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RADIATIVE TRANSFER IN MULTIPLY LAYERED MEDIA

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN PHYSICS,
THE UNIVERSITY OF AUCKLAND, FEBRUARY 2006

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Abstract

The theory of radiative transfer is applied to the problem of multiple wave scattering in a one-dimensional multilayer. A new mathematical model of a multilayer is presented in which both the refractive index and width of each layer are randomized. The layer widths are generated by a new probability distribution which allows for strong layer width disorder. An expression for the transport mean free path of the multilayer is derived based on its single-scattering properties. It will be shown that interference between the field reflected from adjacent layer interfaces remains significant even in the presence of strong layer width disorder. It will be proven that even when the scattering is weak, the field in a random multilayer localizes at certain frequencies. The effect of increasing layer width randomization on this form of localization is quantified.

The radiative transfer model of time-harmonic scattering in multilayers is extended to narrow-band pulse propagation in weakly scattering media. The tendency of pulses to broaden in this medium is discussed. A radiative transport model of the system is developed and compared to numerical solutions of the wave equation. It is observed that pulse broadening is not described by simple transfer theory. The radiative transfer model is extended by the addition of a Laplacian term in an attempt to model the effect of ensemble average pulse broadening. Numerical simulation results in support of this proposal are given, and applications for the theory suggested.

Finally, the problem of acoustic wave scattering by planar screens is considered. The study was motivated by the idea that multiple scattering experiments may prove possible in a medium composed of such scatterers. Successful multiple scattering in a medium of planar scatterers will depend on the scattering cross-section at angles
away from normal incidence. The scattering cross-section is calculated for a circular disc using a new technique for solving the acoustic wave equation on planar surfaces. The method is validated by comparison with available analytic solutions and the geometric theory of diffraction.
Acknowledgments

I would like to thank my principal supervisor Professor Chris Tindle for agreeing to take on this role. Professor Tindle’s advice and insight helped at many stages to guide this work to a conclusion, and his careful reading of the draft through a number of iterations was particularly appreciated.

I would also like to thank Professor Richard Weaver for a comprehensive review of an early draft of this work. In particular, his suggestion that the Bragg resonances apparent in some of the early results were worthy of further exploration led to much of the work that makes up the third chapter. I am also grateful to Professor Weaver for pointing out a mathematical error in the treatment of the boundary conditions of the convective-diffusive two-stream equation given in the fourth chapter.

Finally, I would like to thank my family and friends for their kind support and encouragement over the course of this work.
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