Directed Search without Price Directions

Benoit Julien*   John Kennes[†]

Ian King[‡]

*Australian Graduate School of Management,

[†]University of Copenhagen,

[‡]University of Auckland, ip.king@auckland.ac.nz

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Abstract

This paper presents a very simple directed search model of the labour market in which no wage announcements are made. Wages, instead, are determined by an *ex post* bidding mechanism: an auction without a reserve price. We characterize the properties of the equilibrium of the model, and examine its implied Beveridge curve. We show that this wage determination mechanism induces efficient job entry in equilibrium. A dynamic version of the model is calibrated to the US labour market. The model can account for observed vacancy rates, given parameters that are chosen to match the average wages and the natural rate of unemployment. In the limit, as the time between offer rounds in the model approaches zero, the equilibrium approaches the Walrasian competitive equilibrium.

Key words: search, matching, unemployment theory

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Benoît Julien: Australian Graduate School of Management, Sydney, Australia
bjulienz@hotmail.com

John Kennes: Department of Economics, University of Copenhagen, Denmark
john@johnkennes.dk

Ian King: Department of Economics, University of Auckland, New Zealand
ip.king@auckland.ac.nz

INTRODUCTION

Recent work in search theory has uncovered a simple coordination problem that can explain the co-existence of vacancies and unemployment, and the apparent constant returns to scale in estimated matching functions, without imposing these as *a priori* restrictions. Unemployment may exist even if there are equal numbers of identical buyers and sellers in the labour market, because it is possible, and indeed likely, that two buyers may inadvertently approach the same seller and consequently leave another seller without a buyer. According to this theory, this friction is enough to generate significant unemployment and vacancy rates, even when workers and firms are fully aware of each others' locations and prices. Moreover, in this type of setting, given an appropriate pricing mechanism, the equilibrium allocation is constrained-efficient in the sense that a planner could not improve on the allocation unless the planner was somehow able to reduce the coordination friction itself. This theory has come to be known as directed search theory.

The existence of capacity constraints is central to most of directed search theory. For example, building on earlier work by Peters (1984) and Montgomery (1991), in Julien, Kennes and King (2000) we modeled workers as being capacity-constrained in the sense that they can supply only a finite amount of labour in any time period. Firms are also restricted in the sense that, although they know where all workers are, they move *simultaneously* when approaching workers and must give workers some time to consider their offers. When choosing which job candidates to approach, firms face a strategic situation with many pure strategy equilibria, but only one symmetric equilibrium – where firms randomize. Although, from the point of view of the firms, the pure strategy equilibria all dominate the mixed strategy one, the unique symmetric (mixed strategy) equilibrium is arguably a focal point. In the usual sense then, a coordination problem exists.

Burdett, Shi and Wright (2001) (hereafter BSW) followed a similar approach, but where the roles of worker and firm are reversed: as in Montgomery (1991), in their model, firms are capacity-constrained in the sense that they have only a finite number of jobs to fill, and workers are constrained in by the fact that they can make only one job application.[1] Here, workers face precisely the same coordination problem as firms in the above setting and, once again, attention is focused on the unique symmetric (mixed strategy) equilibrium. In both settings, a matching function emerges as an equilibrium phenomenon due to the randomization in the mixed strategies.

In the usual interpretation, search is "directed" in these models (and in other directed search models)[2] by the prices that are announced by whichever side of the market is selling. In BSW, for example, firms post wages, which all workers observe. In Julien, Kennes and King (2000), workers post reserve wages, which all firms observe. In either case, buyers in the labour market can observe the locations of all the sellers along with their price announcements, and can approach each one costlessly.

Several criticisms of this approach to modeling labour market frictions are immediately apparent. First, as BSW acknowledge, in their model the assumption that each worker can make only one application is key to the existence of the friction. Even in Shi's (2001) dynamic framework, where workers make one application per period, it is natural to ask how long the relevant time period may be before workers are able to apply elsewhere. Albrecht, Gautier, and Vroman (2002) tackled this issue directly, and showed that, if workers are able to make *more than one* offer per period in the BSW model, and firms can choose which applicant to pursue, then the structure of the game is similar to that in Julien, Kennes, and King (2000) – where firms make offers to workers. This can buy more time since firms often give workers some length of time to consider offers.[3].

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[1] Shi (2001) extends the BSW model, making it dynamic, where each worker can make one application per period.
[2] See, for example, Moen (1997) and Acemoglu and Shimer (2000).
[3] In New Zealand, for example, labour law requires that workers are given at least a week to consider job offers.

Another criticism of this theory, though, is that it is relatively rare for firms to commit to wages in their job advertisements – and even rarer for workers to commit to reserve wages when they apply for jobs. This criticism strikes at the heart of all directed search models in the literature so far.

In this paper we present a very simple directed search model in which no price announcements are made at all. In this model, workers sell their labour, but announce no reserve wage. In effect, we show that common knowledge of each worker's outside option is sufficient to "direct" search. A key element in this model is the *ex post* pricing mechanism: a bidding game where, through Bertrand competition, each worker is paid his outside option. In this case, workers that are approached by only one firm are paid precisely the value of being unemployed in the subsequent period. Workers that are approached by more than one firm are paid the value of the outside option they face: the best value offered by the other firms that have approached them.[4]

This *ex post* pricing mechanism makes the model much easier to solve than those with *ex ante* prices. The complication in an *ex ante* pricing model is these prices must be determined by the solution of a non-cooperative game. This game-theoretic approach is necessary because optimal ex ante prices are not invariant to changes in the economic environment (for example, labour productivity). Alternatively, an *ex post* selling mechanism is always optimal for any parameterisation. Our basic finding is that such a pricing mechanism exists in which the key properties of directed search models are preserved. Not surprisingly, this *ex post* pricing rule yields a much simpler exposition of directed search theory because the complicated game theoretic elements are removed. Also, this rule can rationalized as the auction mechanism that emerges in the limit large market considered in Julien, Kennes, and King (2000).[5]

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[4] As we discuss elsewhere (Julien, Kennes and King (2002)) in many settings, this mechanism generates the same outcomes as those using the Mortensen Rule (Mortensen, 1982) where the surplus of the match is awarded to the initiator of the match.

[5] In Julien, Kennes and King (2000) we show that the equilibrium *ex ante* reserve wage converges to the worker's outside option as the scale of the market increases. In Julien, Kennes and King (2001) we show that, in finite-sized markets, when workers can choose how to sell their labour, they would prefer to use an auction with a reserve wage rather than a posted wage. However, as market size increases, the expected payoffs to workers from the three different selling mechanisms converges. See, also, Kultti (1999).

We show that the unique symmetric mixed strategy equilibrium of this model is constrained efficient in the above sense, and we provide an analytical solution for the Beveridge curve implied by the theory. The Beveridge curve is shifted by the length of time that each offer round takes. We also note that, as the length of this time approaches zero, in the limit, the equilibrium approaches the Walrasian (frictionless) competitive equilibrium.

The remainder of the paper is structured as follows. In Section 1, the static model is presented, its equilibrium properties are analysed, and we provide a discussion of the relationship between this model and others in the literature. In Section 2 we consider a discrete time dynamic model, and draw out the implications for the equilibrium Beveridge curve implied by the theory, and the Walrasian limit. We also present the results from two numerical simulations of the stationary equilibrium of the model, calibrated to the US economy. Section 3 then concludes and provides some suggestions for future work.

1. THE STATIC MODEL

Consider a simple economy with a large number N of identical, risk neutral, job candidates where each candidate has one indivisible unit of labor to sell.[6] There are $M = \phi N$ vacancies, where $\phi \geq 0$, and is determined by free entry. The output of a worker is $y_0 = 0$ if unemployed and $y_1 = y > 0$ if employed. It costs an amount k to create a vacancy, where $0 < k < y$. Each vacancy can approach only one candidate.

The order of play is as follows. Given N job candidates, M vacancies enter the market. Once the number of entrants has been established, vacancies choose which candidate to approach. Once vacancies have been assigned to candidates, wages are determined. We solve the model using backwards induction.

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[6] In this type of environment the economy can be closely approximated by the limit economy where $N \to \infty$. We maintain the assumption that N is finite to keep aggregate surplus finite but use the limit probabilities because of their simplicity. The results should therefore be taken as approximations.

Ex Post Wage Determination

Here, we take as given the number of entrants, and the assignment of vacancies to candidates. Let w_i^j denote the wage earned by a worker who is employed in a job with output y_i $i \in \{0,1\}$, whose *outside* option is employment in a job with output y_j. Notice that, for notational convenience, we have classified an unemployed worker as earning a wage w_0^0. Through the ex post bidding game, equilibrium wages have the following structure:

$$w_i^j = y_j \tag{1.1}$$

Thus, if a worker is approached by exactly one firm, his outside option is $y_0 = 0$, and so his wage is $w_1^0 = y_0 = 0$. If a worker is approached by two or more firms then his outside option, when negotiating with any of these firms, is (through Bertrand competition) the output from any of the other firms: $y_1 = y$. In this case $w_1^1 = y_1 = y$. Notice, also, that *ex post*, a *vacancy* will receive the payoff y if she is alone when approaching a candidate, and zero otherwise.

As mentioned in the Introduction, this wage structure has an alternative interpretation: the "Mortensen Rule". Once vacancies have been created, the *surplus* of a match is the output from the match minus the opportunity cost. In the case of a bilateral match, the opportunity cost is $y_0 = 0$, so the surplus from the match is y. In the case of a multilateral match, the opportunity cost is $y_1 = y$ -- the output that would have been produced if the candidate were matched, instead, with one of the alternative vacancies that were assigned to that candidate. In this case, the surplus from the match is zero. Using Mortensen's (1982) terminology, firms "initiate" the match in this environment, because they choose which candidates to approach. Applying the rule that the initiator of the match receives the surplus of the match, where the payoff to the non-initiator (the candidate) $w(s)$ is simply y minus the payoff to the initiator, one obtains (1.1).

The Assignment of Vacancies to Workers

Here, we consider the choices made by the different vacancies about particular candidates to approach. This strategic decision is modeled, with some care, in Julien, Kennes, and King (2000), and we refer the reader to section 2.2 in that paper for a detailed analysis. In this paper, we follow the tradition of restricting attention to the unique symmetric mixed strategy equilibrium, in which vacancies randomize over candidates with equal probability. Let p_i^j denote the probability that a candidate earns wage w_i^j. Consequently, in a large market such as this, the probability distribution of wages facing candidates (for any ϕ, and using (1.1)) is approximated by:

$$w_i^j, p_i^j = \begin{cases} w_0^0 = 0 & p_0^0 = e^{-\phi} \\ w_1^0 = 0 & p_1^0 = \phi e^{-\phi} \\ w_1^1 = y & p_1^1 = 1 - e^{-\phi} - \phi e^{-\phi} \end{cases} \tag{1.2}$$

Here, as in all the static models of this type, the unemployment rate is given by:

$$u = e^{-\phi} \tag{1.3}$$

and the number of matches or hires (the matching function) is given by:

$$H(M, N) = N(1 - e^{M/N}) \tag{1.4}$$

Clearly, from (1.4), the matching function is increasing in both of its arguments, and has constant returns to scale. Figure 1 presents a graphical representation of this matching function, for given values of *N* and *M*.
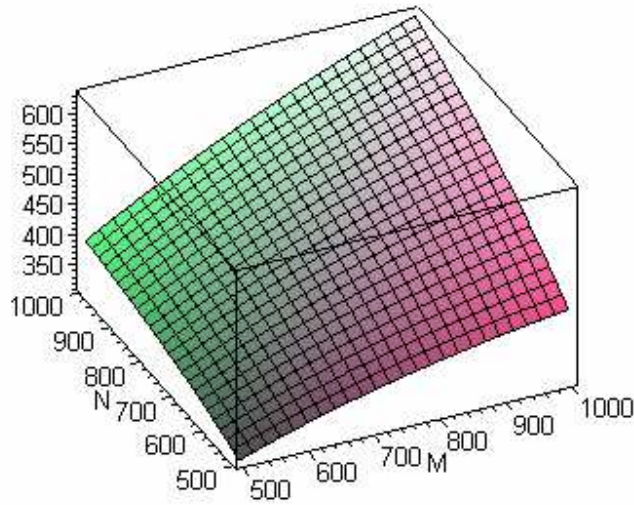
Figure 1: The Equilibrium Matching Function

We now turn the determination of the value of *M,* given *N*, through entry.

Vacancy Entry

If a vacancy is able to hire a candidate then the profit from creating a vacancy is equal to its output (*y*) minus the cost from creating it (*k*) and minus the wage paid to the worker (*w*). The probability of hiring a candidate, and the wage paid to the candidate depends on the assignment of other vacancies, and firms create vacancies as long as the expected profit from doing so is positive. Let *q* denote the probability that the firm is alone when approaching the candidate, so that 1 - *q* is the probability that at least one other firm approaches the candidate. It is easy to show that:

$$q = e^{-\phi} \quad\quad\quad (1.5)$$

By (1.2), whenever a vacancy is assigned to a worker with at least one other firm present, the worker is paid the full output from the match: $w_1^1 = y$. Hence, the only situation in which the vacancy stands to make a positive profit is when the firm is alone

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when approaching a candidate. Thus, expected profits from a vacancy are: $\pi = q(y - w_1^0) - k$. Using (1.2) and (1.5), we then have:

$$\pi = e^{-\phi} y - k \tag{1.6}$$

With competitive entry, we have the additional condition:

$$\pi = 0 \tag{1.7}$$

Using (1.6) and (1.7), we can now determine the equilibrium value of ϕ from the equation:

$$e^{-\phi} = k / y \tag{1.8}$$

or:
$$\phi = \ln y - \ln k$$

With ϕ determined in equation (1.8) all of the endogenous variables are determined. Equation (1.8) also tells us the equilibrium unemployment rate in this model: simply the ratio of the cost to the output from a vacancy:

$$u = k / y \tag{1.9}$$

Similarly, equilibrium unfilled vacancies can be found:

$$v = \frac{M - N(1 - e^{-\phi})}{N} = \phi - 1 + e^{-\phi}$$

and, using (1.8):
$$v = k / y + \ln y - \ln k - 1 \tag{1.10}$$

Using (1.9) and (1.10), we can also derive the Beveridge curve:
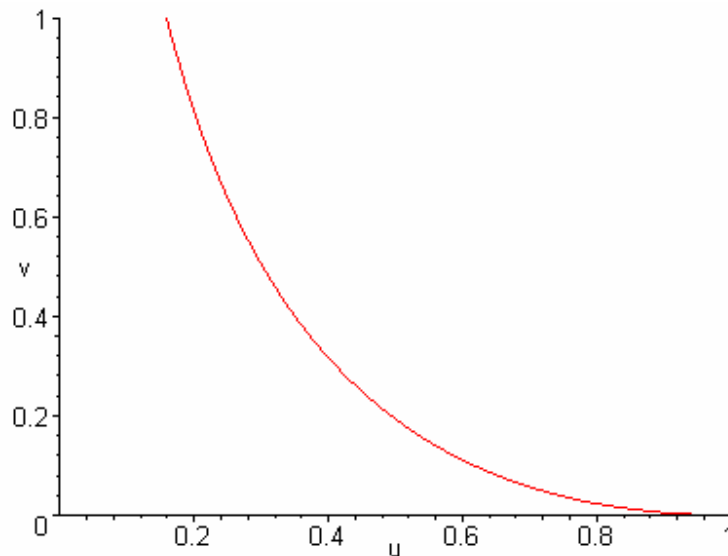
$$v = u - 1 - \ln u \qquad (1.11)$$

Figure 2: The Beveridge Curve in the Static Model

It is worthwhile to note some of the features of this equilibrium. First, from (1.8) and (1.11), we can see that the unemployment rate and the vacancy rate are determined purely by the *ratio* k/y. For example, if productivity and vacancy costs increased proportionately (as is typically assumed in balanced growth models) then unemployment and vacancies would be constant along this path. Also, in this simple static model, nothing shifts the Beveridge curve: changes in either of the two parameters (*y* and k) simply move the economy along the curve. Quite naturally, in the limit as $k \to y$, the ratio of vacancies to candidates $\phi \to 0$, the unemployment rate $u \to 1$, and the unfilled vacancy rate $v \to 0$. The equilibrium also has some wage dispersion – the exact amount of which could now be computed from (1.2).

Constrained Efficiency

Consider a planner that is able to control entry, but still faces the same coordination friction as the private agents. The planner then chooses ϕ to maximize total expected surplus:

$$S = N\left((1-e^{-\phi})y - \phi k\right)$$

Clearly, the first order condition from this maximization problem is exactly the condition (1.8) above. The concavity of the surplus function therefore ensures that the decentralized equilibrium is constrained efficient.

A Numerical Example

If $y=1$ and $k=1/2$ then the equilibrium unemployment rate is 50%, the ratio of vacancies to candidates is 0.6931, and the unfilled vacancy rate is 19.31%. (Clearly, these unemployment and unfilled vacancy rates are very high, reflecting the fact that vacancies have only one chance to hire workers in this static model. More realistic rates occur in the dynamic model, which we analyze in Section 2 below.) The percentage of candidates that find employment, but are paid only their outside option (zero), is 34.655%, leaving 15.345% earning the top wage (1). This implies an average wage, among employed workers, of 0.307. The standard deviation of the wage distribution is 0.213. The wage distribution is also skewed to the right, as in most empirical studies.[7]

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[7] The equilibrium wage distribution in this model is skewed to the right for any value of $y/k < 3.513$, and skewed to the left for any higher value of y/k.

A Comparison with Other Directed Search Models

Given the remarkable simplicity of this model, and the ease with which one can compute the equilibrium allocation, at this point, it is worthwhile to consider the key similarities and differences of this model with others in the directed search literature. In particular, those in papers by Montgomery (1991), BSW, Julien, Kennes and King (2000), and Albrecht, Gautier and Vroman (2002). In Figure 2 we present the extensive form game from the model analyzed in Montgomery (1991) and BSW.

Figure 3: Montgomery (1991) and Burdett, Shi and Wright (2001) Extensive Form
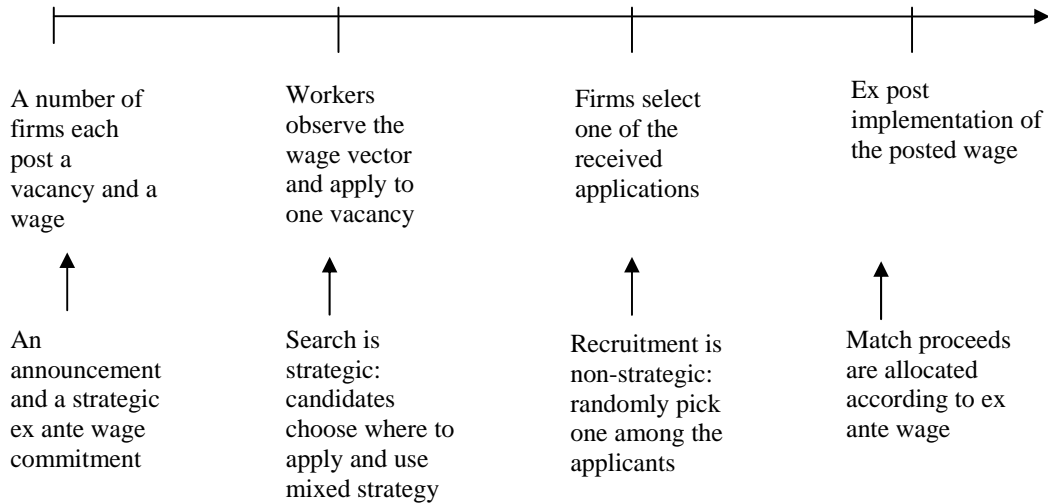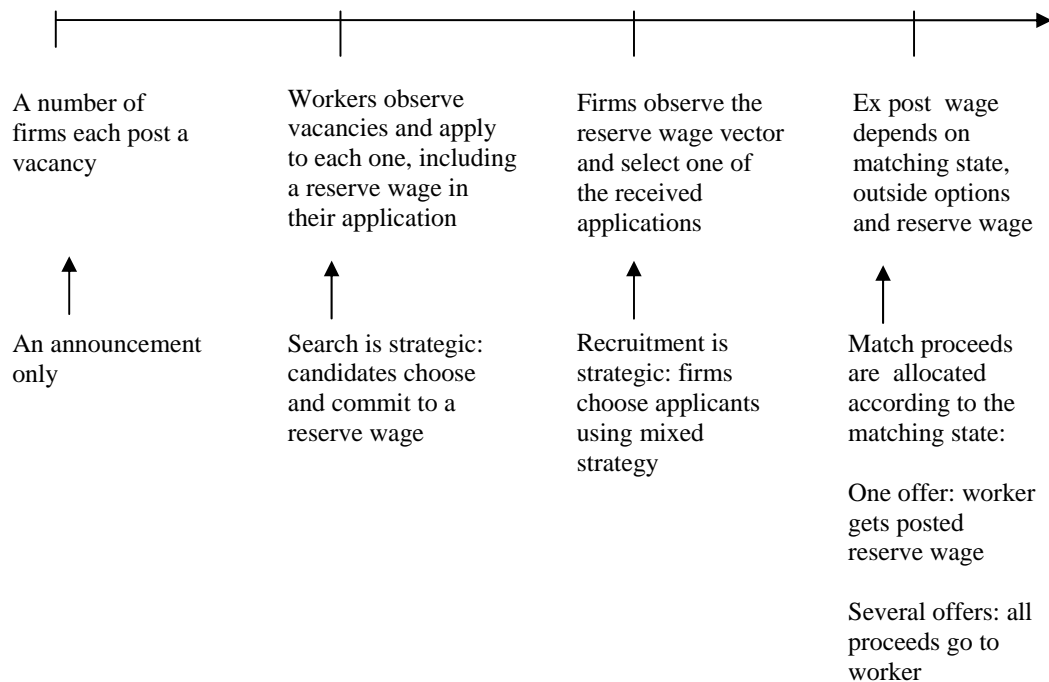

In both Montgomery (1991), and BSW, there are two strategic decisions to be made. First, each firm chooses a wage to post, simultaneously with all other firms. Once this is done, the *search* decision by workers is strategic:[8] each worker chooses a firm to visit, simultaneously with all other workers. Here, workers play mixed strategies. Once both these strategic decisions have been made, then each firm's recruitment decision is non-strategic: each firm simply randomly selects a worker from its pool of applicants.

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[8] We find it useful to make a distinction between the *search* decision, made by workers, and the *recruitment* decision, made by firms. These are, of course, just the two sides of search in two-sided search models like this.
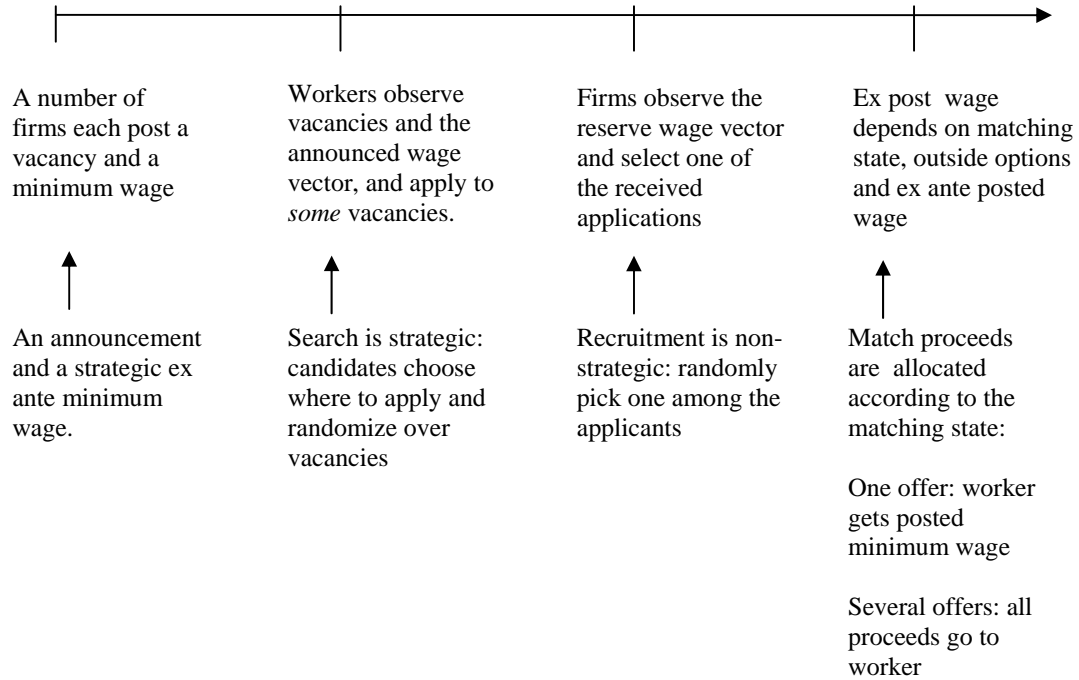
Finally, once applicants have been selected in this way, they work and are paid the *ex ante* announced wage. This last step is not strategic.

Figure 4: Julien, Kennes and King (2000) Extensive Form

A number of firms each post a vacancy	Workers observe vacancies and apply to each one, including a reserve wage in their application	Firms observe the reserve wage vector and select one of the received applications	Ex post wage depends on matching state, outside options and reserve wage
An announcement only	Search is strategic: candidates choose and commit to a reserve wage	Recruitment is strategic: firms choose applicants using mixed strategy	Match proceeds are allocated according to the matching state: One offer: worker gets posted reserve wage Several offers: all proceeds go to worker

In Julien, Kennes and King (2000), there are also two strategic decisions to be made. However, in that model, firms do not post a wage when announcing the existence of their vacancies. Also, the search decision by workers is strategic not because of their decision about where to apply, (in that model, workers apply to every job) but rather, their decision about the appropriate reserve wage to commit to. Here, unlike in BSW, the firms' recruitment decision *is* strategic – firms choose which applicant to approach. Firms play mixed strategies here. Once the firms have been allocated to workers, the auction mechanism determines the wage. Workers who are approached by one firm get paid their reserve wage. (In finite sized markets, the equilibrium reserve wage is positive; in the limit large economy, it is zero.) Workers that are lucky enough to be approached by more than one firm have their wage driven up to the full value of the output.
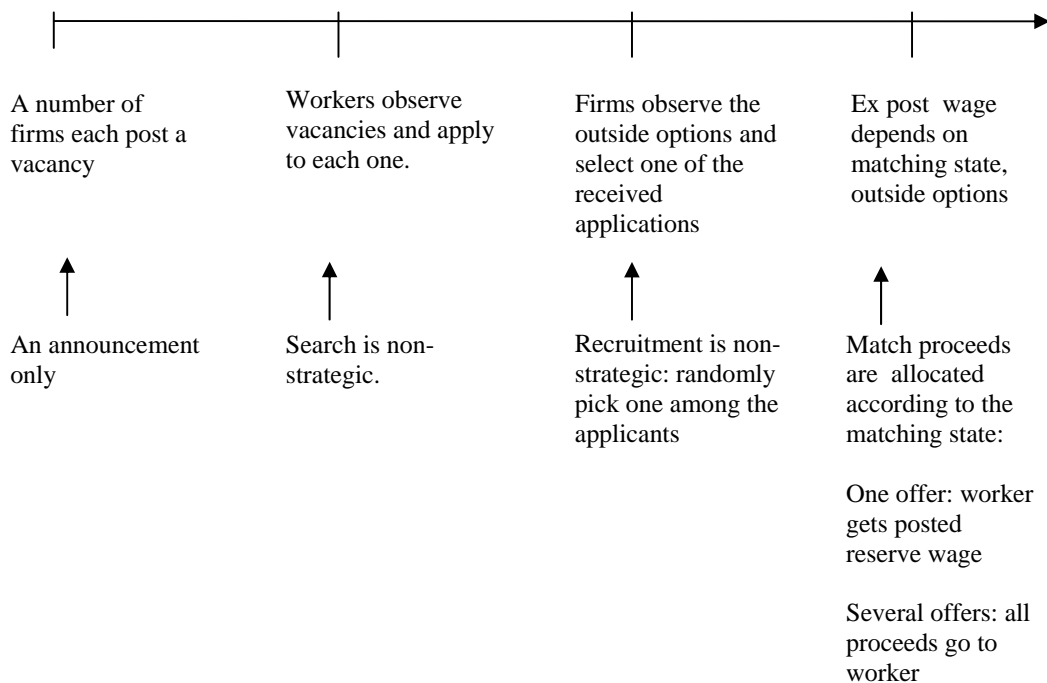
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Figure 5: Albrecht, Gautier and Vroman (2002) Extensive Form

| A number of firms each post a vacancy and a minimum wage | Workers observe vacancies and the announced wage vector, and apply to *some* vacancies. | Firms observe the reserve wage vector and select one of the received applications | Ex post wage depends on matching state, outside options and ex ante posted wage |

An announcement and a strategic ex ante minimum wage.

Search is strategic: candidates choose where to apply and randomize over vacancies

Recruitment is non-strategic: randomly pick one among the applicants

Match proceeds are allocated according to the matching state:

One offer: worker gets posted minimum wage

Several offers: all proceeds go to worker

Albrecht, Gautier and Vroman (2002) present a model that combines elements of both BSW and Julien, Kennes and King (2000). In their framework, as in the other two, there are two strategic decisions made. First, as in BSW, firms post *ex ante* wages to attract workers, in a simultaneous game with other firms. However, unlike in the BSW model, these are simply *minimum* wages that the firms commit to. Firms allow for the possibility of *ex post* competition for workers to drive wages above the announced minimum level. After observing the vector of minimum wages, workers then apply to some vacancies. Albrecht, Gautier and Vroman introduce a parameter here: a – the number of applications that workers make. The number of applications is not a strategic choice for workers, but the *distribution* of these applications is; they focus on the mixed strategy equilibrium. Once firms receive the applications, they then randomly pick an applicant to deal with. This randomization is not modeled as a strategic decision. The randomization, however, is entirely defensible as the outcome of a strategic game, with a mixed strategy equilibrium, as in Julien, Kennes and King (2000), as long as workers are homogeneous. Once the firms are allocated to workers, as in Julien, Kennes and King (2000), *ex post* wage determination takes place. Albrecht, Gautier and Vroman show that,

in the special case where $a = 1$ (i.e., each worker applies to only one vacancy) then the model has exactly the same solution as BSW. However, for any value of a greater than one, ex post wages in the model mimic those in Julien, Kennes and King (2000). In particular, for $a = 2,3,4,...$ the equilibrium posted minimum wage becomes zero, and Bertrand competition between firms, when several approach the same candidate, drives the wage up to the value of the output.

Figure 6: The Extensive Form in This Model

A number of firms each post a vacancy	Workers observe vacancies and apply to each one.	Firms observe the outside options and select one of the received applications	Ex post wage depends on matching state, outside options
An announcement only	Search is non-strategic.	Recruitment is non-strategic: randomly pick one among the applicants	Match proceeds are allocated according to the matching state:
			One offer: worker gets posted reserve wage
			Several offers: all proceeds go to worker

In this model, there are no explicitly strategic decisions. Firms post vacancies, but do not need to compute any *ex ante* equilibrium wage or minimum wage to commit to. Workers apply freely, but do not need to compute any *ex ante* reserve wages to commit to. When firms pick from among the applicants we assume here, as in Albrecht, Gautier, and Vroman (2002), that firms simply randomize. As mentioned above, this is defensible as a mixed strategy equilibrium as long as workers are homogeneous. Once vacancies have been allocated to candidates then the auction determines equilibrium wages in a

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straightforward mechanical fashion, yielding the same wage structure as in the large market case in Julien, Kennes and King (2000), and Albrecht, Gautier and Vroman (2002). As mentioned above, this coincides also with the Mortensen Rule. Because of the absence of any explicit strategic decision-making in this model, the equilibrium is very easy to find and characterize.

Directed and Undirected Search

This model could also be interpreted as one of *undirected* search. If for, example, we interpret the random allocation of vacancies over candidates, according to the matching function (1.3), as a matching *technology* in the usual sense (for example, Pissarides (2000)), rather than as the outcome of a mixed strategy equilibrium, then this model fits into the category of undirected search. There are, however, two key differences between this interpretation of the model and the standard Pissarides model. First, the matching technology here allows for multilateral matches, according to an urn-ball process, while the standard model restricts matches to be pair-wise. Secondly, while both wage determination mechanisms (here and in the standard model) are *ex post*, here we use a simple Bertrand pricing rule rather than the Nash bargaining solution favored in the standard model.

To assess the empirical relevance of this model, we now present a dynamic version, and examine its steady state equilibrium.

2. THE DYNAMIC MODEL

There is large number, *N*, of identical risk neutral workers facing an infinite horizon, perfect capital markets, and a common discount factor $\beta \in (0,1)$. In each time period, each worker has one indivisible unit of labor to sell. At the start of each period $t = 0,1,2,3,...$, there are E_t employed workers, with productivity $y_1 = y$, and $(N - E_t)$ unemployed workers, with productivity $y_0 = 0$. Hence $(N - E_t)$ is the number of agents in the labor force that are actively searching for employment. Also, at the beginning of each period, there are $M_t = \phi_t (N - E_t)$ vacancies. In each period a vacant job has a capital cost of k. Any match in any period may dissolve in the subsequent period with fixed probability $\rho \in (0,1)$. In each period, any vacant job can enter negotiations with at most one worker. However, unemployed workers apply to all firms.

Within each period, the order of play is the same as in the static model: at the beginning of the period, given the state, new vacancies enter. Once the number of entrants has been established, vacancies choose which workers to approach. Once new vacancies have been assigned to candidates, wages are determined through the *ex post* bidding mechanism. We now consider the determination of the equilibrium, using backward induction, within a representative period *t*.

Wage Determination

Let Λ_{it} denote the expected discounted value of a match between a worker and a job of productivity y_i once vacancies have been assigned in period *t*, for $i = 0,1$. Thus, Λ_{0t} represents the value of a "match" between a worker and the unemployment state. Here, through the bidding mechanism, the value of a worker's contract W_{it}^j is equal to the expected discounted value Λ_{jt} of a match between the worker and the worker's second best available alternative:

$$W_{it}^j = \Lambda_{jt} \tag{2.1}$$

The Assignment of Vacancies to Workers

As in the static model, we restrict attention to the unique symmetric mixed strategy equilibrium in which each vacancy randomises over all candidates with equal probability. Let p_{it}^j denote the probability that a candidate receives a contract worth W_{ti}^j. Consequently, the probability distribution of contracts facing candidates (for any ϕ_t, and using (2.1)) is approximated by:

$$W_{it}^j, p_{it}^j \ = \ \begin{cases} W_{0t}^0 = \Lambda_{0t} & p_{0t}^0 = e^{-\phi_t} \\ W_{1t}^0 = \Lambda_{0t} & p_{1t}^0 = \phi_t e^{-\phi_t} \\ W_{1t}^1 = \Lambda_{1t} & p_{1t}^1 = 1 - e^{-\phi_t} - \phi_t e^{-\phi_t} \end{cases} \tag{2.2}$$

Whereas, in the static model, the expression $e^{-\phi}$ represents the unemployment rate; here, in the dynamic model, $e^{-\phi_t}$ represents, in any period *t*, the fraction of candidates that remain unemployed at the end of the period. In discrete time models such as this, the beginning and end of period unemployment rates (and vacancy rates) differ. Researchers have traditionally analysed beginning-of-period rates (see, for example, Shi (2001)) however, end-of-period rates are most directly comparable with their static counterparts. For this reason, we consider both in this paper. Since the number of candidates at the beginning of a period is $(N - E_t)$, the unemployment rate, at the *beginning* of any period *t* is given by

$$u^* = (N - E_t)/N \tag{2.3a}$$

whereas unemployment at the *end* of the period, which is given by:

$$u = u^* e^{-\phi} = \frac{N - E_t}{N} e^{-\phi_t} \tag{2.3b}$$

The number of new hires H_t is given by the matching function:

$$H_t = (N - E_t)(1 - e^{-\phi_t})$$ (2.4)

Using (2.2), we can now express V_t, the value of being a candidate at the beginning of the period, in the following way:

$$V_t = (e^{-\phi_t} + \phi_t e^{-\phi_t})\Lambda_{0t} + (1 - e^{-\phi_t} - \phi_t e^{-\phi_t})\Lambda_{1t}$$ (2.5)

Moreover, a candidate's outside option Λ_{0t} can now be expressed as simply the value of being a candidate in the subsequent period:

$$\Lambda_{0t} = \beta V_{t+1}$$ (2.6)

Vacancy Entry

If a vacancy is able to hire a candidate then the profit from creating the vacancy is equal to the value of the match (Λ_{1t}) minus the cost from creating the vacancy (k) and minus the value of the contract paid to the worker (W_t). The total expected value of the match is equal to the current output y plus the expected discounted future flows of output from the match, and the outside options for the worker if the match separates:[9]

$$\Lambda_{1t} = y + \beta[\rho V_{t+1} + (1 - \rho)y] + \beta^2 (1 - \rho)[\rho V_{t+2} + (1 - \rho)y] + ...$$ (2.7)

The value of the contract paid to the worker depends on the assignment of vacancies to workers. Let q_t denote the probability, in period t, that a vacancy is alone when approaching a worker. This probability is:

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[9] Expected profits for vacancies are driven down to zero, in equilibrium, so future profits for the firm do not appear in (2.7).

$$q_t = e^{-\phi_t} \tag{2.8}$$

By equation (2.2), whenever a candidate is approached by more than one vacancy, the value of the candidate's contract is bid up to the entire value of the match: $W_{1t}^1 = \Lambda_{1t}$. Hence, as in the static model, firms can earn positive profits only when they are alone when approaching candidates. Thus, expected profits are: $\pi_t = q_t(\Lambda_{1t} - W_{1t}^0) - k$. Using (2.2) and (2.8) we get:

$$\pi_t = e^{-\phi_t}(\Lambda_{1t} - \Lambda_{0t}) - k \tag{2.9}$$

Competitive entry implies:

$$\pi_t = 0 \tag{2.10}$$

Equations (2.9) and (2.10), together, imply:

$$e^{-\phi_t}(\Lambda_{1t} - \Lambda_{0t}) = k \tag{2.11}$$

Employment Dynamics

New matches are created according to the matching function (2.4) and broken apart at rate ρ. This leads to employment dynamics:

$$E_{t+1} = (1 - \rho)(E_t + H_t) \tag{2.12}$$

where H_t is given in (2.4).

In this paper, we focus on the stationary equilibrium, where all flows are constant over time.

The Stationary Equilibrium

In the stationary equilibrium, equations (2.4), (2.5), (2.6), (2.7), (2.11),and (2.12) become:

$$H = (N - E)(1 - e^{-\phi})$$ (2.13)

$$V = (e^{-\phi} + \phi e^{-\phi})\Lambda_0 + (1 - e^{-\phi} - \phi e^{-\phi})\Lambda_1$$ (2.14)

$$\Lambda_0 = \beta V$$ (2.15)

$$\Lambda_1 = \frac{y + \beta\rho V}{1 - \beta(1 - \rho)}$$ (2.16)

$$e^{-\phi}(\Lambda_1 - \Lambda_0) = k$$ (2.17)

$$E = (1 - \rho)(E + H)$$ (2.18)

Equations (2.13)-(2.18) are six equations in six unknowns: H, E, ϕ, V, Λ_0, and Λ_1. This system is block recursive, with equations (2.14)-(2.17) determining ϕ, V, Λ_0, and Λ_1 then, once ϕ is determined, equations (2.13) and (2.18) determine E and H. Once these variables are determined, then the solutions for all the other endogenous variables can be found in the stationary equilibrium.

Solving this set of equations, one obtains the following equation, which uniquely determines the value of market tightness, ϕ.

$$e^{-\phi} = [1 - (1-\rho)\beta(1+\phi)e^{-\phi}]\frac{k}{y} \tag{2.19}$$

With ϕ determined in (2.19), the following values are determined:

$$\Lambda_0 = \frac{y - (1 - \beta(1-\rho))e^{\phi}k}{(1-\beta)(1-\rho)} \tag{2.20}$$

$$\Lambda_1 = \frac{y - \rho e^{\phi}k}{(1-\beta)(1-\rho)} \tag{2.21}$$

$$V = \frac{y - (1 - \beta(1-\rho))e^{\phi}k}{\beta(1-\beta)(1-\rho)} \tag{2.22}$$

$$E = \frac{(1-\rho)(1-e^{-\phi})}{1-(1-\rho)e^{-\phi}}N \tag{2.23}$$

$$H = \frac{\rho(1-e^{-\phi})}{1-(1-\rho)e^{-\phi}}N \tag{2.24}$$

Using (2.23) in (2.3a,b), we get the equilibrium unemployment rates at the beginning and end of the period, respectively:

$$u^* = \frac{\rho}{1-(1-\rho)e^{-\phi}} \tag{2.25a}$$

$$u = \frac{\rho e^{-\phi}}{1-(1-\rho)e^{-\phi}} \tag{2.25b}$$

The vacancy rate at the beginning of a period is given by $v^* = M/N = \phi(N-E)/N$. Using (2.23) we get:

$$v^* = \frac{\rho\phi}{1-(1-\rho)e^{-\phi}} \tag{2.26a}$$

At the end of a period, the unfilled vacancy rate $v = (M - H)/N$ can be found, using $M = \phi(N - E)$, together with (2.23) and (2.24) to get:

$$v = \frac{\rho(e^{-\phi} - 1 + \phi)}{1-(1-\rho)e^{-\phi}} \tag{2.26b}$$

Using (2.25a,b) and (2.26a,b), we can now find an expression for the ratios v^*/u^* and v/u in terms of ϕ:

$$v^*/u^* = \phi \tag{2.27a}$$

$$v/u = (e^{-\phi} - 1 + \phi)e^{\phi} \tag{2.27b}$$

At this point, it is instructive to compare (2.27b) with the Beveridge curve in the static model (1.11). In both the static and dynamic models, $e^{-\phi}$ represents the fraction of candidates that go unmatched in equilibrium. In the static model, this is the unemployment rate. Thus, in the static model, (2.27b) reduces to (1.11). However, in the dynamic model, the number of candidates in a period is significantly smaller than the number of workers (the workforce) N, and the Beveridge curve is somewhat different.

Beveridge Curves in the Dynamic Model

Beveridge curves for both the beginning and end of periods can be found in the following way. From (2.25a) we have:

$$\phi = \ln(1-\rho) - \ln(1-\rho/u*)$$

Using this in (2.27a), we have the *beginning-of-period Beveridge curve*:

$$v* = u*\big(\ln(1-\rho) - \ln(1-\rho/u*)\big) \tag{2.28a}$$

Similarly, (2.25b), can be re-written as:

$$e^{\phi} = \rho/u + 1 - \rho$$

Or:

$$\phi = \ln(\rho/u + 1 - \rho)$$

Using these in (2.27b), and collecting terms, we have the *end-of-period Beveridge curve*:

$$v = u\big((\rho/u + 1 - \rho)(\ln(\rho/u + 1 - \rho) - 1) + 1\big) \tag{2.28b}$$

Notice that in the dynamic model, unlike the static model, a shift parameter (ρ) appears in the Beveridge curves. Figures 7a and 7b, below, illustrate the beginning-of-period and end-of-period Beveridge curves, respectively, where ρ has been set equal to 0.01 in each case.
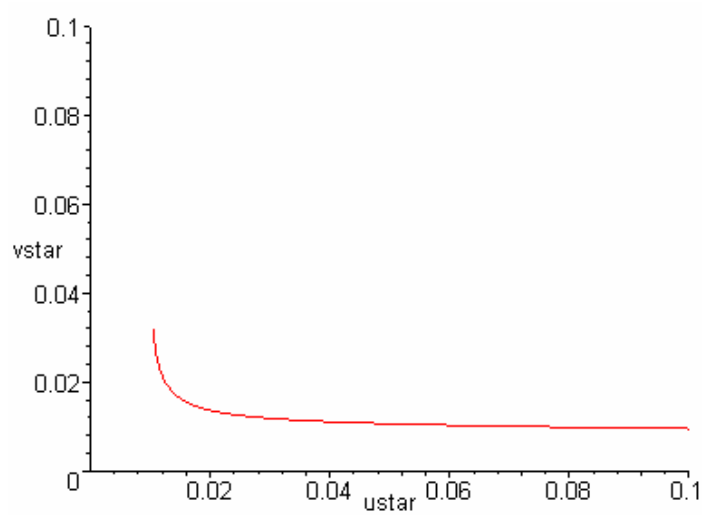
Figure 7a: The Beginning-of-Period Beveridge Curve in the Dynamic Model
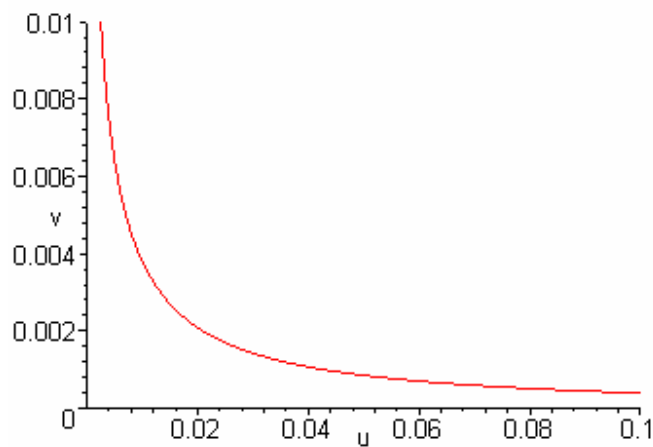

Figure 7b: The End-of-Period Beveridge Curve in the Dynamic Model

Two things are worth noting about these Beveridge curves. First, the beginning-of-period curve is undefined for values of the unemployment rate below the separation rate ρ (for reasons that are clear in equation (2.28a)). Second, in each case, the curves shift outward as ρ increases. Moreover, in the limit, as $\rho \to 0$, both unemployment and vacancies also approach zero.

Constrained Efficiency in the Dynamic Model

As in the static model, consider a planner that is able to control entry in each period, but faces the same coordination problem as individual agents. The planner chooses a sequence $\{\phi_t\}_{t=0}^{\infty}$ with the following objective:

$$\underset{\{\phi_t\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \left\{ y[E_t + H_t] - kM_t \right\}$$

subject to:

(i) $E_{t+1} = (1 - \rho)[E_t + H_t]$

(ii) $H_t = (1 - e^{-\phi_t})(N - E_t)$

(iii) $M_t = \phi_t(N - E_t)$

In the appendix we prove that this objective is met if and only if equation (2.19) is satisfied in the steady state. That is, the planner chooses the same steady state ratio of vacancies to candidates as in the decentralized equilibrium – the equilibrium is constrained-efficient.

Calibrated Examples

There are only four parameters in this model: (β, ρ, y, k). In this paper we consider two different calibrations of the stationary equilibrium. In each case, parameters are chosen so that endogenous variables match data from Katz and Autor's (1999) study of the US labour market for 1995. The two calibrations differ only in the following sense: in the first, parameters are chosen so that the *beginning-of-period* unemployment rate matches the estimated natural rate of unemployment in that year (3.9%), whereas in the second, the *end-of-period* unemployment rate matches this figure. Since Katz and Autor consider weekly data, the discount factor is tied down to $\beta = 0.999$. Similarly, using Kuhn and

Sweetman's (1998) estimate of a 4% monthly separation rate, we set $\rho = 0.01$ (as in the graphical representations of the Beveridge curves, in Figures 7a,b above). (This also implies that the length of time that workers have to consider offers is one week.) We therefore choose values of y and k to match the weekly average real wage in 1982 dollars ($255) and the natural rate of 3.9%. The model then gives us implied values of the vacancy rate and measures of wage dispersion, which can be compared with those in the data.

The following table summarizes the results from the first calibration, matching the beginning-of-period unemployment rate.

Parameter Values: $\beta = 0.999$, $\rho = 0.01$; $y = 339$, $k = 5800$

Statistic	Model	Data
Mean Wage	255	255
Unemployment Rate	3.9	3.9
Vacancy Rate	1.1	1.0
Standard Deviation of Log Wage	0.01	0.616

Table 1: Matching Unemployment at the Beginning of the Period

The first thing to note from Table 1 is that the required cost of vacancies k is quite high, when considering weekly costs. The reason for this is that, in this model, these costs terminate once a vacancy is filled – and vacancies are filled quite quickly in equilibrium. In reality, there are fixed costs when creating jobs and these costs can be quite high. Following Pissarides (2000), though, to keep the state vector as small as possible, we model all costs as flow costs. As a consequence, to balance the expected present value of the infinite stream of the benefits from a job, the costs appear large.

The model generates a beginning-of-period vacancy rate that is quite close to the figure used (from the OECD Main Economic Indicators) – a slight overestimation of the vacancy rate: 1.1% versus 1.0% in the data. Also some wage dispersion is present in the model (the two weekly wages are \$254.26 and \$266.58) but much less than in the data: the model generates a standard deviation of the log wage of only 0.01, compared to the figure of 0.616 reported by Katz and Autor for US data. The model's wage distribution is also skewed to the right with 86.5% of the employed earning the lower wage.

Table 2 summarizes the corresponding figures when matching the end-of-period unemployment rate.

Parameter Values: $\beta = 0.999$, $\rho = 0.01$; $y = 391$, $k = 10000$

Statistic	Model	Data
Mean Wage	255	255
Unemployment Rate	3.9	3.9
Vacancy Rate	0.1	1.0
Standard Deviation of Log Wage	0.0145	0.616

Table 2: Matching Unemployment at the End of the Period

Clearly, to achieve an unemployment rate of 3.9% at the end of the period, we must have a higher unemployment rate at the beginning of the period. Through the Beveridge curve relationships in Figures (7a,b), (which are stable as values of y and k vary) higher unemployment rates will reduce vacancy rates. In this case, the end-of-period vacancy rate is only 10% of the figure in the data. The wage dispersion figure, as in Table 1, is also significantly below that from the Katz and Autor study.

Interpretations

When considering beginning-of-period unemployment and vacancy rates, this model generates a plausible number for the vacancy rate. However, this number falls significantly when end-of-period rates are used. This raises the question of which rates are more appropriate to be used. Estimated vacancy rates use measures of vacancy advertisements. It seems reasonable to interpret these as occurring at the beginning of a period – that is, we expect the number of *unfilled* vacancies within any time period to be less than the number of *advertised* vacancies. For the purposes of policy, though, it is the unfilled vacancies, and unsuccessful candidates that are most relevant.

While this theory generates some wage dispersion, even in the best case, the model can only explain 2% of the dispersion found in the data. If one were to try to explain more of this dispersion using this framework, some other ingredient would be required. For example, in Julien, Kennes and King (2003) we allow for two different types of jobs to be created by firms – one with higher productivity and costs than the other.

A Note on The Walrasian Limit

The key friction in this framework is the time it takes for a firm to approach a worker with a job offer. In the static model firms can approach only one worker, so the time it takes to approach another is effectively infinite. In the dynamic model, as we have calibrated it, this takes one week. Clearly, as this length of time shrinks, the friction disappears. This is reflected in the equilibrium unemployment and vacancy rates derived above. As the length of time shortens, the relevant value of ρ falls. For any finite value of ϕ, this means that u, u^*, v, and v^* all approach zero as $\rho \to 0$ (as can be seen from equations 25a,b and 26a,b). One interpretation of the results in this paper, therefore, is that a week between offers is sufficient to explain the unemployment and vacancy figures we observe – but not the wage dispersion.

3. CONCLUSION

One goal of this paper has been to gain a better understanding why ex ante prices are often not communicated between buyers and sellers in the labour market. The simple answer presented here is that sellers do not have to advertise prices in order for the market to behave efficiently. A related game theoretic answer is found in Julien, King and Kennes (2000). That paper shows that competition (as the scale of the market increases) actually forces the equilibrium reserve prices to equal the sellers' outside options. Moreover, as shown in Kultti (1999) and elsewhere, expected payoffs to agents in the limit large economy are identical whether sellers post prices or reserve prices. Thus competition implies that ex ante prices do not play any significant allocative role in equilibrium.

We believe that our findings help to explain why negotiations in the labour market follow strict bargaining procedures. For example, on the buyers' side of the labour market, if a candidate accepts an employer's offer, then the employer is committed to giving the candidate the job. On the seller's side, however, candidates can apply to multiple vacancies since applying for a job does not carry a commitment to accept if it is offered. These negotiation procedures give workers significant bargaining power in the event of multiple offers because the worker can easily play competing two firms off against each other. What we have shown is that these formal bargaining procedures are an effective substitute for selling mechanisms that attempt to set the bargaining agenda with ex ante price advertisements.

The use of ex post pricing mechanisms by sellers may have important advantages over ex ante pricing mechanisms. For example, they may play a role in markets where unemployment is of sufficient duration and where the use of posted prices is hindered by the existence of frequent changes in the external environment. Therefore, it seems possible to construct a wide range of models in which technological uncertainty and costly price changes imply an advantage for invariant ex post pricing mechanisms. An interesting topic for future research is to explore how menu costs affect the properties of

decentralized selling mechanisms in stochastic environments where the money supply and technology, for example, are subject to change.

The directed search model presented in this paper is extremely simple and easy to solve. Here, as in most directed search models, a very basic coordination problem motivates the existence of unemployment and vacancies in equilibrium. This particular model is easy to work with because it has one less stage of the game than others: there is no need to compute ex ante price (or reserve price) announcements. Despite its simplicity, however, this model is capable of generating plausible unemployment and vacancy rates in equilibrium, along with intuitive comparative static properties. Some wage dispersion is also generated in equilibrium, but only approximately 2% of observed values. Using this type of model to account for observed wage dispersion would require some sort of heterogeneity – either for workers or firms.

The equilibrium in this model is also constrained-efficient is the usual sense – unless a planner can somehow solve the coordination problem, he or she cannot increase aggregate expected output or utility by influencing decisions made by agents. However, this result importantly assumes risk neutrality. With risk aversion, some sort of income insurance would be required to achieve constrained efficiency of this type. Fortunately, due to the simplicity of the framework, analysing policy questions of this type should be relatively straightforward.

Appendix

Solution to the Social Planner's Problem:

The social planner's problem is

$$\underset{\{\phi_t\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \{y[E_t + H_t] - kM_t\}$$

subject to

(i) $E_{t+1} = (1-\rho)[E_t + H_t]$

(ii) $H_t = (1-e^{-\phi_t})(N-E_t)$

(iii) $M_t = \phi_t(N-E_t)$

Rewriting constraints (i) and (ii) as

$$\frac{E_{t+1}}{(1-\rho)} = (1-e^{-\phi_t})(N-E_t) + E_t$$

Define $a_t = \dfrac{E_{t+1}}{(1-\rho)}$ as a control variable. Then rewrite the constraints as

$e^{-\phi_t} = \dfrac{N-a_t}{N-E_t}$ or $\phi_t = \ln[N-E_t] - \ln[N-a_t]$. Since $M_t = \phi_t(N-E_t)$, substituting for ϕ_t, the planner's problem becomes:

$$\underset{\{a_t\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \{a_t y - (\ln(N-E_t) - \ln(N-a_t))k(N-E_t)\}$$

Define the function:

$$f(E_t, a_t) = a_t y - [\ln(N-E_t) - \ln(N-a_t)]k(N-E_t)$$

where

$$f_{a_t}(E_t, a_t) = y + \frac{k(N-E_t)}{(N-a_t)}$$

The Bellman equation is written as

$$V(E_t) = \underset{\{a_t\}_{t=0}^{\infty}}{Max}\{f(E_t, a_t) + \beta V(E_{t+1})\}$$

Since $E_{t+1} = (1-\rho)a_t$, rewrite

$$V(E_t) = \underset{\{a_t\}_{t=0}^{\infty}}{Max}\{f(E_t, a_t) + \beta V((1-\rho)a_t)\}$$

The first-order condition yields

$$f_{a_t}(E_t, a_t) + \beta V_{a_t}((1-\rho)a_t) = 0$$

Updating the Planner's objective function one period yields

$$f(E_{t+1}, a_{t+1}) = a_{t+1}y - [\ln(N - E_{t+1}) - \ln(N - a_{t+1})]k(N - E_{t+1})$$

or

$$f((1-\rho)a_t, a_{t+1}) = a_{t+1}y - [\ln(N - (1-\rho)a_t) - \ln(N - a_{t+1})]k(N - (1-\rho)a_t)$$

Using the Benveniste-Scheinkman equation yields

$$V_{a_t}(E_{t+1}) = f_{a_t}((1-\rho)a_t, a_{t+1}) = k(1-\rho)(1+\phi_t)$$

Using this in the first-order condition yields

$$y + \frac{k(N - E_t)}{(N - a_t)} + k(1-\rho)(1+\phi_t) = 0$$

Which under steady state with appropriate substitution yields:

$$e^{-\phi}y = k(1 - \beta(1-\rho)(1+\phi)e^{-\phi} = 0$$

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