Bayesian Analysis of Financial Time Series

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Abstract

In the last decades new ideas in the analysis of time series which are based on the assumption that trading is performed continuously in time have been developed. This idea has its strength in its flexibility and can overcome many theoretical and practical difficulties experienced with discrete time analysis. Due to Itôs Lemma continuous time models are mathematically tractable, however in practice, statistical analysis is generally difficult since observations are available only at discrete times. Modelling continuous time models using stochastic differential equations including Brownian motions is a natural approach owing to the diffusion type motions of the models involved. I will introduce the most widely used processes for analysing financial models based on continuous time series and review common discretization methods, as well as approximation and estimation techniques.

I will then concentrate on stochastic volatility models, which can also be derived from continuous time models and are nowadays commonly applied to analyse stock market data or exchange rates. Additionally, stochastic volatility models offer an alternative to the Black-Scholes model as a basis for pricing options. Latest advances in estimation methods for this demanding model will be reviewed. I will focus on several ideas on performing a Bayesian analysis for

parameter estimation, one of them is based on automatic differentiation and the extended Kalman filter. This technique is compared with other Bayesian estimation procedures in terms of efficiency with simulated and real data sets.

The need for reliable model checking criteria within the class of stochastic volatility models is revealed to be a complicated issue. Many of the classical or Bayesian standard approaches can not be adapted to this task. Therefore, a new idea on how to perform Bayesian model checking is taken up and further developed for the needs of comparing stochastic volatility models. The resulting deviance information criterion is compared with the more popular but far more computationally demanding Bayes factors and the easy to derive, but less accurate harmonic means.

Furthermore I will give an overview of interest spot-rate models, which have their origin in continuous time. After introducing the latest literature on interest rate specific models, a new estimation technique based on latent observations for discretely observed diffusions using Bayesian methods is reviewed and a prior sensitivity analysis has been performed to assess estimation bias in the Vasicek model. Several case studies and simulations are carried out and results are reported at the end of each chapter.

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