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Economic Progress and Skill Obsolescene

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Abstract

We construct an OLG model of skill vintages with complementarities to examine skill obsolescence when individuals can choose vintages. We find that the problem of excessive progress can exist only in the absence of coordination and transfers among those currently alive. However, too little progress can occur in equilibrium, even in the presence of coordination and transfers. Moreover, allowing coordination or transfers may reduce aggregate surplus. Equilibria with too little progress can take the form of either cycles or stagnation. The introduction of outside debt can eliminate the cyclical equilibrium, leading to a Pareto-improving increase in the rate of progress.

Key words: Skill vintages, technological change.

JEL codes: 041, J24, O33

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Introduction

Skill obsolescence is one of the major problems that come with economic progress. Skills that individuals learn when young can become devalued when new technologies arrive that require a different set of skills. This can be a problem particularly for older workers. In traditional societies, where the pace of technological change is relatively slow, elders are valued because the skills in which they have experience are still relevant. In contrast, in societies with a fast pace of technological change, many seniors become marginalized. The introduction of new techniques can force older people to either learn the new techniques or face obsolescence, since the productivity of their skills is largely a function of the number of other people who have complementary skills. This externality, inherent in skill choices, can potentially lead to intergenerational conflict and suboptimal outcomes.

Recent analyses of this problem, such as Krusell and Rios-Rull (1996) and Aghion and Howitt (1998, chapter 9) have used the vintage human capital model developed by Chari and Hopenhayn (1991) to argue that this can lead to political action by older generations to suppress new technologies. In their framework, uniform technology choices are determined by majority voting. This gives an intergenerational conflict interpretation of Olson's (1982) "vested interests" argument: inefficient periodic occurrences of technological slowdowns may occur due to intergenerational conflicts, through democratic government policy.

In this paper, we argue that similar effects can occur without recourse to government policy, when individuals of different generations can freely choose vintages of skill sets to learn. Under certain circumstances, the problem disappears when individuals can coordinate and exchange transfers. However, under other circumstances, it may persist even in the presence of coordination and transfers, leading to inefficient cycles or stagnation. We present an overlapping generations model of human capital vintages, similar in spirit to those in the above studies, but with a different approach to modeling skill complementarities, which allows for a relatively simple analysis of individual skill choices.

The literature on human capital vintages, following Chari and Hopenhayn (1991), has typically modeled skill complementarity in the following way. Production is a function of "skilled" and "unskilled" labour, with positive and diminishing marginal products, but positive cross-partials. Each production vintage has these properties, where skills learned in one vintage are not applicable in others. With overlapping generations in this environment, agents choose which vintage to work in. Young agents who work as unskilled workers in a particular vintage have the opportunity to work in that same vintage as skilled workers when old. Chari and Hopenhayn (1991) characterize the equilibrium stationary distribution of worker populations in different vintages. Diminishing marginal products ensure that the stationary distribution is non-trivial (many vintages operate simultaneously) and Pareto efficient.

We identify two problems with this approach. First, if the interpretation of "skilled" and "unskilled" workers is taken literally, it is difficult to justify the co-existence of diminishing marginal products and positive cross products in the production technology. If unskilled workers simply know less than skilled ones then skilled workers should be absolutely superior to unskilled ones. That is, skilled workers should be able to perform all unskilled tasks and more. The joint assumption of diminishing marginal products and positive cross products can only be really justified if "unskilled" workers have *different* skills than the "skilled" workers. For example, unskilled workers may have more brute strength than skilled ones. However, this interpretation does not fit well with the maintained assumption that old workers, who worked in a particular vintage when young, can choose whether to be skilled in that vintage or unskilled in another.

The second problem with the standard approach is, to some extent, its very success at generating a rich distribution of simultaneously active vintages. This is a problem simply because characterizing this distribution can be very difficult. Subsequent researchers using this framework (for eg., Krusell and Rios-Rull (1996) and Aghion and Howitt (1998)) have found it convenient to impose the restriction that only one vintage operate at any one time, to keep equilibria tractable. This restriction, while useful in the context of social choices with voting, effectively precludes any analysis of individual choice over technological vintage.

In this paper we model skill complementarities in a different way, which eliminates the distinction between "skilled" and "unskilled" workers, and which allows for a tractable analysis of individual vintage choice. We focus on the commonality of technological vintage as the source of skill complementarity. Here, we model different workers that use a particular technology as having complementary skills, which are unrelated to worker age. The productivity of each vintage is assumed to be an increasing function of the number of individuals who also use that technology, and hence have compatible skills. This modelling approach draws upon the "network effect" literature, pioneered by Katz and Shapiro (1985), in the context of product complementarities.[1] We argue that vintages of human skills are also subject to network effects.

Most forms of production involve extensive interaction between agents, and the productivity of that interaction depends to a considerable degree on the compatibility of the skills of the agents involved. This is not to say that productive matches require individuals to have the *same* skills; on the contrary, interaction is often most valuable when it brings together individuals with different expertise. However, if different skill sets are to complement each other, then they must be compatible. For example, an engineer and an architect interact most productively if they are both familiar with computer-aided design. We believe that these network effects are equally important at a more primitive level. The ability of agents to interact productively relies on a commonality of language, not just in terms of words spoken, but more

[1] Katz and Shapiro (1994) also consider the notion of "forming systems": collections of compatible individual goods (such as nuts and bolts) that work together to provide services. Network effects pervade such systems. Languages are another important area where network effects arise: the usefulness of a language increases with the number of people who speak it. (Church and King (1993)).

fundamentally, in terms of the way they think and the methods they use. It is from this perspective that we view vintages of human skills; we think of a technological vintage as akin to a language, in the broadest sense of that word.

Introducing network effects into the production function, in this way, also generates a tendency for workers to cluster together, rather than disperse themselves across vintages. As a result, the number of active vintages in stationary equilibria is very small: in the overlapping generations model with two periods of life, at most two vintages are active in any period. This feature allows the equilibrium, with individual vintage choices, to be characterized with relative ease.

In any model with network effects, multiple equilibria and coordination problems exist. In this paper, we take the approach of eliminating the multiplicity of equilibria by introducing successively increasing degrees of coordination. We start by considering some of the many equilibria that exist in the absence of any coordination. We then allow all agents within each generation to coordinate. In this case, there are regions in the parameter space in which there exists a unique, surplus maximizing equilibrium. However, there also exist regions in which the unique equilibrium does not maximize surplus, and regions with multiple equilibria and cycles. Next, we allow all agents currently alive (young and old) to coordinate. This reduces the size of the region in which multiple equilibria exist, but does not eliminate it; nor does it eliminate the regions with suboptimal equilibria or cycles. Going further, allowing costless transfers among agents currently alive, eliminates the multiplicity of equilibria and eliminates equilibria with too much progress, but does not eliminate equilibria with too little progress. Moreover, equilibria with too little progress are the only cyclical equilibria. We then demonstrate that introducing intertemporal transfer payments, through government debt, allows the implementation of a Pareto improvement from the suboptimal cyclical equilibrium to a surplus-maximizing one with steady progress.

While progressive degrees of coordination make current generations no worse off, this is not true for future generations or aggregate surplus. We show that the introduction of some coordinating mechanism may, in fact, move the economy from one equilibrium to another in which future generations are made worse off and aggregate surplus is reduced. Similar effects can also occur when introducing the possibility of costless transfers. In all these cases, the economy moves to an equilibrium with a lower rate of progress.

The rest of the paper is organized as follows. Section 1 introduces the model. Section 2 characterizes the surplus-maximizing allocations. Section 3 then presents the equilibrium outcomes, for different degrees of coordination. In section 4, we compare the equilibrium and surplus-maximizing allocations. Finally, in section 5, we discuss the results and the conclusions we draw from them. Proofs of all the propositions are given in the appendix.

1. The Model

We examine an economy with overlapping generations in which agents live for two periods, and each generation has N (initially identical) agents. When young, agents must choose which technology to learn. When old, they must choose whether to retain the technology they learned when young, or re-tool by learning another technology. The payoff from using a particular technology is subject to a network effect. In particular, the payoff $y_t(t)$ to an agent using a particular technology t in period t, when $x_t(t)$ agents use that same technology, is

$$y_t(t) = \begin{cases} g^{t} x_t(t) & \text{for} \quad x_t(t) \geq 2 \\ 0 & \text{for} \quad x_t(t) < 2 \end{cases} \qquad (1.1)$$

where $g > 1$ is a parameter reflecting the productivity of the technology. At least two agents must use a technology for that technology to be productive, and beyond that the payoff to using a technology is proportional to the number of agents using it.[2]

[2] The discontinuity at $x_t(t) = 2$ has no bearing on the substantive results; it serves only to simplify the analysis of the uncoordinated equilibria.

Technologies evolve in the following way. If technology t is learned by the young in period t, then a new technology $t+1$ becomes available in period t+1. Otherwise, technology t remains the most productive technology available. The following learning-by-doing process underlies this evolution: only when agents adopt a new technology do they discover new ways in which it can be improved. Moreover, technology will only progress if some young agents learn the new technology, since only they have any incentive to develop further technologies.[3]

We assume that all old agents enter period 0 endowed with technology $t=0$. Technology $t=1$ becomes available in period 0. If any young agents adopt technology $t=1$, then technology $t=2$ becomes available in period 1, in addition to the existing technologies, $t=0$ and $t=1$. Otherwise, only technologies $t=0$ and $t=1$ are available in period 1. This evolution continues with a new, more advanced technology becoming available each time any young agents adopt the previous new technology. Agents can choose any technology from those available.

Learning costs are a key element of our model. We assume that learning is costless for young agents but re-tooling is costly for the old.[4] In particular, any old agent who re-tools must devote a fraction $k \in [0,1]$ of his available productive time to learning.[5] Thus, an old agent in period t who re-tools with technology t receives a payoff equal to $(1-k)y_t(t)$ in that period.

Preferences take the following form:

$$u(c_1,c_2) = c_1 + bc_2 \tag{1.2}$$

[3] Individual incentives to develop new technologies are not modeled explicitly in this paper. The specification here (that progress is only made if young agents adopt the latest technology) endogenizes the technological progress in a very simple way, in the spirit of the learning-by-doing approach used by Jovanovic and Nyarko (1996).

[4] It makes no difference if learning is costly for the young, provided learning costs are independent of which technology they learn.

[5] Our use of the male pronoun throughout was decided by a coin toss.

where c_1 denotes consumption when young, c_2 denotes consumption when old, and $b \in [0,1]$ is the discount factor. We assume throughout that $bg < 1$.

2. Surplus Maximization

One of our primary goals is to understand the conditions under which equilibrium rates of economic progress are efficient. To this end, we begin by characterizing the allocations in this economy that maximize surplus. A planner who wishes to maximize the discounted surplus in this economy from period 0 forward, chooses from four alternative stationary allocations. Each is described in turn.

2.1 Eternal Stagnation

In this allocation, all agents use the initial technology $t = 0$, and no technological progress occurs. From equation (1.1), the surplus produced by each young agent in period 0 is $2N$ (since $g^0 = 1$). Thus, the total surplus produced by young agents in period 0 is $2N^2$. Similarly, the surplus produced by each old agent is $2N$, so the total surplus produced by old agents in period 0 is $2N^2$. Since no technological change occurs, this allocation repeats itself in every period. Using b to discount future surplus, total discounted surplus in the Eternal Stagnation (*ES*) allocation is

$$S(ES) = \sum_{t=0}^{\infty} b^t [2N^2 + 2N^2] = \frac{4N^2}{1-b} \qquad (2.1)$$

2.2 Partial Learning 1

In this allocation, each agent learns the new technology when young, but does not re-tool with the next new technology when old. Thus, each generation works with its own particular technology. Technological progress occurs at rate $g-1$. In period 0, the surplus produced by each young agent is gN, and by each old agent is N. In period 1, the surplus produced by each young agent is $g^2 N$, and by each old agent is gN, and so on. Thus, total discounted surplus under Partial Learning 1 ($PL1$) is

$$S(PL1) = \sum_{t=0}^{\infty} b^t [g^t N^2 + g^{t+1} N^2] = \frac{N^2(1+g)}{1-bg} \tag{2.2}$$

2.3 Partial Learning 2

In this allocation, each agent learns the old technology when young, and re-tools with the new technology when old. No technological progress occurs (since the young do not learn the new technology). In each period, each young agent produces N, and each old agent produces $gN(1-k)$. Thus, the total discounted surplus under Partial Learning 2 ($PL2$) is

$$S(PL2) = \sum_{t=0}^{\infty} b^t [gN^2(1-k) + N^2] = \frac{N^2[g(1-k)+1]}{1-b} \tag{2.3}$$

2.4 Complete Learning

In this allocation, each agent learns the new technology when young and, re-tools with the next new technology when old. Technological progress occurs at rate $g-1$. In each period, all agents concurrently alive use the same technology. Thus, in period t, each young agent produces $g^{t+1} 2N$, and each old agent produces $g^{t+1} 2N(1-k)$. Therefore, total discounted surplus in the Complete Learning (CL) allocation is

$$S(CL) = \sum_{t=0}^{\infty} b^t [g^{t+1} 2N^2(1-k) + g^{t+1} 2N^2] = \frac{2(2-k)gN^2}{1-bg} \tag{2.4}$$

The following proposition describes the surplus-maximizing allocations in this economy.

Proposition 1

Discounted surplus is maximized by Complete Learning if and only if

$$g \geq \frac{2}{2-k(1-b)}$$

Otherwise, discounted surplus is maximized by Eternal Stagnation.

This result states that discounted surplus is maximized by the two extremes of Complete Learning or Eternal Stagnation. The result is illustrated in Figure 1. The horizontal axis measures $k \in [0,1]$, the fraction of time that each old agent must spend learning if he is to adopt the new technology. The vertical axis measures $g > 1$, the productivity increment associated with each new technology. Given b, these two parameters represent the costs and benefits of economic progress. The parameter space is partitioned by the threshold relationship between g and k from proposition 1. If g is large relative to k, then discounted surplus is maximized by Complete Learning and associated steady growth. However, if the cost of change is too high, then Eternal Stagnation maximizes surplus.

It is intuitively clear that Partial Learning 2 (when the old learn the new technology but the young do not) can never maximize surplus, since it is always dominated by Partial Learning 1 (where the young learn the new technology but the old do not): the young can learn the new technology at no higher cost than the old technology, and by doing so they generate future growth; in contrast, the old must incur a re-tooling cost to learn the new technology, and that learning does not lead to further advances. That Partial Learning 1 is in turn dominated by either

Complete Learning or Eternal Stagnation reflects the network effect in this economy: there are substantial productivity gains if all agents use the same technology.[6]

3. Equilibria

As one would expect, the network effects in this economy give rise to multiple equilibria. Our intention is not to exhaustively characterize all of the equilibria that can occur. Rather, our purpose is to characterize the outcomes with respect to the rate of economic progress under increasing degrees of coordination among agents. We consider the following hierarchy of coordination. First, we briefly examine *uncoordinated equilibria* in which young and old agents individually make technology choices without coordinating their choices with any other agents. We then examine *intragenerational coordination equilibria*, in which all agents within a generation can coordinate on a technology choice. The associated equilibrium outcomes in this case highlight the conflicts that can arise between generations over the rate of economic progress. We then examine the extent to which these conflicts can be resolved through *intergenerational coordination*, in which all agents alive within any period can coordinate, both with and without transfers.

3.1 Uncoordinated Equilibria

In this section we briefly characterize the equilibria associated with uncoordinated individual learning choices. Since our purpose here is mainly to provide a benchmark for the

[6] However, it should be noted that the network effect itself is not enough to rule out partial learning. The result relies to some extent on the linearity of the production function with respect to the number agents in the network, and on the assumption of a steady population. Strict concavity in the production function, or a

coordination equilibria, and to develop the basic structure of the technology choice problem facing individual agents, we confine our attention to stationary equilibria.

The network effect in this economy means that all agents *within* a generation will make the same technology choice even in the absence of coordination.[7] This means that in any period there will only be two technologies in play: the old technology, learned by all current old agents when they were young (technology t), and the new technology, $t+1$. The choice problem for an old agent who enters period t knowing technology t is to retain technology t, or re-tool with technology $t+1$. The payoff from re-tooling for an old agent is $g^{t+1}x_t(t+1)(1-k)$; the payoff to retaining technology t is $g^t x_t(t)$. Thus, the old agent will choose to re-tool if and only if[8]

$$g \geq \frac{x_t(t)}{x_t(t+1)(1-k)}$$

The choice problem for a young agent in period t is to learn technology t, or learn technology $t+1$. In making that choice, the young agent is forward looking: he takes into account how his choice when young will affect the choices available and their associated payoffs when he is old.[9] The payoff to a young agent from learning technology t in period t is:

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difference in the size of old and young populations, could imply that Partial Learning 1 dominates Complete Learning if k is relatively large.

[7] To see this, consider a candidate equilibrium in which some old agents in period 1 retain technology $t=0$, and some re-tool and learn technology $t=1$. The returns to each choice for an old agent must be equal in the candidate equilibrium; but then a deviation by one old agent from $t=0$ to $t=1$ would raise the return to the $t=1$ technology, and reduce the return to the $t=0$ technology (because of the network effect), thereby making the deviation worthwhile. Any candidate equilibrium in which not all old agents make the same choice can be similarly broken. The same logic applies to choices by the young agents; all young agents make the same choice in equilibrium.

[8] We assume throughout that an agent who is indifferent between the new technology and the old technology will choose the new technology. It makes no difference if we assume the converse except along the boundaries of critical parameter regions.

[9] We assume that young agents cannot commit when young to a particular learning choice when old. This assumption has no bearing on the set of uncoordinated equilibria because each agent acting as an individual has no strategic power: the best response of young agents in period t to the learning choices of

$$v_t(t) = g^t x_t(t) + b \max\{g^t x_{t+1}(t), g^{t+1} x_{t+1}(t+1)(1-k), qg^{t+2} x_{t+1}(t+2)(1-k)\} \quad (3.1)$$

where $q \in \{0,1\}$ is an index such that $q = 1$ if at least some young agents in period t learn technology $t+1$, and $q = 0$ otherwise. (Recall that technology $t+2$ is only available if at least some young agents in period t learn technology $t+1$). The second term in (3.1) represents the payoff to the agent when old. It reflects the fact that when old in period t+1, this agent can choose to retain technology t that he learned when young, or re-tool with technology $t+1$, or re-tool with technology $t+2$, if it is available.

In comparison, the payoff to the young agent from learning technology $t+1$ in period t is

$$v_t(t+1) = g^{t+1} x_t(t+1) + b \max\{g^{t+1} x_{t+1}(t+1), g^{t+2} x_{t+1}(t+2)(1-k)\} \quad (3.2)$$

The second term in (3.2) reflects the fact that when old in period t+1, this agent can choose to retain technology $t+1$, or re-tool with technology $t+2$ (which will be available if the agent has chosen technology $t+1$ when young in period t). A young agent in period t will learn technology $t+1$ if and only if $v_t(t+1) \geq v_t(t)$; otherwise he will learn technology t.

The following proposition summarizes the stationary equilibria.[10]

Proposition 2

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the old in period t is invariant to a deviation by any individual old agent. However, the assumption is important for intragenerational coordination equilibria where young agents, acting as a collective, would have strategic power if commitment was possible.

(a) Eternal Stagnation is an uncoordinated equilibrium for all $g \geq 1$ and $k \in [0,1]$.

(b) Complete Learning is an uncoordinated equilibrium for all $g \geq 1$ and $k \in [0,1]$.

(c) Partial Learning 1 (where the young learn the new technology but the old do not re-tool) is an uncoordinated equilibrium if and only if

$$k > \frac{(1+bg^2)(N+1) - g(1+b)N}{bg^2(N+1)}$$

(d) Partial Learning 2 (where the young learn the old technology and the old re-tool) is not an equilibrium.

The most important feature of these equilibria, for the purposes of this paper, is co-existence. There exist at least two stationary equilibria everywhere in the parameter space and, if k is relatively large, there exist three stationary equilibria. This multiplicity of equilibria is due largely to the absence of coordination. The network effect means that a unilateral deviation by one agent from what all other agents are doing is never worthwhile. So if it happens that all agents but one make a particular technology choice, then the remaining agent cannot do better than to make that choice too. On the other hand, when there are two networks functioning simultaneously, one using the old technology and one using the new, a unilateral switch by one agent from one to the other can be worthwhile under some circumstances; this limits the parameter space in which partial learning can be an equilibrium.

3.2 Intragenerational Coordination

An equilibrium is an intragenerational coordination equilibrium if there does not exist a deviation strategy for any coalition of agents within a generation that yields a higher payoff to all members of the coalition than the equilibrium payoff. It is straightforward to show, and intuitively clear, that the network effect in this economy means that any coalition of young agents

[10] There also exist many non-stationary equilibria, in which technological change occurs in some periods and not in others. While these equilibria are interesting, we delay consideration of non-stationary equilibria until our discussion of coordinated equilibria, where the set to be considered is smaller.

will always do best by including all young agents, and any coalition of old agents will always do best by including all old agents. We therefore need to consider only two coalitions: one containing all young agents, and one containing all old agents. We can think of these two coalitions as two single players, each making a technology choice on behalf of all the agents within their coalition. We will henceforth refer to these two players as the "young coalition" and the "old coalition".[11]

It will prove useful to introduce some additional notation to describe the game between these two coalitions. Let $i_t \in \{0,1\}$ denote the technology choice for the young coalition in period *t*, where $i_t = 1$ indicates choosing the new technology, and $i_t = 0$ indicates choosing the old technology. Similarly, let $j_t \in \{0,1\}$ denote the technology choice for the old coalition in period t. We begin our analysis with a characterization of stationary equilibria, in which $\{i_t, j_t\} = \{i_{t+1}, j_{t+1}\} \quad \forall t$, and then consider nonstationary equilibria, in which $\{i_t, j_t\} \neq \{i_{t+1}, j_{t+1}\}$ for some t. We confine attention to deterministic equilibria.

(a) Stationary equilibria

The payoff to a representative member of the old coalition in period t if it chooses the old technology is $g^t[N + (1 - i_t)N]$. The payoff to a representative member if the old coalition re-tools with the new technology is $g^{t+1}(N + i_t N)(1 - k)$. Hence, the old coalition will choose to re-tool if and only if

[11] The game between these two players is not a simple two-person simultaneous move game. The payoff to the old coalition in period t depends only on the choices made by that old coalition and the young coalition in period t. However, the lifetime payoff to the young coalition in period t depends on the choices made in period t and on the choices made in period t+1 by itself (when old) and by the young coalition in period t+1. This part of the game has a sequential aspect to it. The young coalition in period t makes its choice in period t knowing that this choice will affect the choice set available to the young coalition in period t+1: if the young coalition in period t chooses technology t over technology $t+1$, then technology $t+2$ is not available to the young coalition in period t+1. It turns out that this has no bearing on the outcome of the

$$g \geq \frac{2-i_t}{(1+i_t)(1-k)}$$

The payoff to a representative member of the young coalition in period t, if it chooses the old technology is

$$v_t(t) = g^t[N + (1-j_t)N] + b \max\{g^t[N + (1-i_{t+1})N],\ g^{t+1}(N + i_{t+1}N)(1-k)\}$$

(Note that technology $t+2$ is not available in period t+1 if the young in period t choose $i_t = 0$). The payoff to a representative member if the young coalition in period t chooses the new technology is

$$v_t(t+1) = g^{t+1}(N + j_t N) + b \max\{g^{t+1}[N + (1-i_{t+1})N],\ g^{t+2}(N + i_{t+1}N)(1-k)\}$$

The young coalition will choose the new technology if and only if $v_t(t+1) \geq v_t(t)$.

The stationary intragenerational coordination equilibria in this economy are described in the following proposition, and illustrated in Figure 2.

Proposition 3

(a) Eternal Stagnation is an intragenerational coordination equilibrium if and only if

$$g < [2(1+b)/(1+2b)]$$

(b) Complete Learning is an intragenerational coordination equilibrium if and only if

$$g \geq [1/2(1-k)]$$

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game because the *ordering* of the payoffs to the old and new technologies in any period is independent of the history of the game.

(c) Partial Learning 1 (where the young learn the new technology but the old do not re-tool) is an intragenerational coordination equilibrium if and only if

$$g < [1/2(1-k)] \text{ and } g \geq [(2+b)/(1+b)]$$

(The region labeled "PL1" in figure 2).

(e) There does not exist a stationary intragenerational coordination equilibrium if

$$g < [1/2(1-k)] \text{ and } [2(1+b)/(1+2b)] \leq g < [(2+b)/(1+b)]$$

(The region labeled "Cycle" in figure 2).

These results have two noteworthy properties. First, the conditions for each equilibrium are more restrictive than in the absence of coordination. This reflects the network effect: if agents act as a coalition and so preserve the benefits of operating within a network, then there are more opportunities for deviation from a candidate equilibrium than for any single agent acting alone. This reduces the region of the parameter space in which multiple equilibria can occur. In particular, only when g and k are both small can multiple equilibria occur under intragenerational coordination, in which case both Eternal Stagnation and Complete Learning are equilibria (the bottom-left region of Figure 2). A higher value of g (for a given k) will cause the young to deviate from Eternal Stagnation, leaving Complete Learning as the only equilibrium; a higher of k (for a given g) will cause the old to deviate from Complete Learning, leaving Eternal Stagnation as the only equilibrium.

Second, no stationary equilibrium exists in the region of Figure 2 labeled "Cycle". In this region, g is too high to support Eternal Stagnation as an equilibrium because it is worthwhile for the *current* young to deviate from that candidate equilibrium and learn the new technology, if the *next* young are expected to learn that same technology, even if the current old do not re-tool. That is, $i_t = 1$ is the best response to $i_{t+1} = 0$ and $j_t = 0$. Thus, the current young are willing to sacrifice the benefits of working with a universal technology today *if* they expect the new technology they learn today will be used universally when they are old. However, this expectation cannot be fulfilled in a stationary equilibrium; the next young generation have exactly the same incentive to deviate from Eternal Stagnation. Similarly, Partial Learning 1 is not an equilibrium in this region because if the young in period t expect the young in period t+1 to learn the new technology in that period, then it is not worthwhile for the young in period t to learn the new technology, given that the old in period t do not re-tool. That is, in the "Cycle" region, the best response to $j_t = 0$ and $i_{t+1} = 1$, is $i_t = 0$; the $i_t = 1$ response can only be supported at a higher value of g (in which case Partial Learning 1 is the equilibrium). Thus, in the "Cycle" region, neither Eternal Stagnation nor Partial Learning 1 are equilibria.[12] What can be said about technological change in this region? As our labeling in Figure 2 suggests, this region has a nonstationary equilibrium. We discuss non-stationary equilibria next.

(b) Non-stationary equilibria

A non-stationary equilibrium is a time path of technology adoption choices that includes a sequence $\{i_t, j_t\} \rightarrow \{i_{t+1}, j_{t+1}\}$ such that $i_t \neq i_{t+1}$ and/or $j_t \neq j_{t+1}$. It turns out that there is only one such equilibrium. It is described in the following proposition.

Proposition 4

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[12] It is worth noting the role of the discount factor in this non-existence result. If $b = 0$ then the expected actions of the next young are completely irrelevant to the current young. Moreover, if $b = 0$ then the lifetime payoff from learning a more advanced technology when young is sharply reduced; given that the old do not retool, learning the old technology becomes a more attractive choice for the young. These results can be seen in Figure 2. As b approaches zero, the upper and lower boundaries of the "Cycle" region both approach $g = 2$; the "Cycle" region vanishes and the non-existence of stationary equilibria vanishes with it.

(a) Retention of the old technology by all old agents in every period, and adoption of the new technology by all young agents in every *second* period (Periodic Partial Learning) is an intragenerational coordination equilibrium if and only if

$$g < [1/2(1-k)] \text{ and } [2(1+b)/(1+2b)] \leq g < [(2+b)/(1+b)]$$

(The region labeled "Cycle" in figure 2).

(b) No other non-stationary equilibria exist.

The existence of an equilibrium with Periodic Partial Learning in the "Cycle" region of Figure 2 relates directly to the non-existence of a stationary equilibrium in that region. Recall that in the "Cycle" region, the optimal choice for the young coalition in period t is to learn the new technology $t+1$ when the old retain technology t, only if the young coalition in period t+1 also learn technology $t+1$. The cost of working in a smaller network when young is only justified by the benefits of using a more advanced technology if that technology is used within a universal network when old. So in order for the adoption of a new technology to be worthwhile, there must be a period of stagnation after its adoption in which that technology is used universally; otherwise those who adopt the new technology cannot reap a return high enough to warrant the initial opportunity cost.

3.3 Intergenerational Coordination without Transfers

We now consider the possibility of coordination within, and across, generations concurrently alive. We begin with coordination without transfers; in the next subsection we allow transfers between young and old. Intergenerational coordination (without transfers) allows two generations concurrently alive to choose one equilibrium from a set of multiple equilibria if both generations prefer it to all the others in the set. Therefore, the only region of the parameter space where full coordination can change the outcome relative to intragenerational coordination is in that region of Figure 2 where Eternal Stagnation and Complete Learning co-exist as intragenerational coordination equilibria. Proposition 5 summarizes this change.

Proposition 5

Intergenerational coordination equilibria without transfers coincide with the intragenerational equilibria described in Propositions 3 and 4, except that Eternal Stagnation is not a full coordination equilibrium where $g \geq [1/(1-k)]$.

Figure 3 illustrates the intergenerational coordination equilibria without transfers. Notice that this is identical to Figure 2, except for the partitioning of the bottom-left region by the $g = [1/(1-k)]$ line. In the region to the left of this line, k is small enough relative to g that both the old and the young prefer Complete Learning over Eternal Stagnation; that is, Complete Learning Pareto dominates Eternal Stagnation for those currently alive. For higher values of k (in the region to the right of the $g = [1/(1-k)]$ line), the young and old disagree over which outcome is better; the young prefer Complete Learning, while the old prefer Eternal Stagnation. The source of this intergenerational conflict is the cost to the old of re-tooling.[13]

3.4 Intergenerational Coordination with Transfers

Coordination on its own cannot eliminate the multiplicity of equilibria in this economy, because there is disagreement between young and old over which outcome is best. This raises a natural question: under what conditions can this conflict be overcome through transfers between the young and the old? That is, when can one generation "buy off" the other, and so move to its preferred outcome? The following proposition answers this question by characterizing the intergenerational coordination equilibria supported with transfers.

[13] Of course, once the current young become old, they too will prefer stagnation to re-tooling; but the discounted lifetime benefit of adopting the new technology when young more than offsets the anticipated future re-tooling costs associated with the Complete Learning equilibrium.

Proposition 6

(a) Eternal Stagnation is an intergenerational coordination equilibrium supportable with transfers if and only if

$$g < [(2+b)/(2+b-k)]$$

(b) Complete Learning is an intergenerational coordination equilibrium supportable with transfers if and only if $g \geq \hat{g}(k)$, where

$$\hat{g}(k) = \{g : bg^2(1-k) + g[1+(1-k)(1-b)] = 2\}$$

(c) Alternation between no learning and complete learning (Periodic Complete Learning) is an intergenerational coordination equilibrium supportable with transfers if and only if

$$[(2+b)/(2+b-k)] \leq g < \hat{g}(k)$$

(d) No other intergenerational coordination equilibria exist when transfers are possible.

Figure 4 illustrates the intergenerational coordination equilibria supported with transfers. These equilibria have two noteworthy properties. First, there is a unique equilibrium in every region of the parameter space; the multiplicity of equilibria observed under coordination without transfers is eliminated when transfers are possible.

Second, even with coordination supported with transfers, there still exists a nonstationary equilibrium with an endogenous cycle. In this cycle, all agents learn the new technology in one period, and no agents learn the new technology in the next. This cycle is importantly distinct from the cycle that can occur in coordinated equilibria without transfers. In that case the old never re-tool; the learning of the new technology alternates only between one young generation and the next. This means that in every second period the young and the old use different technologies. This splitting of the economy into two competing networks never occurs in coordinated equilibria with transfers: all agents alive in any given period use the same technology. The transfer opportunity between young and old ensures that the productivity benefits of a universal network are fully exploited.

While the nature of the cycle, when transfers are allowed, is distinct from that when they are not, the source of the cycle is essentially the same. For values of g between the two threshold lines in figure 4, re-tooling by the old is to their advantage only if they are partly compensated by the young for the costs of that re-tooling. It is in the interests of the young to make that compensating transfer to the old only if they do not have to incur re-tooling costs when they are old themselves. That is, investing in the new technology, and paying off the old, is worthwhile for the young only if a period of stagnation follows in which the rewards of that investment can be reaped when they are old.

If g is high enough then investment in the new technology becomes worthwhile even in the absence of a respite from change. Thus, above the upper threshold in figure 4, the equilibrium has Complete Learning. Conversely, if g is too low then the gains associated with the new technology are not worth the costs of change; thus, equilibrium below the lower threshold in figure 4 involves Eternal Stagnation.

Before comparing the various equilibria with the surplus maximizing allocations, it is interesting to compare briefly the equilibrium growth rates with and without transfers. Allowing transfers between generations can lead to either an increase or decrease in the equilibrium rate of progress. In particular, there are circumstances under which young agents can compensate old agents for the costs of learning, and thereby gain their support for technological change. Of course, the other side of this coin is the possibility of a slowdown: old agents can "bribe" young agents not to adopt new technologies, and thereby slow down the pace of technological change to their mutual benefit.[14]

4. Equilibrium and Surplus-Maximizing Allocations

It is important to recognize that while all *living* agents would benefit (or at least do no worse) if able to coordinate, future agents may suffer. In particular, future agents are always made worse off if the current rate of technological progress decreases and better off if the current rate of progress increases. These effects on future generations do not enter the calculus of equilibrium. Thus, there is no reason to expect that equilibrium and surplus-maximizing allocations will necessarily coincide. In this section we characterize the circumstances under which they do and don't, and the role played by coordination and transfers.

Uncoordinated Equilibria

Recall that, according to Proposition 1, surplus maximization requires Complete Learning when g is high relative to k and Eternal Stagnation otherwise. This is illustrated in Figure 1. According to Proposition 2, in the absence of any coordination, both Complete Learning and Eternal Stagnation are equilibria everywhere in the parameter space. (Other equilibria also exist.) Hence, in the absence of coordination, agents in the economy may be *lucky* in the sense that their equilibrium maximizes surplus, or *unlucky* if not. At any point in the parameter space above the demarcation line in Figure 1, Complete Learning maximizes surplus. Complete Learning is also an uncoordinated equilibrium at that point. However, other (suboptimal) equilibria also exist. For example, Eternal Stagnation could occur where Complete

[14] It is straightforward to show that $g = \hat{g}(k)$ intersects $g = [1/2(1-k)]$ at $g = [(3+b)/(2+b)]$. This intersection lies strictly below $g = [2(1+b)/(1+2b)]$ for $b < 1$, and coincides with that line when $b = 1$. Thus, there exist regions with both faster and slower growth with transfers.

Learning maximizes surplus if individuals choose technologies in an uncoordinated way. Similarly, Complete Learning could occur when Eternal Stagnation maximizes surplus.

Intergenerational coordination without transfers

When coordination is allowed, without transfers, we have unique equilibria in some, but not all, regions of the parameter space. (This is summarized by Propositions 3, 4, 5, and represented in Figure 3.) Figure 5 combines Figures 1 and 3, to represent both surplus-maximizing solutions and coordination equilibria without transfers on the same diagram. In the unshaded regions of this diagram, unique equilibria exist which also maximize surplus. For example, in region I, Complete Learning maximizes surplus and is the unique equilibrium. Similarly, in region V, Eternal Stagnation maximizes surplus and is the unique equilibrium. Hence, in these unshaded regions, introducing coordination among agents currently alive would never reduce surplus, and may increase it if the original equilibrium without coordination was not the surplus-maximizing one.

However, in this case of coordination without transfers, there also exist regions of the parameter space in which the unique equilibrium does not maximize surplus. For example, in regions II, III and IV in Figure 5, Complete Learning maximizes surplus but is not an equilibrium. In region IV, Eternal Stagnation is the only equilibrium. Similarly, in region III, the unique equilibrium has a cycle, and in region II, the unique equilibrium has only partial learning. In each of these three regions, the introduction of coordination among agents currently alive may actually *reduce* surplus if the uncoordinated equilibrium was Complete Learning. In this case, coordination would also reduce the rate of economic progress in regions III and IV. In region III it would induce an inefficient cycle; in region IV it would stop progress altogether.

In regions VI and VII in Figure 5, both Complete Learning and Eternal Stagnation exist as equilibria, (these are the only two equilibria). In region VII, Complete Learning maximizes surplus. In region VI, Eternal Stagnation maximizes surplus. As with the uncoordinated equilibria, agents in economies such as this may or may not be lucky enough to be in the surplus-maximizing equilibrium. Introducing coordination in economies such as this may change the nature of the equilibrium (if the uncoordinated equilibrium was anything other than Complete Learning or Eternal Stagnation) but would not necessarily lead to the surplus-maximizing allocation.

Intergenerational coordination with transfers

When both coordination and costless transfers are allowed, as characterized in Proposition 6 and Figure 4, each point in the parameter space has a corresponding unique equilibrium. Figure 6 combines Figures 1 and 4 to compare surplus-maximizing with equilibrium allocations in this case. In the unshaded regions, once again, the unique equilibrium allocations maximize surplus. In these regions, allowing both coordination and transfers among agents currently alive would never reduce surplus, and may increase it if the original equilibrium without coordination or transfers was not the surplus-maximizing one. It is also straightforward to show that there exists a region in which two equilibria exist in the absence of transfers (Complete Learning and Eternal Stagnation) but only the Complete Learning (the surplus-maximizing) equilibrium exists when transfers are allowed. Thus, allowing transfers in this region could move the economy from suboptimal Eternal Stagnation to surplus-maximizing Complete Learning.

In the two shaded regions of Figure 6, however, the unique equilibrium allocation is suboptimal and the rate of progress is too slow. In both regions, Complete Learning maximizes surplus. In the lower region Eternal Stagnation is the unique equilibrium outcome. In the upper region, the unique equilibrium has the cycle described in section 3.4. In both of these two regions, in the absence of transfers or coordination, Complete Learning also exists as an equilibrium. Thus, if an economy has parameter values in this region, and if the agents in this economy are lucky enough to have Complete Learning as their equilibrium in the absence of coordination or transfers, then allowing the introduction of coordination with transfers would move the economy away from a surplus-maximizing equilibrium to an equilibrium with a slower rate of economic progress.

Intuitively, the overlapping generations structure of the model is responsible for the existence of regions in the parameter space where the coordinated equilibrium with transfers is suboptimal. The source of the problem is that the benefits accruing to future agents when technology progresses today are not taken into account by agents currently living when technology adoption choices are made. Note that, in contrast to the equilibrium without transfers, there are no circumstances under which the equilibrium with transfers can involve too much progress. The reason is straightforward: too much progress is only ever a problem for agents currently living (since future agents always benefit from progress today); any progress that is not in the collective interests of agents currently living is eliminated when those agents can coordinate and exchange transfers.

The fact that future agents would always benefit from an increased rate of progress, while agents currently living may prefer a slower rate of progress, or no progress at all, raises a natural question: can future agents compensate current agents in return for a faster rate of growth? Any such compensation scheme must, by necessity, involve passing obligations from current generations to future ones; that is, debt.

The size of any debt payments from young agents to old agents in any given period is constrained in this economy by the aggregate product of the young. That is, the young cannot transfer more to the old than they produce. This means that it may not be possible to implement the surplus-maximizing solution as a Pareto improvement over the coordination equilibrium with transfers, even when this equilibrium fails to maximize surplus. That is, even if the benefits to future agents outweigh the costs to agents currently alive, it may not be feasible to translate that *potential* Pareto improvement into an actual Pareto improvement. The following proposition describes the role for Pareto-improving debt in this economy.

Proposition 7

Debt allows the surplus-maximizing solution to be implemented as a Pareto improvement over the coordination equilibrium with transfers if and only if

$$[(2+b)/(2+b-k)] \leq g < \hat{g}(k);$$

that is, if and only if the equilibrium is cyclical.

There are two points to note in relation to this result. First, the result indicates that there is a role for debt to facilitate an *increase* in the rate of economic progress. That role stems from the fact that future generations are significant beneficiaries of any technological progress today, but the cost of that progress is borne by agents currently living. Debt allows those costs and benefits to be spread more evenly across generations. Second, the role for debt is limited to instances where equilibria would otherwise involve a cycle; debt cannot implement the surplus-maximizing solution if the coordination equilibrium with transfers involves Eternal Stagnation. The reason is that the productivity increment from technological change, in the region where Eternal Stagnation is a coordination equilibrium with transfers, is not high enough to facilitate the necessary transfers. While a move from Eternal Stagnation to Complete Learning would in some circumstances yield a higher total surplus, current agents would be made worse off by such a move, and there is no way to compensate them.

5. Conclusions

In this simple framework with network effects, only two possible allocations maximize surplus: Complete Learning, where all agents learn the new technology in every period, and Eternal Stagnation, where nobody learns. The appropriate choice depends on the relative costs and benefits of learning but, in either case, the network effect implies that all agents use the same technology in every period. The network effect also implies that equilibrium allocations can have too much, too little, or just the right amount of progress relative to the surplus-maximizing solution. By examining degrees of coordination, however, we can say more than this.

The problem of too much progress exists only in the absence of coordination or transfers. With no coordination or transfers, multiple equilibria exist everywhere in the parameter space. For example, where Eternal Stagnation maximizes surplus, both Eternal Stagnation and Complete Learning exist as equilibria. Allowing coordination among agents, but not transfers, eliminates the multiplicity of equilibria in many, but not all, regions of the parameter space. Wherever equilibria are unique in this case, there is never too much progress in equilibrium. However, there still exists a region in which Eternal Stagnation maximizes surplus but both Eternal Stagnation and Complete Learning are equilibria. Thus, too much progress can still occur in equilibrium in that region. Allowing both coordination and transfers eliminates the multiplicity of equilibria entirely. In this case, there exists a unique equilibrium at every point in the parameter space. Moreover, at no equilibrium is progress excessive.

The problem of too little progress can be removed with coordination and transfers in some, but not all, regions of the parameter space. With no coordination at all, Eternal Stagnation (for example) exists as an equilibrium everywhere in the parameter space, including the region where Complete Learning maximizes surplus. With coordination but no transfers, some regions of the parameter space have unique equilibria. In some of these regions, the equilibrium maximizes surplus. However, in some others, the unique equilibrium allocation has too little learning. This can take the form of Eternal Stagnation, a cycle, or the right rate of progress but with not enough agents learning. Moreover, the introduction of coordination could actually *worsen* the outcome, by moving the economy from a lucky uncoordinated equilibrium to a suboptimal unique one. Also, with coordination but no transfers, there still exist some regions in the parameter space where Complete Learning maximizes surplus but both Complete Learning and Eternal Stagnation exist as equilibria.

When both coordination and transfers are allowed, although each point in the parameter space has an associated unique equilibrium, this does not eliminate the problem of too little learning in certain regions of the parameter space. In this case, there exist regions where Complete Learning maximizes surplus but the unique equilibrium is cyclical. Also, regions exist where Complete Learning maximizes surplus but the only equilibrium has Eternal Stagnation. Again, in these regions, the introduction of coordination and transfers may actually worsen the equilibrium allocation if the uncoordinated equilibrium happened to maximize surplus.

Going further, allowing transfers across time through debt, eliminates the cyclical equilibria. In this case, each point in the parameter space has a unique equilibrium of either one of two types: Complete Learning or Eternal Stagnation. The region with a cyclical equilibrium, in the absence of debt, now has Complete Learning as its only equilibrium, which also maximizes surplus. Thus, in this region, introducing debt may facilitate a Pareto-improving move from a cycle to steady progress with Complete Learning. Unfortunately, however, there still exists a region where the unique equilibrium has too little progress. In this region, Complete Learning maximizes surplus, but the equilibrium condemns all agents to Eternal Stagnation.

Appendix

Proof of Proposition 1

By equations (2.2) and (2.3), $S(PL1) > S(PL2)$ everywhere in the parameter space. By equations (2.2) and (2.4), $S(CL) > S(PL1)$ everywhere in the parameter space. By equations (2.1) and (2.4), $S(CL) > S(ES)$ if and only if the condition in the proposition holds. ■

Proof of Proposition 2

(a) The payoff to an old agent in period t in the candidate equilibrium is $g^{t}2N$. The payoff to an old agent from deviating unilaterally and re-tooling with the new technology is zero. Therefore, no old agent will deviate unilaterally from the candidate equilibrium.

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t) = g^{t}2N + bg^{t}2N$$

The payoff to a young agent from deviating unilaterally and learning the new technology in period t is

$$v_t(t+1) = 0 + bg^{t}2N(1-k)$$

where the payoff to this agent in period t+1 reflects the fact that his best response to the candidate equilibrium choices in period t+1, after having deviated in period t, is to re-tool with the *old* technology. Any other choice yields a payoff of zero. Since $v_t(t) > v_t(t+1)$, no young agent will deviate unilaterally from the candidate equilibrium.

(b) The payoff to an old agent in period t in the candidate equilibrium is $g^{t+1}2N(1-k)$. The payoff to an old agent from deviating unilaterally and retaining the old technology is zero. Therefore, no old agent will deviate unilaterally from the candidate equilibrium.

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t+1) = g^{t+1}2N + bg^{t+2}2N(1-k)$$

The payoff to a young agent from deviating unilaterally in period t and learning the old technology is

$$v_t(t) = 0 + bg^{t+2}2N(1-k)$$

where the payoff to this agent in period t+1 reflects the fact that his best response to the candidate equilibrium choices in period t+1, after having deviated in period t, is to re-tool with the new technology. Any other choice yields a payoff of zero. Since $v_t(t+1) > v_t(t)$, no young agent will deviate unilaterally from the candidate equilibrium.

(c) The payoff to an old agent in period t in the candidate equilibrium is $g^t N$. The payoff to an old agent from deviating unilaterally and learning the new technology is

$$g^{t+1}(N+1)(1-k)$$

Therefore, the payoff to the old agent is greater in the candidate equilibrium if $k > k_o$, where k_o is given by

$$k_o = \frac{(N+1)g - N}{(N+1)g}$$

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t+1) = g^{t+1}N + bg^{t+1}N$$

The payoff to a young agent from deviating unilaterally in period t and learning the old technology is

$$v_t(t) = g^t(N+1) + b\max\{g^{t+1}N(1-k),\, g^{t+2}(N+1)(1-k)\}$$

where the payoff to this agent when old in period t+1 reflects the fact that his best response to the candidate equilibrium choices in period t+1 is either to retain technology $t+1$ (and network with the old), or re-tool with technology $t+2$ (and network with the young). Choosing an earlier technology yields a payoff of zero. The second choice in period t+1 clearly dominates the first, so $v_t(t)$ becomes

$$v_t(t) = g^t(N+1) + bg^{t+2}(N+1)(1-k)$$

The payoff in the candidate equilibrium is greater than the payoff from deviating if and only if $u_t(t+1) > u_t(t)$, which in turn implies that $k > k_y$, where k_y is given by

(A1) $$k_y = \frac{(N+1)(1+bg^2)-(1+b)Ng}{bg^2(N+1)}$$

Thus, young agents will not deviate from the candidate equilibrium if and only if $k > k_y$, and old agents will not deviate if and only if $k > k_o$. Since $k_y > k_o$, it follows that no agent will deviate from the candidate equilibrium if and only if $k > k_y$.

(d) The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t) = g^t N + bg^{t+1}N(1-k)$$

The payoff to a young agent from deviating unilaterally in period t and learning the new technology is

$$v_t(t+1) = g^{t+1}(N+1) + b\max\{g^t N(1-k), g^{t+1}N\}$$

where the payoff to this agent when old in period t+1 reflects the fact that his best response to the candidate equilibrium choices in period t+1 is either to re-tool with the *old* technology (and network with the young in period t+1), or retain technology $t+1$ (and network with the old). Any other choice yields a payoff of zero. It is clear that $v_t(t+1) > v_t(t)\ \ \forall g > 1$. Thus, a young agent will always choose to deviate unilaterally from the candidate equilibrium. Therefore, it cannot be an equilibrium. ■

Proof of Proposition 3

(a) The payoff to an old agent in period t in the candidate equilibrium is $g^t 2N$. The payoff to an old agent if the old coalition deviates and re-tools with the new technology is $g^{t+1}N(1-k)$. Therefore the old coalition will not deviate from the candidate equilibrium if and only if

(A2) $$g < [2/(1-k)]$$

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t) = g^t 2N + bg^t 2N$$

The payoff to a young agent if the young coalition deviates and learns the new technology in period t is

$$v_t(t+1) = g^{t+1}N + b\max\{g^{t+1}2N, g^{t+2}N(1-k)\}$$

where the payoff to this agent when old in period t+1 reflects the fact that the best response for the old coalition in period t+1 to the candidate equilibrium choice of the young coalition in period t+1 (which is to learn technology $t+1$)[15], is either to retain technology $t+1$, or re-tool with technology $t+2$. The deviation is worthwhile for the young in period t if and only if $v_t(t+1) \geq v_t(t)$. Since $g < [2/(1-k)]$, or else the old in period t will deviate, it follows that

$$\max\{g^{t+1}2N, g^{t+2}N(1-k)\} = g^{t+1}2N$$

So the young in period t will not deviate from the candidate equilibrium if and only if

(A3) $\quad g > [2(1+b)/(1+2b)]$

If condition (A3) holds, then condition (A2) also holds $\forall k \in [0,1]$. So condition (A3) is necessary and sufficient for existence of the candidate equilibrium.

(b) The payoff to an old agent in period t in the candidate equilibrium is $g^{t+1}2N(1-k)$. The payoff to an old agent if the old coalition deviates and retains the old technology is $g^t N$. Therefore the old coalition will not deviate from the candidate equilibrium if and only if

(A4) $\quad g \geq [1/2(1-k)]$

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t+1) = g^{t+1}2N + bg^{t+2}2N(1-k)$$

[15] Note that if the young in period t deviate from the candidate equilibrium and learn technology $t+1$ (which is the new technology in period t), then the old technology in period t+1 will be technology $t+1$. This old technology is the technology that the young in period t+1 learn in the candidate equilibrium.

The payoff to a young agent if the young coalition deviates and learns the old technology in period t is

$$v_t(t) = g^t N + b \max\{g^t N, g^{t+1} 2N(1-k)\}$$

where the payoff to this agent when old in period t+1 reflects the fact that the best response for the old coalition in period t+1 to the candidate equilibrium choice of the young coalition in period t+1 (which is to learn technology $t+1$), is either to retain the old technology t, or re-tool with the new technology $t+1$. The deviation is worthwhile for the young in period t if and only if $v_t(t) > v_t(t+1)$. Since $g \geq [1/2(1-k)]$, or else the old in period t will deviate, it follows that

$$\max\{g^t N, g^{t+1} 2N(1-k)\} = g^{t+1} 2N(1-k)$$

Thus, $v_t(t+1) \geq v_t(t) \quad \forall g \geq 1$ and $k \in [0,1]$. That is, the young will never deviate when condition (A4) holds. Therefore, condition (A4) is necessary and sufficient for existence of the candidate equilibrium.

(c) The payoff to an old agent in period t in the candidate equilibrium is $g^t N$. The payoff to an old agent if the old coalition deviates and re-tools with the new technology is $g^{t+1} 2N(1-k)$. Therefore the old coalition will not deviate from the candidate equilibrium if and only if

(A5) $\qquad g < [1/2(1-k)]$

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t+1) = g^{t+1} N + bg^{t+1} N$$

The payoff to a young agent if the young coalition deviates and learns the old technology in period t is

$$v_t(t) = g^t 2N + b \max\{g^t N, g^{t+1} 2N(1-k)\}$$

where the payoff to this agent when old in period t+1 reflects the fact that the best response for the old coalition in period t+1 to the candidate equilibrium choice of the young coalition in period t+1 (which is to learn technology $t+1$), is either to retain the old technology t, or re-tool with the new technology $t+1$. The deviation is worthwhile for the young in period t if and only if $v_t(t) > v_t(t+1)$. Since $g < [1/2(1-k)]$, or else the old in period t will deviate, it follows that

$$\max\{g^t N, g^{t+1} 2N(1-k)\} = g^t N$$

So the young in period t will not deviate from the candidate equilibrium if and only if

(A6) $$g \geq [(2+b)/(1+b)]$$

So conditions (A5) and (A6) are necessary and sufficient for existence of the candidate equilibrium.

(d) The payoff to an old agent in period t in the candidate equilibrium is $g^{t+1}N(1-k)$. The payoff to an old agent if the old coalition deviates and retains the old technology is $g^t 2N$. Therefore the old coalition will not deviate from the candidate equilibrium if and only if

$$g < [2/(1-k)]$$

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t) = g^t N + bg^{t+1} N(1-k)$$

The payoff to a young agent if the young coalition deviates and learns the new technology in period t is

$$v_t(t+1) = g^{t+1} 2N + b \max\{g^{t+1} 2N, g^{t+2} N(1-k)\}$$

where the payoff to this agent when old in period t+1 reflects the fact that the best response for the old coalition in period t+1 to the candidate equilibrium choice of the young coalition in period t+1 (which is to learn technology $t+1$)[16], is either to retain technology $t+1$, or re-tool with technology $t+2$. The deviation is worthwhile for the young in period t if and only if

[16] See previous footnote.

$v_t(t+1) \geq v_t(t)$. Since $g < [2/(1-k)]$, or else the old in period t will deviate, it follows that

$$\max\{g^{t+1}2N, g^{t+2}N(1-k)\} = g^{t+2}N(1-k)$$

Thus, $v_t(t+1) \geq v_t(t)$ $\forall g \geq 1$ and $k \in [0,1]$. Therefore, the young will always deviate from the candidate equilibrium. Therefore, it cannot be an equilibrium.

(e) This result follows directly from parts (a) to (d). ■

Proof of Proposition 4

(a) The candidate equilibrium comprises the sequence $\{1,0\} \rightarrow \{0,0\} \rightarrow \{1,0\} \rightarrow \{0,0\}$ etc. To fix period notation, let $\{i_t, j_t\} = \{1,0\}$. We need to show that young and old agents in period t, and young and old agents in period t+1, will not deviate from the candidate equilibrium. (The choice problems for agents in period t+l and t+l+1 are equivalent to those in t and t+1 $\forall l \geq 2$).

The payoff to an old agent in period t in the candidate equilibrium is $g^t N$. The payoff to an old agent in period t if the old coalition deviates in period t and re-tools with the new technology is $g^{t+1}2N(1-k)$. Therefore, the old coalition in period t will not deviate from the candidate equilibrium if and only if

(A7) $\quad g < [1/2(1-k)]$

The payoff to an old agent in period t+1 in the candidate equilibrium is $g^{t+1}2N$. The payoff to an old agent in period t+1 if the old coalition deviates in period t+1 and re-tools with the new technology is $g^{t+2}N(1-k)$. Therefore, the old coalition in period t+1 will not deviate from the candidate equilibrium if and only if $g < [2/(1-k)]$. This condition holds if condition (A7) holds.

The lifetime payoff to a young agent in period t in the candidate equilibrium is

$$v_t(t+1) = g^{t+1}N + bg^{t+1}2N$$

The payoff to a young agent in period t if the young coalition in period t deviates and learns the old technology in period t is

$$v_t(t) = g^t 2N + b \max\{g^t 2N, g^{t+1} N(1-k)\}$$

The deviation is worthwhile for the young in period t if and only if $v_t(t) > v_t(t+1)$. Since $g < [2/(1-k)]$, or else the old in period t+1 will deviate, it follows that

$$\max\{g^t 2N, g^{t+1} N(1-k)\} = g^t 2N$$

So the young in period t will not deviate from the candidate equilibrium if and only if

(A8) $$g \geq [2(1+b)/(1+2b)]$$

The lifetime payoff to a young agent in period t+1 in the candidate equilibrium is

$$v_{t+1}(t+1) = g^{t+1} 2N + bg^{t+1} N$$

The lifetime payoff to a young agent in period t+1 if the young coalition in period t+1 deviates and learns the new technology in period t+1 is

$$v_{t+1}(t+2) = g^{t+2} N + b \max\{g^{t+2} N, g^{t+3} 2N(1-k)\}$$

The deviation is not worthwhile for the young in period t+1 if and only if $v_{t+1}(t+1) \geq v_{t+1}(t+2)$. If condition (A7) holds, then

$$\max\{g^{t+2} N, g^{t+3} 2N(1-k)\} = g^{t+2} N$$

So the young in period t+1 will not deviate from the candidate equilibrium if and only if

(A9) $$g < [(2+b)/(1+b)]$$

Thus, conditions (A7), (A8) and (A9) are necessary and sufficient for existence of the candidate equilibrium.

(b) The complete proof is long and tedious; we provide only an outline of it here. A sequence of technology adoption choices can be an equilibrium if and only if every three-period subsequence of the form $\{i_t, j_t\} \rightarrow \{i_{t+1}, j_{t+1}\} \rightarrow \{i_{t+2}, j_{t+2}\}$ that forms part of the candidate equilibrium sequence, is itself an *equilibrium* subsequence. There are 64 distinct three-period subsequences to consider. We examined each one, and found that only 32 of these can be equilibrium subsequences. We then examined which of these three-period subsequences could be joined to form a five-period subsequence such that every three-period subsequence in that five-period subsequence could simultaneously be an equilibrium subsequence. By proceeding in this way, we were able to eliminate all non-stationary sequences except the one identified in part (a) of the proposition.

We first provide an example of how a candidate three-period subsequence can be eliminated as part of an equilibrium sequence. We then provide an example of how a five-period subsequence can be eliminated.

Consider a three-period subsequence $A = \{0,0\} \rightarrow \{1,1\} \rightarrow \{1,0\}$. To fix notation, let the first period in this subsequence be period t, and let the old technology in that period be t. Then the payoff to an old agent in period t+1 in subsequence A is $g^{t+1} 2N(1-k)$. The payoff to an old agent in period t+1 if the old coalition deviates in period t+1 and retains the old technology is $g^{t} N$. Therefore, the old coalition in period t+1 will not deviate from subsequence A if and only if

(A10) $g \geq [1/2(1-k)]$

The payoff to an old agent in period t+2 in subsequence A is $g^{t+1} N$. The payoff to an old agent in period t+2 if the old coalition deviates in period t+1 and re-tools with the new technology is $g^{t+2} 2N(1-k)$. Therefore, the old coalition in period t+2 will not deviate from the candidate equilibrium if and only if

(A11) $g < [1/2(1-k)]$

Conditions (A10) and (A11) contradict; so subsequence A cannot from part of am equilibrium.

Next consider $B = \{1,0\} \rightarrow \{0,0\} \rightarrow \{1,1\}$ and $C = \{1,1\} \rightarrow \{1,0\} \rightarrow \{0,0\}$. It can be shown that both B and C can be equilibrium subsequences. Now suppose A and B are joined to from a five period subsequence $D = \{1,0\} \rightarrow \{0,0\} \rightarrow \{1,1\} \rightarrow \{1,0\} \rightarrow \{0,0\}$. This contains subsequence A, which cannot be an equilibrium subsequence. ■

Proof of Proposition 5

By Propositions 3 and 4, multiple intragenerational coordination equilibria exist only where $g < [2(1+b)/(1+2b)]$ and $g \geq [1/2(1-k)]$. Thus, only in this region can additional coordination eliminate equilibria. In this region, by Proposition 3, there are exactly two equilibria: Eternal Stagnation, and Complete Learning. In any period, given the old technology t, the payoff to each old agent under Eternal Stagnation is $g^{t}2N$, and under Complete Learning is $g^{t+1}2N(1-k)$. Comparing these two payoffs, the old agents would prefer Complete Learning if and only if

(A12) $\qquad g \geq [1/(1-k)]$

In any period, given the old technology t, the payoff to each young agent under Eternal Stagnation is $(1+b)g^{t}2N$, and under Complete Learning is $g^{t+1}2N + bg^{t+2}2N(1-k)$. Comparing these two payoffs, the young agents prefer Complete Learning if and only if

(A13) $\qquad g + bg^{2}(1-k) \geq 1+b$

Since (A12) is sufficient for (A13), and since full coordination requires agreement, the result follows immediately. ■

Proof of Proposition 6

We begin by deriving the best technology choices for agents alive in period t in response to each of the possible expected choices of agents alive in period t+1. These are described in the following four lemmas.

Lemma 1. If agents alive in period t expect $\{i_{t+1}, j_{t+1}\} = \{0,0\}$, then the best choice for agents in period t is

(i) $\{i_t, j_t\} = \{0,0\}$ if and only if $g < [(2+b)/(2+b-k)]$

(ii) $\{i_t, j_t\} = \{1,1\}$ if and only if $g \geq [(2+b)/(2+b-k)]$

Proof. We begin by showing that under $\{i_t, j_t\} = \{1,1\}$, old agents in period t can be at least as well-off, and young agents in period t can be strictly better-off, than under $\{i_t, j_t\} = \{1,0\}$ or $\{i_t, j_t\} = \{0,1\}$, $\forall g > 1$ and $k \leq 1$.

First, consider the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{1,0\}$. The payoff to an old agent in period t under $\{i_t, j_t\} = \{1,0\}$ is $g^t N$. To receive this same payoff under $\{i_t, j_t\} = \{1,1\}$, old agents would have to receive a transfer T from the young:

$$T = g^t N - g^{t+1} 2N(1-k)$$

(Note that T could be negative).

The lifetime payoff to a young agent in period t under $\{i_t, j_t\} = \{1,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$, is

$$v_t(\{1,0\} \to \{0,0\}) = g^{t+1} N + bg^{t+1} 2N$$

The lifetime payoff to a young agent in period t under $\{i_t, j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \to \{0,0\}) = g^{t+1} 2N + bg^{t+1} 2N$$

It is *feasible* for the young to transfer T to the old, since $g^{t+1} 2N \geq T \;\; \forall g > 1$ and $k \leq 1$. So old agents can be at least as well-off, and young agents strictly better-off under $\{i_t, j_t\} = \{1,1\}$ relative to $\{i_t, j_t\} = \{1,0\}$ if and only if

$$v_t(\{1,1\} \to \{0,0\}) - T \geq v_t(\{1,0\} \to \{0,0\})$$

This condition is satisfied $\forall g > 1$ and $k \leq 1$.

Next consider the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,1\}$. The payoff to an old agent in period t under $\{i_t, j_t\} = \{0,1\}$ is $g^{t+1} N(1-k)$. The payoff to an old agent under $\{i_t, j_t\} = \{1,1\}$ is $g^{t+1} 2N(1-k)$. Therefore, old agents are better-off under $\{i_t, j_t\} = \{1,1\}$ $\forall g > 1$ and $k \leq 1$.

The lifetime payoff to a young agent in period t under $\{i_t, j_t\} = \{0,1\}$ when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$, is

$$v_t(\{0,1\} \to \{0,0\}) = g^t N + bg^t 2N$$

The lifetime payoff to a young agent in period t under $\{i_t,\ j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \to \{0,0\}) = g^{t+1}2N + bg^{t+1}2N$$

Therefore, young agents are also better-off under $\{i_t,\ j_t\} = \{1,1\}$ $\forall g > 1$ and $k \leq 1$.

Since $\{i_t, j_t\} = \{0,1\}$ and $\{i_t, j_t\} = \{1,0\}$ are both dominated $\{i_t, j_t\} = \{1,1\}$, we can focus on the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$. The payoff to an old agent in period t under $\{i_t, j_t\} = \{0,0\}$ is $g^t 2N$. To receive this same payoff under $\{i_t, j_t\} = \{1,1\}$, old agents would have to receive a transfer T' from the young:

$$T' = g^t 2N - g^{t+1}2N(1-k)$$

(Note that T' could be negative).

The lifetime payoff to a young agent under $\{i_t, j_t\} = \{0,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$ is

$$v_t(\{0,0\} \to \{0,0\}) = g^t 2N + bg^t 2N$$

The lifetime payoff to a young agent under $\{i_t,\ j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \to \{0,0\}) = g^{t+1}2N + bg^{t+1}2N$$

It is *feasible* for young agents to transfer T' to the old, since $g^{t+1}2N \geq T'$ $\forall g > 1$ and $k \leq 1$. So old agents can be at least as well-off, and young agents better-off, under $\{i_t,\ j_t\} = \{1,1\}$ if and only if

$$v_t(\{1,1\} \to \{0,0\}) - T' \geq v_t(\{0,0\} \to \{0,0\})$$

This is the condition in lemma 1. ■

Lemma 2. If agents alive in period t expect $\{i_{t+1},\ j_{t+1}\} = \{1,1\}$, then the best choice for agents in period t is

(i) $\{i_t,\ j_t\} = \{0,0\}$ if and only if $bg^2(1-k) + g(2 + bk - k - b) < 2$

(ii) $\{i_t,\ j_t\} = \{1,1\}$ if and only if $bg^2(1-k) + g(2 + bk - k - b) \geq 2$

Proof. Using the same approach as for the first part of lemma 1, it is straightforward to show that under $\{i_t,\ j_t\} = \{1,1\}$, old agents in period t can be at least as well-off, and young agents strictly better-off, than under $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{1,0\}$ $\forall g > 1$ and $k \le 1$. We therefore focus on the choice between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$.

Note that the expected choices of agents in period t+1 have no direct effect on the old in period t. Their payoffs under $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{1,1\}$ are the same as when $\{i_{t+1}, j_{t+1}\} = \{0,0\}$. So from the proof of lemma 1, we know that old agents are indifferent between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$ if under $\{i_t, j_t\} = \{1,1\}$ they receive a transfer T' from the young:

$$T' = g^t 2N - g^{t+1} 2N(1-k)$$

(Again note that T' could be negative).

The lifetime payoff to a young agent under $\{i_t, j_t\} = \{0,0\}$ when $\{i_{t+1}, j_{t+1}\} = \{1,1\}$ is

$$v_t(\{0,0\} \to \{1,1\}) = g^t 2N + bg^{t+1} 2N(1-k)$$

The lifetime payoff to a young agent under $\{i_t, j_t\} = \{1,1\}$ is

$$v_t(\{1,1\} \to \{1,1\}) = g^{t+1} 2N + bg^{t+2} 2N(1-k)$$

Therefore, old agents can be at least as well-off, and young agents strictly better-off, under $\{i_t, j_t\} = \{1,1\}$ if and only if

$$v_t(\{1,1\} \to \{1,1\}) - T' \ge v_t(\{0,0\} \to \{1,1\})$$

This is the condition in lemma 2. ■

Lemma 3. If agents alive in period t expect $\{i_{t+1}, j_{t+1}\} = \{1,0\}$, then the best choice for agents in period t is either $\{i_t, j_t\} = \{1,1\}$ or $\{i_t, j_t\} = \{0,0\}$.

Proof. Using the same approach as for the first part of lemma 1, it is straightforward to show that under $\{i_t, j_t\} = \{1,1\}$, old agents in period t can be at least as well-off, and young agents strictly better-off, as under $\{i_t, j_t\} = \{0,1\}$ or $\{i_t, j_t\} = \{1,0\}$ $\forall g > 1$ and $k \le 1$. So the choice for agents in period t reduces to that between $\{i_t, j_t\} = \{1,1\}$ and $\{i_t, j_t\} = \{0,0\}$. ■

Lemma 4. If agents alive in period t expect $\{i_{t+1}, j_{t+1}\} = \{0,1\}$, then the best choice for agents in period t is either $\{i_t, j_t\} = \{1,1\}$ or $\{i_t, j_t\} = \{0,0\}$.

Proof. As for Lemma 3.

We can now prove the four parts of Proposition 6.

(a) This follows directly from part (i) of Lemma 1.

(b) This follows directly from part (ii) of Lemma 2.

(c) This follows directly from part (ii) of Lemma 1 and part (i) of Lemma 2.

(d) From Lemmas 1 to 4 together, we know that neither $\{i_t, j_t\} = \{0,1\}$ nor $\{i_t, j_t\} = \{1,0\}$ are best choices in period t for *any* given expected choices in period t+1. Therefore, neither choice will be observed in equilibrium in any period. The only possible equilibrium choices are $\{i_t, j_t\} = \{1,1\}$ or $\{i_t, j_t\} = \{0,0\}$, and these are characterized fully in parts (a) to (c). ■

Proof of Proposition 7

Consider the payoffs in the Complete Learning allocation when an old agent in period t receives from a young agent in period t, a debt transfer equal to a fraction $a_t \in [0,1]$ of the production by that young agent in that period. Under this scheme, the payoff to an old agent in period 1 is

$$g\,2N(1-k) + a_1 g\,2N$$

and the lifetime payoff to a representative young agent in period t is

$$g^t\,2N(1-a_t) + b[g^{t+1}2N(1-k) + a_{t+1}g^{t+1}2N]$$

In comparison, the payoff to an old agent in period 1 in the Periodic Complete Learning equilibrium is

$$g\,2N(1-k)$$

and the lifetime payoff to a representative young agent in period t in that equilibrium is

$$g^{\frac{t+1}{2}}2N + bg^{\frac{t+1}{2}}2N$$

for $t \geq 1$ odd, and

$$g^{\frac{t}{2}} 2N + bg^{\frac{t+2}{2}} 2N(1-k)$$

for $t \geq 2$ even. Thus, the Complete Learning allocation, with debt transfers, Pareto dominates the Periodic Complete Learning equilibrium if and only if the following conditions hold:

(A14) $$g\,2N(1-k) + a_1 g\,2N \geq g\,2N(1-k)$$

and

(A15) $$g^{t} 2N(1-a_t) + b[g^{t+1} 2N(1-k) + a_{t+1} g^{t+1} 2N] \geq g^{\frac{t+1}{2}} 2N + bg^{\frac{t+1}{2}} 2N$$

for $t \geq 1$ odd; and

(A16) $$g^{t} 2N(1-a_t) + b[g^{t+1} 2N(1-k) + a_{t+1} g^{t+1} 2N] \geq g^{\frac{t}{2}} 2N + bg^{\frac{t+2}{2}} 2N(1-k)$$

for $t \geq 2$ even. First consider condition (A14). This is clearly satisfied for any $a_1 \geq 0$. So without loss of generality, set $a_1 = 0$. Then evaluating (A15) at $t = 1$, we have

(A17) $$a_2 \geq \overline{a} \equiv \frac{1}{g} + k - 1$$

This is feasible ($\overline{a} \leq 1$) if and only if $g \geq [1/(2-k)]$, which holds everywhere in the parameter space. So without loss of generality, set $a_2 = \overline{a}$. Next consider (A15) for $t \geq 3$. Evaluating (A15) at $a_t = a_{t+1} = \overline{a}$, we have

(A18) $$g^{\frac{t-1}{2}}[2 + b - k - \frac{1}{g}] \geq 1 + b \quad \text{for } t \geq 3 \text{ odd}$$

The LHS of (A18) is increasing in t, so a necessary and sufficient condition for (A18) is that it hold at $t = 3$. Setting $t = 3$, and rearranging yields

$$g \geq [(2+b)/(2+b-k)]$$

which is the lower boundary of the Periodic Complete Learning equilibrium. (See proposition 6). Finally, consider (A16). Evaluating (A16) $a_t = a_{t+1} = \overline{a}$, we have

(A19) $$g^{\frac{t}{2}}[2 + b - k - \frac{1}{g}] \geq 1 + bg(1-k) \quad \text{for } t \geq 2 \text{ even}$$

Since the LHS of (A19) is increasing in t, a necessary and sufficient condition for (A19) is that it hold at $t = 2$. Setting $t = 2$, and rearranging yields

$$g \geq (2 / [2 - k(1 - b)])$$

which is the lower boundary of the surplus-maximizing Complete Learning allocation. (See Proposition 1). Thus, there exists a feasible set of transfers to implement Complete Learning from the Periodic Complete Learning equilibrium.

Next consider the Eternal Stagnation equilibrium. The payoff to an old agent in period 1 in this equilibrium is $2N$, and the lifetime payoff to a representative young agent is $2N(1+b)$. Thus, the Complete Learning allocation with debt transfers Pareto dominates the Eternal Stagnation equilibrium if and only if the following hold:

(A20) $$g\,2N(1-k) + a_1 g\,2N \geq 2N$$

and

(A21) $$g^t\,2N(1-a_t) + b[g^{t+1}2N(1-k) + a_{t+1}g^{t+1}2N] \geq 2N(1+b) \quad \forall t \geq 1$$

Rearranging (A20) yields

$$a_1 \geq \bar{a} \equiv \frac{1}{g} + k - 1$$

Evaluating (A21) at $a_t = a_{t+1} = \bar{a}$, we have

(A22) $$g^t[2 + b - k - \frac{1}{g}] \geq 1 + b \quad \forall t \geq 1$$

A necessary and sufficient condition for (A22) is that it hold at $t = 1$. Setting $t = 1$, and rearranging yields

$$g \geq [(2+b)/(2+b-k)]$$

Eternal Stagnation is not a coordination equilibrium with transfers for this range of g. (See Proposition 6). ■

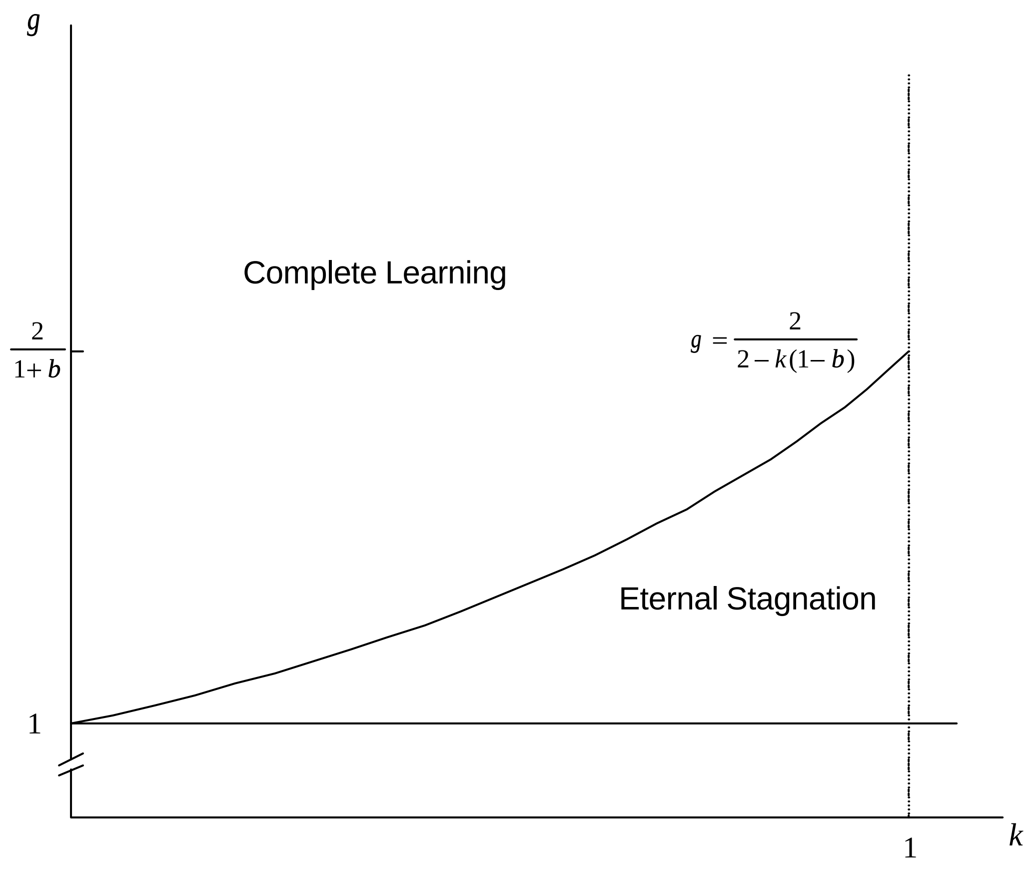

Figure 1
Surplus maximization

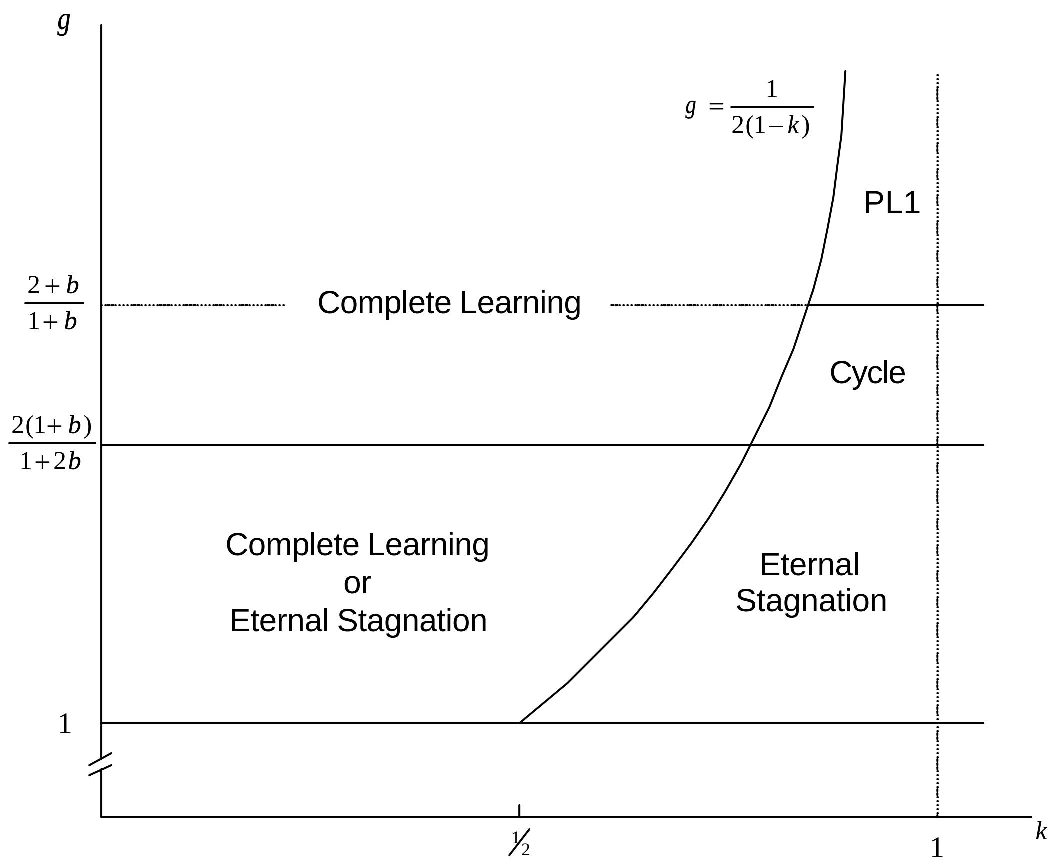

Figure 2
Intragenerational coordination

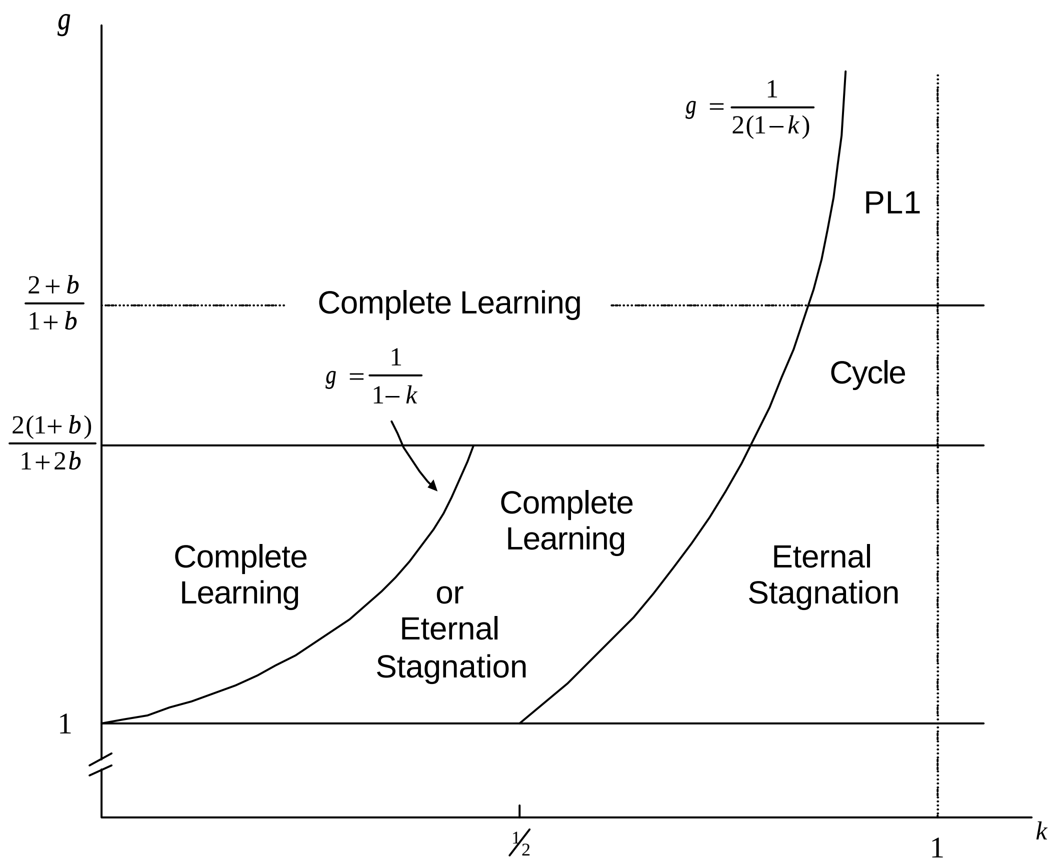

Figure 3
Intergenerational coordination without transfers

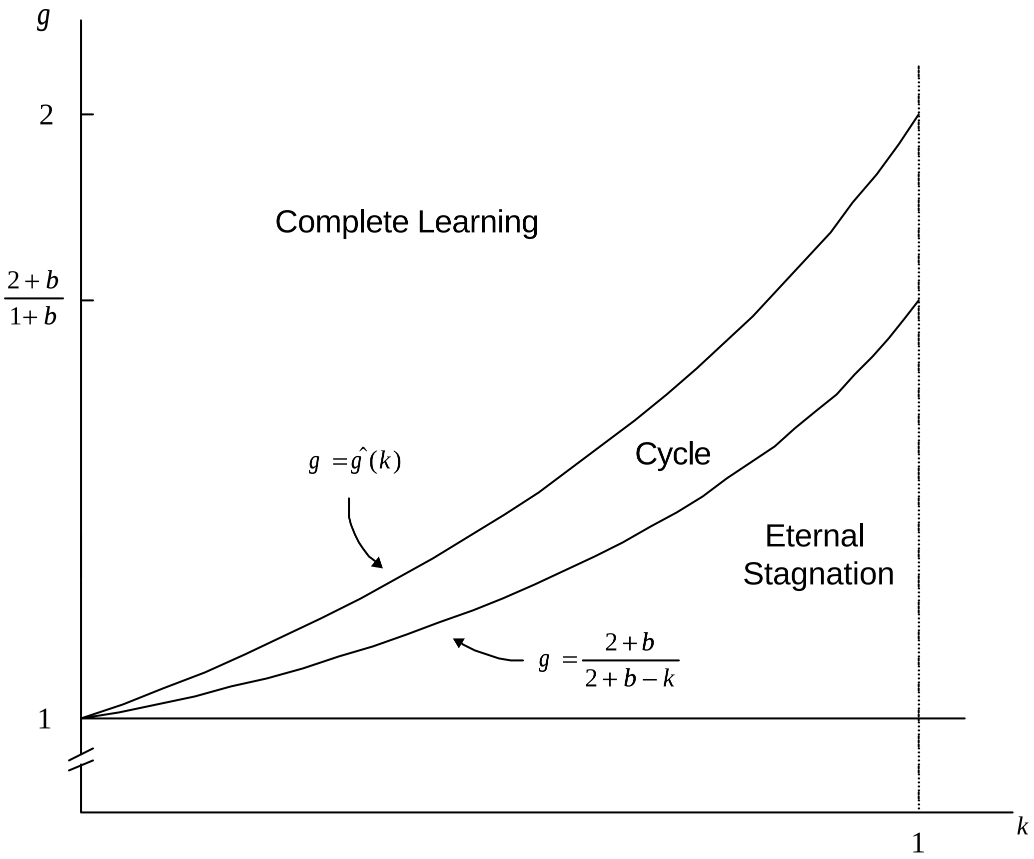

Figure 4
Intergenerational coordination
with transfers

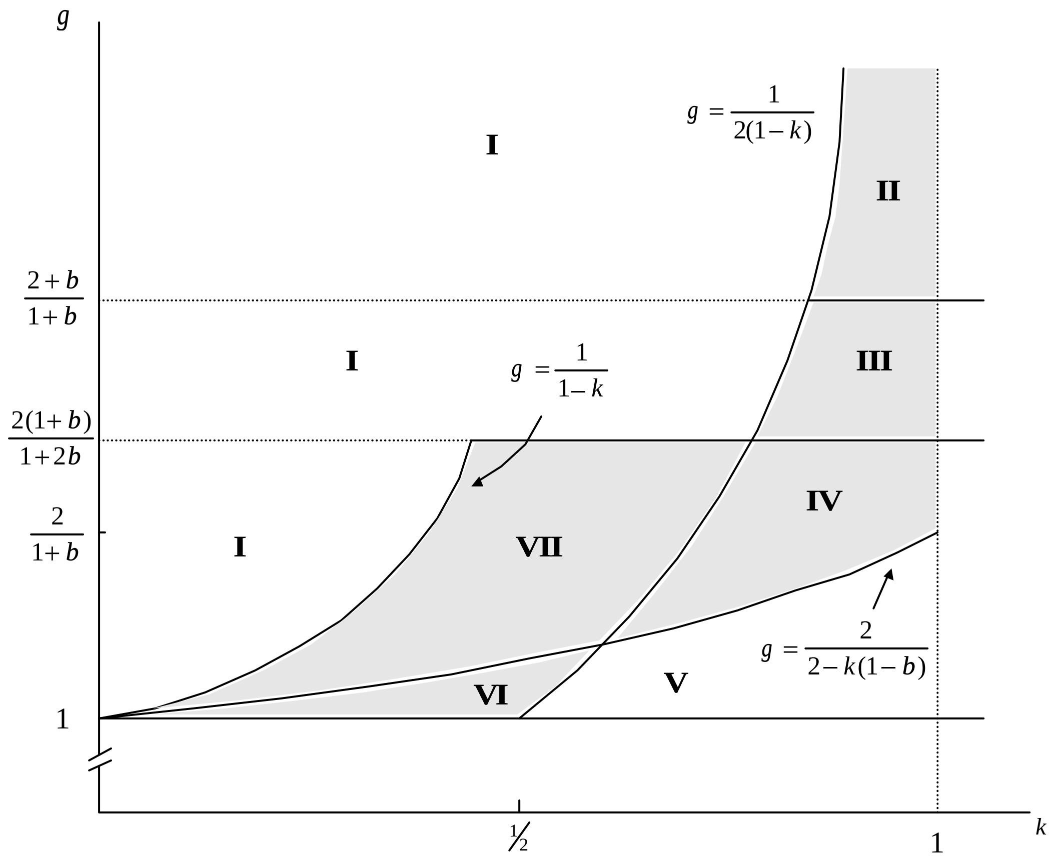

Figure 5
Intergenerational coordination
without transfers vs. surplus maximization

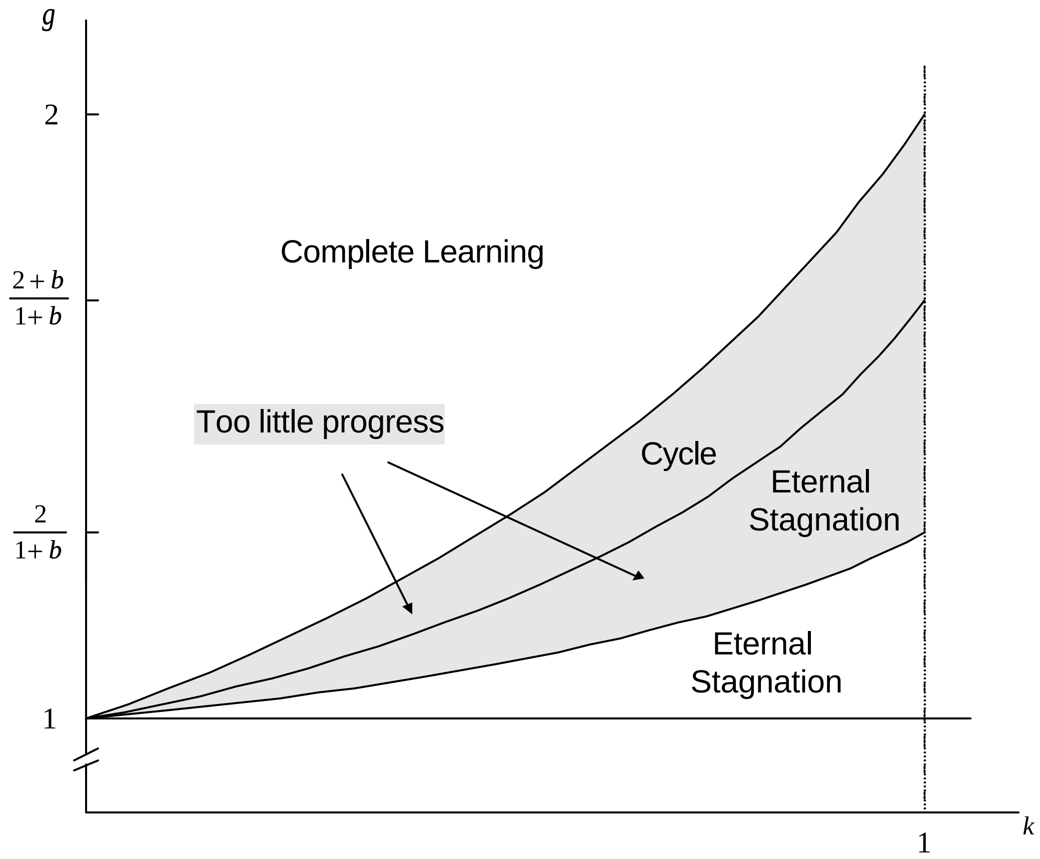

Figure 6
Intergenerational coordination
with transfers vs. surplus maximization

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