diaphragms reported in Chapter 7 were developed in SAP2000 for monotonic and cyclic analysis using the proposed methodology presented in Section 8.1. The models of full-scale diaphragms 1a-PARA, 2a-PARA, 1a-PERP, and 2a-PERP were developed using the configuration details listed in Table 7-1 and illustrated in Figure 7-1, an example of which is provided in Figure 8-18, which shows a screen-capture of modelled full-scale diaphragm 1a-PARA. Joist and floorboard frame elements were assigned the sectional properties reported in Section 7.1.1, while representative sectional properties were assigned to the cross-bracing frame elements using the recommendations presented in Section 8.1.3. No mechanical testing was performed on the full-scale diaphragm timber framing members, so published values were used to characterise material modulus of elasticity (MOE), $E$. The MSG8 timber used for the joists and cross-bracing has a design $E$ of 8 GPa (SANZ 1993), which was directly assigned to the relevant frame elements. The floorboard timber was not formally machine stress graded by the manufacturer, so standardised MOE values were not available for this product. However Fortune (2010) performed MOE tests on two hundred and fifty six similar New Zealand Radiata Pine floorboard profiles, and from which determined a mean $E$ of 10.6 GPa. Allowing for some conservatism, an $E$ of 10 GPa was subsequently assigned to the floorboard frame elements, which was considered suitably reflective of the comparatively higher quality of the floorboard timber compared with the joist timber.
The boundary conditions for parallel-to-joist analysis were modelled in accordance with the recommendations presented in Sections 8.1.6.1. Given the uncertainty surrounding pocketed joist fixity, the influence of boundary condition idealisation on perpendicular-to-joist diaphragm analysis was explored by using different joist end restraints. Diaphragms 1a-PERP and 2a-PERP were modelled firstly with pinned restraints (as detailed in Section 8.1.6.2), and then with fully-fixed restraints. Two different procedures were then developed to model the pocketed joist sequential pinned-to-partially fixed boundary conditions, which are described hereafter.

The first method to idealise pocketed joist behaviour involved the combined application of joint restraints and link elements to the ends of the modelled joists. As depicted in Figure 8-19, pinned restraints were assigned to each joist end to exhibit the initial free-rotation of the pocketed joists up until end rotations of $\alpha_{jp}$ were reached. To automatically convert the joist end conditions from pinned to partially fixed during deformation analysis, one-joint link elements were assigned to the joist ends to provide rotational restraint when end
Chapter 8: Finite Element Modelling

Figure 8-19: Schematic of the pinned and rotational spring boundary conditions adopted for tested diaphragms 1a-PERP and 2a-PERP

Figure 8-20: Generalised moment-rotation characteristic representing joist pinching within URM wall pockets

rotations exceeded $\alpha_{jp}$, as depicted in Figure 8-19a. The link elements were characterised with a multi-linear elastic moment-rotation data set of the form depicted in Figure 8-20, which was assigned to the R1 degree-of-freedom while fixing all remaining degrees-of-freedom ($U1$, $U2$, $U3$, $R2$, and $R3$). Figure 8-20 illustrates that the link elements initially provided zero rotational stiffness to preserve the pinned end restraints up until joist rotations of $\pm\alpha_{jp}$ were reached. However once joist rotations of $\pm\alpha_{jp}$ were exceeded, the
**8.2 Model Validation**

*link* element rotational stiffness \( K_{rot} \) was characterised to increase rotational restraint proportional to increasing diaphragm lateral loading. From Sections 7.1.1 and 7.2.2: \( d_p = 49 \text{ mm}, \ t_j = 45 \text{ mm}, \text{ and } e = 110 \text{ mm} \), which by substituting into Equation 8-5, \( \alpha_{jp} \) was found to be 0.036 rad. Rotational stiffness \( K_{rot} \) was initially trialled with a relatively rigid value of 10,000 kNm/rad but was found to considerably over-predict response, as shown in Section 8.2.4.1. Rotational stiffness was subsequently empirically calibrated to \( K_{rot} = 20 \text{ kNm/rad} \), which was shown to generate accurate analysis results (see Section 8.2.4.1).

The second method to idealise pocketed joist behaviour involved explicitly modelling the embedded lengths of the joists, \( e \), as illustrated in Figure 8-21. In this procedure, initial free-rotation was captured by assigning *pinned restraints* to the joists at the inside dimension of the URM side walls, and by assigning *roller restraints* to the embedded joist end joints. Instead of using rotational springs to exhibit joist pinching, one-joint *gap* elements were assigned to the ends of the embedded joists. *Gap* elements in SAP2000 are intended to represent an initial gap opening by exhibiting zero stiffness up until the assigned *open* dimension is closed, at which point the *gap* elements can be programmed to exhibit a linear stiffness. With reference to Figure 8-21, the *gap* elements were assigned an initial opening of \( g_e = d_p - t_j = 49 - 45 = 4 \text{ mm} \) to the \( U_2 \) degree-of-freedom, while all remaining degrees-of-freedom \( (U_1, U_3, R_1, R_2, \text{ and } R_3) \) were freed from restraint. The *gap* element embedment stiffness \( K_g \) was originally set to the relatively rigid value of 10,000 kN/m but was found to considerably over-predict response, as shown in Section 8.2.4.1. *Gap* element stiffness was subsequently empirically calibrated to a value of \( K_e = 2000 \text{ kN/m} \), which was shown to generate accurate analysis results (see Section 8.2.4.1). It should be noted that the described method was only used for monotonic analysis, if cyclic analysis was desired, then one-joint *hook* elements could also be assigned to the ends of the embedded joists. *Hook* elements in SAP2000 exhibit the same behaviour as *gap* elements except that the assigned linear stiffness is not engaged until the initial dimension is opened, not closed.

The steel load distribution frame described in Sections 7.2.1 and 7.2.2 was modelled explicitly for representative lateral load application. For parallel-to-joist analysis, the
primary truss and secondary loading beams were modelled as individual frame elements that were pin-jointed together to mimic the physical hinged connections. The secondary beams were connected to the appropriate joists (see Figure 7-8) using pinned restraints to represent the pin-jointed joist loaders, as depicted in Figure 8-18. For perpendicular-to-joist analysis, the secondary beams were removed (as described in Section 7.2.2), and the primary truss frame element was connected directly to the appropriate joists (see Figure 7-11) using pinned restraints. A comparatively rigid elastic modulus ($1000 \times E_{\text{timber}}$) was assigned to the loading frame material property. The loading frame joints were assigned roller restraints to provide vertical support while allowing free lateral translation.

### 8.2.4.1 Monotonic analysis

For monotonic diaphragm analysis, the nonlinear link elements representing the floorboard-to-joist nail connections were characterised with the backbone curve developed in Section 5.5.3 for New-NZ nail connections (refer to Appendix B for data points). A monotonic pushover analysis was performed on each developed diaphragm FE model by applying increasing displacements to the modelled loading frames, as depicted in Figure 8-
18. To validate FE modelling accuracy, force-displacement responses and displacement profiles were generated for comparison against corresponding experimental results.

Inspecting Figures 8-22 and 8-23, it is evident that the FE models developed for full-scale diaphragms 1a-PARA and 2a-PARA generally captured force-displacement response with good accuracy. Predicted performance was shown to be typically within 5% of the corresponding test data, and was particularly accurate for all positive displacement amplitudes. However, the performances of 1a-PARA and 2a-PARA were shown to be slightly under-predicted by the FE models for negative displacements - by up to 9% and 13%, respectively, at the maximum negative displacement amplitudes reached during testing. Given that the FE model force-displacement outputs are symmetrical about the x and y axes, and are shown to accurately capture experimental performance for positive displacements, it is likely that the observed discrepancies were caused by experimental procedure rather than poor FE model accuracy. It is postulated that the loading frame could have contributed to diaphragm stiffness for negative displacements, which would have ultimately improved overall load resistance.

Figures 8-24a and 8-25a illustrate the significant effect that boundary condition idealisation has on perpendicular-to-joist diaphragm analysis. The FE models of full-scale diaphragms 1a-PERP and 2a-PERP with fixed end restraints were shown to over-predict stiffness and strength for all displacement amplitudes. FE models with pinned end restraints were shown to accurately predict response up until displacements of approximately ±50 mm (±2% drift), but to under-predict response for all subsequent displacements where joist pinching effects were observed to commence during testing (see Section 7.2.4.1). The accurate prediction of initial performance using pinned boundary conditions confirms the assumption of free joist rotation within the URM wall pockets up until joist pinching occurs, while the observed deviation in predicted performance above displacements of ±50 mm indicates that joist pinching considerably improves diaphragm performance. Figures 8-24a and 8-25a also show that joist pinching only generates partial rotational fixity because diaphragm performance was significantly over-predicted when rotational spring stiffness ($K_{rot}$) or embedment spring stiffness ($K_g$) was set to a relatively rigid value (see Section 8.1.6.2). The confirmation that joist pinching does not generate
Figure 8-22: Comparison of FE model monotonic output with full-scale diaphragm 1a-PARA

Figure 8-23: Comparison of FE model monotonic output with full-scale diaphragm 2a-PARA
8.2 Model Validation

(a) Effect of boundary conditions

(b) Calibrated FE models

Figure 8-24: Comparison of FE model monotonic output with full-scale diaphragm

1a-PERP
Chapter 8: Finite Element Modelling

(a) Effect of boundary conditions

(b) Calibrated FE models

Figure 8-25: Comparison of FE model monotonic output with full-scale diaphragm

2a-PERP
complete rotational fixity demonstrates the complexity of this mechanism and endorses the need for further research in this subject area. Figures 8-24b and 8-25b illustrate that once the FE models developed with sequential pinned-to-partially fixed boundary conditions (described in Section 8.2.4) were appropriately calibrated, the FE model output accurately captured full-scale diaphragm response for all positive and negative displacement amplitudes. The two developed methods for modelling typical intermediate pocketed joist end conditions can therefore capably capture the sequential free-rotation to partially fixed mechanism, although a procedure to correlate spring stiffness with rotational restraint is required to generalise the proposed boundary condition idealisation procedures.

Displacement profiles were generated for diaphragm FE models 1a-PARA and 1a-PERP based on midspan displacement amplitudes of ±50 mm, ±100 mm, and ±150 mm, to verify whether behaviour was suitably captured across diaphragm span. Figure 8-26 shows that the predicted displacement profiles reasonably match the displacement transducer readings which were recorded during the corresponding full-scale diaphragm tests.

### 8.2.4.2 Cyclic analysis

Having demonstrated the effectiveness of the developed FE models to predict monotonic diaphragm response, cyclic analysis was performed on full-scale diaphragms 1a-PARA and 1a-PERP to verify that hysteretic behaviour could be appropriately captured. In accordance with the recommendations presented in Section 8.1.5.2, the backbone curve used for nonlinear link element characterisation for monotonic analysis (Section 8.2.4.1) was appropriately modified for cyclic analysis to exhibit effective yield points of [-0.11 mm, -0.28 kN] and [0.11 mm, 0.28 kN]. The link element hysteretic pivot model was also suitably assigned the parameters $\alpha_1 = \alpha_2 = 1000$ and $\beta_1 = \beta_2 = 1$. For diaphragm 1a-PERP, only the FE model with calibrated rotational spring end conditions was considered. Cyclic time-history analyses were performed on the modelled full-scale diaphragms by applying the displacement schedule illustrated in Figure 7-13 to the modelled loading frames. The generated force-displacement responses are provided in Figure 8-27, where they are shown to compare well with corresponding experimental data. The general shape of the hysteretic responses is suitably captured, apart from stiffness degradation on repeated loading cycles to the same displacement amplitude, which is
slightly under-predicted. Consequently, the developed diaphragm FE models are shown to slightly under-predict hysteretic pinching and in turn, slightly over-predict hysteretic energy dissipation.

![Comparison of FE model displacement profiles with experimental as-built full-scale diaphragm displacement profiles](image)

**Figure 8-26:** Comparison of FE model displacement profiles with experimental as-built full-scale diaphragm displacement profiles
8.3 Conclusions

A methodology to model heritage timber floor diaphragms in the structural analysis software SAP2000 (CSI 2004) was presented. The method was formulated based upon the assumption that timber framing members remain linear-elastic, while all diaphragm nonlinearity is consigned to the primary nail connections. Mechanical interaction between the timber framing members is assumed to be negligible. The basic modelling premise is to
recreate the diaphragm geometrical configuration using elastic frame elements modelled at the true centreline locations of the timber framing members. Each floorboard-to-joist nail connection is modelled using nonlinear link elements, which are assigned representative load-slip characteristics to appropriately capture diaphragm nonlinear behaviour.

Boundary conditions for parallel-to-joist diaphragm analysis were shown to be relatively straightforward, for which recommendations have been made to suitably idealise these conditions for modelling. Three basic boundary condition types were identified for perpendicular-to-joist diaphragm analysis: (1) oversized pockets, in which pocket width is sufficiently large such that joists are able to freely rotate within the pocket without pinching occurring, (2) intermediate pockets, in which pocket width is sufficiently small such that joist pinching will occur after the limiting rotation of $\alpha_{jp}$ is reached, and (3) tight pockets, in which no lateral clearance exists between the joist and adjacent bricks, and pinching occurs for any possible rotation. It is recommended that oversized joist pockets be modelled as pinned restraints to exhibit free-joist rotation. Intermediate and tight joist pocket configurations were found to correspond to complex boundary conditions that generate uncertain levels of rotational restraint. The accurate determination of joist pinching restraint requires experimental confirmation and extensive analysis, which is beyond the scope of this study and is recommended as the subject of future research.

FE model formulation was shown to be consistent with the analytical diaphragm model developed in Chapter 3. The force-displacement responses and displacement profiles that were generated from each diaphragm model using standard diaphragm parameters were shown to be almost identical in both principal loading directions. The consistency of the FE model and the analytical model provides a two-way confirmation of their representative mechanical basis.

The proposed FE modelling method was applied to the tested racking assemblages and sub-component diaphragms reported in Chapter 6, and to the tested as-built full-scale diaphragms reported in Chapter 7. Overall the developed FE models were shown to capably capture both monotonic and cyclic force-displacement response, and diaphragm displacement profile. The FE models were unable to capture degrading sub-component
diaphragm stiffness caused by progressive floorboard cracking due to the limitation of elastic frame elements in SAP2000, which cannot exhibit this type of fractured behaviour. This model limitation is considered unimportant as it was proven in Chapter 7 that floorboard flexural cracking does not affect the performance of realistic diaphragm configurations. The FE models were also unable to capture the significantly enhanced initial stiffness exhibited by the salvaged sub-component diaphragms because the observed composite actions generated through floorboard interface adhesion cannot be captured by the proposed FE modelling technique, which assumes no mechanical interaction to occur between floorboards. If desirable for application, the development of a methodology to model floorboard cracking and inter-floorboard bonding effects is recommended as the subject of future research.

Boundary condition idealisation was shown to considerably influence perpendicular-to-joist diaphragm analysis. The FE models of tested full-scale diaphragms 1a-PERP and 2a-PERP developed with pinned boundary conditions were shown to accurately predict response up until joist pinching was observed to occur, which confirms the assumption of free joist rotation for oversized pocket configurations. Joist pinching was shown to improve diaphragm performance, with pinned FE models under-predicting response for displacements in which joist pinching was observed to occur. Based on these findings, it is concluded that pinned end restraints are a suitably conservative assumption for perpendicular-to-joist diaphragm analysis.

Based on the analyses presented in this chapter and the summary discussion provided in this section, it is concluded that the proposed FE modelling method suitably captures timber floor diaphragm behaviour for realistic configurations that are free of floorboard adhesive products.
Chapter 9

PARAMETRIC ANALYSIS OF DIAPHRAGM BEHAVIOUR

The full-scale diaphragm testing program reported in Chapter 7 successfully satisfied two primary objectives for this research; it experimentally verified the in-plane deformation mechanics of heritage timber diaphragms in both principal loading directions, and it successfully validated the FE modelling procedure outlined in Chapter 8. However because the tested diaphragms were constructed with new timber and new nails, the performance parameters determined in Section 7.2.4 cannot be directly adopted for the seismic assessment of existing timber diaphragms in URM buildings, which are comprised of outdated construction materials that have undergone decades of deterioration. In response to this limitation, the performance of a representative heritage timber diaphragm is established in the first section of this chapter. An FE model of the *standard* diaphragm configuration (see Chapter 1) was developed using the methodology described in Chapter 8, and was programmed with realistic timber material properties and heritage nail connection load-slip data, which was determined from the salvaged nail connection testing reported in Chapter 5.
Using the developed *standard* diaphragm FE model, a comprehensive parametric analysis of diaphragm behaviour is presented in the second section of this chapter. The analysis concentrates on realistic variations in diaphragm construction and diaphragm geometry in order to determine the effect that key configuration parameters have on diaphragm performance. Modification factors are subsequently established for selected configuration parameters to appropriately adjust *standard* diaphragm performance for seismic assessment purposes. Where a single modification factor was unable to suitably adjust *standard* diaphragm performance, procedures to account for possible diaphragm construction or geometric differences during seismic assessments have been proposed.

### 9.1 Heritage Diaphragm Performance

Due to a complete lack of opportunities to extract full-scale diaphragms from existing URM buildings for testing (ignoring the multitude of inherent practical constraints), the experimental determination of truly representative heritage diaphragms was unachievable for the current study. Consequently the only feasible method to establish the performance of heritage timber floor diaphragms was to firstly validate an FE modelling procedure with experimental data, and to then use this procedure to develop a diaphragm FE model programmed with representatively heritage timber material properties and nail connection load-slip data. Development of such an FE model is presented hereafter, followed by the characterisation of predicted performance in each principal loading direction.

Using the methodology presented in Section 8.1, an FE model of the *standard* diaphragm configuration was developed in the structural analysis software SAP2000 (CSI 2004). The *standard* diaphragm represents the most prevalent configuration existing in heritage URM buildings throughout New Zealand. Complete details of the *standard* diaphragm can be found in Chapter 1, but in general the *standard* diaphragm has dimensions of 12.0 m × 8.0 m, with the joists orientated in the direction of the 8.0 m dimension. Joists spanning greater than 6 m would typically be discontinuous due to manufacturing limitations (as outlined in Chapter 1), but for the purpose of establishing basic diaphragm performance, the joists are assumed to span continuously across the 8.0 m length of the diaphragm. The
influence of discontinuous joists is addressed during diaphragm parametric analysis in Section 9.2 below. As outline in Chapter 1, floorboard arrangement in existing URM buildings was found to be variable and without any consistently distinguishable pattern, although discontinuities were almost always positioned at joist locations. To provide a logical floorboard arrangement for modelling the standard diaphragm, floorboard discontinuities were required to be at least 800 mm apart for adjacent courses, and could only occur on the same joist if separated by two or more floorboard courses. The standard diaphragm timber elastic modulus of $E = 10 \text{ GPa}$ was assigned to all modelled timber members. The adopted $E$ value was considered suitably average for the typical timber species used in heritage diaphragm construction, such as Kauri, Rimu, and Matai. The nonlinear link elements were characterised with the averaged backbone curve that was developed in Section 5.5.3 for Salvaged-Parnell nail connections, the data for which is provided in Appendix B for reference. The assignment of the Salvaged-Parnell nail connection data was the most important characterisation for standard diaphragm modelling, as these nail connections were extracted from a real URM building and thus are truly representative of heritage timber floor diaphragm performance.

Boundary conditions for parallel-to-joist analysis were modelled in accordance with the recommendations presented in Section 8.1.6.1 (see Figure 8-9). For perpendicular-to-joist analysis, boundary conditions were shown in Chapter 8 to considerably influence diaphragm performance; specifically pinching effects, which generate variable rotational restraint at the ends of pocketed joists. Due to the difficulty of establishing a typical pocketed joist configuration and the associated uncertainty surrounding rotational fixity, it was considered appropriately conservative to assume oversized joist pockets and therefore adopt pinned joist end restraints for all perpendicular-to-joist analysis (see Figure 8-11).

A monotonic pushover analysis was performed on the standard diaphragm FE model in each principal loading direction, by assigning increasing unidirectional loads to the relevant joist frame elements. To mimic realistic out-of-plane wall inertia loads, the applied forces were distributed using the parabolic expression recommended by ASCE 41-06 (2007) for flexible diaphragms. For parallel-to-joist analysis, the parabolic load distribution was appropriately discretised to assign tributary point-loads to the modelled
joist ends. For perpendicular-to-joist analysis, the parabolic load distribution was again discretised and tributary loads were assigned to regularly spaced joints along the leading joist, because parabolic load distributions cannot be directly assigned to frame elements in SAP2000. It was intended to continue the pushover analyses until strength degradation was observed. However it was found that computation time dramatically increased at large displacement amplitudes relative to diaphragm span. Consequently the pushover analyses were performed for as long as possible until computational efficiency became unmanageable, at which point the analyses were terminated. As detailed in the following sections, the displacements reached during the pushover analyses were sufficiently large to enable the suitable characterisation of standard diaphragm performance, despite the computational limitation that was encountered.

9.1.1 Analysis results

The force-displacement plots generated from the standard diaphragm pushover analysis in each principal loading direction are presented in Figure 9-1. Although the full-scale diaphragms reported in Chapter 7 were constructed with slightly different geometry (10.4 m × 5.535 m), the positive displacement backbone curves of tested diaphragms 1a-PARA and 1a-PERP are plotted on the relevant figures to gauge the effect of deteriorated nail connection performance on overall diaphragm force-displacement response.

The results demonstrate that midspan displacements of 687 mm in the parallel-to-joist direction and 334 mm in the perpendicular-to-joist direction were reached during analysis, which correspond to drift values of 11.5% and 8.4%, respectively, where diaphragm drift ($\theta_d$) is defined by Equation 9-1 below.

$$\theta_d = \frac{2\Delta_d}{L} \times 100$$  \hspace{1cm} (9-1)

where $\Delta_d$ is diaphragm midspan displacement, and $L$ is diaphragm span. In comparison to the drifts reached during diaphragm testing (3.8% in parallel-to-joist direction and 5.4% in perpendicular-to-joist direction), the FE analyses captured significantly more diaphragm performance despite being prematurely terminated due to computational limitation.
The force-displacement curves presented in Figure 9-1 suggest that diaphragm performance is generally reduced when comprised of representatively deteriorated nail connections, particularly initial stiffness which appears to decrease substantially. This performance reduction is consistent with the observations reported in Chapter 5 for nail connection testing, in which the salvaged connections used to characterise the standard diaphragm were shown to have reduced stiffness and strength when compared with newly constructed connections. The FE analyses also further verified the significant flexibility of
heritage timber diaphragms by exhibiting no strength degradation, even at the excessive displacement amplitudes reached during analysis.

9.1.2 Characterisation of heritage diaphragm performance

As outlined in Section 7.2.4.4, diaphragm behaviour requires characterisation to quantify essential performance parameters such as stiffness, strength, and ductility for seismic assessment purposes. It was demonstrated in Section 7.2.4.4 that a bilinear idealisation based on hysteretic energy conservation suitably captures the experimental force-displacement responses of newly constructed diaphragms – a procedure that has also been successfully employed by researchers to characterise past diaphragm test data (Peralta 2003). However upon interpreting the standard diaphragm analysis results presented in Figure 9-1, whilst giving consideration to permissible URM building deformations during an earthquake, it is apparent that previously adopted idealisation procedures are not actually suitable for heritage diaphragm performance characterisation. Accordingly, a revised methodology for diaphragm performance characterisation is proposed hereafter.

Derakhshan (2011) determined that URM walls laterally loaded in their out-of-plane direction are stable up to mid-height displacements equal to 70% of wall thickness. As outlined in Chapter 1, the standard diaphragm configuration represents the diaphragms found in typical one- to three-storey URM buildings, which make up approximately 85% of New Zealand’s URM building stock (Russell 2010). Russell determined that the walls in such URM buildings almost never exceed a thickness of three bricks. If typical brick width is 110 mm and the mortar between the bricks is assumed to be 10 mm thick, then a typical three-leaf URM wall will have a total thickness of $t_w = 3 \times 110 + 2 \times 10 = 350$ mm. Therefore based on the research of Derakhshan, out-of-plane wall stability for the majority of New Zealand URM buildings will be compromised when diaphragm displacements exceed $\Delta_{cr} = 245$ mm. Comparing this critical displacement ($\Delta_{cr}$) with the displacements reached during the standard diaphragm analysis (see Figure 9-1), it is clear that diaphragm displacement capacity grossly exceeds that which is permissible for URM wall stability. Diaphragm performance is therefore governed by out-of-plane wall deformation limits and should be suitably characterised to reflect these limitations.
To account for realistic variations in diaphragm configuration, diaphragm performance must be characterised with geometrical independence. To achieve this independence the established critical displacement of 245 mm can be converted to a critical drift, \( \theta_{cr} \), by using Equation 9-1 with an appropriate value for diaphragm span, \( L \). Published earthquake reconnaissance reports (Dizhur et al. 2010; Ingham and Griffith 2011) indicate that if a perimeter wall of a one- to three-storey URM building has collapsed during an earthquake, it has almost always occurred in the out-of-plane direction and along the longest dimension of the building. Based on these observations, the critical direction of loading for URM buildings associated with the standard diaphragm configuration will invariably be parallel-to-joists, for which diaphragm span is 12.0 m. Substituting \( L = 12.0 \text{ m} \) and \( \Delta_{cr} = 245 \text{ mm} \) into Equation 9-1, the critical diaphragm drift for typical New Zealand URM buildings is calculated to be 4.1%. Consequently, \( \theta_{cr} = 4.1\% \) is established as an appropriate limiting criterion for diaphragm performance characterisation.

For the standard diaphragm configuration, the 4.1% drift limit corresponds to critical displacements of 245 mm and 164 mm in the parallel-to-joist and perpendicular-to-joist directions, respectively. Locating the critical displacements on the relevant force-displacement curves in Figure 9-1, it is evident that due to substantially reduced initial stiffness, heritage diaphragm response within permissible drift limits is effectively linear-elastic in both principal loading directions, as illustrated in Figure 9-2. The implication of this observation is that heritage diaphragms can be considered effectively elastic for the purposes of seismic assessment. Within permissible URM wall deformations, diaphragm performance is therefore most suitably characterised by an effective elastic stiffness \( K_d \), a corresponding effective shear stiffness \( G_d \), and a ductility of \( \mu = 1.0 \). Any reserve diaphragm strength and displacement capacity beyond the critical drift is consequently redundant because it cannot be generated before the URM building itself is likely to have collapsed.

Using the characterisation rationale described above, heritage diaphragm performance parameters in each principal loading direction were established. The limiting forces \( F_{cr} \) corresponding to the critical diaphragm displacements were determined from the FE analysis force-displacement data and were used to calculate effective elastic stiffness,
Chapter 9: Parametric Analysis of Diaphragm Behaviour

(a) Parallel-to-joist

(b) Perpendicular-to-joist

Figure 9-2: Effective linear-elastic response of standard diaphragm

where $K_d = \frac{F_{cr}}{\Delta_{cr}}$. Diaphragm shear stiffness ($G_d$) was evaluated using the idealised shear beam analogy presented in Chapter 4, which was also developed based upon the ASCE 41-06 parabolic load distribution. By rearranging Equation 4-38 and substituting in $K_d = \frac{F_d}{\Delta_{cr}}$, diaphragm effective shear stiffness can be determined as,

$$G_d = \frac{3K_dL}{32B}$$  \hspace{1cm} (9-2)
where $B$ is diaphragm width. As already discussed, because the displacement capacity of heritage diaphragms cannot be engaged within realistic URM building deformation limits, diaphragm ductility must be defined as $\mu = 1.0$. Accordingly, heritage diaphragm overstrength ratio must therefore also be defined as $\Omega = 1.0$ (refer to Equation 7-16). For comparative purposes only, the effective diaphragm shear strength was calculated by substituting the $F_{cr}$ values into Equation 7-5. For additional descriptions of the determined diaphragm performance parameters, refer to Sections 7.2.4.5 to 7.2.4.7.

The effective linear-elastic characterisation of the standard diaphragm analysis results is illustrated in Figure 9-2, and the determined performance parameters are presented in Table 9-1 below. The performance parameters determined in Chapter 7 for newly constructed full-scale diaphragms are also provided in Table 9-1 for comparative discussion.

<table>
<thead>
<tr>
<th>Loading direction</th>
<th>Diaphragm</th>
<th>$F_{cr}$</th>
<th>$\Delta_{cr}$</th>
<th>$K_d$</th>
<th>$R_d$</th>
<th>$G_d$</th>
<th>$\mu$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>kN</td>
<td>mm</td>
<td>kN/m</td>
<td>kN/m</td>
<td>kN/m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallel-to-joist</td>
<td>1a-PARA</td>
<td>-</td>
<td>-</td>
<td>644</td>
<td>1.6</td>
<td>198</td>
<td>7.2</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>Standard</td>
<td>51.0</td>
<td>245</td>
<td>208</td>
<td>3.2</td>
<td>30</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Perpendicular-to-joist</td>
<td>1a-PERP</td>
<td>-</td>
<td>-</td>
<td>1605</td>
<td>1.3</td>
<td>134</td>
<td>8.8</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>Standard</td>
<td>64.0</td>
<td>164</td>
<td>391</td>
<td>2.6</td>
<td>23</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### 9.1.3 Discussion

A brief discussion highlighting the effects that deteriorated nail connections had on timber diaphragm performance is presented. It is acknowledged that the performance parameters listed in Table 9-1 for heritage diaphragms and for new diaphragms are not strictly comparable, having been characterised by different procedures. Nonetheless, the comparison provides an indication of the importance of performing a diaphragm analysis using salvaged nail connection data.
Inspecting Table 9-1 it is evident that diaphragm performance is substantially reduced when comprised of representatively deteriorated nail connections. Heritage diaphragm shear stiffness in each principal loading direction was shown to be between 15% and 17% of the corresponding values determined for newly constructed diaphragms. Although heritage diaphragm displacement capacity was shown to be significant, the revised characterisation procedure (which is based upon URM wall displacement limitations) has translated into dramatic reductions in ductility and overstrength allowance.

The orthotropic behaviour of heritage diaphragm configurations determined from the standard diaphragm FE analysis was shown to be consistent with the conclusions of Chapter 7. With reference to Table 9-1, shear stiffness in the perpendicular-to-joist direction was 23% less than the corresponding value in the orthogonal direction. This finding further emphasises the need to address both principal loading directions during diaphragm seismic assessments.

9.2 Parametric Analysis

A comprehensive parametric analysis of diaphragm performance is presented in this section. The primary objective of the analysis was to determine the effect that key configuration parameters have on diaphragm performance. Based on the standard diaphragm performance characterisation presented in Section 9.1, it is evident that the most important performance parameter for diaphragm seismic assessment is effective shear stiffness ($G_d$), which governs both predicted displacements and the corresponding shear forces generated at these displacements. Accordingly, the parametric analysis was performed with a specific focus on the effect that configuration variations have on diaphragm shear stiffness.

The premise of the parametric analysis was to establish the standard diaphragm as the reference configuration and to then consider realistic variations in selected configuration parameters to determine their effect on diaphragm performance. Unless it is otherwise
stated, each analysis was formulated by considering a range of five values for the relevant parameter: (1) an upper bound value, (2) an upper middle value, (3) a mean value, (4) a lower middle value, and (5) a lower bound value. The standard diaphragm configuration parameters were logically set as the mean values, while realistic upper and lower bound values were taken from the diaphragm characterisation survey presented in Chapter 1. The upper middle and lower middle parameter values were subsequently determined as the midway values between the upper and lower bound values, and the mean values, respectively. To perform each analysis, the standard diaphragm FE model was appropriately modified to account for the selected parameter variation and saved as a new model. A monotonic pushover analysis was subsequently performed on each modified FE model in both principal loading directions, by assigning increasing unidirectional loads to the relevant joist frame elements. As was the case for the standard diaphragm analysis, parabolic load distributions (ASCE 2007) were used to simulate realistic out-of-plane wall inertia loads tributary to the diaphragm. Force-displacement responses were exported from the FE analysis results and were characterised using the procedure outlined in Section 9.1.2 to determine diaphragm effective shear stiffness \( G_d \). The force-displacement response generated from each analysis is provided in Appendix E for reference.

To establish the net effect of each configuration parameter variation on diaphragm performance, the determined shear stiffness values were plotted against the relevant configuration parameter values to illustrate any obvious trend in performance. Having established the necessary performance trends, the relationships between the configuration parameters and corresponding effective shear stiffnesses required suitable quantification in order to address possible diaphragm configuration variations that could be encountered during seismic assessment. To simplify the assessment procedure as much as possible, the performance relationships were specifically quantified to obtain modification factors that would appropriately adjust standard diaphragm effective shear stiffness to account for the relevant configuration difference. To determine the modification factors, the shear stiffness values were firstly normalised with respect to the standard diaphragm effective shear stiffness using Equation 9-3.
where $\bar{G}_d$ is normalised shear stiffness, $G_d$ is diaphragm shear stiffness, and $G_{d(\text{mean})}$ is the standard diaphragm shear stiffness. The normalised shear stiffness values were subsequently plotted against the corresponding parameters and lines-of-best-fit were determined using MATLAB’s linear and nonlinear regression functionality (The Mathworks 2011). Because the lines-of-best-fit were normalised to the standard diaphragm configuration, the desired modification factors are defined by the solutions to the line equations. It is acknowledged that the determined empirical equations are only valid between the upper and lower bound values that were established as realistic diaphragm configuration possibilities (see Chapter 1).

The parametric analysis commenced with realistic variations in diaphragm construction details such as timber elastic modulus, floorboard size, joist size, joist spacing, and the presence of discontinuous joists. The second component of the parametric analysis considered realistic variations in diaphragm geometry, such as aspect ratio and size, and the presence of diaphragm penetrations. Because this research is primarily concerned with diaphragm configurations contained within four URM perimeter walls (which make up approximately 85% of New Zealand’s URM building stock (Russell 2010)) these geometrical variations were considered for rectangular diaphragm configurations only.

### 9.2.1 Parametric analysis of diaphragm construction details

The influence of diaphragm construction parameters on effective shear stiffness ($G_d$) is explored in this section. The ranges of parameter values that were selected for analysis are presented in Table 9-2 for reference. A range of values was not necessary for the analysis of discontinuous joists, which considers only the presence of non-continuously spanning joists seated on intermediate cross-beams without a reliable mechanical connection.
Table 9-2: Diaphragm construction detail variations for parametric analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower bound</td>
</tr>
<tr>
<td>Timber elastic modulus, $E$ (GPa)</td>
<td>4.0</td>
</tr>
<tr>
<td>Floorboard thickness, $t_f$ (mm)</td>
<td>15</td>
</tr>
<tr>
<td>Floorboard width, $d_f$ (mm)</td>
<td>100</td>
</tr>
<tr>
<td>Joist thickness, $t_j$ (mm)</td>
<td>40</td>
</tr>
<tr>
<td>Joist depth, $d_j$ (mm)</td>
<td>150</td>
</tr>
<tr>
<td>Joist spacing, $\ell$ (mm)</td>
<td>300</td>
</tr>
</tbody>
</table>

9.2.1.1 Influence of timber elastic modulus

Timber elastic modulus, $E$, refers to the flexural bending stiffness of timber, which dictates the deformation resistance of diaphragm framing members (Forest Products Laboratory 1999). Figures 9-3a and 9-4a demonstrate that diaphragm shear stiffness is linearly proportional to the timber elastic modulus of the framing members in both the parallel-to-joist loading direction and perpendicular-to-joist loading direction. This observation is consistent with the deformation mechanics presented in Chapter 3, which mathematically described the linear relationship between diaphragm deformation and the flexural stiffness of the timber floorboards (for parallel-to-joist loading) and the timber joists (for perpendicular-to-joist loading).

Inspecting the normalised shear stiffness values plotted in Figures 9-3b and 9-4b, it is evident that timber elastic modulus altered diaphragm shear stiffness by between approximately -20% when $E = 4$ GPa, and approximately +20% when $E = 15$ GPa. To avoid unnecessarily complicating the component of the seismic assessment procedure which addresses variations in timber elastic modulus (see Chapter 10), the equations determined from the regression analysis of the normalised shear stiffness values in each principal loading direction (see Figures 9-3b and 9-4b) were averaged to produce a single equation. The unified equation to calculate the shear stiffness modification factor for variations in timber elastic modulus is presented in Equation 9-4.
Chapter 9: Parametric Analysis of Diaphragm Behaviour

Figure 9-3: Parametric analysis of timber elastic modulus, $E$, for parallel-to-joist loading

(a) Shear stiffness values
(b) Shear stiffness values normalised to standard diaphragm

Figure 9-4: Parametric analysis of timber elastic modulus, $E$, for perpendicular-to-joist loading

(a) Shear stiffness values
(b) Shear stiffness values normalised to standard diaphragm

$$c_E = 0.03E + 0.70, \quad 4.0 \leq E \leq 15.0 \quad (9-4)$$

where $E$ is timber elastic modulus (in GPa) of the floorboards, for parallel-to-joist assessment, or of the joists, for perpendicular-to-joist assessment.
9.2.1.2 Influence of floorboard size

With reference to the diaphragm deformation mechanics presented in Chapter 3, floorboard size primarily influences diaphragm behaviour when loaded parallel-to-joist, so accordingly the parametric analysis of floorboard size was only performed in the parallel-to-joist direction. Both floorboard thickness \( (t_f) \) and floorboard width \( (d_f) \) were individually analysed to isolate their effect on diaphragm performance. The floorboard thickness parametric analysis was performed by simply altering the thickness of the floorboard frame element section property in SAP2000. However the parametric analysis regarding floorboard width could not be achieved by simply altering the floorboard frame element section property because floorboard width dictates the number of floorboards which can fit within the relevant diaphragm dimension, and also the nail couple spacing \( (s) \). Consequently for each floorboard width listed in Table 9-2, the standard diaphragm FE model was appropriately reconfigured with the correct number of floorboards, while consistently taking nail couple spacing as \( s = 2/3 \times d_f \).

Figures 9-5a and 9-6a demonstrate that diaphragm shear stiffness is linearly proportional to floorboard thickness and to floorboard width in the parallel-to-joist loading direction. The linear relationship between floorboard thickness and diaphragm shear stiffness is logical with respect to the second moment of inertia of the rectangular floorboard sections;

\[
I_f = \frac{t_f d_f^3}{12} \tag{9-5}
\]

However it is interesting to observe that diaphragm shear stiffness is only linearly proportional to floorboard width, which based on Equation 9-5, should theoretically have a relationship that is approximately cubically proportional. It is likely that while floorboard bending stiffness increases cubically with increasing floorboard width, the overall effect on diaphragm shear stiffness is counter-balanced by the corresponding changes in the number of floorboards and also nail couple spacing.
Figure 9-5: Parametric analysis of floorboard thickness, $t_f$, for parallel-to-joist loading

Figure 9-6: Parametric analysis of floorboard width, $d_f$, for parallel-to-joist loading

Figure 9-5b indicates that floorboard thickness has a slight effect on diaphragm performance, having changed effective shear stiffness by between -15% and +7% for the lower and upper bound thickness values, respectively. Floorboard width is shown in Figure 9-6b to have comparatively more influence on diaphragm performance, demonstrating shear stiffness changes of between -17% and +11%. From the regression analysis
performed on the normalised shear stiffness values, the following equations were developed to determine the modification factors for variations in floorboard thickness ($c_{t_f}$) and floorboard width ($c_{d_f}$), respectively.

$$c_{t_f} = 14.42t_f + 0.64, \quad 0.015 \leq t_f \leq 0.03 \quad (9-6)$$

$$c_{d_f} = 7.56d_f - 0.02, \quad 0.100 \leq d_f \leq 0.150 \quad (9-7)$$

where $t_f$ is floorboard thickness in metres, and $d_f$ is floorboard width in metres.

### 9.2.1.3 Influence of joist size

The diaphragm deformation mechanics presented in Chapter 3 indicate that joist size primarily influences diaphragm performance in the perpendicular-to-joist loading direction. Accordingly the parametric analysis regarding variations in joist size concentrated solely on the perpendicular-to-joist loading direction. Both joist thickness ($t_j$) and joist depth ($d_j$) were individually analysed to isolate their effect on diaphragm performance. The parametric analyses were achieved by simply altering the relevant section property of the joist frame elements of the standard diaphragm FE model.

The shear stiffness values plotted in Figure 9-7a indicate that a power relationship exists between joist thickness and diaphragm shear stiffness ($G_d \propto t_j^n$) in the perpendicular-to-joist loading direction. The second moment of inertia of the rectangular joist sections described in Equation 9-8 suggests that $n \to 3$, which was further evaluated through the normalised shear stiffness regression analysis presented below.

$$I_j = \frac{d_j t_j^3}{12} \quad (9-8)$$

Figure 9-8a demonstrates that diaphragm shear stiffness is linearly proportional to joist depth in the perpendicular-to-joist loading direction, which is suitably consistent with the second moment of inertia definition provided in Equation 9-8.
Diaphragm performance is shown in Figures 9-7b and 9-8b to be influenced more significantly by realistic variations in joist thickness than by variations in joist depth. Although the shear stiffness values associated with the lower bound joist thickness and joist depth values were comparable, the upper bound joist thickness was shown to increase...
diaphragm shear stiffness by approximately 37%, compared with only 10% increase when the FE model was configured with the upper bound joist depth.

As previously discussed, the relationship between diaphragm shear stiffness and joist thickness should theoretically be cubically proportional with respect to Equation 9-8. Although this presumption was found to be accurate through the regression analysis of the normalised shear stiffness values presented in Figure 9-7b, it was also found that a quadratic polynomial adequately captured the relationship. With the aim of simplifying diaphragm seismic assessment, the quadratic polynomial presented in Equation 9-9 was selected to define the modification factor that is used to adjust standard diaphragm shear stiffness to account for variations in joist thickness.

\[ c_{t_j} = 381.65t_j^2 - 19.60t_j + 1.03, \quad 0.040 \leq t_j \leq 0.065 \]  

(9-9)

where \( t_j \) is joist thickness in metres. The equation describing the line-of-best-fit shown in Figure 9-8b is presented in Equation 9-10 below. The equation determines the modification factor that suitably adjusts standard diaphragm shear stiffness to account for variations in joist depth.

\[ c_{d_j} = d_j + 0.70, \quad 0.150 \leq d_j \leq 0.400 \]  

(9-10)

where \( d_j \) is joist depth in metres.

### 9.2.1.4 Influence of joist spacing

Joist spacing was shown in Chapter 3 to affect diaphragm deformation resistance in both principal loading directions. For parallel-to-joist analysis, joist spacing dictates the number of floorboard-to-joist nail connections, which ultimately governs the overall strength and stiffness of the diaphragm. For perpendicular-to-joist analysis, joist spacing determines the loading that is carried by each joist – greater joist spacing corresponds to a lower number of joists and therefore reduced load sharing which reduces diaphragm deformation resistance, whereas the opposite is true for closer joist spacing. The parametric analysis
regarding joist spacing variations was carried out by appropriately reconfiguring the *standard* diaphragm FE model in accordance with the values listed in Table 9-2. The original joist spacing values selected for parametric analysis were 300 mm, 350 mm, 400 mm (*standard* diaphragm), 525 mm, and 650 mm, but were later modified to the nearest whole multiple of the *standard* diaphragm span to ensure that $L = 12.0$ m did not significantly vary.

The shear stiffness and normalised shear stiffness values generated from the parametric analysis in each principal loading direction are presented in Figures 9-9 and 9-10. It is evident from the plots that a power relationship exists between diaphragm shear stiffness and joist spacing in both principal loading directions, although perpendicular-to-joist shear stiffness appears to have greater sensitivity. From the regression analysis of the normalised shear stiffness data, the following equations were established, which define the observed relationships for parallel-to-joist loading and for perpendicular-to-joist loading, respectively.

$$
\ell_{(para)} = -6.78\ell + 12.31\ell^2 - 8.04\ell + 2.68, \quad 0.300 \leq \ell \leq 0.667 \quad (9-11)
$$

$$
\ell_{(perp)} = -9.82\ell^3 + 18.29\ell^2 - 12.41\ell + 3.66, \quad 0.300 \leq \ell \leq 0.667 \quad (9-12)
$$

where $\ell$ is joist spacing in metres. Equations 9-14 and 9-15 define the modification factors to adjust *standard* diaphragm shear stiffness to account for variations in joist spacing.

### 9.2.1.5 Influence of discontinuous joists

As outlined in Chapter 1, when floor diaphragms were required to span between URM perimeter walls spaced greater than approximately 6.0 m, the joists were typically discontinuous and supported on intermediate steel or timber cross-beams. It was shown in Chapter 7 that discontinuous joists have no detrimental effect on diaphragm performance when reliable mechanical connections are provided (such as bolted lapped connections). However it remained unknown whether discontinuous joists without mechanical connection would impact negatively on diaphragm performance. Accordingly a
parametric analysis was performed by modifying the standard diaphragm FE model in accordance with the recommendations presented in Section 8.1.1 for modelling joists without mechanical connection. The analysis was performed in the perpendicular-to-joist direction only because joist flexural continuity was shown in Chapter 3 to primarily influence diaphragm response in this principal loading direction.
A comparison between the force-displacement outputs from the standard diaphragm FE analysis and the discontinuous joist FE analysis is presented in Figure 9-11a. It is evident that when diaphragm joists are discontinuous without a reliable mechanical connection, diaphragm strength and stiffness is reduced due to a loss of joist flexural continuity. This reduced performance was appropriately reflected in the corresponding shear stiffness values, which are shown in Figure 9-11b to reduce from 23.4 kN/m to 18.7 kN/m for the continuous joist configuration and the discontinuous joist configuration, respectively. The diaphragm comprising discontinuous joists without mechanical connections was therefore shown to exhibit approximately 75% of the shear stiffness of the corresponding diaphragm configuration with continuous joists.

The influence of discontinuous joists on diaphragm performance is addressed explicitly in the seismic assessment procedure developed in Chapter 10, so a modification factor to adjust standard diaphragm performance was therefore not required.

### 9.2.2 Parametric analysis of diaphragm geometry

Current seismic assessment documents (ASCE 2007; NZSEE 2006) assume that diaphragm shear stiffness is a constant performance parameter, regardless of diaphragm geometry. Considering that diaphragm size and aspect ratio vary considerably between URM buildings, and that diaphragms almost always contain some kind of penetration, it is important to verify whether diaphragm effective shear stiffness ($G_d$) is sensitive to relative changes in geometrical configurations.

#### 9.2.2.1 Influence of diaphragm aspect ratio and size

A rectangular diaphragm with arbitrary dimensions $x_d$ and $y_d$ is depicted in Figure 9-12, along with an illustration of the two principal loading directions. Recall that diaphragm span ($L$) is the dimension orthogonal to the applied load, and diaphragm width ($B$) is the dimension parallel to the applied load. Individual changes to the $x_d$ and $y_d$ dimensions therefore have a different effect on diaphragm performance in each principal loading direction, and thus require separate analysis.
9.2 Parametric Analysis

(a) Force-displacement comparison of standard diaphragm with and without discontinuous joists

(b) Shear stiffness values

Figure 9-11: Parametric analysis of the presence of discontinuous joists for perpendicular-to-joist loading

Diaphragm aspect ratio was investigated by individually changing diaphragm span and diaphragm width in each principal loading direction to isolate their effect on effective shear stiffness. The span and width values that were selected for the parametric analysis are listed in Table 9-3. The upper bound and lower bound values were taken from Russell...
Chapter 9: Parametric Analysis of Diaphragm Behaviour

(2010) as realistic limiting dimensions of the target one- to three-storey URM buildings in New Zealand. Intermediate values were subsequently determined by taking regular intervals between the limiting values and the standard (mean) diaphragm geometry. Unlike the previous parametric analyses in which two intermediate values were adopted, multiple intermediate values were required to adequately validate the relationship between effective shear stiffness and the relevant diaphragm dimension.

Table 9-3: Diaphragm geometry variations for parametric analysis

<table>
<thead>
<tr>
<th>Parameter being investigated</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower bound</td>
</tr>
<tr>
<td>Parallel-to-joist analysis</td>
<td>Span, L (m)</td>
</tr>
<tr>
<td></td>
<td>Width, B (m)</td>
</tr>
<tr>
<td>Perpendicular-to-joist analysis</td>
<td>Span, L (m)</td>
</tr>
<tr>
<td></td>
<td>Width, B (m)</td>
</tr>
<tr>
<td>Size, x_d × y_d (m)</td>
<td>6.0 × 4.0</td>
</tr>
</tbody>
</table>

The shear stiffness values determined from the parametric analysis of diaphragm span and diaphragm width in each principal loading direction are presented in Figure 9-13. It is clear from the plotted data that changes in diaphragm span have a significantly greater impact on effective shear stiffness than do identical changes in diaphragm width, for either principal direction of loading. The standard diaphragm effective shear stiffness was shown to increase by up to 280% at the lower bound span of L = 4.0 m, and decrease by up to 30% at the upper bound span of L = 24.0 m. In contrast, diaphragm effective shear stiffness only varied by ±1.5% at the upper and lower bound width limits, indicating that variations in diaphragm width have a negligible effect on diaphragm performance. The shear stiffness results suggest that the influence of diaphragm width on diaphragm performance can be ignored during seismic assessment, whilst likely variations in diaphragm span must be explicitly addressed when evaluating effective shear stiffness. Accordingly, the shear stiffness values determined from the parametric analysis of diaphragm span were normalised to the corresponding standard diaphragm value in order to determine
9.2 Parametric Analysis

appropriate modification factors for seismic assessment. The normalised shear stiffness values calculated for each principal loading direction are plotted against diaphragm span in Figure 9-14. From inspection the data points appear to be hyperbolic, and can therefore be related by an equation of the form:

\[ c_L = Y_L + \frac{1}{Z_L L - X_L} \]  

\hspace*{1cm} (9-13)
where \( c_L \) is the shear stiffness modification factor for diaphragm span (equal to the normalised shear stiffness value), and \( X_L, Y_L, \) and \( Z_L \) are equation constants. To fit the trial hyperbolic equation to each of the data sets, the unknown coefficients \( X_L, Y_L, \) and \( Z_L \) were optimised by minimising the sum of the squares of the differences between the analysis data points and the data points predicted by the assumed model (Equation 9-13). The optimised coefficients determined for each principal loading direction were substituted into Equation 9-13 to produce the following expressions for parallel-to-joist loading and for perpendicular-to-joist loading, respectively:

\[
c_L(\text{para}) = 0.39 + \frac{1}{(0.16L - 0.35)}, \quad 4.0 \leq L \leq 24.0
\]

\[
c_L(\text{perp}) = 0.62 + \frac{1}{(0.45L - 1)}, \quad 4.0 \leq L \leq 24.0
\]

where \( L \) is diaphragm span in metres. The developed hyperbolic functions not only suitably capture the relationship between shear stiffness and diaphragm span, but they also appear to be representative of diaphragm deformation mechanics. That is, when diaphragm span tends towards zero, diaphragm shear stiffness will theoretically tend towards infinity,
producing an asymptote. Such an asymptote was shown to occur at approximately $L = 2.2 \text{ m}$ for both principal directions of loading, as defined by Equations 9-14 and 9-15.

To verify whether the individual effects of changing diaphragm span and diaphragm width are uncoupled, the determined modification factors were used to predict the shear stiffness of diaphragms with simultaneous changes to both span and width. With reference to Figure 9-12, a range of realistic diaphragm sizes were chosen for analysis by establishing a set of $x_d$ values whilst maintaining a constant aspect ratio of $x_d/y_d = 1.5$ to determine a corresponding value of $y_d$. The five selected diaphragm sizes (including the standard diaphragm configuration) are listed in Table 9-3 for reference. The shear stiffness values determined from the FE analysis of the different diaphragm sizes in each principal loading direction are compared in Table 9-4 against the corresponding shear stiffness values determined by applying the relevant modification factor to the standard diaphragm shear stiffness value. The predicted shear stiffness values are shown to differ by less than 5%, which suggests that any rectangular diaphragm geometry within the limiting values can be evaluated by addressing diaphragm span variations only.

<table>
<thead>
<tr>
<th>Diaphragm size, $x_d \times y_d$ (m)</th>
<th>Parallel-to-joist analysis</th>
<th>Perpendicular-to-joist analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_d$ (FEM) kN/m</td>
<td>$G_d$ (Predict) kN/m</td>
</tr>
<tr>
<td></td>
<td>$G_d$ (FEM) kN/m</td>
<td>$G_d$ (Predict) kN/m</td>
</tr>
<tr>
<td>6.0 × 4.0</td>
<td>62.4</td>
<td>60.1</td>
</tr>
<tr>
<td>8.0 × 5.2</td>
<td>42.9</td>
<td>42.7</td>
</tr>
<tr>
<td>12.0 × 8.0</td>
<td>29.8</td>
<td>29.9</td>
</tr>
<tr>
<td>16.0 × 10.8</td>
<td>24.0</td>
<td>24.5</td>
</tr>
<tr>
<td>20.0 × 13.2</td>
<td>21.7</td>
<td>21.6</td>
</tr>
</tbody>
</table>

9.2.2.2 Influence of penetrations

It was shown in Chapter 7 that diaphragm performance is not significantly degraded by the presence of a typical single-case stairwell penetration. However penetrations associated
with lift shafts or with more complex stairwell configurations are typically larger in area, indicating that penetration size can vary considerably between URM buildings and thus motivating further investigation into the effects of penetration size on diaphragm performance. To establish a relationship between penetration area and diaphragm performance, the standard diaphragm FE model was reconfigured to comprise different penetration configurations and analysed in each principal loading direction. As outlined in Table 9-5 and illustrated in Figure 9-15, the parametric analysis consisted of five increasing penetration sizes that were positioned at two basic diaphragm locations: (1) the corner of the diaphragm, and (2) the centre-side of the long axis of the diaphragm. The selected penetration sizes and locations were intended to be a manageable representation of

<table>
<thead>
<tr>
<th>Penetration size, ( l_p \times b_p ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>3.2 × 1.08</td>
</tr>
</tbody>
</table>

![Figure 9-15: Schematic of penetrations used for parametric analysis](image-url)
the innumerable possibilities of penetration geometry and position within a diaphragm foot print. It is acknowledged that the larger penetration areas (4 and 5) are somewhat unrealistic but were nonetheless included in the analysis to adequately verify any observed performance trends. It should be noted that for perpendicular-to-joist analysis, the ends of the modelled joists adjacent to the penetrations were assigned roller restraints to simulate the vertical support provided by the timber framing surrounding the penetration, whilst accounting for the loss of lateral restraint due to detachment from the URM perimeter wall.

The parametric analysis results presented in Figure 9-16 indicate that diaphragm effective shear stiffness considerably reduces with increasing penetration area, regardless of location or loading direction. For both parallel-to-joist analysis and perpendicular-to-joist analysis, the corner penetrations were shown to have a slightly greater effect on diaphragm performance, reducing effective shear stiffness by up to 5% more than did the centrally located penetrations with identical geometry. It is possible that the stress concentrations generated by the penetrations can be more efficiently redistributed when the penetration is located closer to the diaphragm centre.

To establish a method for addressing penetrations during diaphragm seismic assessment, the determined shear stiffness values were normalised using Equation 9-3 and plotted against corresponding area ratios that were defined as:

![Diagrams showing shear stiffness values for increasing penetration size](image)

**Figure 9-16: Shear stiffness values for increasing penetration size**
Area ratio \( \Lambda = \frac{A_{d(\text{net})}}{A_{d(\text{gross})}} \)  

where \( A_{d(\text{gross})} \) is the gross area of the diaphragm (i.e. ignoring the area reduction caused by the penetration), and \( A_{d(\text{net})} \) is the net area of the diaphragm, defined as the gross area of the diaphragm less the area of the penetration. The area ratio appropriately normalised the penetration areas so that the results could be applied to different sized diaphragms with any realistic penetration geometry. Figure 9-17 shows the normalised shear stiffness values to be linearly proportional to the area ratio for parallel-to-joist loading, and to have a slight power relationship with the area ratio for perpendicular-to-joist loading. Despite the slight proportionality difference, diaphragm effective shear stiffness is shown in Figure 9-17 to reduce by approximately 30% in either principal loading direction when the area ratio reaches values of approximately 0.75. Considering that diaphragm performance does not appear to be sensitive to penetration location, in order to simplify the seismic assessment procedure reported in Chapter 10 as much as possible, only one modification factor was developed for each principal direction of loading. The modification factors were determined using the corner penetration data because this data conservatively captured the effects of centrally located penetrations also. The following expressions were determined

![Figure 9-17: Relationship between normalised shear stiffness and the ratio between diaphragm net area and diaphragm gross area](image-url)
9.3 Conclusions

The performance of timber floor diaphragms in heritage New Zealand URM buildings was established by programming the developed standard diaphragm FE model with realistic timber material properties and heritage nail connection load-slip data, which was determined from the salvaged nail connection testing reported in Chapter 5. The force-displacement responses generated from the FE analyses were shown to have considerably reduced initial stiffness in both principal loading directions when compared with newly constructed diaphragm test data (see Chapter 7). Despite the reduced stiffness, heritage diaphragms still exhibited significant displacement capacity, reaching drifts of 11.5% and 8.4% in the parallel-to-joist and perpendicular-to-joist directions, respectively, without any signs of strength degradation.

The performance characterisation methodology adopted in Chapter 7 and in previous studies (Peralta 2003) for newly constructed diaphragms was shown to be unsuitable for heritage timber diaphragms due to reduced initial stiffness and displacement incompatibility with the enclosing perimeter walls. Accordingly a new characterisation rationale was developed based upon permissible URM building deformations. Derakhshan (2011) determined that URM walls laterally loaded in their out-of-plane direction are stable up to mid-height displacements equal to 70% of wall thickness. In accordance with Derakhshan’s research, it was determined that the walls of typical one- to three-storey New Zealand heritage URM buildings were stable up to displacements that were 11.5% and 8.4% in the parallel-to-joist and perpendicular-to-joist directions, respectively.

The lines generated from the equations are plotted in Figure 9-17 to indicate accuracy.

\[
c_{p(\text{para})} = 1.21\Lambda - 0.21, \quad \Lambda \geq 0.75 \tag{9-17}
\]

\[
c_{p(\text{perp})} = 1.67\Lambda^2 - 1.76\Lambda - 1.09, \quad \Lambda \geq 0.75 \tag{9-18}
\]

9.3 Conclusions

The performance of timber floor diaphragms in heritage New Zealand URM buildings was established by programming the developed standard diaphragm FE model with realistic timber material properties and heritage nail connection load-slip data, which was determined from the salvaged nail connection testing reported in Chapter 5. The force-displacement responses generated from the FE analyses were shown to have considerably reduced initial stiffness in both principal loading directions when compared with newly constructed diaphragm test data (see Chapter 7). Despite the reduced stiffness, heritage diaphragms still exhibited significant displacement capacity, reaching drifts of 11.5% and 8.4% in the parallel-to-joist and perpendicular-to-joist directions, respectively, without any signs of strength degradation.

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Zealand heritage URM buildings cannot exceed deformations of approximately $\Delta_{cr} = 245$ mm without the potential of collapse. By correlating the determined critical displacement ($\Delta_{cr}$) to the most vulnerable dimension of the standard diaphragm configuration, a critical diaphragm drift of $\theta_{cr} = 4.1\%$ was established as an appropriate limiting criterion for diaphragm performance characterisation.

Upon locating the critical drift limits on the standard diaphragm force-displacement responses shown in Figure 9-2, it was concluded that heritage diaphragm performance is most suitably characterised by a linear-elastic response with an effective shear stiffness of $G_d$ up until the critical drift limit ($\theta_{cr}$).

It was shown that heritage diaphragm strength and displacement capacity cannot be developed within permissible URM building deformation limits, and are therefore redundant performance parameters that should be ignored for seismic assessment. The implication is that diaphragms should be assessed with a ductility of $\mu = 1.0$ and with a corresponding overstrength factor of $\Omega = 1.0$. The shear force resisted by the diaphragm, $R_d$, should be evaluated as a function of diaphragm effective shear stiffness ($G_d$) and expected deformation magnitude ($\Delta_d$).

Using the updated characterisation procedure described above, heritage diaphragms were found to have an effective shear stiffness of 30 kN/m in the parallel-to-joist direction, and 23 kN/m in the perpendicular-to-joist direction. Comparing these shear stiffnesses with the corresponding values of 198 kN/m and 134 kN/m that were determined in Chapter 7 for newly constructed diaphragms, it is evident that the reduced nail connection performance and updated characterisation rationale have had a significant effect on diaphragm assessment parameters.

Using the developed standard diaphragm FE model, a comprehensive parametric analysis was performed to determine the effect that key configuration parameters have on diaphragm performance. The premise of the parametric analysis was to establish the standard diaphragm as the reference configuration and to then consider realistic variations in selected configuration parameters to determine their effect on diaphragm performance.
Individual variations in diaphragm construction details were shown to alter diaphragm shear stiffness by up to ±40%, which indicates that the combined effect of multiple construction detail changes could have a marked impact on diaphragm performance.

Diaphragm size variations can be addressed by considering changes to diaphragm span alone – changes to diaphragm width were shown to have an insignificant effect on diaphragm performance and can therefore be ignored. Diaphragm effective shear stiffness increased by up to 280% for the lower bound span of $L = 4.0$ m, and decreased by up 30% for the upper bound span of $L = 24.0$ m. Diaphragm performance was shown to consistently reduce with increasing penetration area. Corner penetrations were found to have a slightly greater detrimental effect on diaphragm performance than did centrally located penetrations.

Using the parametric analysis results, modification factors were determined for the analysed parameters to appropriately adjust standard diaphragm performance for the seismic assessment procedure that is developed in the following chapter.
Chapter 10

SEISMIC ASSESSMENT OF TIMBER FLOOR DIAPHRAGMS

A procedure to assess the seismic performance of timber floor diaphragms in URM buildings is presented. Diaphragm evaluation is a crucial component within the wider framework of URM building seismic assessment, as predicted diaphragm performance significantly influences global design loads and perimeter wall displacements. Diaphragm seismic assessment requires accuracy to ensure that predicted performance is representative, yet must remain relatively straightforward so that it is not overly time consuming and can be implemented without requiring complex analysis. Such a procedure is necessary because the majority of URM buildings in New Zealand are one to three storeys in height, and of comparatively low financial value (Russell 2010), meaning that assessments involving finite element modelling or similarly complicated analysis are seldom justifiable.

Communication from engineering practitioners indicates that current assessment documents (ASCE 2007; NZSEE 2006) are insufficiently prescriptive and subsequently are difficult to follow. Such observations are compounded by a lack of confidence regarding published performance parameters, which have not been validated through
sufficient experimental testing and suitable numerical analysis. Recognition of these limitations has motivated the development of an improved assessment procedure, which has been the overarching objective for this research.

A critique of the existing diaphragm assessment procedures that are available to engineering practitioners is presented. A discussion regarding the necessary improvements to the existing procedures is subsequently provided to establish a framework with which to develop a revised assessment procedure. Using the developed framework, and the experimental data and numerical analysis reported in this thesis, a harmonised seismic assessment procedure for timber floor diaphragms in URM buildings is presented. Finally, three realistic diaphragm configurations are seismically assessed to suitably demonstrate application of the developed procedure.

10.1 Existing Assessment Procedures

It is understood that New Zealand structural engineers currently refer to the NZSEE (2006) and ASCE 41-06 (2007) documents to perform detailed seismic assessments of heritage timber floor diaphragms. These documents are considered to be the current state-of-the-art in seismic assessment yet the specific origin of the guidelines published for timber diaphragms is unknown. A brief summary of the assessment procedure offered in each document is presented in the following sections. Following correspondence with eminent New Zealand structural engineers, a critique of the presented guidelines is provided to highlight any limitations that require consideration.

10.1.1 NZSEE (2006) assessment method

Diaphragm strength is conventionally calculated as shear strength per lineal metre width of diaphragm, \( R_d \). Appendix 11B of NZSEE describes a methodology to determine diaphragm shear strength from first principles using Equation 10-1 below:
\[ R_d = \frac{Q_n s}{\ell d_f} \]  

(10-1)

where \( Q_n \) is nominal nail capacity, \( s \) is nail couple spacing, \( \ell \) is joist spacing, and \( d_f \) is floorboard width. The NZSEE document does not provide explicit guidance or a recommended reference for the determination of \( Q_n \), but it is understood that designers typically utilise the New Zealand Timber Structures Standard NZS 3603:1993 (SANZ 1993). Once \( Q_n \) is established, the value of \( R_d \) can be readily determined using the known diaphragm configuration parameters. As shown in Appendix D.1 for the mock assessment of the tested full-scale diaphragms reported in Chapter 7, diaphragm shear strength is calculated as approximately \( R_d = 1.4 \text{ kN/m} \) using the described first principles methodology.

A simple alternative to the method described above is offered by NZSEE in the form of default shear strength values corresponding to different diaphragm configurations. The published table of strength values for horizontal diaphragms is reproduced in Table 10-1 for reference. Upon inspection of the most relevant value for timber floor diaphragms in URM buildings (Item 1c), it is evident that the published shear strength value grossly exceeds the corresponding value determined from first principles. It is not understood why such a large discrepancy exists, but it is obvious that such an inconsistency is an undesirable ambiguity associated with the NZSEE assessment procedure.

<table>
<thead>
<tr>
<th>Item</th>
<th>Materials</th>
<th>Strength values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a Roofs with straight sheathing (sarking) and roofing applied directly to the sheathing</td>
<td>6 kN/m</td>
</tr>
<tr>
<td></td>
<td>b Roofs with diagonal sheathing and roofing applied directly to the sheathing</td>
<td>15 kN/m</td>
</tr>
<tr>
<td></td>
<td>c Floors with straight tongue and groove sheathing</td>
<td>6 kN/m</td>
</tr>
<tr>
<td></td>
<td>d Floors and roofs with sheathing and existing plaster renailed to the joists or rafters</td>
<td>6 kN/m</td>
</tr>
</tbody>
</table>
Diaphragm stiffness $K_d$ is not directly determined using the NZSEE guidelines, but rather diaphragm deformation is firstly evaluated followed the determination of stiffness using the following relationship:

$$K_d = \frac{F_d}{\Delta_d} \quad (10-2)$$

where $F_d$ is total diaphragm force, and $\Delta_d$ is midspan diaphragm displacement. Midspan displacement is calculated from first principles using the methodology detailed in Appendix 11A of the NZSEE document. The methodology assumes that flexible diaphragm deformations are primarily governed by nail slip, and thus $\Delta_d$ is evaluated based upon the governing equation:

$$\Delta_d = \frac{Le_n}{2s} \quad (10-3)$$

where $L$ is diaphragm span, and $e_n$ is nail slip resulting from the applied diaphragm shear force $V_d$. The contributions of diaphragm flexural deformation and diaphragm shear deformation are assumed to be negligible, and are therefore ignored. This seemingly simple equation is complicated by the determination of $e_n$, which is unclear, and for which no explicit guidelines are provided. Engineering practitioners have indicated that $e_n$ is typically calculated at the nominal nail capacity using NZS 3603:1993, and then related back to a corresponding diaphragm shear force using Equation 10-1. The displacement and corresponding stiffness of the diaphragm at its strength capacity can subsequently be calculated. It is recognised that $K_d$ is sensitive to the selection of $F_d$ and is thus subject to the structural engineer’s interpretation, further complicating the application and consistency of the NZSEE procedure. An example of diaphragm displacement and stiffness evaluation using the NZSEE procedure is provided in Appendix D.3.

The NZSEE document provides no specific guidance for timber floor diaphragm ductility. It is understood that engineering practitioners typically adopt a nominal ductility of $\mu = 4$ that is suggested in NZSE 3603:1993 for new timber structures.
10.1.2 ASCE 41-06 (2007) assessment method

Diaphragm shear strength \( (R_d) \) in the ASCE 41-06 document is determined from default values that are tabulated for different diaphragm configurations. The default expected strength values for wood diaphragms listed in ASCE 41-06 Table 8-2 are reproduced in Table 10-2 for reference. The imperial units employed by ASCE have been converted to SI units for convenience.

<table>
<thead>
<tr>
<th>Diaphragm type</th>
<th>Shear stiffness, ( G_d ) (kN/m)</th>
<th>Shear strength, ( R_d ) (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single straight sheathing</td>
<td>350</td>
<td>1.75</td>
</tr>
<tr>
<td>Double straight sheathing</td>
<td>Chorded¹</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>Unchorded</td>
<td>1200</td>
</tr>
<tr>
<td>Single diagonal sheathing</td>
<td>Chorded</td>
<td>1400</td>
</tr>
<tr>
<td></td>
<td>Unchorded</td>
<td>700</td>
</tr>
<tr>
<td>Double sheathing with straight sheathing or flooring above</td>
<td>Chorded</td>
<td>3200</td>
</tr>
<tr>
<td></td>
<td>Unchorded</td>
<td>1600</td>
</tr>
<tr>
<td>Double diagonal sheathing</td>
<td>Chorded</td>
<td>3100</td>
</tr>
<tr>
<td></td>
<td>Unchorded</td>
<td>1600</td>
</tr>
</tbody>
</table>

¹ Tension and compression forces are generated when a diaphragm deforms laterally under seismic loading. Chords are structural members of diaphragms that are orientated perpendicular to the imposed seismic loading, and are used to resist the induced tension and compression forces.

Rather than providing a methodology to calculate diaphragm stiffness \( (K_d) \) from first principles, default shear stiffness \( (G_d) \) values are published in ASCE 41-06, which are intended to be independent of diaphragm geometry (see Table 10-2). The shear stiffness values are used to determine diaphragm midspan displacement at yield \( (\Delta_y) \) in accordance with the following equation:
\[ \Delta_y = \frac{v_y L}{2G_d} \]  

(10-4)

where \( v_y \) is diaphragm shear force per lineal metre width at yield in the direction under consideration. Recognising that \( v_y = F_y / 2B \) and that diaphragm stiffness can be determined using Equation 10-2, Equation 10-4 can be rearranged for an expression of diaphragm stiffness:

\[ K_d = \frac{4BG_d}{L} \]  

(10-5)

An example application of Equation 10-5 is provided in Appendix D.1.

ASCE 41-06 addresses diaphragm ductility capacity using component modification factors (\( m \)-factors) that account for the expected level of ductility at different structural performance limit states. The \( m \)-factors associated with the life safety limit state are analogous to the conventional structural ductility factor \( \mu \) adopted for most design procedures. For single layer straight-sheathed diaphragms, the published \( m \)-factor is 2.0 for span to width ratios of less than three, which, from the research of Russell (2010), applies to almost all diaphragm geometries in New Zealand.

### 10.1.3 Discussion

Heritage straight-sheathed timber diaphragms in New Zealand and in the United States are practically indistinguishable. Although minor material property variations undoubtedly exist, given that the URM building stock in both countries are of similar age, the deteriorated performance of heritage diaphragms would ultimately be comparable. Despite the similarities of the target structure, the New Zealand assessment document (NZSEE 2006) and American assessment document (ASCE 2007) were shown to predict vastly different diaphragm performance parameters. To illustrate the identified inconsistencies, a summary of the determined parameters is provided in Table 10-3. It is evident that severe discrepancy exists between the shear strength, shear stiffness and ductility values determined using each assessment document. It is also evident that inconsistency exists
within the NZSEE document itself, which offers two separate methods for diaphragm strength determination that produce very different results (see Table 10-3). It is not understood why such a large discrepancy exists between the published methods, but nonetheless, a lack of consistency is evident.

Table 10-3: Summary of predicted performance parameters

<table>
<thead>
<tr>
<th>Assessment guide</th>
<th>Shear strength, $R_d$</th>
<th>Shear stiffness, $G_d$</th>
<th>Ductility, $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NZSEE (2006)</td>
<td>(i) 1.40 kN/m</td>
<td>97 kN/m</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>(ii) 6.00 kN/m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ASCE 41-06 (2007)</td>
<td>1.75 kN/m</td>
<td>350 kN/m</td>
<td>2.0</td>
</tr>
</tbody>
</table>

1. Diaphragm shear stiffness is not actually determined as part of the NZSEE assessment procedure, but was determined from the corresponding $G_d$ value using Equation 10-5 so that a comparison could be made between the ASCE 41-06 and NZSEE procedures.

Communication from New Zealand structural engineers indicates that the NZSEE procedure summarised in Section 10.1.1 is insufficiently prescriptive, rendering much of the assessment process difficult to follow and subject to interpretation. This observation was demonstrated by the lack of recommendations provided for the determination of nail connection nominal capacity ($Q_n$), nail connection slip ($e_n$), and an appropriate diaphragm ductility ($\mu$), which consequently required best engineering judgement to evaluate, and to apply to the prescribed diaphragm assessment equations.

It was shown in Chapter 4 that the ASCE 41-06 procedure to evaluate diaphragm midspan displacement is based upon an incorrect diaphragm idealisation. Equations 10-4 and 10-5 derive from a shear beam analogy with a centralised point load, which does not suitably represent the out-of-plane wall loads that are tributary to floor diaphragms during an earthquake. Accordingly, a revised diaphragm idealisation was developed in Chapter 4 for the assessment of diaphragm performance. The midspan displacement equation produced from the updated idealisation should be included in any future assessment procedure (viz. Equation 4-38).
A seminal conclusion from the experimental program reported in Chapter 7 and the finite element analysis presented in Chapter 9 was the highly orthotropic behaviour of heritage timber diaphragms. Shear strength and shear stiffness values were shown to be significantly different in each principal loading direction, yet the current assessment procedures offer no provisions to address this behaviour. In order to improve the transparency and accuracy of the assessment procedures, diaphragm performance parameters should be explicitly provided for each principal loading direction.

NZSEE and ASCE 41-06 both state that: ‘The presence of any but small openings in wood diaphragms will cause a reduction in the stiffness and yield capacity of the diaphragm due to a reduced length of diaphragm available to resist lateral forces’. The effect of diaphragm penetrations (i.e. openings) is clearly recognised in each assessment guide, but no provisions are offered to suitably quantify these effects. Considering that almost all floor diaphragms must contain a penetration to enable floor access, explicit guidance on the influence of penetrations is required.

Diaphragm configuration variations are not addressed in the existing assessment procedures. Individual variations in diaphragm construction details were shown in Chapter 9 to alter diaphragm shear stiffness by up to ±40%, which indicates that the combined effect of multiple construction detail changes could have a marked impact on diaphragm performance.

Lastly, structural engineers have requested better guidance on appropriate structural ductility factors and diaphragm deformation limits (Oliver 2010a).

It is evident from the limitations of NZSEE (2006) and ASCE 41-06 (2007) identified above that a harmonised and consolidated seismic assessment procedure is required for timber floor diaphragms. It is intended that such a procedure will encourage transparency and consistency between international assessment documents. Using the framework established in this section, and the representative diaphragm performance parameters reported in Chapter 9, a revised assessment procedure is developed in the following section.
10.2 Revised Assessment Procedure

As previously discussed, the overarching objective of this research was to develop an accurate assessment procedure for timber floor diaphragms in unreinforced masonry buildings, for engineering practitioners to utilise during independent seismic assessments. Using the experimental and numerical analysis presented in this thesis, the described principal objective is achieved in this section. The developed procedure is embodied in a formalised assessment document entitled *Assessment and Improvement of Unreinforced Masonry Buildings for Earthquake Resistance* (NZSEE 2011) that was prepared as part of the umbrella research project that the current study falls under.

The revised diaphragm assessment procedure is presented in this section exactly as it appears in NZSEE (2011), except for section numbering and cross-references, which have been appropriately modified to suit this thesis. Commentary is also provided wherever necessary, to explain the origin or derivation of specific procedural details. The commentary format differs from the formal assessment manual, in which the commentary was published in a companion document. The proposed procedure is divided into two sections: (1) Section 10.2.1 – Material properties of flexible timber floor diaphragms, and (2) Section 10.2.2 – Deformation assessment of flexible timber floor diaphragms.

It should be noted that the development of a formalised retrofit procedure for timber diaphragms was outside the scope of this research. However in collaboration with eminent structural engineers (Oliver 2010a), a set of diaphragm retrofit guidelines was compiled and published in the updated NZSEE (2011) document. The plywood overlay and stapled sheet metal blocking system reported in Chapter 7 was recommended as a cost-effective retrofit solution that can be implemented into the current URM building stock whilst preserving heritage diaphragm construction. The retrofitted diaphragm test results were also published so that designers had a validated indication of expected retrofit performance.

The revised diaphragm fundamental period equation developed in Chapter 4 is also not included in the developed procedure. The evaluation of diaphragm period is central to the
determination of URM building earthquake loads, which has been investigated by Knox (2012). The outcomes of the analysis presented in Chapter 4, namely Equation 4-37, has therefore been incorporated into the procedure for URM building design action determination developed by Knox (2012).

10.2.1 Material properties of flexible timber floor diaphragms

10.2.1.1 Notation

- \( A_{d(\text{gross})} \): Gross area of the diaphragm, \( m^2 \), equal to total diaphragm area including penetration(s)
- \( A_{d(\text{net})} \): Net area of the diaphragm, \( m^2 \), equal to gross diaphragm area less penetration(s)
- \( B \): Diaphragm width, m
- \( c_E \): Modification factor for timber elastic modulus
- \( c_{f_f} \): Modification factor for floorboard thickness
- \( c_{d_f} \): Modification factor for floorboard width
- \( c_{t_j} \): Modification factor for joist thickness
- \( c_{d_j} \): Modification factor for joist depth
- \( c_{t(\text{para})} \): Modification factor for joist spacing, for parallel-to-joist loading
- \( c_{t(\text{perp})} \): Modification factor for joist spacing, for perpendicular-to-joist loading
- \( c_{L(\text{para})} \): Modification factor for diaphragm span, for parallel-to-joist loading
- \( c_{L(\text{perp})} \): Modification factor for diaphragm span, for perpendicular-to-joist loading
- \( c_{p(\text{para})} \): Modification factor for penetrations, for parallel-to-joist loading
- \( c_{p(\text{perp})} \): Modification factor for penetrations, for perpendicular-to-joist loading
- \( d_f \): Floorboard width, m
- \( d_j \): Joist depth, m
- \( E \): Timber elastic modulus, GPa
- \( G_d \): Diaphragm shear stiffness, kN/m
- \( K_d \): Diaphragm stiffness, kN/m
- \( L \): Diaphragm span, m
- \( A \): Penetration area ratio
10.2 Revised Assessment Procedure

\[ \alpha_c \] Modification factor for diaphragm configuration

\[ \alpha_g \] Modification factor for diaphragm geometry

\[ \alpha_p \] Modification factor for diaphragm penetrations

\[ \ell \] Joist spacing, m

10.2.1.2 Scope

This section sets forth recommendations for the determination of material properties of timber diaphragms in URM buildings. Due to the age of URM structures and the difficulty of quantifying decay and workmanship, the material properties of timber diaphragms are highly variable. Whenever possible, the testing of materials to establish their properties is recommended. If resources do not permit testing then typical properties are presented.

A visual assessment of the existing diaphragm shall be conducted in accordance with Section 10.2.1.4 before determining diaphragm characteristic values and diaphragm stiffness in Sections 10.2.1.5 and 10.2.1.6, respectively.

The recommendations set forth in this document are restricted to rectangular (or four-sided) diaphragm geometries. Special engineering assessment is recommended for diaphragms with greater complexity.

10.2.1.3 Classification

Diaphragms in New Zealand URM buildings shall be classified as flexible unless the maximum lateral deformation of the diaphragm is less than half the average interstorey drift of the vertical lateral-force-resisting elements of the associated storey. In the latter case, the recommendations set forth in this document are not applicable, and a suitable analysis procedure that considers either rigid or stiff diaphragms must be adopted.

C10.2.1.3 Classification

Timber floor diaphragms are routinely classified as flexible using the quantitative definition published in ASCE 41-06 (2007):
‘Diaphragms shall be classified as flexible when the maximum horizontal deformation of the diaphragm along its length is more than twice the average interstorey drift of the vertical lateral-force resisting elements of the storey immediately below the diaphragm’

Given that almost all New Zealand URM buildings comprise timber diaphragms, and that these diaphragms consistently satisfy the ASCE 41-06 definition of ‘flexible’, it is appropriate to default to a flexible diaphragm classification, unless it is proven otherwise.

10.2.1.4 Visual assessment

10.2.1.4.1 Configuration assessment

The configuration details outlined in Table 10-4 shall be determined from physical inspection and compared against the relevant values. If one or more configuration values fall outside the suggested range, the diaphragm properties determined in Sections 10.2.1.5 and 10.2.1.6 may not be representative and special engineering assessment is recommended.

Table 10-4: Diaphragm configuration limits

<table>
<thead>
<tr>
<th>Joist size, ( t_j \times d_j ) mm</th>
<th>Joist spacing, ( \ell ) mm</th>
<th>Floorboard(^1) size, ( t_f \times d_f ) mm</th>
<th>Floorboard arrangement</th>
<th>Nail configuration</th>
<th>Cross-bracing</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 – 65 ( \times ) 150 - 400</td>
<td>300 – 650</td>
<td>15 – 30 ( \times ) 100 - 150</td>
<td>Single layer, laid perpendicular-to-joists</td>
<td>Two nail couple spaced at two thirds floorboard width(^2)</td>
<td>Any size</td>
</tr>
</tbody>
</table>

1. Either straight-edge or tongue & groove
2. Nail spacing may vary

C10.2.1.4.1 Configuration assessment

Diaphragm configuration limits were established from the diaphragm characterisation survey reported in Chapter 1.
10.2 Revised Assessment Procedure

10.2.1.4.2 Condition assessment

A condition assessment of the existing diaphragm shall be performed as specified in this section. With respect to diaphragm performance, the integrity of the connections between floorboards and joists is the most critical factor. The condition of at least 50% of the diaphragm and its primary nail connections must be inspected and compared with the qualitative criteria outlined in Table 10-5.

Table 10-5: Diaphragm condition assessment criteria

<table>
<thead>
<tr>
<th>Condition rating</th>
<th>Condition description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>Timber free of Borer; little separation of floorboards; no signs of past water damage; little or no nail rust; floorboard-to-joist connection tight, coherent and unable to wobble</td>
</tr>
<tr>
<td>Fair</td>
<td>Little or no Borer; less than 3 mm of floorboard separation; little or no signs of past water damage; some nail rust but integrity still fair; floorboard-to-joist connection has some but little movement; small degree of timber wear surrounding nail</td>
</tr>
<tr>
<td>Poor</td>
<td>Considerable Borer; floorboard separation greater than 3 mm; water damage evident; nail rust extensive; significant timber degradation surrounding nails; floorboard-to-joist connection appears loose and able to wobble</td>
</tr>
</tbody>
</table>

C10.2.1.4.2 Condition assessment

From the inspection of numerous URM buildings and timber diaphragms during this research, diaphragm condition was observed to vary considerably. The condition assessment rating was designed to provide structural engineers with greater accuracy during characteristic value determination in Section 10.2.1.5, with respect to the specific condition of the diaphragm under consideration. The qualitative condition criteria presented in Table 10-5 were established from observation and in consultation with eminent New Zealand structural engineers. The criteria focus on diaphragm conditions that most affect the performance of the primary nail connections, which have been shown throughout this research to govern overall diaphragm performance.

If the structural engineer is uncertain which condition category the subject diaphragm falls within, it is recommended that a conservative condition rating be selected.
10.2.1.4.3  **Floorboard interface assessment**

An inspection of the existing diaphragm floorboard interfaces shall be performed, with a specific focus on establishing the presence of adhesives. If adhesives are identified within the floorboard interfaces, then suitable modification of the characteristic values published in Section 10.2.1.5 is permitted, although it is conservative to apply no modification.

**C10.2.1.4.3  Floorboard interface assessment**

It was shown in Chapter 6 that varnish, resin or other glue-like products applied to floor diaphragms for floorboard protection can measurably improve diaphragm stiffness by generating composite actions between floorboards. The characteristic values published in Section 10.2.1.5, which derive from diaphragm analysis assuming no mechanical interaction between the floorboards, can therefore be increased to reflect this improved performance, if warranted. Modification of the characteristic values shall be based on adhesive condition and reliability, and the subsequent effect on diaphragm performance using best engineering judgement. A comprehensive analysis of inter-floorboard adhesion effects is outside the scope of this study, but further research is recommended to suitably quantify these effects for inclusion in this assessment procedure.

10.2.1.5  **Characteristic values**

10.2.1.5.1  **Shear stiffness**

Default expected shear stiffness values for timber diaphragms shall be taken from Table 10-6. The selection of diaphragm characteristic values shall be performed with the following considerations:

(a) The direction of earthquake loading shall be considered in each principal direction: (1) parallel-to-joists, and (2) perpendicular-to-joists. Values presented for (1) can be applied when the side joists have adequate connection to the appropriate shear resisting elements (e.g. the perimeter URM walls). Values presented for (2) above may only be applied when the joists have some level of lateral in-plane support at
their ends (e.g. when the joists are pocketed or embedded into the perimeter URM walls). For the case where joists are supported on a perimeter wall ledge, the level of diaphragm engagement is uncertain, and a unique analysis to determine characteristic values is recommended.

(b) For parallel-to-joist loading, diaphragm characteristic values are not influenced by the presence or absence of discontinuous joists. For perpendicular-to-joist loading, the presence of discontinuous joists without mechanical connections shall be considered separately from diaphragms with either continuous joists or with discontinuous joists having reliable mechanical connections.

(c) Diaphragm characteristic values shall be selected in conjunction with the condition rating determined in Section 10.2.1.4.2.

(d) Diaphragm shear stiffness values do not include the contribution of intermediate support systems, such as cross-beams or columns.

---

**C10.2.1.5.1 Shear stiffness**

The default expected shear stiffness values provided in Table 10-6, corresponding to a condition rating of *fair* and *continuous* joist continuity, were taken from the *standard* diaphragm FE analysis reported in Sections 9.1.2 (see Table 9-1).

Heritage timber floor diaphragms were proven in Chapters 7 and 9 to be highly orthotropic. Accordingly shear stiffness values have been provided for each principal loading direction to enable greater refinement during seismic assessment.

The shear stiffness value published for diaphragms having discontinuous joists and being loaded perpendicular-to-joists was taken from Section 9.2.1.5. Joist continuity primarily influences the perpendicular-to-joist performance of diaphragms, so an explicit value has been provided to account for discontinuous joists in this loading direction. The presence of discontinuous joists for parallel-to-joist loading does not significantly affect diaphragm performance, and as such, specific values for continuous and discontinuous joists was not necessary.
As detailed in Section C10.2.1.4.2, diaphragm condition ratings were established to ensure that diaphragm condition was accounted for during seismic assessment. The shear stiffness values corresponding to a condition rating of *good* were determined by multiplying the relevant *fair* shear stiffness values by approximately 1.15 to appropriately reflect improved performance. The shear stiffness values corresponding to a condition rating of *poor* were determined by multiplying the relevant *fair* shear stiffness values by approximately 0.75 to appropriately reflect reduced performance.

10.2.1.5.2 *Ductility*

Diaphragm ductility capacity shall be taken as $\mu = 1.0$ for all diaphragm configurations and condition ratings.

C10.2.1.5.2 *Ductility*

Heritage diaphragm force-displacement response within permissible URM building displacements was shown in Section 9.1.2 to be most appropriately characterised by a linear-elastic response. Because diaphragm displacement capacity cannot be generated within the displacement limits of the URM building, diaphragm ductility must be assigned a value of $\mu = 1.0$.

10.2.1.5.3 *Damping*

Damping ratio shall be taken as $\zeta = 0.05$ (5%) for all diaphragm configurations and loading directions.

C10.2.1.5.3 *Damping*

The impact excitation free-vibration test results reported in Section 7.3.3 provided no evidence against the inherent 5% damping that is conventionally assumed for dynamically responding structures. A damping value of $\zeta = 0.05$ (5%) has therefore been recommended for heritage timber floor diaphragms.
Although diaphragm damping is not utilised during detailed seismic assessment, details have been provided in order to assist structural engineers undertaking independent finite element time-history analyses, whenever such complex analysis is required.

Table 10-6: Default expected shear stiffness values for single-layer straight-sheathed timber diaphragms

<table>
<thead>
<tr>
<th>Direction of loading</th>
<th>Joist continuity</th>
<th>Condition rating</th>
<th>Shear stiffness, $G_d$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel-to-joists</td>
<td>Continuous or discontinuous joists</td>
<td>Good</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fair</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poor</td>
<td>23</td>
</tr>
<tr>
<td>Perpendicular-to-joists</td>
<td>Continuous joists or discontinuous joists with reliable mechanical connection</td>
<td>Good</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fair</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poor</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Discontinuous joists without mechanical connection</td>
<td>Good</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fair</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poor</td>
<td>14</td>
</tr>
</tbody>
</table>

10.2.1.6 Diaphragm stiffness

Diaphragm stiffness shall be calculated in accordance with Equation 10-6.

\[ K_d = \alpha_g \alpha_p \alpha_c \frac{32BG_d}{3L} \quad (10-6) \]

C10.2.1.6 Diaphragm stiffness

Equation 10-6 derived from the diaphragm idealisation analysis presented in Chapter 4, and more specifically, the rearrangement of Equation 9-2 presented in Section 9.1.2. The modification factors $\alpha_g$, $\alpha_p$, and $\alpha_c$ appropriately adjust diaphragm stiffness to account for variations in diaphragm geometry, penetrations, and diaphragm construction details, respectively.
Chapter 10: Seismic Assessment of Timber Floor Diaphragms

10.2.1.6.1 Diaphragm geometry

The influence of diaphragm geometry on diaphragm stiffness shall be accounted for by the modification factor $\alpha_g$, defined by Equation 10-7.

$$\alpha_g = \begin{cases} \alpha_{L(paral)}, & \text{for parallel-to-joist loading} \\ \alpha_{L(perp)}, & \text{for perpendicular-to-joist loading} \end{cases}$$  

(10-7)

where $\alpha_{L(paral)}$ is the modification factor for diaphragm span in the parallel-to-joist loading direction determined in accordance with Equation 10-8, and $\alpha_{L(perp)}$ is the modification factor for diaphragm span in the perpendicular-to-joist loading direction determined in accordance with Equation 10-9.

Modification factors $\alpha_{L(paral)}$ and $\alpha_{L(perp)}$ shall be determined as follows:

$$\alpha_{L(paral)} = 0.39 + \frac{1}{(0.16L - 0.35)}, \quad 4.0 \leq L \leq 24.0$$  

(10-8)

$$\alpha_{L(perp)} = 0.62 + \frac{1}{(0.45L - 1)}, \quad 4.0 \leq L \leq 24.0$$  

(10-9)

where $L$ is diaphragm span in metres.

Note that an allowance is not required for variations in diaphragm width, which has been shown by research to not significantly influence diaphragm shear stiffness.

C10.2.1.6.1 Diaphragm geometry

The modification factors $\alpha_{L(paral)}$ and $\alpha_{L(perp)}$ that account for diaphragm geometry were derived from the diaphragm parametric analysis presented in Section 9.2.2.1.
10.2 Revised Assessment Procedure

10.2.1.6.2 Diaphragm penetrations

The influence of penetrations on diaphragm stiffness shall be accounted for by the modification factor $\alpha_p$, defined by Equation 10-10.

$$\alpha_p = \begin{cases} 
 c_{p(para)}, & \text{for parallel-to-joist loading} \\
 c_{p(perp)}, & \text{for perpendicular-to-joist loading} 
\end{cases} \quad (10-10)$$

where $c_{p(para)}$ is the modification factor for penetrations in the parallel-to-joist loading direction determined in accordance with Equation 10-11, and $c_{p(perp)}$ is the modification factor for penetrations in the perpendicular-to-joist loading direction determined in accordance with Equation 10-12.

Modification factors $c_{p(para)}$ and $c_{p(perp)}$ shall be determined as follows:

$$c_{p(para)} = 1.21\Lambda - 0.21, \quad \Lambda \geq 0.75 \quad (10-11)$$

$$c_{p(perp)} = 1.67\Lambda^2 - 1.76\Lambda + 1.09, \quad \Lambda \geq 0.75 \quad (10-12)$$

where $\Lambda$ is the penetration area ratio determined in accordance with Equation 10-13.

The penetration area ratio $\Lambda$ shall be determined as follows:

$$\Lambda = \frac{A_{d(net)}}{A_{d(gross)}} \quad (10-13)$$

where $A_{d(gross)}$ is the gross area of the diaphragm, m$^2$, equal to total diaphragm area including penetration(s), and $A_{d(net)}$ is the net area of the diaphragm, m$^2$, equal to gross diaphragm area less penetration(s).
Note that diaphragm penetrations cause stress concentrations which can lead to poor diaphragm behaviour if the penetrations are not addressed during assessment and retrofit design. Methods to mitigate stress concentrations generated by diaphragm penetrations are discussed in Section 12.7 of NZSEE (2011).

### C10.2.1.6.2 Diaphragm penetrations

The modification factors $c_{p(par)}$ and $c_{p(perp)}$ that account for diaphragm penetrations were derived from the diaphragm parametric analysis presented in Section 9.2.2.2.

### 10.2.1.6.3 Diaphragm configuration

The influence of diaphragm configuration variations on diaphragm stiffness, within the scope outlined in Section 10.2.1.4.1, shall be accounted for by the modification factor $\alpha_c$, determined in accordance with Equation 10-14.

$$\alpha_c = \begin{cases} 
    c_E c_t f c_d f c_\ell(par) , & \text{for parallel-to-joist loading} \\
    c_E c_t f c_d j c_\ell(perp) , & \text{for perpendicular-to-joist loading}
\end{cases}$$

(10-14)

where $c_E$ is the modification factor for timber elastic modulus determined in accordance with Equation 10-15, $c_t f$ is the modification factor for floorboard thickness determined in accordance with Equation 10-16, $c_d f$ is the modification factor for floorboard width determined in accordance with Equation 10-17, $c_t j$ is the modification factor for joist thickness determined in accordance with Equation 10-18, $c_d j$ is the modification factor for joist depth determined in accordance with Equation 10-19, $c_\ell(par)$ is the modification factor for joist spacing in the parallel-to-joist loading direction determined in accordance with Equation 10-20, and $c_\ell(perp)$ is the modification factor for joist spacing in the perpendicular-to-joist loading direction determined in accordance with Equation 10-21.
The modification factors required to determine $\alpha_c$ shall be determined using the following equations:

\[ c_E = 0.03E + 0.7, \quad 4.0 \leq E \leq 15.0 \]  \hspace{1cm} (10-15)

where $E$ is timber elastic modulus in GPa.

\[ c_{tf} = 14.42t_f + 0.64, \quad 0.015 \leq t_f \leq 0.03 \]  \hspace{1cm} (10-16)

where $t_f$ is floorboard thickness in metres.

\[ c_{df} = 7.56d_f - 0.02, \quad 0.100 \leq d_f \leq 0.150 \]  \hspace{1cm} (10-17)

where $d_f$ is floorboard width in metres.

\[ c_{tj} = 381.65t_j^2 - 19.60t_j + 1.03, \quad 0.040 \leq t_j \leq 0.065 \]  \hspace{1cm} (10-18)

where $t_j$ is joist thickness in metres.

\[ c_{dj} = d_j + 0.70, \quad 0.150 \leq d_j \leq 0.400 \]  \hspace{1cm} (10-19)

where $d_j$ is joist width in metres.

\[ c_{\ell(\text{para})} = -6.78\ell^3 + 12.31\ell^2 - 8.04\ell + 2.68, \quad 0.300 \leq \ell \leq 0.667 \]  \hspace{1cm} (10-20)

\[ c_{\ell(\text{perp})} = -9.82\ell^3 + 18.29\ell^2 - 12.41\ell + 3.66, \quad 0.300 \leq \ell \leq 0.667 \]  \hspace{1cm} (10-21)

where $\ell$ is joist spacing in metres.

---

**C10.2.1.6.3 Diaphragm configuration**

The modification factors provided in Section 10.2.1.6.3 that account for diaphragm
configuration variations were derived from the diaphragm parametric analysis presented in Section 9.2.1.

### 10.2.2 Deformation assessment of flexible timber floor diaphragms

#### 10.2.2.1 Notation

- \( B \) Diaphragm width, m
- \( F_d \) Total diaphragm seismic load, kN
- \( K_d \) Diaphragm stiffness, kN/m (see Section 10.2.1.6)
- \( F_{res} \) Diaphragm force resistance, kN
- \( t_w \) Wall thickness in the out-of-plane loading direction, m
- \( \Delta_d \) Midspan diaphragm deformation, m
- \( \Delta_{max} \) Maximum midspan diaphragm deformation, m
- \( \mu \) Ductility capacity (see Section 10.2.1.5.2)
- \( \%NBS \) Percentage new building standard

#### 10.2.2.2 Scope

This section sets forth recommendations for the determination of lateral deformation and force resistance of timber floor diaphragms in URM buildings.

Equations 10-22 and 10-24 shall only be directly applied to diaphragms that fall within the configuration limits outlined in Section 10.2.1.4.1. The principles associated with Equations 10-22 and 10-24 may be applied to diaphragms outside these limits, but only at the designer’s discretion.

#### 10.2.2.3 Lateral deformation

Floor diaphragm midspan deformation, \( \Delta_d \), shall be determined in accordance with Equation 10-22 for each principal direction of loading.
\[
\Delta_d = \mu \frac{F_d}{K_d}
\]  
(10-22)

where \( \mu = 1.0 \) (as per Section 10.2.1.5.2), and \( F_d \) is the total seismic load applied to the diaphragm.

10.2.2.3.1 Deformation limits

Floor diaphragm lateral deformations shall not exceed 70% of the thickness of the supported out-of-plane URM perimeter walls. Maximum diaphragm deformation, \( \Delta_{max} \), shall therefore be determined in accordance with Equation 10-23.

\[
\Delta_{max} = 0.7 \times t_w
\]  
(10-23)

C10.2.2.3.1 Deformation limits

It was shown in Section 9.1.2 that heritage diaphragm displacement capacity grossly exceeds permissible URM building displacements. The maximum allowable diaphragm deformation \( \Delta_{max} \) was based upon the research of Derakhshan (2011), who determined that URM walls laterally loaded in their out-of-plane direction become unstable at mid-height displacements exceeding 70% of the wall thickness.

10.2.2.4 Force resistance

The force resistance provided by floor diaphragms shall be determined as the product of expected diaphragm midspan deformation and diaphragm stiffness in the direction under consideration, in accordance with Equation 10-24.

\[
F_{res} = K_d \Delta_d
\]  
(10-24)

C10.2.2.4 Force resistance

It was shown in Section 9.1.2 that heritage diaphragms cannot generate their strength capacity within permissible URM building deformation limits. Diaphragm force resistance,
$F_{res}$, corresponds to the force resisted by the diaphragm at the expected midspan displacement amplitude. While force resistance is not necessary for determining diaphragm performance adequacy (displacement limits will always govern), the expected resistance force of the existing diaphragm can be utilised during retrofit design. The maximum force resistance provided by the diaphragm is determined as $F_{res} = K_d \Delta_{max}$.

### 10.2.2.5 %NBS

%NBS shall be considered using Equation 10-25.

$$\%NBS = \frac{\Delta_{max}}{\Delta_d} \times 100$$

(10-25)

### 10.3 Diaphragm Seismic Assessment Examples

The revised seismic assessment procedure presented in Section 10.2 is used to assess the performance of three diaphragm configurations, to demonstrate correct application of the procedure. The predicted performance parameters are compared against corresponding values determined from FE analysis to verify the accuracy of the procedure.

#### 10.3.1 Diaphragm assessment example one

The URM building configuration associated with diaphragm assessment example one is illustrated in Figure 10-1. The overall dimensions of the diaphragm are indicated, as well as joist orientation. The diaphragm does not contain a penetration and has an approximate aspect ratio of 1.5.
Figure 10-1: URM building configuration for diaphragm assessment example one

**Step 1  Define diaphragm and URM wall parameters**

- East-west dimension of diaphragm, $x_d$ 15.4 m
- North-south dimension of diaphragm, $y_d$ 10.3 m
- Joist thickness, $t_j$ 40 mm
- Joist depth, $d_j$ 200 mm
- Floorboard thickness, $t_f$ 30 mm
- Floorboard width, $d_f$ 150 mm
- Nail couple spacing, $s$ 105 mm
- Joist spacing, $\ell$ 550 mm
- Timber elastic modulus, $E$ 8.0 GPa
- URM wall thickness, $t_w$ 330 mm
Step 2  Determine diaphragm seismic loading

The procedure to determine diaphragm seismic loading is reported in Knox (2012). For the purpose of demonstrating the diaphragm assessment procedure, assume the following seismic loads for the north-south and the east-west directions, respectively:

\[ F_{d,NS} = 85 \text{ kN} \]
\[ F_{d,EW} = 65 \text{ kN} \]

Step 3  Configuration and condition assessment

The diaphragm configuration parameters outlined in Step 1 appropriately fall within the configuration limits prescribed in Section 10.2.1.4.1, and thus the proposed assessment procedure is applicable.

From a comprehensive inspection of the diaphragm, the following condition details were reported:

- Some Bora observed in the floorboard and joist timber, but integrity of timber appears to be OK.
- No significant separation of the tongue and groove floorboards evident.
- No signs of present or past water damage.
- Ceiling partially removed. Inspection of underfloor showed slight gapping (< 0.5 mm) between floorboards and joists, with a small amount of movement possible by applying force to joists with hands.
- Individual nails could not be recovered. Nail rusting unknown.

Through comparison of the diaphragm condition details presented above with the condition assessment criteria outlined in Table 10-5, the diaphragm has been assigned a condition rating of \textit{fair}.
Step 4  Determination of diaphragm characteristic values

From visual inspection the diaphragm joists appear to be tightly pocketed into the perimeter URM walls. Given that the $y_d = 10.3$ m dimension of the diaphragm in which the joists are orientated exceeds 6.0 m, the joists are discontinuous and supported on a single midspan timber cross-beam. The discontinuous joists are lapped over the cross-beam without a mechanical connection. The following characteristic values were subsequently established:

\[
\begin{align*}
G_{d,NS} & = 30 \text{ kN/m} \\
G_{d,EW} & = 19 \text{ kN/m} \\
\mu & = 1.0 \\
\zeta & = 0.05
\end{align*}
\]

Step 5  Determination of diaphragm stiffness

Consider north-south direction:

Determine modification factor for diaphragm geometry using Equations 10-7 and 10-8:

\[
c_{L(\text{para})} = 0.39 + \frac{1}{(0.16 \times 15.4 - 0.35)} = 0.86
\]

\[
\alpha_d = 0.86
\]

Determine modification factor for diaphragm geometry using Equations 10-14, 10-15, 10-16, 10-17, and 10-20:

\[
c_E = 0.03 \times 8.0 + 0.7 = 0.94
\]

\[
c_{t_f} = 14.42 \times 0.03 + 0.64 = 1.07
\]
\[ c_{df} = 7.56 \times 0.150 - 0.02 = 1.11 \]
\[ c_{(para)} = -6.78 \times 0.55^3 + 12.31 \times 0.55^2 - 8.04 \times 0.55 + 2.68 = 0.85 \]
\[ \alpha_c = 0.94 \times 1.07 \times 1.11 \times 0.85 = 0.95 \]

Determine diaphragm stiffness in north-south direction using Equation 10-6:

\[ K_{d,NS} = 0.86 \times 1.0 \times 0.95 \times \frac{32 \times 10.3 \times 30}{3 \times 15.4} = 175 \text{ kN/m} \]

Consider east-west direction:

Determine modification factor for diaphragm geometry using Equations 10-7 and 10-9:

\[ c_{L(perp)} = 0.62 + \frac{1}{(0.45 \times 10.3 - 1)} = 0.90 \]
\[ \alpha_g = 0.90 \]

Determine modification factor for diaphragm geometry using Equations 10-14, 10-15, 10-18, 10-19, and 10-21:

\[ c_E = 0.03 \times 8.0 + 0.7 = 0.94 \]
\[ c_{tj} = 381.65 \times 0.04^2 - 19.60 \times 0.04 + 1.03 = 0.86 \]
\[ c_{dj} = 0.200 + 0.70 = 0.9 \]
\[ c_{(perp)} = -9.82 \times 0.55^3 + 18.29 \times 0.55^2 - 12.41 \times 0.55 + 3.66 = 0.73 \]
\[ \alpha_c = 0.94 \times 0.86 \times 0.9 \times 0.73 = 0.53 \]

Determine diaphragm stiffness in east-west direction using Equation 10-6:

\[ K_{d,EW} = 0.90 \times 1.0 \times 0.53 \times \frac{32 \times 15.4 \times 19}{3 \times 10.3} = 145 \text{ kN/m} \]
Step 6  Diaphragm deformation assessment

Consider north-south direction:

Determine diaphragm deformation limit using Equation 10-23:

\[ \Delta_{\text{max,NS}} = 0.7 \times 0.330 = 0.231 \text{ m} \]

Determine diaphragm deformation in the north-south direction using Equation 10-22:

\[ \Delta_{d,NS} = 1.0 \times \frac{85}{175} = 0.486 \text{ m} > 0.231 \text{ m} \quad \therefore \text{NOT OK} \]

Consider east-west direction:

Determine diaphragm deformation limit using Equation 10-23:

\[ \Delta_{\text{max,EW}} = 0.7 \times 0.330 = 0.231 \text{ m} \]

Determine diaphragm deformation in the east-west direction using Equation 10-22:

\[ \Delta_{d,EW} = 1.0 \times \frac{65}{145} = 0.448 \text{ m} > 0.231 \text{ m} \quad \therefore \text{NOT OK} \]

Step 7  Determination of diaphragm force resistance

Consider north-south direction:

\[ F_{\text{res,NS}} = 1.0 \times 175 \times 0.231 = 40.4 \text{ kN} \]

Consider east-west direction:

\[ F_{\text{res,EW}} = 1.0 \times 145 \times 0.231 = 33.5 \text{ kN} \]
Step 8  Consideration of percentage new building standard

Consider north-south direction:

\[ \%NBS = \frac{0.231}{0.486} \times 100 = 47.5\% \]

Consider east-west direction:

\[ \%NBS = \frac{0.231}{0.448} \times 100 = 51.6\% \]

10.3.2  Diaphragm assessment example two

The URM building configuration associated with diaphragm assessment example two is illustrated in Figure 10-2. The overall dimensions of the diaphragm are indicated, as well as joist orientation. The diaphragm contains a penetration measuring 4.0 m × 1.5 m and has an approximate aspect ratio of 3.0.

Figure 10-2: URM building configuration for diaphragm assessment example two
### Step 1 Define diaphragm and URM wall parameters

East-west dimension of diaphragm, $x_d$ 18.0 m  
North-south dimension of diaphragm, $y_d$ 6.0 m  
Penetration size, $l_p \times b_p$ 4.0 m × 1.5 m  
Joist thickness, $t_j$ 65 mm  
Joist depth, $d_f$ 400 mm  
Floorboard thickness, $t_f$ 20 mm  
Floorboard width, $d_f$ 100 mm  
Nail couple spacing, $s$ 70 mm  
Joist spacing, $\ell$ 300 mm  
Timber elastic modulus, $E$ 15.0 GPa  
URM wall thickness, $t_w$ 330 mm

### Step 2 Determine diaphragm seismic loading

The procedure to determine diaphragm seismic loading is reported in Knox (2012). For the purpose of demonstrating the diaphragm assessment procedure, assume the following seismic loads for the north-south and the east-west directions, respectively:

- $F_{d,NS}$ 100 kN
- $F_{d,EW}$ 70 kN

### Step 3 Configuration and condition assessment
The diaphragm configuration parameters outlined in Step 1 appropriately fall within the configuration limits prescribed in Section 10.2.1.4.1, and thus the proposed assessment procedure is applicable.

From a comprehensive inspection of the diaphragm, the following condition details were reported:

- No signs of Bora.
- No significant separation of the tongue and groove floorboards evident.
- No signs of present or past water damage.
- No ceiling present - condition of underfloor appears very good.
- Floorboard-to-joist nail connections removed as part of the enlargement of penetration (for stairwell). Connections tight and unable to wobble with applied force.
- Salvaged nails appear to have minimal rust.

Through comparison of the diaphragm condition details presented above with the condition assessment criteria outlined in Table 10-5, the diaphragm has been assigned a condition rating of good.

Step 4  Determination of diaphragm characteristic values

From visual inspection the diaphragm joists appear to be loosely pocketed into the perimeter URM walls with a lateral clearance of approximately 5 mm. Given that the $y_d = 6.0$ m dimension of the diaphragm in which the joists are orientated equals 6.0 m, the joists span continuously between the URM perimeter walls. The following characteristic values were subsequently established:

\[
G_{d,NS} = 35 \text{ kN/m} \\
G_{d,EW} = 27 \text{ kN/m}
\]
10.3 Diaphragm Seismic Assessment Examples

\[ \mu = 1.0 \]
\[ \zeta = 0.05 \]

**Step 5  Determination of diaphragm stiffness**

Consider north-south direction:

Determine modification factor for diaphragm geometry using Equations 10-7 and 10-8:

\[ c_{L_{\text{para}}} = 0.39 + \frac{1}{(0.16 \times 18.0 - 0.35)} = 0.79 \]
\[ \alpha_{g} = 0.79 \]

Determine modification factor for diaphragm penetration using Equations 10-10, 10-11, and 10-13:

\[ A = \frac{18.0 \times 6.0 - 4.0 \times 1.5}{18.0 \times 6.0} = 0.94 \]
\[ c_{p_{\text{para}}} = 1.21 \times 0.94 - 0.21 = 0.93 \]
\[ \alpha_{p} = 0.93 \]

Determine modification factor for diaphragm configuration using Equations 10-14, 10-15, 10-16, 10-17, and 10-20:

\[ c_{E} = 0.03 \times 15.0 + 0.7 = 1.15 \]
\[ c_{t_{f}} = 14.42 \times 0.02 + 0.64 = 0.93 \]
\[ c_{d_{f}} = 7.56 \times 0.100 - 0.02 = 0.74 \]
\[ c_{l_{\text{para}}} = -6.78 \times 0.3^{3} + 12.31 \times 0.3^{2} - 8.04 \times 0.3 + 2.68 = 1.19 \]
\[ \alpha_c = 1.15 \times 0.93 \times 0.74 \times 1.19 = 0.94 \]

Determine diaphragm stiffness in north-south direction using Equation 10-6:

\[ K_{d,NS} = 0.79 \times 0.93 \times 0.94 \times \frac{32 \times 6.0 \times 35}{3 \times 18.0} = 86 \text{ kN/m} \]

Consider east-west direction:

Determine modification factor for diaphragm geometry using Equations 10-7 and 10-9:

\[ c_{L(\text{perp})} = 0.62 + \frac{1}{(0.45 \times 6.0 - 1)} = 1.21 \]

\[ \alpha_g = 1.21 \]

Determine modification factor for diaphragm penetration using Equations 10-10, 10-12, and 10-13:

\[ \Lambda = \frac{18.0 \times 6.0 - 4.0 \times 1.5}{18.0 \times 6.0} = 0.94 \]

\[ c_{p(\text{perp})} = 1.67 \times 0.94^2 - 1.76 \times 0.94 + 1.09 = 0.91 \]

\[ \alpha_p = 0.91 \]

Determine modification factor for diaphragm configuration using Equations 10-14, 10-15, 10-18, 10-19, and 10-21:

\[ c_E = 0.03 \times 15.0 + 0.7 = 1.15 \]

\[ c_{t_j} = 381.65 \times 0.065^2 - 19.60 \times 0.065 + 1.03 = 1.37 \]

\[ c_{d_j} = 0.400 + 0.70 = 1.1 \]

\[ c_{l(\text{perp})} = -9.82 \times 0.3^3 + 18.29 \times 0.3^2 - 12.41 \times 0.3 + 3.66 = 1.32 \]
$\alpha_e = 1.15 \times 1.37 \times 1.1 \times 1.32 = 2.29$

Determine diaphragm stiffness in east-west direction using Equation 10-6:

$$K_{d,EW} = 1.21 \times 0.91 \times 2.29 \times \frac{32 \times 18.0 \times 27}{3 \times 6.0} = 2179 \text{ kN/m}$$

Step 6  Diaphragm deformation assessment

Consider north-south direction:

Determine diaphragm deformation limit using Equation 10-23:

$$\Delta_{max,NS} = 0.7 \times 0.330 = 0.231 \text{ m}$$

Determine diaphragm deformation in the north-south direction using Equation 10-22:

$$\Delta_{d,NS} = 1.0 \times \frac{100}{86} = 1.163 \text{ m} > 0.231 \text{ m} \quad \therefore \text{ NOT OK}$$

Consider east-west direction:

Determine diaphragm deformation limit using Equation 10-23:

$$\Delta_{max,EW} = 0.7 \times 0.330 = 0.231 \text{ m}$$

Determine diaphragm deformation in the east-west direction using Equation 10-22:

$$\Delta_{d,EW} = 1.0 \times \frac{70}{2179} = 0.032 \text{ m} < 0.231 \text{ m} \quad \therefore \text{ OK}$$
Step 7  Determination of diaphragm force resistance

Consider north-south direction:

\[ F_{res,NS} = 1.0 \times 86 \times 0.231 = 19.9 \text{ kN} \]

Consider east-west direction:

\[ F_{res,EW} = 1.0 \times 2179 \times 0.231 = 503.3 \text{ kN} \]

Step 8  Consideration of percentage new building standard

Consider north-south direction:

\[ \%NBS = \frac{0.231}{1.163} \times 100 = 19.9\% \]

Consider east-west direction:

\[ \%NBS = \frac{0.231}{0.032} \times 100 = 721.9\% \]

10.3.3  Diaphragm assessment example three

The URM building configuration associated with diaphragm assessment example three is illustrated in Figure 10-3. The overall dimensions of the diaphragm are indicated, as well as joist orientation. The diaphragm contains a corner penetration measuring 6.0 m × 4.0 m and has an approximate aspect ratio of 1.0.
Figure 10-3: URM building configuration for diaphragm assessment example three

Step 1  Define diaphragm and URM wall parameters

East-west dimension of diaphragm, $x_d$  
20.25 m

North-south dimension of diaphragm, $y_d$  
20.25 m

Penetration size, $l_p \times b_p$  
6.0 m \times 4.0 m

Joist thickness, $t_j$  
50 mm

Joist depth, $d_j$  
300 mm

Floorboard thickness, $t_f$  
25 mm

Floorboard width, $d_f$  
135 mm
Nail couple spacing, $s$  
Joist spacing, $\ell$  
Timber elastic modulus, $E$  
URM wall thickness, $t_w$

### Step 2  Determine diaphragm seismic loading

The procedure to determine diaphragm seismic loading is reported in Knox (2012). For the purpose of demonstrating the diaphragm assessment procedure, assume the following seismic loads for the north-south and the east-west directions, respectively:

\[
F_{d,NS} = 110 \text{ kN} \\
F_{d,EW} = 95 \text{ kN}
\]

### Step 3  Configuration and condition assessment

The diaphragm configuration parameters outlined in Step 1 appropriately fall within the configuration limits prescribed in Section 10.2.1.4.1, and thus the proposed assessment procedure is applicable.

From a comprehensive inspection of the diaphragm, the following condition details were reported:

- No signs of Bora.
- Slight ($< 2$ mm) separation of the straight-edge floorboards.
- South-east corner of diaphragm appears to have suffered some water damage from past roof leak.
- No ceiling present - condition of underfloor appears OK.
- Floorboard-to-joist nail connections removed as part of the enlargement of penetration (for stairwell). Slight movement of connection possible.
• Slight rusting apparent on salvaged nails.

Through comparison of the diaphragm condition details presented above with the condition assessment criteria outlined in Table 10-5, the diaphragm has been assigned a condition rating of *fair*.

### Step 4 Determination of diaphragm characteristic values

From visual inspection the diaphragm joists appear to be loosely pocketed into the perimeter URM walls with a lateral clearance of approximately 5 mm. The joists are discontinuous and supported on intermediate cross-beams at third span locations. The discontinuous joists are lapped over the cross-beams and appear to have been retrofitted with a four-bolt connection sometime in the past. The following characteristic values were subsequently established:

\[
G_{d,NS} = 23 \text{ kN/m} \\
G_{d,EW} = 30 \text{ kN/m} \\
\mu = 1.0 \\
\zeta = 0.05
\]

### Step 5 Determination of diaphragm stiffness

Consider north-south direction:

Determine modification factor for diaphragm geometry using Equations 10-7 and 10-9:

\[
c_{L(\text{perp})} = 0.62 + \frac{1}{(0.45 \times 20.25 - 1)} = 0.74 \\
\alpha_g = 0.74
\]
Determine modification factor for diaphragm penetration using Equations 10-10, 10-12, and 10-13:

\[ A = \frac{20.25 \times 20.25 - 6.0 \times 4.0}{20.25 \times 20.25} = 0.94 \]

\[ c_{p(\text{perp})} = 1.67 \times 0.94^2 - 1.76 \times 0.94 + 1.09 = 0.91 \]

\[ \alpha_p = 0.91 \]

Determine modification factor for diaphragm configuration using Equations 10-14, 10-15, 10-18, 10-19, and 10-21:

\[ c_E = 0.03 \times 12.0 + 0.7 = 1.06 \]

\[ c_{t_j} = 381.65 \times 0.05^2 - 19.60 \times 0.05 + 1.03 = 1.00 \]

\[ c_{d_j} = 0.30 + 0.70 = 1.00 \]

\[ c_{l(\text{perp})} = -9.82 \times 0.45^3 + 18.29 \times 0.45^2 - 12.41 \times 0.45 + 3.66 = 0.88 \]

\[ \alpha_c = 1.06 \times 1.00 \times 1.00 \times 0.88 = 0.93 \]

Determine diaphragm stiffness in east-west direction using Equation 10-6:

\[ K_{d,NS} = 0.74 \times 0.91 \times 0.93 \times \frac{32 \times 20.25 \times 23}{3 \times 20.25} = 154 \text{kN/m} \]

Consider east-west direction:

Determine modification factor for diaphragm geometry using Equations 10-7 and 10-8:

\[ c_{L(\text{para})} = 0.39 + \frac{1}{0.16 \times 20.25 - 0.35} = 0.74 \]

\[ \alpha_g = 0.74 \]
Determine modification factor for diaphragm penetration using Equations 10-10, 10-11, and 10-13:

\[
A = \frac{20.25 \times 20.25 - 6.0 \times 4.0}{20.25 \times 20.25} = \frac{0.94}{0.94} = 0.94
\]

\[
c_{p(\text{para})} = 1.21 \times 0.94 - 0.21 = 0.93
\]

\[
\alpha_p = \left(\frac{1.21}{0.94}\right) = 0.93
\]

Determine modification factor for diaphragm configuration using Equations 10-14, 10-15, 10-16, 10-17, and 10-20:

\[
c_E = 0.03 \times 12.0 + 0.7 = 1.06
\]

\[
c_{f} = 14.42 \times 0.025 + 0.64 = 1.00
\]

\[
c_{d} = 7.56 \times 0.135 - 0.02 = 1.00
\]

\[
c_{\text{t(\text{para})}} = -6.78 \times 0.45^3 + 12.31 \times 0.45^2 - 8.04 \times 0.45 + 2.68 = 0.94
\]

\[
\alpha_c = 1.06 \times 1.00 \times 1.00 \times 0.94 = 1.00
\]

Determine diaphragm stiffness in north-south direction using Equation 10-6:

\[
K_{d,EW} = 0.74 \times 0.93 \times 1.00 \times \frac{32 \times 20.25 \times 30}{3 \times 20.25} = \frac{220 \text{ kN/m}}{220 \text{ kN/m}}
\]

**Step 6 Diaphragm deformation assessment**

Consider north-south direction:

Determine diaphragm deformation limit using Equation 10-23:

\[
\Delta_{\text{max,NS}} = 0.7 \times 0.330 = 0.231 \text{ m}
\]
Determine diaphragm deformation in the north-south direction using Equation 10-22:

\[ \Delta_{d,NS} = 1.0 \times \frac{110}{154} = 0.714 \text{ m} > 0.231 \text{ m} \quad \therefore \text{NOT OK} \]

Consider east-west direction:

Determine diaphragm deformation limit using Equation 10-23:

\[ \Delta_{\text{max},EW} = 0.7 \times 0.330 = 0.231 \text{ m} \]

Determine diaphragm deformation in the east-west direction using Equation 10-22:

\[ \Delta_{d,EW} = 1.0 \times \frac{95}{220} = 0.432 \text{ m} > 0.231 \text{ m} \quad \therefore \text{NOT OK} \]

**Step 7  Determination of diaphragm force resistance**

Consider north-south direction:

\[ F_{\text{res,NS}} = 1.0 \times 154 \times 0.231 = 35.6 \text{ kN} \]

Consider east-west direction:

\[ F_{\text{res,EW}} = 1.0 \times 220 \times 0.231 = 50.8 \text{ kN} \]

**Step 8  Consideration of percentage new building standard**

Consider north-south direction:

\[ \%NBS = \frac{0.231}{0.714} \times 100 = 32.4\% \]
Consider east-west direction:

\[
\%NBS = \frac{0.231}{0.432} \times 100 = 53.5\%
\]

### 10.3.4 FE model validation

Having been developed directly from the parametric analysis reported in Section 9.2, the modification factors provided in the revised seismic assessment procedure are inherently capable of appropriately adjusting standard diaphragm shear stiffness to account for the relevant configuration difference. However in order to verify whether the individual effect of each diaphragm configuration parameter is uncoupled and that the revised seismic assessment procedure is capable of appropriately adjusting standard diaphragm shear stiffness to account for simultaneous changes to multiple diaphragm configuration parameters, the assessment examples reported in Section 10.2.2.5 must be compared against corresponding FE models. To perform this comparison, FE models of the three diaphragm configurations presented in Section 10.2.2.5 were developed in SAP2000 (CSI 2004). Monotonic pushover analyses were performed on each model in each principal loading direction, from which force-displacement responses were generated. The critical drift limit \(\theta_{cr} = 4.1\%\) was located on each response and the corresponding effective linear-elastic stiffness \(K_d\) was determined (see Figure 10-4). The stiffness values determined from the FE analyses are compared in Table 10-7 against the corresponding

<table>
<thead>
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<th>Assessment example number</th>
<th>Parallel-to-joist analysis</th>
<th>Perpendicular-to-joist analysis</th>
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<td>(K_d) (FEM)</td>
<td>(K_d) (Predict)</td>
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<td>86</td>
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<tr>
<td>Three</td>
<td>215</td>
<td>220</td>
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</tbody>
</table>
Figure 10-4: Force-displacement output data from FE analyses of diaphragm seismic assessment examples
values determined using the detailed assessment procedure, as presented in the preceding section. It is evident from the comparison that less than 5% error exists between the relevant shear stiffness values, indicating that the revised assessment procedure capably predicts the seismic performance of timber diaphragms by appropriately accounting for simultaneous variations in diaphragm geometry, penetration size, and diaphragm configuration.

10.4 Conclusions

New Zealand structural engineers currently refer to assessment documents NZSEE (2006) and ASCE 41-06 (2007) to perform detailed seismic assessments of heritage timber floor diaphragms. A critique of the published assessment procedures has highlighted several limitations that require consideration for the improved assessment of diaphragm seismic performance. The NZSEE and ASCE 41-06 procedures were shown to be inconsistent, offering vastly different performance parameter values for essentially identical diaphragm configurations. Both procedures do not address the orthotropic behaviour of heritage diaphragms, nor do they allow for variations in diaphragm geometry and configuration. The influence of penetrations on diaphragm performance is acknowledged in both documents, but explicit provisions that quantify these effects are not provided. From communication with eminent structural engineers, the procedures are considered to be insufficiently prescriptive, and better guidance has been requested on appropriate ductility factors and diaphragm deformation limits.

Based on the framework established from the identified limitations of the current assessment procedures, a revised seismic assessment procedure was developed for heritage timber floor diaphragms. The procedure was populated with the representative heritage diaphragm performance parameters determined in Section 9.1, and incorporates the modification factors developed in Section 9.2 to account for variations in diaphragm geometry, penetration size, and diaphragm configuration. The developed procedure was formulated based upon the shear beam idealisation presented in Chapter 4, and considers
the diaphragm to respond linear-elastically with a ductility of $\mu = 1.0$, which was shown in Chapter 9 to be the most appropriate characterisation of diaphragm performance. Based upon the work of companion researcher Derakhshan (2010), a suitable diaphragm deformation limitation has also been prescribed, which is based upon the stability of the out-of-plane URM perimeter walls. The procedure also appropriately reflects the conclusion from Chapter 9 that diaphragm stiffness completely governs response because diaphragm strength capacity cannot be generated within permissible URM building displacement limits.

Three worked examples have been provided to demonstrate suitable application of the revised assessment procedure – each for a different diaphragm configuration. The accuracy of the procedure was verified by comparing predicted diaphragm stiffness against the stiffness determined from detailed finite element analysis. The revised assessment procedure was shown to be by highly accurate, with less than 5% error evident between the relevant shear stiffness values.

The revised seismic assessment procedure has been embodied in a formal assessment document entitled *Assessment and Improvement of Unreinforced Masonry Buildings for Earthquake Resistance* (NZSEE 2011), which accompanies the current NZSEE (2006) assessment document for structural engineers to utilise during independent URM building seismic assessments.
Chapter 11

SUMMARY AND CONCLUSIONS

The principal objective of this research was to develop a procedure for the detailed seismic assessment of timber floor diaphragms in URM buildings. Through an integrated experimental and analytical research program investigating the in-plane performance of heritage diaphragms, the described principal objective was successfully achieved in Chapter 10.

An extensive literature review has indicated that currently there is a lack of published research pertaining to the seismic response of heritage timber diaphragms, despite considerable evidence highlighting the importance of diaphragm performance on the overall seismic response of URM buildings. Diaphragm orthotropic behaviour has been largely ignored, and the influence of penetrations on diaphragm performance has not been suitably quantified. The effect of deterioration and out-dated construction materials on diaphragm performance has remained completely unknown.

To gain a deeper understanding of straight-sheathed diaphragm behaviour, a new generalised analytical model was developed which mathematically describes the mechanics of diaphragm deformations in both principal loading directions. The model can be used to determine the linear-elastic or nonlinear monotonic force-displacement response of a diaphragm, and to predict diaphragm deformation at any span location. Using the
analytical model, the idealisation of diaphragm behaviour for fundamental period
determination and for displacement determination in current assessment documents ASCE
41-06 (2007) and NZSEE (2006) were found to be incorrectly formulated. The analytical
model was subsequently used to develop a revised assessment rationale, which is based
upon a suitable shear beam idealisation with an applied parabolic load distribution.

Nail connection testing was performed on newly constructed connections and connections
salvaged from the diaphragms of two URM buildings in New Zealand. A substantial
reduction in stiffness, load carrying capacity, and displacement capacity was observed for
the salvaged connections, indicating that deterioration has a significant effect on nail
connection performance.

A series of racking tests was performed on small-scale diaphragm assemblages to
investigate inter-floorboard friction effects. Frictional resistance between either straight-
edge floorboards or tongue and groove floorboards was found to be insignificant, and
therefore it was concluded that inter-floorboard friction effects should be ignored during
diaphragm seismic assessments. Following the racking tests, eight newly constructed
small-scale diaphragms and two small diaphragm sections extracted from a heritage New
Zealand URM building were pseudo-statically tested in the direction parallel-to-joists. Test
results from newly constructed specimens confirmed that floorboard type is unlikely to
affect full-scale diaphragm performance. Despite being comprised of deteriorated nail
connections, the extracted diaphragm sections were shown to have considerably greater
stiffness than did the newly constructed small-scale diaphragms. It was determined that an
applied floorboard surface treatment created adhesion between the floorboards, generating
composite action which significantly improved performance.

As a capstone to the experimental component of this research, a series of full-scale
diaphragms were pseudo-statically tested. Four as-built diaphragms were constructed, two
of which were tested parallel-to-joists, while the other two were tested perpendicular-to-
joists. After each test was completed, the diaphragms were retrofitted with a plywood
overlay and stapled sheet metal blocking system, and subsequently retested. All as-built
diaphragms exhibited highly flexible behaviour and remained completely serviceable at the
conclusion of testing. The orthotropic behaviour of heritage diaphragm construction was shown to be significant, with up to 32\% reduction in shear stiffness for diaphragms loaded perpendicular-to-joists. Test results indicated that a small stairwell penetration does not significantly reduce diaphragm performance, and that if a reliable mechanical connection is provided for discontinuous joists, then diaphragm performance is practically unaffected by the presence of discontinuous joists. The applied retrofit dramatically improved as-built diaphragm strength and stiffness, and the retrofitted diaphragm was shown to remain satisfactorily serviceable at the demand displacements of a 1/500 year design earthquake. Consequently, the plywood overlay and stapled sheet metal blocking system is recommended as a cost-effective retrofitting technique that can be instituted within New Zealand’s current URM building stock without requiring significant modification or removal of the existing diaphragm.

A methodology to model heritage timber floor diaphragms in the finite element (FE) analysis software SAP2000 (CSI 2004) was developed to enable greater versatility for diaphragm parametric analysis. The basic modelling premise is to recreate the diaphragm geometrical configuration using elastic frame elements at the true centreline locations of the timber framing members. Each floorboard-to-joist nail connection is modelled using nonlinear link elements, which are assigned representative load-slip characteristics to appropriately capture diaphragm nonlinear behaviour. Using the proposed methodology, FE models of the small-scale and full-scale diaphragm tests were developed and programmed with the relevant nail connection load-slip backbone curves that were determined from nail connection testing. Overall the developed FE models demonstrated good agreement with experimental data, thus verifying the suitability of the modelling method. Boundary conditions were shown to considerably influence perpendicular-to-joist diaphragm analysis due to the partial fixity generated through joist pinching within the URM wall pockets. Two methods were proposed to suitably cater for the observed pinching effects, although specific research is required to quantify these effects in a generalised manner. However it was concluded that pinned end restraints are a suitably conservative assumption for perpendicular-to-joist diaphragm analysis.
The performance of timber floor diaphragms in heritage New Zealand URM buildings was established by developing an FE model of a representative diaphragm configuration and programming the model with realistic timber material properties and heritage nail connection load-slip data, which was determined from salvaged nail connection testing. Pushover analysis results showed that diaphragm initial stiffness is considerably reduced in both principal loading directions when compared against the force-displacement response data generated from full-scale diaphragm testing. This is a result that is consistent with the nail connection test observations reported in Chapter 5. A drift of $\theta_{cr} = 4.1\%$ was established as a limiting criterion for diaphragm performance characterisation because out-of-plane wall stability is likely to be severely comprised beyond this limit. It was concluded that due to reduced initial stiffness, heritage diaphragm behaviour is effectively linear-elastic within permissible URM building displacement limits. Diaphragm force-displacement response can therefore be characterised with an effective shear stiffness, $G_d$. It was shown that heritage diaphragm strength and displacement capacity cannot be developed within the critical drift limit. The implication is that diaphragms should be assessed with a ductility of $\mu = 1.0$ and with a corresponding overstrength factor of $\Omega = 1.0$. Using the described characterisation procedure, heritage diaphragms were found to have an effective shear stiffness of $G_d = 30$ kN/m in the parallel-to-joist direction and of $G_d = 23$ kN/m in the perpendicular-to-joist direction.

A comprehensive parametric analysis of diaphragm behaviour was performed. Individual variations in diaphragm construction details were shown to alter diaphragm shear stiffness by up to $\pm 40\%$, which indicates that the combined effect of multiple construction detail changes could have a marked impact on diaphragm performance. Diaphragm width ($B$) was shown to insignificantly influence shear stiffness, while diaphragm span ($L$) had a significant effect. Diaphragm shear stiffness was shown to reduce proportional to increasing penetration area, with corner penetrations having greater detrimental effect than did centrally located penetrations. Modification factors were determined for incorporation into the revised seismic assessment procedure to appropriately adjust diaphragm performance with respect to variations in diaphragm geometry, penetration area, and diaphragm configuration.
Several limitations of the existing assessment documents ASCE 41-06 (2007) and NZSEE (2006) employed by New Zealand structural engineers for timber floor diaphragm seismic assessment were identified. Based on the framework established from the identified limitations, a revised assessment procedure was developed. The major contributions of the updated procedure are described hereafter:

- Performance characterisation appropriately revised to linear-elastic response with shear stiffness of $G_d$ and ductility of $\mu = 1.0$.
- Procedure formulated based upon correct shear beam idealisation with a suitable parabolic load distribution that simulates the out-of-plane wall inertia loads tributary to the diaphragm during an earthquake.
- Published performance parameters that are truly representative of timber floor diaphragms in heritage New Zealand URM buildings.
- Framework to address the orthotropic behaviour of diaphragms.
- Modification factors to appropriately adjust diaphragm performance for potential variations in diaphragm geometry, penetration size, and diaphragm configuration.

The revised assessment procedure is considered to be sufficiently prescriptive and logically arranged to enable the accurate and time-effective assessment of timber floor diaphragm seismic performance. The revised seismic assessment procedure has been embodied in a formal assessment document entitled *Assessment and Improvement of Unreinforced Masonry Buildings for Earthquake Resistance* (NZSEE 2011), which accompanies the current NZSEE (2006) assessment document for structural engineers to utilise during independent URM building seismic assessments.

Although the objectives of the current study were achieved, there remains considerable scope for research to advance the understanding of diaphragm seismic performance and seismic assessment. To facilitate future research direction, a set of recommendations for further research is provided in the following section.
11.1 Future Research

The analytical model developed in Chapter 3 was successfully used to describe diaphragm deformation mechanics and to establish the most suitable idealisation of diaphragm behaviour for assessment purposes. However potential exists to commercialise the model for engineering practitioners to utilise during the assessment of diaphragm seismic performance. Such a model would offer a time-effective alternative to evaluate the nonlinear performance of heritage diaphragms. Appropriate calibration of the nail connection input parameters is required, as well as the development of a user-friendly interface, which could be developed using MATLAB (The Mathworks 2011) or a similarly sophisticated computer software.

In the absence of suitable testing opportunities, heritage diaphragm performance was established from this research by firstly developing a validated FE model of a representative diaphragm configuration, and subsequently programming the model with salvaged nail connection load-slip test data. There remains considerable motivation to test full-scale timber diaphragms from heritage URM buildings, either by extraction or by testing in-situ. Pseudo-static and dynamic testing are both warranted. Test results would provide the necessary data to further validate the heritage diaphragm performance parameters determined from this research and published in the updated assessment procedure.

In addition to the pseudo-static or dynamic testing described above, snap-back testing of heritage diaphragms is recommended to experimentally quantify diaphragm fundamental period, which is central to the determination of URM building earthquake design loads. Snap-back test results could be used to validate the fundamental period assessment equation proposed in Chapter 4, and which has been adopted by Knox (2012).

The stiffnesses of two diaphragm sections extracted from a heritage URM building were shown to be significantly higher than expected due to inter-floorboard adhesion, which was observed to generate composite floorboard actions. A comprehensive analysis of inter-
floorboard adhesion effects was outside the scope of this research, but further testing and analysis is recommended to suitably quantify the influence of applied floor varnishes or resins (or similar) on diaphragm performance. Such research could contribute to the proposed assessment procedure by providing quantifiable allowances for improved diaphragm stiffness due to inter-floorboard bonding if adhesives are present.

The plywood overlay and sheet metal blocking system reported in Chapter 7 was shown to suitably improve diaphragm strength and stiffness based on current design procedures. However, further research is required to further develop cost-effective retrofit solutions for heritage diaphragms with a specific focus on optimising diaphragm stiffness for overall URM building performance. Such research has begun to take place (Brignola 2009; Giongo et al. 2011), but requires additional attention in order to develop a formalised performance-based framework for structural engineers to design suitable diaphragm retrofits, which are guaranteed to improve the seismic performance of the URM building.

The current study provides updated and representative data for diaphragm-only performance. Diaphragm stiffness may be improved substantially by the stiffness contribution of the out-of-plane walls, if effectively connected. Significant contribution would be made to URM building seismic assessment by exploring the system-level effect of combined diaphragm and out-of-plane wall stiffness. Additionally, such research could also provide further validation of the diaphragm displacement limit ($\Delta_{\text{max}} = 0.7t_{\text{wall}}$) that was adopted for this research.

Pocketed joists were shown in Chapter 8 to be a complex boundary condition to idealise for diaphragm FE modelling. In order to include the effects of joist pinching in future timber diaphragm performance analysis, further research is recommended to quantify the rotational fixity generated within the URM wall pockets. Such research might include isolated tests on laterally loaded joists within URM wall pockets, and 3D solid modelling to capture the effects of localised brick and timber crushing, brick movement, and slippage of the joist within the wall pocket. By establishing a generalised modelling procedure to capture pocketed joist restraint, further parametric analysis of diaphragm performance
could be undertaken to develop a suitable procedure to address improved diaphragm performance in the direction perpendicular-to-joists during seismic assessments.

The scope of this research was limited to rectangular (four-sided) diaphragm geometries because these configurations constitute the majority of existing floor diaphragms in New Zealand URM buildings (Russell 2010). Using the developed FE modelling methodology, it is recommended that analysis be conducted on more complex diaphragm geometries such as acute trapezoidal and L-shaped diaphragms, which have also been shown to occur throughout New Zealand (see Russell 2010).

Roof diaphragms in URM buildings typically consist of timber trusses and corrugated roof panelling. A ceiling of timber floorboards was sometimes fastened to the underside of the trusses, but typically the trusses were left exposed. Due to the considerable differences in floor and roof diaphragm construction, the research presented this thesis is not directly applicable to roof diaphragms in URM buildings. In order to develop a corresponding seismic assessment procedure for roof diaphragms, an integrated experimental and analytical research program is required for roof diaphragms.
Chapter 12

REFERENCES


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Appendix A

MATLAB PROGRAMMING FOR DIAPHRAGM ANALYSIS

A.1 MATLAB Code to Determine Linear-Elastic Diaphragm Deformation Profile

% This m.file determines the displacement profile of any straight-
% sheathed diaphragm, given a uniformly distributed load applied in
% either principal direction and a linear-elastic nail connection
% stiffness

% Author: Aaron Wilson
% Date: 18 August 2011

clc
clear all
close all

%% Input values
% Geometric parameters
L = 10400; % Span of diaphragm (mm)
B = 5535; % Depth of diaphragm (mm)
nj = 27; % Number of joists
fb_t = 18; % Floorboard thickness (mm)
fb_d = 135; % Floorboard width (mm)
j_t = 45; % Joist thickness (mm)
j_d = 290; % Joist depth (mm)
s = 95; % Nail spacing (mm)
E = 8000; % Elastic Modulus of timber (N/mm^2)

% Loading direction (Parallel-to-joist=PAR, Perpendicular-to-joist=PERP)
LD = 'PAR';

% Total applied UDL
W = 3; % Uniformly distributed load (N/mm)

% Effective linear-elastic nail stiffness
k = 900; % Effective stiffness (N/mm)

% Position to consider
x = L/2; % Position along floorboard under consideration

%% Automatically generated parameters
I_fb = (fb_t*fb_d^3)/12; % Second moment of floorboard (mm^4)
I_j = (j_d*j_t^3)/12; % Second moment of joist (mm^4)

beta = k*s^2/2; % Stiffness parameter

xnorm = x/L; % Normalised x position

% Parameters specific to loading direction. IF-statements test whether the applied load is parallel-to-joist or perpendicular-to-joist, and subsequently determines the relevant parameters
if strcmp(LD,'PAR') == 1
    l = L/(nj-1); % Joist spacing (centres) (mm)
    nb = floor(B/fb_d); % Number of floorboards
    m = floor(nj/2); % Number of nail connections within half diaphragm span
    w = W/nb; % UDL per floorboard (N/mm)
    betan = beta*L/(E*I_fb); % Normalised stiffness parameter
    if ceil(x/l) <= m
        i = ceil(x/l); % Section of floorboard under consideration
    else
        i = m;
    end
elseif strcmp(LD,'PERP') == 1
    l = B/(nj-1); % Joist spacing (centres) (mm)
    nb = floor(L/fb_d); % Number of floorboards
    m = floor(nb/2); % Number of nail connections within half diaphragm span
    w = W/(nj-1); % UDL per joist (N/mm)
    betan = beta*L/(E*I_j); % Normalised stiffness parameter
    if ceil(x/fb_d) <= m

A.1 MATLAB Code to Determine Linear-Elastic Diaphragm Deformation Profile

i = ceil(x/fb_d);            % Section of floorboard under
% consideration
else
i=m;
end

%% Solve for unknowns using system of equations

% The following IF-statements tests which principal direction the
% diaphragm is being loaded in, then whether there is an even number of
% members across the span of the diaphragm. The relevant code is
% subsequently executed.

% For loading Parallel-to-joist
if strcmp(LD,'PAR') == 1
% For an even number of members
if rem(nj,2)== 0
% Set up coefficient matrix
G = zeros(3*m);

G(1,3) = 1;  
G(2,(3*m-2)) = 1;  
G(2,(3*m-1)) = 1;

r=3;
for n=1:m
G((r),(3*n-2)) = (2*(m-1/2))/betan - (n-1);
G((r),(3*n-1)) = -(m-1/2);
if n>1
for p=1:(n-1)
G((r),(3*p-2)) = -(p-1);
G((r),(3*p-1)) = -(m-1/2);
end
end
r=r+1;
end
if m>1
for j=2:m
G(r,(3*j-5)) = (j-1)^2/(4*(m-1/2)^2);  
G(r,(3*j-4)) = (j-1)/(2*(m-1/2));  
G(r,(3*j-3)) = 1;  
G(r,(3*j-2)) = -(j-1)^2/(4*(m-1/2)^2);  
G(r,(3*j-1)) = -(j-1)/(2*(m-1/2));  
G(r,(3*j)) = -1;
end
r=r+1;
end

if m>1
for j=2:m
G(r,(3*j-5)) = (j-1)/(m-1/2);  
G(r,(3*j-4)) = 1;  
G(r,(3*j-2)) = -(j-1)/(m-1/2);  
G(r,(3*j-1)) = -1;
end

% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
Appendix A: MATLAB Programming for Diaphragm Analysis

```matlab
r=r+1;
end

clear j n p r

% Set up solution matrix
H = zeros(3*m,1);

H(1) = 0;
H(2) = 1/24;

r=3;
for n=1:m
    H(r) = 1/96*(n-1)/(m-1/2)*(n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
    r=r+1;
end

for p=r
    H(p) = 0;
    r=r+1;
end

clear n p r

% For an odd number of members
else
    % Set up coefficient matrix
    G = zeros(3*m);

    G(1,3) = 1;
    G(2,(3*m-2)) = 1;
    G(2,(3*m-1)) = 1;

    r=3;
    for n=1:m
        G((r),(3*n-2)) = (2*m)/betan - (n-1);
        G((r),(3*n-1)) = -m;
        if n>1
            for p=1:(n-1)
                G((r),(3*p-2)) = -(p-1);
                G((r),(3*p-1)) = -m;
            end
        end
        r=r+1;
    end

    if m>1
        for j=2:m
            G(r, (3*j-5)) = (j-1)^2/(4*m^2);
            G(r, (3*j-4)) = (j-1)/(2*m);
            G(r, (3*j-3)) = 1;
            G(r, (3*j-2)) = -(j-1)^2/(4*m^2);
            G(r, (3*j-1)) = -(j-1)/(2*m);
            G(r, (3*j)) = -1;
        end
    end
```

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A.1 MATLAB Code to Determine Linear-Elastic Diaphragm Deformation Profile

```matlab
r=r+1;
end

if m>1
    for j=2:m
        G(r,(3*j-5)) = (j-1)/m;
        G(r,(3*j-4)) = 1;
        G(r,(3*j-2)) = -(j-1)/m;
        G(r,(3*j-1)) = -1;
        r=r+1;
    end
end

clear j n p r

% Set up solution matrix
H = zeros(3*m,1);

H(1) = 0;
H(2) = 1/24;

r=3;
for n=1:m
    H(r) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1));
    r=r+1;
end

for p=r
    H(p) = 0;
    r=r+1;
end

clear n p r

end

% For loading Perpendicular-to-joist
elseif strcmp(LD,'PERP') == 1
    % For an even number of members
    if rem(nb,2)== 0
        % Set up coefficient matrix
        G = zeros(3*m);

        G(1,3) = 1;
        G(2,(3*m-2)) = 1;
        G(2,(3*m-1)) = 1;

        r=3;
        for n=1:m
            G((r),(3*n-2)) = (2*(m-1/2))/betan - (n-1);
            G((r),(3*n-1)) = -(m-1/2);
            if n>1
                for p=1:(n-1)
                    G((r),(3*p-2)) = -(p-1);
                end
            end
        end
```
```matlab
G((r),(3*p-1)) = -(m-1/2);
end
end
r=r+1;
end

if m>1
  for j=2:m
    G(r,(3*j-5)) = (j-1)^2/(4*(m-1/2)^2);
    G(r,(3*j-4)) = (j-1)/(2*(m-1/2));
    G(r,(3*j-3)) = 1;
    G(r,(3*j-2)) = -(j-1)^2/(4*(m-1/2)^2);
    G(r,(3*j-1)) = -(j-1)/(2*(m-1/2));
    G(r,(3*j)) = -1;
    r=r+1;
  end
end
if m>1
  for j=2:m
    G(r,(3*j-5)) = (j-1)/(m-1/2);
    G(r,(3*j-4)) = 1;
    G(r,(3*j-2)) = -(j-1)/(m-1/2);
    G(r,(3*j-1)) = -1;
    r=r+1;
  end
end

clear j n p r

% Set up solution matrix
H = zeros(3*m,1);
H(1) = 0;
H(2) = 1/24;
r=3;
for n=1:m
  H(r) = 1/96*(n-1)/(m-1/2)*(n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
  r=r+1;
end
for p=r
  H(p) = 0;
  r=r+1;
end

clear n p r

% For an odd number of members
else
% Set up coefficient matrix
G = zeros(3*m);
G(1,3) = 1;
```
A.1 MATLAB Code to Determine Linear-Elastic Diaphragm Deformation Profile

\[
\begin{align*}
G(2,(3*m-2)) &= 1; \\
G(2,(3*m-1)) &= 1;
\end{align*}
\]

\[
r=3; \\
\text{for } n=1:m \\
\quad G((r),(3*n-2)) = (2*m)/\text{betan} - (n-1); \\
\quad G((r),(3*n-1)) = -m; \\
\quad \text{if } n>1 \\
\quad \quad \text{for } p=1:(n-1) \\
\quad \quad \quad G((r),(3*p-2)) = -(p-1); \\
\quad \quad \quad G((r),(3*p-1)) = -m; \\
\quad \text{end} \\
\quad r=r+1; \\
\text{end} \\
\text{if } m>1 \\
\quad \text{for } j=2:m \\
\quad \quad G((r),(3*j-5)) = (j-1)^2/(4*m^2); \\
\quad \quad G((r),(3*j-4)) = (j-1)/(2*m); \\
\quad \quad G((r),(3*j-3)) = 1; \\
\quad \quad G((r),(3*j-2)) = -(j-1)^2/(4*m^2); \\
\quad \quad G((r),(3*j-1)) = -(j-1)/(2*m); \\
\quad \quad G((r),(3*j)) = -1; \\
\quad \quad r=r+1; \\
\quad \text{end} \\
\text{end} \\
\text{end} \\
\text{clear } j \ n \ p \ r \\
\%	ext{ Set up solution matrix} \\
H = \text{zeros}(3*m,1); \\
H(1) = 0; \\
H(2) = 1/24; \\
\]

\[
r=3; \\
\text{for } n=1:m \\
\quad H((r)) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1)); \\
\quad r=r+1; \\
\text{end} \\
\text{for } p=r \\
\quad H((p)) = 0; \\
\quad r=r+1; \\
\text{end}
\]
Appendix A: MATLAB Programming for Diaphragm Analysis

clear n p r
end
end

% Determine matrix of unknown coefficients, Ai, Bi, and Ci
X = G\H;

% Displacements

if strcmp(LD,'PAR') == 1
    if x >= (i-1)*l && x <= i*l || x <= L/2
        v = w*L^4/(E*I_fb)*(1/24*xnorm^4 - 1/12*xnorm^3 ... 
        + X(3*i-2)*xnorm^2 + X(3*i-1)*xnorm + X(3*i));
    end
else
    if strcmp(LD,'PERP') == 1
        if x >= (i-1)*fb_d && x <= i*fb_d || x <= L/2
            v = w*L^4/(E*I_j)*(1/24*xnorm^4 - 1/12*xnorm^3 ... 
            + X(3*i-2)*xnorm^2 + X(3*i-1)*xnorm + X(3*i));
        end
    end
end

disp(['Displacement at span location ',num2str(x),'mm: ','v = ',... 
    num2str(v),'mm']);

% Displacement profile

if strcmp(LD,'PAR') == 1
    % Consider half diaphragm span
    incr = 50;                         % Number of increments per section
    x_prof = ((0:l/incr:L/2)/L)';      % Normalised span location vector
    vn = zeros(1,length(x_prof));      % Set up displacement vector
    for j=1:length(x_prof)             % Loop to determine displacements
        if j/incr <= m
            n=ceil(j/incr);
        else
            n=m;
        end
        vn(j) = w*L^4/(E*I_fb)*(1/24*x_prof(j)^4 - 1/12*x_prof(j)^3 ... 
            + X(3*n-2)*x_prof(j)^2 + X(3*n-1)*x_prof(j) + X(3*n));
    end
    clear j n incr
else
    if strcmp(LD,'PERP') == 1
        % Consider half diaphragm span
        incr = 15;                         % Number of increments per section
        x_prof = ((0:fb_d/incr:L/2)/L)';    % Normalised span location vector
        vn = zeros(1,length(x_prof));      % Set up displacement vector
        for j=1:length(x_prof)             % Loop to determine displacements
            if j/incr <= m
                n=ceil(j/incr);
            end
            vn(j) = w*L^4/(E*I_j)*(1/24*x_prof(j)^4 - 1/12*x_prof(j)^3 ... 
                + X(3*n-2)*x_prof(j)^2 + X(3*n-1)*x_prof(j) + X(3*n));
        end
    end
end
A.2 MATLAB Code to Determine Linear-Elastic Diaphragm Force-Displacement Response

% This m.file determines the force-displacement response of any straight-
% sheathed diaphragm, given a uniformly distributed load applied in
% either principal direction and a linear-elastic nail connection
% stiffness

% Author: Aaron Wilson
% Date: 18 August 2011

clc
clear all
close all

%% Input values

% Geometric parameters
L = 10400; % Span of diaphragm (mm)
B = 5535; % Depth of diaphragm (mm)
nj = 27;  % Number of joists
fb_t = 18; % Floorboard thickness (mm)
fb_d = 135; % Floorboard width (mm)
j_t = 45; % Joist thickness (mm)
j_d = 290; % Joist depth (mm)
s = 95; % Nail spacing (mm)
E = 8000; % Elastic Modulus of timber (N/mm^2)
% Loading direction (Parallel-to-joist=PAR, Perpendicular-to-joist=PERP)
LD = 'PAR';

% Total applied UDL schedule (must start from a non-zero positive value)
Lincr = 0.1;                        % Loading increment (N)
Wmax = 3.5;                         % Maximum applied UDL (N/mm)
W = (0:Lincr:Wmax)';                % UDL schedule (N/mm)
clear Lincr

% Effective linear-elastic nail stiffness
k = 900;                            % Effective stiffness (N/mm)

% Position to consider
x = L/2;                            % Position along floorboard under
% consideration

%%% Automatically generated parameters
I_fb = (fb_t*fb_d^3)/12;            % Second moment of floorboard (mm^4)
I_j = (j_d*j_t^3)/12;               % Second moment of joist (mm^4)

beta = k*s^2/2;                     % Stiffness parameter

xnorm = x/L;                        % Normalised x position

% Parameters specific to loading direction. IF-statements test whether
% the applied load is parallel-to-joist or perpendicular-to-joist, and
% subsequently determines the relevant parameters
if strcmp(LD,'PAR') == 1
    l = L/(nj-1);                   % Joist spacing (centres) (mm)
    nb = floor(B/fb_d);             % Number of floorboards
    m = floor(nj/2);                % Number of nail connections within
% half diaphragm span
    w = W(nb);                      % UDL per floorboard (N/mm)
    betan = beta*L/(E*I_fb);        % Normalised stiffness parameter
    if ceil(x/l) <= m
        i = ceil(x/l);               % Section of floorboard under
    % consideration
    else
        i=m;
    end
elseif strcmp(LD,'PERP') == 1
    l = B/(nj-1);                   % Joist spacing (centres) (mm)
    nb = floor(L/fb_d);             % Number of floorboards
    m = floor(nb/2);                % Number of nail connections within
% half diaphragm span
    w = W/(nj-1);                   % UDL per joist (N/mm)
    betan = beta*L/(E*I_j);         % Normalised stiffness parameter
if ceil(x/fb_d) <= m
    i = ceil(x/fb_d);           % Section of floorboard under
    % consideration
else
    i=m;
end

%% Solve for unknowns using system of equations

% The following FOR loop determines the displacement of the span position
% under consideration for each loading increment
v = zeros(length(w),1);
for q=1:length(w)
    % The following IF-statements tests which principal direction the
    % diaphragm is being loaded in, then whether there is an even number of
    % members across the span of the diaphragm. The relevant code is
    % subsequently executed.
    % For loading Parallel-to-joist
    if strcmp(LD,'PAR') == 1
        % For an even number of members
        if rem(nj,2)== 0
            % Set up coefficient matrix
            G = zeros(3*m);
            G(1,3) = 1;
            G(2,(3*m-2)) = 1;
            G(2,(3*m-1)) = 1;
            r=3;
            for n=1:m
                G((r),(3*n-2)) = (2*(m-1/2))/betan - (n-1);
                G((r),(3*n-1)) = -(m-1/2);
                if n>1
                    for p=1:(n-1)
                        G((r),(3*p-2)) = -(p-1);
                        G((r),(3*p-1)) = -(m-1/2);
                    end
                end
                r=r+1;
            end
            if m>1
                for j=2:m
                    G((r),(3*j-5)) = (j-1)^2/(4*(m-1/2)^2);
                    G((r),(3*j-4)) = (j-1)/(2*(m-1/2));
                    G((r),(3*j-3)) = 1;
                    G((r),(3*j-2)) = -(j-1)^2/(4*(m-1/2)^2);
                    G((r),(3*j-1)) = -(j-1)/(2*(m-1/2));
                    G((r,(3*j))) = -1;
                end
            end
        end
    end
end
Appendix A: MATLAB Programming for Diaphragm Analysis

```matlab
if m>1
    for j=2:m
        G(r,(3*j-5)) = (j-1)/(m-1/2);
        G(r,(3*j-4)) = 1;
        G(r,(3*j-2)) = -(j-1)/(m-1/2);
        G(r,(3*j-1)) = -1;
        r=r+1;
    end
end
clear j n p r

% Set up solution matrix
H = zeros(3*m,1);
H(1) = 0;
H(2) = 1/24;

r=3;
for n=1:m
    H(r) = 1/96*(n-1)/(m-1/2)*(n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
    r=r+1;
end
for p=r
    H(p) = 0;
    r=r+1;
end
clear n p r

% For an odd number of members
else
    % Set up coefficient matrix
    G = zeros(3*m);
    G(1,3) = 1;
    G(2,(3*m-2)) = 1;
    G(2,(3*m-1)) = 1;
    r=3;
    for n=1:m
        G((r),(3*n-2)) = (2*m)/betan - (n-1);
        G((r),(3*n-1)) = -m;
        if n>1
            for p=1:(n-1)
                G((r),(3*p-2)) = -(p-1);
                G((r),(3*p-1)) = -m;
            end
        end
        r=r+1;
    end
end
```
A.2 MATLAB Code to Determine Linear-Elastic Diaphragm Force-Displacement Response

```matlab
if m>1
    for j=2:m
        G(r,(3*j-5)) = (j-1)^2/(4*m^2);
        G(r,(3*j-4)) = (j-1)/(2*m);
        G(r,(3*j-3)) = 1;
        G(r,(3*j-2)) = -(j-1)^2/(4*m^2);
        G(r,(3*j-1)) = -(j-1)/(2*m);
        G(r,(3*j)) = -1;
        r=r+1;
    end
end

if m>1
    for j=2:m
        G(r,(3*j-5)) = (j-1)/m;
        G(r,(3*j-4)) = 1;
        G(r,(3*j-2)) = -(j-1)/m;
        G(r,(3*j-1)) = -1;
        r=r+1;
    end
end

clear j n p r

% Set up solution matrix
H = zeros(3*m,1);

H(1) = 0;
H(2) = 1/24;

r=3;
for n=1:m
    H(r) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1));
    r=r+1;
end

for p=r
    H(p) = 0;
    r=r+1;
end

clear n p r

end

elseif strcmp(LD,'PERP') == 1
    % For an even number of members
    if rem(nb,2) == 0
        % Set up coefficient matrix
        G = zeros(3*m);
        G(1,3) = 1;
    end
```
Appendix A: MATLAB Programming for Diaphragm Analysis

\[ G(2, (3m-2)) = 1; \]
\[ G(2, (3m-1)) = 1; \]

\[ r=3; \]
\[ \text{for } n=1:m \]
\[ G((r), (3n-2)) = (2(m-1/2))/\text{betan} - (n-1); \]
\[ G((r), (3n-1)) = -(m-1/2); \]
\[ \text{if } n>1 \]
\[ \text{for } p=1:(n-1) \]
\[ G((r), (3p-2)) = -(p-1); \]
\[ G((r), (3p-1)) = -(m-1/2); \]
\[ \text{end} \]
\[ r=r+1; \]
\[ \text{end} \]
\[ \text{if } m>1 \]
\[ \text{for } j=2:m \]
\[ G([r], (3j-5)) = (j-1)^2/(4(m-1/2)^2); \]
\[ G([r], (3j-4)) = (j-1)/(2(m-1/2)); \]
\[ G([r], (3j-3)) = 1; \]
\[ G([r], (3j-2)) = -(j-1)^2/(4(m-1/2)^2); \]
\[ G([r], (3j-1)) = -(j-1)/(2(m-1/2)); \]
\[ G([r], (3j)) = -1; \]
\[ r=r+1; \]
\[ \text{end} \]
\[ \text{end} \]

\[ \text{clear } j \ n \ p \ r \]

\% Set up solution matrix
\[ H = \text{zeros}(3m, 1); \]

\[ H(1) = 0; \]
\[ H(2) = 1/24; \]

\[ r=3; \]
\[ \text{for } n=1:m \]
\[ H(r) = 1/96*(n-1)/(m-1/2)*n^2*(n-1)/(2*(m-1/2)) - n*(2*n-1)); \]
\[ r=r+1; \]
\[ \text{end} \]

\[ \text{for } p=r \]
\[ H(p) = 0; \]
\[ r=r+1; \]
\[ \text{end} \]
clear n p r

% For an odd number of members
else

% Set up coefficient matrix
G = zeros(3*m);

G(1,3) = 1;
G(2,(3*m-2)) = 1;
G(2,(3*m-1)) = 1;

r=3;
for n=1:m
    G((r),(3*n-2)) = (2*m)/betan - (n-1);
    G((r),(3*n-1)) = -m;
    if n>1
        for p=1:(n-1)
            G((r),(3*p-2)) = -(p-1);
            G((r),(3*p-1)) = -m;
        end
    end
    r=r+1;
end

if m>1
    for j=2:m
        G(r,(3*j-5)) = (j-1)^2/(4*m^2);
        G(r,(3*j-4)) = (j-1)/(2*m);
        G(r,(3*j-3)) = 1;
        G(r,(3*j-2)) = -(j-1)^2/(4*m^2);
        G(r,(3*j-1)) = -(j-1)/(2*m);
        G(r,(3*j)) = -1;
        r=r+1;
    end
end

if m>1
    for j=2:m
        G(r,(3*j-5)) = (j-1)/m;
        G(r,(3*j-4)) = 1;
        G(r,(3*j-2)) = -(j-1)/m;
        G(r,(3*j-1)) = -1;
        r=r+1;
    end
end

clear j n p r

% Set up solution matrix
H = zeros(3*m,1);

H(1) = 0;
H(2) = 1/24;
Appendix A: MATLAB Programming for Diaphragm Analysis

r=3;
for n=1:m
    H(r) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1));
    r=r+1;
end

for p=r
    H(p) = 0;
    r=r+1;
end

clear n p r

% Determine matrix of unknown coefficients, Ai, Bi, and Ci
X = G\H;

%% Displacements
if strcmp(LD,'PAR') == 1
    if x >= (i-1)*l && x <= i*l || x <= L/2
        v = w*L^4/(E*I_fb)*(1/24*xnorm^4 - 1/12*xnorm^3 + X(3*i-2)*xnorm^2 + X(3*i-1)*xnorm + X(3*i));
    end
elseif strcmp(LD,'PERP') == 1
    if x >= (i-1)*fb_d && x <= i*fb_d || x <= L/2
        v = w*L^4/(E*I_j)*(1/24*xnorm^4 - 1/12*xnorm^3 + X(3*i-2)*xnorm^2 + X(3*i-1)*xnorm + X(3*i));
    end
end
disp(v);

disp(['Maximum displacement at span location ',num2str(x),'mm: ',num2str(v(end)),'mm']);

%% Force-Displacement Plot

% Total applied load
P = W*L/1000;

figure
plot(v,P,'Color','r');
xlabel('Diaphragm displacement (mm)'); ylabel('Total applied load (kN)');
title('Linear-Elastic Force-Displacement');
A.3 MATLAB Code to Determine Nonlinear Diaphragm Deformation Profile

```
% This m.file determines the displacement profile of any straight-
% sheathed diaphragm, given a uniformly distributed load applied in
% either principal direction and a nonlinear nail connection load-slip
% relationship

% Author: Aaron Wilson
% Date: 22 August 2011

clc
clear all
close all

%% Input values
% Geometric parameters
L = 10400;                          % Span of diaphragm (mm)
B = 5535;                           % Depth of diaphragm (mm)
nj = 27;                            % Number of joists
fb_t = 18;                          % Floorboard thickness (mm)
fb_d = 135;                         % Floorboard width (mm)
j_t = 45;                           % Joist thickness (mm)
j_d = 290;                          % Joist depth (mm)
s = 95;                             % Nail spacing (mm)
E = 8000;                           % Elastic Modulus of timber (N/mm^2)

% Loading direction (Parallel-to-joist=PAR, Perpendicular-to-joist=PERP)
LD = 'PAR';

% Total applied UDL
W = 3.5;                            % Uniformly distributed load(N/mm)

% Effective linear-elastic nail stiffness
k = 900;                            % Effective stiffness(N/mm)

% Nonlinear load-slip characteristic coefficients for nails, taken from
% Dolan & Madsen (1992) for the load-slip relationship developed by
% Foschi (1974)
K0 = 1182;                           % Initial stiffness (N/mm)
F0 = 920;                            % Intitial force (N)
K1 = 50;                             % Secondary stiffness (N/mm)

% Position to consider
x = L/2;                             % Position along floorboard under
% consideration

tol = 1e-10;

%% Automatically generated parameters
```
Appendix A: MATLAB Programming for Diaphragm Analysis

\[ I_{fb} = \frac{(fb_t*fb_d^3)}{12}; \quad \text{Second moment of floorboard (mm}^4)\]

\[ I_{j} = \frac{(j_d*j_t^3)}{12}; \quad \text{Second moment of joist (mm}^4)\]

\[ \beta = \frac{k*s^2}{2}; \quad \text{Stiffness parameter} \]

\[ b_1 = F_0; \quad \text{Simplified coefficients} \]

\[ b_2 = K_1*s/2; \]

\[ b_3 = K_0*s/(2*F_0); \]

\[ x_{norm} = \frac{x}{L}; \quad \text{Normalised x position} \]

% Parameters specific to loading direction. IF-statements test whether
% the applied load is parallel-to-joist or perpendicular-to-joist, and
% subsequently determines the relevant parameters

if strcmp(LD,'PAR') == 1
    \[ l = L/(nj-1); \quad \text{Joist spacing (centres) (mm)} \]
    \[ nb = \text{floor}(B/fb_d); \quad \text{Number of floorboards} \]
    \[ m = \text{floor}(nj/2); \quad \text{Number of nail connections within} \]
    \[ \text{half diaphragm span} \]
    \[ w = \frac{W}{nb}; \quad \text{UDL per floorboard (N/mm)} \]
    \[ Q = \frac{w*L^3}{(E*I_{fb})}; \quad \text{Simplified coefficients} \]
    \[ b_0 = \frac{s}{(E*I_{fb})}; \]
    \[ \beta_{norm} = \frac{\beta*L}{(E*I_{fb})}; \quad \text{Normalised stiffness parameter} \]
    \[ \text{if } \text{ceil}(x/l) \leq m \]
        \[ i = \text{ceil}(x/l); \quad \text{Section of floorboard under consideration} \]
    \[ \text{else} \]
        \[ i = m; \]
    \[ \text{end} \]
elseif strcmp(LD,'PERP') == 1
    \[ l = B/(nj-1); \quad \text{Joist spacing (centres) (mm)} \]
    \[ nb = \text{floor}(L/fb_d); \quad \text{Number of floorboards} \]
    \[ m = \text{floor}(nb/2); \quad \text{Number of nail connections within} \]
    \[ \text{half diaphragm span} \]
    \[ w = \frac{W}{(nj-1)}; \quad \text{UDL per joist (N/mm)} \]
    \[ Q = \frac{w*L^3}{(E*I_j)}; \quad \text{Simplified coefficients} \]
    \[ b_0 = \frac{s}{(E*I_j)}; \]
    \[ \beta_{norm} = \frac{\beta*L}{(E*I_j)}; \quad \text{Normalised stiffness parameter} \]
    \[ \text{if } \text{ceil}(x/fb_d) \leq m \]
        \[ i = \text{ceil}(x/fb_d); \quad \text{Section of floorboard under consideration} \]
    \[ \text{else} \]
        \[ i = m; \]
    \[ \text{end} \]
end
%% Solve for elastic coefficients for the initial estimate of unknown nonlinear coefficients

% The following IF-statements tests which principal direction the diaphragm is being loaded in, then whether there is an even number of members across the span of the diaphragm. The relevant code is subsequently executed.

% For loading Parallel-to-joist
if strcmp(LD,'PAR') == 1
  % For an even number of members
  if rem(nj,2)== 0
    % Set up coefficient matrix
    R = zeros(3*m);

    R(1,3) = 1;
    R(2,(3*m-2)) = 1;
    R(2,(3*m-1)) = 1;
    r=3;
    for n=1:m
      R((r),(3*n-2)) = (2*(m-1/2))/betan - (n-1);
      R((r),(3*n-1)) = -(m-1/2);
      if n>1
        for p=1:(n-1)
          R((r),(3*p-2)) = -(p-1);
          R((r),(3*p-1)) = -(m-1/2);
        end
        r=r+1;
      end
    end
    if m>1
      for j=2:m
        R((r),(3*j-5)) = (j-1)^2/(4*(m-1/2)^2);
        R((r),(3*j-4)) = (j-1)/(2*(m-1/2));
        R((r),(3*j-3)) = 1;
        R((r),(3*j-2)) = -(j-1)^2/(4*(m-1/2)^2);
        R((r),(3*j-1)) = -(j-1)/(2*(m-1/2));
        R((r),(3*j)) = -1;
        r=r+1;
      end
    end
  end
  if m>1
    for j=2:m
      R((r),(3*j-5)) = (j-1)/(m-1/2);
      R((r),(3*j-4)) = 1;
      R((r),(3*j-2)) = -(j-1)/(m-1/2);
      R((r),(3*j-1)) = -1;
      r=r+1;
    end
  end
end

clear j n p r
% Set up solution matrix
G = zeros(3*m,1);
G(1) = 0;
G(2) = 1/24;

r=3;
for n=1:m
    G(r) = 1/96*(n-1)/(m-1/2)*(n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
    r=r+1;
end
for p=r
    G(p) = 0;
    r=r+1;
end
clear n p r

% For an odd number of joists
else
    % Set up coefficient matrix
    R = zeros(3*m);
    R(1,3) = 1;
    R(2,(3*m-2)) = 1;
    R(2,(3*m-1)) = 1;

    r=3;
    for n=1:m
        R((r),(3*n-2)) = (2*m)/betan - (n-1);
        R((r),(3*n-1)) = -m;
        if n>1
            for p=1:(n-1)
                R((r),(3*p-2)) = -(p-1);
                R((r),(3*p-1)) = -m;
            end
        end
        r=r+1;
    end
    if m>1
        for j=2:m
            R(r,(3*j-5)) = (j-1)^2/(4*m^2);
            R(r,(3*j-4)) = (j-1)/(2*m);
            R(r,(3*j-3)) = 1;
            R(r,(3*j-2)) = -(j-1)^2/(4*m^2);
            R(r,(3*j-1)) = -(j-1)/(2*m);
            R(r,(3*j)) = -1;
            r=r+1;
        end
    end
    if m>1
for j=2:m
  R(r,(3*j-5)) = (j-1)/m;
  R(r,(3*j-4)) = 1;
  R(r,(3*j-2)) = -(j-1)/m;
  R(r,(3*j-1)) = -1;
  r=r+1;
end
end

clear j n p r

% Set up solution matrix
G = zeros(3*m,1);

G(1) = 0;
G(2) = 1/24;

r=3;
for n=1:m
  G(r) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1));
  r=r+1;
end

for p=r
  G(p) = 0;
  r=r+1;
end

clear n p r

end

% For loading Perpendicular-to-joist
elseif strcmp(LD,'PERP') == 1
  % For an even number of floorboards
  if rem(nb,2)== 0
    % Set up coefficient matrix
    R = zeros(3*m);
    R(1,3) = 1;
    R(2,(3*m-2)) = 1;
    R(2,(3*m-1)) = 1;

    r=3;
    for n=1:m
      R((r),(3*n-2)) = (2*(m-1/2))/betan - (n-1);
      R((r),(3*n-1)) = -(m-1/2);
      if n>1
        for p=1:(n-1)
          R((r),(3*p-2)) = -(p-1);
          R((r),(3*p-1)) = -(m-1/2);
        end
      end
      r=r+1;
    end
  end
end
if m>1
    for j=2:m
        R(r,(3*j-5)) = (j-1)^2/(4*(m-1/2)^2);
        R(r,(3*j-4)) = (j-1)/(2*(m-1/2));
        R(r,(3*j-3)) = 1;
        R(r,(3*j-2)) = -(j-1)^2/(4*(m-1/2)^2);
        R(r,(3*j-1)) = -(j-1)/(2*(m-1/2));
        R(r,(3*j)) = -1;
        r=r+1;
    end
end

if m>1
    for j=2:m
        R(r,(3*j-5)) = (j-1)/(m-1/2);
        R(r,(3*j-4)) = 1;
        R(r,(3*j-2)) = -(j-1)/(m-1/2);
        R(r,(3*j-1)) = -1;
        r=r+1;
    end
end

clear j n p r

% Set up solution matrix
G = zeros(3*m,1);

G(1) = 0;
G(2) = 1/24;

r=3;
for n=1:m
    G(r) = 1/96*(n-1)/(m-1/2)*(n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
    r=r+1;
end

for p=r
    G(p) = 0;
    r=r+1;
end

clear n p r

% For an odd number of floorboards
else
    % Set up coefficient matrix
    R = zeros(3*m);

    R(1,3) = 1;
    R(2,(3*m-2)) = 1;
    R(2,(3*m-1)) = 1;

    r=3;
    for n=1:m

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A.3 MATLAB Code to Determine Nonlinear Diaphragm Deformation Profile

\[
\begin{align*}
R((r),(3*n-2)) &= (2*m)/\text{betan} - (n-1); \\
R((r),(3*n-1)) &= -m; \\
\text{if } n > 1 \\
&\quad \text{for } p=1:(n-1) \\
&\quad \quad R((r),(3*p-2)) = -(p-1); \\
&\quad \quad R((r),(3*p-1)) = -m; \\
&\quad \text{end} \\
&\quad r=r+1; \\
&\text{end} \\
\text{if } m > 1 \\
&\quad \text{for } j=2:m \\
&\quad \quad R(r,(3*j-5)) = (j-1)^2/(4*m^2); \\
&\quad \quad R(r,(3*j-4)) = (j-1)/(2*m); \\
&\quad \quad R(r,(3*j-3)) = 1; \\
&\quad \quad R(r,(3*j-2)) = -(j-1)^2/(4*m^2); \\
&\quad \quad R(r,(3*j-1)) = -(j-1)/(2*m); \\
&\quad \quad R(r,(3*j)) = -1; \\
&\quad \quad r=r+1; \\
&\quad \text{end} \\
&\text{end} \\
\text{clear } j \ n \ p \ r \\
\%	ext{ Set up solution matrix} \\
G = \text{zeros}(3*m,1); \\
G(1) = 0; \\
G(2) = 1/24; \\
r=3; \\
\text{for } n=1:m \\
G(r) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1)); \\
r=r+1; \\
\text{end} \\
\text{for } p=r \\
G(p) = 0; \\
r=r+1; \\
\text{end} \\
\text{clear } n \ p \ r \\
\text{end} \\
\text{end}
% Determine matrix of unknown linear coefficients
u_elastic = R\G;

clear R G

% Solve for the unknowns coefficients of the nonlinear system of % equations

% The Newton-Raphson algorithm is used to numerically determine the % unknown coefficients Ai, Bi, and Ci

% The following IF-statements tests which principal direction the % diaphragm is being loaded in, then whether there is an even number of % members across the span of the diaphragm. The relevant code is % subsequently executed.

% For loading Parallel-to-joist if strcmp(LD,'PAR') == 1 % For an even number of joists if rem(nj,2)== 0

% Specify initial U-matrix. The U-matrix contains values for the % unknown coefficients Ai, Bi, and Ci which are determined % iteratively using the Newton-Raphson algorithm
Uold = zeros(3*m,1);
Unew = u_elastic;               % The linear-elastic coefficients are % used for the initial estimate of % Unew

itr = 0;
while abs(Unew-Uold) > tol*ones(3*m,1)...
| abs(Unew-Uold) == zeros(3*m,1) %#ok<OR2>

% Construct R-matrix. The R-matrix contains the governing % nonlinear equations
R = zeros(3*m,1);
R(1) = Unew(3);
R(2) = Unew(3*m-2)+Unew(3*m-1)-1/24;

r=3;
for n=1:m
R(r) = 2*(m-1/2)*Unew(3*n-2)/(b0*L) - b2*(n-1)*Unew(3*n-2)...
- (m-1/2)*b2*Unew(3*n-1) + ((m-1/2)*b1/Q...
+ b2*(n-1)^3/(48*(m-1/2)^2) - b2*(n-1)^2/(16*(m-1/2))...
+ b2*(n-1)*Unew(3*n-2) + (m-1/2)*b2*Unew(3*n-1))...
*exp(-b3*Q*((n-1)^3/(48*(m-1/2)^3)
-((n-1)^2/(16*(m-1/2))^2)+(n-1)/(m-1/2)*Unew(3*n-2)...+Unew(3*n-1)) - n*(m-1/2)*b1/Q...
- b2*(n-1)/(96*(m-1/2))*(n^2*(n-1)/(2*(m-1/2))...
-n*(2*n-1));
if n>1 for p=1:(n-1)
R(r) = R(r) + (-b2*(p-1)*Unew(3*p-2)...
A.3 MATLAB Code to Determine Nonlinear Diaphragm Deformation Profile

```
- (m-1/2)*b2*Unew(3*p-1) + ((m-1/2)*b1/Q... 
+ b2*(p-1)^3/(48*(m-1/2)^2)... 
- b2*(p-1)^2/(16*(m-1/2))... 
+ b2*(p-1)*Unew(3*p-2)... 
+ (m-1/2)*b2*Unew(3*p-1))... 
*exp(-b3*Q*{(p-1)^3/(48*(m-1/2)^3)... 
- (p-1)^2/(16*(m-1/2)^2)... 
+ (p-1)/(m-1/2)*Unew(3*p-2)+Unew(3*p-1))));
end
end
r=r+1;
end

clear n p

if m>1
  for n=2:m
    R(r) = (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-5)... 
    + (n-1)/(2*(m-1/2))*Unew(3*n-4) + Unew(3*n-3)... 
    - (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-2)... 
    - (n-1)/(2*(m-1/2))*Unew(3*n-1) - Unew(3*n); 
    r=r+1;
  end
end

clear n

if m>1
  for n=2:m
    R(r) = (n-1)/(m-1/2)*Unew(3*n-5) + Unew(3*n-4)... 
    - (n-1)/(m-1/2)*Unew(3*n-2) - Unew(3*n-1); 
    r=r+1;
  end
end

clear n r

% Construct Tangent Matrix
KT = zeros(3*m);
KT(1,3) = 1;
KT(2,3*m-2:3*m-1) = 1;

r=3;
for n=1:m
  KT(r,3*n-2) = 2*(m-1/2)/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)... 
    - b2*b3*(n-1)^4/(48*(m-1/2)^3)*Q... 
    + b2*b3*(n-1)^3/(16*(m-1/2)^2)*Q + b2*(n-1)... 
    - b2*b3*(n-1)^2*Unew(3*n-2)/(m-1/2)*Q... 
    - b2*b3*Unew(3*n-1)*Q)... 
    *exp(-b3*Q*{(n-1)^3/(48*(m-1/2)^3)... 
    - (n-1)^2/(16*(m-1/2)^2)... 
    + (n-1)/(m-1/2)*Unew(3*n-2)+Unew(3*n-1)));
end
```

Appendix A: MATLAB Programming for Diaphragm Analysis

- \( b_2 \* b_3 \* (n-1) \* U_{new}(3 \* n-2) \* Q + (m-1/2) \* b_2 \ldots \\
- (m-1/2) \* b_2 \* b_3 \* U_{new}(3 \* n-1) \* Q \ldots \\
- \exp(-b_3 \* Q* ((n-1)^3/(48*(m-1/2)^3) \ldots \\
-(n-1)^2/(16*(m-1/2)^2) + (n-1)/(m-1/2) \* U_{new}(3 \* n-2) \\
+ U_{new}(3 \* n-1)))

for \( p = 1 \) to \( (n-1) \)

\( KT(r, 3 \* p-2) = -b_2*(p-1) + (-b_1*b_3*(p-1) \\
- b_2*b_3*(p-1)^4/(48*(m-1/2)^3) \* Q \\
+ b_2*b_3*(p-1)^3/(16*(m-1/2)^2) \* Q + b_2*(p-1) \\
- b_2*b_3*(p-1)^2*U_{new}(3 \* p-2)/(m-1/2) \* Q \\
- b_2*b_3*U_{new}(3 \* p-1) \* Q) \ldots \\
\exp(-b_3 \* Q* ((p-1)^3/(48*(m-1/2)^3) \ldots \\
-(p-1)^2/(16*(m-1/2)^2) + (p-1)/(m-1/2) \* U_{new}(3 \* p-2) \\
+ U_{new}(3 \* p-1)))

\( KT(r, 3 \* p-1) = -(m-1/2) \* b_2 + (-m-1/2) \* b_1*b_3 \\
- b_2*b_3*(p-1)^3/(48*(m-1/2)^2) \* Q \\
+ b_2*b_3*(p-1)^2/(16*(m-1/2)) \* Q \\
- b_2*b_3*(p-1)*U_{new}(3 \* p-2) \* Q + (m-1/2) \* b_2 \\
- (m-1/2) \* b_2*b_3*U_{new}(3 \* p-1) \* Q) \ldots \\
\exp(-b_3 \* Q* ((p-1)^3/(48*(m-1/2)^3) \ldots \\
-(p-1)^2/(16*(m-1/2)^2) + (p-1)/(m-1/2) \* U_{new}(3 \* p-2) \\
+ U_{new}(3 \* p-1)))

end

r=r+1;
end

clear n p

if \( m > 1 \)

for \( n = 2 \) to \( m \)

\( KT(r, 3*n-5) = (n-1)^2/(4*(m-1/2)^2) \)
\( KT(r, 3*n-4) = (n-1)/(2*(m-1/2)) \)
\( KT(r, 3*n-3) = 1 \)
\( KT(r, 3*n-2) = -(n-1)^2/(4*(m-1/2)^2) \)
\( KT(r, 3*n-1) = -(n-1)/(2*(m-1/2)) \)
\( KT(r, 3*n) = -1 \)

r=r+1;
end
end

clear n

if \( m > 1 \)

for \( n = 2 \) to \( m \)

\( KT(r, 3*n-5) = (n-1)/(m-1/2) \)
\( KT(r, 3*n-4) = 1 \)
\( KT(r, 3*n-2) = -(n-1)/(m-1/2) \)
\( KT(r, 3*n-1) = -1 \)

r=r+1;
end
end

clear n r

delt_u = -inv(KT)*R;
Uold = Unew;
A.3 MATLAB Code to Determine Nonlinear Diaphragm Deformation Profile

Unew = Uold + delt_u;

% Code to terminate loop if convergence does not occur to within
% the specified tolerance
itr=itr+1;
if itr == 10000            % Specify maximum number of iterations
    disp('Maximum iterations reached')
    return
end

% For an odd number of joists
else

    % Specify initial U-matrix. The U-matrix contains values for the
    % unknown coefficients Ai, Bi, and Ci which are determined
    % iteratively using the Newton-Raphson algorithm
    Uold = zeros(3*m,1);  % The linear-elastic coefficients are
    Unew = u_elastic;       % used for the initial estimate of
    % Unew

    itr = 0;
    while abs(Unew-Uold) > tol*ones(3*m,1)...
        | abs(Unew-Uold) == zeros(3*m,1)  %#ok<OR2>

        % Construct R-matrix. The R-matrix contains the governing
        % nonlinear equations
        R = zeros(3*m,1);

        R(1) = Unew(3);
        R(2) = Unew(3*m-2)+Unew(3*m-1)-1/24;

        r=3;
        for n=1:m
            R(r) = 2*m*Unew(3*n-2)/(b0*L) - b2*(n-1)*Unew(3*n-2)...
                - m*b2*Unew(3*n-1) + (m*b1/Q + b2*(n-1)*3/(48*m^2)... 
                - b2*(n-1)^2/(16*m) + b2*(n-1)*Unew(3*n-2)...
                + m*b2*Unew(3*n-1))*exp(-b3*Q*((n-1)^3/(48*m^3)...
                - (n-1)^2/(16*m^2)+(n-1)/m*Unew(3*n-2)+Unew(3*n-1)))...
                - n*m*b1/Q - b2*(n-1)/(96*m)*(n^2*(n-1)/(2*m)-n*(2*n-1));
            if n>1
                for p=1:(n-1)
                    R(r) = R(r) + (-b2*(p-1)*Unew(3*p-2)...
                        - m*b2*Unew(3*p-1) + (m*b1/Q)...
                        + b2*(p-1)*Unew(3*p-2) + m*b2*Unew(3*p-1))... 
                        *exp(-b3*Q*((p-1)^3/(48*m^3)-(p-1)^2/(16*m^2)... 
                            +(p-1)/m*Unew(3*p-2)+Unew(3*p-1)));
                end
            end
            r=r+1;
        end
Appendix A: MATLAB Programming for Diaphragm Analysis

clear n p

if m>1
    for n=2:m
        R(r) = (n-1)^2/(4*m^2)*Unew(3*n-5)...  
            + (n-1)/(2*m)*Unew(3*n-4) + Unew(3*n-3)...  
            - (n-1)^2/(4*m^2)*Unew(3*n-2)...  
            - (n-1)/(2*m)*Unew(3*n-1) - Unew(3*n);    
        r=r+1;
    end
end

clear n

if m>1
    for n=2:m
        R(r) = (n-1)/m*Unew(3*n-5) + Unew(3*n-4)...  
            - (n-1)/m*Unew(3*n-2) - Unew(3*n-1);    
        r=r+1;
    end
end

clear n r

% Construct Tangent Matrix
KT = zeros(3*m);

KT(1,3) = 1;
KT(2,3*m-2:3*m-1) = 1;

r=3;
for n=1:m
    KT(r,3*n-2) = 2*m/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)...  
        - b2*b3*(n-1)^4/(48*m^3)*Q + b2*b3*(n-1)^3/(16*m^2)*Q...  
        + b2*(n-1) - b2*b3*(n-1)^2*Unew(3*n-2)/m*Q...  
        - b2*b3*Unew(3*n-1)*Q)*exp(-b3*Q*((n-1)^3/(48*m^3)...
        -(n-1)^2/(16*m^2)+(n-1)/m*Unew(3*n-2)+Unew(3*n-1));
    KT(r,3*n-1) = -m*b2 + (-m*b1*b3 - b2*b3*(n-1)^3/(48*m^2)*Q...  
        + b2*b3*(n-1)^2/(16*m)*Q - b2*b3*(n-1)*Unew(3*n-2)*Q...  
        + m*b2 - m*b2*b3*Unew(3*n-1)*Q)...  
        *exp(-b3*Q*((n-1)^3/(48*m^3)-(n-1)^2/(16*m^2)... 
        +(n-1)/m*Unew(3*n-2)+Unew(3*n-1)));    
    for p=1:(n-1)
        KT(r,3*p-2) = -b2*(p-1) + (-b1*b3*(p-1)...  
            - b2*b3*(p-1)^4/(48*m^3)*Q...  
            + b2*b3*(p-1)^3/(16*m^2)*Q + b2*(p-1)...  
            - b2*b3*(p-1)^2*Unew(3*p-2)/m*Q...  
            - b2*b3*Unew(3*p-1)*Q)*exp(-b3*Q*((p-1)^3/(48*m^3)...  
            -(p-1)^2/(16*m^2)+(p-1)/m*Unew(3*p-2)+Unew(3*p-1));
        KT(r,3*p-1) = -m*b2 + (-m*b1*b3...  
            - b2*b3*(p-1)^3/(48*m^2)*Q...  
            + b2*b3*(p-1)^2/(16*m)*Q...  
            - b2*b3*(p-1)*Unew(3*p-2)*Q + m*b2...  
            - m*b2*b3*Unew(3*p-1)*Q)...  
            *exp(-b3*Q*((p-1)^3/(48*m^3)...  
            -(p-1)^2/(16*m^2)+(p-1)/m*Unew(3*p-2)+Unew(3*p-1)));
end
r=r+1;
end

clear n p

if m>1
  for n=2:m
    KT(r,3*n-5) = (n-1)^2/(4*m^2);
    KT(r,3*n-4) = (n-1)/(2*m);
    KT(r,3*n-3) = 1;
    KT(r,3*n-2) = -(n-1)^2/(4*m^2);
    KT(r,3*n-1) = -(n-1)/(2*m);
    KT(r,3*n) = -1;
    r=r+1;
  end
end

clear n

if m>1
  for n=2:m
    KT(r,3*n-5) = (n-1)/m;
    KT(r,3*n-4) = 1;
    KT(r,3*n-2) = -(n-1)/m;
    KT(r,3*n-1) = -1;
    r=r+1;
  end
end

clear n r

delt_u =-inv(KT)*R;
Uold = Unew;
Unew = Uold + delt_u;

% Code to terminate loop if convergence does not occur to within % the specified tolerance
itr=itr+1;
if itr == 10000  % Specify maximum number of iterations
  disp('Maximum iterations reached')
  return
end
end
end

% For loading Perpendicular-to-joist
elseif strcmp(LD,'PERP') == 1
  % For an even number of floorboards
  if rem(nb,2)== 0

    % Specify initial U-matrix. The U-matrix contains values for the % unknown coefficients Ai, Bi, and Ci which are determined % iteratively using the Newton-Raphson algorithm
    Uold = zeros(3*m,1);

Appendix A: MATLAB Programming for Diaphragm Analysis

Unew = u_elastic; % The linear-elastic coefficients are % used for the initial estimate of % Unew

itr = 0;
while abs(Unew-Uold) > tol*ones(3*m,1)...  
    | abs(Unew-Uold) == zeros(3*m,1)   %#ok<OR2>

    R = zeros(3*m,1);
    R(1) = Unew(3);
    R(2) = Unew(3*m-2)+Unew(3*m-1)-1/24;

r=3;
for n=1:m
    R(r) = 2*(m-1/2)*Unew(3*n-2)/(b0*L) - b2*(n-1)*Unew(3*n-2)...  
        - (m-1/2)*b2*Unew(3*n-1) + ((m-1/2)*b1/Q...  
        + b2*(n-1)^3/(48*(m-1/2)^2) - b2*(n-1)^2/(16*(m-1/2))...  
        + b2*(n-1)*Unew(3*n-2) + (m-1/2)*b2*Unew(3*n-1))...  
        *exp(-b3*Q*((n-1)^3/(48*(m-1/2)^3)...  
        - (n-1)^2/(16*(m-1/2)^2) + (n-1)/(m-1/2)*Unew(3*n-2)...  
        + Unew(3*n-1)) - n*(m-1/2)*b1/Q...  
        - b2*(n-1)/(96*(m-1/2))...  
        *(n^2*(n-1)/(2*(m-1/2)) - n*(2*n-1));
    if n>1
        for p=1:(n-1)
            R(r) = R(r) + (-b2*(p-1)*Unew(3*p-2)...  
                - (m-1/2)*b2*Unew(3*p-1) + ((m-1/2)*b1/Q...  
                + b2*(p-1)^3/(48*(m-1/2)^2) - b2*(p-1)^2/(16*(m-1/2))...  
                + b2*(p-1)*Unew(3*p-2) + (m-1/2)*b2*Unew(3*p-1))...  
                *exp(-b3*Q*((p-1)^3/(48*(m-1/2)^3)...  
                - (p-1)^2/(16*(m-1/2)^2) + (p-1)/(m-1/2)*Unew(3*p-2)...  
                + Unew(3*p-1)) - (p-1)/(m-1/2)*Unew(3*p-2)+Unew(3*p-1));
        end
    end
end
r=r+1;
end

clear n p

if m>1
    for n=2:m
        R(r) = (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-5)...  
            + (n-1)/(2*(m-1/2))*Unew(3*n-4) + Unew(3*n-3)...  
            - (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-2)...  
            - (n-1)/(2*(m-1/2))*Unew(3*n-1) - Unew(3*n);  
        r=r+1;
    end
end

clear n
A.3 MATLAB Code to Determine Nonlinear Diaphragm Deformation Profile

```matlab
if m>1
    for n=2:m
        R(r) = (n-1)/(m-1/2)*Unew(3*n-5) + Unew(3*n-4) - (n-1)/(m-1/2)*Unew(3*n-2) - Unew(3*n-1);
        r=r+1;
    end
end

clear n r

% Construct Tangent Matrix
KT = zeros(3*m);
KT(1,3) = 1;
KT(2,3*m-2:3*m-1) = 1;

r=3;
for n=1:m
    KT(r,3*n-2) = 2*(m-1/2)/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)...
                  - b2*b3*(n-1)^4/(48*(m-1/2)^3)*Q...
                  + b2*b3*(n-1)^3/(16*(m-1/2)^2)*Q + b2*(n-1)...
                  - b2*b3*(n-1)^2*Unew(3*n-2)/(m-1/2)*Q...
                  - b2*b3*Unew(3*n-1)*Q)...
                  *exp(-b3*Q*((n-1)^3/(48*(m-1/2)^3)...
                  - (n-1)^2/(16*(m-1/2)^2)...
                  + (n-1)/(m-1/2)*Unew(3*n-2) + Unew(3*n-1)));
    KT(r,3*n-1) = -(m-1/2)*b2 + (-(m-1/2)*b1*b3...
                  - b2*b3*(n-1)^3/(48*(m-1/2)^2)*Q...
                  + b2*b3*(n-1)^2/(16*(m-1/2))*Q...
                  - b2*b3*(n-1)*Unew(3*n-2)*Q + (m-1/2)*b2...
                  - (m-1/2)*b2*b3*Unew(3*n-1)*Q)...
                  *exp(-b3*Q*((n-1)^3/(48*(m-1/2)^3)...
                  - (n-1)^2/(16*(m-1/2)^2) + (n-1)/(m-1/2)*Unew(3*n-2)...
                  + Unew(3*n-1)))
    for p=1:(n-1)
        KT(r,3*p-2) = - b2*(p-1) + (-b1*b3*(p-1)...
                      - b2*b3*(p-1)^4/(48*(m-1/2)^3)*Q...
                      + b2*b3*(p-1)^3/(16*(m-1/2)^2)*Q + b2*(p-1)...
                      - b2*b3*(p-1)^2*Unew(3*p-2)/(m-1/2)*Q...
                      - b2*b3*Unew(3*p-1)*Q)...
                      *exp(-b3*Q*((p-1)^3/(48*(m-1/2)^3)...
                      - (p-1)^2/(16*(m-1/2)^2) + (p-1)/(m-1/2)*Unew(3*p-2)...
                      + Unew(3*p-1)))
        KT(r,3*p-1) = -(m-1/2)*b2 + (-(m-1/2)*b1*b3...
                      - b2*b3*(p-1)^3/(48*(m-1/2)^2)*Q...
                      + b2*b3*(p-1)^2/(16*(m-1/2))*Q...
                      - b2*b3*(p-1)*Unew(3*p-2)*Q + (m-1/2)*b2...
                      - (m-1/2)*b2*b3*Unew(3*p-1)*Q)...
                      *exp(-b3*Q*((p-1)^3/(48*(m-1/2)^3)...
                      - (p-1)^2/(16*(m-1/2)^2) + (p-1)/(m-1/2)*Unew(3*p-2)...
                      + Unew(3*p-1)));
    end
end
r=r+1;

clear n p
```
if m>1
    for n=2:m
        KT(r,3*n-5) = (n-1)^2/(4*(m-1/2)^2);
        KT(r,3*n-4) = (n-1)/(2*(m-1/2));
        KT(r,3*n-3) = 1;
        KT(r,3*n-2) = -(n-1)^2/(4*(m-1/2)^2);
        KT(r,3*n-1) = -(n-1)/(2*(m-1/2));
        KT(r,3*n) = -1;
        r=r+1;
    end
end

clear n

if m>1
    for n=2:m
        KT(r,3*n-5) = (n-1)/(m-1/2);
        KT(r,3*n-4) = 1;
        KT(r,3*n-2) = -(n-1)/(m-1/2);
        KT(r,3*n-1) = -1;
        r=r+1;
    end
end

clear n r

delt_u = -inv(KT)*R;
Uold = Unew;
Unew = Uold + delt_u;

% Code to terminate loop if convergence does not occur to within % the specified tolerance
itr=itr+1;
if itr == 10000 % Specify maximum number of iterations
    disp('Maximum iterations reached')
    return
end

% For an odd number of floorboards
else

    % Specify initial U-matrix. The U-matrix contains values for the % unknown coefficients Ai, Bi, and Ci which are determined % iteratively using the Newton-Raphson algorithm
    Uold = zeros(3*m,1);
    Unew = u_elastic;               % The linear-elastic coefficients are % used for the initial estimate of % Unew

    itr = 0;
    while abs(Unew-Uold) > tol*ones(3*m,1)...             %#ok<OR2>
        | abs(Unew-Uold) == zeros(3*m,1)
% Construct R-matrix. The R-matrix contains the governing nonlinear equations
R = zeros(3*m,1);

R(1) = Unew(3);
R(2) = Unew(3*m-2)+Unew(3*m-1)-1/24;

r=3;
for n=1:m
    R(r) = 2*m*Unew(3*n-2)/(b0*L) - b2*(n-1)*Unew(3*n-2)...
    - m*b2*Unew(3*n-1) + (m*b1/Q + b2*(n-1)^3/(48*m^2)...
    - b2*(n-1)^2/(16*m) + b2*(n-1)*Unew(3*n-2)...
    + m*b2*Unew(3*n-1))*exp(-b3*Q*((n-1)^3/(48*m^3)...
    - (n-1)^2/(16*m^2)+(n-1)/m*Unew(3*n-2)+Unew(3*n-1))...
    - n*m*b1/Q - b2*(n-1)/(96*m)*(n^2*(n-1)/(2*m)-n*(2*n-1));
    if n>1
        for p=1:(n-1)
            R(r) = R(r) + (-b2*(p-1)*Unew(3*p-2)...
            - m*b2*Unew(3*p-1) + (m*b1/Q...
            + b2*(p-1)^3/(48*m^2) - b2*(p-1)^2/(16*m)...
            + b2*(p-1)*Unew(3*p-2) + m*b2*Unew(3*p-1))...
            *exp(-b3*Q*((p-1)^3/(48*m^3)-(p-1)^2/(16*m^2)...
            +(p-1)/m*Unew(3*p-2)+Unew(3*p-1)));
        end
    end
    r=r+1;
end

clear n p

if m>1
    for n=2:m
        R(r) = (n-1)^2/(4*m^2)*Unew(3*n-5)...
        + (n-1)/(2*m)*Unew(3*n-4) + Unew(3*n-3)...
        - (n-1)^2/(4*m^2)*Unew(3*n-2)...
        - (n-1)/(2*m)*Unew(3*n-1) - Unew(3*n);
        r=r+1;
    end
end

clear n

if m>1
    for n=2:m
        R(r) = (n-1)/m*Unew(3*n-5) + Unew(3*n-4)...
        - (n-1)/m*Unew(3*n-2) - Unew(3*n-1);
        r=r+1;
    end
end

clear n r

% Construct Tangent Matrix
KT = zeros(3*m);
Appendix A: MATLAB Programming for Diaphragm Analysis

\[ KT(1,3) = 1; \]
\[ KT(2,3*m-2:3*m-1) = 1; \]

\[ r=3; \]
\[ \text{for } n=1:m \]
\[ \text{KT}(r,3*n-2) = 2*m/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)... \]
\[ - b2*b3*(n-1)^4/(48*m^3)*Q + b2*b3*(n-1)^3/(16*m^2)*Q... \]
\[ + b2*(n-1) - b2*b3*(n-1)^2*Unew(3*n-2)/m*Q... \]
\[ - b2*b3*Unew(3*n-1)*Q*exp(-b3*Q*((n-1)^3/(48*m^3)... \]
\[ - (n-1)^2/(16*m^2)+(n-1)/m*Unew(3*n-2)+Unew(3*n-1))); \]
\[ KT(r,3*n-1) = -m*b2 + (-m*b1*b3 - b2*b3*(n-1)^3/(48*m^2)*Q... \]
\[ + b2*b3*(n-1)^2/(16*m)*Q - b2*b3*(n-1)*Unew(3*n-2)*Q... \]
\[ + m*b2 - m*b2*b3*Unew(3*n-1)*Q... \]
\[ *exp(-b3*Q*((n-1)^3/(48*m^3)-(n-1)^2/(16*m^2)... \]
\[ +(n-1)/m*Unew(3*n-2)+Unew(3*n-1))); \]
\[ \text{for } p=1:(n-1) \]
\[ \text{KT}(r,3*p-2) = - b2*(p-1) + (-b1*b3*(p-1)... \]
\[ - b2*b3*(p-1)^4/(48*m^3)*Q... \]
\[ + b2*b3*(p-1)^3/(16*m^2)*Q + b2*(p-1)... \]
\[ - b2*b3*(p-1)^2*Unew(3*p-2)/m*Q... \]
\[ - b2*b3*Unew(3*p-1)*Q*exp(-b3*Q*((p-1)^3/(48*m^3)... \]
\[ -(p-1)^2/(16*m^2)+(p-1)/m*Unew(3*p-2)+Unew(3*p-1))); \]
\[ KT(r,3*p-1) = -m*b2 + (-m*b1*b3... \]
\[ - b2*b3*(p-1)^3/(48*m^2)*Q... \]
\[ + b2*b3*(p-1)^2/(16*m)*Q... \]
\[ - b2*b3*(p-1)*Unew(3*p-2)*Q + m*b2... \]
\[ - m*b2*b3*Unew(3*p-1)*Q... \]
\[ *exp(-b3*Q*((p-1)^3/(48*m^3)... \]
\[ -(p-1)^2/(16*m^2)+(p-1)/m*Unew(3*p-2)+Unew(3*p-1))); \]
\[ \text{end} \]
\[ r=r+1; \]
\[ \text{end} \]

\text{clear n p} \]
\[ \text{if } m>1 \]
\[ \text{for } n=2:m \]
\[ \text{KT}(r,3*n-5) = (n-1)^2/(4*m^2); \]
\[ KT(r,3*n-4) = (n-1)/(2*m); \]
\[ KT(r,3*n-3) = 1; \]
\[ KT(r,3*n-2) = -(n-1)^2/(4*m^2); \]
\[ KT(r,3*n-1) = -(n-1)/(2*m); \]
\[ KT(r,3*n) = -1; \]
\[ r=r+1; \]
\[ \text{end} \]
\[ \text{end} \]

\text{clear n} \]
\[ \text{if } m>1 \]
\[ \text{for } n=2:m \]
\[ KT(r,3*n-5) = (n-1)/m; \]
\[ KT(r,3*n-4) = 1; \]
\[ KT(r,3*n-2) = -(n-1)/m; \]
\[ KT(r,3*n-1) = -1; \]
A.3 MATLAB Code to Determine Nonlinear Diaphragm Deformation Profile

```matlab
r=r+1;
end
clear n r
delt_u = -inv(KT)*R;
Uold = Unew;
Unew = Uold + delt_u;
end
end
end
end
end

%% Displacements
if strcmp(LD,'PAR') == 1
if x >= (i-1)*l && x <= i*l || x <= L/2
v = w*L^4/(E*I_fb)*(1/24*xnorm^4 - 1/12*xnorm^3 ...
+ Unew(3*i-2)*xnorm^2 + Unew(3*i-1)*xnorm + Unew(3*i));
elseif strcmp(LD,'PERP') == 1
if x >= (i-1)*fb_d && x <= i*fb_d || x <= L/2
v = w*L^4/(E*I_j)*(1/24*xnorm^4 - 1/12*xnorm^3 ...
+ Unew(3*i-2)*xnorm^2 + Unew(3*i-1)*xnorm + Unew(3*i));
end
end
disp(['Displacement at span location ',num2str(x),'mm: ','v = ',...
num2str(v),'mm']);
endif strcmp(LD,'PAR') == 1
% Consider half diaphragm span
incr = 50;                         % Number of increments per section
x_prof = ((0:1/incr:L/2)/L)';      % Normalised span location vector
vn = zeros(1,length(x_prof));      % Set up displacement vector
for j=1:length(x_prof)             % Loop to determine displacements
    if j/incr <= m
        n=ceil(j/incr);
    else
        n=m;
    end
    vn(j) = w*L^4/(E*I_fb)*(1/24*x_prof(j)^4 - 1/12*x_prof(j)^3 ...
+ Unew(3*n-2)*x_prof(j)^2 + Unew(3*n-1)*x_prof(j)...
+ Unew(3*n));
end
```

- 435 -
end
clear j n incr

elseif strcmp(LD,'PERP') == 1
  % Consider half diaphragm span
  incr = 15;
  x_prof = ((0:fb_d/incr:L/2)/L)';
  vn = zeros(1,length(x_prof));

  for j=1:length(x_prof)
      if j/incr <= m
          n=ceil(j/incr);
      else
          n=m;
      end
      vn(j) = w*L^4/(E*I_j)*(1/24*x_prof(j)^4 - 1/12*x_prof(j)^3 ... + Unew(3*n-2)*x_prof(j)^2 + Unew(3*n-1)*x_prof(j)... + Unew(3*n));
  end
  clear j n incr
end

% Plot full profile
vn=vn';
vplot = [-vn;-flipud(vn)];
xplot = [x_prof*L;(x_prof*L+L/2)];
clear xprof

figure
plot(xplot,vplot,'Color','r');
set(gca,'XLim',[0 L],'XTick',0:L/2:L);
xlabel('Span location (mm)'); ylabel('Displacement (mm)');
title('Nonlinear Displacement Profile');

A.4 MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

% This m.file determines the force-displacement response of any
% straight-sheathed diaphragm, given a uniformly distributed load applied
% in either principal direction and a nonlinear nail connection load-slip
% relationship

% Author: Aaron Wilson
% Date: 22 August 2011

clc
clear all

close all

%% Input values
% Geometric parameters
L = 10400;                          % Span of diaphragm (mm)
B = 5535;                           % Depth of diaphragm (mm)
nj = 27;                            % Number of joists
fb_t = 18;                          % Floorboard thickness (mm)
fb_d = 135;                         % Floorboard width (mm)
j_t = 45;                           % Joist thickness (mm)
j_d = 290;                          % Joist depth (mm)
s = 95;                             % Nail spacing (mm)
E = 8000;                           % Elastic Modulus of timber (N/mm^2)

% Loading direction (Parallel-to-joist=PAR, Perpendicular-to-joist=PERP)
LD = 'PAR';

% Total applied UDL schedule (must start from a non-zero positive value)
Lincr = 0.1;                        % Loading increment (N)
Wmax = 3.5;                         % Maximum applied UDL (N/mm)
W = (1e-10:Lincr:(Wmax+1e-10))';    % UDL schedule(N/mm)
clear Lincr

% Effective linear-elastic nail stiffness
k = 900;                            % Effective stiffness(N/mm)

% Nonlinear load-slip characteristic coefficients for nails, taken from
% Dolan & Madsen (1992) for the load-slip relationship developed by
% Foschi (1974)
K0 = 1182;                          % Initial stiffness (N/mm)
F0 = 920;                           % Intital force (N)
K1 = 50;                            % Secondary stiffness (N/mm)

% Position to consider
x = L/2;                            % Position along floorboard under
% consideration

% Specify tolerance for Newton-Raphson convergence
tol = 1e-10;

%% Automatically generated parameters
I_fb = (fb_t*fb_d^3)/12;           % Second moment of floorboard (mm^4)
I_j = (j_d*j_t^3)/12;             % Second moment of joist (mm^4)

beta = k*s^2/2;                     % Stiffness parameter
b1 = F0;                           % Simplified coefficients
b2 = K1*s/2;
b3 = K0*s/(2*F0);
xnorm = x/L;

% Parameters specific to loading direction. IF-statements test whether
% the applied load is parallel-to-joist or perpendicular-to-joist, and
%% subsequently determines the relevant parameters
if strcmp(LD,'PAR') == 1
  l = L/(nj-1); % Joist spacing (centres) (mm)
  nb = floor(B/fb_d); % Number of floorboards
  m = floor(nj/2); % Number of nail connections within % half diaphragm span
  w = W/nb; % UDL per floorboard (N/mm)
  Q = w*L^3/(E*I_fb); % Simplified coefficients
  b0 = s/(E*I_fb);
  betan = beta*L/(E*I_fb); % Normalised stiffness parameter
  if ceil(x/l) <= m
    i = ceil(x/l); % Section of floorboard under % consideration
  else
    i=m;
  end
elseif strcmp(LD,'PERP') == 1
  l = B/(nj-1); % Joist spacing (centres) (mm)
  nb = floor(L/fb_d); % Number of floorboards
  m = floor(nb/2); % Number of nail connections within % half diaphragm span
  w = W/(nj-1); % UDL per joist (N/mm)
  Q = w*L^3/(E*I_j); % Simplified coefficients
  b0 = s/(E*I_j);
  betan = beta*L/(E*I_j); % Normalised stiffness parameter
  if ceil(x/fb_d) <= m
    i = ceil(x/fb_d); % Section of floorboard under % consideration
  else
    i=m;
  end
end

%% Solve for elastic coefficients for the initial estimate of unknown % nonlinear coefficients

%% The following IF-statements tests which principal direction the % diaphragm is being loaded in, then whether there is an even number of % members across the span of the diaphragm. The relevant code is % subsequently executed.

%% For loading Parallel-to-joist
if strcmp(LD,'PAR') == 1
  % For an even number of members
  if rem(nj,2)== 0
    % Set up coefficient matrix
A.4 MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

R = zeros(3*m);
R(1,3) = 1;
R(2,(3*m-2)) = 1;
R(2,(3*m-1)) = 1;

r=3;
for n=1:m
    R((r),(3*n-2)) = (2*(m-1/2))/betan - (n-1);
    R((r),(3*n-1)) = -(m-1/2);
    if n>1
        for p=1:(n-1)
            R((r),(3*p-2)) = -(p-1);
            R((r),(3*p-1)) = -(m-1/2);
        end
    end
    r=r+1;
end

if m>1
    for j=2:m
        R(r,(3*j-5)) = (j-1)^2/(4*(m-1/2)^2);
        R(r,(3*j-4)) = (j-1)/(2*(m-1/2));
        R(r,(3*j-3)) = 1;
        R(r,(3*j-2)) = -(j-1)^2/(4*(m-1/2)^2);
        R(r,(3*j-1)) = -(j-1)/(2*(m-1/2));
        R(r,(3*j)) = -1;
        r=r+1;
    end
end

if m>1
    for j=2:m
        R(r,(3*j-5)) = (j-1)/(m-1/2);
        R(r,(3*j-4)) = 1;
        R(r,(3*j-2)) = -(j-1)/(m-1/2);
        R(r,(3*j-1)) = -1;
        r=r+1;
    end
end

clear j n p r

% Set up solution matrix
G = zeros(3*m,1);
G(1) = 0;
G(2) = 1/24;

r=3;
for n=1:m
    G(r) = 1/96*(n-1)/(m-1/2) * (n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
    r=r+1;
end
for p=r
    G(p) = 0;
    r=r+1;
end

clear n p r

% For an odd number of joists
else
    % Set up coefficient matrix
    R = zeros(3*m);
    R(1,3) = 1;
    R(2,(3*m-2)) = 1;
    R(2,(3*m-1)) = 1;

    r=3;
    for n=1:m
        R((r),(3*n-2)) = (2*m)/betan - (n-1);
        R((r),(3*n-1)) = -m;
        if n>1
            for p=1:(n-1)
                R((r),(3*p-2)) = -(p-1);
                R((r),(3*p-1)) = -m;
            end
        end
        r=r+1;
    end
    if m>1
        for j=2:m
            R(r,(3*j-5)) = (j-1)^2/(4*m^2);
            R(r,(3*j-4)) = (j-1)/(2*m);
            R(r,(3*j-2)) = 1;
            R(r,(3*j-1)) = -(j-1^2)/(4*m^2);
            R(r,(3*j)) = -(j-1)/(2*m);
            R(r,(3*j+1)) = -1;
            r=r+1;
        end
    end

    if m>1
        for j=2:m
            R(r,(3*j-5)) = (j-1)/m;
            R(r,(3*j-4)) = 1;
            R(r,(3*j-2)) = -(j-1)/m;
            R(r,(3*j-1)) = -(j-1)/(2*m);
            r=r+1;
        end
    end

clear j n p r

% Set up solution matrix
G = zeros(3*m,1);
A.4 MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

G(1) = 0;
G(2) = 1/24;

r=3;
for n=1:m
    G(r) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1));
    r=r+1;
end

for p=r
    G(p) = 0;
    r=r+1;
end

clear n p r

end

% For loading Perpendicular-to-joist
elseif strcmp(LD,'PERP') == 1
    % For an even number of floorboards
    if rem(nb,2)== 0
        % Set up coefficient matrix
        R = zeros(3*m);

        R(1,3) = 1;
        R(2,(3*m-2)) = 1;
        R(2,(3*m-1)) = 1;

        r=3;
        for n=1:m
            R((r),(3*n-2)) = (2*(m-1/2))/betan - (n-1);
            R((r),(3*n-1)) = -(m-1/2);
            if n>1
                for p=1:(n-1)
                    R((r),(3*p-2)) = -(p-1);
                    R((r),(3*p-1)) = -(m-1/2);
                end
            end
            r=r+1;
        end

        if m>1
            for j=2:m
                R(r,(3*j-5)) = (j-1)^2/(4*(m-1/2)^2);
                R(r,(3*j-4)) = (j-1)/(2*(m-1/2));
                R(r,(3*j-3)) = 1;
                R(r,(3*j-2)) = -(j-1)^2/(4*(m-1/2)^2);
                R(r,(3*j-1)) = -(j-1)/(2*(m-1/2));
                R(r,(3*j)) = -1;
                r=r+1;
            end
        end
    end

end
if m>1
    for j=2:m
        R(r,(3*j-5)) = (j-1)/(m-1/2);
        R(r,(3*j-4)) = 1;
        R(r,(3*j-2)) = -(j-1)/(m-1/2);
        R(r,(3*j-1)) = -1;
        r=r+1;
    end
end

clear j n p r

% Set up solution matrix
G = zeros(3*m,1);
G(1) = 0;
G(2) = 1/24;
r=3;
for n=1:m
    G(r) = 1/96*(n-1)/(m-1/2)*(n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
    r=r+1;
end
for p=r
    G(p) = 0;
    r=r+1;
end

clear n p r

% For an odd number of floorboards
else
    % Set up coefficient matrix
    R = zeros(3*m);
    R(1,3) = 1;
    R(2,(3*m-2)) = 1;
    R(2,(3*m-1)) = 1;
r=3;
for n=1:m
    R((r),(3*n-2)) = (2*m)/betan - (n-1);
    R((r),(3*n-1)) = -m;
    if n>1
        for p=1:(n-1)
            R((r),(3*p-2)) = -(p-1);
            R((r),(3*p-1)) = -m;
        end
    end
    r=r+1;
end
if m>1
    for j=2:m

A.4 MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

```matlab
R(r,(3*j-5)) = (j-1)^2/(4*m^2);
R(r,(3*j-4)) = (j-1)/(2*m);
R(r,(3*j-3)) = 1;
R(r,(3*j-2)) = -(j-1)^2/(4*m^2);
R(r,(3*j-1)) = -(j-1)/(2*m);
R(r,(3*j)) = -1;
r=r+1;
end
end
if m>1
  for j=2:m
    R(r,(3*j-5)) = (j-1)/m;
    R(r,(3*j-4)) = 1;
    R(r,(3*j-2)) = -(j-1)/m;
    R(r,(3*j-1)) = -1;
    r=r+1;
  end
end

clear j n p r

% Set up solution matrix
G = zeros(3*m,1);
G(1) = 0;
G(2) = 1/24;

r=3;
for n=1:m
  G(r) = 1/96*(n-1)/m*(n^2*(n-1)/(2*m)-n*(2*n-1));
  r=r+1;
end
for p=r
  G(p) = 0;
  r=r+1;
end

% Determine matrix of unknown linear coefficients
u_elastic = R\G;

clear R G

%% Solve for the unknowns coefficients of the nonlinear system of % equations

% The Newton-Raphson algorithm is used to numerically determine the % unknown coefficients A1, B1, and C1 for each applied UDL increment
```
The following FOR loop determines the displacement of the span position
under consideration for each loading increment

\[ v = \text{zeros}(\text{length}(w),1); \]
\[ \text{for } q=1:\text{length}(w) \]

The following IF-statements tests which principal direction the
diaphragm is being loaded in, then whether there is an even number of
members across the span of the diaphragm. The relevant code is
subsequently executed.

For loading Parallel-to-joist
\[ \text{if strcmp(LD,'PAR') == 1} \]
\[ \text{if rem(nj,2)== 0} \]

Specify initial U-matrix. The U-matrix contains values for the
unknown coefficients Ai, Bi, and Ci which are determined
iteratively using the Newton-Raphson algorithm
\[ U_{old} = \text{zeros}(3*m,1); \]
\[ U_{new} = u_{\text{elastic}}; \]
\[ \text{itr = 0; while abs(U_{new}-U_{old}) > tol*ones(3*m,1)...} \]
\[ | \text{abs(U_{new}-U_{old})} == \text{zeros(3*m,1)} \text{ } \text{ } \text{#ok<OR2>} \]

Construct R-matrix. The R-matrix contains the governing
nonlinear equations
\[ R = \text{zeros}(3*m,1); \]
\[ R(1) = U_{new}(3); \]
\[ R(2) = U_{new}(3*m-2)+U_{new}(3*m-1)/24; \]
r=3;
\[ \text{for } n=1:m \]
\[ R(r) = 2*(m-1/2)*U_{new}(3*n-2)/(b0*L) - b2*(n-1)*U_{new}(3*n-2)\ldots \]
\[ - (m-1/2)*b2*U_{new}(3*n-1) + ((m-1/2)*b1/Q(q)\ldots \]
\[ + b2*(n-1)^3/(48*(m-1/2)^2) - b2*(n-1)^2/(16*(m-1/2))\ldots \]
\[ + b2*(n-1)*U_{new}(3*n-2) + (m-1/2)*b2*U_{new}(3*n-1)\ldots \]
\[ *\exp(-b3*Q(q)*((n-1)^3/(48*(m-1/2)^3)\ldots \]
\[ -(n-1)^2/(16*(m-1/2)^2)+((n-1)/(m-1/2)*U_{new}(3*n-2)\ldots \]
\[ +U_{new}(3*n-1)) - n*(m-1/2)*b1/Q(q)\ldots \]
\[ - b2*(n-1)/(96*(m-1/2)^2)*(n^2*(n-1)/(2*(m-1/2)-n*(2*n-1)); \]
\[ \text{if } n>1 \]
\[ \text{for } p=1:n-1 \]
\[ R(r) = R(r) + (-b2*(p-1)*U_{new}(3*p-1)\ldots \]
\[ - (m-1/2)*b2*U_{new}(3*p-1) + ((m-1/2)*b1/Q(q)\ldots \]
\[ + b2*(p-1)^3/(48*(m-1/2)^2)\ldots \]
\[ - b2*(p-1)^2/(16*(m-1/2))\ldots \]
\[ + b2*(p-1)*U_{new}(3*p-2)\ldots \]
\[ + (m-1/2)*b2*U_{new}(3*p-1)\ldots \]
\[ *\exp(-b3*Q(q)*((p-1)^3/(48*(m-1/2)^3)\ldots \]
\[ -(p-1)^2/(16*(m-1/2)^2)\ldots \]
\[ +(p-1)/(m-1/2)*U_{new}(3*p-2)+U_{new}(3*p-1)); \]
A.4 MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

```matlab
end
end
r=r+1;
end

clear n p

if m>1
    for n=2:m
        R(r) = (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-5)...
            + (n-1)/(2*(m-1/2))*Unew(3*n-4) + Unew(3*n-3)...
            - (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-2)...
            - (n-1)/(2*(m-1/2))*Unew(3*n-1) - Unew(3*n);
        r=r+1;
    end
end

clear n

if m>1
    for n=2:m
        R(r) = (n-1)/(m-1/2)*Unew(3*n-5) + Unew(3*n-4)...
            - (n-1)/(m-1/2)*Unew(3*n-2) - Unew(3*n-1);
        r=r+1;
    end
end

clear n r

% Construct Tangent Matrix
KT = zeros(3*m);
KT(1,3) = 1;
KT(2,3*m-2:3*m-1) = 1;

r=3;
for n=1:m
    KT(r,3*n-2) = 2*(m-1/2)/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)...
        - b2*b3*(n-1)^4/(48*(m-1/2)^3)*Q(q)...
        + b2*b3*(n-1)^3/(16*(m-1/2)^2)*Q(q) + b2*(n-1)...
        - b2*b3*(n-1)^2*Unew(3*n-2)/(m-1/2)*Q(q)...
        - b2*b3*Unew(3*n-1)*Q(q))*exp(-b3*Q(q)...
        *((n-1)^3/(48*(m-1/2)^3) - (n-1)^2/(16*(m-1/2)^2)...
        + (n-1)/(m-1/2)*Unew(3*n-2)+Unew(3*n-1));
    KT(r,3*n-1) = -(m-1/2)*b2 + (-(m-1/2)*b1*b3...
        - b2*b3*(n-1)^3/(48*(m-1/2)^2)*Q(q)...
        + b2*b3*(n-1)^2/(16*(m-1/2))^2)*Q(q)...
        - b2*b3*(n-1)*Unew(3*n-2)*Q(q) + (m-1/2)*b2...
        - (m-1/2)*b2*b3*Unew(3*n-1)*Q(q)...
        *exp(-b3*Q(q) * ((n-1)^3/(48*(m-1/2)^3)...
        - (n-1)^2/(16*(m-1/2)^2)+(n-1)/(m-1/2)*Unew(3*n-2)...
        +Unew(3*n-1));
    for p=1:(n-1)
        KT(r,3*p-2) = - b2*(p-1) + (-b1*b3*(p-1)...
            - b2*b3*(p-1)^4/(48*(m-1/2)^3)*Q(q)...
            + b2*b3*(p-1)^3/(16*(m-1/2)^2)*Q(q) + b2*(p-1)...
```
Appendix A: MATLAB Programming for Diaphragm Analysis

\[- b2*b3*(p-1)^2*Unew(3*p-2)/(m-1/2)*Q(q)...\]
\[- b2*b3*Unew(3*p-1)*Q(q))...\]
\[-exp(-b3*Q(q)*((p-1)^3/(48*(m-1/2)^3)...\]
\[-(p-1)^2/(16*(m-1/2)^2)+(p-1)/(m-1/2)*Unew(3*p-2)...\]
\[-Unew(3*p-1))};\]

KT(r,3*p-1) = -(m-1/2)*b2 + (-(m-1/2)*b1*b3...
- b2*b3*(p-1)^3/(48*(m-1/2)^2)*Q(q)...
+ b2*b3*(p-1)^2/(16*(m-1/2))*Q(q)...
- b2*b3*(p-1)*Unew(3*p-2)*Q(q) + (m-1/2)*b2...
- (m-1/2)*b2*b3*Unew(3*p-1)*Q(q)...
*exp(-b3*Q(q)*((p-1)^3/(48*(m-1/2)^3)...\]
\[-(p-1)^2/(16*(m-1/2)^2)+(p-1)/(m-1/2)*Unew(3*p-2)...\]
\[-Unew(3*p-1))};\]

end
r=r+1;
end

clear n p

if m>1
    for n=2:m
        KT(r,3*n-5) = (n-1)^2/(4*(m-1/2)^2);
        KT(r,3*n-4) = (n-1)/(2*(m-1/2));
        KT(r,3*n-3) = 1;
        KT(r,3*n-2) = -(n-1)^2/(4*(m-1/2)^2);
        KT(r,3*n-1) = -(n-1)/(2*(m-1/2));
        KT(r,3*n) = -1;
        r=r+1;
    end
end
clear n

if m>1
    for n=2:m
        KT(r,3*n-5) = (n-1)/(m-1/2);
        KT(r,3*n-4) = 1;
        KT(r,3*n-2) = -(n-1)/(m-1/2);
        KT(r,3*n-1) = -1;
        r=r+1;
    end
end
clear n r

delt_u =-inv(KT)*R;
Uold = Unew;
Unew = Uold + delt_u;

% Code to terminate loop if convergence does not occur to within
% the specified tolerance
itr=itr+1;
if itr == 10000
    % Specify maximum number of iterations
    disp('Maximum iterations reached')
    return
end
% For an odd number of joists
else

% Specify initial U-matrix. The U-matrix contains values for the % unknown coefficients Ai, Bi, and Ci which are determined % iteratively using the Newton-Raphson algorithm
Uold = zeros(3*m,1);
Unew = u_elastic;               % The linear-elastic coefficients are % used for the initial estimate of % Unew

itr = 0;
while abs(Unew-Uold) > tol*ones(3*m,1)...
    | abs(Unew-Uold) == zeros(3*m,1)  %#ok<OR2>

% Construct R-matrix. The R-matrix contains the governing % nonlinear equations
R = zeros(3*m,1);

R(1) = Unew(3);
R(2) = Unew(3*m-2)+Unew(3*m-1)-1/24;

r=3;
for n=1:m
    R(r) = 2*m*Unew(3*n-2)/(b0*L) - b2*(n-1)*Unew(3*n-2)...
    - m*b2*Unew(3*n-1) + (m*b1/Q(q) + b2*(n-1)^3/(48*m^2)...
    - b2*(n-1)^2/(16*m) + b2*(n-1)*Unew(3*n-2)...
    + m*b2*Unew(3*n-1))*exp(-b3*Q(q)*((n-1)^3/(48*m^3)...
    -(n-1)^2/(16*m^2)+(n-1)/m*Unew(3*n-2)+Unew(3*n-1)))...
    - n*m*b1/Q(q)...  
    - b2*(n-1)/(96*m)*(n^2*(n-1)/(2*m)-n*(2*n-1));
    if n>1
        for p=1:(n-1)
        R(r) = R(r) + (-b2*(p-1)*Unew(3*p-2)...
            - m*b2*Unew(3*p-1) + (m*b1/Q(q)...
            + b2*(p-1)^3/(48*m^2) - b2*(p-1)^2/(16*m)...
            + b2*(p-1)*Unew(3*p-2) + m*b2*Unew(3*p-1))...
            *exp(-b3*Q(q)*((p-1)^3/(48*m^3)...
            -(p-1)^2/(16*m^2)...) 
            +(p-1)/m*Unew(3*p-2)+Unew(3*p-1))));
        end
    end
    r=r+1;
end
if m>1
for n=2:m
    R(r) = (n-1)^2/(4*m^2)*Unew(3*n-5)...
        + (n-1)/(2*m)*Unew(3*n-4) + Unew(3*n-3)...
        - (n-1)^2/(4*m^2)*Unew(3*n-2)...
end

end

clear n p

if m>1
for n=2:m
    R(r) = (n-1)^2/(4*m^2)*Unew(3*n-5)...
        + (n-1)/(2*m)*Unew(3*n-4) + Unew(3*n-3)...
        - (n-1)^2/(4*m^2)*Unew(3*n-2)...
end

end

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\[ - \frac{(n-1)}{2m} \cdot U_{new}(3n-1) - U_{new}(3n); \]
\[ r = r + 1; \]
end
end
clear n
if m > 1
  for n = 2:m
    R(r) = \frac{(n-1)}{m} \cdot U_{new}(3n-5) + U_{new}(3n-4)\ldots
    - \frac{(n-1)}{m} \cdot U_{new}(3n-2) - U_{new}(3n-1); \]
    r = r + 1;
  end
end
clear n r

% Construct Tangent Matrix
KT = zeros(3*m);
KT(1, 3) = 1;
KT(2, 3:m-2:3*m-1) = 1;
r = 3;
for n = 1:m
  KT(r, 3*n-2) = 2*m/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)\ldots
  - b2*b3*(n-1)^4/(48*m^3)*Q(q)\ldots
  + b2*b3*(n-1)^3/(16*m^2)*Q(q) + b2*(n-1)\ldots
  - b2*b3*(n-1)^2*U_{new}(3n-2)/m*Q(q)\ldots
  - b2*b3*U_{new}(3n-1)*Q(q)\ldots
  \times \exp(-b3*Q(q) \times ((n-1)^3/(48*m^3)\ldots
  - (n-1)^2/(16*m^2) + (n-1)/m*U_{new}(3n-2) + U_{new}(3n-1))); \]
  KT(r, 3*n-1) = -m*b2 + (-m*b1*b3\ldots
  + b2*b3*(n-1)^3/(48*m^2)*Q(q)\ldots
  - b2*b3*(n-1)^2/(16*m)*Q(q)\ldots
  - b2*b3*(n-1)*U_{new}(3n-2)*Q(q)\ldots
  + m*b2 - m*b2*b3*U_{new}(3n-1)*Q(q)\ldots
  \times \exp(-b3*Q(q) \times ((n-1)^3/(48*m^3)\ldots
  - (n-1)^2/(16*m^2) + (n-1)/m*U_{new}(3n-2) + U_{new}(3n-1))); \]
  for p = 1:(n-1)
    KT(r, 3*p-2) = - b2*(p-1) + (-b1*b3*(p-1)\ldots
    - b2*b3*(p-1)^4/(48*m^3)*Q(q)\ldots
    + b2*b3*(p-1)^3/(16*m^2)*Q(q) + b2*(p-1)\ldots
    - b2*b3*(p-1)^2*U_{new}(3p-2)/m*Q(q)\ldots
    - b2*b3*U_{new}(3p-1)*Q(q)\ldots
    \times \exp(-b3*Q(q) \times ((p-1)^3/(48*m^3)\ldots
    - (p-1)^2/(16*m^2) + (p-1)/m*U_{new}(3p-2) + U_{new}(3p-1))); \]
    KT(r, 3*p-1) = -m*b2 + (-m*b1*b3\ldots
    - b2*b3*(p-1)^3/(48*m^2)*Q(q)\ldots
    + b2*b3*(p-1)^2/(16*m)*Q(q)\ldots
    - b2*b3*(p-1)*U_{new}(3p-2)*Q(q) + m*b2\ldots
    - m*b2*b3*U_{new}(3p-1)*Q(q)\ldots
    \times \exp(-b3*Q(q) \times ((p-1)^3/(48*m^3)\ldots
    - (p-1)^2/(16*m^2) + (p-1)/m*U_{new}(3p-2) + U_{new}(3p-1))); \]
  end
end
r = r + 1;
end

clear n p

if m>1
    for n=2:m
        KT(r,3*n-5) = (n-1)^2/(4*m^2);
        KT(r,3*n-4) = (n-1)/(2*m);
        KT(r,3*n-3) = 1;
        KT(r,3*n-2) = -(n-1)^2/(4*m^2);
        KT(r,3*n-1) = -(n-1)/(2*m);
        KT(r,3*n) = -1;
        r=r+1;
    end
end

clear n

if m>1
    for n=2:m
        KT(r,3*n-5) = (n-1)/m;
        KT(r,3*n-4) = 1;
        KT(r,3*n-2) = -(n-1)/m;
        KT(r,3*n-1) = -1;
        r=r+1;
    end
end

clear n r

delt_u =-inv(KT)*R;
Uold = Unew;
Unew = Uold + delt_u;

% Code to terminate loop if convergence does not occur to within % the specified tolerance
itr=itr+1;
if itr == 10000 % Specify maximum number of iterations
    disp('Maximum iterations reached')
    return
end
end
end

% For loading Perpendicular-to-joist
elseif strcmp(LD,'PERP') == 1
    % For an even number of floorboards
    if rem(nb,2)== 0

        % Specify initial U-matrix. The U-matrix contains values for the % unknown coefficients Ai, Bi, and Ci which are determined % iteratively using the Newton-Raphson algorithm
        Uold = zeros(3*m,1);
        Unew = u_elastic; % The linear-elastic coefficients are % used for the initial estimate of
% Unew

itr = 0;
while abs(Unew-Uold) > tol*ones(3*m,1)...
  | abs(Unew-Uold) == zeros(3*m,1) %#ok<OR2>

% Construct R-matrix. The R-matrix contains the governing
% nonlinear equations
R = zeros(3*m,1);
R(1) = Unew(3);
R(2) = Unew(3*m-2)+Unew(3*m-1)-1/24;

r=3;
for n=1:m
  R(r) = 2*(m-1/2)*Unew(3*n-2)/(b0*L) - b2*(n-1)*Unew(3*n-2)...
    - (m-1/2)*b2*Unew(3*n-1) + ((m-1/2)*b1/Q(q)...
      + b2*(n-1)^3/(48*(m-1/2)^2) - b2*(n-1)^2/(16*(m-1/2))...
      + b2*(n-1)*Unew(3*n-2) + (m-1/2)*b2*Unew(3*n-1))...
    *exp(-b3*Q(q)*((n-1)^3/(48*(m-1/2)^3)...
      -(n-1)^2/(16*(m-1/2)^2)+(n-1)/(m-1/2)*Unew(3*n-2)...
      +Unew(3*n-1))) - n*(m-1/2)*b1/Q(q)...
    - b2*(n-1)/(96*(m-1/2))*(n^2*(n-1)/(2*(m-1/2))-n*(2*n-1));
  if n>1
    for p=1:(n-1)
      R(r) = R(r) + (-b2*(p-1)*Unew(3*p-2)...
        - (m-1/2)*b2*Unew(3*p-1) + ((m-1/2)*b1/Q(q)...
        + b2*(p-1)^3/(48*(m-1/2)^2)...
        - b2*(p-1)^2/(16*(m-1/2))...
        + b2*(p-1)*Unew(3*p-2)...
        + (m-1/2)*b2*Unew(3*p-1))...
      *exp(-b3*Q(q)*((p-1)^3/(48*(m-1/2)^3)...
        -(p-1)^2/(16*(m-1/2)^2)...
        +(p-1)/(m-1/2)*Unew(3*p-2)+Unew(3*p-1))));
    end
  end
  r=r+1;
end

clear n p

if m>1
  for n=2:m
    R(r) = (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-5)...
      + (n-1)/(2*(m-1/2))*Unew(3*n-4) + Unew(3*n-3)...
      - (n-1)^2/(4*(m-1/2)^2)*Unew(3*n-2)...
      - (n-1)/(2*(m-1/2))*Unew(3*n-1) - Unew(3*n);
    r=r+1;
  end
end

clear n

if m>1
  for n=2:m
    R(r) = (n-1)/(m-1/2)*Unew(3*n-5) + Unew(3*n-4)...
end
% A.4 MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

% (n-1)/(m-1/2)*Unew(3*n-2) - Unew(3*n-1);

r=r+1;
end
end

clear n r

% Construct Tangent Matrix
KT = zeros(3*m);

KT(1,3) = 1;
KT(2,3*m-2:3*m-1) = 1;

r=3;
for n=1:m
  KT(r,3*n-2) = 2*(m-1/2)/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)...
  - b2*b3*(n-1)^3/(48*(m-1/2)^3)*Q(q)...
  + b2*b3*(n-1)^3/(16*(m-1/2)^2)*Q(q) + b2*(n-1)...
  - b2*b3*(n-1)^2*Unew(3*n-2)/(m-1/2)*Q(q)...
  - b2*b3*Unew(3*n-1)*Q(q)...
  *exp(-b3*Q(q)*((n-1)^3/(48*(m-1/2)^3)...
  - (n-1)^2/(16*(m-1/2)^2)...
  + (n-1)/(m-1/2)*Unew(3*n-2)+Unew(3*n-1)));
  KT(r,3*n-1) = -(m-1/2)*b2 + (-(m-1/2)*b1*b3...
  - b2*b3*(n-1)^3/(48*(m-1/2)^2)*Q(q)...
  + b2*b3*(n-1)^2/(16*(m-1/2))*(q) + (m-1/2)*b2...
  - (m-1/2)*b2*b3*Unew(3*n-1)*Q(q)...
  *exp(-b3*Q(q)*((n-1)^3/(48*(m-1/2)^3)...
  - (n-1)^2/(16*(m-1/2)^2)+(n-1)/(m-1/2)*Unew(3*n-2)...+
  +Unew(3*n-1)))
  for p=1:(n-1)
    KT(r,3*p-2) = - b2*(p-1) + (-b1*b3*(p-1)...
    - b2*b3*(p-1)^3/(48*(m-1/2)^3)*Q(q)...
    + b2*b3*(p-1)^3/(16*(m-1/2)^2)*Q(q) + b2*(p-1)...
    - b2*b3*(p-1)^2*Unew(3*p-2)/(m-1/2)*Q(q)...
    - b2*b3*Unew(3*p-1)*Q(q)...
    *exp(-b3*Q(q)*((p-1)^3/(48*(m-1/2)^3)...
    - (p-1)^2/(16*(m-1/2)^2)+(p-1)/(m-1/2)*Unew(3*p-2)...+
    +Unew(3*p-1)));
    KT(r,3*p-1) = -(m-1/2)*b2 + (-(m-1/2)*b1*b3...
    - b2*b3*(p-1)^3/(48*(m-1/2)^2)*Q(q)...
    + b2*b3*(p-1)^2/(16*(m-1/2))^2)*Q(q) + (m-1/2)*b2...
    - (m-1/2)*b2*b3*Unew(3*p-1)*Q(q)...
    *exp(-b3*Q(q)*((p-1)^3/(48*(m-1/2)^3)...
    - (p-1)^2/(16*(m-1/2)^2)+(p-1)/(m-1/2)*Unew(3*p-2)...+
    +Unew(3*p-1)));
  end
  r=r+1;
end

clear n p

if m>1
  for n=2:m...
KT(r,3*n-5) = (n-1)^2/(4*(m-1/2)^2);
KT(r,3*n-4) = (n-1)/(2*(m-1/2));
KT(r,3*n-3) = 1;
KT(r,3*n-2) = -(n-1)^2/(4*(m-1/2)^2);
KT(r,3*n-1) = -(n-1)/(2*(m-1/2));
KT(r,3*n) = -1;
r=r+1;
end
end

clear n
if m>1
   for n=2:m
      KT(r,3*n-5) = (n-1)/(m-1/2);
      KT(r,3*n-4) = 1;
      KT(r,3*n-2) = -(n-1)/(m-1/2);
      KT(r,3*n-1) = -1;
      r=r+1;
   end
end

clear n r

delt_u =-inv(KT)*R;
Uold = Unew;
Unew = Uold + delt_u;

% Code to terminate loop if convergence does not occur to within % the specified tolerance
itr=itr+1;
if itr == 10000 % Specify maximum number of iterations
    disp('Maximum iterations reached')
    return
end

% For an odd number of floorboards
else
    % Specify initial U-matrix. The U-matrix contains values for the % unknown coefficients Ai, Bi, and Ci which are determined % iteratively using the Newton-Raphson algorithm
    Uold = zeros(3*m,1);
    Unew = u_elastic; % The linear-elastic coefficients are % used for the initial estimate of % Unew

    itr = 0;
    while abs(Unew-Uold) > tol*ones(3*m,1)... %#ok<OR2>
        | abs(Unew-Uold) == zeros(3*m,1)
    % Construct R-matrix. The R-matrix contains the governing % nonlinear equations

    % Code to terminate loop if convergence does not occur to within % the specified tolerance
    itr=itr+1;
    if itr == 10000 % Specify maximum number of iterations
        disp('Maximum iterations reached')
        return
    end

end

% End of MATLAB Programming for Diaphragm Analysis
MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

R = zeros(3*m,1);
R(1) = Unew(3);
R(2) = Unew(3*m-2)+Unew(3*m-1)-1/24;

r=3;
for n=1:m
    R(r) = 2*m*Unew(3*n-2)/(b0*L) - b2*(n-1)*Unew(3*n-2)...
        - m*b2*Unew(3*n-1) + (m*b1/Q(q) + b2*(n-1)^3/(48*m^2)...
        - b2*(n-1)^2/(16*m) + b2*(n-1)*Unew(3*n-2)...
        + m*b2*Unew(3*n-1))*exp(-b3*Q(q)*((n-1)^3/(48*m^3)...
        - (n-1)^2/(16*m^2)+(n-1)/m*Unew(3*n-2)+Unew(3*n-1)))...
        - n*m*b1/Q(q)...
        - b2*(n-1)/(96*m)*(n^2*(n-1)/(2*m)-n*(2*n-1));
    if n>1
        for p=1:(n-1)
            R(r) = R(r) + (-b2*(p-1)*Unew(3*p-2)...
                - m*b2*Unew(3*p-1) + (m*b1/Q(q)...
                + b2*(p-1)^3/(48*m^2) - b2*(p-1)^2/(16*m)...
                + b2*(p-1)*Unew(3*p-2) + m*b2*Unew(3*p-1))...
                *exp(-b3*Q(q)*((p-1)^3/(48*m^3)...
                - (p-1)^2/(16*m^2)...
                + (p-1)/m*Unew(3*p-2)+Unew(3*p-1))));
        end
    end
    r=r+1;
end
clear n p

if m>1
    for n=2:m
        R(r) = (n-1)^2/(4*m^2)*Unew(3*n-5)...
            + (n-1)/(2*m)*Unew(3*n-4) + Unew(3*n-3)...
            - (n-1)^2/(4*m^2)*Unew(3*n-2)...
            - (n-1)/(2*m)*Unew(3*n-1) - Unew(3*n);
        r=r+1;
    end
end
clear n

if m>1
    for n=2:m
        R(r) = (n-1)/m*Unew(3*n-5) + Unew(3*n-4)...
            - (n-1)/m*Unew(3*n-2) - Unew(3*n-1);
        r=r+1;
    end
end

% Construct Tangent Matrix
KT = zeros(3*m);
Appendix A: MATLAB Programming for Diaphragm Analysis

\[ KT(1,3) = 1; \]
\[ KT(2,3m-2:3m-1) = 1; \]

\[ r=3; \]
\[ \text{for } n=1:m \]
\[ \begin{align*}
KT(r,3n+2) &= 2m/(b0*L) - b2*(n-1) + (-b1*b3*(n-1)...
- b2*b3*(n-1)^4/(48*m^3)*Q(q)...
+ b2*b3*(n-1)^3/(16*m^2)*Q(q)...
+ b2*(n-1) - b2*b3*(n-1)^2*Unew(3n-2)/m*Q(q)...
- b2*b3*Unew(3n-1)*Q(q))... 
*exp(-b3*Q(q)*((n-1)^3/(48*m^3)... 
-(n-1)^2/(16*m^2)+(n-1)/m*Unew(3n-2)+Unew(3n-1))); \\
KT(r,3n+1) &= -m*b2 + (-m*b1*b3...
- b2*b3*(n-1)^3/(48*m^2)*Q(q)... 
+ b2*b3*(n-1)^2/(16*m)*Q(q)... 
- b2*b3*(n-1)*Unew(3n-2)*Q(q)... 
+ m*b2 - m*b2*b3*Unew(3n-1)*Q(q))... 
*exp(-b3*Q(q)*((n-1)^3/(48*m^3)... 
- (n-1)^2/(16*m^2)+(n-1)/m*Unew(3n-2)+Unew(3n-1))); \\
\end{align*} \]
\[ \text{for } p=1:(n-1) \]
\[ \begin{align*}
KT(r,3p+2) &= -b2*(p-1) + (-b1*b3*(p-1)...
- b2*b3*(p-1)^4/(48*m^3)*Q(q)... 
+ b2*b3*(p-1)^3/(16*m^2)*Q(q) + b2*(p-1)... 
- b2*b3*(p-1)^2*Unew(3p-2)/m*Q(q)... 
- b2*b3*Unew(3p-1)*Q(q))... 
*exp(-b3*Q(q)*((p-1)^3/(48*m^3)... 
-(p-1)^2/(16*m^2)+(p-1)/m*Unew(3p-2)+Unew(3p-1))); \\
KT(r,3p+1) &= -m*b2 + (-m*b1*b3...
- b2*b3*(p-1)^3/(48*m^2)*Q(q)... 
+ b2*b3*(p-1)^2/(16*m)*Q(q)... 
- b2*b3*(p-1)*Unew(3p-2)*Q(q) + m*b2...
- m*b2*b3*Unew(3p-1)*Q(q))... 
*exp(-b3*Q(q)*((p-1)^3/(48*m^3)... 
-(p-1)^2/(16*m^2)+(p-1)/m*Unew(3p-2)+Unew(3p-1))); \\
\end{align*} \]
\[ \text{end} \]
\[ r=r+1; \]
\[ \text{end} \]
\[ clear n p \]
\[ \text{if } m>1 \]
\[ \text{for } n=2:m \]
\[ \begin{align*}
KT(r,3n-5) &= (n-1)^2/(4*m^2); \\
KT(r,3n-4) &= (n-1)/(2*m); \\
KT(r,3n-3) &= 1; \\
KT(r,3n-2) &= -(n-1)^2/(4*m^2); \\
KT(r,3n-1) &= -(n-1)/(2*m); \\
KT(r,3n) &= -1; \\
r=r+1; \]
\[ \text{end} \]
\[ \text{end} \]
\[ clear n \]
\[ \text{if } m>1 \]
\[ \text{for } n=2:m \]
A.4 MATLAB Code to Determine Nonlinear Diaphragm Force-Displacement Response

```matlab
KT(r,3*n-5) = (n-1)/m;
KT(r,3*n-4) = 1;
KT(r,3*n-2) = -(n-1)/m;
KT(r,3*n-1) = -1;
r=r+1;
end
end
clear n r
delt_u =-inv(KT)*R;
Uold = Unew;
Unew = Uold + delt_u;

% Code to terminate loop if convergence does not occur to within % the specified tolerance
itr=itr+1;
if itr == 10000            % Specify maximum number of iterations
    disp('Maximum iterations reached')
    return
end
end
end

%% Displacements
if strcmp(LD,'PAR') == 1
    if x >= (i-1)*l && x <= i*l | | x <= L/2
        v(q) = w(q)*L^4/(E*I_fb)*(1/24*xnorm^4 - 1/12*xnorm^3 ...
         + Unew(3*i-2)*xnorm^2 + Unew(3*i-1)*xnorm + Unew(3*i));
    end
elseif strcmp(LD,'PERP') == 1
    if x >= (i-1)*fb_d && x <= i*fb_d | | x <= L/2
        v(q) = w(q)*L^4/(E*I_j)*(1/24*xnorm^4 - 1/12*xnorm^3 ...
         + Unew(3*i-2)*xnorm^2 + Unew(3*i-1)*xnorm + Unew(3*i));
    end
end
disp(["Maximum displacement at span location ",num2str(x),"mm: ",...
    "v(max) = ",num2str(v(end)),"mm"]);

%% Force-Displacement Plot
% Total applied load
P = W*L/1000;
figure
plot(v,P,'Color','r');
xlabel('Diaphragm displacement (mm)'); ylabel('Total applied load (kN)');
title('Nonlinear Force-Displacement');
```

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Appendix B

NAIL CONNECTION TEST RESULTS

B.1 Force-Displacement Response

The measured force-displacement responses of the nail connections described in Chapter 5 are presented. The results from: (1) six nail connections built with the same materials as used for sub-component diaphragm testing (see Chapter 6) are presented in Figure B-1, (2) twelve nail connections constructed with the same materials as used for full-scale diaphragm testing (see Chapter 7) are presented in Figure B-2, (3) twelve nail connections prepared from timber floor sections extracted from a URM building in Parnell, Auckland, are presented in Figure B-3, and (4) twenty two nail connections prepared from timber floor sections removed from the heritage URM T Adair building in the central business district of Gisborne are presented in Figure B-4.

B.2 Averaged Backbone Data

An outline of the lower bound, mean, and upper bound backbone data for each connection type is presented in Tables B-1 to B-4 for nonlinear spring calibration in the diaphragm FE models reported in Chapter 8. In each case the data is based upon application of Equation 5-2 to the relevant data population.
Appendix B: Nail Connection Test Results

Figure B-1: New-USA connection force-displacement responses
Figure B-2: New-NZ connection force-displacement responses
Appendix B: Nail Connection Test Results

Figure B-2 continued
Figure B-3: Parnell connection force-displacement responses
Figure B-3 continued
B.1 Force-Displacement Response

Figure B-4: T Adair connection force-displacement responses
Appendix B: Nail Connection Test Results

Figure B-4 continued
B.1 Force-Displacement Response

Figure B-4 continued
Appendix B: Nail Connection Test Results

Figure B-4 continued
### Table B-1: Backbone data for New-USA nail connections

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### Appendix B: Nail Connection Test Results

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### Appendix B: Nail Connection Test Results

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B.2 Averaged Backbone Data

Table B-4: Backbone data for Salvaged-T Adair nail connections

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### Appendix B: Nail Connection Test Results

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Appendix C

SUPPLEMENTARY FULL-SCALE DIAPHRAGM TEST RESULTS

C.1 Force-Displacement Response

The force-displacement responses of the tested full-scale diaphragms are plotted in Figure C-1 with relevant backbone curves, which were developed using the methodology outlined in Section 7.2.4.3.
Figure C-1: Force-displacement response with backbone curves
Figure C-1 continued

(c) 2a-PARA

(d) 2b-PARA

Figure C-1 continued
Figure C-1 continued
(g) 2a-PERP

(h) 2b-PERP

Figure C-1 continued
C.2 Side Frame Deformation

Figures C-2 to C-5 show the in-plane (S_{IP1} and S_{IP2}) and out-of-plane (S_{OP1} to S_{OP4}) deformations of the steel side frames relative to applied load for diaphragms 1a-PARA, 1b-PARA, 2a-PARA and 2b-PARA, respectively. Refer to the illustration provided in Figure 7-8 for transducer locations.

Figure C-2: Diaphragm 1a-PARA side frame displacements
C.2 Side Frame Deformation

Figure C-2 continued

(e) $S_{OP3}$

(f) $S_{OP4}$
Figure C-3: Diaphragm 1b-PARA side frame displacements
Figure C-4: Diaphragm 2a-PARA side frame displacements
Appendix C: Supplementary Full-Scale Diaphragm Test Results

Figure C-5: Diaphragm 2b-PARA side frame displacements

(a) $S_{IP1}$  
(b) $S_{IP2}$  
(c) $S_{OP1}$  
(d) $S_{OP2}$  
(e) $S_{OP3}$  
(f) $S_{OP4}$
C.3 Performance Degradation

C.3.1 Stiffness degradation

Effective shear stiffness for each loading cycle to the same displacement amplitude is plotted against cycle number in Figure C-6 for as-built and retrofitted diaphragms. Effective shear stiffness values were normalised by the effective shear stiffness of the first loading cycle of the relevant displacement amplitude.

Figure C-7 and Figure C-8 show effective shear stiffness plotted against displacement amplitude for as-built and retrofitted diaphragms, respectively.

Figure C-6: Shear stiffness degradation between cycles
Figure C-6 continued
Figure C-7: As-built diaphragms effective shear stiffness at each displacement amplitude
Figure C-8: Retrofitted diaphragm effective shear stiffness at each displacement amplitude
C.3.2 Energy degradation

The energy dissipated during each loading cycle to the equivalent displacement amplitude is plotted against cycle number in Figure C-9 for as-built and retrofitted diaphragms. Dissipated energy values were normalised by the energy dissipated during the first loading cycle of the relevant displacement amplitude.

Figure C-10 and Figure C-11 show the dissipated energy at increasing displacement amplitude for as-built and retrofitted diaphragms, respectively.

Figure C-9: Dissipated energy degradation between cycles at equivalent displacement amplitudes
Appendix C: Supplementary Full-Scale Diaphragm Test Results

Figure C-9 continued
C.3 Performance Degradation

Figure C-10: As-built diaphragms dissipated energy at each displacement amplitude
Figure C-11: Retrofitted diaphragms dissipated energy at each displacement amplitude
C.4 Modal Analysis

The fundamental frequency values and associated mode shapes as well as corresponding modal assurance criteria plots identified from experimental modal testing and system identification of tested full-scale diaphragms are provided in Tables C-1 to C-15. All as-built and retrofitted diaphragms were tested before and after pseudo-static loading, except for diaphragm 1b-PERP-after pseudo-static testing, which was not tested. Generally six sets of results are provided for each series of modal tests performed.
### Table C-1: 1a-PARA modal analysis results before cyclic testing

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<th>Mode #1 F=23.5103 Hz</th>
<th>Mode #1 F=23.4949 Hz</th>
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<tr>
<td>SSI2</td>
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</tr>
</tbody>
</table>

| **Test 5** | ![Mode #1 F = 23.5599 Hz](image) | ![Mode #1 F = 23.5599 Hz](image) |
| PP     | 23.44       |                          |
| FDD    | 23.44       |                          |
| EFDD   | 23.44       |                          |
| ERA    | 23.38       |                          |
| SSI1   | 23.52       |                          |
| SSI2   | 23.56       |                          |

| **Test 6** | ![Mode #1 F = 23.647 Hz](image) | ![Mode #1 F = 23.647 Hz](image) |
| PP     | 23.44       |                          |
| FDD    | 23.44       |                          |
| EFDD   | 23.44       |                          |
| ERA    | 23.65       |                          |
| SSI1   | 23.50       |                          |
| SSI2   | 23.51       |                          |
# Appendix C: Supplementary Full-Scale Diaphragm Test Results

## Table C-2: 1a-PARA modal analysis results after cyclic testing

<table>
<thead>
<tr>
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<th>Fundamental frequency</th>
<th>Mode shapes</th>
<th>Modal assurance criteria</th>
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<td>Method</td>
<td>f (Hz)</td>
<td>PP</td>
</tr>
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<td>Test 1</td>
<td>PP</td>
<td>17.58</td>
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<td>Test 1</td>
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**Graphs**: 
- Mode #1, F = 18.3933 Hz
- Mode #1, F = 18.5275 Hz
- Mode #1, F = 18.2632 Hz

**Node Number**

**Normalized Modal Amplitude**

<table>
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<table>
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<th>F= 18.2632 Hz</th>
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<th>ERA</th>
<th>SSI</th>
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Table C-2 continued

<table>
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<th>Modal Amplitude</th>
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<td>EFDD</td>
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<td>ERA</td>
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Appendix C: Supplementary Full-Scale Diaphragm Test Results

**Table C-3: 1b-PARA modal analysis results before cyclic testing**

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<th>Method</th>
<th>$f$ (Hz)</th>
<th>Modal frequency</th>
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<th>Modal assurance criteria</th>
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<td>PP</td>
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<td>FDD</td>
<td>27.34</td>
<td>FDD</td>
<td>ERA, SSI2</td>
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<tr>
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<td>EFDD</td>
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<td>ERA, SSI2</td>
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<tr>
<td></td>
<td>ERA</td>
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<td>FDD, EFDD</td>
<td>SSI1, SSI2</td>
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<td>SSI1</td>
<td>FDD, EFDD</td>
<td>ERA, SSI2</td>
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<td>SSI2</td>
<td>FDD, EFDD</td>
<td>ERA, SSI1</td>
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![Graphs for Test 1, 2, and 3 showing modal analysis results and MAC values]

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C.4 Modal Analysis

Table C-3 continued

<table>
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Test 4

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<td>FDD</td>
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<td>EFDD</td>
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![Graph](image1.png)

Test 5

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<tr>
<td>FDD</td>
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<td>EFDD</td>
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![Graph](image2.png)

Test 6

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![Graph](image3.png)
### Table C-4: 1b-PARA modal analysis results after cyclic testing

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### Table C-4 continued

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<td><img src="image11" alt="Mode #1" /></td>
<td><img src="image12" alt="Modal Assurance" /></td>
</tr>
</tbody>
</table>

| 5    | PP     | 17.58          | ![Mode #1](image13) | ![Modal Assurance](image14) |
|      | FDD    | 17.58          | ![Mode #1](image15) | ![Modal Assurance](image16) |
|      | EFDD   | 17.58          | ![Mode #1](image17) | ![Modal Assurance](image18) |
|      | ERA    | 17.53          | ![Mode #1](image19) | ![Modal Assurance](image20) |
|      | SSI1   | 17.37          | ![Mode #1](image21) | ![Modal Assurance](image22) |
|      | SSI2   | 17.41          | ![Mode #1](image23) | ![Modal Assurance](image24) |

| 6    | PP     | 17.58          | ![Mode #1](image25) | ![Modal Assurance](image26) |
|      | FDD    | 17.58          | ![Mode #1](image27) | ![Modal Assurance](image28) |
|      | EFDD   | 17.58          | ![Mode #1](image29) | ![Modal Assurance](image30) |
|      | ERA    | 17.73          | ![Mode #1](image31) | ![Modal Assurance](image32) |
|      | SSI1   | 17.58          | ![Mode #1](image33) | ![Modal Assurance](image34) |
|      | SSI2   | 17.78          | ![Mode #1](image35) | ![Modal Assurance](image36) |
### Table C-5: 2a-PARA modal analysis results before cyclic testing

<table>
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<th>Modal Assurance Criteria</th>
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</tbody>
</table>

![Mode #1 F= 19.5312 Hz](image1)

![Mode #1 F= 20.1772 Hz](image2)

![Mode #1 F= 19.5313 Hz](image3)

| Test 2 |
|--------|-----------------------|-------------|--------------------------|
| PP     | 19.53                 |             |                          |
| FDD    | 19.53                 |             |                          |
| EFDD   | 19.53                 |             |                          |
| ERA    | 20.18                 |             |                          |
| SSI1   | 19.00                 |             |                          |
| SSI2   | 18.85                 |             |                          |

| Test 3 |
|--------|-----------------------|-------------|--------------------------|
| PP     | 19.53                 |             |                          |
| FDD    | 19.53                 |             |                          |
| EFDD   | 19.53                 |             |                          |
| ERA    | 18.88                 |             |                          |
| SSI1   | 19.08                 |             |                          |
| SSI2   | 18.71                 |             |                          |
### Table C-5 continued

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<tbody>
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Appendix C: Supplementary Full-Scale Diaphragm Test Results

Table C-7: 2b-PARA modal analysis results before cyclic testing

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For each test, the mode shapes and modal assurance criteria are shown in graphical form.
### Table C-7 continued

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![Mode #1 F = 26.0938 Hz]

![Modal Assurance Criteria]

| **Test 5**            |             |                          |
| **Method**            | f (Hz)      |                          |
| PP                    | 25.39       |                          |
| FDD                   | 25.39       |                          |
| EFDD                  | 25.39       |                          |
| ERA                   | 26.17       |                          |
| SSII                  | 26.13       |                          |
| SSII2                 | 26.43       |                          |

![Mode #1 F = 26.2473 Hz]

![Modal Assurance Criteria]

| **Test 6**            |             |                          |
| **Method**            | f (Hz)      |                          |
| PP                    | 25.39       |                          |
| FDD                   | 25.39       |                          |
| EFDD                  | 25.39       |                          |
| ERA                   | 26.24       |                          |
| SSII                  | 26.10       |                          |
| SSII2                 | 26.10       |                          |

![Mode #1 F = 26.2359 Hz]

![Modal Assurance Criteria]
Table C-8: 2b-PARA modal analysis results after cyclic testing

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### Appendix C: Supplementary Full-Scale Diaphragm Test Results

#### Table C-9: 1a-PERP modal analysis results before cyclic testing

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<tr>
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<tr>
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Table C-9 continued

<table>
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**Test 4: Right**

<table>
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<tr>
<th>Method</th>
<th>(f)(Hz)</th>
</tr>
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<tbody>
<tr>
<td>PP</td>
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</tr>
<tr>
<td>FDD</td>
<td>11.72</td>
</tr>
<tr>
<td>EFDD</td>
<td>11.72</td>
</tr>
<tr>
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</tr>
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<td>SSI2</td>
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<tr>
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</table>

**Test 5: Right**

<table>
<thead>
<tr>
<th>Method</th>
<th>(f)(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP</td>
<td>11.72</td>
</tr>
<tr>
<td>FDD</td>
<td>11.72</td>
</tr>
<tr>
<td>EFDD</td>
<td>11.72</td>
</tr>
<tr>
<td>ERA</td>
<td>11.26</td>
</tr>
<tr>
<td>SSI2</td>
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</tr>
<tr>
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**Test 6: Right**

<table>
<thead>
<tr>
<th>Method</th>
<th>(f)(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP</td>
<td>11.72</td>
</tr>
<tr>
<td>FDD</td>
<td>11.72</td>
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### Table C-10: 1b-PERP modal analysis results before cyclic testing

<table>
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<th>Test 3: Left</th>
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</thead>
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<td><strong>Method</strong></td>
<td><strong>f (Hz)</strong></td>
<td><strong>f (Hz)</strong></td>
</tr>
<tr>
<td>PP</td>
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<td>21.48</td>
</tr>
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<td>21.48</td>
<td>21.48</td>
</tr>
<tr>
<td>EFDD</td>
<td>21.48</td>
<td>21.48</td>
</tr>
<tr>
<td>ERA</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SSI1</td>
<td>22.20</td>
<td>21.86</td>
</tr>
<tr>
<td>SSI2</td>
<td>-</td>
<td>22.73</td>
</tr>
</tbody>
</table>

**Mode shapes**

- **Mode #1**: F = 22.734 Hz
- **Mode #2**: F = 22.1965 Hz
- **Mode #1**: F = 21.4844 Hz
- **Mode #2**: F = 21.4884 Hz

**Modal assurance criteria**

- **MAC** values for each mode and method are shown in the images.
Table C-10 continued

<table>
<thead>
<tr>
<th>Fundamental frequency</th>
<th>Mode shapes</th>
<th>Modal assurance criteria</th>
</tr>
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</table>

**Test 4: Right**

<table>
<thead>
<tr>
<th>Method</th>
<th>$f$(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP</td>
<td>21.48</td>
</tr>
<tr>
<td>FDD</td>
<td>21.48</td>
</tr>
<tr>
<td>EFDD</td>
<td>21.48</td>
</tr>
<tr>
<td>ERA</td>
<td>-</td>
</tr>
<tr>
<td>SSI1</td>
<td>21.55</td>
</tr>
<tr>
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<td>-</td>
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**Test 5: Right**

<table>
<thead>
<tr>
<th>Method</th>
<th>$f$(Hz)</th>
</tr>
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<tbody>
<tr>
<td>PP</td>
<td>21.48</td>
</tr>
<tr>
<td>FDD</td>
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<td>EFDD</td>
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<td>ERA</td>
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**Test 6: Right**

<table>
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<tr>
<th>Method</th>
<th>$f$(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP</td>
<td>21.48</td>
</tr>
<tr>
<td>FDD</td>
<td>21.48</td>
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<tr>
<td>EFDD</td>
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</tr>
<tr>
<td>ERA</td>
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</tr>
<tr>
<td>SSI1</td>
<td>22.13</td>
</tr>
<tr>
<td>SSI2</td>
<td>-</td>
</tr>
</tbody>
</table>
Table C-11: 1b-PERP modal analysis results after cyclic testing

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<th>Mode shapes</th>
<th>Modal assurance criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test 1: Left</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method</td>
<td>f (Hz)</td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td>13.67</td>
<td></td>
</tr>
<tr>
<td>FDD</td>
<td>13.67</td>
<td></td>
</tr>
<tr>
<td>EFDD</td>
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</tr>
<tr>
<td>ERA</td>
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<tr>
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<table>
<thead>
<tr>
<th>Node Number</th>
<th>Normalized Modal Amplitude</th>
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<tbody>
<tr>
<td>1.5</td>
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</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>2.5</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
</tr>
<tr>
<td>3.5</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
</tr>
<tr>
<td>4.5</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

| **Test 2: Left** |                                   |
| Method    | f (Hz)     |                                   |
| PP        | 13.67      |                                   |
| FDD       | 13.67      |                                   |
| EFDD      | 13.67      |                                   |
| ERA       | 14.42      |                                   |
| SSI1      | 13.78      |                                   |
| SSI2      | 13.87      |                                   |

<table>
<thead>
<tr>
<th>Node Number</th>
<th>Normalized Modal Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.1</td>
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<tr>
<td>4</td>
<td>0.6</td>
</tr>
<tr>
<td>4.5</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

| **Test 3: Left** |                                   |
| Method    | f (Hz)     |                                   |
| PP        | 13.67      |                                   |
| FDD       | 13.67      |                                   |
| EFDD      | 13.67      |                                   |
| ERA       | 13.82      |                                   |
| SSI1      | 13.67      |                                   |
| SSI2      | 13.13      |                                   |

<table>
<thead>
<tr>
<th>Node Number</th>
<th>Normalized Modal Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
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<td>2</td>
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<td>3.5</td>
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<tr>
<td>4</td>
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</table>
C.4 Modal Analysis

Table C-11 continued

<table>
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<tr>
<th>Fundamental frequency</th>
<th>Mode shapes</th>
<th>Modal assurance criteria</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Test 4: Right</td>
<td></td>
</tr>
<tr>
<td>Method  ( f ) (Hz)</td>
<td>PP 13.67</td>
<td>FDD 13.67</td>
</tr>
<tr>
<td></td>
<td>Test 5: Right</td>
<td></td>
</tr>
<tr>
<td>Method  ( f ) (Hz)</td>
<td>PP 13.67</td>
<td>FDD 13.67</td>
</tr>
<tr>
<td></td>
<td>Test 6: Right</td>
<td></td>
</tr>
<tr>
<td>Method  ( f ) (Hz)</td>
<td>PP 13.67</td>
<td>FDD 13.67</td>
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### Table C-12: 2a-PERP modal analysis results before cyclic testing

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<tr>
<td>Test 1: Left</td>
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</tr>
<tr>
<td><strong>Method</strong></td>
<td><strong>f (Hz)</strong></td>
<td><strong>MAC</strong></td>
</tr>
<tr>
<td>PP</td>
<td>19.53</td>
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<td>FDD</td>
<td>19.53</td>
<td>0.95</td>
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<tr>
<td>EFDD</td>
<td>19.53</td>
<td>0.95</td>
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<tr>
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<td>SSI1</td>
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</tr>
<tr>
<td>SSI2</td>
<td>19.87</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Test 2: Left

| **Method** | **f (Hz)** | **MAC** |
| PP | 19.53 | 0.95 |
| FDD | 19.53 | 0.95 |
| EFDD | 19.53 | 0.95 |
| ERA | 19.78 | 0.95 |
| SSI1 | 19.73 | 0.95 |
| SSI2 | 19.78 | 0.95 |

Test 3: Left

| **Method** | **f (Hz)** | **MAC** |
| PP | 19.53 | 0.95 |
| FDD | 19.53 | 0.95 |
| EFDD | 19.53 | 0.95 |
| ERA | 19.38 | 0.95 |
| SSI1 | 19.76 | 0.95 |
| SSI2 | 19.83 | 0.95 |
Table C-12 continued

<table>
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<tr>
<th>Fundamental frequency</th>
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</tr>
</thead>
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</tr>
<tr>
<td>Method</td>
<td>f(Hz)</td>
<td></td>
</tr>
<tr>
<td>PP</td>
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<td></td>
</tr>
<tr>
<td>FDD</td>
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<td></td>
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<tr>
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<tr>
<td>Test 5: Right</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method</td>
<td>f(Hz)</td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td>19.53</td>
<td></td>
</tr>
<tr>
<td>FDD</td>
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<tr>
<td>Method</td>
<td>f(Hz)</td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td>19.53</td>
<td></td>
</tr>
<tr>
<td>FDD</td>
<td>19.53</td>
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</table>
Table C-13: 2a-PERP modal analysis results after cyclic testing

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<th>Test 2: Left</th>
<th>Test 3: Left</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>f (Hz)</td>
<td>Method</td>
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<tr>
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<td>15.63</td>
<td>PP</td>
</tr>
<tr>
<td>FDD</td>
<td>15.63</td>
<td>FDD</td>
</tr>
<tr>
<td>EFDD</td>
<td>15.63</td>
<td>EFDD</td>
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<td>SSI1</td>
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<td>15.91</td>
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Table C-13 continued

<table>
<thead>
<tr>
<th>Fundamental frequency</th>
<th>Mode shapes</th>
<th>Modal assurance criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 4: Right</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method</td>
<td>f (Hz)</td>
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<tr>
<td>PP</td>
<td>15.63</td>
<td></td>
</tr>
<tr>
<td>FDD</td>
<td>15.63</td>
<td></td>
</tr>
<tr>
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<td>15.63</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>SSI2</td>
<td>15.35</td>
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</tr>
</tbody>
</table>

![Graph showing Mode #1 for Test 4]

| Test 5: Right         |             |                          |
| Method                | f (Hz)      |                          |
| PP                    | 15.63       |                          |
| FDD                   | 15.63       |                          |
| EFDD                  | 15.63       |                          |
| ERA                   | 15.26       |                          |
| SSI1                  | 15.24       |                          |
| SSI2                  | 15.29       |                          |

![Graph showing Mode #1 for Test 5]

| Test 6: Right         |             |                          |
| Method                | f (Hz)      |                          |
| PP                    | 15.63       |                          |
| FDD                   | 15.63       |                          |
| EFDD                  | 15.63       |                          |
| ERA                   | 16.64       |                          |
| SSI1                  | 15.25       |                          |
| SSI2                  | 15.30       |                          |

![Graph showing Mode #1 for Test 6]
Table C-14: 2b-PERP modal analysis results before cyclic testing

<table>
<thead>
<tr>
<th>Test 1: Left</th>
<th>Fundamental frequency</th>
<th>Mode shapes</th>
<th>Modal assurance criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>( f (\text{Hz}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td>29.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDD</td>
<td>29.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EFDD</td>
<td>29.30</td>
<td></td>
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</tr>
<tr>
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<tr>
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<tr>
<td>SSI2</td>
<td>28.87</td>
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</tr>
</tbody>
</table>

![Graph](image1)

![Graph](image2)

![Graph](image3)

<table>
<thead>
<tr>
<th>Test 2: Left</th>
<th>Fundamental frequency</th>
<th>Mode shapes</th>
<th>Modal assurance criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>( f (\text{Hz}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td>29.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDD</td>
<td>29.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EFDD</td>
<td>29.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERA</td>
<td>28.67</td>
<td></td>
<td></td>
</tr>
<tr>
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</tbody>
</table>

![Graph](image4)

![Graph](image5)

![Graph](image6)

<table>
<thead>
<tr>
<th>Test 3: Left</th>
<th>Fundamental frequency</th>
<th>Mode shapes</th>
<th>Modal assurance criteria</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( f (\text{Hz}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td>29.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDD</td>
<td>29.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EFDD</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ERA</td>
<td>28.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSI1</td>
<td>29.00</td>
<td></td>
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![Graph](image7)

![Graph](image8)

![Graph](image9)
Table C-14 continued

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**Test 4: Right**

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## Table C-15: 2b-PERP modal analysis results after cyclic testing

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### Fundamental frequency

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### Modal assurance criteria

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Appendix D

DIAPHRAGM ASSESSMENT CALCULATIONS

D.1 Diaphragm Assessment and Retrofit Design

The following calculations were used for a mock seismic assessment of the tested as-built diaphragms and to subsequently design appropriate retrofit details.

Assume the following for the assessment:

- Diaphragm configuration exactly that of 1a-PARA.
- Two-storey Typology D URM building (Russell 2010), located in Wellington, New Zealand and founded on Class D sub-soil (SANZ 2004).
- Ground floor height, 4.0 m.
- First floor height, 3.0 m.
- Building dimensions identical to diaphragm.
- Three brick leafs at ground storey and two brick leafs at first floor.
- Unit weight of brick, $\gamma = 18$ kN/m$^3$.
- Side walls and back wall to have no penetrations.
• Front wall to comprise typical Typology D street frontage penetrations (Russell 2010).
• Strengthen to 67% of current building standard requirements for 1/500 return period earthquake.

The mock URM building configuration is illustrated in Figure D-1 for future reference.

![Figure D-1: Schematic of mock URM building](image)

**D.1.1 Seismic weights**

Assume general office use from NZS 1170.5:2004 Table 3.1 (SANZ 2004). Ignore effects of cross-beams, openings and columns.

Diaphragm self weight = 17.8 kN

Expected live load = 

\[3.0 \times 10.4 \times 5.535\]  

\[= 172.7 \text{ kN}\]
NZS 1170.4:2004 Equation 4.2(i):

\[ w_i = G_i + \sum \psi E Q_i \]

\[ \psi_a = 0.3 + \frac{3}{\sqrt{115.1}} = 0.58 \]

\[ \psi_E = 0.3 \]

\[ \therefore w_L = 0.3 \times 0.58 \times 172.7 = 30.0 \text{ kN} \]

Determine area of penetrations:

Ground floor \[ A_g = 3 \times \left( \frac{4}{2} - (4 - 3) \right) = 3.0 \text{ m}^2 \]

First floor \[ A_f = 0.8 \times \left( \frac{3}{2} - 1 \right) = 0.8 \text{ m}^2 \]

Determine seismic weights:

Grid line A wall ⇒

\[ V_{u,A} = \left( 5.535 \times \frac{4.0}{2} - 3.0 \right) \times 0.360 \times 18 + \left( 5.535 \times \frac{3.0}{2} - 0.8 \right) \times 0.240 \times 18 \]

\[ V_{u,A} = 85 \text{ kN} \]

Grid line B wall ⇒

\[ V_{u,B} = 5.535 \times \frac{4.0}{2} \times 0.360 \times 18 + 5.535 \times \frac{3.0}{2} \times 0.240 \times 18 \]
Appendix D: Diaphragm Assessment Calculations

\[ V_{u,b} = 108 \text{ kN} \]

Grid lines 1 and 2 walls ⇒

\[ V_{u,1,2} = 10.4 \times \frac{4.0}{2} \times 0.360 \times 18 + 10.4 \times \frac{3.0}{2} \times 0.240 \times 18 \]

\[ V_{u,1} = V_{u,2} = 202 \text{ kN} \]

D.1.2 Diaphragm fundamental period

Using the provisions of ASCE 41-06 (2007),

\[ T_1 = \sqrt{3.07\Delta_d} \]

where \( \Delta_d \) = maximum in-plane deflection due to a lateral load of 1.0g in metres.

\[ \Delta_d = \frac{V_u}{K_d} \]

where \( V_u \) is diaphragm lateral load (kN) and \( K_d \) is diaphragm stiffness (kN/mm).

\[ K_d = \frac{4BG_d}{L} \]

where \( B \) is diaphragm width (m), \( L \) is diaphragm span, and \( G_d \) is diaphragm shear stiffness from Table 8-2.

Consider north-south direction:

\[ V_{u,NS} = 17.8 + 202 + 202 + 30 \]

\[ V_{u,NS} = 452 \text{ kN} \]

\[ K_d = \frac{4 \times 5.535 \times 350}{10.4} = 745 \text{ kN/m} \]
:\[ \Delta_d = \frac{452}{745} = 0.61 \text{ m} \]

\[ \therefore T_{1,NS} = \sqrt{3.07 \times 0.61} = 1.37 \text{ s} \]

Consider east-west direction:

\[ V_{u,EW} = 17.8 + 85 + 108 + 30 \]

\[ V_{u,EW} = 241 \text{ kN} \]

\[ K_d = \frac{4 \times 10.4 \times 350}{5.535} = 2631 \text{ kN/m} \]

\[ \therefore \Delta_d = \frac{241}{2631} = 0.09 \text{ m} \]

\[ \therefore T_{1,EW} = \sqrt{3.07 \times 0.09} = 0.53 \text{ s} \]

### D.1.3 Diaphragm seismic loading

Using the provisions of NZS 1170.5:2004 and recommendations offered by Oliver (2010a), determine the design seismic loads.

Consider north-south direction:

\[ V_{D,NS} = C_1 C_3 C_d(T_1) W_D \]

\[ C_d(T_1) = \frac{C(T_1) S_p}{k_\mu} \]

\[ C(T_1) = C_h(T) ZRN(T, D) \]
Appendix D: Diaphragm Assessment Calculations

\[ C(T_1) = 1.56 \times 0.4 \times 1.0 \times 1.0 \]

\[ C(T_1) = 0.62 \text{g} \]

Take \( S_p = 0.7 \) and \( \mu = 4 \)

\[ C_d(T_1) = \left( \frac{0.62 \times 0.7}{4} \right) = 0.11 \text{g} \]

Determine seismic weight for 67% capacity of current standards.

\[ V_{D,NS} = 0.67 \times 1.0 \times 1.0 \times 0.11 \times 452 \]

\[ V_{D,NS} = 33 \text{kN} \]

**Consider east-west direction:**

\[ C(T_1) = 3.0 \times 0.4 \times 1.0 \times 1.0 \]

\[ C(T_1) = 1.20 \text{g} \]

As before, take \( S_p = 0.7 \) and \( \mu = 4 \)

\[ C_d(T_1) = \left( \frac{1.2 \times 0.7}{4} \right) = 0.21 \text{g} \]

\[ V_{D,EW} = 0.67 \times 1.0 \times 1.0 \times 0.21 \times 241 \]

\[ V_{D,EW} = 34 \text{kN} \]
D.1.4 Diaphragm deformation assessment

Diaphragm midspan displacement is typically limited to the lesser of: (1) 150 mm, or (2) half the thickness of the supporting wall, which in this case is 120 mm.

**Consider north-south direction:**

\[
\Delta_{d,NS} = \mu \frac{V_u}{K_d} = 4 \times \frac{33}{745} = 177 \text{ mm} > 120 \text{ mm} \quad \therefore \text{Not OK}
\]

**Consider east-west direction:**

\[
\Delta_{d,EW} = \mu \frac{V_u}{K_d} = 4 \times \frac{34}{2631} = 52 \text{ mm} < 120 \text{ mm} \quad \therefore \text{OK}
\]

Diaphragm deformation exceeds imposed limits in N-S direction. Diaphragm is therefore excessively flexible and requires retrofitting to improve stiffness.

D.1.5 Existing sheathing strength capacity assessment

(i) From Appendix 11B of NZSEE (2006):

\[
\phi R_b = \frac{\phi Q_n s}{\ell b_s}
\]

where \( \phi = 1 \), \( Q_n \) is nail capacity (kN), \( s \) is nail couple spacing (m), \( \ell \) is joist spacing, and \( b_s \) is floorboard width.

\( Q_n \) may be evaluated using the New Zealand timber design standard, NZS 3603:1993 (SANZ 1993).

\[
\phi Q_n = k Q_k
\]

where,
Appendix D: Diaphragm Assessment Calculations

\[ k = 1.0 \times 1.0 \times 1.0 \times 1.0 \times 1.0 \times 1.3 = 1.3 \]

\[ Q_k = \]

\[ \therefore Q_n = 1.3 \times 0.631 = 0.820 \text{ kN} \]

\[ \therefore \phi R_n = \frac{1.0 \times 0.82 \times 0.095}{0.4 \times 0.135} = 1.44 \text{ kN/m} \]

(ii) From ASCE 41-06 (2007) Table 8-2:

\[ \phi R_n = 1.75 \text{ kN/m} \]

Hence NZSEE (2006) provisions more conservative. Take:

\[ \phi R_n = 1.44 \text{ kN/m} \]

D.1.6 Retrofit details

It was shown in Section D.1.4 that ‘existing’ diaphragm stiffness was insufficient and retrofitting was required. Trial an appropriate retrofit design and check its suitability. Try the following retrofit details:

- F8 grade 15 mm \( \times \) 1200 mm \( \times \) 2400 mm plywood overlay
- Use 75 mm \( \times \) 24 gauge zinc plated sheet metal with ECKO SF-9222 staples (1.24 mm \( \times \) 1.00 mm rectangular cross-section leg length at least 16 mm) @ 100 mm centres to block the plywood
- Provide chords along grid lines 1 and 2. Chords to be 40 mm \( \times \) 6 mm mild steel flat.
D.1.7 Revised diaphragm fundamental period

Consider north-south direction:

Determine yield strength of plywood overlay:

Holmes Solutions found a single ECKO SF-9222 staple fastening 24 gauge sheet metal to a plywood panel to have a capacity of 0.6 kN. Hence for the proposed 100 mm staple spacing ⇒

\[ Q_n = \frac{0.6}{0.1} = 6 \text{ kN/m} \]

\[ V_y = 2 \times Q_n \times B \]

\[ \vdash V_{y,NS} = 2 \times 6 \times 5.535 = 66.4 \text{ kN} \]

Plywood diaphragm yield displacement can be evaluated using NZS 3603:1993. From which, displacement is calculated as:

\[ \Delta_d = \Delta_1 + \Delta_2 + \Delta_3 \]

where \( \Delta_1 \) is flexural displacement, \( \Delta_2 \) is shear displacement, \( \Delta_3 \) is displacement due to nail slip.

\( \Delta_1 \) is typically negligible for plywood diaphragms and is considered zero.

Shear deformation:

\[ \Delta_2 = \frac{V_{y,NS}L}{8Gb_t} \]

\[ \vdash \Delta_2 = \frac{66.4 \times 10.4}{8 \times 0.455 \times 5.535 \times 0.015} = 0.0023 \text{ m} \]
Appendix D: Diaphragm Assessment Calculations

\[ \Delta_2 = 2.3 \text{ mm} \]

Deformation due to nail slip:

\[ \Delta_3 = \frac{(1 + a)me_n}{2} \]

where \( a = 2.0 \) and \( m = 5 \)

Nail slip \( e_n \) can be taken direction from test results (Oliver 2008),

\[ e_n = 1.8 \text{ mm} \]

\[ \therefore \Delta_3 = \frac{(1 + 2.0) \times 5 \times 1.8}{2} = 13.5 \text{ mm} \]

\[ \therefore \Delta_y = \Delta_1 + \Delta_2 + \Delta_3 = 2.3 + 13.5 = 15.8 \text{ mm} \]

Determine combined diaphragm stiffness:

\[ K_d = K_{existing} + K_{overlay} \]

\[ \therefore K_d = 745 + \frac{66.4}{0.0158} = 4948 \text{ kN/m} \]

Therefore elastic deformation of retrofitted diaphragm with 1.0g of lateral loading is:

\[ \Delta_d = \frac{452}{4948} = 0.091 \text{ m} \]

\[ \therefore T_{1,NS} = \sqrt{3.07 \times 0.091} = 0.53 \text{s} \]

Consider east-west direction:

\[ V_{y,EW} = 2 \times 6 \times 10.4 = 124.8 \text{ kN} \]
Therefore elastic deformation of retrofitted diaphragm with 1.0g of lateral loading is:

\[ \Delta_d = \frac{241}{18,231} = 0.013 \text{ m} \]

\[ \therefore T_{1,EW} = \sqrt{3.07 \times 0.013} = 0.20 \text{ s} \]

**D.1.8 Revised diaphragm seismic load calculation**

Given the increased stiffness and reduced fundamental period, the diaphragm seismic loads must be re-evaluated.

**Consider north-south direction:**

\[ C(T_1) = 2.91 \times 0.4 \times 1.0 \times 1.0 = 1.16 \text{g} \]

Take \( S_p = 0.7 \) and \( \mu = 4.0 \) for the plywood overlay

\[ k_\mu = \frac{\mu - 1}{0.7} \text{T}_1 + 1 \quad \text{for } T_1 < 0.7 \text{s} \]
Appendix D: Diaphragm Assessment Calculations

\[ k_\mu = \frac{(4 - 1) \times 0.53}{0.7} + 1 = 3.27 \]

\[ \therefore C_d(T_1) = \frac{1.16 \times 0.7}{3.27} = 0.25g \]

Hence can calculate seismic load equal to 67% of current design standards:

\[ V_{d,NS} = 0.67 \times 1.0 \times 1.0 \times 0.25 \times 452 = 75.7 \text{ kN} \]

**Consider east-west direction:**

\[ C(T_1) = 3.0 \times 0.4 \times 1.0 \times 1.0 = 1.2g \]

\[ k_\mu = \frac{(4 - 1) \times 0.2}{0.7} + 1 = 1.9 \]

\[ \therefore C_d(T_1) = \frac{1.2 \times 0.7}{1.9} = 0.44g \]

\[ C_1 = 1.0 + (0.5 - 0.2) \times \frac{(1.5 - 1.0)}{(0.5 - 0.1)} = 1.36 \]

\[ V_{d,EW} = 0.67 \times 1.36 \times 1.0 \times 0.44 \times 451 = 96.6 \text{ kN} \]
D.1.9 Revised diaphragm shear strength assessment

Consider north-south direction:

\[ V_{\text{max,NS}} = \frac{V_d}{2B} = \frac{75.7}{2 \times 5.535} = 6.84 \text{ kN/m} \]

\[ \phi R_n = 1.44 + 6.0 = 7.44 \text{ kN/m} \]

\[ \phi R_n(7.44) > V_{\text{max,NS}}(6.84) \quad \therefore \text{OK} \]

Consider east-west direction:

\[ V_{\text{max,EW}} = \frac{96.6}{2 \times 10.4} = 4.64 \text{ kN/m} > 7.44 \text{ kN/m} \quad \therefore \text{OK} \]

Therefore ECKO SF-9222 staples @ 100 mm centres OK.

D.1.10 Revised diaphragm deformation assessment

Consider north-south direction:

Before evaluating north-south deformations, the actual ductility required, \( k_{\mu,\text{act}} \), must be determined. This can be achieved by back calculating as follows:

\[ k_{\mu,\text{act}} = \frac{0.67 \times C_1 \times C_3 \times C(T_2) \times S_p \times V}{2 \times \phi R_n \times B} \]

\[ k_{\mu,\text{act}} = \frac{0.67 \times 1.0 \times 1.0 \times 1.16 \times 0.7 \times 452}{2 \times 7.44 \times 5.535} = 3.0 \]

Recall that,
Appendix D: Diaphragm Assessment Calculations

\[ k_\mu = \frac{(\mu - 1)T_1}{0.7} + 1 \]

which by rearranging becomes,

\[ \mu = \frac{(k_\mu - 1)0.7}{T_1} + 1 \]

\[ \therefore \mu_{act} = \frac{(3.0 - 1) \times 0.7}{0.53} + 1 = 3.64 \]

Hence,

\[ \Delta_{d,NS} = \frac{\mu_{act} V_d}{K_d} = 3.64 \times \frac{75.7}{4948} = 55.7 \text{ mm} \]

\[ \Delta_{d,NS}(55.7 \text{ mm}) < 150 \text{ mm or 120 mm} \therefore \text{OK} \]

Consider east-west direction

Again, calculate \( \mu_{act} \)

\[ k_{\mu,act} = \frac{0.67 \times 1.39 \times 1.0 \times 1.2 \times 0.7 \times 241}{2 \times 7.44 \times 10.4} = 1.22 \]

\[ \mu_{act} = \frac{(1.22 - 1) \times 0.7}{0.20} + 1 = 1.77 \]

\[ \therefore \Delta_{d,EW} = 1.77 \times \frac{96.6}{18231} = 9.4 \text{ mm} \]

\[ \Delta_{d,EW}(9.4 \text{ mm}) < 150 \text{ mm or 120 mm} \therefore \text{OK} \]
D.1.11 Plywood thickness check

Consider north-south direction:

\[ V_{\text{max,NS}} = \frac{V_d}{2} = \frac{75.7}{2} = 38 \text{ kN} \]  
\( \text{Shear demand} \)

\[ M_{\text{max,NS}} = \frac{5V_dL}{32} = \frac{5 \times 75.7 \times 10.4}{32} = 123 \text{ kNm} \]  
\( \text{Moment demand} \)

The plywood strength may be evaluated using the provisions of NZS 3603:1993, as follows. Bending strength is determined as:

\[ \phi M_{ni,NS} = \phi k_1 k_8 k_{14} k_{15} f_{pb} t_e \frac{d^2}{6} \]

\[ \therefore \phi M_{ni,NS} = 0.8 \times 1.0 \times 1.0 \times 1.0 \times 0.68 \times 22.5 \times 0.006 \times \frac{5.535^2}{6} \]

\[ \phi M_{ni,NS} = 375 \text{ kNm} > M_{\text{max,NS}} (123 \text{ kNm}) \therefore \text{OK} \]

Shear strength is determined as:

\[ \phi V_{ni,NS} = \phi \frac{2}{3} k_1 k_8 k_{14} k_{15} f_{ps} t d \]

\[ \therefore \phi V_{ni,NS} = 0.8 \times \frac{2}{3} \times 1.0 \times 1.0 \times 1.0 \times 1.0 \times 0.89 \times 4.2 \times 0.0015 \times 5.535 \]

\[ \phi V_{ni,NS} = 166 \text{ kN} > V_{\text{max,NS}} (38 \text{ kN}) \therefore \text{OK} \]

Consider east-west direction:

\[ V_{\text{max,EW}} = \frac{96.6}{2} = 43.3 \text{ kN} \]  
\( \text{Shear demand} \)
Appendix D: Diaphragm Assessment Calculations

\[ M_{\text{max,EW}} = \frac{5 \times 96.6 \times 5.535}{32} = 83.5 \text{ kNm} \quad \text{(Moment demand)} \]

Strengths:

\[ \phi M_{ni,EW} = 0.8 \times 1.0 \times 1.0 \times 1.0 \times 0.68 \times 22.5 \times 0.006 \times \frac{10.4^2}{6} \]

\[ \phi M_{ni,EW} = 1324 \text{ kNm} > M_{\text{max,EW}} (83.5 \text{ kN}) \checkmark \text{OK} \]

\[ \phi V_{ni,EW} = 0.8 \times \frac{2}{3} \times 1.0 \times 1.0 \times 1.0 \times 1.0 \times 0.89 \times 4.2 \times 0.0015 \times 10.4 \]

\[ \phi V_{ni,EW} = 311 \text{ kN} > V_{\text{max,EW}} (43.3 \text{ kN}) \checkmark \text{OK} \]

Therefore, plywood thickness shown to be adequate.

D.1.12 Diaphragm chord design

Design diaphragm chords to remain elastic using capacity design principles.

Consider north-south direction:

Overstrength shear capacity of the retrofitted diaphragm is determined as follows.

\[ R_{n,os} = \phi_{os} R_n \]

where \( \phi_{os} \) is nail overstrength factor taken from NZS 3606:1993 as 1.6, and \( R_n \) is shear capacity of the retrofitted diaphragm equal to 7.44 kN/m.

\[ \therefore R_{n,os} = 1.6 \times 7.44 = 11.9 \text{ kN/m} \]

Diaphragm overstrength factor, \( \phi_{os,NS} \), can subsequently be calculated:
\[ \phi_{os,NS} = \frac{2R_{n,os}B}{V_d} \]

\[ \therefore \phi_{os,NS} = \frac{2 \times 11.9 \times 5.535}{75.7} = 1.74 \]

From which the maximum overstrength bending moment, \( M_{os,NS} \), is determined as,

\[ M_{os,NS} = \frac{5\phi_{os,NS}V_dL}{32} \]

\[ \therefore M_{os,NS} = \frac{5 \times 1.74 \times 75.7 \times 10.4}{32} = 214 \text{ kNm} \]

Maximum elastic chord forces, \( P_{os,NS} \), are therefore,

\[ P_{os,NS} = \frac{M_{os,NS}}{B} = \frac{214}{5.535} = 38.7 \text{ kN} \]

Design chord members. Trial a 40 mm × 6 mm mild steel flat fastened along each 10.4 m edge of the diaphragm with 75 mm × 10 gauge screws @ 100 mm centres. Provide double blocking between joists for screw embedment and also plywood edge nailing (detailed later).

Minimum cross-sectional area required:

\[ A_{g,min} = \frac{38.7}{1.0 \times 0.3} = 129 \text{ mm}^2 \]

\[ A_g (240 \text{ mm}^2) > A_{g,min} (129 \text{ mm}^2) \therefore \text{OK} \]

Check screw size and spacing:
Appendix D: Diaphragm Assessment Calculations

\[ N_{NS}^* = \frac{2P_{os,NS}}{L} = \frac{2 \times 38.7}{10.4} = 7.44 \text{ kN/m} \]

Using Section 4.3.2 of NZS 3603:1993 screw capacity may be evaluated.

\[ \phi Q_n = \phi nk Q_k \]
\[ Q_k = 0.854 \text{ kN}, \phi = 1.0, \text{ and } n = 1.0 \]
\[ k = 1.0 \times 1.0 \times 1.0 = 1.0 \]
\[ \phi Q_n = \frac{1.0 \times 0.854}{0.1} = 8.54 \text{ kN/m} > N_{NS}^* (7.44 \text{ kN/m}) \therefore \text{OK} \]

Chord design for north-south loading adequate.

**Consider east-west direction:**

\[ \phi_{os,EW} = \frac{2 \times 11.9 \times 10.4}{96.6} = 2.56 \]
\[ M_{os,EW} = \frac{5 \times 2.56 \times 96.6 \times 5.535}{32} = 214 \text{ kNm} \]
\[ P_{os,EW} = \frac{M_{os,EW}}{B} = \frac{214}{10.4} = 20.6 \text{ kN} \]

Design chord members. Because continuous joists exist along the 5.535 m edges of the diaphragm, these can be used as chord members if sufficient edge nailing is provided. Try 75 mm × 3.15 mm DIA power driven nails @ 100 mm centres.

\[ N_{EW}^* = \frac{2P_{os,EW}}{L} = \frac{2 \times 20.6}{5.535} = 7.44 \text{ kN/m} \]
\[ \phi R_n = \frac{0.8 \times 1.3 \times 1.4 \times 0.631}{0.1} = 9.2 \text{ kN/m} > N_{EW}^* (7.44 \text{ kN/m}) \therefore \text{OK} \]
Check tension capacity of the MSG8 290 mm × 45 mm joists:

\[ \phi N_t = \phi k_1 k_4 f_t A \]

\[ \therefore \phi N_t = 0.8 \times 1.0 \times 1.0 \times 6000 \times 0.290 \times 0.045 \]

\[ \therefore \phi N_t = 62.6 \text{ kN} > N_{EW}^* \ (7.44 \text{ kN/m}) \therefore \text{OK} \]

Chord design for east-west loading adequate.

**D.1.13 Sub-diaphragm requirements**

For each loading direction a reliable load path must exist into the diaphragm. This was achieved through sufficient nailing of the loaded joists (parallel-to-joist) or loaded blocking lines (perpendicular-to-joists).

**Consider north-south direction:**

Determine force required to be transferred by loaded joists:

\[ V_{u, \text{per joist}} = \frac{75.7}{4} = 18.9 \text{ kN} \]

Try 75 mm × 3.15 mm DIA power driven nails @ 150 mm centres ⇒

\[ \phi Q_n = 0.8 \times 1.22 \times 1.4 \times 0.631 \times \left(\frac{5.535}{0.150}\right) \]

\[ \phi Q_n = 31.0 \text{ kN} > V_{u, \text{per joist}} \ (18.9 \text{ kN}) \therefore \text{OK} \]

**Consider east-west direction:**
Appendix D: Diaphragm Assessment Calculations

Determine force required to be transferred by loading blocking lines:

\[
V_{u, per\, joist} = \frac{96.6}{2} = 48.3 \text{ kN}
\]

Again, try 75 mm × 3.15 mm DIA power driven nails @ 150 mm centres ⇒

\[
\phi Q_n = 0.8 \times 1.3 \times 1.4 \times 0.631 \times \left(\frac{10.4}{0.150}\right)
\]

\[
\phi Q_n = 63.4 \text{ kN} > V_{u, per\, joist} (18.9 \text{ kN}) \therefore \text{OK}
\]

D.1.14 Diaphragm edge nailing

Sufficient nailing around the diaphragm perimeter must be provided to effectively transfer shear forces out of the diaphragm.

For both loading directions:

Recall that retrofitted diaphragm has \(\phi R_n = 7.44 \text{ kN/m}\), so sufficient nail must be provided to resist this shear capacity.

Try 75 mm × 3.15 mm DIA power driven nails @ 100 mm centres around all diaphragm edges ⇒

\[
\phi Q_n = 0.8 \times 1.3 \times 1.4 \times 0.631 \times \frac{1}{0.100}
\]

\[
\phi Q_n = 9.2 \text{ kN/m} > \phi R_n (7.44 \text{ kN/m}) \therefore \text{OK}
\]
D.2 Determination of Elastic Earthquake Loads

The following calculations were used to determine the design elastic loads used in Section 7.2.5.2. The calculations are based on 1/500 return period earthquake, assuming $\mu = 1.0$, using the provisions of NZS 1170.5:2004.

Consider north-south direction:

$$C(T_1) = 2.91 \times 0.4 \times 1.0 \times 1.0 = 1.16g$$

Take $S_p = 1.0$ and $\mu = 1.0$

$$\therefore C_d(T_1) = \frac{1.16 \times 1.0}{1.0} = 1.16g$$

$$V_{D,e,NS} = 1.0 \times 1.0 \times 1.16 \times 452 = 524 \text{kN}$$

Consider east-west direction:

$$C(T_1) = 3.0 \times 0.4 \times 1.0 \times 1.0 = 1.2g$$

Take $S_p = 1.0$ and $\mu = 1.0$

$$\therefore C_d(T_1) = \frac{1.2 \times 1.0}{1.0} = 1.2g$$

$$V_{D,e,EW} = 1.0 \times 1.0 \times 1.2 \times 241 = 289 \text{kN}$$
D.3 NZSEE (2006) Procedure to Determine Diaphragm Displacement and Diaphragm Stiffness

The following calculations demonstrate the application of the provisions published in Appendix 11A of NZSEE (2006) for the determination of diaphragm displacement and diaphragm stiffness.

For straight sheathed diaphragms, midspan displacement is determined using:

\[ \Delta_d = \frac{Le_n}{2s} \]

where \( L \) is diaphragm span, \( e_n \) is nail slip resulting from the applied diaphragm shear force \( V_d \), and \( s \) is nail couple spacing.

Structural engineers typically use the provisions of NZS 3603:1993 Section 4.2.2.3 to determine expected nail slip, \( e_n \), generated from nail force \( F_n \). Diaphragm displacement and nail slip are typically evaluated at strength capacity \( R_n \).

Recall that when a diaphragm is at strength capacity, individual nail force is equal to its nominal capacity \( Q_n \). Clause 4.2.2.3(a) states that ‘A load equal to 1.25 times the nominal short term strength of a single nail gives and average slip of 2.5 mm,’ and Equation 4.4 defines nail slip up to values of 0.5 mm. When nail force is equal to \( Q_n \), nail slip lies between 0.5 mm and 2.5 mm, and linear interpolation between Clause 4.2.2.3(a) and Equation 4.4 is required. Determine nail load when \( \delta_n = 0.5 \) mm:

\[ F_n @ 0.5 \text{ mm} = \sqrt{\frac{0.5 \times 0.82^2}{1.0 \times 0.8}} = 0.648 \text{ kN} \]

\[ \therefore \text{Determine } e_n \text{ for an applied load of } F_n = Q_n = 0.82 \text{ kN} \]
\[ \delta = \frac{(2.5 - 0.5)}{(1.25 \times 0.820 - 0.648)} \times (0.820 - 0.648) + 0.5 = 1.4 \text{ mm} \]

Determine midspan diaphragm displacement and diaphragm stiffness, assuming that \( s = 95 \text{ mm}, B = 5.535 \text{ m}, L = 10.4 \text{ m}, \ell = 400 \text{ mm}, \) and \( d_f = 135 \text{ mm} \) ⇒

\[ \Delta_a = \frac{Le_n}{2s} = \frac{10.4 \times 1.4}{2 \times 95} = 0.077 \text{ m} \]

Recall that diaphragm load \( F_d \) can be determined using the nominal capacity of the nail connections:

\[ F_d = \frac{2 \times Q_n \times s \times B}{\ell d_f} = \frac{2 \times 0.82 \times 0.095 \times 5.535}{0.4 \times 0.135} = 16.0 \text{ kN} \]

\( \therefore \) Diaphragm stiffness \( \Delta_d \) ⇒

\[ K_d = \frac{F_d}{\Delta_d} = \frac{16.0}{0.077} = 207 \text{ kN/m} \]

And the corresponding diaphragm shear stiffness using ASCE 41-06 equation ⇒

\[ G_d = \frac{K_dL}{4B} = \frac{131.7 \times 10.4}{4 \times 5.535} = 97 \text{ kN/m} \]
Appendix D: Diaphragm Assessment Calculations
Appendix E

DIAPHRAGM PARAMETRIC ANALYSIS
RESULTS

Force-displacement data was exported from each of the finite element pushover analyses performed as part of the parametric analysis of diaphragm behaviour reported in Chapter 9. The generated force-displacement plots are presented in Figures E-1 to E-18 for reference, where force is total applied load and displacement is midspan displacement in the direction of loading. The plots are grouped with respect to the configuration parameter that was analysed, and are thus labelled accordingly.
Figure E-1: Parallel-to-joist analysis for variations in timber elastic modulus, $E$
Appendix E: Diaphragm Parametric Analysis Results

Figure E-2: Perpendicular-to-joist analysis for variations in timber elastic modulus, $E$

(a) $E = 4.0$ GPa
(b) $E = 7.0$ GPa
(c) $E = 10.0$ GPa
(d) $E = 12.5$ GPa
(e) $E = 15.0$ GPa
Appendix E: Diaphragm Parametric Analysis Results

Figure E-3: Parallel-to-joist analysis for variations in floorboard thickness, $t_f$
Appendix E: Diaphragm Parametric Analysis Results

Figure E-4: Parallel-to-joist analysis for variations in floorboard width, $d_f$

(a) $d_f = 100$ mm
(b) $d_f = 118$ mm
(c) $d_f = 135$ mm
(d) $d_f = 143$ mm
(e) $d_f = 150$ mm
Appendix E: Diaphragm Parametric Analysis Results

Figure E-5: Perpendicular-to-joist analysis for variations in joist thickness, $t_j$

(a) $t_j = 40$ mm  
(b) $t_j = 45$ mm  
(c) $t_j = 50$ mm  
(d) $t_j = 58$ mm  
(e) $t_j = 65$ mm
Appendix E: Diaphragm Parametric Analysis Results

Figure E-6: Perpendicular-to-joist analysis for variations in joist depth, $d_j$

(a) $d_j = 150$ mm

(b) $d_j = 225$ mm

(c) $d_j = 300$ mm

(d) $d_j = 350$ mm

(e) $d_j = 400$ mm
Appendix E: Diaphragm Parametric Analysis Results

Figure E-7: Parallel-to-joist analysis for variations in joist spacing, \( \ell \)

- (a) \( \ell = 300 \) mm
- (b) \( \ell = 350 \) mm
- (c) \( \ell = 400 \) mm
- (d) \( \ell = 522 \) mm
- (e) \( \ell = 667 \) mm
Appendix E: Diaphragm Parametric Analysis Results

Figure E-8: Perpendicular-to-joist analysis for variations in joist spacing, $\ell$

(a) $\ell = 300$ mm

(b) $\ell = 350$ mm

(c) $\ell = 400$ mm

(d) $\ell = 522$ mm

(e) $\ell = 667$ mm
Appendix E: Diaphragm Parametric Analysis Results

Figure E-9: Parallel-to-joist analysis for variations in diaphragm span, $L$

(a) $L = 4.0$ m
(b) $L = 8.0$ m
(c) $L = 10.0$ m
(d) $L = 12.0$ m
(e) $L = 18.0$ m
(f) $L = 24.0$ m
Appendix E: Diaphragm Parametric Analysis Results

Figure E-10: Parallel-to-joist analysis for variations in diaphragm width, $B$
Appendix E: Diaphragm Parametric Analysis Results

Figure E-11: Perpendicular-to-joist analysis for variations in diaphragm span, $L$

(a) $L = 4.0$ m

(b) $L = 6.0$ m

(c) $L = 8.0$ m

(d) $L = 12.0$ m

(e) $L = 18.0$ m

(f) $L = 24.0$ m
Figure E-12: Perpendicular-to-joist analysis for variations in diaphragm width, $B$
Appendix E: Diaphragm Parametric Analysis Results

Figure E-13: Parallel-to-joist analysis for variations in diaphragm size, $L \times B$

(a) $x_d \times y_d = 6.0 \times 4.0$ m

(b) $x_d \times y_d = 8.0 \times 5.2$ m

(c) $x_d \times y_d = 12.0 \times 8.0$ m

(d) $x_d \times y_d = 16.0 \times 10.8$ m

(e) $x_d \times y_d = 20.0 \times 13.2$ m
Appendix E: Diaphragm Parametric Analysis Results

(a) $x_d \times y_d = 6.0 \text{ m} \times 4.0 \text{ m}$

(b) $x_d \times y_d = 8.0 \text{ m} \times 5.2 \text{ m}$

(c) $x_d \times y_d = 12.0 \text{ m} \times 8.0 \text{ m}$

(d) $x_d \times y_d = 16.0 \text{ m} \times 10.8 \text{ m}$

(e) $x_d \times y_d = 20.0 \text{ m} \times 13.2 \text{ m}$

Figure E-14: Perpendicular-to-joist analysis for variations in diaphragm size, $L \times B$
Appendix E: Diaphragm Parametric Analysis Results

Figure E-15: Parallel-to-joist analysis for corner penetrations
Appendix E: Diaphragm Parametric Analysis Results

(a) No penetration

(b) \( l_p \times b_p = 3.2 \text{ m } \times 1.08 \text{ m} \)

(c) \( l_p \times b_p = 4.0 \text{ m } \times 1.755 \text{ m} \)

(d) \( l_p \times b_p = 4.8 \text{ m } \times 2.430 \text{ m} \)

(e) \( l_p \times b_p = 5.6 \text{ m } \times 3.105 \text{ m} \)

(f) \( l_p \times b_p = 6.4 \text{ m } \times 3.915 \text{ m} \)

Figure E-16: Parallel-to-joist analysis for central penetrations
Appendix E: Diaphragm Parametric Analysis Results

Figure E-17: Perpendicular-to-joist analysis for corner penetrations

(a) No penetration
(b) \( l_p \times b_p = 3.2 \text{ m} \times 1.08 \text{ m} 
(c) \( l_p \times b_p = 4.0 \text{ m} \times 1.755 \text{ m} 
(d) \( l_p \times b_p = 4.8 \text{ m} \times 2.430 \text{ m} 
(e) \( l_p \times b_p = 5.6 \text{ m} \times 3.105 \text{ m} 
(f) \( l_p \times b_p = 6.4 \text{ m} \times 3.915 \text{ m} 

Appendix E: Diaphragm Parametric Analysis Results

Figure E-18: Perpendicular-to-joist analysis for central penetrations