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Oligopoly

Gregory Goering John Boyce

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Gregory E. Goering*
John R. Boyce**

*Associate Professor
Department of Economics
University of Alaska Fairbanks
Fairbanks, Alaska USA 99775–6080
(907) 474–5572 (voice)
(907) 474–5219 (fax)
ffgeg@uaf.edu

**Senior Lecturer
Department of Economics
University of Auckland
Private Bag 92019
Auckland, New Zealand
64 9 373 7599, ext. 5252 (voice)
64 9 373 7000 (fax)
j.boyce@auckland.ac.nz

Abstract: This paper examines the use of taxation to control external damage due to pollution when product durability is endogenously determined. A special form of the emissions function is also examined which is equivalent to an excise tax on output. The dynamic oligopoly model indicates that many conventional results in the durability and taxation literature need not hold when durability is endogenously determined. In particular, the analysis shows durability may not be independent of industry structure nor will firms minimize their manufacturing costs of providing service. In addition, the second-best tax on imperfectly competitive firms is not necessarily less than the tax on a competitive firm when firms choose their product's durability.

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Key words: product durability, Pigouvian taxes, oligopoly. *JEL classification:* L1, H2

I. Introduction

Many studies have examined the impact of taxation on a firm's output choice. In particular, the use of taxes to mitigate and control externalities (such as pollution) in imperfectly competitive environments has received considerable attention in the past several decades. For example, Buchanan [1969], Barnett [1980], and Innes *et al.* [1991] examine the second-best tax on monopolistic firms that generate externalities. They conclude that the optimal tax on a monopolist is less than a competitive firm since the monopolist restricts output, implying that the market power distortion works in the opposite direction of the negative externality distortion. Other authors, such as Levin [1985], show that the optimal tax in oligopolistic markets is even more complicated than monopolistic markets due to firm asymmetries and strategic behavior.

However, these studies have ignored the fact that firms have other variables under their control such as the durability or quality of their product. There is evidence that this is an important omission. For example, Asch and Seneca [1976] found that many of the most heavily polluting industries in the U. S. are both highly concentrated and manufacture durable products. Examples include the aircraft, automobile, steel, and refrigeration industries, which all manufacture durable products and have relatively large pollution abatement expenditures. The influence of emissions taxation on the durability of the firm's product, as well as the optimal second-best tax when durability is endogenously determined has not received much attention in the literature.

This paper develops and analyzes a game theoretic model of product durability and emissions taxes. An infinite horizon oligopoly model is developed where the firms, as a byproduct of

manufacturing durable goods, produce environmental pollutants which are taxed. The firms' emissions are assumed to be a function of both the durability of their output and the number of units they produce in each period. We also examine two special forms of the emissions tax function where the tax depends only upon output. An extreme version of this is a per unit sales or an excise tax. The results indicate that many of the conventional results in both the durability choice and taxation literature fail to hold.

When durability is included, there are three possible sources of market failure: 1) over-production due to a pollution externality; 2) under-production due to market power; and 3) in-appropriate durability levels (e.g., planned obsolescence) due to producers choosing a durability which does not minimize the social costs of providing a given service level. Buchanan's [1969] paper and much of the subsequent optimal taxation literature has focused on the first two externalities. Coase [1972] and much of the subsequent durability literature focused on the third. We combine all three possible market failures into a positive model of firm response to taxation and consider the normative implications for optimal taxation in a second-best world.

In terms of the durability results, the model shows that durability will not generally be independent of the number of firms (market structure) when emissions taxes are in place. This is in contrast to Swan's [1970, 1971] conclusion that the firm's durability choice is independent of market structure.¹ Furthermore, the failure of the independence result is shown to occur even in rental markets in contrast to Swan's [1981] and Raviv and Zemel's [1977] finding that taxation, in particular a corporate income tax, only upsets the independence result if a monopsonistic producer's output is sold. The reason for this difference in results stems from the fact that emissions or excise taxes depend on the level of output, thus affecting marginal production decisions, whereas profit taxes do not affect marginal production decisions.

In addition, since firms minimize the sum of pure production costs plus emission tax costs, we show that if emissions are a function only of output (not durability), an emissions tax will induce a renting firm to lower its periodic output and increase product durability.² By doing so a renting firm can

provide the same service flow at a lower total cost (manufacturing plus emissions tax costs).

In terms of taxation, we show that the second-best optimal tax on imperfectly competitive firms is not necessarily less than the tax on a competitive firm. This is in contrast to the conventional wisdom that the tax on an imperfectly competitive firm, such as a monopolist, is necessarily less than a competitive firm due to the distortion (reduction) of market output levels. This unconventional result is due to the fact that an imperfectly competitive firm's durability choice is influenced by the tax. Depending upon the cost of durability at the margin and the form of the emission and demand functions this distortion may move the firm closer to the socially optimal durability or farther away from it. In particular, when the durability cost function exhibits increasing returns, demand is linear, and survival function for durability is linear in durability choice, the optimal tax on an oligopolistic or monopolistic market may be greater than that placed on a competitive industry when durability is endogenous.

II. The Basic Oligopoly Durability Model

Suppose that the durable goods industry of interest is comprised of n producers.³ At each instant these producers select the durability or quality of their product and the number of units to manufacture. Thus firm i chooses a durability $\delta_i(t)$ and output level $q_i(t)$. The stock of available goods for use by firm i at any time t is then given by (cf. Muller and Peles [1990]):

$$(1) \quad Q_i(t) = \int_{\tau}^t \phi[t-s, \delta_i(s)] q_i(s) ds + Q_i(\tau),$$

where $Q_i(\tau) = \int_{-\infty}^{\tau} \phi[t-s, \delta_i(s)] q_i(s) ds$ is the firm's initial stock at the beginning of the planning

horizon τ and $\phi(t-s, \delta_i(s))$ is the fraction of a unit of durable production manufactured at time s that is available for use at time t .⁴ The "survival" function ϕ is assumed to satisfy all the usual conditions, i.e., ϕ is increasing in durability $\delta_i(t)$ ($\delta_i > 0$) and decreasing in product age $t-s$ ($\delta_i < 0$). It is also normalized such that $\phi[0, \delta_i(s)] = 1$, and $\phi(t-s, 0) = 0$ for all $t-s > 0$.

If we let $Q_{-i}(t)$ represent the total stock of all the remaining $n-1$ firms [i.e., $Q_{-i}(t) = \sum_{j \neq i} Q_j(t)$], then

the total industry stock of goods at any point in time is given by $y(t) = Q_i(t) + Q_{-i}(t)$. The inverse demand for services is assumed to be stationary over time and a function of the total industry stock, implying $p(t) = f[y(t)]$. This demand function is assumed to be twice continuously differentiable with $f''(y) < 0$. As is standard in oligopoly models it is also assumed that an increase in the stock of one firm will decrease all other firm's marginal revenue, i.e., $f''(y(t)) + f''(y(t))Q_i(t) < 0$ for all i (see Hahn [1962]).

On the cost side, producers face a constant marginal cost with respect to output (constant returns to scale) but marginal durability costs may be increasing or decreasing. Hence, total manufacturing costs at each instant are $c[\delta_i(t)]q_i(t)$, where c is twice differentiable with $c'(\delta) > 0$. In addition each firm must pay an emissions tax bill which depends on the form of the emissions function as well as the tax rate. An increase in output or durability usually implies that more resources are used in the production process indicating emissions would tend to increase. Thus the model supposes that the effluents emitted during the manufacturing process may be due to either the product's durability or the number of units produced, implying firm i 's total emissions can be defined as $\varepsilon[\delta_i(t), q_i(t)]$. Emissions are assumed to be non-decreasing in output ($\varepsilon_q \geq 0$) and product durability ($\varepsilon_\delta \geq 0$).⁵ The total tax bill in each period is then given by $\varepsilon[\delta_i(t), q_i(t)]w$, where w is the emissions tax rate faced by all producers in all periods. Note that when $\varepsilon[\delta_i(t), q_i(t)] = q_i(t)$, w can be interpreted as simple per unit (excise) tax on output, indicating that the model can also be used to analyze excise or sales taxes impact on the firm's durability choice.

The goal of each firm is the maximization of discounted profits over the planning horizon. Letting r equal the common discount rate among all producers and consumers, this implies the i 'th firm will seek to maximize

$$(2) \quad \Pi_i = \int_{\tau}^{\infty} \{f[Q_i(t)+Q_{-i}(t)]Q_i(t) - c[\delta_i(t)]q_i(t) - w\varepsilon[\delta_i(t),q_i(t)]\} e^{-r(t-\tau)} dt,$$

subject to the state equation (1) for all $i = 1, \dots, n$, through its choice of durability $\delta_i(t)$ and quantity $q_i(t)$.

Before moving to the next section where the optimal paths for durability and output are examined, it is necessary to explain our choice of solution. First, as various authors have noted the profit function shown in (2) and subsequent maximization is valid only if the firm *rents* its output or if it sells its output, that it can pre-commit to current buyers to take the value of their stock of durable goods into account.⁶ If the firm sells its output without any commitment ability, the solution for this problem will be dynamically inconsistent—firms will have an incentive to sell more output in the future at the expense of current purchasers since the capital loss on the existing stock is born by buyers, not the firm. Consequently, (2) is only valid for a renting firm which internalizes this capital loss since it owns the entire stock of its production $Q_i(t)$ at every point in time or for a selling firm with commitment ability with its customers.⁷ Second, in theory, several solution procedures are available to solve this differential game. However, for tractability the integral game developed below supposes the firms use open-loop strategies, which in effect implies the firms can commit both to rival producers and consumers. This avoids any time consistency problems but the resulting equilibria are not necessarily subgame perfect.⁸ An example of where this assumption may be innocuous is the case where the firms offer “best-price” provisions [e.g., Butz (1990)]. In this case, firms are able to commit to consumers via the best-price provision by assuring them of the best-price for future actions in the market. For example, this can take the form of rebates to consumers if prices fall. Such mechanisms have been used extensively in electric turbogenerating industry (General Electric and Westinghouse used best-price guarantees from 1963-1977 for these durables, see Butz [1990], pg. 1071). Commercial refrigeration units, sources of CFC emissions, are often rented. This is especially true for standardized models, though less so for custom jobs.

The standard optimal control method cannot be used to solve the differential game for the optimal durability and output paths applicable since the constraint (1) is an integral constraint. However, this

problem does satisfy the assumptions of Kamien and Muller [1976] for the solution of integral state equation optimal control problems (see also Kamien and Schwartz [1991] Section 20). The Hamiltonian for the representative firm i can be defined as:⁹

$$(3) \quad H_i = [f(Q_i + Q_{-i})Q_i - c(\delta_i)q_i - \varepsilon(\delta_i, q_i)w]e^{-r(t-\tau)} + \int_t^{\infty} \phi[s-t, \delta_i(t)] q_i(t) \mu_i(s) ds \\ + \sum_{j \neq i} \left\{ \int_t^{\infty} \phi[s-t, \delta_j(t)] q_j(t) \mu_j^i(s) ds \right\},$$

where μ_i and μ_j^i (for all $j \neq i$) are the costate variables. Since firm i 's control variables $\delta_i(t)$ and $q_i(t)$ do not directly influence any other firm's state equation (1), the relevant first-order (necessary) conditions for the maximization of (3) are:

$$(4) \quad \frac{\partial H_i}{\partial q_i} = 0 = -[c(\delta_i) + \varepsilon_q(\delta_i, q_i)w]e^{-r(t-\tau)} + \int_t^{\infty} \phi[s-t, \delta_i(t)] \mu_i(s) ds,$$

$$(5) \quad \frac{\partial H_i}{\partial \delta_i} = 0 = -[c'(\delta_i)q_i + \varepsilon_\delta(\delta_i, q_i)w]e^{-r(t-\tau)} + \int_t^{\infty} \phi_\delta[s-t, \delta_i(t)] \mu_i(s) ds,$$

$$(6) \quad \frac{\partial H_i}{\partial Q_i} = \mu_i(t) = [f(Q_i + Q_{-i}) + f'(Q_i + Q_{-i})Q_i]e^{-r(t-\tau)},$$

for all $i = 1, \dots, n$ (Kamien and Schwartz [1991, pp. 250-253]). Using (6) to eliminate μ_i from equations (4) and (5), and exploiting the symmetry of the model implies the following symmetric necessary conditions for output and durability respectively:

$$(7) \quad \int_t^{\infty} \phi[s-t, \delta(t)] \{f[nQ(s)] + f'(nQ(s))Q(s)\} e^{-r(s-t)} ds - c(\delta) - \varepsilon_q(\delta, q)w = 0,$$

$$(8) \quad \int_t^{\infty} \phi_\delta[s-t, \delta(t)] \{f[nQ(s)] + f'(nQ(s))Q(s)\} e^{-r(s-t)} ds - c'(\delta)q - \varepsilon_\delta(\delta, q)w = 0,$$

where the firm subscripts are suppressed.¹⁰ Both (7) and (8) indicate that if the second-order

conditions are satisfied (i.e., the Hamiltonian is strictly concave in the state and control variables) the firms optimally equate the stream of discounted revenues generated by a unit of output or durability with the sum of their respective marginal manufacturing and emissions tax costs.¹¹

III. Long-Run Durability with Taxation

In section II it was assumed that all n firms face identical demand function, cost, and emissions functions which are also stable over time. This suggests as long as the product survival function ϕ is sufficiently smooth and continuous the firms will approach a long-run steady-state symmetric equilibrium.¹² Let the firms' constant symmetric long-run output and durability be given by \bar{q} and $\bar{\delta}$, respectively. Further, let $\bar{Q} = \beta(\bar{\delta})\bar{q}$ equal the steady-state output stock of each firm, where $\beta(\bar{\delta}) =$

$\int_t^\infty \phi(s-t, \bar{\delta}) ds$ is the (undiscounted) stream of service provided by a unit of durable output. Then

the necessary conditions for the maximization of firm discounted profits given by (7) and (8) simplify to:

$$(9) \quad [f(n\bar{Q}) + f'(n\bar{Q})\bar{Q}]\rho(\bar{\delta}) - c(\bar{\delta}) - \varepsilon_q(\bar{\delta}, \bar{q})w = 0,$$

$$(10) \quad \{[f(n\bar{Q}) + f'(n\bar{Q})\bar{Q}]\rho'(\bar{\delta}) - c'(\bar{\delta})\}\bar{q} - \varepsilon_\delta(\bar{\delta}, \bar{q})w = 0,$$

where $\rho(\bar{\delta}) = \int_t^\infty \phi(s-t, \bar{\delta}) e^{-r(s-t)} ds$ is the discounted stream of service provided by a unit of durable

output. Note that $\rho'(\bar{\delta}) > 0$ since $\phi_\delta > 0$.

We can use the symmetric conditions (9) and (10) to ascertain how the firm's long-run durability $\bar{\delta}$ is influenced by changes in industry structure or the tax rate. The easiest way to do this is to implicitly differentiate (9) and (10) with respect to the number of firms and tax rate, and then utilize Cramer's

rule to find $\partial \bar{\delta} / \partial n$ and $\partial \bar{\delta} / \partial w$. Using $\partial \bar{Q} / \partial \bar{\delta} = \rho'(\bar{\delta}) \bar{q}$ and $\partial \bar{Q} / \partial \bar{q} = \rho(\bar{\delta})$ with (9) and (10) and simplifying gives (see Appendix A):¹³

$$(11) \quad \frac{\partial \bar{\delta}}{\partial n} = \frac{w \bar{Q}(f' + f'' \bar{Q})[\rho' \bar{q} \epsilon_{qq} + \rho(\epsilon_{\delta} \bar{q} - \epsilon_{q\delta})]}{|J|},$$

$$(12) \quad \frac{\partial \bar{\delta}}{\partial w} = \frac{\rho[(n+1)f' + f'' n \bar{Q}](\rho \epsilon_{\delta} - \rho' \bar{q} \epsilon_q) - [(\epsilon_q \epsilon_{\delta}) / \bar{q} + \epsilon_{\delta} \epsilon_{qq} - \epsilon_q \epsilon_{q\delta}] w}{|J|},$$

where $|J|$ is the determinant of the Jacobian matrix. Since we have assumed that each firm's own marginal revenue is decreasing in another firm's output, both $(n+1)f' + f'' n \bar{Q}$ and $f' + f'' \bar{Q}$ are negative. The equilibria are also assumed to be "stable" so that the sign of the Jacobian determinant is the same as that of the Hessian matrix of the Hamiltonian, implying that J is negative definite ($|J| > 0$).¹⁴ With this in mind (11) and (12) yield some interesting results.

PROPOSITION ONE: i) *Long-run durability is independent of the number of firms when the tax is zero ($w = 0$), but this will not be true in general for $w > 0$; ii) long-run durability is decreasing (increasing) in the number of firms if emissions depend only on output, and there are decreasing (increasing) returns to scale in emissions; iii) long-run durability is decreasing in the number of firms if emissions depend only on durability.*

Proof: i) When $w = 0$, the numerator of (11) vanishes, implying that long-run durability is independent

of the market structure (n). ii) When emissions are a function only of output (11) becomes $\frac{\partial \bar{\delta}}{\partial n} =$

$\frac{w \bar{Q}(f' + f'' \bar{Q}) \rho' \bar{q} \epsilon_{qq}}{|J|} \leq 0$ as $\epsilon_{qq} \geq 0$. iii) When emissions are a function only of product durability [$\epsilon(\bar{\delta}, \bar{q}$

$) = \epsilon(\bar{\delta})$], (11) becomes $\frac{\partial \bar{\delta}}{\partial n} = \frac{w \bar{Q}(f' + f'' \bar{Q}) \rho(\epsilon_{\delta} \bar{q})}{|J|} < 0$. •

Similarly, we can state:

PROPOSITION TWO: i) *Long-run durability is increasing in the emissions tax rate when emissions depend only on output*; ii) *long-run durability is decreasing in emissions tax rate when emissions depend only on the durability level*.

Proof: i) When emissions depend only on the output level [i.e., $\varepsilon(\bar{\delta}, \bar{q}) = \varepsilon(\bar{q})$], (12) becomes $\frac{\partial \bar{\delta}}{\partial w} =$

$$\frac{-(n+1)f' + f''n\bar{Q}][\rho\rho'\bar{q}\varepsilon_q}{|J|} > 0; \text{ ii) When emissions depend only on durability } [\varepsilon(\bar{\delta}, \bar{q}) = \varepsilon(\bar{\delta})], (12)$$

$$\text{becomes } \frac{\partial \bar{\delta}}{\partial w} = \frac{[(n+1)f' + f''n\bar{Q}]\rho^2\varepsilon_\delta}{|J|} < 0. \quad \bullet$$

Note that in the absence of taxation ($w = 0$) the firms will set their durability to minimize the manufacturing cost of providing a unit for service. From (9) and (10) this implies the firm's optimal long-run durability is defined by:

$$(13) \quad c'(\bar{\delta})/c(\bar{\delta}) = \rho'(\bar{\delta})/\rho(\bar{\delta}),$$

which is independent of the firm's market share or output level. Thus (13) indicates the firm's long-run cost-minimizing durability $\bar{\delta}$ is independent of the market structure (number of firms n) in this case since $\bar{\delta}$ does not depend on output or market share. Thus, with constant returns to scale in output (linear production costs) and no taxation Swan's independence of market structure and durability holds [i.e., $\partial \bar{\delta}/\partial n = 0$ when $w = 0$ in (11)].¹⁵ On the other hand, when $w > 0$ the firms will not choose the durability which satisfies (13), but will instead choose a durability that minimizes manufacturing plus tax costs. For example, an excise or per unit sales tax where $\varepsilon(\bar{\delta}, \bar{q}) = \bar{q}$ would imply that firms choose their durability to satisfy:

$$(14) \quad c'(\bar{\delta})/[c(\bar{\delta}) + w] = \rho'(\bar{\delta})/\rho(\bar{\delta}).$$

Thus the cost-minimizing durability which minimizes the pure manufacturing costs defined by (13) will not be chosen due to the distortion of the tax. In this case differentiating (14) with respect to w or by examining (12) we see that when a simple excise tax is in place $\partial \bar{\delta} / \partial w > 0$, implying firms will have an incentive to increase product durability above the cost-minimizing level defined by (13). Therefore, as one would expect, excise taxes will force firms to make the product more durable in comparison with the durability defined by (13). However, both (11) and (14) indicate that durability will still be independent of industry structure in this case. Thus there are forms of the emissions function $\varepsilon(\bar{\delta}, \bar{q})$ where the independence result holds but the firms will not minimize the pure production costs of providing service to customers. In other words, although the industry structure does not influence durability choice in these cases the tax rate does.

Finally, note that when $w > 0$ Proposition One indicates the independence result will not generally hold. For example, suppose that the external damage or emissions are due solely to the number of units produced so that $\varepsilon(\bar{\delta}, \bar{q}) = \varepsilon(\bar{q})$, with $\varepsilon_{qq} > 0$. Then we see from (11) that $\partial \bar{\delta} / \partial n < 0$, implying that a monopolist would manufacture the most durable product. On the other hand, if there are economies of scale in emissions reduction $\varepsilon_{qq} < 0$ (perhaps a more likely scenario), then (11) shows that at $\partial \bar{\delta} / \partial n > 0$ and a monopolist would manufacture the least durable product. This suggests that claims of “planned obsolescence” may not be relevant when durable goods industries face taxation since $\partial \bar{\delta} / \partial n$ can be of any sign depending upon the form of the emissions function.

Our analysis suggests the independence of durability with respect to the number of firms will not in general hold in rental markets if firms are taxed. This result is in contrast to the findings of Raviv and Zemel [1977] and Swan [1981] which conclude that the independence result is only upset by corporate income taxation if the monopolist *sells* all its output. The current model suggests that emissions taxes or other forms of Pigouvian taxes will upset the independence result in rental as well as sales markets under certain conditions. The difference in results is due to the tax structure assumptions. Raviv and

Zemel [1977] and Swan [1981] use corporate income tax structures based on Feldstein and Rothschild [1974]. In rental markets with corporate taxes the firm's optimal (cost-minimizing) durability does not depend on market share or output level. Since the optimal product durability does not depend upon output with corporate taxes it is independent of market structure (which affects output levels). In contrast, Pigouvian emissions taxes cause a renting firm's cost-minimizing durability to depend directly on the firm's market share or production level. Thus as the industry structure changes so does the firm's long-run durability with these types of taxes.¹⁶

Proposition Two states that when emissions taxes rise, firms whose emissions depend primarily on output will increase the durability of their goods. In 1993, the Clinton Administration backed down on the 1987 Montreal Protocol (signed by over 130 nations) banning ozone-destroying chlorofluorocarbons (CFC's) such as Freon used in air conditioning and refrigeration units.¹⁷ The original ban would have prevented further production after December 31, 1995 and imposed a tax of \$4.35 per pound (which was selling for about one dollar per pound in 1987) on Freon in the interim. The leading U. S. manufacturer, Du Pont, had planned to end production in 1994, but was persuaded by the Clinton Administration to continue production. The reason was that a large portion of the stock of refrigeration and air-conditioning units in the U. S. had not switched to the new ozone-friendly substitutes.¹⁸ In 1994, only approximately 25% of the 80,000 large air-conditioning systems had switched, and there were about 140 million vehicle air-conditioners, 160 million home refrigerators, and five million commercial refrigerators still using CFCs.¹⁹ It is not clear that this resulted directly from an increase in durability of the products, but our model predicts that increased durability may be the outcome of an increase in a pollution tax on a durable goods producer.²⁰

IV. Optimal (Second-Best) Pigouvian Tax in Long-Run

We now turn to the normative implications of our model. What makes the calculation of an optimal tax interesting in the imperfectly competitive durable goods case is that there are *three* possible types of misallocation which can occur in durable goods industries: 1) the misallocation of resources due to the pollution externality, 2) the misallocation due to the under-provision of the service level (stock) of

the durable good, and 3) the misallocation due to producers choosing a durability which does not minimize the social costs of providing a given service level. Thus, a first-best solution would require firms to charge a competitive price and for each firm to choose its output and durability to minimize the sum of production plus pollution costs for a given industry output.

Only the first two types of distortions are considered in standard second-best optimal tax models such as Buchanan [1969] and Barnett [1980]. In these models, it is assumed that the social planner only has one instrument with which to correct all of the misallocation problems.²¹ We show that the existence of the third possible type of misallocation (inappropriate durability levels) can reverse the conventional wisdom that the second-best tax on a monopolistic industry is less than a tax on competitive industry.

To see this note that a social planner would seek to maximize the discounted stream of net surplus *recognizing* that (9) and (10) will optimally determine the oligopolists' long-run durability and output, i.e., firm durability and output are functions of the tax rate w . In the steady-state the net surplus (given the existence of a symmetric n firm oligopolistic industry) would be:²²

$$(15) \quad V = S[n\bar{Q}(w)] - nc[\bar{\delta}(w)]\bar{q}(w) - E[n\epsilon(\bar{\delta}(w), \bar{q}(w))],$$

where $S[n\bar{Q}(w)] = \int_0^{n\bar{Q}(w)} f(g) dg$ is the area under the inverse industry demand curve at each instant,

and $E(n\epsilon)$ is the industry emissions damage function, assuming that the emissions damage function can be written as $E(n\epsilon) = n\epsilon$.²³ Thus the firms' emissions functions measure the social cost of the firms' pollution. The social planner would maximize (15) with respect to w , which gives:

$$(16) \quad \frac{dV}{dw} = [f(n\bar{Q})p'(\bar{\delta})\bar{q} - c'(\bar{\delta})\bar{q} - \epsilon_{\delta}(\bar{\delta}, \bar{q})]\frac{\partial \bar{\delta}}{\partial w} + [f(n\bar{Q})p(\bar{\delta}) - c(\bar{\delta}) - \epsilon_q(\bar{\delta}, \bar{q})]\frac{\partial \bar{q}}{\partial w} = 0,$$

where it is recalled that $\partial \bar{Q}/\partial \bar{\delta} = p'(\bar{\delta})\bar{q}$, and $\partial \bar{Q}/\partial \bar{q} = p(\bar{\delta})$ are the appropriate derivatives.

Substituting from (9) and (10), and then solving for the optimal tax rate w^* yields the implicit equation

(see appendix B):

$$(17) \quad w^* = \frac{(f\phi \bar{q}\bar{Q} + \varepsilon_\delta) \frac{\partial \bar{\delta}}{\partial w} + (f\phi \bar{Q} + \varepsilon_q) \frac{\partial \bar{q}}{\partial w}}{\varepsilon_\delta \frac{\partial \bar{\delta}}{\partial w} + \varepsilon_q \frac{\partial \bar{q}}{\partial w}},$$

where derivatives $\partial \bar{\delta} / \partial w$ and $\partial \bar{q} / \partial w$ are obtained by implicitly differentiating the firm's long-run necessary conditions (9) and (10). Thus, they are functions of w^* , as can be seen by (12) which defines $\partial \bar{\delta} / \partial w$.

First, note that (17) implies $w^* = 1$, since in a competitive industry firms behave as though $f\phi = 0$, which yields $w^* = 1$. Hence the tax on a competitive industry has been normalized so w^* equals unity.

Now, suppose that durability is *exogenously* specified (i.e., $\partial \bar{\delta} / \partial w = 0$). Then we may state:

PROPOSITION THREE: *When durability is exogenous (i.e., $\partial \bar{\delta} / \partial w = 0$), the optimal tax on oligopolistic firms is less than the optimal tax if the industry were perfectly competitive.*

Proof: When durability is exogenous, so that $\partial \bar{\delta} / \partial w = 0$, (17) collapses to

$$(17') \quad w^* = 1 + \frac{f\phi \bar{Q}}{\varepsilon_q} < 1,$$

where the inequality comes from $f\phi < 0$. •

Thus we find that in the event that durability is exogenous, the standard Buchanan [1969] result that the optimal tax on an imperfectly competitive firm is less than the tax on a competitive firm holds.

This is because when $\bar{\delta}$ is exogenous, firms choose only output and since they are oligopolistic, they have market power and, consequently, under-provide the good. Thus w^* is optimally set less than one since the market power distortion is in the opposite direction of the pollution externality distortion.

On the other hand, when the product's durability is endogenously determined (17) suggests that w^*

is not necessarily less than one. As the tax rate changes the firms may change the durability of the product as well as their output levels complicating the choice of w^* . To illustrate that w^* can be greater than one if product durability is endogenous, suppose that the demand and product survival functions are linear ($f'' = \rho'' = 0$), and that the emissions are purely due to output levels so that $\varepsilon(\bar{\delta}, \bar{q}) = \varepsilon(\bar{q})$.

PROPOSITION FOUR: *When demand and product survival functions are linear (i.e., $f'' = \rho'' = 0$), emissions depend only upon output levels [i.e., $e(\bar{d}, \bar{q}) = e(\bar{q})$], then so long as stability and second-order conditions hold, i) durability increases as the tax increases and, ii) output decreases as the tax increases.*

Proof: From (9), and (10) (or (12)), we find that:

$$(18) \quad \frac{\partial \bar{q}}{\partial w} = \frac{[(n+1)(\rho \bar{q})^2 f' - c'' \bar{q}] \varepsilon_q}{|J|},$$

$$(19) \quad \frac{\partial \bar{\delta}}{\partial w} = \frac{-(n+1) f' \rho \bar{q} \varepsilon_q}{|J|}.$$

If the equilibria are stable ($|J| > 0$) and second-order conditions are satisfied, (18) and (19) indicate that

$$\frac{\partial \bar{q}}{\partial w} < 0 \text{ and } \frac{\partial \bar{\delta}}{\partial w} > 0. \quad \bullet$$

The intuition behind the result that firms will increase durability and decrease output as the tax rate increases if the tax is only levied on output is due to the fact that the firm can avoid part of its future tax burden by supplying units for service through durability rather than production \bar{q} . Thus if policy makers place a tax only on the firm's output, the tax may have the unintended effect of increasing the firm's product durability or quality. This also suggests w^* is not necessarily less than one.

PROPOSITION FIVE: *When demand and product survival functions are linear (i.e., $f'' = \rho'' = 0$)*

and if emissions depend only upon output levels [$e(\bar{d}, \bar{q}) = e(\bar{q})$], then if stability and second order conditions hold, i) increasing returns to durability [$c\alpha < 0$] imply that $w^* > 1$, ii) constant returns to durability [$c'' = 0$] imply $w^* = 1$, and iii) decreasing returns to durability [$c\alpha > 0$] imply that $w^* < 1$.

Proof: Substituting (18) and (19) into (17) implies:

$$(20) \quad w^* = 1 - \frac{f\phi\bar{Q}c\bar{\alpha}\bar{q}}{\kappa\epsilon_q},$$

where $\kappa = (n+1)(\rho'\bar{q})^2 f\phi - c\bar{\alpha}\bar{q}$. As long as the equilibria are stable we know that $\kappa < 0$. Thus, (20) implies that w^* will be less than, equal to, or greater than one depending upon the sign of $c\alpha$. Hence if there are increasing returns to product durability ($c\alpha < 0$) then the optimal tax on a monopolist is greater than one. If there are constant returns with respect to durability ($c\alpha = 0$) the model indicates that the optimal tax is one, the same as a non-durable goods competitive industry. And if there are decreasing returns to durability ($c\alpha > 0$), then the optimal tax is less one.²⁴ •

The result with respect to increasing returns to durability do not appear in the literature, and goes against the Buchanan [1969] result. This result is due to the firm's choice of durability coupled with the assumed emissions function where only output causes external damage. Recall from Proposition Four that if the firm's emission are purely due to output, an emission tax will cause the firm to increase the durability of the product to reduce emissions to reduce the subsequent tax bill. Without the tax the firm has no incentive to increase durability in this manner and only looks at its own pure production costs. In this sense the firm's product durability is below the socially efficient level. If $c\alpha < 0$ it implies increasing returns with respect to durability. In this case as the tax is increased the firms increase durability which not only reduces the emission damage but also has the added benefit of reducing their marginal cost of durability. It is the reduction in marginal durability cost which causes the optimal $w^* > 1$. Equation (20) shows that if $c\alpha > 0$ the optimal $w^* < 1$ since the marginal cost of durability is

increased as the tax is increased. Here as the tax is increased the product durability still increases but this increases the marginal durability cost. Thus a social planner will set a lower tax if $c < 0$. This example illustrates that if a tax is used to correct the pollution externalities, firms may manufacture products with *higher* durability depending upon the sign of c .

V. Discussion and Conclusions

Since many of the industries which pollute are oligopolistic and manufacture durable products a model is presented which combines each of these elements. We examined an infinite horizon oligopoly model where each durable goods oligopolist causes damages external to the industry through the emission of pollutants. The model shows that when durability is endogenously determined many of the conventional results of the optimal second-best taxation literature and the product durability literature do not hold if Pigouvian or excise taxes are levied on the industry.

In terms of the durability results, the model shows that the independence of market structure and durability does not necessarily prevail in the long-run even if firms rent their output. Furthermore, it is possible that a monopolistic industry may produce the most durable product, suggesting that the notion of “planned obsolescence” may not be relevant in durable goods industries subject to taxation. The analysis further suggests that an increase in an excise tax placed on output tends to increase the product’s durability. It is also shown that if the majority of the pollutants emitted are due to production rather than durability levels, or if the tax is an excise tax, the oligopolists will increase their product’s durability. The reason is that by increasing durability, firms can avoid taxes based on output. This suggests that an unintended effect of an increase in an emissions tax may be that firms begin to make their products more durable. Thus, one would expect, for example, that automobiles would become more durable as a consequence of automobile manufacturers being forced to internalize more of the costs of pollution generated during manufacture. In the event that firms are originally producing the automobiles at the optimal level of durability (i.e., that which minimizes durability costs per unit of services provided), then this means that automobiles will become too durable. A possible example of this is the increase in luxury car (which are generally more durable) sales as a result of the expansion

of environmental laws from the 1970s forward, which raised the costs of production for manufacturers.²⁵

Similarly, the U. S. Environmental Protection Agencies' (EPA) ban (in effect a very large tax increase) on chlorofluorocarbon use in refrigeration units produced after the year 2,000 (U. S. Environmental Protection Agency [1995]), originally set for 1995, may have the unintended consequence that presently produced refrigeration units may be too durable, implying that instead of reducing CFC use, the EPA may have actually increased the length of time old technology refrigerators are in service.²⁶ Whether this is true depends upon whether or not the increase in durability is greater than the decrease in current production from Proposition Four. If the regulators did anticipate this increase in durability of current production (which we doubt), it would probably be viewed as a good result since the perception is that increased durability has the positive secondary effect of reduced post-service waste. Nevertheless, the original ban set for 1995 was postponed in large part due to the durability of the existing stock. The Clinton Administration actually had to ask the leading U. S. manufacturer of CFCs, Du Pont, to continue production beyond 1994 to service the existing stock of refrigeration and air-conditioning units.²⁷

In addition, we show that the second-best Pigouvian tax is not necessarily lower for an imperfectly competitive industry than for a competitive industry. The standard result due to Buchanan [1969] is that the optimal second best tax on pollution in the presence of market power in the output market is less than the optimal tax in a competitive market. This is due to the fact that there are two market failures, over-provision due to the pollution externality and under-provision due to market power. Our results show that when one adds durability into the equation, the standard result may be reversed. The conventional result of a lower tax on imperfectly competitive firms does hold when durability is exogenously specified, but may not hold when it is endogenous. The form of the emission and durability cost function is of primary importance in determining whether or not an oligopolistic industry should be taxed less than a comparable competitive industry. When the pollution externality is corrected using a tax, firms will increase the durability of their product if emissions are primarily due to output levels. In

the event that there are increasing returns in durability production, we show that the standard result is reversed.

While the present model considers the dynamic aspects of firm behavior in a durable goods oligopoly with pollution externalities, it does not take into account strategic behavior. However, our own attempts at considering the feed-back solutions suggest that such a solution may be intractable, even under much simpler assumptions than in the present model. Thus it is not clear whether our current results are fully general.

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Endnotes

¹ Swan's result has spawned a large body of literature examining the conditions required for independence. Kamien and Schwartz [1974], for example, argue that a monopolist may produce a less durable good than pure competitor given the monopolist is restricted to only one plant (see, however, Swan's [1977] comment on their finding). In general, rental markets and constant returns to scale are sufficient for independence. See Schmalensee [1979] for a review of the early durability literature and Goering [1992, 1993] for an examination of the independence result in oligopoly markets and with learning economies.

² Any tax on output only has this property.

³ Note that the number of firms n is parametrically specified. This parametric specification along with other simplifications outlined below allows us to examine the influence of market structure (i.e., number of firms in the market) on the firm's long-run durability choice under taxation. This parametric approach, among other things, has the disadvantage of implicitly ruling out entry (or exit) in the industry even in the long-run. However, we conduct comparative statics exercises on the effect the number of firms has on the industry. Thus indirectly we can also address entry and exiting issues by assuming an exogenous rate of return available to like capital and by examining how changes in that rate of return affect the equilibrium number of firms, and how that in turn affects output and durability choices of the firms. However, even in this case, we are not able to analyze the welfare effects during the transition period.

⁴ The continuous time infinite horizon complexity allows us to examine long-run steady state issues that depend upon the form of the decay function (as Swan [1970, 1971] originally addressed) and to analyze the time path for durability choice as in Muller and Peles [1988, 1990]. Note that the simpler two-period durability frameworks (e.g. Bulow [1982, 1986]) are a collapsed form of "short-run" analysis and cannot be used to address these issues. These simpler frameworks do have the advantage, however, of being able to address rental versus sales issues that cannot be examined in our durability choice model.

⁵ Inclusion of the cases $\varepsilon_\delta = 0$ or $\varepsilon_q = 0$ allows the examination of industries where emissions are exclusively due to the number of units produced or product durability respectively. Note also, as pointed by an anonymous referee, in reality increased durability may decrease future pollution damage. This may occur due to less accumulation of worn out products at landfills, for example. The model as it is currently specified is not general enough to capture this type of effect. Indeed, what would be required is the addition of another state (i.e., stock) variable for pollution. This would allow changes in durability or output to impact not only the instantaneous pollution damage but also the future stock of emissions and, consequently the future amount of pollution damage. While certainly of interest, we leave such an extension for future research.

⁶ This was first noted by Coase [1972]. A variety of studies have addressed this time consistency issue and mechanisms, such as best-price provisions or reputation, which the firm can use to mitigate its commitment problem (see Ausubel and Deneckere [1987, 1989], Gul [1987], and Butz [1990]).

⁷ In a related fashion, as is well known, differential games can be solved in several different ways depending upon the firm's commitment ability with each other, i.e., the strategies firms use. The two common strategies utilized are: 1) open-loop (Nash) strategies which in essence assume the firms can commit to a certain action or time path for the entire planning horizon, and 2) feedback (subgame perfect) strategies that assume no pre-commit and allow the firm to choose an action based on the current state of the system as well as the time. Thus open-loop strategies assume commitment ability while feedback strategies do not. Kamien and Schwartz [1991, pp. 272-288] provide a discussion of these strategies in standard differential games. It should be noted, however, that the durable goods model developed is not a standard differential game (as defined by Kamien and Schwartz) since the state equations or constraints given by (1) are integral equations. Thus the durable goods oligopoly model is probably more aptly titled a differentiable "integral game". See Fershtman *et al.* [1992] for a discussion of these types of games. However, they only provide the necessary conditions for the open-loop case.

⁸ The authors are unaware of any paper that solves for the feedback (subgame perfect) equilibria of an integral game. However, Goering [1992] has shown in a simple two-period durable goods oligopoly model that if firms rent and produce output in all periods the open-loop equilibrium is in fact a degenerate feedback equilibrium (i.e., is subgame perfect). This suggests that the assumption of open-loop strategies (commitment ability) may only influence the results if sales are assumed.

⁹ To simplify the notation the time variable is suppressed in the remainder of the paper except where it would cause undue confusion.

¹⁰ The analysis focuses on symmetric interior solutions since the firms have identical cost and emission functions. However, since both durability and output are endogenously determined asymmetric equilibria may also exist. Additionally, corner or boundary solutions may be possible (e.g., $\delta_i(t) = 0$ for some t) if the Hamiltonian is not strictly concave. Due to the complexity of durability model and to facilitate comparison to previous durability studies (which implicitly assume symmetric interior solutions) only symmetric interior solutions are examined in the integral game.

¹¹ See the *Journal's* editorial Web site for a brief analysis of the short-run optimal product durability, as well as all appendices referenced below.

¹² Although the steady-state collapses the dynamics into a simpler form, it still contains many dynamic elements including the initial stock conditions and the entire infinite time path of the stock of durable goods, among others.

¹³ The long-run condition \bar{Q} cannot be differentiated directly. To do so assumes the firms can change their initial stock $Q_i(\tau)$, which is obviously not true, and consequently, will lead to erroneous results.

Sieper and Swan [1973] demonstrate that even though $\bar{Q} = \mathbf{b}(\bar{\delta})\bar{q}$ is the correct long-run equilibrium relationship, explicitly taking the initial conditions into account implies $\partial\bar{Q}/\partial\bar{\delta} = \rho'(\bar{\delta})\bar{q}$ and $\partial\bar{Q}/\partial\bar{q} = \rho(\bar{\delta})$, are the appropriate derivatives to use.

¹⁴ Basically, stability implies that the firms' reaction functions must cross "correctly" which is closely related to second-order curvature conditions (see Dixit [1986]). Most commonly use demand, cost, and

emissions functions would ensure this. For example, with linear demand, $\rho'' = c'' = \varepsilon_\delta = 0$, and $\varepsilon_{qq} > 0$,

the determinant $|J| = -w(n+1)\varepsilon_{qq}f\rho^2\bar{q}^2 > 0$, which implies sufficient curvature.

¹⁵ As Schmalensee [1979] and Abel [1983] note, Swan's independence result is sensitive to the assumption of linear production costs. With convex costs of production the firm's cost-minimizing durability $\bar{\delta}$ will depend upon output (i.e., equation (13) will depend upon \bar{q}) and, consequently, $\bar{\delta}$ will not be independent of market structure.

¹⁶ Among other things, this means that endogenous entry, which we are ignoring, would affect the long-run durability levels since entry affects the number of firms in the industry and hence market share or output per firm.

¹⁷ CFCs used in most home and commercial refrigeration units cause pollution in proportion to the number produced because the pollution occurs when the CFCs are released as the unit is being disposed of. Automotive air-conditioning units are an exception as they are prone to leak, and so need recharging.

¹⁸ HFC-134a is more chemically volatile so that it combines with other elements before rising to the ozone layer.

¹⁹ See Jack Cheevers, "Industry Shivers at Ban on CFC Refrigerants," *Los Angeles Times*, November 1, 1994, Business section, p. 3.

²⁰ Short-run optimal product durability analysis (available at the *Journal's* editorial Web site) also reveals that the form of the emissions function is critical in determining whether or not durability increases or decreases along the optimal approach path to the steady-state. If, for example, the majority of the pollutants emitted are due to the volume of production and not the durability of the product, the firm's durability will decline over time when an emissions tax is in place. This declining durability path is also shown to hold with excise taxes. Interestingly, White [1971] notes that the durability of all US makes of automobiles tended to decrease through the 1950's and 1960's (automobile durability has likely increased in recent decades). Using monopoly frameworks Muller and

Peles [1988, 1990] and Goering [1993] show declining durability could occur if the cost of durability at the margin decreases or if the marginal cost of durability is independent of knowledge accumulation when learning economies are present. The current oligopoly model suggests that the observed pattern of declining quality may also be due in part to emissions controls or excise taxes since these taxes influence the firm's marginal cost.

²¹ In a perfect (first-best) world another instrument would be available to correct the distortion due to market power. The current model supposes that it is not possible to directly correct the misallocation due to market power, implying the optimal emissions tax w^* will be second-best.

²² In general, net surplus is the area under the market demand curve (total surplus) less any production costs and emission damage costs. In the steady-state this implies the discounted surplus depends upon the infinite time-paths of firm output, durability, and the discount rate. Thus the discounted net surplus V is a function of the number of firms n , and the durability and output paths chosen $[\bar{\delta}(w) \text{ and } \bar{q}(w)]$ and the resulting stock path $\bar{Q}(w)$.

²³ This supposes that the marginal damage of the industry is simply additive in firm emissions and normalizes the marginal damage to one. It is also worth mentioning that the model implicitly assumes that once a durable unit of production “wears out” there is no added disposal or waste fee to remove the unit. If a durable unit

²⁴ Note that with this parameterization and constant (or increasing) returns to durability, the second-order conditions need not hold for a perfectly competitive industry, *unless* the firms recognize that only a certain size and number of price-taking firms can fit in the market. Thus, strictly speaking, we cannot interpret $w^* = 1$ as the “competitive tax” where there is an infinite number of arbitrarily small price-taking firms. However, we can interpret $w^* = 1$ in this example as a “competitive industry” where a set number of n firms equate marginal cost and price. In other words, the model is comparing an equilibrium of n firms setting price equal to marginal cost to a market structure with the same number of oligopolistic firms.

²⁵ Other factors, such as rising incomes and falling (real) gasoline prices, were also at work.

²⁶ In the case of the relatively non-leaking commercial and home refrigeration units, this may not have affected pollution levels except through output. But for relatively leaky automotive air-conditioning units, this may not be true.

²⁷ See note 19, above.

²⁸ The Hessian of second-order conditions and the Jacobian to the system of equations (A.3') and (A.4') are not identical. (Although they will have the same sign of their determinants with stability as noted in the text of the paper.) That this general statement works is most easily seen in a simple example with a linear demand and constant marginal cost n -firm oligopoly. Let profits be denoted as:

$$\pi_i = (a - bQ)q_i - cq_i,$$

where $Q = \sum_{j=1}^n q_j$. The first-order condition is:

$$\frac{\partial \pi_i}{\partial q_i} = a - c - b \frac{\partial Q}{\partial q_i} q_i - bQ = a - c - bq_i - bQ = 0 = a - c - b(n+1)q.$$

The second line holds only if firms are identical (i.e., in the symmetric equilibrium). The second-order condition assuming Cournot conjectural variations is:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -2b,$$

for each firm. However, in the symmetric equilibrium a change in a affects output of each firm as:

$$\frac{\partial q}{\partial a} = \frac{1}{\partial a} \{ (a - c) / [-b(n+1)] \} = \frac{1}{b(n+1)}.$$

Thus the Jacobian matrix involves $(n+1)$ rather than 2 in the symmetric equilibrium, showing the difference between the Jacobian matrix of the symmetric equilibrium and the Hessian matrix.

²⁹ The analysis for $\partial q / \partial n$ and $\partial q / \partial w$ can be done in a similar fashion. Since these derivatives are not used directly in the analysis, they are not reported here.