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**The  
Atomic  
Trampoline  
Cavity**

Gregory John Liston

A thesis  
submitted in partial fulfillment  
of the requirements for the degree  
of the  
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at the  
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# Abstract

Using an intense evanescent light wave as the lower mirror, and the gravitational force as the upper mirror, a vertical cavity for storing atoms can be constructed. Transverse confinement is obtained by totally internally reflecting the light off a concave as opposed to a planar crystal surface, which results in any atom reflected by the evanescent potential at a position away from the central axis receiving an impulse towards it.

After a cursory discussion of atom optics and atomic cavities, we outline the configuration of the atomic trampoline cavity described above, and present analysis of the motion of atoms within it. A discussion of the classical dynamics and quantum modes in the cavity is given, together with other complicating factors which act as loss mechanisms out of the cavity. Various aspects of obtaining experimental realizations and applications of the cavity are considered.

A detailed study of the quantum dynamics of atoms in the three dimensional cavity reveals that the dispersion can be adequately described in the transverse directions using a simulation involving a classical distribution of point-like atoms, where the probability density of finding an atom at a particular position in the simulation corresponds to the probability density of the atomic wavefunction. The classical simulations, however, significantly underestimate the spreading in the vertical direction.

By calculating the modes of the atomic trampoline cavity, both in and out of the evanescent potential, the proportion of each of the modes in the excited state, and hence the decay rate, or linewidth due to spontaneous emission can be calculated. We found that even when the effect of the evanescent potential was included, the modes obtained correspond to those calculated by Wallis, Dalibard and Cohen-Tannoudji [Appl. Phys. B 54, 407 (1992)], who treated the bottom potential as infinitely steep and not exponentially decaying. In contrast to an optical Fabry-Pérot cavity, the linewidth was found to be strongly dependent on energy. Various other cavity parameters (finesse and  $Q$ ) which depend on the loss due to spontaneous emission were also calculated.

Using a ring cavity rather than a laser traveling wave to provide the light that totally internally reflects off the internal surface of the dielectric crystal, we can accumulate the phase change due to the single atom bouncing into and out of the evanescent wave and altering the refractive index of the cavity. A measurement of the phase of this light will reveal information about the atom. We found that the measurement did not significantly alter the mean or standard deviation of the atomic energy distribution across the modes of the cavity, as to first order the phase change of the light in the cavity is independent of the energy of the atom. The significant change in the energy distribution was the introduction of oscillations, which occurred when the phase measured was significantly different from the expected mean. The reason for these oscillations is that the measurement implies the weighting of modes just entering or leaving the evanescent wave should be increased or decreased. Ways of bringing this currently infeasible experiment closer to being achievable using novel design mechanisms are also discussed.



# Acknowledgments

*I dread success. To have succeeded is to have finished one's business on earth, like the male spider, who is killed by the female the moment he has succeeded in his courtship. I like a state of continual becoming, with a goal in front and not behind.*

*George Bernard Shaw*

Coming to the end of a long period of work on a single subject, I'm told, is always an emotionally turbulent time. While not dreading success like Mr Shaw, I can certainly relate to his ideal of always striving, always reaching for a goal in front and never resting on the merits of the goals completed. The end of the Ph.D. seems to be not just the writing of a book, but the close of a season. In its closure, I want to thank not only those who have directly helped with the thesis, but also those who have made the season memorable.

To Professor Dan Walls, who has made this group one of the most successful in Australasia and indeed the world, goes my admiration and sincere thanks.

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Particular acknowledgment and recognition should go to those whom I've lived with. Thanks to Rob and Suzanne, who rescued me from a bad situation, and have always been there with a prayer, a hand or a sympathetic ear when it was needed. I hope one day I have a chance to repay your kindness. To Phil and Fraser (who both had the pure audacity to go and get married (to different women of course), and then kick me out of the house all within the space of a month and a half just because of some obscure desire for marital bliss, as if you couldn't have marital bliss and a flatmate), goes my sincere respect and gratitude. I learned a lot from living with both of you. Finally, to Nick, Graeme, Nicola and Lauren, a special vote of thanks for putting up with me, especially during the last few months when the pressure was on and I wasn't handling it well. I shall always remember your friendship, sincerity and support. I cannot wait to see Lauren in seven or eight years and say "I lived with you when you were *this* big - you were noisy."

To my friends at church, the infamous D.J.'s who still haven't changed their name after I don't know how many years, especially Angela, Rachel, Duane and Julianne (even though she left), thanks for the

good times and the fun. To Pastor John, for your wisdom and for investing time in me. Your effort is very much appreciated. To my friends in Australia who have kept contact; Julien, Kathy, Tony and Chris, your friendship means a great deal to me. To Jo, the loveliest person I have ever known, thankyou for making the last few months bearable. I hope all your dreams come true; your strength and compassion make me feel very humble.

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Finally, and most importantly, thankyou to Jesus, for always being there, for holding me up when I'm falling down, for showing me undeserved and unstoppable love, for giving me some ability to understand your incredible creation, and the opportunity to study it, for teaching me more and more what it means to know and love you. This thesis is dedicated to the glory of God.

*Greg Liston*  
*January 1996*

# Preface

*Science has “explained” nothing; the more we know the more fantastic the world becomes and the profounder the surrounding darkness.*

*Aldous Huxley*

*I assert that the cosmic religious experience is the strongest and noblest driving force behind scientific research.*

*Albert Einstein*

A work of magnitude is often begun with a history of the subject to date; a thesis on light then, has a long history to record.

In the beginning ..... God said let there be light. *Genesis 1:1*

My lecturer in third year cosmology suggested in contrast that God had actually said “Let there be quantum field theory”. From the relative safety of across the Tasman I would dispute this, not least because I very much doubt whether there are Hebrew words for “quantum field theory”, and if there were Moses probably would not have known how to spell them. Certainly no Hebrew scholar I have ever discussed the subject with has been utterly convinced.

Nevertheless, ever since then people have been trying to work out exactly what God meant. Everyone used to think they knew what “In the beginning” was trying to get across though lately people have started to dispute that there ever was a beginning; no-one ever claimed to know what “God” meant, at least not in public; “said”, “let” and “there” are relatively obvious and not worthy of discussion, and there has been little interest in “be”. “Light” is enclosed in the subject of this thesis.

Between the time God said this and now, even though there is little chance of comparison, we claim a lot of progress has been made. One of the clever things that physicists have done is to change the name of light. The advantage of changing the name of some physical property is that it gives the impression to people who know little about it that you actually know what you are talking about. A classic example is gravity. Gravity is the force that makes two masses attract one another, and physicists can tell you a lot about it. They can tell you that rocks fall and that planets go around the sun and many other things, but they cannot tell you why. As one observer noted,

People used to think that angels pushed the planets in their orbit around the sun. With gravity the only thing that has happened is that the angels are pushing in another direction. *Author unknown and quoted with considerable license.*

It is quite incredible that after all these years of watching rocks fall we honestly still don't know why they do it. Obviously physicists got annoyed with never being able to answer the always asked question “Why do rocks fall?”, and so they invented a new name for the force of attraction called gravity. Now when people deftly avoid the falling debris from a nearby volcano or construction site, and turn to a

physicist to ask “Why?”, the physicist can deftly reply “Because of gravity”, which translated into real English means “Rocks will do what rocks will do”. Of course physicists can explain how the rocks fall, and on seeing a rock thrown by a tyre on an asphalt road, if they’re quick enough they can calculate whose windscreen it’s going to smash, but this is an altogether different question to why the rock did it. Description and understanding are not the same thing.

As for light, physicists changed the name of light to photons, so that when anyone asks “What is light anyway”, they can immediately reply “Light is made up of little particles, called photons”, which sounds a whole lot better, but means little more than “Light is light”. Einstein is reported to have said “Every physicist thinks he knows what a photon is. I spent my life trying to find out what a photon is, and I still don’t know.” Whilst we might berate Einstein for his use of sexist language, we still must admire his honesty.

Some people, and especially physicists anxious to defend the homefront will think I have overstated my case. Perhaps they would point to the sentence above describing light as particles, and claim that we have added to our understanding of the world; before we didn’t know light was a particle, now we do. While I am more than content with saying that we can describe a bit of nature we couldn’t describe before, I would claim it is nonsensical to suggest that we have increased understanding by describing one thing (light) in terms of something else (particles) that we also don’t understand. To be able to describe something we need to say what it does, to be able to understand something we need to say why it doesn’t do something else.

Lord Rutherford, who did all of his work in England and yet is still a great New Zealand scientist, said, “All science is either physics or stamp collecting”. I disagree. I claim all science including physics is stamp collecting. A physicist’s job, like other scientists is to record and describe nature; not to explain it, and most certainly not, as has been the wont of recent famous physicists, to explain it away.

I love physics. The knowledge and description of the universe brings fulfillment and enduring satisfaction. We do a disservice to physics and all of science when we make out that it is something it is not. It is important to describe, to test, to analyze, but a simple look at the scientific method shows that it is impossible for it to explain. Layers of description can be replaced by deeper layers of description, but understanding is always one step beyond where science can reach.

Anyway, to conclude with the world’s worst metaphor, come and wander through God’s amazing stamp collection. Notice a new and colourful design, an unusual style of paper, or the way that when you hold a particular stamp up to the light you can see a rift of watercolour adding depth to the original simplicity. I hope you feel a small sense of awe that I do when I think of the wonderful collection that God has made, and the privilege it is to study it.

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