Optimal Income Tax in the Presence of Status Effects

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The classical optimal income tax problem does not reveal many general properties except for the well-known tendency for marginal tax rates to reduce for high ability types, and in fact to become zero for the top type. The existence of distortions from individuals competing to attain social status by using consumption signals justifies some measure of income tax. The question posed here is whether it also constitutes a reason for a more progressive income tax schedule. The answer is found to be broadly negative if progressivity is interpreted as increasing marginal tax rates. On the other hand, status-seeking makes the optimal tax schedule steeper so that redistribution is increased. Broadly, the analysis of status-seeking based on a signalling approach confirms and strengthens the existing view of an optimal tax schedule.

JEL: H2; Status, signalling, income tax

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1. Introduction

People live within a social context, and care about how they are judged by others. Thus individuals make economic decisions over consumption, savings and labour supply in part to affect such judgements. Early work by Hirsch (1976) used the concept of “positional goods” as those where consumption levels are observed and used to rank people in terms of social status. Frank (1984, 1985a, b) has analysed the distortions involved in responding to the possibility of improving one’s social status by increased consumption of positional goods. Taxes on positional goods may be Pareto Optimal since they counter over-consumption and hence reduce distortions[1] . However, the taxing of positional goods may not be feasible. A number of reasons exist. The main ones are that the particular goods in question may change rapidly, and that they may be entangled with goods which are not affected by status contests. For instance, it would be difficult to tax a particular brand of sports shoe higher than other brands. After all, a fashionable sports shoe is still a sports shoe. Finally, the tax of certain goods at a discriminatory level may be outlawed by trade rules.

The difficulty of taxing positional goods has led to the suggestion that either general expenditure or income is taxed, or more correctly that such taxes can be justified at least in part as correcting factors for distortions that arise from status contests. The argument is that since higher income (or expenditure) involves higher tax, the real cost (eg in terms of amount of time spent working) of spending on status-seeking is higher, and this leads to lower

[1] See Ireland, 1994, Bernheim, 1994, and Seidman 1987, 1988 among other contributions.

quantities of "positional goods" being purchased. A number of approaches to the analysis of status-seeking activities conclude that an argument exists for a positive marginal income tax rate in order to reduce distortions which would otherwise be present. One approach (Ireland, 1997) models the process within a signalling game among a continuum of "ability" types. The key result is that a small linear income tax can provide a Pareto optimal improvement. Basically, those on low incomes gain from the redistribution of the tax as benefits, while those on high incomes gain from the lower expenditure necessary to retain their social standing within the new signalling equilibrium. The Pareto improvement is possible because the income tax acts counter to a distortion in labour supply: status seeking prompts individuals to over supply labour in order to fund more expenditure on status-yielding goods. In a signalling equilibrium, no social gain is achieved by such expenditure since all types are revealed, and the externalities of forcing others to spend more to signal their types are not included in individuals' calculations. Hence an equilibrium with lower status expenditures is a less-distorted allocation.

It is probable that the above result is robust to different approaches to modelling status-seeking, and a number of other approaches have been used. [2] What is not so clear is whether the argument extends to a justification for a progressive income tax (one where the marginal tax rate for higher income individuals is higher than that for lower income individuals). A simple claim would be that high income people spend more on status-seeking

[22] See for example Cole et al (1992). Also Bagwell and Bernheim (1996) who take a signalling approach to status contests but show that if vertical product differentiation exists and individual types are very different then no wasteful signals are necessary. However, as types become closer it becomes too easy for the lower types to mimic the higher types and so more aggressive (that is more wasteful) signalling becomes necessary. Other externalities have been considered. For example Konrad and Lommerud (1993) suggest that excessive risk taking may arise in a model where status is important. Other signalling motivations include charitable donations. See for example Glazer and Konrad, (1991).

(and have to in order to keep up their social standard), and thus these individuals should be taxed at a higher marginal tax rate since they have a bigger distortion to correct. Such a claim is particularly interesting since economics has some difficulty in making a theoretical case for increasing marginal tax rates. Classic papers (Mirrlees, 1971, Seade, 1977, Sadka 1976, Weymark, 1986) all combine to present a complex picture of an optimal income tax schedule. See for example the discussion in Myles, 1995, Chapter 5. A key point in this picture is that the top of the tax schedule should be flat, so that the highest type of individual does not distort her labour supply. [3]The intuition behind this result is that this highest type would increase aggregate tax paid only trivially if her marginal tax rate was raised, whereas a distortion in labour supply would be created. On the other hand, for a type further down the distribution a positive marginal tax rate would increase tax paid both by that type and all individuals higher up in the income distribution, and would only cause a distortion in labour supply for that type. Further, distortions on the lower type may make the higher type less attracted to mimicking the lower type in order to avoid tax on income. These incentive compatibility constraints put a limit on how fast taxes can increase with income and still be implemented. The shape of the optimal tax schedule has few general properties other than its regressive nature at the top. Simulation results (see for example Atkinson and Stiglitz, 1980) indicate some initial increase in marginal tax rates then a falling off to low levels for top types. One problem is that comparison of gains and losses are made more complicated by the number of individuals at different levels.

[3] See particularly, Sadka, 1976, Theorem 4.

In this paper we seek to build a simple optimal income tax model which incorporates a social status signalling equilibrium mechanism. Such a task is ambitious since we start from the knowledge that, absent any social signalling, the optimal tax schedule has few general properties. Thus we need to assess the impact on the optimal tax schedule of including social status signalling, asking whether the "top type" result still holds, and whether there is a general case for positive marginal tax rates for lower types. We also need to assess the relative impact of status on the marginal tax rates faced by different types, in order to see whether the change induced by a greater status effect is toward or away from a more progressive income tax schedule.

In section 2, we describe the model as one including four basic incentive compatibility constraints. We show in section 3 how a separability simplification allows us to consider a corresponding but much simpler problem. Section 4 demonstrates an explicit solution for a quasi-linear utility function. In section 5 we present an overall assessment. The approach of the paper is to consider the model as one relating to individual consumers. However, it should be realised that status is not a preserve of individuals. Firms, banks and governments also behave in status seeking ways, and minor adjustments of the model would yield analyses of income tax and status seeking for a wide group of economic agents.

2 The Model

Optimal income tax within a finite economy has previously been studied, for example a finite economy is considered in Weymark (1986)[4]. Our approach is also to study a finite economy, and so we will assume that there are I + 1 types of individual. The typical individual of type i ($i = 0,1,2,\ldots,I$) has an *effective* time allocation of x_i, and $x_{i+1} > x_i$ so that the best endowed individual (most able individual) is type I and the worst endowed is type 0.[5] An individual has a fundamental utility, which is the objective function in a complete information setting, of $u(c_i,h_i)$, which is a concave function, strictly concave in at least one argument, and with positive marginal utilities, and where $c_i = y_i - t_i - s_i$ and $h_i = x_i - y_i$. Thus c_i is the level of consumption and h_i the level of leisure. Leisure is defined as endowed effective time minus labour supply time y_i. Consumption is defined as income equal to labour supply y_i (that is the wage per hour of labour is set to 1) minus tax t_i minus a wasteful expenditure incurred in signalling of s_i.This wasteful expenditure can be considered as the extra expenditure necessary to make the consumption portfolio have a given degree of observability: the greater the degree required, the greater the expenditure. Alternatively it can be interpreted as expenditure on products which are bought purely to impress. The latter interpretation is obviously more limited. The former requires a notion of a consumption technology where one product characteristic is its observability by others.[6]

[4] See also Guesnerie and Seade, 1982, for a discussion of the importance of considering an economy without a continuum of individuals. Here, the matter is largely dictated by the need to keep track of multiple constraints between types.

[5] See Hall (1975) for a discussion of this approach to heterogeneity of individuals.

[6] The requirement is for a consumption technology (Lancaster, 1966), with observability as one characteristic produced by products in different amounts.

There are two information transmission processes taking place. First, the individual can choose from a menu of various (y_i , t_i) pairs. The menu is set by the government and this is the tax payment problem. Second, each individual can choose from among various levels of s. That is the individual can choose to make a signal in order to cause his/her social group to infer that he/she is of a particular type. This is the type-signalling problem with social status as the goal. By making a choice about income level (and thus tax to be paid), the individual is telling the government about his/her type using a self-selection mechanism private to the individual and the government. By making expenditures s the individual is sending information to his/her social group which is not observable by the government. Thus the two information transmission mechanisms are separate in their destinations, but each destination agent knows that the other signal/screening mechanism is active. Thus the wasteful expenditure s cannot be directly taxed since it is not observed by government. Also the wasteful expenditure s would probably not be needed as a signal if the amount of tax paid was public knowledge. A conflict arises as the individual would wish to avoid tax by reducing labour supply, but this would make social status harder to pay for.

We define α as the degree of incomplete information in the social group (Ireland and Yamashige, 1998). Thus α is the chance or relative frequency of events where the social group is going to infer the status of the individual from actions that can be observed. The objective function of the individual in this incomplete information setting is a weighted average of "fundamental" utility and that inferred by others in the social group.[7] It can be interpreted as an expected utility function or simply as a weighted average:

[7] The weighted average can be thought to be a result of subjective probabilities (Anscombe and Aumann, 1963). Alternatively, it is just an assessment of the importance of others' opinions.

$$U(x_i) = (1-\alpha)\, u(c,h) + \alpha\, v(s) \qquad (1)$$

where v(s) is the utility level inferred from s. In a separating equilibrium the inference will be correct and $U(x_i) = u(c,h) = v(s)$. We will postulate that v(s) is of the form:

$$v(s) = u(y_i - t_i - s, x_i - y_i) \quad \text{if } s_i \le s < s_{i+1} \qquad (2)$$

so that the social group infers type from observing s and then applies the "right" income level and tax paid. Thus we will make extensive use of the shorthand notation $v(s_i) \equiv u(y_i - t_i - s_i, x_i - y_i)$ Both this and the government's calculation of income tax charges depend on all relevant incentive compatibility conditions holding. A number of modes of imitating other types are possible and at least four need explicit consideration. These are

IC1: $u(y_i - t_i - s_i, x_i - y_i) \ge (1-\alpha)\, u(y_{i-1} - t_{i-1} - s_i, x_i - y_{i-1}) + \alpha\, u(y_i - t_i - s_i, x_i - y_i) \quad \forall i > 0$

or, using (2),

ICI: $v(s_i) \ge (1-\alpha)\, u(y_{i-1} - t_{i-1} - s_i, x_i - y_{i-1}) + \alpha\, v(s_i) \quad \forall i > 0$ (3)

IC2: $u(y_i - t_i - s_i, x_i - y_i) \ge (1-\alpha)\, u(y_{i-1} - t_{i-1} - s_{i-1}, x_i - y_{i-1}) + \alpha\, u(y_{i-1} - t_{i-1} - s_{i-1}, x_{i-1} - y_{i-1})$

or

IC2: $v(s_i) \ge (1-\alpha)\, u(y_{i-1} - t_{i-1} - s_{i-1}, x_i - y_{i-1}) + \alpha\, v(s_{i-1}) \quad \forall i > 0$ (4)

!C3: $u(y_i - t_i - s_i, x_i - y_i) \geq (1-\alpha)\, u(y_i - t_i - s_{i+1}, x_i - y_i) + \alpha\, u(y_{i+1} - t_{i+1} - s_{i+1}, x_{i+1} - y_{i+1})$

or

IC3 $v(s_i) \geq (1-\alpha)\, u(y_i - t_i - s_{i+1}, x_i - y_i) + \alpha\, v(s_{i+1}) \quad \forall i < I$ (5)

IC4: $u(y_i - t_i - s_i, x_i - y_i) \geq (1-\alpha)\, u(y_{i+1} - t_{i+1} - s_{i+1}, x_i - y_{i+1}) + \alpha\, u(y_{i+1} - t_{i+1} - s_{i+1}, x_{i+1} - y_{i+1})$

or

IC4: $v(s_i) \geq (1-\alpha)\, u(y_{i+1} - t_{i+1} - s_{i+1}, x_i - y_{i+1}) + \alpha\, v(s_{i+1}) \quad \forall i < I$ (6)

Conditions IC1 and IC2 relate to the individual's temptation to pretend to be a poorer type to the government. This is done by making only y_{i-1} income and thus paying only t_{i-1} tax. In IC1 the individual still funds status at level i by reducing consumption further. Thus the right-hand side of IC1 is a weight of $1-\alpha$ on fundamental utility when paying the lower tax, plus a weight of α on the inferred utility level $v(s_i)$. The constraint means that imitating to the government but not the social group must not be better than sending all the correct signals. In the RHS of IC2 he/she also imitates the lower type to the social group by not maintaining level i signals, and again this must not be better than truth revelation.

Conditions IC3 and IC4 relate to the temptation to imitate a higher type in signalling to the social group. Thus IC3 states that the individual should not want to pretend to be a higher type to the social group while revealing type truthfully to the government. IC4 is a more extreme form of pretence where the individual is willing to pay more tax to earn more income from which to finance imitation of a higher type to the social group: the individual is constrained not to want to imitate a higher type to both the social group and the government.

The four conditions relate to eliminating temptations to pretend to be a lower type to either the government or to both, or to pretend to be a higher type to either the social group or to both. Of course other possibilities might remain but are unlikely given the structure which we will put on the preference function. For example, to pretend to act poor to the government and rich to the social group (low y and high s) would imply a very low level of real consumption, and thus would be unlikely to be an attraction.

The government sets I+1 pairs (y_i, t_i) from which the individual chooses. There will also be I+1 minimal signal expenditures set by the social group. Of course higher signal expenditures than the minimal might be required (signalling equilibria are rarely unique, see Spence (1973) and Mailath, (1987)). However, there are attractions in choosing the lowest signals to produce a separating equilibrium since otherwise our analysis would be susceptible to the charge that other equilibria exist which have less impact. We thus have I+1 triplets (y_i, t_i, s_i) to find, such that the the social group separates its members and the government maximises a welfare function. The objective of the government is to maximise by choice of (y_i, t_i, s_i) a welfare function of the form

$$W = \sum_{i=0}^{I} n_i \mu_i u(s_i) \qquad (7)$$

subject to constraints (3)-(6), plus

$$g = \sum_{i=0}^{I} n_i t_i \quad \text{(a government budget constraint)} \qquad (8)$$

and

$s_i \geq 0, \forall i$ (a constraint that signals have to be non-negative)   (9)

where n_i is the number of individuals of type i and μ_i is the relative welfare weight of each such individual in the welfare function, and we normalise the scale of these weights so that $\sum_{i=0}^{I} n_i\mu_i = \sum_{i=0}^{I} n_i = N$. We assume that $\mu_i \geq \mu_{i+1}, \forall i < I$.

The problem is very complex and a certain amount of structure has to be imposed to make reasonable progress. We will therefore concentrate on examining the incentive-compatibility constraints when the fundamental utility function is strictly separable. Also, a quasi-linear special case is of considerable interest since it permits the tax and signal variables to be found explicitly from the constraints.

3 Incentive Compatibility when Utility is Additively Separable

Consider the fundamental utility function to be of the form:

$$u(c, h) = g(c) + f(h) \tag{10}$$

where g(c) and f(h) are concave in c and h respectively, with at least one sub-function strictly concave, and thus u(.,.) is concave. The expected utility function is then

$$U(x_i) = (1-\alpha)\,(g(y - t - s) + f(x_i - y)) + \alpha\, v(s) \tag{11}$$

We begin our analysis with two lemmas.

Lemma 1: Assume fundamental utility has the form of (10). Then (i) if IC1 and IC3 hold as equalities then IC2 holds as an equality;(ii) if IC2 and IC3 hold as equalities then IC1 holds as an equality.

Proof: write IC1-3, inequalities (3)-(5), for i>0, as

$$\text{IC1: } v(s_i) - v(s_{i-1}) \geq g(y_{i-1} - t_{i-1}- s_i) - g(y_{i-1} - t_{i-1}- s_{i-1}) + f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1}) \qquad (3')$$

$$\text{IC2: } v(s_i) - v(s_{i-1}) \geq (1-\alpha)(f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1})) \qquad (4')$$

$$\text{IC3: } v(s_i) - v(s_{i-1}) \leq (-(1-\alpha)/\alpha)\,(g(y_{i-1} - t_{i-1}- s_i) - g(y_{i-1} - t_{i-1}- s_{i-1})) \qquad (5')$$

(i) multiply IC1 as an equality by (1-α) and IC3 as an equality by α and add to obtain

$$v(s_i) - v(s_{i-1}) = (1-\alpha)(f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1}))$$

which is IC2 as an equality.

(ii) multiply IC2 as an equality by $(1-\alpha)^{-1}$ and IC3 as an equality by $-(\alpha(1-\alpha)^{-1})$ and sum to obtain

$$v(s_i) - v(s_{i-1}) = g(y_{i-1} - t_{i-1} - s_i) - g(y_{i-1} - t_{i-1} - s_{i-1}) + f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1})$$

which is IC1 as an equality

Lemma 2: Again assume the fundamental utility function has the the form of (10). If the solution has $y_i > y_{i-1}$, all i> 0, then IC2 holding as an equality, all i, implies that IC4 holds. If y_i, > y_{i-1} , all >0, then IC4 holding as an equality implies that IC2 holds. There is no solution to the problem with y_i decreasing in i, some i if f(h) is strictly concave.

Proof: Write IC2, (4), as

$$v(s_i) - v(s_{i-1}) \geq (1-\alpha)(f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1})) \tag{4'}$$

and IC4, (6), as

$$v(s_i) - v(s_{i-1}) \leq (1-\alpha)(f(x_i - y_i) - f(x_{i-1} - y_i)) \tag{6'}$$

The RHS of (6’) is greater than the RHS of (4’) iff $y_i > y_{i-1}$, since $(f(x_i - y) - f(x_{i-1} - y)$ is non-decreasing in y due to the concavity of f(h). Finally, if f(h) is strictly concave then $(f(x_i - y) - f(x_{i-1} - y)$ is increasing in y and IC2 and IC4 cannot both be satisfied if $y_i < y_{i-1}$ for some i, and the lemma is proved.

The implication of the lemmas is that if we take one (either one) of IC1 and IC2, plus IC3, and ignore the other 2 incentive compatibility constraints, then the simplified problem will have a solution which is feasible in the original problem, provided the optimal y_i are increasing in i: that is that the relatively well-endowed do not take more than their extra endowment as leisure. Additionally, we know that if we take a simplified problem with IC4 binding, and therefore not IC2, then only one of IC3 and IC1 can bind.[8] Now in any interesting tax problem, one of IC1 and IC2 should bind else tax redistribution is not constrained. We know that IC2 will not be binding if IC4 is binding. Also we show below that IC1 and IC4 cannot both be binding in a solution: hence IC4 cannot bind.

Lemma 3: If IC1 and IC4 are binding then IC3 is not satisfied if f(h) is strictly concave.

Proof: Rewrite IC4, (6’), as an equality

$$v(s_i) - v(s_{i-1}) = (1-\alpha)(f(x_i - y_i) - f(x_{i-1} - y_i)) \qquad (6'')$$

[8] If both IC1 and IC3 were binding then by Lemma 1 so would LC2 bind. But this is not compatible with IC4 binding by Lemma 2.

and IC1 (3’) as an equality

$$v(s_i) - v(s_{i-1}) = g(y_{i-1} - t_{i-1} - s_i) - g(y_{i-1} - t_{i-1} - s_{i-1}) + f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1}) \qquad (3'')$$

Substitute (3’’) into IC3 (5’):

$$v(s_i) - v(s_{i-1}) \leq (-(1-\alpha)/\alpha) \; (v(s_i) - v(s_{i-1}) - f(x_i - y_{i-1}) + f(x_{i-1} - y_{i-1}))$$

or

$$v(s_i) - v(s_{i-1}) \leq (1-\alpha) \; (f(h_i - y_{i-1}) - f(x_{i-1} - y_{i-1})) < (1-\alpha) \; (f(x_i - y_i) - f(x_{i-1} - y_i)) \qquad (12)$$

since $f(x_i - y) - f(x_{i-1} - y)$ is increasing in y, and see that (12) contradicts (6’’).

The importance of these lemmas is that a solution to the simplified problem with IC1 (or IC2) plus IC3 binding is also a feasible and optimal solution to the original problem. Without the separability condition on fundamental preferences, this is not possible to state. The choice of which of IC1 and IC2 are binding will depend on whether consumption and leisure are complements or substitutes (here they are neither). More importantly IC4 cannot be so easily disregarded. We can now state a proposition which demonstrates that the basic results from optimal income tax theory need not be changed by the addition of status effects.

Proposition 1: If the fundamental utility is additively separable as in (10) then (i) type I’s labour supply equates the marginal utilities of consumption and leisure while (ii) *for i< I, type i's labour supply leaves the marginal utility of consumption higher than the marginal utility of leisure.*

Proof: the optimal income tax problem can be written as a Lagrangean, and first-order conditions stated. We have from (3), (5), (7) and (8), the Lagrangean

$$L = \sum_{i=0}^{I} n_i\mu_i\, v(s_i) - \theta(g - \sum_{i=0}^{I} n_i t_i) - \sum_{i=1}^{I} (\lambda_i(u(y_{i-1} - t_{i-1} - s_i, x_i - y_{i-1}) - v(s_i))) -$$
$$\sum_{i=0}^{I-1} (\delta_i((1-\alpha)\, u(y_i - t_i - s_{i+1}, x_i - y_i) + \alpha\, v(s_{i+1}) - v(s_i))) \qquad (13)$$

First-order conditions with respect to the y_i yield the necessary information as to the level of marginal tax rates. A higher marginal tax rate is reflected in a higher marginal rate of substitution of leisure for consumption. The multipliers are all non-negative from standard Kuhn-Tucker theory. For the top type's income or labour supply, y_I, we have

$$\partial L/\partial y_I = (n_I\mu_I + \lambda_I - \alpha\delta_{I-1})\, (g'(y_I - t_I - s_I) - f'(x_I - y_I)) = 0 \qquad (14)$$

and for other types, labour supply y_i, we have

$$\partial L/\partial y_i = (n_i\mu_i + \lambda_i - \alpha\delta_{i-1} + \delta_i)\, (g'(y_i - t_i - s_i) - f'(x_i - y_i)) - \lambda_{i+1}((g'(y_i - t_i - s_{i+1}) - f'(x_{i+1} - y_i))) -$$
$$\delta_i((1-\alpha)(g'(y_i - t_i - s_{i+1}) - f'(x_i - y_i))) = 0 \qquad (15)$$

It is easily seen that

$$g'(y_I - t_I - s_I) - f'(x_I - y_I) = 0 \quad (16)$$

$$g'(y_i - t_i - s_i) - f'(x_i - y_i) > 0 \quad (17)$$

The condition (17) comes directly from noting that (15) is negative at that y_i where $g'(y_i - t_i - s_i) - f'(x_i - y_i) = 0$. Thus labour supply must be less than this level to yield the first-order condition and hence (17) must hold.

It is apparent that the addition of the status effect does not change the result that the top type faces a zero marginal tax rate and thus equates the marginal utilities of consumption and leisure, while all other types under-supply labour in the sense that the marginal utility of leisure is less than the marginal utility from consumption. There is no way of ascertaining if the income tax schedule is actually made less progressive by the addition of the status effect without more structure. We therefore turn to a more specific case.

4. Quasi-linear Case

A special case of strong separability is when the utility function has g(c) linear.[9] In this quasi-linear case, we can solve explicitly for taxes and signals and obtain results about the shape of the optimal tax schedule. We use the binding incentive compatibility constraints to find t_i and $t_i + s_i$ for all $i > 0$, and substitute into the government's budget constraint to eliminate t_0. We then use the efficiency condition (minimal signals) to set s_0 to zero. The result is an unconstrained maximisation problem to be maximised with respect to the individual type income levels.

We note that the quasi-linear case is such that, under full information for both the social group and the government, no signals would be made and taxes would transfer all income to the types with highest welfare weights. Such a scenario is scarcely interesting. However, under incomplete information any redistribution is limited by the high types' informational rent and the status signals will affect this. We can consider the welfare function as a linear approximation to a non-linear problem if we wish.

Lemma 4: If IC1, IC3 hold as binding constraints for all i then

$$t_i = t_0 + \sum^{i}_{j=1} H_j \qquad (18)$$

$$t_i + s_i = t_0 + s_0 + \sum^{i}_{j=1} K_j \qquad (19)$$

where

[9] Note that the alternative of setting f(h) as linear has the implication that, for example, IC1 and IC3 as equalities implies that IC2 and IC4 are also equalities. However the taxes and signals cannot be found explicitly and so there does not seem to be an associated Proposition 3. Thus we only consider g(c) as linear in this paper.

$H_j = y_j - y_{j-1} + f(x_j - y_j) - f(x_j - y_{j-1})$

$K_j = y_j - y_{j-1} + f(x_j - y_j) - (1-\alpha)f(x_j - y_{j-1}) - \alpha\, f(x_{j-1} - y_{j-1})$

Proof: write IC1 and IC3 as equalities in the quasi-linear form to obtain

$$t_i - t_{i-1} = y_i - y_{i-1} + f(x_i - y_i) - f(x_i - y_{i-1}), \qquad \forall i > 0 \qquad (3'')$$

$$s_i - s_{i-1} = \alpha\{ f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1}) \}, \forall i > 0 \qquad (5'')$$

find t_i and s_i respectively in terms of t_{i-1} and s_{i-1} from (3'') and (5''), and then use repeated substitution.

Lemma 5: The taxes given by (18) can be substituted into the government budget constraint to yield an expression for t_0:

$$t_0 = (g - \Sigma_{k=1}^{I}(n_k\Sigma_{j=1}^{k} H_j))/N \qquad (20)$$

and minimal signals are obtained by setting

$s_0 = 0$, from (11).

Proof: find t_0 by substitution of (18) into (10).

Lemma 6: Using Lemma 4 and Lemma 5, the welfare function (7) can be written

$$W = \Sigma_{i=0}^{I} (n_i\mu_i(y_i + f(x_i - y_i)) - (g - \Sigma_{k=1}^{I}(n_k\Sigma_{j=1}^{k} H_j)) - \Sigma_{k=1}^{I} (n_k\mu_k \Sigma_{j=1}^{k}K_j) \qquad (21)$$

as a function of $y_0,\dots,y_I$ only.

Proof: straightforward substitution.

Proposition 2: By Lemmas 1-6, the first-order conditions for maximising welfare (7) subject to IC1, IC2, IC3 and IC4, plus the budget constraint and non-negative signals constraint, are those for maximising (21) with respect to $y_0,\dots,y_I$, provided the optimal choice has the property that $y_i>y_{i-1}$, all i. The first-order conditions for the simplified problem are

$$y_i:\ J_{i+1}\,(f'(x_{i+1} - y_i) - f'(x_i - y_i)) + n_i(1-f'(x_i - y_i)) = 0 \qquad i<I \qquad (22)$$

$$y_I:\ N_I(1-f'(x_I - y_I)) = 0 \qquad (23)$$

where

$J_{i+1} = N_{i+1} - (1-\alpha)N^*_{i+1}$

$N_i = \sum_{k=i}^{I} n_k$

$N^*_i = \sum_{k=i}^{I} n_k\ \ \mu_k$

and $N_0 = N^*_0 = N$ by our normalisation.

Proof : see Appendix

A series of results can be drawn from Proposition 2, as corollaries.

Corollary 1: The highest type supplies the efficient level of labour, $f'(h_I) = 1$.

Corollary 2: Maximum welfare is a decreasing function of status importance, $dW/d\alpha$ < 0.

Proof: $dW/d\alpha = \partial W/\partial\alpha$ from the envelope theorem. Thus

$dW/d\alpha = - \; \sum_{i=1}^{I} (n_i\mu_i \sum_{j=1}^{i} (f(x_j - y_{j-1}) - f(x_{j-1} - y_{j-1})) < 0$

since $x_j > x_{j-1}$, and f(.) is an increasing function.

Corollary 3: An increase in status importance (α) will decrease optimal labour supply for all types other than the highest type I.

Proof: Note that dW/dy_i is not a function of any y_j , $j \neq i$. Hence trivial comparative statics implies that

sign $(dy_i/d\alpha) = \text{sign}(f'(x_{i+1} - y_i) - f'(x_i - y_i))$

which is negative given the concavity of f(.).

Corollary 4: For any $\alpha \geq 0$, each type $i < I$ supplies less than the efficient level of labour.

Proof: at y_i such that $f'(x_i - y_i) = 1$, (22) is negative since $f'(x_{i+1} - y_i) < 1$ and $J_i > 0$.

For Proposition 2 and the corollaries 1-4 above to hold we need to have $y_i > y_{i-1}$, all i, as a property of the solution. The results confirm that $y_I > y_{I-1}$, but we cannot be assured that labour supply is generally increasing in i without even more structure. It is likely that if the solution to our simplified problem was found to have $y_i < y_{i-1}$ for some i then the signalling constraints we have incorporated are miss-specified. After all, the case where incomes are negatively correlated with ability and yet individuals are keen to spend money wastefully to gain social status makes little sense.

We now consider the central question of the paper. This is whether the presence of status effects leads the government to impose a more or less progressive tax system. We have already shown that the marginal tax at the top of the income distribution will continue to be zero with status effects. More generally, does the impact of status effects change the marginal tax on poorer individuals more or less than that on richer individuals? The answer to this question is complex, just as the issue of general progressivity of tax schedules is elusive. From (22) and the interpretation of the Marginal Rate of Tax for type i (MRT_i) as 1 - $f'(x_i - y_i)$, we have

$$MRT_i = (J_{i+1}/n_i)\, B_i$$

where $B_i = -(f'(x_{i+1} - y_i) - f'(x_i - y_i)) > 0$

If B_i can be assumed constant (for example if f(.) has the quadratic form $\beta h - h^2/2$ and x increases in constant amounts φ), then whether MRT increases or decreases from i-1 to i still has no general answer. However, it is possible to make a clear statement about how the tax distortions will respond to an increase in the size of the status effect. The impact of increasing α can be calculated by constructing the proportional effect on MRT as

$$(dMRT_i/d\alpha)/MRT_i = N^*_{i+1} / J_{i+1}$$

Now we can provide an illustrative proposition of some weight:

Proposition 3: In the quasi-linear case with B_i above constant and equal to j, the proportional impact on the marginal rate of income tax, which implements the optimal tax schedule, of an increase in a is greater the lower is i.

Proof: Note that for i=I there is no impact on the marginal tax rate of an increase in α. For i < I we have

$$N^*_{i+1} / J_{i+1} = N^*_{i+1} / (N_{i+1} - (1-\alpha)N^*_{i+1}) < N^*_i / (N_i - (1-\alpha)N^*_i) = N^*_i / J_i \quad (24)$$

The inequality (24) must hold since, considering the middle terms and cross-multiplying:

$$(N^*_i - \mu_i n_i)(N_i - (1-\alpha)N_i^*) < N^*_i(N_i - n_i - (1-\alpha)N^*_i + (1-\alpha)\mu_i n_i)$$

or, after cancellations,

$$N^*_i < \mu_i N_i$$

which holds given our assumption that the welfare weights μ_i decrease with i.

The importance of Proposition 3 is that it gives a strong indication that increasing status importance leads to proportionately greater increases in marginal tax rates for low types than for high types. Some care is however needed. Income is an endogenous variable, and so a better indication of the effect on progressivity is to take the extra tax paid by the i rather than i-1 ability classes, as this takes account of the shift in income earning which any change in tax schedules will prompt. Thus again use the quadratic form of f(.) and the constant changes in ability of amount φ, and also assume that there are the same number of individuals in each group I. Using these simplifications , the equilibrium income levels can now be found from (22) as

$$y_i = x_i - \beta + 1 - \varphi(I - i - (1-\alpha)z_i)$$

where

$$z_i = \mu_{i+1} + \mu_{i+2} + ... + \mu_I$$

and we note that for y to increase everywhere with i we need that $\mu_i < 2/(1-\alpha)\ \forall$ I. Substitute the equilibrium income levels into the tax constraints (3''). We obtain

$$t_i - t_{i-1} = \varphi^2(2 - (1-\alpha)\mu_i)[I - i -(1-\alpha)z_i + (2 - (1-\alpha)\ \mu_i)/2] \qquad \forall i > 0 \qquad (3^*)$$

Now from (3*) it is clear that an increase in α has three positive effects on the tax difference. Thus higher status importance increases the tax differential for each type. The bottom type 0 thus clearly benefits due to the increased tax revenue in the budget. However the amount of the extra tax paid by i, over that paid by i - 1, when i is small relative to when i is large, is not clear. Suppose for example that μ_i and μ_{i-1} are virtually the same, then the difference in (3*) as i increases is dictated by the change in the expression I - i -(1-α)z_i. This is

$$(-1 + (1-\alpha)\mu_i)$$

and implies an increase in t_i - t_{i-1} as i increases of

$$\varphi^2(-1 + (1-\alpha)\,\mu_i\,)\,(2 - (1-\alpha)\mu_i).$$

Thus if $\mu_i > 1/(1-\alpha)$ (true for low levels of ability) this is positive and the tax incidence is progressive. An increase in α then has an ambiguous effect. If $\mu_i < 1/(1-\alpha)$ (true for higher levels of ability) the expression is negative and the tax incidence is not progressive since the extra tax declines with i. This decline is then accentuated by an increase in α. Finally, increasing α makes the region of progressivity, at the lower end of the distribution of types, shrink. Thus the conclusion is that the distribution becomes locally less progressive while the poor benefit from generally increasing marginal tax rates due to flat rate handouts (higher values of -t_0) from the surplus created. An indicative example of how the optimal tax schedule changes as α changes from zero to a positive number is sketched in Figure 1. One way of viewing this is to see the schedule in the absence of status effects as being similar to that predeicted by other studies. Status effects permits a justification for higher marginal taxes and thus shifts the schedule steeper (and in that sense makes tax more progressive) but also reduces the region of increasing marginal tax. If the society had been limited to linear tax systems then the effect of the status parameter would be to make the intercept more negative and the slope steeper. A similar picture is seen here.

Some more feel for the effect of status in the absence of other factors can be obtained by setting all the μ_i coefficients equal to one, thus adopting a welfare function which gives equal weight to all types. The tax difference (3*) then becomes

$$t_i - t_{i-1} = \varphi^2(1+\alpha)[\alpha(I - i) + (1+\alpha)/2] \qquad \forall i > 0 \qquad (3^{**})$$

which is positive (income tax distortion to counteract the status effect) but decreases in i (so that the tax schedule is not locally progressive, and becomes less progressive as α increases. On the other hand, tax increments are larger as α increases, and so the government budget allows t_0 to be lower. [10]

Intuition for both Proposition 3 and the results from this limited example is easily seen from IC3 which states that signal levels are sufficient to separate from the adjacent lower type:

$$s_i - s_{i-1} \geq \alpha\{ f(x_i - y_{i-1}) - f(x_{i-1} - y_{i-1}) \}, \forall i > 0 \qquad (5'')$$

[10] If α is zero then positive tax differences will still be optimal, but this is a result of the finite economy. If φ becomes small, the number of types I - i will increase but the other term will not and will tend to zero as φ tends to zero. Thus in the absence of both status effects and redistributional objectives the tax schedule will become flat as $\varphi \to 0$.

The expression $f(x_i - y) - f(x_{i-1} - y)$ is increasing in y, so that the extra signal necessary to separate from the next lowest type is higher the higher that type's labour supply (income level). This is the *extra* signal and thus restricting the i type's labour supply reduces the signal cost to all higher types, at the cost of an additional labour supply distortion for the i types alone. When i is near I there is less signal gain from imposing a further distortion, and at i = I there is no gain at all. Thus the impact of greater importance of social signalling is just the same as the impact of incomplete information in the optimal income tax problem. The gains from causing a distortion are probably greater at the bottom of the distribution of types simply because the benefits of lessening the ability to mimic higher groups is then aggregated over the larger number of higher types who have to spend less to separate.

5. Conclusion

The literature on optimal income taxation has stressed the importance of asymmetric information and has found that there is little support from the theory for a general recommendation for progressive tax schedules. The extension of the analysis to incorporate the signalling of social status by making "wasteful" expenditures has brought new factors into the model. The key intuition, however, is found to replicate the intuition of the optimal tax model. The notion that there is a strong reason from status seeking to introduce more progressive taxation has been found to have little support, albeit within a very specific and limited case. The intuition is that signalling is driven by the desire of lower types to mimic higher types and achieve an unwarranted social status. If the lower types pay more taxes then they cannot mimic so easily and separating signals are lower. Thus taxes can to some extent replace wasteful consumption by making mimicking more costly. The externality is transmitted upwards, in a cumulative way, each type having to pay to separate from the adjacent lower type. A reduction in willingness to signal by type ι will lead to lower separating signals by all higher types i > ι. The well-known result of optimal income tax theory, that the highest type should not face a distorting marginal income tax carries over into our model. The intuition is that there is no point in introducing a distortion on tax grounds since the additional tax revenue would be very small as only the top type would pay the tax: better to increase the marginal tax of a lower type which would be paid by that type *and* all higher types while only distorting the lower type's labour supply. The ratio of benefit to distortion cost is higher, and the distortion for lower types will make mimicking them less attractive for higher types. Further, we have found that there is no justification for increased progressivity in terms of status inefficiency since it would not decrease the lower type's ability to mimic and hence the cost of maintaining status for the higher type. On the other hand we have found that a steeper tax scheduleis optimal in the presenceof status seeking and this in itself has strong redistributional properties. Our argument above raises doubts only over the prescription for increasing marginal tax rates over the set of consumer types.

Of course, we have only been considering a particular type of fully separating equilibrium. It may be the case that other outcomes exist, including pooling equilibria, and that such alternatives increase as income tax schedules change. We can thus make only limited claims for our conclusions. It may well be that we should think of our analysis as limited to fairly small changes in income tax schedules. Perhaps large ones would cause social status to be dropped entirely from individual preferences and prompt a shift to a totally different mode of behaviour. We have however found some support for the current results of optimal income tax theory within a status-seeking world.

A final comment relates to the interpretation of the model when “individuals” are replaced by other agents. Most obvious alternative applications relate to firms and to nation states. In the former, the acquisition of prestigious headquarters facilities or large public relations budgets may well seek to increase the prestige of the company and its valuation if circumstances arise where it is judged by banks, other customers seeking prestigious connections of their own, or shareholders deciding on voting behaviour. Again relative position is all-important, and again the circumstances where such status is likely to be an issue may not be realised (α not equal to unity). Similarly, nations equip national airlines with expensive airplanes, build palaces and monuments and defend undefendable exchange rates. In so far as firms’ incomes are taxed and nations are allocated development funds according to poverty, the existence of these status expenditures do not justify a prescription of progressivity. Within a signalling model it is necessary to curtail the mimicker’s ability to mimic in order to obtain a general reduction is status expenditure.

Appendix: Proof of Proposition 2

The first part of the Proposition is straightforward since Lemmas 1-6 permit substitution of sufficient constraints that the problems are equivalent.

The second part is obtained by differentiating (21) by y_i. First note that the derivatives of H_j and K_j with respect to y_i are given by

$\partial H_j/\partial y_i = 0$ if $j \neq i, i+1$

$\partial H_j/\partial y_i = 1 - f'(x_i-y_i)$ if $j=i$

$\partial H_j/\partial y_i = -(1 - f'(x_{i+1}-y_i)$ if $j=i+1$

and

$\partial K_j/\partial y_i = 0$ if $j \neq i, i+1$

$\partial K_j/\partial y_i = 1 - f'(x_i-y_i)$ if $j=i$

$\partial K_j/\partial y_i = -1 + (1-\alpha) f'(x_{i+1}-y_i) + \alpha f'(x_i-y_i)$ if $j=i+1$

Summing over j yields

$\Sigma_{j=1}^{k} \partial H_j/\partial y_i = 0$ if $i > k$

$\Sigma_{j=1}^{k} \partial H_j/\partial y_i = 1 - f'(x_i-y_i)$ if $i=k$

$\Sigma_{j=1}^{k} \partial H_j/\partial y_i = f'(x_{i+1}-y_i) - f'(x_i-y_i)$ if $i<k$

and

$\Sigma_{j=1}^{k} \partial K_j/\partial y_i = 0$ if $i>k$

$\Sigma_{j=1}^{k} \partial K_j/\partial y_i = 1 - f'(x_i-y_i)$ if $i=k$

$\Sigma_{j=1}^{k} \partial K_j/\partial y_i = (1-\alpha)(f'(x_{i+1}-y_i) - f'(x_i-y_i))$ if $i<k$

Now the general form of the derivative of (21) with respect to y_i is

$$\partial W/\partial y_i = n_i\mu_i(1- f'(x_i-y_i)) + \Sigma_{k=1}^{I} n_k \Sigma_{j=1}^{k} \partial H_j/\partial y_i - \Sigma_{k=1}^{I} n_k\mu_k \Sigma_{j=1}^{k} \partial K_j/\partial y_i$$

As we have shown above that three possible values occur for the summations over j (for each of the H and K terms), and one of these is zero, the summations over k are simple. Collecting the coefficients of each term yields

$$\partial W/\partial y_i = n_i\mu_i(1- f'(x_i-y_i)) +$$
$$n_i(1- f'(x_i-y_i)) + N_{i+1}(f'(x_{i+1}-y_i)) - f'(x_i-y_i)) -$$
$$\{ n_i\mu_i(1- f'(x_i-y_i)) + N_{i+1}*(1-\alpha)(f'(x_{i+1}-y_i)) - f'(x_i-y_i)) \}$$

and so

$$\partial W/\partial y_i = n_i(1- f'(x_i-y_i)) + J_{i+1}(f'(x_{i+1}-y_i)) - f'(x_i-y_i))$$

Now when $i < I$, J_{i+1} is non-zero and the first order condition is (22). When $i=I$, J_{i+1} is zero and the first-order condition is (23).

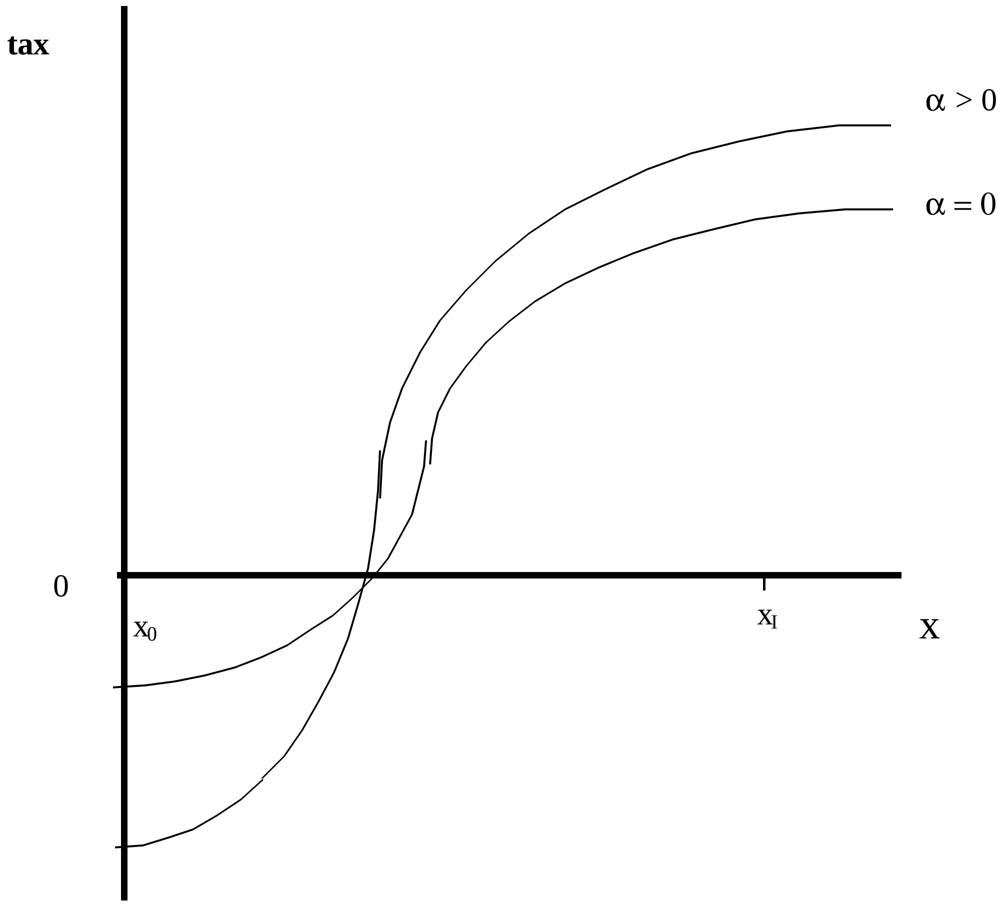

Figure 1: The tax schedule becomes steeper and more redistributive, but is concave over a greater range of x.

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