Economics Department

Economics Working Papers

The University of Auckland *Year* 2002

Auction Beats Posted Prices in a Small Market

Benoit Julien* John Kennes†

Ian King‡

*University of Miami,

†University of Auckland, j.kennes@auckland.ac.nz

‡University of Auckland, ip.king@auckland.ac.nz

Auctions Beat Posted Prices in a Small Market

Benoît Julien John Kennes Ian King

March 2002

Abstract

In a model with two buyers and sellers we consider the choice of sales mechanism from three possibilities: posted prices, and auctions with and without reserve prices. With homogenous goods, sellers' expected revenues are highest when both sellers auction with reserve prices – 33% higher than if posting prices and 100% higher than if auctioning without reserve prices. When sellers can choose their mechanism before choosing prices, both sellers auction with a reserve price in the dominant strategy equilibrium. With heterogenous goods, the equilibrium with posted prices is inefficient (Montgomery (1991)) but the equilibria with both types of auctions are efficient.

Key words: Coordination, price-posting, competing auctions, efficiency, directed search.

JEL Codes: D43, D44, D45, D83

Benoît Julien: Department of Economics, University of Miami, USA
bjulien@miami.edu

John Kennes: Department of Economics, University of Auckland, New Zealand
j.kennes@auckland.ac.nz

Ian King: Department of Economics, University of Auckland, New Zealand
ip.king@auckland.ac.nz

This research is funded, in part, by a grant from the Social Sciences and Humanities Research Council of Canada. We would like to thank Jan Eeckhout, Merwan Engineer, Jeremy Greenwood, Robert Jones, Boyan Jovanovic, Robert E. Lucas Jr., Nicolas Schmitt, Shouyong Shi, Robert Shimer, and Randy Wright for helpful discussions.

1. INTRODUCTION

Auctions and posted prices are two commonly used selling mechanisms for a wide variety of products. The usage of auctions has traditionally been motivated by the existence of a monopoly seller possessing imperfect information about the buyers' valuations of the object for sale. (See, for example, Maskin and Riley (1984) and McAfee and McMillan (1987).) An auction features a bidding process that allocates a scarce resource to the most willing buyer and extracts the most rent for the seller. The market for rare art objects is a classic example of where auctions are commonly used. By way of contrast, price-posting has usually been motivated by the sale of goods whose value is commonly understood, no bidding takes place, and the good may be rationed according to some rule (Cournot (1887), Kreps and Sheinkman (1983)). The sale of groceries at a supermarket is but one example where this mechanism is used. Arguably, until recently, auctions have been more costly to operate than price-posting. However, with recent improvements in communication technologies (particularly the internet) the relative costs of these mechanisms have started to converge. Due to the existence of websites such as eBay and Half.com, sellers can now choose the selling mechanism, for almost any product that one can imagine, at a similar cost. Understanding this choice therefore becomes an important issue for many different markets.

Theoretical work on this issue initially considered the problem of a monopolist facing a fixed arrival rate of buyers (Wang (1993)). However, the development of competing auction theory (McAfee (1993), Julien, Kennes and King (2000)) and price-posting among competing sellers (Peters (1984), Montgomery (1991)),[1] has also allowed for consideration of sellers making this choice when faced with other sellers acting strategically. In large markets with informational asymmetries, McAfee (1993) argued that sellers choose auctions in equilibrium, assuming that buyers are aware of the mechanism chosen by sellers before they approach the sellers. Peters (1994) showed, in a similar environment, that sellers would choose to post prices if buyers were unaware of the pricing mechanisms before choosing sellers, and discount factors were large enough.

[1] See, also, Burdett, Shi and Wright (2001).

In large markets with *homogeneous* buyers and sellers, Kultti (1999) showed that the expected payoffs to sellers are identical under the two different pricing mechanisms. With homogeneity but *small* markets, Julien, Kennes and King (2001a) showed that sellers' expected payoffs are higher if all sellers auction, in any finite-sized market and the difference in the payoffs declines monotonically with market size. Thus, even in the absence of any informational asymmetries, it pays to auction in small markets.[2] In the smallest (2-by-2) market, (where the sellers' premium from the auction is at its highest – 33%) we also showed that the equilibrium choice of selling mechanism depends importantly on the *sequencing* of decisions: if sellers choose both mechanism and price simultaneously then equilibria exist with all different combinations of mechanisms. However, if sellers are able to commit to a mechanism before the price then the unique dominant strategy equilibrium has both sellers auctioning.

In this paper, we extend this work in two ways. First, motivated by the work of Coles and Eeckhout (2000), we consider a larger class of mechanisms for sellers to choose over. In particular, we allow three different mechanisms: price-posting, auctioning with a reserve price, and auctioning without a reserve price.[3] As shown in Julien, Kennes, and King (2001b), the expected payoffs from these mechanisms are identical in large markets. However, we show here that they differ substantially in small markets. For example, in the 2-by-2 market with homogeneous buyers and sellers, sellers *double* their expected payoffs if they auction with a reserve price rather than without one. Moreover, when sellers can commit to a mechanism before a price, auctioning with a reserve price is the dominant strategy equilibrium, and maximizes the expected payoffs to sellers.

[2] Analyzing competing auctions in small markets with informational asymmetries poses several technical difficulties that are not present in large markets (Julien (1997)). McAfee (1993), for example, imposed key assumptions (i) and (ii) that effectively preclude the application of his results to small markets.
[3] An astute referee pointed out that we do not consider the choice across all feasible pricing mechanisms. This more general question is beyond the scope of this paper. However, we believe that the three different mechanisms considered here are the most commonly observed ones for many goods.

We also consider how things change when sellers are heterogenous. Montgomery (1991) showed that, in a small market when sellers offer goods of different quality, the equilibrium prices are inefficient if sellers post prices. Here, we show that this inefficiency disappears if sellers auction – either with a reserve price or without. Thus, we argue, auctions beat posted prices in small markets.

Auctions are also attractive, from a theoretical point of view, because of their simplicity. With heterogeneity of this type, closed-form solutions are not available for the price-posting game but are easily found when sellers auction. In particular, we find that the equilibrium reserve price is simply half the buyers' valuation of the seller's commodity.[4]

The paper is organized as follows. In Section 2, we present a simple model of the decision to auction or post prices under the simplifying assumption that sellers are homogeneous. This section has four subsections. In the first three subsections, we develop the three essential subgames of the model: both sellers posting prices, both sellers auctioning, and one seller posting a price while the other auctions. In the final subsection we use the expected payoffs from the three subgames to generate a simple normal form game for the decision to auction or to post a price. In section 3 we introduce heterogeneous sellers and demonstrate the efficiency result. In the final section of the paper we present our conclusions and consider some directions for future research.

[4] Interestingly, this yields the same payoffs as in Kultti and Niinimaki (2002), where it assumed that the buyer and seller enter a standard bilateral bargaining process if only one buyer approaches a seller.

2. THE MODEL

The market consists of two identical risk neutral sellers and two identical risk neutral buyers. Each buyer wants to buy one unit of an indivisible good, and is willing to pay up to his reservation price, which is constant across buyers, and normalized to 1. If he purchases at price p then he obtains utility $1-p$. If he is unable to buy, his payoff is zero. Each seller has one unit of the good to sell. If she sells at price p, she obtains payoff p; if she does not sell, she obtains a payoff of zero.

Exchange in this market is a four stage game. In the first stage, each seller chooses whether to post a price, auction with a reserve price, or auction without a reserve price. In the second stage, the sellers set posted / reserve prices: a reserve price in the case of an auction, and a posted price otherwise. In the third stage, buyers observe the posted prices and reserve prices of the sellers, and choose which seller to visit. In the final stage, the good is sold in a bidding game in the case of an auction and by a rationing rule in the case of a posted price. Equilibria in this model are solved by backwards induction: first, the bidding/rationing game, second, the buyers' choice of seller to visit, third, the sellers' choice of posted/reserve price, and finally, the sellers' choice of whether to post a price or auction with or without a reserve price. The analysis of this problem is facilitated by the sequential development of three key subgames in which (i) both sellers post prices, (ii) both sellers auction, and (iii) one seller posts a price and the other auctions. Once the payoffs of each of these three subgames have been characterized, the equilibrium choice of mechanism can be analysed in a simple 3 by 3 normal form game, which we solve.

2.1 *Both Sellers Posting Prices*

The posted-price subgame follows Montgomery (1991) and Burdett, Shi and Wright (2001). Here, each seller chooses a posted price s_j where $j \in \{1,2\}$ is used to index sellers. Each buyer then chooses to visit either seller based on the posted price of each seller and the expected behavior of the other buyer. The probability that a buyer visits seller j is given by π_i^j where i is used to index buyers. Buyer i's expected utility from visiting seller j is given by:

$$U_i^j = (1-\pi_{-i}^j)(1-s_j) + \pi_{-i}^j (1-s_j)/2 \qquad (2.1)$$

where π_{-i}^j is the probability that seller i is not the only potential buyer visiting seller j, in which case the good is rationed according to a symmetric rationing rule. As is standard in this literature, attention here is focused on equilibria in which buyers play mixed strategies.[5] A mixed strategy equilibrium exists in this game such that each buyer is indifferent about which seller he visits. Moreover, since each buyer faces the same vector of posted prices, the probability that each buyer visits seller j is the same. Thus, $U_i^1 = U_i^2$ and $\pi_i^j = \pi^j$ for all i. Using these conditions, the probability π^j that any particular buyer visits seller j is solved as follows:

[5] Two asymmetric pure strategy equilibria also exist: where buyer 1 approaches seller 1 and buyer 2 approaches seller 2 with probability one, and where buyer 1 approaches seller 2 and buyer 2 approaches seller 1 with probability one. From the point of view of buyers, these two pure strategy equilibria are strictly preferred to the mixed one. Thus, buyers face a pure coordination game when making their location choice. In settings with the same number *N* of buyers and sellers, there are *N*! such asymmetric pure strategy equilibria but still only one symmetric mixed strategy equilibrium. While, in the simplest 2-by-2 game, one might imagine that the two buyers may find a way to coordinate, this becomes less plausible in only slightly larger markets. The unique mixed strategy equilibrium arguably represents a natural focal point (see Binmore et al, (1993)) in these environments, and has become the focus of the analysis in the literature. It is also worth noting that sellers play pure strategies when choosing prices in this framework. This contrasts with work by Kreps and Sheinkman (1983), and Peters (1984) who focus on mixed pricing strategies in similar capacity-constrained environments. Here, because buyers play mixed strategies when making their location choices, sellers face simple continuous expected demand functions – even though the number of buyers in the market is discrete, which allows for pure strategies in prices in a natural way. For an intuitive interpretation of mixed strategy equilibria, see Reny and Robson (2001).

$$\pi^j = \frac{1-2s_j + s_{-j}}{2-s_j - s_{-j}} \tag{2.2}$$

Expression (2.2) is the reaction function of buyers to the posted prices of sellers. The probability that seller j is able to sell her good is:

$$q(s_j, s_{-j}) = 2\pi^j(1-\pi^j) + (\pi^j)^2$$

where $2\pi^j(1-\pi^j)$ is the probability that the seller j faces one potential buyer and $(\pi^j)^2$ is the probability that she faces two (in which case, she can sell only to one). We can think of $q(s_j, s_{-j})$ as the demand function for good j. A key feature of this demand function is that it is continuous and differentiable. Moreover, under posted prices, the seller has the same payoff if one or two potential buyers come to visit. Therefore, the expected payoff to seller j from offering posted price s_j is given by:

$$R(s_j, s_{-j}) = q(s_j, s_{-j})s_j \tag{2.3}$$

The solution to each seller's problem is found simply by differentiating her payoff function with respect to her posted price, and setting this derivative equal to zero. In this way we obtain the following reaction function for seller j:

$$s_j(s_{-j}) = \frac{(s_{-j}+1)(s_{-j}-2)}{5s_{-j}-7} \tag{2.4}$$

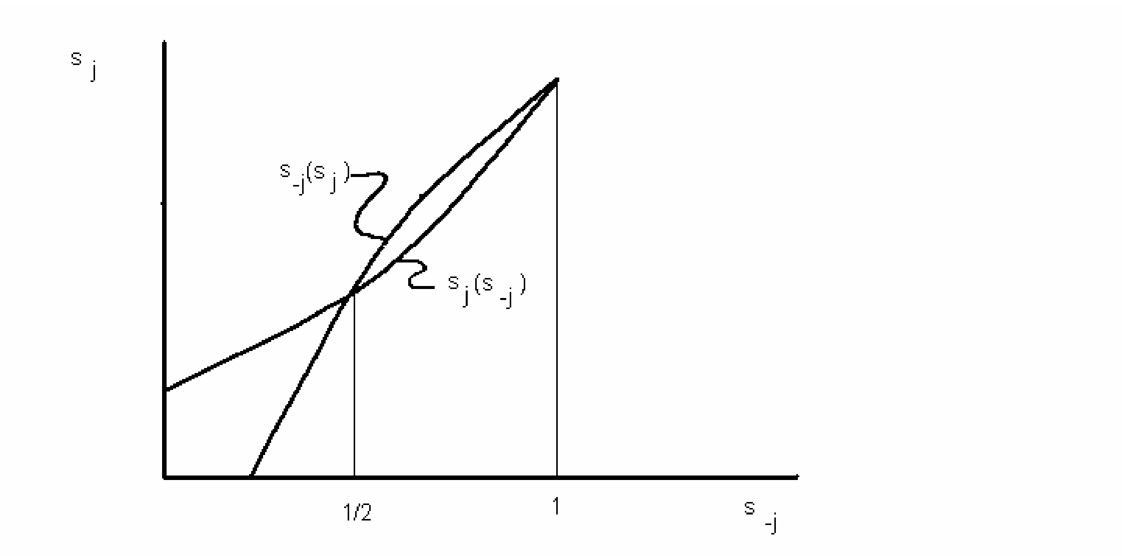

Figure 1: Reaction Functions When Both Sellers Post Prices

The intersection of the sellers' reaction functions, illustrated in Figure 1, gives the equilibrium posted price of $s_j^* = 1/2$ for both sellers. Substituting these values into (2.2) yields: $\pi^j = 1/2$ which then implies that $q(s_j^*, s_{-j}^*) = 3/4$. The equilibrium payoff to sellers in this subgame is therefore, from (2.3):

$$R(s_j^*, s_{-j}^*) = 3/8 \tag{2.5}$$

From Figure 1, it can also be observed that a price of unity for both sellers also satisfies the two reaction functions. This price is not an equilibrium, however, because if seller j chooses a posted price of unity then the utility that any buyer would get from visiting that seller is zero, no matter how the other buyer behaves. Therefore, the other seller can charge a slightly lower price, get the whole market (i.e., a buyer with certainty), and leave the first seller with an incentive to cut the price. The payoff function of seller j does not have this discontinuity if the other seller posts a price equal to one half. However, it is worth noting that seller j gets no customers if she posts a price greater than 3/4 when the other seller posts a price equal to one half. In this case, the buyers then visit the other seller with certainty and take their chances with that seller's rationing rule.

2.2 *Both Sellers Auctioning*

In this subsection we consider two cases. In the first, each seller chooses a *reserve* price r_j as a best response to the reserve price of the other seller. In the second case, each seller sets her reserve price equal to zero. In either case, each buyer then chooses to visit either seller based on the reserve price of each seller and the expected behavior of the other buyer.

Buyer i's expected utility from visiting seller j is given by:

$$U_j^i = (1-\pi_{-i}^j)(1-r_j) \tag{2.6}$$

where $(1-\pi_{-i}^j)$ is the probability that the buyer is alone at this auction. If the buyer is not alone, then competitive bidding ensures that the sale price of the good is unity, in which case the buyer does not obtain any utility from purchasing the good. An equilibrium in mixed stategies also exists for this subgame, such that each buyer is indifferent which seller he visits. Moreover, since each buyer faces the same vector of reserve prices, the probability that any particular buyer visits seller j is the same. Therefore, $U_i^1 = U_i^2$ and $\pi_i^j = \pi^j$ for all i. Consequently, the probability π^j that any particular buyer visits seller j is given by:

$$\pi^j = \frac{1-r_j}{2-r_j-r_{-j}} \tag{2.7}$$

This expression is the reaction function of the buyers to the reserve prices of the sellers. When choosing their reserve prices, sellers exploit this function. The probability that seller j has exactly one potential buyer visit is given by $q^1(r_j, r_{-j}) = 2\pi^j(1-\pi^j)$, while the probability that she has two potential buyers visit is $q^2(r_j, r_{-j}) = (\pi^j)^2$. Whereas in the posted price case, the number of potential buyers who visit has no bearing on the price at which the good is sold (as long as this number is not zero), in the auction case, this is not true. In particular, the good is sold at the reserve price r_j if only one potential buyer visits seller j, but sold at the price of unity (the buyer's valuation) if more than one potential buyer visits. Thus, the payoff $R(r_j, r_{-j})$ to seller j offering reserve price r_j is given by:

$$R(r_j, r_{-j}) = q^1(r_j, r_{-j})r_j + q^2(r_j, r_{-j}) \tag{2.8}$$

When choosing the reserve price, sellers simply differentiate the payoff function (2.8) with respect to r_j and set the result equal to zero. Doing so, we find the following reaction function for seller j:

$$r_j(r_{-j}) = 1/2 \tag{2.9}$$

The non-responsiveness of this reaction function to the reserve price of the other seller deserves some comment. A higher reserve price by the other seller clearly raises the probability that a buyer will visit seller j. However, seller j chooses not to raise her reserve price in response because she would then discourage potential buyers from visiting her own auction. When the number of buyers equals the number of sellers (as in this 2 by 2 case) these effects exactly cancel out.[6]

The expected payoff function of the seller (2.8) is continuous and concave, given the other seller is choosing the equilibrium reserve price. Therefore, the vector of reserve prices satisfying the reaction functions of the sellers is an equilibrium of this subgame.

[6] More generally, when the number of buyers and sellers differ, these reaction functions have a positive slope. See Julien, Kennes, and King (2000).

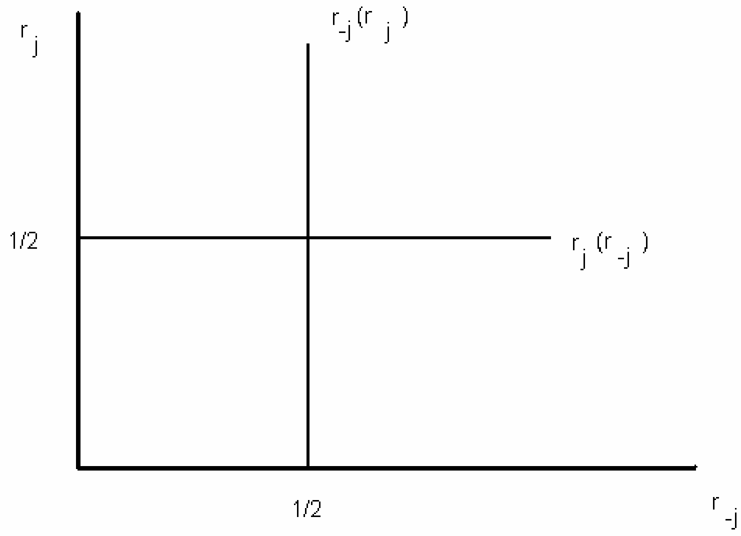

Figure 2: Reaction Functions When Both Sellers Auction

The two reaction functions of the two sellers, illustrated in Figure 2, determine the equilibrium reserve price for each seller at $r_j^* = 1/2$. Using this in (2.7) implies that $\pi^j = 1/2$, which then implies that $q^1(1/2,1/2) = 1/2$ and $q^2(1/2,1/2) = 1/4$. These choices can now be substituted back into the payoff function (2.8) to obtain the equilibrium expected payoff to each seller when both choose the equilibrium reserve price:

$$R(r_j^*, r_{-j}^*) = 1/2. \tag{2.10}$$

Since the choice of the reserve price in (2.9) is independent of the reserve price of the other seller, we can now also find the payoff to a seller who chooses a reserve price when the other seller auctions but with a reserve price. Setting $r_j = 1/2$ and $r_{-j} = 0$ in (2.7) yields: $\pi^j = 1/3$. This implies that $q^1(1/2,0) = 4/9$ and $q^2(1/2,0) = 1/9$. Hence, by (2.8) we obtain:

$$R(r_j^*, 0) = 1/3 \tag{2.11}$$

Similarly, setting $r_j = 0$ and $r_{-j} = 1/2$ in (2.7) yields: $\pi^j = 2/3$. This implies that $q^1(0,1/2) = 4/9$ and $q^2(0,1/2) = 4/9$. Equation (2.8) then implies:

$$R(0, r_j^*) = 4/9 \tag{2.12}$$

We can also find the equilibrium payoffs to sellers if both auction without a reserve price. Setting $r_j = r_{-j} = 0$ in (2.7) gives $\pi^j = 1/2$. This implies that $q^1(0,0) = 1/2$ and $q^2(0,0) = 1/4$. Then, from (2.8):

$$R(0,0) = 1/4 \tag{2.13}$$

2.3 *One Seller Posting a Price, the Other Auctioning*

The auction and posted-price subgame, developed here, is similar to the previous two subgames, except that one seller posts a price while the other conducts an auction. We denote the posted price seller by her posted price s, and the auction seller by her reserve price r. Again, we consider two possible reserve prices – the best response and simply setting the reserve equal to zero. The probability that buyer i visits the posted price seller is given by π_i^s. The expected utility that buyer i receives if he visits the posted price seller is given by:

$$U_i^s = (1 - \pi_{-i}^s)(1 - s) + \pi_{-i}^s (1 - s)/2 \tag{2.14}$$

while the expected utility from visiting the seller who auctions is given by:

$$U_i^r = \pi_{-i}^s (1 - r) \tag{2.15}$$

As in the previous two subgames, an equilibrium in mixed strategies exists for this subgame such that each buyer is indifferent about which seller to visit. Also, since each buyer faces the same vector of prices, the probability that any particular buyer will visit a particular seller is the same. Therefore, $U_i^s = U_i^r$ and $\pi_i^s = \pi^s$ for all i. These two conditions allow us to solve for the probability π^s that any particular buyer will visit the posted price seller:

$$\pi^s = \frac{1-s}{3/2 - r - s/2} \tag{2.16}$$

Equation (2.16) expresses the reaction function of the buyers to the reserve and posted prices of the two sellers. Both sellers exploit this function when making their posted/reserve price decisions. The expected payoff to a seller offering posted price s is given by:

$$R(s,r) = q(s,r)s \tag{2.17}$$

where $q(s,r) = 2\pi^s(1-\pi^s) + (\pi^s)^2$ is the probability that the posted price seller receives a visit from at least one buyer. The expected payoff of the seller offering the reserve price r is given by:

$$R(r,s) = q^1(r,s)r + q^2(r,s) \tag{2.18}$$

where $q^1(r,s) = 2\pi^s(1-\pi^s)$ is the probability that one buyer will visit the auctioning seller, and $q^2(r,s) = (1-\pi^s)^2$ is the probability that two will.

Differentiating the payoff functions of each seller with respect to the relevant decision variable and setting these derivatives equal to zero gives two reaction functions. First, for the price-posting seller, we have:

$$s(r) = \frac{2r-3}{4r-5} \qquad (2.19a)$$

and second, for the auctioning seller, we have:

$$r(s) = \frac{1+s}{6} \qquad (2.19b)$$

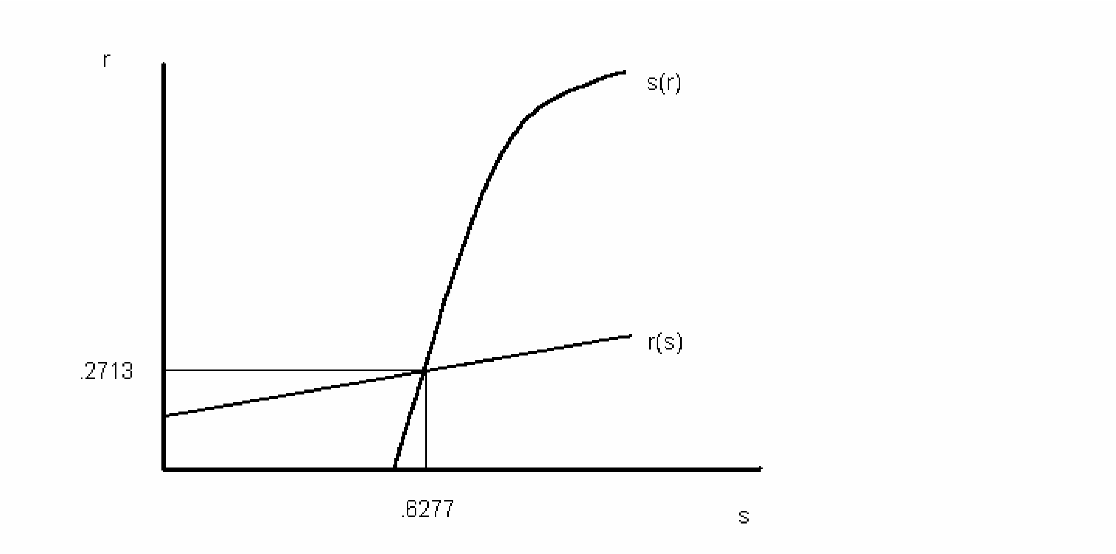

Figure 3: Reaction Functions When One Seller Posts and One Auctions

The intersection of these two reaction functions, illustrated in Figure 3, yields the equilibrium reserve price $r^* = 0.271286$ and posted price $s^* = 0.6277$. Substituting these choices into (2.13) yields:

$$\pi^s(s^*, r^*) = 0.4069 \qquad (2.20a)$$

Similarly, when the auctioning seller sets the reserve price equal to zero, from (2.19a), we have $s = 3/5$. Using this in (2.16) we find:

$$\pi^s(3/5, 0) = 1/3 \qquad (2.20b)$$

The payoff to the price-posting seller can now be found in these different cases. Using (2.20a) and (2.20b), the probabilities that this seller will receive a visit from at least one buyer are equal to:

$$q(s^*, r^*) = 0.6482 \qquad q(3/5,0) = 5/9$$

respectively in each case. Now, using (2.17), we find:

$$R(s^*, r^*) = 0.4069 \tag{2.21a}$$

$$R(3/5,0) = 1/3 \tag{2.21b}$$

as the payoffs to the price-posting seller when the other seller auctions with the equilibrium reserve price and the zero reserve price respectively.

The payoffs to the sellers that auction can be found by first calculating the probabilities of one and two buyers visiting in the different cases:

$$q^1(r^*, s^*) = 0.4827 \qquad q^2(r^*, s^*) = 0.3518$$

$$q^1(0,3/5) = 4/9 \qquad q^2(0,3/5) = 4/9$$

Using these values in (2.18), we can now calculate the expected payoffs to sellers that auction in the two cases:

$$R(r^*, s^*) = 0.4827 \tag{2.22}$$

$$R(0,3/5) = 4/9 \tag{2.23}$$

2.4 *Equilibrium*

The three subgames, in sections 2.1, 2.2, and 2.3, determined the expected payoffs to each seller from either auctioning or posting a price, given the choice of selling mechanism of the other seller. These payoffs now allow us to characterize the equilibrium choice of selling mechanism by considering the following simple normal form game, which is constructed using the payoffs from (2.5), (2.10), (2.11), (2.12), (2.13), (2.21a), (2.21b), (2.22), and (2.23):[7]

	Post Prices	Auction With Reserve	Auction With No Reserve
Post Prices	0.375, .0.375	0.407, 0.483	0.333, 0.444
AuctionWith Reserve	0.483, 0.407	0.5, 0.5	0.444, 0.333
Auction With No Reserve	0.444, 0.333	0.333, 0.444	0.25, 0.25

Table 1

Inspection of Table 1 reveals that this game has a unique Nash equilibrium – where both sellers auction with a reserve price. Moreover, this choice is strictly dominant for each seller. Hence auctions with reserve prices maximize the sellers' joint surplus. Because the game is symmetric, the probability that the good gets sold is the same (1/2) whenever both sellers use the same mechanism. Consequently, rents are simply transferred from buyers to sellers if the sellers choose to auction using a reserve price rather than use either of the other two mechanisms.

Table 1 lists the mechanisms in declining order of price commitment from the point of view of sellers. The highest level of commitment occurs where sellers commit to the posted price, no matter how many buyers arrive *ex post*. When sellers auction with a

[7] For ease of comparison, all numbers in this table have been expressed in decimal form.

reserve price, they commit only to a price that would apply if only one buyer arrives: if both arrive then the object goes to the highest bidder. When sellers do not commit to a particular reserve price, the price is determined purely by *ex post* competition. This analysis shows that the expected revenue for sellers is maximized in the intermediate case – auctioning with a reserve price. Posting prices is the second-best mechanism, and auctioning without a reserve price is the worst here, for sellers. The premium that sellers get from auctioning with a reserve price is also very significant in this simple 2-by-2 case: by doing this, they would receive an expected payoff that is 33% higher than what they would get if both sellers posted prices, and 100% higher than their payoffs if both auctioned without a reserve price.

3. HETEROGENEOUS SELLERS AND EFFICIENCY

In this section we analyse the equilibrium outcome in an auction market when sellers have heterogeneous goods. In general, heterogeneous goods imply price dispersion as buyers choose different probabilities of approaching the different sellers in mixed strategy equilibria. As mentioned above, Montgomery (1991) found that, in this environment, the equilibrium probabilities are not constrained-efficient when sellers post prices. Here, we examine whether or not this is true when sellers auction with or without a reserve price. To do this, we compare the equilibrium probabilities in both cases with those that would be chosen by a planner seeking to maximize total expected surplus.

As in Montgomery's framework, the market consists of identical buyers, but sellers hold goods of different quality. Let $j \in \{h, l\}$ denote the two sellers, where seller h has a high quality indivisible good worth y_h to each buyer and that seller l has a low quality indivisible good worth $y_l < y_h$ to each buyer. Each buyer is willing to pay up to his reservation price, which is equal to the worth of each good. If a buyer purchases the good at price p, he obtains utility $y_j - p$, $j = h, l$. If the seller sells her good at price p she obtains payoff p; otherwise, she has payoff zero.

3.1 *The Planner's Problem*

The planner chooses the probabilities π^j that any particular buyer will visit seller j, to maximize total expected output:

$$V = \max_{\pi^h, \pi^l} \{(1-(1-\pi^h)^2) y_h + (1-(1-\pi^l)^2 y_l\} \tag{3.1}$$

where $\pi^h + \pi^l = 1$. It is straightforward to show that the planner's choice, denoted $\overline{\pi}^j$, is given by:

$$\overline{\pi}^j = \frac{y_j}{y_h + y_l} \quad j = h, l \tag{3.2}$$

Thus, for example, if the value of the high quality good is $\theta > 1$ times the value of the low quality good, then the optimal probability that each buyer visits the high quality seller is θ times the probability that each buyer visits the low quality seller.

3.2 The Equilibrium with Both Sellers Auctioning

As in section 2.2 above, we consider cases where each seller chooses a reserve price r_j strategically, and where no reserve is set. Buyers choose probabilities of visiting sellers, given the reserve prices and the expected behavior of the other buyer. The utility of visiting seller j is given by:

$$U_j^i = (1-\pi_{-i}^j)(y_j - r_j) \tag{3.3}$$

As in the homogeneous seller case, a mixed strategy equilibrium exists where each buyer faces the same vector of reserve prices and the probability that any particular buyer visits seller j is the same. Thus, $U_i^h = U_i^l$ and $\pi_i^j = \pi^j$ for all i. This implies:

$$\pi^j = \frac{y_j - r_j}{y_h + y_l - r_h - r_l} \tag{3.4}$$

Sellers exploit this reaction function when making their strategic reserve price decision. The payoff $R(r_j, r_{-j})$ to a seller offering a reserve price r_j is given by:

$$R(r_j, r_{-j}) = 2\pi^j(1-\pi^j)r_j + (\pi^j)^2 y_j \tag{3.5}$$

where π^j is given in (3.4), $j = h, l$. Once again, the solution to seller j's reserve price problem is found by choosing the r_j that maximizes $R(r_j, r_{-j})$, given r_{-j}. This yields the reaction function:

$$r_j(r_{-j}) = \frac{y_j}{2} \tag{3.6}$$

As in the homogeneous case analysed in section 2.2, the strategic reserve price for seller j is independent of the reserve price of the other seller. Comparing (3.6) with (2.9) reveals that this is a very straightforward generalization of the homogenous good case: sellers announce reserve prices that are precisely one half of the value of the surplus.

The intersection of the reaction functions yields the Nash equilibrium reserve prices:

$$r_j^* = \frac{y_j}{2} \tag{3.7}$$

When both sellers choose their reserve prices strategically, then substitution of (3.7) into (3.4) yields:

$$\pi^{*j} = \frac{y_j}{y_h + y_l} \tag{3.8}$$

which is identical to the planner's $\bar{\pi}^j$ in (3.2). Also, when neither seller has a reserve price, substitution of $r_h = r_l = 0$ into (3.4) produces exactly the same result. Thus, whenever both sellers have access to the same auction mechanism, the equilibrium choices will be constrained efficient.

4. CONCLUSIONS AND DISCUSSION

The usage of auctions, rather than simple posted prices, has traditionally been motivated by the existence of informational asymmetries. In this paper we have examined a very simple setting with two buyers and two sellers, but with no informational asymmetries at all. We have shown that, when the product for sale is homogenous, it pays for sellers to auction with a reserve price. As shown in our earlier work, auctioning with a reserve price gives sellers a 33% premium in expected revenues over what they would get if they posted prices. We also show here how important reserve prices are: in this setting, if sellers auction *without* a reserve price then they receive an expected payoff that is 33% *lower* than what they could get by posting prices. Put another way, auctioning with a reserve price yields sellers an expected revenue that is *double* the amount they would get if they auctioned without a reserve price.

In this setting, when sellers can sequence their decisions – choosing a sales mechanism before choosing a specific (reserve) price – then both sellers choosing to auction with a reserve price is the unique Nash equilibrium and the dominant strategy equilibrium. Thus, sequencing their decisions in this way allows sellers to find the equilibrium that maximizes their expected payoffs. Although we did not study the problem directly in this paper, we conjecture that, without this sequencing, multiple equilibria exist.[8]

---

[8] As mentioned above, multiple equilibria abound in this type of framework without sequencing. See, for example, Coles and Eeckhout (2000) and Julien, Kennes, and King (2001a).

In environments where sellers have different qualities of good to sell, Montgomery (1991) has shown that price-posting does not produce an efficient allocation of resources. We have shown here that this changes when sellers auction – either with or without a reserve price. In both cases, the efficient allocation is achieved in equilibrium. Examining the equilibrium choice of sales mechanism in this setting would, of course, be a natural next step. However, analyzing this question with heterogeneous goods is much more difficult than with homogeneous goods because no closed-form solutions are available for the price-posting equilibrium, although they are available for the auction equilibria. We therefore leave this question for future research.

In this study, we examined different mechanisms with different degrees of pricing commitment. However, it also true that we assumed that each seller is able to commit in the same way to each mechanism that she chooses. That is, sellers announce that they will be selling via a particular mechanism and are committed to using that mechanism once buyers arrive. While it is true that, in many markets, it may be difficult to identify the particular commitment mechanism that makes this possible, it is also true that mechanisms do exist in many other markets. For example, when sellers use auction houses they typically sign agreements that they are willing to respect the outcome of the auction process. Internet-based sales sites such as eBay and Half.com also require similar agreements. The explosive growth of the volume and value of goods sold through these sites, we believe, makes this analysis increasing relevant.

As has become commonplace in this literature, we have focused here on the unique symmetric mixed strategy equilibrium when buyers make their location choices. Asymmetric pure strategy equilibria also exist which, in fact, Pareto dominate the equilibrium where buyers mix. Thus, a coordination problem lies at the heart of all this work. Experimental evidence (Ochs (1990)) has shown that buyers do have trouble coordinating, even in very small markets.[9] Further experimental work, exploring behavior in simple perfect information environments such as this, seems worthwhile.

[9] One of us (King), together with Bob Forsythe and Tom Reitz at the University of Iowa, has also conducted preliminary experiments in this precise framework, finding similar results.

Risk is another aspect that has been abstracted away from here. Given that auctions are inherently more risky, it may be possible to change some of the results if risk aversion were introduced. (However, preliminary investigations, using simple square root utility functions for sellers in the homogenous case, did not change any substantive results.) Again, experimental work could be helpful.

Finally, the role of public policy in coordinating trade between relatively small numbers of buyers and sellers could be explored. A key feature of our model is that the decentralized equilibrium is Bertrand in strategies but Cournot in outcomes. This raises interesting questions about the efficiency of investment in small markets because, like Bertrand, we find that the existence of two sellers is enough for efficiency. This result suggests that this framework could be used to shed new light on an old theoretical problem – inefficient capacity precommitment with Bertrand competition.

REFERENCES

Binmore, K., J. Swierzbinsi, S. Hsu, and C. Proulx, (1993) "Focal Points and Bargaining", *International Journal of Game Theory*, 22, 381-409.

Burdett, K., S. Shi, and R. Wright, (2001) "Pricing and Matching with Frictions", *Journal of Political Economy*, 109, 1060-1085.

Coles, M., and J. Eeckhout, (2000) "Indeterminacy in Directed Search", University of Pennsylvania manuscript.

Cournot, A., (1887), *Recherches sur les Principes Mathematiques de la Theorie des Richesses*, [English edition: *Researches into the Mathematical Principles of the Theory of Wealth*], edited by N. Bacon. London: McMillan, 1987.

Julien, B., (1997) "Essays in Resource Allocation Under Informational Asymmetries", unpublished PhD dissertation, University of Western Ontario.

Julien, B., J. Kennes, and I. King (2000) "Bidding for Labor" *Review of Economic Dynamics*, 3, 619-649.

Julien, B., J. Kennes, and I. King (2001a) "Auctions and Posted Prices in Directed Search Equilibrium", *Topics in Macroeconomics*, 1, Article 1.

Julien, B., J. Kennes, and I. King (2001b) "Directed Search without Price Directions", University of Auckland manuscript.

Kreps, D., and J. Sheinkman, (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics*, 14, 326-37

Kultti, K., (1999) "Equivalence of Auctions and Posted Prices", *Games and Economic Behaviour*, 27, 106-113.

Kultti, K., and J-P. Niinimaki, (2002) "About Equilibrium Wages", Helsinki School of Economics manuscript.

Maskin, E., and J. Riley, (1984) "Monopoly with Incomplete Information", *Rand Journal of Economics*, 15, 171-196.

McAfee, R.P., (1993) "Mechanism Design by Competing Sellers", *Econometrica*, 61, 1281-1312.

McAfee, R.P., and J. McMillan, (1987) "Auctions and Bidding", *Journal of Economic Literature*, 25, 699-738.

Montgomery, J., (1991) "Equilibrium Wage Dispersion and Interindustry Wage Differentials", *Quarterly Journal of Economics*, 106, 163-179.

Ochs, J., (1990) “The Coordination Problem in Decentralized Markets: An Experiment”, *Quarterly Journal of Economics*, 105, 545-559.

Peters, M., (1984) “Bertrand Equilibrium with Capacity Constraints and Restricted Mobility”, *Econometrica*, 52, 1117-1129.

Peters, M., (1994) “Equilibrium Mechanisms in a Decentralized Market”, *Journal of Economic Theory*, 64, 390-423.

Reny., P., and A. Robson, (2001) “Reinterpreting Mixed Strategy Equilibria: A Unification of the Classical and Bayesian Views”, University of Chicago manuscript.

Wang, R., (1993) “Auctions Versus Posted-Price Selling”, *American Economic Review*, 83, 838-851.