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Matching Foundations

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Abstract

We compare equilibrium allocations in directed search models where prices are determined alternatively by posting and by competing auctions. Sellers' expected payoffs are higher when all sellers auction, but the difference in the payoffs decreases rapidly with market size and vanishes in the limit "large" economy. In this large economy, buyer and seller payoffs are different, but entry of both buyers and sellers is constrained-efficient. When sellers can choose whether to post prices or auction in the 2-buyer 2-seller case, then the equilibrium choice depends on whether or not sellers can commit. If both sellers can commit, then the dominant strategy equilibrium has both sellers auctioning. If neither seller can commit, then all possible combinations are equilibria.

Key words: Matching, directed search, coordination, price-posting, competing auctions.

JEL: E24, J31, J41, J64, D44

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INTRODUCTION

Individuals are constrained by the number of hours in a day that they have to work with. Also, employers are often constrained by the number of jobs that they have to fill. These facts make the analysis of equilibria with capacity constraints, developed by Peters (1984), particularly relevant to the labor market. Recent work by Montgomery (1991), Burdett, Shi, and Wright (1997), Shi (1999) and Julien, Kennes, and King (2000) has explored the implications of this insight. One major implication is that, when buyers randomize in their decision about which seller to approach, the equilibrium matching function generated by this process in large economies shares many properties with the matching function commonly used in macroeconomics (for example, Pissarides (1990)). Thus, these "directed search" models can be used to analyse factors that influence this matching process.

Although the models presented in these papers all imply the same matching function, there are some significant differences in the models. In particular, in Montgomery (1991), Burdett, Shi, and Wright (1997) and Shi (1999), firms play the role of sellers -- selling jobs to workers. In Julien, Kennes, and King (2000), workers sell labor to firms. Also, in the first three papers, sellers post prices (wages) that they commit to, regardless of the number of buyers that approach them. In the fourth paper, instead, sellers commit to a bidding game where the good (labor) is sold to the highest bidder. (This is known as the "competing-auction" framework.) In general, the expected payoffs to buyers and sellers are not invariant to these distinctions.

In this paper, we explore the extent and the implications of these differences. In a general model with arbitrary numbers of agents on each side of the market we compare the expected payoffs for buyers and sellers under both price-posting and competing-auction assumptions. We find that, for any finite number of agents, the expected payoffs for sellers are always higher in the competing-auction game than in the price-posting game. This difference in payoffs is significant in the 2-by-2 case, but gets small quite quickly as either the scale of the economy increases or as the buyer-seller ratio increases, and vanishes in both limits. Thus, in markets of significant size, sellers will be approximately indifferent about the sales mechanism used.

We also find that the difference in the expected payoffs between buyers and sellers does not vanish either as market size increases or as the buyer-seller ratio increases. Hence, in the labor market context, it matters to workers and firms whether they are buyers or sellers. However, entry by both buyers and sellers is constrained-efficient in the large economy.

Finally, we consider the problem of sellers choosing which sales mechanism to use (price-posting or competing-auction). Here, we restrict attention to the 2-by-2 case, where the difference in the payoffs is most significant. We find that the equilibrium choice depends crucially on the degree of commitment on the part of the sellers. If sellers can commit to a mechanism before prices (or reserve prices) are announced, then the dominant strategy equilibrium has both sellers choosing to auction. In the absence of this commitment, however, all possible combinations are equilibria. In particular, without commitment, both price-posting and auctioning can co-exist in equilibrium.

The remainder of the paper is organized as follows. The general environment, with arbitrary numbers of buyers and sellers is laid out in section 1. Sections 2 and 3, respectively, present the results in the price-posting and competing-auction models with arbitrary numbers of buyers and sellers. Section 4 compares the expected payoffs for the different agents in the different models, both in large and small economies. Section 5 then considers the 2-by-2 game where sellers can choose sales mechanisms. Finally, Section 6 presents the conclusion and some discussion.

1. THE PLAYERS

The market consists of $N \geq 2$ identical risk neutral sellers and $M \geq 2$ identical risk neutral buyers. Each seller has one unit of the good to sell. If she sells at price p then she obtains payoff *p*; if she does not sell, she receives a payoff of zero. Each buyer wants to buy one unit of the good and is willing to pay up to his reservation price, which is normalized to 1. If a buyer purchases at price p, then his payoff is $1-p$. If he is unable to buy, his payoff is zero. Each buyer has only one opportunity to buy, and must choose only one seller to visit.[1]

2. THE PRICE-POSTING GAME

The price-posting game has the following structure. First, each seller announces her price and rationing rule. The seller is committed to selling at the announced price if at least one buyer makes an offer at that price. If $m \geq 1$ buyers offer to buy from her at that price, each buyer will be able to purchase from her with probability $1/m$. Second, after observing all of the announced prices, each buyer chooses which seller to visit (that is, which seller to make an offer of purchase to). Finally, once buyers have been allocated across sellers, the good is allocated to buyers according to the rationing rule. Equilibria are computed by backward induction.

There exist multiple asymmetric pure strategy equilibria in this model but, as in Montgomery (1991), Burdett, Shi and Wright (1997) and Shi (1999), attention is focussed here on the unique symmetric mixed strategy equilibrium in which each seller charges the same price and buyers randomize over sellers -- visiting each seller with the same probability $p \in (0,1)$.[2] In this section, we simply report the equations derived in Burdett, Shi and Wright (1997).

[1] For dynamic versions of the price-posting and competing-auction games, where buyers can visit one seller in each period, see Cao and Shi (1999) and Julien, Kennes and King (2000) respectively.

[2] This is usually justified by pointing out the high degree of coordination among buyers required to implement the pure strategy equilibria. Experimental evidence also suggests that buyers have trouble coordinating in this way, even in very small markets (Ochs, 1990). See Cao and Shi (1999) for further discussion on this point.

The probability that each buyer assigns to visiting each seller is:

$$p = 1/N \tag{2.1}$$

The expected number of matches (or the "matching function") is given by:

$$x(N,M) = N(1-(1-1/N)^{M}) \tag{2.2}$$

As mentioned above, this matching function shares many of the properties of the Cobb-Douglas matching function that is commonly used in macroeconomics. Figure 1 presents a graphical representation of this function, for $N = 2, 3, \ldots, 30$ and $M = 2, 3, \ldots, 30$.

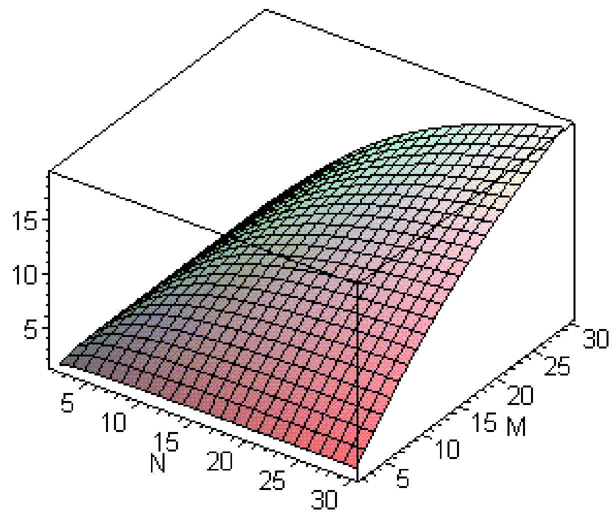

Figure 1: The Equilibrium Matching Function

The equilbrium price of the good is:

$$p = p_P(N,M) = \frac{N - N\left(1 + \frac{M}{N-1}\right)(1-1/N)^M}{N - \left(N + \frac{M}{N-1}\right)(1-1/N)^M} \tag{2.3}$$

The expected payoff to each seller is:

$$S_P(N,M) = \left(1-(1-1/N)^M\right)p_P(N,M) \tag{2.4}$$

The expected payoff to each buyer is:

$$B_P(N,M) = \frac{N}{M}\left(1-(1-1/N)^M\right)\left(1-p_P(N,M)\right) \tag{2.5}$$

A key feature to note in this equilibrium is that the sellers receive the price $p_P(N,M)$ when *at least one* buyer approaches them. This occurs with probability $(1-(1-1/N)^M)$ where $(1-1/N)^M$ is the probability of no buyers approaching. The expected payoff to the seller is simply the product of these two values. The expected payoff for sellers has an analogous interpretation.

3. THE COMPETING-AUCTION GAME

The game has the following structure. First, each seller announces a reserve price and a rationing rule. The seller is committed to selling the good in a bidding game with the following rules. If only one buyer approaches the seller, then she will sell the good to that buyer at the reserve price. If more than one buyer approaches, then the good will be sold to the highest bidder. If this price is offered by $m \geq 2$ buyers then each of these buyers will receive the good with probability $1/m$. Second, once buyers have observed all announced reserve prices, each buyer chooses which seller to visit. Finally, once buyers have been allocated across sellers, the good is allocated according to the bidding game. Again, equilibria are computed by backwards induction, and attention is restricted to the unique symmetric mixed strategy equilibrium. In this equilibrium each seller announces the same reserve price r and buyers randomize over sellers, visiting each seller with the same probability $p \in (0,1)$.

From Julien, Kennes and King (2000), this equilibrium has the following properties. Exactly as in the price-posting game, the probability that each buyer assigns to each seller is given in equation (2.1). Also, the equilibrium matching function is given in equation (2.2). However, in the competing-auction game, there is price dispersion in equilibrium: sellers that have only one buyer visit them receive only the reserve price, while sellers that have more than one buyer visit are able to receive the entire value of the surplus (unity). The equilibrium reserve price of the good is:

$$r = r(N, M) = \frac{M - 1}{M + N(N - 2)} \tag{3.1}$$

The equilibrium price of the good is:

$$p_A(N, M) = \begin{cases} r(N, M) & if \quad m = 1 \\ 1 & if \quad m \geq 2 \end{cases} \tag{3.2}$$

The expected payoff to each seller is:

$$S_A(N,M) = \frac{M}{N}(1-1/N)^{M-1} r(N,M) + \left(1-(1-1/N)^M - \frac{M}{N}(1-1/N)^{M-1}\right)$$

This is simply the weighted sum of the prices the seller receives, where the weights are the probabilities of each event. If no buyers arrive, the seller receives nothing. This occurs with probability $(1-1/N)^M$.[3] As in equation (3.2), if one buyer arrives, the price is $r(N,M)$. The probability of this happening is $(M/N)(1-1/N)^{M-1}$. If at least two arrive then the price is unity. The probability associated with this event is $1-(1-1/N)^M-(M/N)(1-1/N)^{M-1}$. Collecting terms, one obtains:

$$S_A(N,M)=1-(1-1/N)^M-\frac{M}{N}(1-1/N)^{M-1}\left(1-r(N,M)\right) \tag{3.3}$$

The expected payoff to each buyer is:

$$B_A(N,M)=(1-1/N)^{M-1}(1-r(N,M)) \tag{3.4}$$

Here, a buyer receives a positive payoff, *ex post*, if and only if he is the only buyer to approach the seller, in which case he receives $1-r(N,M)$. The probability of this event is $(1-1/N)^{M-1}$. The expected payoff is therefore the product of these expressions.

4. COMPARING EQUILIBRIUM OUTCOMES IN THE TWO GAMES

To compare the prices and expected payoffs in these two games, it is useful to re-write these expressions in a way that allows a simple analysis of the effects of scale and market tightness. Let b denote the ratio of buyers to sellers, (a measure of market tightness) then we can write M as $M=bN$, and express the prices and payoffs given in (2.3), (2.4), (2.5), (3.2), (3.3), and (3.4) respectively as:

$$p_P(N,b)=\frac{N-N\left(1+\frac{bN}{N-1}\right)(1-1/N)^{bN}}{N-\left(N+\frac{bN}{N-1}\right)(1-1/N)^{bN}} \tag{4.1}$$

[3] When workers are sellers, this probability is the expected unemployment rate in this game.

$$S_P(N,b) = \left(1-(1-1/N)^{bN}\right)p_P(N,b) \tag{4.2}$$

$$B_P(N,b) = b\left(1-(1-1/N)^{bN}\right)(1-p_P(N,b)) \tag{4.3}$$

$$r(N,b) = \frac{bN-1}{bN+N(N-2)} \tag{4.4}$$

$$S_A(N,b) = 1-(1-1/N)^{bN} - b(1-1/N)^{bN-1}\left(1-r(N,b)\right) \tag{4.5}$$

$$B_A(N,b) = (1-1/N)^{bN-1}\left(1-r(N,b)\right) \tag{4.6}$$

4.1 *Posted Prices and Reserve Prices*

We first consider the relative sizes of the equilibrium posted prices and reserve prices in the two games. The results in the 2-by-2 case and the limit cases (where $N \to \infty$, and $b \to \infty$) are given in the following proposition.

Proposition 1:

a) In the 2-by-2 economy, both posted and reserve prices equal 1/2.

b) For any given buyer-seller ratio b, in the limit as the scale of the market $N \to \infty$:

$$p_P(b) = \frac{e^b - b - 1}{e^b - 1} \tag{4.7}$$

$$r(b) = 0 \tag{4.8}$$

c) For any given scale N, in the limit as buyer-seller ratio $b \to \infty$, both posted and reserve prices equal 1.

Proof:

a) Substituting $N = 2$ and $b = 1$ in (4.1) and (4.4) produces the first result.

b,c) Holding b constant and taking limits in (4.1) and (4.4) respectively yields (4.7) and (4.8). Holding N constant and taking limits of (4.1) and (4.4) respectively yields $p = r = 1$. ■

Before discussing these results, it is useful to consider the values of $p_P(N,b)$ and $r(N,b)$ for more general finite values of N and b. Figure 2, below, illustrates $p_P(N,b)$ for $N = 2, \ldots, 30$ and $b = 1, \ldots, 30$.

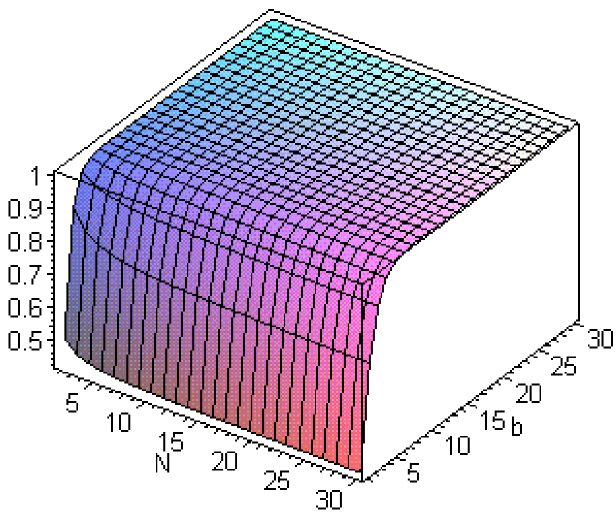

Figure 2: Posted Prices

Clearly, from Figure 2, posted prices converge monotonically to their limit values. For example, when b =1, and N = 2, the price equals 1/2 (at the bottom left corner of

Figure 1). Holding N fixed at 2, but increasing b, we observe that the posted price rises sharply at first, then asymptotically approaches the limit value 1. This is quite intuitive -- as the ratio of buyers to sellers increases, each seller has a higher probability of receiving at least one buyer, and can price at a level that allows her to get close to being able to extract the entire surplus. Notice that, once $b \geq 10$, with $N = 2$, the price is more than 99% of the value of the surplus.

Holding b fixed at 1, but increasing N, Figure 2 also shows that the posted price falls from 1/2 to its limit value (approximately 0.418 in this case) quite rapidly. Thus, even though sellers have a first-mover advantage in this game, with equal numbers of buyers and sellers, the equilibrium price is only 1/2 in the 2-by-2 case, and erodes down to a value below 1/2 as the scale of the market increases. This result follows from the particular sequence of events assumed here (i.e., sellers move first in this game, by choosing their prices) and the assumption that sellers can costlessly announce their price to all buyers, while buyers are restricted to making offers to only one seller. With randomization, buyers place less weight on approaching each seller than each seller places on approaching each buyer. (Each seller approaches each buyer with certainty, but each buyer approaches any seller with some probability less than one.) Thus, as the market gets large, keeping the ratio of buyers to sellers constant, there are asymmetric influences of increasing M and N: for each seller, the probability of being approached decreases more quickly (as N increases) than the analogous probability for buyers (as M increases). This then erodes the first-mover advantage that sellers have when announcing their reserve prices.

It is also interesting to note that the *positive* effect of increasing b outweighs the *negative* effect from increasing N commensurately. This is illustrated, in Figure 2, by the fact that the surface flattens out at the value 1, for large values of b.

We now consider the shape of the reserve prices $r(N, b)$ in equilibrium, illustrated in Figure 3, below, for $N = 2, \ldots, 30$ and $b = 1, \ldots, 30$.

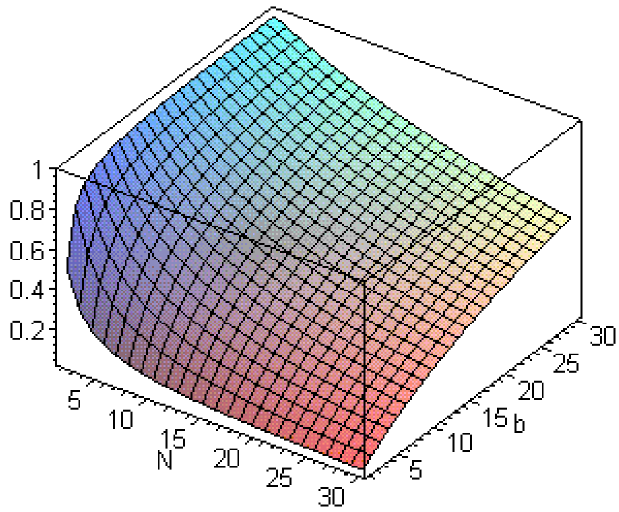

Figure 3: Reserve Prices

Reserve prices also converge monotonically to their limit values. As with posted prices, when $b = 1$ and $N = 2$, the reserve prices equal 1/2. Fixing N, reserve prices increase as the buyer-seller ratio b increases -- eventually reaching unity. The reasoning behind this is completely analogous to that with posted prices. Similarly, fixing b but increasing N decreases reserve prices. However, whereas posted prices converge to a positive value as N increases, reserve prices converge to zero. By increasing the size of the market, because of the asymmetry discussed above, sellers face more competition and cut prices and reserve prices. Posted price sellers care only about generating at least one offer while competing-auction sellers care about generating at least two offers in order to extract the surplus. For this reason, sellers using the auction are more aggressive at reducing equilibrium reserve prices. Notice also that, contrary to the posted price equilibrium, it is not true that the positive effect on the reserve price from increasing b dominates the negative effect from increasing N commensurately. In fact, when $N = b$, increasing both leads the reserve price to converge to 1/2, as in the 2-by-2 case.

Figure 4 illustrates the difference between the equilibrium posted and reserve prices $p_B(N,b) - r(N,b)$, for N = 2, …, 30 and b = 1, …, 30. This difference is zero where N = 2 and b = 1, but is positive everywhere else. In general, posted prices are no smaller than reserve prices.

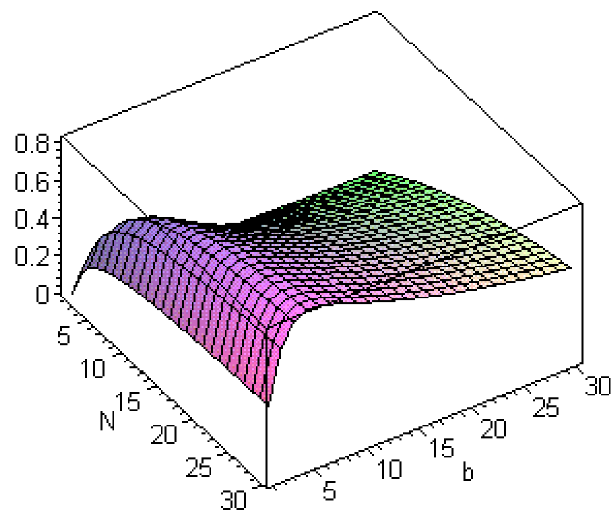

Figure 4: Posted Prices minus Reserve Prices

4.2 *Comparing Expected Payoffs*

In this section, we compare the expected payoffs for buyers and sellers under the two different sales mechanisms. We first compare these payoffs in the respective limits, as the scale of the market $N \to \infty$ and the buyer-seller ratio $b \to \infty$.

4.2.1 *Expected Payoffs in the Limits*

Proposition 2 summarizes the main result in this subsection.

Proposition 2:

a) In the limit, as the scale of the market $N \to \infty$:

   i) the sellers' expected payoffs in both the price-posting and competing-auction games converge to

$$S_P(b) = S_A(b) = S(b) = 1 - e^{-b} - be^{-b} \tag{4.9}$$

   ii) the buyers' expected payoffs in both the price-posting and competing-auction games converge to

$$B_P(b) = B_A(b) = B(b) = e^{-b} \tag{4.10}$$

b) In the limit, as the ratio of buyers to sellers $b \to \infty$:

   i) the sellers' expected payoffs in the price-posting and competing-auction games converge to 1

   ii) the buyers' expected payoffs in the price-posting and competing-auction games converge to 0

Proof: Follows from taking the appropriate limits of (4.2), (4.3), (4.5) and (4.6) ■

Part (b) of the above proposition is entirely straightforward and intuitive: in either game, as the number of buyers per seller gets arbitrarily large, each seller is able to extract the entire surplus.

Part (a) is less straightforward. As N increases, this erodes the first-mover advantage that sellers have when announcing their reserve prices. In finite-sized markets, where the reserve price is positive, the auction gives sellers the advantage of being able to exploit the *ex post* allocation of buyers, when more than one buyer approaches the seller.

(For example, in the 2-by-2 case, both r and p_P equal 1/2 but sellers in the competing-auction receive a premium if more than one buyer approaches the seller -- the price equals one.) As the market increases, however, the reserve price is driven down to zero (while the posted price converges to a positive number: $(1-e^{-b}-be^{-b})/(1-e^{-b})$), for reasons described in Section 4.1, and the advantage of being able to exploit *ex post* opportunities disappears.

Notice that, even in large markets, expected payoffs to buyers and sellers are not equal (in general) in either the price-posting or competing-auction equilibria. That is, while sellers receive the same expected payoff in price-posting and competing-auction equilibria when markets are large, these payoffs are not the same as those received by buyers in either case. Using (4.9) and (4.10), we can compute the difference in these payoffs as:

$$S(b)-B(b)=1-(2+b)e^{-b} \tag{4.11}$$

This leads to the following corollary.

Corollary: In the large economy:

a) $sign(S(b)-B(b))=sign(b-b_0)$ where b_0 solves: $e^{b_0}-b_0=2$.

b) With equal numbers of buyers and sellers, expected payoffs are greater for buyers than for sellers.

Proof:

Define b_0 where $S(b_0)-B(b_0)=0$. From (4.11), one obtains $e^{b_0}-b_0=2$. Also, since $S(b)-B(b)$ is strictly decreasing in *b*, the first result follows. Since $b_0\approx 1.1462>1$, the second result follows. ■

We now examine the efficiency properties of these equilibria in large markets.

4.2.2 *Firm Entry and Constrained Efficiency in Large Markets*

Suppose firms can enter into these markets, where the cost of entry is $c_S > 0$ if the firm is a seller and $c_B > 0$ if the firm is a buyer. We consider these cases separately. When firms are sellers, using (4.9), the entry condition implies that the equilibrium buyer-seller ratio in the economy (b_S) is determined by:

$$1 - e^{-b_S} - b_S e^{-b_S} = c_S \tag{4.12}$$

Alternatively, when firms are buyers, using (4.10), the following entry condition determines the equilibrium buyer-seller ratio b_B:

$$e^{-b_B} = c_B \tag{4.13}$$

We now consider a planner's problem, where the planner is constrained in that he faces the matching friction as a technological constraint (in the tradition of Hosios' (1990)). Given the number of workers in the economy, the planner chooses the number of firms that enter to maximize expected surplus.

If firms are sellers then, given *M*, the planner chooses *N* to maximize:

$$V_S = N(1 - e^{M/N}) - Nc_S \tag{4.14}$$

If firms are buyers then, given N, the planner chooses *M* to maximize:

$$V_B = N(1 - e^{M/N}) - Nc_B \tag{4.15}$$

Proposition 3 summarizes the result.

Proposition 3:

In the large economy, firm entry is constrained-efficient both if firms are buyers and if firms are sellers.

Proof:

Differentiating (4.14) and (4.14) with respect to N and M respectively yields equations (4.12) and (4.13) respectively. Concavity of the objective functions ensures that the first order conditions are sufficient for a maximum. ■

Proposition 3 has particular relevance in the labor market. This implies that firm entry is constrained-efficient, in large economies, in both the models where firms are sellers of jobs (Montgomery (1991), Burdett, Shi and Wright (1997), Shi (1999)) and where firms are buyers of labor (Julien, Kennes, and King (2000)). One implication of this result is that the local market structure imposed by Moen (1997) is not necessary for constrained efficiency when firms post wages.

4.2.3 Expected Payoffs in Finite Markets

The results in Propositions 2 and 3 hold only in the respective limits considered in each case. In general, outside these limits, agents' expected payoffs are different in competing-auction and price-posting markets. Also, the constrained-efficiency result does not hold in small markets.[4] It is useful, however, to ask how quickly the equilibrium expected payoffs in small markets converge to their large market values. The answer is that they converge very quickly. Figure 5 illustrates the difference $S_A(N,b)-S_P(N,b)$, using equations (4.2) and (4.5).

[4] See Julien, Kennes and King (2000), for a discussion of why small markets are not constrained-efficient with competing auctions.

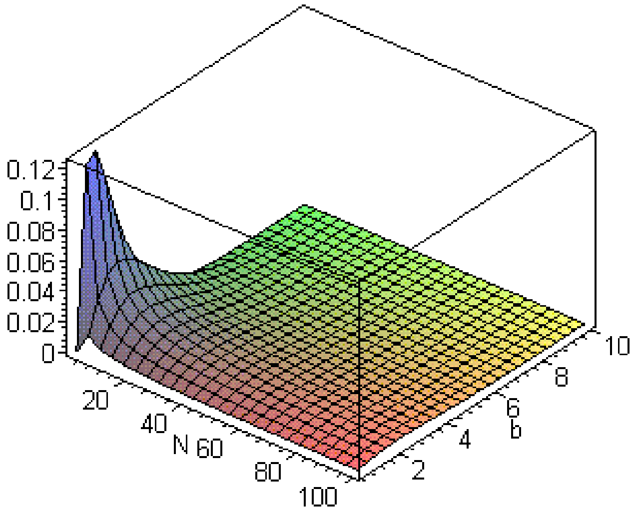

Figure 5

This figure shows that, for all finite N and b, where $N \geq 2$ and $M \geq 2$, sellers' expected payoffs in the competing-auction game are strictly larger than in the price-posting game. This difference is maximized in the 2-by-2 case (where N = 2, b = 1) and drops off precipitously as either N or b increase. This difference is less than 1% of the value of the surplus for all N > 30 and all b > 5 and converges uniformly to zero as $N \to \infty$ and as $b \to \infty$.

5. CHOOSING A SALES MECHANISM IN THE 2-BY-2 ECONOMY

In this section, we allow sellers in a market to choose whether to post a price or to auction their good. We confine our attention to the 2-by-2 case, where the differences in the payoffs that agents receive are the most dramatic. We analyse the problem by the sequential development of three key subgames in which (i) both sellers post prices, (ii) both sellers auction, and (iii) one seller posts a price and the other auctions. The subgames (i) and (ii) are identical to the games considered in sections 2 and 3 respectively above, with M and N set equal to 2. As will become clear below, it is useful to work through the 2-by-2 cases because the reaction functions will be used when considering the equilibrium choices of sales mechanism.[5]

5.1 *Both Sellers Posting Prices*

Here, each seller chooses a posted price p_j where $j \in \{1,2\}$ is used to index sellers. Each buyer then chooses to visit either seller based on the posted price of each seller and the expected behavior of the other buyer. The probability that a buyer visits seller j is given by p_i^j where i is used to index buyers. Buyer i's expected payoff from visiting seller j is given by:

$$B_P(i,j) = (1 - p_{-i}^j)(1 - p_j) + p_{-i}^j (1 - p_j)/2 \tag{5.1}$$

where p_{-i}^j is the probability that seller i is not the only potential buyer visiting seller j, in which case the good is rationed according to the symmetric rationing rule. In the mixed strategy equilibrium each buyer is indifferent about which seller he visits. Moreover, since each buyer faces the same vector of posted prices, the probability that each buyer visits seller j is the same. Thus, $B_P(i,1) = B_P(i,2)$ and $p_i^j = p^j$ for all i. Using these conditions, together with (5.1), the probability p^j that any particular buyer visits seller j becomes:

$$p^j = \frac{1 - 2p_j + p_{-j}}{2 - p_j - p_{-j}} \tag{5.2}$$

[5] At this point, it is useful to make a slight change in notation. In the previous sections of this paper, the arguments in the functions emphasized the numbers of players on each side of the market. (Attention was restricted entirely to symmetric equilibria, it was unnecessary to index particular agents.) In this section, the numbers of players on each side of the market are set at two, but we allow for asymmetric equilibria. For this reason, the arguments in the functions are used to index particular agents.

Expression (5.2) is the reaction function of buyers to the posted prices of sellers. The probability that seller j is able to sell her good is:

$$q(p_j, p_{-j}) = 2p^j(1-p^j) + (p^j)^2$$

where $2p^j(1-p^j)$ is the probability that the seller j faces one potential buyer and $(p^j)^2$ is the probability that she faces two (in which case, she can sell only to one). We can think of $q(p_j, p_{-j})$ as the demand function for good j. A key feature of this demand function is that it is continuous and differentiable. Moreover, under posted prices, the seller has the same payoff if one or two potential buyers come to visit. Therefore, the expected payoff to seller j from offering posted price p_j is given by:

$$S_P(p_j, p_{-j}) = q(p_j, p_{-j})\, p_j \tag{5.3}$$

The solution to each seller's problem is found simply by differentiating her payoff function with respect to her posted price, and setting this derivative equal to zero. In this way we obtain the following reaction function for seller j:

$$p_j(p_{-j}) = \frac{(p_{-j}+1)(p_{-j}-2)}{5p_{-j}-7} \tag{5.4}$$

The intersection of the sellers' reaction functions gives the equilibrium posted price of $p_j^* = 1/2$. The equilibrium prices of this subgame can then be substituted into the payoff functions of each seller to give the equilibrium payoff of:[6]

$$S_P(p_j^*, p_{-j}^*) = 0.375. \tag{5.5}$$

[6] Notice that, using the notation from Section 2, from (2.4) we get: $S_P(2,2) = .375$.

5.2 *Both Sellers Auctioning*

As in section 3 above, each seller chooses a *reserve* price r_j. Each buyer then chooses to visit either seller based on the reserve price of each seller and the expected behavior of the other buyer. Buyer i's expected utility from visiting seller j is given by:

$$B_A(i, j) = (1 - p^{j}_{-i})(1 - r_j) \tag{5.6}$$

where $(1 - p^{j}_{-i})$ is the probability that the buyer is alone at this auction. If the buyer is not alone, then competitive bidding ensures that the sale price of the good is unity, in which case the buyer does not obtain any utility from purchasing the good. In the mixed strategy equilibrium, each buyer is indifferent about which seller he visits. Moreover, since each buyer faces the same vector of reserve prices, the probability that any particular buyer visits seller j is the same. Therefore, $B_A(i,1) = B_A(i,2)$ and $p^{j}_{i} = p^{j}$ for all i. Consequently, using (5.6), the probability p^{j} that any particular buyer visits seller j is given by:

$$p^{j} = \frac{1 - r_j}{2 - r_j - r_{-j}} \tag{5.7}$$

This expression is the reaction function of the buyers to the reserve prices of the sellers. Sellers exploit this function in their reserve price decision. The probability that seller j has exactly one potential buyer visit is given by $q^1(r_j, r_{-j}) = 2p^{j}(1 - p^{j})$, while the probability that she has two potential buyers visit is $q^2(r_j, r_{-j}) = (p^{j})^2$. Whereas in the posted price case, the number of potential buyers who visit has no bearing on the price at which the good is sold (as long as this number is not zero), in the auction case, this is not true. In particular, the good is sold at the reserve price r_j if only one potential buyer visits seller j, but sold at the price of unity (the buyer's valuation) if more than one potential buyer visits. Thus, the payoff $S_A(r_j, r_{-j})$ to seller j offering reserve price r_j is given by:

$$S_A(r_j, r_{-j}) = q^1(r_j, r_{-j})r_j + q^2(r_j, r_{-j}) \tag{5.8}$$

The solution to the seller's problem is found by differentiating the seller's payoff function (5.8) with respect to r_j and setting the result equal to zero. Doing so, we find the following reaction function for seller j:[7]

$$r_j(r_{-j}) = 1/2 \tag{5.9}$$

The expected payoff function of the seller (5.8) is continuous and concave, given the other seller is choosing the equilibrium reserve price. Therefore, the vector of reserve prices satisfying the reaction functions (5.9) is an equilibrium of this subgame. The reaction functions determine the equilibrium reserve price for each seller at $r_j^* = 1/2$. These choices can now be substituted back into the payoff functions to obtain the equilibrium expected payoff to each seller:[8]

$$S_A(r_j^*, r_{-j}^*) = 1/2. \tag{5.10}$$

5.3 *One Seller Posting a Price, the Other Auctioning*

The auction and posted-price subgame, developed here, is similar to the previous two subgames, except that one seller posts a price while the other conducts an auction.

[7] The non-responsiveness of this reaction function to the reserve price of the other seller deserves some comment. A higher reserve price by the other seller clearly raises the probability that a buyer will visit seller j. However, seller j chooses not to raise her reserve price in response because she would then discourage potential buyers from visiting her own auction. When the number of buyers equals the number of sellers (as in this 2 by 2 case) these effects exactly cancel out. More generally, when the number of buyers and sellers differ, these reaction functions have a positive slope

We denote the posted price seller by her posted price p, and the auction seller by her reserve price r. The probability that buyer i visits the *posted price* seller is denoted by p_i. The expected utility that buyer i receives if he visits the posted price seller is given by:

$$B_{PA}(i, P) = (1 - p_{-i})(1 - p) + p_{-i}(1 - p)/2 \qquad (5.11)$$

while the expected utility from visiting the seller who auctions is given by:

$$B_{PA}(i, A) = p_{-i}(1 - r) \qquad (5.12)$$

As in the previous two subgames, an equilibrium in mixed strategies exists for this subgame such that each buyer is indifferent about which seller to visit. Also, since each buyer faces the same vector of prices, the probability that any particular buyer will visit a particular seller is the same. Therefore, $B_{PA}(i, P) = B_{PA}(i, A)$ and $p_i = p$ for all i. These two conditions allow us to solve for the probability p that any particular buyer will visit the posted price seller:

$$p = \frac{1 - p}{3/2 - r - p/2} \qquad (5.13)$$

Equation (5.13) expresses the reaction function of the buyers to the reserve and posted prices of the two sellers. Both sellers exploit this function when making their posted/reserve price decisions. The expected payoff to a seller offering posted price s is given by:

$$S_{PA}(p, r) = q(p, r)p \qquad (5.14)$$

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[8] Using the notation of Section 3, from (3.3) this payoff is: $S_A(2,2) = 1/2$.

where $q(p,r) = 2p(1-p) + (p)^2$ is the probability that the posted price seller receives a visit from at least one buyer. The expected payoff of the seller offering the reserve price r is given by:

$$S_{PA}(r, p) = q^1(r, p)r + q^2(r, p) \tag{5.15}$$

where $q^1(r, p) = 2p(1-p)$ is the probability that one buyer will visit the auctioning seller, and $q^2(r, p) = (1-p)^2$ is the probability that two will.

Differentiating the payoff functions of each seller with respect to the relevant decision variable and setting these derivatives equal to zero gives two reaction functions. First, for the price-posting seller, we have:

$$p(r) = \frac{2r-3}{4r-5} \tag{5.16}$$

and second, for the auctioning seller, we have:

$$r(p) = \frac{1+p}{6} \tag{5.17}$$

The intersection of these two reaction functions yields the equilibrium reserve price $r^* = 0.2713$ and posted price $p^* = 0.6277$. Substituting these choices into the sellers' expected payoff functions yields the expected payoff for the price-posting seller as:

$$S_{PA}(p^*, r^*) = 0.4069 \tag{5.18}$$

and the expected payoff for the auctioning seller as:

$$S_{PA}(r^*, p^*) = 0.4827 \tag{5.19}$$

5.4 Equilibrium

The three subgames presented in sections 5.1 - 5.3 determine the reaction functions of each seller, and the equilibrium prices and payoffs, given the selling mechanism of both sellers. In this section, we determine the choice of selling mechanism under two alternative assumptions about commitment. First, we consider the equilibrium choice under the assumption that both sellers commit to a particular selling mechanism prior to choosing a price (or reserve price). We then consider equilibria in which sellers make a simultaneous choice of both mechanism and price.

5.4.1 Equilibrium with Commitment

Using equations (5.5), (5.10), (5.18) and (5.19), which give the payoffs to sellers under the assumption that the sales mechanism is predetermined, we obtain the following payoff matrix.

	Auction	Price Post
Auction	.5, .5	.4827, .4069
Price Post	.4069, .4827	.375, .375

Table 1: Payoffs Under Commitment

Proposition 3:

When both sellers commit to a sales mechanism at a prior stage before prices are chosen, then *both sellers choosing to auction* is the dominant strategy equilibrium.

Proof:

This follows from direct observation of the payoff matrix in Table 1. ■

5.4.2 *Equilibrium Without Commitment*

Here, sellers can choose both mechanism and price simultaneously. There are four candidate equilibria: post-post, auction-auction, post-auction, and auction-post. The following proposition summarizes the result.

Proposition 4:

> When neither seller can commit to a sales mechanism before prices are chosen, then *all possible combinations* (post-post, auction-auction, post-auction, and auction-post) are equilibria.

Proof:

We consider deviations from each candidate equilibrium in turn. If both sellers post and choose the equilibrium price in that case, then the equilibrium prices and payoffs for both sellers are as given in Section 5.1: $p_j^* = 1/2$, $S_P(p_j^*, p_{-j}^*) = 0.375$. Consider now a seller who considers deviating by choosing to auction and picking the reserve price that is the optimal response to the other seller posting $p = 1/2$. From the reaction function (5.17) we have: $r(1/2) = 1/4$. Now substituting p = 1/2 and r = 1/4 into equation (5.13) one obtains: $p = 1/2$. Using this in equation (5.15) yields: $S_{PA}(r = 1/4, p = 1/2) = .375$. Hence, deviation by either player from this candidate equilibrium will not increase the payoff for either player, so this is an equilibrium.

If both sellers auction and choose the equilibrium reserve price in that case, then the equilibrium reserve prices and payoffs for both sellers are as given in Section 5.2: $r_j^* = 1/2$, $S_A(r_j^*, r_{-j}^*) = 1/2$. Consider now a seller who considers deviating by choosing to post and picking the posted price that is the optimal response to the other seller using $r = 1/2$. From the reaction function (5.16) we have: $p(1/2) = 2/3$. Now substituting $p = 2/3$ and $r = 1/2$ into equation (5.13) one obtains: $p = 1/2$. Using this in equation (5.14) yields: $S_{PA}(p = 2/3, r = 1/2) = 1/2$. Hence, deviation by either player from this candidate equilibrium will not increase the payoff for either player, so this is an equilibrium.

If one seller auctions and the other posts a price, using the equilibrium prices, then the equilibrium reserve prices, posted prices, and payoffs are as given in Section 5.3: $r^* = .2713$, $p^* = .6277$, $S_{PA}(r^*, p^*) = .4827$, $S_{PA}(p^*, r^*) = .4096$,. Suppose that the auctioning seller deviates by choosing to post, assuming that the other seller continues to post, but at the posted price in the candidate equilibrium. From the reaction function (5.4) we have: $p(.6277) = .5785$. Using this in (5.2) we obtain: $p = .5931$. Using this in (5.3), yields: $S_P(.5785,.6277) = .4827$. Thus, the auctioning seller is made no better off by deviating. Similarly, suppose the posting seller chooses to deviate by auctioning, assuming that the other seller continues to auction, but at the reserve price in the candidate equilibrium. From the reaction function (5.9) we have: $r(.2713) = 1/2$. Using these reserve prices in (5.7), we obtain: $p = .4069$. Using this in (5.8) yields: $S_A(.5,.2713) = .4069$. Thus, the posting seller is made no better off by deviating. ■

In the absence of a commitment mechanism, therefore, one could observe all possible combinations, including the co-existence of price-posting and auctioning. Notice that, relative to posting prices, both sellers auctioning maximizes the sellers' joint surplus. Due to the symmetry of the game, the probability that the good is sold is the same in the subgames where both sellers auction and both sellers post prices (1/2). Consequently, rents are simply transferred from buyers to sellers if the sellers choose to auction rather than post prices. However, sellers can be assured of this outcome in equilibrium only if they have some way to commit.

6. CONCLUSIONS AND DISCUSSION

Both of the directed search models that we have considered here (with price-posting and competing auctions) are consistent with the same matching function, which has plausible properties. In markets of any significant size, sellers' expected payoffs under these two sales mechanisms are approximately the same. Significant differences in the payoffs exist in the 2-by-2 case, but these differences drop away precipitously as market size increases and fall to zero in the limit. Moreover, when sellers choose which sales mechanism to use, even in the 2-by-2 case, both price-posting and auctioning can coexist in equilibrium in the absence of a mechanism that commits sellers to their announced sales method. The equilibrium that maximizes sellers' expected payoffs is the one in which both sellers auction. This equilibrium is not unique without this type of commitment.

The expected payoffs for buyers and sellers in these models are not the same either in the small or large market cases but, in either case, entry is constrained efficient in large markets. One implication is that, in the context of the labor market, constrained-efficient entry by firms will be obtained whether they are modelled as buyers of labor (as in Julien, Kennes, and King (2000)) or sellers of jobs (as in Montgomery (1991), Burdett, Shi and Wright (1997) or Shi (1999)). Another implication is that the local market mechanism used by Moen (1997) is not necessary to achieve constrained efficiency.

The analysis in this paper ties in with a related literature on mechanism selection. Lu and McAfee (1996) consider a similar environment, but with two separate large markets. In one market, prices are determined by auction; in the other, by bilateral bargaining. Both buyers and sellers can choose which market to enter. The authors show that two equilibria exist: one where all sales are conducted through bargaining, and another where all sales are conducted using the auction. However, only the auction equilibrium is evolutionary-stable. They argue that, in this sense, auctions "drive out" bargaining. Kultti (1999) makes a similar argument, but where prices are determined by price-posting in one market and bargaining in the other. In this case, price-posting "drives out" bargaining. When comparing price-posting and auctions, Kultti also shows that many equilibria exist with different proportions of auctioning and posting sellers. In this way, he argues that auctions and price-posting are "practically equivalent". Our work in this paper, in effect, shows that this last result is robust to relaxing the assumptions of large and mutually exclusive markets.

Clearly, both price-posting and auctioning are observed in the real world. As is consistent with the theory here, auctions tend to be observed more in small markets. In the context of the labor market, high-skilled workers in small markets tend to auction their labor while wage-posting by firms is typically observed in larger, less specialized, markets. The analysis here suggests that both of these structures can be approximately constrained-efficient.

The real world is also rich with heterogeneity and informational imperfections. As Montgomery (1991) points out, the presence of heterogeneity generates an interesting conflict between two possible public policy objectives in this type of environment. Surplus maximization implies longer expected queues for workers of higher quality. However, minimizing unemployment implies an equalization of queue lengths. If surplus maximization is chosen as the goal then, given the basic coordination problem underlying this framework, this implies an optimal level of unemployment that is higher than its minimum level. Montgomery demonstrates that the wage-posting equilibrium will achieve this level if and only if the market is large. In Julien, Kennes and King (1999), we show that auctioning labor will achieve this optimal level, regardless of market size. Shi (1998) shows that vacancy entry will also be constrained-efficient in large economies of this type. Finally, in independent work, Coles and Eeckhout (2000) and Hillas, Julien, and King (2000) make the point that, when the number of agent types equals the number of agents, then two-sided heterogeneity can solve the coordination problem, moving the economy to the first-best allocation. The impact of informational asymmetries has yet to be fully understood.

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