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Failure of Sandwich Honeycomb Panels in Bending

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A thesis submitted in partial fulfillment of the requirements for a degree of Doctor of Philosophy at the University of Auckland

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February 2006
Abstract

This thesis investigates failure in sandwich panels due to bending, specifically localised buckling or wrinkling, a predominant failure mechanism for thin gauge honeycomb sandwich panels loaded in bending or compression. Over the past 60 years, considerable work has been devoted to understanding wrinkling and trying to predict failure loads in damaged and undamaged panels accurately. Existing wrinkling expressions were shown to over-estimate failure loads by over 100%. Discrepancies between wrinkling expressions and experimental failure loads were previously attributed to imperfections and irregularities in the structure. The aim of this thesis is to investigate this problem and try to accurately predict failure loads and understand the underlying failure mechanisms in damaged and undamaged panels, using a combination of numerical and analytical techniques.

Classical wrinkling models use a continuum core to model complex cellular honeycomb cores. This type of model reduces complex cellular geometry to a series of effective properties that provide constant support to the face sheet. In reality, honeycomb cores provide support around the periphery of the cell walls and not across the entire surface of the face sheet. Due to the nature of wrinkling and the size of the wavelength, incorrect representation of the core could affect the failure loads and model. This study made direct comparisons between linear buckling loads of a discrete-cored sandwich panel and a continuum-cored sandwich panel. Discrete properties were converted to continuum properties within a Finite Element package. The result conclusively showed that both models predict the same linear failure loads, disproving the theory that the core representations contribute to the difference between experimental and analytical models. It was also shown that existing wrinkling models can accurately predict linear wrinkling loads. These linear model loads do not necessarily match the collapse strength of the physical panel and in most cases predict a significantly higher value.

The research then moves on to developing expressions to convert cellular geometry into continuum properties accurately. Expressions are developed for honeycomb structures with fillets in their junctions. Both out-of-plane and in-plane modulus properties are reviewed and the models are verified against Finite Elements models and experimental results. Studies showed that the restrained in-plane modulus can be up to ten times stiffer than the commonly used free modulus value. This has a significant effect on the wrinkling stress. By using the correct value, the discrete model and continuum models predict the same loads. The classical wrinkling expressions also predict the same wrinkling stress as the Finite Element models.

After establishing that the core representation is not the cause of the prediction error, the thesis turns to non-linear Finite Element models to predict failure loads and failure mechanism of thin-gauge sandwich honeycomb structures loaded in bending. A continuum three-dimensional non-linear Finite Element model, with bilinear plasticity, is compared with a set of experiments that use
different types of Nomex cores and face sheets. The models show that the panels fail prematurely due to core crushing because of wrinkles forming in the face sheets. Experimental results indicate similar trends.

The final section examines the affect of impact damage in honeycomb sandwich structures. Due to the thin face sheets and thick cores used on many aircraft and marine components, sandwich panels offer little resistance to impact events. Resulting damage usually consists of a layer of crushed core and a shallow dent in the face sheet. This type of damage often leads to a significant reduction in the load-carrying capacity of the panel through a full range of damage sizes. Finite element and analytical models were developed to accurately predict and capture the localised wrinkling failure mechanism which occurs in the impacted area. Models were directly compared to experimental results, with a high degree of correlation. The numerical and analytical models showed that impact damaged panels were failing due to wrinkling instability and not due to premature core crushing, which is the case with undamaged panels. They showed that two factors influence the wrinkling failure load: damage depth and damage diameter.
Dedication

To my parents

Jan & Jennie Staal

In return for all your guidance and support

Thank you
Acknowledgements

I would first and foremost like to thank my supervisors Gordon Mallinson, and Krishnan Jayaraman. Your support, guidance and motivation during the course of this study have been immeasurable.

I would like to thank my mentor and friend Damian Horrigan, for your support, advice and encouragement during this research. Without your assistance, certain aspects of this research may not have materialised.

I must also acknowledge certain staff at Fisher and Paykel for their support during the first two years of this PhD. Special thanks must go to Ian Holt, my immediate manager, for organising financial support and giving me the freedom to undertake this research. I must also acknowledge and thank Steven Mansell, who offered me guidance with Ansys and other aspects of this work during the early stages of this project.

Thank you to those people who have made this exercise possible through non-academic contributions. Thank you Mum and Dad, for extending your parental duties yet again and providing a home whilst I completed work on this thesis. Thank you to my fellow student and friend Tem Southward for his advice and encouragement during the duration of this work. Thank you to the support staff in Mechanical Engineering Strength of Materials laboratory, especially to Jos Geurts, Rex Halliwell, Barry Fullerton for the numerous experimental solutions they provided.

I would like to express my gratitude to Air New Zealand, in particular Howard King, Mike Pervin and Andrew Freese from Technical Services, for your advice during the course of this work and for organising the materials and facilities needed to manufacture the test panels.

To Catherine Jensen I give my sincere thanks. I am in your debt for the marvellous support you have offered me over the years and the personal sacrifices you have made. Also a special thanks to Nicola Anstice, for her friendship and invaluable assistance during the editing phase of this thesis.

I also wish to thank Peter Morrow and Ben Smit for reviewing this thesis before final submission.

Lastly I wish to thank my friends for creating the continued enthusiasm required for this project through enjoyable relaxation and leisure time.
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List of Symbols and Glossary of Terms

Notation

Note: all pressure units are in MPa and length units in mm unless otherwise stated

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{x0}, G_c$</td>
<td>MPa</td>
<td>Continuum shear modulus in X direction (loading direction)</td>
</tr>
<tr>
<td>$G_{yz}$</td>
<td>MPa</td>
<td>Continuum shear modulus in y direction</td>
</tr>
<tr>
<td>$E_{z}, E_c$</td>
<td>MPa</td>
<td>Core modulus in the out-of-plane direction</td>
</tr>
<tr>
<td>$G_i$</td>
<td>MPa</td>
<td>Shear modulus of the ith cellular wall</td>
</tr>
<tr>
<td>$E_f$</td>
<td>MPa</td>
<td>Face sheet modulus</td>
</tr>
<tr>
<td>$E_s$</td>
<td>MPa</td>
<td>Cell wall modulus</td>
</tr>
<tr>
<td>$G_i$</td>
<td>MPa</td>
<td>Cell wall shear modulus</td>
</tr>
<tr>
<td>$E_x$</td>
<td>MPa</td>
<td>Continuum in-plane modulus in x direction</td>
</tr>
<tr>
<td>$E_y$</td>
<td>MPa</td>
<td>Continuum in-plane modulus in y direction</td>
</tr>
<tr>
<td>$t_c$</td>
<td>mm</td>
<td>Sandwich panel thickness (thin gauge) / core thickness</td>
</tr>
<tr>
<td>$E_1$</td>
<td>MPa</td>
<td>Damaged continuum core modulus in the z direction (region 1 – damaged core)</td>
</tr>
<tr>
<td>$E_2$</td>
<td>MPa</td>
<td>Continuum core modulus in the z direction (region 2 – undamaged core)</td>
</tr>
<tr>
<td>$t_f$</td>
<td>mm</td>
<td>Face sheet thickness.</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td></td>
<td>Compressive strength of the core in the Z direction</td>
</tr>
<tr>
<td>$\sigma_{sc}$</td>
<td></td>
<td>Compressive strength of the parent material cell walls</td>
</tr>
<tr>
<td>$h$</td>
<td>mm</td>
<td>Depth of damage zone</td>
</tr>
<tr>
<td>$\theta$</td>
<td></td>
<td>Internal angle</td>
</tr>
<tr>
<td>$v_f, v_f$</td>
<td></td>
<td>Poisson’s ratio of the face sheet</td>
</tr>
<tr>
<td>$v_s$</td>
<td></td>
<td>Poisson’s ratio of the cell wall (parent material)</td>
</tr>
<tr>
<td>$b$</td>
<td>mm</td>
<td>Cell wall length</td>
</tr>
<tr>
<td>$t_s$</td>
<td>mm</td>
<td>Cell wall thickness</td>
</tr>
<tr>
<td>$\sigma_{cr}$</td>
<td>MPa</td>
<td>Critical wrinkling facesheet stress / failure stress</td>
</tr>
<tr>
<td>$P_{cr}$</td>
<td>N</td>
<td>Critical wrinkling load</td>
</tr>
<tr>
<td>$\tau_{zx}$</td>
<td>MPa</td>
<td>Continuum shear strength</td>
</tr>
<tr>
<td>$\tau_{zy}$</td>
<td>MPa</td>
<td>Continuum shear strength</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>ton/m$^3$</td>
<td>Density of the continuum</td>
</tr>
<tr>
<td>Symbol</td>
<td>Unit</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td>-------------</td>
</tr>
<tr>
<td>( \rho )</td>
<td>ton/m (^3)</td>
<td>Density of the parent material (cell walls)</td>
</tr>
<tr>
<td>( P )</td>
<td>MPa</td>
<td>Maximum graduated pressure load</td>
</tr>
<tr>
<td>( A_o )</td>
<td>mm</td>
<td>Amplitude of the initial wrinkle / facesheet waviness</td>
</tr>
<tr>
<td>( \sigma_t )</td>
<td>MPa</td>
<td>Tensile cut-of-stress of the core</td>
</tr>
<tr>
<td>( \tau_c )</td>
<td>MPa</td>
<td>Core shear strength</td>
</tr>
<tr>
<td>( \gamma_a )</td>
<td></td>
<td>Ultimate shear strain of the adhesive</td>
</tr>
<tr>
<td>( L_{cr}, L )</td>
<td>mm</td>
<td>Critical wrinkling half wavelength (wrinkling expressions)</td>
</tr>
<tr>
<td>( L_{dam} )</td>
<td>mm</td>
<td>Damage Diameter / size</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>mm</td>
<td>Critical wrinkling wavelength (wrinkling expressions)</td>
</tr>
<tr>
<td>( \nu_{zx}, \nu_{zy}, \nu_{31} )</td>
<td></td>
<td>Poisson’s ratio of the core in the out-of-plane loading direction</td>
</tr>
<tr>
<td>( \nu_{xy} )</td>
<td></td>
<td>In-plane Poisson’s ratio of the core for loading in the X direction</td>
</tr>
<tr>
<td>( \nu_{yx} )</td>
<td></td>
<td>In-plane Poisson’s ratio of the core for loading in the Y direction</td>
</tr>
<tr>
<td>( \gamma_i )</td>
<td>mm(^3)</td>
<td>Shear strain in the ith cellular walls</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>mm(^3)</td>
<td>Shear strain in the continuum block</td>
</tr>
<tr>
<td>( V )</td>
<td>mm(^3)</td>
<td>Volume of the continuum block</td>
</tr>
<tr>
<td>( V_i )</td>
<td>mm(^3)</td>
<td>Volume of the ith cellular wall</td>
</tr>
<tr>
<td>( U_{curved} )</td>
<td>Nmm</td>
<td>Strain energy curved section of wall (fillet)</td>
</tr>
<tr>
<td>( U_{angle} )</td>
<td>Nmm</td>
<td>Strain energy in the angled wall</td>
</tr>
<tr>
<td>( U_{block} )</td>
<td>Nmm</td>
<td>Strain energy in an equivalent block which covers the same area as the unit cell.</td>
</tr>
<tr>
<td>( \varepsilon_x )</td>
<td>Strain of face sheet in x direction</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_y )</td>
<td>Strain of face sheet in y direction</td>
<td></td>
</tr>
<tr>
<td>( L )</td>
<td>mm</td>
<td>( L = b - b \cdot \psi \cdot 2 ) (in-plane and out-of-plane continuum conversion models)</td>
</tr>
<tr>
<td>( E_{cc} )</td>
<td>MPa</td>
<td>Constrained in-plane modulus in central area (either x or y direction)</td>
</tr>
<tr>
<td>( E_{cf} )</td>
<td>MPa</td>
<td>Constrained in-plane modulus at the core/face sheet interface (either x or y direction)</td>
</tr>
<tr>
<td>( R )</td>
<td>mm</td>
<td>( r = \frac{\psi \cdot b}{\tan \left( \frac{\theta}{2} \right)} )</td>
</tr>
<tr>
<td>( \psi )</td>
<td></td>
<td>Fillet ratio (percentage of wall length b)</td>
</tr>
<tr>
<td>( K )</td>
<td>Nmm</td>
<td>Face sheet stiffness</td>
</tr>
<tr>
<td>( P_x )</td>
<td>N</td>
<td>Face sheet load</td>
</tr>
<tr>
<td>( q )</td>
<td></td>
<td>Reaction pressure of the foundation</td>
</tr>
</tbody>
</table>
**Glossary of terms**

**Out-of-plane** - Properties or loading in the $Z$ direction (Figure 1-5)

**In-plane** – Properties or loading in the $X$-$Y$ direction (Figure 1-5)

**Discrete** – Core modelled as a detailed cellular core with properties of the cell walls and cellular geometry

**Continuum** – Core modelled as a solid block with effective properties in the three principal directions

**Wrinkling** – Local buckling instability in sandwich panels

**Ribbon direction** – Inline with the double walls – $X$ direction

**Transverse direction** – Normal to the ribbon direction – $Y$ direction

**Thin gauge** – Sandwich panel consisting of thin facesheets and a comparatively thick core

**Finite Element** – Finite Element

**BVID** – Barely visible impact damage / dent

**Linear wrinkling stress** – Applied facesheet stress at the eigenvalue buckling load (the instability point)

**Non-linear wrinkling stress** – Applied facesheet stress at the non-linear buckling load – point where the model becomes unstable (does not take into account material non-linearity)

**Non-linear failure stress** – Applied facesheet stress at the non-linear failure load – point where the model becomes unstable (can take into account material non-linearity)

**Undamaged stress** – Failure stress of the undamaged panel

**Damaged stress** – Failure stress of the damaged panel

**Restrained in-plane modulus** – Modulus calculated when the core is restrained by the facesheets

**Free in-plane modulus** – Modulus calculated with the facesheets detached

**3D Continuum model** – Three dimensional model based on brick and shell elements using continuum core properties

**2D Plane stress model** – Two dimensional solid model based on plane stress elements using continuum core properties

**3D Discrete model** – Accurate model of a sandwich honeycomb panel, developed using shell elements with cellular properties and geometry

**Natural / critical wrinkling wavelength** – Wavelength that wrinkles will form at the minimum load
Chapter 1: Introduction

This doctoral thesis examines wrinkling failure in honeycomb sandwich panels, in an attempt to explain why existing models over-predict experimental failure loads. The effect of representing the behaviour of complex cellular geometry by simplified effective properties in classical linear wrinkling models is examined. Overall failure of undamaged and damaged sandwich panels is investigated using non-linear Finite Element models, linear wrinkling models and experimental results. Tools that can accurately predict failure loads in undamaged panels and panels with a large section of sub-surface core damage are developed. A better understanding of wrinkling failure and other mechanisms that govern the collapse of thin-gauge sandwich panels in bending is gained.
Section 1.1 Background

Marine, automotive and aerospace industries are continually striving to optimise material performance in terms of strength and weight. Success has been achieved through the growth of high performance materials including fibrous composites such as carbon fibre composites, ceramics, new alloys, and through the use of structural concepts such as sandwich construction.

Sandwich panels consist of three layers: two high-modulus high-density face sheets bonded to a low density core. The core is designed to keep the face sheets a desired distance apart and carry the entire transverse shear load. The face sheets carry the in-plane tensile and compressive loads.

The faces of sandwich panels are usually manufactured from carbon or glass fibre reinforced composite, while the core typically consists of a hexagonal honeycomb structure manufactured from Nomex fibre or aluminium. Sandwich construction has high bending stiffness and, in many cases, results in a high strength to weight ratio. The panels are designed so that every component is placed under maximum stress, making weight-saving possible and sandwich panels popular in high performance applications. Nomex honeycomb panels with a thin glass or carbon face have specialised use in external and internal aircraft fuselage components and racing yachts. These panels, however, are particularly prone to wrinkling modes of failure when under compressive stress.

Figure 1-1 – Loading paths in sandwich panels

Figure 1-2 – Cross section of impact damaged sandwich panel showing face sheets and Nomex honeycomb core

Aircraft manufacturers use composites and sandwich technology to manufacture many components. For example the Boeing 777 comprises almost 40% composite panels by volume (see Figure 1-3). The choice of a composite for a particular structural application depends on the complex interaction of many factors, including maximum loading strength, stiffness criteria, damage tolerance, reparable, ease of manufacture, and cost.
Section 1.2: Numerical and analytical models

Higher material and certification costs, and a lack of understanding by structural designers are cited as three reasons why US manufacturers do not make more use of composites on large commercial aircraft [2]. Currently sandwich panels are typically used for secondary structures such as flaps, control panels, fairings and interior panels. Primary structures of the wings and the fuselage are still almost entirely manufactured from aluminium extrusions and castings, due to the complex behaviour of sandwich structures and the difficulty in predicting their failure modes.

Section 1.2 Numerical and analytical models

Extensive work has been carried out on the development of computational models for studying the response, life and failure of sandwich panels and shells in an attempt to make their use more widespread. Noor [2] categorised existing computational models for sandwich plates and shells into four types: detailed, three dimensional (3D), two dimensional (2D) and simplified. Varying forms of these models have been developed throughout this study, to investigate the failure of sandwich panels in bending. Table 1-1 provides details of these models.
### Table 1-1 – Categories of sandwich models [2] and location of model types in the thesis

For a more detailed overview and references to various models refer to Burton and Noor [2]. Descriptions of some of these models can also be found in the literature reviews specific to each chapter. The simplified, three dimensional and detailed models were also used for analytical and Finite Element work.
1.2.1 Failure modes of sandwich panels in bending

Unlike traditional materials such as aluminium, composite sandwich structures exhibit a number of complex failure modes. Failure modes and failure loads are often hard to predict, and in many cases not adequately explained by theory or experiment [3]. The main bending/compressive failure mechanisms are shown in Figure 1-4 and described below.

1. **Tensile and compressive face sheet failure**: The face sheet reaches the yield or fracture stress of the material and fails due to excessive bending stress through wrinkling of the face sheet and collapse of the core, or conversely through excessive edgewise compressive stress.

2. **Localised buckling (wrinkling)** of the face sheet: This is due to the compressive instability of the face sheet, as the core can no longer support the face sheet’s normal load, which leads to inwards or outwards buckling depending on the core and bond properties.
3. Dimpling: This consists of localised indentation of the face sheet into individual cells and occurs in panels with relatively large cells and thin facing. This form of compressive instability is similar to wrinkling.

4. Global buckling: This is caused by overall buckling of the sandwich beam and usually occurs in panels with relatively stiff face sheets and thin cores.

Localised buckling or wrinkling is the predominant failure mode for Nomex honeycomb panels with thin glass or carbon faces loaded under axial compression or bending. Wrinkling, a localised instability consisting of wavelengths smaller than the overall face length is determined by the vertical and shear stiffness of the core and the stiffness of the face sheet. Wrinkling will often lead to core crushing and fracturing of the face sheet due to excessive bending or compressive stress in the face sheet/core.

1.2.2 Wrinkling models

Numerous studies of wrinkling have been completed over the past 60 years and most of these have led to the same conclusions regarding the modelling and derivation of the models [4]. The typical form of the simplified plane stress model, which predicts wrinkling instability for loading in the $X$ direction (Figure 1-5 gives the cell orientation), is:

$$\sigma_{wr} = C \left( \frac{E_f E_c}{G_{fc}} \right)^{\frac{1}{3}}$$

(1-1)

The constant $C$ in Equation (1-1) varies depending on the decay function, which determines how the wrinkle dampens through the core depth. Thus using a linear function (Hoff and Mautner [5]), $C = 0.91$ (1947). With the exponential function (derived by Plantema [4] and Zenkert [3]), $C = 0.85$. In the current model (Chapter 2), using the equilibrium controlling decay function, $C = 0.825$.

The critical wrinkling stress in (1-1) is calculated using an assumed sinusoidal buckling shape and a derivation similar to a beam on an elastic foundation. This expression assumes that the core is a continuum: a core that provides continuous support to the facings, and the same expression is used to model wrinkling in both cellular and foam cores.
Section 1.3 Objectives

The aims of this research were to understand why existing wrinkling expressions over-predict failure loads in thin gauge sandwich panels subject to bending, and to develop models that accurately predict failure loads in damaged and undamaged states.

The existing wrinkling models, shown in Equation (1-1), appear to overestimate the failure loads of panels loaded in compression or bending in almost all situations. In certain cases, the models have been up to 100% inaccurate. Hoff and Mautner [5] considered this and developed an alternative expression, $\sigma_{\text{cr}} = 0.5(E/E_z G_{zz})^{\frac{1}{3}}$, where the coefficient of 0.5 (originally 0.91) is a correction factor based on experimental results. This revised expression has become the industry standard for predicting the failure loads of sandwich panels loaded in compression or bending.

Some of this difference between the analytical and experimental values has been attributed to manufacturing defects and imperfections in the structure. Various authors ([4], [6, 7]) have investigated this problem and shown that these issues account for less than a 10-20% drop in failure loads and not the 40-50% discrepancies between the experiments and predictions by Equation (1-1).

The central research question of this research is: “Can linear wrinkling models predict failure in thin gauge sandwich panels under bending?”

A flow diagram of the objectives of this thesis is shown in Figure 1-7.

Section 1.4 A brief overview of this thesis

To understand why the existing models over-predict failure loads, the first part of this research re-investigated existing linear wrinkling expressions. It looked at the implications of using discrete and continuum core representations to model localised bending failure mechanisms such as wrinkling.
(Figure 1-6 shows both core models.) It was postulated that any discrepancy between the analytical models and experimental failure stresses was caused by a simplified representation of the core. Due to the large number of hexagonal cells that form honeycomb cores, engineers traditionally use smeared or continuum properties to calculate deformations, moments and stresses in sandwich panels. Continuum properties are also used to model failure mechanisms such as wrinkling and to develop wrinkling expressions. With continuum properties, complex cellular properties are reduced to a set of orthotropic properties in the principal directions.

With a continuum core model, face sheets are supported across an entire surface, compared with discrete core models where the face sheets are supported only around the periphery of individual cells. In most cases replacing a discrete core with an equivalent continuum core is a reasonable approximation, as a typical discrete core consists of thousands of individual cells. In the case of overall modes of deformation, the tiny cells might equally be a solid.

However, with localised failure modes such as wrinkling, this approximation may not be valid. With thin face sheet Nomex honeycomb panels, wrinkling wavelengths are in the order of two to three cell widths in length. With such small wavelengths it was thought that wrinkling could be influenced at a cellular level rather than a global level such as overall bending deformation. At a cellular level the wrinkling deformation becomes reliant on single cells and their interaction with neighbouring cells. To test this theory a number of detailed numerical Finite Element (Finite Element) models and three dimensional (3D) continuum models were developed to compare the wrinkling stresses directly.

Findings from this test showed that both models predict the same wrinkling loads, meaning that either model can be used, providing accurate continuum properties are extracted from the discrete model. The research then develops new closed form analytical expressions to convert discrete properties into continuum properties for both the in-plane and out-of-plane moduli. These models are verified using numerical Finite Element models and experimental data. Because the core representation is neither influencing the failure loads nor contributing to the error, non-linear Finite Element models are then used to predict failure loads and modes of bending in sandwich panels.
Non-linear Finite Element models are used to examine the progression of failure and internal stresses in the structure and to develop ways to accurately predict failure stresses, thus allowing engineers to design optimised structures in terms of their strength and weight.

Various models are developed to predict failure loads, and to investigate the progression of wrinkling failure and subsequent modes of failure in a damaged and undamaged state. Each model is verified against experimental results.

As an extension of the work on undamaged panels, the research examines the effects of impact damage on sandwich structures. Impact damage can potentially ground aircraft until satisfactory repairs have been performed. It is a relatively common phenomenon on aircraft due to the susceptible locations of many of the sandwich panels and the low damage resistance of honeycomb cores under localized transverse loading. Due to their inherently thin faces, thin gauge sandwich structures offer little resistance to impact damage. Impact strikes often create large areas of barely visible impact damage (BVID) that consists of crushed core and a shallow crater in the face sheet. BVID can have a large bearing on the residual strength of the structure: Aitken [8] found that impacted sandwich panels failed less than half of their equivalent load of an undamaged panel in a wrinkling type failure. As impact damage has a significant effect on the failure stress, it presents a challenge to continued airworthiness.

While examining damaged panels, this research studies the complex behaviour of damaged sandwich structures and tries to predict the failure modes and loads accurately. Various models have attempted to predict failure stresses in panels damaged by impact; however, impact damage in thin walled sandwich structures is still misunderstood, and not accurately predicted by existing failure models.

All of these objectives are broken into self-contained chapters. Within each chapter are a literature review, numerical and analytical work and experiments relevant to the chapter. In addition, each chapter includes a discussion and relevant conclusions.
Section 1.5  Part 1 - Linear wrinkling models (Chapters 2 – 3 – 4)

The first aim of the research was to find out why existing linear wrinkling expressions do not correlate with experimental results. Part 1 determines if linear wrinkling can be accurately accounted for using a simplified representation of the core (continuum core), or whether complex cellular geometry (discrete core) is needed. It was theorised that the core representation may contribute to errors between experimental results and existing analytical models. This enquiry involves developing both numerical and analytical models to represent the complex cellular construction by continuum data. Models used to convert properties from discrete to continuum cores were verified against collected experimental data and Finite Element models. An in-depth review of the in-plane and out-of-plane moduli was also undertaken as part of this work.

1.5.1 Chapter 2 – Linear wrinkling stresses in discrete and continuum cored honeycomb panels

Chapter 2 examines the effect of modelling wrinkling with discrete versus continuum cores. Continuum cores are the traditional way to represent the complex cellular structure. Chamis [9] looked at the implications of modelling honeycomb cores as discrete structures in relation to
The first part of this chapter reviews some of the existing analytical continuum models and then
continues by developing three dimensional continuum and detailed discrete Finite Element buckling
models. For comparison, a discrete honeycomb structure is converted to a continuum structure,
using a unit cell Finite Element model which is displaced in various directions to calculate the
effective continuum properties.

Through a direct comparison of linear buckling loads it was found that both core representations
produced similar results, suggesting that either could be used to represent this failure mode. In
addition, all of the continuum Finite Element models were verified against existing linear wrinkling
expressions. Regression expressions were also created using the discrete and continuum Finite
Element models.

1.5.2 Chapter 3 – Out-of-plane continuum properties of honeycomb cores

Chapter 3 develops expressions to convert discrete cellular properties into out-of-plane continuum
properties (properties in the $Z$ direction shown in Figure 1-5). Unlike previous models, these newly
developed expressions include fillets in the junctions of the corners. Finite element models,
manufacturers’ and experimental data were used to validate the models.

The latter part of this chapter is dedicated to determining the discrete cellular properties for three
types of Nomex core experimentally. These properties include wall thickness, cell length, cellular
modulus and average cellular geometry. Cellular properties are needed to create discrete models
and verify the conversion models and as a guide in the discrete Finite Element models developed in
Chapter 2. No prior information existed on discrete properties, due to the complexity of finding
these values and the fact that most people deal with continuum properties.

1.5.3 Chapter 4 – In-plane continuum properties of honeycomb cores

Chapter 4 investigates the in-plane modulus (properties in the $X$ and $Y$ direction shown - Figure
1-5) of Nomex cores, properties which are traditionally neglected in all expressions for wrinkling.
Vonach and Rammerstorfer [1] were the first authors to consider the effect of this modulus on the
wrinkling stress. They showed that low values of in-plane modulus can have a large effect on the
wrinkling stress. With low values, the core is able to displace sideways, allowing wrinkles to form
and grow at lower loads. Traditional calculations assume that there is negligible deformation in the
in-plane direction and all deformation is restricted to the out-of-plane / through thickness direction.
This assumption is equivalent to saying that the core is infinitely stiff in the in-plane direction.

Most of the existing expressions which calculate the in-plane modulus assume that the core is in a
free state, where in reality cores are always attached to face sheets. Unlike the out-of-plane
modulus, these face sheets have a pronounced effect on the in-plane modulus. They provide
additional restraint at the top and bottom surface of the core, stopping it from expanding freely like
a large spring.
Becker [10] was one of the first authors to consider the “thickness effect”, that is the effect of having the face sheets attached to the core. This chapter continues Becker’s work and develops an expression to calculate the restrained in-plane modulus for non-filleted and filleted honeycomb cells. Finite Element models were used to verify this work. When these verified expressions were used in the continuum Finite Element model, it was found that the models have much better correlation with the existing analytical wrinkling models, and also the discrete wrinkling models developed in Chapter 2.

Section 1.6 Part 2 - Non-linear wrinkling models (Chapters 5 – 6)

Part 2 develops a method to predict failure stresses in damaged and undamaged panels loaded in bending which is the second aim of the research. It also contributes to the first aim, by using non-linear Finite Element models to show how panels can fail at lower loads than those predicted by linear expressions. The models used for this study are similar to models used in Section 1, except that they are solved using a non-linear, buckling type approach, with plasticity models introduced to account for core crushing and face sheet fracture. As part of this study a comprehensive experimental program was undertaken to verify the failure modes and loads at the Finite Element models. With undamaged panels a range of panel configurations were tested, but with damaged panels, one type of panel configuration was tested over a complete range of damage sizes to examine the reduction in the load carrying capacity of the structure, as damage size increased.

Figure 1-9 – Flow chart showing Part 2 of this research – This part investigates failure of undamaged and damaged panels using combinations of non-linear numerical models, linear wrinkling expressions and experimental results
1.6.1 Chapter 5 – Predicting failure loads of undamaged sandwich panels subject to bending

Chapter 5 develops accurate expressions and models that predict failure loads in undamaged sandwich beams loaded in bending. Current models are inadequate and do not predict undamaged failure loads with any degree of certainty. Failure was found to be caused by wrinkles growing in amplitude and progressively crushing the core. This induces large bending stresses in the face sheets leading ultimately to the final collapse of the panel. A high degree of correlation was found between the results of the non-linear models, the newly developed analytical model, and experimental results. The traditional linear buckling models were shown to be inadequate for this situation.

1.6.2 Chapter 6 – Impact damage honeycomb sandwich panels

This chapter investigates impact damage in thin gauge Nomex sandwich structures and the effect of damage on the residual strength of panels when loaded in bending. The study was carried out over a range of damage sizes, ranging from 0 mm through to 150 mm diameter.

The chapter uses a combination of three dimensional and plane-stress Finite Element models, and analytical models based around the classical wrinkling formulations to predict failure stresses and failure modes, with results verified against experimental test data. The tests are constructed to simulate bird and hail stone strikes on trailing edge wedges of wing flaps on commercial aircraft. The test panels are manufactured using the same material and lay-up as the physical wedge of the aircraft flaps. In the larger size impacts, damage is reproduced using a large pneumatic gun with either a solid wooden ball or rubber ball full of gelatine being used as a projectile. With the smaller damage sizes, the panel was damaged by hitting a smaller solid ball with a hammer. The damaged panels were then loaded in four point bending to failure.

In all cases, the failure mode consisted of wrinkle type deformation in the barely visible impact zone. Impact damage wrinkles were found to form at comparatively low loads due to the layer of
crushed core below the surface.

Analytical and non-linear Finite Element models were developed to predict the failure mode and stress. Both of these models showed good correlation with the experimental results, and gave a better idea of the complex failure modes of impact-damaged sandwich panels.

**Section 1.7 Part 3 – Conclusions, Review and Recommendations (Chapter 7)**

This final chapter summarises all of the results found throughout the thesis. It also elaborates on the significant findings of this research, and how these findings have added to the current body of knowledge.
Chapter 2: Linear wrinkling stresses in discrete and continuum cored honeycomb panels

This chapter examines the implications of modelling wrinkling in sandwich honeycomb cores using a simplified continuum representation of the core. Localised wrinkling in sandwich honeycomb panels is traditionally modelled using a continuum core. In a continuum core, the support provided to the face sheet is constant, while in reality it is discrete with a cellular core. Classical linear wrinkling expressions based on this continuum core assumption over-predicted the observed stresses. This chapter aims to assess whether the method of representing the core is causing this discrepancy, or whether there are other reasons for the difference. The problem is reviewed using a series of complex linear Finite Element models. One model is based on a cellular core, while the other is based on a continuum core. Properties used in the continuum model are directly extracted from the discrete Finite Element model. Direct comparisons are made between the two linear buckling loads. Comparisons are also made between existing analytical models and continuum Finite Element models. It was shown that both types of models predict the same wrinkling stress, providing accurately converted properties are used.
Section 2.1 Introduction

When thin skinned sandwich panels are loaded in bending they will invariably fail due to wrinkling / localised buckling rather than any other failure mode.

Various models ([5],[11],[12],[4],[3]) have been developed to predict the onset of wrinkling in sandwich structures. The majority are based around linear buckling theory and assume that the core provides continuous support to the facings (continuum core representation). Most of these models have led to the same result as regards the predicted wrinkling stresses and the derivation [4]. Most assume a state of plane stress, use a predefined wrinkling shape and represent the core as a continuum. Slight variations exist due to treatment of the wrinkle’s displacement through the core depth, the chosen method of solving the problem and the way the core’s in-plane deformation is dealt with. The worst problem of these models is possibly that they tend to over-predict failure loads. During experiments, the panels fail at about 60% of the wrinkling load predicted by the linear models [5]. This discrepancy has previously been attributed to imperfections and irregularities in the structure.

This chapter reconsiders the underlying continuous core support assumption in respect to wrinkling failure. Using a simplified continuum core to model a strictly discrete structure could lead to error between linear wrinkling models and experimental results. Wrinkles form at a natural wavelength which is dependent on the foundation and face sheet stiffness. With foam cores the ratio cell size to wrinkle wavelength is small, meaning that there are enough cells per wrinkle to represent the foundation as a continuum. With Nomex honeycomb type cores this ratio approaches unity or at least one cell per wrinkle. For these types of cellular cores it is possible that the discrete core could have significant influence on the buckling load and mode. It is also possible for the face sheet and the cellular walls to buckle locally, an event that is not represented by the continuum core. The objective of this chapter is to determine how well continuum core models can represent failure in honeycomb panels. Due to the complexity of the cellular geometry, this is completed using Finite Element analysis.

Section 2.2 Review of Wrinkling models

The earliest model which attempted to predict localised buckling was presented by Timoshenko [13]). Timoshenko assumed that the through thickness stiffness of the core is infinite. This led to an equation which predicts buckling for short wavelengths.

\[
\frac{1}{P_{cr}} = \frac{1}{P_e} + \frac{1}{t_i w G_c}
\]

(2-1)

Where \( P_{cr} \) is the critical buckling load, \( P_e \) is the Euler buckling load, \( t_i \) is the thickness of the sandwich, \( w \) is the width of the strut, and \( G_c \) is the transverse shear modulus of the core.

With through thickness stiffness factored in, short wavelength buckling modes appear, usually known as face sheet wrinkling. Wrinkling is generally either symmetric or anti-symmetric about the middle surface of the sandwich. Symmetric wrinkling can be predicted using the simple model of a
beam resting on an elastic foundation. Anti-symmetric wrinkling is found while accounting for through the thickness and transverse core flexibilities (Z direction - Figure 1-5).

The classical wrinkling models can be split into three main groups, plane stress: plane strain, three dimensional and a sub category of plane stress called in-plane.

**Plane stress models**

A large proportion of published models for the prediction of the onset of wrinkling are based on a mathematical model of a uni-axially loaded, two dimensional sandwich strut. Most of these models assume an isotropic face and core. All the models represent the core as a continuum and can be categorised into two main groups [3]. The first group use energy methods to derive the governing equations, while the second group use equilibrium, continuity conditions and Airy’s stress functions to derive governing differential buckling equations.

Models based on the energy methods assume that the faces take all the compressive membrane force and the vertical and shear support is contained in the core, which acts as an elastic foundation. The faces are assumed to undergo a sinusoidal displacement that damps out through the thickness of the core. The buckling loads are found by enforcing the requirement that the work done by the compressive force is equal to the sum of the strain energy stored in the face material due to bending, and the strain energies from direct and shearing stresses developed in the core.

The general expressions for wrinkling stress in an isotropic sandwich beam under plane stress is given by Hoff and Mautner [5] as

$$\sigma_{cr} = C(E_f E_c G_c)^{\frac{1}{3}}$$

(2-2)

The value of $C$ changes depending on the decay function and depth of the wrinkle. The traditional model of Hoff and Mautner [5] used a linear decay function which gave a $C$ value of 0.91. Plantema [4] used an exponential decay function which results in a value of $C$ of 0.85.

Hoff and Mautner’s [5] derivation considered symmetric and anti-symmetric wrinkling separately. Their model was similar to that of Gough et al. [12] in that they ignored shear traction on the face core interface and used an isotropic core model. They used predetermined shapes for both the symmetric and skew wrinkles. They assumed that the face sheet displacements are damped out linearly to a depth zero. If the core thickness is large enough the mid-plane deformation is considered to be negligible, for both anti-symmetric and symmetric wrinkling.

Strain energy techniques were used to minimize the potential energy stored in the faces and the core. They showed that anti-symmetric wrinkling was dominant for thin cores. When the core thickness increases, symmetric wrinkling becomes the dominating mode shape.

Zenkert [3] showed that symmetric wrinkling is more likely to occur when the ratio of core to face thickness $\frac{t_c}{t_f} > 17$ and anti-symmetric when $\frac{t_c}{t_f} < 17$. For panels with thin face sheets and thick
cores or for any panels in pure bending, symmetric wrinkling models should be used. Panels in pure bending should also be modelled with symmetric wrinkling. Symmetric wrinkling is the only mode being investigated in this thesis.

Extensional strain energy in the face sheets and core strain energies were neglected in Hoff and Mautner’s formulation. The model assumed that there were no in-plane movements. Hoff and Mautner concluded that the expression for a thick core and symmetric wrinkling provides a reasonable estimate of stress in all cases. The value for wrinkling stress is given by

\[
\sigma_{wr} = 0.91(E_f E_c G_c)^{\frac{1}{3}}
\]  

(2-3)

It is important to note that these equations are independent of geometry and are purely a function of core and face material properties.

Plantema [4] presented a similar model to that of Hoff and Mautner [5] but used a different equation for the face sheet displacement.

\[
w = W e^{-kz} \sin \frac{n \pi x}{L}
\]  

(2-4)

This assumes that the waves decay exponentially through the thickness. This compares to the equation \( w = \frac{W(h-z)}{h} \sin \frac{\pi x}{L} \) of Hoff and Mautner [5], which assumes a linear damping of the core displacements and that the buckling of one face has no effect on the face sheet on the other side. Using this exponential displacement function Platema showed that the critical wrinkling load is simply expressed as:

\[
P_{cr} = \frac{3}{2} (2D_f E_c G_c)
\]  

(2-5)

Where, if the facesheet is isotropic

\[
D_f = \frac{E_f t_f^3}{12} \text{ for a narrow beam, or}
\]  

(2-6)

\[
D_f = \frac{E_f t_f^3}{12(1-\nu_f^2)} \text{ for a plate in cylindrical bending}
\]  

(2-7)

Inserting equation (2-6) and (2-7) into (2-5) gives:

\[
\sigma_{wr} = 0.85(E_f E_c G_c)^{\frac{1}{3}} \text{ for the plate}
\]  

(2-8)

\[
\sigma_{wr} = 0.825(E_f E_c G_c)^{\frac{1}{3}} \text{ for a beam}
\]  

(2-9)

In most cases, authors found that the correlation of their models with experimental data was very poor and attributed this to small initial imperfections that trigger a core compression or core to face sheet failure. Plantema [4] suggested that initial irregularities are likely to reduce the wrinkling stress by up to 20%.
Hoff and Maunter tested five different flat regular sandwich panels. They suggested a $C$ of 0.5 be used in practical situations as this is a more conservative formula, compared with their calculated coefficient $C$ of 0.91. It also matches their experimental results more closely.

Wan [6] argued that the wrinkling stress in the facings is related to initial waviness that exists in the surface, which led to a development by Yusuff [14] of an expression which included a provision for waviness.

An alternative approach to the energy method is to make a better approximation of the actual core stress distribution by using Airy stress functions, based on a predetermined sinusoidal displacement. The expression for critical face sheet stress is derived by using the core stress function and minimising the governing differential equation solution for a beam on an elastic foundation. Using this approach Gough and de Bruyne [12] found a $C$ of 0.79. Allen [11] and Zenkert [3] calculated a value for $C$ of 0.78 using the same approach. This coefficient of 0.78 represents an upper bound for sandwich wrinkling stresses and is valid when the core is sufficiently thick. Allen calculated a theoretical lower bound value $C$ of 0.63 for a core which is sufficiently thin.

Equation (2-7) the governing differential equation for a beam on an elastic foundation, which is specialized for unidirectional loading.

$$K' \frac{\partial^4 w}{\partial x^4} + P_x \frac{\partial^2 w}{\partial x^2} + q = 0 \quad (2-10)$$

The aim of the analytical solutions of Equation (2-7) is to determine $\hat{q}$, the reaction stress with respect to the plate. From the Airy stress function the stress required to deform the core in this manner is given by Equation (2-11)(Zenkert [3])

$$\sigma_c = -\frac{a}{l} W \sin \frac{\pi x}{l} \quad \text{with} \quad a = \frac{2\pi E_c}{(3 - \nu_c)(1 + \nu_c)} \quad (2-11)$$

Allen [11] used Airy’s stress functions and the differential equation approach to develop his wrinkling model. The model assumed that the core was isotropic with $G_c = \frac{E_c}{2(1 + \nu_c)}$

$\nu_c = 0.3$ giving the expression

$$\sigma_{cr} = 0.78(E_f E_c G_c)^{\frac{1}{3}} \quad (2-12)$$

Using this the stress function approach Gough, Elam and de Bruyne [12] produced a model that assumes that the face sheets are inextensible and the compressive stresses in the direction of the applied load can be ignored. In their model there was no assumed deformation pattern for the face sheet and the thickness of the sandwich was assumed to be large enough to preclude any interactions between the face sheets. The buckling loads were found by solving core stress functions and solving the corresponding bi-harmonic equations of elasticity. They found that when

Section 2.2: Review of Wrinkling models 19
the core was significantly thick, \( \frac{t_f}{t_c} \left( \frac{E_f}{E_c} \right)^{1/3} < 0.2 \). The stress at which wrinkling will occur is then given as

\[
\sigma_{wr} = 0.79(E_f E_c G_t)^{1/3} \tag{2-13}
\]

Williams, Leggett and Hopkins [15] developed a theory that unified overall buckling and wrinkling. Their models included transverse behaviour of the core by finding a suitable stress function. They were the first to investigate the effects of symmetric and anti-symmetric buckling. They retained the assumption that the effect of the core axial stress could be neglected. Their model accounts for transverse shear and through thickness flexibilities of the core as well as stretching and interaction of the face sheets. They concluded that anti-symmetric wrinkling occurs at a lower load than symmetric wrinkling in sandwich structures with solid isotropic cores. They also suggested that Gough Elam and de Bruyne [12] (Expression (2-13)) produced accurate estimates of the small wave length face sheet wrinkling loads.

**Anti-plane assumption**

The plane stress assumption allows the in-plane core stress in the direction of the applied load to be non-zero, albeit small. Williams [16] used the anti-plane assumption that forces the stress to be zero and derived Equation (2-14) for the critical wrinkling stress.

\[
\sigma_{wr} = \frac{0.83}{(1 - v_f^2)^{1/3}}(E_f E_c G_t)^{1/3} \tag{2-14}
\]

Hemp [17] argued that this result violated equilibrium conditions associated with the anti-plane assumption and derived Equation (2-15)

\[
\sigma_{wr} = \frac{0.83}{(1 - v_f^2)^{1/3}} \left( \frac{E_f t_f}{E_c t_c} \right)^{1/2} \tag{2-15}
\]

Yusuff [14] combined the anti-plane assumptions of Williams [16] and Hemp [17] with the wrinkling model of Hoff and Mautner [5] to estimate symmetric wrinkling stresses. The models are related to a calculated value \( W \), the depth of the fully decayed wrinkle. It was derived by equating strain and shear energy terms to energy stored in an equivalent elastic spring. The critical wrinkling stress is found by minimising the spring expression with respect to \( W \).

\[
\sigma_{wr} = 0.961(E_f E_c G_t)^{1/3} \tag{2-16}
\]

For thick cores, \( W < 0.5 t_c \)

\[
\sigma_{wr} = 0.82(E_f E_c G_t)^{1/3} \tag{2-17}
\]

For cores where, \( W = 0.5 t_c \)
Where \( W \) is given by

\[
W = 0.72 t_f \left( \frac{E_f E_c}{G_c^2} \right)^{\frac{1}{5}}
\]  

(2-18)

**Plane Strain**

Norris et al. [18] derived an expression for the critical stress assuming a state of plane strain:

\[
\sigma_{cr} = Q_f \frac{E_f E_c}{(1-v_f^2)(1-v_c^2)} \frac{1}{G_c}
\]

(2-19)

Where \( Q_f \) is 0.72, \( v_f \) is 0.3 and \( v_c \) is 0. They also suggested that the expression proposed by Hoff and Mautner could be used as long as the plane strain modulus is used as an alternative.

Norris et al. [18] showed that by assuming the ratio of out-of-plane core modulus to in-plane core modulus to be significantly large, Equation (2-19) simplifies to the equation developed by Hemp [17]. Norris et al’s model extended on Gough et al. [12] model to include a core in a state of plane stress or plane strain. Goodier and Neou [19] evaluated the effect of core axial stress on the wrinkling of the faces and verified that it was small compared to the effects of the out-of-plane modulus. This led them to the conclusion that the in-plane modulus could be ignored.

**In-plane modulus**

Classical papers such as those of Hoff and Mautner, and Yusuff are based on a simplified assumption regarding the influence of the in-plane stiffness of the core.

In the differential equation approach, the boundary conditions are simplified by assuming that the core stiffness in the in-plane direction is infinity. This leads to in-plane strain \( \varepsilon_x = 0 \) in the whole core. In a similar way the strain energy approach assumes the in-plane displacement is zero and the in-plane stiffness can be ignored. This is similar to the differential equation approach. Vonach and Rammerstorfer [1] showed that the in-plane stiffness has a large effect on the wrinkling load and believed that it could account for some of the differences between analytical and experimental wrinkling stress. They developed equations that include the in-plane modulus.
\[ \sigma_{xz} = 0.85 \sqrt{E_f \left( \frac{X_4 - X_3 \mu_2}{\mu_2} \right) + \nu_{zc} \left( X_4 \mu_2 - X_3 \mu_1 \right) \left[ \frac{E_x}{E_z} \right]^{-1} \left( \frac{E_z}{E_x} \right)^2 } \] (2-20)

Where

\[ X_3 = \frac{\mu_1^2 + \nu_{zc}^e \frac{D_{zc}}{D_x}}{\mu_1^2 - \mu_2^2} \quad \text{and} \quad X_4 = \frac{\mu_1^2 + \nu_{zc}^e \frac{D_{zc}}{D_x}}{\mu_1^2 - \mu_2^2} \]

\[ \mu_1 = \sqrt{\left( \frac{\sqrt{D_x^e D_x^c}}{D_{zc}} \right) + \left( \frac{\sqrt{D_z^e D_z^c}}{D_{zc}} \right)^2 - 1}, \quad \mu_2 = \sqrt{\left( \frac{\sqrt{D_y^e D_y^c}}{D_{zc}} \right) - \left( \frac{\sqrt{D_z^e D_z^c}}{D_{zc}} \right)^2 - 1} \]

and

\[ D_z^e = E_z^e, \quad D_x^e = \frac{2E_x^e G_{xc}^e}{(E_x^e - 2\nu_x^e G_{xc}^e)} \]

Wadsworth and Horrigan [20] extended Vonach and Rammerstorfer's model to include a layer of subsurface damage. They split the core into two distinct regions to allow degradation of core properties. The model takes into account shear traction between the face sheet and core. They showed that the effect of the core shear traction decreases as the degree of orthotropy increases in the core. Their work also suggested that including the in-plane modulus term would account for the differences between analytical and experimental wrinkling stress. Their conclusions are based on the unrestrained in-plane modulus value. Later in this chapter it will be shown that the in-plane modulus can gain significant stiffness from attached face sheets and the new value found using the restrained modulus should be used instead. When this is used the wrinkling stress increases back up to the classical models.

Three dimensional models

Subsequently, various authors [21], [22], [23], [24] have reviewed the problem of wrinkling in three dimensional sandwich plates using more complex numerical techniques, especially in their methods for treating the rigidities of the facings about the mid plane, material anistropopy, transverse and normal shear deformations and the face-to-core interface conditions.

Benson and Mayers [21] investigated general instability and face wrinkling through variational energy methods applied to a three dimensional case. The model analysed wrinkling simultaneously with overall buckling, with symmetric and anti-symmetric wrinkling modes being dealt with separately.

Hadi and Mathews [22] developed Benson and Mayer’s model further to solve the case of simultaneous symmetric and anti-symmetric wrinkling. They included orthotopic and anti-symmetric cross ply, as well as an orthotropic core. They compared their model directly to
experimental and analytical results of Webber et al. [25] and Peace and Webber [23]. They found good agreement for both sandwich columns and sandwich panels.

Kim and Hong [24], analysed the buckling of unbalanced anti-symmetric sandwich plates with an adhesive layer of finite stiffness. Their work considered bending, stretching, stretch-shearing, and bending coupling actions simultaneously using the Rayleigh Ritz method.

Peace and Webber [23] studied the overall buckling and facing wrinkling of sandwich panels with carbon fibre reinforced plastic facings and honeycomb cores subject to uniaxial compression. Their theoretical predictions were compared with experimental results.

Webber et al. [25] developed this theory further to include unbalanced laminate cross ply facings. They showed that the inclusion of an adhesive layer increased the wrinkling loads substantially. They modelled the adhesive layer as an additional laminate where Kim and Hong [24] had represented the bond line with correction factors at the face/core interface.

The theory was extended for flexural wrinkling of beams by Gutierrez and Webber [26] to include non-linear behaviour of the facings. Their model includes a tangent modulus in the face equations to account for loss of stiffness. A double integration scheme was used to calculate the wrinkling loads and the facing strains.

Aiello and Ombres [12] investigated local wrinkling by modelling compression faceplates as thin unsymmetrical laminates resting on, two parameter elastic foundations. These two parameters are independent of the core material and must be defined for every solution. Their model was based on first order shear deformation theory. Using this theory means that global and local buckling must be solved separately. This assumption also assumes that there is no interaction between the two faceplates.

Starlinger and Rammerstorfer [27] developed a three dimensional model which would calculate wrinkling loads on sandwich panels that are not loaded on their axes of orthotropy.

Zenkert [3] considered the effects of bi-harmonic loading and showed that the buckling loads in the $X$ and $Y$ directions were independent of each other. For design purposes, he suggests that the two dimensional model should be used and the smaller of the $X$ and $Y$ direction wrinkling loads be used as the limiting load.

When looking at the relevant merits of all models, it is interesting to note that all the models are based on a continuum core assumption and most are solved using linear buckling models. There is also a generally weak correlation between experimental results and the models, and most of the models are either two dimensional plane strain models or plane stress models similar to Hoff and Mautner’s [5].
Section 2.3 Revised analytical wrinkling models

Below are derivations of two linear wrinkling models. The first uses the strain energy approach and the second model uses a differential type approach to derive similar wrinkling expressions. The technique used in the second model can be adapted for use in a multi-core model or a model that represents impact damage. Both models use equilibrium conditions to define the governing wrinkling decay function. In contrast, other authors define these decay functions prior to solving their models. It is felt that these two models are the most representative simplified expressions for linear wrinkling loads, so are called the revised models. They will also form the foundation for future comparisons to the Finite Element expressions.

2.3.1 Hoff and Mautner’s derivation

It is assumed that the deformed shape of a face sheet will consist of a sinusoidal displacement, which linearly decays in the z direction. This is shown in Figure 2-1 and is given by the following Equation (2-21);

\[
w = \frac{W_c}{h} \sin \left( \frac{\pi x}{L} \right)
\]  

(2-21)

To find the critical buckling load it is assumed that the work done by the compressive force during the displacement of the face is equal to the strain energy stored in the face material and the core.

A general expression for the critical stress is found by equating the three components of strain energy and rearranging for the critical stress.

\[
\sigma_{cr} = \frac{E_c L^2}{\pi^2 t_f h} + \frac{h G_c}{3 t_f} + \frac{\pi^2 E_f}{12} \left( \frac{t_f}{L} \right)^2
\]

(2-22)
The critical stress in this equation depends on the parameters $L$ (1/2 wrinkle wavelength) and $h$ (depth of affected core). The actual values of these parameters are those which make the critical stress a minimum. An equation for the critical wavelength can be obtained by setting the partial derivatives of Equation (2-22) to zero with respect to the parameters $L, h$. The expression for the critical wavelength $\lambda$ is:

$$\lambda = 2L = 1.65t_f \sqrt{\frac{E_f^2}{E_c G_c}}$$  \hspace{1cm} (2-23)

where the natural wavelength $\lambda$ is twice the critical half wavelength $L$.

Substitution in the equation for critical stress yields

$$\sigma_{cr} = 0.91(E_f E_c G_c)^{\frac{1}{3}}$$  \hspace{1cm} (2-24)

### 2.3.2 Revised analytical wrinkling model: Strain energy approach using a derived hyperbolic decay function (Modified Hoff and Mautner)

This revised model is based on a similar methodology to Hoff and Mautner’s model, except it uses a derived exponential decay function to satisfy equilibrium conditions and the compatibility equations.

The assumed deflected shape of the face sheet has the form:

$$w_f(x, z) = W \sin \frac{\pi x}{L}$$  \hspace{1cm} (2-25)

The displacement of the core is assumed to be related to (2-25) so:

$$w_c(x, z) = Wf(z) \sin \frac{\pi x}{L}$$  \hspace{1cm} (2-26)

The function $f(z)$ is an unknown function which relates the face sheet displacement to the core. It is assumed that the wrinkle dampens out at depth $h$ below the core.

An equilibrium condition must be maintained in the vertical direction, so:

$$\frac{\partial \sigma_c}{\partial z} + \frac{\partial \tau_{cz}}{\partial x} = 0 \Rightarrow \frac{\partial^2 w}{\partial z^2} + \frac{G_c}{E_c} \frac{\partial^2 w}{\partial x^2} = 0$$  \hspace{1cm} (2-27)

The solution to this partial differential equation is

$$w(x, z) = W \left( \frac{\sinh \alpha z}{\sinh \alpha h} \right) \sin \left( \frac{\pi x}{L} \right) \text{ where } \alpha = \frac{\pi}{L} \sqrt{\frac{G_c}{E_c}}$$  \hspace{1cm} (2-28)

Using this deflected shape, the strain energy in the core and face are found using the technique of Hoff and Mautner[5]. To find the critical buckling load, the sum of the strain energies in the core and face are equated to the work done by the applied load and the equation rearranged to give
By assuming that \( \cosh(\pi\alpha) = \sinh(\pi\alpha) \)

\[
P_{\text{crit}} = \frac{\pi^2 E_f t_f^3}{12L^2} + \frac{L\sqrt{E_f G_c}}{\pi} \frac{\cosh\pi\alpha}{\sinh\pi\alpha}
\]  
(2-29)

The above assumption is valid (less than 1% error) when \( \pi\alpha > 3 \) or when \( L < 15\text{mm} \). Values that fall within these ranges are common for most Nomex honeycomb panels. By differentiating \( P_{\text{crit}} \) with respect to \( L \), an expression for the critical wrinkling half wavelength is found.

\[
L = 1.73t_f \left( \frac{E_f^2}{E_c G_c} \right)^{\frac{1}{6}}
\]  
(2-31)

Substituting Equation (2-31) back into Equation (2-30) gives the minimum wrinkling stress

\[
\sigma_{\text{cr}} = 0.825(E_f/E_c G_c)^{\frac{1}{3}}
\]  
(2-32)

A similar expression is found using the differential equation approach. Instead of using Airy’s stress equations to develop the governing stress expression for the core, a similar approach is to derive a sinusoidal displacement function using equilibrium and compatibility equations. This function is then used to minimize the governing differential equation of elasticity.

### 2.3.3 Revised analytical wrinkling model: Differential equation approach using a derived exponential decay function

This approach is very similar to Allen [11] except it can deal with a finite thickness of core. This generic approach is used for solving wrinkling in undamaged and damaged sandwich panels (Chapter 6).

Combining the constraint \( \sigma_z = 0 \) (valid for honeycomb cores with low in-plane modulus) with the stress relations for a beam on an elastic foundation, gives the governing equilibrium equation for displacement in the \( Z \) direction:

\[
\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \frac{\partial}{\partial z} \left( E_z \frac{\partial w}{\partial z} \right) + \frac{\partial}{\partial x} \left[ G_{xc} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right] = 0
\]  
(2-33)

which simplifies to

\[
\frac{\partial^2 w}{\partial z^2} + \left( \frac{G_{xc}}{E_z - \nu_{za} G_{xc}} \right) \frac{\partial^2 w}{\partial x^2} = 0
\]  
(2-34)

The displacement function is:

\[
w(x, z) = Wf(z) \sin \left( \frac{\pi \alpha}{L} \right)
\]  
(2-35)

Where \( f(z) \) is an unknown decay function which is solved by satisfying Equation (2-34)
\[ W \sin \left( \frac{\pi x}{L} \right) \left[ \frac{\partial^2 f(z)}{\partial z^2} - \left( \frac{G_{sc}}{E_z - v_{sc} G_{sc}} \right) \left( \frac{\pi}{L} \right)^2 f(z) \right] = 0 \]  

(2-36)

The general solution for the above differential equation is

\[ w(x, z) = W \sin \left( \frac{\pi x}{L} \right) \left( Ae^{\alpha \left( \frac{\pi}{L} \right) z} + Be^{-\alpha \left( \frac{\pi}{L} \right) z} \right) \]  

(2-37)

where \( \alpha^2 = \frac{G_{sc}}{E_z - v_{sc} G_{sc}} \)

This generalised displacement (Equation (2-37)) function can be used to find wrinkling expressions for an undamaged and damaged core. Below is the summary of the undamaged core model. A similar approach is used to find an expression for a damaged core (Chapter 6).

**Case 1: No damage, infinitely deep core**

By assuming that the core is infinitely deep at the face sheet \( z = 0 \), \( w(x, 0) = W \sin \left( \frac{\pi x}{L} \right) \) \( \Rightarrow B = 1 \)

at \( z = \infty \), \( w(x, \infty) \Rightarrow 0 \) \( \Rightarrow A = 0 \)  

(2-38)

The governing displacement function becomes

\[ w(x, z) = W e^{\alpha \left( \frac{\pi}{L} \right) z} \sin \left( \frac{\pi x}{L} \right) \]  

(2-39)

Stress in the face sheet can be found:

\[ \sigma_z = WE_z - \alpha \left( \frac{\pi}{L} \right) e^{\alpha \left( \frac{\pi}{L} \right) z} \sin \left( \frac{\pi x}{L} \right) \]  

(2-40)
The equilibrium equation from a beam on an elastic foundation evaluated at the face sheet is

$$D \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} - \sigma_z = D \left( \frac{\pi}{L} \right)^4 - P \left( \frac{\pi}{L} \right)^2 + E \alpha \left( \frac{\pi}{L} \right) = 0$$  \hspace{1cm} (2-41)

Rearranging (2-39) in terms of $P$ gives

$$P = D \left( \frac{\pi}{L} \right)^2 + E \alpha \left( \frac{L}{\pi} \right) = 0$$  \hspace{1cm} (2-42)

and minimising in terms of $L$ gives the critical wrinkling half wavelength

$$L_{cr} = \frac{\pi}{2D} \left( \frac{E}{\alpha} \right)^{\frac{1}{3}}$$  \hspace{1cm} (2-43)

Substituting $L_{cr}$ into $P$ and dividing by $t_f$ the face sheet thickness gives the critical wrinkling stress

$$\sigma_{cr} = 0.8255 \left( \frac{E_f E_z G_{xy}}{1 - \nu_{xy} \left( \frac{G_{xy}}{E_z} \right)} \right)^{\frac{1}{3}}$$  \hspace{1cm} (2-44)

and $\sigma_{cr} \approx 0.8255 \left( E_f E_z G_{xy} \right)^{\frac{1}{3}}$ since $1 - \nu_{xy} \left( \frac{G_{xy}}{E_z} \right) \approx 1$ as $\nu_{xy} = 0$ for honeycomb cores.

The result as expected is comparable with the new modified Hoff and Mautner’s expression

$$\sigma_{cr} = 0.825 \left( E_f E_z G_{xy} \right)^{\frac{1}{3}}$$  \hspace{1cm} (2-45)

This compares well with Hoff and Mautner’s [5] expression of $\sigma_{cr} = 0.91 \left( E_f E_z G_{xy} \right)^{\frac{1}{3}}$ which is calculated using a linear decay function that defies equilibrium conditions.

The same expression was found by Yusuff [14] (Equation (2-17)) and Plantema [4] (Equation (2-9), using a different approach. Yusuff’s model uses a linear decay function. The 0.825 coefficient comes from two averaged expressions: one was based on an infinitely deep core and a calculated $W$ (wrinkle decay depth) and the other from an expression found when $W$ was equal to $\frac{t_c}{2}$.

Plantema’s model assumed that the core dampened out through an exponential decay.

Williams’s [16] anti-plane model also produces a similar wrinkling stress despite using different solution procedures. However, unlike the existing models, this new model can deal with a finite core depth and is used when subsurface damage is present (Chapter 6).
Section 2.4 Development of continuum and discrete Finite Element models

The large majority of research in the past two years has focused on the areas of impact resistance, fatigue and fracture analysis, which are the areas of greatest concern to the aerospace industry. In the past twenty years the emphasis has shifted from theoretical research to optimisation of laminates with Finite Element analysis being used as a design tool for panel analysis problems [3]. As a result the development of analytical sandwich models has declined, principally due to the difficulty in obtaining exact solutions. Finite element techniques utilise specially designed sandwich elements, which allow accurate analysis of sandwich design problems. In general these are more accurate than many existing analytical solutions, which require several approximations and in some cases the use of numerical methods to solve the differential equations.

Finite elements are particularly useful when modelling complex discrete cellular geometry such as honeycomb cores as they require fewer assumptions. Finite Element models can also offer greater insight into the failure mechanisms. With analytical modelling, in particular wrinkling, the buckling mode is assumed, while with Finite Element modelling the buckling mode is a result of the applied loads and geometry and not a pre-determined shape. Having said this, this research has used both numerical and analytical tools where possible. Analytical models are still useful in certain cases as they provide a simple means to check against different modes of failure such as wrinkling, and still assist in understanding the mechanics behind the failure mechanism.

The question asked in this current research is whether a continuum core can be used in place of complex discrete honeycomb cores to accurately capture localised failure modes such as wrinkling. Finite element models are used to answer this question as the detailed discrete models are too complex to solve analytically.

In the past, various authors have investigated discrete sandwich structures. Kim and Hong [24] and Zhang and Ashby [28], for example, have looked at ways to convert the discrete properties to continuum properties. (This is covered in Chapter 3 and Chapter 4.) These authors have used a combination of Finite Element and analytical models to achieve the conversion. Goswami and Becker [29] looked at debonding of face sheets from Nomex cores, through stress concentrations in a cellular Finite Element model. Burton and Noor [30] developed detailed discrete models to assess the free vibration response of an infinitely long sandwich structure. The discrete models were then compared to equivalent continuum models. Chamis et al. [9] directly compared discrete to continuum models. Their work investigated the difference between the thermal properties of the two models, using Finite Element analysis to find structural continuum properties.

Most of the existing discrete Finite Element models are simplified representations of the physical structure; for example they use plate elements instead of shell elements, ignoring the bending degrees of freedom (DOF). Others use a reduced number of elements and approximate boundary conditions to represent a larger structure. All the models use straight walls and model the joint of the walls with a sharp intersection.

Previous papers based around a continuum core assume that the core provides continuous support
to the facings. With continuum cores, orthotropic core properties are found by converting the cellular to continuum properties using numerical or analytical models, or by measuring them directly from experiments. Continuum properties are then used in either the analytical models or Finite Element models.

As already noted, one of the aims of this chapter is to compare the wrinkling stresses from a discrete cellular model to those from a continuum solid model. This is a relatively straightforward procedure that consists of developing a discrete core model and finding eigenvalue, then converting discrete cellular properties into orthotropic continuum properties through a single cell Finite Element model. The same values can be found using closed form analytical expressions. After establishing the continuum properties, a continuum model is built using converted properties. The eigenvalue from the continuum model is then compared directly to the eigenvalue from the discrete model.

**Figure 2-3 - Procedure for comparison of discrete and continuum wrinkling models**

### 2.4.1 Discrete Finite Element model

In this model the individual cells are modelled using shell elements for walls, faces and end plates. Shell elements are used to model structures where one dimension (the thickness) is much smaller than the other dimensions. As a result of this assumption the stresses transverse to the surface of the shell are assumed to be negligible. In general shell elements follow shell and plate theory covered in most basic mechanics texts. Shell elements offer six degrees of freedom at each corner node (both rotation and displacement), making them ideally suited for modelling three dimensional (3D) structures with thin walls.

All models are developed using the Ansys classic Finite Element environment and the Ansys Parametric Data Language (APDL).
Figure 2-4 and Figure 2-5 show a typical discrete cell Ansys model. The walls were constructed with map meshed quad elements and the top faces are a combination of quad and triangular free mesh elements. SHELL 181 elements, a linear four-noded element that allows large rotation and strain, composite lay-ups and support for a large number of material models elements, were used in the discrete model.

The core consists of a number of cells made from shell walls, and linked together to form a hexagonal shape. Attached to the top and bottom of the core is another shell wall that represents the face. At either end of the core are two more shell faces, used for constraints and loading surfaces. The face sheets and end walls are attached to the cells by coincident nodes. The panels are loaded with a pure bending moment applied to one of the end plates. The load is applied by means of a linearly graduating pressure load starting at a unit pressure of 1MPa at the top face sheet and finishing at -1MPa at the bottom face sheet. The resulting moment loads the bottom and top faces into pure tension and compression respectively. The end plates are 5mm thick shell walls and made of high modulus material, and are designed to remain stiff under the graduating pressure load and to distribute this evenly to the face sheets.

The cellular walls have different thickness depending on the location of the wall. The walls in line with the ribbon / X direction, are double the thickness of the angled walls (see Figure 2-4). The walls are assigned the modulus of the parent core material, for example the combined modulus of Nomex paper and phenolic resin, or aluminium.

The faces are assigned isotropic properties, as the fibre orientation has been omitted and it has been assumed that the lay-up is symmetrical in the principal orientations. The sandwich panel is also symmetric about its centre line; both faces are of the same thickness.

*Figure 2-4: Plan view of the cells showing double wall thicknesses in the line with the ribbon or X direction*
Symmetry constraints were used in discrete model to reduce the number of cells in the transverse direction. These constraints were applied to the faces down the sides of the panel. This type of constraint restricts movement in the transverse plane, and stops unrestrained cell faces, down the sides of the panel from bulging outwards (Figure 2-6). Symmetry constraints help to minimise the number of cells in the width direction by replicating the behaviour of central cells in an unconstrained larger model. By doing this, more cells can be added to in the ribbon direction to capture longer wrinkling wavelengths.

Table 2-1 compares two different models: one with symmetry constraints and the model without. From this table we can see that both models predict similar wrinkling stresses when there are at least 4 cells in the width direction. This is because the two inside cells are restrained by their neighbouring cells and have similar stiffness in this area of the model. In this case, the bulging of cell walls on the outer faces only have a small effect on the stiffness and wrinkling stress as the stiffness is dominated by the upright walls. When the number of cells in the width direction decreases to one, the model without symmetry constraints shows a completely different deformation pattern and wrinkling stress. In this configuration, none of the walls remain upright and all bulge outwards. This bulging or warping of the cells reduces the stiffness of the core, and therefore the wrinkling stress.
Figure 2-6 - Plan view comparing core deflections from a linear wrinkling analysis. The model on the left has symmetry constraints down its edge, while the model on the right is unconstrained. Symmetry conditions force the outside cells to have the same deformation as the inside cells. This stops the cell walls from bulging outwards, which reduces the core stiffness.

<table>
<thead>
<tr>
<th>Number of cells (ribbon direction)</th>
<th>Number of cells (width direction)</th>
<th>Symmetry constraint</th>
<th>Unconstrained model</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td>350.0</td>
<td>138.9</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>343.1</td>
<td>312.3</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>338.3</td>
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<td>1</td>
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</tr>
<tr>
<td>4</td>
<td>2</td>
<td>365.4</td>
<td>338.7</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>363.0</td>
<td>344.1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>361.9</td>
<td>347.7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>359.8</td>
<td>353.0</td>
</tr>
</tbody>
</table>

Table 2-1 - Table comparing wrinkling stresses in different sized discrete models, with symmetry and non-symmetry boundary conditions. When symmetry conditions are used, the results suggest that wider models can be replaced with narrower models. This is because symmetry constraints force the cells to behave in the same way, and have similar stiffness to the inner most cells in an unconstrained larger model.

At least eight cells were used in the ribbon direction to allow a complete wrinkle to form in the facings. The element density and the number of cells in the discrete core were verified in a convergence study. To compare discrete to continuum models directly, the same restraints and boundary conditions were used on the continuum model. All wrinkling models were solved using the eigenvalue solver.

Fillets were added at the intersection of the cellular walls, which is more representative of actual Nomex core (Figure 2-5). In the first iteration the junction was set with straight walls, the traditional and simplest way to model discrete honeycomb structures. Unfortunately, the limitations of such models became apparent when extracting eigenvalue buckling modes and loads.

When the straight intersection joint was used, the models tended to wrinkle adjacent to the joint
When fillets were integrated into the same model, the structure buckled in the top face sheet at approximately twice the stress of the straight-walled model, due to the relative stiffness of the joint. In straight walled intersections, the joint is geometrically stiff, so it resists bending of the cellular structure. The weakest area is a single wall around the joint which wrinkles. The correct buckling mode is only found when fillets are added to the corners. These fillets allow the intersection to bend and flex, thus allowing a wrinkle to form in the face. The buckling load that is captured in the straight-walled model is not the failure buckling load. When the cell walls buckle the panel still has residual stiffness and does not collapse until it loses complete stiffness, which is the case when the faces buckle. This can be shown in non-linear buckling models, with both types of models buckling at similar loads in the faces. Non-linear buckling solutions continue until the entire model becomes unstable and collapses.

This study is the first to consider wrinkling in discrete cores and to integrate fillets into the core. It was interesting to note that fillets only affect linear buckling loads but do not affect non-linear buckling loads.

Material/element type, cell size, fillet size, internal cellular angles, material properties, cell wall and face sheet thickness, core depth, number of cells in the core and mesh density can be adjusted through APDL files in the discrete model.

The majority of the models are set up with the following parameters (Table 2-2). These values were established in a convergence study which found the best combinations of cell numbers and element density to give the best results in terms of solution and solving time.

<table>
<thead>
<tr>
<th>Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells (ribbon direction)</td>
<td>8</td>
</tr>
<tr>
<td>Number of cells (width direction)</td>
<td>1</td>
</tr>
<tr>
<td>Mesh density (height)</td>
<td>15</td>
</tr>
<tr>
<td>Mesh density (cell wall length)</td>
<td>7</td>
</tr>
</tbody>
</table>

*Table 2-2 - Standard parameters used in the discrete model*
2.4.2 Continuum Finite Element model

The continuum panels were modelled with a combination of shell and solid elements. SHELL 181 elements were used for the faces and loading plates on each end of the panel. SOLID 186 elements, a 20-noded brick element with a parabolic basis function, was used to model the core. A reduced integration element formulation was used to avoid volumetric locking. Volumetric locking is an overly stiff response which occurs in fully integrated elements when the material behavior is nearly or fully incompressible (Poisson’s ratio approaches or equals 0.5). It is caused by spurious pressure stresses developing in the element, which cause the element to have an “over stiffness” for deformations that should not cause any volume change.

Shell elements were used for the face sheets because thin brick elements tend to be artificially stiff and overestimate the buckling load. This also duplicated the discrete model, so the only difference between the models lies in the core. All the other parameters were the same.

An orthotropic material model was used in the continuum core. Isotropic material models were used for the face sheet and the two end plates.

The continuum model was constrained and loaded in the same way as the discrete model, with a pure varying pressure load on one end and fixed constraint on the other. The varying pressure load puts the panel into pure bending by applying a pure moment to the end plate. Again, symmetry constraints were used on the side walls.

In addition to the solid continuum model, a plane stress continuum model was developed to predict the buckling load. This model was used in a regression study to produce an equation which was
based on numerical results.

Unlike the traditional analytical expressions, this Finite Element model includes the in-plane modulus and sandwich geometry (face/core thickness).

This regression model was constructed from around 2000 individual Finite Element runs. The stress is calculated using the following formula for all three Finite Element models:

\[
\sigma_f = \frac{P t_f}{6t_j}
\]  

(2-46)

Section 2.5 Regression Model / Non Dimensional Study

Equations were developed directly from the Finite Element models using a combination of a regression and a non-dimensional study, which groups variables into dimensionless numbers. This is a technique used to reduce the number of variables, and therefore number of Finite Element models run.

The Buckingham \( \Pi \) theorem forms the basis of these dimensionless studies. The Buckingham \( \Pi \) theorem proves that, in a physical problem including \( n \) quantities where there are \( m \) dimensions, the quantities can be rearranged into \( n-m \) independent dimensionless parameters. Streeter and Wylie [31] present a detailed summary of the Buckingham \( \Pi \) theorem and dimensional analyses.
### Table 2-3 - The Dimensional Numbers for the discrete model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_f$</td>
<td>Face sheet modulus</td>
<td>N/m² kg/m·s²²</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Core wall modulus</td>
<td>N/m² kg/m·s²²</td>
</tr>
<tr>
<td>$t_f$</td>
<td>Face thickness</td>
<td>m m</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Wall thickness</td>
<td>m m</td>
</tr>
<tr>
<td>$b$</td>
<td>Cell wall length</td>
<td>m m</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Cell angle</td>
<td>Rad Rad</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Sandwich panel thickness</td>
<td>m m</td>
</tr>
<tr>
<td>$\sigma_{cr}$</td>
<td>Wrinkling Stress</td>
<td>N/m² kg/m·s²²</td>
</tr>
</tbody>
</table>

Equation (2-49) is the non-dimensional equation that comes from taking each input back to its base units. For example $E_f$ = face modulus (units of stress $\frac{N}{m^2}$) reduces to $\frac{kg}{m·s^2} = \frac{kg}{m·s^2}$

If all the non-dimension terms are expressed in terms of wrinkling stress, the non-dimensional expression becomes

$$\frac{\sigma_{cr}}{E_f} = f\left[\left(\frac{t_c}{b}\right)^a \left(\frac{t_f}{b}\right)^d \left(\frac{t_s}{b}\right)^e \left(\frac{E_s}{E_f}\right)^{\frac{c}{2}}\right] \theta$$

(2-47)

To simplify the expression, theta, the internal angle of the core is removed from the expression. Theta is the only non-dimensional number that does not satisfy a power law relationship, so will cause problems during the regression analysis. Removing theta from the expression has limited effect as the majority of honeycomb cores have internal angles of around 60 degrees.

$$\frac{\sigma_{cr}}{E_f} = f\left[\left(\frac{t_c}{b}\right)^a \left(\frac{t_f}{b}\right)^d \left(\frac{t_s}{b}\right)^e \left(\frac{E_s}{E_f}\right)^{\frac{c}{2}}\right]$$

(2-48)

When the numerical data is fitted using a regression analysis, the expression will take the form of:

$$\frac{\sigma_{cr}}{E_f} = A \left(\frac{t_c}{b}\right)^b \left(\frac{t_f}{b}\right)^d \left(\frac{t_s}{b}\right)^e \left(\frac{E_s}{E_f}\right)^{\frac{c}{2}}$$

(2-49)

The non-dimensional parameters were verified by adjusting all six physical parameters, while keeping the non-dimensional numbers the same as the previous run. In theory, if the correct non-dimensional numbers were chosen and if their values were unchanged between runs, the answer should be the same regardless of physical parameters. For this exercise every number except $E_c$ and $E_f$ were changed.
Table 2-4 - Validation of the non-dimensional parameters

Table 2-4 shows that the non-dimensional numbers are correct, because regardless of which parameter is used we get the same Eigenvalue.

### 2.5.1 Discrete regression model

A regression analysis was used to find the power coefficients in the non-dimensional model using a specific program written and implemented in Matlab. This program takes the raw data, in this case the discrete Finite Element results and the non-dimensional parameters, and fits power curves to it. The end result is an expression similar to

\[
\sigma_{xz} = \frac{4.11}{E_f} \left( \frac{t_f}{b} \right)^{1.31} \left( \frac{t_s}{b} \right)^{0.63} \left( \frac{t_c}{b} \right)^{-1.08} \left( \frac{E_s}{E_f} \right)^{0.54}
\]  

(2-50)

This is a non-dimensional wrinkling expression which was developed for discrete geometry. The power coefficients are found using a regression analysis and about 300 individual Finite Element runs. The residual for the model was 99.3% indicating a very good fit to the numerical data.

This expression is based on a fillet ratio of 20%, which means that 40% of a single wall is fillets and 60% is a straight-walled section. It also assumes the honeycomb cells have an internal angle of 64 degrees, a common internal angle for Nomex type cores.

### 2.5.2 Plane stress continuum regression model

A similar expression can be found for solid continuum models. This expression uses a plane stress model and was created from about 2000 individual runs. As with the discrete regression model, the residual is as high as 99.7%, indicating an extremely good fit to the data.

\[
\sigma_{xz} = \frac{1.256}{E_f} \left( \frac{t_f}{t_f} \right)^{-0.170} \left( \frac{G_{xz}}{E_f} \right)^{0.143} \left( \frac{E_s}{E_f} \right)^{0.424} \left( \frac{E_c}{E_f} \right)^{0.068}
\]  

(2-51)

### 2.5.3 Existing model

\[
\sigma_{xz} = \frac{0.825}{E_f} \left( \frac{G_{xz}}{E_f} \right)^{0.33} \left( \frac{E_s}{E_f} \right)^{0.33}
\]  

(2-52)

Some distinct differences exist between these two expressions. The regression expression includes three additional terms: thickness of the sandwich, face sheet thickness and the in-plane modulus,
changing some of the power relationships, to balance the equation on both sides. Interestingly, the shear modulus is not as prevalent and the out-of-plane modulus is more dominant in the regression equation. Despite this, both models predict very similar wrinkling loads when the restrained in-plane modulus is used.

Section 2.6 Equivalent continuum honeycomb core properties

When comparing the buckling loads and modes in the discrete and continuum models, the properties must be converted from one form to another. In the discrete model, the properties required to construct a model are cell size, internal angles, wall thickness and wall modulus. In the solid model effective continuum properties are needed for the core, which are found through experimentation, or by converting discrete properties to continuum properties with analytical expressions or Finite Element models. As we are comparing two Finite Element models directly, the latter is the preferred option.

There are two types of properties required in the continuum models. Out-of-plane properties are those properties that act in the through thickness direction ($G_{xz}$, $G_{yz}$, $E_z$) while in-plane properties ($E_x$, $E_y$) act in-line with the sandwich panel. The out-of-plane properties are the most important to the functionality of the core, as they resist the shear stresses and out-of-plane loading. The in-plane properties are usually ignored completely as the core is considered to have negligible stiffness in the X and Y planes; all of the in-plane compression and tensile forces are carried by the faces.

This is a reasonable assumption for the majority of the failure modes and global deflections but is inadequate for predicting local wrinkling failure. When most authors have considered the in-plane modulus, they have considered it in a free state (when the core has no face sheets attached to each surface). Without face sheets the core is like a large spring and has about 1000th of the stiffness of the out-of-plane directions. For example, with 1/8-3.0 Nomex core the in-plane modulus value is approximately 200KPa. This compares to 132MPa ($E_z$), 40 MPa ($G_{xz}$) and 20MPa ($G_{yz}$).

When face sheets are attached to the top and bottom edges of the core, the core must follow the same deformation pattern as the face sheets. This means that the core is not free to expand and contract in the in-plane direction and the additional restraint of the face sheet increases the effective in-plane stiffness by a factor of 10. This is significant in terms of the wrinkling stress, as there is an asymptotic relationship between the wrinkling stress and the in-plane modulus [1]. With the correct in-plane modulus, the results tend towards the traditional wrinkling models outlined above. For common types of Nomex cores this in-plane modulus in the ribbon direction is approximately 3-5MPa. Out-of-plane and in-plane properties are covered in detail in chapters 3 and 4 respectively.

The continuum properties used in this study come directly from a unit cell finite element model. The model takes the discrete data and converts it through to continuum properties using reaction forces and the overall dimensions of the model.

Constraint equations were used to force the core into pure shear for the calculation of the shear
moduli and hold the sidewalls upright.

**Section 2.7 Comparison of discrete and continuum wrinkling models**

Seventeen different configurations, most of these based around standard Nomex cores with glass and carbon faces were used in this test. As the whole conversion process is done within the Finite Element package, no experimental continuum properties were used.

Figure 2-10, Table 2-5 and Table 2-6 show direct comparisons between the discrete and continuum wrinkling stresses for a range of panel configurations. There was a 2.3% average difference between the discrete and continuum models for the majority of the cases, proving there is no difference between the wrinkling stresses of the discrete and continuum model. This is a significant finding, as the two models have completely different element types. The only thing that unites them is the conversion of the properties within the conversion models. With this level of correlation, it is safe to use a continuum core to represent this failure mode. It has also been shown that the type of core chosen would not account for the difference between the experimental and wrinkling model results.

![Comparison between discrete and continuum models](image)

*Figure 2-10 - Comparison of wrinkling stresses for the two different model types*
### Table 2-5 - Shows the run numbers and the discrete data selected for each run. The properties are loosely based on common Nomex cores with glass or carbon face sheets

<table>
<thead>
<tr>
<th>Run Number</th>
<th>( b ) (mm)</th>
<th>( t_f ) (mm)</th>
<th>( t_c ) (mm)</th>
<th>( E_f ) (MPa)</th>
<th>( E_s ) (MPa)</th>
<th>( \theta ) angle (mm)</th>
<th>( \psi ) fillet ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.92</td>
<td>0.47</td>
<td>0.048</td>
<td>25500</td>
<td>3811</td>
<td>25.5</td>
<td>64.2</td>
</tr>
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<td>2</td>
<td>1.92</td>
<td>0.2</td>
<td>0.048</td>
<td>25500</td>
<td>3811</td>
<td>25.5</td>
<td>64.2</td>
</tr>
<tr>
<td>3</td>
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<td>0.6</td>
<td>0.048</td>
<td>25500</td>
<td>3811</td>
<td>25.5</td>
<td>64.2</td>
</tr>
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<td>4</td>
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<td>0.8</td>
<td>0.048</td>
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</tr>
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<td>0.048</td>
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<td>3811</td>
<td>25.5</td>
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</tr>
<tr>
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<td>64.2</td>
</tr>
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<td>25500</td>
<td>3811</td>
<td>25.5</td>
<td>64.2</td>
</tr>
<tr>
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<td>0.048</td>
<td>40000</td>
<td>3811</td>
<td>25.5</td>
<td>64.2</td>
</tr>
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<td>0.47</td>
<td>0.048</td>
<td>72000</td>
<td>3811</td>
<td>25.5</td>
<td>64.2</td>
</tr>
<tr>
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<td>0.47</td>
<td>0.048</td>
<td>48750</td>
<td>4406</td>
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<td>11</td>
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<td>0.47</td>
<td>0.048</td>
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<td>5000</td>
<td>25.5</td>
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</tr>
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<td>8000</td>
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</tr>
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<td>0.47</td>
<td>0.048</td>
<td>25500</td>
<td>3811</td>
<td>10.0</td>
<td>64.2</td>
</tr>
<tr>
<td>14</td>
<td>1.92</td>
<td>0.47</td>
<td>0.048</td>
<td>25500</td>
<td>3811</td>
<td>15.0</td>
<td>64.2</td>
</tr>
<tr>
<td>15</td>
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<td>0.47</td>
<td>0.048</td>
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<td>3811</td>
<td>30.0</td>
<td>64.2</td>
</tr>
<tr>
<td>16</td>
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<td>0.02</td>
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<td>3811</td>
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<td>64.2</td>
</tr>
<tr>
<td>17</td>
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<td>0.47</td>
<td>0.06</td>
<td>25500</td>
<td>3811</td>
<td>25.5</td>
<td>64.2</td>
</tr>
</tbody>
</table>
Table 2-6 - Table showing the calculated continuum properties from the Finite Element model and a comparison between discrete and continuum wrinkling stresses.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>$E_z$ (MPa)</th>
<th>$G_{xz}$ (MPa)</th>
<th>$G_{yz}$ (MPa)</th>
<th>$E_x$ (MPa)</th>
<th>$E_y$ (MPa)</th>
<th>Stress discrete (MPa)</th>
<th>Stress continuum (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>142.9</td>
<td>28.5</td>
<td>16.4</td>
<td>1.69</td>
<td>1.37</td>
<td>350.6</td>
<td>347.6</td>
</tr>
<tr>
<td>2</td>
<td>142.9</td>
<td>28.5</td>
<td>16.4</td>
<td>1.69</td>
<td>1.37</td>
<td>265.2</td>
<td>244.0</td>
</tr>
<tr>
<td>3</td>
<td>142.9</td>
<td>28.5</td>
<td>16.4</td>
<td>1.69</td>
<td>1.37</td>
<td>368.7</td>
<td>354.7</td>
</tr>
<tr>
<td>4</td>
<td>142.9</td>
<td>28.5</td>
<td>16.4</td>
<td>1.69</td>
<td>1.37</td>
<td>388.7</td>
<td>371.5</td>
</tr>
<tr>
<td>5</td>
<td>47.6</td>
<td>9.6</td>
<td>5.5</td>
<td>0.07</td>
<td>0.06</td>
<td>141.0</td>
<td>155.3</td>
</tr>
<tr>
<td>6</td>
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<td>14.3</td>
<td>8.3</td>
<td>0.22</td>
<td>0.19</td>
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<td>202.4</td>
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<td>7</td>
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<td>7.3</td>
<td>4.2</td>
<td>0.03</td>
<td>0.02</td>
<td>98.8</td>
<td>131.0</td>
</tr>
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<td>142.9</td>
<td>28.5</td>
<td>16.4</td>
<td>1.69</td>
<td>1.37</td>
<td>423.6</td>
<td>409.0</td>
</tr>
<tr>
<td>9</td>
<td>142.9</td>
<td>28.5</td>
<td>16.4</td>
<td>1.69</td>
<td>1.37</td>
<td>533.9</td>
<td>510.3</td>
</tr>
<tr>
<td>10</td>
<td>165.2</td>
<td>33.0</td>
<td>19.0</td>
<td>1.96</td>
<td>1.59</td>
<td>499.9</td>
<td>481.9</td>
</tr>
<tr>
<td>11</td>
<td>187.5</td>
<td>37.4</td>
<td>21.6</td>
<td>2.22</td>
<td>1.80</td>
<td>410.7</td>
<td>414.4</td>
</tr>
<tr>
<td>12</td>
<td>300.0</td>
<td>59.9</td>
<td>34.5</td>
<td>3.56</td>
<td>2.88</td>
<td>549.8</td>
<td>563.1</td>
</tr>
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<td>16.6</td>
<td>1.69</td>
<td>1.37</td>
<td>383.1</td>
<td>372.5</td>
</tr>
<tr>
<td>14</td>
<td>143.2</td>
<td>28.6</td>
<td>16.5</td>
<td>1.69</td>
<td>1.37</td>
<td>362.0</td>
<td>356.9</td>
</tr>
<tr>
<td>15</td>
<td>142.8</td>
<td>28.5</td>
<td>16.4</td>
<td>1.69</td>
<td>1.37</td>
<td>349.9</td>
<td>347.0</td>
</tr>
<tr>
<td>16</td>
<td>59.2</td>
<td>11.9</td>
<td>6.8</td>
<td>0.13</td>
<td>0.11</td>
<td>195.0</td>
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</tr>
<tr>
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<td>179.2</td>
<td>35.7</td>
<td>20.6</td>
<td>3.19</td>
<td>2.54</td>
<td>411.2</td>
<td>414.0</td>
</tr>
</tbody>
</table>

In models 2, 5, and 7 there was no direct correlation between the wrinkling stresses. In these cases a second failure mode appears, producing a difference as high as 33%. This failure mode is a combination of wrinkling and dimpling that the author has termed 'wrinkling/dimpling failure'. In this buckling mode the cell walls remain vertical and a localised dimple forms inside each cell at the trough of each wrinkle.

The only way to capture this failure mode is to model the discrete geometry, as the dimpling mode needs cells to form within. The reason for the wrinkling stress in the discrete model being lower is that the dimpling failure mode absorbs some of the energy, which is otherwise used to form the wrinkle in the face sheet and bend the cellular walls.

To get a wrinkling wavelength which is in the order of one cell width, the core must be much stiffer than the face sheet. In practical situations this failure mode is unlikely to occur as thin face sheets are never used on relatively dense cores, or cores with large cell sizes.

This dimpling and wrinkling failure modes are shown in Figure 2-11 and Figure 2-12.
2.7.1 Comparison of the wrinkling stress models: Finite Element and analytical

The purpose of this study was to compare the wrinkling stresses and buckling modes in discrete and continuum cored models.

To add to this study the two continuum Finite Element models were compared to existing analytical expressions: the “revised” analytical expression of

$$
\sigma_{cr} = 0.825 \left( \frac{E_f E_z G_{x\gamma}}{z f} \right)^{\frac{1}{2}}
$$

(2-53)

and Vonach and Rammerstorfer’s [1] model (Equation (2-20)).

As discussed in the text above, the in-plane modulus of the core material can have a significant effect on the wrinkling stress and is the one property that is omitted from many classical expressions. With the current analytical model the in-plane extension terms are excluded from the derivation. Excluding this term assumes that there is minimal deformation in the in-plane direction. To get the same effect in the Finite Element model, the in-plane stiffness must be large enough to stop any in-plane movement.

With the Finite Element and Vonach and Rammerstorfer’s [1] models the in-plane modulus is factored and is calculated with the face sheets attached to core.

Table 4 shows a comparison between the wrinkling stresses of the four different models.
Table 2-7 - Comparison of different wrinkling models

<table>
<thead>
<tr>
<th>$t_c$</th>
<th>$t_f$</th>
<th>$E_f$</th>
<th>$G_{xz}$</th>
<th>$E_x$</th>
<th>Wrinkling Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Revised analytical model</strong></td>
</tr>
<tr>
<td>25.53</td>
<td>0.47</td>
<td>25500</td>
<td>28.5</td>
<td>16.4</td>
<td>142.9</td>
</tr>
<tr>
<td>25.53</td>
<td>0.2</td>
<td>25500</td>
<td>28.5</td>
<td>16.4</td>
<td>142.9</td>
</tr>
<tr>
<td>25.53</td>
<td>0.6</td>
<td>25500</td>
<td>28.5</td>
<td>16.4</td>
<td>142.9</td>
</tr>
<tr>
<td>25.53</td>
<td>0.8</td>
<td>25500</td>
<td>28.5</td>
<td>16.4</td>
<td>142.9</td>
</tr>
<tr>
<td>25.53</td>
<td>0.47</td>
<td>25500</td>
<td>9.6</td>
<td>5.5</td>
<td>47.6</td>
</tr>
<tr>
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<td>8.3</td>
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<td>35.8</td>
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The plane stress Finite Element model predicts almost identical wrinkling stresses to the “revised” analytical model, except in the instance where the thickness/height ratio of the sandwich is reduced. This is because the analytical model assumes that there is no interaction between the top and the bottom faces. The most notable difference is between the two Finite Element models: the plane stress and 3D continuum models. The wrinkling stress of the 3D solid Finite Element model is approximately 15% lower than the plane stress Finite Element model, due to the different element formulations. There are two reasons for this. Plane stress solid elements are stiffer than three dimensional higher order elements, due to an additional degree of freedom in the depth direction. The face sheets are also made from shell elements in the solid model, which are more flexible than equivalent solid elements with the same thickness.

Rammerstorfer’s results were, on average, even lower than the solid continuum model, an interesting finding considering the model is based around a plane stress assumption, so it should be similar to the plane stress Finite Element model. Unlike the “revised model”, it makes allowance for deformation in the in-plane direction and assumes that this in-plane deformation dampens out at an infinitely deep depth. For a beam loaded in bending this is clearly not the case and could explain the difference.
From these findings it was concluded that the wrinkling stress in the modified analytical model closely matches the wrinkling stress in the plane stress Finite Element model when the correct restrained in-plane modulus was used.

It is likely that the stress values with the current analytical model and the plane stress model are too high, and the most accurate linear wrinkling stress can be found with the solid model. With the 3D solid model, there is no plane stress assumption and the result also closely matches the discrete Finite Element model, which is most representative of the linear buckling load. While this is the case, a reasonable estimate of the linear buckling stress can be found from any of the plane stress models including the continuum regression model, which is based on plane stress Finite Element results.

Unfortunately the linear stresses are not representative of the expected experimental values and over predict failure stresses. When non-linear models are used, the stresses are generally about 10-20% lower than these values. This type of model takes into account initial perturbations in the structure. The stresses are further reduced by premature fracturing and crushing of the core before the structure reaches the wrinkling stress. This is the actual cause of the error between the experimental and existing linear wrinkling models. The only way to accurately capture failure stress due to wrinkling is to adopt a non-linear scheme or account for core crushing in the linear analytical model (Chapter 5).

Section 2.8 Conclusion

Through this study it was found that a simplified continuum model predicts the same wrinkling stress as an equivalent discrete model. The agreement between the two models was quite
surprising considering the only properties that tie them together are those effective continuum properties extracted from discrete data. There was one case when the models predicted different wrinkling stresses. This is when the wrinkling wavelength approaches the cell size, in which case a discrete model is needed to capture an additional dimpling mode. The close agreement between the continuum and discrete models proves that any discrepancy between the experimental and analytical models could not be attributed to core representation. The significance of this finding means that continuum cores can be used to capture localised failure in discrete honeycomb panels, thus removing the need to model complex cellular geometry.

It was also shown that revised analytical model of

\[
\sigma_{xx} = 0.825 \left( \frac{E_f E_z G_{xz}}{\sigma_c} \right)^{1/3}
\]  

(2-54)

that excludes the in-plane modulus, offers a good estimation of the linear wrinkling stress. This is assuming that the restrained in-plane modulus (modulus calculated with face sheet attached) is used and the model is being compared with an equivalent plane stress model. For models which require the in-plane modulus, a good average value for Nomex cores is between 3 to 5MPA (Chapter 4). As an alternative to using the existing closed form expressions, the plane stress regression model could be used to predict an accurate liner wrinkling stress.
Chapter 3: Out-of-plane continuum properties of honeycomb cores

In this chapter, analytical expressions to convert discrete honeycomb properties into out-of-plane continuum properties are developed. These properties are used in the continuum wrinkling models. Unlike previous models, the new expressions accurately model honeycomb Nomex cores with fillets at the intersection of the walls. The equations are verified using Finite Element models. The equations are also tested against manufacturers’ data, using sets of experimentally determined discrete properties from three common types of Nomex cores.
Section 3.1 Introduction

Aluminium or Nomex honeycomb cores are used extensively in modern sandwich panels. With sandwich panels the out-of-plane shear and through thickness-Z direction loads are generally carried by the core whilst the in-plane (X-Y direction) compression / tensile loads are generally carried by the face sheets. This is because honeycomb cores are very flexible in the in-plane direction and stiff in the out-of-plane direction because thin walled sections being much stiffer in direct extension and compression than in bending.

A range of analytical models exist for determining the out-of-plane and in-plane properties of honeycomb structures. These models convert the complex cellular geometry into a set of effective continuum properties that can be used for most sandwich calculations. Two of the more notable authors are Ashby and Gibson [32], whose work is used for calculating failure strengths and continuum properties for most types of cellular structures. Continuum properties are also found through experimentation and supplied by manufacturers of honeycomb cores.

This current study examines out-of-plane properties in honeycomb cellular cores. It develops expressions to convert discrete properties of honeycomb cores. This is the first known study to verify these expressions against detailed Finite Element models and to include fillets at the wall intersections.

Modelling honeycomb with straight joints is a valid approximation for large cell aluminium cores, but less valid for Nomex cores, as fillets are present at the wall intersections. When comparing differences between continuum and discrete cores (Chapter 2), fillets were found to have a large influence on localised buckling loads and modes. Because of the geometry, the joints tended to be overly stiff and resist bending, giving an invalid buckling load without fillets. As fillets are present in Nomex and aluminium cellular cores, it is a logical step to include these in the conversion expressions.

This chapter also develops a full set of discrete properties (wall modulus, geometry) for three types of Nomex cores through experimentation. Discrete properties for aluminium and Nomex-type cores are difficult to obtain due to the irregularity of the material and the size of each individual cell and wall. To validate these established discrete properties, the cellular properties were converted to continuum properties by using the developed analytical models, and verified against manufacturers’ and in-house experimental continuum values.

Section 3.2 Review of out-of-plane models

Several models have been developed to calculate out-of-plane continuum properties based on discrete geometry. Zhang and Ashby [28] developed a set of simple mechanics equations to describe an idealised honeycomb structure as a continuum of two principal shear moduli, compressive modulus, crush strength and shear stress in the cellular walls.
Out-of-plane loading behaviour

Zhang and Ashby [28] describe the behaviour of honeycomb cores under out-of-plane loading as follows. If honeycomb cores are loaded in the out-of-plane / \( Z \) direction, the cell walls are put into tension, compression or shear. Out-of-plane compressive loading usually results in elastic or plastic buckling of the thin cell walls (see Figure 3-1 for cell orientations).

For elastic / plastic materials such as aluminium, the cell walls will buckle in compression and then yield, giving a long, almost flat plateau in the stress-strain curve.

For a brittle material such as Nomex, the deformation is dominated by bending of the cell walls until a critical failure or collapse stress is reached. As loading increases under a displacement-controlled experiment, the cell walls will buckle or crush and the load drop to a steady state crushing value. Within this region, the material will unload and load elastically with a reduced modulus. This behaviour is demonstrated in the flatwise compression experiments in Section 3.5.4.

Collapse strength

The compressive collapse strength, \( \sigma_c \), can be estimated from the ‘rule of mixtures’ honeycomb expression (equation (3-1)):

\[
\frac{\sigma_c}{\sigma_{sc}} = \frac{2}{(1 + \cos \theta) \sin \theta} \frac{t_s}{b} = \frac{\rho_c}{\rho_s}
\]  

(3-1)

It should be noted that the original cell notation has been modified to keep the existing expressions consistent with the models developed in this chapter. Figure 3-1 shows the cell notation used in the new and original models by other authors.
Out-of-plane modulus and shear strengths

Two of the most important properties of honeycomb core are the out-of-plane shear and compressive moduli. Zhang and Ashby [28] showed that the out-of-plane shear and compressive moduli of honeycombs are independent of the height of the cells. Honeycomb cores exhibit slight anisotropy in their out-of-plane shear strength and stiffness due to a set of doubled walls. These double thickness walls are created during the manufacturing process and are aligned in the ribbon direction. Information pertaining to these walls and the manufacturing process of Nomex honeycomb cores is found in appendix C.

By using simple mechanics models based on an array of regular hexagons, and by considering the double wall effect, approximate expressions for the continuum shear stress are derived. These continuum shear stress models are based on plates in pure shear or compression and are shown in equation (3-2).

\[
\frac{\tau_{x_2}}{E_{sc}} = 1.7 \left( \frac{\rho_s}{\rho_r} \right)^{\frac{1}{3}} \quad \text{and} \quad \frac{\tau_{y_2}}{E_{sc}} = 2.6 \left( \frac{\rho_s}{\rho_r} \right)^{\frac{1}{3}} \quad (3-2)
\]

In these models the honeycomb structures are considered to be an interconnected network of plates. Kelsey et al. [33] and Zhang and Ashby [28] developed a series of expressions, based on this theory, that predict the two principal shear moduli and the out-of-plane compression modulus. Their models assumed that the stress distribution is uniform through each wall, while in fact this stress distribution is unknown. Kelsey et al. [33] developed limits for the shear moduli using variable and constant stress distributions. The upper and lower bounds are based on potential energy and complementary potential energy methods. A unit displacement method gives the upper bound by applying a kinematically compatible strain field (variable stress field). A force method gives a lower bound using a statically compatible uniform stress field (constant stress field). The two transverse shear moduli are found by applying the following inequalities to the unit cell:

\[
\frac{1}{2} \frac{\tau_{x_2}^2 V}{G_{sc}} \leq \frac{1}{2} \sum_i \left( \frac{\tau_i^2}{G_i} V_i \right) , \quad (\alpha = x, y) \quad \text{Lower bound} \quad (3-3)
\]

\[
\frac{1}{2} G_{sc} \gamma_{x_2}^2 V \leq \frac{1}{2} \sum_i \left( G_i \gamma_i^2 V_i \right) , \quad (\alpha = x, y) \quad \text{Upper bound} \quad (3-4)
\]

Where \(\gamma_i\) and \(\tau_i\) are assumed uniform equilibrium strain and stress states in the \(i\)th walls of the representative cell segments, \(V_i\) is the volume of the \(i\)th wall, \(G_i\) is the shear modulus of the walls, and \(G_{sc}\), \(\tau_{sc}\), \(\gamma_{sc}\), \(V\) are shear stress, shear strain, shear modulus and volume of the respective replacement continuum volume. (See Noor [2] or Kelsey et al. [33] for a full description of this method.) In practice Kelsey et al. [33] suggests that the actual shear modulus should fall between these two values.

The following equations are presented in Ashby and Gibson [32], Noor [2] and Kelsey [33]:
Shear modulus in the ribbon / X direction

\[
\frac{1 + \sin(\theta)}{2\cos(\theta)} G_s \frac{t_s}{b} \leq G_{xz} \leq \frac{1 + \sin^2(\theta)}{(1 + \sin(\theta))\cos(\theta)} \frac{t_s}{b} G_s
\]  

(3-5)

where the lower and upper bounds for shearing in the ribbon direction from equation (3-5) are

\[
G_{xzLB} = \frac{1 + \sin(\theta)}{2\cos(\theta)} G_s \frac{t_s}{b} \quad \text{Lower bound}
\]

(3-6)

and

\[
G_{xzUB} = \frac{1 + \sin^2(\theta)}{(1 + \sin(\theta))\cos(\theta)} \frac{t_s}{b} G_s \quad \text{Upper bound}
\]

(3-7)

Shear modulus in the transverse / Y direction

The shear modulus in the Y direction is given by:

\[
G_{yz} = \frac{\cos(\theta)}{1 + \sin(\theta)} G_s \frac{t_s}{b}
\]

(3-8)

Compression modulus Z direction

The Young’s modulus in the out-of-plane Z direction is also given by a rule of mixtures expression.

\[
E_z = \frac{2}{(1 + \cos(\theta))\sin(\theta)} \frac{t_s}{b} E_s
\]

(3-9)

Poisson’s Ratio

The two out-of-plane Poisson’s ratios can be found from equation (3-10)

\[
v_{xz} = \frac{E_s}{E_z} v_s \approx 0 \quad v_{yz} = \frac{E_s}{E_z} v_s \approx 0
\]

(3-10)

This is for loading in the in-plane direction and movement in the Z (out-of-plane) direction.

For loading in the Z (out-of-plane) direction and movement in the in-plane direction, the Poisson’s ratio can be predicted from:

\[
v_{xz} = v_{yz} = v_s
\]

(3-11)

Alternative models

Grediac [34] showed that the core thickness affects the shear modulus \( G_{zx} \) by developing an equation which finds an average value of the modulus based on the thickness of the sandwich. This expression uses the upper and lower bounds of Kelsey et al. [33] and the sandwich height.

\[
G_{xz} = G_{xzLB} + \frac{0.78h}{b} (G_{xzUB} - G_{xzLB})
\]

(3-12)

Zenkert [3] presented approximate solutions to define the two out-of-plane shear moduli, \( G_{xz} \) and \( G_{yz} \) as
\[ G_{zz} = \frac{16t}{30D} G_s \text{ and } G_{xz} = \frac{4t}{3D} G_s \]  

(3-13)

where \( D \) is the average width across the flats of the cell.

Complete equations, along with full derivations, are found in Zhang and Ashby [28]. Noor [30] and Gibson and Ashby [32] also give a thorough overview of the literature on cellular materials and describe many experimental results for honeycomb panels.

**Experimental validation**

In some of these earlier works (Kelsey et al. [33] and Zhang and Ashby [28]) attempts were made to validate the models against experimental results. Kelsey et al. [33], for example, examined different ways to experimentally determine the shear modulus of aluminium foil honeycomb panels. They concluded that the experimental three point bending test gave the most satisfactory results, which are closest to the predictions from the analytical expressions. The values found in a three point bend test were generally lower than those from a single block shear test, due to expected bending of the plates. In the Kelsey et al. test, the plate ends were restrained against bending, whilst in the experiments described in this chapter, knife edges were cut into either end of the plate and used to restrain the plate ends. These knife edges are equivalent to a pin support and provide no rotational restraint. In contrast, Patras et al. [35] suggest that the block shear test yields the best result as it measures the shear modulus directly.

**Finite element validation**

More recently Finite Element models have been used to verify models. Meraghni and Benzeggagh [36], for example, used Finite Element models and analytical solutions to find the continuum properties and review the stresses within cellular cores. They found good correlation between both methods; however their models produced values which were significantly lower than the expressions presented above (equation (3-6) and (3-7)).

This chapter uses combinations of experimental and Finite Element models to validate the analytical expressions.

**Section 3.3 New out-of-plane continuum expressions**

This section develops new expressions for the two principal shear moduli, \( G_{xy} \) and \( G_{yz} \), and the out-of-plane modulus, \( E_z \) which includes fillets.

The following analytical models are based on similar assumptions to those of Kelsey et al. [33]:

The adhesive joint provides a perfect bond between the common faces and does not add to the stiffness of the sandwich.

The core is perfectly manufactured with all walls being identical in length, and symmetrical about the longitudinal and transverse centre lines.

The individual cell walls are only capable of carrying shear stress in the plane of each individual
wall. The shear stress is uniform, except for the fillets where it may vary. In the case of $E_z$ only vertical compressive loads are calculated.

The unit displacement method is used to derive the conversion equations. This approach was believed to be more representative of the physical test, as the core experiences a prescribed displacement during plate shear testing. Expressions were developed using strain-energy relationships, since the strain in each wall was a known constant. By using simple expressions for plates in shear, different strain energies are found for each wall. The strain energy in the fillet is found by integrating a strain energy expression for the angled wall in terms of $d\theta$. Each of these individual strain energy components are added to give the overall strain energy in the unit cell.

To find the continuum shear modulus, the discrete core is simply replaced by a solid block under the same shear strain as the sidewalls and has the equivalent strain energy of all of the walls combined. Solving this expression gives the shear modulus.

The general form of this derivation is shown in Equation (3-14) where the strain energy in a continuum volume is equated to the combined strain energy in the cellular walls.

$$\frac{1}{2} G_{xz} \gamma^{\nu} V = \frac{1}{2} \sum_i G_i \gamma_i^{\nu} V_i$$  \hspace{1cm} (3-14)
The models are based on a \( \frac{1}{4} \) section of a cell shown in Figure 3-2. All of the notation used in the following derivation is based on this model. The cellular wall is broken into three parts: straight walled section (Points A-B, E-F), curved walled section (Points B-C, D-E) and angled walled section (Points C-D).

### 3.3.1 Calculating the continuum shear modulus \( G_{xz} \)

The procedure used to calculate the shear modulus of \( G_{xz} \) is outlined below.

**Calculate strain energy in each of the walls**

The first step in this derivation is to find strain energy expressions for the three wall sections:

\[
U_{\text{straight}} = \frac{G_s \gamma \xi t_s L_t}{2}
\]

**Straight walled section**

\[
U_{\text{angle}} = \frac{1}{2} G_s \gamma \xi t_s L_t \left( \frac{1}{2} \cos(2\theta) + \frac{1}{2} \right) = \frac{1}{2} G_s \gamma \xi t_s L_t \cos^2(\theta)
\]

**Angled walled section**

To calculate the strain energy in the curved wall \( U_{\text{curved}} \), \( L \) is replaced by \( r \theta \) in the angled walled expression then integrated with respect to \( \theta \) to give:

\[
U_{\text{curved}} = \int \frac{1}{2} G_s \gamma \xi t_s r_t \left( \frac{1}{2} \cos(2\theta) + \frac{1}{2} \right) \theta \theta = \frac{1}{4} (\cos(\theta) \sin(\theta) + \theta) G_s \gamma \xi t_s r_t
\]

**Curved walled section**

**Calculate strain energy in an equivalent continuum block**

The next step is find an expression for strain energy in a continuum block:

\[
U_{\text{block}} = \frac{G_{xz} \gamma \xi}{2} (1 + \cos(\theta)) b^2 \sin(\theta) t_c
\]

**Equate the strain energy terms and solve for the continuum modulus**

The strain energy in an equivalent volume is now equated to the sum of the strain energies in each of the walls. This term is then rearranged and solved for \( G_{xz} \):

\[
U_{\text{block}} = 2U_{\text{curved}} + U_{\text{angle}} + U_{\text{straight}} \text{ or a } \frac{1}{4} \text{ section of core}
\]

Solving for \( G_{xz} \) gives:

\[
G_{xz} = G_t \frac{r \cos(\theta) \sin(\theta) + r \theta + L \cos^2(\theta) + L}{b^2 \sin(\theta) (1 + \cos(\theta))}
\]
3.3.2 Calculating the continuum shear modulus $G_{yz}$ and compression modulus $E_z$

Expressions for $G_{yz}$ and $E_z$ are found in a similar way.

The derivation of $G_{yz}$ is similar to $G_{xz}$ except

$$\frac{\cos(2\theta) + \frac{1}{2}}{2}$$

is replaced by

$$-\frac{\cos(2\theta) + \frac{1}{2}}{2}$$

in Equation (3-17) to give

$$G_{yz} = -G_{t_s} r \frac{\cos(\theta) \sin(\theta) - r \theta + L \cos^2(\theta) - L}{b^2 \sin(\theta)(1 + \cos(\theta))^2}$$

Equation (3-21)

$E_z$ is the simplest expression to find as it is based on plates in pure compression.

$$E_z = 2E_s t_s \frac{r \theta + L}{b^2 \sin(\theta)(1 + \cos(\theta))^2}$$

Equation (3-22)

Equations (3-20),(3-21),(3-22) are new expressions that predict out-of-plane continuum properties from discrete geometry. These models include fillets at the junctions of the walls.

Section 3.4 Finite Element Models

Finite element models were developed to validate the analytical expressions that convert discrete to continuum properties. Noor [2] built two discrete Finite Element models to examine the effect of cell numbers on the natural frequency and found that three dimensional solid elements and two dimensional plate shear elements gave similar results. In his study the edges of his panels were simply supported. When he compared discrete models against continuum models, he concluded that the size of the structure has a large influence on the free vibration natural frequency and showed that both continuum and discrete models produced similar vibration responses.

Credic [34] developed a Finite Element model to compare the difference between analytical conversion models and Finite Element results. He studied the shear stress distribution within the core, but this was deliberately omitted from the current study as this research is concerned with out-of-plane continuum modulus values and not the stress distribution. Credic’s Finite Element model consists of a small section of a unit cell which was half the overall height of the full core and contains half the inclined wall and straight wall (giving $\frac{1}{16}$th of the unit cell). This compares with the model used in this chapter which describes $\frac{1}{4}$ of the full unit cell. Credic showed that $G_{xz}$ has two limits and the true value depends on the ratio of core thickness to cell wall length. He concluded that the shear modulus in the $X$ direction decreased as the thickness of the sandwich increases, explained by the way the walls deform. In Credic’s model the deformation of the wall is a combination of rotation and shear. The model developed for this chapter shows a similar type of behaviour when the element’s rotational degrees of freedom are released.

The model used in this chapter consists of a repeating $\frac{1}{4}$ cellular unit. Ansys’s SHELL 181
elements were used to construct the model and find the two out-of-plane shear moduli $G_{xz}$ and $G_{yz}$ and the out-of-plane modulus $E_z$. To find the moduli the top and bottom faces were displaced by a prescribed amount and the reaction forces were found. These force values were combined, converted into a stress, and then divided by the strain to give the respective modulus value.

A relatively dense mesh was used, even though a convergence study showed that the mesh density had less than a 1% effect on the final modulus values over a full range of useable mesh densities. Single wall thicknesses were used throughout the model, as the double thickness walls in line with the ribbon direction are on the split line of the repeating unit cell. On the top and the bottom of the cellular walls are two sections of face sheets. The bottom face sheet was restrained and the top face sheet had a prescribed displacement.

Since shell elements were used, certain degrees of freedom were restrained. Shell elements will bend, extend and shear due to their three translational and three rotational degrees of freedom. To calculate the shear modulus, the bending degrees of freedom were restrained leading to pure shearing of the elements; this was achieved through the use of constraint equations. In this model each horizontal row of nodes was forced to have the same vertical displacement. Symmetry constraints were used on the straight side walls of the model, resulting in a section of core that could only displace sideways in pure shear (Figure 3-4, 3-5).

![Figure 3-3 - Discrete Finite Element model in pure shear for calculation of $G_{xz}$](image)
A similar approach was used to find the out-of-plane modulus $E_c$. For this model the face sheet was removed and pure vertical displacements were applied to the nodes. Constraint equations were added to the side walls to account for symmetry conditions. The face sheets were removed to allow the core to compress vertically and to eliminate local bending around the face areas, for compatibility with the analytical model. A test confirmed that this made approximately 1-2% difference in the result but was needed to check the validity of the analytical models.

### 3.4.1 Comparison between analytical and Finite Element model

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<th>$t_c$ (cell wall thickness)</th>
<th>$E_w$ (cell wall modulus)</th>
<th>$h$ (core depth)</th>
<th>$\theta$ (Internal angle)</th>
<th>$\psi$ (fillet ratio)</th>
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### Table 3-1 – Comparison of results between the Finite Element models and analytical model

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<th>$E_z$ (Analytical) (MPa)</th>
<th>$G_{xz}$ (Ansys) (MPa)</th>
<th>$G_{xz}$ (Analytical) (MPa)</th>
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</tbody>
</table>

The results of the Finite Element model and analytic results are within 0.04%; an excellent result considering the Finite Element model is a numerical approximation and the analytical models are based on plates in pure shear and compression. It is important to note the differences between Models 3 and 4: the Models are identical except Model 4 has no fillets at the wall intersection. The result from the last model can be predicted using the equations developed for this paper with a fillet ratio of zero, or using the classical models of Zhang and Ashby [28], Kelsey et al. [33] and Noor [2]. The results suggest that fillets can be excluded as the two shear modulus values are similar in both examples. From this comparison the only property affected by the fillet is the out-of-plane modulus, $E_z$.

### Section 3.5 Experiments

The aim of the research reported in this section is to find a set of properties that could be used to validate the analytical models. Finding continuum properties for various Nomex cores is a simple task as these are readily supplied by the manufacturers. In contrast, discrete properties are not as simple to find. Parameters such as cell wall thickness, cell wall lengths, internal angles and wall modulus are needed to model cores as a discrete structure. For Nomex cores the cells wall thickness and geometry can vary between adjoining cells. The effective modulus of the walls may also vary as this is dependent on the volume fraction of resin to Nomex paper. For these reasons, the values found in the preceding section are only approximate values and may change from cell to cell.

#### 3.5.1 Cellular geometry

Wall thickness and cell geometry are two of the more important properties in discrete cellular cores, but both of these can vary significantly from cell to cell. The objective of these experiments was to find some “nominal” values for three different types of cores. Random sections of core (cut from a larger section) were used to manufacture microscope test specimens. Each section of Nomex was set in a thermosetting resin within a vacuum chamber to ensure that the specimen was free of air bubbles. The surface of the specimen was then ground flat (to a few microns). By using a calibrated microscope, the thickness of each wall was measured. All thickness measurements were taken around the centre of each wall. Figure 3-5 shows magnified views of the cellular walls and
the test samples for 1/8”-3.0 cores.
Cellular dimensions were calculated from measurements across the diagonal single-thickness walls and measurements in the width direction across the double thickness walls. Measurements were taken using digital callipers, across a minimum of 20 cells to find an average dimension. With the standard cell layout, it was assumed that the diagonal and horizontal walls were the same length. Table 3-2 and Figure 3-6 are results from these experiments.
3.5.1.1 The specimen

![Figure 3-6 – Measurement key for discrete cellular geometry](image)

<table>
<thead>
<tr>
<th>Dimensions (mm)</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HRH-10-1/8-3.0</td>
</tr>
<tr>
<td>(b) (mm)</td>
<td>1.93</td>
</tr>
<tr>
<td>(\theta)</td>
<td>61.54</td>
</tr>
<tr>
<td>(e) (mm)</td>
<td>3.32</td>
</tr>
<tr>
<td>(\psi) (mm)</td>
<td>3.4</td>
</tr>
<tr>
<td>(\mu) (mm)</td>
<td>3.77</td>
</tr>
<tr>
<td>(t_s), single wall (mm)</td>
<td>0.0569</td>
</tr>
<tr>
<td>(t_s), double wall (mm)</td>
<td>0.108</td>
</tr>
</tbody>
</table>

*Table 3-2 – Measurements of three types of Nomex cores*

3.5.2 Cell wall modulus tests

Perhaps the hardest property to define in a Nomex cellular core is the modulus of the cellular walls. Manufacturers release information on the continuum properties of cellular cores, but there is limited information on Nomex cellular walls. The properties include the wall thickness, Nomex grade, resin type and wall strength and modulus. Because of their importance in a discrete model, precise values are needed.
The following section outlines three ways to find an approximate value for this cell wall modulus; only two of these were used. The simplest way is to reverse the conversion expressions of Ashby and Gibson [32], or those developed in this chapter. Instead of using the discrete core modulus and discrete cellular geometry to find the effective continuum properties, the discrete modulus can be found from the experimentally determined continuum modulus. To reduce the amount of error in this calculation, precise discrete properties (cell size, wall thickness, internal angles) are needed. Due to the irregularity of the structure it was felt that the two following methods would yield a better result.

The wall modulus can be found through the rule of mixtures if the constituent properties of the paper and resin are known. The overall modulus can be found by averaging the properties of the paper and resin.

The modulus can also be found through direct measurement using a tensile test or a flexural modulus test. Tensile tests have a number of problems associated with them, for example, finding a specimen large enough to place between jaws. Nomex type paper coated with resin also tends to be very brittle and can fracture in the jaws. A flexural modulus test is a better alternative to a tensile test in this application and is typically used in applications where the specimens are very small. This test is specifically designed to find the flexural properties of reinforced plastics including high modulus composites and electrical plastics.

### 3.5.2.1 Flexural modulus test

The flexural modulus test is based on ASTM D790-98 “Standard Test for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials” [37]. Instead of using a tensile specimen, the stiffness and strength properties are determined from a three point bending test. From the ASTM specification, the recommended specimen size and span width for material smaller than 1.6mm in thickness is 2” in length by ½” wide, with a 1” support span. The specimens were cut from a core with walls smaller than ½”, so the specimen sizes were different from the recommended values, a problem which was unavoidable.

A 100N load cell was used in the extensometer and the load was applied at a rate of 0.5mm/min.

The flexural modulus was calculated from:

\[
E_b = \frac{L^3 m}{4bd^3}
\]

where:

- \(L\) = support span, mm
- \(b\) = width of beam tested, mm
- \(d\) = depth of beam tested, mm
- \(m\) = slope of tangent to the initial straight-line portion of the load-deflection curve, N/mm of deflection.
### Specimen:

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$m$</th>
<th>$L$</th>
<th>$b$</th>
<th>$d$</th>
<th>Flexural modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.35</td>
<td>40.00</td>
<td>11.49</td>
<td>0.50</td>
<td>3933.94</td>
</tr>
<tr>
<td>2</td>
<td>0.31</td>
<td>40.00</td>
<td>11.26</td>
<td>0.50</td>
<td>3510.46</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>40.00</td>
<td>4.82</td>
<td>0.22</td>
<td>3989.74</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>3811.38</strong></td>
</tr>
</tbody>
</table>

| Table 3-3 – Flexural modulus Test Results |

There are four main issues with these measured values:

1. The value is sensitive to error in the thickness measurements of each specimen. If the thickness is varied by 0.02mm (approximately 4%) it can change the modulus by 400MPa (approximately 10%).
2. The thickness of paper and the volume fraction of resin used on the test specimens was different for larger sized cores than for the smaller cell cores used in the sandwich panels; a larger core was used to produce these test specimens.
3. The sample size of three specimens was small.
4. The dimensions of the samples were outside the recommendations of the ASTM standard.

Although problems exist with this type of test, this result still provides an indicative value for the Nomex / phenolic wall modulus.

![Figure 3-7 – Flexural Modulus Test using a section of wall from an oversize honeycomb core](image)

### 3.5.2.2 Rule of mixtures approach

The cell wall modulus can be found using the rule of mixtures combined with volume fractions given by Hexcel, the manufacture of Nomex core. The constituent properties of the Nomex paper and the phenolic resin were found using some indicative values from Matweb’s material database.

Contact was made with DuPont, the manufacturer of Nomex paper, to find information on the type of paper used in honeycomb Nomex cores.

They provided the following information:
- 1/8 3.0 uses 2 mil T412 Nomex paper
- 1/8 4.0 can be 2 or 3 mil T412 Nomex paper
- 3/16 3.0 can be 2 or 3 mil T412 Nomex paper

(Note: 1mil = 0.0254mm)

Hexcel typically use 2 mil paper for 1/8 3.0 and 3/16 3.0 cores and 3mil paper for 1/8 4.0.

The properties of T410 Nomex paper (similar to T412) were sourced from www.matweb.com. The modulus values are calculated directly from film tensile break strengths and elongation values at breaking. These are listed in Table 3-4:

<table>
<thead>
<tr>
<th>T410</th>
<th>2.0</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Thickness (mils)</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Density (g/cm³)</td>
<td>0.72</td>
<td>0.8</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.0508</td>
<td>0.0762</td>
</tr>
<tr>
<td>Tensile modulus - transverse direction (MPa)</td>
<td>537</td>
<td>508</td>
</tr>
<tr>
<td>Tensile modulus - machine direction (MPa)</td>
<td>776</td>
<td>751</td>
</tr>
<tr>
<td>Average tensile modulus (MPa)</td>
<td>656</td>
<td>629</td>
</tr>
</tbody>
</table>

Table 3-4 – Nomex paper properties supplied by Matweb

Some indicative values for resole phenolic resin were found from www.matweb.com

<table>
<thead>
<tr>
<th>Phenolic resin (electrical grade)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus (MPa)</td>
<td>8500</td>
</tr>
<tr>
<td>Density (g/cm³)</td>
<td>1.38</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.447</td>
</tr>
</tbody>
</table>

Table 3-5 - Phenolic Resin Properties supplied by Matweb for modulus, density and Poisson’s ratio

Using approximate volume fractions of resin and nomex paper supplied by Hexcel the following effective cell wall modulus was found. These values are listed in Table 3-6. It should be noted that the volume fractions specified were calculated for the whole core and may vary in individual walls. The modulus values for the core and resin were also estimated and may change slightly in practice. This volume fraction is based on the dry and wet weights of paper before and after the core is dipped in resin.
### Table 3-6 - Cell wall modulus found using rule of mixtures

<table>
<thead>
<tr>
<th>Modifier</th>
<th>Units</th>
<th>1/8&quot;-3.0</th>
<th>1/8&quot;-4.0</th>
<th>3/16-3.0</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume Fraction</td>
<td>%</td>
<td>33.3%</td>
<td>28.3%</td>
<td>33.3%</td>
<td></td>
</tr>
<tr>
<td>Nomex (2 mil)</td>
<td>MPa</td>
<td>656</td>
<td>629</td>
<td>656</td>
<td></td>
</tr>
<tr>
<td>Resol Phenolic Resin</td>
<td>MPa</td>
<td>8500</td>
<td>8500</td>
<td>8500</td>
<td></td>
</tr>
<tr>
<td>Overall Wall Modulus ($E_w$)</td>
<td>MPa</td>
<td>3268</td>
<td>2857</td>
<td>3268</td>
<td>3131</td>
</tr>
</tbody>
</table>

This rule of mixtures approach yields an average value of approximately 3150MPa which is in the range of the 3800MPa found using the flexural modulus test. It was thought that this calculated value contained less errors than the flexural modulus tests, and was therefore used to compare the numerical models to the physical results. In practice both properties will produce similar values when applied to the Finite Element and analytical model, as the modulus is directly proportional to the continuum values.

It was interesting to see that the flexural test and the rule-of-mixtures moduli, were very different to the value found by Zhang and Ashby [28]. Their experimental value based on tensile tests was 9000MPa compared to approximately 3150MPa found as reported in Table 3-6. This value of 9000MPa is closer to the modulus of resin by itself.

When 9000MPa is used in the conversion expression the resulting continuum modulus values are approximately double what they should be, suggesting that the values found in this work are more indicative of the actual wall modulus.

The Poisson’s ratio of the material is assumed to be 0.4 in both cases and the crush strength is taken as 60 MPa. This value of 60MPa was determined from the internal wall stress in the finite element model using the manufacturers’ crush strength value. The same value was found using the volume fractions of resin and Nomex paper and the tensile and flexural strengths of Nomex paper and resole resin respectively. The calculation for the cell wall strength is outlined in Table 3-7. This compares to a wall strength value of 80MPa listed by Zhang and Ashby [28], which came from a tensile test.
## Modulus Calculations for Core walls

<table>
<thead>
<tr>
<th></th>
<th>Units</th>
<th>1/8”-3.0 2mil Nomex</th>
<th>1/8”-4.0 3mil Nomex</th>
<th>3/16-3.0 2mil Nomex</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume Fraction</td>
<td></td>
<td>33.3%</td>
<td>28.3%</td>
<td>33.3%</td>
<td></td>
</tr>
<tr>
<td>Maximum tensile strength</td>
<td>MPa</td>
<td>70</td>
<td>83</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>(Nomex)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum tensile strength</td>
<td>MPa</td>
<td>32</td>
<td>40.6</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>(Nomex)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average tensile strength</td>
<td>MPa</td>
<td>51</td>
<td>62</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>(Nomex)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average flexural strength of resole phenolic resin</td>
<td>MPa</td>
<td>71</td>
<td>71</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>Overall strength of cell wall</td>
<td>MPa</td>
<td>58</td>
<td>64</td>
<td>58</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3-7 – Cell wall strength found using rule of mixtures

### 3.5.3 Shear Modulus Tests

The core has two main functions. The first is to keep the faces separated, thus increasing the flexural stiffness of the sandwich, and the second is to carry shear force in the structure. Shear stiffness is therefore one of the most critical values for sandwich honeycomb cores.

As a reference check of the manufacturer’s data, two different cores have been tested using a plate shear test; the number of test specimens was, however, limited to four due to a lack of available material. The HRH10-1/8”-3.0 core was tested once in both the transverse and the ribbon directions, whilst the HRH10-3/16”-3.0 was tested twice in the ribbon direction. This test was based on ASTM C273 [38] “Test for shear in sandwich structures”.

The specimens measured 55mm wide by 330mm in length and bonded to either face sheet was a rigid 12mm steel plate. The load was applied compressively to knife edges machined on the ends of the plates, with a load rate of 0.5mm/min. After 1.5mm of deflection the test was stopped, as this deflection produced a sufficiently linear load versus deflection curve to find the shear stiffness.

Although there were a limited number of specimens, the results for the HRH10-1/8-3.0 core were similar to the manufacturer’s data. In contrast, results for HRH10-3/16-3.0 were different to the manufacturer’s specification. The average value of the two specimens tested was 19.90 MPa, about half of the manufacturer’s value.

This discrepancy could be explained by different test methods. The tests in this research measure the shear modulus directly from a plate shear test, while other authors have suggested that Hexcel / Ciba use a flatwise bending test to obtain their values [35].
### Table 3-8 – Shear tests results: Show a large discrepancy between the manufacturer’s and experimental shear modulus value for HRH10-3/16-3.0 core

<table>
<thead>
<tr>
<th>Core</th>
<th>Orientation</th>
<th>Width (mm)</th>
<th>Length (mm)</th>
<th>Plate Shear Modulus (MPa)</th>
<th>Manufacturer’s modulus value (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH10-3/16-3.0</td>
<td>Ribbon (L direction)</td>
<td>55</td>
<td>330</td>
<td>19.75</td>
<td></td>
</tr>
<tr>
<td>HRH10-3/16-3.0</td>
<td>Ribbon (L direction)</td>
<td>55</td>
<td>330</td>
<td>20.04</td>
<td></td>
</tr>
<tr>
<td>HRH10-1/8-3.0</td>
<td>Width (W direction)</td>
<td>55</td>
<td>320</td>
<td>23.32</td>
<td>25</td>
</tr>
<tr>
<td>HRH10-1/8-3.0</td>
<td>Ribbon (L direction)</td>
<td>55</td>
<td>320</td>
<td>39.06</td>
<td>41</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>55</td>
<td>320</td>
<td>19.90</td>
<td>41</td>
</tr>
</tbody>
</table>

3.5.4 Stabilised compression tests

The stabilised compression test was used to determine the out-of-plane modulus, $E_z$, based upon a common ASTM test procedure, ASTM C 365 “Standard Test for Flatwise Compressive Strength of Sandwich Cores” [38]. Under this procedure a specimen must be less than $10000\text{mm}^2$ in area, (100mm per side), and greater than $645\text{mm}^2$ (25.4mm per side). The total number of specimens must not be less than 5. In the test, each core must be supported at the loading surfaces to prevent the top and bottom faces of the core from buckling locally and transferring the load through the centre of the core. In addition the loading pads are required to be self-aligning and have a crosshead speed of 0.1mm/min. Each of the specimens was square in plan view and cut out of the standard panels that were used for the wrinkling tests in Chapter 5. The panels were loaded until they reached crushing load.

The specimens were loaded between a set of self-aligning plates, to ensure that the load would be
applied evenly across the loading surfaces. Furthermore, all test specimens were square in plan and measured 55mm on each side.

Failure consisted of buckling of the walls in all specimens. The location of the buckling varied between successive specimens and was randomly distributed through the depth. Figure 3-10 and Figure 3-11 show the test apparatus.

<table>
<thead>
<tr>
<th>Specimen Numbers</th>
<th>HRH-10-1/8-3.0</th>
<th>HRH-10-1/8-4.0</th>
<th>HRH-10-3/16-3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment Average $E_z$ (MPa)</td>
<td>131.5</td>
<td>185.6</td>
<td>123.8</td>
</tr>
<tr>
<td>Manufacturer’s Spec $E_z$ (MPa)</td>
<td>137.8</td>
<td>193</td>
<td>137.8</td>
</tr>
</tbody>
</table>

Table 3-9 – Summary of out-of-plane experimental results for $E_z$

These results were similar to the manufacturer’s specification, suggesting that the test results are accurate.
Section 3.6: Comparison of experimental values to Finite Element /analytical models

### 3.5.5 Summary of discrete / continuum experimental values

<table>
<thead>
<tr>
<th>Core</th>
<th>$b$</th>
<th>$t_s$</th>
<th>$E_s$</th>
<th>$\theta$</th>
<th>$\Psi$ Fillet ratio</th>
<th>$G_{xz}$ (Hexcel)</th>
<th>$G_{yz}$ (Hexcel)</th>
<th>$E_z$ (Hexcel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH-10-1/8-3.0</td>
<td>1.915</td>
<td>0.056</td>
<td>3150</td>
<td>62</td>
<td>0.2</td>
<td>41.38</td>
<td>24.1</td>
<td>138</td>
</tr>
<tr>
<td>HRH-10-3/16-3.0</td>
<td>2.78</td>
<td>0.052</td>
<td>3150</td>
<td>65</td>
<td>0.2</td>
<td>44.83</td>
<td>23.44</td>
<td>138</td>
</tr>
<tr>
<td>HRH-10-1/8-4.0</td>
<td>2.13</td>
<td>0.086</td>
<td>3150</td>
<td>62</td>
<td>0.2</td>
<td>59.31</td>
<td>32.4</td>
<td>193</td>
</tr>
</tbody>
</table>

*Table 3-10 – Summary of out-of-plane experimental properties*

Table 3-10 shows a summary of discrete and continuum properties for three Nomex cores. Manufacturers’ continuum properties were used in place of the experimental values as these values are available in a design environment.

Section 3.6 Comparison of experimental values to Finite Element /analytical models

<table>
<thead>
<tr>
<th>Core</th>
<th>$b$</th>
<th>$t_s$</th>
<th>$E_s$</th>
<th>$\theta$</th>
<th>$\Psi$ Fillet ratio</th>
<th>$G_{xz}$ model</th>
<th>$G_{yz}$ model</th>
<th>$E_z$ model</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH-10-1/8-3.0</td>
<td>1.915</td>
<td>0.056</td>
<td>3150</td>
<td>62</td>
<td>0.2</td>
<td>31.8</td>
<td>17.48</td>
<td>136.8</td>
</tr>
<tr>
<td>HRH-10-3/16-3.0</td>
<td>2.87</td>
<td>0.052</td>
<td>3150</td>
<td>66</td>
<td>0.2</td>
<td>18.57</td>
<td>11.7</td>
<td>85</td>
</tr>
<tr>
<td>HRH-10-1/8-4.0</td>
<td>2.13</td>
<td>0.086</td>
<td>3150</td>
<td>62</td>
<td>0.2</td>
<td>43.32</td>
<td>24.29</td>
<td>189.6</td>
</tr>
</tbody>
</table>

*Table 3-11 – Discrete properties measured experientially*

<table>
<thead>
<tr>
<th>Core</th>
<th>$G_{xz}$ Exp., (Hexcel)</th>
<th>$G_{yz}$ Exp., (Hexcel)</th>
<th>$E_z$ Exp., (Hexcel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH-10-1/8-3.0</td>
<td>39.06 (41.38)</td>
<td>20.3 (24.14)</td>
<td>131.5 (138)</td>
</tr>
<tr>
<td>HRH-10-3/16-3.0</td>
<td>19.9 (44.83)</td>
<td>(23.44)</td>
<td>123.8 (138)</td>
</tr>
<tr>
<td>HRH-10-1/8-4.0</td>
<td>(59.31)</td>
<td>(32.4)</td>
<td>185.6 (193)</td>
</tr>
</tbody>
</table>

*Table 3-12 – Comparison between experimental and analytical continuum properties, using the measured discrete values*

In this study, the experimentally determined discrete properties (Table 3-11) are used in the conversion models and directly compared against the experimental values (Sections 3.5.3 and 3.5.4) and manufacturers’ data.
Table 3-12 shows the results of this comparison between the experimental results, manufacturers’ specifications and the analytical / Finite Element models. Manufacturers’ values, which are shown in brackets beside experimental results, are similar to the values found by the models/discrete experimental properties in most cases.

Nevertheless, Table 3-12 shows some differences between the experimental and analytical results. Some of the difference can be attributed to manufacturing tolerances, irregularities and determination of discrete properties. As previously described, finding accurate and average discrete information for Nomex cores is an almost impossible task, as cells are irregular in shape and the volume fraction of resin changes along the cellular walls.

With Nomex type cores, large clumps of resin tend to form at the intersection of the walls. These are not accounted for in either the analytical or the Finite Element models and could explain why the analytical models under-predict the shear modulus out-of-plane values. Figure 3-5 highlights resin accumulating in the junction of the walls. This stops the section of wall encapsulated by the pool of resin from shearing. It is plausible that the only section of core that shears is the section of wall that is clear of this clump of resin, which leads to a reduction in the effective shear area and an increase in the effective shear modulus. This is only applicable for $G_{xz}$ and $G_{yz}$ and not $E_z$. With $E_z$ all elements are loaded; however in some cases the loading distribution may be uneven, with more being carried at the junctions of the walls than the thin cell walls.

To investigate this explanation the effective cell length, $b$, was reduced by 20% (10% on either end) to account for the clumping of resin, an estimated value based on the microscopic views of the cells (done for $G_{xz}$ and $G_{yz}$ and not $E_z$). With these reduced lengths, analytical and experimental values are almost identical for $\frac{1}{8}\ "$ cells but the correlation was still poor for the $\frac{3}{16}\ "$ cell. The results from this study are shown in Table 3-13.

The large difference between the experimental result and manufacturers’ result of 3/16-3.0 core is an interesting discovery to come from this study. The manufacturer published a $G_{xz}$ value which was double that found experimentally and also by the analytical model, suggesting that there could be error in the manufacturers’ values.

This problem is not new; Petras [39] and other authors (Ashby [28], Noor [30]) have discovered similar problems. Petras found Hexcel / Ciba use a short beam test to measure shear values indirectly, even though they state on their current data sheet that they use a plate shear test. If these values were found using a beam flexural test, it is probable that there would be a difference in the results. Hexcel also use a 0.5" specimen while all tests in this paper were done on 1" specimens. While this should not affect the result, it cannot be excluded entirely without a more thorough investigation.

The other problem is that analytical and Finite Element models are free of imperfections, which can affect the load shearing between the walls and thus have a bearing on the final result. It is still unclear why there is such a difference and further work should be done to find out. However, based
on the convincing results of the other two \( \frac{1}{8}'' \) cores, it could be argued that the models predict the Nomex core properties with reasonable accuracy.

<table>
<thead>
<tr>
<th>Honeycomb models</th>
<th>( G_{xz} ) Model (MPa)</th>
<th>( G_{yz} ) model (MPa)</th>
<th>( E_z ) model (MPa)</th>
<th>( G_{xz} ) Exp.,(hexcel) (MPa)</th>
<th>( G_{yz} ) Exp.,(hexcel) (MPa)</th>
<th>( E_z ) Exp.,(hexcel) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH-10-1/8-3.0</td>
<td>39</td>
<td>21.86</td>
<td>136.8</td>
<td>39.1 (41.3)</td>
<td>20.3 (24.14)</td>
<td>131.5 (138)</td>
</tr>
<tr>
<td>HRH-10-3/16-3.0</td>
<td>23.22</td>
<td>14.69</td>
<td>85</td>
<td>19.9 (44.3)</td>
<td>(23.44)</td>
<td>123.8 (138)</td>
</tr>
<tr>
<td>HRH-10-1/8-4.0</td>
<td>54.16</td>
<td>30.37</td>
<td>189.6</td>
<td>(59.31)</td>
<td>(32.4)</td>
<td>185.6 (193)</td>
</tr>
</tbody>
</table>

*Table 3-13 – Comparison between experimental and analytical continuum properties, using a reduced wall length to allow for the resin clumping at the wall intersections*

**Section 3.7 Conclusions**

A new set of closed form analytical expressions were developed to convert discrete honeycomb properties into out-of-plane continuum properties. Each model was verified against an equivalent Finite Element shell model and includes fillets at the wall intersections. The equations are:

\[
G_{xz} = G_{xz} \frac{r \cos(\theta) \sin(\theta) + r \theta + L \cos^2(\theta) + L}{b^2 \sin(\theta)(1 + \cos(\theta))} \text{ Out-of-plane shear modulus in the X direction}
\]

\[
G_{yz} = -G_{yz} \frac{r \cos(\theta) \sin(\theta) - r \theta + L \cos^2(\theta) - L}{b^2 \sin(\theta)(1 + \cos(\theta))} \text{ Out-of-plane shear modulus in the Y direction}
\]

\[
E_z = 2E_z \frac{r \theta + L}{b^2 \sin(\theta)(1 + \cos(\theta))} \text{ Out-of-plane compressive modulus in the Z direction}
\]

The study successfully found discrete data for three common Nomex honeycomb cores (1/8”-3.0, 3/16-3.0, 1/8”-4.0) and used the analytical expressions to compare these properties directly to the manufacturers’ results and with continuum test data. It was shown that when the effective length of the cell wall was reduced, to take into account the pool of resin which forms in the junction, the degree of correlation was good for two of the three cores. With the larger cores, the manufacturers’ shear properties for approximately double the values found in both the analytical and numerical models and in-house experimental results. It is unclear why this is the case and more tests are needed to check the manufacturers’ experimental values.
Chapter 4: In-plane continuum properties of honeycomb cores

The previous chapter presented a review of out-of-plane honeycomb properties. Sets of new expressions were developed to convert discrete properties into continuum properties. These expressions were verified using 3D Finite Element models and experimental results. A set of discrete properties were also developed for three common types of cores. The current chapter continues this work by developing expressions for the in-plane modulus.

Traditionally, researchers have used an in-plane modulus value which ignored the face sheets and their interaction with the core (“free modulus”). When face sheets are attached, the core gains significant in-plane stiffness (typically 10x for Nomex cores). This work develops useable expressions for the “constrained modulus” and re-examines the existing free modulus expressions to include fillets. All values are verified against Finite Element models. The results show that when the correct in-plane modulus is used, there is a high degree of correlation between discrete and continuum linear wrinkling models and also between continuum Finite Element models and classical analytical models. In comparison, there was limited correlation with the free modulus models.
Section 4.1 Introduction

This chapter examines the in-plane modulus and its effect on wrinkling in sandwich structures. Traditionally, the in-plane modulus is calculated neglecting the influence of the face sheets, the 'free state' (unconstrained) assumption.

The primary function of the honeycomb core is to separate the face sheets and to provide out-of-plane stiffness to carry through-thickness compression and shear loads. The core also increases the overall in-plane stiffness of the structure (although less so than face sheets, so this contribution is, in most cases, ignored). However, with localised wrinkling, the in-plane stiffness support is in fact significant, as it prevents the core from bowing outwards as wrinkles form in the facings. With low in-plane core stiffness the panel will tend to wrinkle even at low loads. Without face sheets, a core behaves like a spring in extension and has no significant stiffness. With face sheets added, the effective in-plane stiffness increases significantly.

Where the face sheet is glued to the core, the cellular walls have the same deformation as the face sheet. This additional face restraint stops the core from acting like a spring and geometrically stiffens it. Because the surfaces of the core are restrained, the side walls must have the same displacement as the face sheet (shown in Figure 4-1), thereby altering the problem from one of pure bending, to a combination of extension and bending. This effect can be quite pronounced, for example, for small cell Nomex cores such as HRH10-1/8”-3, the in-plane modulus can change from a free state value of 0.15 MPa to the restrained state value of 3 MPa, an increase of approximately 20 times.

Figure 4-1 - Displacement plot from the Finite Element model showing the thickness effect (the effect that the face sheet and cell depth have on the in-plane deformation). The top and bottom of core has the same displacement as the face sheet, while the core towards the center of the cell is less restrained.

This chapter examines the in-plane modulus of honeycomb structures, and the effects of restraining the core with the face sheets and adding fillets at the junctions of the walls. In Chapter 2 and
Chapter 3 it was shown that fillets have a large effect on both the buckling mode and the continuum properties. The current chapter extends previous models that have only examined straight walls and no fillets, by adding fillets to the free-state in-plane expressions.

The first section of this chapter reviews the existing in-plane “free” models and models that include fillets at the intersection points of the walls. The second section develops models that include the additional face sheet restraint. The final section reviews wrinkling failure and the effect that the in-plane modulus has on this failure mode.

Section 4.2 Review of in-plane literature

Without face sheets the in-plane deformation of a cell is straight forward. When honeycomb cores are loaded in-plane, the cell walls bend, giving linear elastic behaviour. Bending will persist to strains as large as 10% (which is possible because the honeycomb behaves like a spring) and the geometry allows large distortion of the structure with only small strains in the members. After the 10% strain is reached the core begins to exhibit non-linear behaviour with the load / deflection curve increasing rapidly in a non-linear fashion.

Models by Ashby and Gibson

Ashby and Gibson [32], and Burton and Noor [30] presented similar expressions for the “free” in-plane modulus; their models are based on a combination of bending and extension terms.

Burton and Noor’s models

\[
E_{noor} = E_s \left( \sin(\theta) \left( \frac{\sin^2(\theta)}{(1 + \cos(\theta))(t_x/b)} + \frac{1 + \cos^2(\theta)}{(1 + \sin(\theta))(t_x/b)} \right) \right)^{-1} \tag{4-1}
\]

\[
E_{\nuoor} = E_s \left( \sin(\theta) \left( \frac{(1 + \cos(\theta))\cos^2(\theta)}{(t_x/b)^2} + \frac{\sin^2(\theta)}{(t_x/b)} \right) \right)^{-1} \tag{4-2}
\]

\[
G_{12noor} = E_s \left( \frac{t_x}{b} \right)^3 \frac{(1 + \cos(\theta))}{\frac{5}{4} \sin(\theta)} \tag{4-3}
\]
\[ v_{12\text{noor}} = -\cos(\theta) \begin{bmatrix} \frac{1}{(t_s/b)^3 + \left(\frac{t_s}{b}\right)} & \frac{1}{(t_s/b)^3}\cos(\theta) \\ \sin^2(\theta) & \cos^2(\theta) \end{bmatrix} \begin{bmatrix} \frac{1}{(t_s/b)^3(1 + \cos(\theta))} & \left(\frac{t_s}{b}\right)(1 + \sin(\theta)) \end{bmatrix} \] (4-4)

**Ashby and Gibson's models**

\[ E_{sash} = \frac{\sin(\theta)}{(1 + \sin(\theta))\cos^2(\theta)} \left(\frac{t_s}{b}\right)^3 E_s \] (4-5)

\[ E_{sash} = \frac{(1 + \cos(\theta))}{\sin^3(\theta)} \frac{(t_s/b)^3}{E_s} \] (4-6)

\[ v_{12\text{ash}} = \frac{(1 + \cos(\theta))\cos(\theta)}{\sin^2(\theta)} \] (4-7)

\[ v_{21\text{ash}} = \frac{\sin^2(\theta)}{(1 + \cos(\theta))\cos(\theta)} \] (4-8)

\[ G_{12\text{ash}} = \frac{(1 + \cos(\theta))}{17\sin(\theta)} \left(\frac{t_s}{b}\right) E_s \] (4-9)

A full list of the in-plane and out-of-plane properties can be found in Ashby and Gibson [32], including a full set of derivations for the in-plane property models. Most of these expressions were derived by balancing forces. The cell notation used by Ashby and Gibson [32], and Burton and Noor [30] in their models was adjusted slightly, to keep the terminology consistent with the models presented in this chapter. The only terms that differ slightly between Noor et al. [30] and Ashby et al. [32], is \( G_{12} \), the in-plane shear modulus. In practice, either model works as both expressions yield values close to zero.

**Modifications of Ashby and Gibson’s models**

Masters and Evans [40] developed expressions for the in-plane properties, including stretching, hinging and flexure terms, and representations of fillets. Their work is an extension of the Ashby and Gibson and Burton and Noor models, which are based around the flexure and stretching terms. While the concepts used in the Masters and Evans paper are similar to the expressions developed in the “free” modulus section (Section 4.3) of this chapter, the final results are very different. The models developed in this chapter revert to the expressions by Noor and Ashby and also perfectly match the Finite Element results, identifying probable flaws in Masters and Evans’s models.

**In-plane collapse strength and crushing predictions**

Other authors have examined collapse strengths and the effects of non-linear deformation on the
honeycomb cores under in-plane loading. Zhu and Mills [41] examined non-linear behaviour of honeycombs under in-plane compression with a range of core materials, considering both elastic and elastic-plastic responses. They also examined the different deformation and buckling mechanisms. This current investigation considers only small linear deformation so overall strains below 10%.

Kyriakides and Papka [42] investigated the mechanisms governing in-plane crushing of honeycomb, using full-scale numerical Finite Element models to examine the post-buckling behaviour of metallic honeycombs. They found that crushing begins in a small localised zone and rapidly extends to neighbouring cells. The load deflection curve is initially stiff and linear until it reaches the crush zone. They showed that the underlying crushing mechanism was the same in all specimens, even though the crush strength may vary.

Guo and Gibson [43] examined the elastic buckling strength and the plastic collapse strength of regular honeycombs with defects in a large deformation Finite Element study. They showed that single isolated defects in the structure reduce the collapse strength and the modulus.

**The “thickness effect”**

Becker has published many papers on homogenising discrete properties. In papers [44], [45] he developed a generalised method to convert random geometry into continuum properties. Becker’s work discusses the thickness effect: the effect of having the face sheet bonded to the surface of the core. By minimising the internal energy of the structure, he developed a closed form analytical model [10] that predicted the in-plane moduli for various core thicknesses. In this model he showed that as the core thickness decreases, the effective in-plane modulus increases as the restraint from the facings becomes more prominent.
Section 4.3 Free in-plane modulus model

This section develops expressions to predict free in-plane modulus values of $E_x$, $E_y$ and Poisson’s ratio $\nu_{xy}$. In Chapter 3, fillets were integrated into the existing discrete/continuum conversion expressions for out-of-plane properties. For completeness, fillets were included in free-state in-plane modulus expressions of Ashby and Gibson [32], and Burton and Noor [30].

In a free state (face sheets removed), the core is free to move perpendicular to the loading direction. As an example, the core will move in the transverse direction if loaded in the ribbon direction and vice versa. Figure 4-2 shows a model loaded in the $X$ (ribbon) direction and displacing in the $Y$ (transverse) direction through extension and bending of the walls.

Section 4.4 Calculation of the free modulus properties ($E_x$, $E_y$, $\nu_{xy}$)

The method used to calculate the free-state in-plane modulus in the $X$ and $Y$ directions and the Poisson’s ratio is outlined in brief below. For the complete derivation refer to the appendices.

The derivation of the two in-plane moduli ($E_x$ and $E_y$) and the Poisson’s ratio $\nu_{xy}$ were found using Castigliano’s Theorem, an approach based on energy methods.
Figure 4-3 is a diagram showing the basic construction and notations used in the model. The model consists of three main parts: straight wall, fillet / curved wall and an inclined wall, representing a $\frac{1}{4}$ section of a complete cell.

Figure 4-3 includes every force and displacement to find the range of properties in the free state models and the constrained central core models (Section 4.7). A brief outline of the generic procedure is given below.

Step 1: The bending and extension strain energy expressions are found for all three walls (inclined, straight, fillet), and added together.

Step 2: The energy expression is differentiated with respect to the applied load, to find the displacement at the location of the load. (Castigliano's Theorem)

Step 3: The displacement from the Castigliano Theorem is equated to the displacement at the loading point (from the applied stain); this expression is rearranged and solved for the unknown quantity, which is the force needed to displace the model by the prescribed amount.

Step 4: The continuum modulus is found by dividing the calculated stress by the maximum strain at...
the loading point (where the stress is force / area perpendicular to loading direction).

The only variation between calculations of $E_x$ and $E_y$ is the direction of load and displacements, and the size of the area used in the stress calculation in step 5. In the formulation, one of the forces ($W$ or $P$, Figure 4-3) is set and the other force is omitted from the calculation or set to zero depending on whether the modulus or Poisson’s ratio is being calculated. The Poisson’s ratio is found by dividing the applied strain in the one direction by the resultant transverse strain in the opposite direction. This Poisson’s strain is found by setting the force in the transverse direction to zero and finding the resultant displacement. Below are the two simplified expressions (Equations (4-10),(4-11)) for the “free” in-plane moduli. For the complete derivation, refer to appendix B

\[ E_x = -\left(1 + \cos(\theta)\right)E_s \left(\frac{t_x^2}{\left[36r^3 \cos(\theta) \sin(\theta) - 12r^2 \theta - 24r^2 L \sin^2(\theta) \right]} - \frac{24r^3 \cos^2(\theta) + 24r^2 \cos(\theta) L \sin(\theta) \theta}{-6rL^2 \sin^2(\theta) \theta - L^3 \sin^2(\theta) \theta} \right) \sin(\theta) \]  

\[ E_y = b \sin(\theta)E_s \left(\frac{t_x^2}{\left[-48r^3 \sin^2(\theta) - 24r^2 L \cos(\theta) + 24r^3 \sin^2(\theta) \theta + 36r^3 \sin(\theta) \cos(\theta) + 24r^2 \sin(\theta) L \cos(\theta) \theta + 12r^3 \theta + \right]} + \frac{24r^2 L \cos^2(\theta) + 6rL^2 \cos^2(\theta) \theta + L^3 \cos^2(\theta)}{b + b \cos(\theta)} \right) \]  

(4-11)

4.4.1 Verification of the “free” modulus expressions

Comparative tests were made between Finite Element models and the analytical expressions. To keep the models consistent, both types of models were developed using beam theory. The Finite Element model was created within Ansys, using a high order parabolic element (BEAM189).

Two different models sizes were created for this comparison study; a $\frac{1}{8}$ section of the cell and $\frac{1}{4}$ of the cell. The $\frac{1}{8}$ section mirrors the design of the analytical model. A study was undertaken to compare both sizes of models, and, as expected, both produced identical results, indicating that either could be used for this study.

<table>
<thead>
<tr>
<th>$b$ (mm)</th>
<th>$t_s$ (mm)</th>
<th>$E_x$ (MPa)</th>
<th>$\Psi$ Internal Angle</th>
<th>$\Psi$ Ratio of Fillet</th>
<th>$E_x$ Analytical (MPa)</th>
<th>$E_y$ Analytical (MPa)</th>
<th>$\nu_{xy}$ Analytical</th>
<th>$\nu_{xy}$ Ansys</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.07</td>
<td>5000</td>
<td>60</td>
<td>0.3</td>
<td>0.222</td>
<td>0.223</td>
<td>0.086</td>
<td>0.086</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>3800</td>
<td>30</td>
<td>0.2</td>
<td>3.492</td>
<td>3.504</td>
<td>0.060</td>
<td>0.060</td>
</tr>
<tr>
<td>2</td>
<td>0.055</td>
<td>3500</td>
<td>65</td>
<td>0.2</td>
<td>0.076</td>
<td>0.076</td>
<td>0.072</td>
<td>0.072</td>
</tr>
<tr>
<td>2</td>
<td>0.055</td>
<td>3500</td>
<td>65</td>
<td>0</td>
<td>0.139</td>
<td>0.139</td>
<td>0.258</td>
<td>0.259</td>
</tr>
</tbody>
</table>

*Table 4-1 – Comparison between the analytical “free model” and an equivalent Finite Element model*
The modulus was calculated using reaction forces, in which reaction forces were divided by half the overall length or width of the cell, depending on which modulus is calculated. This stress was subsequently divided by the strain to give an effective continuum modulus. The Poisson’s ratio was calculated by dividing the prescribed strain by the resultant strain perpendicular to loading. Fixed constraints and prescribed displacements were added at either end of the model (Points A and D), and constraint equations were used on the edge walls to enforce a symmetry condition with the neighbouring cell.

Table 4-1 shows a direct comparison between the analytical and the Finite Element models. This comparative study shows an almost perfect correlation between both models, suggesting that the analytical models are accurate based on an assumption of beams in bending and extension.

An interesting comparison occurs between the two final cell iterations, where the only difference is a fillet in the first of these two models. When the fillet length was set to zero the model developed in this chapter reduces to the expressions developed by Ashby and Gibson [32] and Noor and Burton [30], the classical formulas. When fillets are added it appears that the stiffness in all directions reduces significantly, and the Poisson’s ratio increases.

This is explained by the way the structure deforms. With fillets, most of the bending and extension will occur in the fillet. Because fillets are curved in shape they will naturally unravel, providing little resistance to this mode of deformation. Without the fillets the angled walls act like a cantilever beam, built in at one end and under a bending load at the opposite end. Instead of the fillet unwinding, the beam must bend to absorb the axial extension of the honeycomb structure. In the non-fillet model, more energy is required to achieve the same overall deformation, so the resulting modulus is higher.

Section 4.5 Comparison of the “free” in-plane modulus model to experiments

A set of tests were conducted to investigate how the physical core properties compare to the analytical models above.

The experimental values can be found using an adaptation of ASTM C 363 [46], which tests delamination strength of honeycomb core.

A total of seven specimens were cut from honeycomb. The last 30mm of each section was filled with EC3524 blue filler. This provides a solid section for mounting the specimen to the extensometer and applying tensile load to the remainder specimen. EC3524 is used typically to stabilise honeycomb in areas of impact damage (core crushing) or around the periphery of a panel where there is a transfer of load.

Each specimen was strained to about 10% of its original length before terminating the test. After 10% strain the specimen exhibits non-linear trends, with the stiffness increasing exponentially.

The modulus was calculated from the direct strain and the effective force in the cross section.
<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Orientation</th>
<th>Core</th>
<th>Gauge length (mm)</th>
<th>Crosshead Rate (mm/min)</th>
<th>Width (mm)</th>
<th>Young's Modulus (KPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Transverse $E_y$</td>
<td>HRH10-3/16-3.0</td>
<td>230</td>
<td>10</td>
<td>60</td>
<td>747.8</td>
</tr>
<tr>
<td>2</td>
<td>Transverse $E_y$</td>
<td>HRH10-3/16-3.0</td>
<td>240</td>
<td>10</td>
<td>70</td>
<td>743.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average 745.5</td>
</tr>
<tr>
<td>3</td>
<td>Ribbon $E_x$</td>
<td>HRH10-3/16-3.0</td>
<td>217</td>
<td>10</td>
<td>70</td>
<td>241.9</td>
</tr>
<tr>
<td>4</td>
<td>Ribbon $E_x$</td>
<td>HRH10-3/16-3.0</td>
<td>195</td>
<td>10</td>
<td>70</td>
<td>293.4</td>
</tr>
<tr>
<td>5</td>
<td>Ribbon $E_x$</td>
<td>HRH10-3/16-3.0</td>
<td>190</td>
<td>10</td>
<td>70</td>
<td>285.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average 273.8</td>
</tr>
<tr>
<td>6</td>
<td>Transverse $E_y$</td>
<td>HRH10-1/8-3.0</td>
<td>155</td>
<td>10</td>
<td>50</td>
<td>290.2</td>
</tr>
<tr>
<td>7</td>
<td>Ribbon $E_x$</td>
<td>HRH10-1/8-3.0</td>
<td>155</td>
<td>10</td>
<td>50</td>
<td>443.5</td>
</tr>
</tbody>
</table>

Table 4-2 – In-plane modulus test results

4.5.1 Results from the models

Materials properties for discrete structures are derived from Chapter 3. The experimental results are compared to free in-plane modulus equations developed in the preceding section.
### Table 4-3 – Discrete Properties – Modified for the in-plane modulus (0.1 fillet and 80% of the wall length to account for resin in the joints)

<table>
<thead>
<tr>
<th>Honeycomb models</th>
<th>$b$ (mm)</th>
<th>$t_s$ (mm)</th>
<th>$E_x$ (MPa)</th>
<th>$\theta$ (Internal Angle)</th>
<th>$\Psi$ (Ratio of Fillet)</th>
<th>$E_x$ Experiment (MPa)</th>
<th>$E_y$ Experiment (MPa)</th>
<th>$E_x$ Model (MPa)</th>
<th>$E_y$ Model (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH-10-1/8-3.0</td>
<td>1.532</td>
<td>0.056</td>
<td>3150</td>
<td>62</td>
<td>0</td>
<td>0.443</td>
<td>0.290</td>
<td>0.32</td>
<td>0.408</td>
</tr>
<tr>
<td>HRH-10-3/16-3.0</td>
<td>2.296</td>
<td>0.052</td>
<td>3150</td>
<td>66</td>
<td>0</td>
<td>0.273</td>
<td>0.745</td>
<td>0.067</td>
<td>0.143</td>
</tr>
</tbody>
</table>

#### 4.5.2 Discussion

Table 4-3 shows a high degree of variation between experimental results and equivalent results from the model. This may be due to the experimental method or the structure of honeycomb.

When the cores were stretched, the deformation in the central section was greater than in the area closest to the joints, indicating strain is significantly higher in the centre than nearer the anchoring points. The calculation for the average in-plane modulus was based on an average strain calculated from the total displacement between the jaws divided by specimen length. (To get an accurate modulus the strain in the centre of the core should be measured. Strain gauging the centre is complicated with a honeycomb structure.)

Another possible explanation relates to the pool of resin which accumulates in the corners of the walls. This simulates a stiff joint, stopping the fillets from unravelling and reduced the effective wall length. The calculations used to estimate the experimental values took this into account by reducing the effective wall length and removing fillets. However, large differences still exist in the results, especially with HRH10-3/16-3.0 core which caused some problems as described in chapter 3, and again has some unexplained behaviour here. The experimental modulus values are significantly higher in the HRH10-3/16-3.0 core than HRH10-1/8-3.0, despite the cores having the same density and the HRH10-1/8-3.0 having smaller cells, which in theory should be stiffer as smaller cells limit the amount of bending.

In conclusion, further work is needed to investigate the abnormalities observed in the experimental results. However as the majority of this thesis involves comparing analytical and numerical Finite Element models, it is those results that have been focused upon.

#### Section 4.6 Constrained modulus “Thickness Effect”

Becker’s paper [10] looked at the implications of the “thickness effect” and showed that cores gain significant stiffness when attached to facings. Because the core is glued to face sheets, top and bottom core surfaces must follow the same deformation pattern as the faces. Under in-plane-loading this will be pure extension or compression of walls. As we move towards the centre of the core, the face sheet restraint reduces and the core is free to bend and stretch in the angled walls,
and extend freely in the two straight double thickness walls. (see Figure 4-6 and Figure 4-7)

Finite element models were created to investigate the thickness effect. These models were constructed using SHELL 181 elements, and consist of a section of cell with face sheets attached at the top and bottom edges.

The core was loaded through a displacement applied to one end and a restraint on the opposite end. The two side walls of the core are constrained to have the same displacement as the face sheet, invoking a symmetry condition. It is this symmetry condition which provides most of the additional in-plane stiffness. Because these side walls are essentially fixed, the core is more resilient to strains in the in-plane direction, and the angled walls must now extend, as well as bend, for the core to have in-plane extension.

Figure 4-8 shows the in-plane modulus plotted against the thickness. This graph was plotted from results calculated from the Finite Element model and was based around the nodal reaction forces on the displaced edge. The effective in-plane modulus at each node is found by dividing the reaction force with the sliver of area between the neighbouring nodes. This is in turn divided by the overall strain to get the local modulus.

Figure 4-6 - Thickness effect with face sheets removed – shows reference points A and D

Figure 4-7 – Plan view showing the thickness effect – shows reference points A and D

Figure 4-8 shows two distinct areas: the constrained face region, a high modulus area where the core connects to the facings, and the central core region. In the constrained face region the face sheet follows the same strain pattern as the faces, with each wall being in pure extension.
As we move away from the faces towards the centre of the core, this restraint condition relaxes and the walls start to bend and twist out of plane, resulting in a reduced modulus, the "constrained central region". Between these two regions is a transition zone. The average modulus is found by summing the nodal forces and dividing by the overall thickness, or averaging the modulus values. It is important to mention that as the thickness of the core decreases, the effective in-plane modulus increases and tends to the constrained value at the core / face sheet interface. This is where the term "thickness effect" originates from.

Determining an experimental value for this modulus is almost impossible. The predominant reason for experimental error is that when face sheets are added, the overall in-plane stiffness of the core is drowned by the modulus of the face sheets, which are thousands of times stiffer. For this reason, all comparisons and verifications are made using the finite package and analytical models, which do not contain experimental error.

Figure 4-6 and Figure 4-7 show a complex deformation pattern, especially through the transition area where the modulus changes from the restrained-state to the free-state. Mathematically evaluating the in-plane modulus for all three deformation areas simultaneously is daunting. Becker [10] broke this problem down by first determining the strain energy in the constrained region, and then finding the strain energy in the free section. The overall restrained modulus was then
calculated by averaging the energy expressions through the transition area and minimising this overall energy expression.

The analytical models developed here differ slightly from Becker's work as his model is based on non-fillet geometry and excludes any bending terms in the derivation. Interestingly, bending is the most dominate mode of deformation in the unrestrained area. Unlike Becker’s model, the current model functions with any internal angle compared with Becker’s fixed 60°.

Instead of trying to minimise the energy expression, the overall modulus is calculated using an averaging function, which takes the constrained face modulus and the constrained central core modulus and computes an average value, based on a prescribed depth of the transition area.

The following two sections develop expressions for these two distinct areas, (constrained face and constrained central core moduli), followed by a section which examines averaging the two values to find the overall effective continuum modulus.

**Section 4.7 “Constrained central core” modulus model**

Figure 4-8 shows that the “constrained central core” modulus is approximately uniform through the entire depth of the core until it reaches the transition areas. Because of the uniformity of this region, the modulus can be modelled using beams and assuming that the value is the constraint through the affected depth.

Two main constraints must be enforced in the central core area. From Figure 4-9 we can deduce that the deformations at point A and D in the cell walls must be the same as in the face sheet, because the deformation at the top of the core (attached directly to the facesheet) is the same as the deformation in the centre of the core. Furthermore, the horizontal wall must have the same transverse displacement as the face sheets and because the model is symmetrical about points A and D, only 1/8th of the repeating unit is used in the analytical derivation.

The analytical derivation of this constrained central area is similar to the derivation of the free core. The main difference lies at point D; instead of this point having a prescribed displacement in one direction and being free in the other, both directions are forced to have the same displacement of the face sheet. This leads to an expression which must solve reaction forces $W$ and $P$ (see Figure 4-3) simultaneously while satisfying the given face sheet displacements.

Due to the sheer size of the expressions, this mathematical procedure has only been outlined in the appendix B, but is easily implemented on a symbolic mathematical package such as Mathcad or Mapel.
4.7.1 Comparison with Finite Element models

Two different sizes of Finite Element models were created to verify this analytical work. The model shown in Figure 4-10 represents ¼ of the overall cell and was developed using beam elements. A similar 1/8th cell model, matching the size of the analytical model, was also created, producing identical results to the ¼ cell model. In the Finite Element model, one end point is displaced by the strain of the face sheet, consisting of a pure displacement in one direction and Poisson’s displacement of the face sheet in the other direction.
Table 4-4 shows a direct comparison between the Finite Element and analytical models. For the 60° and 65° internal angle models, the percentage difference in modulus values is between 0.15% and 3.5%. With the 30° model this difference increases to 20.5%; this is of little concern, as standard cores have internal angles of approximately 65°, making 30° an outlying case. It was also noted that correlation was better in the free model than the current restrained model, as the deformation is in two directions and consists of a complex interaction of extension and bending strains.

<table>
<thead>
<tr>
<th>Run</th>
<th>b cell wall length (mm)</th>
<th>t cell wall thickness (mm)</th>
<th>( E_w ) cell wall modulus (MPa)</th>
<th>( \theta ) Internal Angle</th>
<th>( \frac{\nu}{\nu_{\text{fillet}}} )</th>
<th>( E_x ) Ansys model (MPa)</th>
<th>( E_y ) Ansys model (MPa)</th>
<th>( \nu_{xy} ) Analytical (MPa)</th>
<th>( \nu_{xy} ) Analytical (MPa)</th>
<th>( \nu_{xy} ) core = ( \nu_{xy} ) face</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.07</td>
<td>5000</td>
<td>60</td>
<td>0.3</td>
<td>3.62</td>
<td>3.75</td>
<td>0.91</td>
<td>0.94</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.05</td>
<td>3800</td>
<td>30</td>
<td>0.2</td>
<td>128.00</td>
<td>154.20</td>
<td>2.88</td>
<td>3.49</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.055</td>
<td>3500</td>
<td>65</td>
<td>0.2</td>
<td>1.59</td>
<td>1.64</td>
<td>1.51</td>
<td>1.56</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.055</td>
<td>3500</td>
<td>65</td>
<td>0.2</td>
<td>1.13</td>
<td>1.17</td>
<td>1.06</td>
<td>1.09</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.055</td>
<td>3500</td>
<td>65</td>
<td>0</td>
<td>13.61</td>
<td>13.59</td>
<td>33.44</td>
<td>33.39</td>
<td>0.3</td>
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</table>

Table 4-4 – Comparison between analytical “constrained central core” model and equivalent Finite Element model

The two final iterations (runs 4 & 5) directly compare a fillet and non-fillet model. It appears that the fillets have a significant influence on the effective modulus because when the fillets are removed the non-fillet modulus is up to 25 times greater than the fillet modulus. This difference is attributed to the shift from a bending dominated problem in the fillet model, to an extension dominated problem in the non-fillet case. The fillet model is also believed to be more accurate and representative of a physical core, as the joints in a physical core are not as rigid as in a model with fillets removed.

Section 4.8 “Constrained core/face sheet” modulus model

The effective modulus at the core/face sheet interface is easier to calculate than the constrained central core model (Section 4.7). At this interface the core is subject to pure extension and compression of cell walls as there is no bending of the cellular walls. The magnitude of the strains is controlled by the face sheets because the core is glued directly of the face sheet at this point. Energy methods were used to derive this model and this derivation is also outlined in detail in Appendix B.

4.8.1 Comparison of “constrained core/face sheet” modulus to the Finite Element model

To verify the constrained analytical model, a similar model was created in Ansys. The model consists of a shell plate, representing the face sheet and a set of link elements which form the cell walls. The link elements are attached to shell elements, such that any movement of the facings are superimposed on the cell walls. Link elements have been chosen over beam elements because they only have tension and compression degrees of freedom, whilst beam elements can carry bending
moments. In the derivation of the analytical model it was assumed that the walls carry only compression and tensile forces. All of the bending terms were excluded because these are negligible compared to the extension terms. When link elements are used no artificial forces and moments are transferred from the faces to the cellular walls.

Ansys’s SHELL 181 elements were used for the face sheet and LINK180 elements were used for the cellular walls.

<table>
<thead>
<tr>
<th>b</th>
<th>t</th>
<th>E_s</th>
<th>θ</th>
<th>Ψ</th>
<th>ν_x</th>
<th>E_x, Ansys model (MPa)</th>
<th>E_x, Analytical (MPa)</th>
<th>ν_x, Analytical (MPa)</th>
<th>ν_y, Analytical core = ν_y, face</th>
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</thead>
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<tr>
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<td>60</td>
<td>0.3</td>
<td>0.3</td>
<td>126.69</td>
<td>126.69</td>
<td>40.28</td>
<td>44.35</td>
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<td>136.69</td>
<td>135.75</td>
<td>50.27</td>
<td>51.19</td>
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<td>3811</td>
<td>65</td>
<td>0.2</td>
<td>0.3</td>
<td>74.13</td>
<td>74.60</td>
<td>40.90</td>
<td>37.85</td>
</tr>
<tr>
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<td>0.055</td>
<td>3800</td>
<td>40</td>
<td>0.2</td>
<td>0.3</td>
<td>251.90</td>
<td>253.30</td>
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<tr>
<td>2</td>
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<td>65</td>
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<td>0.3</td>
<td>68.17</td>
<td>68.47</td>
<td>34.81</td>
<td>37.58</td>
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<tr>
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<td>3500</td>
<td>65</td>
<td>0.2</td>
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<td>72.72</td>
<td>73.69</td>
<td>39.35</td>
<td>39.74</td>
</tr>
<tr>
<td>2</td>
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<td>3500</td>
<td>65</td>
<td>0</td>
<td>0</td>
<td>74.48</td>
<td>77.03</td>
<td>50.00</td>
<td>50.00</td>
</tr>
</tbody>
</table>

Table 4-5 – Comparison between analytical “constrained face/core” model and equivalent Finite Element model

Table 4-5 shows a high degree of correlation between the analytical and Finite Element models. This result validates the analytical models and proves that it can be used to predict the constrained core / face modulus.

**Section 4.9 Finding the effective modulus**

The following section investigates averaging the “constrained face/core” modulus with the “constrained central core” modulus to find an average value to calculate the overall constrained modulus. Becker [10], in his derivation of the constrained in-plane modulus, used an averaging function which attempted to replicate the changing modulus function shown in Figure 4-8. This
work employs a similar concept. 

The following function, $E_{\text{new}}$, defines the modulus at any point through the depth of the core. Lambda $(\lambda)$ represents a number which varies the severity of decay from the constrained modulus to the unconstrained modulus. The resulting curve is shown in Figure 4-12.

The function curve is defined by Becker [10] as:

$$ f(\xi) = 1 - \frac{\cosh(\lambda \xi)}{\cosh(\frac{\lambda t_c}{2})} $$

This function estimates the form of the transition between the face sheet and the central core areas.

Adapting the function to this model gives:

$$ E_{\text{new}} = E_{cf}(1 - f(\xi)) + f(\xi)E_{cc} $$

(4-13)

Where $E_{cf}$ is the constrained modulus (face sheet) and $E_{cc}$ is the constrained modulus (central core).

$$ E_{\text{new}} = (E_{cf} - E_{cc}) \left( \frac{\cosh(\lambda \xi)}{\cosh(\frac{\lambda t_c}{2})} \right) + E_{cc} $$

(4-14)

The average modulus $E_{\text{average}}$ is now calculated from equation (4-14) by finding the area under this function and averaging it. The dashed line in Figure 4-12 shows $E_{\text{average}}$

$$ E_{\text{average}} = \frac{2}{t_c} \int_0^h E_{\text{new}} \, d\xi $$

(4-15)

$$ E_{\text{average}} = \frac{1}{2} E_{cf} e^{\frac{1}{2} \lambda t_c} - \frac{1}{2} E_{cf} e^{-\frac{1}{2} \lambda t_c} - \frac{1}{2} E_{cc} e^{\frac{1}{2} \lambda t_c} + \frac{1}{2} E_{cc} e^{-\frac{1}{2} \lambda t_c} + \frac{1}{2} E_{cc} t_c \lambda \cosh \left( \frac{1}{2} \lambda t_c \right) $$

(4-16)
Figure 4-12 – Plot of the decay function of Ex through half the core depth. This starts at the constrained face/core value of 80 MPa and damps quickly to the constrained central core value of 5 MPa. The dashed line shows the average value of both constrained modulus values.

Table 4-6 – Calculation of the average constrained modulus and comparison to Finite Element shell model

Table 4-6 shows a comparison between the Finite Element model and the analytical models. A unit cell Finite Element model was used (the same as the shell model used in Chapter 2). The model consists of thin shell element walls. To find the value analytically, the constrained modulus from the different sections of the cell are inserted into Equation (2-45). The value is then adjusted using different values of lambda (\( \lambda \)). For Nomex type cores a value of 3 appears to give a good correlation with the Finite Element model. The two different models shown here are 1/8-3.0 cells, with and without fillets at the junction of the walls.
4.9.1 Significance of the in-plane modulus in terms of wrinkling

In Chapter 2 it was found that the in-plane modulus influences the wrinkling instability stress. This is possibly the only situation where this in-plane modulus plays a part in determining the final solution in thin gauge honeycomb sandwich panels. In wrinkling, unlike other failure modes, the waves attempt to deform the core sideways. The in-plane modulus is the only property that resists this mode of deformation. In traditional models this modulus is usually excluded from the solution by making the assumption that the stress or strain is zero in that direction. Vonach and Rammerstorfer[47] introduced an alternative solution which considered the effect of this modulus. Wadsworth et al. [20] extended their work to include a provision for a layer of subsurface damage.

Figure 4-13 shows the in-plane modulus versus the wrinkling stress for various wrinkling models. This figure shows a direct comparison between Ramerstorfer’s model, different Finite Element models and the revised Hoff and Mautner’s wrinkling expression (Chapter 2). The results plotted are based on a panel with a 1/8-3.0 Nomex core and glass facings (the classical model of Hoff and Mautner [5] assumes that there is negligible deformation in the in-plane direction).

It can be seen that for low values of the in-plane modulus (1MPa or less) there is significant reduction in the wrinkling stress.

In Chapters 2 and 3, the difference between the classical wrinkling formulas and some experimental results were discussed. Previous research often attributes this large discrepancy to imperfections and irregularities in the structure. Later authors such as Ramerstorfer et al. [1] and Wadsworth et al [20] suggest these differences are a result of excluding the in-plane modulus in the classical formulations.

Vonach and Rammerstorfer [1] and Wadsworth et al. [20] based their reasoning on the free modulus which is calculated without the face sheets. With a very small value of the in-plane modulus (in the order of 100Kpa), the wrinkling stresses in the Finite Element models and Vonach and Rammerstorfer [1] model tend towards the experimental failure loads (shown by the lowest horizontal line).

When the restrained value is reintroduced into the expression, the wrinkling stress begins to approach the classical formula of Hoff and Mautner [5] and the revised Hoff and Mautner equation (4-17), thus suggesting that their assumption of negligible in-plane deformation is reasonable in the case of linear wrinkling. The stress begins to approach the classical model because the deformation in the in-plane direction ceases as the value climbs above 2MPa. This deformation is clearly shown in Figure 4-14 and Figure 4-15.

\[
\sigma_{cc} = 0.825(E_y E_z G_{xz})^{1/3} \quad \text{(Chapter 2)} \quad (4-17)
\]

The work of Vonach and Rammerstorfer [1] also appears to underpredict the linear wrinkling stresses, as analysed in Chapter 2, which compared wrinkling stresses of the various numerical and analytical models with correct in-plane modulus values.
To demonstrate that the “thickness effect” exists, a final comparison was made between discrete and continuum cored wrinkling models. In Chapter 2 a compressive study was completed to directly compare the wrinkling stresses of the two core representations and the validity of using continuum cores to model this localised failure mode. To directly compare the models, the various continuum properties are needed, which are extracted from a discrete cell.

Results showed that when the restrained in-plane modulus is used in the continuum model the wrinkling stresses are approximately equal between the continuum and discrete models. Conversely, when the free in-plane modulus is used in the model there is no correlation between
the wrinkling stresses (illustrated in Figure 4-16).

With the discrete model, none of these effective properties are assumed. The response of the structure is solely dependent on the cell structure and the stiffness of the walls. Therefore, the only way that the discrete and continuum core models align is if there is some form of interaction occurring between the face sheet and core, restraining the deformation in the in-plane direction.

<table>
<thead>
<tr>
<th>Converted Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrinkling stress discrete model = 393.4 MPa</td>
</tr>
<tr>
<td>Wrinkling stress = 387.3 MPa</td>
</tr>
<tr>
<td>(Using 2.1 MPa EX &quot;Constrained central core model&quot;) Face sheets attached</td>
</tr>
<tr>
<td>Wrinkling stress = 327.6 MPa</td>
</tr>
<tr>
<td>(Using 0.2 MPa EX &quot;Free modulus model&quot;) No face sheets</td>
</tr>
</tbody>
</table>

**Figure 4-16 – Effect of in-plane modulus on discrete and continuum wrinkling models**

Results also show that using the averaged value of the constrained face and constrained central core modulus will give a reasonable level of correlation; however the best results are found when the “central constrained modulus” is used.

Figure 4-15 highlights that most of the deformation is occurring in the centre of the core, meaning that the area which is absorbing most of the in-plane energy is the “constrained central” region. For this reason the constrained central value is better to use when calculating failure than the averaged value for wrinkling. For all other modes of deformation an average value is better. However, the differences between the averaged value and the constrained modulus are minimal.
Section 4.10 Conclusions / Discussion

This chapter developed sets of expressions to convert discrete properties into continuum in-plane properties (properties in-line with the core in the X and Y directions). The study investigated the “thickness effect”, which is when the modulus is calculated with the faces attached to the core. This was the first study to develop usable expressions for a restrained in-plane modulus and verify these against linear Finite Element models.

Results of this study demonstrated that in-plane stiffness increases significantly when face sheets are added to the core. When face sheets are attached to the core the effective modulus changes through the depth of the core. At the core/face sheet interface the modulus is the highest. In this area the cellular walls are under pure extension and compression as they are forced to have the same strain pattern of the facings. As we move away from the faces the modulus drops significantly through a transition zone until it reaches a second region. This is usually between 2-3mm below the face sheets. This area is under a combination of bending and extension, as part of its deformation is controlled by the displacement of the face sheet. The other section is free to move. Expressions have been developed to predict the modulus in these two regions and an averaging function was created to average the two values and find an overall effective modulus.

It was shown that the in-plane modulus has a significant effect on the linear wrinkling stress, especially with low values of these properties. There is an exponential relationship between wrinkling stress and in-plane modulus. With low values of the modulus the wrinkling loads approach experimental loads, which can be as low as 50% of the predicted wrinkling stress of the classical wrinkling models. Vonach and Rammerstorfer [1] and Wadsworth et al. [20] suggest that this discontinuity between the experimental results and the classical models was a result of excluding these properties in the calculations. This was demonstrated to be incorrect. When the restrained modulus is used instead of the free modulus, the wrinkling stress from the continuum cored Finite Element models tends towards the analytical models.

The thickness effect was verified using the comparison between continuum and discrete analytical models. When the restrained in-plane modulus was used, there was an almost perfect correlation between two models. Conversely, when the free modulus was used with the continuum model, the wrinkling stress was significantly lower. This proved that the thickness effect does exist and needs to be accounted for when modelling wrinkling instability accurately.

For ease of use, the modulus calculated for the central section of the restrained core can be used in the majority of calculations because there is very little difference in the result between the overall averaged constrained modulus and the central area.

A set of analytical models were developed to estimate the in-plane modulus when the core is in a free state, (without face sheets attached). These models add to previous works by Ashby and Gibson [32], and Burton and Noor [30] by including fillets in the junctions of the walls. Fillets were found to have a significant effect on the modulus by decreasing the modulus by 75% (for the
values in the study).

There was poor agreement between the free modulus experimental tests and the calculated free modulus values. This was attributed to error in the experimental testing procedure.
Chapter 5: Predicting failure loads of undamaged sandwich honeycomb panels subject to bending / compression

This chapter describes a new method to accurately predict failure loads in undamaged sandwich panels subject to bending. The current method of predicting failure loads using linear wrinkling models is limited; there is a general lack of correlation between experimental and analytical / numerical wrinkling models. In some cases the linear wrinkling models have overestimated wrinkling failure loads in undamaged panels by 100%. Various authors have tried to account for this discrepancy by modifying their linear models to take into account imperfections or irregularities in the structure, or in some cases by using correction factors to get them to agree.

The new method is an extension of existing linear wrinkling analysis. Non-linear Finite Element wrinkling models have also been developed and used to find failure stresses in undamaged panels and to track the failure mode and determine the failure mechanism. Based on the Finite Element failure mechanism, a revised analytical expression was developed. These numerical and analytical models were then verified against four point bend tests for a range of panel configurations. Comparisons were also made with existing models. The results showed that the panels collapse due to localised core crushing as a consequence of face sheet wrinkling. By using correct core failure criteria an almost perfect correlation was found between experimental, numerical and analytical results.
Section 5.1 Introduction

Despite many theoretical and experimental attempts to predict failure due to face sheet wrinkling, the structural designer still lacks an adequate, analytical tool to handle the task [48, 49]. In almost all cases existing models over predict the wrinkling stresses.

The aim of this study was to accurately predict failure loads and to determine the failure mechanism causing them to fail below the wrinkling stress. Non-linear Finite Element models were used to track the growth of the wrinkle and examine stresses and strains in the core and face sheet for possible failure. This work extended Kassapoglou et al.’s [50] and Yussuff’s [7, 51] linear wrinkling models, which incorporate failure in the core and face sheet due to extreme stress and strain in the core. These models, along with a model developed using the same approach, were verified against Finite Element models and experimental loads.

5.1.1 Current Literature for damage failure of undamaged panels

Hoff and Mautner [5], as an example, completed a number of experiments and found a large discrepancy between their analytical and experimental results. They presented an expression to predict failure loads in sandwich panels due to bending:

$$\sigma_{cr} = 0.5(E_f E_z G_{xz})^{\frac{1}{3}}$$

This expression was modified from their original expression of $$\sigma_{cr} = 0.91(E_f E_z G_{xz})^{\frac{1}{3}}$$. The adjustment of the coefficient of 0.91 to 0.5 was based on experimental results.

This equation is widely used in the industry for predicting wrinkling failure in sandwich structures. (Hexcel [52], Zenkert [3]).

Poor theoretical and experimental correlation is generally attributed in manufacturing imperfections [53]. These small initial facesheet imperfections trigger a flatwise core or core-to-facesheet failure. Various authors have tried to modify some of the existing wrinkling expressions, in order to account for this poor theoretical-experimental correlation. These models account for such things as core crushing resulting from initial perturbations, facesheet failure, and adhesive joint failure before the panels reach the wrinkling instability loads.

Reducing core properties to account for differences between experimental and analytical results

Daniel and Abbot [54] conducted an experimental programme to determine the wrinkling loads of panels in pure compression, three and four point bending and cantilever loading. Beams were manufactured with carbon / epoxy facings, with aluminium honeycomb cores and closed cell PVC foam cores. They showed that core failure reduces core support and precipitates wrinkling at lower loads, suggesting that a reduced through thickness and shear modulus be used for the continuum properties to get a better approximation to the failure load.
Including initial waviness or imperfection

Other authors ([7],[6],[18],[55]) have tried to account for the poor correlation between theoretical and experimental models, by incorporating waviness or imperfections in the structure. Initial waviness was thought to occur during the manufacturing process. Yusuff [7], Wan [6], and two Forest Product Laboratory reports (FPL) by Norris et al. [18], [55], have included waviness parameters in their linear wrinkling derivations.

In the FPL models the amplitude of the initial waviness is the unknown parameter. Wan [6] suggested that the initial waviness amplitude is proportional to the square of the critical buckle wavelength and inversely proportional to the face sheet stress. The equations used by Norris et al. [18] are based on the amplitude being proportional to the core thickness, while Norris, Ericksen and Voss [55] present equations that assume that initial wave height is proportional to the wave length. The models by Norris et al. were developed under the presumption that the core fails in tension.

Under these conditions the shear modulus terms cancel, giving Equation (5-2)

$$P_{cr} = \frac{0.82E_f}{1 + \delta^0 \left( \frac{E_c}{\sigma_c t_c} \right)^2} \frac{E_c t_f}{E_f t_c} \left( \frac{E_c t_f}{E_f t_c} \right)_{\delta^0}$$  (5-2)

where

$$\delta^0 = \frac{K_e t, \sigma_c}{E_c}, \text{(Norris et al.)}$$  (5-3)

$$\delta^0 = \frac{K_e L_{cr}}{\pi}, \text{(Norris, Boller and Voss)}$$  (5-4)

$$\delta^0 = \frac{K_e L_{cr}}{\pi t_f}, \text{(Wan)}$$  (5-5)

$$L_{cr} = \pi \left( \frac{t_c D_{f}}{2E_c} \right)$$

$$K_e = \text{Experimentally determined constant}$$

The problem with the FPL models is that they require extensive destructive testing to determine the constant $K_e$. This constant is specific for each individual core and panel configuration, meaning its estimation can be a costly process.

Plantema [4] added an additional term into their wrinkling expressions to account for the waviness. He demonstrated that the effect was worst when the imperfection wavelength approached the wrinkling wavelength. Plantema [4] suggested that in practical terms, initial irregularities could account for around 80% of the original linear stress, which is lower than the 60% figure proposed by Hoff and Mautner in their reduced experimental equation. Plantema’s value of 80% is closer to the non-linear wrinkling stress of 90% shown in the preceding section. This non-linear stress takes into account waviness in the structure but does not account for facesheet or core failure before the non-linear wrinkling stress.
Models that include core crushing and tensile failure of the face sheet

Yusuff [7, 51] also developed a model with a provision for waviness. Instead of using the buckling equations proposed by Norris et al. [18], [55] (which were developed under the assumption of core tension failure and no core modulus), the face wrinkling equations by Yusuff simulate the core stress distribution and include the core shear modulus. His derivation was similar to the classical works by Hoff and Mautner [5], in that he considered the face layer as an infinite beam on an elastic foundation. In this model he assumes that the face has initial perturbation that takes the form of

$$w_0 = A_0 \sin \frac{\pi x}{L}$$

He then went on to solve the governing equations for a beam on an elastic foundation. The model considers face sheet and core fracture by examining strains in the sandwich due to buckling of the facings. This is possible in a linear solution, because the non-linear strains cancel out of the governing differential equation with perturbation added.

For core failure stress

$$\sigma_{cr} = \left( \frac{2}{3} \frac{E_f E_c}{t_f t_c} \right)^{1/2} \frac{1}{1 + \frac{2A_0}{\sigma_c t_c}}$$

for $t_{core} < 2h$ for thin core

$$\sigma_{cr} = \frac{0.96 \left( E_f E_c G_c \right)^{1/2}}{1 + \frac{E_f A_0}{h \sigma_c}}$$

for $t_{core} > 2h$ for thick cores

Where $h = 0.72 t_f \left( \frac{E_f E_c}{G_c} \right)^{1/3}$ is the zone of displacement of the core.

A similar expression is found for core shear stress

$$\tau_{cr} = \frac{0.96 \left( E_f E_c G_c \right)^{1/3}}{1 + \frac{\pi G_c A_0}{L_c \tau_c}}$$

Kassapoglou et al. [50], argued that if the wrinkle amplitude was large enough or the wrinkle wavelength was small enough, then it is probable that the panel could fail due to a failure mechanism other than wrinkling. This could be core crushing or face sheet bending as shown in the non-linear study.

Kassapoglou et al. [50] built on previous works, in particular Hoff and Mautner’s [5] and Yusuff’s [7, 51], by developing expressions that predict failure due to core crushing, shear failure, adhesive failure and face sheet bending. In the same way as Yusuff [7, 51], Kassapoglou’s modified
wringling model used an assumed initial amplitude to calculate the respective stresses. This type of approach gives more insight into stiffness and failure strengths of various components that affect the failure load of the sandwich. Thus, it facilitates material selection, gives guidelines for improved materials, and makes possible more efficient designs. They compared their models against experimental tests with a mixed degree of success. In some cases there was up to 30% difference between the predicted value and experimental value. This difference was attributed to limited material data for failure strength values, the multiplicity of the materials (three types of cores and face sheets lay-ups and different adhesives) and noise present in the waviness measurements. Despite some disagreement, as a whole they considered the agreement between experimental and theory to be good. For thick cores, the expressions to predict the four different failure modes are:

\[ \sigma_{cr} = \frac{\sigma_m}{1 + \frac{E_c A_o}{h \sigma_c}} \]  
Failure due to core in compression \hspace{1cm} (5-10)

\[ \sigma_{cr} = \frac{\sigma_m}{1 + \frac{E_c A_o}{h \sigma_t}} \]  
Failure due to tensile fracture of the core \hspace{1cm} (5-11)

\[ \sigma_{cr} = \frac{\sigma_m}{1 + \frac{\pi G_c A_o}{L_{cr} \tau_c}} \]  
Failure due to shearing of the core \hspace{1cm} (5-12)

\[ \sigma_{cr} = \frac{\sigma_m}{1 + \frac{\pi A_o}{L_{cr} \gamma_a}} \]  
Failure due to failure of the face / core interface adhesive \hspace{1cm} (5-13)

Where \( \sigma_m \) is the critical wrinkling stress, which is the same as Hoff and Mautner’s [5]. For \( t_c > 2h \) For thick cores

\[ \sigma_m = 0.91 \left( E_f, E_c, G_c \right)^{1/3} \]  
\hspace{1cm} (5-14)

\[ L_{cr} = 1.65 t_f \left( \frac{E_f^2}{E_f G_c} \right)^{1/6} \]  
\hspace{1cm} (5-15)

\[ h = 0.91 t_f \left( \frac{E_f E_c}{G_c} \right)^{1/3} \]  
\hspace{1cm} (5-16)

for \( t_c < 2h \) For thin core

\[ \sigma_m = \frac{0.817 \left( \frac{E_f E_c t_f}{t_c} \right)^{1/2} + 0.167 G_c t_{core}}{t_f} \]  
\hspace{1cm} (5-17)

\[ L_{cr} = 1.42 \left( \frac{E_f t_f^3 t_c}{E_c} \right)^{1/2} \]  
\hspace{1cm} (5-18)
\[ h = \frac{r_c}{2} \]

If known waviness data is not available they suggest a value of \( \frac{A_h}{t_f} = 0.1 \), which works out to be 0.05mm, based on the 0.5 mm face sheets used in this study. This also ties in perfectly with the observed waviness value of 0.05mm measured on the coordinate measuring machine in Section 5.2.1. for thin face sheet thicknesses.

The models of Kassapoglou et al. [50] and Yussuff [7, 51] are compared to experimental results and a new model is developed at the end of this chapter (Section 5.7).

**In-plane modulus effect on the low failure loads**

Wadsworth et al. [20], Vonach and Rammerstorfer [1] showed that the differences between experimental and analytical wrinkling loads were a result of the exclusion of the in-plane modulus from the existing classical models. The discussion in Chapter 4 has shown that the in-plane modulus will affect the wrinkling stress, but not to the extent suggested in papers [20] and [1]. When a constrained in-plane modulus value is used (which includes interaction with face sheets), as opposed to the free value (no face sheet interaction), the wrinkling stress tends towards the existing wrinkling models and not the experimental stress.

**Experimental observations**

Webber et al. [56] studied overall buckling and wrinkling in sandwich panels with carbon faces and Nomex cores. Their work combined theoretical predictions with experimental tests. Test panels were simply supported at either end and subjected to an end compressive load. During their tests they had difficulty identifying the failure mechanism because failure occurred almost instantaneously and catastrophically. They assumed that the failure load corresponded to the wrinkling stress. Strain gauges mounted on the surfaces of the panels indicated some form of local instability during loading and prior to failure. In most cases their theoretical models overpredicted the wrinkling stress, which is contrary the majority of other authors’ findings.

Norris et al. [18] tested hundreds of sandwich struts made from different combinations of face and core material. The authors observed four distinct models of failure during there tests.

1. Elastic wrinkling of facesheets at stresses below the proportional limit of the facesheet material
2. Core failure due to “initial imperfections in the facesheets”
3. Core failure at stresses above the proportional limit of the facesheet material
4. Compressive strength failure of the facesheets at stresses insufficient to cause wrinkling failure

Summaries of some of the studies of wrinkling work and the associated experiments have been compiled by Zenkert [3], Plantema [4] and Allen [11]. More recently Ley et al. [53] collated and summarised these studies in a NASA document entitled “Face sheet Wrinkling in Sandwich Construction”.

\[ 5-19 \]
Section 5.2 Determining material properties

Materials properties are taken from Hexcel’s manufacturer’s specification sheet for the different configurations of cores.

The face sheet thicknesses were measured off assembled face sheets. The in-plane core modulus $E_x$ and $E_y$ were fixed at 3MPa for all four types of Nomex cores used in this study. This is an average value for the “constrained” in-plane modulus of Nomex cores described in the previous chapter.

5.2.1 Finding accurate perturbation levels

Surface scans were taken of a panel to measure the flatness of the face sheet. A coordinate measuring machine (CMM) was used to digitise the surface and produce a surface plot. (See Figure 5-1). A large indentation in the centre of the plot is obvious because the surface scan was done on an impact damaged panel, but the results from the undamaged area are still applicable for this study. In this undamaged area the plot shows up to 0.05mm face sheet perturbation. This value was used as the maximum value by which to perturb the mesh in the following non-linear Finite Element models.

![Figure 5-1 – Surface scan of impact damaged panel showing displacements in mm on the key](image)

Section 5.3 Finite Element Models

This study used two different types of Finite Element models: a discrete core model and a continuum core model. The discrete model consisted of a core made with cellular geometry and the continuum model had a core modelled as an orthotropic solid block. Non-linear Finite Element models were used to capture and visualise the failure modes as these occurred, and determine the magnitude of the wrinkle and the resulting stresses in the structure.
5.3.1 Continuum Non-linear Finite Element model

The continuum structure used in this study was identical to the model developed for Chapter 2, and used to compare linear wrinkling stresses in a continuum core panel with a discrete cellular core panel.

The sandwich beams were modelled with SHELL 181 elements on the face sheets and loading plates on either end of the sandwich. The core was modelled with SOLID 186 reduced integration elements (to stop volumetric locking). Solid 186 is a 20-noded brick element with a parabolic basis function. Different types of elements were tested, but this combination gave the fastest solution time without compromising the quality of the solution. Shell elements were used for the face sheets because thin brick elements tend to be artificially stiff and make it likely that the buckling load will be overestimated. An orthotropic material model was used in the continuum core, and isotropic material models were used for the face sheet and the two end plates.

The continuum model was loaded with a pure varying pressure load on one end and fixed constraint on the other end. The varying pressure load puts the panel into pure bending by applying a moment to the end plate. Symmetry constraints were used on the side walls to stop the side walls from splaying outwards.

Plane stress models were also developed with PLANE 183 full integration elements, and compared to solid element continuum models. Results were found to be identical to three dimensional continuum models, meaning either model could be used.

5.3.2 Discrete model

The discrete model which was originally designed for Chapter 2 was extended to show that this theory was transferable between discrete and continuum core representations. This discrete model was developed from SHELL 181 elements, and consisted of a number of cells which were made from cell walls, positioned in a hexagonal pattern. The walls in line with the ribbon direction had double wall thickness. On each end of the panel were two walls which acted as the constraining wall and the load application wall. The face sheets were also made from shell plates. All of the walls were held together by coincident nodes and assigned an isotropic material property. (For more information on this model refer to Chapter 2.)

5.3.3 Determining failure loads in Finite Element models

In Finite Element analysis, buckling is dealt with as quasistatic phenomena either by finding the eigenvalue (load factor) or by following the large deflection solution using the non-linear incremental implicit scheme. The eigenvalue approach is particularly quick and simple, as it requires only two steps: a linear stress analysis, then eigenvalue extraction of the lowest eigenvalue. In a strictly mathematical sense, buckling is a singularity that comes in two forms: limit points and bifurcations, which simply means the deflected shape suddenly changes. As an example, a strut which is shrinking under an axial compressive load may suddenly start to bend. The shape that it takes at the critical point will determine whether the structure becomes unstable and rapidly
develops large deflections or will continue to load with moderate deflections. With linear buckling
the deflection in the transverse direction remains zero until the model has buckled (Figure 5-2).
While this approach can be quick there are some drawbacks. The buckling modes and loads that
are found may not necessarily match the collapse strength of the panels. The eigenvalue solver
could be anticipating earlier buckling modes that are not critical to the overall integrity of the
structure. Moreover, with eigenvalues, the absolute displacement values and the stresses that come
from the modal shapes predicted by the eigenvalue analysis are irrelevant.

To overcome some of these problems a non-linear incremental model was used (Figure 5-3). A
structure will often exhibit initial imperfections or irregularities and start to bend well before the
bifurcation point. Using a non-linear technique, the model can include features such as initial
imperfections and large-deflection response. From a non-linear analysis, accurate stresses and
deflections can be extracted from the solution; for this reason, a non-linear buckling analysis is
usually more accurate, and recommended for design or evaluation of actual structures.
Summary of two Types of Finite Element buckling analysis

Linear (Eigenvalue), Non-Linear Newton-Raphson approach

A bifurcation point is a point in load history where two branches of the solution are possible.

5.3.3.1 Non-linear buckling solution procedure

In a non-linear analysis a structure is perturbed to initiate failure. If the structure is free of imperfection, it will fail at a marginally higher load than the eigenvalue failure load. In reality no structure is free of imperfections and irregularities. The question is, how large are these...
imperfections and how do they influence the buckling load?

To insert imperfection into the Finite Element model, the first step is to run a linear eigenvalue buckling analysis. The second step is to morph some of the buckled shape into the Finite Element model, using a function in Ansys called “upgeom” to perturb each node by a given amount.

Once the buckling load has been established and perturbation has been added to the solution, the panel is loaded beyond the eigenvalue load. The load will keep increasing until the solution becomes unstable and fails to converge. For this type of run, the large deflection option is turned on. This option forces the model to update the stiffness matrix after each iteration, ensuring that the geometric stiffening effects are accounted for. Modified Newton-Raphson is used for the solution procedure.

Within this solution procedure, the Auto-time stepping option is activated. If the solution becomes unstable and fails to converge, Ansys will automatically decrease the load. It will then progressively increase and decrease the load until it finds the critical point where the solution becomes unstable. The load at this critical point is taken as the failure load.

An alternative to using Auto-time stepping is to use the Arc-length method. The Arc-length method follows post-buckling behaviour and will identify snap through and snap back behaviour. It can also find the peak of the load / deflection curve or the buckling load. It is slightly different from the conventional Newton-Raphson technique in that it tracks the solution using arcs instead of step increments. (Figure 5-4 and Figure 5-5)

The Ansys online documentation suggests that the Arc-length technique should be used to establish an estimate of the buckling load, and Auto-time stepping used to improve the estimate as it yields more accurate results. Personal experience has shown that the two methods produce similar results, but the Arc-length method takes longer.
5.3.4 Continuum model results

As expected when these panels are loaded up in the non-linear incremental scheme, wrinkles start to form at the beginning of the load step. As the panel approaches the wrinkling instability point, the waves increase rapidly in magnitude. The wrinkling instability load occurs at between 80% and 95% of the eigenvalue load, depending on the size of the imperfection.

When wrinkles form in the face sheet, they will cause the face sheet to bend, producing bending stress. Once this bending stress is added to the in-plane compressive stress, the overall stress can be high and in some cases exceed the fracture strength of the face material. The core is also being compressed as the face sheets wrinkle. At the troughs of each wrinkle these stresses can be large and can cause the core to crush locally.

Table 5-1 and Figure 5-6 are the results from a continuum non-linear Finite Element, which was based on a panel manufactured with HRH-10-1/8-3.0 core and BMS-79 two layer glass facings.

<table>
<thead>
<tr>
<th>HRH10-1/8-3.0-1&quot; core 2.3 MPa crush strength) &amp; 2 ply BMS-79 facings (517MPa fracture strength) (220MPa Experimental Failure Stress)</th>
<th></th>
<th></th>
<th></th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied face sheet &quot;Wrinkling&quot; stress (MPa)</td>
<td>Percentage of eigen load</td>
<td>Core stress (MPa)</td>
<td>Face stress (MPa)</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>10%</td>
<td>0.1</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>127</td>
<td>35%</td>
<td>0.7</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>182</td>
<td>50%</td>
<td>1.4</td>
<td>233</td>
<td></td>
</tr>
<tr>
<td>225</td>
<td>62%</td>
<td>2.3</td>
<td>317</td>
<td>Core crushing and panel failure</td>
</tr>
<tr>
<td>237</td>
<td>65%</td>
<td>2.6</td>
<td>341</td>
<td></td>
</tr>
<tr>
<td>237</td>
<td>65%</td>
<td>5.4</td>
<td>517</td>
<td>Face sheet Fracture</td>
</tr>
<tr>
<td>291</td>
<td>80%</td>
<td>5.5</td>
<td>536</td>
<td></td>
</tr>
<tr>
<td>329</td>
<td>91%</td>
<td>13.4</td>
<td>951</td>
<td></td>
</tr>
<tr>
<td>337</td>
<td>93%</td>
<td>17.7</td>
<td>1158</td>
<td>Non-Linear buckling load (limit point - Figure 5-3)</td>
</tr>
<tr>
<td>364</td>
<td>100%</td>
<td></td>
<td></td>
<td>Eigenvalue Buckling</td>
</tr>
</tbody>
</table>

Table 5-1 - Stresses in non-linear model

The wrinkling stress was calculated by converting the moment applied to each end of the panel into a stress on each face. It was assumed that the entire moment is carried by the face sheets as a tensile and compressive stress. The following expression was used to convert the eigenvalues and non-linear loads (output from Ansys) into useable face sheet stresses.

\[
\sigma_f = \frac{P_t}{6t_f} \tag{5-20}
\]

From Table 5-1 and Figure 5-6 we see that at the beginning of the load curve, at 10% of the eigenvalue load (wrinkling stress of 36MPa), the face sheet and the core stresses are 40MPa and 0.1MPa respectively. As the load increases, the wrinkle depth and the bending stress start to
increase in amplitude. At the non-linear buckling load (93% of the eigenvalue stress wrinkling stress of 337MPa), the face sheet and core stresses are 1158MPa and 17.7MPa respectively. The difference between the face sheet stress and the wrinkling stress is due to the additional bending stress caused by wrinkles forming in the face sheets. These values would be comparable if the face sheets were free of wrinkles.

Although the panel should collapse at an applied eigenvalue face sheet stress of 337MPa due to wrinkling instability, the panel never reaches this type of load. The compressive strength of the face material is 517MPa and the core crush strength is 2.3MPa. By the time the panel reaches the limiting wrinkling collapse stress, the face sheet and the core would have failed due to excessive compressive stress, through combinations of compression and bending.

At only 62% of the instability eigenvalue load, the crush strength of the core is exceeded and core starts to collapse locally under the face sheet, setting up a catastrophic chain of events. At first the face sheet dips into the core and experiences an increase in bending stress. Once the stress in the facesheet reaches its fracture strength, the face sheet will fail and the entire panel will collapse.

It is interesting to note that the collapse stress from the non-linear Finite Element model (62% of the eigenvalue buckling stress) matches the original design equation by the Hoff and Mautner [5] and experimental results. Despite this agreement, failure is not caused by wrinkling instability but is due to another failure mode: localised core crushing as a result of wrinkles forming in the face.

To accurately capture the failure mechanism and load, a non-linear incremental model is needed to trace the growth of the wrinkle up until the collapse stress. A linear model similar to Yussuff [7] and Kassapoglu [57] with core crushing added can also approximate this load. A new improved version of their models is derived in section 5.6.1.
10% Eigenvalue Stress (36MPa)
Face stress = 40 MPa
Core stress = 0.1 MPa

65% Eigenvalue Stress (237MPa)
Face stress = 341 MPa
Core stress = 2.6 MPa

93% Eigenvalue Stress (337MPa)
Face stress = 1158 MPa
Core stress = 17.7 MPa

Figure 5-6 – Stress plots showing the stresses in the core and face sheet over the range of loading
5.3.5 Discrete model results

From the continuum model it was shown that the panel can fail before it reaches the wrinkling stress due to core crushing. To investigate this further, a discrete core model was developed to see if discrete models fail in a similar way to continuum models. With the discrete model, the modulus and the fracture strength of the Nomex-phenolic resin walls were used. The discrete geometric and material properties used for 1/8”-3.0 Nomex core were determined in Chapter 3. These include cell wall thickness, wall modulus and nominal geometry measurements. The model was loaded in the same way as its equivalent continuum model.

Finding the fracture strength of the Nomex-phenolic resin walls was approximated using a discrete cell Finite Element model, and loading it to the continuum crush strength. It was determined that the fracture strength of the Nomex walls is approximately 50MPa.

Using the same approach as the continuum model, the discrete model reaches the core’s fracture strength of 60MPa at a wrinkling stress of 220MPa, with 0.05mm of initial perturbation. A 220MPa failure stress is almost identical to the continuum model and the experimental results, supporting the view that either type of model could be used to determine the failure loads. A plot showing the stress distribution is shown in Figure 5-7

![Stress plots showing the stresses and buckling mode in the non-linear discrete model](image)

**Figure 5-7** – Stress plots showing the stresses and buckling mode in the non-linear discrete model

### Section 5.4 Wrinkling Experiment

Undamaged panels were loaded to failure to determine failure stresses and validate Finite Element models. Seven different configurations of panels were manufactured to gather a large enough sample of experimental results. The first six panels consist of combinations of three different Nomex cores (HRH10-3/16”-3.0 -1”, HRH10-1/8”-3.0 -1”, HRH10-1/8”-5.0 -1”) with three different types of face material (BMS-79, a glass satin weave prepreg and BMS 8-256, a carbon fibre prepreg). To increase the number of samples the face sheet layers were varied from one to two
Panel 7 was a pre-manufactured panel from Norbond (manufacturers’ designation of 221-0133-500.), constructed from glass-fibre/phenolic resin faces and a 1/8”-3.0 Nomex core. It is typically found in floor panels and internal dividing walls in commercial aircraft.

Table 5-2 shows a summary of the experimental panel configurations:

<table>
<thead>
<tr>
<th>Panel</th>
<th>Core</th>
<th>Faces sheet</th>
<th>Number of layers</th>
<th>Sandwich Thickness (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HRH10 1/8”-3.0 -1”</td>
<td>BMS 8-79</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>HRH10 1/8”-3.0 -1”</td>
<td>BMS 8-256, BMS 5-101 adhesive</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>HRH10 3/16&quot;-3.0 -1”</td>
<td>BMS 8-256, BMS 5-101 adhesive</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>HRH10 3/16&quot;-3.0 -1”</td>
<td>BMS 8-79</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>HRH10 1/8”-3.0 -1”</td>
<td>BMS 8-79</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>HRH10 1/8”-4.0 -1”</td>
<td>BMS 8-79</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>HRH10 1/8”-3.0 -0.5”</td>
<td>L528-7781 Phenolic</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

*Table 5-2 – Summary of the panel configuration for the undamaged wrinkling experiments*

Most of these panels were manufactured using a vacuum bag technique which is one of the more common manufacturing methods. The method is outlined in Appendix C. Each panel was made to measure 720mm x 320mm.

*Figure 5-8 - Experimental test panels before being cut into test specimens*

To find wrinkling failure loads at least one face sheet must be put into pure compression. This was achieved with a four point bend test that puts the centre of the panel into pure bending, producing a constant compression force in the face. Spacings on the inside and outside supports and loading
bars were varied depending on the specimen size. The test was largely based around ASTM C393 [58].

The procedure is used to determine flexural and shear stiffness, in particular, shear modulus and shear strength of the core, or compressive or tensile strength of the facings. C393 requires the specimen to be rectangular, with a width not less than twice the specimen thickness nor greater than half the length. The length of the span changes depending on the type of test. The specimen length is generally less for a shear test than a flexural test. Length is deliberately chosen to increase the moment, so that a panel will fail due to compressive instability as opposed to shear failure. For this experiment, it is ideal to have a long slender specimen which would allow the face sheet to reach the buckling load.

For these tests the specimens were cut from 720mmx320mm panels.

The specimen length was reduced to 350mm, to maximise the number of samples. A CNC mill and a knife were used to cut dog-bone shaped legs into each specimen. This increased the compressive stress in the face and decreased the flexural rigidity of the panel, so the panel would fail due to compressive instability and not core shear failure.

The majority of the specimens were cut in a similar way to the one illustrated in Figure 5-9. A constant cross head speed of 2 mm/min was applied to the panel using an extensometer. This rate was based on the ASTM standard which stipulates that the panel should fail in three minutes. Figure 5-10 to Figure 5-13 are photographs from the tests.
5.4.1 Experimental observations

Some initial cracking sounds were heard prior to failure of each specimen. A small wave (detected in most of the specimens) also formed along the edge of the specimen. On some of the specimens strain gauges were positioned in the centre of the failure zone. These behaved normally at the beginning of the load curve but as the load approached the failure load, the strain measurements either stayed the same or even decreased, indicating some form of strain relaxation or wrinkle in the surface. Webber et al. [56] noted a similar trend, indicating some form of local instability during loading and prior to failure. In all cases the final visual failure mode was face sheet fracture. It is difficult to pinpoint the exact failure mode. As with most composites, failure is an almost instantaneous event, where most of the evidence of failure is destroyed by the panel while it is collapsing.

It was surmised that the wrinkles first formed along the edge of the panel where the bond between the core and face is the weakest. As the load increased the whole surface started to deform into minute waves. As these increased in amplitude, they caused the core to start crushing locally under the face sheet and the face to fracture, once the combination of the bending stress and in-plane stress exceeded the compression strength of the material.
Section 5.5 Results

5.5.1 Comparison between experimental and Finite Element models

Material properties used in the Finite Element model were taken directly from the manufacturer’s data sheet. (Table 5-3, Table 5-4) The face sheets thickness was measured directly from fabricated face sheets using digital callipers.

<table>
<thead>
<tr>
<th>Core</th>
<th>$G_{xz}$</th>
<th>$G_{yz}$</th>
<th>$E_z$</th>
<th>$\sigma_c$</th>
<th>Stabilised crush strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td></td>
</tr>
<tr>
<td>HRH-10-1/8-3</td>
<td>40</td>
<td>24</td>
<td>138</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>HRH-10-3/16-3</td>
<td>40</td>
<td>24</td>
<td>138</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>HRH-10-1/8-4</td>
<td>60</td>
<td>32</td>
<td>193</td>
<td>3.9</td>
<td></td>
</tr>
</tbody>
</table>

Table 5-3 – Core properties

<table>
<thead>
<tr>
<th>Face Material</th>
<th>$E_f$</th>
<th>face sheet modulus</th>
<th>$\sigma_f$</th>
<th>Failure strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-256</td>
<td></td>
<td>50300</td>
<td></td>
<td>745</td>
</tr>
<tr>
<td>8-79</td>
<td></td>
<td>25500</td>
<td></td>
<td>517</td>
</tr>
<tr>
<td>I528-7781 Phenolic</td>
<td></td>
<td>23000</td>
<td></td>
<td>517</td>
</tr>
</tbody>
</table>

Table 5-4 - Face sheet properties

In the failure models, a bilinear-isotropic-plasticity model was added to the isotropic and orthotropic material models. When the stress exceeded the crush and fracture strength of the core and facesheet materials, the modulus and resulting material stiffness was reduced to zero. This type of bilinear material behaviour represents an elastic, perfectly plastic material. At this point the model stopped converging as it became numerically unstable. This was due to a rapid loss of stiffness in the core which led to an overall loss of stiffness and instability of the panel. The load at which the model failed to converge was taken as the failure load. Table 5-5 compares the experimental failure stress to the predicted stress from the Finite Element model. The predicted load (Finite Element model) was closer to the experimental failure loads for most configurations of panels than the existing model by Hoff and Mautner [5] Equation (5-1), the industry standard for predicting failure due to wrinkling. Some difference was found between experimental values and predicted Finite Element values in Models 2 and 6. These two panels were built using the HRH-10-3/16”-3 core, the same core that had an inconsistency in shear modulus values as discussed in Chapter 3.
Table 5-5 – Comparisons between undamaged Finite Element model and experimental results

In Chapter 3 it was shown that the measured and calculated shear modulus value of 20MPa was half of the manufacturer’s value of 40MPa. One probable cause for this difference is the use of an incorrect modulus value. In specimen 2b and 6b the modulus was dropped from 40MPa to the measured value of 20MPa. This gave a better correlation with experimental results, reinforcing the statement (Chapter 3) that the manufacturer’s shear modulus is incorrect.

Some interesting observations were made while comparing the experimental results to the plane-stress and solid Finite Element models.

The first observation concerns the in-plane modulus. The discussion in Chapter 2, Chapter 4 and the current chapter showed that the in-plane modulus had a significant effect on the linear buckling loads and non-linear buckling loads (when linear material models are used). Both linear and non-linear buckling models were equally affected by changing the in-plane modulus.

The results in this chapter indicate that when non-linear plasticity models are used (the stiffness degrades once a limiting stress values are reached), the non-linear buckling loads were different (with different in-plane moduli) but the linear eigenvalue buckling loads were the same. This is because failure loads are controlled by crushing and collapse of the core and not instability of the structure (which is the case in standard non-linear buckling models). With the crush models, the stress which causes the core to fail is controlled by the out-of-plane stiffness ($E_z$) and the resulting strain ($\varepsilon_z$) from the wrinkle. The strain that occurs in the in-plane direction due to the in-plane stiffness has no bearing on the vertical collapse of the core and resulting failure of the panel. This is
shown in Table 5-6 which shows the crush strength for two different configurations of panels with a range of in-plane moduli.

<table>
<thead>
<tr>
<th>$t_f$ (mm)</th>
<th>$E_z$ (MPa)</th>
<th>$G_{xz}$ (MPa)</th>
<th>$E_x$ (MPa)</th>
<th>$E_f$ (MPa)</th>
<th>$\sigma_c$ Core crush strength (MPa)</th>
<th>$\sigma_t$ Fracture strength face (MPa)</th>
<th>Linear wrinkling stress (MPa)</th>
<th>Non-linear failure stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.47</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>25500</td>
<td>2.3</td>
<td>517</td>
<td>363.5</td>
<td>236.3</td>
</tr>
<tr>
<td>0.47</td>
<td>138</td>
<td>40</td>
<td>5.0</td>
<td>25500</td>
<td>2.3</td>
<td>517</td>
<td>386.6</td>
<td>241.6</td>
</tr>
<tr>
<td>0.4</td>
<td>138</td>
<td>40</td>
<td>0.2</td>
<td>50300</td>
<td>2.3</td>
<td>745</td>
<td>372.9</td>
<td>297.6</td>
</tr>
<tr>
<td>0.4</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>50300</td>
<td>2.3</td>
<td>745</td>
<td>456.9</td>
<td>299.7</td>
</tr>
</tbody>
</table>

*Table 5-6 – Comparison of linear and non-linear failure stress with different in-plane moduli. The linear failure stresses are affected by a changing in-plane modulus, whilst the non-linear failure stresses are not. This is due to core crushing which controls the failure load in the non-linear models.*

This comparison was done using both two dimensional and three dimensional models. Both of these predict the same failure stresses with non-linear material properties activated.

Sources of error between the theoretical and experimental results can include:

1. Sample size – in most cases only seven specimens were used.
2. Measurements of the face sheets and differences in properties – any variation between the average value material thickness and the actual material thicknesses could affect the calculated experimental failure loads. The Finite Element models are affected by variations between the properties supplied by the manufacturers and the physical properties of the material used on the panels.
3. Influences from the edges – the edges of the test specimens can effect the buckling loads.
4. Size of the perturbation – the perturbation is estimated at 0.05mm. However it can vary slightly. This value has a significant effect in the predicted non-linear buckling loads and actual variation may account for some difference between the experimental and numerical results.

Despite some error, with the level of correlation shown in Table 5-5 between experimental and Finite Element results it can be reasoned that panels made with Nomex honeycomb cores, with glass or carbon “thin” faces, are failing due to core crushing as a result of wrinkling. In the original theory [5], it was suggested that panels are failing at lower loads than the analytical models due to imperfections and irregularities. These trigger core crushing and facesheet-to-core failure. This theory is true to a degree. Core crushing and facesheet failure is caused by wrinkles forming in the facesheet, which will occur with or without imperfection. Imperfection will allow these to occur earlier and will also reduce the linear wrinkling instability load by up to 20%, not the full 40-50% seen in experiments.

**Section 5.6 Waviness models (Kassapoglous et al. and Yussuff Models)**

The model by Kassapoglou [50] (summarised by Harris et al. [49]) and Yussuff [7] built on the work by Hoff and Mautner [5], by going beyond a simple waviness model and examining the effect
of other failure modes caused by wrinkles forming in the facings. These include core compression, tensile stress and shear failure, adhesive failure, tension and shear and face sheet failure due to bending. Their models were based on the Hoff and Mautner [5] wrinkling model and included a function which decreases the wrinkling stress based on a wave shape of a known amplitude and wavelength. Both authors argued that if the wrinkle amplitude was large enough or the wrinkle wavelength was small enough then it is probable that the panel could fail due to a failure mechanism other than wrinkling. This could be core crushing or face sheet bending as shown in the non-linear study.

These analytical models are still based on a linear analysis using failure criteria of the core and face sheet. This is possible because the non-linear terms cancel out of the governing differential equation when an initial perturbation is added, leaving only linear terms in the expression. Therefore, the only parameters used to determine the strain in the core and face sheet are the amplitude of the initial perturbation and the failure stress of the core. Based on these values the maximum permissible size of the wrinkle is determined and thus the strain in the core. The non-linear models in the previous section are now replaced by linear models, and a similar solution becomes available.

### 5.6.1 Revised analytical failure model

The following analytical model predicts failure stress due to core crushing in panels with initial imperfection. It tries to duplicate the behaviour found in the non-linear models using the same principles as the Yussuff [7] and Kassapoglou [57] models, but deviates slightly in the way the wrinkling stress and critical strains are calculated. This model is based on the linear wrinkling stresses models developed in this thesis. The first of these is found in Chapter 2 and predicts the linear wrinkling stress of an unperturbed, undamaged panel. The second model is found in Chapter 6 and predicts the failure stress for an infinitely large size impact dent with a layer of subsurface damage. To find the limiting stress due to core crushing the following expression can be used

![General wrinkling model notation](image-url)
In Chapter 2 and Chapter 6 it is shown that the wrinkle wavelength is

\[ w = W \sin \left( \frac{\pi x}{L} \right) e^{-a \left( \frac{x}{L} \right)^{z}} \]  

(5-21)

where

\[ \alpha^{2} = \frac{G_{xz}}{E_{z} - v_{xz} G_{xz}} \]  

(5-22)

This is found by making a generic displacement function \( w(x, z) = W f(z) \sin \left( \frac{\pi x}{L} \right) \) satisfy the following equilibrium condition

\[ \frac{\partial^{2} w}{\partial z^{2}} + \left( \frac{G_{xz}}{E_{z} - v_{xz} G_{xz}} \right) \frac{\partial^{2} w}{\partial x^{2}} = 0 \]  

(5-23)

Equation (5-21) defines the displacement of the wrinkle through the depth of the core.

When Equation (5-21) is substituted in the governing differential equation, (Equation (5-24)) for a beam on an elastic foundation

\[ D \frac{d^{4} w}{dx^{4}} + P \frac{d^{2} w}{dx^{2}} + E_{z} \frac{d^{2} w}{dz} = 0 \]  

(5-24)

the critical wrinkling wavelength and the critical wrinkling stress is found

\[ \sigma_{cr} = 0.825 \left( E_{f} E_{z} G_{xz} \right)^{1/3} \]  

(5-25)

where

\[ L = \pi \left( \frac{2t_{f}^{1/3}}{12} \frac{12}{E_{z} \alpha} \right) \]  

(5-26)

If an initial perturbation is added to the face sheet in the form

\[ w_{0} = A_{0} \sin \left( \frac{\pi x}{L} \right) \]  

(5-27)

and if each face is subjected to axial force \( P \), the deflection and curvature of the face is defined by \( (w_{0} + w) \). The differential equation of buckling of a face of unit width, elastically supported, becomes

\[ D \frac{d^{4} w}{dx^{4}} + P \frac{d^{2} w}{dx^{2}} + E_{z} \frac{d^{2} w}{dz} = -P \frac{d^{3} w_{0}}{dx^{3}} \]  

(5-28)

and the solution to this equation at the face sheet is
\[ w = \frac{A_0}{\left(\frac{P_m}{P} - 1\right)} \sin\left(\frac{\pi x}{L}\right) \]  \quad (5-29)

in which

\[ P_m = D\left(\frac{\pi}{L}\right)^2 + E_z \alpha \left(\frac{L}{\pi}\right) \]  \quad (5-30)

and the amplitude of the deflection is

\[ W = \frac{A_0}{\left(\frac{P_m}{P} - 1\right)} \]  \quad (5-31)

At the critical wavelength, \( P_m \) is a minimum and \( W \) is a maximum; hence the core stress is a maximum. This is shown in Equation (5-31). If \( A_0 = 0 \) then \( W \) becomes indeterminate and out-of-plane deflections are possible. In the case of no waviness \( P_m \) (maximum wrinkling load) can be expressed as \( \sigma_{cr} = 0.825 \left( \frac{E_f E_z G_{zz}}{\sigma^3} \right)^{\frac{1}{3}} \) (critical wrinkling stress with no imperfection) if the wrinkle amplitude is expressed as

\[ W = \frac{A_0}{\left(\frac{\sigma_m}{\sigma_{cr}} - 1\right)} \]  \quad (5-32)

This can be rearranged to give the new critical wrinkling stress

\[ \sigma_{cr} = \frac{\sigma_m}{\left(\frac{A_0}{W} + 1\right)} \]  \quad (5-33)

and

\[ \sigma_m = 0.825 \left( \frac{E_f E_z G_{zz}}{\sigma^3} \right)^{\frac{1}{3}} \]

the maximum amplitude is bounded by the maximum permissible stress in the core at the point of failure, which for core crushing is the compression strength of the core. This stress can be used to define the amplitude \( W \), which is usually indeterminate in a linear wrinkling problem. This is why a linear wrinkling model can be used in this special case to find \( W \).

To find the stress in the core at the face sheet the following expression is used:

\[ \sigma_z = E_z \frac{\partial w}{\partial z} \]  \quad (5-34)

Using the displacement function defined in Equation (5-21),

\[ \sigma_z = W E_z \alpha \left(\frac{\pi}{L}\right) e^{-\frac{\pi}{L}} \sin\left(\frac{\pi x}{L}\right) \]  \quad (5-35)
At the face sheet when \( z = 0 \)
\[
\sigma_0 = -WE_e \alpha \left( \frac{\pi}{L} \right) \sin \left( \frac{\pi x}{L} \right)
\]
and \( w_0 = -W \sin \left( \frac{\pi x}{L} \right) \) (5-36)

The maximum compression stress is found when \( x = \frac{3L}{2} \) where \( \sin \left( \frac{\pi x}{L} \right) = -1 \)

The maximum core stress is given by
\[
\sigma_{\text{max}} = WE_e \alpha \left( \frac{\pi}{L} \right)
\]
(5-37)

Rearranging in terms of maximum amplitude,
\[
W = \frac{\sigma_{\text{max}}}{\left( E_e \alpha \left( \frac{\pi}{L} \right) \right)}
\]
(5-38)

where half the critical wrinkling wavelength is
\[
L = \frac{\pi}{\sqrt{\frac{2t_f^3}{12E_e \alpha}}} \quad \text{and equation (5-22) becomes } \alpha = \sqrt{\frac{G_{\text{sc}}}{E_z}} \quad \text{because } \nu_{xz} = 0 \text{ for honeycomb cores}
\]

Substituting Equation (5-38) back into Equation (5-33) and simplifying gives new critical failure
stress based on the compression strength of core, the initial perturbation, and the maximum critical
wrinkling stress
\[
\sigma_{cr} = \frac{0.825 \left( E_f E_z G_{sc} \right)^{\frac{1}{3}}}{A_0 \sqrt{E_z G_{sc}} \left( \frac{\pi}{L_{cr}} \right)} + \frac{1}{\sigma_c}
\]
(5-39)

where
\[
L_{cr} = \pi \left( \frac{E_f}{6 \sqrt{E_z G_{sc}}} \right)^{\frac{1}{3}}
\]
(5-40)

In general terms
\[
\sigma_{cr} = \frac{0.825 \left( E_f E_z G_{sc} \right)^{\frac{1}{3}}}{A_0 \sqrt{E_z G_{sc}} \left( \frac{\pi}{L_{cr}} \right)} + \frac{1}{\sigma_c}
\]
where \( L_{cr} = \pi \left( \frac{E_f}{6 \sqrt{E_z G_{sc}}} \right)^{\frac{1}{3}} \)
(5-41)
This compares to Kassapoglou [57] and Yussuff [7, 51]

\[
\sigma_{cr} = \frac{0.96 \left( E_f E_c G_c \right)^{\frac{1}{3}}}{A_h E_c + 1} \quad [7, 51],
\]

\[
\sigma_{cr} = \frac{0.91 \left( E_f E_c G_c \right)^{\frac{1}{3}}}{A_h E_c + 1} \quad [57]
\]

\[
h = 0.72 t_f \left[ \frac{E_f E_c}{G_c^2} \right]^{\frac{1}{3}} \quad [7, 51],
\]

\[
h = 0.9 t_f \left[ \frac{E_f E_c}{G_c^2} \right]^{\frac{1}{3}} \quad [57]
\]

These expressions are solved for compression of the core. Similar expressions can be found for shearing of the core, face sheet fracture and interface bond failure. Refer to Yussuff [7].

**Section 5.7 Comparison of existing models to the revised failure model**

Equation (5-41) is now compared to the non-linear Finite Element models and the models of Yussuff [7] and Kassapoglou [57].

Figure 5-17 plots initial perturbation against wrinkling stress and includes results from the new model, a non-linear plane-stress Finite Element model with core crushing, Kassapoglou’s model and Yussuff’s model. All three models exhibit similar behaviour of decreasing failure stress for increasing initial perturbation. Kassapoglou’s and Yussuff’s models predict similar values with the new model predicting values with failure stresses between 10-50% lower than the original models of Kassapoglou and Yussuff. For perturbations less than 0.1mm, the Finite Element results were similar to the newly developed model. For larger perturbations the Finite Element results tended towards the existing models of Yussuff and Kassapoglou. This change may be caused by a variation in the core strains. Figure 5-15 and Figure 5-16 show the non-linear failure strain plots for 0.05mm and 0.15mm of initial perturbation respectively. It appears that the affected area of the core is deeper in the 0.05mm model (indicating higher strains) than in the 0.15mm model, even though the crush strengths / maximum strains are similar. In the analytical models, these strains depths are the same, regardless of initial perturbation, as wrinkles are forced to dampen out at the same depth. Higher strains in the non-linear Finite Element model would explain why it tends to predict a lower relative stress value when there is minimal waviness. This trend is shown in Figure 5-17, where the non-linear Finite Element results jumps from a low relative failure stress (smaller initial perturbation) to higher relative failure stresses (larger initial perturbations).

This result suggests that the final size, shape and resultant strain distribution in the core is dependent on the size of the perturbation. The strain distribution appears to be non-linear, which means that the final shape of the wrinkle is different from the initial wave shape. Accurate prediction of collapse strains can only be achieved using a non-linear analytical model, although the new model provides a good estimate of failure stresses for smaller perturbations.
Section 5.8: Comparison of models (Finite Element / Analytical) to experimental results

A direct comparison is made between experimental results and the various models in Table 5-8. The calculated failure stresses are based on an initial perturbation of 0.05mm and the material properties found in Table 5-7.

The tabulated results show that the new model predicts the closest values to the experimental results. The average error between experimental values and the new model is 0.7% for the new model compared to up to 17.7% for the existing model of Hoff and Mautner. If the amplitude $A_o$ is
unknown then an approximate value of \( \frac{A_0}{l_f} = 0.1 \) [57] should be used. For the types of cores and face sheets used in this chapter, this value is between 0.047mm and 0.04mm (close to the 0.05mm used in this chapter).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Core</th>
<th>Face Material</th>
<th>( t_c )</th>
<th>( t_f )</th>
<th>( E_z )</th>
<th>( G_{xz} )</th>
<th>( \sigma_c )</th>
<th>( E_f )</th>
<th>( \sigma_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>hrh-10-1/8-3</td>
<td>8-256</td>
<td>25.4</td>
<td>0.4</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>50300</td>
<td>745</td>
</tr>
<tr>
<td>2</td>
<td>hrh-10-3/16-3</td>
<td>8-256</td>
<td>25.4</td>
<td>0.4</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>50300</td>
<td>745</td>
</tr>
<tr>
<td>2B</td>
<td>hrh-10-3/16-3</td>
<td>8-256</td>
<td>25.4</td>
<td>0.4</td>
<td>138</td>
<td>20</td>
<td>2.2</td>
<td>50300</td>
<td>745</td>
</tr>
<tr>
<td>3</td>
<td>hrh-10-1/8-3</td>
<td>8-79</td>
<td>25.4</td>
<td>0.47</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>25500</td>
<td>517</td>
</tr>
<tr>
<td>4</td>
<td>hrh-10-1/8-3</td>
<td>8-79</td>
<td>25.4</td>
<td>0.47</td>
<td>193</td>
<td>60</td>
<td>3.6</td>
<td>25500</td>
<td>517</td>
</tr>
<tr>
<td>5</td>
<td>hrh-10-1/8-3</td>
<td>I528-7781</td>
<td>12.7</td>
<td>0.508</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>23000</td>
<td>517</td>
</tr>
<tr>
<td>6</td>
<td>hrh-10-3/16-3</td>
<td>8-79</td>
<td>25.4</td>
<td>0.47</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>25500</td>
<td>517</td>
</tr>
<tr>
<td>6b</td>
<td>hrh-10-3/16-3</td>
<td>8-79</td>
<td>25.4</td>
<td>0.47</td>
<td>138</td>
<td>20</td>
<td>2.2</td>
<td>25500</td>
<td>517</td>
</tr>
<tr>
<td>7</td>
<td>hrh-10-1/8-3</td>
<td>8-79 1x</td>
<td>25.4</td>
<td>0.26</td>
<td>138</td>
<td>40</td>
<td>2.2</td>
<td>25500</td>
<td>517</td>
</tr>
</tbody>
</table>

Table 5-7 – Material properties for the 7 test panels (2B and 6B have a reduced shear modulus)
### Table 5-8 – Comparisons between undamaged Finite Element model, Kassapoglou’s and Yussuff’s models and experimental failure stresses, including average error between the models and experimental results.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Core</th>
<th>Face Material</th>
<th>Average Experimental</th>
<th>Non-linear Finite Element model</th>
<th>Hoff and Mauer’s failure model</th>
<th>Revised model</th>
<th>Kassapoglou model</th>
<th>Yussuff model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>hrh-10-1/8-3</td>
<td>8-256</td>
<td>300</td>
<td>294</td>
<td>326</td>
<td>286</td>
<td>388</td>
<td>376</td>
</tr>
<tr>
<td>2</td>
<td>hrh-10-3/16-3</td>
<td>8-256</td>
<td>269</td>
<td>294</td>
<td>326</td>
<td>286</td>
<td>388</td>
<td>376</td>
</tr>
<tr>
<td>2B</td>
<td>hrh-10-3/16-3</td>
<td>8-256</td>
<td>269</td>
<td>280</td>
<td>259</td>
<td>274</td>
<td>354</td>
<td>350</td>
</tr>
<tr>
<td>3</td>
<td>hrh-10-1/8-3</td>
<td>8-79</td>
<td>223</td>
<td>236</td>
<td>260</td>
<td>221</td>
<td>303</td>
<td>292</td>
</tr>
<tr>
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<td>8-79</td>
<td>263</td>
<td>287</td>
<td>333</td>
<td>282</td>
<td>387</td>
<td>373</td>
</tr>
<tr>
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<td>251</td>
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</tr>
<tr>
<td>7</td>
<td>hrh-10-1/8-3</td>
<td>8-79</td>
<td>1 LAYER</td>
<td>162</td>
<td>178</td>
<td>260</td>
<td>159</td>
<td>234</td>
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| Overall Average Error (Model – Experiment) | 5.69% | 17.91% | 0.69% | 37.12% | 32.45% |

### Section 5.9 Further Development

One possibility is to develop a non-linear analytical model that accounts for the strain in the core, core crushing and face sheet fracture. Kuhhorn et al. [59] developed a general non-linear theory for sandwich shells with seven kinematic degrees of freedom which are able to describe global and local buckling. Frostig [60], [61-65] described a refined core model. Within his work he examined wrinkling and also non-linear incremental loading with plasticity models. There is a potential to extend the Frostig and Kuhhorn et al. [59] models and develop a standalone non-linear analytical model which can accurately predict failure loads for a range sandwich configurations.

The wrinkling experiments could also be reviewed. It is possible to use optical methods to measure initial perturbation and growth of wrinkles in undamaged faces. For future work an optical test device such as this would provide invaluable information on the failure process, and displacement information could be used to validate non-linear Finite Element models.
Section 5.10 Conclusions

Bending failure loads in undamaged sandwich panels were predicted accurately using non-linear Finite Element models. These models show that the panels are failing due to core crushing as a result of wrinkles forming in the faces. As the core crushes it allowed the face sheet to deform further, eventually leading to face sheet fracture and collapse of the panel. These results were verified against the experimental data.

The result also demonstrates that traditional linear analytical wrinkling models (Chapter 2 – Current expression $\sigma_{cr} = 0.825(E_f/E_cG_c)^{\frac{1}{3}}$) cannot be used to predict the failure loads as they do not account for non-linear material behaviour. The statement Hoff and Mautner [5] and other authors made, suggesting that lower failure loads are caused by initial imperfections and irregularities, is only true to an extent. The 0.05mm of initial perturbation only accounts for a 10%-20% reduction in stresses from the linear buckling result, and not the full 40%-50% difference between experimental and analytical models. This difference is attributed to the non-linear behaviour such as face sheet fracture, wrinkle growth and core crushing. Non-linear behaviour will occur more rapidly in panels with initial manufacturing imperfection as wrinkles form more readily when facesheets are initially perturbed.

The existing reduced expression by Hoff and Mautner [5] of $\sigma_{cr} = 0.5(E_f/E_cG_c)^{\frac{1}{3}}$ will give an approximate failure stress; however, a non-linear Finite Element model or the revised failure expression below provides the best estimate of failure loads. In the series of tests reported in this chapter it was found that the new analytical model Equation (5-42) produced the best estimate of failure stress based on 0.05mm of initial perturbation. As an estimate $A_0$ (initial perturbation) the following expression $A_0/t_f = 0.1$ [57] could be used.

$$\sigma_{cr} = \frac{0.825(E_f/E_cG_c)^{\frac{1}{3}}}{A_0\sqrt{E_cG_c} \left( \frac{\pi}{L_{cr}} \right) + 1}$$

where $L_{cr} = \pi_f \left( \frac{E_f}{6\sqrt{E_cG_c}} \right)^{\frac{1}{3}}$

(5-42)

It was found that the new wrinkling model above produced a more realistic failure stress than Yussuff’s [7] and Kassapoglou’s [57] equivalent core crushing models.

Unlike linear wrinkling models and non-linear models (with no core crushing) it was found that the in-plane modulus has no effect on the failure stress. This is controlled by the crush strength / out-of-plane compressive modulus of the core and the vertical displacement of the wrinkle.
Chapter 6: Impact damaged honeycomb sandwich panels

Chapter 6 examines the effect of impact damage on sandwich structures. This type of damage can cause panels to fail at significantly lower loads than predicted.

For this reason, impact damage in sandwich structures has been a focus for numerous studies in the past, including those which examine damage tolerance of structures and also the post-impact residual strength of structures. This study specifically examines thin gauge honeycomb panels and was concerned only with barely visible impact damage (BVID), the type of damage that is found on aircraft components.

This chapter presents an overview of BVID on panels, including an in-depth examination of the failure modes and failure loads of damaged panels. Non-linear finite element models were used to accurately predict failure loads, then these models were used to develop a simplified closed-form analytical model which predicts failure stresses through a complete range of damage sizes and depths. This is the first study to develop a useable simplified tool to estimate reductions in load carrying capacity for a range of damage sizes. It presents a detailed account of the failure mechanism governing the collapse of damaged panels.
Section 6.1 Introduction

During normal operation, the trailing edge wing flaps (Figure 6-1) and other flight control surfaces on aircraft are subject to impact events from numerous sources, including, hailstones bird strikes, and debris lifted from the undercarriage during take-off and landing.

Two types of damage occur in these typically sandwich components: impacts that puncture the skin and must be repaired straight away, and impacts that crush the core directly below the face sheet and cause a large, barely visible impact dent (BVID) with no face sheet failure. This second type of damage is more common on aerodynamic panels and is investigated in this research. Figure 6-2 shows a typical cross section of non-metallic sandwich honeycomb panel with BVID.

Figure 6-1 – Aft wing flaps on a 747-400 aircraft under repair

Impact damage has been shown to greatly affect the load carrying capacity of the component, causing panels to fail at lower loads than expected. Aitken [8] showed that impact damage larger than 60mm reduces the panels load carrying capacity by 50%. For this reason, engineers are currently bound by manufacturers’ allowable damage limits that state that if damage sites are too
close together, or larger than 1”, then an aircraft must be grounded until suitable repairs are made [66]. As a result, flights may be delayed and revenue lost due to non-operational aircraft. For example, seventeen of Qantas Airways’ international fleet were grounded for over a week after a freak hail storm on a Sydney tarmac (1999). Figure 6-3 and Figure 6-5 illustrate typical hailstone damage to a flight control surface on a Boeing 767’s outside low speed aileron (wing panel) that was hit by hailstones in the Sydney storm. Figure 6-4 and Figure 6-6 show BVID on engine fairings due to hailstone strikes and tyre debris damage on the trailing edge wing flaps on a 747.

Wing flaps and flight control panels are typically designed for stiffness and not strength. Because these panels are designed to remain stiff under large aerodynamic loading, it is plausible that aircraft can still fly with large damage areas, even though load carrying capacity is significantly reduced. Zenkert et al. [67] also suggest that panels that sustain impact damage and have reduced load carrying capacity may still meet their design requirements, as most components are designed for stiffness and not strength.

Because of the costs associated with repairing BVID, it is important that engineers have a good understanding of impact damage and its effect on the structural integrity of the panel. This would
help engineers determine whether an aircraft is airworthy and can continue to fly until permanent repairs are made. It could also facilitate in increasing the current allowable damage limits, which have a direct and significant effect on the operational costs of the fleet.

This chapter uses Finite Element models and a simplified linear model to examine panels with BVID and their respective failure modes and failure loads. These models were verified against full-scale test panels which were damaged then loaded to failure.

**Summary of objectives**

1. To understand how impact damage affects the load-carrying capacity of aircraft-type panels through a complete range of damage sizes. Damage ranged from 0mm to 150mm in diameter, on panels designed to simulate the trailing-edge-wedge of a Boeing 747-400 aircraft flap.
2. To develop a method to predict the failure stress of BVID impact damaged panels.
3. To examine experimentally the reduction of the core support in an area of crushed core.
4. To investigate solid and soft body impacts on thin gauge sandwich structures.
5. To develop a simplified analytical model which can be used to check failure loads quickly.

### Section 6.2 Current literature on impact damage in sandwich panels

Minuget [68] suggests that the problem of impact damage should be broken into two parts: damage resistance and damage tolerance. Damage resistance is concerned with the creation of the damage due to a specific impact event. The variables include material lay-up of the face sheet, thickness of the face sheet, type of core material, and boundary conditions of the sandwich structure. Damage tolerance is concerned with the structural response and integrity of the panels after an impact event. For damage tolerance, attempts are made to predict the residual strength of the laminate for a given state of damage, for example amount of delamination or BVID size.

#### 6.2.1 Damage resistance

Failure due to an impact event can consist of a complex combination of failure modes. The damage initiation thresholds and damage sizes depend on a multitude of factors: face sheet configuration and thickness, fabrication techniques, material, geometry, interface properties between the face and the core, impact velocity and energy, indenter shape, temperature, boundary conditions, and environmental factors [69].

**Effect of face sheet thickness on damage type**

Panels with thicker face sheets tend to offer greater levels of resistance to impact events. With thicker facings the energy is absorbed by the face sheet due to its increased bending stiffness. Damage usually consists of fibre cracking and delamination, either within the face sheet lay-up or at the core/face sheet interface. When the face sheet is thin, most of the energy is absorbed by the core in the form of localised core crushing. In Nomex cores this crushing/buckling mechanism consists of multiple fractures and crease lines through the damage depth (BVID).

Davies et al. [70] compared the response to impact damage of two types of composite panels
manufactured from carbon epoxy skins and aluminium honeycomb. The first of these panels was manufactured with thick face sheets and thin honeycomb core and found to withstand impact damage better than the second panel, which with a thin face sheet and thick honeycomb core (thin-gauge), provided more limited resistance to impact damage. The thick skinned, thin cored panel absorbed 76% of the 120J impact energy and its compressive strength post-impact was reduced to 66% of the undamaged panel. In contrast, the thin face sheet, thick cored panel absorbed 93% of the incident energy and the residual strength dropped to 32% of the undamaged value. This finding reiterates the fact that thin face sheet composite panels are more susceptible to impact damage. In addition to experiments, numerically explicit Finite Element models were developed to compare the energy absorption and the force history during the impact event. The force deflection curve was accurately captured by the model, as were the permanent deformations.

Herup and Palazotto [71] also investigated low velocity impact and damage thresholds. Their work looked at sandwich structures consisting of 4-48 ply graphite/epoxy cross ply-laminate face sheets and Nomex honeycomb cores. Static and low velocity impacts were compared using energy histories. They showed that static tests have lower damage thresholds than low velocity tests. These differences were more evident in panels with thicker face sheets, with the first major drop in stress appearing in the static test well before the dynamic test. For the range of parameters they found that this initial drop in stress was associated with core failure. In practical situations they suggest that dynamic loading cannot be modelled as a quasi-static load event when panels are manufactured with thick face sheets. To support the Herup and Palazotto [71] findings, in Section 6.6.1.2 (crush tests) of this thesis, it is noted that static loading caused less damage to the core than impact loading, along with a very different crushing mechanism.

**Low velocity impacts and BVID**

A number of studies have been conducted to examine the effect of low velocity impact damage on sandwich beams consisting of carbon/epoxy face sheets and honeycomb cores. For low velocity impact, the type of impact events that commonly occur on aircraft, most of the damage occurs at the top of the panel, either within the face sheet, at the core/facing interface, or directly under the face sheet in the core. Damage usually consists of one or more of the following five different failure mechanisms:

1) Core buckling or crushing within honeycomb cores
2) Delaminating of the impacted face sheet
3) Core cracking (with foam cores)
4) Matrix cracking
5) Fibre breakage in facings

Much of the experimental work on this topic has been summarised by Abrate [72] and Tomblin [69]. Aitken [8] has also presented an extensive review on soft and rigid body impact damage.
Aitken [8] demonstrated that soft body impact can cause significant subsurface damage when thin
gauge sandwich panels are used. Soft body impact events on aircraft typically come from tyre tread
being flicked up from the undercarriage or through bird strikes on body panels, while rigid body
impact events typically come from tools being dropped during maintenance or from hail stones.
Impacts from rigid events usually result in damage that has the same profile as the impactor, while
soft body impact consists of a uniform depth of damage across the same region. Aitkin suggests
that the type of damage, soft or rigid, creates different post-impact failure loads. In this chapter it
is shown that the depth of damage has a major influence on failure load, with the deepest part of
the damage governing the residual strength of the panel. This means that either type of damage
can produce the same failure load providing the maximum core depths are similar.

Tomblin et al. [73] showed that low velocity blunt objects can produce fairly widespread Nomex®
core crushing with little residual face sheet indentation or visibly detectable face sheet damage.
Depending on the size of the impactor, the stiffness of the face sheet and impact energy, other
forms of damage may occur, including delamination (disbonding between the core and face sheet).
Matrix cracking may also develop at the impact site.

Schoeppner and Abrate [74] investigated low velocity impact load levels for delamination in
composite sandwich laminates. Using the Airforce Research laboratory database consisting of
approximately 500 low velocity impact-load histories, they examined the delamination threshold
loads for a full range of composite laminates ranging from 9 to 96 plies and manufactured from
three types of materials. For damage resistance design calculations, they suggest that if the
predicted peak load is below the experimentally determined delamination threshold then the
laminate is not expected to incur significant damage. With thin gauge sandwich structures, face
sheets are usually too thin to delaminate and will usually bend under the impact. Hence, for thin
gauge sandwich panels such as those being studied in this research, BVID is more common than
delamination.

6.2.2 Damage tolerance

One challenge of aircraft design is to adequately predict the residual strength of the damaged
composite sandwich structure. Current literature shows the need for greater simplicity in the
models to predict failure loads in BVID. This requires a better understanding of the failure
mechanisms associated with BVID.

Local wrinkling / buckling in BVID panels

Localised wrinkling/buckling is still the most common form of failure for thin gauge impact damaged
sandwich panels loaded under compression [3]. This compressive loading can be through either
bending or pure in-plane compression. Figure 6-7 shows a typical wrinkle which consists of a crater-
like deformation pushing down into the core in the damage site. As the load increases, the wrinkles
typically grow towards the edge of the panel and cause final failure and collapse of the structure.
The size of the wrinkle and the eventual failure load depends on the extent of the damaged core below the surface. Varied levels of core crushing will occur depending on the energy of the impact and its absorption by the face sheet, and the stiffness properties of the core. Wherever crushed core exists, there is a reduction in the effective out-of-plane core modulus. Any reduction to this modulus directly affects the foundation stiffness and allows the damaged area to wrinkle and become unstable at comparatively low loads compared with an undamaged section. For the Nomex® panels examined in this project, the reduction in modulus can be as high as 92%. Experimentally, Gadke and Kirschke [75] found that under a compressive loading the section of skin containing core damage failed due to wrinkling at a much lower load than an undamaged panel.

Aitken [8] conducted a literature review covering a wide range of impact and damage tolerance testing on thin gauge sandwich panels. From this review he established that practical impact testing data was limited. Consequently, he decided to study the effects of impact damage on the trailing wedge of the aft wing flap on Boeing 747-200 and 747-400 aircraft. This study involved damaging a number of test panels with soft body projectiles travelling at high velocities to simulate bird strikes (Figure 6-1 shows the trailing edge wing flap examined by Aitken).

To understand the behaviour of impacted honeycomb sandwich components, a series of damaged panels were loaded in four point bending to apply an in-plane compression load to the damaged area. As the load was applied, Aitken [8] noted three clearly visible in-plane wrinkles forming across the width of the panel in the damaged area. The largest wrinkle was in the centre of the panel, with two smaller wrinkles adjacent. As the load increased he found that the wrinkles grew rapidly in depth and width as the panel approached the collapse strength. As the wrinkle deepened the wrinkle tips began to crush the undamaged core at the edges of the damaged region. This allowed the wrinkles to propagate rapidly towards the outside boundaries of the panel, causing the panel to finally fail as a result of face sheet fracture and instability of the whole panel. Aitken [8] found that the wrinkling of the face sheet reduced the load carrying capacity of the panel by approximately 50% over a range of damage sizes.
Minguet [68] observed a similar pattern of wrinkle formation while investigating the behaviour of impacted minimum gauge sandwich panels loaded in compression.

**Critical damage size and natural wrinkling wavelength**

The occurrence of multiple wrinkles (as observed by Aitken [8]) in larger damaged areas is explained by Equation (6-1) [3] and Equation (6-2) [5]. Wrinkles will tend to form at a critical / natural wavelength which is controlled by core and facesheet stiffness. The natural wavelength is the wavelength needed to form wrinkles at the lowest load. Equation (6-1) predicts the natural wavelength of a sandwich panel for a general case and equation (6-2) predicts the half wavelength of a sandwich panel. Equation (6-2) gives the natural half-wavelength and was found while deriving Hoff and Mautners’ [5] critical wrinkling stress equation.

\[
\lambda = 5.33t_f \sqrt{\frac{E_t}{E_c}} \quad \text{Zenkert's simplified equation [3] (6-1)}
\]

\[
\lambda = 2L = 2 \left( 1.73t_f \sqrt{\frac{E_t}{E_c}} \right) \quad \text{(6-2)}
\]

Half the critical / natural wavelength from the wrinkling expression developed in Chapter 2.

Equation (6-2) was developed using sandwich core model rather than assuming a beam on an elastic foundation and is therefore, slightly more accurate than equation (6-1). Using equation (6-1) the natural wavelength of Aitken’s [8] panels was 31.7mm, based on the damaged properties in Table 6-1 and an infinitely deep damaged core. From Equation (6-2) the natural wavelength was found to be 25.1mm. Because Aitken’s [8] damage sizes ranged from 71mm to 141mm, three wrinkle wavelengths can form in the damaged area (based on a wavelength of 25.1mm).

**Open hole model**

Various authors have attempted to predict the failure load for impact damaged panels. Kassapoglou et al. [76], [57], [77], [50] published several papers that related to damage tolerance of composite laminates and thin-gauge sandwich panels, where damage consists of indentation, disbond and delamination. He modelled the impact area as a delamination. (This compares to an inclusion of reduced stiffness in this chapter.) The calculation of the failure stress used the Von-Mises failure criteria and failure was determined when the ultimate stress reached a critical value at the edge of the damage site.

Kassapoglou [77] examined the effect of low speed impact on the compressive strength of graphite/epoxy sandwich panels. He found that the impact damage decreased the load-carrying capacity of the panels by up to 33%. A one-parameter model was developed to predict the failure stress of impacted panels. This theoretical model was found to be in agreement with all experimental results. The model consisted of an area of damage which in turn included a patch of reduced stiffness. He postulated that a damage area consisting of complex geometry, matrix cracking, indentation and delamination can be modelled as single delamination under compression.
The size of the delamination could be related to an analytically determined damage size. The stress concentration was calculated using a classical elasticity approach and the derivation of this model can be found in his later papers [57] [50].

Recently Zenkert et al. [67] investigated the effect of impact damage of sandwich structures in marine applications. The overall aim of Zenkert et al’s project was to accurately predict failure loads in damaged panels with barely visible impact damage (BVID) caused by blunt objects, and visible impact damage (VID) caused by sharp objects. Zenkert adopted existing analytical models by Soutis and Curtis [78], which used two approaches. One used microbuckling type damage and assumed an equivalent hole representing BVID, while the other used an equivalent crack model when micro cracks are present in the face sheet for VID. In addition to this damage prediction model, Zenkert et al. [67] developed a full scale realistic 3D solid model of a BVID panel (similar to the solid models used in this chapter). The failure criterion of this model was based on a maximum allowable “far field strain” (strains outside the damage area). Based on the empirical model, a compressive damage assessment scheme which allows designers to determine if a panel needs immediate repair was developed.

Lacy and Hwang [79] developed a semi-elliptical numerical model for predicting the residual strength and the damage progression of impact damaged sandwich structures. Their model consists of a large scale non-linear Finite Element model that included relevant details of sandwich geometry and damage morphology. The influence of face sheet property degradation and core degradation on the stress redistribution in damaged sandwich panels was examined. Shell elements were used to model the face sheets and a 20-noded solid element was used to model the core. A single element instead of multiple elements was used to capture the non-linear response of the core. This is because his experimentally determined core response was averaged over the full depth of the core so only one element was needed to capture the core response. The model works by adjusting the stiffness properties of the face sheet in the elliptical damage area. These are varied from 0% (a hole in the face sheet) to 100% (undamaged). This is similar to Kassapoglu’s [77] model, which also uses reduced face sheet properties. Lacy and Hwang found that their residual predictions were consistent with their experimental results. They also showed that Finite Element models have the potential to provide conservative estimates of the residual strength.

It is interesting to note that the models used by Lacy and Hwang [79], Zenkert et al. [67], and Kassapoglu [77] do not implicitly model BVID. In all these models the face sheet properties were degraded to get an equivalent hole that represents BVID. With BVID the face sheet remains largely intact and has no significant reduction in stiffness; only a localised area of core has reduced stiffness. For this reason it was felt that the open hole method was not representative of BVID in thin gauge sandwich panels. To add to this argument, it is noted that Lacy and Hwang averaged their damaged core response over the full core thickness, while with BVID panels, the damaged core consists of a thin layer below the surface.
**Non-linear wrinkling models**

Minguet [68] presented a different approach to modelling the residual strength of damaged sandwich structures; he used the indentation depth as a parameter for evaluating the buckling load. His core used a bi-linear elastic orthotropic plasticity model. In this model the stress is linear until the core crushing stress is reached; beyond this point the core will support only up to the crush stress. The core in the damaged region is assumed to have zero stiffness. The undamaged face is restrained out-of-plane to force localised wrinkling in the damaged area. The model was solved using a non-linear incremental scheme. Minguet found that his model accurately predicts the deformation mode under compressive loading and shows a growth of the initial wrinkle perpendicular to the compressive load. It also has good correlation with the existing experimental data. He concluded that the panels were failing due to growth of the initial dent followed by unstable propagation of the wrinkle towards the edge of the panel.

Recently Shipsha, Hallstrom and Zenkert published the results of experimental [80] and modeling studies [81] of impact damage of sandwich structures. They addressed the effect of low velocity impact on foam cored sandwich panels. The impact damage consists of an indentation of the core with a sub-interface cavity, surrounded by a layer of crushed core and no visible damage to the facesheet. 2D non-linear Finite Element models were developed to capture the post impact failure. Two cases were examined; shear loading of the damaged area and geometric non-linear behavior such as local buckling or wrinkling. With the shear model, bridging behaviour in a crack was considered at the edge of the cavity, using point stress criterion and crack kink angle to predict failure loads. In the geometric case, non-linear Finite Element analysis was employed to capture localized buckling and the resulting instability load. In both cases models demonstrated good agreement with their experimental results.

For the thin gauge panels with BVID examined in this research, Minguet's [68] model is more appropriate than Kassapoglou's [57], [50] model, because, Minguet's model represents the failure mode as a face sheet indentation and core damage and neglects the effects of delaminations and matrix failure. However there are limitations with Minguets model. For example, he models the section of damage core as a full thickness block with zero stiffness. This model was also solved using a non-linear solver to resolve the complex system of equations.

**Testing model types against experiments**

The objective of Cvitkovich and Jackson's [82] work was to experimentally investigate the failure mechanisms of compressive face sheet wrinkling combined with either impact damage or an open hole. Specimens of two, three and four layers of carbon epoxy were bonded to Nomex cores. Three different levels of barely visible impact damage were tested on the impacted specimens. Strain gauges were placed around the periphery of the damaged area in all tests. The strain distributions were examined at several stress levels. It was found that in the open hole the normalised strain distribution and load ratios were nearly constant until just prior to failure. For the impacted specimens the normalised strains changed over a wide range of stresses before failure, indicating non-elastic behaviour and stable damage growth. The strains increased more rapidly at high levels,
while the percentage of load carried by the damaged area decreased. Cvitkovich and Jackson observed two distinct failure modes for impacted specimens. The two layer specimen exhibited inwards buckling failure (wrinkle into the panel), whilst the four layered panel exhibited outwards type failure (wrinkle out of the panel). The three layer panel failed in either mode. With the inward type buckle, the buckle increased slowly in height and width and just prior to failure the impact dent increased rapidly leading to failure. (The same trend was observed by Aitken [8] on his thin gauge impact damaged sandwich beams.) With the open holed specimens and the outward buckled specimens, the failure mode consisted of the fibres bucking outwards over a very small region around the periphery of the damage site. This was followed by a zigzag fracture which was initiated from the damage site and grew horizontally across the face sheet. Cvitkovich and Jackson found that wrinkling analysis of Minguet [68] appeared to predict the correct failure mode but needed some modification to predict the influence of face sheet thickness and core material strength. For impact damage specimens they showed that Minguet’s model, which consists of an initial face sheet indentation on a damaged core, appeared to be the most suitable analysis for the inward buckle type failure mechanism. They suggest that Kassapoglou’s [57], [50] model, modelling the indentation as an initial elliptic inclusion of reduced stiffness, may have the potential to predict failure due to outward buckling of the face sheet. This method is also appropriate on open holed specimens with relatively thick face sheets.

Other models

Davies et al. [70] used an explicit numerical method to predict the post impact failure load of their impact damaged panels. During their experiments they noted that the face sheet dipped into the damaged area as a compressive load was applied. The final collapse of the panel occurred as the wrinkle grew rapidly towards the edge of the panel.

Strain gauges were mounted around the outside of the damage site and in the centre of the impact zone. As the compressive stress increased, the strains on the outside of the damage area appeared to increase linearly with the load, suggesting that local buckling is confined to the damaged region. The displacement histories show that the failure initiates from the local indentation, caused by the impact event, and the collapse of the panel is due to instability in the damaged region. Davies et al. classified the failure as being similar to classical micro buckling in a monolithic laminate, supported by anti-buckling guides.

They observed up to a 66% decrease in the undamaged failure load through impact damage for thin-skinned, thick-cored panels. Their numerical model appeared to predict the growth of a localised buckle in the damaged area and also skin fibre damage at failure, as the local wrinkle propagates as a narrow band right across the panel. It was interesting to note that Davies et al. used an explicit Finite Element code for their post-impact damage models, while the current study uses an implicit code. Explicit codes are designed for transient impact type models, and therefore not suitable for static loading situations used in post-impact damage models.

Turnbull [83] and Van Bokhoven [84] developed a numerical model using energy methods and a
linear buckling analysis. Their model consisted of a plate resting on an elastic foundation; in the
centre of the plate there was a damaged area with reduced foundation stiffness. The resultant
governing equations were solved using Matlab. From their model they were able to obtain a plot of
damage size versus wrinkling stress. These showed a rapid drop in residual strength from an
undamaged state to around the critical damage size, and then a constant stress past the critical
damage size. They found that their model under predicted experimental failure loads and suggested
that this was due to the way the core was modelled and the relationship between the core and the
face.

Wadsworth [20] developed a plane strain model that was an extension of Ramastorfer’s [1] model
which was developed for a pure wrinkling case. Wadsworth’s model included two distinct core
areas: a sub-surface damage area and an undamaged area. The failure stress was based on a
linear buckling analysis.

Despite many theoretical and numerical attempts to model wrinkling instability in thin or thick
gauge BVID sandwich panels, the failure process and mechanisms that control the collapse of
panels are in most cases not well documented or understood. Most of the existing models do not
specifically model BVID. They either reduce the face sheet stiffness in the damage zone to facilitate
instability at lower loads or do not model the layer of sub-surface damage with reduced core
properties. Moreover, the existing analytical models are too complex and still require some form of
numerical solution to solve the governing differential equations. Simple expressions are required to
calculate the residual strength of thin-gauge sandwich panels for a range of damage sizes and
depths. Also a better understanding of the failure process that governs the collapse of BVID panels
is needed. To this end, significant emphasis was placed in this impact damage study on
understanding the failure process and using this knowledge to develop concise analytical
expressions to predict residual strengths; these were verified using Finite Element and experimental
results.

**Section 6.3 Impact damage Finite Element models (damage resistance)**

Experimental results showed that a soft body BVID consists of a shallow crater in the face sheet
with a thin (constant thickness) layer of damage beneath the crater. Aitken [8] examined the
problem of soft body impact damage on aircraft flaps, using panels configured in the same way as
wing flaps on 747 aircraft (The same panels are used in this impact damage study).

To gain a greater understanding of the impact event, he developed a non-linear explicit Finite
Element model. He noted that in both real life experiments and the numerical model, damage
consisted of shallow crater-like deformation.

**6.3.1 Current impact model**

A similar explicit model was developed using Ansys / LS-Dyna, a package which solves transient
dynamic problems, in particular impact events. LS-Dyna is used extensively in the automotive
industry to simulate car impact tests.
The following model was developed using axi-symmetric PLANE 162 elements. A crushed foam core model was used to simulate crushing of the core. A 100mm diameter projectile, of similar density to ice, was fired at the panel at 33m/s. Figure 6-8 to Figure 6-13 show comparisons between rigid and soft body impact, using Ls-Dyna and experiments. As noted in section 6.2.1, rigid body impact tends to leave a defined crater in the surface and a parabolic shaped crushed core beneath the face sheet. In contrast, soft body impact leaves a constant depth of damage with a shallow crater in the face sheet. In practical terms, damage depth and damage diameter (Section 6.4.6) have the greatest influence on the failure loads, meaning in reality both rigid and soft body impact damage can produce similar post-impact failure loads providing the depth of damage and damage size are comparable.

For further diagrams showing the complete impact event (using Ansys modelling) refer to Appendix F.
Section 6.4 Predicting residual strengths using analytical and Finite Element models (damage tolerance)

This section develops analytical and numerical models to predict the load carrying capacity of flap panels through a range of impact damage sizes. Before these analytical and numerical models are developed, it is important to understand how the load carrying capacity is expected to change with damage size. The best indication of this comes from existing research by Aitken [8]. Panels identical to those used in this project failed due to wrinkling at approximately half the load of undamaged panels. Damage sizes ranged in diameter from 71mm to 141mm ($3960\text{mm}^2$ – $15604\text{mm}^2$ area) and the respective reductions in the load capacities ranged from 53% to 45% of the undamaged panel. When a best fit trend line is constructed through the data points, it shows a gradual decrease in load carrying capacity for increasing damage sizes. By extrapolating this line to zero damage size, it should cross at 100% (the load carrying capacity of an undamaged panel), but in fact gives a value of approximately 58%. This implies a change between 71mm damage diameter sizes and undamaged panels. The next important question to answer is, how do the rapid onset (below critical damage size) and gradual (above critical damage size) decrease in residual strength co-exist in the residual strength plot.
Section 6.4: Predicting residual strengths using analytical and Finite Element models (damage tolerance)

6.4.1 The effect of damage size (diameter)

The literature review showed that panels have a critical wavelength that can change depending on the damage depth. For this argument we assume an infinite depth of damage. This wavelength is approximately found (for infinite damage depth) by minimizing the critical wrinkling stress (Equation (6-2)) in-terms of wrinkle length and using damaged core properties. In most situations the wrinkle wavelength will tend towards the critical wavelength since this requires the least amount of energy or load. This will happen only when the damage size is greater than or equal to twice the critical wavelength. This damage diameter is known as the critical damage size, which is equivalent to the natural wavelength of the panel.

![Figure 6-15 - Three classifications of damage through the range of damage sizes](image-url)
Figure 6-16 – Formation of wrinkles in two different damage sizes. This figure shows how a decreasing damage size will reduce the wrinkle wavelength and increase the residual strength of the panel. When damage is larger than the critical wavelength multiple wrinkles form with minimal resistance. As the damage size decreases to approximately the critical wavelength, the buckling load increases due to more edge restraint.

6.4.1.1 Damage larger than the critical size

When the damage is sufficiently large enough the damage area behaves like an infinitely long beam and multiple wrinkles form at the critical wavelength and at the minimum possible load. The boundary at the damage edge site has a minimal affect the formation of the wrinkles. Considering the boundary conditions on the centre most wrinkle, the wrinkle is essentially pin supported as it forms as part of a repeating sine wave. Because this internal boundary condition stays the same over a large range of damage sizes and because this wrinkle is only marginally affected by the boundary of the damage site the wrinkling load is constant in this region. This wrinkling load will start to increase as the damage size reduces towards the critical wrinkling wavelength surrounding undamaged area resists formation of the wrinkle.

6.4.1.2 Damage near the critical size

As the damage nears the critical damage size only one wrinkle will form at a full wavelength. Due to the surrounding undamaged facesheet the wrinkle starts to see rotational restraint and the wrinkles edges become partially supported. The increased edge resistance counteracts any bending of the facesheet and increases the wrinkling load.

6.4.1.3 Damage smaller than the critical size

Once the damage becomes smaller than the critical wavelength the wrinkle will still try to form at the critical wavelength even though it maybe being forced to take-up a smaller wavelength than ideal. This increases the wrinkling load significantly because the effective core support increases. As the damage size decreases the rotation of the boundary will increase and the edge support softens. While wrinkling is the predominant form of failure, if the damage is too small the panel can fail due to another mechanism. This will be a combination of wrinkling / core crushing in the undamaged regions and wrinkling in the damaged area. Core crushing is caused by excessive deformation of the undamaged core around the periphery of the damaged area as a result of the increased moment generated as the damage size decreases. This type of failure is prevalent in undamaged panels, where panels are shown to fail at half the wrinkling instability stress as a result of local core crushing.
6.4.2 The effect of damage depth

The second contributing factor is the damage depth, which has a large bearing on the wrinkling stress. As the damage size decreases, the undamaged core beneath the crushed core layer starts to contribute to the stiffness of the foundation. For larger size damage (with damage depths up to 5mm) this effect is small. When the damage depth approaches 2mm this effect becomes prominent; for example the increased foundation stiffness may double the wrinkling stress found when the damage layer is full core depth.

6.4.3 Summary of residual strength behaviour

The above situations can explain the observed trend in the data of a rapid onset then gradual decrease in residual strength with increasing damage sizes. Where damage is lower than the critical damage size, wrinkles are forced to form at shorter wavelengths than the critical wavelength because of the edge restraint from the undamaged area. This requires much higher energy and therefore increases the residual strength of the panel. The damage depths also tend to be smaller in this region, which also increase the residual stress. Where damage is larger than the critical damage size, the wrinkling load is essentially constant because the wrinkle wavelengths are similar to the critical wavelength and the wrinkles are not restrained by the edge of the damage site. The low failure loads observed in this area can be attributed to the central controlling wrinkles forming at the critical wavelength for the range of damage sizes. The marginal decrease in the stress can be attributed to a slight decrease in support from the undamaged area and a small increase in the depth of the damage layer.

Having established the possible behaviour of the damaged panels when loaded, the failure loads must then be determined. This has been achieved using a simplified analytical solution, which is verified against more complex numerical non-linear Finite Element models.

6.4.4 Damage tolerance (residual strength) Finite Element models

Non-linear Finite Element models were successfully used in Chapter 5 to understand the failure mode and to predict the failure loads of undamaged panels. Similar types of models were used in this section, again developed in Ansys. These models were set up using Ansys’s internal programming language (APDL), which allows various parameters such as damage size to be changed and multiple configurations to be run with ease.

Predicting the failure stress was completed using two different types of models. The first model was based around solid / shell three dimensional elements and geometry, and was most representative of the physically impacted panel. The second type of model used two dimensional plane-stress elements. This type of model assumes that this impact dent is through width and the impact area controls the failure stress of the panel.

Both types of models consisted of a section of composite panel with an impact dent of a known diameter and depth. Beneath the impact dent was an area of crushed core, consisting of a section
of core with a reduced stiffness.

This reduced core stiffness was originally investigated by Aitken [8] through a series of flatwise compression experiments, where he found that the stiffness of this crushed region was about 40% of the original stiffness. Experiments in this chapter showed that this value lies between 7.5-10%, (considerably less than the values found by Aitken) and has a significant effect on the overall failure and buckling stresses. Shipsha et al. [80] showed the value for Rohacell WF51 foam core can be as low as 3%, compared to the 7.5-10% value for Nomex honeycomb core.

When these panels exhibit soft body impact damage, it is common to have a small dent left in the surface. Various studies completed within the University of Auckland ([8];[85];[84];[83]) have shown that surface dents can be as much as 0.5mm.

Both types of models have the ability to incorporate an impact crater into the upper surface of the model. Using a function within Ansys called “upgeom”, the impact dent can be superimposed into the impacted surface of the model. The initial shape of the dent was taken from a uniformly distributed pressure load applied to the area of damage.

The same function was used to apply perturbation to the structure from an eigenvalue buckling solution, a necessity when solving the model using a non-linear buckling approach.

### 6.4.4.1 Solid three dimensional model

The panels were loaded using a pure bending moment which put the top face sheet into pure compression and the bottom face sheet into pure tension. This loading scenario was similar to the loading experienced by a panel during a four point bend test, where a pure moment is applied between the two inner supports of the loading rig. The bending moment was applied to the Finite Element model through a varying pressure load on one end, and fixed restraint on the opposite end. Both models had steel plates fastened to either end to transfer the load through to the panel and facings.

The solid model has been designed to replicate closely a physically impacted damaged panel, with circular damage, and an impact dent in the surface skin. The model consisted of a circular region of damaged core surrounded by undamaged core, and was constructed using map meshing. Both the damaged and undamaged cores are defined using orthotropic material properties. For damaged core, these properties are calculated by multiplying the undamaged properties by a reduction factor, in this case 8% for HRH10-1/8-3.0 Nomex core (Section 6.6.1.2).

SOLID 185 linear elements with enhanced strain formulation were used to model the core. Enhanced Strain Formulation (Incompatible Modes, Assumed Strain) add internal degrees of freedom to lower-order quad/hex elements. The displacement gradient tensor is modified with these extra 'enhanced' terms, hence the name “Enhanced Strain”. Enhanced Strain elements are useful when shear locking is encountered (e.g., bending-dominated problems), which is common in low order elements. Enhanced Strain Formulation removes the need for higher order elements, which allows a high degree of freedom model to be solved using a non-linear solver on a standard work station.
The face sheets and end plates were modelled with SHELL 181 elements, a lower order shell element; these were meshed over the top of the SOLID 185 elements, on the upper and lower surfaces of the panel and on either end of the panel to model the face sheets and end plates respectively.

**Figure 6-17 – Three dimensional Finite Element damage model showing a section of damage core of finite depth in amongst undamaged core**

### 6.4.4.2 Two Dimensional Model

The two dimensional model was based around plane-stress solid elements. The PLANE 183 element is a higher order two dimensional solid element which works in plane-strain, plane stress or axi-symmetric. This model was similar to the two dimensional undamaged model used for the regression study in Chapter 2.

The sandwich beam model consisted of two face sheets bonded to an orthotropic core. Within the core was a layer of damage, just below the top face sheet in the centre of the panel. This layer had degraded properties to represent a layer of crushed core. For the HRH10-1/8-3.0 Nomex core used in this chapter, the damaged core modulus values were again degraded to 8% of their original stiffness (Section 6.6.1.2). At either end of the panel were two solid blocks, designed to transfer the load through to the structure. In a similar fashion to the solid model, impact dents could be added to the structure using Ansys's “upgeom” command. The shape is taken from a pressure loading applied to the damaged area.

The structure was loaded by a pure bending moment applied to the panel through a variable pressure load on one of the solid blocks. On the other end was a built in constraint. Figure 6-18 and Figure 6-19 are plots of linear and non-linear wrinkling in plane-stress impact models.
6.4.4.3 General solution procedure for both models

A pressure load was first applied to the impact area and some of this deformed shape was used to tweak the elements and create an impact crater in the damage area. Eigenvalues were then extracted from the model and some of this buckled shape was used to perturb the existing Finite Element mesh for the non-linear solution.

Non-linear material properties were assigned to the face sheet and the undamaged core. These were bilinear-isotopic-plasticity models (elastic and perfectly plastic zones) which represent crushing and fracturing of the core and face sheets respectively.

The model was solved using a Newton-Raphson iterative solver with Auto-time stepping activated. The final failure stress was determined from this solution at the point where the model fails to converge.

6.4.4.4 The material properties

Core properties were based on Hexcel HRH-10-1-8-3.0 Nomex honeycomb core and face sheet properties are based on BMS-79 prepreg satin weave (see Table 1).
### Properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_f$</td>
<td>25.5 GPa</td>
</tr>
<tr>
<td>$G_{xz}$ or $G_c$</td>
<td>41 MPa</td>
</tr>
<tr>
<td>$G_{yz}$</td>
<td>24 MPa</td>
</tr>
<tr>
<td>$E_z$ or $E_c$</td>
<td>138 MPa</td>
</tr>
<tr>
<td>$E_x$</td>
<td>3 MPa</td>
</tr>
<tr>
<td>$E_y$</td>
<td>1.5 MPa</td>
</tr>
<tr>
<td>$G_{xz}$ (Damaged core - 8% of undamaged)</td>
<td>3.28 MPa</td>
</tr>
<tr>
<td>$G_{yz}$ (Damaged core - 8% of undamaged)</td>
<td>1.92 MPa</td>
</tr>
<tr>
<td>$E_z$ (Damaged core - 8% of undamaged)</td>
<td>11.04 MPa</td>
</tr>
<tr>
<td>$E_x$ (Damaged core - 8% of undamaged)</td>
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</tr>
<tr>
<td>$E_y$ (Damaged core - 8% of undamaged)</td>
<td>0.12 MPa</td>
</tr>
<tr>
<td>$t_f$ (Thickness of face sheet)</td>
<td>0.47mm</td>
</tr>
<tr>
<td>$t_c$ (Thickness of core)</td>
<td>25.5 mm</td>
</tr>
<tr>
<td>$v_{xy}$ (Poisson’s ratio face sheet)</td>
<td>0.3</td>
</tr>
<tr>
<td>$v_{xy}$ (Poisson’s ratio core)</td>
<td>0.49</td>
</tr>
<tr>
<td>$\sigma_{crush}$ (Stabilized Crush strength of core)</td>
<td>2.3 MPa</td>
</tr>
<tr>
<td>$\sigma_{crit}$ (Compressive failure strength of face sheet)</td>
<td>517 MPa</td>
</tr>
</tbody>
</table>

### Table 6-1 – Properties for HRH10-1/8-3.0 core and BMS 8-79 1581 glass face sheet

### 6.4.4.5 Results

Table 6-2 and Figure 6-3, show non-linear and linear stresses results for the plane stress and the solid impact models. These results demonstrate failure stresses over a range of damage sizes from 0mm (undamaged) to 150mm diameter. The solid and plane stress non-linear wrinkling stresses were also plotted against damage size in Figure 6-20. The damage depth was based on an experimental value for larger damage, and on estimated values for sub-critical damage. Some of the main results are listed below for the two main regions of damage.

**Damage larger than twice the critical wavelength**

- A 7% difference between the linear and non-linear wrinkling stresses exists in the plane stress model, with damage larger than 50mm. This difference is due to 0.05mm of perturbation.
- The linear and non-linear wrinkling stresses were identical in the three dimensional solid model when damage was larger than 50mm.

**Damage smaller than twice the critical wavelength**

- The percentage difference between the linear and non-linear wrinkling stresses increased in both models as the damage size got smaller than twice the critical wavelength.
Wrinkles no longer form at their ideal wavelength due to more resistance from the outlying undamaged area. This led to a greater amount of instability in the system and a reduction in the non-linear or physical failure load.

As the damage size nears the undamaged state, the stresses induced on the periphery of the damage sites increases. This led to crushing of the core and premature failure as shown in the undamaged beams in Chapter 5.

**Plane stress non-linear results**

<table>
<thead>
<tr>
<th>Damage Width</th>
<th>Damage depth</th>
<th>Dip</th>
<th>Perturbation</th>
<th>Damage core ratio</th>
<th>Linear buckling stress</th>
<th>Non-linear buckling stress</th>
<th>% of linear buckling stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td></td>
<td>(MPa)</td>
<td>(MPa)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td>389.4</td>
<td>245.3</td>
<td>63%</td>
</tr>
<tr>
<td>15</td>
<td>1.5</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>199.9</td>
<td>178.3</td>
<td>89%</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>175.2</td>
<td>159.5</td>
<td>91%</td>
</tr>
<tr>
<td>25</td>
<td>2.2</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>161.0</td>
<td>145.4</td>
<td>90%</td>
</tr>
<tr>
<td>30</td>
<td>2.5</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>150.4</td>
<td>139.8</td>
<td>93%</td>
</tr>
<tr>
<td>40</td>
<td>2.7</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>141.7</td>
<td>131.7</td>
<td>93%</td>
</tr>
<tr>
<td>50</td>
<td>3</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>133.1</td>
<td>123.7</td>
<td>93%</td>
</tr>
<tr>
<td>75</td>
<td>3.5</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>123.1</td>
<td>115.2</td>
<td>94%</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>116.2</td>
<td>108.0</td>
<td>93%</td>
</tr>
<tr>
<td>150</td>
<td>4.5</td>
<td>0.1</td>
<td>0.05</td>
<td>0.08</td>
<td>110.7</td>
<td>102.9</td>
<td>93%</td>
</tr>
</tbody>
</table>

*Table 6-2 – Results for two dimensional plane stress non-linear damaged model*

**3D Solid model non-linear results**

<table>
<thead>
<tr>
<th>Damage Diameter/Width</th>
<th>Damage depth</th>
<th>Dip</th>
<th>Perturbation</th>
<th>Damage core ratio</th>
<th>Linear buckling stress</th>
<th>Non-linear buckling stress</th>
<th>% of linear buckling stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td></td>
<td>(MPa)</td>
<td>(MPa)</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>192.2</td>
<td>166.9</td>
<td>87%</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>178.0</td>
<td>154.5</td>
<td>87%</td>
</tr>
<tr>
<td>30</td>
<td>2.5</td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>157.8</td>
<td>151.2</td>
<td>96%</td>
</tr>
<tr>
<td>50</td>
<td>3</td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>135.3</td>
<td>135.3</td>
<td>100%</td>
</tr>
<tr>
<td>75</td>
<td>3.5</td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>119.7</td>
<td>119.7</td>
<td>100%</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>118.9</td>
<td>118.9</td>
<td>100%</td>
</tr>
<tr>
<td>150</td>
<td>4.5</td>
<td>0</td>
<td>0.05</td>
<td>0.08</td>
<td>117.6</td>
<td>117.6</td>
<td>100%</td>
</tr>
</tbody>
</table>

*Table 6-3 – Results for three dimensional non-linear damaged model*
Differences between the model types

The results have shown that the three dimensional and the two dimensional plane-stress models behave in very similar ways, and predict similar linear and non-linear failure loads. This is because the collapse of the structure is controlled by instability in the damaged area and the centre most wrinkle. Regardless of whether the damage is full width or contained within a circular area, the centre most wrinkle will trigger overall instability of the structure and the collapse of the panel. This wrinkle triggers instability in the panel at approximately the same time with both model types. Slight differences in the loads can be attributed to the way that wrinkles form and the resulting instability in the structure. With circular damage, the outer most wrinkles in the wrinkle pattern are more restricted by the edges and are smaller because wrinkles only form in a circular area. This is more stable than a structure with full-width damage and, as a consequence, requires slightly more energy to create overall instability in the structure. In reality, the panel should fail just before the linear load due to imperfections in the structure. A comparison is made between the two 3D solid models and 2D plane stress models in Figure 6-23.
Chapter 6: Impact damaged honeycomb sandwich panels

3D circular damage model
Standard configuration, 100mm dia damage- 4mm deep
Linear buckling load – 119MPa
Non-linear buckling load - 118MPa (99% of linear load)

3D full width damage model
Standard configuration, 100mm damage length- 4mm deep
Linear buckling load – 117MPa
Non-linear buckling load - 100MPa (86% of linear load)

2D plane stress damage model
Standard configuration, 100mm damage length- 4mm deep
Linear buckling load – 116MPa
Non-linear buckling load - 108MPa (93% of linear load)

Figure 6-21 – Direct comparison of wrinkling loads (liner and non-linear) for three different configurations of Finite Element models

Due to the similarity in the results, the two dimensional plane stress model can be used instead of the more complex three dimensional model because failure loads are controlled by the damage diameter and depth and not the damage shape. It should be noted that with a plane stress element formulation, the damage is assumed to be full width and not circular in shape. The plane stress model was used for all comparative studies from this point onwards because:

- It had a faster solution time
- It showed the same trend as the 3D circular damage model.
The solid circular model failed at the linear buckling load, which in theory is marginally too high. The linear wrinkling loads and the non-linear wrinkling loads are approximately the same in all three cases, suggesting that the failure is caused by instability, and is controlled by the length of the damaged area and not influenced by the shape of the damage. The plane stress model uses higher-order elements while the solid model uses lower-order brick elements with reduced integration. The solid model is more susceptible to numerical issues such as volumetric locking. The plane stress model lies between the two solid models so will give an average value. The plane stress model can be compared directly to the analytical models.

**Summary of key results / observations from the finite element models**

**Linear and non-linear failure loads**

Perhaps the most significant finding from this study was the fact that both linear and non-linear wrinkling loads are approximately the same, as the internal stresses are not high enough to cause core crushing or some other non-linear behaviour caused by wrinkles straining the core or face sheet further. This would suggest that the panel is failing due to instability in the damaged area and not due to core crushing as discovered with undamaged panels in Chapter 5. This was confirmed by examining the internal stresses in the panel, at the non-linear failure load. These stresses were found to be lower than the collapse stress of the core and the face sheet. This finding means that linear wrinkling models can be used to find failure loads of BVID panels.

Another significant finding relates to the wrinkle which controls the failure load. The central wrinkle was found to govern the collapse of the panel. This wrinkle will tend to form at the lowest load and will trigger the instability. This is because it generally forms above the deepest area of crushed core (least foundation support) and the furthest distance away from the surrounding undamaged area (least resistance). In most situations the deepest section of crushed core will be in the centre of the damaged area, as solid impact damage is typically parabolic in shape. This finding also means that the edge effects on wrinkles (these include an increase in core support around the periphery of the damage site, and an increase in bending resistance from the undamaged facesheet around the boundaries.) from the surrounding undamaged area are minimal in certain situations. For example, when multiple wrinkles form in larger damage sizes, the edge effects are inconsequential as these do not affect the central wrinkle. It is only when the damage size approaches the critical wrinkle wavelength that edges of the damage site, and the increased relative stiffness of the facesheet to damage length, affect the load that the wrinkle forms at.

Another finding relates to the impact dent, which was added to the model before loading. Through testing it was found that this initial impact dent had minimal effect on the overall wrinkling stress, providing the final thickness of the damage was comparable to the non-dented model. This dent actually reduces the thickness of the damage zone. Because of this thickness change and increased foundation support, the wrinkling stress increased. However, the same load was found by simply leaving the dent out and reducing the thickness of the damage core by the depth of the dent. For
this reason, the dent was left out of most of the models and instability in the non-linear models was triggered by perturbing the mesh using the eigenvalue mode shape. This finding shows that model is most sensitive to the depth of damage in the centre of the damage site and not shallow deformities in the surface.

Effects of different damage profiles

A comparison is made between different damage profiles to see to how much damage profile affects the wrinkling stress. In this study the depth at the centre of the damage site was held constant in the three damage profiles shown in Figure 6-22. The results in Table 6-4 show a 5% difference between these profiles. This slight difference is attributed to a small increase in foundation stiffness for the wrinkles surrounding the central wrinkle. With such a small increase in load the result confirms that wrinkling loads are controlled by the deepest section of damage and the single wrinkle that forms over the top of it. Because the constant damage depth model accounts for both soft and ridged body impact, and predicts the lowest possible wrinkling stress, this damage profile was used for predicting failure loads and has formed the basis of all analytical models in this chapter.

![Figure 6-22 – Comparison of different damage profiles. The constant thickness model is used in this chapter as it gives a conservative estimate of wrinkling stress](image-url)
Table 6-4 – Comparison of wrinkling stresses from the three damage profiles shown in Figure 6-22. Results show a 5% difference in the results caused by increased foundation stiffness on the outlying wrinkles. This result suggests that the center most wrinkling dominates the failure load.

Wrinkle wavelength

Displacement plots of the linear buckling modes are shown in Figure 6-23. These plots prove that the wrinkling wavelength decreases with decreasing damage sizes. This is caused by two factors:

1. As the damage size is reduced, the wrinkle is forced to form at smaller wavelengths than its critical-natural wavelength; this leads to an increased wrinkling stress.
2. The second cause is more subtle and is related to the damage depth. As the damage depth decreases with decreasing damage diameters, the effective foundation stiffness increases. With thinner layers of damage, the support from the undamaged core is more pronounced, giving a higher effective modulus value. The effective foundation modulus is controlled by a combination of damaged and undamaged cores, with the relative proportions of each giving this effective value.

Failure mechanisms

The failure mechanism in BVID panels is a classic case of wrinkling instability. It is a direct result of the wrinkle increasing in amplitude, and the damaged area becoming unstable and losing stiffness. Davies et al. [70] also noted that the collapse of the panel was due to instability and this instability was confined to the damaged area.

The theorised failure mechanism is described below:

1. In most cases the wrinkle starts small and then grows rapidly as it reaches the collapse strength. As the wrinkle grows in amplitude the wrinkle tips begin to crush the undamaged core
which surrounds the damaged area. The combination of both of these causes the damaged area to become unstable and decrease rapidly in stiffness. A similar type of behaviour was noted by Aitken [8], Turnbull [83] and Van Bokhoven [84] in their experimental work, which measured vertical displacements as load was ramped up. In experiments Zenkert et al. [67] noted a sudden increase in out-of-plane displacement in the BVID damaged area with no significant change in load. This would suggest a large instability in the centre of the panel due to gross deformation.

2. This loss of stiffness leads to the peripheral area of the panel bearing extra load and a rapid growth of the wrinkle tip across the panel. This leads to fracturing of the face sheet and final collapse of the structure due to gross deformation.
Section 6.4: Predicting residual strengths using analytical and Finite Element models (damage tolerance)

Figure 6-23 - Illustrations of the various eigenvalue solutions for the range of damage. Notice how the wrinkling wavelength changes with small damage sizes.

Figure 6-24 – Non-linear buckling three dimensional damaged model
6.4.5 Effect of damage depth and damage size on wrinkling stresses

From the Finite Element work it was established that damage size has a significant effect on the wrinkling stress. Failure stresses are approximately 50% of the undamaged panel for damage larger than twice the critical wrinkling size. The failure stress climbs rapidly back to the undamaged stress for damage smaller than this size. Two main factors influence the wrinkling stress: damage diameter and wrinkling wavelength. A simple plane stress model (Figure 6-25) was created to investigate the effects of these two parameters. The model was loosely based around the classical wrinkling models, which consist of one face sheet and a section of core with a symmetry condition on the base. Unlike previous Finite Element models, this model was loaded in direct compression to mirror the loading conditions of the analytical models. Two end plates were mounted on either end of the panel to transfer the load evenly through the structure. The model had an inbuilt layer of damage which could be adjusted in depth. The depth of the core was also increased until there was no further influence on the wrinkling stress. At this point it was considered to be infinitely deep. The same results are also found using the plane stress bending model in section 6.4.4.2.

*Figure 6-25 – Plane-stress Finite Element half model used to validate the theory*
Section 6.4: Predicting residual strengths using analytical and Finite Element models (damage tolerance)

Table 6-5 – Shows the breakdown of the problem into a components of wrinkling stress: damage depth and damage diameter (for 50mm dia and 4mm damage depth)

<table>
<thead>
<tr>
<th>Damage Dia</th>
<th>Damage Depth</th>
<th>Wrinkling Stress (FE Model)</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mm)</td>
<td>(mm)</td>
<td>(MPa)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>200 (infinite)</td>
<td>71.9</td>
<td>Starting point</td>
</tr>
<tr>
<td>2</td>
<td>200 (infinite)</td>
<td>114.1</td>
<td>Contribution damage depth</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>77.9</td>
<td>Contribution damage size</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>121.2</td>
<td>Combined</td>
</tr>
</tbody>
</table>

Table 6-5 shows typical results from this study. This is illustrated in Figure 6-26. The study looked at the wrinkling stress for a panel with a 50mm diameter damage and 4mm deep damage area. This combination gives a wrinkling stress of 120MPa.

The wrinkling stress is also found by splitting the problem into two parts. The first row shows the wrinkling stress for a beam with infinitely thick damage and a 200mm damage size, (infinitely long). This is the lowest possible stress for this combination of materials, so it is used as the starting point for these models. Row 2 shows the contribution due to the damage depth. In this case it is 43MPa.
This is based on an infinitely long beam with a finite damage depth. The third row shows the
centration due to damage diameter, which is 6MPa. This is the result for a finite panel length and
infinitely deep damage. Adding the two contributions to the starting point gives a stress of 120MPa.
There is a 1MPa difference when both parts are solved simultaneously, meaning that the problem
can be broken into two parts, (damage length and damage depth). These components can be
added together, making the analytical derivation simpler (Figure 6-26).

Table 6-6 and Table 6-7 are more examples showing how damage can be split into two parts and
combined to give the overall solution. Table 6-6 is for 30mm damage diameter and 2mm damage
depth. Table 6-7 is for 100mm damage diameter and 7mm damage depth.

<table>
<thead>
<tr>
<th></th>
<th>Damage Dia</th>
<th>Damage Depth</th>
<th>Wrinkling Stress (FE Model)</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mm)</td>
<td>(mm)</td>
<td>(MPa)</td>
<td>(MPa)</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>200 (infinite)</td>
<td>Infinite</td>
<td>71.9</td>
<td>Starting point</td>
</tr>
<tr>
<td>2</td>
<td>200 (infinite)</td>
<td>2</td>
<td>146.8</td>
<td>74.83 = (A2-A1) Contribution damage depth</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>Infinite</td>
<td>89.7</td>
<td>17.8 = (A3-A1) Contribution damage size</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>2</td>
<td>164.5</td>
<td>164.5 = (A1+B2+B3) Combined</td>
</tr>
</tbody>
</table>

Table 6-6 – Shows the break down of the problem into components of wrinkling stress: damage depth and damage diameter (for 30mm dia and 2mm damage depth)

<table>
<thead>
<tr>
<th></th>
<th>Damage Dia</th>
<th>Damage Depth</th>
<th>Wrinkling Stress (FE Model)</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mm)</td>
<td>(mm)</td>
<td>(MPa)</td>
<td>(MPa)</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>200 (infinite)</td>
<td>Infinite</td>
<td>71.9</td>
<td>Starting point</td>
</tr>
<tr>
<td>2</td>
<td>200 (infinite)</td>
<td>2</td>
<td>94.7</td>
<td>22.7 = (A2-A1) Contribution damage depth</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>Infinite</td>
<td>73.2</td>
<td>1.3 = (A3-A1) Contribution damage size</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>7</td>
<td>96</td>
<td>96.1 = (A1+B2+B3) Combined</td>
</tr>
</tbody>
</table>

Table 6-7 – Shows the break down of the problem into components of wrinkling stress: damage depth and damage diameter (for 100mm dia and 7mm damage depth)
Figure 6-27 – Comparison of wrinkling stress contributions from damage depth and damage size. The results are based on the panels used in Table 6-2.

Figure 6-27 shows the contribution of damage depth and damage diameter on the wrinkling stress. The test took the combinations of panels shown in Table 6-2. For these combinations of damage depth and size, the damage depth has the most influence on the buckling stress. Damage size only starts to contribute to the failure stress below twice the critical wrinkling wavelength. This is due to the central wrinkle being forced into an unnatural smaller wavelength and being influenced by the surrounding undamaged area.

6.4.6 Predicting residual strength using analytical models

This section develops a simple yet effective closed form analytical model to predict impact failure loads across a range of damage sizes. In the previous section it was found that the Finite Element results can be split into two parts: damage depth and impact size. These parts can be added at a later stage to give the final combined solution, making the formulation of the new model much simpler as the problem can be broken down.

Added to this, the non-linear models in Section 6.4.4 showed that linear models can be used to predict failures stresses for impact damage, because the panels were failing due to instability. This is a significant finding as linear models are much simpler to solve, and can be developed analytically without needing an iterative solver.

With these two vital pieces of information, two different linear models were developed. One
investigates the contribution to the wrinkling stress due to damage depth, and the other examines the contribution due to damage size.

6.4.6.1 Contribution due to damage size

Hetenyi [86] was one of the first authors to investigate the response of beams on elastic foundations. His book covered various loading scenarios including buckling instability of beams under a range of boundary conditions.

Hetenyi showed that the buckling load for an infinitely long beam, on an elastic foundation, can be expressed as

\[ N_{cr} = 2\sqrt{kEI} \]  \hspace{1cm} (6-3)

This is simply the Winkler foundation model.

where: \( N_{cr} \) is the buckling load, \( k \) is the foundation stiffness, \( E \) is the modulus of the face sheet, \( I \) is the second moment of area of the face sheet.

A similar expression was found for a finite length beam with built-in-ends:

\[ N_{cr} = \frac{4\pi^2 EI}{l^2} + 2\sqrt{kEI} \]  \hspace{1cm} (6-4)

Both expressions are similar except for an additional term in front of the Winkler foundation model is the Euler buckling term for a beam with built-in ends. The second half of the expression is the standard wrinkling expression for an infinitely long beam on an isotropic core. Hetenyi's model reiterates the findings in the Finite Element models, by showing that the impact damage problem can be split into two parts. The first half of Equation (6-4) calculates the foundation support and wrinkling for an infinitely long structure, while the second half calculates the shortening of the damage size (which can be expressed simply with the Euler buckling expression). This is verified at the end of the analytical section.

From Equation (6-4) the contribution due to the end-fixity or the shortening of the wavelength is given by:

\[ \sigma_{\text{damage, size}} = \frac{\pi^2 E I_f^2}{3l_{\text{dam}}^2} \]  \hspace{1cm} (6-5)

This is simply an Euler buckling term for a beam with built-in ends.
6.4.6.2 Contribution due to damage depth

This model follows the classical wrinkling models and is based on an infinitely long beam with a finite damage depth. As the contribution due to damage depth can be split from damage length, the effect of the end restraint through decreasing damage sizes is removed from this formulation. The model below is a continuation of the undamaged differential equation model developed in Chapter 2. For the complete derivation of both models refer to Appendix A.

Hooks law for the core is assumed for the orthotropic core material

$$
\left( \begin{array}{c} 
\varepsilon_x \\
\varepsilon_z 
\end{array} \right) = \left( \begin{array}{cc} 
\frac{1}{E_x} & -\frac{v_{xz}}{E_z} \\
-\frac{v_{xz}}{E_z} & \frac{1}{E_z} 
\end{array} \right) \left( \begin{array}{c} 
\sigma_x \\
\sigma_z 
\end{array} \right)
$$

(6-6)

where it is assumed that \( \sigma_x = 0 \) in the core. This is a valid constraint for Nomex type core.

Since \( \sigma_x \approx 0 \Rightarrow \sigma_z = E_z \varepsilon_z \) and \( \varepsilon_x \approx -\frac{v_{xz}}{E_z} \sigma_z \)

(6-7)

$$
\sigma_z = E_z \frac{\partial w}{\partial z} \quad \text{and} \quad \varepsilon_z = -v_{xz} E_z \Rightarrow \frac{\partial u}{\partial x} = -v_{xz} \frac{\partial w}{\partial z}
$$

(6-8)

Substituting Hooke’s law into the vertical equilibrium equation yields

$$
\frac{\partial^2 w}{\partial z^2} + \left( \frac{G_{xz}}{E_z - v_{xz} G_{xz}} \right) \frac{\partial^2 w}{\partial x^2} = 0
$$

(6-9)

\( w(x, z) \) is assumed to be sinusoidal and decay to 0 at \( z \to \infty \). \( f(z) \) is a decay function derived through equilibrium conditions

$$
w(x, z) = Wf(z) \sin \left( \frac{\pi z}{L} \right) \quad \text{Substituting into equation (6-9) gives}
$$

(6-10)

$$
W \sin \left( \frac{\pi x}{L} \right) \left[ \frac{\partial^2 f(z)}{\partial z^2} - \left( \frac{G_{xz}}{E_z - v_{xz} G_{xz}} \right) \left( \frac{\pi}{L} \right)^2 f(z) \right] = 0
$$

(6-11)

denoting \( \alpha^2 = \frac{G_{xz}}{E_z - v_{xz} G_{xz}} \)

This leads to a general displacement function

$$
w(x, z) = W \sin \left( \frac{\pi x}{L} \right) \left( A e^{-\frac{\pi x}{L}} + B e^{-\frac{\pi z}{L}} \right)
$$

(6-12)
Case 2: Damage to depth h, infinite core

![Diagram](image)

**Figure 6-28 – Case 2: Damage to depth h, infinite core**

(Case 1 – undamaged wrinkling model is described in Chapter 2)

A damaged wrinkling model based on infinitely large damage area and a finite damage depth is used. This expression was specifically developed to predict failure in damaged sandwich panels.

The general expressions for the core deformation in the two regions are:

**Region 1:**

\[ w(x_1, z_1) = W \sin \left( \frac{\pi x_1}{L} \right) \left( A_1 e^{a \frac{z_1}{L}} + B_1 e^{-a \frac{z_1}{L}} \right) \]  \hspace{1cm} (6-13)

**Region 2:**

\[ w(x_2, z_2) = W \sin \left( \frac{\pi x_2}{L} \right) \left( A_2 e^{a \frac{z_2}{L}} + B_2 e^{-a \frac{z_2}{L}} \right) \]  \hspace{1cm} (6-14)

Note that with the assumption of isotropic damage, the resulting ratio \( \alpha \) is the same in 1 and 2.

The boundary conditions for the core are: (note: \( \eta = \frac{E_1}{E_2}, \eta \leq 1 \))

1. As \( z \to \infty \Rightarrow w_1(x, \infty) = 0 \)
2. As \( z \to 0 \Rightarrow w_1(x, 0) = W \sin \left( \frac{\pi x}{L} \right) \)
3. Displacement at the boundary \( w_1(x, h) = w_2(x, h) \)
4. Direct stress compatibility at the boundary \( \frac{\partial w_1(x, h)}{\partial z} = \left( \frac{E_2}{E_1} \right) \frac{\partial w_2(x, h)}{\partial z} \)

Solving the above simultaneously gives the core displacement equations in the two regions.
\[
w(x_1, z_1) = W \sin \left( \frac{\pi x}{L} \right) \left[ \frac{\frac{\alpha \pi}{e^L}}{1 + \left( \frac{\eta + 1}{\eta - 1} \right) e^{\frac{2 \pi h}{L}}} + \frac{\frac{\alpha \pi}{e^L}}{1 + \left( \frac{\eta + 1}{\eta - 1} \right) e^{\frac{2 \pi h}{L}}} \right]
\]

(6-15)

\[
w(x_2, z_2) = W \sin \left( \frac{\pi x}{L} \right) \left[ \frac{2 \left( \frac{\eta}{\eta + 1} \right) e^{\frac{\alpha \pi}{L}}}{1 + \left( \frac{\eta - 1}{\eta + 1} e^{\frac{2 \pi h}{L}}} \right) \right]
\]

(6-16)

The core stress in the damaged area 1 is \( \sigma = E_1 \frac{\partial w}{\partial z} \)

\[E_i = \text{Compressive modulus } E_i \text{ of the damaged core}\]

at the face sheet ( \( z = 0 \) )

\[
\sigma_0 = W \sin \left( \frac{\pi x}{L} \right) E_i \left( \frac{\alpha \pi}{L} \right) \left[ \frac{1 + \left( \frac{\eta + 1}{\eta - 1} e^{\frac{2 \pi h}{L}}} \right) \right] - \left[ 1 + \left( \frac{\eta - 1}{\eta + 1} e^{\frac{2 \pi h}{L}}} \right) \right]^{-1}
\]

\[
\Rightarrow \quad \sigma_0 = W \sin \left( \frac{\pi x}{L} \right) E_i \left( \frac{\alpha \pi}{L} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) \text{ Where } \gamma = \left( \frac{\eta + 1}{\eta - 1} e^{\frac{2 \pi h}{L}}} \right)
\]

(6-17)

Equilibrium of the face sheet is defined by

\[
D \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} - \sigma_0 = 0
\]

\[
D \left( \frac{\pi}{L} \right)^4 - P \left( \frac{\pi}{L} \right)^2 - E_i \left( \frac{\alpha \pi}{L} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) = 0
\]

(6-18)

\[
P = D \left( \frac{\pi}{L} \right)^2 - E_i \left( \frac{\alpha \pi}{L} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right)
\]

The minimum wrinkling load occurs when \( \frac{dP}{dL} = 0 \)

\[
\frac{dP}{dL} = -2 \left( D \left( \frac{\pi}{L} \right)^3 - E_i \left( \frac{\alpha \pi}{L} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) - E_i \left( \frac{\alpha \pi}{L} \right) \frac{\partial}{\partial L} \left( \frac{1 - \gamma}{1 + \gamma} \right) \right)
\]

\[
= -2 \left( D \left( \frac{\pi}{L} \right)^3 - E_i \left( \frac{\alpha \pi}{L} \left( \frac{1 - \gamma}{1 + \gamma} \right) - 4E_i \left( \frac{\alpha^2 \pi}{L} \right) \left( \frac{h}{\pi} \right) \frac{\gamma}{(1 + \gamma)^2} \right) = 0
\]

(6-19)

Unfortunately solving (6-19) directly for \( L \) proved to be too difficult within the time constraints of this project. Therefore it was necessary to use an iterative solver in Mathcad to minimize \( P \) with respect to \( L \) to find the critical wrinkling half wavelength \( L_{cr} \), and then find the critical wrinkling stress by back substituting \( L_{cr} \). The critical wrinkling stress is given by
\[ \sigma_{cr} = \frac{E_f}{12} \left( \frac{\pi}{L_{cr}} \right)^2 - E_i \left( \frac{\alpha L_{cr}}{\pi f} \right) \left( 1 - \gamma \right) \left( 1 + \gamma \right) \] (6-20)

where \( L_{cr} \) is the critical wrinkling half-wavelength found numerically for each case.

where \( \gamma = \left( \frac{\eta + 1}{\eta - 1} \right)^{\frac{2 \pi h}{L_{cr}}} \)

\( \eta = \frac{E_1}{E_2}, \eta \leq 1 \) Stiffness ratio of damaged to undamaged core. \( (E_f = \text{damaged core modulus in the Z direction}, E_i = \text{undamaged core modulus in the Z direction}) \)

In this study the value was experimentally determined as 0.08.

### 6.4.6.3 Comparison of analytical models to Finite Element models

This section tests the two analytical models (damage depth, damage diameter) against the Finite Element models using a linear buckling analysis (the same method used by the analytical model).

Comparisons were made using the plane stress Finite Element half model, which was most representative of the analytical expressions. This model was described in Section 6.4.5. The main features are: single face sheet, symmetry boundary condition on the bottom, pure compression load, finite depth of damage which goes full width and two loading pads on either end.

This design of structure mimics the analytical model which predicts the wrinkling stress due to damage depth. Figure 6-25 shows an illustration of the model.

<table>
<thead>
<tr>
<th>Damage width</th>
<th>Damage depth</th>
<th>Wrinkling Stress Finite Element half model</th>
<th>Wrinkling Stress Analytical model</th>
<th>Critical wrinkling Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mm)</td>
<td>(mm)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(mm)</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>30</td>
<td>77</td>
<td>80</td>
<td>26.3</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>10</td>
<td>88</td>
<td>86</td>
<td>23.6</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>5</td>
<td>108</td>
<td>102</td>
<td>20.4</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>4.5</td>
<td>112</td>
<td>105</td>
<td>20.0</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>4</td>
<td>116</td>
<td>109</td>
<td>19.5</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>3.5</td>
<td>122</td>
<td>114</td>
<td>18.9</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>3</td>
<td>130</td>
<td>121</td>
<td>18.3</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>2.5</td>
<td>139</td>
<td>130</td>
<td>17.6</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>2</td>
<td>150</td>
<td>141</td>
<td>16.9</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>1.5</td>
<td>167</td>
<td>158</td>
<td>15.9</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>1</td>
<td>193</td>
<td>184</td>
<td>14.8</td>
</tr>
<tr>
<td>200 (inf.)</td>
<td>0</td>
<td>426</td>
<td>429</td>
<td>11.4</td>
</tr>
</tbody>
</table>

Table 6-8 – Contribution due to damage depth based on equation (6-20)

Table 6-8 shows the effect of damage depth on the wrinkling stress, with a direct comparison between the analytical model (Equation (6-20)) and the Finite Element model. Through this study it
was found that the analytical and Finite Element models predict similar wrinkling stresses for changing damage depths. This result validates the analytical model and proves that the model can be used to predict wrinkling stresses due to changing damage depths. It is interesting to see how the wrinkling stress increases with decreasing damage depths, from 77MPa at 30mm to 426MPa at 0mm. These limits are also found from the modified wrinkling expression (Chapter 2) with damaged and undamaged properties respectively. The critical wrinkling wavelength from the analytical model is also displayed as a function of damage depth. From these results it appears that the critical wrinkle wavelength reduces with damage depth; this is due to the foundation stiffness increasing with decreasing damage depth. Most of these values are lower than the 25.1mm critical wavelength predicted by equation (6-2) which is based on infinitely thick damaged depth, a value comparable to 26.3mm in Table 6-8.

In Table 6-9 the core’s influence on the wrinkling load was removed (by making the core effectively infinitely deep), which leaves just the increase in stress due to the damage size reducing. The analytical and Finite Element results are comparable down to 25mm of damage diameter. Below this point the Finite Element model predicts a lower stress contribution. An explanation of this difference is given in Section 6.4.6.4.

<table>
<thead>
<tr>
<th>Damage width (mm)</th>
<th>Damage depth (mm)</th>
<th>Wrinkling Stress (MPa)</th>
<th>Damage diameter contribution - Finite Element model (MPa)</th>
<th>Damage diameter contribution - analytical model (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>30 (inf.)</td>
<td>137.9</td>
<td>61.37</td>
<td>82.36</td>
</tr>
<tr>
<td>20</td>
<td>30 (inf.)</td>
<td>111.9</td>
<td>35.38</td>
<td>46.33</td>
</tr>
<tr>
<td>25</td>
<td>30 (inf.)</td>
<td>105.1</td>
<td>28.56</td>
<td>29.65</td>
</tr>
<tr>
<td>30</td>
<td>30 (inf.)</td>
<td>94.0</td>
<td>17.46</td>
<td>20.59</td>
</tr>
<tr>
<td>50</td>
<td>30 (inf.)</td>
<td>82.4</td>
<td>5.85</td>
<td>7.41</td>
</tr>
<tr>
<td>75</td>
<td>30 (inf.)</td>
<td>79.0</td>
<td>2.47</td>
<td>3.29</td>
</tr>
<tr>
<td>100</td>
<td>30 (inf.)</td>
<td>77.8</td>
<td>1.28</td>
<td>1.85</td>
</tr>
<tr>
<td>150</td>
<td>30 (inf.)</td>
<td>76.8</td>
<td>0.33</td>
<td>0.82</td>
</tr>
<tr>
<td>200</td>
<td>30 (inf.)</td>
<td>76.5</td>
<td>0.00</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Table 6-9 – Contribution due to damage size based on equation (6-5)

6.4.6.4 Combined analytical model

The components representing the influence of damage size and damage depth are now added to give the overall model

\[
\sigma_{cr,\text{damaged}} = \left[ \frac{E_f}{12} \left( \frac{\pi_f}{L_{cr}} \right)^2 - E_l \left( \frac{\alpha L_{cr}}{\pi_f} \right) \left( 1 - \gamma \right) \right] + \frac{\pi^2 E_l t_f^2}{M_{dam}^2} \tag{6-21}
\]

This section directly compares the combined analytical solution to the complete plane stress bending model and the simplified plane stress compression half model. Equation (6-21) is used for this comparison.
Figure 6-29, Figure 6-30 and Table 6-10 compare the difference between the analytical model and two Finite Element models (bending model and compression model).

**Figure 6-29** – Figure showing a comparison between the combined analytical damage model (6-21) and the compression (Section 6.4.5) and bending (Section 6.4.4.2) Finite Element models (for damage < 50mm).

**Figure 6-30** – Figure showing a comparison between the combined analytical damage model (6-21) and the compression (Section 6.4.5) and bending (Section 6.4.4.2) Finite Element models (for all damage sizes).
Table 6-10 – Shows a comparisions between the combined analytical damaged model (6-21) and the compression (section 6.4.5) and bending (section 6.4.4.2) Finite Element models

<table>
<thead>
<tr>
<th>Damage Width / Diameter</th>
<th>Damage depth</th>
<th>Finite Element plane stress half model</th>
<th>Finite Element plane stress bending model</th>
<th>Analytical model</th>
<th>Critical Wavelength (mm) of Analytical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mm)</td>
<td>(mm)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
<td>(MPa)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>425.7</td>
<td>389.4</td>
<td>429</td>
<td>11.4</td>
</tr>
<tr>
<td>15</td>
<td>1.5</td>
<td>252.2</td>
<td>199.9</td>
<td>240.4</td>
<td>15.9</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>189.5</td>
<td>175.2</td>
<td>187.3</td>
<td>16.9</td>
</tr>
<tr>
<td>25</td>
<td>2.2</td>
<td>174.5</td>
<td>161.0</td>
<td>165.6</td>
<td>17.2</td>
</tr>
<tr>
<td>30</td>
<td>2.5</td>
<td>156.7</td>
<td>150.4</td>
<td>150.6</td>
<td>17.6</td>
</tr>
<tr>
<td>50</td>
<td>3</td>
<td>136.2</td>
<td>133.1</td>
<td>128.4</td>
<td>18.3</td>
</tr>
<tr>
<td>75</td>
<td>3.5</td>
<td>125.2</td>
<td>123.1</td>
<td>117.3</td>
<td>18.9</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>118.1</td>
<td>116.2</td>
<td>110.9</td>
<td>19.5</td>
</tr>
<tr>
<td>150</td>
<td>4.5</td>
<td>112.3</td>
<td>110.7</td>
<td>105.8</td>
<td>20.0</td>
</tr>
<tr>
<td>200</td>
<td>5</td>
<td>108.0</td>
<td>99.1</td>
<td>102.5</td>
<td>20.4</td>
</tr>
</tbody>
</table>

Both the plane-stress half model and the analytical model produce similar results, as they both assume flatwise compression, infinite core depth for an undamaged core and finite depth for the damaged core. They also assume that the face sheet is built in at the edge of the damage site. For these two linear models there was a maximum error of 5%.

Errors start to increase between the plane-stress bending model and the other two models when the diameter of damage reduced below 25mm or approximately 1.5x the natural wavelength (wavelength varies depending on the damage depth). Here, the wrinkle is being forced to take unnatural smaller wavelengths due to the support provided at the periphery of the model. With both the analytical and plane-stress half model the support represents a built-in end. In the bending model the face sheet can still rotate and displace vertically into the edge of the damage site. This decreases the amount of energy needed to form the wrinkle and consequently the wrinkling load. It also allows the wrinkle to form closer to the natural wavelength. Figure 6-33 and Figure 6-34 clearly illustrate this point by showing differences in the way linear wrinkles form with a 15mm length of damage. The buckling mode is quite different between the two Finite Element models; the face sheet is displaced and rotated at the edge of the damage site in the bending model but fixed in the half model.

This is not the case when the damage size is greater than twice the natural wrinkling wavelength (50mm in this case) (Figure 6-31 and Figure 6-32) as multiple waves can form at their natural wavelength. As the wrinkling loads are controlled by the central wrinkle, (not influenced by the edges) all three models predict similar loads for large damage.

To calculate accurate failures loads (similar to the bending Finite Element model) for smaller damage sizes (<1.5x wrinkle wavelength), the analytical model must account for rotation at the edges. The severity of the rotation / moment at the edge varies depending on the diameter.
damage. The resulting formulation is complex, requiring a jump in stiffness at the edges and the
two contributions - damage depth and damage length - to be solved simultaneously. This has been
left for future work as the current model is accurate down to the critical damage sizes. For values
below the critical damages size the results could found by extrapolating back to the undamaged
failure stress, which is calculated using Equations (6-24) or (6-25).

6.4.6.5 Predicting failure stresses in real panels using the combined analytical
damaged model

In reality damaged panels will fail at just below the linear buckling load. This is shown in Table 6-3,
which compares linear with non-linear wrinkling stresses. The non-linear stresses are approximately
90% of the linear stresses. The simplest way to predict the failure load is to multiply the linear
analytical model by 0.95 to give a reduced expression.
For damaged panel:

\[
\sigma_{cr, damaged} = 0.95 \left[ \frac{E_f}{12} \left( \frac{\pi f}{L_{cr}} \right)^2 - E_i \left( \frac{\alpha L_{cr}}{\pi f} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) + \frac{\pi^2 E_f t_f^2}{3 L^2_{dam}} \right]
\]  (6-22)

where \( L_{cr} \) is the critical wrinkling wavelength found by minimising the critical wrinkling expression below in terms of \( L \).

\[
\sigma_{cr, depth} = \frac{E_f}{12} \left( \frac{\pi f}{L} \right)^2 - E_i \left( \frac{\alpha L}{\pi f} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right)
\]  (6-23)

For undamaged panels:

As an estimate of the undamaged stress the following expression can be used. For an in-depth review of wrinkling in undamaged panels, refer to Chapter 5

\[
\sigma_{cr, undamaged} = 0.5 \left( E_f E_{zx} G_{zz} \right)^{\frac{1}{3}} \text{ Using undamaged core properties}
\]  (6-24)

Or a more accurate expression is

\[
\sigma_{cr} = \frac{0.825 \left( E_f E_{zx} G_{zz} \right)^{\frac{1}{3}}}{A_0 \sqrt{E_x G_{zz}} \left( \frac{\pi}{L_{cr}} \right)} \left( \frac{E_f}{6 \sqrt{E_x G_{zz}}} \right)^{\frac{1}{3}} + 1
\]  (6-25)

and an estimate of \( A_0 \) (initial imperfection) is \( \frac{A_0}{t_f} = 0.1 \) [57]

6.4.6.6 Comparison of results between the non-linear Finite Element model and analytical model

The overall correlation between the non-linear wrinkling stresses and the analytical model (6-22) is good. The validity reduces when the damage size is below 20mm or approximately the critical wrinkling wavelength (evaluated for 5mm of damage), due to built-in end problem as stated in Section (6.4.6.4). These models are compared with experimental results at the end of this chapter. Figure 6-35 shows these comparisons.
Comparisons between FE non-linear and analytical failure stresses

Figure 6-35 - Comparison between analytical damage model (equation (6-22)) and the non-linear results (Table 6-2) Ranging from 0mm to 3145mm (200mm Ø)

Section 6.5 Issues with previous research at the University of Auckland

Wadsworth [20, 85] developed a two-layer closed form analytical expression which includes a separate damage region on top of an undamaged section of core. This model was an extension of a model completed by Vonach and Rammerstorfer [1] that predicted wrinkling loads using a single layer of core. Some of the ideas developed by Wadsworth [20, 85], Vonach and Rammerstorfer [1] were different from those found in this chapter. Much of their findings and conclusions were based around the free in-plane modulus (modulus computed with the face sheets detached – Chapter 4). Like Vonach and Rammerstorfer [1], Wadsworth concluded that the discrepancy between existing analytical expressions and experimental results was caused by the exclusion of this in-plane modulus. Chapters 2 and 4 disputed this argument and showed the existing wrinkling models match analytical expressions and not experimental results, when the restrained in-plane modulus was used (modulus computed with the face sheets on). These chapters along with Chapter 5 suggested that the lower than expected failure loads were not caused by the exclusion of the in-plane modulus, but were in fact caused by core crushing as a result of waviness in the facings. Added to this, when the correct restrained in-plane modulus was included in Vonach and Rammerstorfer [1] expressions, the wrinkling stress was about 20% less than the analytical model and equivalent plane-stress Finite Element model. This would suggest that further models (such as Wadsworth’s [20, 85] models) developed using the same principles as Vonach and Rammerstorfer [1] underestimate the wrinkling stress.

Another discrepancy with previous research relates to the damaged core modulus. Aitken [8] suggested that the core modulus should be reduced by 60% (or 40% of the undamaged modulus) in the damaged region to account for reductions in stiffness due to core crushing. Experimental
results within this chapter suggest (Section 6.6.1.2) this value should have been around 90% (or 10% of the undamaged modulus). Differences of this magnitude have a pronounced affect on the overall wrinkling/ failure stress. This value of 40% core stiffness was used by Wadsworth [20, 85] in his numerical and analytical models to compare against experimental values. Using this value of 40%, Wadsworth’s model seemed to match previous experimental results. However, it is thought that this was a coincidence and due to using low in-plane modulus values, a stiffer damaged core value and a model that underestimates linear failure loads, rather than accurate modelling of BVID failure loads. These issues were rectified in the current study.

**Section 6.6 Experimental Work**

A number of experimental laboratory tests were completed to compare theoretical predictions of damage size effects with actual results. The next section will describe the testing equipment, procedure and the results.

### 6.6.1 Test Panels

To simulate impact damage to structures typical of a flap trailing edge, a total of thirty test panels were manufactured. The test panels used were designed to replicate actual flaps and were made using the same materials and manufacturing technique. They were identical to the panels used by Aitken [6].

These panels along with the panels used in the previous chapters (different material lay-up) were fabricated at Air New Zealand Engineering Services, with the support of the composite shop and Air New Zealand, who supplied all materials and the facilities required to fabricate them. The panels were made from the following lay-up and aircraft grade materials:

<table>
<thead>
<tr>
<th>Core</th>
<th>Hexcel HRH 10-1/8-3.0-1”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face sheet</td>
<td>BMS 8-79 1581 Epoxy glass pre-impregnated ply</td>
</tr>
<tr>
<td>Lay-up</td>
<td>Two laminate plies on each side of a Nomex® core. The grain orientation was 0 degrees for the two outside plies and 90 degrees for the two inside plies</td>
</tr>
<tr>
<td>Size</td>
<td>720mm long by 320mm wide</td>
</tr>
<tr>
<td>Manufacturing procedure</td>
<td>Vacuum bag technique</td>
</tr>
</tbody>
</table>

*Table 6-11 – Panel configuration used in the residual strength impact damage tests*

For more details on manufacturing procedures and panel data refer to Appendix C.

### 6.6.1.1 Creating impact damage

To create realistic damage of similar shape and size to actual damage, a large pneumatic cannon was used (Figure 6-36). The cannon consisted of a 35 litre tank which was capable of holding a maximum charge of 150psi. The tank was filled from the main air-supply line and when the desired firing pressure was reached, a 50mm exhaust valve was released and the barrel was supplied with high pressure air. Three different sized barrels with matching sized balls were manufactured to get
a range of damage sizes. The barrels had inside diameters of 30mm, 50mm and 100mm. The majority of the damage was created using the 30mm barrel. Impact balls were made from an Indonesian hardwood which has a similar density to hailstones.

For the purpose of this project a hard body projectile was used compared with a soft projectile used by Aitken [6]. There are a number of reasons for this. Firstly, for the smaller size damage being evaluated, hard body impacts (for example hailstones) are more common than soft body damage. Secondly, it was shown that damage area and damage depth are more important than the profile of damage, as these control the failure load. Finally, a small hard body impactor is simpler to manufacture than a soft body equivalent.

![Impact cannon with small barrel and test panel support frame](image)

The end of the gun was placed 3m away from the front of the panels at an incident angle of 7 degrees. Attached to the end of the barrel were infrared timing lights which were used to measure the speed of the projectile. When the projectile passed through the timing lights, two energy spikes were measured and recorded on a linked oscilloscope. The projectile’s speed was calculated from the elapsed time between the spikes and the distance between the sensors.

The test panels were fixed by their edges in a steel frame to simulate a built-in end condition. The steel frame structure was fabricated from 25x50mm RHS sections. The flaps on an aircraft are supported around three edges with a number of linkages to the wing, with aluminium reinforcement around the periphery. The four edges were clamped in this case because test panels have less stiffness than a large flap section. The panels were bolted in place with 13 bolts around the perimeter of the frame and were held in an upright position (Figure 6-36). The position of the frame relative to the gun was adjusted to get a number of damage sites on the front of each panel.

A number of tests were carried out on the impacted panels to find appropriate impact velocities and pressures to create a certain size damage. The pressures needed to create damage between 20mm and 40mm in diameter ranged from 2psi to 7psi for the smaller barrelled gun. After a number of preliminary tests, it was found that the gun could not create small enough damage. Smaller
damage was created by placing a wooden ball on top of the panel and hitting it with a hammer. This technique was very efficient as it gave easily repeatable damage areas as small as 11mm. A coin was taped on the panel to find the extent of the damage. Where sub-surface damage is present there is a distinct difference in the tapping sound between an undamaged area and a damaged area. By working around the damage site and listening for this difference in sound it was possible to plot the damage site.

### 6.6.1.2 Core Crushing

A cross section of the damage revealed parabolic core crushing. The maximum damage depth for a ridged impactor was located in the centre of the defect and tapered to low levels of damage in the surrounding area. This is in contrast to a soft body impact, which tapers rapidly at the edges and is almost a constant depth through the remaining damaged area. After impact, the impacted face sheet had returned to its original position. Figure 6-46 shows a cross section of an impacted test panel which has a layer of crushed core below the surface.

### 6.6.2 Compression and four point bend testing

#### 6.6.2.1 Specimen preparation

A total of 26 manufactured panels were used for the tests. A number of damage sites were created on each panel to reduce the amount of wastage. In a preliminary study which looked at the test specimen size, it was found that panels needed to be as long as possible so that failure occurred through compressive instability and not core shear failure at the load points. The final test specimens were 720mm long and were approximately 106mm, 150mm, 210mm and 320mm wide. The width of each panel was determined by the size and number of damage sites with a damage size to specimen width ratio of 30% [87]. After the impacts were created, the panels were cut into the final specimens. A total of 65 test specimens were made.

*Figure 6-37 - Impact test specimens*
6.6.3 **Crush tests – determining modulus values**

This section develops an experimental method to determine the core’s out-of-plane properties in the damaged area. To find the reduction in the effective stiffness a simple test was performed. First an area of barely visible impact damage was created in the specimen by hitting the panel with the device shown in Figure 6-38.

![Device that was hit against panel to create BVID](image1)

![Picture showing BVID (Aitken [8])](image2)

A small probe was then pushed into the damaged area and a force deflection curve was recorded (Figure 6-40). The same probe was pushed into an area of undamaged core (Figure 6-41) and a second force deflection curve was recorded. The reduction in stiffness was calculated from difference in the gradients of the damaged and undamaged force deflection plots (Figure 6-42).

![Probe being pushed into damaged core](image3)

![Probe being pushed into undamaged core](image4)
Damaged / Undamaged Stiffness

Shipsha et al. [80] employed a slightly different technique while examining crushed core properties for WF51 foam core. For their tests they placed a block of foam in between two aluminium chucks, crushed it fully, then reloaded it and compared the load-deflection curve of the undamaged core to the reloaded crushed core curve. In the crushing load direction, the compressive modulus dropped to 3% of the undamaged modulus.

While there are slight differences in the test methods, both techniques measure the crushed core directly, without the additional stiffness contributions from undamaged core, and should produce similar results.

6.6.3.1 Results

The average difference in the gradients was 92% or 8% of the original stiffness. This value of 8% was used for all Finite Element and analytical models in this chapter. This compares to a value of 3% for foam core [80].

6.6.3.2 Previous values

Aitken [8] used a different approach when finding this value. Instead of probing the panel in a crushed core area, he recreated the crushed core using a flatwise compression. This test creates a completely different kind of damage to the type of damage found when crushing the core in an impact scenario. Instead of the damage consisting of multiple fracture and crease lines, as in the case of an impacted panel, the damage consisted of a single crease or fracture line around the
centre of the panel. This type of crushed core is much stiffer than an equivalent barely visible impact damaged region, so leads to a much higher damaged modulus value. This is because the damaged core is surrounded by undamaged core, so the compression modulus is an average of undamaged and damaged properties and not a directly measured property.

The stiffness was measured as 40% (value calculated from the reloading curve) compared to the 8% found above. A typical load deflection curve for the Aitken [8] experiment is shown in Figure 6-42. This incorrect value was used by Wadsworth, and Staal and Southward [20, 85, 88, 89]) in their numerical and analytical models.

Figure 6-43 – Compression test, showing a fracture line through specimen

![Compression test, showing a fracture line through specimen](image)

Figure 6-44 – Fore / deflection plot from the flatwise compression test

![Fore / deflection plot from the flatwise compression test](image)
It was also found that Wadsworth [85], Turnbull [83] and Bokhovens [84] damaged their test panels by applying a slow compressive load in an extensometer instead of a rapid impact load. This produces a similar crushing mechanism as Aitken's [8] flatwise compression tests.

Their resulting damaged failure stresses were approximately 50% of the values found by Staal [89] and Southward [88], and Aitken [8]. It is thought that any difference in failure loads was directly attributed to the way damage was created. This theory still needs to be proved. When damage is created slowly the honeycomb appears to fracture in a clean line a few millimetres below the face sheet. When the same damage is created using an impact event the core in the crushed region crumbles with crushing starting at the face sheet and finishing some millimetres below the face sheet.

![Fracture line](image1)

**Figure 6-45 - Damaged created slowly in an Instron**  **Figure 6-46 - Damage created using an impact**

### 6.6.3.3 Four Point Bending

The best way to study damage tolerance for thin gauge sandwich panels is to apply an in-plane compression load to the damaged area. This is achieved using a series of static four point bend tests on an assortment of impact damaged panels. This test is used because it puts the centre of the panel into pure bending, thereby applying a constant compression load to the face sheet. For this test a four point bending rig was manufactured. Supports and loading points were made from 25mm diameter steel bars. The outside and inside support bars were spaced 670mm and 300mm apart respectively. This test is designed to conform to the ASTM C393 testing procedure for sandwich panels. The same testing procedure was used for all tests. A similar test was performed on the undamaged wrinkling experiments in Chapter 5.
Displacement gauges were used to measure the overall deflection of the panel between the central load points and the localised displacement of the damaged area. Three gauges were set up along the edge of the panel at even spaces between the central loading bars. A fourth gauge was placed in the centre of the damaged area. The edge gauges were used to measure the curvature and check the bending moment of the panel. The gauge at the damage site was used to detect the onset of wrinkling and the way in which the damage site deformed as the load was applied. The panels were loaded at a rate of 2mm/min. Measurements of the load and the panels’ displacements were taken at various stages in the test. The first measurement was recorded when the wrinkling started, i.e. when the displacements of the damage gauge and the central edge gauge began to move at different rates to each other. Another measurement was taken when the wrinkle reached the edge of the original damage site. A series of other loads and displacements were noted throughout the test. The final load was recorded when the panel failed (Figure 6-8).

The results from the tests were normalised for the panel widths so different sized test panels could be compared. This load was then converted into a bending moment using span between the load
and support points. Using a simple substitution this was converted into a face sheet stress. This particular calculation assumes that all of the in-plane compressive and tensile stresses are carried through the face sheet. The strength percentages were calculated using a stress value for an undamaged panel of 218MPa, experimentally obtained with identical panels by Aitken [6]. The results are displayed in tabulated form in Appendix E.

### 6.6.4 Experimental Observations

Several key events were observed during the loading of the damaged panels. Initially the damaged site and the remainder of the panel deformed at uniform rates. As the compressive load began to increase, the profile of the damaged region started to change. This non-uniform deformation was the first sign of wrinkling and consistently occurred early in the test (Figure 6-48, Figure 6-49). In all test cases the face sheet started to push down into the damaged core to form a deep crater in the damaged region.

As the load continued to climb, a deep trough or wrinkle developed across the crater’s width in a perpendicular direction to the applied loading. A further increase in loading saw the wrinkle get deeper while its length remained largely unchanged. For larger sized damage, wrinkles deepened almost instantaneously, while with smaller damage sizes the growth of the wrinkle was slower and only started to accelerate as the panel approached the collapse strength. This was caused by the added restraint imposed by the undamaged area on smaller size damage (Turnbull [83] and Bokhoven [84] and Zenkert et al. [67] showed a similar trend in their work). Digital callipers were used to measure the growth of the wrinkle.

By the time the wrinkle had stopped growing in length, the wrinkle tip was usually just outside the original damaged region (Figure 6-50).

![Figure 6-49 - Starting of wrinkling](image1.png)  
![Figure 6-50 - Wrinkle outside the damage area](image2.png)
For damage less than 20mm, no further wrinkle propagation was observed until the sheet failed. For larger damage sizes, the wrinkle began to propagate at a steady rate. As the wrinkle tip approached the edge, the panels became unstable due to the resulting decrease in stiffness. This localised instability led to the buckling of the face sheet either into or out of the panel. On some of the panels the face sheet cracked through the damage site and tore away from the adjacent core. For some very small damage sizes (around 10mm) the failure occurred underneath the load point, or in some cases the core sheared due to excessive shear force. These results have been removed from the main pool of information as failure did not occur in the damage zone.

6.6.5 Failure and Wrinkle stress plot

Failure stress plot

Results from the bending test are plotted to show the post-impact load carrying capacity of the panels as a function of damage size. (Figure 6-53)
These results follow a similar pattern to the theoretical models with two distinct trends and a large curved transition zone between them. The failure stress initially drops sharply from the undamaged strength of 100% down to 70% and then veers and gradually decays to an approximately constant failure stress of 50%. If a vertical line is drawn through the point at 70% the damage size would correspond to a critical damage size of 20 mm. A similar value is shown in Table 6-8 for damage in the range of 3mm to 4mm in depth.

This plot also agrees well with Aitken’s existing data, with residual strengths tapering down to 50% for damage areas greater than 3960mm² (71mm).

Cyclic loading

In practical situations, panels with impact damage are usually subjected to some form of cyclic loading. In the case of aircraft flaps this could be through simply retracting and extending the flaps or vibrations during flight. Aitken [8] investigated cyclic loading on damaged sandwich panels. In his first test he set the maximum load to 80% of the predicted failure load, the minimum to 20% of the predicted failure load and mean load to 50% of the predicted failure load. In all cases the tested panels failed due to wrinkle growth, with some failing between 8654 and 15967 cycles. In another test he kept the mean at 50% but reduced the maximum load to 70%. In this case all the panels survived the full 90000 cycles without failure. He found that if the maximum load was too close to the failure load then you could expect further crushing and degradation of the panel and eventual collapse without reaching the static failure loads.

If panels are expected to see high cyclic loading, the results from Aitken’s [8] study would suggest using a safety factor on the failure stresses to take into account different loading scenarios.
However, if panels are designed for stiffness and not strength, and only operate at 20% of their ultimate failure load, it is still conceivable that the component will retain long service life without further growth of the damage, so the failure stresses may be used as limiting value. Based on experiments completed by Aitken it is probable that a safety factor of 0.7 for a heavily loaded panel will suffice. Further investigation of this is needed on a range of test panels.

**Wrinkling initiating stress plot**

A plot of the initiating wrinkling stress versus damage area gives a better understanding of the wrinkle formation. This gives a stress at the point where wrinkle growth was first observed.

![Wrinkling initiating stress vs. Damage Area](image)

**Figure 6.10 Wrinkle initiating stress plot**

The wrinkling initiating stress plot shows a more defined region than the failure plot where the load curve turns sharply upwards with decreasing damage sizes. In a similar way to the failure stress plots, the critical damage size can be found by putting best fit lines through the data points and dropping a vertical line at the point where the data turns away from the initial steep trend line. This gives an estimate value of 22mm (approximately the wrinkling wavelength evaluated at 5mm damage depth).

What the wrinkling initiation stress shows is a threshold for wrinkle growth. Before this stress it is guaranteed that no degradation of the core will occur, as wrinkles only form once the panel reaches this load. These wrinkling initiation stress values are significantly lower than the 70% threshold found experimentally by Aiken [8] and in practice are impractical. Aiken [8] showed that even though wrinkles had formed in the damaged area it does not mean that they will continue to grow. It was only when the load reached 70% of the static failure load that further degradation of the undamaged core could be expected.
Figure 6-54 - Combined failure stress and wrinkle initiation plot

Figure 6-54 shows a plot of wrinkling initiation stress plotted against failure stress. As expected, both sets of data show similar trends, as wrinkling instigates the final instability failure of the panels. The difference between the stresses is attributed to the gradual growth of wrinkles in the damage site and the propagation of wrinkles to the edges. The curve of the transition zones can be explained using the analogy of a simply supported wrinkle and a fixed supported damage area (Section 6.4). When damage approaches the critical damage size and the restraint from the undamaged region increases, both the wrinkling and failure loads will be affected. This extra restraint is represented by the gradual increase in loads from Aitken’s large size damage to the critical damage size.

This plot also shows a difference in width in the two transition zones of the curves. When wrinkles initially form, the undamaged region surrounding the damage zone has the least influence on the wrinkle formation. It is not until the wrinkles deepen and approach the failure load that the undamaged face sheet will start to resist the deflection. This effect is very distinct for damage around the critical damage size. For this reason, the transition zone in the failure plot is wider, as the effect of the side restraint is more prominent just prior to failure than at the beginning of loading.

Variation in the data / limitations of this study

In the failure stress plots and the wrinkling initiating plots there is some scatter in the experimental data. Explanations of probable sources of this scatter are given below:
1. In cases where the damage was very small it was difficult to accurately locate the subsurface damage using a tap test. If the damage diameters are slightly out, the resulting data point will be incorrectly placed. The only way to accurately measure damage non-destructively is to perform a test using ultrasound equipment.

2. Manufacturing differences in the panels can contribute to small changes in the residual stress. These can include slight variations in material batches (three different cores and five different rolls of glass were used to manufacture the panels) and different oven cures.

3. If panels are unsymmetrical then the panel can be loaded unevenly. This affects the way the panel deforms under load and could give a slight fluctuation in the failure loads.

4. Anti-clastic curvature effects can also influence the failure loads. This became an issue on wider panels and caused strain gauges to move at different rates and the central displacement to be larger than the edge displacement. This made it hard to assess when wrinkling began and could lead to error in the wrinkle initiation stress plot.

5. Some of the scatter around the critical damage size can be attributed to the wrinkling stress increasing and being forced into smaller buckling modes. As a result there is increased instability in this area.

6. The proximity of the edges to the damage area can also influence the propagation rate of the wrinkle and the final instability and failure of the face sheet. This was investigated using three similar sized impact dents on three different width panels (panels (10/2, 13/1, 24/1) were used – see Appendix E). The final failure stresses were different, indicating that the position of the damage area relative to the panel edge can have some effect on the failure stress.

**Section 6.7 Comparison between theoretical and actual data**

Referring to Figure 6-55 to Figure 6-58, it is clear that the experimental failure stress is in close agreement with the predicted stress from the derived models.

The non-linear plane stress model produces the best result, although the knocked down analytical model (equation (6-22)) predicts very similar loads. The analytical model over-predicts the failure load when the damage is less than the critical wrinkle wavelength. This is due to the assumption that the face sheet is built in at the edge of the damage, where in reality, it can still rotate and displace vertically. The more complete explanation is given in Section 6.4.6.4.
Figure 6-55 - Experimental failure stresses plotted against Analytical and Finite Element failure stresses (Stress vs Damage Area) reduced data set

Figure 6-56 - Experimental failure stresses plotted against Analytical and Finite Element failure stresses (Stress vs Damage Area) Full data set
Figure 6-57 - Experimental failure stresses plotted against Analytical and Finite Element failure stresses (Stress vs Damage Diameter) reduced data set

Figure 6-58 - Experimental failure stresses plotted against Analytical and Finite Element failure stresses (Stress vs Damage Diameter) Full data set
**Section 6.8 Discussion and Conclusion**

When thin gauge sandwich panels are impacted by projectiles, the damage typically consists of a thin layer of crushed core beneath the impact site and a shallow crater in the face sheet surface. The extent of the damage depends on the energy and the shape of the projectile and the relative stiffness of the core and face sheet. This type of damage allows panel to wrinkle and fail at much lower loads than an equivalent undamaged area. Failure is caused by instability and loss of stiffness in the damaged area.

Equations were successfully developed to predict the load carrying capacity of panels through a range of damage sizes. It was shown that the problem of impact damage can be broken into two parts (the effect of damage size and the effect of damage depth). These components can be added together to give an overall failure stress. Because failure is through instability and not core failure or face sheet fracture, linear buckling models can be used. The following expression can be used to predict the failure loads:

For damaged panels

\[
\sigma_{cr, \text{damaged}} = 0.95 \left( \frac{E_f}{12} \left( \frac{\pi f}{L} \right)^2 - E_i \left( \frac{\alpha L}{\pi f} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) \right) + \frac{\pi^2 E_f f_f^2}{3 d_{dam}} \tag{6-26}
\]

Where \( L \) is the critical wrinkling wavelength found by minimising the critical wrinkling below in terms of \( L \).

\[
\sigma_{cr, \text{depth}} = \frac{E_f}{12} \left( \frac{\pi f}{L} \right)^2 - E_i \left( \frac{\alpha L}{\pi f} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) \tag{6-27}
\]

As an estimate of the undamaged panel failure stress the following expression can be used

\[
\sigma_{cr, \text{undamaged}} = 0.5 \left( \frac{E_f E_z G_{sc}}{E_t} \right)^{\frac{1}{3}} \tag{6-28}
\]

Using undamaged core properties

Or more accurate expression is

\[
\sigma_{cr, \text{undamaged}} = 0.825 \left( \frac{E_f E_z G_{sc}}{E_t} \right)^{\frac{1}{3}} \left( \frac{E_f}{6 \sqrt{E_z G_{sc}}} \right)^{\frac{1}{3}} \tag{6-29}
\]

Where \( L_{cr} = \frac{E_f}{6 \sqrt{E_z G_{sc}}} \), and an estimate of \( A_0 \) (initial imperfection) is \( \frac{A_0}{t_f} = 0.1 \) [57]

This model works well down to the point where the damage length is equivalent to the critical damage size. Below this value the experimental and analytical results can deviate by up to 20%, due to the built-in-end condition of the face sheet. However, as most damage encountered is larger than these critical sizes, this model would work well in most situations. Furthermore, the remaining results (from critical damage size to undamaged) could be approximated by extrapolated a linear
line back from the critical damage size stress to the stress value of the undamaged panel.

Non-linear Finite Elements that incorporate a layer of damaged core successfully predict the failure stress through the complete range of damage sizes. It was proved that a plane-stress model could be used to accurately capture the failure stress through a study which compared a three dimensional model and a plane-stress model.

Experiments showed that the damaged area should be modelled with 8% of undamaged core modulus, compared with 40% used by previous authors [8, 20, 83-85, 88, 89].

The Finite Element models and experiments suggest that the failure mechanism consists of a complex combination of wrinkling instability and core crushing. As the damage size gets smaller (< critical wrinkle wavelength), wrinkles are forced to form at smaller wavelengths in the damaged area. The formation and the growth of these waves requires more energy than areas that support full wrinkle wavelengths. This results in a rapid increase in failure stress to the undamaged panel stress. The stress is also affected by a sudden change in damage depth as it shifts from a damaged panel to an undamaged panel. The eventual collapse of the panel occurs when the panel becomes unstable and the wrinkle propagates instantaneously into the undamaged area. The reduction in the overall stiffness/instability of the damaged region can be caused through further core crushing in the adjacent undamaged core or just local instability in the damaged region. The panel will eventually fail when there is enough energy in the unstable damaged area to cause instantaneous core crushing/wrinkle growth across the entire width of the panel.

If the damage is large enough (> critical wrinkling wavelength), the panels will fail due to pure wrinkling instability. Failure is a result of the damaged area losing stiffness, leading to wrinkle growth and collapse of the undamaged area adjacent to the damage region. The failure stress plateaus to a constant value of about 50% of the undamaged failure stress in this region. In this area multiple wrinkles can form at their natural wavelength. If the damage area is large enough any small change in damage size has limited effect on the failure load.

It was shown that the central wrinkle controls the failure load. This will form at the lowest load, due to it having least support from the surrounding undamaged area. In damage areas larger than the critical wrinkle size the central wrinkle is free to form at will, unlike the wrinkles directly beside the edges of the damage site which are restrained by the undamaged area. As the damage approaches the critical damage size this wrinkle will still form at the minimum load as this is the only wrinkle to form; however the wrinkle will be affected by the undamaged area. Because this wrinkle usually forms on top of the deepest section of crushed core (which has the least amount of foundation support) it will form at the lowest load.

Cyclic loading has a potential to reduce the maximum failure stress. If the stress constantly fluctuates above a threshold level, over a period of time the undamaged core surrounding the damage site could soften due to repeated manipulation. This would lead to an increase in the damage area and potentially a lower failure load. Aitken [8] found that this threshold value/safety factor is around 70% of the maximum static load case for the types of panels investigated in this chapter. More experiments are needed to examine this more closely.
Chapter 7: Conclusions, Review and Recommendations

The aim of this research was to investigate loading of thin gauge honeycomb sandwich panels in bending, and to see if existing wrinkling expressions can be used to capture the resulting failure mode. Previous work has shown that existing models over-predict failure loads.

This was achieved through a comprehensive study which examined all aspects of failure in bending. The research showed that existing wrinkling expressions, based on some simplified assumptions with regards to the core model, predict linear wrinkling failure in undamaged panels accurately, assuming that a restrained in-plane modulus is used (calculated with the face sheets attached).

This led to work that examined the in-plane continuum modulus and its effect on the wrinkling stress, the out-of-plane continuum modulus, and experiments to determine discrete properties for Nomex cores.

This linear study was followed by a non-linear study and the development of a further analytical model to predict the physical failure loads of undamaged panels. It was shown that the panels never reach the linear wrinkling stress values and fail prematurely due to core crushing or face sheet fracture, depending on the ratio of skin to core stiffness.

Having established that the existing linear models cannot predict failure loads in undamaged panels the thesis turned to barely visible impact damage (BVID) panels and the effect that damage has on the wrinkling failure mode and residual strength of the panel. Simple linear analytical models were developed to predict the loss of strength due to impact damage. Unlike undamaged panels, it was found that linear wrinkling models could be used in this special case as the panels fail due to instability of the damaged area caused by wrinkles forming at comparatively low loads.

A comprehensive experimental programme was undertaken as part of this damage study.

Every analytical model was checked against Finite Element models, and, where practical, against experimental results.
Section 7.1 Summary of major findings

- Showed that continuum cores can be used to model wrinkling failure in discrete (honeycomb) cellular cored panels. This also proved that the core representation was not responsible for errors between experimental results and analytical expressions.
- Found that existing models accurately predict linear wrinkling loads in sandwich panels if correct continuum properties are used (specifically the in-plane modulus).
- Developed new expressions to calculate the out-of-plane continuum modulus. These were verified against manufacturers’ and Finite Element values, with sets of discrete properties found for three common Nomex cores.
- Showed that the in-plane modulus gains significant stiffness when the face sheet restraint is considered. New expressions were developed to calculate this restrained value.
- Through Finite Element analysis and experiments, it was found that thin gauge honeycomb panels were failing due to core crushing before reaching the linear / non-linear (90% of linear wrinkling load) instability wrinkling loads. Failure occurs at approximately 60% of linear load. A new analytical model was developed to account for core crushing.
- Found that when a layer of sub-surface core damage exists the failure load can drop to 50% of the undamaged failure load. Two factors influence this reduction in load: damage depth and damage diameter. Unlike undamaged panels, damaged panels reach the instability buckling limit due to the significant drop in stiffness in the crushed core area. New linear analytical expressions were developed to predict the failure loads through a range of damage sizes and depths. These were verified against experimental results and non-linear Finite Element buckling models.

Section 7.2 Chapter 2

Chapter 2 examined linear wrinkling failure and the difference between modelling wrinkling in honeycomb panels using a continuum core approximation as compared with a detailed discrete core model. This is the first study to perform this comparison.

It was hypothesised that error between experimental results and existing wrinkling expressions is caused by the core representation and the way complex cellular cores are converted to simplified continuum models. By using a continuum core, it is implied that the core provides continuous face sheet support, where, in reality, it is discrete with a cellular core. With such small wrinkle wavelengths (order of two cells long for standard thin gauge honeycomb panels) it is feasible that cells will influence the failure load and buckle internally. This deformation is not captured using continuum core properties. To solve this problem a better three dimensional model of the core is possibly needed. The only way to accurately do this is to model each individual wall and cell, and only then would the model truly account for the core’s complex cellular shape and accurately represent the core support.

There is, however, an obvious disadvantage with modelling the sandwich panel accurately, i.e.
increased computational time. As the number of cells in the core increases, so does the number of degrees of freedom in the model. Computation time generally increases at a rate of degrees of freedom squared. This is one of the reasons why this type of model is never used.

A complicated set of Finite Element models were created to compare discrete and continuum wrinkling models and to validate some of the classical linear wrinkling models. Because all cores regardless of their structure are typically modelled using a continuum representation, the results from this chapter were significant.

When the properties were accurately converted from a discrete to an equivalent continuum core the two linear wrinkling models correlated well, suggesting that either type of core could be used to model this failure mode. A perfect match also existed between classical analytical wrinkling models and continuum Finite Element models, proving that the classical models predict accurate instability failure loads based on their linear assumptions. This finding has shown that the core representation is not responsible for differences between the current analytical models and experimental results. The true reason for this difference is related to the solution procedure, in particular the use of linear buckling models which do not account for non-linear material behaviour and the progressive growth of the wrinkle. To predict accurate failure loads a non-linear model is needed or a modified linear wrinkling model with an allowance for core crushing (Chapter 5).

7.2.1 Major findings / conclusions

1. Discrete and continuum cored wrinkling models predict the same buckling loads, providing correct conversion properties are used. These conversion properties are the properties needed to model an orthotropic core in the continuum model. They can be calculated directly from a discrete cell, using either a Finite Element model in the case of this chapter, or conversion expressions found in Chapter 3 and Chapter 4. In this study it was shown that the in-plane modulus must be modelled using the constrained model (Chapter 4).

2. Fillets are required at the junctions of the walls in the discrete linear wrinkling models to allow the core to bend and twist out of plane and wrinkle in the facings.

3. These models were solved using a linear eigenvalue analysis. The only way to truly capture the failure load of undamaged panels is to use a non-linear buckling solution. This will take into account imperfections and irregularities in the panel. When such a procedure is used we would expect a drop of between 10-20% in the wrinkling stress. If core crushing and face sheet fracture is factored into the equation it can be significantly more.

4. An additional failure mode was found which is a combination of wrinkling and dimpling. This occurs when the wrinkling wavelength, which is controlled by the ratio of out-of-plane core stiffness and face sheet stiffness, approaches the cell size. When this happens the face sheet will dip down into individual cells at the same time as wrinkling globally. Wrinkling/dimpling appeared to have a lower failure load than the equivalent continuum model. Fortunately it is unlikely to occur as relatively thin face sheets and extremely stiff cores are needed for the wavelength to approach the cell size. Panels fitting these requirements are impractical and
never manufactured. The discrete model is needed to capture the failure mechanism, which is a lot more complicated that a simple continuum model.

5. A modified version of the classical wrinkling models was presented. The same expression was derived using a differential equation approach and energy methods. This model formed the base for future comparisons to other types of models. The technique used in the differential model is similar to the the multiple layer damage wrinkling model used in Chapter 6. The modified linear wrinkling expression is

$$\sigma_{cr} = 0.825 \left( E_f E_g G_{sc} \right)^{\frac{1}{3}}$$

6. Regression expressions where developed from the plane-stress solid continuum model and the discrete model. These are

$$\frac{\sigma_{cr}}{E_f} = 4.11 \left( \frac{t_f}{l} \right)^{1.31} \left( \frac{t_s}{l} \right)^{0.63} \left( \frac{t_c}{l} \right)^{-1.08} \left( \frac{E_s}{E_f} \right)^{0.54} \quad \text{discrete model} \quad (7-1)$$

$$\frac{\sigma_{cr}}{E_f} = 1.256 \left( \frac{t_c}{t_f} \right)^{-0.170} \left( \frac{G_{sc}}{E_f} \right)^{0.143} \left( \frac{E_c}{E_f} \right)^{0.424} \left( \frac{E_s}{E_f} \right)^{0.068} \quad \text{continuum model} \quad (7-2)$$

This compares to the modified classical expression of

$$\frac{\sigma_{cr}}{E_f} = 0.825 \left( \frac{G_{sc}}{E_f} \right)^{0.33} \left( \frac{E_c}{E_f} \right)^{0.33} \quad (7-3)$$

7. Comparisons where made between the two classical analytical models and the two Finite Element models (3D continuum and 2D plane-stress Finite Element models). When the restrained in-plane modulus was used there was an almost perfect correlation between the modified classical expression and the 2D plane-stress model or continuum regression model. This proved that the existing wrinkling expressions accurately predict instability wrinkling loads, but not necessarily the failure loads (Chapter 5.) The solid model produced marginally lower wrinkling stresses than the plane stress model. This is due to the increased flexibility with the 3D elements. Vonach and Ramerstorfer’s [1] model was shown to produce stresses which where about 20% lower than the equivalent plane stress model, indicating a problem with their formulation.

### 7.2.2 Future work

- Examine the wrinkling / dimpling mode and see if this can be replicated experimentally.
- Develop a better understanding of why certain discrete properties have more effect on wrinkling failure than other properties.

### Section 7.3 Chapter 3

In Chapter 2, discrete and continuum wrinkling loads were compared by converting cellular properties to continuum properties within a Finite Element model. Results showed that both continuum and discrete Finite Element models produced the same linear wrinkling loads, suggesting that a simple analytical conversion model could be used to convert cellular properties into continuum properties and then a classical analytical wrinkling model could be used to find the linear
wringling load. In theory should produce the same wringling stress as the more complex discrete continuum cored numerical models.

This chapter examined out-of-plane continuum properties and ways to convert discrete cellular properties to continuum properties without going through lengthy Finite Element solutions. This included redeveloping the existing models that convert discrete properties to continuum properties, by adding fillets in the cell corners to accurately represent Nomex type cores. These were verified against Finite Element shell models.

Experiments were used to develop sets of discrete properties and find continuum properties for three commercially available Nomex cores. These properties were used to validate the analytical models and to check manufacturers’ data sheets. Most of this information is difficult to obtain due to the cell sizes and the reluctance of manufacturers to release manufacturing information.

### 7.3.1 Major findings / conclusions

1. This study developed a set of closed form analytical expressions that convert discrete cellular properties into out-of-plane properties. The expressions were explicitly developed for honeycomb cores by including fillets at the intersection of the walls. The expressions were accurately verified using Finite Element shell models. These are

\[
G_{xz} = G_{t}t_{s} \frac{r \cos(\theta) \sin(\theta) + r \theta + L \cos^{2}(\theta) + L}{b^{2} \sin(\theta)(1 + \cos(\theta))} \tag{7-4}
\]

Out-of-plane shear modulus in the X direction

\[
G_{yz} = -G_{t}s \frac{r \cos(\theta) \sin(\theta) - r \theta + L \cos^{2}(\theta) - L}{b^{2} \sin(\theta)(1 + \cos(\theta))} \tag{7-5}
\]

Out-of-plane shear modulus in the Y direction

\[
E_{z} = 2E_{t}s \frac{r \theta + L}{b^{2} \sin(\theta)(1 + \cos(\theta))} \tag{7-6}
\]

Out-of-plane compressive modulus in the Z direction

2. The study successfully found discrete data for three common Nomex honeycomb cores (1/8”-3.0, 3/16-3.0, 1/8”-4.0) and used the conversion models to test these properties directly against the manufacturer’s properties and in-house continuum test properties. It was shown that when the effective length of the cell wall was reduced to take into account the pool of resin which forms in the junction, the degree of correlation was high for two of the three cores.

3. Finding average discrete properties for three types of cores was a complicated process and something that had the potential for large error. The properties of greatest concern were the modulus value of the cellular walls and the thickness measurements of the walls. When correct properties were used in the conversion models, the calculated continuum modulus was similar to the experimental values, suggesting that these discrete properties are reasonably accurate.
4. With the larger 3/16” core the manufacturer’s shear properties were about double the values found in both the analytical and numerical models. It is unclear why this is the case and more tests are needed to check the manufacturer’s experimental values.

7.3.2 Future work
- Re-examine the modulus of Nomex cores. It was felt that tests should be done to determine a modulus value for Nomex paper and phenolic resin. These properties were supplied as indicative values by manufacturers, as they are uncommon and never tested for.
- Further tests are needed to check the manufacturer’s continuum shear properties for a range of cores. The large difference found in one of the tests conducted as part of this research is a concern and needs further investigation.

Section 7.4 Chapter 4

Chapter 4 continues on from Chapter 3 by developing expressions for the in-plane core modulus; the modulus in the $X$ and $Y$ orientations of the panel. Until recently, most classical wrinkling models ignored the in-plane modulus and assumed that there is negligible deformation or stress in the in-plane direction. This indirectly leads to another assumption, namely of a high in-plane modulus to preclude in-plane strain. In reality the in-plane modulus is orders of magnitude lower than its equivalent out-of-plane properties. It also has a large bearing on the wrinkling stress as it stops wrinkles from displacing sideways, and should be calculated correctly when used in Finite Element models or any other model which requires this value.

The traditional way to calculate the in-plane modulus is with face sheets detached. This leads to a very low in-plane modulus as the core behaves like a spring and is free to extend through bending of the walls. This chapter showed that when this modulus is calculated with the face sheet attached, it is significantly higher than the free value. This is because the face sheet provides additional restraint to the top and bottom faces of the core, which forces the face sheet to extend and bend for any in-plane movements. When this value was used in the continuum Finite Element models, the linear wrinkling stress matches the existing continuum analytical models. If the free modulus was used, then the Finite Element model underpredicts the wrinkling stress. This chapter is the first study to develop simple useable expressions for the restrained in-plane modulus and to examine its importance in wrinkling. All models were verified using equivalent Finite Element models.

7.4.1 Major findings / conclusions
1. This chapter investigated the “thickness effect”, which is when the modulus is calculated with the faces attached to the core. Results demonstrated that the in-plane stiffness increases significantly when face sheets are added to the core. When face sheets are attached to the core the effective modulus changes through the depth of the core. At the core / face sheet interface, the modulus is the highest. In this area the cellular walls are under pure extension
and compression as the core follows the strain pattern of the facings. As we move away from the faces, it drops significantly through a transition zone until it reaches a second region. This area is under a combination of bending and extension as part of its deformation and is controlled by the displacement of the face sheet. The other section is free to move. New expressions were developed to predict the modulus in these two regions and an averaging function was created to average the two values and find an overall effective modulus.

2. It was shown that the in-plane modulus has a significant effect on the linear wrinkling stress, especially with low values of these properties. There is an exponential relationship between wrinkling stress and in-plane modulus. With low values of the modulus the wrinkling loads approach experimental loads, which can be as low as 50% of the predicted wrinkling stress of the classical wrinkling models. Vonach and Rammerstorfer [1, 20], and Wadsworth et al. [1, 20] suggest that this discontinuity between the experimental results and the classical wrinkling expressions was a result of excluding these properties in the calculations. This was proved to be incorrect. When the restrained modulus is used instead of the free modulus, the wrinkling stress from the Finite Element models tends towards the classical analytical models.

3. The thickness effect was verified using a comparison between continuum and discrete analytical models. When the restrained in-plane modulus was used there was an almost perfect correlation between the two models. Conversely, when the free modulus was used with the continuum model the wrinkling stress was significantly lower. This proved that the thickness effect exists and needs to be accounted for when modelling wrinkling instability accurately.

4. The modulus in the central section of the restrained core can be used in the majority of calculations. This is because most of the in-plane deformation specifically caused by wrinkling is isolated to the central area of the core, meaning there is little difference in the result between the averaged constrained modulus value and the constrained central core value.

5. A new set of analytical models were developed to estimate the in-plane modulus when the core is in a free state (without face sheets attached). These expressions add to previous work and include fillets in the junctions of the walls.

6. There was poor agreement between the free modulus experimental tests and the calculated free modulus values. This was attributed to error in the experimental testing procedure.

**Section 7.5 Chapter 5**

This study used non-linear Finite Element models to track the progressive growth of the wrinkle in a sandwich panel under a bending load. This helped to pinpoint the failure mode and to explain why existing linear models overpredict experimental failure loads even though they were found to accurately predict wrinkling instability failure in the previous three chapters. Through these non-linear wrinkling models it was shown that the panels never reach the instability failure load and fail prior to that due to core crushing as a result of wrinkles forming in the facings. As wrinkles form at the beginning of the loading, the stresses induced in the core by the wrinkled face sheet reach a point where the core collapses locally underneath the face sheet. This additional deformation causes the face sheets to fracture and the panel to collapse. Experiments were completed and
compared directly to the Finite Element prediction and a specifically developed analytical model. The degree of correlation was good between the experimental results and the results from the models. In all cases, this was better than the knock-down version of the classical wrinkling expression currently used in design.

### 7.5.1 Major findings / conclusions

1. Failure stresses were accurately predicted using non-linear Finite Element models and Equation (7-7). The collapse of the panels is caused by core crushing as a result of wrinkles forming in the face sheet. These progressively grow until the compression strength of the core is exceeded, at which time the core collapses and the face sheet fractures due to gross deformation and bending stress.

2. The result also demonstrates that traditional linear analytical wrinkling models (Chapter 2) cannot be used to predict the failure loads as they do not account for non-linear material behaviour.

3. A new model (based on the work of Kassapoglou [50] and Yussuff [7]) was developed to predict this failure load. This model accounts for core crushing as a result of wrinkles forming in the face. It also produces the best result: 0.7% average error with experimental values. The Finite Element model produced values with an average error of 5.7%. In contrast the existing expression by Hoff and Mautner [5] of 

\[ \sigma_{cr} = 0.5 \left( \frac{E_f}{E_{cr}} \right)^{\frac{1}{3}} \]

has a 17.9% error. The revised expression is

\[ \sigma_{cr} = \frac{0.825 \left( \frac{E_f}{E_{cr}} \right)^{\frac{1}{3}}}{A_0 \sqrt{E_{cr} \left( \frac{\pi}{L_{cr}} \right)^{\frac{1}{3}}} + 1} \]

If the initial perturbation is unknown, an estimate \( A_0 \) is found using the following expression:

\[ \frac{A_0}{t_f} = 0.1 \] [57]

4. Unlike linear wrinkling models and non-linear models (with no core crushing), it was found that the in-plane modulus has no effect on the failure stress. This is controlled by the crush strength / out-of-plane compressive modulus of the core and the vertical displacement of the wrinkle.

### 7.5.2 Future work

- Use photo-optics with a minimum resolution of 0.02mm to examine initial waviness in the face sheet. Also track the wrinkle growth in the face sheet using the same tool and produce displacement time history surface plots. These can be used to validate the non-linear Finite Element results.
- Develop a non-linear analytical model to work in a similar way to the Finite Element model and capture the true strain field for different sizes of initial perturbation.
Section 7.6 Chapter 6

Chapter 6 completes this wrinkling study by examining damaged panels and the effect that damage has on the residual strength of the structure. When thin gauge honeycomb sandwich panels are hit by an impactor, rigid or soft, it results in BVID (barely visible impact damage). Damage of this type typically consist of a thin layer of crushed core beneath the surface and a shallow dent on the surface. An area of crushed core has reduced stiffness, and so allows wrinkles to form at lower loads. The reduction in load carrying capacity for BVID was shown to be as high as 50% of the undamaged strength, depending on the size of the damage. For this reason a study which examines the failure mechanics is very important.

With the use of non-linear Finite Element models the growth of the wrinkle was tracked and the resulting failure mechanism was found. Unlike undamaged panels, damaged panels fail due to wrinkling instability in the damaged area and not core failure. It was shown that the problem can be broken into two parts: the effect that damage size has on the wrinkling stress and the effect that damage depth has on the wrinkling stress. The depth of the damage was found to have the most significant effect on the wrinkle load, with the region of deepest damage (usually the centre of the dent) controlling the failure stress. The damage size also becomes important as the damage approaches the critical wrinkling wavelength. These two stress components can be added to give the final solution. In terms of modelling the failure, a simple linear model can be used with damaged panels, because this is an instability problem and the linear wrinkling stress is reached in the damaged section.

Extensive experimental tests were carried out in this study. This involved creating realistic damage in manufactured panels and then loading these panels to failure to find their post-impact residual strength. The results were checked against numerical Finite Element models and the developed closed form analytical models. In both cases, the degree of correlation was impressive. Further tests were done to look at the change in effective core stiffness between damaged and undamaged cores.

This is believed to be the first study to look at this problem in two parts and develop a relatively simple model which can capture the failure load through a range of damage sizes. Other models which try to predict failure loads in BVID panels must be solved numerically. It is also one of the first studies to track the failure mode using non-linear Finite Element models and illustrate how the structures are failing.

7.6.1 Major findings

1. Impact damage in thin gauge honeycomb panels typically consists of a thin layer of crushed core beneath the impact site and a thin crater in the face sheet surface. The extent of the damage depends on the energy and the shape of the projectile and the relative stiffness of the core and face sheet. This type of damage allows panels to wrinkle and fail at much lower loads than an equivalent undamaged panel. Unlike an undamaged area, failure is caused by
instability and loss of stiffness in the damaged area as opposed to core crushing due to wrinkle growth in the face sheet.

2. Expressions were developed to predict the load carrying capacity of panels through a range of damage sizes. The problem of impact damage can be broken into two parts (the effect of damage size and the effect of damage depth). These components can be added together to give an overall failure stress. Because failure is through instability and not core failure or face sheet fracture (as is the case with undamaged panels), linear buckling models can be used. The following expression can be used to predict the failure loads:

\[
\sigma_{cr, damaged} = 0.95 \left[ \frac{E_f}{12} \left( \frac{\pi f}{L_{cr}} \right)^2 - E_F \left( \frac{\alpha L_{cr}}{\pi f} \right) \left( 1 - \frac{\gamma}{1 + \gamma} \right) + \frac{\pi^2 E_f t_f^2}{3 L_{dam}^2} \right] \tag{7-8}
\]

where \( L_{cr} \) is the critical half wrinkling wavelength found by minimising the critical wrinkling below in terms of \( L \).

\[
\sigma_{cr, depth} = \frac{E_f}{12} \left( \frac{\pi f}{L} \right)^2 - E_F \left( \frac{\alpha L}{\pi f} \right) \left( 1 - \frac{\gamma}{1 + \gamma} \right) \tag{7-9}
\]

**For damaged panels**

As an estimate of the undamaged stress the following expression can be used

\[
\sigma_{cr, undamaged} = 0.5 \left( \frac{E_f Z G_{xz}}{E_f Z' G_{xz}} \right)^{1/3} \text{ Using properties of undamaged cores} \tag{7-10}
\]

or a more accurate expression is

\[
\sigma_{cr, undamaged} = \frac{0.825 \left( E_f Z G_{xz} \right)^{1/3}}{A_0 \sqrt{E_f Z' G_{xz}}} \left( \frac{\pi f}{L_{cr}} \right)^{1/3} \text{ where } L_{cr} = \frac{E_f}{6 \sqrt{E_f Z' G_{xz}}} \tag{7-11}
\]

Equation (7-8) matches experimental values down to the critical damage size. Below this value the experimental and analytical results deviate by up to 20%, due to the built-in-end condition of the face sheet. As most damage falls outside these critical sizes, this model works well in most practical situations.

3. Non-linear Finite Elements that incorporate a layer of damage core successfully predict the failure stress through the complete range of damage sizes. It was proved that a plane-stress model could be used to accurately capture the failure stress through a study that compared a three dimensional model with the plane-stress model.
4. Experiments showed that the damaged area should be modelled with 8% of undamaged core modulus, compared with 40% used by previous authors [8, 20, 83-85, 88, 89].

5. Plots of failure stress against damage size show that the stress drops off rapidly from the undamaged stress at 100% and plateaus to a constant value of 50% at approximately twice the critical damage size.

6. The experiments and models suggest that failure mechanism consists of a complex combination of both wrinkling instability and core crushing. Wrinkles are forced to form at smaller wavelengths within the damaged area when damage is small (< critical wrinkle wavelength). The formation and the growth of these waves requires more energy than areas that support full wrinkle wavelengths. This is one reason why the failure stress increases with damage smaller than the critical wavelength. This stress is also increased due a rapid change in damage depth as the panel moves from a damaged to an undamaged state. The eventual collapse of the panel occurs when the panel becomes unstable and the wrinkle propagates instantaneously into the undamaged area. The reduction in the overall stiffness/instability of the damaged region can be through further core crushing in the adjacent undamaged core or just local instability in the damaged region. The panel will eventually fail when there is enough energy in the unstable damaged area to cause instantaneous core crushing/wrinkle growth across the entire width of the panel.

7. If the damage is large enough (> critical wrinkling wavelength), the panels will fail due to pure wrinkling instability. Failure is a result of the damaged area losing stiffness, leading to wrinkle growth and collapse of the undamaged area adjacent to the damage region. The failure stress plateaus to a constant value of about 50% of the undamaged failure stress in this region. In this area multiple wrinkles can form at their natural wavelength. If the damage area is large enough any small change in damage size has limited effect on the failure load.

8. It was shown that the central wrinkle controls the failure load. The central wrinkle should usually wrinkle at the lowest load, due to its formation in the deepest section of damage and having the least support from the surrounding undamaged area. If the damage is larger than the critical wrinkle size the wrinkle can form freely, unlike the wrinkles directly beside the edges of the damage site which are restrained by the undamaged area. If the damage size approaches the critical damage size only one wrinkle will form, but at a higher stress due to support from the undamaged area.

9. Cyclic loading has a potential to reduce the maximum failure stress. If the stress constantly fluctuates above a threshold level, over a period of time the undamaged core surrounding the damage site could soften due to repeated manipulation. This would lead to an increase in the damage area and potentially a lower failure load. Aitken [8] found that this threshold value/safety factor is around 70% for the types of panels investigated in this chapter. More experiments are needed to examine this more closely.
7.6.2 Future work

- Further experiments should be done using different material configurations to check that the theory works across a range of panel configurations.
- In a similar way to undamaged panels, use photo optics to scan the surfaces and track the wrinkle as it progressively grows and propagates across the face sheet.
- Develop a model that incorporates face sheet rotation at the edge the damage site where the core changes from a damaged to an undamaged state. This should lead to more precise estimates of failure stress for small damage sizes.
- Minimise the damaged wrinkling expression and find a closed form analytical solution for the critical wrinkling wavelength.
### References


52. Hexcel Composites, *Hexweb honeycomb sandwich design technology*, in AGU 07803861.


APPENDICES
A Wrinkling Analytical models

This appendix details the derivations used to develop the wrinkling models used in chapter 2 and 6.

A.1 Hoff and Mautner’s Derivation [5]

This is the original model by Hoff and Mautner

\[ w_x = W \sin \frac{\pi x}{L} \]  
(A-1)

The displacement in the core is assumed to damp out linearly to \( h' \), as

\[ w = W \frac{z}{h} \sin \frac{\pi x}{L} \]  
(A-2)

To find the critical buckling load it is assumed that the work done by the compressive force during the displacement is equal to the strain energy stored in the face material and the core.

Assuming the core to have low (or zero) in-plane modulus the tensile/compressive stress in the core can be written \( \sigma_z = E_c \frac{\partial w}{\partial z} \) and if the deflections are in the Z direction only, the core shear stress can be written \( \tau_{xz} = G_c \frac{\partial w}{\partial x} \). Hence the strain energy stored in the core over the length \( L \) is

\[ U_c = \frac{1}{2E_c} \int_0^L \int_0^h \sigma_z^2 \, dx \, dz + \frac{1}{2G_c} \int_0^L \int_0^h \tau_{xz}^2 \, dx \, dz = \frac{E_c W^2 L}{4h} + \frac{G_c \pi^2 W^2 h}{12L} \]  
(A-3)

The energy stored in the face due to bending is
\[ U_f = \frac{D}{2} \int_0^L \left( \frac{d^2 w_f}{dx^2} \right)^2 dx = \frac{W^2 \pi^4 t_f^3 E_f}{48L^3} \] (A-4)

The work done by the applied load is
\[ V = \frac{P}{2} \int_0^L \left( \frac{dw_f}{dx} \right)^2 dx = \frac{\pi^2 W^2}{4L} - P = \frac{\pi^2 W^2 t_f}{4L} \sigma_{cr} \] (A-5)

Now, solving the energy equation \( V = U_c + U_f \) and rearranging yields
\[ \sigma_{cr} = \frac{E_c L^2}{\pi^2 t_f} + \frac{hG_c}{3t_f} + \frac{\pi^2 E_f}{12} \left( \frac{t_f}{L} \right)^2 \] (A-6)

This critical stress depends on the variables \( L \) and \( h \). The values of these are those that make the critical stress a minimum. Therefore they are found by differentiating equation (A-6) with respect to each variable and setting the respective equations equal to zero.
\[ \frac{d\sigma_{cr}}{dh} = -\frac{E_c L^2}{\pi^2 t_f h^2} + \frac{G_c}{3t_f} = 0 \] (A-7)

and
\[ \frac{d\sigma_{cr}}{dL} = \frac{2LE_c}{\pi^2 t_f h} - \frac{\pi^2 E_f t_f^2}{6L^3} = 0 \] (A-8)

Rearranging then yields the critical value of each:
\[ h = 0.91 t_f \sqrt[3]{\frac{E_f E_c}{G_c}} \quad \text{and} \quad L = 1.65 t_f \sqrt[5]{\frac{E_f^2}{E_c G_c}} \]

Substituting these values into equation (A-6) gives the critical face stress as
\[ \sigma_{cr} = 0.91 \left( E_f E_c G_c \right)^{\frac{1}{5}} \] (A-9)

**A.2 Revised energy method model (Modified Hoff and Mautner)**

The displacement of the face sheet is the same as that used by Hoff and Mautner [5]:
\[ \overline{w}_f = W \sin \frac{\pi x}{L} \] (A-10)

Assuming that the displacement of the core is to be related to (A-10), then:
\[ w(x, z) = Wf(z) \sin \left( \frac{\pi x}{L} \right) \] (A-11)

\( f(z) \) is an unknown function which relates the face sheets displacement to the core, where \( f(h) = 1 \) and \( f(1) = 0 \) as the core deflection varies from the face sheet through the depth.
An equilibrium condition must be maintained in the vertical direction.

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = 0$$  \hspace{1cm} (A-12)

Assuming, as Hoff and Mautner did, that

$$\sigma_z = E_c \frac{\partial w}{\partial z} \quad \text{and} \quad \tau_{xz} = G_c \frac{\partial w}{\partial x}$$

then (A-12) is rewritten in terms of deflection

$$\frac{\partial}{\partial z} \left[ E_c \frac{\partial w}{\partial z} \right] + \frac{\partial}{\partial x} \left[ G_c \frac{\partial w}{\partial x} \right] = 0$$  \hspace{1cm} (A-13)

This is the same as

$$\frac{\partial^2 w}{\partial z^2} + G_c \frac{\partial^2 w}{\partial x^2} = 0$$  \hspace{1cm} (A-14)

This is where the shortcomings of Hoff and Mautner's [5] equation are presented. They assumed a deflected shape of

$$w = W \frac{z}{h} \sin \frac{\pi x}{L} \quad \text{where} \quad f(z) = \frac{z}{h} \quad \text{(linear constant)}.$$  

In Hoff and Mautner's [5] deflected shape, the second partial derivatives are $$\frac{\partial^2 w}{\partial x^2} = 0$$ and

$$\frac{\partial^2 w}{\partial x^2} = -W \frac{z}{h} \frac{\pi^2}{L} \sin \frac{\pi x}{L}.$$  

Clearly these cannot add together to satisfy (A-14) so the initial linear decay function of $$\frac{z}{h}$$ must be incorrect. The function $$f(z)$$ can be determined by making it obey equilibrium in the core (A-14).

Substituting (A-11) into (A-14) gives

$$W \left[ \frac{d^2 f}{dz^2} - \frac{G_c}{E_c} \left( \frac{\pi}{L} \right)^2 f(z) \right] \sin \frac{\pi x}{L} = 0 \quad \text{let} \quad \alpha^2 = \frac{G_c}{E_c} \left( \frac{\pi}{L} \right)^2$$

Therefore the general solution

$$f(z) = A_i \sinh(\alpha z) + B_i \cosh(\alpha z)$$  \hspace{1cm} (A-15)

describes the displacement decay through the core. The following boundary conditions are used to solve this general solution.

(a) $$w(x,0) = 0$$

(b) $$w(x,h) = W$$

From (a),

$$B1 = 0$$  \hspace{1cm} (A-16)
and from (b),

\[ W(A, \sinh(\alpha h)) \sin \left( \frac{\pi z}{L} \right) = W \]

\[ \Rightarrow A_i = \frac{1}{\sinh(\alpha h)} \quad (A-17) \]

Substituting equations (A-15), (A-16) and (A-17) into (A-14) gives the final equation for the core displacement

\[ w(x, z) = W \left( \frac{\sin \alpha}{\sinh \alpha h} \right) \sin \left( \frac{\pi z}{L} \right) \quad \text{where} \quad \alpha = \frac{\pi}{L} \sqrt{\frac{G_c}{E_c}} \quad (A-18) \]

Strain energy methods are now used and the strain energies in the core and face sheet are equated with the work done by the applied load. The strain energy in the face sheet is

\[ U_f = \frac{D_c}{2} \int_0^L \left( \frac{d^2 w_f}{dx^2} \right)^2 dx = \frac{W^2 \pi^4 t_f^3 E_f}{48L^3} \quad (A-19) \]

The strain energy in the core is

\[ U_c = \frac{E_c}{2} \int_0^h \left( \frac{\partial w}{\partial z} \right)^2 dx dz + \frac{G_c}{2} \int_0^h \left( \frac{\partial w}{\partial x} \right)^2 dx dz \]

\[ = \frac{E_c}{2} \int_0^h \left( \frac{\alpha^2 \cosh \alpha z}{\sinh \alpha h} \right) dx dz + \frac{G_c}{2} \int_0^h \left( \frac{\pi^2}{L} \right)^2 \sinh \alpha h \left( \frac{1}{2} \right) dz \]

\[ = \frac{W^2}{2} \left[ E_c \left( \frac{L}{2} \right) \left( \frac{\alpha^2}{\sinh^2 \alpha h} \right) \left( \frac{1}{2} + \frac{\cosh 2\alpha z}{2} \right) \right] + \frac{\pi^2 G_c}{L} \left[ \frac{1}{4\alpha} \sinh 2\alpha h - \frac{h}{2} \right] \]

\[ = \frac{W^2}{2} \left[ E_c \left( \frac{L}{2} \right) \left( \frac{\alpha^2}{\sinh^2 \alpha h} \right) \left( \frac{1}{2} + \frac{\cosh 2\alpha \pi}{2} \right) \right] + \frac{\pi^2 G_c}{L} \left[ \frac{1}{4\alpha} \sinh 2\alpha h + \frac{\pi G_c}{E_c} \left( \frac{1}{E_c} \right) \left( \frac{1}{G_c} \right) \right] \]

\[ = \frac{W^2}{2} \left[ \frac{1}{4} \sinh 2\alpha h \left( \frac{\sqrt{E_c G_c}}{E_c} \right) \left( \frac{\sqrt{G_c}}{E_c} \right) \right] \]

\[ = \frac{W^2}{2} \left[ \frac{\sqrt{E_c G_c}}{E_c} \right] \left( \frac{\cosh \pi \alpha}{\sinh \pi \alpha} \right) \left( \frac{1}{E_c} \right) \left( \frac{1}{G_c} \right) \]

\[ = \frac{\pi W^2}{4} \left( \frac{\cosh \pi \alpha}{\sinh \pi \alpha} \right) \sqrt{E_c G_c} \]

where \( \alpha = \left( \frac{\pi}{L} \right) \sqrt{\frac{G_c}{E_c}} \)

The work done by the applied load is
\[ V = \frac{P}{2} \int_0^L \left( \frac{dw_f}{dx} \right)^2 dx = \frac{\pi^2 W^2}{4L} \]
\[ P = \frac{\pi^2 W^2 L_f}{4L} \sigma_{cr} \quad \text{(A-21)} \]

Letting \( V = U_f + U_c \) and rearranging gives

\[ P_{cr} = \frac{\pi^2 E_f t_f^3}{12L^2} + \frac{L(E_c G_c)^{\frac{1}{6}}}{\pi} \frac{\text{Cosh}(\pi \alpha)}{\text{Sinh}(\pi \alpha)} \quad \text{(A-22)} \]

It can be shown that \( \text{cosh}(\pi \alpha) = \text{sinh}(\pi \alpha) \) for values of \( L \) less than 15mm by a plot of \( \frac{\text{cosh}(\pi \alpha)}{\text{sinh}(\pi \alpha)} \) against \( L \).

This makes the differentiation and rearrangement to follow much simpler. On the assumption that \( L \) is less than 15mm \( \frac{\text{cosh}(\pi \alpha)}{\text{sinh}(\pi \alpha)} \approx 1 \)

\[ P_{cr} = \frac{\pi^2 E_f t_f^3}{12L^2} + \frac{L(E_c G_c)^{\frac{1}{6}}}{\pi} \quad \text{(A-23)} \]

Equating the derivative of equation (A-14), with respect to \( L \), to zero and rearranging gives the value of \( L \) corresponding to the minimum load. This value of \( L \) is half the natural wavelength

\[ L_{cr} = \frac{\lambda}{2} = \frac{\pi f}{6^3} \left( \frac{E_f}{E_c G_c} \right)^{\frac{1}{6}} \approx 1.73 f \left( \frac{E_f}{E_c G_c} \right)^{\frac{1}{6}} \quad \text{(A-24)} \]
Hoff and Mautner’s equivalent is 

\[ L = t_f 1.65 \left( \frac{E_f^2}{E_c G_c} \right)^{\frac{1}{6}} \]

The difference 

\[ \frac{L_{\text{new}}}{L_{\text{hoff}}} = \frac{1.73}{1.65} = 1.05 \]

This equation differs by 5% from the value for ‘\( L \)’ given by Hoff and Mautner [6].

When this value of \( L \) is inserted into equation (A-24), the minimum/critical load is obtained.

Dividing this by the face thickness gives the critical stress

\[ \sigma_{cr} = 0.825 \left(E_f E_c G_c\right)^{\frac{1}{3}} \] (A-25)

### A.3 Differential equation approach using a derived hyperbolic decay function (Chapter’s 2, 6)

Hooks law for the core is assumed for the orthotropic core material

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_z
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{E_x} & -\nu_{xz} \\
-\nu_{zx} & \frac{1}{E_z}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_z
\end{bmatrix}
\] (A-26)

It is assumed that \( \sigma_x \approx 0 \) for honeycomb cores with low in-plane modulus.

Since \( \sigma_x \approx 0 \Rightarrow \sigma_z = E_z \varepsilon_z \) and \( \varepsilon_x \approx -\nu_{xz} \sigma_z \)

\[
\Rightarrow \sigma_z = E_z \frac{\partial w}{\partial z} \text{ and } \varepsilon_x \approx -\nu_{xz} \varepsilon_z \Rightarrow \frac{\partial u}{\partial z} \approx -\nu_{xz} \frac{\partial w}{\partial z}
\] (A-27)

Substituting Hooke’s law into the vertical equilibrium equation

\[
\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} = \frac{\partial}{\partial z} \left( E_z \frac{\partial w}{\partial z} \right) + \frac{\partial}{\partial x} \left[ G_{xz} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right] = 0
\] (A-28)

Substituting (A-27) into (A-28) yields

\[
\frac{\partial^2 w}{\partial z^2} + \left( \frac{G_{xz}}{E_z - \nu_{xz} G_{xz}} \right) \frac{\partial^2 w}{\partial x^2} = 0
\] (A-29)

\( w(x, z) \) is assumed to be sinusoidal and decay to \( 0 \) at \( z \approx \infty \). \( f(z) \) is the decay function derived through equilibrium conditions

\[
w(x, z) = Wf(z) \sin \left( \frac{\pi x}{L} \right)
\]

\[
\Rightarrow W \sin \left( \frac{\pi x}{L} \right) \left[ \frac{\partial^2 f(z)}{\partial z^2} \right] - \left( \frac{G_{xz}}{E_z - \nu_{xz} G_{xz}} \right) \left( \frac{\pi}{L} \right)^2 f(z) = 0
\] (A-30)
Hence to satisfy equilibrium we see that
\[
\frac{\partial^2 f(z)}{\partial z^2} - \alpha^2 \left(\frac{\pi}{L}\right)^2 f(z) = 0
\]
Denoting \( \alpha^2 = \frac{G_{xz}}{E_z - \nu \alpha G_{xz}} \)
\[
\Rightarrow w(x, z) = W \sin \left(\frac{\pi x}{L}\right) e^{-\alpha \left(\frac{\pi}{L}\right)^2 z} + B e^{-\alpha \left(\frac{\pi}{L}\right)^2 z}
\]
(A-31)

A.3.1 Case 1: No damage, infinite deep core (Chapter 2)

This is the simplest case and was used to verify that the method works and produces the same result as the strain energy method used in the modified Hoff and Mautner expression. This particular case is outlined in Chapter 2.

By assuming that the core is infinitely deep

at the face sheet \( z = 0 \), \( w(x, 0) = W \sin \left(\frac{\pi x}{L}\right) \Rightarrow B = 1 \)

at \( z = \infty \), \( w(x, \infty) \Rightarrow 0 \Rightarrow A = 0 \)

\[
\Rightarrow w(x, z) = W e^{-\alpha \left(\frac{\pi}{L}\right)^2 z} \sin \left(\frac{\pi x}{L}\right)
\]
(A-32)

\[
\sigma_z = E_z \frac{\partial w}{\partial z}
\]

\[
\Rightarrow \sigma_z = W E_z e^{-\alpha \left(\frac{\pi}{L}\right)^2 z} \sin \left(\frac{\pi x}{L}\right)
\]
(A-33)

The stress at the face sheet is given by

at \( z = 0 \), \( \sigma_z = -W E_z e^{-\alpha \left(\frac{\pi}{L}\right)^2 z} \sin \left(\frac{\pi x}{L}\right) \) and \( w(x, 0) = -W \sin \left(\frac{\pi x}{L}\right) \)

(A-34)

Equilibrium equation for a beam on an elastic foundation evaluated at the face sheet is
\[ D \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} - \sigma_z = D \left( \frac{\pi}{L} \right)^4 - P \left( \frac{\pi}{L} \right)^2 + E_z \alpha \left( \frac{\pi}{L} \right) = 0 \]  

(A-35)

Rearranging in terms of \( P \) and minimizing in terms of \( L \) gives the critical wrinkling wavelength

\[ P = D \left( \frac{\pi}{L} \right)^2 + E_z \alpha \left( \frac{L}{\pi} \right) = 0 \]

\[
\begin{align*}
\frac{dP}{dL} &= -2D \left( \frac{\pi^2}{L^3} \right) + \frac{E_z \alpha}{\pi} = 0 \\
\Rightarrow 2D \left( \frac{\pi}{L} \right)^3 &= E_z \alpha \left( \frac{L}{\pi} \right) \\
\Rightarrow L_{cr} &= \pi \left( \frac{2D}{E_z \alpha} \right)^\frac{1}{3}
\end{align*}
\]

(A-36)

Substituting back into \( P \) gives the critical wrinkling stress

\[ P_{cr} = D \left( \frac{E_z \alpha}{2D} \right)^\frac{2}{3} + E_z \alpha \left( \frac{2D}{E_z \alpha} \right)^\frac{1}{3} \]

\[ = D^\frac{1}{3} (E_z \alpha)^\frac{2}{3} \left( \frac{1}{2} + 2^\frac{1}{3} \right) \]

\[ = \left[ \frac{E_f t_f}{12} \right]^\frac{1}{3} E_z^\frac{2}{3} \left( \frac{G_{xz}}{E_z - \nu_{xz} G_{xz}} \right)^\frac{1}{3} \left( \frac{1}{2^\frac{1}{3}} + 2^\frac{1}{3} \right) \]

\[ = (E_f G_{xz})^\frac{1}{3} t_f E_z^\frac{2}{3} \left( \frac{G_{xz}}{E_z - \nu_{xz} G_{xz}} \right)^\frac{1}{3} \left( \frac{1}{12^\frac{1}{3}} + 2^\frac{1}{3} \right) \]

\[ \approx 0.8255 t_f \left[ \frac{E_f E_z G_{xz}}{1 - \nu_{xz} \left( \frac{G_{xz}}{E_z} \right)} \right]^\frac{1}{3} \]

where \( \sigma_{cr} = \frac{P_{cr}}{t_f} \)

\[ \sigma_{cr} = 0.8255 \left[ \frac{E_f E_z G_{xz}}{1 - \nu_{xz} \left( \frac{G_{xz}}{E_z} \right)} \right]^\frac{1}{3} \]  

(A-37)
and \( \sigma_{cr} \approx 0.8255 \left( E_1 E_2 G_{xz} \right)^{\frac{1}{3}} \) since \( 1 - v_{xz} \left( \frac{G_{xz}}{E_z} \right) \approx 1 \) as \( v_{xz} \approx 0 \) for honeycomb cores

### A.3.2 Case 2: Damage to depth \( h \), infinite core

Damaged wrinkling model based on infinitely large damage area and a finite damage depth. This expression was specifically developed to predict failure in damaged sandwich panels and was outlined in Chapter 6.

**Case 2: Damage to depth \( h \), infinite core**

Region 1: \( w(x_1, z_1) = W \sin \left( \frac{\pi x}{L} \right) \left( A_1 e^{-\left( \frac{\pi z}{L} \right)^2} + B_1 e^{-\left( \frac{\pi z}{L} \right)^2} \right) \)  \hspace{1cm} (A-38)

Region 2: \( w(x_2, z_2) = W \sin \left( \frac{\pi x}{L} \right) \left( A_2 e^{-\left( \frac{\pi z}{L} \right)^2} + B_2 e^{-\left( \frac{\pi z}{L} \right)^2} \right) \)  \hspace{1cm} (A-39)

Note that with the assumption of isotropic damage, the resulting ratio \( \alpha \) is the same in 1 and 2.

The boundary conditions for the core are: (note: \( \eta = \frac{E_1}{E_2}, \eta \leq 1 \))

1. \( z \to \infty \) \( \Rightarrow \) \( w_2(x, \infty) = 0 \) \( \Rightarrow \) \( A_2 = 0 \)
2. \( z = 0 \) \( \Rightarrow \) \( w_1(x, 0) = W \sin \left( \frac{\pi x}{L} \right) \) \( \Rightarrow \) \( A_1 + B_1 = 1 \)
3. Continuity of the displacement at the boundary \( w_1(x, h) = w_2(x, h) \)
   \[ A_1 e^{-\left( \frac{\pi h}{L} \right)^2} + B_1 e^{-\left( \frac{\pi h}{L} \right)^2} = A_2 e^{-\left( \frac{\pi h}{L} \right)^2} + B_2 e^{-\left( \frac{\pi h}{L} \right)^2} \quad \Rightarrow \quad A_1 e^{-\left( \frac{\pi h}{L} \right)^2} + B_1 = B_2 \]
4. Vertical equilibrium at the boundary \( \frac{\partial w_1(x, h)}{\partial z} = \left( \frac{E_2}{E_1} \right) \frac{\partial w_2(x, h)}{\partial z} \)
   \[ A_1 e^{-\left( \frac{\pi h}{L} \right)^2} - B_1 e^{-\left( \frac{\pi h}{L} \right)^2} = \left( \frac{1}{\eta} \right) B_2 e^{-\left( \frac{\pi h}{L} \right)^2} \quad \Rightarrow \quad A_1 e^{-\left( \frac{\pi h}{L} \right)^2} - B_1 = -\left( \frac{1}{\eta} \right) B_2 \]
Solving the above simultaneously gives

\[ A_i = \left[ 1 + \frac{\eta + 1}{\eta - 1} e^{\frac{2 \alpha x}{L}} \right]^{-1} \]  
\[ (A-40) \]

\[ B_2 = \frac{2 \left( \frac{\eta}{\eta + 1} \right)}{1 + \left( \frac{\eta + 1}{\eta - 1} \right) e^{\frac{2 \alpha x}{L}}} \]  
\[ (A-41) \]

\[ B_i = \left[ 1 + \frac{\eta - 1}{\eta + 1} e^{\frac{2 \alpha x}{L}} \right]^{-1} \]  
\[ (A-42) \]

\[ w(x_1, z_i) = W \sin \left( \frac{\alpha \pi}{L} \right) \left[ \frac{\alpha x}{L} e^{\frac{2 \alpha x}{L}} + \frac{e^{\frac{2 \alpha x}{L}}}{1 + \left( \frac{\eta + 1}{\eta - 1} \right) e^{\frac{2 \alpha x}{L}}} \right] \]  
\[ (A-43) \]

\[ w(x_2, z_2) = W \sin \left( \frac{\alpha \pi}{L} \right) \left[ \frac{2 \left( \frac{\eta}{\eta + 1} \right) e^{\frac{2 \alpha x}{L}}}{1 + \left( \frac{\eta - 1}{\eta + 1} \right) e^{\frac{2 \alpha x}{L}}} \right] \]  
\[ (A-44) \]

The core stress in area 1 is \( \sigma_z = E_i \frac{\partial w_i}{\partial z} \)

at the face sheet ( \( z = 0 \))

\[ \sigma_0 = W \sin \left( \frac{\alpha \pi}{L} \right) E_i \left( \frac{\alpha \pi}{L} \right) \left[ 1 + \left( \frac{\eta + 1}{\eta - 1} \right) e^{\frac{2 \alpha x}{L}} \right]^{-1} - \left[ 1 + \left( \frac{\eta - 1}{\eta + 1} \right) e^{\frac{2 \alpha x}{L}} \right]^{-1} \]

\[ \sigma_0 = W \sin \left( \frac{\alpha \pi}{L} \right) E_i \left( \frac{\alpha \pi}{L} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) \text{ where } \gamma = \left( \frac{\eta + 1}{\eta - 1} \right) e^{\frac{2 \alpha x}{L}} \]  
\[ (A-45) \]

Equilibrium of the face sheet is defined by

\[ D \frac{\partial^4 w}{\partial x^4} + P \frac{\partial^2 w}{\partial x^2} - \sigma_0 = 0 \]

\[ D \left( \frac{\pi}{L} \right)^4 - P \left( \frac{\pi}{L} \right)^2 - E_i \left( \frac{\alpha \pi}{L} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) = 0 \]

\[ P = D \left( \frac{\pi}{L} \right)^2 - E_i \left( \frac{\alpha L}{\pi} \right) \left( \frac{1 - \gamma}{1 + \gamma} \right) \]  
\[ (A-46) \]
The minimum wrinkling load occurs when \( \frac{dP}{dL} = 0 \)

\[
\frac{dP}{dL} = -2\left(\frac{D}{\pi L}\right)^3 - E_1 \left(\frac{\alpha}{\pi} \left(1 - \frac{1}{1 + \gamma}\right) - E_i \left(\frac{\alpha dL}{\pi} \right) \frac{d}{dL} \left(1 - \frac{1}{1 + \gamma}\right)\right) = 0
\]

\[
\frac{dP}{dL} = -2\left(\frac{D}{\pi L}\right)^3 - E_1 \left(\frac{\alpha}{\pi} \left(1 - \gamma\right) - 4E_i \left(\frac{\alpha^2}{L}\right) \frac{h}{\pi} \frac{\gamma}{(1 + \gamma)^2}\right) = 0
\]

(A-47)

The last two equations were too hard to solve algebraically, so an iterative solver in Mathcad was used to minimize \( P \) (A-46) in respect to \( L \) (the critical wrinkling half-wavelength). The critical wrinkling stress is found by back substituting the critical wrinkling half-wavelength into (A-46)

\[
\sigma_{cr} = \frac{E_f}{12} \left(\frac{\pi f}{L_{cr}}\right)^2 - E_i \left(\frac{\alpha L_{cr}}{\pi f}\right) \left(1 - \frac{1}{1 + \gamma}\right)
\]

(A-48)

Where \( L_{cr} \) is the critical wrinkling half-wavelength found numerically for each case.

and \( \gamma = \left(\frac{\eta + 1}{\eta - 1}\right) e^{\frac{2\pi h}{L_{cr}}}, \eta = \frac{E_1}{E_2}, \eta \leq 1, \alpha = \sqrt{\frac{G_{xz}}{E_z - \nu_{xz} G_{xz}}} \) (same value for region 1 and 2)
B In-plane analytical models (Chapter 4)

1/8th cell showing the notation used in the in-plane analytical models

B.1 Free modulus expressions (Section 4.3)

B.1.1 Calculation of the free modulus in the X direction

**Step 1: Find bending moment expressions for the fillet and angled wall.**

The moment expression for the fillet about a general point C

\[ M_c = W \left( r \cos(\theta - \gamma) - r \cos(\theta) + \frac{L}{2} \sin(\theta) \right) \]  \hspace{1cm} (B-1)

and similarly the moment in the angled wall (point C to D) is

\[ M_x = x \sin(\theta)W \]  \hspace{1cm} (B-2)
Step 2: Calculate strain energy components of bending and extension in each wall.

The strain energy in the fillet (Point B to C) due to bending and extension.

\[
U_{\text{bendfill}} = \int_{0}^{\theta} \frac{M^2}{2EI} r \, d\gamma = \frac{1}{8} W^2 r \left( \frac{-6r^2 \sin(\theta) \cos(\theta) + 2r^2 \theta + 4rL \sin^2(\theta) + 4r^2 \cos^2(\theta) \theta}{E_s I} \right)
\]

(B-3)

\[
U_{\text{extfill}} = \int \frac{1}{2} \left( \frac{\sigma^2}{E_s} - t_c \right) \, d\theta \Rightarrow \frac{1}{2} \left( \frac{1}{2} \cos(\theta) \sin(\theta) + \frac{1}{2} \theta \right) r \frac{W^2}{t_c t_e E_s}
\]

(B-4)

where

\[
\sigma = \frac{W \cos(\theta)}{t_c t_e}, \quad I = \frac{t_c^3}{12}
\]

(B-5)

The strain energy in the angled wall (Point C and D) due to bending and extension is given by

\[
U_{\text{bendang}} = \int_{0}^{\frac{L}{2}} \frac{M^2}{2EI} \, dx \Rightarrow \frac{1}{48} L^3 \sin^2(\theta) \frac{W^2}{E_s I}
\]

(B-6)

\[
U_{\text{extang}} = \frac{1}{2} \frac{\sigma^2}{E_s} t_c t_e \beta \Rightarrow \frac{1}{4} W^2 \frac{\cos^2(\theta)}{t_c t_e E_s} L
\]

(B-7)

where

\[
\sigma = \frac{W \cos(\theta)}{t_c t_e}
\]

(B-8)

and the strain energy in the straight wall (Point A to B)

\[
U_{\text{straight}} = \frac{1}{2} \frac{\sigma^2}{E_s} t_c t_e L \frac{L}{2} \Rightarrow \frac{1}{4} W^2 \frac{L}{t_c t_e E_s}
\]

(B-9)

where

\[
\sigma = \frac{W}{t_c t_e}
\]

(B-10)

Step 3: Sum the strain energy terms together

\[
U_{\text{total}} = U_{\text{bendfill}} + U_{\text{bendang}} + U_{\text{extang}} + U_{\text{straight}} + U_{\text{extfill}}
\]

(B-11)

Step 4: Use Castigliano's theorem to calculate the reaction force \(W\) due to the applied displacement.

Expressions for the X displacement at point D due to load \(W\) is found by differentiating \(U_{\text{total}}\) with respect to load \(W\) (Castigliano).

\[
\mu_{\text{load}} = \frac{dU_{\text{total}}}{dW}
\]

(B-12)
and the deflection at point D due to the prescribed strain is:

$$\mu_{\text{strain}} = \left(\frac{b}{2} + \frac{b}{2} \cos(\theta)\right) \varepsilon_x$$ \hspace{1cm} (B-13)

The deflection at point D from Castigliano (Equation B-12) is equated to the prescribed displacement at the end of the angled walled (Equation B-13). This equation is then rearranged in terms of W, where W is the force needed to displace the system by the given displacement $\mu_{\text{strain}}$

$$\mu_{\text{strain}} = \mu_{\text{load}} \Rightarrow \left(\frac{b}{2} + \frac{b}{2} \cos(\theta)\right) \varepsilon_x = \frac{dU_{\text{total}}}{dW} \text{ solve } W \hspace{1cm} (B-14)$$

**Step 5: Calculate the modulus.**

The modulus is derived from the stress, which is calculated from W acting over the entire width of the cell, divided by the strain in the system.

$$E_x = \frac{W}{b \sin(\theta) \varepsilon_x} \hspace{1cm} (B-15)$$

and because this is a bending dominated problem the solution can be simplified to

$$U_{\text{total}} = U_{\text{bendfill}} + U_{\text{bendang}} \hspace{1cm} (B-16)$$

$$Ex = -12(1 + \cos(\theta))E_x \frac{I}{36r^3t_\perp \cos(\theta) \sin(\theta) - 12r^3t_\perp \sin^2(\theta) - 12r^3t_\perp \cos(\theta) + 24r^2t_\perp \sin(\theta) \theta - 6rt_\perp L^2 \sin^2(\theta) \theta - L^3 \sin^2(\theta) \theta - 12 \cos^2(\theta) LI - 12 LI - 12 r \cos(\theta) \sin(\theta) - 12 rl \theta \sin(\theta)} \hspace{1cm} (B-17)$$

**B.1.2 Calculation of the free modulus in the Y direction**

In this case the model is solved for load P and the moment expression (Equation (B-1)) for the fillet becomes:

$$M_\gamma = P \left( r \sin(\theta) - r \sin(\gamma) + \frac{L}{2} \cos(\theta) \right) \hspace{1cm} (B-18)$$

and the angled wall (B-2) changes to
\[ M_x = P \cos(\theta)x \]  
(B-19)

The strain energy terms change slightly with the stress in the angled and fillet walls (Equation B-5) becoming

\[ \sigma = \frac{P \sin(\theta)}{th} \]  
(B-20)

The summation of the strain energy Equation B-11 becomes

\[ U_{total} = U_{bendfill} + U_{bendang} + U_{extan} + U_{estfil} \]  
(B-21)

Note: there is no straight wall term as this has 0 net force acting on it in this direction.

The deflection in the \( Y \) direction is given by

\[ v_{strain} = \left( \frac{b}{2} \sin(\theta) \right) \varepsilon_y \]  
and \[ v_{load} = \frac{dU_{total}}{dP} \text{ where} \]

\[ v_{strain} = v_{load} \Rightarrow \left( \frac{b}{2} \sin(\theta) \right) \varepsilon_y = \frac{dU_{total}}{dP} \text{ solve } P \]  
(B-22)

The effective continuum modulus in the \( Y \) direction is found using

\[ E_y = \frac{P}{\left( \frac{b + b \cos(\theta)}{h} \right) \varepsilon_y} \]  
(B-24)

\[
E_y = 12b \sin(\theta)E_y I \left[ \frac{t_s}{-48r^3 t_c \sin(\theta) - 24r^2 t_c L \cos(\theta) + 24r^3 t_c \sin^2(\theta)\theta + 36r^3 t_c \sin(\theta)\cos(\theta) + 24r^2 t_c \sin(\theta)L \cos(\theta)\theta + 12r^3 t_c \theta + 24r^2 t_c L \cos^2(\theta) + 6r t_c L^2 \cos^2(\theta)\theta + L^3 \cos^2(\theta) t_c + 12 LI}{b + b \cos(\theta)} \right] + 12r \theta
\]

with the extension terms removed the model simplifies to

\[ U_{total} = U_{bendfill} + U_{bendang} \]  
(B-25)

\[
E_y = 12b \sin(\theta)E_y I \left[ \frac{I}{-48r^3 \sin(\theta) - 24r^2 L \cos(\theta) + 24r^3 \sin^2(\theta)\theta + 36r^3 \sin(\theta)\cos(\theta) + 24r^2 \sin(\theta)L \cos(\theta)\theta + 12r^3 \theta + 24r^2 L \cos^2(\theta) + 6rL^2 \cos^2(\theta)\theta + L^3 \cos^2(\theta)}{b + b \cos(\theta) t_c} \right]
\]

B.1.3 “Free” in-plane Poisson’s Ratio

The Poisson’s ratio is found by adding load \( P \) to the bending moment expressions for the fillet and angled wall. The moment expressions become:
\[ M_c = W \left( r \cos(\theta - \gamma) - r \cos(\theta) + \frac{L}{2} \sin(\theta) \right) - P \left( r \sin(\theta) - r \sin(\theta - \gamma) + \frac{L}{2} \cos(\theta) \right) \]  

(B-26)

\[ M_x = W \sin(\theta) x - P \cos(\theta) x \]  

(B-27)

The strain energy terms are equated in a similar way to Section (Section 4.4):

\[ U_{\text{total}} = U_{\text{bendfill}} + U_{\text{bendang}} + U_{\text{ext, ang}} + U_{\text{ext, straight}} + U_{\text{fill}} \]  

(B-28)

\[ U_{\text{bendang}} = \frac{L}{2} \frac{M_z^2}{2E_sI} dx, U_{\text{bendfill}} = \int_0^\theta \frac{M_z^2}{2E_sI} r \sin(\theta) d\theta, U_{\text{ext, ang}} = \frac{1}{2} \frac{\sigma^2}{E_s} t_s t_c, U_{\text{ext, straight}} = \int \frac{1}{2} \left( \frac{\sigma^2}{E_s} t_s t_c \right) d\theta \]

where

\[ \sigma = \frac{W \cos(\theta) + P \sin(\theta)}{t_s t_c}, \quad \beta = \frac{L}{2} \quad \text{and} \quad U_{\text{ext, straight}} = \frac{1}{2} \frac{\sigma^2 t_s t_c L}{2} \quad \text{where} \quad \sigma = \frac{W}{t_s t_c} \]

Castigliano’s theorem is used to calculate the deflection in the \( Y \) direction at point \( D \).

The term \( \text{dispy} \) contains forces \( W \) and \( P \), where force \( P \) is set to 0, because the only forced displacement is in the \( X \) direction.

\[ \text{dispy} = \frac{dU_{\text{total}}}{dP} \]  

(B-29)

The Poisson’s ratio is given by

\[ v_{xy} = \frac{\text{dispy}}{\frac{b}{2} \sin(\theta)} \]

which is \[ v_{xy} = \frac{\epsilon_y}{\epsilon_x} \]  

(B-30)

**B.2 Constrained in-plane modulus (Section 4.6)**

**B.2.1 \( E_x \) “constrained central core” modulus**

*Step 1: Find the bending moment expressions for the inclined fillet walls.*

The moment expression for the fillet between points \( B \) and \( C \) is:

\[ M_c = W \left( r \cos(\theta - \gamma) - r \cos(\theta) + \frac{L}{2} \sin(\theta) \right) - P \left( r \sin(\theta) - r \sin(\theta - \gamma) + \frac{L}{2} \cos(\theta) \right) \]  

(B-31)

where \( C \) is an arbitrary point in the fillet.
The bending moment expression for the inclined wall between points \(C\) and \(D\) is:

\[
M_x = W \sin(\theta)x - P \cos(\theta)x
\]  

where \(X\) is any arbitrary distance from point \(D\) along the wall.

**Step 2: Calculate strain energy components of bending (bend) and extension (ext) in each wall.**

Strain energy in the fillet is given by:

\[
U_{\text{bendfil}} = \int_{0}^{\theta} \frac{M_c^2}{2EI}x\,d\theta
\]  

\[
U_{\text{exffil}} = \frac{1}{2}r\left(\frac{\sigma^2}{E_s}t_st_c\right)d\theta
\]

where

\[
\sigma = \frac{W \cos(\theta) + P \sin(\theta)}{t_st_c}, \quad I = \frac{t_s^3t_c}{12}
\]

The strain energy in the inclined wall is:

\[
U_{\text{extang}} = \frac{1}{2} \frac{\sigma^2}{E_s}t_st_c \beta
\]

\[
U_{\text{bendang}} = \int_{0}^{\frac{L}{2}} \frac{M_c^2}{2EI}dx
\]

where \(\beta = \frac{L}{2}\)

For the straight walled section between points \(A\) and \(B\) the strain energy is

Note: There is no bending in this wall, only a strain energy component due to extension

\[
U_{\text{extstraight}} = \frac{1}{2} \frac{\sigma^2}{E_s}t_st_c \frac{L}{2}
\]

where \(\sigma = \frac{W}{th}\)

**Step 3: Add the strain energy terms together.**

\[
U_{\text{total}} = U_{\text{bendfil}} + U_{\text{bendang}} + U_{\text{extang}} + U_{\text{extstraight}} + U_{\text{exffil}}
\]

**Step 4: Use Castigliano’s theorem to calculate the reaction force \(W\) and \(P\), due to the applied displacement.**

Expressions for displacements at point \(D\) (in the \(X\) and \(Y\) directions) due to loads \(W\) and \(P\) are
found by differentiating $U_{total}$ with respect to loads $W$ and $P$. Where

$$
\mu_{load} = \frac{dU_{total}}{dW} \quad \text{Displacement (x direction) at point D due to load W} \tag{B-40}
$$

$$
v_{load} = \frac{dU_{total}}{dP} \quad \text{Displacement (Y direction) at point D due to load P} \tag{B-41}
$$

and the deflection at point D due to the prescribed strain is

$$
\mu_{strain} = \left( \frac{b}{2} + \frac{b}{2} \cos(\theta) \right) \epsilon_x, \quad v_{strain} = \left( \frac{b}{2} \sin(\theta) \right) (-\epsilon_y) \tag{B-42}
$$

which are the displacements of the face sheet and core at point D

The prescribed displacement is now equated to the deflection caused by load $W$ and $P$

$$
\left( \frac{b}{2} + \frac{b}{2} \cos(\theta) \right) \epsilon_x = \frac{dU_{total}}{dW} \tag{B-43}
$$

$$
\left( \frac{b}{2} \sin(\theta) \right) (-\epsilon_y) = \frac{dU_{total}}{dP} \tag{B-44}
$$

The reaction forces $(W, P)$, which are currently unknowns, are used to calculate the modulus. These two expressions can be solved simultaneously to find $W$ which is used to calculate the modulus in the $X$ direction.

$$
E_x = \frac{W}{b \sin(\theta) \epsilon_x} \tag{B-45}
$$

**B.2.2 $E_y$ “constrained central core” modulus**

The modulus in the $Y$ direction can be found in a similar way to the modulus in the $X$ direction.

Equations B-44 and B-45 become;

$$
\left( \frac{b}{2} + \frac{b}{2} \cos(\theta) \right) (-\epsilon_y) = \frac{dU_{total}}{dW} \tag{B-46}
$$

$$
\left( \frac{b}{2} \sin(\theta) \right) (\epsilon_y) = \frac{dU_{total}}{dP} \tag{B-47}
$$

The modulus is found using

$$
E_y = \frac{P}{(b + b \cos(\theta)) \epsilon_y} \tag{B-48}
$$
B.3 “Constrained core/face sheet” modulus (Section 4.8)

B.3.1 “Constrained core/face sheet” modulus in the $X$ direction

This modulus is found using strain energy expressions.

The expression of strain in any inclined wall at an angle is given by

$$
\varepsilon_{\text{ang}} = \frac{E_s (1 + v)}{2} \cos(2\theta) + \frac{E_s (1 - v)}{2}
$$

(B-49)

Where $\varepsilon_s$ is the strain in the face sheet

The strain energy terms are given by

$$
U_{\text{straight}} = \frac{E_s \varepsilon_s^2 t_s L_t}{2}
$$

(B-50)

$$
U_{\text{angle}} = \frac{E_s \varepsilon_{\text{ang}}^2 t_s L_t}{2}
$$

(B-51)

$$
U_{\text{fillet}} = \int \frac{E_s \varepsilon_{\text{ang}}^2 t_{r} L_t}{2} \, d\theta
$$

(B-52)

The strain energy in an equivalent block with the same overall volume as the cell is equated to the combined strain energies in the walls

$$
\frac{E_s \varepsilon_s^2}{2} V_s = \sum_i U_{\text{ai}}
$$

(B-53)

Where: $U_{ai}$ is the strain energy in the $i$th walls, $V_s$ is a volume of an equivalent block which is replacing the entire cell, $\varepsilon_s$ is the strain in the system and $E_s$ is the continuum modulus

Which can be written as,

$$
U_{\text{total}} = U_{\text{straight}} + U_{\text{angle}} + 2U_{\text{fillet}}
$$

where

$$
U_{\text{total}} = \frac{E_s \varepsilon_s^2}{2} (1 + \cos(\theta)) b^2 \sin(\theta) h
$$

(B-54)

Therefore

$$
\frac{E_s \varepsilon_s^2}{2} (1 + \cos(\theta)) b^2 \sin(\theta) h = U_{\text{straight}} + U_{\text{angle}} + 2U_{\text{fillet}}
$$

(B-55)

This expression can be rearranged in terms of the unknown $E_x$ to give:
A similar approach can be taken for the constrained modulus in the $Y$ direction.

### B.3.2 “Constrained core/face sheet” modulus in the $Y$ direction

In the $Y$ direction the function which defines the strain in a wall at any angle becomes

$$
\varepsilon_{\text{ang}} = \frac{E_y}{2} \left[ \frac{(1 + \nu)}{\cos(2\theta + \pi)} + \frac{\nu (1 - \nu)}{2} \right]
$$

The individual strain energy's in the walls are given by

$$
U_{\text{straight}} = E_y \frac{\varepsilon_{\text{ang}}^2 t_s L_t c}{2}
$$

$$
U_{\text{angle}} = E_y \frac{\varepsilon_{\text{ang}}^2 t_s L_t c}{2}
$$

$$
U_{\text{fillet}} = \int \frac{E_y \varepsilon_{\text{ang}}^2 t_s r_t c}{2} d\theta
$$

The strain energy in an equivalent block with same overall volume as the cell is equated to the combined strain energies in the walls

$$
\frac{E_y \varepsilon_{y}^2}{2} V_y = \sum_i U_{y_i}
$$

Where $U_{y_i}$ is the strain energy in the $i$th walls, $V_y$ is a volume of an equivalent block which is replacing the entire cell, $\varepsilon_y$ is the strain in the system and $E_y$ is the continuum modulus. This can which can be expressed as:

$$
\frac{E_y \varepsilon_{y}^2 (1 + \cos(\theta))}{2} b^2 \sin(\theta) h = U_{\text{straight}} + U_{\text{angle}} + 2U_{\text{fillet}}
$$

The in-plane modulus is found by rearranging the expression above

$$
E_y = \frac{1}{8} E_s t_s \frac{\left( 10L^2 + 2L \cos^2(2\theta) + 4L \cos^2(2\theta) v + 4L \cos(2\theta) + 2L \cos^2(2\theta) v^2 \right)}{b^2 \sin(\theta)(1 + \cos(\theta))}
$$

$$
- 4L \cos^2(2\theta) v^2 - 4Lv + 2L v^2 + r \cos(2\theta) \sin(2\theta) + 6r \theta \\
+ 2rv \cos(2\theta) \sin(2\theta) - 4rv \theta + 4r \sin(2\theta) + r v^2 \cos(2\theta) \sin(2\theta) \\
+ 6rv^2 \theta - 4rv^2 \sin(2\theta)
$$
C Test Panels

C.1 Nomex Core

Throughout this study three different size and density cores were used. The type of core used was HRH-10 which is a honeycomb designation widely used for aerospace applications.

HRH-10 is manufactured from Nomex® aramid fibre sheets. Nomex is a Kevlar based paper supplied by Dupont.

HRH-10 is widely accepted throughout the aerospace industry and several commercial areas as a very tough, environmentally resistant core material in sandwich panels. It is designed and used in flat and contoured shapes, with a variety of facing materials and adhesives, and has extensive service in secondary and tertiary structures. Most of the interior panels of commercial jets such as the Boeing 777, 747, 737 and Airbus A320, A340 etc are made from this core material, primarily because of its resilience, small cell size/low density combination and fire resistance. Exterior aircraft panels such as fairings, radomes and helicopter blades are designed with HRH-10.

HRH-10 is constructed from strips of Nomex which are glued together at intervals along their length. This is done with a thermosetting adhesive. Once bonded the sheets are pulled in the transverse direction to produce a hexagonal block. To lock the honeycomb shape the block is dipped into a phenolic resin. The number of times this dipping process takes place determines the density of the block. Once dried the core is trimmed to the desired thickness. Because of this construction method, the honeycomb is anisotropic as particular walls normal to the ribbon direction have two layers of paper, while other walls only have a single layer.

Three types of used were

- HRH 10-1/8-3.0
- HRH 10-1/8-4.0
- HRH 10-3/16-3.0

where

<table>
<thead>
<tr>
<th>HRH-10</th>
<th>designates the honeycomb type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8,3/16</td>
<td>is the cell size in inches</td>
</tr>
<tr>
<td>3.0,4.0</td>
<td>is the nominal density in lbs/in^3</td>
</tr>
</tbody>
</table>
Manufactures Properties – US Core

<table>
<thead>
<tr>
<th>Core</th>
<th>Bare $E_z$ crush Strength (MPa)</th>
<th>Stabilized $E_z$ Crush Strength (MPa)</th>
<th>$G_{xy}$ (MPa)</th>
<th>$G_{yz}$ (MPa)</th>
<th>$E_z$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH-10 - 1/8 - 3.0</td>
<td>2.0</td>
<td>2.2</td>
<td>41.38</td>
<td>24.1</td>
<td>137.9</td>
</tr>
<tr>
<td>HRH-10 - 1/8 - 4.0</td>
<td>3.6</td>
<td>4.0</td>
<td>59.3</td>
<td>32.4</td>
<td>193.1</td>
</tr>
<tr>
<td>HRH-10 - 3/16 - 3.0</td>
<td>1.9</td>
<td>2.2</td>
<td>44.83</td>
<td>23.4</td>
<td>137.9</td>
</tr>
</tbody>
</table>

Nominal values for these types of core (manufactures values)

<table>
<thead>
<tr>
<th>Core</th>
<th>Stabilized $E_z$ crush strength (MPa)</th>
<th>$G_{xy}$ (MPa)</th>
<th>$G_{yz}$ (MPa)</th>
<th>$E_z$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRH-10 – 1/8 - 3.0</td>
<td>2.2</td>
<td>40</td>
<td>24</td>
<td>138</td>
</tr>
<tr>
<td>HRH-10 – 1/8 - 4.0</td>
<td>4.0</td>
<td>60</td>
<td>32</td>
<td>193</td>
</tr>
<tr>
<td>HRH-10 – 3/16 - 3.0</td>
<td>2.2</td>
<td>40</td>
<td>23</td>
<td>138</td>
</tr>
</tbody>
</table>

**C.2 Face sheets**

There were two types of face sheets used throughout this research project. One was a glass epoxy laminate and the other one was a carbon-epoxy laminate. The laminates are uni-directional weave which are pre-impregnated with resin.

Both of these face sheet materials are given Boeing material specifications designations.
Hexcel Composites produce a material system which is compliant to both of the Boeing specifications.

**C.2.1 BMS 8-79 (Hexcel F155)**

Hexcel’s equivalent to BMS 8-79 glass face sheet is its F155™ resin with an 8 harness satin weave.

F155™ is an advanced modified epoxy formulation designed for autoclave curing to offer very high laminate strengths with increased fracture toughness and adhesive properties. The cure cycle of F155™ is 250°F (121°C) for 90 minutes. Typical applications of this controlled flow epoxy resin are co-curing onto honeycomb and bonding to metal. F155 is also combined with carbon and Kevlar matrix fabrics and a variety of tapes.

<table>
<thead>
<tr>
<th>Hexcel Designation</th>
<th>Fibre</th>
<th>Fibre weight g/m²</th>
<th>Weave</th>
<th>Count warp x Fill</th>
</tr>
</thead>
<tbody>
<tr>
<td>1581-38”-F155</td>
<td>150-1/2</td>
<td>303</td>
<td>8 Harness Satin</td>
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</table>

<table>
<thead>
<tr>
<th>Hexcel Designation</th>
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</tr>
</thead>
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<td>Compression Strength</td>
<td>517 MPa</td>
</tr>
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<td>Tensile Strength</td>
<td>483 MPa</td>
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<tr>
<td>Tensile Modulus</td>
<td>23.4 GPa</td>
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<tr>
<td>Compression inter-laminar shear strength</td>
<td>68.9 MPa</td>
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<tr>
<td>F155 Resin Tensile Modulus</td>
<td>3.24 GPa</td>
</tr>
<tr>
<td>Fibre Volume Fraction</td>
<td>45%</td>
</tr>
</tbody>
</table>

**C.2.2 BMS 8-256 (Hexcel F593)**

Hexcel’s equivalent to BMS 8-256 is the F593 resin with a plane weave Carbon Fibre.

F593 is a 350°F curing epoxy system with a very low flow for carbon fabric and tape applications. It provides excellent laminate and sandwich properties. As a low flow resin, F593 lends itself to net resin, zero bleed applications. Because of the low flow content an additional adhesive layer is needed to bond the laminate to the honeycomb core.

<table>
<thead>
<tr>
<th>Hexcel Designation</th>
<th>Fibre</th>
<th>Fibre Weight g/m²</th>
<th>Weave</th>
<th>Count Warp x Fill</th>
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</thead>
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<tr>
<td>W3T282-42”-F593</td>
<td>Toray T-300/3K</td>
<td>193</td>
<td>Plane</td>
<td>12.5x12.5</td>
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</table>
**Hexcel Designation**

<table>
<thead>
<tr>
<th>Hexcel Designation</th>
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</tr>
</thead>
<tbody>
<tr>
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<tr>
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<tr>
<td>F155 Resin Tensile Modulus</td>
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<tr>
<td>Fibre Volume Fraction</td>
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</tr>
</tbody>
</table>

Because of the low resin bleed off, an additional adhesive is used to stick the carbon BMS 8-256 face to the honeycomb surface. The adhesive layer is BMS 5-101.

### C.3 Panel Configurations

<table>
<thead>
<tr>
<th>Panel</th>
<th>Core</th>
<th>Faces sheet</th>
<th>Number of layers</th>
<th>Sandwich thickness (inches”)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>HRH10 1/8&quot;-3.0 -1”</td>
<td>BMS 8-79</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>HRH10 1/8&quot;-3.0 -1”</td>
<td>BMS 8-256, BMS 5-101 adhesive</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>HRH10 3/16”-3.0 -1”</td>
<td>BMS 8-256, BMS 5-101 adhesive</td>
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<td>1</td>
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<tr>
<td>4</td>
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<td>1</td>
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<tr>
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<td>BMS 8-79</td>
<td>2</td>
<td>1</td>
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<tr>
<td>7</td>
<td>HRH10 1/8&quot;-3.0 -0.5”</td>
<td>L528-7781 Phenolic</td>
<td>0.5</td>
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</tbody>
</table>

### C.4 Panel Fabrication

#### C.4.1 Materials

- Hexcel Nomex Honeycomb: HRH-10-1-1-8-3.0-1.0 BMS 8-124 class IV type V grade 3.0.
- Hexcel Nomex Honeycomb: HRH-10-1-1-8-4.0-1.0 BMS 8-124 class IV type V grade 4.0.
- Hexcel Nomex Honeycomb: HRH-10-1-3-16-3.0-1.0 BMS 8-124 class IV type V grade 3.0.
- Hexcel Carbon fibre Prepreg: W3T282-42”-F593 BMS 8-256

Specimen Size: 720mm x 320mm. Ribbon and 0 degree direction equivalent to longest side orientation.

La-yup: Two plies either at 0 degrees and 90 degrees, outer plies in the 0 degree direction.

Bagging
C.4.2 Lay-up procedure

1. A clear release film was placed onto the carbon platen.
2. Two bottom face sheets cloths were assembled and laid on top of the release film.
3. Two steel frames were placed on top of the bottom face sheets. The frames stop the panels crushing when the vacuum is applied.
4. Pre-cut cores were placed into the frames. The top face sheet was then placed on top of the core.
5. A perforated release film was placed on the top face sheet to control bleed-off during curing.
6. A bleeder cloth was put onto the assembly to absorb any epoxy resin bled off from the prepreg during curing phase.
7. A breather cloth that allows air to be sucked out of the vacuum bag was placed onto the assembly.
8. The entire assembly was covered by a vacuum bag which forms a seal with the platen, via an edge sealant tape. The vacuum bag was pulled to a vacuum of approximately 13.5psi.
9. The panels were placed into the oven for curing. The entire cook cycle takes 4.5 hours. The actual cooking (soak) stage takes 90 minutes at 250°F and 350°F for the glass and carbon faces respectively. The balance of time is taken up by the cooling up and cooling down phases.

The requirements were:

- All the bags must pull a minimum of 12.5 psi before the cook cycle.
- Panels are rejected if the pressure falls below 10psi during cooking.
### C.4.3 Panel manufacturing photographs

<table>
<thead>
<tr>
<th>Four panels on the platen after cure</th>
<th>Cutting core out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panels being vacuum bagged</td>
<td>Laying top face sheet over core</td>
</tr>
<tr>
<td>Vacuum hose which supplies a vacuum to the bag</td>
<td></td>
</tr>
</tbody>
</table>
D Ansys Elements (from Ansys’s online documentation)

D.1 SHELL181 Element Description

SHELL181 is suitable for analyzing thin to moderately-thick shell structures. It is a 4-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. (If the membrane option is used, the element has translational degrees of freedom only). The degenerate triangular option should only be used as filler elements in mesh generation.

SHELL181 is well-suited for linear, large rotation, and/or large strain non-linear applications. Change in shell thickness is accounted for in non-linear analyses. In the element domain, both full and reduced integration schemes are supported. SHELL181 accounts for follower (load stiffness) effects of distributed pressures.

SHELL181 may be used for layered applications for modeling laminated composite shells or sandwich construction. The accuracy in modeling composite shells is governed by the first order shear deformation theory (usually referred to as Mindlin-Reissner shell theory).

SHELL181 Geometry

S.2 SOLID185 Element Description

SOLID185 is used for the 3-D modeling of solid structures. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has plasticity, hyperelasticity, stress stiffening, creep, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials.

A higher-order version of the SOLID185 element is SOLID186.

SOLID185 Geometry
D.3 SOLID186 Element Description

SOLID186 is a higher order 3-D 20-node structural solid element. SOLID186 has quadratic displacement behavior and is well suited to modeling irregular meshes (such as those produced by various CAD/CAM systems).

The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. SOLID186 may have any spatial orientation. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials.

SOLID186 Geometry
D.4 PLANE183 Element Description

PLANE183 is a higher order 2-D, 8-node element. PLANE183 has quadratic displacement behavior and is well suited to modeling irregular meshes (such as those produced by various CAD/CAM systems).

This element is defined by 8 nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element (plane stress, plane strain and generalized plane strain) or as an axisymmetric element. This element has plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials. Initial stress import is supported. Various printout options are also available.

PLANE183 Geometry

D.5 LINK 180 Element Description

LINK180 is a spar that can be used in a variety of engineering applications. This element can be used to model trusses, sagging cables, links, springs, etc. This 3-D spar element is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal x, y, and z directions. As in a pin-jointed structure, no bending of the element is considered. Plasticity, creep, rotation, large deflection, and large strain capabilities are included. By default, LINK180 includes stress stiffness terms in any analysis with large deformation activated. Elasticity, isotropic hardening plasticity, kinematic hardening plasticity, Hill anisotropic plasticity, Chaboche non-linear hardening plasticity, and creep are supported.

LINK180 Geometry
D.6 PLANE162 Element Description

PLANE162 is used for modeling 2-D solid structures in ANSYS LS-DYNA. The element can be used either as a planer or as an axisymmetric element. The element is defined by four nodes having six degrees of freedom at each node: translations, velocities, and accelerations in the nodal x and y directions. A three-node triangle option is also available, but not recommended.

The element is used in explicit dynamic analyses only. When using this element, the model must only contain PLANE162 elements - you cannot mix 2-D and 3-D explicit elements in the same model. Furthermore, all PLANE162 elements in the model must be the same type (plane stress, plane strain, or axisymmetric).

PLANE162 Geometry
### E Experimental values

#### E.1 Undamaged experimental tests (Chapter 5)

<table>
<thead>
<tr>
<th>Face sheet Material</th>
<th>Core Material</th>
<th>Panel width (mm)</th>
<th>Sandwich Thickness (mm)</th>
<th>( t_f ) (mm)</th>
<th>( h ) (mm)</th>
<th>Failure load (N)</th>
<th>Internal Span (mm)</th>
<th>Overall Span (mm)</th>
<th>Failure stress (MPa)</th>
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**F Numerical models**

**F.1 LS-dyna impact damage model – Chapter 6**

Key Parameters – LS-Dyna model
- Crush stress 2.2MPa
- Out-of-plane modulus 138MPa
- Facesheet modulus = 25500MPa
- Core thickness = 25.5 mm
- Facesheet thickness = 0.47mm
- LS-dyna core model - Foam core crush model – isotropic core (E = 138MPa)
- LS-dyna ball model - Fluid material
- Speed of projectile 33m/s
- Density of fluid 1000kg/m3

**F.1.1 Rigid body impact**

![Images of rigid body impact at different time intervals]
$12e^{-4}$ sec

$13.2e^{-4}$ sec

$14.4e^{-4}$ sec

$15.6e^{-4}$ sec
F.1.2 Soft Body impact
$1.44\times10^{-4}$ sec

$15.6\times10^{-4}$ sec