

# Conditions of Calculating Amplifier Cut-Off Frequency by Time Constant

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**Abstract:** This paper mathematically proves the condition of estimating an amplifier cut-off frequency by the time constant method. Under the condition of unrelated capacitor loop circuit, a simple one to one relationship exists between each pole and time constant. The pole can therefore be determined by its corresponding time constant.

**Keywords** time constant, pole, dominant pole, cut-off frequency.

## 1. INTRODUCTION

An amplifier has a lower and an upper cut-off frequency. To find the cut-off frequencies precisely, the transfer method can be used. The procedure of the transfer function method for the lower cut-off frequency can be summarized as:

- (1) Find the small signal equivalent circuit at the lower frequency band.
- (2) Find the transfer function of the equivalent circuit.
- (3) Find all the poles and zeros of the transfer function.
- (4) Determine the cut-off frequency from the poles and zeros either by Bode plot or the dominant pole method.

The procedure of the transfer function method for the

upper cut-off frequency is the similar but need to use small signal equivalent circuit at upper frequency band.

For the amplifier that has only one single capacitor, the transfer function method is not too difficult to use. For an amplifier that has multiple capacitors, in theory, it is also possible to calculate the cut-off frequencies by using transfer function. However, the calculation becomes very complex and difficult as the circuit is topologically complex. This paper presents a simple and easy method that can be used in engineering estimation of the cut-off frequency – named the time constant method.

The time constant method is an engineering approximation. It does not require a precise mathematical analysis of the amplifier. The

approximation however satisfies the engineering requirements. Particularly for the multiple capacitor amplifier, when it is impossible to determine the dominant pole, the time constant method can be very effective way to estimate the cut-off frequency.

In this paper, the relationships between transfer function poles, time constants, and cut-off frequencies are developed and discussed. The condition and procedure of using time constant method is also presented.

## 2. THEORETICAL ANALYSIS

### 2.1 Lower cut-off frequency

Consider Figure 1 of two-port linear network. Assume the capacitor  $C_1$  and  $C_2$  are independent from each other and there are no other capacitors and independent sources in the network. The two-port network may consist of resistors and dependent sources, but free of any independent sources and capacitors or inductors. When  $v_2=0$ , the short circuit equivalent admittance  $Y_{1s}$  is in parallel with  $C_1$ . When  $v_1=0$ , the short circuit equivalent admittance  $Y_{2s}$  is in parallel with  $C_2$ . The transfer admittances are  $Y_{12}$  and  $Y_{21}$ .

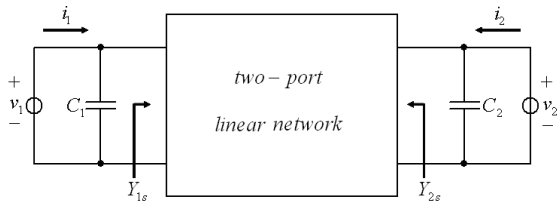


Figure 1. Network model for short-circuit time constant

Y – Parameter matrix is given by

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} Y_{1s} + sC_1 & Y_{12} \\ Y_{21} & Y_{2s} + sC_2 \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} \quad (1)$$

$$|Y| = \begin{vmatrix} Y_{1s} + sC_1 & Y_{12} \\ Y_{21} & Y_{2s} + sC_2 \end{vmatrix} \quad (2)$$

Let the determinant  $|Y| = 0$ . We can find the root  $p_1$  and  $p_2$ . When frequency  $f = p_1$  or  $p_2$ ,  $I_1(s) = 0$  and  $I_2(s) = 0$ , even if  $V_1(s) \neq 0$  or  $V_2(s) \neq 0$ . Therefore,

$p_1$  and  $p_2$  are the natural frequencies and the poles at lower frequency.

Assume K a constant,

$$K(s - p_1)(s - p_2) = |Y| \quad (3)$$

$$K(s - p_1)(s - p_2) = (Y_{1s} + sC_1)(Y_{2s} + sC_2) - Y_{12}Y_{21} \quad (4)$$

$$K[s^2 - (p_1 + p_2)s + p_1p_2] = C_1C_2s^2 + (C_1Y_{2s} + C_2Y_{1s})s + Y_{1s}Y_{2s} - Y_{12}Y_{21} \quad (5)$$

Equating the coefficients,

$$K = C_1C_2 \quad (6)$$

$$-K(p_1 + p_2) = C_1Y_{2s} + C_2Y_{1s} \quad (7)$$

$$Kp_1p_2 = Y_{1s}Y_{2s} - Y_{12}Y_{21} \quad (8)$$

Solving (6), (7), and (8), simultaneously gives

$$\begin{aligned} -(p_1 + p_2) &= Y_{1s}/C_1 + Y_{2s}/C_2 = 1/R_{1s}C_1 + 1/R_{2s}C_2 \\ &= 1/\tau_{1s} + 1/\tau_{2s} \end{aligned} \quad (9)$$

$$p_1p_2 = (Y_{1s}Y_{2s} - Y_{12}Y_{21})/C_1C_2 \quad (10)$$

If  $C_1$  loop circuit and  $C_2$  loop circuit are independent from each other, i.e. the short-circuit time constants are independent, the transfer admittances  $Y_{12} = 0$ , and  $Y_{21} = 0$ , then,

$$-(p_1 + p_2) = 1/R_{1s} + 1/R_{2s} = 1/\tau_{1s} + 1/\tau_{2s} \quad (11)$$

$$p_1p_2 = G_{1s}G_{2s}/C_1C_2 = 1/\tau_{1s} \cdot 1/\tau_{2s} \quad (12)$$

where  $\tau_{1s}$  and  $\tau_{2s}$  are the short-circuit time constant,  $\tau_{1s} = R_{1s}C_1$ ,  $\tau_{2s} = R_{2s}C_2$ .

$$\text{And, } p_1 = -1/\tau_{1s}, p_2 = -1/\tau_{2s}. \quad (13)$$

From (13), it states that the two poles  $p_1$  and  $p_2$  are directly related to two short-circuit time constants  $\tau_{1s}$  and  $\tau_{2s}$  and can be calculated simply through (13).

Conclusion statement 1: If  $C_1$  loop circuit and  $C_2$  loop circuit are independent, the two poles  $p_1$  and  $p_2$  are directly related to two short-circuit time constants  $\tau_{1s}$  and  $\tau_{2s}$  and can be calculated simply through (13).

After finding the poles, the lower cut-off frequency  $\omega_L$  can be found easily through these poles.

Due to linearity of the circuit, same approach can be used to prove the following conclusion statement two.

Conclusion statement 2: For a circuit with N capacitors, if the N capacitor loops are independent, the poles are directly related to the corresponding time constants and can be calculated simply through  $p_i = -1/\tau_{is}$ ,  $i = 1, 2, \dots, N$ .

## 2.2 Upper cut-off frequency

Consider a two-port linear circuit of Figure 2. The circuit has two independent capacitors  $C_1$  and  $C_2$ . There are no other capacitors or inductors in the circuit. The two-port network may consist of resistors and dependent sources, but free of any independent sources and capacitors or inductors. When  $i_2=0$ ,  $C_1$  is in series with input resistor  $R_{1o}$ . When  $i_1=0$ ,  $C_2$  is in series with output resistor  $R_{2o}$ . The transfer resistances are  $R_{12}$  and  $R_{21}$ .

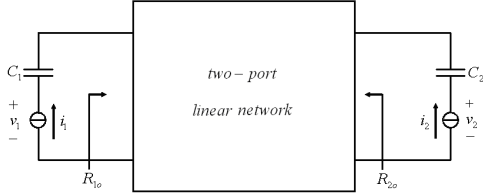


Figure 2. Network model for open-circuit time constant

Z- Parameter matrix of the circuit is given by

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} R_{1o} + \frac{1}{sC_1} & R_{12} \\ R_{21} & R_{2o} + \frac{1}{sC_2} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \quad (14)$$

$$|Z| = \begin{vmatrix} R_{1o} + \frac{1}{sC_1} & R_{12} \\ R_{21} & R_{2o} + \frac{1}{sC_2} \end{vmatrix} \quad (15)$$

Let the determinant  $|Z| = 0$ . We can find the root  $p_1$  and  $p_2$  of the equation. When frequency  $f = p_1$  or  $p_2$ ,  $V_1(s) = 0$  or  $V_2(s) = 0$ , even if  $I_1(s) \neq 0$  and  $I_2(s) \neq 0$ . Therefore,  $p_1$  and  $p_2$  are the natural frequencies and the poles at the upper frequency band.

Assume K a constant,

$$K(s - p_1/s)(s - p_2/s) = |Z| \quad (16)$$

$$K(s - p_1/s)(s - p_2/s) = (R_{1o} + 1/sC_1)(R_{2o} + 1/sC_2) - R_{12}R_{21} \quad (17)$$

Multiply (17) with  $s^2$ ,

$$K[s^2 - (p_1 + p_2)s + p_1p_2] = (R_{1o}R_{2o} - R_{12}R_{21})s^2 + (R_{1o}/C_2 + R_{2o}/C_2)s + 1/C_1 C_2 \quad (18)$$

Equate the coefficients,

$$K = R_{1o}R_{2o} - R_{12}R_{21} \quad (19)$$

$$-K(p_1 + p_2) = R_{1o}/C_2 + R_{2o}/C_2 \quad (20)$$

$$K p_1 p_2 = 1/C_1 C_2 \quad (21)$$

Divide (20) by (21),

$$-(1/p_1 + 1/p_2) = R_{1o} C_1 + R_{2o} C_2 = \tau_{1o} + \tau_{2o} \quad (22)$$

$$p_1 p_2 = 1/C_1 C_2 (R_{1o}R_{2o} - R_{12}R_{21}) \quad (23)$$

Where  $\tau_{1o}$  and  $\tau_{2o}$  are the open-circuit time constant,

$$\tau_{1o} = R_{1o}C_1, \tau_{2o} = R_{2o}C_2.$$

When two capacitor loop circuits are independent, i.e., the two open-circuit time constants are not related, the transfer impedance  $R_{12}=0$ , and  $R_{21}=0$ .

And

$$-(1/p_1 + 1/p_2) = \tau_{1o} + \tau_{2o} \quad (24)$$

$$p_1 p_2 = 1/C_1 C_2 R_{1o}R_{2o} = 1/\tau_{1o} \cdot 1/\tau_{2o} \quad (25)$$

$$\text{Therefore, } p_1 = -1/\tau_{1o}, p_2 = -1/\tau_{2o} \quad (26)$$

From (26), it states that the two poles  $p_1$  and  $p_2$  are directly related to two open-circuit time constants  $\tau_{1o}$  and  $\tau_{2o}$  and can be calculated simply through (26)

Conclusion statement 3: If  $C_1$  and  $C_2$  loop circuits are independent, the two poles  $p_1$  and  $p_2$  are directly related to two open-circuit time constants  $\tau_{1o}$  and  $\tau_{2o}$  and can be calculated simply through (26).

Same approach can be used to prove the following conclusion statement.

Conclusion statement 4: For a circuit with N capacitors, if the N capacitor loop circuits are independent, the poles are directly related to the corresponding open-circuit time constants and can be calculated simply through  $p_i = -1/\tau_{io}$ , where  $i = 1, 2, \dots, N$ .

After finding the poles, the upper cut-off frequency

$\omega_u$  can be found easily through the poles.

### 3. CONCLUSION

From the analysis in section 2, the following conclusion can be drawn.

(1) When the capacitance loops are independent from each other, i.e., the time constant are not related, a pole is determined exclusively by its corresponding time constant. This is expressed by  $p_i = -1/\tau_i$ ,  $i = 1, 2, \dots N$ . The lower and upper cut-off frequencies can then be determined by Bode plot or the dominant pole method.

(2) When the capacitance loops are not independent from each other, i.e., the time constant are related, a pole can not be determined exclusively by its corresponding time constant.

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