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Distribution of Human Capital and Economic Growth

Debasis Bandyopadhyay
University of Auckland, d.bandyopadhyay@auckland.ac.nz

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Debasis Bandyopadhyay[1]
Department of Economics, University of Auckland
Private Bag 92019, Auckland, New Zealand
Ph: 64-9-373-7599; x7439, Fax: 64-9-373-7427
E-Mail: debasis@auckland.ac.nz

ABSTRACT

This paper provides an empirically tractable model of economic growth where the distribution of human capital is central to understanding the key issues. Long run growth is possible only if the distribution of human capital belongs to a known class such that investment in education, the model's engine of growth, exceeds inter-generational depreciation of human capital. The model contributes to understanding of the puzzle of growth disparities among countries by exhibiting multiple steady states under alternative paradigms of growth. It provides a purely neoclassical model to explain why a lower income inequality may correspond to a higher rate of growth.

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Key Words: Human Capital Distribution, Cross-country Growth Disparities, Growth-inequality Debate

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1. INTRODUCTION

There has been a sudden outburst of literature on income distribution and growth based on human capital theory (e.g., Galor and Zeira, 1993). Those literature add a new dimension to macroeconomic dynamics by making the distribution of human capital a fundamental determinant of the macroeconomic aggregates. Concurrently, there have been renewed interests in the controversies surrounding alternative paradigms of growth (e.g., Solow, 1994) facing the fact of cross-country growth disparities (e.g., Quah, 1996) and the fact of a growth-inequality relationship (e.g., Chang, 1994). Can we exploit the new dimension of macroeconomics to interpret those facts with a single empirically tractable model of economic growth? This paper does just that. In particular, it offers a single model connecting distribution of human capital, growth and income inequality to provide a purely neoclassical explanation of the growth diversities and the growth-inequality relationship. In the model, human capital is the sole capital input and both skilled and unskilled labor are essential for the production of a single perishable consumption good. The quality of the labor force and the intensity of innovative activities in the economy influence the production technology through their external effects. There is no credit market following the absence of a tangible capital. Within this model world, the paper offers a hypothesis, with empirical support, that the diversity of human capital distribution alone can significantly explain the observed diversities of growth.

The paper builds its hypothesis by linking growth disparities and unequal distribution of earning opportunities. Adult individuals with unequal endowments of human capital face unequal earning opportunities in innovative activities that reward human capital or skill. An adult's human capital depends partly on her parent's human capital due to an externality associated with her family upbringing and partly on what her parent allocates for her schooling. Only adults with human capital higher than a basic skill level can earn a skill premium by pursuing innovative activities. In this environment, parents provide for schooling only if they themselves have human capital higher than a threshold level. The basic skill and the threshold levels are determined endogenously by the distribution of human capital among the respective groups. Consequently, the bias or unevenness in the distribution of human capital among the adults determines the relative proportion of adults who pursue those innovative activities, which in turn determines the skill premium, returns to parental investment in schooling, rate of accumulation of human capital and growth. In general, there are multiple steady states displaying multiple paths of growth and different degrees of income inequality. The path of development including its long run steady state, therefore, depends on the history and, in particular, on the distribution of economic opportunity determined by the initial distribution of human capital. The conclusions survive whether or not there is endogenous growth in the model. In particular, with alternative parameter specifications the model displays no endogenous growth as in Solow (1956) or constant growth as in Lucas (1988) or increasing growth as in Romer (1986). The possibility of multiple steady states with identical preference and technology enhances its ability to account for the multifaceted growth experiences across countries in all three cases.

Barro and Lee (1993) provide distributions of highest educational attainment among the adults of age 25 and above. Comparisons of those distributions in 1960 for the countries with the highest and the lowest per capita income in 1990 or for the countries with the highest and the lowest average annual growth rates between 1960 and 1990, reveal significant differences (e.g., see Figures 1a-b, 2a-b) offering important clues to understanding the puzzle of growth disparities. Moreover, it is quite surprising to discover (see, Tables 1A-C) that a model with only the relative proportions HQ and NQ of adults respectively with high and no educational qualification can explain about 75% of disparities of income per adult compared to 50% done by Solow (1956) and 68% done by Mankiw, et al (1992). It is also interesting to note in Table 1B that the variables *HQ* and *NQ* can reasonably explain the key variables of the Solow (1956) model. These findings suggest that the distribution of human capital is more fundamental than variables that are conventionally used for explaining income disparities. Following this motivating result, the paper develops an empirically tractable growth model and calibrates its parameters to match the reported growth observations. After calibrating the parameters to fit the growth data, the model offers an even stronger implication regarding the growth inequality relationship. In particular, a higher income-inequality implies a lower rate of growth. This conclusion coincides with Persson and Tabellini (1994) but with an important distinction. It arises in a non-political and a purely neoclassical environment and, therefore, reinforces the findings of Clarke (1995) and Perotti (1996) that the relationship is more fundamental than the political system of a country.

The paper also offers a new perspective on the growth versus equality debate. There is an old hypothesis (see e.g., Cline, 1974) that higher inequality is necessary for faster growth and, therefore, provides a development dilemma. The dilemma follows from the presumption that while saving is good for growth, the poor save too little and, therefore, a redistribution from the rich to poor would reduce saving and hence would retard growth. This paper argues that, on the contrary, a rapid growth and an equitable income distribution are compatible. Indeed our model retains the characteristics that the rich save a higher fraction of their income than the poor and that an increase in the saving rate leads to higher growth. Nevertheless, a more unequal distribution of income between the skilled and unskilled population corresponds to a lower rate of saving and hence a lower rate of growth. This occurs because a more equitable distribution of human capital releases constraints for a higher fraction of the adults to pursue innovative activities increasing the relative supply of skilled workers, and fostering both equity and growth.

The paper distinguishes between unequal opportunity and unequal ability. The former arises from the credit market imperfection and the dynasty specific intergenerational altruism while the latter is assigned by the nature or the history at the initial date. It is the former that pushes the model economy to an inefficient state where the growth rate is below its maximum and income inequality is above its minimum possible limit. Interestingly, even when agents initially have a perfectly equal distribution of human capital as well as identical preferences and technology, income inequality arises endogenously as a precondition for growth and persists in the long run. The equal distribution of human capital, however, generates the minimum possible degree of income inequality and the maximum possible growth.

The paper has five sections. Section 2 provides a description of the model's environment. Section 3 discusses the equilibrium. Section 4 examines the properties, existence and multiplicity of the model's steady state. Section 5 includes a discussion on calibration and simulation for evaluating the model's prediction regarding the cross-country growth disparities and the growth-inequality relationship. Section 6 summarizes the contribution of the paper.

2. THE MODEL

Consider an environment with variable human capital, labor, and a single perishable consumption good. An agent lives two periods, one as a child being attached to an adult and one as an adult when she receives a child of her own. There is a continuum of dynasties with measure one and at each date $t \geq 0$, a typical dynasty consists of an adult and a child. The adult has one unit of labor and $h \geq 0$ units of human capital. She earns her income by choosing between the occupations of manager and worker and then divides her income between current consumption and investment in her child's education. Preferences display intergenerational altruism, and so the adult maximizes the present discounted value of consumption of her dynasty. Dynasties differ only in terms of the adult's endowment of human capital at t=0. At $t \geq 0$, Ψ_t denotes the cumulative distribution of human capital among the date t adults. The history specifies the initial distribution Ψ_0.

Production is done by groups, each of which consists of a manager and one or more workers. The real world counterpart of the model's managers are scientists, engineers, business supervisors and other professionals whose occupations provide them with the autonomy to implicitly maximize their own profits by allowing them to specialize into innovative and entrepreneurial activities and to employ outside labor to carry out routine work complementary to their innovations. The output q of a group depends on the manager's human capital h, the number n^d of workers she employs and the technology level $A>0$ accessible to the manager such that $q = Ah^{1-a}(n^d)^a$, where $0<a<1$ measures the output elasticity of worker. Assume that the technology level A depends, following Lucas (1988), on a knowledge spillover process that increases with the quality of labor measured by the economy's average human capital stock H and, following Romer (1990), on the stock A_0 of non-rival knowledge. Moreover, assume that the knowledge spillover increases with the intensity of innovative activities in the economy measured by the proportion m of adults who are managers. In particular, assume that $A = A_0 m^{\theta} H^b$, where $b>0$ is a parameter measuring the degree of externality and $\theta \geq 0$ is a parameter measuring the institutional barriers to spillover of knowledge. If $\theta=0$, there is no such barrier. Note managers serve as the only conduit, although not the only source, of such spillovers of knowledge. To summarize, at each date $t \geq 0$ the output q_t of a manager is given by

(1) $$q(h, n_t^d; H_t, m_t) = A_0 m_t^{\theta} H_t^b h^{1-a} (n_t^d)^a, \qquad t=0, 1, 2....$$

Observe that there are m firms per capita in this economy; each is run by one manager. On average, there are $(1-m)/m$ workers per firm. Note that $\theta>0$ implies that the total factor productivity in the economy decreases with the average firm size measured in employees per manager. This feature is consistent with the idea that "small is beautiful" and big is unmanageable and, therefore, results in diseconomies of scale due to fixedness of the manager's input. To further motivate the assumption take the fraction HE of adults with high levels of education as a proxy for m and then note from Figure 3 a positive relationship between HE and the per capita income.

At each date $t \geq 0$ given the wage rate w_t of a worker and the two external factors H_t and m_t, a manager with h units of human capital employs n_t^d number of workers so as to

(2) $$\underset{n_t^d > 0}{Maximize} \left[q(H_t, m_t, n_t^d, h) - w_t n_t^d \right], \qquad t=0, 1, 2....$$

The first order condition of (2) yields $w_t = aA_0 H_t^b m_t^{\theta} h^{1-a} (n_t^d)^{a-1}$, or, equivalently, the optimal number $n_t^d(h)$ of workers employed by a manager with h units of human capital is

(3) $$n_t^d(h) = \left(\frac{a m_t^{\theta} H_t^b}{w_t} \right)^{\frac{1}{1-a}} h, \qquad t=0, 1, 2....$$

By (1)-(3) at each date $t \geq 0$ the indirect profit of a manager is proportional to her human capital stock h and is given by $r_t h$, where,

(4) $$r_t = (1-a) A_0 m_t^{\theta} H_t^b (aA_0 m_t^{\theta} H_t^b / w_t)^{a/(1-a)}, \qquad t=0,1, 2,....$$

The Basic Skill Level

At each date $t \geq 0$, x_t denotes the level of *basic skill* such that an adult with x_t units of human capital earns an equal amount either as a manager or as a worker. By (4), x_t satisfies

(5) $$w_t = r_t x_t, \qquad t=0,\ 1,\ 2,\ldots$$

The adult's occupation $n_t(.)$ is an indicator function such that if she is a worker, $n_t(.)=1$, otherwise, if she is a manager, $n_t(.)=0$. At each date $t \geq 0$, her occupational choice $n_t(.)$ and the resulting income $y_t(.)$ as functions $h \geq 0$ are

(6) $$n_t(h) = 1, \text{ if } h < x_t;\ \ n_t(h) = 0, \text{ if } h > x_t;\ \ n_t(h) = 1 \text{ or } 0, \text{ if } h = x_t,$$

(7) $$y_t(h) = n_t(h)w_t + (1 - n_t(h))r_t h.$$

Figure 4 illustrates how the basic skill level divides the adults into two occupational groups, workers and managers, according to their individual stock of human capital.

An adult's human capital h_{t+1} at a date $t+1$ is positively related to her parent's human capital h_t and the investment s_t in her schooling made by her parent at date t. In particular,

(8) $$h_{t+1} = (1-\delta)h_t + s_t, \quad 0 < \delta < 1 \qquad t=0,\ 1,\ 2,\ ..$$

The above formulation presumes an externality $\delta > 0$ associated with family upbringing in the tradition of Benabou (1996). It also assumes that without investment in schooling the current generation can transfer only a fraction $\delta < 1$ of existing knowledge to the future generation. Consequently, knowledge is maintained or accumulated only if a generation acquires them through parental investment in schooling. This feature is similar to Mankiw, et al. (1992) but the motivation comes from the history of the middle ages: Knowledge held by the ancient civilizations are lost today, because during the middle ages few invested in teaching the ancient languages to transfer the codes of those civilizations to the following generations.

Following Barro (1974) assume intergenerational altruism such that at each date $t \geq 0$, the utility v_t of the adult as function of her family's consumption c_t and her child's utility v_{t+1} as a grown-up adult at the following date $t+1$ satisfies

(9) $$v_t = V(c_t, v_{t+1}) = u(c_t) + \beta v_{t+1}, \qquad t=0, 1, 2, \ldots$$

We assume that $u(.)$ is strictly concave, bounded above, $u(0)=-\infty$, $u'(0)=\infty$, $0<\beta<1$, such that $v_0 = \sum_{t=0}^{\infty} \beta^t u(c_t)$. Also, assume that there is a function $f(.)$ of the ratio of c_{t+1} to c_t such that

(10) $$\frac{u'(c_t)}{u'(c_{t+1})} = f\left(\frac{c_{t+1}}{c_t}\right), \quad f'(1) < \frac{1-\beta(1-\delta)}{\delta}, \quad f'' \leq 0, \quad t=0, 1, 2, \ldots$$

The adult with h units of human capital chooses a suitable occupation $n_t(h)$ following (6) and divides her income $y_t(h)$, given by (7), between consumption c_t and investment s_t such that

(11) $$c_t + s_t \leq y_t(h) \qquad t=0, 1, 2, \ldots$$

At $t=0$ the optimization problem of the adult with $h \geq 0$ units of human capital is to choose a sequence $\{(c_t(h) \geq 0,\ s_t(h) \geq 0,\ n_t(h) \in \{0, 1\})\}_{t=0,1,2,\ldots}$ given $\{w_t, r_t\}_{t=0,1,2,\ldots}$ so as to

(12) $$\textit{Maximize} \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ subject to (6), (7), (8) and (11).}$$

Threshold Level of Human Capital for Investment

Adults with low human capital do not invest in schooling. In particular, by (12), at each date $t \geq 0$, there is a *threshold level* $h_t^* > 0$ of human capital such that an adult with $h < h_t^*$, finds the opportunity cost of investment in schooling to be higher than the present value of all possible future benefits from it. In other words,

$$(13)\qquad u'[y_t(h_t^*)] \geq \sum_{k=1}^{\infty} \beta^k (1-\delta)^{k-1} [1-n_{t+k}(h_t^*)] r_{t+k} u'[c_{t+k}(h_t^*)], \qquad t=0, 1, 2, \ldots,$$

where at the date $t+1$ the sequence $\{c_{t+k}(h_t^*), n_{t+k}(h_t^*)\}$ solves the optimization problem of the adult with $(1-\delta)h_t^*$ units of human capital. All adults with $h > h_t^*$ invest $s_t(h)>0$ in their child's education such that $s_t(h)$ satisfies the following first order condition for optimization:

$$(14)\qquad u'(y_t(h) - s_t(h)) = \sum_{k=1}^{\infty} \beta^k (1-\delta)^{k-1} (1-n_{t+k}(h)) r_{t+k} u'(c_{t+k}(h)), \; t=0, 1, 2, \ldots.$$

Lemma 1. At $t \geq 0$, if $h > h_t^*$ and for all $k \geq 1$, $n_{t+k}(h)=0$, then for all $h \geq x_t$, $s_t(h)$ increases proportionately with h and for all $h < x_t$, $s_t(h)$ increases as a diminishing proportion of h.

Proof. See Appendix A.

3. EQUILIBRIUM

The set of sequences $\{(c_t(h), s_t(h), n_t(h), n_t^d(h) : h \geq 0; x_t, r_t, m_t, H_t, w_t\}_{t=0,1,2,..}$ and the initial distribution Ψ_0 describe the model's *equilibrium* such that at each $t \geq 0$, the labor demand $n_t^d(.)$ satisfies (2), the implicit rental price r_t of human capital satisfies (4), the basic skill x_t satisfies (5), the sequence $\{(c_t(h), s_t(h), n_t(h)\}_{t=0,1,2,..}$ satisfies (12), and $\{H_t, m_t\}_{t \geq 0}$ coincides with the same generated by the optimal sequence $\{s_t(h), n_t(h)\}_{t=0,1,2,..}$, such that

$$(15)\qquad m_t = \int_{\{h: n_t(h)=0\}} d\Psi_t(h),$$

(16) $$H_{t+1} = (1-\delta)\int h d\Psi_t(h) + \int s_t(h) d\Psi_t(h), \quad H_0 = \int h d\Psi_0(h),$$

the labor market clears such that at each date *t=0, 1, 2,...,*

(17) $$\int_{\{h: n_t(h)=0\}} n_t^d(h, w_t; H_t, m_t) d\Psi_t(h) = \int_{\{h: n_t(h)=1\}} d\Psi_t(h)$$

and the goods market clears such that at each date *t=0, 1, 2..,*

(18) $$\int_{h\geq 0} (c_t(h) + s_t(h)) d\Psi_t(h) = \int_{\{h: n_t(h)=0\}} q_t(h, n_t^d(h); H_t, m_t) d\Psi_t(h).$$

The above definition yields a sequence $\{H_t, m_t, H_{mt}\}_{t\geq 0}$.. of state variables that characterize the equilibrium, where H_{mt} denotes the total human capital of managers such that

(19) $$H_{mt} = \int_{\{h: n_t(h)=0\}} h d\Psi_t(h), \qquad t=0, 1, 2,....$$

By (3), (15) and (17) the equilibrium wage rate w_t is given by

(20) $$w_t = aA_0 m_t^{\theta} H_t^b H_{mt}^{1-a} (1-m_t)^{a-1} \qquad t=0, 1, 2,...$$

By (20) the wage rate of workers increases with the economy's average human capital and the relative proportion of managers. The former positively influences the productivity of workers through an external effect while the latter augments the relative scarcity of workers. By (4) and (20) the implicit rental r_t price of human capital is given by

(21) $$r_t = (1-a) A_0 H_t^b H_{mt}^{-a} m_t^{\theta} (1-m_t)^a \qquad t=0, 1, 2,...$$

By (21) the price r_t of human capital and hence the rate of return $r_t + 1 - \delta$ from the investment in schooling is an inverted-U shaped function of m_t. A new manager generates an external benefit to other managers with her innovative activities. She, however, augments the relative scarcity of workers and hence the wage rate, a cost to all managers. For a low value of m, additional benefits are disproportionately higher than additional costs and, therefore, returns to schooling increases with additional managers in the economy. A high value of m, however, turns the balance in the opposite direction. By (5), (20) and (21) it follows that,

(22) $$x_t = \frac{aH_{mt}}{(1-a)(1-m_t)}, \qquad t=0, 1, 2,...$$

The basic skill level is independent of the workers' human capital, which has no market value. An economy with more managers and with a higher human capital of managers has a higher relative price of workers and requires a higher level of basic skill from a manager.

Lemma 2. At each date t=0, 1, 2,.., for any arbitrary distribution Ψ_t such that $\Psi_t(0)\neq 1$, there exists a unique level of the basic managerial skill $0<x_t<hmax_t<\infty$, where $\Psi_t(hmax_t)=1$.

Proof. See Appendix A.

Lemma 3 At each date t=0,1,2,..., there exists a unique value of $h_t^*>0$ and $0<\Psi_t(h_t^*)<1$.

Proof. See Appendix A.

Proposition 1. There exists a unique equilibrium in this model.

Proof. See Appendix A.

Growth Disparities

By (1), (3) and (20) at equilibrium the per capita output Y_t and its growth rate γ_t are given by

(23) $$Y_t = A_0 m_t^{\theta} H_t^{1+b} H_{mt}^{-a}(1-m_t)^a, \qquad t=0, 1, 2,....$$

(24) $$\gamma_t = \left(\frac{H_{t+1}}{H_t}\right)^{1+b}\left(\frac{H_{mt+1}}{H_{mt}}\right)^{-a}\frac{m_{t+1}^{\theta}(1-m_{t+1})^{a}}{(1-m_t)^{a}m_t^{\theta}}-1, \quad t=0,\ 1,\ 2,....$$

Note that by (23) and (24) the per capita output and its growth rate depends non-trivially on the distribution of human capital between workers and managers. For example, a country with the same average human capital and the same manager to worker ratio as others could still have a higher per capita income if its workers possess a higher share of its total human capital than the workers in those other countries. Indeed workers are only able to sell their raw labor and hence do not receive any return on their human capital but their untapped human capital raises the overall productivity in the economy and that makes the difference. Countries could experience differences also in the growth rate of per capita output even when they share a common growth rate of average human capital. By (23)-(24), it is possible to theoretically account for those differences by examining the cross-country differences in the growth rates of human capital of workers and managers separately. Note that the presence of both too many managers and too few managers in an economy yield too little output. This feature follows from the assumption that both skilled and unskilled labor are essential to production. We now focus on the steady state, explore the possibility of multiple steady states and identify a set of distinct equivalence classes of distributions of human capital to characterize the growth diversities that this model exhibits.

4. STEADY STATE

At each date t, the state variables of the model are H_t, m_t and H_{mt}. In the steady state,

(25) $$m_t = m, \qquad m = 1-\Psi_0(0), \qquad t=0, 1, 2,.....$$

(26) $$H_t = H_{mt} = (1+\gamma)^t H \qquad H = \int h d\Psi_0, \qquad t=0, 1, 2,.....$$

where, $0<m<1$, $H>0$, $\gamma \geq 0$, and Ψ_0 denotes the initial distribution of human capital. It follows, therefore, from (20), (22), (25) and (26) that at each date $t \geq 0$ the wage rate w_t and the basic skill x_t grow at the stationary rate γ from their respective initial states w and x where,

$$(27) \qquad w = \frac{am^{\theta} A_0 H^{1+b-a}}{(1-m)^{1-a}},$$

$$(28) \qquad x = \frac{aH}{(1-a)(1-m)}.$$

Denote by $w(m, H)$ and $x(m, H)$ the values of w and x that respectively satisfy (27) and (28) for any given values of m and H. By (21), (25) and (26) the price r_t of human capital satisfies,

$$(29) \qquad r_t = (1-a)A_0 H^{b-a} m^{\theta} (1-m)^{a} (1+\gamma)^{(b-a)t} \qquad t=0,1,2,..$$

Proposition 2. In the steady state, at each date $t \geq 0$, the implicit rental price r_t of human capital must be equal to a time invariant constant $r>0$ and either $b=a$ or $\gamma=0$.

Proof. See Appendix A.

Consequently, by (7), (8), (10) and (14), a manager's investment $s_t(h)$ is a time invariant fraction i of her human capital stock h and the investment rate given by the fraction i satisfies

$$(30) \qquad \gamma = i - \delta,$$

$$(31) \qquad i = f^{-1}(\beta(r+1-\delta)) - 1 + \delta,$$

where, $f^{-1}(.)$ denotes the inverse of function $f(.)$, defined by (10). The assumed strict concavity of u implies that $f' > 0$ and, therefore, by the Inverse Function Theorem f^{-1}

exists. If $b \neq a$, by Proposition 2, $\gamma=0$, and hence by (30) $i=\delta$ and by (10) and (31) r is independent of m. If, however, $b=a$, by (29)-(31) r, i and γ are functions $r(.)$, $i(.)$ and $\gamma(.)$ of m such that $\gamma=\gamma(m)$ satisfies (30) given $i=i(m)$, and $i=i(m)$ satisfies (31) given $r=r(m)$, where,

$$(32) \qquad r(m) = (1-a)A_0 m^{\theta}(1-m)^{a}, \qquad \text{if } b=a$$

$$= \beta^{-1} - 1 + \delta, \qquad \text{if } b \neq a.$$

By (23), (25) and (26), it follows, therefore, that the per capita output Y_t satisfies

$$(33) \qquad Y_t = A_0 H^{1+b-a} m^{\theta}(1-m)^{a}(1+\gamma(m))^{(1+b-a)t} \qquad t=0, 1, 2, \ldots,$$

by (26), (29), (32) and (33) the steady state output-(human) capital ratio is given by a function $z(.)>0$ of m, if $b=a$, or by a constant $g>0$, if $b \neq a$, where,

$$(34) \qquad z(m) = A_0 m^{\theta}(1-m)^{a}; \qquad g = \frac{\beta^{-1} - 1 + \delta}{1-a},$$

and by (27)-(29) workers and managers respectively receive aY_t and $(1-a)Y_t$ units of output.

Existence and Multiplicity of Steady States

In this section we discuss the existence of a steady state such as described above following Proposition 3 which summarizes the behavior of individual dynasties in the steady state.

Proposition 3. In the steady state, given $0<m<1$ and $H>0$: (a) a worker has no human capital, consumes her wage which grows at a rate $\gamma(m) \geq 0$ from its initial state $w(m, H)$ and does not invest in her child's education; (b) a manager's human capital, income and consumption grow at the rate $\gamma(m) \geq 0$ from their respective initial states $h>0$, $r(m)h$ and $(r(m)-i(m))h$ and at each date she invests a constant fraction $i(m)$ of her human capital in *her child's education such that* $\delta \leq i(m) < r(m)$.

By Proposition 3, all managers invest a fraction $i \geq \delta > 0$ of their human capital at all dates. By (14), such choice is optimal if and only if the initial human capital of each manager exceeds a threshold level $h^* = (r-i)^{-1}w$ such that the marginal benefit from such investment for adults with $h > h^*$ exceeds the marginal cost. Given $1 > m > 0$ and $H > 0$ define $h^*(m, H)$ as

$$(35) \qquad h^*(m,H) = \frac{w(m,H)}{r(m) - i(m)}.$$

It follows, therefore, that in the steady state, the distribution of human capital must satisfy

$$(36) \qquad \Psi_t(h^*(m,H)(1+\gamma(m))^t) = 1 - m = \Psi_t(0), \qquad t = 0,\ 1,\ 2,$$

...

By (31)-(34), the fraction of income a manager saves is, a function $s_m(.)$ of m such that

$$(37) \qquad s_m(m) = i(m)/r(m),\ 1 > s_m(m) \geq s_{min} = \delta\beta(1-\delta+\delta\beta)^{-1}, \qquad 0 < m < 1.$$

Denote by $\phi(m)$ the ratio of the average income of workers to managers such that

$$(38) \qquad \phi(m) \equiv \left(\frac{aY_t/(1-m)}{(1-a)Y_t/m}\right) = \frac{am}{(1-a)(1-m)},\ \phi(0)=0,\ \phi' > 0,\ 0 < m < 1,$$

and $(\phi(m)^{-1} - 1)$ equals the skill premium relative to the wage rate. By (26) and (35) it follows that $H \geq mh^*$ and, therefore, by (27)-(28), (31)-(32) and (35),

$$(39) \qquad (1 - s_m(m)) \geq \phi(m).$$

By (39) a positive saving rate, or $s_m > 0$ implies $\phi(m) < 1$, or equivalently, a positive skill premium. By (38), it follows that $m<1-a$. Proposition 4 sets a tighter upper bound for m.

Proposition 4. In the steady state, $m \leq m_U < 1-a$, *where* $m_U = \phi^{-1}(1-s_{min})$. *If* $\delta=0$, $m_U=1-a$.

If b≠a, then $\gamma = 0$, or equivalently, $i = \delta$. Consequently, by (29) and (34), m and H satisfy

$$(40) \qquad H^{b-a} z(m) = g \Leftrightarrow H = H(m) \equiv \left(\frac{\beta^{-1} + 1 - \delta}{(1-a) z(m)} \right)^{\frac{1}{b-a}}.$$

Lemma 4. Given b=a, $\gamma(m) \geq 0$, *or equivalently,* $i(m) \geq \delta$, *if and only if* $z(m) \geq g$.

Proof. See Appendix A.

Let m^* denote the argmax of $z(m)$ and $\hat{m} = \min\{m^*, m_U\}$. By (34), (37) and Proposition 4,

$$(41) \qquad m^* = \frac{\theta}{a+\theta}, \quad \hat{m} = \min\left\{ \frac{\theta}{a+\theta}, \frac{(1-a)(1-\beta)}{1-\beta+\delta\beta a} \right\}$$

Note that if $\theta>0$ and $z(m^*) \geq g$, the steady state value of m has a lower bound $m_L>0$ such that $m_L<m^*$ and $z(m_L)=g$. If $\theta>0$, the rate of knowledge spillover decreases as m decreases. If the value of m falls below the critical level m_L, the implied market returns to schooling becomes too little to ensure a rate of investment in schooling that exceeds the rate of depreciation.

Lemma 5. $m_U \geq m_L$ if and only if $z(\hat{m}) \geq g$.

Proof. See Appendix A.

For any given $m_L \geq m \geq m_U$ and $H>0$ define the equivalence class $D(m, H)$ of distributions Ψ of human capital such that $\Psi \in D(m,H)$, if and only if

$$(42) \qquad \Psi(0) = 1-m, \quad \int h d\Psi = H, \text{ and } \Psi(h^*(m,H)) = \Psi(0).$$

Lemma 6. The equilibrium with any initial distribution $\Psi \in D(m, H)$ is a distinct steady state if and only if m satisfies (39) and either b=a or H satisfies (40).

Proof. See Appendix A.

Does there exist such a steady state? Are there multiple steady states? Proposition 5 provides positive answers but with assumptions that the set $[m_L, m_U]$ is neither empty nor a singleton.

Proposition 5. If $z(\hat{m}) \geq z(m_L) = g$ there exists a steady state. If, in addition, $z(\hat{m}) > g$ there are multiple steady states such that if b=a, the growth rate varies with m *across the steady states, or if $b \neq a$, the per capita output* Y, *vary across steady states as functions of* m.

The set of steady states may not be connected. In particular, if there are multiple steady states but $z(m_U)<g$ and m^* does not satisfy (39), the set of steady state values of m is disconnected. The following Lemma describes how we can rule out that possibility.

Lemma 7. If $z(m_U)>g$ then $m_U>m_L$ and there exists a critical fraction $m_c \in (m_L, m_U)$ such that $z(m) \geq g$ and $(1-s_m(m)) \geq \phi(m)$ if and only if $m \in [m_L, m_c]$.

Ignoring the case $b>a$ when the steady states are unstable and restricting the set of steady states to $[m_L, m_c]$, explore why some countries are rich while others are poor and why some poor countries get even poorer over time. Figures 5-6 and Proposition 6 provide some clues:

Proposition 6. If $m_c \leq m^*$, then $\gamma'(m)>0$ and $H'(m)>0$ for all $m \in [m_L, m_c]$. If $m_c>m^*$, then m^* maximizes either the growth rate, if b=a, or the per capita output, if b<a; and the growth rate, if b=a, or the per capita output, if b<a, is an inverted U shaped function of m.

Consequently, by (34), the (human)capital-output ratio varies across steady states as follows:

Proposition 7. In case $b=a$, if $m_c \leq m^*$, then output-capital ratio $z(m)$ increases with m, otherwise if $m_c > m^*$, then it is an inverted U shaped function of m such that $z(m) \leq z(m^*)$. In case $b \neq a$, the output-capital ratio is independent of m and is equal to the constant $g>0$.

Data on cross-country variations of the human capital-output ratio could, therefore, contribute to our understanding of the empirical relevance of models with or without long run growth.

Growth Disparities in Alternative Paradigms

Note that by (33) the elasticity of per capita output with respect to the average human capital is $(1+b-a)$. It follows, therefore, that if $b<a$, there is economy-wide diminishing returns to (human) capital per capita as in Solow (1956), while $b>a$ corresponds to increasing returns as in Romer (1986) and $b=a$ corresponds to constant returns as in Lucas (1988). Our model exhibits a balanced long run growth in per capita output only if there is economy wide constant returns to capital per capita. This is consistent with the recent observations made in Solow (1994) regarding the endogenous growth models. Interestingly, the constant return, or $b=a$, is only a necessary but not a sufficient condition for growth in this model. Instead, the distribution of human capital that determines the steady state value of m has the final key to ensuring a non-zero growth rate according to this model. In particular, long run growth requires that the value of m is be greater than m_L such that the investment rate $i(m)$ exceeds the intergenerational rate δ of depreciation. We now compare the model's equilibrium near the steady state with that of the three alternative paradigms of growth mentioned above.

If $b=a$, the equilibrium describes a balanced growth path with $\gamma>0$ similar to Lucas (1988) with an important exception, however. The long run growth rate γ, by Proposition 5, varies across economies with identical technology and preferences. Like Lucas (1988), the rate of investment in human capital, by (30), determines the growth rate and hence, by (31), there is a connection between the thriftiness parameter β and growth but with an important distinction. By Proposition 6, two countries with the same thriftiness may invest in human capital at different rates and hence grow at different rates, if their initial distributions of human capital do not belong to the same equivalence class defined by (42). The model also explains some puzzling non-linear growth dynamics within this paradigm. For example, it is difficult to explain why some countries like India and Sri Lanka initially experience a negative growth rate but instead of getting trapped into a no growth state, grow forever in the long run. The model offers a clue. Consider the initial distribution that puts a positive mass in the interval (0, $h^*(m, H)$) such that

$$H_{wt} = \int_0^{h^*(m,H_t)} h d\Psi_t \text{ and } H_{mt} = \int_{h>h^*(m,H_t)} h d\Psi_t , \qquad t=0, 1, 2,... \tag{43}$$

The dynasties with the initial human capital $h<h^*(m, H)$ are of workers and do not invest while others are managers and invest at the rate $i(m)>\delta$. It follows, therefore, that the growth rate of human capital per capita is a weighted average of $-\delta H_{wt}$ and $(i(m)-\delta)H_{mt}$. It is possible for an economy with a low ratio of H_{mt} to H_{wt} to experience a negative growth rate of average human capital and, hence output per capita, at the initial phase of the development process. Over time growth rates become positive and then the economy grows for ever.

Consider the other cases, $b<a$ and $b>a$: If $b<a$, the model has no long run growth as in Solow (1956). By (42) pick an initial distribution $\Psi \in D(m, H)$. Find the long run value $H(m)$ of per capita human capital by (40) and the long run per capita output by (33). If the initial per capita stock $H<H(m)$, then by (29), for all $t \geq 0$ the implicit price r_t, exceeds its steady state value and, hence, by (31) the investment rate $i_t>\delta$. Consequently, the per capita stock H_t of human capital increases over time exhibiting short run growth as in Solow (1956).

As H_t increases, by (29), $b<a$ implies that the price r_t of human capital decreases and, therefore, by (31) the rate i_t of investment in human capital decreases to its steady state value δ. Consequently, the growth rate of human capital and output per capita diminishes over time and disappears in the long run as in Solow (1956) but with one important difference. This model exhibits multiple steady states. By Proposition 5, therefore, it partially explains cross-country variations of per capita income. If $b>a$, there are increasing returns with respect to H as in Romer (1986) but there are multiple but unstable steady states. Consider the distribution $\Psi \in D(m, H)$. By (29) and (31), if $H > H(m)$, there is ever increasing growth; but if $H < H(m)$, there is a perpetual decline at the rate $-\delta$. Along the path of growth (or decay) the price elasticity of human capital remains constant and is given by $(b-a)$. If by chance $H=H(m)$, the economy gets stuck in that unstable steady state. This knife-edge property, as Solow (1994) points out, is common to all growth models with increasing returns. There is one difference, however. The critical value around which we observe this knife-edge property depends on the initial distribution of human capital. The equilibrium path near the steady state too depends on the history represented by the initial state. Suppose that initially $m<m_c<m^*$ such that, by Proposition 7, $z'(m)>0$. Consider an exogenous shock that increases the value of m. By Lemma 1, the investment rate i_t is independent of a manager's individual stock h and, by (31), the investment rate varies directly with the price r_t of human capital. Consequently, by (30), the path of H_t satisfies

$$H_{t+1} = H_t + (i_t - \delta)H_t, \qquad t=0, 1, 2, \ldots.. \tag{44}$$

Figure 7 illustrates the path of convergence near the steady state. If $b<a$, there is a short run growth spur and the per capita human capital and income converges to a higher state in the long run. If $b>a$, there is accelerating growth. Instead, if initially, $m^*<m<m_c$ such that

$z'(m) < 0$, then a sudden increase in m brings down the long run income per capita, when $b<a$, or leads to a perpetual decline in per capita income, when $b>a$. Therefore, the initial state is an important determinant of the future outcome.

Diversity in Economic Growth and the value of θ

Possibility of multiple steady states in our model under alternative paradigms of growth is the key to its contribution to our understanding of the economic growth disparities across countries. We now examine the implications of alternative values of θ on the specific nature of this multifaceted growth in this model with the help of the following Lemma.

Lemma 8. If $z(m_U)>g$, then there exists a unique value $\theta^* \in (0, 1-a)$ such that $m_c<m^*$ if and only if $\theta \geq \theta^*$, where $\theta^*=(1-a)(1-s_m(\theta^*/(a+\theta^*)))$.

Proof. See Appendix A.

For simplicity we restrict our attention to the case $z(m_U)>g$ so as to ensure by Lemma 6, that the complete set of steady states is connected and given by $[m_L, m_c]$. Quah (1996) reports that despite growth disparities across countries, there is no strong evidence of disjoint partitioning in the data. By Proposition 6 and Lemma 7, therefore, if $\theta \geq \theta^*$, a higher value of m corresponds to either a higher long run growth rate γ, if $b=a$, or a higher per capita income, if $b<a$. Otherwise, if $\theta<\theta^*$, the growth rate, if $b=a$, or the per capita income, if $b<a$, is an inverted-U shaped function of m. If $\theta=0$, the growth rate or the per capita income strictly decreases with m. Murphey, et al (1991) report a positive relationship between the relative proportion of engineering graduates in the labor force and the growth rates. By definition, the relative proportion m of the model's managers can be identified with the proportion of high-skilled individuals such as scientists and engineers in the population. A value of $\theta \geq \theta^*>0$ is, therefore, consistent with the growth observation reported by Murphey, et al (1991) under the Lucas (1988) interpretation of the model with $b=a$. The Barro and Lee (1993) database provides information on the fraction *HE* of the adults with high education and, in particular, of adults with some secondary level education for several countries. Figure 3 exhibits a positive relationship between the fraction *HE* and the per capita income across countries. By assuming a positive association between the fractions *HE* and m, we conclude that $\theta \geq \theta^* > 0$ is consistent with the above fact under Solow (1956) interpretation of our model with $b<a$.

The Growth-Inequality Relationship

Assuming a value of $\theta \geq \theta^* > 0$ consistent with the growth observations, we now explore its implication on the growth-inequality relationship across the steady states. We use two separate notions of inequality: (a) the skill premium (e.g, Juhn, et al, 1993) associated with the relative price of skilled labor and (b) the Gini-coefficient. The skill premium depends on the distribution of adults between the two occupations. The initial allocation of human capital among the adults determines that distribution. In the absence of public education and a viable credit market, the children receive unequal human capital from their parents and, therefore, face ex-ante unequal opportunity as grown-up adults. If this unequal opportunity impedes a fraction of the adults to pursue high skill managerial occupations, then an increase in the relative price of skill may be undesirable from the equity aspect. On the other hand a higher rate of investment in human capital that precipitates a faster growth requires a higher skill premium as an incentive. This argument may seem similar to how Kaldor (1956) poses as the growth-equity trade-off and a development dilemma for the politician. Unlike in Kaldor, however this paper argues that, a lower income inequality may foster growth and hence there is possibly no such dilemma. At the steady state, for any given value of $m \in [m_L,$

$m_c]$ define the income inequality ratio $I(m)$ to be equal to the skill premium, $[\phi(m)]^{-1}-1$, such that

(47) $$I_t = \frac{(r_t H_{mt} / m_t) - w_t}{w_t}, \qquad t=0, 1, 2,.....$$

By (25)-(32), (37)-(39) and (47) express this inequality as a function $I(.)$ of m as follows:

(48) $$I(m) = \left(\frac{1}{\phi(m)} - 1 \right) \geq \frac{s_m(m)}{1 - s_m(m)} = I_{min} \geq \frac{\delta\beta}{1-\beta} > 0.$$

Proposition 8. All steady states exhibit income inequality and a higher saving rate requires a higher value of the minimum possible income inequality across the steady states.

We now express the growth rate γ as a function of I. By (38) and (48), get m as a function $m(.)$ of I and then use (30)-(32) to get γ as a function $G(.)$ of I, when $b=a$, such that

(49) $$\gamma = G(I) \equiv i(m(I)) - \delta, \text{ where } i(m) \equiv f^{-1}(\beta((1-a)z(m) + 1 - \delta)) - 1 + \delta.$$

Following the growth observations that rule out the possibility of a disconnected set of steady states assume, $z(m_U)>g$. By Lemma 7, it follows that the set of steady states is $[m_L, m_c]$. Following the conclusions regarding reasonable values of the parameter θ, assume $\theta \geq \theta^* > 0$. By Lemma 8, therefore, it follows that $m^*>m_c$, where m^* is the argmax of $z(m)$ and, therefore, by (34), $z'(m) > 0$. Consequently, applying the Inverse Function Theorem and the Chain Rule of differentiation on (39), (48) and (49) we conclude that $G'(I) < 0$.

Proposition 9. A higher income inequality implies a lower rate of growth.

Intuitively, the absence of a credit market for the acquisition of human capital accounts for the persistence of income inequality at a level higher than its minimum possible limit. The higher level of inequality corresponds to a lower fraction of adults with high skill necessary for conducting innovative activities. A lower intensity of innovative activities implies a lower rate of spillover of knowledge and that, in turn, implies a lower rate of growth. Using the Gini coefficient g as another measure of inequality we can also get a growth-inequality relationship such as described in Proposition 9. For example, the equal distribution of human capital among the managers implies that in the steady state

$$g = 0.5(1 - a - m). \tag{50}$$

By (50) m and g are negatively related. It is interesting to note that even if all adults have identical stock of human capital, by Proposition 4, $m < 1 - a$ and hence, by (50), $g>0$ indicating positive income inequality at the steady state. Also, following steps similar to the derivation of Proposition 9, note that the Gini-coefficient is negatively related to the growth rate. Interestingly, however, the growth rate does not depend on the relative distribution of human capital among the managers, even though the Gini coefficient does. In other words, given m and H, a change of distribution of human capital may change the value of g but will not necessarily change the growth rate. Therefore, the inequality I is more relevant than the Gini coefficient g for empirically accounting the cross-country diversities of income inequality and growth. The former arises endogenously from unequal opportunity due to family connections, as suggested by Benabue (1996) and due to market imperfections, such as suggested by Loury (1981) and affects growth. The latter arises from the random allocation of human capital by history or nature as modeled in Loury (1981) but does not affect growth. Consequently, the policies should aim at equating opportunities rather than equating income of people with different abilities. This paper suggests that the absence of a public education system combined with the absence of private credit market gives rise to unequal opportunity among the future generation when they evaluate alternative forms of occupation. Under this constraint the utility maximizing choice of occupation by the adults is not always consistent with the most efficient growth path available in the economy. Therefore, a more equitable opportunity that frees up the family and market related constraints for a greater proportion of adults implies faster growth with lower income inequality. Define an index *EER* as a measure of equality of educational and associated occupational opportunity among the adults by the ratio of the fraction *HE* of adults with access to some secondary education to the fraction *NQ* of adults with no education. Assume that the fraction m of adults who are the model's managers is a non-decreasing function of *EER*. Table 1A shows that the above index of equality influences the per capita income positively and explains its diversity more significantly than the secondary school enrollment ratio in the growth model of Mankiw, et al. (1992).

Note even if we start from an equal situation with identical agents, the model generates inequality of income endogenously and converges to a steady state $m=m_c$ and the resulting income inequality $I(m_c)>0$ persists for ever. This result is similar to Bandyopadhyay (1993) and Freeman (1996) but has an important qualification: Equal distribution of human capital leads to the steady state with minimum income inequality and maximum rate of growth.

Policy Implications

Consider a country, stuck in a low growth and high inequality steady state. Can the government of that country do anything to promote growth with equity? While, this paper does not formally examine the answer to that question, the properties of the model's steady states allow us to make a few policy conjectures. Suppose that $m<m^*<m_c$. In this

case, to maximize growth the government needs to design a policy to encourage a fraction (m^*-m) of the population to switch from low skill manual work to high skill managerial activities by investing in their children's education. This can be done, for example by conducting a lottery among the fraction $(1-m)$ of the population who are workers. The government can use the lottery to offer e units of education to each of the fraction $(m^*-m)/(1-m)$ of the lucky workers who win the lottery. At the steady state, by (35), $e \geq h^*(m^*, H(e,m))$ such that all future adults of the dynasty of each new manager find it optimal to invest sufficiently in their children's education for ever and $H(e,m) = e(m^*-m)+H$, where, H is the human capital per capita before the lottery. At the new steady state the skill premium of the managers would be lower, since the wage of unskilled labor increases more than proportionately relative to the manager's profit but the consumption of each adult irrespective of her occupational type would be higher than the previous steady state. Thus, the policy would lead to lower income inequality and a higher rate of growth. We can also achieve similar results by introducing financial intermediaries such as discussed in Jovanovic and Greenwood (1990) or public education financed by education bonds. The government could sell those bonds to adults as an alternative to investment in private education. Those bonds could pay returns higher than the market returns on education corresponding to an inefficient steady state. The policy would lead towards a Pareto improvement over the competitive allocation. Glomm and Ravikumar (1992) and Eckstein and Zilcha (1994) provide alternative views regarding the impact of public education on income inequality and growth in a similar growth model.

5. CALIBRATION AND SIMULATION

Parameters of the model are a, b, A_0, β, δ and θ. By (27) and (33), identify the parameter a with the income share of unskilled labor. Either $b=a$, or, by (29), $\gamma=0$ and estimate b using the long run output elasticity $(1+b-a)$ of human capital from (33). Estimate A_0 to match the average per capita income, if $b<a$, or the average growth rate, if $b=a$, across the model's steady states with the corresponding cross-country average per capita income or growth rates. With capital immobility under borrowing constraint such as Barro, et al. (1995), interpret the model's capital broadly to include both human and physical capital to get a measure of output-capital ratio z. An alternative measure of z is the ratio of output to the average years of schooling multiplied by a suitable productivity index. The observed rate s of saving is identified with the weighted average of the rate s_m of saving by the managers who receive a fraction $(1-a)$ of national income and that of workers who do not save. Given the world average rate of saving $s=(1-a)s_m$ and the output-capital ratio z, by (30) and (34), estimate $\delta = sz-\gamma$. Estimate β that satisfies (31) given the pre-determined values of s, z, a, δ and a specific utility function. Finally, estimate the model's idiosyncratic parameter θ using (49) and data on long run skill premium I, long run growth rate and the values of other parameters determined earlier. For example, with a logarithmic utility function, considering the case when $b=a$, rewrite (49) to specify the growth-inequality relationship as follows:

$$(51) \qquad \gamma = A_0\beta(1-a)\left(\frac{1-a}{1+aI}\right)^{\theta}\left(\frac{a(1+I)}{1+aI}\right)^{a} + \beta(1-\delta)-1.$$

Alternatively, by assuming $b<a$, we can specify the relationship between the level of per capita income Y and the inequality index I using (32), (33), (34), (40) and (48) such that

$$(52) \qquad Y = A_0 g^{\frac{1+b-a}{b-a}}\left(\frac{1-a}{1+aI}\right)^{\frac{\theta}{a-b}}\left(\frac{a(1+I)}{1+aI}\right)^{\frac{a}{a-b}},$$

where g is given by (34). The equations (51) and (52) constitute two empirically tractable growth-inequality relationships that this model offers for examining the cross-country observations in a purely neoclassical framework. Note that even if $\theta=0$, still income inequality and growth are related by (51) or (52) in this model. One can estimate θ by using appropriate data on the index I of income inequality. Instead, calibrate a suitable value for θ to replicate the observed growth disparities and then check its implication on the model's growth-inequality relationship. Calibrate the other parameters to match a 35% output elasticity of labor, a 70% output elasticity ($1+b-a$) of human capital, a cross-country average saving rate of 10%, an average output-capital ratio of 0.4325, an average growth rate of 3%. Given the set of parameter values consistent with the above observations, compute the values of m_U and m^* by Proposition 4 and (41), the value of $m_L<m^*$ such that $z(m_L)=g$, using (34) and (41), m_c by Lemma 6, (38) and (39), and θ^* by Lemma 7. If $b<a$, the set $(0, m_U)$ describes the steady state values of the relative proportion m of skilled managers in the adult population, there is no long run growth and the equations (33) and (34) determine per capita income $Y(.)>0$ as a function of $m\in(0, m_U)$. If $b=a$ and $z(m_U)>g$, then the connected set $[m_L, m_c]$ describes the steady state values of m and the equations (30)-(32) determine the long run growth rate $\gamma(.)$ as a function of m. If, in addition, $\theta\geq\theta^*$, then the model's income inequality and growth rate are negatively related. There is no obvious data source for estimating the values of m and then testing how well the model explains the observed growth disparities. The relative proportion HE of adults with high education, calculated from the Barro and Lee (1993) database is, however, a reasonable approximation for the fraction m of adults who are the model's managers. Define HE such that it equals the fraction of adults with a minimum of some secondary school level education. However, examine only a restricted set of countries such that the values of HE, the proxy for m, belong to the set that the model identifies as steady states for the chosen set of parameter values. Table 2 and Figure 8 present a comparison between the model's predicted value of per capita income and the observed data. The model "fits the data" well for $\theta=0.73$ in the following sense: The correlation coefficient between the model's predicted value and the data is about 0.81 and the model generated coefficient of variation of per capita income is about 0.92, which is about the same as the coefficient of variation of 0.93 of per capita income that we observe across the set of countries listed in Table 2. Table 3 and Figure 9 present a comparison between the model's predicted value of growth rates and the observed data. The correlation coefficient between the predicted value and the observation is about 0.53. In addition, Figure 9 demonstrates a negative relationship between the model's inequality I, calculated using the *HE* values in 1990 as proxies for the values of m in conjunction with the equations (38) and (48), and the average annual growth rate of real GDP per capita between 1965 and 1990 calculated from the Penn World Table (Mark 5.6). In particular, the regression of the observed growth rates on the model's inequality shows that a one unit increase in the income inequality I corresponds to a 0.012 percentage point reduction in the growth rate.

6. CONCLUDING REMARKS

The distinctive contribution of this paper in the recently popular literature on distribution and growth such as Galor (1993) and Freeman (1996) is that it follows a unified approach to examine two important puzzles of macroeconomics:(i) observed diversities of per capita income and its growth rates, and (ii) the reported growth-inequality relationship. Moreover, unlike others, it offers an empirically tractable neoclassical model of economic growth where the distribution of human capital plays the central role. It also explains the data reasonably well compared to Mankiw, et al (1992) under the Solow(1956) interpretation of the model. In its endogenous growth specification, the ultimate determinant of growth comes from the specification of preference and technology parameters as in Lucas (1988) but with an important exception: It lays down an algorithm to identify a set of equivalence classes of distributions of human capital necessary for long run growth. Only a restricted class of distributions of human capital can fuel the engine of growth, by ensuring that the investment in human capital exceeds the intergenerational depreciation rate of human capital. The model contributes to our understanding of the so-called neoclassical puzzle of cross-country variation of per capita income and growth rates by generating multiple steady states under alternative paradigms of growth.

It also provides a purely neoclassical explanation of the observed negative relationship between income inequality and the rate of growth. In particular, a more equitable distribution of human capital provides incentives to a higher fraction of population to acquire skill and to conduct innovative activities which leads to a higher relative supply of managers and a higher rate of knowledge spillover and that, in turn, foster both equity and growth. The model offers explicitly two testable restrictions on the growth-inequality data: one within an endogenous growth model and the other one within a model without long run growth. It is a theoretical algorithm for pursuing future empirical research involving distribution of human capital, income inequality and economic growth. The paper also puts a new perspective to the growth-equity debate by offering a model where a human capital led growth is fostered by a more equal distribution of human capital. There is an optimal distribution of human capital that maximizes growth. In the absence of a credit market and public education, the allocation of human capital based on parental investment alone may be sub-optimal and may hinder the economy from reaching its maximum potential growth path. While the introduction of credit market brings down the inequality to its minimum possible value and maximizes growth, it is not viable, since human capital cannot be used as a collateral. The model, however, provides insights for other policies that too may generate growth with equity. Lotteries among the workers and government issued education bonds for financing schooling of the children of the poorly educated adults are examples of policies that may achieve a country's dual objective of fostering growth with greater equity. The analysis of such policies is, however, beyond the scope of this paper but should be an important topic for future research.

APPENDIX A

<u>Proof of Lemma 1</u>: By (10) and (14), $f\left(\frac{y_{t+1}(h)-s_{t+1}(h)}{y_t(h)-s_t(h)}\right)=\beta(r_{t+1}+1-\delta)$. Denote $f^{-1}(\beta(r_{t+1}+1-\delta)$ by R_{t+1} and the ratio $\frac{s_t(h)}{h}$ by $i_t^*(h)$, where, h denotes an adult's human capital stock at $t\geq 0$. By (7)-(8), $\{i_t^*(h)\}_{t\geq 0}$ satisfies the following difference equation:

$$(53) \qquad i_t^*(h)-\frac{i_{t+1}^*(h)}{R_{t+1}+r_{t+1}}=\frac{R_{t+1}}{R_{t+1}+r_{t+1}}\frac{y_t(h)}{h}-\frac{(1-\delta)r_{t+1}}{R_{t+1}+r_{t+1}}, \qquad t=0,\,1,\,2,\ldots$$

Or, equivalently, using forward expansion with inverse lag operator L^{-1} we can write

$$(54) \qquad i_t^*(h)\left[1-\frac{1}{R_{t+1}+r_{t+1}}L^{-1}\right]=\frac{R_{t+1}}{R_{t+1}+r_{t+1}}\frac{y_t(h)}{h}-\frac{(1-\delta)r_{t+1}}{R_{t+1}+r_{t+1}}, \qquad t=0,\,1,\,2,\ldots$$

By assumption, we are considering the set of adults with human capital $h>h_t^*$ such that $i_t(h)>0$. Clearly, the set is non-empty if and only if $R_{t+1}r_t>(1-\delta)r_{t+1}$. Also, $R_{t+1}r_t>(1-\delta)r_{t+1}$ implies $w_tR_{t+1}>(1-\delta)r_{t+1}x_t$, since $w_t=r_tx_t$. By (54), it follows, therefore, (i) for all $h<x_t$ such that $y_t(h)=w_t$ the investment rate $i_t(h)$ decreases with h; (ii) for all $h\geq x_t$ such that $y_t(h)=r_th$, $i_t(h)$ is independent of h. Q.E.D.

Proof of Lemma 2: Omit subscript t for notational simplicity. Denote by $p(x)$ the fraction of adults that has exactly x units of human capital. Denote by $\varphi(x)$ the fraction of adults with x units of human capital that supplies unskilled labor. By (6), (15) express the fraction $m(.)$ of adults that conducts managerial work and by (6) and (19) their total human capital $H_m(.)$ as functions of x such that

$$(55)\qquad H_m(x) \equiv \left(\int_{h \geq x} h d\Psi(h) - \varphi(x) x p(x) \right), \qquad \varphi(x) \in [0, 1]$$

$$(56)\qquad m(x) \equiv 1 - \Psi(x) + p(x)(1 - \varphi(x)), \qquad \varphi(x) \in [0, 1]$$

By (22) and (55)-(56) we have the fixed point problem in x as follows:

$$(57)\qquad x = \frac{aH_m(x)}{(1\text{-}a)(1\text{-}m(x))} \ ,$$

Let us define $\Gamma: R_+ \ X\,[0, 1] \longrightarrow R$ as follows.

$$(58)\qquad \Gamma(x, \varphi(x)) = \frac{a\left(\int_{h \geq x} h d\Psi - \varphi(x) x p(x) \right)}{(1-a)(\Psi(x) - p(x)(1 - \varphi(x)))} - x, \ \varphi(x) \in [0, 1].$$

Think of $\Gamma(x, \varphi(x))$ as a correspondence of x and seek for a pair of numbers $x^*>0$ and $\varphi(x^*) \in [0, 1]$ such that $\Gamma(x^*, \varphi(x^*))=0$. Note that Γ, as defined above, is a convex valued, (since $\varphi(x) \in [0, 1]$), continuous, (since Ψ is right continuous), and strictly monotone decreasing correspondence of x. By (19), there is $hmax < \infty$ such that $\Psi(hmax) = 1$. By (58), it follows, therefore, $\Gamma(hmax, 1)<0$ and $\Gamma(0, 1)>0$. It follows, therefore, that there exists $x^* \in [0, hmanx]$ such that $\Gamma(x^*, 0) \geq 0$ and $\Gamma(x^*, 1) \leq 0$. Find a real number $\varphi(x^*) \in [0, 1]$ such that $\Gamma(x^*, \varphi(x^*))=0$. The solution $x^*>0$ is unique, since Γ is strictly monotone in x. Q.E.D.

Proof of Lemma 3: At any date $t \geq 0$, given a distribution Ψ_t, determine x_t uniquely by Lemma 2, determine w_t, r_t by (20)-(21) and $y_t(h)$ for all $h \geq 0$ by (6) and (7). If at some date t, there is no $h>0$ such that (13) holds, then all adults choose $s_t(h)>0$. By Lemma 1, (6) and (14), it follows, therefore, that there must be a date $t+k$, such that $n_{t+k}(.)=0$ for all adults. By (17), however, that cannot be an equilibrium. By (6)-(7) income $y_t(h)$ is non-decreasing in h and by assumption $u(.)$ is strictly concave. By (9)-(10), it follows, therefore, that if (13) holds for some $h_1>0$, it must hold with strict inequality for all $h'<h_1$ and if (13) fails for some h_2, then for all $h' \geq h_2$ (14) holds. Consequently, the threshold level of human capital stock $h_t^* = max\{h>0: s_t(h)=0\} > 0$. Q.E.D.

Proof of Lemma 4: By (31), $i(m) \geq \delta \Leftrightarrow f^{-1}(\beta(r(m)+1-\delta)) \geq 1$. By (10), $f(1)=1$, strict concavity of $u(.)$ implies $f'>0$. By the Inverse Function Theorem, therefore, $f^{-1}(1)=1$ and $(f^{-1})'>0$. By (32) it follows, $r(m) \geq \beta^{-1} - 1 + \delta$ and, therefore, by (34), $z(m) \geq g$. Q.E.D.

Proof of Lemma 5: By (34) $m_u \geq m_L$ implies that there is $m_u \geq m \geq m_L$ such that $z(m) \geq z(m_L)=g$. By (41), $z(m^*) \geq z(m) \geq g$. Also, by (41), if $m^*<m_U$, $m^*=\hat{m}$ and $z(\hat{m}) \geq g$; otherwise, if $m^*>m_U$, $m_U=\hat{m}$ but by (34), $z'(m)>0$, and therefore, $z(\hat{m})=z(m_U) \geq z(m_L)=g$. On the other hand, $z(\hat{m}) \geq g = z(m_L)$ implies by (34) that $m_U \geq \hat{m} \geq m_U$. Q.E.D.

Proof of Lemma 6: If the initial distribution $\Psi \in D(m, H)$ and m satisfies (39), then in a balanced growth state, by (13)-(14), all adults with initial human capital $h>h^*(m, H)$ invest in schooling at all dates while no adults with initial human capital $h=0<h^*(m, H)$ do. Consequently, (25) holds. Also, by construction, $m_L \leq m \leq m_U$. By (41) and Lemma 5, it follows that $z(m) \geq g$ and, therefore, by Lemma 4, if $b=a$, then $i(m) \geq \delta$ and, therefore, by (8)

and (42), $\gamma(m)\geq 0$. Consequently, (26) holds. If $b\neq a$, then by Proposition 2 $\gamma=0$. By (29) and (34), it follows, that H must satisfy (40). Also, if H satisfies (40) when $b\neq a$, (29) holds and, therefore, (26) holds. Consequently, the equilibrium with $\Psi\in D(m, H)$ as the initial distribution is a distinct steady state. Q.E.D.

<u>Proof of Lemma 7</u>: By (34), $z(m_U)>g$ implies that the set $[m_U, m_L]$ is non-empty and by (10), (31), (32) and (34) $s_m'(m)=\beta(1-a)z'(m)f^{-1'}(.)\geq 0$ if and only if $m_L< m\leq m^*$. Also, by (37), $(1-s_m(m))\geq(1-s_{min})$. By (38), $\phi'(m)>0$ and that implies $\phi(m_U)>\phi(m_L)$. By Proposition 4, it follows that $\phi(m_L)<(1-s_m(m_L))$. At the same time, since $z(m_U)>g$, by Proposition 4 and (34), $\phi(m_U)>(1-s_m(m_U))$. By continuity of $s_m(.)$ and $\phi(.)$, it follows, therefore, that there exists a fraction $m_c\in(m_L, m_U)$ such that $\phi(m_c)=(1-s_m(m_c))$; and by concavity of $z(.)$ and $\phi(.)$ along with the condition $z(m_U)>g$ it implies that the fraction $m_c\in(m_L, m_U)$ is unique. Given $z(m_U)>g$, it follows that $m_U>m\geq m_L$ implies $z(m)\geq g$. Also, $m\leq m_c$ implies $(1-s_m(m))\geq\phi(m)$. If, however, $m>m_c$, by (37) and (38), $(1-s_m(m))<\phi(m)$ and if $m<m_L$, by (34), $z(m)<g$. Q.E.D.

<u>Proof of Lemma 8:</u> By Lemma 6, $z(m_U)>g$ implies that $[m_L, m_c]$ represents the set of steady states. By (39), $m_c\leq m^*$ if and only if $\phi(m^*)\geq(1-s_m(m^*))$, or, by (38) and (41), if and only if $\theta\geq c(\theta)\equiv(1-a)(1-s_m(\theta/(a+\theta)))$. By (37) $0<s_m(.)<1$ and, therefore, $0<c(\theta)<1$. Consequently, $\theta=0$ implies $\theta<c(\theta)$ while $\theta\geq 1-a$ implies $\theta>c(\theta)$. By (10), (31), (32), (34) and (37), for all $\theta\geq 0$, $c'(\theta)>0$. It follows, therefore, that there is $\theta^*\in(0, 1-a)$ such that if and only if $\theta\geq\theta^*$, $\theta\geq c(\theta)$; or equivalently, $\phi(m^*)\geq(1-s_m(m^*))$; or equivalently, $m_c\leq m^*$. Q.E.D.

<u>Proof of Proposition 1</u>: At each date $t\geq 0$, given the aggregate state variable Ψ_t, determine

x_t uniquely by Lemma 2. Consequently, the budget set of every adult defined by (11) is convex and compact and, therefore, the optimization problem (12) is well defined. By Lemma 3 determine h_t* uniquely given any conjectured distribution Ψ^c_{t+1}. By (6)-(8) and (14) define an operator T that maps Ψ^c_{t+1} uniquely to Ψ_{t+1}. By Schauder Fixed-Point Theorem[2] establish the existence of a unique distribution Ψ_{t+1} such that $T(\Psi_{t+1})=\Psi_{t+1}$. Therefore, the equilibrium sequence $\{\Psi_{t+1}\}$ is uniquely determined given $\Psi_0 \neq 1$. Q.E.D.

Proof of Proposition 2: If $b=a$, by (29) r_t is a time invariant constant r, which is a function of m. We claim that even if $b \neq a$, in the steady state r_t is a constant but its value does not depend on m. In the steady state $\gamma_t=\gamma$ implies that $i_t=i$ and $h_{t+1}=(1+\gamma)h_t$. Therefore, at a date t, a manager with h_t units of human capital earns $r_t h_t$ units of output and her consumption is $(r_t - i)h_t$ units. By (10) the First Order Condition of the optimization problem (12) implies

(59) $$f\left(\frac{(r_{t+1}-i)(1+\gamma)}{r_t - i}\right) = \beta(r_{t+1}+1-\delta), \qquad t=0,\ 1,\ 2,\ldots$$

By (29), $r_{t+1}/r_t=(1+\gamma)^{b-a}$. It follows, therefore, from I

(60) $$f\left(\frac{((1+\gamma)^{b-a}-i/r_t)(1+\gamma)}{1-i/r_t}\right) = \beta(r_{t+1}+1-\delta), \quad t=0,1,\ 2\ldots$$

It follows from (29) that unless , (i) $\gamma=0$, $b>a$ implies that the RHS approaches infinity while the LHS approaches a finite limit $f((1+\gamma)^{1+b-a})$; (ii) unless $\gamma=0$, $b<a$ implies that the LHS

---

[2] "Let X be a bounded subset of R^1, and let C(X) be the space of bounded continuous function of X, with sup norm. Let F be a non-empty, closed, bounded and convex subset of C(X). If the mapping T: F→F is continuous and the family T(F) is equicontinuous, then T has a fixed point in F." Ref: *Recursive Methods in Economics Dynamics* (P. 520), Stokey, Lucas, Prescott.

approaches $f((1+\gamma))>1$ (since by (10), $f(1)=1$ and $f'>0$) but the RHS approaches $\beta(1-\delta)<1$, and, hence, we get a contradiction. On the other hand, if $b\neq a$, then $\gamma=0$, or, equivalently, $i=\delta$ and $r_t=r=\beta^{-1}-1+\delta$ constitute a solution consistent with the steady state conditions. Q.E.D.

REFERENCES

Bandyopadhyay, D., 1993, Distribution of Human Capital and Economic Growth, Ph. D. Thesis (University of Minnesota, Minneapolis, Minnesota).

Barro, R. J. and J. Lee, 1993, International Comparisons of Educational Attainment, Journal of Monetary Economics 32, 363-94.

Barro, R. J., 1974, Are Government Bonds Net Wealth?, Journal of Political Economy 82, 1095–1117.

Barro, R. J., Mankiw, N. G. and X. Sala-I-Martin, 1995, Capital Mobility in Neoclassical Models of Growth, American Economic Review 85, 103-115.

Benabou, R., 1996, Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance, American Economic Review 86, 585-609.

Chang, R., 1994, Income Inequality and Economic Growth: Evidence and Recent Theories, Federal Reserve Bank of Atlanta Economic Review 79, 1-10.

Clarke, George R. G., 1995, More evidence on income distribution and growth, Journal of Development Economics 47, 403-427.

Cline, W.R., 1974, Distribution and Development: A Survey of Literature, Journal of Development Economics, 1, 359-400.

Eckstein, Z. and I. Zilcha, 1994, The effect of compulsory schooling on growth, income distribution and welfare, Journal of Public Economics 54, 339-59.

Freeman, S., 1996, Equilibrium Inequality among Identical Agents, Journal of Political Economy 104, 1047-1063.

Galor, O. and J. Zeira, 1993, Income Distribution and Macroeconomics, Review of Economic Studies 60, 35-52.

Glomm, G. and B. Ravikumar, 1992, Public versus private investment in human capital: Endogenous growth and Income Inequality, Journal of Political Economy 100, 818-34.

Greenwood, Jeremy and B. Jovanovic, 1990, Financial Development, Growth, and the Distribution of Income, Journal of Political Economy 98, 1076-1107.

Juhn, C., Murphy, K. M. and B. Pierce, 1991, Wage Inequality and the Rise in Returns to Skill, Journal of Political Economy 101, 410-442.

Kaldor, N., 1956, Alternative Theories of Distribution, Review of Economic Studies 23, 83-100.

Lucas, R. E., 1988, On the Mechanics of Economic Development, Journal of Monetary Economics 22, 3-42.

Loury, G., 1981, "Intergenerational Transfers and the Distribution of Earnings," Econometrica 49, 843-867.

Mankiw, N. G., Romer, D. and Weil, D. N., 1992, "A Contribution to the Empirics of Economic Growth," Quarterly Journal of Economics, 107, 407-37.

Murphy, K. M., Shleifer, A. and Vishny, R. W., 1991, The Allocation of Talent: Implications for Growth, Quarterly Journal of Economics 106, 503-530.

Perotti, R., 1996, Growth, Income Distribution, and Democracy: What the Data Say, Journal of Economic Growth (June), 149-187.

Persson, T. and G. Tabellini, 1994, Is Inequality Harmful for Growth?, American Economic Review 84, 600-621.

Quah, D., 1996, Convergence Empirics Across Economies with (Some) Capital Mobility," Journal of Economic Growth (March), 95-123.

Romer, P., 1986, Increasing Returns and Long-Run Growth, Journal of Political Economy 94, 1002-1037.

Romer, P., 1990, Endogenous Technological Change, Journal of Political Economy 98, S71-S102.

Solow, R. M., 1956, A Contribution to the Theory of Economic Growth, Quarterly Journal of Economics 70, 65-94.

Solow, Robert M., 1994, Perspectives On Growth Theory, Journal of Economic Perspectives 8, 45-54.

Stokey, L. N. and Lucas, R. E., with E. C. Prescott, 1989, Recursive Methods in Economic Dynamics (Harvard University Press, Cambridge, Massachusetts).

Summers, R. and Heston, A., 1989, The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988, Quarterly Journal of Economics CVI, 328-368.

APPENDIX B

Figure 1a: HCD in 1960 for the Lowest Income Quintile Countries in 1990

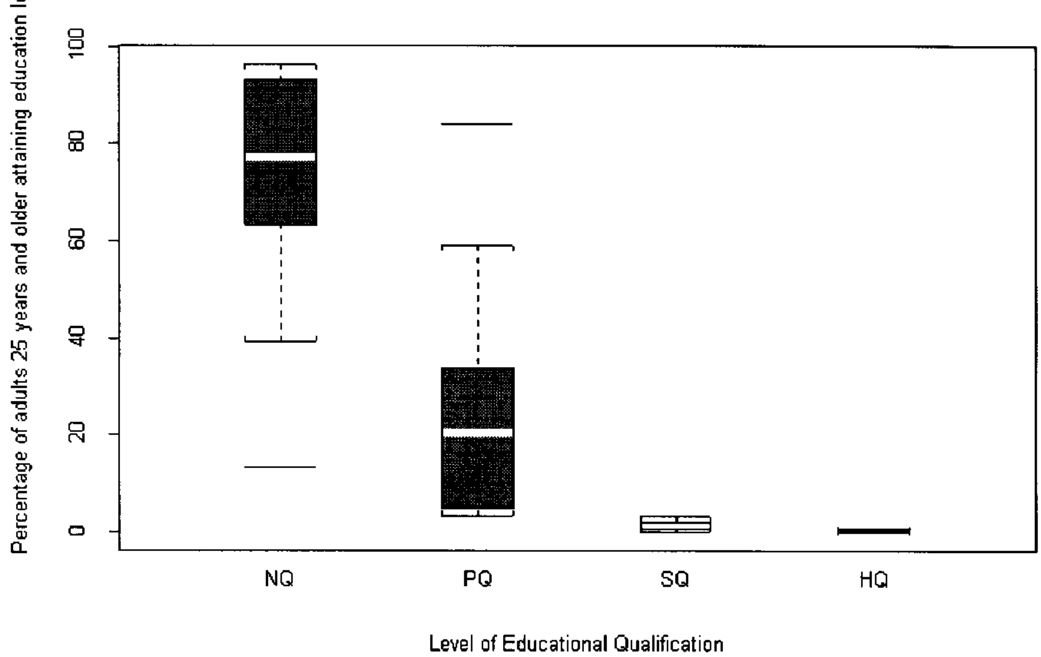
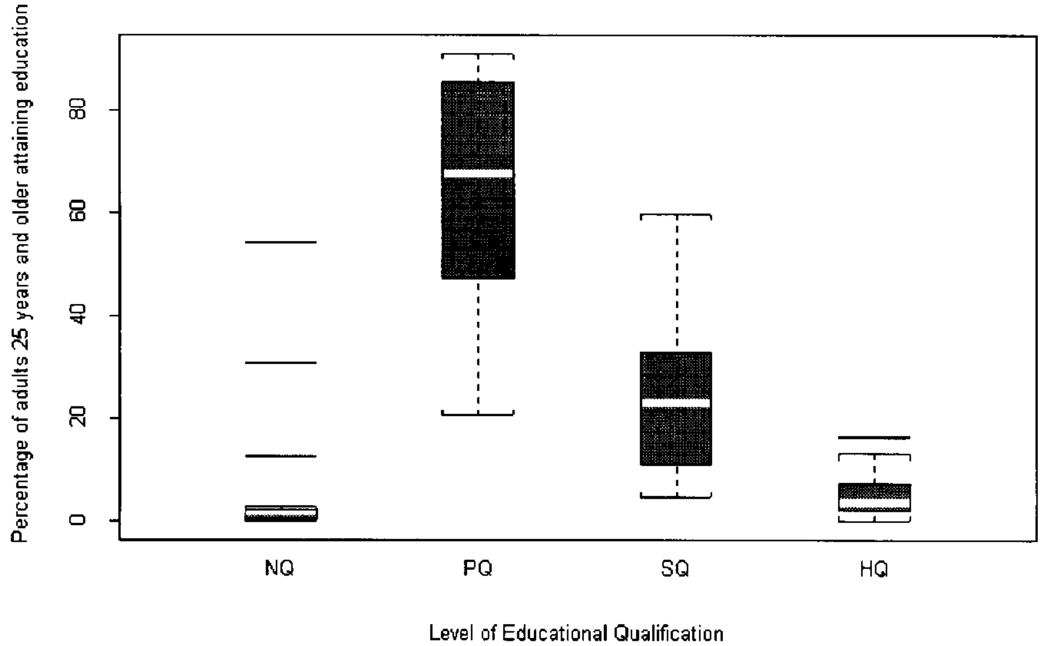


Figure 1b: HCD in 1960 for the Highest Income Quintile Countries in 1990



Data source: Barro & Lee (1993)

Population: Individuals of age 25 years and above (adults)	
NQ	Percentage of adults with no formal educational qualifications
PQ	Percentage of adults with some primary level educational qualifications
SQ	Percentage of adults with some secondary level educational qualifications
HQ	Percentage of adults higher than secondar level educational qualifications

HCD = Human Capital Distribution: Box Plot; RGDP=Real GDP per capita
The range of income (RGDP) per capita in '85 US$ from Penn Table (Mark 5.6)
The Lowest Income Quintile Countries: (518,1226)
The Highest Income Quintile Countries: (11513,18055)

Figure 2a: HCD in 1960 for the Bottom Quintile of Long Run Growth Rate

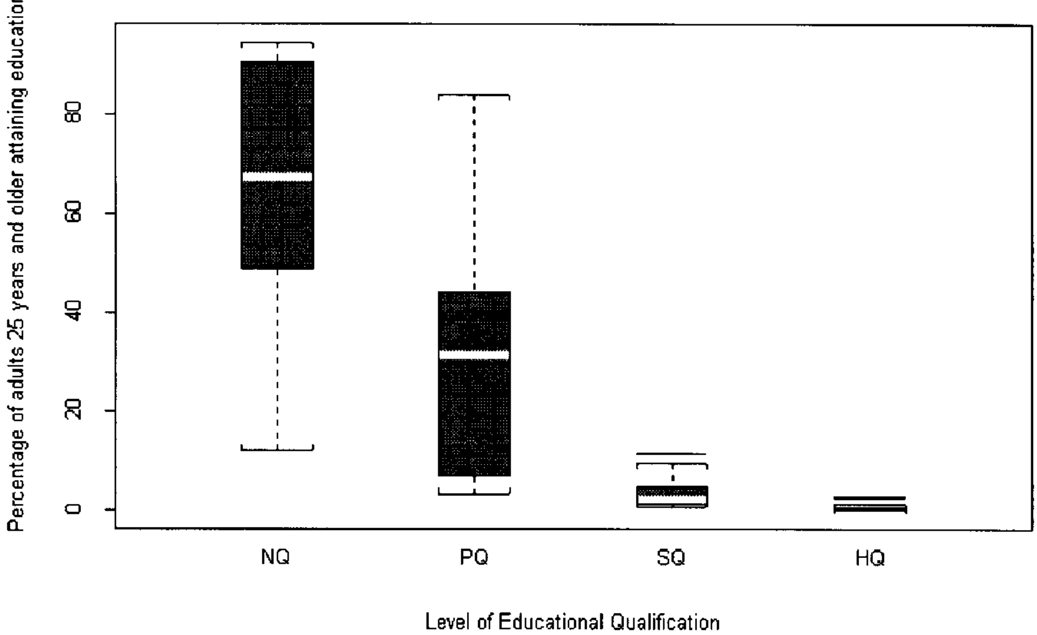
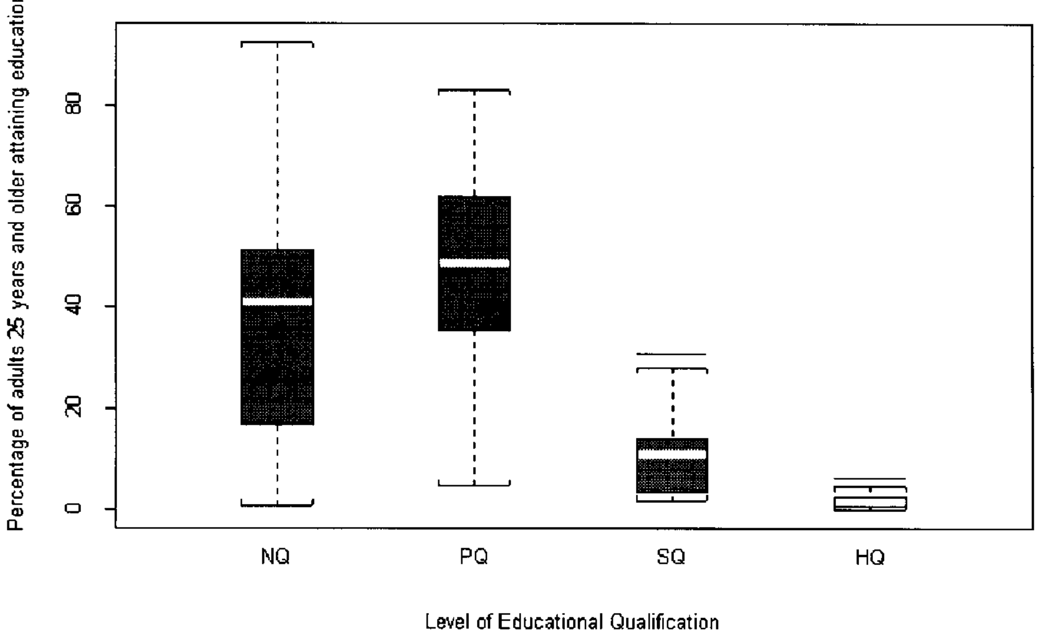


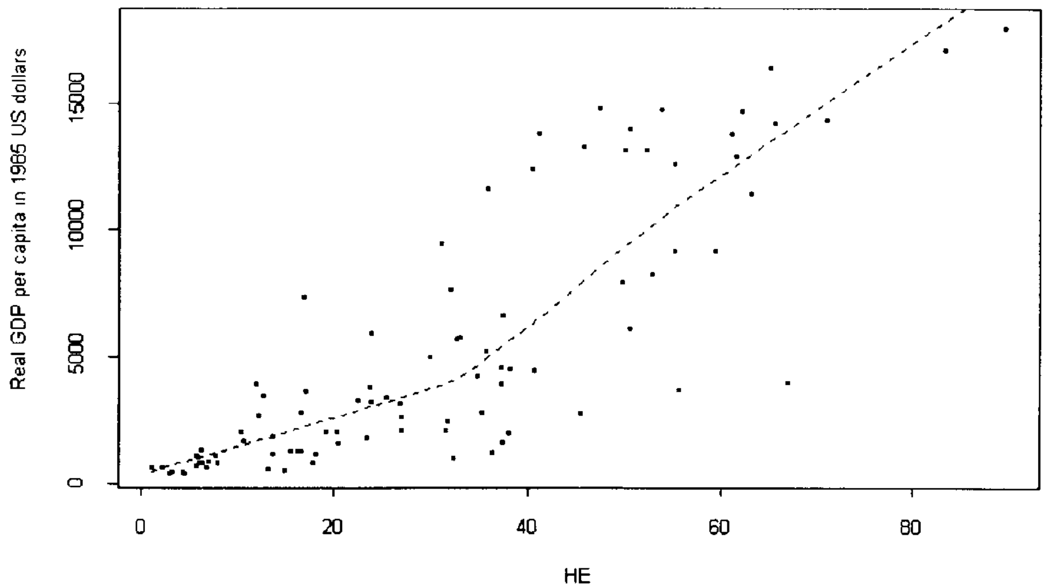
Figure 2b: HCD in 1960 for the Top Quintile of Long Run Growth Rate



Data Source: Barro & Lee (1993)

HCD = Human Capital Distribution: Box-Plot
Long Run Growth Rate = Average Annual Growth Rate of RGDP per capita between 1965-90
The range of long run growth rates:
The Bottom Quintile Countries: (-2.22%,0.39%)
The Top Quintile Countries: (3.08%,7.36%)

Figure 3 : RGDP per capita vs Percentage of Adults with Higher Education



HE	Percentage of adults 25 years and older with a minimum of some secondary level education: *HE=HQ+SQ* Data source: Derived from Barro & Lee (1993)
RGDPC	Real GDP per capita in 1985 US dollars Data source: Penn World Tables (Mark 5.6)

Figure 4

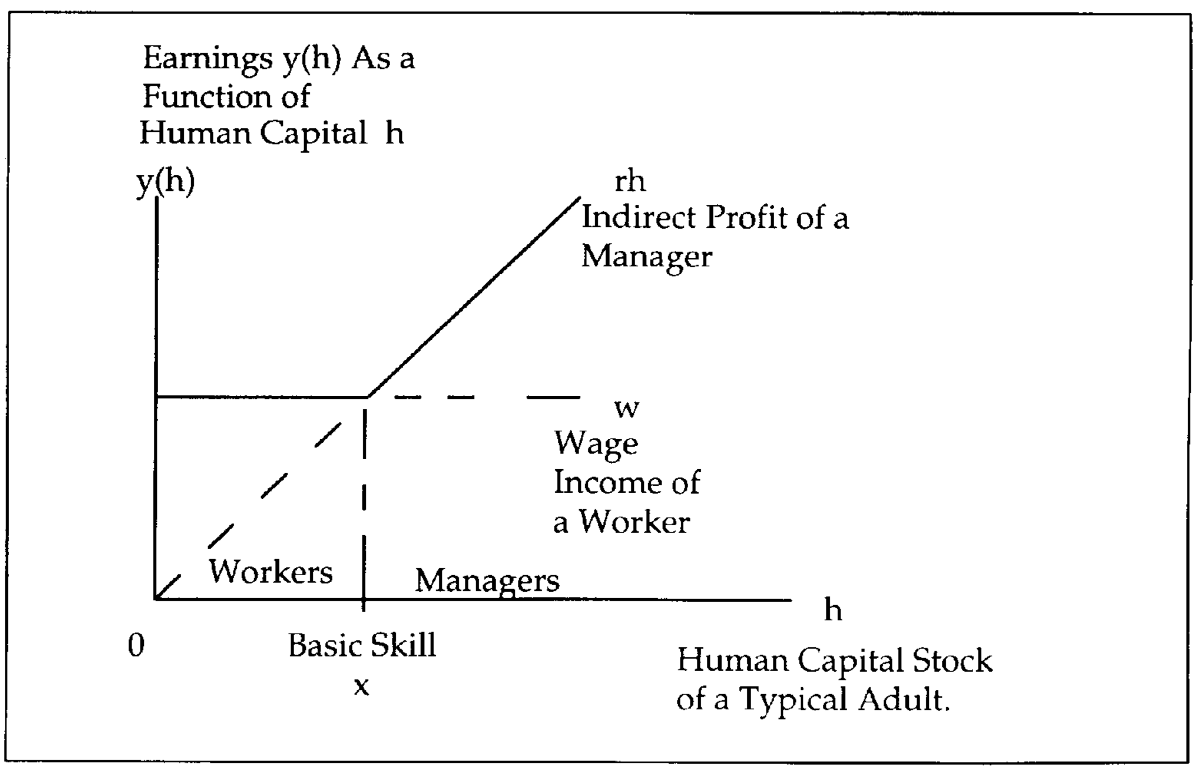


Figure 5

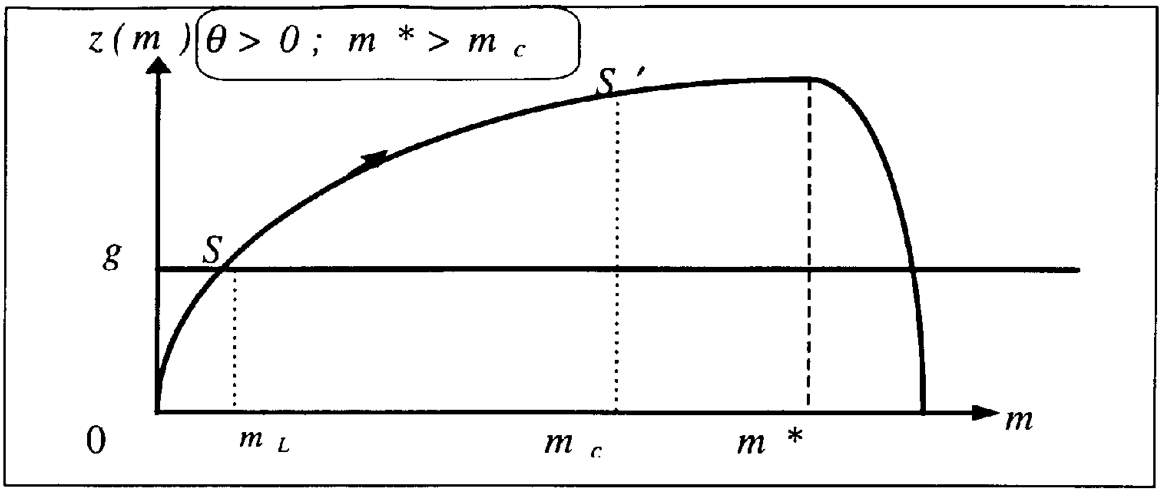


Figure 6

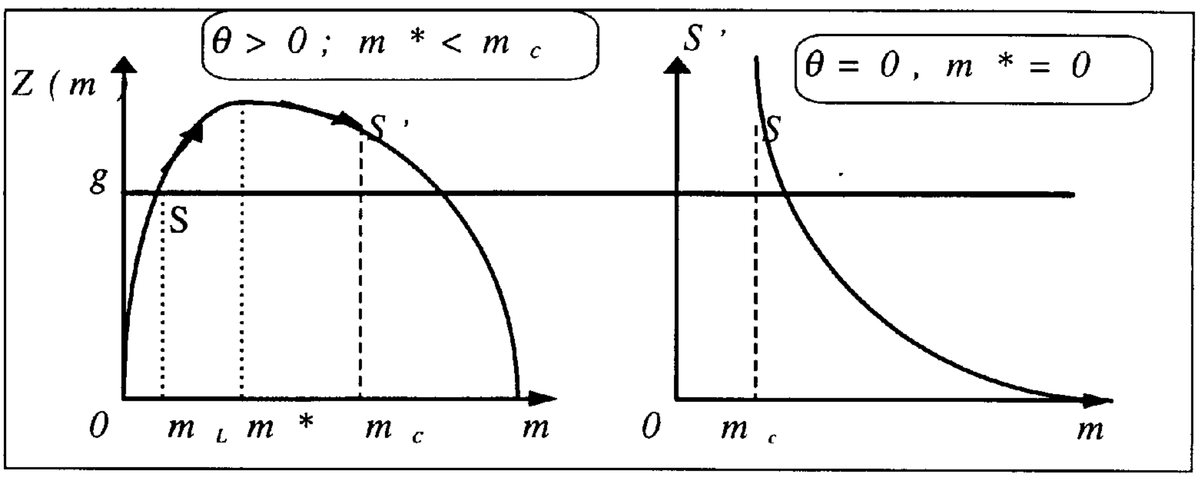


Figure 7

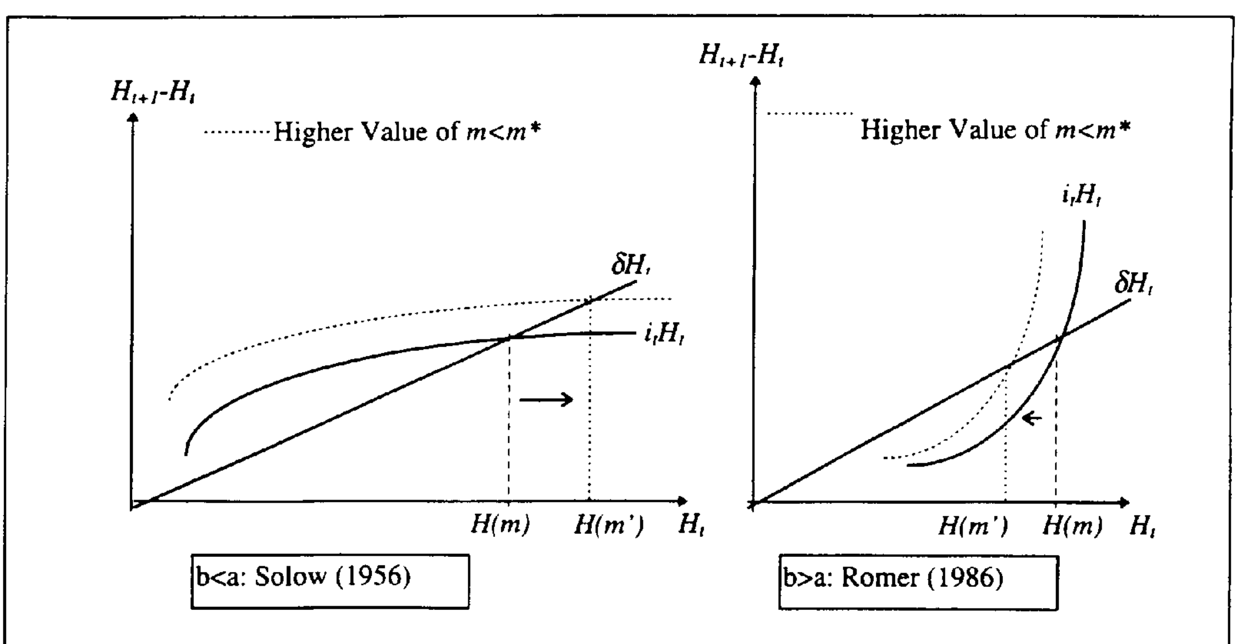
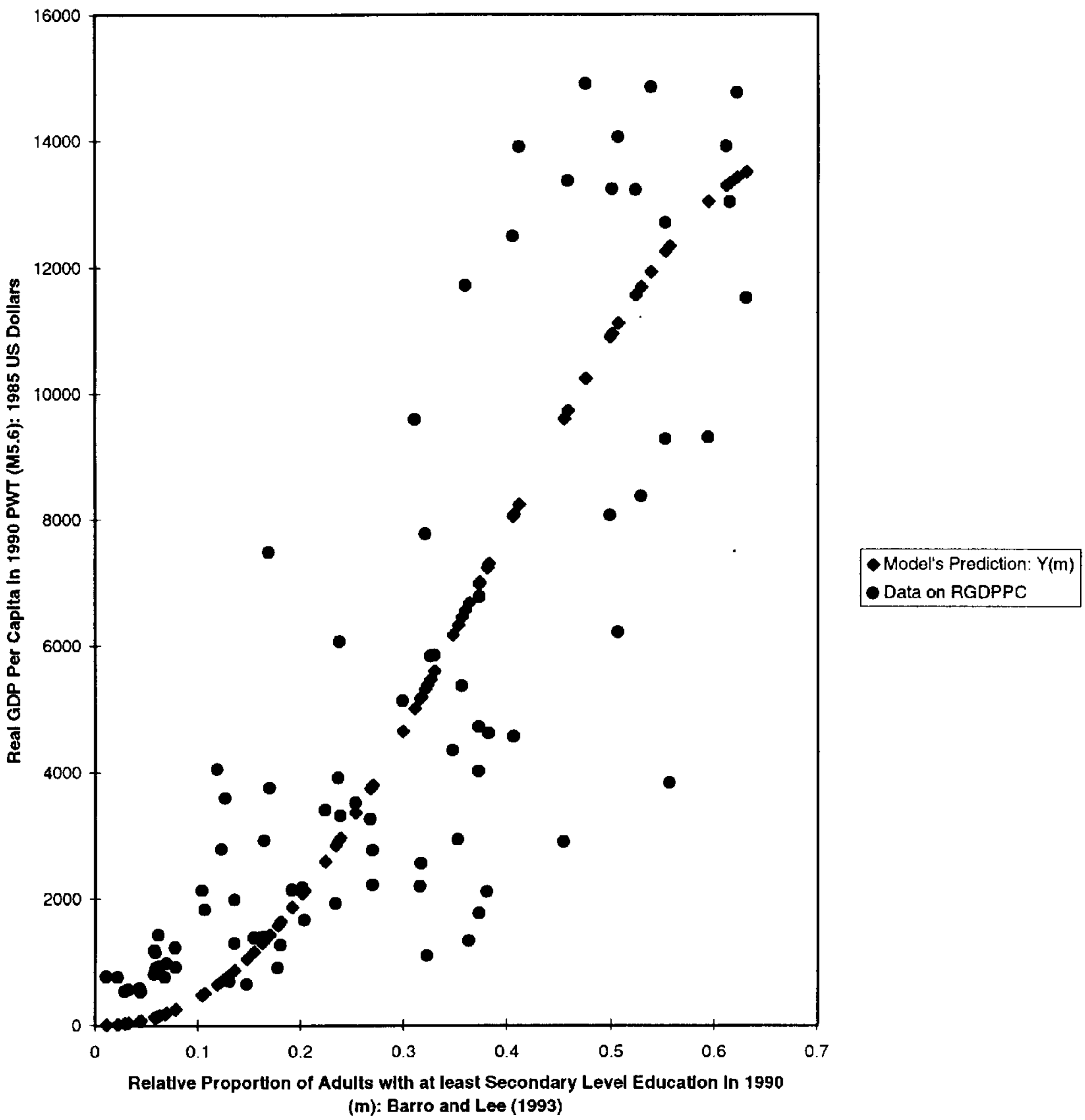


Figure 8

Cross-Country Disparities of Per Capita Income in 1990 and the Model's Prediction

Parameters: a=0.35, b=0.05, beta=0.81, delta=0.01, Theta=0.73, A_0=17.36;

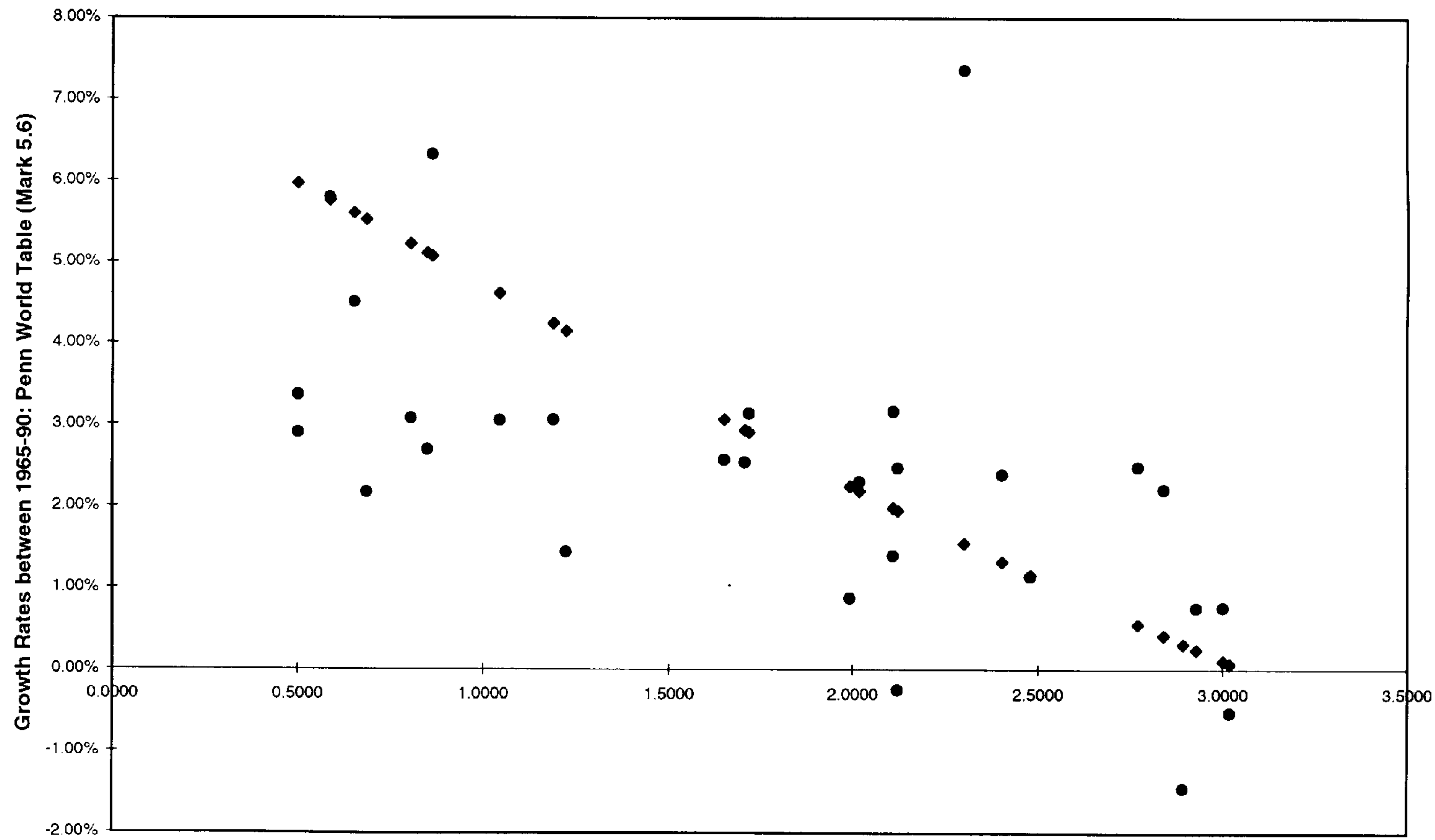


Coefficient of Variation of RGDPPC: Model=0.92, Data=0.93;
Correlation Coefficient between Y(m) and RGDPPC90 =0.81

Figure 9

Parameters: a=b=0.35, β=0.81, δ=0.01, θ=0.73, A_0=1.

Growth-Inequality Relationship: Model's Prediction and Data



Income Inequality (I) or Skill Premium generated by the Model using Barro and Lee (1993) Database

$$\gamma = 0.0459 - 0.0120\, I$$

(0.0070) (0.0036)

S.E. in Parenthesis; R-Square (Adjusted)=0.2634; N=29

Table 1A

Dependent Variable: LN Real GDP Per Adult in 1985 (Mankiw, et al, 1992)

Models / Independent Variable	Estimated Coefficients (Standard Error) Number of Observations:87*			
	Solow (1956)	Mankiw-Romer-Weil (1992)	Model 1	Model 2
LN (*I/Y*)	1.50 (0.18)	0.68 (0.19)	0.53 (0.15)	****
LN (*n*+0.05)	–0.74 (0.60)	–0.80 (0.48)	****	****
LN (SCHOOL)	****	0.75 (0.11)	****	****
LN (*HQ*)	****	****	****	0.46 (0.06)
LN (NQ)	****	****	****	–0.27 (0.05)
LN EER	****	****	0.31 (0.03)	****
Intercept	2.09 (1.48)	3.04 (1.19)	9.12 (1.21)	8.44 (0.22)
Standard Error	0.74	0.59	0.50	0.52
F-Stat	44	63	101	133
R-Squared (Adjusted)	0.50	0.68	0.78	0.75

*The regression exercise includes all countries for which both Mankiw, et al (1992) and Barro and Lee (1993) provide relevant data. *I/Y*=investment (=saving) rate, *n*=population growth rate and SCHOOL=percentage of working age population in secondary school averaged between 1960-85: Data Source, Mankiw, et al (1992). *NQ*=percentage with no education, SQ=percentage with some secondary level education and *HQ*=percentage with higher than secondary level education of the adult people of age 25 and above, Data Source, Barro and Lee (1993); EER≡(HQ+SQ)/NQ.

Table 1B

VARIABLES:	**Dependent Variables: Average Investment Rate (*I/Y*) and Population Growth Rate (*n*) '60-'85 (Mankiw, et al, 1992)** Estimated Coefficients (Standard Error) Number of Observations:87*			
Dependent / Independent	***I/Y*** (1)	***I/Y*** (2)	***n+0.05*** (3)	***n+0.05*** (4)
LN (*HQ*)	0.23 (0.03)	0.15 (0.04)	****	0.04 (0.01)
LN (NQ)	****	–0.11 (0.04)	0.06 (0.01)	0.09 (0.01)
Intercept	2.52 (0.05)	2.95 (0.15)	–2.82 (0.03)	–2.95 (0.04)
Standard Error	0.37	0.35	0.11	0.11
F-Stat	57	36	55	38
R-Squared (Adjusted)	0.39	0.45	0.39	0.46

Table 1C

Correlation Matrix	Ln (SCHOOL)	Ln(*n+0.05*)	Ln(*I/Y*)	Ln(EER)	Ln(*NQ*)	Ln(*HQ*)
Ln (SCHOOL)	1.00					
Ln(*n+0.05*)	–0.22	1.00				
Ln(*I/Y*)	0.65	–0.37	1.00			
Ln(EER)	0.83	–0.47	0.68	1.00		
Ln(*NQ*)	–0.66	0.63	–0.61	–0.92	1.00	
Ln(*HQ*)	0.88	–0.22	0.63	0.91	–0.68	1.00
Ln(GDP/Adult)	0.78	–0.35	0.71	0.87	–0.76	0.83

Table 2

Parameters and Critical Variables:									
a	***b***	β	δ	θ	A_0	***r***	***H(m)/Y(m)***	m_U	***m****
0.35	0.05	0.81	0.01	0.73	17.36	0.2446	2.6577	0.6404	0.6759

COUNTRY	*m-90: Barro & Lee, 93*	*Y(m): Model*	*RGDP PC 90: Penn Table*	COUNTRY	*m-90: Barro & Lee, 93*	*Y(m): Model*	*RGDP PC 90 (Penn Table)*	COUNTRY	*m-90: Barro & Lee, 93*	*Y(m): Model*	*RGDP PC 90 (Penn Table)*
Mozambique	0.0110	2	760	Portugal	0.169	1410	7478	China	0.364	6678	1324
Rwanda	0.0220	12	756	Turkey	0.17	1429	3741	Fiji	0.373	6970	4007
Mali	0.0290	23	531	Ghana	0.178	1580	902	Argentina	0.373	6970	4706
Uganda	0.0330	32	554	India	0.181	1638	1264	Philippines	0.374	7002	1763
C. African R.	0.0440	63	579	Paraguay	0.192	1862	2128	Greece	0.374	7002	6768
Malawi	0.0450	66	519	Domin Rep.	0.202	2076	2166	Sri Lanka	0.381	7230	2096
Zimbabwe	0.0580	121	1182	Bolivia	0.204	2120	1658	Uruguay	0.383	7295	4602
Gambia	0.0580	121	799	Iran, I.R. of	0.224	2584	3392	Italy	0.406	8043	12488
Senegal	0.0590	126	1145	Egypt	0.234	2831	1912	Yugoslavia	0.407	8075	4548
Sierra Leone	0.0600	131	901	Syria	0.237	2907	3897	France	0.412	8237	13904
P.N.G.	0.0620	142	1425	Venezuela	0.238	2932	6055	Panama	0.455	9599	2888
Benin	0.0630	147	920	Colombia	0.239	2958	3300	Iceland	0.459	9721	13362
Sudan	0.0680	176	757	Costa Rica	0.254	3351	3499	Norway	0.476	10233	14902
Lesotho	0.0700	188	972	S. Africa	0.268	3735	3248	Taiwan	0.499	10892	8063
Cameroon	0.0780	243	1226	Ecuador	0.27	3791	2755	Belgium	0.501	10947	13232
Kenya	0.0790	250	911	Congo	0.27	3791	2211	Finland	0.507	11111	14059
Guatemala	0.1040	472	2127	Malaysia	0.299	4635	5124	Bulgaria	0.507	11111	6203
El Salvador	0.1070	504	1824	Spain	0.311	4999	9583	U.K.	0.524	11557	13217
Brazil	0.1190	643	4042	Peru	0.316	5153	2188	Cyprus	0.529	11682	8368
Algeria	0.1230	693	2777	Jamaica	0.317	5184	2545	Hong Kong	0.539	11925	14849
Thailand	0.1270	745	3580	Trind & To	0.321	5308	7764	Ireland	0.553	12244	9274
Zambia	0.1310	799	689	Guyana	0.323	5370	1094	Austria	0.553	12244	12695
Nicaragua	0.1360	870	1294	Mexico	0.326	5464	5827	Poland	0.557	12330	3820
Indonesia	0.1360	870	1974	Mauritius	0.33	5590	5838	Israel	0.595	13040	9298
Togo	0.1480	1051	641	Chile	0.348	6162	4338	Denmark	0.612	13284	13909
Honduras	0.1550	1165	1377	Jordan	0.353	6323	2919	Netherlands	0.616	13334	13029
Bangladesh	0.1630	1302	1390	Hungary	0.357	6452	5357	Sweden	0.623	13415	14762
Tunisia	0.1650	1338	2910	Singapore	0.36	6548	11710	New Zealand	0.632	13505	11513
Pakistan	0.1650	1338	1394	• Coefficient of Variation of RGDPPC Model: 0.9176 Data: 0.9304				• Correlation Coefficient between the Predicted Value and the Data=0.8107			

Table 3

Parameters and Critical Variables:										
a	b	β	δ	θ	A_0	m_L	m_U	m^*	m_c	θ^*
0.35	0.35	0.81	0.01	0.73	1.00	0.3130	0.6404	0.6759	0.5530	0.5077

COUNTRY	*SEC25 +HIGH25 in 1990: m (Barro & Lee, 93)*	*Price of Human Capital: r(m)*	*Skill Premium or Income Inequality: I(m)*	*Inequality needed for Growth: $I_{min}(m)$*	*Model's Predicted Growth Rate: $\gamma(m)$*	*PCRGDP Growth RATE:65-90 (Penn Table)*	*Saving Rate Per Capita: $(1-a)^* i(m)/r(m)$*	*Human Capital to Output Ratio: $z(m)^{-1}$*
Peru	0.3160	0.2454	3.0199	0.0456	0.07%	-0.53%	2.84%	2.6482
Jamaica	0.3170	0.2459	3.0014	0.0471	0.11%	0.76%	2.93%	2.6435
Trinidad & Tobago	0.3210	0.2476	2.9283	0.0531	0.25%	0.76%	3.28%	2.6248
Guyana	0.3230	0.2485	2.8925	0.0561	0.32%	-1.46%	3.45%	2.6156
Mexico	0.3260	0.2498	2.8396	0.0604	0.42%	2.21%	3.70%	2.6021
Mauritius	0.3300	0.2515	2.7706	0.0662	0.56%	2.49%	4.04%	2.5844
Chile	0.3480	0.2590	2.4795	0.0913	1.17%	1.14%	5.44%	2.5099
Jordan	0.3530	0.2610	2.4039	0.0980	1.33%	2.39%	5.80%	2.4906
Singapore	0.3600	0.2637	2.3016	0.1072	1.55%	7.35%	6.29%	2.4645
Argentina	0.3730	0.2687	2.1218	0.1236	1.96%	-0.26%	7.15%	2.4188
Fiji	0.3730	0.2687	2.1218	0.1236	1.96%	2.47%	7.15%	2.4188
Greece	0.3740	0.2691	2.1085	0.1249	1.99%	3.17%	7.21%	2.4155
Philippines	0.3740	0.2691	2.1085	0.1249	1.99%	1.40%	7.21%	2.4155
Sri Lanka	0.3810	0.2717	2.0172	0.1334	2.20%	2.30%	7.65%	2.3924
Uruguay	0.3830	0.2724	1.9918	0.1358	2.26%	0.87%	7.77%	2.3860
Italy	0.4060	0.2805	1.7171	0.1621	2.91%	3.14%	9.06%	2.3171
Yugoslavia	0.4070	0.2809	1.7059	0.1632	2.94%	2.55%	9.12%	2.3143
France	0.4120	0.2825	1.6505	0.1686	3.08%	2.58%	9.38%	2.3006
Panama	0.4550	0.2958	1.2245	0.2108	4.15%	1.44%	11.32%	2.1974
Iceland	0.4590	0.2969	1.1889	0.2144	4.24%	3.06%	11.47%	2.1891
Norway	0.4760	0.3015	1.0444	0.2288	4.61%	3.05%	12.10%	2.1557
Taiwan	0.4990	0.3072	0.8646	0.2465	5.08%	6.32%	12.85%	2.1156
Belgium	0.5010	0.3077	0.8497	0.2480	5.11%	2.69%	12.92%	2.1124
Finland	0.5070	0.3091	0.8059	0.2522	5.23%	3.08%	13.09%	2.1030
UK	0.5240	0.3127	0.6870	0.2635	5.52%	2.17%	13.56%	2.0784
Cyprus	0.5290	0.3138	0.6535	0.2666	5.60%	4.50%	13.68%	2.0717
Hong Kong	0.5390	0.3157	0.5884	0.2725	5.76%	5.79%	13.92%	2.0589
Austria	0.5530	0.3182	0.5012	0.2802	5.96%	2.90%	14.23%	2.0427
Ireland	0.5530	0.3182	0.5012	0.2802	5.96%	3.36%	14.23%	2.0427

- Correlation Coefficient between the Predicted and the Observed Growth Rates=0.5327.

APPENDIX C (Not to be Included in the Paper)

(Algebra or Graphs for Propositions:4-6 & Equations: 31, 37, 39, 51-53)

- Proposition 4: An Illustration

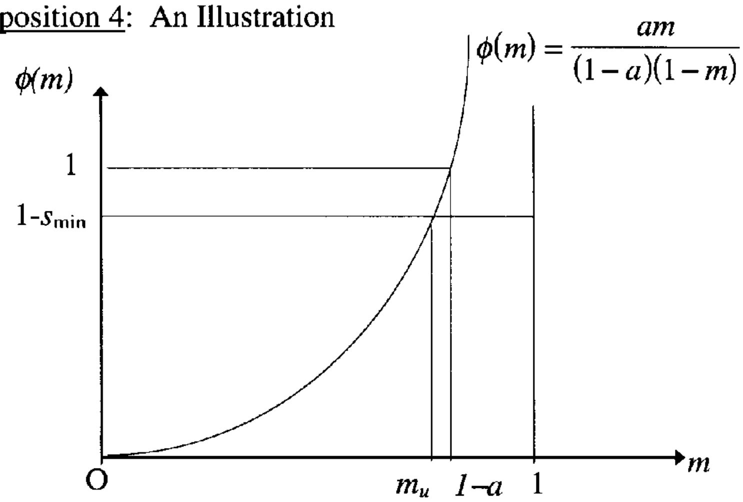


Figure 10

By (37), $s_m(m) \geq s_{\min}$

$\Leftrightarrow (1\text{-}s_m(m)) \leq (1\text{-}s_{\min})$

By definition $\phi(m_u) = (1 - s_{\min})$

$\Leftrightarrow \phi(m_u) \geq (1 - s_m(m))$

In a steady state

$\phi(m) \leq (1 - s_m(m)) \leq \phi(m_u)$

Also $\phi' > 0$

$\therefore m \leq m_u < 1 - a$ Q.E.D.

- Derivation of the formula (41) for m_U explicitly:

Definition: $s_{\min}=i_{\min}/r_{\min}$; $i_{\min}=\delta$, $r_{\min}=(1\text{-}a)g$.

$\therefore s_{\min} = \dfrac{\delta}{(1-a)g}$; By definition, $\phi(m_u)=(1-s_{\min})$.

It follows, therefore, $\dfrac{am_u}{(1-a)(1-m_u)} = \dfrac{(1-a)g-\delta}{(1-a)g} \overbrace{=\dfrac{1\text{-}\beta}{1\text{-}\beta+\delta\beta}}^{\text{By (34)}}$.

$\Leftrightarrow \dfrac{a(1-\beta+\delta\beta)}{(1-a)(1-\beta)} = \dfrac{1-m_u}{m_u} = \dfrac{1}{m_u} - 1$.

$\Leftrightarrow \dfrac{1}{m_u} = \dfrac{(1-\beta)-a(1-\beta)+a(1-\beta)+a\delta\beta}{(1-a)(1-\beta)} \Leftrightarrow m_u = \dfrac{(1-a)(1-\beta)}{1-\beta+a\delta\beta}$. Q.E.D.

- Proposition 5: An Illustration

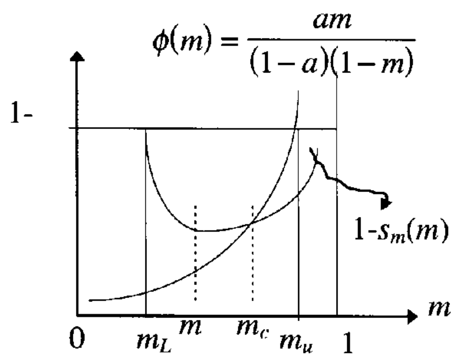


Figure 11

By Lemma 5, $m_u \geq m_L$. $\Leftrightarrow \phi(m_u) \geq \phi(m_L)$. By definition of m_L, if $m=m_L$, $i(m)=\delta$, $r(m)=(1\text{-}a)g$ and, therefore, $s_m(m_L)=s_{\min}$. Also, by definition of m_u, $\phi(m_u)=1-s_{\min}$. It follows, therefore, $\phi(m_L)\leq(1-s_m(m_L))$. Consequently, there exists $m\in[m_L, m_u]$ such that $\phi(m)\leq(1-s_m(m))$. Use such m to define $D(m, H)$ such that $H>0$, if $b=a$ and $H=H(m)$ that satisfies (40) if $b\neq a$. By Lemma (6) the C.E. with $\Psi\in D(m, H)$ is a steady state. Also, by Lemma 6, if $z(\hat{m}) > z(m_L)$, $m_u>m_L$ and there exits $m_c\leq m_u$ such that $m_L<m_c$ and all $m\in[m_L, m_c]$ satisfy the condition (39) and, therefore, there are multiple steady states. Q.E.D.

- Proposition 6: An Illustration

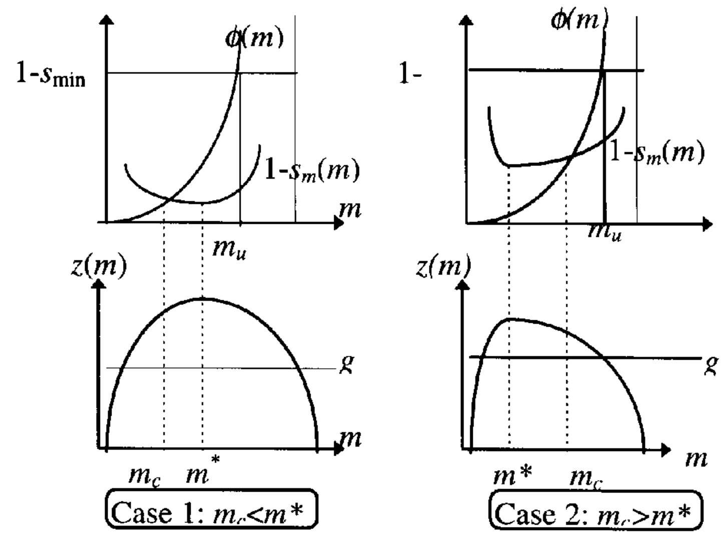


Figure 12

(i) If $b=a$, by (30)-(32),

$$\gamma(m) = i(m) - \delta$$

$$= f^{-1}(\beta(1-a)z(m)+1-\delta)-1+\delta$$

By the Chain Rule of differentiation, and the Inverse Function Theorem, it follow, therefore,

$$\gamma'(m) = \frac{\beta(1-a)z'(m)}{f'(\cdot)}$$

By (10), $f'(\cdot)>0$ and, hence, by (41), $\gamma'(\cdot)\geq 0 \Leftrightarrow z'(\cdot)\geq 0 \Leftrightarrow m\leq m^*$. It follows, therefore, that if $m_c\leq m^*$, $\gamma'(\cdot)\geq 0$; otherwise, if $m_c>m^*$, m^* maximizes the growth rate.

(ii) If $b<a$, by Proposition 2, $\gamma=0$ and, therefore, by (33) and (34), $Y = H^{1+b-a}z(m)$. By (40), $H^{b-a}z(m)=g$. For all $m\in[m_L, m_c]$ define $H(.)$ and $Y(.)$ as functions of m such that (33) and (40) are satisfied such that $Y(m)=gH(m)= g^{\frac{1+b-a}{b-a}} z(m)^{\frac{1}{a-b}}$. Consequently, $Y'(m)\geq 0 \Leftrightarrow z'(m)\geq 0$. By (41), it follows, therefore, if $m_c\leq m^*$, $Y'(\cdot)\geq 0$; otherwise, if $m_c>m^*$, m^* maximizes the per capita income. Q.E.D.

1. Equation (31):

If r_t=r, and n_{t+k}=0 for all k=1, 2, 3,.., by (8) and (14), it follows that:

$$u'(rh - s_t(h)) = \beta(r+1-\delta)u'(r(h_{t+1}(h) - s_{t+1}(h)), \qquad t=0,\ 1,\ 2,..$$

Define the investment rate $i_t(h) \equiv \dfrac{s_t(h)}{h_t(h)}$, for all t=0, 1, 2,... By (10), it follows,

$$f\left(\frac{rh_{t+1}(h) - i_{t+1}(h)h_{t+1}(h)}{rh_t(h) - i_t(h)h_t(h)}\right) = \beta(r+1-\delta)$$

By (7), it follows,

$$= f\left(\frac{(r - i_{t+1}(h))(1-\delta + i_t(h))}{(r - i_t(h))}\right) = \beta(r+1-\delta)$$

Denote $f^{-1}(\beta(r+1-\delta)$ by R. It follows, therefore, the sequence $\{i_t(h)\}$ satisfies the following difference equation:

$$(r - i_{t+1}(h))(1 + i_t(h) - \delta) = R(r - i_t(h))$$

$$\Leftrightarrow i_{t+1}(h) = \frac{(r+R)i_t(h) - r(R+\delta-1)}{1 + i_t(h) - \delta}.$$

Note from the above equation that $\dfrac{di_{t+1}(h)}{di_t(h)} > 0$ and $\dfrac{d^2 i_{t+1}(h)}{di_t(h)^2} < 0$. It follows, therefore, that the above difference equation has two stationary solutions: $0 < i_t(h) = r$ and $0 < i_t(h) = i = R + \delta - 1 < r$. All non-stationary solutions converge to $i_t(h) = r$. Consumer optimality (14), however, implies that for all $t \geq 0$, $i_t(h) < r$. It follows, therefore,

$$i_t(h) = i = f^{-1}(\beta(r+1-\delta)) - 1 + \delta.$$

- <u>Equation (37)</u>

$$s_m = \int \frac{s_t(m,h)}{y_t(m,h)} d\Psi(h) = \int \frac{i(m)h}{r(m)h} d\Psi(h) = \frac{i(m)}{r(m)}$$

Note that $i(m) \geq \delta$ and, by (32) and (34), $r(m)=(1-a)z(m)$, $z(m) \geq g$.

By definition of m_L, $z(m_L)=g$, $r(m_L)=(1-a)g$ and $i(m_L)=\delta$.

Therefore, $s_m(m_L)=\dfrac{\delta}{(1-a)g}=(1-\delta+\delta\beta)^{-1}\delta\beta$ (by 34).

Define $s_{\min} \equiv \delta\beta(1-\delta+\delta\beta)^{-1}$.

<u>Equation (37) claims</u>: $s_m(m) \geq s_m(m_L) \equiv s_{min}$.

Proof. Step 1: $i(m)<(f'(1+i(m)-\delta))^{-1}\beta r(m)$.

Proof. $i(m_L)=\delta$, $r(m_L)=\beta^{-1}+1-\delta$ and, by (10), $f'(1)<\delta^{-1}(1-\beta(1-\delta))$. It follows that the above inequality holds at $m=m_L$. Denote $(1+i(m)-\delta)$ by (.). For all $m>m_L$, by (31), as m increases the L.H.S. of the above inequality increases at a rate $i'(m)=[f'(.)]^{-1}\beta r'(m)$ while the R.H.S. of the inequality increases at a rate $\beta r'(m)[(f'(.))^{-1}+r(f'(.))^{-2}(-f''(.))]$. By (10), $f''<0$. It follows, therefore, that the above inequality holds for all $m \geq m_L$.

Step 2: $s'_m(m)=(1-a)\, z'(m)\left(\dfrac{(f'(1+i(m)-\delta))^{-1}\beta r(m)-i(m)}{(r(m))^2}\right)$

Step 3: $s'_m(m) \geq 0 \Leftrightarrow z'(m) \geq 0$.

Step 4: $m \geq m_L \Leftrightarrow z(m) \geq z(m_L) \Leftrightarrow s_m(m) \geq s_m(m_L) \equiv s_{\min}$.

- Equation (39):

$$\left(1-s_m(m)\right)\geq\phi(m)$$

$$H\geq mh^* = \frac{mw(M,H)}{r(m)-i(m)} = \frac{mx(m,H)}{\left(1-s_m(m)\right)}$$

By (27) and (32)

$$\Leftrightarrow \left(1-s_m(m)\right)H \geq \frac{amH}{(1-r)(1-m)} \equiv \phi(m)H$$

$$\Leftrightarrow \left(1-s_m(m)\right)\geq\phi(m).$$

- Set of equations required for calibration of parameters with U(c)=Lnc

1. $\frac{w.(1-m)}{y}=a$

2. $\frac{\partial \ln y_t}{\partial \ln H_t}=1+b-a \quad by \;\; (33)$

3. $Ln(A_0)=Ln(\overline{Y})-(1+b-a)\overline{H}-\theta Ln(\overline{m})$,

   where $\overline{Y}$, $\overline{H}$, $\overline{m}$ denote the average steady state values of Y, H and m across all countries for which the model identifies a steady state.

4. $s=(1-a)s_m=(1-a)\,{}^{i}\!/_{r}$

   $=(1-a)\beta-\frac{(1-a)(1-\beta)(1-\delta)}{(1-a)z}$, since, $i=\beta r-(1-\beta)(1-\delta)$ and $r=(1-a)z$.

   $s=(1-a)\beta-\frac{(1-\beta)(1-\delta)}{z}$

5. $\delta=i-\gamma=\frac{rs}{(1-a)}-\gamma$, since $s=(1-a)i/r$

$\Leftrightarrow \delta=sz-\gamma.$

6. Equation (51) OR (52):

(i) Derivation of Equation (51):

By (38) and (48), $I = \frac{(1-a)(1-m)}{am} - 1 = \frac{(1-a) - m + am - am}{am}$

$\Leftrightarrow amI = (1-a) - m$

$\Leftrightarrow m(1+aI) = (1-a)$

$$\Leftrightarrow m = \frac{(1-a)}{1+aI} \qquad \text{(I)}$$

$$\Leftrightarrow 1 - m = \frac{1 - aI - 1 + a}{1+aI} = \frac{a(I+1)}{1+aI} \qquad \text{(II)}$$

By (30) and (31) with u(c)=Ln(c), it follows,

$\gamma = i - \delta$

$= \beta(r + 1 - \delta) - 1 + \delta - \delta$

$= (1-a)z\beta + \beta(1-\delta) - 1$

$= A_0\beta(1-a)m^\theta(1-m)^a + \beta(1-\delta) - 1$ by (34).

By (I) and (II) of the previous page, it follows, therefore,

$$\gamma = A_0\beta(1-a)\left(\frac{1-a}{1+aI}\right)^\theta \left(\frac{a(1+I)}{1+aI}\right)^a + \beta(1-\delta) - 1.$$

(ii) Derivation of (52)

By (33) and (34)

$$Y = A_0 H^{1+b-a} m^\theta (1-m)^a = H^{1+b-a} z(m) = \frac{(1-a)}{(1-a)} H^{b-a} z(m) H$$

$$H=\left(\frac{g}{z(m)}\right)^{\frac{1}{b-a}}=g^{\frac{1}{b-a}}A_0^{\frac{1}{1-b}}m^{\frac{\theta}{a-b}}(1-m)^{\frac{\sigma}{a-b}}$$

$$Y=\left(\frac{g}{z(m)}\right)^{\frac{1+b-a}{b-a}}z(m)$$

$$=g^{\frac{1+b-a}{b-a}}z^{\left(1-\frac{1+b-a}{b-a}\right)}$$

$$=g^{\frac{1+b-a}{b-a}}z^{\frac{b-a-1-b+a}{b-a}}$$

$$=g^{\frac{1+b-a}{b-a}}z^{\frac{1}{a-b}}$$

$$=g^{\frac{1+b-a}{b-a}}A_0^{\frac{1}{a-b}}m^{\frac{\sigma}{a-b}}(1-m)^{\frac{a}{a-b}}$$

By (I) and (II) of the previous page, it follows, therefore,

$$Y=g^{\frac{1+b-a}{b-a}}A_0^{\frac{1}{1-b}}\left(\frac{1-a}{1+aI}\right)^{\frac{\sigma}{a-b}}\left(\frac{a(1+I)}{1+aI}\right)^{\frac{a}{a-b}}.$$

- <u>Equation (53):</u>

Note that $i^*_{t+1}(h)=\frac{s_{t+1}(h)}{h}\cdot\frac{(1-\delta+i_l(h))}{(1-\delta+i_t(h))}$

$$=\frac{s_{t+1}(h)}{h_{t+1}(h)}(1-\delta+i_t(h))$$

(III) $$=i_{t+1}(h)(1-\delta+i_t(h))$$

By (14)if $n_{t+k}(h)=0$ for all k=1, 2,..., $y_{t+k}(h)=r_{t+k}h_{t+k}(h)$ and therefore, for all $t\geq 0$,

$$u'\left(y_t(h)-s_t(h)\right)=\beta\left(r_{t+1}+1-\delta\right)u'\left(y_{t+1}(h)-s_{t+1}(h)\right)$$

By (10), it follows,

$$\Leftrightarrow f\left(\frac{y_{t+1}(h)-s_{t+1}(h)}{y_t(h)-s_t(h)}\right)=\beta\left(r_{t+1}+1-\delta\right)$$

$$\Leftrightarrow f\left(\frac{r_{t+1}h_{t+1}(h)-i_{t+1}(h)h_{t+1}(h)}{r_t h-i_t(h)}\right)=\beta\left(r_{t+1}+1-\delta\right),$$

since by (8), $h_{t+1}(h)=(1-\delta)h+s_t$. Define $R_{t+1}\equiv f^{-1}\left(\beta\left(r_{t+1}+1-\delta\right)\right)$. It follows, therefore,

$$\frac{\left(r_{t+1}-i_{t+1}(h)\right)\left(1-\delta+i_t(h)\right)}{\left(r_t-i_t(h)\right)}=R_{t+1}$$

$$\Leftrightarrow (1-\delta)r_{t+1}-(1-\delta)i_{t+1}(h)-i_t(h)i_{t+1}(h)+i_t(h)r_{t+1}=R_{t+1}r_t-i_t(h)R_{t+1}$$

$$\Leftrightarrow i_t(h)\left[R_{t+1}+r_{t+1}\right]=R_{t+1}r_t+i_{t+1}(h)\left(1-\delta+i_t(h)\right)=R_{t+1}r_t+i^*_{t+1}(h)$$, by (III) above.

$$\Leftrightarrow i^*_t(h)-\frac{i^*_{t+1}(h)}{\left[R_{t+1}+r_{t+1}\right]}=\frac{R_{t+1}r_t-(1-\delta)r_t}{R_{t+1}+r_{t+1}}.$$

Appendix D:Not Included in the Paper

Data for Table 1A and Table 1B Compiled from Barro and Lee (1993) and Mankiw-Romer-Weil (1992)

COUNTRYCODE	COUNTRY	SHCODE	NO25 (NQ): %	PRI25 (PQ): %	SEC25 (SQ): %	HIGH25 (HQ): %	HE= HQ+SQ	EER= (HQ+SQ)/NQ	School M R-W: %	GDP/ADULT	n: 60-85	I/Y: 60-85
DZA	Algeria	1	64.9	27.6	5.3	2.3	7.6	0.0354	4.5	4371	2.6	24.1
BEN	Benin	3	85.7	10.5	3.2	0.6	3.8	0.0070	1.8	1071	2.4	10.8
CAF	Central Africa	9	78.8	18.3	2.6	0.4	3	0.0051	1.4	789	1.7	10.5
COG	Congo	12	58.7	21.3	17	3	20	0.0511	3.8	2624	2.4	28.8
EGY	Egypt	13	64.1	16.5	14.8	4.6	19.4	0.0718	7.0	2160	2.5	16.3
GHA	Ghana	17	63.2	20.8	15.3	0.8	16.1	0.0127	4.7	727	2.3	9.1
KEN	Kenya	21	56.4	37.7	5.2	0.7	5.9	0.0124	2.4	1329	3.4	17.4
LSO	Lesotho	22	33.5	60.8	5.2	0.5	5.7	0.0149	2.0	1483	1.9	12.6
LBR	Liberia	23	76.6	13.1	8.6	1.7	10.3	0.0222	2.5	944	3	21.5
MWI	Malawi	25	55	39.8	4.8	0.4	5.2	0.0073	0.6	823	2.4	13.2
MLI	Mali	26	91.3	6.6	1.8	0.3	2.1	0.0033	1.0	710	2.2	7.3
MUS	Mauritius	28	21.1	52.7	23.1	3.1	26.2	0.1469	7.3	2967	2.6	17.1
MOZ	Mozambique	30	78.6	20.4	0.8	0.1	0.9	0.0013	0.7	1035	2.7	6.1
NER	Niger	31	90.3	8.7	0.8	0.2	1	0.0022	0.5	841	2.6	10.3
RWA	Rwanda	33	68.1	29.7	1.9	0.3	2.2	0.0044	0.4	696	2.8	7.9
SEN	Senegal	34	61.3	33.7	3.7	1.4	5.1	0.0228	1.7	1450	2.3	9.6
SLE	Sierra Leone	36	84.4	11	4.1	0.5	4.6	0.0059	1.7	805	1.6	10.9
ZAF	South africa	38	24.4	46.6	26.7	2.3	29	0.0943	3.0	7064	2.3	21.6
SDN	Sudan	39	76.7	18.6	3.9	0.8	4.7	0.0104	2.0	1038	2.6	13.2
TGO	Togo	42	70.7	18	9.8	1.5	11.3	0.0212	2.9	978	2.5	15.5
TUN	Tunisia	43	66.3	18.9	12	2.8	14.8	0.0422	4.3	3661	2.4	13.8
UGA	Uganda	44	65.8	31.3	2.6	0.3	2.9	0.0046	1.1	667	3.1	4.1
ZAR	Zaire	45	62.2	32.2	4.9	0.6	5.5	0.0096	3.6	412	2.4	6.5
ZMB	Zambia	46	43.1	44	12.3	0.7	13	0.0162	2.4	1217	2.7	31.7
ZWE	Zimbabwe	47	36.2	58.6	4.2	1	5.2	0.0276	4.4	2107	2.8	21.1
CAN	Canada	50	1.2	19.9	59.5	19.3	78.8	16.0833	10.6	17935	2	23.3
CRI	Costa Rica	51	14.1	64.1	10.2	11.6	21.8	0.8227	7.0	4492	3.5	14.7
DOM	Dominican R	53	44	39.3	10.3	6.4	16.7	0.1455	5.8	3308	2.9	17.1
SLV	El Salvador	54	35.4	54.6	6.6	3.4	10	0.0960	3.9	1997	3.3	8.0
GTM	Guatemala	56	54.1	36.2	6.2	3.5	9.7	0.0647	2.4	3034	3.1	8.8

Appendix D:Continued

Data for Table 1A and Table 1B Compiled from Barro and Lee (1993) and Mankiw-Romer-Weil (1992)

COUNTRYCODE	COUNTRY	SHCODE	NO25 (NQ): %	PRI25 (PQ): %	SEC25 (SQ): %	HIGH25 (HQ): %	HE= HQ+SQ	EER= (HQ+SQ)/NQ	School M R-W: %	GDP/ADULT	n: 60-85	I/Y: 60-85
HTI	Haiti	57	59.5	30.5	9.3	0.7	10	0.0118	1.9	1237	1.3	7.1
HND	Honduras	58	33.5	51.3	11.9	3.3	15.2	0.0985	3.7	1822	3.1	13.8
JAM	Jamaica	59	2.6	72.7	21.9	2.8	24.7	1.0769	11.2	3080	1.6	20.6
MEX	Mexico	60	31	50	11.7	7.3	19	0.2355	6.6	7380	3.3	19.5
NIC	Nicaragua	61	44.3	44.8	3.4	7.5	10.9	0.1693	5.8	3978	3.3	14.5
PAN	Panama	62	18.8	48.2	21.9	11.1	33	0.5904	11.6	5021	3	26.1
TTO	Trinidad & To	65	3.7	67.9	25.1	3.3	28.4	0.8919	8.8	11285	1.9	20.4
USA	United States	66	1.5	7.4	57.4	33.7	91.1	22.4667	11.9	18988	1.5	21.1
ARG	Argentina	67	6.7	64.7	20.4	8.2	28.6	1.2239	5.0	5533	1.5	25.3
BOL	Bolivia	68	44.1	34.4	14	7.5	21.5	0.1701	4.9	2055	2.4	13.3
BRA	Brazil	69	33.9	54.8	5	6.4	11.4	0.1888	4.7	5563	2.9	23.2
CHL	Chile	70	9	57.2	25.6	8.3	33.9	0.9222	7.7	5533	2.3	29.7
COL	Colombia	71	24.5	53.4	16.4	5.7	22.1	0.2327	6.1	4405	3	18.0
ECU	Ecuador	72	24.5	51	10.9	13.6	24.5	0.5551	7.2	4504	2.8	24.4
PRY	Paraguay	74	13.5	67.9	14.1	4.5	18.6	0.3333	4.4	3914	2.7	11.7
PER	Peru	75	23.8	45.5	18.8	12	30.8	0.5042	8.0	3775	2.9	12.0
URY	Uruguay	77	4.7	58	29.2	8.1	37.3	1.7234	7.0	5495	0.6	11.8
VEN	Venezuela	78	23.4	46.1	20.5	10	30.5	0.4274	7.0	6336	3.8	11.4
BGD	Bangladesh	81	66.6	17.6	14.1	1.7	15.8	0.0255	3.2	1221	2.6	6.8
BUR	Myanmar (Bu	82	55.9	27.8	14.5	1.9	16.4	0.0340	3.5	1031	1.7	11.4
HKG	Hong Kong	84	18.4	35.6	38.3	7.7	46	0.4185	7.2	13372	3	19.9
IND	India	85	67	16.1	13.2	3.7	16.9	0.0552	5.1	1339	2.4	16.8
IDN	Indonesia	86	33	56.1	10.3	0.6	10.9	0.0182	4.1	2159	1.9	13.9
IRN	Iran, I.R. of	87	69.4	12.3	15.7	2.6	18.3	0.0375	6.5	7400	3.4	18.4
IRQ	Iraq	88	67.8	19.5	7.8	4.9	12.7	0.0723	7.4	5626	3.2	16.2
ISR	Israel	89	11.6	29.4	35	24	59	2.0690	9.5	10450	2.8	28.5
JPN	Japan	90	0.4	41.7	41.9	16	57.9	40.0000	10.9	13893	1.2	36.0
JOR	Jordan	91	56.3	18.9	13.9	11	24.9	0.1954	10.8	4312	2.7	17.6
KOR	Korea	92	15.4	27.6	45.3	11.7	57	0.7597	10.2	4775	2.7	22.3
KWT	Kuwait	93	50.3	7.2	29.7	12.7	42.4	0.2525	9.6	25635	6.8	9.8

Appendix D:Continued

Data for Table 1A and Table 1B Compiled from Barro and Lee (1993) and Mankiw-Romer-Weil (1992)

COUNTRYCODE	COUNTRY	SHCODE	NO25 (NQ): %	PRI25 (PQ): %	SEC25 (SQ): %	HIGH25 (HQ): %	HE= HQ+SQ	EER= (HQ+SQ)/NQ	School M R-W: %	GDP/ADULT	n: 60-85	I/Y: 60-85
MYS	Malaysia	94	29.9	44.8	23.3	2	25.3	0.0669	7.3	5788	3.2	23.2
NPL	Nepal	95	88.7	5.7	4.2	1.5	5.7	0.0169	2.3	974	2	5.9
PAK	Pakistan	97	76.3	11.4	10.3	2	12.3	0.0262	3.0	2175	3	12.2
PHL	Philippines	98	10.1	54.7	17.3	17.8	35.1	1.7624	10.6	2430	3	14.9
SGP	Singapore	100	35.6	39.9	20.1	4.3	24.4	0.1208	9.0	14678	2.6	32.2
LKA	Sri Lanka	101	13.6	50.1	34.9	1.4	36.3	0.1029	8.3	2482	2.4	14.8
THA	Thailand	104	21.3	67.5	6.1	5	11.1	0.2347	4.4	3220	3.1	18.0
AUT	Austria	107	1	45.8	47.5	5.7	53.2	5.7000	8.0	13327	0.4	23.4
BEL	Belgium	108	1.2	52.6	35.5	10.7	46.2	8.9167	9.3	14290	0.5	23.4
DNK	Denmark	110	2	44	35.4	18.6	54	9.3000	10.3	16491	0.6	26.6
FIN	Finland	111	1	54.6	30.6	13.8	44.4	13.8000	11.5	13779	0.7	36.9
FRA	France	112	1.1	62.7	25.6	10.5	36.1	9.5455	8.9	15027	1	26.2
DEU	Germany, W.	113	1.3	71.5	19.5	7.7	27.2	5.9231	8.4	15297	0.5	28.5
GRC	Greece	114	10.6	58.1	22.6	8.7	31.3	0.8208	7.9	6868	0.7	29.3
IRL	Ireland	117	3.4	45.1	41.9	9.6	51.5	2.8235	11.4	8675	1.1	25.9
ITA	Italy	118	17.2	45.9	30.2	6.7	36.9	0.3895	7.1	11082	0.6	24.9
NLD	Netherlands	121	3.2	38.1	44.9	13.8	58.7	4.3125	10.7	13177	1.4	25.8
NOR	Norway	122	2.9	54.2	29.4	13.5	42.9	4.6552	10.0	19723	0.7	29.1
PRT	Portugal	124	26.2	58	11.4	4.5	15.9	0.1718	5.8	5827	0.6	22.5
ESP	Spain	125	5.2	70.2	17.6	7	24.6	1.3462	8.0	9903	1	17.7
SWE	Sweden	126	2.6	37.5	43	16.9	59.9	6.5000	7.9	15237	0.4	24.5
CHE	Switzerland	127	5.1	30.2	52.9	11.8	64.7	2.3137	4.8	15881	0.8	29.7
TUR	Turkey	128	46.2	39.5	10.2	4.1	14.3	0.0887	5.5	4444	2.5	20.2
GBR	United Kingdc	129	3.9	47.7	35.7	12.8	48.5	3.2821	8.9	13331	0.3	18.4
AUS	Australia	131	2.1	27.7	48.4	21.8	70.2	10.3810	9.8	13409	2	31.5
NZL	New Zealand	133	1	13.3	55.3	30.3	85.6	30.3000	11.9	12308	1.7	22.5
PNG	Papua New G	134	75.9	18.7	4.9	0.5	5.4	0.0066	1.5	2544	2.1	16.2