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MODELLING WIND FLOW OVER COMPLEX TERRAIN

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF
ENGINEERING SCIENCE
OF THE UNIVERSITY OF AUCKLAND
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

John O'Sullivan
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Abstract

The increasing worldwide use of wind energy means that wind farms are being constructed in areas where the terrain is complex. Two important features of wind flow over complex terrain are flow separation and anisotropic turbulence. The most commonly used simulation approaches for wind flow use either linearised methods or the Reynolds-averaged Navier-Stokes (RANS) equations with a $k$-$\varepsilon$ turbulence closure. Neither of these approaches are capable of estimating separation accurately and they cannot represent anisotropic turbulence.

In the research discussed in this thesis an effective and robust approach for modelling wind flow over complex terrain was developed using two accurate turbulence closures capable of capturing these features. One is the $v^2f$ turbulence closure which has shown good ability to predict flow separation in other applications. The other is the algebraic structure-based turbulence model (ASBM) which can accurately represent anisotropic turbulence. A new algorithm for blending a full Reynolds stress tensor turbulence closure with an eddy-viscosity closure is presented. It blends the $v^2f$ and ASBM closures enabling stable simulations of a number two-dimensional and three-dimensional flows. By gradually increasing the blending factor results are obtained for the ASBM closure alone.

Estimates of the wall-normal fluctuations and the Reynolds stress components adjacent to the ground were required to extend the $v^2f$ and ASBM closures to simulations of wind flow over complex terrain. Using a combination of fundamental physics and previously published results new wall functions were developed that provided these estimates for both smooth surfaces and rough terrain. The same scalings were applied for key parameters as are used in existing standard wall functions to
ensure that new wall functions are consistent. The new wall functions were validated using a number of test cases and are shown to be robust and accurate.

An investigation of boundary conditions is carried out and a method is presented for generating realistic wind profiles for the inflow boundary of wind flow simulations. Combining these techniques simulations are carried out of two-dimensional flows over smooth and rough representative hills and of three-dimensional flows over real hills. Comparisons are made with experimental data and previously published simulation results. For the two-dimensional simulations over a smooth hill the results show that the ASBM closure accurately estimates the mean flow and turbulence and outperforms simulations using other closures. For the rough hill the ASBM closure combined with the new wall functions achieves a very good agreement with the experimental data and accurately represents the anisotropic turbulence.

The three-dimensional simulations of the wind-tunnel experiments of Kettles hill confirm the effectiveness of the approach. The results obtained for the profiles of both the mean wind velocity and the components of the Reynolds stress tensor provide a good match to the experimental data. The full-scale simulations of Askervein hill are the first application of the ASBM closure to a high Reynolds number, three-dimensional, atmospheric flow. The comparisons with field data show that the approach presented in this thesis produces accurate estimates of the wind flow over complex terrain. For the steady parts of the flow the ASBM closure performs as well as or better than previously published solutions obtained using large-eddy simulation (LES) and hybrid RANS/LES.

The success of the approach presented in this thesis for accurately modelling wind flow over complex terrain on the scale of a wind farm makes it a very useful tool. It is envisaged that it will be adopted more widely and more validation of the ASBM closure and the new wall functions will occur.
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Nomenclature

Greek Symbols

\( \alpha \)  
Blending factor between eddy-viscosity and Reynolds stress closures.

\( \beta \)  
Boundary-layer height constant.

\( \delta \)  
Boundary-layer height.

\( \delta_{ij} \)  
Kronecker delta tensor.

\( \varepsilon \)  
Dissipation of turbulent kinetic energy or simply dissipation.

\( \varepsilon_1 \)  
Dissipation in the near-wall cell.

\( \varepsilon_{ij} \)  
Dissipation tensor.

\( \epsilon_{ijk} \)  
Levi-Civita alternating tensor.

\( \kappa \)  
von Karman’s constant.

\( \eta_f \)  
Ratio of frame rotation to mean strain.

\( \eta_m \)  
Ratio of mean rotation to mean strain.

\( \phi \)  
Wall blocking parameter.

\( \phi_c \)  
Latitude of the current position.

\( \Psi_i' \)  
Vector stream function.

\( \nu \)  
Kinematic viscosity.

\( \nu_T \)  
Eddy viscosity.

\( \rho \)  
Density.

\( \sigma_k \)  
Turbulent kinetic energy transport constant.

\( \sigma_\varepsilon \)  
Dissipation transport constant.

\( \tau \)  
Total shear stress.

\( \tau_w \)  
Wall shear stress.
\( \tau_{\text{wrough}} \) \hspace{1cm} Wall shear stress for a rough wall.

\( \omega_c \) \hspace{1cm} Angular velocity of the Earth.

\( \omega_{ci} \) \hspace{1cm} Angular velocity vector of the Earth.

\( \omega'_i \) \hspace{1cm} Fluctuating vorticity vector.

**Roman Symbols**

\( a^2 \) \hspace{1cm} Measure of the anisotropy.

\( \alpha_{ij}^R \) \hspace{1cm} Eddy-axis tensor as a result of pure rotation.

\( \alpha_{ij}^S \) \hspace{1cm} Eddy-axis tensor as a result of pure strain.

\( A_w \) \hspace{1cm} Area of wind flow.

\( B \) \hspace{1cm} Log law constant.

\( B_{\text{rough}} \) \hspace{1cm} Log law constant for a rough wall.

\( B_{ij} \) \hspace{1cm} Wall blockage tensor.

\( c_{\text{ABL}} \) \hspace{1cm} Proportionality constant for ABL height.

\( C_1 \) \hspace{1cm} First constant for Yang’s extended log law model.

\( C_2 \) \hspace{1cm} Second constant for Yang’s extended log law model.

\( C_D \) \hspace{1cm} Dissipation constant for Prandtl’s mixing length model.

\( C_{\varepsilon 1} \) \hspace{1cm} First dissipation constant for \( k-\varepsilon \) model.

\( C_{\varepsilon 2} \) \hspace{1cm} Second dissipation constant for \( k-\varepsilon \) model.

\( C_{f1} \) \hspace{1cm} First elliptic relaxation constant for \( v^2f \) model.

\( C_{f2} \) \hspace{1cm} Second elliptic relaxation for \( v^2f \) model.

\( C_L \) \hspace{1cm} Wall blocking constant.

\( C_{\mu} \) \hspace{1cm} Eddy-viscosity constant for \( k-\varepsilon \) model.

\( C_{\nu} \) \hspace{1cm} Eddy-viscosity constant for Prandtl’s mixing length model.

\( C_{v^2f} \) \hspace{1cm} Constant for \( v^2f \) wall function.

\( d_{ij} \) \hspace{1cm} Normalised dimensionality tensor.

\( D_{ij} \) \hspace{1cm} Dimensionality tensor.

\( f \) \hspace{1cm} Elliptic relaxation variable.

\( f_c \) \hspace{1cm} Coriolis parameter.

\( f_i \) \hspace{1cm} Body force vector.
\( f_{ij} \) Normalised circulicity tensor.
\( F_{ij} \) Circulicity tensor.
\( h \) Height parameter for “Witch of Agnesi” hill.
\( H \) Normalisation length, eg. boundary-layer height or hill height.
\( H_1 \) First height parameter for modified “Witch of Agnesi” hill.
\( H_2 \) Second height parameter for modified “Witch of Agnesi” hill.
\( I \) Turbulent intensity.
\( k \) Turbulent kinetic energy.
\( k_1 \) Turbulent kinetic energy at the near-wall cell.
\( K_s^+ \) Non-dimensional equivalent grain-of-sand roughness.
\( l \) Inner layer depth.
\( \tilde{l} \) Length scale.
\( L \) Length scale.
\( L_a \) Length parameter for “Witch of Agnesi” hill.
\( L_s \) Length of separated flow.
\( N \) Transport constant for wall-normal fluctuations.
\( P \) Production of turbulent kinetic energy.
\( P_1 \) Production of turbulent kinetic energy in near-wall cell.
\( P_{ij} \) Production tensor.
\( p \) Instantaneous pressure.
\( \bar{p} \) Mean pressure.
\( \hat{p} \) Modified mean pressure.
\( p' \) Fluctuating pressure.
\( P_w \) Power available in the wind.
\( PG \) Magnitude of the pressure gradient divide by the density of air.
\( q^2 \) Twice the turbulent kinetic energy.
\( r_{ij} \) Normalised Reynolds stress tensor.
\( R_{ij} \) Reynolds stress tensor \( = u_i'u_j' \).
\( R_{ij}^+ \) Non-dimensional Reynolds stress tensor.
\( R_{ij}' \) Combined viscous and turbulent stress tensor.
\( \mathcal{R}_{ij} \) Pressure-rate-of-strain tensor.
Reynolds number.

The largest eigenvalue of the mean rate-of-strain tensor $S_{ij}$.  

Mean rate-of-strain tensor.

Time variable.

Time scale.

Reynolds stress flux.

Velocity scale.

Friction velocity.

Non-dimensional wall velocity.

Non-dimensional rough wall velocity.

Instantaneous velocity vector.

Fluctuating velocity vector.

Mean velocity vector.

Mean velocity tangential to the wall in the near-wall cell.

Mean free stream velocity.

Reference velocity.

Reynolds stress tensor $= R_{ij}$.  

Reynolds stress tensor in the near-wall cell.

Wall-normal fluctuations.

Wall-normal fluctuations in the near-wall cell.

Non-dimensional wall-normal fluctuations.

Wind velocity.

Geostrophic wind velocity.

Cartesian coordinate, usually the streamwise direction.

Cartesian coordinate vector.

Cartesian coordinate, usually the cross-stream direction.

Cartesian coordinate, usually the vertical direction.

Non-dimensional wall distance.

Aerodynamic roughness length.

The distance of the first cell centre from the wall.

Aerodynamic roughness length in wall units.
$z_i^+$ The distance of the first cell centre from the wall in wall units.

$z_{ref}$ Reference height.

**Abbreviations**

ABL Atmospheric boundary layer.

ASBM Algebraic structure-based model.

BSL Baseline-$\omega$.

CFD Computational fluid dynamics.

DNS Direct numerical simulation.

EVM Eddy-viscosity model.

HT The hill top location for the Askervein hill simulations.

LES Large eddy simulation.

MPI Message passing interface.

PRM Particle representation model.

QUICK Quadratic Upwind Interpolation for Convective Kinematics.

RANS Reynold-averaged Navier-Stokes.

RDT Rapid deformation theory.

RIX Site ruggedness index.


RS The reference site location for the Askervein hill simulations.

SBM Structure-based model.

SGS Subgrid-scales.

SIMPLE Semi-Implicit Method for Pressure-Linked Equations.

SMC Second moment closure.

SSG Speziale-Sarkar-Gatski.

SST Shear stress transport.

T4 The tower 4 location for the Kettles hill simulations.

T9 The tower 9 location for the Kettles hill simulations.

Chapter 1

Introduction

1.1 Motivation

The study of wind flow in the atmospheric boundary layer (ABL) is important in many applications. The dispersion of pollutants and hazardous materials, wind loading on man-made structures, and prediction of local weather conditions are just a few examples. One of the most significant research areas at present related to the ABL is the production of energy from wind. As the world struggles to come to terms with the issues of climate change and the rising cost of fossil fuels, renewable energy has become the focus of much public, political, and scientific interest. Wind farms, once installed, have the potential to provide clean, inexhaustible energy resources that can generate enough electricity to power millions of homes and businesses. The World Wind Energy Association reports that by the end of 2010, 196 Gigawatts of wind power capacity were installed worldwide (WWEA, 2010). This represents approximately 2.5% of global electricity consumption and is an increase of 37 Gigawatts since the end of 2009.

Early wind farms tended to be constructed in areas where the terrain is relatively smooth and simple. Now, as a result of this trend of increasing the worldwide wind energy production, wind farms are being constructed in areas where the terrain is more complex. This is especially true of New Zealand where almost all available terrain is complex. More complex terrain means that predicting important parameters, such as
power production, turbine fatigue life, and peak velocities, is much more complicated and requires the use of more sophisticated computational fluid dynamics (CFD) tools.

However, Palma et al. (2008) note that despite their impact in many areas, such as the automotive or aeronautical engineering, CFD techniques have not yet become common in wind energy engineering. Similarly, Hargreaves & Wright (2007) observe that while CFD packages are now widely used by wind engineers, they are not suitable for ABL flows using their standard settings. In particular, the use of the standard $k-\varepsilon$ model for turbulence closure is not adequate for capturing the important dynamics of wind flow over complex terrain. Undheim et al. (2006) also note that turbulence modelling is still a major challenge, particularly in complex terrain, and that more accurate turbulence models are required for atmospheric flow. Both Eidsvik (2005) and Hanjalic (2005) observe that the inability of the standard turbulence models to accurately reproduce the anisotropy of turbulence leads to limitations in many applications of wind flow modelling.

The inadequacy of current techniques provides the motivation for investigating the modelling of wind flow over complex terrain using CFD techniques. In particular, it motivated the research described in this thesis on more sophisticated turbulence representations that are capable of accurately capturing the effects of separation and anisotropic turbulence.

### 1.2 Selecting an Approach

#### 1.2.1 Historic Wind Flow Modelling

For many years the computational method used most commonly for modelling wind flow has been the wind atlas methodology (Landberg et al., 2003). In simple terms, this method uses linearized flow equations to correct existing long-term measurements for several different effects including sheltering objects, terrain classification, and domain contours. The advantage of this method is that it is well established and simple to use. By far the most widely used application of this method is the WASP computer code developed by the RISO National Laboratory in Denmark.
1.2. SELECTING AN APPROACH

WAsP has enjoyed widespread success because the use of linearized flow equations make it able to predict the wind resource accurately and efficiently when the terrain is sufficiently smooth to ensure attached flows. However, as Bowen & Mortensen (1996) pointed out, WAsP was increasingly being used to predict wind flow over sites that had sufficiently complex terrain as to no longer fall within the operational envelope of the software. In an attempt to quantify the problem, a site ruggedness index (RIX) was proposed as a coarse measure of the terrain complexity and hence the extent of flow separation. The RIX is defined as “the fractional extent of the surrounding terrain which is steeper than a certain critical slope” (Mortensen et al., 2006). Upon revisiting the issue ten years later, despite the addition of corrections based on the RIX, Mortensen et al. (2006) still concluded that it is not in general advisable to apply WAsP in complex terrain. This conclusion is supported by many researchers (Landberg et al., 2003; Palma et al., 2008; Perivolaris et al., 2006; Wood, 2000). In introducing his work Wood (2000) observed that flows over hills of engineering interest are inherently nonlinear. Palma et al. (2008) put the case more strongly stating that linearised methods cannot predict flow separation and are “clearly unsuitable” for modelling wind flow over complex terrain.

1.2.2 CFD for Modelling Wind Flow

When modelling wind flow using CFD techniques two approaches have emerged as the most widely used. The first, solving the Reynolds Averaged Navier-Stokes equations (RANS), has long been the mainstay of industrial CFD (Hanjalic, 2005) and it is now widely used in wind flow applications. Essentially, this approach calculates the mean velocity and pressure fields for the flow and models the effects of the fluctuations about those means. It does not have restrictive grid requirements which leads to it being relatively computationally inexpensive. A range of possibilities are available for modelling the fluctuations which leads to a variety of options when using the RANS approach. A detailed description and discussion of this approach is given in Section 2.2.1. The second approach is Large Eddy Simulation (LES) which directly calculates the motion of larger scale structures and models the effect of the small scale motion.
The scale of the motion that must be modelled is determined by the size of the grid and hence is referred to as the subgrid-scale motion (SGS). The computational requirements of LES are such that it is only relatively recently that it has become a potential alternative to the RANS approach.

In his review of the future of RANS and LES simulations Hanjalic (2005) states that eddy-viscosity models used in most modern RANS methods “have serious fundamental deficiencies and cannot be trusted for predicting genuinely new situations of realistic complexity”. Also, by their very nature, RANS methods average over all turbulence scales and hence lose much of the detailed nature of the turbulence characteristics. For modelling wind farms it is important to represent turbulence accurately as it can affect the electricity production, increase the probability of malfunctions and reduce the equipment life-time (Palma et al., 2008). However, with respect to LES Hanjalic (2005) reports that conventional LES on grids that are too coarse can be very inaccurate and inferior to even simple, conventional RANS calculations, especially in attached flows regions. He also observes that in trying to achieve accurate predictions of wall-bounded flows LES is severely constrained by the near-wall resolution requirements. He goes on to cite the work of Pope (1999) that observe that in the immediate future it is likely that increasing computer power will make possible RANS simulations with better spatial resolution, higher numerical accuracy, larger and better designed grids, and better handling of boundary conditions and turbulence. Luca & Sadiki (2008) agree, arguing that for many engineering and industrial purposes RANS modelling will continue to be the main approach used regardless of the progress of LES. They state that the “necessary computer resources, the rising trends in reducing the costs of ownership and in shortening the time to design, as well as the product development and commercialization will compel this practical way at least for several decades, especially for flows that are strongly affected by viscous near-wall processes”. They go on to observe that “the main thrust of research in recent years has been directed towards simplification and adjustment of lower-order models to encourage the uptake of improved closures in industrial CFD”. Hanjalic (2005) also notes the commercial drive behind continuing research into RANS methods concluding that “RANS will further play an important role, especially in industrial and
environmental computations, and that the further increase in the computing power will be used more to utilize advanced RANS models to shorten the design and marketing cycle rather than to yield the way to LES”.

The identical forms of the governing equations for RANS and LES mean that it is also possible to create hybrid solvers (Hanjalic, 2005; Bechmann et al., 2007a,b; Bechmann & Sørensen, 2010; Palma et al., 2008). This is an exciting new prospect that enables the capture of the often important unsteady characteristics of wind flow without the prohibitive computational cost of LES. It achieves this by using the RANS approach in the near-wall region and the LES approach away from the ground. Bechmann & Sørensen (2010) have developed a method for seamlessly switching between the two regions and have reported good results. However, the requirement to generate and store large amounts of data to resolve unsteady behaviour currently restricts the hybrid approach to smaller targets areas within a wind farm where the effects of unsteady motion may be particularly important. Also, as Hanjalic (2005) and Palma et al. (2008) note, improvements to the RANS approach will feed directly into the hybrid approach.

After careful consideration the RANS approach was selected for the research project discussed in this thesis. The most compelling reason of those discussed above is that currently neither LES nor the hybrid approach are a practical option for modelling wind flow over complex terrain. Other researchers have reduced the scale of the area of interest so that these approaches could be used Bechmann et al. (2007a,b); Bechmann & Sørensen (2010); Jimenez et al. (2008) but for this project it was decided that an approach that could be applied to modelling a whole wind farm was required. The potential for improvements in the RANS approach to contribute to improvements in the LES and hybrid approaches was also an important factor in the decision.
1.2.3 Modelling Turbulence within the RANS Approach

As with most fluid flows, turbulence modelling remains a significant challenge when simulating wind flow using RANS methods. Both Hanjalic (2005) and Eidsvik (2005) observe the inability of standard turbulence models to accurately reproduce the anisotropy of turbulence which is important in many applications of wind flow models. Hanjalic goes on to state that the “limitations of linear eddy-viscosity models (EVMs) have been recognized already in the early days of turbulence modeling and the attention has been turned to the second moment closure SMC that makes the most logical and physically most appropriate RANS modeling framework”. Eidsvik supports this saying that flows over hills may also contain regions with mean velocity maxima and adverse pressure gradients and suggests that full dynamic modelling of the Reynolds stress components may be desirable. In complex terrain particularly, there is a close connection between the mean velocity and the level of turbulence thus requiring more accurate turbulence models (Undheim, 2006).

Section 2.3 gives descriptions of a number of common methods for modelling turbulence within the RANS framework, ranging from simple eddy-viscosity models to complex Reynolds stress models. While they are well established, none of these common models are satisfactory for modelling wind flow over complex terrain. Radhakrishnan et al. (2008) note that whereas eddy-viscosity models are not accurate in predicting complex flows, Reynolds Stress models are difficult to implement numerically and tend to have high computational stiffness. Other authors have also pointed out that most turbulence models have difficulties in predicting flow separation from curved surfaces such as hills (Wang et al., 2004; Loureiro et al., 2008).

These difficulties have provoked a number of projects investigating the capability of various turbulence models. Kobayashi et al. (1994) and Svensson & Haggkvist (1990) both used extended forms of the k-ε model to study wind flow over forested hills and compared results with wind-tunnel experiments carried out by Ruck & Adams (1991). In each case reasonable agreements were obtained for the mean flow but the prediction of the turbulence quantities was poor. Ten different turbulence models were assessed by Hurley (1997) for flow over a hill and they concluded that while the results may be sufficient for meteorological purposes the estimation of the turbulence
1.2. SELECTING AN APPROACH

quantities was again poor. Both Ying & Canuto (1997) and Castro & Apsley (1997) compared simulation results with wind-tunnel data from the RUSHIL experiment (Khurshudyan et al., 1981) and despite using more complex models neither achieved a good agreement for the turbulence. More recently Wang et al. (2004) tested five and Loureiro et al. (2008) tested six turbulence models and their ability to predict separation behind two- and three-dimensional hills. Wang et al. (2004) found that none of the models performed well in three dimensions and that only one performed satisfactorily in two dimensions. Loureiro et al. (2008) reported that three of the turbulence models entirely failed to predict the separation and that none of the models captured well both the mean flow and the turbulence quantities.

The failure of more commonly used turbulence models to accurately predict wind flow over complex terrain motivated the investigation of alternative approaches. The approach investigated in the present work is based on the work of Radhakrishnan et al. (2008) who used a combination of the $v^2f$ model and the algebraic structure-based turbulence model (ASBM) to simulate flows over a backwards facing step and in an asymmetric diffuser. The results they reported compared well with previously published experimental data and direct numerical simulations (DNS). While these flows are relatively simple they contain many of the features of wind flow in complex terrain including separation and reattachment. The $v^2f$ model is capable of more accurately capturing the dynamics of separation and the ASBM has the ability to represent the Reynolds stress anisotropy at relatively low computational cost and complexity. The ASBM is one of a family of Structure Based Models (SBMs) that represent the structure of turbulence (Reynolds, 1989; Kassinos & Reynolds, 1995; Kassinos et al., 2001, 2006). Descriptions of each of these models are given in Section 2.3.
1.3 Objective

The objective of this work was to develop a RANS approach for modelling wind flow over complex terrain that is able to represent the anisotropy of turbulence and to capture the dynamics of separation accurately. The approach was validated by comparing computational results with those obtained from experiments, field tests and other simulations.

1.4 Contributions

The most important contributions of this work are as follows:

1. The development of a new algorithm for blending an eddy-viscosity turbulence closure with a full Reynolds stress closure that greatly improves the stability and convergence of the solution.

2. The development of a new wall function that accurately estimates the Reynolds Stress tensor adjacent to a wall. The wall function is consistent with existing standard wall functions and it can also be used for rough surfaces.

3. The development of a new wall function that accurately estimates the wall-normal fluctuations adjacent to a wall for the $v^2f$ turbulence closure.

4. Improvements to the implementation of the ASBM turbulence closure algorithm increasing the stability of the solution process.

5. The first known successful application of the ASBM turbulence closure to three-dimensional, high Reynolds number, wall-bounded atmospheric flows.

6. The demonstration and quantification of errors induced by the use of inconsistent boundary conditions when simulating wind flow in the atmospheric boundary layer.
1.5 Overview

A description of the theory applied throughout this work is given in Chapter 2. The theory of atmospheric flows is described and the governing equations presented. Next the development of the RANS equations is given and the resulting closure problem is discussed. A number of common turbulence closures are presented and their deficiencies described. The $v^2f$ turbulence closure is defined and its ability to represent some of the anisotropy of near-wall turbulence is detailed. Next the history and development of Structure Based models is given and the ASBM presented. In the forms presented neither the $v^2f$ turbulence closure nor the ASBM closure could be applied to wind flow over complex terrain as both require resolving the viscous length scale at the ground. To overcome this problem new wall functions were developed which estimate the wall-normal fluctuations and the Reynolds stress tensor in the log law region adjacent to the ground. Thus Chapter 2 includes a discussion of wall functions and the standard wall functions are defined and discussed. The derivations and definitions for the new wall functions are the presented: first the $\overline{v^2}$ wall function and then a wall function for the Reynolds stress tensor. Finally a summary of the combinations of turbulence closures and near-wall treatment used in this work is given.

Chapter 3 begins with a brief discussion of available RANS software and their relative merits. It follows with a description of the selected software OpenFOAM and its structure. The changes implemented as part of this project to include global pressure gradients and Coriolis forces thus extending the standard code to make the application atmosFOAM are described next. The details of the solution process are then presented and the parameters used in this work discussed. The algorithms for the turbulence closures follows and the innovations for the ASBM closure are developed and presented. Finally brief descriptions of the grid generation and post processing software are given.

In chapter 4 the results for a number of preliminary two-dimensional simulations are presented. A method for generating inflow profiles that are consistent with ground and top boundary conditions is developed. Using these profiles a horizontally homogeneous boundary layer is simulated successfully. A number of simulations of boundary
layers are then presented that quantify the errors introduced by commonly used inconsistent boundary conditions. The same errors are quantified for simulations that include an idealised ridge and a real ridge. Next the method for generating inflow profiles is extended to the generation of realistic Ekman wind profiles. A comparison is made between the results of simulations of the real ridge using Ekman inflow profiles and log law inflow profiles. The chapter ends with a discussion of a number of simulations investigating the grid dependence of the wall functions that have been developed as part of this work. It is established that the new wall functions have grid independence that is satisfactory and similar to that for the standard wall functions.

Chapter 5 presents the results of a number of two-dimensional simulations of flows over representative hills. The details of experiments carried out by Loureiro et al. (2007, 2009) are given as these provide the data for comparison. The simulation configurations and the computational domains are described and the boundary condition are given. The results from the low $Re$ smooth hill simulations show that the ASBM and $v^2f$ closures outperform the $k$-$\omega$-SST closure and that the ASBM closure results agree very well with the experimental data. Comparisons with other turbulence closures used in simulations carried out by Loureiro et al. (2008) reveal that the ASBM closure also performs better than any tested in their work. The results from high $Re$ smooth hill simulations are presented and establish that the agreement is good between wall-resolved simulations and wall-function simulations. The high $Re$ rough hill simulations are then compared to the experimental data and the agreement is found to be very good. Finally the results of the two-dimensional simulations are summarised.

A number of fully three-dimensional simulations are presented in Chapter 6. It begins with preliminary simulations of wind flow over a ridge based on the techniques discussed in Chapter 4. A methodology is developed and described for carrying out large scale three-dimensional simulations of atmospheric flows using the $k$-$\varepsilon$ closure. The results are compared with measurements made using a sodar device and are used to explain a measurement discrepancy. The next section discusses simulations of the wind-tunnel experiment of Kettles hill using the ASBM turbulence closure and the new wall functions. Comparisons between the simulation results and the experimental
results show that the ASBM closure accurately predicts the fully three-dimensional, high $Re$ flow over the model terrain. The final section presents the results for simulations of wind flow over the well-known Askervein hill using the ASBM closure and the new wall functions. Comparisons are made between the simulation results, the field data and simulations carried out by other researchers. These show that the ASBM closure and the new wall functions accurately predict the field data and compare well with the previous simulations using RANS, LES and hybrid RANS/LES approaches. This is the first known successful application of the ASBM turbulence closure to a full-scale, high $Re$, three-dimensional atmospheric flow.

Chapter 7 presents the results from work carried out investigating two complex three-dimensional benchmark problems. The first is an asymmetric diffuser that has been studied previously both experimentally and numerically at Stanford University. The results of simulations using the ASBM closure are presented and discussed. The second problem of flow through a square duct is the presented and the simulations that were carried out are described. An in-depth analysis of the flow is conducted and comparisons are made with DNS data. The important insights gained through the analysis are presented and an area where the ASBM closure may be improved is identified.

The key results and conclusions are summarised in Chapter 8 and finally in Chapter 9 future work is discussed.
Chapter 2

Theory

2.1 Atmospheric Flows

Wind flow is fundamentally caused by the solar heating of the Earth’s atmosphere. While little of the incident radiation from the sun is directly absorbed, the heat reflected and radiated by the Earth’s surface significantly influences the atmosphere (Stull, 1988). The non-uniform nature of this heating, both in space and time, and other phenomena such as condensation and evaporation cause the atmosphere to have a non-uniform pressure distribution. In a more local sense, wind flow is caused by pressure differences at a given elevation (Kaimal & Finnigan, 1994).

2.1.1 Coriolis Forces and Geostrophic Wind

Acting alone a horizontal pressure gradient will cause air to move from a high pressure region to a low pressure region. However the rotation of the Earth also imposes a Coriolis force upon the moving air. In the absence of other influences a frictionless wind balance is established between the pressure force and the Coriolis force (Simiu & Scanlan, 1986). The velocity of the wind once the balance has been established is called the geostrophic wind velocity and is given in equation (2.1).
\[ V_g = \frac{PG}{f_c} \]  

(2.1)

In equation (2.1) \( PG \) is the magnitude of the pressure gradient divided by the density of the air and \( f_c \) is the Coriolis parameter. The definition of the Coriolis parameter is:

\[ f_c = 2\omega_c \sin \phi_c \]  

(2.2)

where \( \omega_c \) is the angular velocity of the Earth and \( \phi_c \) is the latitude of the current position. Because of the frictionless wind balance, the direction of the geostrophic wind is along the isobars.

### 2.1.2 The Atmospheric Boundary Layer

In the lowest part of the atmosphere the Earth’s surface exerts a horizontal drag force on the moving air. This has two important effects, it causes the creation of a boundary layer and it also disrupts the frictionless wind balance. This boundary layer is called the Atmospheric Boundary Layer (ABL). Unlike many boundary layers, the ABL does not continue to grow as the flow advances because the pressure gradient above the influence of the frictional force generated at the surface “re-energises” (Simiu & Scanlan, 1986) the flow. Indeed, the point at which the effect of the surface of the earth is negligible is one definition of the height of the ABL (Simiu & Scanlan, 1986). Alternatively Kaimal & Finnigan (1994) define the ABL as the “lowest 1-2 km of the atmosphere, the region most directly influenced by the exchange of momentum, heat, and water vapour at the earth’s surface”. And Stull (1988) defines the boundary layer as “that part of the atmosphere that is directly influenced by the presence of the earth’s surface, and responds to surface forcings with a timescale of about an hour or less”. Stull refers to examples of the surface forcings that include frictional drag, heat and pollutant transfer, evaporation, and the effects of topography.
2.1. ATMOSPHERIC FLOWS

2.1.3 Ekman Spirals

The surface of the Earth disrupts the frictionless wind balance because as the air is slowed, the Coriolis force is reduced. This causes the wind closer to the ground to turn back towards the direction of the pressure gradient. As this effect is dependent upon the elevation above the ground, the resulting velocity profile is a spiral. This spiral is known as the *Ekman spiral* and an example of one calculated as part of this project is shown in Figure 2.1.

2.1.4 Stratification and Stability

The same effects which cause a non-uniform pressure distribution in the air above the Earth’s surface also cause variations in the density and temperature. In certain circumstances the density variation in the air is arranged by gravity into layers ranging from the denser air close to the surface and the less dense air above. This is called *stratification*. When density and temperature variations exist and a volume of air
moves vertically, it may experience a force acting upon it due to buoyancy effects. If the force it experiences pushes it further from its original position, the conditions are said to be *unstable*. If the force attempts to return it to its original position the conditions are said to be *stable*. Under *neutrally stable* conditions the volume does not experience a buoyancy force when it moves. For a more detailed explanation of this phenomenon refer to the references (Kaimal & Finnigan, 1994; Plate, 1971; Simiu & Scanlan, 1986; Stull, 1988).

2.1.5 The Neutral Atmospheric Boundary Layer

The study of wind flow for wind energy production is primarily concerned with moderately strong winds. In these conditions, the turbulence created by the interaction with the Earth’s surface dominates the effects of heat convection. This leads to thorough turbulent mixing which in turn produces neutral stratification and the ABL can be referred to as neutral. As the wind speeds are significantly less than the speed of sound, wind flows may be assumed to be incompressible. Under these circumstances it can be shown that the height of the ABL, $\delta$, can be estimated using the relationship (Stull, 1988):

$$\delta = c_{ABL} \frac{u_*}{f_c}$$  \hspace{1cm} (2.3)

In equation (2.3) $c_{ABL}$ is a proportionality constant with a value usually in the range 0.25-0.3 and $u_*$ is the friction velocity. The friction velocity is defined as:

$$u_* = \sqrt{\frac{\tau_w}{\rho}}$$  \hspace{1cm} (2.4)

where $\tau_w$ is the shear stress at the ground and $\rho$ the density of air. Also, by using similarity considerations analogous to those used in two-dimensional boundary-layer flow theory, a relationship for the mean horizontal velocity in the lowest 100m of the ABL can be obtained and is given in equation (2.5).
\[ u(z) = \frac{u_\ast}{\kappa} \ln \frac{z}{z_0} \]  

This is the well-known logarithmic law where \( \kappa \) is von Karman’s constant and \( z_0 \) is the aerodynamic roughness length or simply roughness length. The roughness length is a measure of the roughness of the terrain concerned. Roughness has the effect increasing the drag of the surface of the Earth thus retarding the flow further. This can be seen from equation (2.5), the greater the roughness length the slower the mean velocity at a given height. Table 2.1 gives approximate roughness lengths for various types of terrain.

Table 2.1: Aerodynamic Roughness Length for Various Terrain

<table>
<thead>
<tr>
<th>Type of Terrain</th>
<th>( z_0 ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muddy terrains, wetlands, icepack</td>
<td>( 1 \times 10^{-5} - 3 \times 10^{-5} )</td>
</tr>
<tr>
<td>Water</td>
<td>( 1 \times 10^{-4} - 2 \times 10^{-4} )</td>
</tr>
<tr>
<td>Sand</td>
<td>( 1 \times 10^{-4} - 1 \times 10^{-3} )</td>
</tr>
<tr>
<td>Snow</td>
<td>( 0.001 - 0.006 )</td>
</tr>
<tr>
<td>Mown grass, Airport runway areas</td>
<td>( 0.001 - 0.01 )</td>
</tr>
<tr>
<td>Low grass, farmland with few trees</td>
<td>( 0.01 - 0.04 )</td>
</tr>
<tr>
<td>High grass, farmland with trees</td>
<td>( 0.04 - 0.1 )</td>
</tr>
<tr>
<td>Pine forest (mean height of trees 15m)</td>
<td>( 0.9 - 1.0 )</td>
</tr>
<tr>
<td>Sparsely built-up suburbs</td>
<td>( 0.2 - 0.4 )</td>
</tr>
<tr>
<td>Densely built-up suburbs</td>
<td>( 0.8 - 1.2 )</td>
</tr>
<tr>
<td>Cities</td>
<td>( 1 - 4 )</td>
</tr>
</tbody>
</table>
2.2 Governing Equations

Wind flow in the neutral ABL is governed by the incompressible Navier-Stokes equations. Using the Einstein summation notation these equations can be written in Cartesian coordinates as follows:

\[
\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] + f_i \tag{2.6}
\]

\[
\frac{\partial u_i}{\partial x_i} = 0 \tag{2.7}
\]

where \(x_i\) are the components of the coordinates system, \(u_i\) represents the components of velocity, \(\nu\) is the kinematic viscosity, \(f_i\) represents body forces such as Coriolis and gravity and \(p\) is the pressure. In most cases these equations cannot be solved analytically and so numerical methods must be used. Even using numerical methods, the ranges of length scales and time scales that exist in wind flow are so great that the computational cost of resolving them all is prohibitively high. Instead, a reduced form of equations (2.6) and (2.7) is derived which can be solved numerically. Following the discussion in Chapter 1, the reduced form of the equations selected for this work are the Reynolds Averaged Navier-Stokes (RANS) equations.

2.2.1 Reynolds Averaged Navier-Stokes Equations

In this classical approach (Reynolds, 1895) each of the field variables are decomposed into a mean quantity and a fluctuating part:

\[
u_i = \bar{u}_i + u_i' \quad p = \bar{p} + p' \tag{2.8}
\]

In equations (2.8) and hereafter the overline indicates the mean quantity and the prime the fluctuating part. Substituting these into the Navier-Stokes equations (2.6) and (2.7) and applying averaging appropriately, the Reynolds Averaged Navier-Stokes
2.2. GOVERNING EQUATIONS

equations are obtained and can be written in the form:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial u'_i u'_j}{\partial x_j}$$

(2.9)

$$\frac{\partial u_i}{\partial x_i} = 0$$

(2.10)

Following the discussion in chapter 1, only the steady state case is considered in this work. This reduces the equations further to:

$$\frac{u_j}{u_i} \frac{\partial \overline{p_i}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial u'_i u'_j}{\partial x_j}$$

(2.11)

$$\frac{\partial \overline{p_i}}{\partial x_i} = 0$$

(2.12)

In equation (2.11) the term $u'_i u'_j$ is the one point correlation of the fluctuating components of the velocity and is referred to as the Reynolds stress tensor. The six components of this symmetric tensor along with the mean pressure and three components of the mean velocity bring the total number of unknowns in the equations to 10. As there are only four partial differential equations to solve, the system is not closed. This is referred to as the closure problem of turbulence (Pope, 2000) and in order solve equations (2.11) and (2.12) the Reynolds stresses must be determined. Different methods for determining the Reynolds stresses may be collectively referred to as turbulence closures.
2.3 Turbulence Closures

2.3.1 The Boussinesq Approximation

In 1877 Boussinesq introduced the turbulent-viscosity hypothesis which states that the deviatoric Reynolds stress is proportional to the mean rate-of-strain (Pope, 2000). This can be written as follows:

\[-u'_i u'_j = 2\nu_T S_{ij} - \frac{2}{3} k \delta_{ij}\] (2.13)

where the mean rate-of-strain \( S_{ij} \) is given by:

\[S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\] (2.14)

and \(\nu_T\) is referred to as the eddy viscosity. The turbulent kinetic energy \(k\) is equal to half the trace of the Reynolds stress tensor:

\[k = \frac{1}{2} u'_i u'_i\] (2.15)

The Boussinesq approximation is directly analogous to the relationship between stress and rate-of-strain in a Newtonian fluid and provides a convenient closure to the RANS equations. Substituting equation (2.13) into the momentum equation (2.11) gives:

\[\frac{\partial u_i}{\partial x_j} - \frac{\partial}{\partial x_j} \left[ (\nu + \nu_T) \frac{\partial u_i}{\partial x_j} \right] = -\frac{\partial \hat{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial u_i}{\partial x_j} \right)\] (2.16)

where \(\hat{p}\) is the modified mean pressure which has absorbed the density and the hydrostatic part of the turbulent kinetic energy as shown in equation (2.17).
\[ \dot{p} = \frac{\nabla p}{\rho} + \frac{2}{3} k \]  

(2.17)

### 2.3.2 Eddy-Viscosity Closures

The RANS equations written in the form of (2.16) can be solved once the eddy viscosity is determined throughout the field. There are many different methods for determining the eddy viscosity of varying degrees of complexity and accuracy. Prandtl (1925) developed one of the earliest turbulence models by reasoning that the eddy viscosity at any point in the turbulent field can be represented by a single velocity and length scale.

\[ \nu_T = \tilde{l} \tilde{u} \]  

(2.18)

Prandtl showed that for a two-dimensional flow the velocity scale can be calculated using:

\[ \tilde{u} = C_\nu \tilde{l} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]  

(2.19)

Smagorinsky (1963) proposed an extension to this model which is applicable to three-dimensional flows:

\[ \nu_T = C_\nu \tilde{l}^2 S \]  

(2.20)

where \( S \) is given by:

\[ S = \sqrt{2S_{ij}S_{ij}} \]  

(2.21)

The major drawback of this model is that it requires knowledge of the mixing length scale \( \tilde{l} \) throughout the field. This is only possible for simple flows and is inevitably dependent on the geometry of the flow (Pope, 2000). This model also fails to account for the transport of turbulence quantities (Versteeg & Malalasekera, 1999) which led
Kolmogorov (1942) and Prandtl (1945) to independently suggest that it is better to base the velocity scale on the turbulent kinetic energy:

\[ \nu_T = C_\nu \tilde{l} k^{1/2} \]  \hspace{1cm} (2.22)

Kolmogorov and Prandtl went on to suggest that a transport equation could be solved for \( k \) which led to this approach being called the \textit{one-equation} model. An exact evolution equation can be derived for \( k \) using the Navier-Stokes equations (2.6) and (2.7), the Reynolds decomposition (2.8) and its definition (2.15). A model version of the \( k \) evolution equation can then be obtained by modelling the energy flux term (Pope, 2000). The steady version of this model equation is the \( k \) transport equation which can be written in the form:

\[ \frac{\partial (k \bar{u}_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \nu_T \frac{\partial k}{\sigma_k \partial x_j} \right] + P - \varepsilon \] \hspace{1cm} (2.23)

where the production of turbulent kinetic energy \( P \) is given by:

\[ P = \nu_T S^2 \] \hspace{1cm} (2.24)

In the one-equation model the dissipation of turbulent kinetic energy, hereafter simply dissipation, \( \varepsilon \) is calculated directly from \( k \) and \( \tilde{l} \):

\[ \varepsilon = C_D \frac{k^{3/2}}{\tilde{l}} \] \hspace{1cm} (2.25)

While the inclusion of the turbulent kinetic energy equation in the one-equation model offers advantages over the mixing length model (Pope, 2000), the requirement remains that the length scale \( \tilde{l} \) must be specified throughout the flow. To overcome this Launder & Spalding (1974) proposed that a second transport equation for \( \varepsilon \) should be solved and the length scale constructed.
2.3. TURBULENCE CLOSURES

2.3.3 The $k-\varepsilon$ Closure

The $k-\varepsilon$ closure belongs to a class of two-equation models. This is because two transport equations are solved for turbulence quantities which allow the construction of a length scale $L$ and time scale $T$ for the entire field:

\[ L = \frac{k^{3/2}}{\varepsilon} \quad (2.26) \]
\[ T = \frac{k}{\varepsilon} \quad (2.27) \]

The eddy viscosity is calculated directly from the turbulent kinetic energy and the dissipation.

\[ \nu_T = C_\mu \frac{k^2}{\varepsilon} \quad (2.28) \]

The transport equation that must be solved for the dissipation is different in its origins to the turbulent kinetic energy equation. It is essentially an empirical equation (Pope, 2000) and can be written in its steady form as:

\[ \frac{\partial (\varepsilon u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \nu_T \frac{\partial \varepsilon}{\sigma_\varepsilon \partial x_j} \right] + C_\varepsilon \frac{p \varepsilon}{k} - C_{\varepsilon^2} \frac{\varepsilon^2}{k} \quad (2.29) \]

The constants in equations (2.23) and (2.28)-(2.29) have been well established for boundary-layer flows by fitting data for a wide range of turbulent flows (Launder & Sharma, 1974). Several authors (Hagen et al., 1981; Beljaars et al., 1987; Richards & Hoxey, 1993; Palma et al., 2008; Solazzo et al., 2009) have proposed modifications to the constants in an effort to better simulate the ABL. However, because the present work is concerned with making comparisons with the “industry standard” turbulence closure, the standard values for the constants were used and are given in Table 2.2.
Table 2.2: Standard Constants for the $k$-$\varepsilon$ Closure (Launder & Sharma, 1974)

| $C_\mu$ | 0.09 |
| $\sigma_k$ | 1.0 |
| $\sigma_\varepsilon$ | 1.3 |
| $C_{\varepsilon 1}$ | 1.44 |
| $C_{\varepsilon 2}$ | 1.92 |

2.3.4 Reynolds Stress Closures

By definition, the Boussinesq approximation that is central to eddy-viscosity closures means that the turbulence is modelled as isotropic. As discussed in chapter 1, in wall-bounded flows such as wind flow, the near-wall turbulence is strongly anisotropic (Durbin, 1991). An alternative to the Boussinesq approximation is to retain the full Reynolds stress tensor and solve equations to determine each of its components. This leads to another class of closures called *Reynolds stress models*. Using the Reynolds decomposition and following the same process that led to the RANS equations (2.9) and (2.10), exact transport equations for the Reynolds stresses can be obtained:

$$\frac{\partial u'_i u'_j}{\partial t} + u_k \frac{\partial u'_i u'_j}{\partial x_k} + \frac{\partial}{\partial x_k} T_{kij} = P_{ij} + R_{ij} + \varepsilon_{ij} \quad (2.30)$$

In equation (2.30) the mean-flow convection term and the production tensor $P_{ij}$ can be calculated from the mean flow variables and the components of the Reynolds stress tensor. However the Reynolds stress flux $T_{kij}$, pressure-rate-of-strain tensor $R_{ij}$ and the dissipation tensor $\varepsilon_{ij}$ must all now be modelled to close the equations. Launder et al. (1975) are often cited as developing the first full model (LRR model) and since then a number of Reynolds stress models have been developed. In particular, modelling the pressure-rate-of-strain tensor accurately is extremely difficult (Bradshaw et al., 1987; Pope, 2000).
2.3. TURBULENCE CLOSURES

The difficulties in modelling these terms and the complexity of the systems of tensor equations have led researchers to seek simplifications. One approach is to approximate the transport terms in equation (2.30) and replace it by a set of algebraic equations. The closure then becomes an algebraic stress model and closures of this type have had varying degrees of success (Kassinos et al., 2006). The advantage is that the Reynolds stresses are determined locally as functions of $k$, $\varepsilon$ and the mean velocity gradients. However, the approximations required to develop the algebraic expression mean that this approach is inherently less general and less accurate than Reynolds stress models (Pope, 2000). A second alternative approach is to use an elliptic relaxation model to represent the pressure-rate-of-strain term. Durbin (1991) proposed a model of this form which has proved to be successful in modelling wall-bounded flows.

2.3.5 The $v^2f$ Closure

The $v^2f$ closure, initially called the $k$-$\varepsilon$-$v^2$ model (Durbin, 1991), is based on the idea of including turbulence anisotropy in near-wall region of the flow. Durbin observed that by using the wall-normal fluctuations in $\overline{v^2}$ to scale $\nu_T$, it was suppressed in the correct manner in the near-wall region. Rather than using equation (2.28), $\nu_T$ is defined as:

$$\nu_T = C_\mu \overline{v^2}T$$

(2.31)

where the time scale $T$ is determined by also using the limiting Kolmogorov scale. Following the version of the $v^2f$ closure presented in Kalitzin et al. (2005) $T$ is calculated by:

$$T = \min \left[ \max \left( \frac{k}{\varepsilon}, 6 \left( \frac{k}{\varepsilon} \right)^{1/2} \right), \frac{0.6k}{\sqrt{3} \overline{v^2} C_\mu S} \right]$$

(2.32)

Durbin (1991) also determined that the value of $C_\mu$ needed to be changed for the correct behaviour of $\nu_T$ to be observed near the wall. The steady transport equation
for $\overline{v^2}$ is written:

$$\overline{\frac{\partial v^2}{\partial x_i}} = k f - N \frac{\overline{v^2}}{k} \varepsilon + \frac{\partial}{\partial x_j} \left( \nu + \nu_T \frac{\partial \overline{v^2}}{\partial x_i} \right)$$ \hspace{1cm} (2.33)

where $f$ is the elliptic relaxation term that introduces the non-local effects. The form of equation (2.33) has been determined using the unsimplified version of the pressure-rate-of-strain term from the LRR model Launder et al. (1975); Durbin (1991). The elliptic relaxation equation is:

$$f - L^2 \frac{\partial^2 f}{\partial x_k \partial x_k} = (C_{f1} - 1) \left( \frac{2/3 - v^2/k}{T} \right) + C_{f2} \frac{P}{k} + (N - 1) \frac{v^2}{kT}$$ \hspace{1cm} (2.34)

The calculation of the turbulent length scale also includes information from the Kolmogorov scale:

$$L = C_L \max \left[ \min \left( \frac{k^{3/2}}{\varepsilon}, \frac{k^{3/2}}{\sqrt{3} v^2 C_\mu S} \right), C_\nu \frac{v^{3/4}}{\varepsilon^{1/4}} \right]$$ \hspace{1cm} (2.35)

The constants for the $v^2 f$ closure are given in Table 2.2 below.

| $C_\mu$ | 0.22 |
| $C_\varepsilon$ | 1.9 |
| $C_{f1}$ | 1.4 |
| $C_{f2}$ | 0.3 |
| $C_L$ | 0.23 |
| $C_\nu$ | 70 |
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Note that the constant $C_{\varepsilon_1}$ in equation (2.29) must be calculated using:

$$C_{\varepsilon_1} = 1.4 \left( 1 + 0.050 \sqrt{k/v^2} \right)$$  \hspace{1cm} (2.36)

While the $v^2f$ closure accounts for the near-wall turbulence anisotropy without the need to solve the full set of Reynolds stress transport equations, it still fundamentally relies upon the Reynolds stress tensor alone for representing the turbulence field. It shares this feature with both eddy-viscosity closures and Reynolds stress closures. An alternative approach to closing the RANS equations is to use a broader representation of the turbulence field. The structure based models discussed in the next section are examples of this approach.

2.3.6 Structure Based Models

The Reynolds stress tensor only provides information about the magnitude of each of the components of the turbulent fluctuations. This can be referred to as the componentality of the turbulence. Since the early 90’s Reynolds and co-workers (Reynolds, 1989; Kassinos & Reynolds, 1995; Kassinos et al., 2001, 2006) have argued that information about the turbulence structure is also required to accurately characterise the turbulence field. They introduce the concepts of structure dimensionality and structure circulicity to provide this information. The dimensionality tensor gives the level of two-dimensionality of the turbulence while the circulicity describes the large-scale structure of the vorticity field (Kassinos et al., 2001). The important distinction between the componentality and dimensionality of turbulence is described clearly by Kassinos & Reynolds (1995) using the concept of three basic, idealised eddies: jettal, vortical and helical, and by considering two important examples of rapid deformation theory (RDT).

Collectively the Reynolds stress $R_{ij}$, dimensionality $D_{ij}$ and circulicity $F_{ij}$ tensors are referred to as the structure tensors. They can all be defined in terms of the vector
stream function $\Psi_i'$:

$$R_{ij} = u'_iu'_j = \epsilon_{ist} \epsilon_{jqp} \frac{\partial \Psi'_i}{\partial x_s} \frac{\partial \Psi'_q}{\partial x_p}$$

(2.37)

$$D_{ij} = \frac{\partial \Psi'_k}{\partial x_i} \frac{\partial \Psi'_k}{\partial x_j}$$

(2.38)

$$F_{ij} = \frac{\partial \Psi'_i}{\partial x_k} \frac{\partial \Psi'_j}{\partial x_k}$$

(2.39)

The vector stream function itself is defined as:

$$u'_i = \epsilon_{its} \frac{\partial \Psi'_s}{\partial x_t}$$

$$\frac{\partial \Psi'_i}{\partial x_i} = 0$$

$$\frac{\partial^2 \Psi'_i}{\partial x_k \partial x_k} = -\omega'_i$$

(2.40)

By considering homogeneous turbulence it can be shown that the contractions of the structure tensors are equal to twice the turbulent kinetic energy:

$$R_{kk} = D_{kk} = F_{kk} = q^2 = 2k$$

(2.41)

This can be used to define normalised structure tensors as follows:

$$r_{ij} = R_{ij}/q^2$$

(2.42)

$$d_{ij} = D_{ij}/q^2$$

(2.43)

$$f_{ij} = F_{ij}/q^2$$

(2.44)
Kassinos & Reynolds (1995) went on to show that a constitutive relationship exists between the structure tensors. In the case of homogeneous turbulence this relationship can be written in the form of equation (2.45).

\[ r_{ij} + d_{ij} + f_{ij} = \delta_{ij} \]  

(2.45)

The relationship shows that two of the tensors are linearly independent which supports the idea that modelling turbulence in terms of the Reynolds stress tensor alone can only work in specific situations (Kassinos et al., 2006). The representation of turbulence in terms of its structure tensors opens the way for a whole family of structure based models to be developed. Kassinos et al. (2000) and Poroseva (2002) present models within this family and go on to develop a number of transport equations for the different structure tensors. The additional transport equations increase the complexity and computational requirements, and introduce new closure issues that must be overcome.

One of the alternatives to solving the Reynolds stress transport equations described in Section 2.3.4 is to model the Reynolds stress tensor as an algebraic function of one or more different tensors. In an analogous way, rather than solving transport equations for the structure tensors, Kassinos & Reynolds (1995) developed algebraic expressions to model their behaviour.

### 2.3.7 The Algebraic Structure Based Model (ASBM)

Kassinos & Reynolds (1995) showed that RDT calculations can be made in a practical and exact manner by using a particle representation model (PRM). The PRM is based on the idea that by following the evolution of a set of “particles”, the statistics of the ensemble of that set can be determined and then used to model the properties of the evolving field that contains the particles. A set of evolution equations is developed for the properties of the hypothetical particles that emulates the exact equations for the evolution of the field properties.

In the case of the algebraic structure based model, the “particles” are hypothetical eddies comprised of a jettal component and a vortical component. Each eddy has an
eddy-axis vector which is parallel to the jettal component and perpendicular to the vortical component. The eddies are also allowed to be flattened in the direction normal to the eddy axis. By averaging over the ensemble of these hypothetical eddies constitutive equations can be developed that relate the normalised structure tensors to the statistics of the ensemble. These constitutive equations are written in terms of the eddy-axis tensor $a_{ij}$, the eddy-flattening tensor $b_{ij}$ and a set of structure scalars.

The structure scalars are in turn based on three quantities:

- $\eta_m$ - the ratio of mean rotation to mean strain
- $\eta_f$ - the ratio of frame rotation to mean strain
- $a^2$ - a measure of the anisotropy

These three quantities and the eddy-axis and eddy-flattening tensors are calculated directly from the mean strain rate, the mean rotation, the total rotation and a time scale of the turbulence. Detailed descriptions of the ASBM, its development and the full set of model equations can be found in Reynolds (1992); Kassinos & Reynolds (1995); Kassinos et al. (2000, 2001, 2006).

**Wall Blocking**

For wall-bounded flows such as wind flow, wall blocking must be taken into account. As the flow approaches the wall, viscous forces drive the velocity to zero so that the no-slip condition is satisfied at the wall itself. The component of velocity normal to the wall is driven to zero faster than the other components of velocity. This is because wall blocking acts at length scales larger than the viscous scales (Radhakrishnan et al., 2008). As a result the eddies are re-oriented to be parallel to the wall as they approach it. In the ASBM this effect is achieved by including a blockage tensor $B_{ij}$ which acts upon the eddy-axis tensor. The definition of the the blockage tensor was presented by Kassinos et al. (2006) and is given in equation (2.46).
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\[ B_{ij} = \frac{\partial \phi / \partial x_i \partial x_j}{\partial \phi / \partial x_k \partial x_k} \] (2.46)

In equation (2.46) \( \phi \) is an elliptic-relaxation parameter (Durbin, 1991) and calculated by solving the modified Helmholtz equation:

\[ L^2 \frac{\partial^2 \phi}{\partial x_k \partial x_k} = \phi \] (2.47)

\( L \) is calculated from:

\[ L = C_L \max \left( \frac{k^{3/2}}{\varepsilon}, C_\nu \sqrt{\frac{\nu^3}{\varepsilon}} \right) \] (2.48)

The constants in equation (2.48) are \( C_L = 0.80 \) and \( C_\nu = 0.17 \) following Radhakrishnan et al. (2008). For boundary conditions \( \phi = 1 \) at solid walls and \( \partial \phi / \partial x_n = 0 \) where \( x_n \) is the wall-normal direction.

2.4 Wall Functions

Wall functions apply boundary conditions at some distance from the wall, which removes the requirement to solve the governing equations all the way to the wall. This has two important benefits in simulations of wind flow. First, it significantly reduces the computational resource required by allowing the use of a much coarser grid near the wall. Second, by applying the boundary conditions at some point away from the wall the exact geometry of the wall does not need to be fully resolved. This allows the complex geometries caused by terrain effects, vegetation, etc., to be modelled by including an equivalent roughness in the wall functions.
2.4.1 The Standard Wall Function

The wall functions used in this work are based on the standard wall function model presented by Launder & Spalding (1974) and the rough wall model derived from it. It is assumed that the mean flow is approximately parallel to the wall and thus log law relations can be applied as boundary conditions at the first cell center away from the wall. Also, the friction velocity is modelled using the turbulent kinetic energy in the near-wall cell (Wilcox, 1998):

\[ u^* = \frac{C_1}{4} \frac{\mu}{k^{1/2}} \]  

(2.49)

where \( C_\mu \) is a constant taken from the turbulence closure. This means that the non-dimensional wall distance \( z^+ \) can be approximated by:

\[ z^+ = \frac{C_1^{1/4} k^{1/2} z_1}{\nu} \]  

(2.50)

where \( z_1 \) is the normal distance from the wall to the center of the first cell. For smooth walls, the non-dimensional wall velocity \( u^+ \) is given by:

\[ u^+ = \frac{1}{\kappa} \ln (z^+) + B \]  

(2.51)

where \( \kappa \) is von Karmen’s constant and \( B \) is the log law constant, which is equal to 5.2 (Pope, 2000). The log law approximations are not applied directly to the velocity variables but instead are used to adjust the eddy viscosity at the wall.

\[ \nu_T = \nu \left( \frac{z^+}{u^+} - 1 \right) \]  

(2.52)
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Similarly, the turbulent kinetic energy is not defined by the wall function at the cell nearest to wall, but instead the wall shear stress is approximated using the log law and this is included in the production term in the turbulent kinetic energy equation:

\[ P_1 = \tau_w \frac{\partial u}{\partial z} \bigg|_{1}, \text{ with: } \tau_w = C_\mu^{1/2} k_1^{1/2} \overline{u}_1 \]

(2.53)

where \( \overline{u}_1 \) is the mean velocity tangential to the wall in the cell nearest to the wall. For the dissipation however, the value is defined directly by the wall function method and is given in equation (2.54).

\[ \varepsilon_1 = C_\mu^{3/4} k_1^{3/2} \kappa z_1 \]

(2.54)

2.4.2 The Rough Wall Function

The effects of roughness are easily introduced into wall functions of this form by adjusting the non-dimensional wall velocity accordingly:

\[ u_{\text{rough}}^+ = \frac{1}{\kappa} \ln \left( \frac{z^+}{K_s^+} \right) + B_{\text{rough}} \]

(2.55)

where \( K_s^+ \) is the non-dimensional equivalent grain-of-sand roughness height which is determined by the surface. \( B_{\text{rough}} \) is the rough wall log law constant and for the present work a value of 8.5 was used (Pope, 2000). It is important to note that a requirement for using the wall function boundary conditions is that the point of application must be inside the log law region. For fully turbulent flows this means that grids must be selected such that this point is at \( z^+ = 30 \) or greater. This issues is examined in more detail in Section 4.3.
2.4.3 A New $v^2f$ Wall Function

Using the $v^2f$ turbulence closure introduces the extra variables $v^2$ and $f$. It also requires that the grid is refined sufficiently for the first cell center to lie inside the viscous sub-region at approximately $z^+ = 1$. Unsurprisingly using the $v^2f$ model on coarse grids with the wall functions described in 2.4 produces poor results. This is because on a coarse grid the strong gradients in the flow variables near the wall cannot be accurately resolved. It is particularly important to accurately represent the gradient of $\nu_T$ because equation (2.16) shows that it is responsible for contributing the effects of turbulence to the mean flow. In the $v^2f$ closure $\nu_T$ is calculated directly from $v^2$ through equation (2.31) and thus it follows that it is important to accurately estimate the wall normal gradient of $v^2$ in any wall function.

In previous work it was found that it was possible to accurately estimate the mean velocity without using a wall function for $v^2$ by carefully choosing the height of the near-wall cell (O’Sullivan et al., 2010). However, this is essentially a manual method of setting the gradient of $v^2$ and clearly this approach requires a priori knowledge of the flow solution. It also requires that the value of $z^+$ remains effectively constant throughout the domain. Neither of these conditions are realistic in practical applications.

To calculate the gradient of $v^2$ in the near-wall cell the result from Kalitzin et al. (2005) is used:

$$\frac{\partial v_2^+}{\partial z^+} = \frac{C_{v^2}}{\kappa z^+}$$

(2.56)

where $C_{v^2}$ is a constant and the reported value of 0.193 was used. Note that $z$ again represents the wall-normal distance. In dimensional form this can be rewritten as:

$$\frac{\partial v^2}{\partial z} = \frac{u^2 C_{v^2}}{\kappa z}$$

(2.57)
Applying the resolved grid boundary condition of $\overline{u^2} = 0$ at the wall and using a first order approximation of the gradient, an equation for the value of $\overline{v^2}$ in the near-wall cell is obtained:

$$\overline{v^2}_1 = \frac{\tau_w C_{v^2}}{\rho \kappa}$$  \hspace{1cm} (2.58)

where equation (2.4) has been used to replace the friction velocity.

To calculate the wall shear stress $\tau_w$, equation (2.53) is used. This makes the wall function consistent with the standard approach used for $\overline{u_i}$ and $\nu_T$ and also increases its robustness. This consistency leads to the $\overline{v^2}$ wall function having similar grid-independence properties as the standard wall functions. Further discussion of this is presented in Section 4.3. The other significant advantage of using $\tau_w$ to scale $\overline{v^2}_1$ is that it provides a simple, consistent mechanism for introducing roughness into the wall function by using the standard rough wall approximation for wall shear stress:

$$\left( \frac{\overline{v^2}}{\nu_1} \right)_{\text{rough}} = \frac{\tau_{w, \text{rough}} C_{v^2}}{\rho \kappa}$$  \hspace{1cm} (2.59)

where

$$\tau_{w, \text{rough}} = C_{\mu}^{1/4} k_{\text{r}}^{1/2} \frac{\overline{u}_1}{u_{\text{r, rough}}}$$  \hspace{1cm} (2.60)

Note that the effect of the estimate for $f$ in the near-wall cell is much less significant and given a good estimate for $\overline{v^2}$, a reasonable value for $f$ is obtained using the standard boundary condition of $f = 0$ at the wall without the use of a wall function.

### 2.4.4 A New Reynolds Stress Wall Function

When the ASBM closure is used for estimating the components of the Reynolds stress tensor, the domain must again be resolved all the way to the viscous sublayer. Applying the ASBM closure on a coarse grid gives poor results for the mean flow and all the variables in the near-wall cell. In particular, investigations have shown it
overestimates the shear component of the Reynolds stress tensor in the near-wall cell which leads to large overestimations in the mean velocity and subsequently all of the other variables. To correct this, the shear component of the Reynolds stress tensor in the near-wall cell is estimated by assuming that locally the flow approximates channel flow. While this is not the case in most flows, the same assumption is made in the derivation of the standard wall functions presented in Section 2.4.1 and it has proved to be satisfactory. Pope (2000) gives the result for the total shear stress in a channel flow as:

$$\tau = \rho \nu \frac{\partial \bar{u}}{\partial z} - \rho \bar{u}' w'$$  \hspace{1cm} (2.61)

Note that in equation (2.61) $\bar{u}$ is the tangential velocity and $z$ the wall-normal distance. Pope also shows this equation can be solved to give an expression for $\tau$ dependent only on $\tau_w$, the wall distance $z$ and the boundary-layer thickness $\delta$:

$$\tau(z) = \tau_w \left(1 - \frac{z}{\delta}\right)$$  \hspace{1cm} (2.62)

By combining equations (2.61) and (2.62) an expression for the shear component of the Reynolds stress tensor can be obtained and applied at the near-wall cell:

$$\bar{u}' w' \bigg|_1 = \nu \left. \frac{\partial \bar{u}}{\partial z} \right|_1 - \frac{\tau_w}{\rho} \left(1 - \frac{z_1}{\delta}\right)$$  \hspace{1cm} (2.63)

As with the $\bar{v}'$ wall function equation (2.53) can be used to calculate $\tau_w$ making the wall function consistent with those used for the other variables, increasing its robustness and providing a simple mechanism for introducing roughness. To calculate the local boundary-layer thickness a log layer is again assumed so that the gradient of $\bar{u}$ tends to zero as $y$ tends to $\delta$. 


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From this $\delta$ can be approximated as the point at which the gradient of $\overline{u}$ drops below a certain value $\beta$:

$$\delta = \frac{u^*}{\kappa \beta}$$

(2.64)

For the present simulations a value of 0.01 was used initially for $\beta$. However studies of channel flows of known boundary-layer height suggested that $\beta = 0.5$ gave better estimates of $\delta$. This value was used for all the subsequent simulations though testing showed that simulation results were not sensitive to the choice of $\beta$.

Once the shear component of the Reynolds stress tensor has been calculated using equation (2.63) it is important to apply it correctly. This requires rotating the Reynolds stress tensor so that it is aligned with the wall-normal direction and the tangential velocity, correcting the shear component in this orientation, then rotating the corrected tensor back into the grid aligned coordinates. At each point care must be taken to ensure that the components of $u'_i u'_j$ maintain the correct sign, especially at areas of complex flow such as separation and re-attachment points. Note that following Pope (2000) there is also a small adjustment in the calculation of $\tau$ in the rough wall case:

$$\tau(z)_{\text{rough}} = \tau_{w_\text{rough}} \left(1 - \frac{z - z_0}{\delta}\right)$$

(2.65)

where the offset $z_0$ is the aerodynamic roughness height.

In this work the new Reynolds stress wall function is applied to a tensor obtained using the ASBM closure. However the manner in which the Reynolds stress tensor is obtained is independent of the calculation of the wall function. Applying it in conjunction with other closures that calculate the full Reynolds stress tensor should also be possible.
2.5 Summary of Turbulence Closures and Wall Treatments

A number of different simulations have been carried out as part of this work. Depending on the flow that is being investigated, different turbulence closures are selected from Section 2.3 for comparison. Also, different near-wall treatments from Section 2.4 are selected to use with each of the turbulence closures. The different flows that are investigated can be broken down into three categories, low $Re$ flows over smooth surfaces, high $Re$ flows over smooth surfaces and high $Re$ flows over rough surfaces. Tables 2.4-2.6 summarise the combinations of turbulence closures and near-wall treatments used in this work. The wall functions that have been developed as part of this work are highlighted.

Table 2.4: Low $Re$ Smooth Surface Simulations

<table>
<thead>
<tr>
<th>Turbulence closure</th>
<th>Wall treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$-$\omega$-SST</td>
<td>All variables fully resolved</td>
</tr>
<tr>
<td>$v^2f$</td>
<td>All variables fully resolved</td>
</tr>
<tr>
<td>ASBM</td>
<td>All variables fully resolved</td>
</tr>
</tbody>
</table>
### Table 2.5: High $Re$ Smooth Surface Simulations

<table>
<thead>
<tr>
<th>Turbulence closure</th>
<th>Wall treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$-$\varepsilon$</td>
<td>$\nu_T$ smooth wall function</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon$ wall function</td>
</tr>
<tr>
<td></td>
<td>$\overline{\nu}$ no-slip</td>
</tr>
<tr>
<td></td>
<td>$k, \hat{\rho}$ zero-gradient</td>
</tr>
<tr>
<td>$\nu^2$ $f$</td>
<td>$\overline{\nu^2}$ smooth wall function - developed in this work</td>
</tr>
<tr>
<td></td>
<td>$\nu_T$ rough wall function</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon$ wall function</td>
</tr>
<tr>
<td></td>
<td>$\overline{\nu}$ no-slip</td>
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<td></td>
<td>$k, \hat{\rho}$ zero-gradient</td>
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<td>$f$ 0</td>
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<tr>
<td>ASBM</td>
<td>$\overline{u'u'}$ smooth wall function - developed in this work</td>
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<td></td>
<td>$\overline{\nu^2}$ smooth wall function - developed in this work</td>
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<td>$\nu_T$ rough wall function</td>
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<td>$\varepsilon$ wall function</td>
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<td>$\overline{\nu}$ no-slip</td>
</tr>
<tr>
<td></td>
<td>$k, \hat{\rho}$ zero-gradient</td>
</tr>
<tr>
<td></td>
<td>$f$ 0</td>
</tr>
<tr>
<td></td>
<td>$\phi$ 1</td>
</tr>
</tbody>
</table>
Table 2.6: High $Re$ Rough Surface Simulations

<table>
<thead>
<tr>
<th>Turbulence closure</th>
<th>Wall treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k-$\varepsilon</td>
<td>$\nu_T$ rough wall function</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon$ wall function</td>
</tr>
<tr>
<td></td>
<td>$\bar{u}$ no-slip</td>
</tr>
<tr>
<td></td>
<td>$k$, $\hat{p}$ zero-gradient</td>
</tr>
<tr>
<td>$v^2 f$</td>
<td>$\bar{v}^2$ rough wall function - developed in this work</td>
</tr>
<tr>
<td></td>
<td>$\nu_T$ rough wall function</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon$ wall function</td>
</tr>
<tr>
<td></td>
<td>$\bar{\pi}$ no-slip</td>
</tr>
<tr>
<td></td>
<td>$k$, $\hat{p}$ zero-gradient</td>
</tr>
<tr>
<td></td>
<td>$f$ 0</td>
</tr>
<tr>
<td>ASBM</td>
<td>$u_i' u_j'$ rough wall function - developed in this work</td>
</tr>
<tr>
<td></td>
<td>$\bar{v}^2$ rough wall function - developed in this work</td>
</tr>
<tr>
<td></td>
<td>$\nu_T$ rough wall function</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon$ wall function</td>
</tr>
<tr>
<td></td>
<td>$\bar{\pi}$ no-slip</td>
</tr>
<tr>
<td></td>
<td>$k$, $\hat{p}$ zero-gradient</td>
</tr>
<tr>
<td></td>
<td>$f$ 0</td>
</tr>
<tr>
<td></td>
<td>$\phi$ 1</td>
</tr>
</tbody>
</table>
Chapter 3

Numerical Method

There are a wide variety of numerical methods that can be used to solve the steady RANS equations given in Section 2.2.1. Most approaches fall into one of the following four categories; finite difference, finite element, finite volume or spectral methods. Each of these approaches set up discrete forms of the governing equations that can then be solved numerically. A detailed description of each approach is given in Versteeg & Malalasekera (1999).

Chapter 1 established that the main objective of this research is to investigate turbulence models and near-wall treatments that can be used with the RANS equations to improve wind flow prediction over complex terrain. Therefore the most important factor in selecting a numerical method was that it was implemented in a well-established, robust and reliable software package. This removed the need for expending effort on implementing the numerical method and allowed the research to be focused on the development and testing of turbulence models and near-wall treatments.

A number of other criteria were also considered when selecting the software package. First, due to the large range of length scales in a steady simulation of wind flow over complex terrain, parallel computational capability was key. Second, extendability was required to enable the implementation of new physical models within the framework of the existing numerical method. Finally, practical considerations such as access and license issues also influenced the selection process.
Several different commercial CFD packages were assessed such as ANSYS CFX, FLUENT and COMSOL. A number of bespoke codes including SIMRA (Eidsvik, 2005), VANE, VENTOS (Palma et al., 2008) and Gerris were also investigated. Each of the packages has advantages and disadvantages but ultimately the open source solution framework OpenFOAM was selected as the best fit with the requirements of this research.

3.1 OpenFOAM

OpenFOAM is an open source, freely available CFD toolbox. At its core, OpenFOAM has a set of efficient C++ modules that are used to make up applications, utilities and libraries. It uses colocated, polyhedral, finite volume numerics that can be applied on unstructured meshes and can be easily extended to run in parallel. More information is given in the OpenFOAM literature (Jasak et al., 2007) and at www.openfoam.org.

3.1.1 simpleFOAM and the SIMPLE Algorithm

SimpleFOAM is the OpenFOAM application that is used to solve the steady, incompressible RANS equations and is an implementation of the SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm. The SIMPLE algorithm is a pressure-correction method that partly decouples the Navier-Stokes equations by taking the divergence of the momentum equation (2.6) and substituting in the continuity equation (2.7). This provides an explicit equation for the pressure that can be solved iteratively in combination with the momentum equation.
A detailed description of the algorithm and its derivation can be found in Ferziger & Peric (2002) but it can be summarised as follows:

1. Set the boundary conditions.

2. Solve the discretised momentum equation for the intermediate velocity field.

3. Compute the mass fluxes at the cells faces.

4. Solve the pressure equation and apply under-relaxation.

5. Correct the mass fluxes at the cell faces.

6. Correct the velocities on the basis of the new pressure field.

7. Update the boundary conditions.

8. Repeat until convergence criteria are met.

Note that the algorithm used in SimpleFOAM also includes additional corrector steps to account for the effects of non-orthogonal grids.

In the simpleFOAM implementation, the RANS momentum equation (2.11) is rewritten as:

\[ \frac{\partial \rho \vec{u}_i}{\partial t} + \frac{\partial R_{ij}^*}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} \]  

(3.1)

where \( R_{ij}^* \) includes the effects from both the viscous terms and turbulence. This form is used directly in the discretised momentum equation and indirectly in the pressure equation. It also provides a convenient mechanism for including various turbulence closures in the algorithm.
3.1.2 atmosFOAM

SimpleFOAM can be used to solve for any incompressible, steady fluid flow and is not specific to atmospheric flows. As discussed in Chapter 2, wind flow is driven by large scale pressure gradients and its exact form is determined by the balance with Coriolis forces. To accurately represent atmospheric flows, these two body forces must be included in the solution algorithm. As part of this research project simpleFOAM was extended to create the atmosFOAM application which includes these effects. The open source nature of OpenFOAM made this extension a simple process. Both the Coriolis force and the pressure gradient force are treated as body forces that contribute to the momentum equation. Including these effects equation (3.1) becomes:

\[
\sum_{j} \frac{\partial u_i}{\partial x_j} + \frac{\partial R_{ij}}{\partial x_j} + P G_i + 2 \epsilon_{ijk} u_k \omega_c \sin \phi_c = - \frac{1}{\rho} \frac{\partial p}{\partial x_i}
\]

(3.2)

where the vector \(PG_i\) is the magnitude of the pressure gradient divided by the density of the air, \(\omega\) is the angular velocity of the Earth and \(\phi\) is the latitude of the current position. All of these parameters are defined in a file which allows them to be controlled easily for each simulation. By setting both body forces to zero the simpleFOAM algorithm is recovered.

3.1.3 Solvers

OpenFOAM has a number of iterative linear solvers implemented that can be used solve the large system of linear equations generated as part of the solution algorithm. This system is of the usual form:

\[
Ax = b
\]

(3.3)

An excellent description of the implementation of solvers in OpenFOAM is given in the report of Behrens (2009) and is summarised here. For more information refer to the original report or the documentation available at www.openfoam.org.
The following linear solvers are those used in this work:

- **GAMG** - Geometric agglomerated algebraic multigrid solver
- **PBiCG** - Preconditioned bi-conjugate gradient solver for asymmetric lduMatrices using a run-time selectable preconditioner
- **PCG** - Preconditioned conjugate gradient solver for symmetric lduMaxtrices using a run-time selectable preconditioner

The term lduMatrix refers to a matrix which has coefficients that can be stored as three different arrays. One for the lower triangle (l), one for the diagonal of the matrix (d) and a third array for the upper triangle (u). The matrix preconditioners are used to improve the convergence performance of the solvers. The preconditioned form of equation (3.3) is:

\[ M^{-1}Ax = M^{-1}b \]  

The choice of the preconditioner \( M \) depends on several factors but usually it is an easily invertible approximation of \( A \) and leads to computationally cheap matrix and vector multiplications. In basic terms, the preconditioner can be thought of as leading to faster propagation of information through the computational mesh. The preconditioners used were:

- **DIC** - Simplified diagonal-based incomplete Cholesky preconditioner for symmetric matrices (symmetric equivalent of DILU). The reciprocal of the preconditioned diagonal is calculated and stored.
- **DILU** - Simplified diagonal-based incomplete LU preconditioner for asymmetric matrices. The reciprocal of the preconditioned diagonal is calculated and stored.
- **FDIC** - Faster version of the DICPreconditioner diagonal-based incomplete Cholesky preconditioner for symmetric matrices (symmetric equivalent of DILU) in which the reciprocal of the preconditioned diagonal and the upper coefficients divided by the diagonal are calculated and stored.
• GAMG - Geometric agglomerated algebraic multigrid preconditioner

As well as the preconditioners, OpenFOAM offers a number of smoothers that can be used to improve performance and reduce the mesh dependency of the convergence. In this work only the Gauss-Siedel smoother was used.

In general, the PCG and PBiCG solvers were used for all one and two-dimensional simulations and for smaller simulations. For larger, three-dimensional simulations GAMG solvers were also used where improvements in convergence speeds could be achieved. In each case the appropriate preconditioners were chosen depending upon the form of the system of equations. Regardless of the choices in solver and preconditioner, OpenFOAM allows control of the convergence criteria through either an absolute value or a relative value. The best results in terms of convergence and stability were achieved by using an absolute convergence criteria for the pressure and a combination of both absolute and relative convergence criteria for the other variables. Because of the potential for numerical errors to accumulate due to the presence of periodic boundary conditions, the convergence criteria were more stringent for one and two-dimensional simulations.

3.1.4 Differencing Schemes

OpenFOAM offers a wide range of differencing schemes by default and additional schemes are easily incorporated because of the open source format of the software. A control file enables the differencing scheme for each derivative to be selected at run time which allows fine level control on a case-by-case basis. Despite the range of options and fine level of control available, the differencing schemes selected for this work were the most basic, well-established and stable options. This enabled the new algorithms and physical models to be assessed in isolation from the effects of more complex differencing schemes. For all derivatives the standard Gaussian finite volume integration was used with linear interpolation from the cell centres to the faces with the exception of the convection terms where upwind interpolation was used. Note that Gaussian finite volume integration with linear interpolation is exactly equivalent to central differencing when using finite difference techniques.
Some initial simulations were run using the Quadratic Upwind Interpolation for Convective Kinematics (QUICK) scheme for the convection terms however this approach was abandoned following difficulties in achieving a stable converged solution once complex terrain was introduced. For the Laplacian terms, the linear interpolation scheme included an explicit non-orthogonal correction for the pressure and velocity. This was also the case for the $f$ and $\phi$ terms from the $v^2f$ and ASBM closures respectively. For the $k$, $\varepsilon$ and $v^2$ terms the Laplacian term included a limited non-orthogonal correction with the limiting value set to 0.5. This limits the non-orthogonal correction to a maximum of 0.5 times the orthogonal contribution. More detailed descriptions of the differencing schemes and their derivations can be found in the OpenFOAM user guide (OpenCFD Limited, 2009).

3.1.5 Parallelisation

The method for running OpenFOAM in parallel is known as domain decomposition and essentially divides the domain of a large simulation into smaller pieces that are run on separate processors. Each smaller simulation is then effectively run as an independent simulation. However, each requires information from its neighbouring simulations to provide the correct boundary conditions and vice versa. In OpenFOAM the coordinated passing of information between these simulations is handled by openMPI which is the public domain implementation of the standard message passing interface (MPI).

The domain decomposition is carried out as a pre-processing activity before the simulation is run using the standard openFOAM utility, decomposePar. There are a number of options available to control the decomposePar utility but for this work only simple domain decomposition was used. This allows the domain to be divided into a specified number of subdomains in each of the three coordinate directions. Given the regular nature of the solution domains, the specification of these subdivisions was a straightforward process.

Once the simulation has been decomposed the application atmosFOAM can then
be run directly using the openMPI command `mpirun`. Upon completion, the simulation must be reassembled using another OpenFOAM utility, `reconstructPar`. Following the reconstruction of the domain, the simulation results appear in exactly the same form as those generated by simulations run on a single cpu and may be post-processed in the same way.

Because of the computationally intensive nature of the algorithms and the large scale of some of the simulation domains, the speed-up achieved through the use of parallel processing was significant. Details of the scaling with the number of processors used is presented in the relevant results sections.

### 3.1.6 Relaxation

As Ferziger & Peric (2002) note, the use of under-relaxation is important in pressure-correction algorithms such as the SIMPLE algorithm. For the steady flows like those studied in this work, the under-relaxation parameter in the momentum equation is equivalent to the time step in unsteady flows and plays an important role in the stability of the solution. For most of the solutions in this work the standard openFOAM values for the under-relaxation parameters were used. However, while solving larger scale, more complex flows the under-relaxation parameters were often reduced at the start of the solution process to allow the solution to evolve more slowly. The standard openFOAM under-relaxation parameters are given in the table below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\bar{u}_i$</td>
<td>0.7</td>
</tr>
<tr>
<td>$k$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
0.5 was used for each of the variables and this was found to be sufficient in most cases.

3.2 Turbulence Closure Algorithms

In both the simpleFOAM algorithm and the atmosFoam algorithm, the effects of turbulence are included through the term $\frac{\partial R_{ij}^*}{\partial x_j}$ in equation (3.1). This term also includes the effects of viscosity which provides a convenient mechanism for introducing an effective viscosity used by eddy-viscosity turbulence closures. In turbulence closures such as the ASBM where the full Reynolds stress tensor is calculated, the viscous and turbulence effects are simply treated separately and combined to calculate $\frac{\partial R_{ij}^*}{\partial x_j}$.

The following subsections give descriptions of the algorithms used by each of the turbulence closures to calculate this term.

3.2.1 The $k$-$\varepsilon$ Closure

The algorithm for calculating $\frac{\partial R_{ij}^*}{\partial x_j}$ using the $k$-$\varepsilon$ closure is implemented in the following way:

1. Calculate the eddy viscosity $\nu_T$ using equation (2.28).
2. Calculate production of turbulent kinetic energy $\mathcal{P}$ using equation (2.24).
3. Solve equation (2.29) for $\varepsilon$.
4. Solve equation (2.23) for $k$.
5. Re-calculate $\nu_T$.
6. Calculate $\frac{\partial R_{ij}^*}{\partial x_j}$ using the following:

$$\frac{\partial R_{ij}^*}{\partial x_j} = -\frac{\partial}{\partial x_j} \left[ (\nu + \nu_T) \frac{\partial \bar{u}_i}{\partial x_j} \right] - \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_j}{\partial x_i} \right)$$

(3.5)

Note that when wall functions are being used as boundary conditions they must be applied at step 3 for $\varepsilon$ but also at steps 1 and 5 for $\nu_T$ as described in Section 2.4.1.
CHAPTER 3. NUMERICAL METHOD

3.2.2 The $v^2f$ Closure

The $v^2f$ closure is also an eddy-viscosity closure and so follows the same basic algorithm for calculating $\frac{\partial R_{ij}}{\partial x_j}$ as above. As described in Section 2.3.5, the difference lies in the calculation of the eddy viscosity. The algorithm is implemented as follows:

1. Calculate the turbulence time scale $T$ using equation (2.32).
2. Calculate the turbulence length scale $L$ using equation (2.35).
3. Calculate the eddy viscosity $\nu_T$ using equation (2.31).
4. Calculate production of turbulent kinetic energy $\mathcal{P}$ using equation (2.24).
5. Solve equation (2.29) for $\varepsilon$.
6. Solve equation (2.23) for $k$.
7. Solve equation (2.33) for $v^2$.
8. Solve equation (2.34) for $f$.
9. Re-calculate $T$.
10. Re-calculate $\nu_T$.
11. Calculate $\frac{\partial R_{ij}}{\partial x_j}$ using equation (3.5)

In some circumstances it is necessary bound the value of $v^2$ to ensure it does not exceed $\frac{2}{3}k$. This is carried out following the calculation of $v^2$ in step 7. Also, in the same manner as the $k-\varepsilon$ closure, if wall functions are used the $v^2$ wall function described in Section 2.4.3 is applied as part of step 7.

3.2.3 The ASBM Closure

The ASBM closure provides the full Reynolds stress tensor ($R_{ij}$) and so its integration into the openFOAM implementation varies significantly from the eddy-viscosity
closures. In theory, the term $\frac{\partial R_{ij}^*}{\partial x_j}$ could be calculated directly once the Reynolds stresses have been determined:

$$
\frac{\partial R_{ij}^*}{\partial x_j} = -\frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial u_i u_j'}{\partial x_j}
$$

(3.6)

However, Kalitzin et al. (2004) found that implementing the ASBM closure directly leads to issues with stability. To overcome this they used a deferred-correction approach that effectively adds an eddy-viscosity contribution at each iteration while subtracting the same contribution calculated at the previous iteration. Thus the term $\frac{\partial R_{ij}^*}{\partial x_j}$ is calculated as:

$$
\frac{\partial R_{ij}^*}{\partial x_j} = -\frac{\partial}{\partial x_j} \left[ (\nu + \nu_T) \frac{\partial u_i}{\partial x_j} \right] + \frac{\partial u_i u_j'}{\partial x_j}
$$

(3.7)

where $n$ and $n+1$ indicate the previous and the current time iteration, respectively. Once a converged solution is obtained, the terms involving the eddy viscosity cancel out leaving only the ASBM Reynolds stress components. As a consequence of the deferred-correction approach, the eddy viscosity must also be calculated as part of the ASBM closure. This is not an additional computational cost as the ASBM closure already requires a turbulence time scale, the turbulent kinetic energy and, in the presence of a wall, a turbulence length scale. Having calculated these quantities previously, the additional computation cost of calculating the eddy viscosity is minimal. For all the simulations in this work carried out with the ASBM turbulence closure, the $v^2f$ closure was used to calculate the eddy viscosity.

While the deferred-correction approach enabled the implementation of a stable algorithm, as more complex flows at higher $Re$ were explored, it was found that stability issues returned. This is largely due to the fact that the ASBM closure is a highly nonlinear and localised model (see Sections 2.3.6 and 2.3.7 and the references therein). While working at the 2010 summer research program at Stanford’s Center for Turbulence Research the author developed a new coupling method that introduces a blending between the Boussinesq approximation and ASBM closure. This original work is one of the important contributions of this project. The term $\frac{\partial R_{ij}^*}{\partial x_j}$ is now
calculated as follows:

\[
\frac{\partial R_{ij}}{\partial x_j} = -\frac{\partial}{\partial x_j} \left[ (\nu + \nu_T) \frac{\partial u_i^{(n+1)}}{\partial x_j} \right] - \frac{\partial}{\partial x_j} \left[ (1 - \alpha) \nu_T \frac{\partial u_j}{\partial x_i} - \alpha \left( \nu_T \frac{\partial u_i}{\partial x_j} + \left( u_i' u_j' - \frac{2}{3} k \delta_{ij} \right) \right) \right]^{(n)}
\]  (3.8)

Here \( \alpha \) is the blending factor and it can be shown that for \( \alpha = 0 \), equation (3.5) is recovered. For \( \alpha = 1 \) and a converged solution (timestep \( n + 1 \to n \)), the Boussinesq approximation cancels out and only the Reynolds stresses and the viscous term remain, recovering equation (3.6). Note that by subtracting the isotropic contribution from the Reynolds stress components, the definition of \( \hat{p} \) is consistent for all values of \( \alpha \). For the simulations presented in this work \( \alpha \) was set initially to 0, then increased to 0.5 and then gradually increased to \( \alpha = 1 \). For each value of \( \alpha \) the simulation was run until a converged solution was obtained.

As described above, using the ASBM closure in this form leads to additional steps in the algorithm for calculating \( \frac{\partial R_{ij}}{\partial x_j} \) rather than an alternative algorithm. Because the \( \nu^2 f \) closure was used to calculate the eddy viscosity, the algorithm in Section 3.2.2 is used where step 11 is replaced with the following algorithm:

11. Solve equation (2.47) for blockage parameter \( \phi \).

12. Calculate the mean strain rate and mean rotation rate.

13. Filter the mean strain rate and mean rotation rate.

14. Calculate the blockage tensor \( B_{ij} \) using equation (2.46).

15. Calculate the eddy-axis tensor as a result of pure strain \( a^{S}_{ij} \).

16. Calculate the eddy-axis tensor as a result of rotation \( a^{R}_{ij} \).

17. Calculate the ratio of mean rotation to mean strain \( \eta_m \).

18. Calculate the ratio of frame rotation to mean strain \( \eta_f \).

19. Filter and relax \( \eta_m \) and \( \eta_f \).
20. Calculate the structure parameters.

21. Calculate the structure tensors including the Reynolds stress tensor.

22. Filter and relax the Reynolds stress tensor.

23. Calculate $\frac{\partial R_{ij}}{\partial x_j}$ using equation (3.8)

The filtering at steps 13, 19 and 22 are necessary because of the highly nonlinear and localised nature of the ASBM. An example of this can be observed by noting that part of the ASBM makes calculations using the gradient of the mean strain rate. In some cases, small numerical errors in the calculation of the velocity gradients that make up the mean strain rate can lead to small errors in its value but opposite signs in its gradient from one cell to the next. This can be clearly seen in Figure 3.1 where the $S_{11}$ component is plotted from a simulation of a two-dimensional hill.

![Figure 3.1: $S_{11}$ calculated at the base of a two-dimensional hill with filtering (---) and without (—). The position of the hill is also shown.](image_url)

At the base of the hill at $x/H = 5$ the change from a non-orthogonal grid to an orthogonal grid causes small errors in the velocity gradient calculations. As the ASBM is applied independently at each cell, this can lead to large differences in the Reynolds stresses calculated in neighbouring cells. The nonlinear nature of the ASBM then causes these errors to propagate such that the final solution can be affected as
shown in Figure 3.2.

Figure 3.2: $\overline{u'w'}$ calculated over a two-dimensional hill using the ASBM closure (a) without filtering and (b) with filtering.

In all the simulations in this work the filtering was implemented using a top hat filter based on the neighboring cells. The filtering of the ratios $\eta_m$ and $\eta_f$ at step 19 is one of the incremental improvements made to the ASBM closure algorithm as part of this work. These ratios play a key role in determining the state of the turbulence in a given cell. Filtering them before calculating the structure tensors reduces the likelihood of calculating different states of turbulence for two neighbouring cells as a result of small differences in the values of $\eta_m$ and $\eta_f$. This in turn increases the stability of the closure. For the same reason, the ASBM closure algorithm has been changed so that the values of $\eta_m$ and $\eta_f$ are stored at each iteration and relaxed. In an analogous way to the spatial filtering, this prevents large changes in $\eta_m$ and $\eta_f$ from one iteration to the next and hence increases the stability of the algorithm.

Two other improvements were made in the algorithm, both concerning the calculation of the eddy-axis tensors $a_{ij}^S$ and $a_{ij}^R$. The first was to retain both tensors throughout the domain from one iteration to the next. The values of the tensors from the previous iteration are then used as the initial values in the Newton-Raphson solve for the tensors in the current iteration. This greatly reduces the occurrence of non-convergence during the Newton-Raphson solve process. Testing showed that even in a large domain a few non-converged values for the these tensors can rapidly lead to unstable solutions. The second improvement is associated with the first as it was the introduction of a simple but effective method for restarting the Newton-Raphson solver with a new initial guess in the case of non-convergence.
Finally note that care must be taken when using the ASBM closure to model turbulence in truly two-dimensional flows. Its sensitivity to numerical errors in velocity gradients means that problems can occur when the velocity in the cross-stream direction is zero. To correct this the cross-stream shear components of Reynolds stress tensor are explicitly set to zero in two-dimensional flows. This technique was used in all of the simulations discussed in chapter 5.

3.3 Grid Generation

In spite of the term “complex terrain”, the geometry of the computational domain for large scale simulations of atmospheric flows are relatively simple when compared to other CFD problems. For all the simulations carried out in this work the computational domain is a large box with the bottom of the box being the ground. In the cases of the simple simulations used to calculate inflow profiles, the bottom of the box is flat and the grid purely orthogonal. For the simulations involving two-dimensional and three-dimensional hills, the bottom of the box follows the contours of the ground. In all the simulations the approach was to create an orthogonal two-dimensional grid in the horizontal directions \(x\) and \(y\), and then lay the grid over the surface of the ground. The contours of the ground then determine the elevation or \(z\) coordinate of each point in the two-dimensional grid. From each of these points on the ground, grid points are created vertically at different stretching ratios such that the top of the domain is flat. Also for all simulations, the horizontal directions are orientated such that the reference wind is aligned with the \(x\) direction. Examples of the grid generated in this manner are shown in Figures 3.3 and 3.3. In both cases blocks of 4 cells are shown as a single cell and the scales on each axis is different to increase the readability of the figures.

As can be seen from the examples, the grids can be stretched in all three directions and the stretching need not be uniform. This allows a high resolution grid to be created in the areas of most interest or where high gradients are known to exist. The simple computational domains with flat ground were created using the OpenFOAM tool \texttt{blockMesh}. This is a basic tool that is quick and convenient for creating grids of
Figure 3.3: Example of two-dimensional computational grid clipped and zoomed for readability.

Figure 3.4: Example of three-dimensional computational grid.
this type. For more information see the OpenFOAM user guide (OpenCFD Limited, 2009). To create the computational grids including complex terrain MATLAB scripts were developed that generate CFX4 grids which can then be converted to OpenFOAM grids using the standard utility \texttt{cfx4ToFoam}. Because of the contours of the ground, these computational grids had areas which were non-orthogonal thus the appropriate corrections needed to be used in the solution algorithm.

### 3.4 Post Processing

The regular nature of the computational grids used for the simulations in this work meant that there were several options available for post-processing results. Initially post-processing was carried out using CFX-Post however this required time consuming and sometimes difficult file-type conversions. Paraview was also used regularly but like CFX-Post, its ability to handle results from a plethora of different CFD simulations actually meant it was somewhat cumbersome for the specific tasks required for this work. The output files created by OpenFOAM can be written in ascii format and for regular grids the indexing is straightforward, and a large number of efficient custom post-processing routines were developed in MATLAB. These routines give a fine level of control when querying the results and a wide range of options for presenting data in a meaningful and instructive manner. One exception to this is the streamline plots which were created using Paraview.
Chapter 4

Preliminary Simulations

Before attempting two- and three-dimensional simulations of wind flow over complex terrain a large number of preliminary simulations were carried out. Some of these simulations were designed as test cases for new approaches and new algorithms. Others were used to compare different options within standard CFD techniques. Several simulations were focused on determining the boundary conditions to be used, particularly at the inflow boundary. The key preliminary simulations are described in the following sections with results presented and discussed. Note that much of the work presented in this chapter has been already published and will only be summarised here. For a more detailed discussion refer to O’Sullivan et al. (2011).

4.1 Boundary Condition Experiments

To determine the most appropriate boundary conditions to apply to simulations over complex terrain a number of numerical experiments were carried out in simpler domains. In this way the effects of different boundary conditions could be assessed and a quantitative comparison carried out. Comparisons of this nature are not present in the current literature and there is not a clear, collective opinion within the community on the correct application of boundary conditions (O’Sullivan et al., 2011). Particular attention was focused on the inflow boundary condition and its parameterisation. To achieve this, a direct comparison of the variation of the velocity profiles
over an empty domain using different inflow boundary conditions was made. Solving simple atmospheric flows in an empty domain in this manner is commonly referred to as simulating the neutrally stable atmospheric boundary layer (Richards & Hoxey, 1993; Hargreaves & Wright, 2007; Yang et al., 2008, 2009).

4.1.1 Simulating the Neutrally Stable Atmospheric Boundary Layer

The importance of accurate simulations of the neutrally stable ABL as a prerequisite for simulations of flows over terrain or obstacles is well established (Richards & Hoxey, 1993; Blocken et al., 2007; Hargreaves & Wright, 2007; Yang et al., 2009). While it is a physically simple flow, it has proved difficult to simulate numerically without erroneous streamwise gradients developing. These gradients have been attributed to a number of causes such as inconsistencies between the inflow profile and other boundary conditions (Blocken et al., 2007; Hargreaves & Wright, 2007; Yang et al., 2009) and inconsistencies between the inflow and the turbulence model (Riddle et al., 2004). For this work a neutrally stable ABL was successfully simulated using the following approach: the appropriate boundary conditions were determined at the ground and at the top of the domain and a one-dimensional simulation was run to generate the profile to use at the inflow. This ensures that the inflow profile is consistent with all other boundary conditions and the turbulence model, regardless of which closure is used.

4.1.2 Inflow Profile Generation

As discussed in O’Sullivan et al. (2011), generating inflow profiles using a one-dimensional simulation is relatively straightforward and has been suggested before (Wright & Easom, 1999; Blocken et al., 2007). A numerical domain is created that resolves the vertical direction but is only a single cell wide in both horizontal directions. The side boundary conditions are then prescribed to be periodic in both directions. For the vertical direction the grid is stretched away from the ground to resolve the steep gradients found there. The boundary condition that is to be used in the full simulation at the ground is also applied in the one-dimensional simulation. Note that this
boundary condition need not be a rough wall function as is required for simulations of the ABL. In following sections smooth wall functions and no slip conditions are also used where appropriate. The remaining boundary condition at the top of the domain determines the nature of the inflow profile.

In O’Sullivan et al. (2011) we argue that a Dirichlet boundary condition is not appropriate at the top boundary condition and that a shear stress condition should be used instead. The shear stress boundary conditions can be calculated easily from the same parameters as those used to determine the more commonly used inflow profile models. For the work in this section two inflow profile models were considered. One suggested by Richards & Hoxey (1993) (hereafter referred to as the RH solution) and the other, its extension, proposed by Yang et al. (2009) (hereafter referred to as the Yang solution). For completeness, the full RH and Yang solutions are given below. For their development please refer to the references.

RH Solution

\[
\begin{align*}
  u &= \frac{u_s}{\kappa} \ln \left( \frac{z + z_0}{z_0} \right) \\
  k &= \frac{u_s^2}{\sqrt{C\mu}} \\
  \varepsilon &= \frac{u_s^3}{\kappa (z + z_0)}
\end{align*}
\]

Yang Solution

\[
\begin{align*}
  k &= \frac{u_s^2}{\sqrt{C\mu}} \sqrt{C_1 \ln \left( \frac{z + z_0}{z_0} \right) + C_2} \\
  \varepsilon &= \frac{u_s^3}{\kappa (z + z_0)} \sqrt{C_1 \ln \left( \frac{z + z_0}{z_0} \right) + C_2}
\end{align*}
\]

As Yang et al. (2009) note, equations (4.2) and (4.3) are special cases of equations (4.4) and (4.5) with \( C_1 = 0 \) and \( C_2 = 1 \). As an extension of the RH solution the Yang
solution uses the same equation (4.1) for the velocity. The shear stress boundary conditions for these profiles are the following:

**RH Solution**

\[
\frac{\partial u}{\partial z} = \frac{u_*}{\kappa (z + z_0)} \tag{4.6}
\]

\[
\frac{\partial k}{\partial z} = 0 \tag{4.7}
\]

\[
\frac{\partial \varepsilon}{\partial z} = \frac{-u_*^3}{\kappa(z + z_0)^2} \tag{4.8}
\]

**Yang Solution**

\[
\frac{\partial k}{\partial z} = \frac{1}{(z + z_0)\sqrt{C_1 \ln \left(\frac{z + z_0}{z_0}\right) + C_2}} \tag{4.9}
\]

\[
\frac{\partial \varepsilon}{\partial z} = \frac{u_*^3}{\kappa(z + z_0)^2} \left[\frac{C_1}{2\sqrt{C_1 \ln \left(\frac{z + z_0}{z_0}\right) + C_2}} - \sqrt{C_1 \ln \left(\frac{z + z_0}{z_0}\right) + C_2}\right] \tag{4.10}
\]

Shear stress boundary conditions drive the flow and generate profiles in an equivalent way to Dirichlet boundary conditions. Figure 4.1 shows inflow profiles generated using the shear stress boundary conditions along with the RH and Yang solutions. All of the profiles are generated using the wind conditions given in Table 4.2 and for Yang solution the values of \(C_1\) and \(C_2\) are set to -0.01 and 1.23 respectively. This gave the best match between the Yang solution and the one-dimensional solution which was known to be consistent with the other boundary conditions.

The agreement for the velocity and dissipation is very good, however both the one-dimensional model solution and the Yang solution differ from the RH solution for \(k\). This is not surprising as Yang solution was specifically designed to capture the spike in \(k\) observed near the ground that is not accounted for in the RH solution (Yang et al., 2009).
Figure 4.1: Inflow profiles for simulations of a neutrally stable ABL using the $k$-$\varepsilon$ closure. (a) Streamwise velocity, (b) turbulent kinetic energy and (c) dissipation.
Having generated an inflow profile using the approach described above, simulating a neutrally stable ABL becomes a trivial task as the the periodic boundary conditions ensure the one-dimensional solution is equivalent to an infinitely long boundary layer. One of the important consequences of accurately modelling the neutrally stable ABL is that this approach enables the quantitative assessment of the impact of using inconsistent boundary conditions.

### 4.1.3 Inconsistent Boundary Conditions

When simulating wind flow in the ABL it is common practice to used a zero-gradient boundary condition at the top of the domain and prescribe a form of the log law given in Section 2.1.5 as the inflow boundary condition. These two boundary conditions are not consistent and lead to erroneous streamwise gradients in the solution (O’Sullivan et al., 2011). Also, many forms of the log law inflow are also inconsistent with the boundary condition at the ground. Having successfully simulated a neutrally stable ABL, the result can be used as a benchmark against which the impact of these inconsistencies can be quantitatively assessed. The wind profile for the ABL was selected to be the same as that used by Hargreaves & Wright (2007) and by Richards & Quinn (2002). Its specifications are given in Table 4.2 in terms of a reference velocity at a reference height. These can be substituted into equations (4.1)-(4.3) to determine the value of $u^*$. Table 4.2 also gives the specifications of the simulation domains.

Six different cases were then run using the same simulation configuration except with different boundary conditions. Three different inflow profiles were used; the profile generated by the one-dimensional model, the standard RH solutions and the more generalised Yang solutions. For each of the profiles two sets of boundary conditions were applied at the top boundary of the domain. The first was the consistent boundary conditions described by equations (4.6)-(4.10) and the second was the zero-gradient boundary conditions. In all cases a zero-gradient boundary condition was used for the pressure at the inlet. Zero-gradient boundary conditions were used at the outlet for all the solution variables except the pressure for which a fixed value
was defined. Table 4.1 summarises the simulation cases and is the reference for the results shown in all of the figures.

### Table 4.1: Simulation Cases for Boundary Condition Experiments

<table>
<thead>
<tr>
<th>Case</th>
<th>Inflow Profile</th>
<th>Top Boundary Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1-D Model Solution</td>
<td>Consistent</td>
</tr>
<tr>
<td>Case 2</td>
<td>1-D Model Solution</td>
<td>Zero-Gradient</td>
</tr>
<tr>
<td>Case 3</td>
<td>RH Solution</td>
<td>Consistent</td>
</tr>
<tr>
<td>Case 4</td>
<td>RH Solution</td>
<td>Zero-Gradient</td>
</tr>
<tr>
<td>Case 5</td>
<td>Yang Solution</td>
<td>Consistent</td>
</tr>
<tr>
<td>Case 6</td>
<td>Yang Solution</td>
<td>Zero-Gradient</td>
</tr>
</tbody>
</table>

Figure 4.2 shows the normalised error for $u$, $k$ and $\varepsilon$ between the inflow profiles and the solution at $x = 4900m$. For all the plots in this section and the following section, the normalised error was calculated using the inflow profiles for each variable.

Note that for Cases 3, 4, 5 and 6, as expected the worst streamwise gradients occur in the cell adjacent to the ground leading to the maximum normalised error occurring at that point. The limits of the axes in Figure 4.2 have been set inside these points so that the trends throughout the domain can be displayed more meaningfully. However, error information is included in Table 4.3 (see Section 4.1.4) which summarises both the maximum and the average normalised error in each of the variables for each of the simulations.

The results of the simulation for Case 1 show that by using the inflow profiles developed using the one-dimensional model in combination with consistent boundary conditions at the top of the domain, a horizontally homogeneous boundary layer can be reproduced to an accuracy determined by the simulation convergence criteria. In Figure 4.2 the results for Case 1 cannot be distinguished from the vertical axis. This
Figure 4.2: Normalised error in (a) streamwise velocity, (b) turbulent kinetic energy and (c) dissipation for simulations in empty domain A (height 500 m).
result is significant because it means that Case 1 may be used as a baseline for comparison in the different simulation configurations in the following sections. Comparing Case 2 to Case 1 allows the error in Case 2 to be attributed entirely to the application of inconsistent, zero-gradient boundary conditions at the top of the domain. The normalised error increases from the top of the domain moving downwards and in the lowest region of the domain the inconsistent top boundary conditions cause an over estimation of the streamwise velocity by more than 5%. Similarly, for $k$, Case 2 shows that the inconsistent top boundary conditions cause errors throughout the domain. For most of the domain the inconsistency suppresses the turbulent kinetic energy, at the top of the domain by as much as 20%. In the tenth of the domain nearest the ground, the level of turbulent kinetic energy is overestimated by almost 10%. Similar discrepancies are caused in $\varepsilon$ for Case 2, with under estimations of about 15% in the upper part of the domain and over estimations of up to 15% near the ground.

The results for Case 3 show that while the RH solutions given in equations (4.1)-(4.3) are not exact solutions for the simulation conditions, if consistent boundary conditions are applied at the top of the domain then relatively good homogeneity can be achieved in the streamwise velocity with normalised errors of less than 1% for most of the domain. As expected, assuming a constant value for the turbulent kinetic energy profile causes streamwise gradients in the solution. Plot (b) in Figure 4.2 shows that the normalised error in the turbulent kinetic energy grows towards the ground and is overestimated by more than 5% for the lower third of the domain. The same effect can be seen in the normalised error for $\varepsilon$ and can be attributed to the flow adjusting from the RH solutions to the solution in equilibrium with the wall function at the ground.

Examining the results for Case 4 shows that the normalised error for each variable is the summation of the normalised errors in Cases 2 and 3. This demonstrates that the errors introduced by using boundary conditions inconsistent with the solution combine linearly. In Case 2, the error is a result of the inconsistency of the top boundary condition. In Case 3, it is the result of the inconsistency between the incoming solution and the ground boundary condition. And in Case 4, it is the summation of the two effects. Hence the resulting streamwise gradients are as expected.
Table 4.2: Simulation Specifications for Empty Domain Experiments

<table>
<thead>
<tr>
<th></th>
<th>Domain A</th>
<th>Domain B</th>
<th>Domain C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain dimensions (m)</td>
<td>$5000 \times 10 \times 500$</td>
<td>$5000 \times 10 \times 1500$</td>
<td>$5000 \times 10 \times 2500$</td>
</tr>
<tr>
<td>Grid dimensions</td>
<td>$500 \times 1 \times 100$</td>
<td>$500 \times 1 \times 150$</td>
<td>$500 \times 1 \times 200$</td>
</tr>
<tr>
<td>First cell’s vertical height (m)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Vertical stretching ratio</td>
<td>1.067</td>
<td>1.050</td>
<td>1.039</td>
</tr>
<tr>
<td>Reference height, $z_{ref}$ (m)</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Reference mean wind speed, $\overline{u}_{ref}$ (m/s)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Roughness length, $z_0$ (m)</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The normalised errors in streamwise velocity increase as $z$ decreases, over estimating the true value by 7% near the ground. Both the turbulent kinetic energy and the dissipation are underestimated by up to 20% in the top three quarters of the domain and overestimated by almost 20% in the lower quarter. Clearly, the use of inconsistent top boundary conditions adds to the errors near the ground that are caused by using the RH solutions as profiles at the inflow.

The results for Case 5 are similar to those for Case 3. However, as has been shown by Yang et al. (2009), the use of a more generalised approximate solution for the inflow can significantly reduce the streamwise gradients in both the turbulent kinetic energy and the dissipation. For almost the entire domain the error remains less than 1%, 3% and 2% for streamwise velocity, turbulent kinetic energy and dissipation, respectively.

As a result of the improvements made by using the Yang solutions as inflow profiles, the errors in Case 6 are also less than those seen in Case 4 and are similar to those seen in Case 2.
4.1.4 Simulations with Increased Domain Height

Having successfully simulated a horizontally homogeneous boundary layer and quantitatively assessed the impact of using inconsistent boundary conditions at both the top of the domain and at the ground, the same six simulations were carried out on two larger domains to determine if the streamwise gradients could be reduced by moving the top boundary away from the area of interest near the ground. This is a common approach and often the justification for using inconsistent boundary conditions at the top of the domain is that they are far enough from the area of interest near the ground to no longer have an impact. Increasing the computational domain size invalidates the assumptions that the Coriolis effects may be ignored and that the shear stress is constant throughout the domain. However, the emphasis of these simulations was to investigate the impact of moving the inconsistent boundary conditions away from the zone of interest rather to produce a physically accurate description of a large portion of the ABL and so the simplifying assumptions were maintained. The setup for the domains is given in Table 4.2. In each domain the vertical height of the first cell next to the ground was kept at 0.05m to ensure consistency between the results. Also, grid refinement tests were carried out to ensure the results were independent of the grid dimensions. For each domain a new one-dimensional solution was obtained on a consistent grid using the approach summarised in 4.1.2.

Figures 4.3 and 4.4 show plots for the normalised error in each of the variables for the two domains. In order to show the trends as the domain height is increased, Table 4.3 presents both the maximum and the average normalised error in each of the variables for each of the simulations.

It is clear from Figures 4.3 and 4.4 that the trends identified in the simulations in domain A also hold for each of the simulations in the larger domains. In each of the variables, inconsistent boundary conditions at the top of the domain cause streamwise gradients throughout the domain which can be observed as normalised errors between the inflow profiles and the solution profiles at $x = 4900m$. In Cases 4 and 6, these errors combine linearly with errors caused by inflow profiles inconsistent with the ground boundary conditions exacerbating the streamwise gradients.
Figure 4.3: Normalised error in (a) streamwise velocity, (b) turbulent kinetic energy and (c) dissipation for simulations in empty domain B (height 1500 m).
Figure 4.4: Normalised error in (a) streamwise velocity, (b) turbulent kinetic energy and (c) dissipation for simulations in empty domain C (height 2500 m).
For the streamwise velocity, regardless of the domain height, inconsistent boundary conditions at the top of the domain cause an over estimation throughout the domain, worsening near the ground. For turbulent kinetic energy and dissipation the plots show that as the domain height is increased, the large errors near the top boundary and the ground boundary remain while an area of relatively small errors develops in the middle region. This suggests that while errors occur throughout the domain, they are more localised than for the streamwise velocity.

As expected, Table 4.3 shows that by moving the top boundary condition away from the ground, the streamwise gradients and hence the normalised errors are reduced. However, while they are reduced, they are not removed and even throughout the largest domain, which is five times the height of the original domain, streamwise gradients are present.

For the streamwise velocity, several other results are clear. For all of the cases except Case 2, the maximum error is largely unaffected by the domain height. This is because the error occurs in the cell closest to the ground and is a result of the inconsistency between the inflow profile and the ground boundary condition. Thus domain height has no impact. For Case 2, while the maximum error occurs near the ground, it is a result of the inconsistent top boundary condition and hence it decreases as the domain height is increased, following the trend for the average error. For Cases 4 and 6, which have consistent top boundary conditions, it can be seen that while the average error is small, it also decreases as domain height increases. For these cases the error is due to the inconsistency between the inflow profiles and the ground boundary condition. This trend is important because it indicates that while the error induced in the streamwise velocity by the inconsistency at the ground is much smaller than that caused by inconsistency at the top boundary condition, it persists throughout the domain and hence diminishes as the domain size is increased.

The results for the turbulent kinetic energy show the same trends. However, unlike the streamwise velocity, the maximum error is affected by the domain height for Cases 2, 4 and 6. This is because the maximum error is contributed to significantly by both the inconsistency at the top boundary and the ground boundary. The contribution from the inconsistency at the top boundary can be clearly seen as a spike in the
Table 4.3: Normalised Error Information for Domain Height Simulations

<table>
<thead>
<tr>
<th>Normalised Error in streamwise velocity, $\overline{u}$</th>
<th>Domain A</th>
<th>Domain B</th>
<th>Domain C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-gradient</td>
<td>Max</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>1-D Model</td>
<td>5.6%</td>
<td>3.3%</td>
<td>2.9%</td>
</tr>
<tr>
<td>RH</td>
<td>21%</td>
<td>3.2%</td>
<td>23%</td>
</tr>
<tr>
<td>Yang</td>
<td>21%</td>
<td>3.3%</td>
<td>23%</td>
</tr>
<tr>
<td>Consistent</td>
<td>Max</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>1-D Model</td>
<td>~ 0%</td>
<td>~ 0%</td>
<td>~ 0%</td>
</tr>
<tr>
<td>RH</td>
<td>25%</td>
<td>0.10%</td>
<td>25%</td>
</tr>
<tr>
<td>Yang</td>
<td>24%</td>
<td>0.089%</td>
<td>24%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normalised Error in turbulent kinetic energy, $k$</th>
<th>Domain A</th>
<th>Domain B</th>
<th>Domain C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-gradient</td>
<td>Max</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>1-D Model</td>
<td>20%</td>
<td>12%</td>
<td>5.6%</td>
</tr>
<tr>
<td>RH</td>
<td>28%</td>
<td>11%</td>
<td>22%</td>
</tr>
<tr>
<td>Yang</td>
<td>29%</td>
<td>11%</td>
<td>13%</td>
</tr>
<tr>
<td>Consistent</td>
<td>Max</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>1-D Model</td>
<td>~ 0%</td>
<td>~ 0%</td>
<td>~ 0%</td>
</tr>
<tr>
<td>RH</td>
<td>17%</td>
<td>3.5%</td>
<td>17%</td>
</tr>
<tr>
<td>Yang</td>
<td>7.5%</td>
<td>1.5%</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normalised Error in dissipation, $\varepsilon$</th>
<th>Domain A</th>
<th>Domain B</th>
<th>Domain C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-gradient</td>
<td>Max</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>1-D Model</td>
<td>16%</td>
<td>11%</td>
<td>7.8%</td>
</tr>
<tr>
<td>RH</td>
<td>83%</td>
<td>11%</td>
<td>72%</td>
</tr>
<tr>
<td>Yang</td>
<td>69%</td>
<td>11%</td>
<td>59%</td>
</tr>
<tr>
<td>Consistent</td>
<td>Max</td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>1-D Model</td>
<td>~ 0%</td>
<td>~ 0%</td>
<td>~ 0%</td>
</tr>
<tr>
<td>RH</td>
<td>60%</td>
<td>2.4%</td>
<td>60%</td>
</tr>
<tr>
<td>Yang</td>
<td>48%</td>
<td>0.62%</td>
<td>47%</td>
</tr>
</tbody>
</table>
normalised error at the ground in Case 2. It is this contribution that reduces as the
domain height is increased.

The reduction in streamwise gradients in the turbulent kinetic energy by using the
Yang solution over the RH solution is also apparent. With consistent top boundary
conditions, the Yang solutions result in a maximum normalised error of around 7%
and an average of 1.5% or less. This compares with the RH solutions resulting in a
maximum error of 17% and an average of 3.5% or less.

The trends in the dissipation mirror those of the turbulent kinetic energy with the
exception that the maximum errors are much larger in Cases 2 to 6. In all cases the
maximum error occurs in the cell closest to the ground and reflects the discrepancy
between the RH and Yang solutions and the value of the dissipation prescribed by
the standard wall function (equation (2.54)).

4.1.5 Impact for Flow Over a Theoretical Ridge

In order to investigate the impact of using inconsistent boundary conditions in the
presence of an obstacle; a simulation domain was created containing a representa-
tive ridge. The profile used for the ridge is the so-called “Witch of Agnesi” profile
which is commonly used to represent hills (Kaimal & Finnigan, 1994). It is described
mathematically by the distribution given in equation (4.11).

\[ z = h \left[ 1 + \left( \frac{x}{L_a} \right)^2 \right]^{-1} \] (4.11)

The values of \( h = 25\text{m} \) and \( L_a = 215.9\text{m} \) were selected such that equation (4.12) is
satisfied.

\[ \frac{z_0}{l} = 10^{-3} \] (4.12)

where the inner layer depth, \( l \), is defined by Jackson & Hunt (1975) using equation
(4.13).

\[ \frac{l}{L_a} \ln \left( \frac{l}{z_0} \right) = 2\kappa^2 \] (4.13)
In simulations of this type a blockage ratio can be defined as the ratio of the height of the obstacle to the domain space above it. The values of $h$ and $L$ were selected to ensure the ridge conforms to the recommended blockage ratio given by Franke et al. (2007) and also to allow a qualitative comparison with the linearised theory results. The grid was designed to follow the topography of the ground given by equation (4.11). The first cell height and the cell expansion ratio were the same as those given in Table 4.2. The top 20 cells were allowed to compress so that the top of the domain was fixed at 500m. This ensured that no streamwise gradients occurred due to changes in the grid structure close to the ground. The wind profile used was the same as that given in Table 4.2. Once again six simulation cases were run as defined by Table 4.1 and the numerical method used was that described in 4.1.2.

Figure 4.5 shows a plot of the speed-up profile obtained at the top of the theoretical ridge for Case 1 using consistent top boundary conditions and inflow profiles developed using a one-dimensional model. This result agrees well with the linearised theory (Jackson & Hunt, 1975; Simiu & Scanlan, 1986; Hunt et al., 1988; Kaimal & Finnigan, 1994) in predicting the value and position of the maximum speed-up at the top of the ridge.

![Figure 4.5: Speed-up at the top of a theoretical ridge](image-url)
Because the results for Case 1 agree well with the theory and the model does not introduce errors due to inconsistent boundary conditions, these results were then used to calculate the normalised error in the variables at the top of the theoretical ridge for the other cases. The results of the calculations are shown in Figure 4.6.
The first significant result is that, despite the flow being complicated by the presence of the theoretical ridge, the normalised errors generated by inconsistent boundary conditions are of the same order as those evident in the simulations of the horizontally homogeneous boundary layer. Also, comparing Figure 4.2 with 4.6 it can be seen that the trends observed in the simulations of the horizontally homogeneous boundary layer occur in the presence of a theoretical ridge. Cases 2, 4 and 6 overestimate the streamwise velocity throughout the domain; overestimate turbulent kinetic energy and dissipation near the ground and underestimate them higher in the domain. Cases 3 and 5 have small normalised errors generally confined to the region near the ground.

It is interesting to note that the normalised error profiles for each of the cases are similar to those seen in Figure 4.3 for the larger domain simulations. Extending the domain in the vertical direction has reduced the proportion of the domain represented by the boundary layer and this is similar to what happens as the flow speeds up to pass over the ridge. The boundary layer at the ground thins, and thus represents a smaller proportion of the vertical space in the domain.

4.1.6 Impact for Flow Over a Real Ridge

To investigate the impact of consistent boundary conditions on the flow over a real ridge a simulation was set up using digitised terrain data and measured data for \( z_0 \) and \( u_* \). The measurements were made using a cup anemometer located at the inflow boundary and the values of \( z_0 \) and \( u_* \) were used to define the wind profile used as the inlet boundary condition. The ridge contained several smaller ridges on the downstream slope and a number of areas where recirculating flow could occur. The simulation was set up so that the domain extended five times the maximum ridge height in the vertical direction. This conforms to the recommendations of Franke et al. (2007) on the maximum blockage ratio. The details of the simulation are given in Table 4.4 and details of the experiment and wind conditions can be found in Behrens (2010).
Table 4.4: Real Ridge Simulation Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain dimensions (m)</td>
<td>$9280 \times 10 \times 2500$</td>
</tr>
<tr>
<td>Grid dimensions</td>
<td>$928 \times 1 \times 90$</td>
</tr>
<tr>
<td>Height of first vertical cell (m)</td>
<td>1</td>
</tr>
<tr>
<td>Vertical stretching ratio</td>
<td>1.076</td>
</tr>
<tr>
<td>Friction velocity, $u_*$ (m/s)</td>
<td>0.7</td>
</tr>
<tr>
<td>Roughness length, $z_0$ (m)</td>
<td>0.31</td>
</tr>
</tbody>
</table>

As with the previous domains, a one-dimensional model was used to develop inflow profiles consistent with the boundary condition at the ground following the procedure described in 4.1.2. Simulations were then run for the six cases described in Table 4.1.

Figure 4.7 shows the streamlines for the solution of the simulation for Case 1. It gives a perspective of the complexity of the terrain and shows the presence of three regions of recirculation. As with the theoretical ridge, the solution for Case 1 was used as the baseline case and it was assumed that flow changes due to the presence of the real ridge in each of the simulations would be equal to that in Case 1. This allows the examination of the impact of the inconsistencies at the top boundary and between the ground boundary condition and the inflow profiles. The point selected for the comparison is at $x = 6700\text{m}$. This is the peak of the ridge and is the most likely site for a potential wind turbine.

For Case 2, the additional complexity of the flow has little effect on the normalised errors shown in Figure 4.8. For each of the variables, the profiles and magnitudes of the normalised error are in line with those shown in Figure 4.6 and Figure 4.2 for the theoretical hill and the horizontally homogeneous boundary layer respectively. This confirms that also in the presence of more complex, realistic flows, inconsistent boundary conditions at the top of the domain will cause erroneous streamwise gradients in the solution.
For Cases 3, 4, 5, and 6, the normalised error profiles of turbulent kinetic energy and dissipation for Cases 3-6 differ from those of the theoretical hill and the horizontally homogeneous boundary layer. Instead of under estimating the turbulent kinetic energy in the upper part of the domain, it is overestimated throughout the domain as a result of the inconsistent boundary conditions. For Cases 5 and 6, the dissipation is underestimated slightly while as in Cases 3 and 4 it is overestimated throughout the domain.

Unlike the previous simulations, for turbulent kinetic energy and dissipation the trends seen between the cases are no longer dominated by the top boundary condition. Figure 4.8 show that Case 3 has similar results to Case 4 and that Case 5 has similar results to Case 6. This indicates that as the complexity of the solution increases, the errors contributed by inconsistent top boundary conditions remain. However, the errors contributed by inconsistencies between the inflow profiles and the ground boundary condition grow and contribute a larger proportion of the overall error. In this regard, Cases 5 and 6 again demonstrate that using the more general Yang solutions introduce less streamwise gradients in both turbulent kinetic energy and dissipation throughout the domain than the standard RH solutions.
Figure 4.8: Normalised error in (a) streamwise velocity, (b) turbulent kinetic energy and (c) dissipation for simulations over a real ridge.
4.1.7 **Discussion**

It is common practice in CFD for wind flow to use zero-gradient boundary conditions at the top of the domain and to use analytical profiles at the inflow boundary. Both of these approximations are inconsistent with the CFD models solved and result in erroneous streamwise gradients in the solution. The objective of this section was to determine consistent boundary conditions for use in simulations of the ABL. To achieve this, two approaches were combined. The first was to use consistent Neumann boundary conditions at the top of the domain that allow a shear stress to be applied to the flow and allow fluid to enter and exit the domain. The second was to use these top boundary conditions to develop one-dimensional solutions for the flow variables that are consistent with the wall function used as the ground boundary condition. Simulations of a large domain using these two approaches showed that a horizontally homogeneous boundary layer could be produced. Quantitative comparisons could then be carried out to between these simulations and simulations using the standard inconsistent boundary conditions.

The results presented show that regardless of the domain size or the presence of an obstacle, using inconsistent zero-gradient boundary conditions at the top of the domain introduces errors in streamwise velocity, turbulent kinetic energy and dissipation throughout the domain. In all of the simulations the errors were most severe in the lowest part of the domain which is usually the zone of engineering interest.

An example of the impact of this systematic error, is shown by the results for the simulations of a real ridge. In the area of interest, the lowest 150m of the domain, the use of inconsistent zero-gradient top boundary conditions caused an artificial over estimation of the streamwise velocity of more than 4%. The well known cubic relationship between the power available in the wind, $P_w$, and the velocity, $v_w$, is given in equation (4.14) where $\rho$ is the density of air and $A_w$ the area of wind flow. An over estimation of 4% in velocity corresponds to an over estimation in the available power by 12.5% and may lead to mistaken assumptions in the planning of a wind farm.
Another important parameter commonly used in wind engineering is the turbulent intensity, $I$, which can be calculated from equation (4.15).

$$I = \frac{2^{2/3}k}{v_w}$$  \hspace{1cm} (4.15)

The effect of using inconsistent zero-gradient top boundary conditions was to underestimate the turbulent intensity by 3% in the lowest 150m of the domain. This was caused by the over estimation of the velocity which was not offset by the over estimation of the turbulent kinetic energy by 2% in the same region. Again, this may be an important consideration when planning engineering structures in areas of high turbulence.

The results also show that using analytical inflow profiles that are not consistent with the ground boundary conditions of the simulation introduces erroneous streamwise gradients. The errors reduce quickly moving away from the ground boundary but do persist to a small degree throughout the domain. It was also shown that by increasing the domain height the impact of the errors can be reduced.

Another finding of the simulations supports the work of Yang et al. (2009). By using the more general profiles they suggested, the streamwise errors can be significantly reduced if the correct values of $C_1$ and $C_2$ are selected.

The work of Hargreaves & Wright (2007) suggests that by adjusting the wall function to agree more closely with the analytic solutions of Richards and Hoxey, the erroneous streamwise gradients may also be reduced. While the results they present following their discussion support this conclusion, it is important to note that they also changed the top boundary condition in their simulations to that of a constant shear stress. By changing both the boundary condition at the top and at the bottom of the domain it is difficult to quantify which contributed most to the improved results. The results of the present work show that using a shear stress boundary condition that is consistent with the inflow profiles could be responsible for the dramatic improvement in their simulation of the horizontally homogeneous boundary condition.
However, their observations about the need to improve the wall functions used for atmospheric boundary-layer flow are well founded. One of the advantages of the approach presented here is that new wall functions can be developed without the need for developing new, consistent analytic inflow profiles. The one-dimensional solution approach using periodic boundary conditions described in 4.1.2 can be applied for any wall function, or for any turbulence closure, and guarantees erroneous streamwise gradients will not be introduced. It is only possible to assess new wall functions and the different turbulence closures as presented in the following sections because the systematic errors caused inconsistent boundary conditions are removed.

4.2 Realistic Ekman Profiles

One of the implications of considering flows on the scale of real ridges and hills is that the height of the domain must increase to ensure no artificial blockage is caused above the peak. In the simulations carried out in Sections 4.1.5 and 4.1.6 a domain height of 2500 m is used to accommodate this. However, as Section 2.1.2 states the ABL normally extends to no more than 1-2 km above the surface. This means that for simulations of this size the top boundary may be well inside the freestream region and the shear stress boundary conditions are no longer valid. Determining the correct boundary conditions at this height is difficult. At the inflow the top of the domain is likely to be in the freestream region where estimates of the freestream velocity can be made and other quantities should have negligible gradients. However, over the peak itself it is possible that the ABL stretches further above the ground, even to the top of the domain.

To solve this problem all of the simulations of large, atmospheric flow carried out in this work use very tall domains. This ensures that the top boundary conditions will always be physically consistent. The actual height of the domain was determined by calculating the point at which the gradient in the turbulent kinetic energy, dissipation and other variables drops to a negligible level.

In the previous section it was clearly established that in order to accurately simulate atmospheric flows the boundary conditions must be consistent. In particular the
inflow profile must be consistent with the top and ground boundary conditions. Once
simulations extend beyond the ABL, the inflow profiles that represent the properties
of the flow above the ABL must be used. Empirical formulae exist for wind flow pro-
files that include the full Ekman form but using these profiles means that the benefits
gained by using a one-dimensional model to generate the inflow profiles are lost.

Indeed, the empirical formulas were found to be unnecessary because with two
small adjustments the approach described in Section 4.1.2 works equally well for
generating Ekman profiles as for log law profiles. The adjustments required were
simply to generate the profiles in the same manner as reality - by driving the wind
with a pressure gradient and including the effects of Coriolis. These changes led to
the development of the atmosFoam application described in Section 3.1.2. Examples
of the profiles generated in this manner are shown in Figure 4.9. As can be seen from
the figure, it is the turbulent kinetic energy which requires such a large domain height
before it attains a negligible gradient.

### 4.2.1 Comparison of Simulations with Ekman and Log Law Inflow
Profiles

Once the method for generating the Ekman profiles with consistent boundary con-
ditions had been developed the next step was to compare the results obtained using
these as boundary conditions against those using log law boundary conditions. To
achieve this another simulation of the real ridge line in Section 4.1.6 was carried out
using a large vertical domain and the Ekman inflow profile. Figure 4.10 shows a com-
parison of the two inflow profiles used. In plot (a) the two profiles are nearly identical.
This is to be expected as plot (a) shows only the first 1000 m of the atmosphere which
corresponds to roughly the height of the ABL under the wind conditions prescribed.
Inside the ABL the log law profile is clearly an accurate representation of the true
Ekman spiral. However, as plot (b) shows, above 1000 m the two profiles diverge.
The Ekman spiral shows the characteristic flattening and a negative gradient above
3000 m which the log law is unable to capture.

The impact of using the incorrect log law in domains that extend above the ABL
4.2. REALISTIC EKMAN PROFILES

Figure 4.9: Realistic Ekman inflow profiles for the (a) streamwise velocity, (b) turbulent kinetic energy and (c) dissipation calculated using the $k$-$\varepsilon$ closure.
is clearly seen in Figure 4.11. Plot (a) shows that the simulation using the log law inflow profile for the entire 2500 m domain causes an under estimation of the velocity at the peak of the ridge by 1 m/s or 6%. By zooming in on the top of the profiles in plot (b) the cause of the error becomes clear. Using the log law inflow profiles and the consistent shear stress boundary condition at the top of the boundary has the effect of retarding the flow at the peak of the ridge. The solution obtained using the Ekman profiles shows that at 2000 m above the peak the streamwise velocity has a negative gradient. By enforcing a positive gradient through the shear stress boundary condition the flow is slowed throughout the domain in order to meet this condition.

![Figure 4.10: Comparison between Ekman (—) and log law (-.-) velocity profiles at the inflow of a simulated two-dimensional ridge. (a) The lowest 1000 m and (b) the lowest 5000 m.](image)

The problem is not localised to the region above the peak of the ridge. Figure 4.12 show contour plots of the streamwise velocity contours of solutions obtained using the log law inflow profiles and the Ekman inflow profiles. Only the first 2400 m of the domain for the Ekman solution is shown for comparison. Above that the flow becomes approximately uniform. Throughout the domain the solution obtained using the realistic Ekman profiles shows a more complex flow pattern. The complex terrain causes steeper gradients and higher velocities. The explanation above for the profile above the peak applies throughout the domain. In two dimensions the effect
4.2. REALISTIC EKMAN PROFILES

of the constant shear stress boundary condition at the top of the domain is to restrict
the flow to be slower and more uniform.

Figure 4.11: Comparison between Ekman (—) and log law (-.-) velocity profiles at
the inflow of a simulated two-dimensional ridge. (a) The lowest 1000 m and (b) the
lowest 5000 m.

Figure 4.12: Streamwise velocity contours for a real two-dimensional ridge calculated
using (a) log law inflow profiles and (b) Ekman inflow profiles.
4.3 Wall Functions and Grid Dependence

Section 2.4 describes the standard wall functions that are commonly used as ground boundary conditions when modelling atmospheric flows. It also describes the new wall functions that have been developed as part of this work for use with the $v^2 f$ and the ASBM turbulence closures. The aim of the work described in this section was to assess the grid dependence of the new wall functions and compare it with that of the standard wall functions. To achieve this several simulations were made of a turbulent boundary layer using each of the turbulence closures and their associated near-wall treatment at various grid resolutions. The boundary layer was calculated over a domain 6 m long and 0.24 m high with $Re = 47720$ and a uniform inflow profile. This boundary layer was selected because it was also required for simulations carried out in Chapter 5. The $Re$ for this flow is many orders of magnitude less than those of the other simulations in this chapter because the accurate experimental data of the near-wall region required for validation is only available for laboratory-scale boundary layers. Figure 4.13 shows a contour of the streamwise velocity and a number of profiles along the domain for the solution using a fully resolved grid and the ASBM turbulence closure. The growth of the turbulent boundary layer is clearly visible.

![Streamwise velocity contours](image)

Figure 4.13: Streamwise velocity contours for a turbulent boundary layer calculated using the ASBM closure. Velocity profiles at $x = 1.16, 2.36, 3.56$ and $4.76$ m also shown.

The test-case simulations carried out are given below in Table 4.5. For each turbulence closure one wall-resolved simulation was carried out without wall functions.
on a grid with a first cell height at $z^+ = 1$. This was used as the baseline against which the accuracy of the solutions obtained with the wall functions were measured. Note that for the $k-\varepsilon$ closure the wall-resolved simulation was carried out using the $k-\omega$-SST closure rather than a low-$Re$ wall model. Also note that in all cases only the smooth wall functions were tested. This is because the wall-resolved baseline simulations can only be carried out for a smooth wall. It is assumed that the grid dependence will be similar for the rough wall functions.

Table 4.5: Wall Function Grid Dependence Test Cases

<table>
<thead>
<tr>
<th>Turbulence Closure</th>
<th>First grid cell height - $z_1^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k-\omega$-SST</td>
<td>1</td>
</tr>
<tr>
<td>$k-\varepsilon$ with wall function</td>
<td>21, 35, 70</td>
</tr>
<tr>
<td>$v^2f$</td>
<td>1</td>
</tr>
<tr>
<td>$v^2f$ with wall function</td>
<td>21, 35, 70</td>
</tr>
<tr>
<td>ASBM</td>
<td>1</td>
</tr>
<tr>
<td>ASBM with wall function</td>
<td>21, 35, 70</td>
</tr>
</tbody>
</table>

Figure 4.14 shows a comparison of the mean velocity profile for the three different wall-function grids and the wall-resolved grid. Comparing plot (a) and plot (b) it can be seen that while increasing the height of the first cell does effect the $v^2f$ solution in the first few cells, the solutions converge in the rest of the domain in a similar way to the $k-\varepsilon$ solution. For all three values of the first grid cell height the solutions agree well with the wall-resolved solution. Figure 4.15 shows that the solution for $v^2$ using wall functions is also almost independent of first grid cell height and agrees well with the wall-resolved grid.

Plot (c) in Figure 4.14 shows a comparison of the solutions using three different wall-function grids and the wall-resolved solution using the ASBM closure. As for the $v^2f$ solution, a few cells away from the wall the solutions using wall functions are
Figure 4.14: Comparison of velocity profiles for a turbulent boundary layer calculated using a wall-resolved grid and three different wall-function grids. Plots show (a) the $k$-$\omega$-SST and $k$-$\varepsilon$ closures, (b) the $v^2f$ closure and (c) the ASBM closure. For all cases $z_1^+ = 21$ (○), $z_1^+ = 35$ (∆) and $z_1^+ = 70$ (×).

almost independent of the first grid cell height and all agree well with the wall-resolved solution. Figure 4.16 shows that the solutions for the components of Reynolds stress using wall functions are also relatively independent of first grid cell height and agree well with the wall-resolved solutions.

From these grid dependence studies it can be concluded that the new wall functions for the $v^2f$ and ASBM closures that have been developed as part of this work, function in much the same way as the standard wall functions. With large first grid cell heights they are slightly less accurate in the first few cells but the error reduces rapidly and the mean flow is correctly predicted.
Figure 4.15: Comparison of $v'^2$ profiles for a turbulent boundary layer calculated using a wall-resolved grid and three different wall-function grids. $z_1^+ = 21$ (○), $z_4^+ = 35$ (△) and $z_7^+ = 70$ (×).

Figure 4.16: Comparison of Reynolds stress component profiles for a turbulent boundary layer calculated using a wall-resolved grid and three different wall-function grids. $u'u' (- -), u'w' (.-) and w'w' (---)$. In all cases $z_1^+ = 21$ (○), $z_4^+ = 35$ (△) and $z_7^+ = 70$ (×).
4.4 Summary

In this chapter the results from a number of preliminary simulations were presented. A method for generating inflow profiles was developed that ensures that the profiles are consistent with both the ground boundary condition and the top boundary condition. Using the inflow profiles generated in this manner it was possible to accurately simulate a horizontally homogeneous boundary layer. By using the successful simulation as a baseline the impact of inconsistent boundary conditions could be investigated. Two sets of analytical profiles were tested at the inflow boundary and the commonly used zero-gradient boundary condition was tested at the top of the domain. The results showed that using the zero-gradient boundary conditions introduced significant errors in the form of streamwise gradients. Using analytical profiles at the inflow boundary was found to introduce relatively small errors if consistent top boundary conditions were used.

The size of the domain was then extended to investigate the effect of moving the top boundary away from the zone of interest near the ground. It was found that for inconsistent boundary conditions this reduced the average errors but that the errors near the ground remained. An idealised ridge was introduced to the domain and the trends observed for the empty boundary layer were again observed. Similarly for simulations of flow over a real ridge systematic errors were caused by inconsistent boundary conditions, particularly those at the top of the domain.

During the simulations of the idealised and real ridge it was observed that the log law that was used at the inflow and top boundary conditions was no longer appropriate given the size of the domain. Instead a new method for developing realistic Ekman wind profiles was developed. A simulation of the real ridge was run using the Ekman profiles at the inflow boundary condition and the results compared with the log law simulation results. The comparison showed that using a log law inflow profile throughout a large domain causes an under estimation of the velocity speed-up over obstacles.

Finally a number of grid dependence simulations were carried out for the new wall functions that have been developed as part of this work. Both the new $v^2f$ wall
function and the new ASBM wall function were found to have similar grid dependence properties to the standard wall functions.

Identifying consistent boundary conditions, developing realistic Ekman inflow profiles and validating the new wall functions were each important preliminary parts of this research project. These techniques were applied throughout the two-dimensional and three-dimensional simulations that are discussed in the following chapters and contributed to the success of the approach described.
Chapter 5

Two-dimensional Simulations

The importance of developing more accurate turbulence closures for modelling wind flow was discussed in Chapter 1. The inability of the $k$-$\varepsilon$ closure to accurately represent wind flow in complex terrain where separation and anisotropic turbulence exist was also noted. In the preliminary simulations in Chapter 4 the presence of separation was observed in several places in the results of the simulations over a real ridge. This simulation would then seem to be the logical choice for testing the $v^2f$ and ASBM closures presented in Chapter 2. However, like most full-scale test cases of wind flow, there is very little accurate data available that can be used to verify turbulence closures satisfactorily. The alternative is to find a suitable data set from a representative experiment. Radhakrishnan et al. (2008) compared simulations with experiments measuring flows over both a backward facing step and an asymmetric planar diffuser with good results. However, in both of these flows the separation point is easily predicted which is not the case for separation occurring in wind flow over complex terrain. The objective of the work presented in this section was to compare simulation results with data from experiments that are more representative of wind flow over complex terrain. The experiments selected were carried out by Loureiro et al. (2007, 2009) and provide an excellent data set for comparison.
CHAPTER 5. TWO-DIMENSIONAL SIMULATIONS

Figure 5.1: Photograph of the experimental setup for measuring flow over a two-dimensional hill Loureiro et al. (2009).

5.1 Experimental Setup

All of the experiments were carried out using one of the water channels of the Hydraulics Laboratory of the Civil Engineering Department at the University of Oporto in Portugal. The channel is open and is 17 m long and has a cross-section of 0.4 m wide by 0.6 m high. The working section is 3 m long and is located 8 m from the channel entrance. Measurements of the mean and fluctuating velocities were made using a Dantec laser-Doppler anemometry system. More detailed information about both the channel and the measuring equipment including the pumping equipment is available in Loureiro et al. (2007). The photograph in Figure 5.1 shows the working section of the channel along with the laser-Doppler equipment and one of the model hills.

The shape of the hill was the same in all of the experiments and is defined using a modified “Witch of Agnesi” profile given in equation (5.1).

\[
z(x) = \begin{cases} 
H_1 \left[1 + \left(\frac{x}{L_a}\right)^2\right]^{-1} - H_2 & -2L_a < x < 2L_a \\
0 & \text{otherwise}
\end{cases}
\]

The values of the constants were \(H_1 = 75\text{mm}\), \(H_2 = 15\text{mm}\), giving a hill height, \(H = H_1 - H_2 = 60\text{mm}\). The characteristic length, \(L_a\), represents the distance from the crest to the half-height point and was 150mm. With these parameters the hill has a maximum slope of 18.6° or 0.325 which is above the critical slope of 0.3 given by
Mortensen et al. (2006) as the point above which terrain can be considered “rugged”.

The experiments considered for this work contributed to data sets for two flow configurations. The first is a smooth hill at a relatively low $Re$ (Loureiro et al., 2007) and the second is a rough hill at a higher $Re$ (Loureiro et al., 2009). In the rough case the roughness was created by applying rubber strips 3 mm high and 3 mm wide to the surface of the hill at 9 mm spacing. The details of the flow conditions for the two cases is given in Table 5.1

### 5.2 Simulation Setup

In order to test the $v^2f$ and ASBM turbulence closures and their associated new wall functions several simulations were carried out. Table 5.2 summarises the simulation cases presented. For the cases where experimental data was available for comparison, simulations using either the $k-\varepsilon$ closure or the $k-\omega$-SST closure were also carried out. As discussed in Chapter 1 these closures are still the industry standard for CFD simulations of environmental flows and it is important to also compare their performance with the experimental data. Rather than attempt to use the $k-\omega$-SST closure for both wall-resolved and wall-function grids it was felt that using the $k-\varepsilon$ closure with wall functions is more representative of the usual approach in industry. For the $Re = 47720$ case note that there was no experimental data available and the intention of these simulations was to make a direct comparison between simulations.
Table 5.2: Simulation Cases for Two-dimensional Hill Studies

<table>
<thead>
<tr>
<th>Turbulence Closure</th>
<th>$Re = 4772$</th>
<th>$Re = 47720$</th>
<th>$Re = 31023$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Smooth</td>
<td>Smooth</td>
<td>Rough</td>
</tr>
<tr>
<td></td>
<td>Wall-resolved</td>
<td>Wall-resolved</td>
<td>Wall Function</td>
</tr>
<tr>
<td>$k$-$\varepsilon$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$k$-$\omega$-SST</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>$v^2f$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>ASBM</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Experimental Data</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

of a complex, separated flow run on both wall-resolved grids and wall-function grids.

### 5.2.1 Computational Domains

The four different computational domains used are summarised in Table 5.3. The domains for $Re = 4772$ and $Re = 31023$ were designed to be equivalent to the experimental set up used in Loureiro et al. (2007) and Loureiro et al. (2009) respectively. The two domains for $Re = 47720$ were designed to be similar to the first two but included a steeper hill to encourage a significant amount of separation. In all cases the height of the domain was $4H$ where $H$ is the height of the hill. Also, all of the domains included a length of $10H$ across the hill itself, $10H$ of outlet section following it and a long inlet section. The length of the inlet section varied for each domain and was selected so that the incident boundary layer measured in the experiments was achieved at the same point in the computational domain and is also given in Table 5.3. The shape of the hill profile given in equation (5.1) determined the bottom of each of the computational domains.

All of the grids were stretched in both the streamwise and vertical direction. In the vertical direction this was to achieve the necessary resolution for the cell nearest
<table>
<thead>
<tr>
<th></th>
<th>( Re = 4772 ) Smooth Wall-resolved</th>
<th>( Re = 47720 ) Smooth Wall-resolved</th>
<th>( Re = 31023 ) Rough Wall Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells</td>
<td>( 320 \times 1 \times 50 )</td>
<td>( 424 \times 1 \times 60 )</td>
<td>( 424 \times 1 \times 40 )</td>
</tr>
<tr>
<td>First cell height</td>
<td>( z^+ \approx 1 )</td>
<td>( z^+ \approx 1 )</td>
<td>( z^+ \approx 30 )</td>
</tr>
<tr>
<td>Minimum streamwise cell</td>
<td>( 0.083H )</td>
<td>( 0.017H )</td>
<td>( 0.017H )</td>
</tr>
<tr>
<td>Hill width parameter ( L_H ) (mm)</td>
<td>150</td>
<td>125</td>
<td>125</td>
</tr>
<tr>
<td>Inflow length</td>
<td>( 40H )</td>
<td>( 60H )</td>
<td>( 60H )</td>
</tr>
<tr>
<td>Free stream velocity ( \bar{u}_\delta ) (m/s)</td>
<td>0.0482</td>
<td>0.482</td>
<td>0.482</td>
</tr>
<tr>
<td>Friction velocity ( u_* ) (m/s)</td>
<td>0.0028</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Roughness length ( z_0 ) (mm)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 5.3: Computational Domains and Boundary Conditions for Two-dimensional Hill Studies
the ground. The expansion ratio was held constant for the first 30 vertical cells throughout the length of the domain and then allowed to change in the larger cells in above to account for the presence of the hill. In the streamwise direction the cells were compressed between the inflow region and the hill and then expanded between a distance of $5H$ downstream of the hill to the outlet. This allowed a high resolution grid over the hill and its wake region without the additional computational cost of unnecessary resolution in the inflow and outflow regions. Note that the resolution of grids was determined after several simulations were carried out to ensure that they were sufficiently refined.

5.2.2 Boundary Conditions

Uniform profiles for $\overline{u}$, $k$, $\varepsilon$, and $v^2$ were applied at the inlet boundary for all of the simulations and the turbulent boundary layer allowed to develop. The values chosen for the profiles were such that the observed boundary layer was predicted at the first measuring station location. Zero-gradient was prescribed for the remaining variables $f$, $\phi$, and $\hat{p}$. An outlet boundary condition was used at the outlet, prescribing a reference value for $\hat{p}$ and zero-gradient for all other variables. At the top of the domain and on the spanwise boundaries symmetry was imposed. For the ground boundary conditions, no-slip was used for the fully resolved simulations, and the wall functions described in Section 2.4 were used for the coarse grids.

5.3 Low $Re$ Smooth Wall Simulations

The objective of the low $Re$ simulations was to compare the results using three different turbulence closures with the experimental data from Loureiro et al. (2007). Preliminary results for the $v^2f$ and ASBM closures have been presented previously (O’Sullivan et al., 2010) and the final results presented here are in the same format. The results herein also include those for the $k$-$\omega$-SST closure. Figure 5.2 shows the streamlines in the region immediately downstream of the hill and exact values for the separation and reattachment points are presented in Table 5.4.
5.3. LOW RE SMOOTH WALL SIMULATIONS

Figure 5.2: Velocity streamlines above a smooth two-dimensional hill $Re = 4772$ calculated using (a) the $k$-$\omega$-SST closure, (b) the $v^2f$ closure and (c) the ASBM closure compared with (d) experimental results from Loureiro et al. (2007)

All three turbulence closures are able to predict the presence of a separated zone however the ASBM closure most closely matches the experimental results. The $k$-$\omega$-SST closure does not capture the separation point or the reattachment point accurately and the the extent of the recirculating region is underestimated. Using the $v^2f$ closure improves the predictions and it estimates the separation point well but still slightly underestimates the point of reattachment and the extent of the recirculating region. The ASBM closure estimates both the separation and reattachment points well and the extent of the recirculation agrees well with the experimental data. For all three closures the center of the recirculation is predicted to be slightly further upstream than the experimental data shows. Also, the shape of the separated region immediately after separation is not predicted correctly by any of the closures. This may be because the simulations were carried out with perfectly smooth walls where as in the experiment a very small but finite roughness of 0.08mm was present. The results for the rough wall simulations in Section 5.5 show more accurate predictions of the area around the separation point.
The plots in Figure 5.3 reinforce the increased accuracy the ASBM closure offers. In plot (a) it can be seen that all three closures capture the undisturbed boundary-layer profile well but at the mid-point of the downslope in plot (b) the $k$-$\omega$-SST and $v^2f$ are no longer able to accurately predict the flow.

A broader set of results for the ASBM closure are presented in Figure 5.4. Note that for each plot in the figure the profiles are scaled by different amounts to increase their readability. The mean streamwise velocity is scaled by 0.5, the $u'u'$ component by 15, the $u'w'$ component by 30 and the $w'w'$ component by 25. Thus whereas each of the components of the Reynolds stresses appear to be similar throughout the domain, when the scaling is removed they are significantly different. This demonstrates the anisotropic nature of the Reynolds stresses in wall-bounded flows and emphasises the need for turbulence closures capable of representing them. For the mean streamwise velocity the agreement with the experimental results is very good. As stated previously the separated region is predicted quite accurately. The qualitative agreement for the Reynolds stresses is good throughout the domain. For all three stresses the
form of the profiles is in good agreement but the magnitudes are underestimated in and around the separated region. The most energy carrying stress, $u'u'$, is the most accurately predicted, though even this is underestimated at the start of the separated region.

To more closely examine the results for the Reynolds stress components Figure 5.5 shows more detailed views of them at (a) the undisturbed boundary layer and also in the center of the separated region (b). The agreement for the incident boundary layer is very good for all of the Reynolds stress components. In the separated region the agreement is still qualitatively very good despite an underestimation of the magnitude. The anisotropy of the Reynolds stress tensor is well predicted. The ASBM closure correctly predicts the small saddle point in $u'u'$ near the ground and the maximums of the other components.

Figure 5.6 shows the Reynolds stress components halfway up the upstream side of the hill. This plot shows most of the same trends with a good qualitative agreement in all of the stress components and a slight underestimation of the magnitude of $u'u'$ away from the wall. However, in the shear stress $u'w'$ near to the ground the ASBM closure is predicting a region of positive shear where the experimental data shows there is none. In this region the flow is experiencing a positive pressure gradient due to the curvature of the boundary. The RDT examples that were used to develop the ASBM closure do not include this type of pressure gradient so it is not surprising that it misinterprets the acceleration due to the pressure gradient as positive shear stress.
Figure 5.4: (a) Streamwise velocity, (b) $u' u'$, (c) $u' w'$ and (d) $w' w'$ for a smooth two-dimensional hill $Re = 4772$ calculated using the ASBM closure compared with experimental results (Loureiro et al., 2007).
5.3. LOW RE SMOOTH WALL SIMULATIONS

Figure 5.5: Reynolds stress components for a smooth two-dimensional hill $Re = 4772$ calculated using the ASBM closure: $\overline{u'u'}$ ( ), $\overline{u'w'}$ ( ) and $\overline{w'w'}$ ( ) compared with experimental results: $\overline{u'u'}$ ( ), $\overline{u'w'}$ ( ) and $\overline{w'w'}$ ( ) from Loureiro et al. (2007). Solutions at (a) $x/H = -12.5$ and (b) $x/H = 2.5$.

Figure 5.6: Reynolds stress components for a smooth two-dimensional hill $Re = 4772$ calculated using the ASBM closure: $\overline{u'u'}$ ( ), $\overline{u'w'}$ ( ) and $\overline{w'w'}$ ( ) compared with experimental results: $\overline{u'u'}$ ( ), $\overline{u'w'}$ ( ) and $\overline{w'w'}$ ( ) from Loureiro et al. (2007) at (a) $x/H = -2.5$. 
5.3.1 Comparison to Loureiro Simulations

Loureiro et al. (2008) also carried out a number of simulations and made comparisons with their experimental results. Their objective was also to test the ability of different turbulence closures to accurately represent the experimental data. However, their approach was to use the CFD package ANSYS CFX and a number of its pre-programmed turbulence closures. The closures they tested included four eddy-viscosity closures:

- $k-\varepsilon$
- RNG $k-\varepsilon$
- $k-\omega$
- $k-\omega$-SST

and two Reynolds stress closures:

- Speziale-Sarkar-Gatski (SSG)
- baseline-$\omega$ (BSL)

These closures are all examples of the turbulence closures described in Chapter 2. Note also that Loureiro et al. (2008) used different near-wall treatments for several of the closures depending on the settings in ANSYS CFX. For a detailed description of the closures and the near-wall treatments refer to Loureiro et al. (2008) and the references therein.

In their results Loureiro et al. (2008) found that the two $k-\varepsilon$ closures and the Reynolds stress SSG closure failed to predict the separated flow. For the remaining four closures the size of the separated zone is summarised using the separation length ($L_s/H$) in Table 5.4. In all cases, both simulated and experimental, the separation and reattachment points were calculated by determining the point at which the $u$ component of velocity changes sign at a distance $z/H = 0.016$ away from the wall. As Loureiro et al. (2008) discuss, none of the closures they tested predict the size of the separated region well. The results for the present work are similar for the $k-\omega$-SST though the separated region is predicted to be slightly further down the back of the
Table 5.4: Length of Separated Region ($L_s/H$) for Various Turbulence Closures

<table>
<thead>
<tr>
<th>Turbulence Closure</th>
<th>Separation Point ($x/H$)</th>
<th>Reattachment Point ($x/H$)</th>
<th>$L_s/H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loureiro et al. (2008)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k-\omega$</td>
<td>0.50</td>
<td>5.60</td>
<td>5.10</td>
</tr>
<tr>
<td>$k-\omega$-SST</td>
<td>0.53</td>
<td>5.53</td>
<td>5.00</td>
</tr>
<tr>
<td>BSL</td>
<td>0.47</td>
<td>5.33</td>
<td>4.86</td>
</tr>
<tr>
<td>Experiments</td>
<td>0.50</td>
<td>6.67</td>
<td>6.17</td>
</tr>
<tr>
<td>Present work</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k-\omega$-SST</td>
<td>0.94</td>
<td>5.82</td>
<td>4.88</td>
</tr>
<tr>
<td>$v^2f$</td>
<td>0.59</td>
<td>6.23</td>
<td>5.64</td>
</tr>
<tr>
<td>ASBM</td>
<td>0.52</td>
<td>6.97</td>
<td>6.45</td>
</tr>
</tbody>
</table>

However, the results for the $v^2f$ and ASBM closures reinforce the findings in the previous section. Both outperform all the other turbulence closures and the ASBM closure predicts the extent of the separated region very accurately with only a slight overestimation of the reattachment point.

More detailed results from Loureiro et al. (2008) are reproduced in Figures 5.7-5.9 with the equivalent results included for the ASBM closure from the present work. Each figure shows the streamwise component of velocity $u$ and the non-zero components of the Reynolds stress tensor plotted at the position of a given measuring station. As Loureiro et al. (2008) discuss, none of the turbulence closures tested in their work predict the behaviour of the flow at the top of the hill accurately. All of the closures overestimate the velocity in the near-wall region and its gradient and completely fail to capture the nature of the Reynolds stress components. In contrast the agreement between the ASBM closure and the experimental data is very good. The streamwise velocity is well matched as are the normal components of the Reynolds stress tensor. Both the streamwise fluctuations and the shear stress component are
underestimated at heights more than $0.2H$ above the top of the hill. The shear component is also estimated as a positive value at higher heights which is not correct. The reasons for this were discussed in the previous section.

Figure 5.7: Profiles of streamwise velocity and Reynolds stress components at $x/H = 0$ calculated using the ASBM closure (- -) compared with other closures and experimental results as indicated (Loureiro et al., 2008).
Near the centre of the separated region at $x/H = 3.75$ the results are less impressive. The ASBM predicts the mean velocity better than the other closures but as discussed previously it underestimates the magnitudes of the Reynolds stress components. Even so, in the near-wall region it still outperforms the other closures and is the only closure to predict the saddle point observed in the streamwise fluctuations.

In the recovery zone beyond the reattachment point the performance of several closures is roughly equal. As Table 5.4 shows, the ASBM solution predicts the largest separated area and as a result has recovered less by the $x/H = 10$ measuring station. However, apart from the vertical fluctuations, the ASBM closure is the most accurate of the closures. Again it is the only closure to qualitatively predict the near-wall behaviour of the streamwise fluctuations.

Figure 5.10 shows a plot of the wall shear stress $\tau_w$ against streamwise position. The ASBM closure performs best of the closures that capture the separation and it is the most accurate upstream and at the top of the hill. In their work Loureiro et al. (2008) suggest that the $k-\omega$, the $k-\omega$-SST and the BSL closures significantly overestimate the peak in $\tau_w$ due to overestimating the near-wall velocity gradients. Figure 5.7 shows that the ASBM closure estimates the streamwise velocity and its gradient very well even at the top of the hill and yet it still predicts a larger peak than recorded in the experiments. This indicates that while the other closures do overestimate the peak, the error may be compounded by an underestimation in the experimental data. Loureiro et al. (2008) acknowledge this possibility by stating that the peak was not well captured in the experiments making comparisons difficult.

In their final remarks Loureiro et al. (2008) state that the $k-\omega$-SST closure performs the best in terms of the mean flow properties and the BSL closure represents the Reynolds stress components most accurately. The figures presented here show that the ASBM closure clearly outperforms all of the other closures with regard to the mean flow. For the Reynolds stress components its superiority is less clear at some points in the flow but in considering the flow as a whole it easily provides the most accurate predictions.

Loureiro et al. (2008) also provide some data on the computational cost of the various turbulence closures. Because of difference in the computational grids, the
Figure 5.8: Profiles of streamwise velocity and Reynolds stress components at $x/H = 3.75$ calculated using the ASBM closure (---) compared with other closures and experimental results as indicated (Loureiro et al., 2008).
Figure 5.9: Profiles of streamwise velocity and Reynolds stress components at $x/H = 10.0$ calculated using the ASBM closure (---) compared with other closures and experimental results as indicated (Loureiro et al., 2008).
Figure 5.10: Comparison of streamwise wall shear stress between the ASBM closure (---), other closures and experimental results as indicated (Loureiro et al., 2008).
convergence criteria and the hardware used it is impossible to make an exact comparison. However, as simulations were carried out using the $k$-$\omega$-SST closure in both their work and the present work, it can be used to make relative comparisons. Loureiro et al. (2008) report that the BSL closure is the most accurate at representing the Reynolds stress tensor and required approximately four times as much computation time on four times as many processors. In contrast Table 5.5 shows the ASBM closure required less than three times the computation time that the $k$-$\omega$-SST closure required, using the same number of processors.

Table 5.5: Simulation Times for Low Re Smooth Hill

<table>
<thead>
<tr>
<th>Turbulence Closure</th>
<th>Iterations</th>
<th>Computational Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$-$\varepsilon$</td>
<td>$\sim 13000$</td>
<td>$\sim 1800$ s</td>
</tr>
<tr>
<td>$v^2f$</td>
<td>$\sim 15800$</td>
<td>$\sim 2800$ s</td>
</tr>
<tr>
<td>ASBM</td>
<td>$\sim 21500$</td>
<td>$\sim 4200$ s</td>
</tr>
</tbody>
</table>

5.4 High Re Smooth Wall Simulations

In order to assess the effectiveness of the wall functions described in Section 2.4 higher Re simulations were necessary. This is because at low Re the boundary layer is thicker and the requirement that the first grid cell center lies at $z^+ \approx 30$ cannot be fulfilled while still resolving the details of the flow. Thus high Re smooth wall simulations were carried out to compare the predictions of a complex flow by (i) resolving the flow all the way to the wall and (ii) using the new wall functions. Following on from the results given in Section 5.3 it was assumed that the predictions using the wall-resolved grid and the ASBM closure gives a good representation of the flow field and thus effectively acts as the benchmark. Figure 5.11 shows the results of the four simulations at four different $x$ locations in the domain.
Figure 5.11: Comparison of streamwise velocity for a smooth two-dimensional hill $Re = 47720$ calculated using the $v^2f$ closure (---) and the ASBM closure (—) on a wall-resolved grid with the $v^2f$ closure (◦) and the ASBM closure (△) on a wall-function grid. Solutions at (a) $x/H = -12.5$, (b) $x/H = -2.5$, (c) $x/H = 2.5$ and (d) $x/H = 15$. 
Considering first only the wall-resolved simulations, the $v^2f$ closure again predicts a smaller separated region than the ASBM closure. At the higher $Re$ this discrepancy appears to be more significant than that shown in Section 5.3. Consequently, downstream of the separated region, the $v^2f$ streamwise velocity recovers more quickly. These differences again highlight the impact of including the full Reynolds stress tensor in the turbulence closure. Comparing the solutions obtained using the wall functions it is clear that overall the agreement is good. The only area of significant disagreement is again in the separated region. Here for both closures the solutions using wall functions show smaller separated regions than the wall-resolved solutions. This is not surprising given that the wall functions are based on an assumption of a local channel flow which is clearly not the case in the separated region.

Figure 5.12 shows the results for the Reynolds stress components obtained from the simulations using the ASBM closure. Again, the agreement throughout the domain is qualitatively very good. The simulation using wall functions represents the anisotropy well and the relative magnitude of the components is correct. In the separated region the magnitudes of the Reynolds stress components are slightly underestimated. Plot (b) shows the issue described in Section 5.3 where the wall-resolved simulation predicts positive shear stress near the wall on the upwind slope of the hill. The wall-function solution also predicts a positive shear stress near the wall at the same point but it is significantly smaller. It is possible that the additional positive shear stress is one of the causes of the discrepancy in the mean solutions as it could have lead to a slight overestimation of the separation region in the wall-resolved simulation. This overestimation would combine with underestimations by the wall-function solution and the $v^2f$ closure solutions to create the total discrepancy. Because the wall-function solutions do not resolve the flow close to the wall they fail to capture the local minima observed in the wall-resolved solution for the Reynolds stresses at $x/H = 2.5$ inside the separated region (plot (c)). However, the positions of other maxima and minima at different point in the flow are well predicted by the wall-function solution.
Figure 5.12: Comparison of Reynolds stress components for a smooth two-dimensional hill $Re = 47720$ calculated using the ASBM closure between a wall-resolved grid $u'w'$ (—), $u'w'$ (—) and $w'w'$ (—) and a wall-function grid $u'w'$ (Δ), $u'w'$ (○) and $w'w'$ (×). Solutions at (a) $x/H = -12.5$, (b) $x/H = -2.5$, (c) $x/H = 2.5$ and (d) $x/H = 15$. 
5.5 Rough Wall Simulations

The results from the rough wall simulations show similar trends to those obtained for the smooth wall simulations. Plot (d) in Figure 5.13 shows that the separation that occurred in the rough wall experiments was smaller than that of the smooth wall. The separation point is further down the downstream slope of the hill and the re-attachment point is closer to its base. The combination of a higher $Re$ but flow retarding roughness increases the complexity of the flow. Plot (a) shows that the $k-\varepsilon$ closure only just captures separation and it starts too close to the top of the hill and only exists in a thin layer adjacent to the downslope of the hill. As a result the mean flow is barely disturbed behind the hill and is thus incorrectly predicted. The $v^2f$ closure performs better and a larger separated region is predicted. However, the separation point is still too close to the top of the hill and the separated region is too thin. The agreement between the ASBM closure using the new wall functions and the experimental data is much better. While the centre of the recirculation in the separated region is not in the correct position, the separation point, reattachment point and size of the separated region are in good agreement.

More detailed plots of the streamwise velocity are given in Figure 5.14. Plot (a) shows the profiles at the first measuring station $x/H = -8.27$ where the flow is an undisturbed boundary layer. Here all three turbulence closures predict the mean flow correctly with the ASBM closure predicting the profile closer to the wall more accurately. The profiles in plot (b) are at $x/H = 2.3$ which is in the centre of the separated region. The superiority of the ASBM closure to capture the size and character of the separated region is clearly demonstrated. The $v^2f$ closure predicts the velocity of the reversed flow near the wall well but underestimates the velocity deficit further from the wall. As shown by the streamline plots, the $k-\varepsilon$ closure is not able to predict the separated region well and subsequently the streamwise velocity estimates are too high.

The plots in Figure 5.15 show the profiles for the streamwise velocity and the Reynold stress components calculated using the ASBM closure and the data obtained
Figure 5.13: Comparison of velocity streamlines above a rough two-dimensional hill $Re = 31023$ calculated using (a) $k$-$\varepsilon$ closure, (b) $v^2f$ closure, (c) ASBM closure and (d) experimental results from Loureiro et al. (2009)

Figure 5.14: Streamwise velocity for a rough two-dimensional hill $Re = 31023$ calculated using the ASBM closure (—), the $v^2f$ closure (- -) and the $k$-$\varepsilon$ closure (-.-) compared with experimental results (Loureiro et al., 2009). Solutions at (a) $x/H = -8.27$ and (b) $x/H = 2.3$. 
in the experiments. As already seen in the streamline plot, the agreement in the velocity is excellent throughout the flow. Like the results for the smooth simulations the qualitative agreement between the calculated Reynolds stress and the experimental results is also very good. Plot (b) shows that a good agreement is achieved for the streamwise fluctuations but that there is a slight underestimation of their magnitude above the separated region. The same is true of the shear stress in plot (b). While the form of the vertical normal stress is predicted quite well its magnitude is underestimated in the same way as occurred in the smooth wall simulations.

The Reynolds stress components calculated using the ASBM closure and recorded in the experiments are shown in more detail at two locations in Figure 5.16. Plot (a) is the undisturbed boundary layer at the first measuring station at \( x/H = -8.27 \). The qualitative agreement is very good with a slight overestimation of the streamwise fluctuations and a slight underestimation of the vertical fluctuations. The same is true of plot (b) which shows the results in the center of the separated region. Like the smooth simulations the agreement is very good and the prediction of the maxima is accurate. The streamwise fluctuations are still slightly overestimated and the vertical fluctuations are slightly underestimated.

In Figure 5.15 all three components of the Reynolds stress tensor display spikes in the near-wall cell on the upstream slope and at the top of the hill. Figure 5.17 shows a more detailed view of the components on the upslope of the hill at \( x/H = -1.33 \). The error occurs because neither the new wall functions presented here nor the standard ones, account for the positive pressure gradient caused by the curvature and instead interpret the velocity speed-up as a higher friction velocity. In the cells further away from the wall, this error reduces and at this stage it does not appear to affect the mean solution significantly. Finally, the computation times for the rough hill simulations is given in Table 5.6. It shows the same trends as the times for the smooth hill simulations with the ASBM closure requiring roughly three times as much computation time.
Figure 5.15: (a) Streamwise velocity, (b) $\overline{u'u'}$, (c) $\overline{u'w'}$ and (d) $\overline{w'w'}$ for a rough two-dimensional hill $Re = 31023$ calculated using the ASBM closure compared with experimental results (Loureiro et al., 2009).
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Figure 5.16: Reynolds stress components for a rough two-dimensional hill $Re = 31023$ calculated using the ASBM closure: $\overline{uu'}$ (—), $\overline{uw'}$ (.-.) and $\overline{ww'}$ (- -) compared with experimental results: $\overline{uu'}$ (△), $\overline{uw'}$ (○) and $\overline{ww'}$ (×) from Loureiro et al. (2009). Solutions at (a) $x/H = -8.27$ and (b) $x/H = 2.3$.

Figure 5.17: Reynolds stress components for a rough two-dimensional hill $Re = 31023$ calculated using the ASBM closure: $\overline{uu'}$ (—), $\overline{uw'}$ (.-.) and $\overline{ww'}$ (- -) compared with experimental results: $\overline{uu'}$ (△), $\overline{uw'}$ (○) and $\overline{ww'}$ (×) from Loureiro et al. (2009) at $x/H = -1.33$. 
Table 5.6: Simulation Times for High $Re$ Rough Hill

<table>
<thead>
<tr>
<th>Turbulence Closure</th>
<th>Iterations</th>
<th>Computational Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$-$\varepsilon$</td>
<td>$\sim 15000$</td>
<td>$\sim 1700$ s</td>
</tr>
<tr>
<td>$v^2f$</td>
<td>$\sim 19000$</td>
<td>$\sim 2900$ s</td>
</tr>
<tr>
<td>ASBM</td>
<td>$\sim 35200$</td>
<td>$\sim 6100$ s</td>
</tr>
</tbody>
</table>

5.6 Summary

A number of simulations were carried out to test the accuracy of the $v^2f$ and ASBM turbulence closures for calculating separated flows over a representative two-dimensional hill. The simulations were setup to the same specifications as the experiments carried out by Loureiro et al. (2007, 2009) so that comparisons could be made. Simulations were also run using the industry standard closures $k$-$\varepsilon$ and $k$-$\omega$-SST to establish the relative merit of the turbulence closures. For the smooth hill simulations comparisons were made with results obtained by Loureiro et al. (2008) using a number of different turbulence closures. For the simulations of flow over a rough hill wall functions were used as the ground boundary condition. The wall functions used for the $v^2f$ and ASBM closures are the new ones that have been developed as part of this project, and were presented in Chapter 2.

The results of the simulations for the smooth hill showed that the ASBM closure is much more accurate in predicting the size and position of the separated region than the $v^2f$ closure which is itself more accurate than the $k$-$\omega$-SST closure. The ASBM closure also predicts the components of the Reynolds stresses very well throughout the flow although their magnitudes are underestimated inside and downstream of the separated region. Neither the $k$-$\omega$-SST or $v^2f$ closures are capable of estimating the components of the Reynolds stress tensor.

The comparisons with other turbulence closures showed that the ASBM closure outperformed all of those tested including two Reynolds stress models. It predicted the mean flow very well and over the entire domain its representation of the Reynolds
stress tensor was more accurate. At the top of the hill it was the only turbulence closure to capture the details of the flow with any accuracy. Relative comparisons showed that it is also less computationally expensive than the most accurate Reynolds stress model.

In the simulations comparing the wall-resolved and wall-function solutions the results show that the new wall functions for both the $v^2f$ and ASBM closures perform well. The ASBM solution using wall functions predicts a smaller separated zone than the wall-resolved solution as do both the $v^2f$ solutions. It may be that there is a slight over-estimation of the size of the separated region by the wall-resolved ASBM solution and a small under estimation by the others.

The rough wall simulations show the same trend as the smooth wall simulations. In a more complex flow the increased accuracy of the ASBM closure is even more marked. Only the ASBM closure was able to capture the size and extent of the separated region though it predicted a centre for the recirculation too close to the top of the hill. The qualitative agreement in the Reynolds stress components was good.

As a result of the simulations carried out in this chapter it is clear that the ASBM closure provides an accurate representation of separated flow behind both smooth and rough two-dimensional hills. This is not achieved by other RANS closures satisfactorily. With this validation completed attention was then turned to three-dimensional test cases and then to fully three-dimensional real flows.
Chapter 6

Three-dimensional Simulations

This chapter presents the results from a number of three-dimensional simulations. After the success of the two-dimensional simulations discussed in the previous chapter, three-dimensional simulations are the natural progression for testing the turbulence closures and the new wall functions. Included in Section 6.1 of this chapter is a discussion of some three-dimensional simulations that were carried out before the two-dimensional simulations discussed in Chapter 5. While they do not address the topic of turbulence closures or near-wall treatments they raise some interesting issues regarding fully three-dimensional simulations. The remainder of this chapter presents the results for simulations of two well-known wind flow experiments; Kettles hill in Alberta, Canada and Askervein hill in South Uist, Scotland.

6.1 Preliminary 3D Wind Flow Simulations

Prior to carrying out the simulations discussed in Chapter 5 the two-dimensional simulations of a real ridge presented in Chapter 4 were extended to three-dimensions. The intention of these simulations was not to assess turbulence closures or wall functions but to develop methods for carrying out large three-dimensional simulations and to investigate a real-world wind problem using CFD. The results show that by applying the techniques developed in Chapter 4, consistent, large-scale simulations of high $Re$ flows can be carried out effectively and some meaningful comparisons to
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Field data can be made.

6.1.1 Background

Apart from the difficulties complex terrain presents in terms of computational modelling of wind flow, there are range of logistical and technical issues that also arise. One example is the collection of wind data. The work in this section was carried out as part of a collaboration with the Physics department at the University of Auckland who initiated a project to collect wind measurements using both cup anemometers and a sodar device at a site on a rugged ridge near Palmerston North in New Zealand. At the same time simulations were carried out as part of this work to compare with the measurements made by the Physics department to help explain their findings.

Because of their portability and relatively low cost, remote sensing systems such as sodar devices are now commonly used alongside cup anemometers for wind energy resource assessments (Behrens, 2010; Behrens et al., 2011). However, Behrens (2010) points out that while sodars have been extensively validated and utilised in homogeneous terrain several studies have found that they underestimate wind speed in complex terrain. Figure 6.1 shows the five-beam sodar device developed at the University of Auckland that was used. The figure also shows an example of the low scrub that covers the ridge behind the device.

Sodar devices utilise acoustic technology to measure wind profiles above terrain. The five-beam sodar device used for this project is a monostatic device meaning that the transmission and reception modules are collocated. The five beams are used to gather information about the wind speed in three-dimensions so that the full wind vector can be calculated using a set of geometric equations. For a detailed description of sodar technology and the specific device refer to Behrens (2010) and Behrens et al. (2011) and the references therein. The schematic in Figure 6.2 shows the configuration of the beams in the device.

When analysing their measurements Behrens (2010) found that there was a discrepancy of 0.14 m/s between the sodar’s four lateral beams at 80 m elevation under
the prevailing wind conditions. They suggested that the discrepancy could be attributed to the effects of the topography because the sodar and cup anemometer mast were placed near the peak of the ridge. This is common practice when evaluating wind resource in complex terrain. To establish the impact of the topography on the wind flow at the peak of the ridge a CFD simulation was carried out as part of the research project described in this thesis.

6.1.2 Simulation Setup

The simulation domain was created using detailed terrain data from the Land Information New Zealand website. First the data was rotated to align the prevailing wind with the $x$ coordinate direction and then transformed to place the anemometer mast at the $(0, 0)$ position. As described in Chapter 3 a numerical grid was then laid over the top of the contours to create the bottom boundary of the domain. The grid was stretched in both the streamwise direction and the vertical direction to achieve a high resolution around the sodar position without excessive computational cost. Figure 6.3 shows a satellite map of the area with the computational domain marked by a red boundary.

Figure 6.1: Sodar device used for measuring wind speeds in complex terrain.
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Figure 6.2: Schematic of the beams from the sodar device.

Figure 6.3: Satellite image of the ridge used for the sodar measurements. Computational domain marked in red. Extent of low scrub bush indicated in green.
Careful inspection of the satellite image reveals that the majority of the terrain upwind of the ridge is cultivated land. The same is true of the land downwind and so a varying aerodynamic roughness length was required for the simulations. For the ridge itself a constant aerodynamic roughness length of 0.31 m was selected following discussions with the team making the sodar measurements. The area for which this value was used is indicated in Figure 6.3 by the bright green boundaries. For the cultivated land a constant value of 0.03 m was used. Both of these values are consistent with the data in Table 2.1.

Developing the inflow boundary conditions was more difficult. The sodar and cup anemometer measurements were only made on and around the ridge and no recordings were made over the same period upwind of the ridge. The approach used can be summarised as follows:

1. Develop inflow profiles using an estimated friction velocity.
2. Run a two-dimensional simulation of the ridge.
3. Compare the calculated velocity profile at the mast location with the measured data.
4. If the agreement is not satisfactory, adjust the friction velocity, develop new inflow profiles and run another two-dimensional simulation.
5. If the agreement is satisfactory, run a three-dimensional simulation.
6. Compare the calculated velocity profile at the mast location with the measured data.
7. If the agreement is not satisfactory, adjust the friction velocity, develop new inflow profiles and run another three-dimensional simulation.

While this approach seems computationally intensive, after the initial simulations, the one-dimensional simulations to generate the inflow profiles, the two-dimensional simulations and even the full three-dimensional simulations all converged very quickly as the incremental change caused by adjusting the reference velocity was small. Once
a converged three-dimensional solution was obtained that accurately estimated the wind profile at the anemometer mast location, the results could be used to analyse the wind speeds at the sodar beam locations. The details of the final converged simulation are given in Table 6.1. Symmetry boundary conditions were used at the side of the domain and consistent top boundary conditions were calculated using equations (4.6)-(4.8). The $k$-$\varepsilon$ turbulence closure was used with rough wall functions used as the ground boundary condition.

Table 6.1: Sodar Comparison Ridge Simulation Specifications

<table>
<thead>
<tr>
<th>Simulation Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain dimensions (m)</td>
</tr>
<tr>
<td>Grid dimensions</td>
</tr>
<tr>
<td>Height of first vertical cell (m)</td>
</tr>
<tr>
<td>Vertical stretching ratio</td>
</tr>
<tr>
<td>Friction velocity, $u_*$ (m/s)</td>
</tr>
<tr>
<td>Roughness length, $z_0$ (m)</td>
</tr>
</tbody>
</table>

6.1.3 Results

While the inflow boundary conditions were selected to achieve a good agreement with the magnitude of the streamwise velocity measured at the anemometer mast, the agreement with the form of the profile in Figure 6.4 is also encouraging. The aerodynamic roughness plays an important role in determining the velocity near the ground and the good match to the profile indicates that the estimates used in the simulation for both the bush and the open terrain were satisfactory.

Figure 6.5 show the impact of the change in aerodynamic roughness on both the streamwise velocity component and the vertical velocity component at three different heights above the ground. The topography of the ridge is also shown and the change
in aerodynamic roughness length indicated. Note that the ridge topography is not to scale. Both plot (a) and plot(b) show the same pattern with the velocity components responding to the topographic effects at all three levels in the regions of low aerodynamic roughness. However, in the region of high aerodynamic roughness the variation in both the streamwise and vertical components of the velocity is suppressed in the flow adjacent to the ground. This is as expected but it is interesting to note that at only 6 m above the ground the impact of the high aerodynamic roughness is much less apparent and the response of the velocity components to the topography is fairly consistent for all values of $x$. 

Figure 6.4: Streamwise velocity profile at anemometer mast position in sodar comparison simulations.
Figure 6.5: Streamwise (a) and vertical (b) velocities plotted at $y = 0$ m and $z = 0.5$ m (blue), 6 m (green) and 34 m (red). High aerodynamic roughness length shown in gray. Topography not to scale.
6.1. PRELIMINARY 3D WIND FLOW SIMULATIONS

The velocity field of the converged solution can be interpolated to calculate the velocity at any given point. The possible sodar beam positions at 80 m elevation can be calculated using the beam angle of 30°. By calculating the velocity at each of these positions a velocity variation above the minimum velocity can be determined for all the positions. The results plotted in Figure 6.6 show that a velocity variation of nearly 0.14 m/s is predicted by the simulation results which agrees well with the measured discrepancy. Behrens (2010) went on to show that by using the simulation velocity field to adjust the sodar measurements for topographic effects the error between the sonic anemometer and the sodar measurements could be reduced from 4.3% to 0.4% at 60 m and from 6 % to 0.1 % at 80 m.

Figure 6.6: Velocity variation around 30° beam angle at 80 m elevation above sodar position.
6.1.4 Discussion

A methodology was developed and a fully three-dimensional simulation was carried out of wind flow over a rugged ridge near Palmerston North in New Zealand. The results for the velocity field were used to confirm that the discrepancy observed between measurements made by different beams of a sodar device could be attributed to topographic effects. Behrens (2010) used the simulation velocity field to adjust the sodar data for topographic effects and significantly reduced the error between the sodar and cup anemometer measurements. While these results are qualitatively good and the methodology for carrying out large three-dimensional simulations proved to be effective, two aspects of the simulation were unsatisfactory.

First, as discussed in Section 4.2, when the simulation domain extends vertically to over 2500 m and the hill over 500 m, enforcing a log law top boundary condition causes errors in the velocity calculated above the hill. In this particular simulation the error introduced in the velocity did not affect the results as the simulation was calibrated using velocity data at the top of the ridge. However, in normal simulations where data at the inflow is used to calibrate the simulation the results could be badly affected. This problem prompted the development of the Ekman profiles and the atmosFoam application described in Chapters 3 and 4.

Second, the lack of available validation data limits the usefulness of the simulation results. As Behrens (2010); Behrens et al. (2011) note, a simple potential flow model can produce similar velocity field estimations at a fraction of the computational cost. The real advantage of CFD simulations is that they can provide detailed information over a whole simulation domain including regions where physical measurements may be difficult or impossible. To achieve this sufficient data must be available to calibrate and validate the model. At this stage the data for the ridge do not meet that criterion and for that reason simulations using more accurate turbulence closures and Ekman inflow profiles were not attempted. The possibility of revisiting simulations of the ridge is addressed in Chapter 9.
6.2 Kettles Hill Simulations

In 1980 a project was started to study boundary-layer flow over a low, isolated, three-dimensional hill (Teunissen & Flay, 1981). The hill selected was Kettles hill in the foothills of the Rocky Mountains in Alberta, Canada. The objective was to compare full-scale wind and turbulence measurements with predictions of a mathematical model and measurements obtained from wind-tunnel experiments. A good summary of the project is provided by Salmon et al. (1988) which concludes by stating that both the mathematical model and the wind-tunnel experiments provided good predictions of the horizontal distribution of normalised wind speeds. Prof. Flay explained (private communication) that accurate and detailed data were recorded for the full Reynolds stress tensor during the wind-tunnel experiments carried out in 1981 (Teunissen & Flay, 1981). Accurate data sets of very high $Re$, three-dimensional, atmospheric flows are rare and while the Kettles Hill Project explicitly avoided separated flow, it is still a useful test case for the ASBM turbulence closure and the new wall functions. Note that simulations were only carried out using the ASBM closure because it is the only closure studied in the present work that is capable of representing the full Reynolds stress tensor. The following section describes the setup for both the wind-tunnel experiments and the numerical simulation. The subsequent sections present the results which show that simulations using the ASBM closure are capable of accurately predicting high $Re$, fully three-dimensional flows.

6.2.1 Experimental and Simulation Setup

The wind-tunnel experiments were carried out in the Atmospheric Environment Service Boundary-Layer Wind Tunnel in Ontario. This is an open-circuit tunnel with a test section 1.83 m high $\times$ 2.44 m wide $\times$ 18.3 m long. For a detailed description of the equipment, measurement techniques and procedures refer to Teunissen & Flay (1981). The length scale of the wind-tunnel model was selected to be 1:1000 and a gradient wind speed of 16 m/s was used. The model was made out of polyurethane foam, cut and sanded to accurately match the contours shown in plot (a) in Figure 6.8. At this scale the similarity requirements for a neutrally stable boundary-layer
flow reduce to geometric similarity, Reynolds-number similarity and approach-flow similarity (Salmon et al., 1988). As Salmon et al. (1988) note all of these criteria were satisfactorily met except for the $Re$ similarity which cannot be satisfied at the wind-tunnel scale. For the present work this factor combined with the lack of detailed, accurate full-scale measurements led to the decision to simulate the wind-tunnel scale hill rather than the full-scale hill. The $Re$ for the wind-tunnel experiments was $2.6 \times 10^5$ which still provides a good test case for the ASBM turbulence closure and the new wall functions.

Figure 6.9 shows a schematic of the model placement in the wind tunnel including a number of the key measurement points. Plot (b) in Figure 6.8 shows the contours of the bottom of the domain used for the computational simulation. The key measuring points of B, G, H and I are indicated on this plot along with T4 and T9 which correspond to tower locations in the full-scale experiment. The colours of the points in this plot are used throughout Section 6.2.2 for clarity. The positions of the other full-scale measuring towers can be seen through careful inspection of plot (a) in Figure 6.8 or more easily by referring to Teunissen & Flay (1981). Note that the computational
Figure 6.8: Contour maps of (a) the wind-tunnel model (Teunissen & Flay, 1981) and (b) the bottom of the computational domain for Kettles hill. Measuring positions B (black), G (green), H (blue), I (red), tower 4 (magenta) and tower 9 (cyan) shown on plot (b).
simulation does not include the full length of the wind tunnel but instead has an inflow boundary condition 2.3 m upstream of the center of the model (position H).

To calculate the correct inflow boundary conditions a two-dimensional simulation of the empty wind tunnel was carried out in a similar manner to the boundary-layer inflow simulations in Chapter 5. Once the two-dimensional simulation converged the profiles of the variables at position B were saved and used as the inflow boundary conditions of the full simulation. The details of the two-dimensional boundary-layer simulation and the full three-dimensional simulation of the model hill are given in Table 6.2

Table 6.2: Kettles Hill Simulation Specifications

<table>
<thead>
<tr>
<th>Simulation Specifications</th>
<th>2D Domain dimensions (m)</th>
<th>3D Domain dimensions (m)</th>
<th>2D Grid dimensions</th>
<th>3D Grid dimensions</th>
<th>Height of first vertical cell (m)</th>
<th>Vertical stretching ratio</th>
<th>Reference velocity, $\overline{u}_{ref}$ (m/s)</th>
<th>Reference height, $z_{ref}$ (m)</th>
<th>Aerodynamic roughness, $z_0$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D Domain dimensions (m)</td>
<td>20 × 2.44 × 1.83</td>
<td>3.9 × 2.44 × 1.83</td>
<td>200 × 1 × 50</td>
<td>240 × 160 × 50</td>
<td>1.5×10^{-3}</td>
<td>1.101</td>
<td>16.78</td>
<td>0.32</td>
<td>3.0×10^{-5}</td>
</tr>
</tbody>
</table>

The aerodynamic roughness caused problems for the wind-tunnel experiments. As Teunissen & Flay (1981) discuss, the intention was to use roughness to remove the effects of the viscous sublayer and hence achieve a better $Re$ similarity. They calculated that to achieve a hydraulically rough flow a grain-of-sand roughness of greater than 1.4 mm was required. However, they rejected this as it scaled to unrealistic objects 1.4 m high on the surface of the full-scale hill and thus would prevent meaningful comparisons. As a compromise they selected a value of 0.5 mm, putting
the flow in the transitionally rough state, and hoped that the viscous effects would be sufficiently reduced. Their calculations were that the steel filings should have produced an aerodynamic roughness in the range 0.02-0.035 mm which was close to the value of $3.0 \times 10^{-5}$ m they calculated for the Masonite surface of the wind tunnel. Unfortunately they observed the formation of an internal boundary layer at the edge of the model hill indicating that the roughness of the Masonite surface was not equal to that of the roughened model hill. Teunissen & Flay (1981) state that they thought this effect was probably more detrimental than any potential viscous effects.

![Figure 6.9: Measurement positions in wind tunnel for Kettles hill experiment (Teunissen & Flay, 1981).](image)

The aim of the simulations carried out in this section was to make comparisons with the wind-tunnel experiments and not the full-scale experiments, hence there was no need to achieve Re similarity. Therefore a single value was selected for the aerodynamic roughness that matched the value calculated at position B by Teunissen & Flay (1981). The possibility of using a different aerodynamic roughness for the area of the domain corresponding to the model hill was considered but later abandoned.
because of a lack of data for the actual roughness change.

The flow near the walls and top of the wind tunnel was not resolved in the simulation. It was considered that these boundaries were sufficiently far from the measurement area to have no impact on the results. As such, symmetry boundary conditions were used at the top and sides of the computational domain. The computational grid was stretched in the vertical direction for both the two-dimensional and three-dimensional simulations. For the three-dimensional simulation it was also stretched in both the streamwise and cross-stream directions. This allowed a high resolution to be achieved over the hill without prohibitive computational expense. The example of a three-dimensional grid given is Figure 3.4 is a portion of the grid used for these simulations.

### 6.2.2 Results

The aim of the simulations discussed in this section was to assess the ability of the ASBM turbulence closure to predict the mean velocity field and Reynolds stress tensor for a large, high $Re$, three-dimensional flow over a real hill. For that reason the results presented in this section consist of profiles of these quantities measured at various points above and around the Kettles hill model rather than the speed-up factors which were the focus of the wind-tunnel experiments. For the purposes of clarity all of the results in this section are presented in the same format. Plot (a) in each figure is the profile of the quantity at point B measured during the wind-tunnel experiments and also calculated by the simulation. Plots (b) and (c) are the wind-tunnel measurements for the quantity at various measurement points scaled to the same boundary-layer height as at point B. Finally, plots (d) and (e) are the simulation results for the quantity at the same measurement points scaled to the same boundary-layer height as at point B in the simulation. Note that in Figure 6.14 there is only one plot each for the wind-tunnel measurements and the simulation results.

The reason the results are presented in this form is to be consistent with the results presented by Teunissen & Flay (1981) so that meaningful comparisons can be made. Their intention was to compare wind-tunnel results with full-scale experiments.
where no boundary-layer growth occurs because of the balance of pressure and Coriolis forces (see Section 2.1). To account for the boundary-layer growth in the wind tunnel, Teunissen & Flay (1981) scaled the vertical axes of the profiles by the ratio of the boundary-layer height to the reference boundary-layer height at position B. As they discuss this has the important implication that profiles at different measurement points are scaled by different amounts. Thus comparisons between the profiles must be made by comparing each with the profile at B. Without access to the full data set, it was simpler for the present work to calculate profiles from the simulation in the same way rather than attempts to “re-scale” the wind-tunnel profiles to their actual height. The boundary-layer height at each position $P$ was calculated by finding the height $z_{\delta P}$ where $\bar{u}_P = 0.995 \, (\bar{u}_{ref})_P$. The measurement positions are shown in Figure 6.8 but can be described as follows: Position B is 2.3 m upwind of the peak of Kettles hill which is position H. Position G is 0.3 m north of H and in the same plane perpendicular to the flow direction. Position I is also in the perpendicular plane except 0.3 m south or at $(0, -0.3)$ in cartesian coordinates. The two tower positions are 0.021 m south of the center line with tower 4 (T4) 0.447 m upstream and tower 9 (T9) 0.218 m downstream of H.

One important result that can been seen by examining the plots marked (a) in Figures 6.10-6.14 is that the simulation slightly underestimates the height of the boundary layer at position B. This was determined to be a direct result of the issues surrounding the aerodynamic roughness described in Section 6.2.1. As noted previously, the aerodynamic roughness of $3.0 \times 10^{-5}$ m corresponds to the transitionally rough flow regime. Teunissen & Flay (1981) give the following criteria:

$$
\begin{align*}
&z_0^+ > 3.5 & \text{fully rough flow}, \\
&0.2 \cdot 0.3 < z_0^+ < 3.5 & \text{transitionally rough flow}, \\
&z_0^+ < 0.2 \cdot 0.3 & \text{fully smooth flow},
\end{align*}
$$

(6.1)

where $z_0^+ = \frac{u_{\tau} z_0}{\nu}$ and was approximately 1.4 for these simulations. Similar criteria are used in the standard wall functions to make an adjustment to the calculation of $u_{\text{rough}}^+$ for transitionally rough flow regimes. While the new $v^2f$ wall function inherits
the transitional roughness treatment through equation (2.60):

$$
\tau_{w,\text{rough}} = C_{1}\frac{\mu}{k}^{1/2} \frac{\overline{u_p}}{u_{\text{rough}}} 
$$

the new ASBM wall function only partially accounts for transitional roughness. This is because although the new ASBM wall function also uses equation (2.60), it also uses equation (2.65):

$$
\tau(z)_{\text{rough}} = \tau_{w,\text{rough}} \left( 1 - \frac{z - z_0}{\delta} \right) 
$$

which does not account for transitional roughness. The result is that the new ASBM wall function underestimates the effect of a transitionally rough surface and hence underestimates the boundary-layer height. The impact of this effect on the results was limited by using a blend of 50% of the ASBM closure and 50% of the $v^2 f$ closure. Apart from the slight underestimation of the boundary-layer height, it can be seen from the plots marked (a) in Figures 6.10-6.14 that the predictions of the mean velocity and Reynolds stress components are very good at position B.

The plots in Figure 6.10 show that the ASBM closure predicts the mean velocity well throughout the flow. In plots (b) and (d) it can be seen that the boundary-layer height and speed-up at positions G, H and I are well predicted. Similarly in plots (c) and (e) the agreement is very good and the simulations capture the crossover of profiles of H and tower 9 at $z = 0.2$ m.

The results for the streamwise fluctuations are presented in Figure 6.10. Plot (a) shows again that at position B the boundary-layer height is slightly underestimated. The magnitude of the fluctuations is also slightly underestimated at all heights apart from adjacent to the ground. Comparing plots (b) and (d) the profiles can be seen to be well predicted by the simulation. Similarly a comparison of plots (c) and (e) shows the accuracy of the simulation results in matching the experimental data. Careful inspection shows that the simulation does capture the high values of $\overline{u' u'}$ near the ground at positions H, G, I, t4 and t9. However, in the simulation these peaks are
Figure 6.10: Comparison of streamwise velocity over Kettles hill calculated using the ASBM closure with experimental data.

(a) at position B simulation (—) and experiment (○)
(b) experiment at position B (black), G (green), H (blue) and I (red)
(c) experiment at position B (black), tower 4 (magenta), H (blue) and tower 9 (cyan)
(d) simulation at position B (black), G (green), H (blue) and I (red)
(e) simulation at position B (black), tower 4 (magenta), H (blue) and tower 9 (cyan)
restricted to the region very close to the ground whereas in the experimental data they extend a short distance above. It is possible that some experimental error contributes to this discrepancy because measurements in the region immediately adjacent to the wall were difficult to make (Teunissen & Flay, 1981). Though as Salmon et al. (1988) notes extreme care was take in making the measurements during the wind-tunnel experiments.

For the vertical fluctuations the agreement between the simulation and the wind-tunnel experiments is also very good. Despite the slight under-estimation of the boundary-layer height, the magnitude of the fluctuations is well predicted except near to the ground where it is slightly overestimated. A comparison of the results for the experiment to those for the simulation shows that the profiles of the vertical fluctuations are well estimated. The larger values for the fluctuations at the ground is correctly predicted for positions G, H and t4. Also the height of 0.1 m where the profile for position t4 crosses the profile for position B is accurately captured.

Figure 6.13 shows the results for the shear stress fluctuations $u'w'$. While the agreement between the simulation and the wind-tunnel results is good, the profiles at G, I and t4 are not as well predicted as for the other components of the Reynolds stress tensor. The profile at position H is well represented with lower values than at position B predicted at all heights apart from at the ground where the spike in magnitude is correctly predicted. The final plots for this section in Figure 6.14 show the cross-stream fluctuation at positions B and H. The fluctuations were only measured at two positions during the wind-tunnel experiments, but for both of them the predictions of the simulation are very good.

6.2.3 Discussion

The detailed and accurate wind-tunnel measurements that were made as part of the Kettles Hill Project by Teunissen & Flay (1981) provide a rare and valuable data set for assessing the capabilities of the ASBM turbulence closure and the new wall functions. The wind-tunnel experiments were carried out at a high $Re$ and measurements were made of all of the important components of the Reynolds stress
6.2. KETTLES HILL SIMULATIONS

Figure 6.11: Comparison of streamwise fluctuations over Kettles hill calculated using the ASBM closure with experimental data.
(a) at position B simulation (—) and experiment (○)
(b) experiment at position B (black), G (green), H (blue) and I (red)
(c) experiment at position B (black), tower 4 (magenta), H (blue) and tower 9 (cyan)
(d) simulation at position B (black), G (green), H (blue) and I (red)
(e) simulation at position B (black), tower 4 (magenta), H (blue) and tower 9 (cyan)
Figure 6.12: Comparison of vertical fluctuations over Kettles hill calculated using the ASBM closure with experimental data.
(a) at position B simulation (—) and experiment (○)
(b) experiment at position B (black), G (green), H (blue) and I (red)
(c) experiment at position B (black), tower 4 (magenta) and H (blue)
(d) simulation at position B (black), G (green), H (blue) and I (red)
(e) simulation at position B (black), tower 4 (magenta) and H (blue)
6.2. KETTLES HILL SIMULATIONS

Figure 6.13: Comparison of shear stress fluctuations over Kettles hill calculated using the ASBM closure with experimental data.
(a) at position B simulation (—) and experiment (○)
(b) experiment at position B (black), G (green), H (blue) and I (red)
(c) experiment at position B (black), tower 4 (magenta) and H (blue)
(d) simulation at position B (black), G (green), H (blue) and I (red)
(e) simulation at position B (black), tower 4 (magenta) and H (blue)
Figure 6.14: Comparison of cross-stream fluctuations over Kettles hill calculated using the ASBM closure with experimental data.
(a) at position B simulation (—) and experiment (○)
(b) experiment at position B (black) and H (blue)
(c) simulation at position B (black) and H (blue)
tensor. Problems arose in the experiment because it was impossible to achieve a fully rough flow regime and maintain surface similarity with the full-scale hill. Also issues occurred because of roughness differences between the model surface and the surface of the wind tunnel itself. In the simulation these problems were avoided by simulating the wind-tunnel experiments and using the actual roughness value. However this created difficulties in the simulation because the roughness of the model was sufficiently low to fall into the transitionally rough flow regime, and while the new $v^2f$ wall function caters for the transitional flow regime, the new ASBM wall function does not. To overcome this problem a blending of 50% ASBM closure and 50% $v^2f$ closure was used. The extension of the new ASBM wall function to include transitionally rough flow regimes is discussed in Chapter 9 but it is not seen as a major obstacle for using the ASBM closure to model atmospheric flows because transitionally rough flow regimes are extremely rare in atmospheric flows.

The results from the simulation are very good. The blended ASBM closure estimates the mean flow very well at all of the measurement points. The agreement for the normal components of the Reynolds stress tensor is also very good. The simulation correctly predicts spikes in the normal fluctuations immediately adjacent to the ground but the thickness of these spikes is underestimated. Because the components of the Reynolds stress are only calculated by the ASBM closure, and not by the $v^2f$ closure it is likely that the issues concerning transitional roughness affect the accuracy of the simulation very near to the ground. For the shear stress component the agreement at positions G, H and t4 is less satisfactory but at positions B and H the agreement is very good.

In summary, the work discussed in this section has shown that the ASBM turbulence closure and the new wall functions developed as part of this research project provide accurate predictions of the mean flow and Reynolds stress tensor for a fully three-dimensional, high $Re$ atmospheric flow. The work discussed in the following section extended this to the full-scale test case of Askervein hill.
6.3 Askervein Hill Simulations

The Askervein Hill Project (Taylor & Teunissen, 1987) is one of the most well-known field studies of wind flow over complex terrain. It was a major cooperative project with the aim of making detailed measurements of the spatial characteristics of the mean wind flow and turbulence over a low hill (Mickle et al., 1988). Askervein hill is located on the Scottish island of South Uist in the Hebrides and is elliptic in shape and approximately 2 km x 1 km in size. It has a maximum height of 116 m above its surroundings and is relatively isolated with flat plains to the south west and some larger hills to the north east. The ocean is roughly 4 km to the west and south west. A photograph of the hill is shown in Figure 6.15 and a map of the hill and the nearby coast is given in Figure 6.16. Also marked on the map in Figure 6.16 are two important points, RS the reference site and HT the highest point of the hill or “hill top”. The surface of the hill and surrounding terrain is largely open grassland and is relatively homogeneous. Taylor & Teunissen (1987) calculated its aerodynamic roughness to be $z_0 \approx 0.3$ m.

![Figure 6.15: Photo of Askervein hill (Bechmann & Sørensen, 2010).](image)
As discussed in several sections previously, detailed and accurate data of real atmospheric flows are rare. The detailed measurements made during the Askervein hill Project have become very well-known because they provide the means for validating numerical models against data for a real, high $Re$, turbulent flow. Within CFD alone there are numerous examples of code validation that have been carried out using the Askervein hill data (Bechmann & Sørensen, 2010; Bechmann et al., 2007a,b; Chow & Street, 2009; Lopes et al., 2007; Beljaars et al., 1987; Eidsvik, 2008, 2005; Undheim et al., 2006; Castro et al., 2003).

Figure 6.16: Map of Askervein hill and nearby coast (source Google Maps).
In most cases the TU-03B data set has been used for model validation and it was also selected for the present work. This data set was taken during a three hour period on October 3rd, 1983 when the atmosphere was approximately neutrally stable and the wind was blowing from the south west at 210°. At the reference site (RS) the wind speed was measured to be 8.9 m/s at a height of 10 m above the ground (Taylor & Teunissen, 1987). Measurements were also made along three lines crossing over the hill. The lines are shown in plot (a) of Figure 6.17 and marked as A, AA and B. Note that the lines A and B intersect at the point HT at the top of the hill. Along line A nine measuring stations were placed recording the mean wind and the turbulence of the flow 10 m above the ground. At both HT and RS cup, sonic, tilted Gill and vertical Gill anemometers were used to record the inflow conditions and the speed-up at the top of the hill.

### 6.3.1 Simulation Setup

The computational domain was selected to be large enough to include the coastal plain upwind of Askervein hill and the larger hills to the north east, downwind. By including the coastal plains and in particular the reference site RS, it was possible to ensure the inflow conditions were the same for the numerical simulation as the field data. The larger downwind hills were included for several reasons. First, Mickle et al. (1988) discuss the possibility of the larger hills having some impact on the flow. Second, part of the aim of the work in this section was to simulate a real, complex, high Re number flow on a wind farm scale. So despite the lack of validation data for the region around the larger hills, it provided a good test for the approach. Finally, to make comparisons with the Hybrid RANS/LES simulations of Bechmann & Sørensen (2010) it was more consistent to use a similar domain size as that used in their work. Like the simulations in the previous sections, the computational domain used a grid mapped to the contours of the topography that was stretched in all three directions. The terrain data was generously supplied by Dr. Bechmann at DTU Risø and some additional high resolution data for the top of the hill was added by digitising the 2 m contour plots from Undheim et al. (2006). A 10 m contour plot of Askervein
6.3. ASKERVEIN HILL SIMULATIONS

The details of the computational domain and the simulation set up are given in Table 6.3. The large vertical dimensions of the domain were used to accommodate the Ekman inflow profiles described in Section 4.2 but because of the stretching in the vertical direction, it did not add significantly to the computational cost. In the horizontal directions cells 100 m x 100 m were used at the edges of the domain and a constant grid of 5 m in the x-direction and 20 m in the y-direction over the hill itself. In both directions the grid was stretched uniformly between the edges and the constant central grid.

Table 6.3: Askervein Hill Simulation Specifications

<table>
<thead>
<tr>
<th>Simulation Specifications</th>
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<tbody>
<tr>
<td>Domain dimensions (km)</td>
</tr>
<tr>
<td>Grid dimensions</td>
</tr>
<tr>
<td>Height of first vertical cell (m)</td>
</tr>
<tr>
<td>Vertical stretching ratio</td>
</tr>
<tr>
<td>Reference velocity at RS, ( \bar{u}_{ref} ) (m/s)</td>
</tr>
<tr>
<td>Reference height, ( z_{ref} ) (m)</td>
</tr>
<tr>
<td>Aerodynamic roughness, ( z_0 ) (m)</td>
</tr>
<tr>
<td>Coriolis parameter, ( f_c ) (rad/s)</td>
</tr>
<tr>
<td>Pressure gradient, ( PG ) (N/m^3)</td>
</tr>
</tbody>
</table>

As discussed in Section 4.2, the Ekman inflow profiles can be generated by creating a physically realistic balance between the Coriolis force and the pressure gradient. Because the domain of the simulations in this section was so large, the same Coriolis force and pressure gradient were applied to the full simulation to ensure that the geostrophic wind direction was maintained over the whole domain. These values are
Figure 6.17: Contour maps of (a) Askervein hill (Undheim et al., 2006) and (b) the bottom of the computational domain.
also given in Table 6.3. To accommodate the Ekman inflow profiles which have both $u$ and $v$ components of velocity, the side boundary conditions were set to be periodic. This in turn required that the topography of the bottom of the domain needed to match at the periodic edge boundaries so that step changes in the grid and their accompanying numerical problems could be avoided. To achieve this the edges of the domain were smoothed to be at the same elevation as the inflow boundary. The same approach was used in Section 6.2 and was found to be satisfactory given the distance between the domain edges and the area of interest around Askervein hill. At the top of the domain zero-gradient was enforced as the boundary condition for all of the variables except the velocity for which a geostrophic wind velocity was applied using the parameters in Table 6.3 and equation (2.1). At the outflow boundary a standard set of outflow boundary conditions were applied: zero-gradient for all variables except a fixed reference value for the pressure.

![Figure 6.18: Contours of the bottom of the computational domain around Askervein hill with flow vectors at $z = 10$ m shown.](image)

One of the implications of applying periodic boundary conditions at the side of the domain is that the incoming flow need not be exactly aligned with the $x$-coordinate of the grid. In fact, when using Ekman inflow profiles, the incoming flow cannot
be perfectly aligned with the grid as the flow direction changes with height above the ground. Because of the numerical schemes used for calculating derivatives (see Section 3.1.4) it is advisable that the flow direction is roughly aligned with the $x$-axis but for moderate wind speeds there is potential to vary the angle of approach. This means that it is possible to predict flows for several wind directions using the same numerical simulation by simply varying the inflow boundary conditions. This was one of the considerations when setting up the simulations presented in this section and so the grid was aligned with line A over Askervein hill rather than the $210^\circ$ wind direction. Figure 6.18 shows a plot of the contours of the ground in the simulation domain immediately around Askervein hill with vectors showing the calculated wind direction 10 m above the ground. Note that for readability vectors are not shown for every computational cell.

To calculate the Ekman profiles for the inflow boundary conditions the approach described in Section 4.2 was used. A one-dimensional simulation was driven by a balance between the Coriolis force and the pressure gradient using periodic boundary conditions in both horizontal directions. The surface roughness was set to $z_0 = 0.03$ m as in the full-scale simulation and Ekman profiles were allowed to develop naturally. The value of the pressure gradient was varied until a good match was achieved for the velocity profile measured at the reference site RS. However, upon inspecting the components of the Reynolds stress tensor it was discovered that the agreement was relatively poor for each of them. With the exception of the streamwise fluctuations the magnitudes of all of the components were overestimated. After consideration it was observed that location of Askervein hill adjacent to a relatively narrow coastal plain means that the turbulence may not have reached an equilibrium with the ground. Because the method used for generating the Ekman profiles essentially models the flow over terrain for a sufficient distance that a steady state is reached, it incorrectly predicted the profiles at the reference site. The likelihood was that aspects of the lower turbulence flow that developed over the open ocean were still present in the flow when it reached RS.

To achieve a better agreement with the profiles at the reference site and a more physically accurate representation of the inflow conditions two additional simulations
were carried out. The first was another one-dimensional simulation this time using the aerodynamic roughness for open water of $z_0 = 2 \times 10^{-4}$ m. The second was a two-dimensional simulation over a change in aerodynamic roughness from $z_0 = 2 \times 10^{-4}$ m to $z_0 = 0.03$ m. An initial pressure gradient of $PG = 0.0017 \text{ N/m}^3$ was selected to drive the simulations because this was the value used by Bechmann & Sørensen (2010) in their simulations. Profiles were then calculated from the two-dimensional simulation at a distance downstream of the roughness change equal to the distance the reference site was downwind of the coast. The intention was to iteratively vary the pressure gradient until the profiles generated at this point gave a satisfactory agreement with the field data. Fortunately, the results for the initial pressure gradient provided a good agreement with the field data and iteration was not required.

The results from the two-dimensional simulation were then used as the inflow boundary conditions for the full-scale three-dimensional simulation. Because of the length of the inflow boundary for the three-dimensional simulation, the distance to the open sea varies along its length. To accurately represent this profiles were saved from the two-dimensional simulation at different distances from the roughness change and these profiles were used as inflow boundary conditions at points at the equivalent distance to the waters edge.

Figure 6.19 shows three different sets of profiles. One is the inflow profiles generated using the initial one-dimensional simulation with an aerodynamic roughness of $z_0 = 0.03$ m. The second set are the profiles generated by the one-dimensional simulation over open water. The final set are the profiles calculated at the reference site RS for the full three-dimensional simulation. Plots (c) and (d) show the overestimation of $\overline{v'w'}$, $\overline{w'u'}$ and $\overline{u'w'}$ by the initial one-dimensional simulation fully developed over grass. In the same plots the agreement achieved by using the two-dimensional model is much better. In particular, the gradients of the profiles that were measured by the sonic anemometers has been captured. This is encouraging because in their discussion of the field measurements Mickle et al. (1988) state that interpolating between the two sonic anemometer measurements is likely to give a good estimate of the fluctuations. The agreement for the slope of the $\overline{u'u'}$ fluctuations was also very good though their magnitude was underestimated by 25%. For the streamwise velocity the
agreement at RS was also very good. It was slightly overestimated above 20 m but this error was deemed to be acceptable.

6.3.2 Results

While the Askervein hill Project was probably one of the most extensive field experiments carried out to measure wind data over terrain, there are still only a relatively small number of comparisons that can be made. Unlike the water channel experiments in Chapter 5 or the wind-tunnel experiments in Section 6.2 it was prohibitively expensive to collect detailed profiles of the mean wind field and turbulence at numerous locations during the field study. Instead the profiles were recorded at a few select locations and measurements were taken at a single height at others. The measurements that were considered most reliable (Mickle et al., 1988) are the ones presented here. They are the same measurements that were used for comparison by Bechmann & Sørensen (2010), Lopes et al. (2007) and Castro et al. (2003) and so comparisons are made between the simulation results of the present work, the previous simulation results and the experimental data. Note that the simulations carried out by Castro et al. (2003) were RANS simulations, those by Lopes et al. (2007) LES while both RANS and hybrid RANS/LES were carried out by Bechmann & Sørensen (2010). Refer to the original works for details of their simulations and details of the models used.

The plots in Figures 6.20 and 6.22 are all comparisons of simulation results with data measured 10 m above the ground along line A over the top of Askervein hill. Line A is shown in Figure 6.17 and for the TU-03B data set it was at an angle of 13.8° to the wind direction. Plot (a) shows the speed-up along line A for the simulations and the experimental data. Note that the hill is shown at the bottom of the plot and is to scale in the \(x\)-direction but not in the \(z\)-direction. The plot shows that all of the simulations achieve a good agreement with the experimental data. The simulation of the present work achieves the best agreement upwind of the hill with the other simulations slightly under estimating the magnitude of the negative speed-up or velocity deficit. This is probably due to the inflow boundary conditions that
6.3. ASKERVEIN HILL SIMULATIONS

Figure 6.19: Profiles of (a) streamwise velocity and Reynolds stress components (b) $u'v'$, (c) $v'w'$, (d) $w'u'$ and (e) $u'w'$ at RS (—) compared to anemometer measurements from Taylor & Teunissen (1987) - cup (○), vertical Gill (□), sonic (+) and titled Gill (×). Profiles fully developed over open water (-.-) and grass (- -) also shown.
Figure 6.20: Comparison of (a) fractional speed-up ratio and (b) turbulent kinetic energy along line A between field measurements (●), LES by Lopes et al. (2007) (—), RANS by Castro et al. (2003) (- -), RANS/LES (...) and RANS (-.-) by Bechmann & Sørensen (2010) and the present work with the ASBM closure (- -)
were used as none of the other simulations used a change in roughness upwind of the inflow or varying profiles across inflow boundary.

Downwind of the hill the best agreement is achieved by the RANS solution of Castro et al. (2003) who used a high-order differencing scheme. The RANS solution of the present work and the one by Bechmann & Sørensen (2010) both underestimate the velocity deficit whereas the LES and hybrid solutions both overestimate it. Bechmann & Sørensen (2010) explain this by pointing out that several authors describe the flow on the lee side of the hill as complex and on the verge of forming a separated region (Taylor & Teunissen, 1983; Raithby et al., 1987; Castro et al., 2003; Lopes et al., 2007). The LES and hybrid solutions predict a separated region which is larger than the intermittent one indicated in the field data which leads to an overestimation of the velocity deficit. The RANS solutions fail to predict significant separation and as a result underestimate the velocity deficit. Figure 6.21 shows a comparison of the separated region predicted by Bechmann & Sørensen (2010) in plot (a). Plot (b) shows the cells around the hill with negative or near-zero streamwise velocities as predicted by the present work. There is good agreement in the distributions and it is clear that the present simulation predicts small reversed flows that have the potential to evolve into intermittent separation in a transient simulation.

The turbulent kinetic energies calculated along line A are shown in plot (b) of Figure 6.20 for each of the simulations. The results are also influenced by the issue of intermittent separation. Here the LES and the hybrid solutions again overestimate the effect of the separation and predict higher values for $k$ on the lee side of the hill. The present work and the RANS solution by Bechmann & Sørensen (2010) both underestimate $k$ as a result of under estimating the separation. The RANS simulation by Castro et al. (2003) predicts the value of $k$ well on the lee side of the hill but this is probably a result of over estimating it throughout the domain. As the plot shows, the change in $k$ predicted by their solution along line A is similar to the other two RANS solutions but on the upwind side of the hill it is overestimated by almost 100%. While the hybrid solution of Bechmann & Sørensen (2010) achieves the best match with the experimental data the ASBM closure from the present work also achieves good agreement, especially at the top of the hill.
Figure 6.21: Comparison of (a) separated zone (indicated in white) calculated by Bechmann & Sørensen (2010) and (b) cells with negative streamwise velocity (●) or near-zero streamwise velocity (●) for the present work with the ASBM closure. Positions of ten meter measuring towers on line A also shown (●).
The results for the components of the Reynolds stress tensor along line A are shown in Figure 6.22. Only results for the present work, the LES by Lopes et al. (2007) and the hybrid solution by Bechmann & Sørensen (2010) are shown as the other RANS simulations are not capable of representing the full Reynolds stress tensor. Three different grid resolutions are presented for the LES simulations. In plot (a) the results for the streamwise fluctuations reflect those for the turbulent kinetic energy with the simulation from the present work unable to capture the intermittent separation and therefore under estimating the turbulence on the lee side of the hill. However, upwind of the hill and in particular at the top of the hill, the ASBM closure in the present work provides the best estimate of the streamwise fluctuations. For the cross-stream fluctuations shown in plot (b) the ASBM closure in the present work achieves a reasonable agreement around the hill top but upwind the magnitude of the fluctuations is overestimated and on the lee side the separation issue also affects the results. The LES on the fine grid and the hybrid solutions both achieve better results. The over estimation upwind of the hill is probably caused by the inflow boundary conditions which Figure 6.19 shows are higher than the measured values for $\overline{uv}$.

Plot (c) of Figure 6.22 shows the results for the vertical fluctuations for which the ASBM closure achieves a good agreement. As expected the effect of the intermittent separation is not captured but for the rest of line A the ASBM closure outperforms the other simulations. The same is true of the results for the shear stress component in plot (d) where the agreement between the ASBM closure and the field data is excellent. The only point the ASBM closure does not match is at the top of the hill where it incorrectly predicts a positive value. This phenomenon was observed at the top of the hill in the simulations in Chapter 5 and while it does not appear to adversely effect the results it is an area for further work, as discussed in Chapter 9. The excellent match over the rest of the plot shows the capabilities of the new wall functions for the ASBM closure as they aim explicitly to achieve a good agreement in the shear stress component.

The remaining figures compare measurements of profiles made at the hill top with estimates from simulations by Bechmann & Sørensen (2010) and Lopes et al. (2007) and results from the present work. Figure 6.23 shows the speed-up at HT for each.
Figure 6.22: Comparison of Reynolds stress components (a) $u'u'$, (b) $v'v'$, (c) $w'w'$ and (d) $u'w'$ along line A between field measurements ($\bullet$), LES by Lopes et al. (2007) on a fine grid ($-$), medium grid (- -), coarse grid (-.-), LES (---) by Bechmann & Sørensen (2010) and the present work with the ASBM closure (---).
All the simulations achieve a good agreement with the field data but the present work using the ASBM closure is slightly better than the others, particularly close to the ground. In Figure 6.24 the profiles of the components of the Reynolds stress tensor are shown. The simulation results are only shown for the Lopes et al. (2007) LES solutions and the present work. For all of the components a reasonable agreement is achieved by each of the simulations given the sparse field data. The exception is the shear stress component where the ASBM closure incorrectly predicts positive values above 5 m as discussed earlier. However, below 5 m it more accurately matches the field data. For the streamwise fluctuations the fine LES simulation achieves the best agreement but for the vertical component the ASBM closure outperforms all of the LES simulations. For the cross-stream components all of the simulations achieve a good match.
Figure 6.24: Comparison of Reynolds stress components (a) $u'^2$, (b) $v'^2$, (c) $w'^2$ and (d) $w'u'$ at HT between field measurements (●), LES by Lopes et al. (2007) on a fine grid (—), medium grid (- -), coarse grid (.-) and the present work with the ASBM closure (--).
6.3.3 Discussion

In this section the method and results have been presented for simulations of wind flow over Askervein hill using the ASBM turbulence closure and the new wall functions. This is the first known application of the ASBM turbulence closure to simulate a full-scale, high $Re$, three-dimensional, real atmospheric flow. The results show that the simulations using the ASBM closure agree very well with the experimental data and compete well with LES and hybrid RANS/LES simulations which are also capable of resolving the Reynolds stress tensor.

The Askervein Hill Project was selected because it is one of the most well-known wind flow test cases and one of the best field data sets. In particular, the TU-03B data set has been used many times previously to validate numerical models of wind flow over terrain (Raithby et al., 1987; Castro et al., 2003; Undheim et al., 2006; Eidsvik, 2005, 2008; Lopes et al., 2007; Bechmann et al., 2007a,b; Bechmann & Sørensen, 2010). Setting up the simulation posed interesting problems and as well as applying the techniques described in Chapters 4 and 5, extra simulations of wind flow over open water and over a roughness change were carried out to achieve a good agreement with experimental data for the inflow profiles. The agreement in the gradients of the Reynolds stress components was particularly good which indicates the value of carrying out these additional simulations.

The results from the comparisons along line A over the top of the hill showed that like the other RANS methods the simulations from the present work could not estimate the intermittent separation identified previously by several authors (Taylor & Teunissen, 1983; Raithby et al., 1987; Castro et al., 2003; Lopes et al., 2007; Bechmann & Sørensen, 2010). This is not a short coming of the ASBM closure or the methods used in this work but rather a property of the RANS approach which can not capture unsteady behaviour. Apart from the impact of the intermittent separation the agreement between the ASBM closure and the experimental results is very good. Upwind of the hill the ASBM provides the best estimate of the speed-up of all the simulations compared. The ASBM closure is the only RANS simulation considered that is capable of capturing the Reynolds stress components. It also outperformed the LES and hybrid RANS/LES simulations at various points along line A. At the top of
the hill the ASBM closure incorrectly estimates a positive shear stress component as was observed in the two-dimensional simulations in Chapter 5. Apart from that point it achieved the best agreement of any of the simulations for the rest of the length of the line A for the shear stress component. This is also significant as it indicates that the new wall function performed very well because it acts explicitly to estimate the shear stress. The agreement between the experimental data and the present simulation result was also good for the profiles at the hill top with the ASBM closure and the LES simulations of Lopes et al. (2007) performing comparably.

The capability of the ASBM closure to estimate the steady parts of the flow as well or better than the LES and hybrid RANS/LES simulations is impressive. While it is more complex than standard RANS models it is still computationally much cheaper than either the LES or hybrid RANS/LES simulations. Details of the computational time required for the simulations by Lopes et al. (2007) and Bechmann & Sørensen (2010) were not published so direct comparisons are not possible. For the present work the full-scale simulation of Askervein hill with almost 9 million cells required

Figure 6.25: Computational speed-up of Askervein hill simulation through parallelisation. BestGRID cluster (+) and Engineering Science HPC (○)
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approximately 18 hours running on 96 processors of the BeSTGRID cluster at the University of Auckland. The cluster uses Intel® Xeon® X5650 processors running at 2.67GHz. Also, because of the computationally intensive nature of the simulations and the domain decomposition parallelisation the computational speed-up achieved was very good as is shown in Figure 6.25. Note that a result from the Engineering Science department’s HPC is also included. This machine has four Dual-Core AMD Opteron™ 8218 processors on each node running at 2.6 GHz. This demonstrates the potential for running similarly sized simulations of entire wind farm sites relatively quickly on large clusters.

Finally it should be emphasised that RANS simulations using the ASBM closure are not seen as a replacement for LES or Hybrid/RANS simulations. Rather there is a good opportunity for the approaches to be used together to provide accurate estimates of mean wind flow and turbulence over wind farm scale sites. A RANS approach using the ASBM closure could be used, as it has been in this section, to carry out steady simulations for a number of key wind directions at an acceptable computational cost. Where the results indicate the flow is steady they may be used directly to estimate mean velocity and all the components of the Reynolds stress tensor. Where the results indicate the possibility of unsteady flow, as on the lee side of Askervein hill, then less computationally expensive LES or Hybrid LES/RANS simulations could be carried out of the smaller area of interest. The results from RANS simulations using the ASBM closure could provide accurate, detailed boundary conditions for LES or Hybrid RANS/LES simulations because it solves for the full Reynolds stress tensor.
Chapter 7

Current Benchmark Simulations

The successful simulations discussed in Chapters 5 and 6 demonstrated that the ASBM closure combined with the new wall functions is capable of accurately predicting wind flow over complex terrain. However, none of the three-dimensional test cases studied in Chapter 6 provide detailed data for very complex flow regions including separation and reattachment. As discussed in Chapter 9 some recent studies have been carried out to provide this sort of validation data. During this project that data was not yet available and so smaller scale three-dimensional benchmark test cases were sought.

The Stanford asymmetric diffuser provides a suitable test case as good experimental and LES data (Cherry et al., 2006; Pecnik & Iaccarino, 2008) is available and the asymmetry generates a genuinely complex, three-dimensional flow. Then Section 7.2 describes the experimental and numerical set up for this test case. The next section presents the results for the simulation using the ASBM closure and discusses the difficulties encountered in simulating the flow in the corners of the diffuser. Attention is then turned to the simpler problem of a square duct flow and simulation results are presented and compared with DNS data. Important aspects of the physics are identified and areas where the ASBM closure can be improved are discussed.
7.1 Experimental and Simulation Setup

The Stanford asymmetric diffuser was originally studied because of its similarity to expansions in gas turbine engines. However, the large separated region and complex secondary flows are equally likely to occur in wind flow over complex terrain. These flow phenomena are particularly difficult or impossible for isotropic turbulence closures to predict (Pecnik & Iaccarino, 2008) which makes the Stanford diffuser a good test case for validating turbulence closures.

The dimensions of the diffuser are shown in plot (a) of Figure 7.1 and plot (b) shows a photo of the experimental equipment. The working fluid for the experiment was water with a gadolinium-based contrast agent. The velocity data was collected using magnetic resonance velocimetry which measures the components of the mean velocity vector in a three-dimensional volume. The $Re$ for the experiments was set to approximately 10000. For more details of the experiment refer to Cherry et al. (2006).

The LES simulations were carried out using Stanford’s unstructured, parallel CFD code CDP using a very large computational grid with 14 million cells. The RANS simulations were carried out on a smaller grid of 1.8 million cells using the commercial CFD software Fluent. More detailed information regarding the numerical simulations is available in Cherry et al. (2006) and Pecnik & Iaccarino (2008).

Because the initial objective was to validate the ASBM turbulence closure and not the new wall functions the grid used for the simulations in present work was the same as those used by Pecnik & Iaccarino (2008) which is a wall-resolved grid. The boundary conditions used were also the same which were no-slip walls and a simple velocity inlet and outflow. The velocity at the inflow was set to a constant value matching the experimental value. The computational domain also reflected the experimental set up in that the diffuser was sufficiently far downstream of the inlet for the flow to be fully developed.
7.1. EXPERIMENTAL AND SIMULATION SETUP

Figure 7.1: The Stanford asymmetric diffuser. (a) Dimensions (Pecnik & Iaccarino, 2008) and (b) photo of experimental equipment (Cherry et al., 2006).
CHAPTER 7. CURRENT BENCHMARK SIMULATIONS

7.2 Results for the Asymmetric Diffuser

As with the simulations in the previous chapters a solution was first obtained using the $v^2f$ closure and then simulations were carried out using a blending factor to increase the contribution of the ASBM closure. The results for the initial $v^2f$ simulations were identical to those obtained by Pecnik & Iaccarino (2008) and the streamwise velocity contours are shown in Figure 7.2. However, as the blending factor was increased to values over 50% it became increasingly difficult to achieve a converged solution. By 70% a converged solution could not be achieved regardless of the size of the blending factor increments. This was disappointing as instantaneous streamwise velocity contours showed that the solution agreed very well with the experimental data and the LES but that unsteady behaviour near the corners of the flow was preventing convergence. Figure 7.2 shows the streamwise velocity contours at several locations throughout the diffuser for the $v^2f$ and LES simulations and the experimental data Cherry et al. (2006); Pecnik & Iaccarino (2008). An instantaneous streamwise velocity contour calculated as part of the present work using 70% of the ASBM closure blended with the $v^2f$ closure is also shown.

Further investigation showed that in fact the unsteady behaviour in the corners developed long before the expansion and was present for much of the inflow duct. The inflow of the diffuser is simply a rectangular duct so the work for this project was then focused on the square duct problem.

7.3 Simulations of a Square Duct

Despite the physical simplicity of the square duct problem it is particularly challenging because of the secondary flows generated in the duct as a result of Reynolds stress imbalance (Pecnik & Iaccarino, 2008). However, it has the advantage that a number of DNS simulations have been carried out of the flow (Huser et al., 1994; Joung et al., 2007) including one by the team at TU Delft (private communication) who have collaborated with parts of the project presented in this thesis.
7.3. SIMULATIONS OF A SQUARE DUCT

ASBM 70 % - Instantaneous Contours

EXPERIMENT

LES

V²F Baseline

Figure 7.2: Comparison of cross-stream contours of streamwise velocity through the asymmetric diffuser’s expansion between different simulations and experimental data (Pecnik & Iaccarino, 2008).
The simulations presented in this section were set up to match the DNS carried out by the TU Deflt team. The dimensions of the domain and the specifications of the simulations are given in Table 7.1. The value of the viscosity was calculated to achieve a friction Reynolds number \( Re_\tau = 360 \) corresponding to a bulk \( Re \approx 5400 \).

The computational domain was uniform in the cross-stream directions and in the streamwise direction periodic boundary conditions were used so it was only a single cell wide.

### Table 7.1: Square Duct Simulation Specifications

<table>
<thead>
<tr>
<th>Simulation Specifications</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain dimensions (m)</td>
<td>0.1 × 1 × 1</td>
</tr>
<tr>
<td>Grid dimensions</td>
<td>1 × 192 × 192</td>
</tr>
<tr>
<td>Height of first cell (m)</td>
<td>0.00521</td>
</tr>
<tr>
<td>Stretching ratio</td>
<td>1.0</td>
</tr>
<tr>
<td>Kinematic viscosity, ( \nu ) (m(^2)/s)</td>
<td>0.00278</td>
</tr>
<tr>
<td>Pressure gradient, ( PG ) (N/m(^3))</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The simulations of the square duct using the ASBM closure show the same problem as experienced with the asymmetric diffuser, namely unsteady behaviour in the corners of the domain. Figure 7.3 shows the streamwise velocity component as calculated by the DNS, the \( v^2f \) closure and two different levels of blending with the ASBM closure. Like all eddy-viscosity closures, the \( v^2f \) closure models the Reynolds stress tensor as isotropic and as a result it is unable to predict the secondary flow (Pecnik & Iaccarino, 2008). In the streamwise velocity contours in plot (b) of Figure 7.3 this is apparent as a lack of streamwise velocity transported towards the corners by the secondary flow. For the converged solution using a 50%/50% blending with the ASBM closure shown in plot (c), the streamwise velocity is in better agreement with the DNS data but already too much secondary flow is calculated. At a 30%/70%
blending the secondary flows had become unsteady and a converged solution could not be obtained. The contour shown in plot (d) of Figure 7.3 is an instantaneous result obtained at this blending level.

Figure 7.3: Comparison of the streamwise velocity component between (a) DNS, (b) $\nu^2 f$ closure, (c) 50%/50% $\nu^2 f$/ASBM and (d) 30%/70% $\nu^2 f$/ASBM - instantaneous contour.

Figure 7.4 shows the cross-stream velocity vectors for the DNS data and simulations using two different blending levels. This demonstrates the problem in another manner as it can be seen that even in the converged solution in plot (b), the secondary flow is more complex than the DNS data calculates. The beginnings of an additional, smaller recirculation are present on either side of the diagonal close to the corner and note that the flow adjacent to the wall does not remain parallel to it. In the
instantaneous plot in (c) the secondary flow has become unsteady and the symmetry that should exist along the diagonal has been completely destroyed.

![Diagram of cross-stream velocity vectors](image)

Figure 7.4: Comparison of cross-stream velocity vectors between (a) DNS, (b) 50%/50% $v^2f$/ASBM and (c) 30%/70% $v^2f$/ASBM - instantaneous plot.

Further investigation suggested that the unsteady secondary flow results from physically incorrect wall blocking in the region close to a corner. To investigate this the ASBM closure was then used to post-process the DNS data so that direct, quantitative comparisons could be made. This was achieved by using the DNS data
as the initial conditions for the RANS simulation, advancing the simulation a single
iteration then inspecting the Reynolds stress components calculated by the ASBM
closure. Figures 7.5 and 7.6 show the results for all of the components of the Reynolds
stress tensor. The agreement for the normal stresses is good with the exception that
the $v'v'$ and $w'w'$ components have maxima which extend too far towards the corners
of the duct. The agreement for the off-diagonal components is also good generally.
However, close examination reveals that the $u'v'$ and $u'w'$ components do not change
sign as they cross the diagonal close to the corners. This means these stresses are
the wrong sign in this region. In the $v'w'$ component similar problems exist. Instead
of weak maxima or minima on the diagonals, the ASBM closure incorrectly predicts
strong maxima or minima on the diagonals and then opposing maxima or minima
immediately off the diagonals near the corners.

In their work Pecnik & Iaccarino (2008) demonstrated the importance of the
difference between the $v'v'$ and $w'w'$ components in determining the secondary flow
in a square duct. Figure 7.7 shows a comparison of the this difference calculated
for both the DNS data and predicted using the ASBM closure. The agreement is
generally good but the ASBM closure predicts that near the corners there is a sharp
change in the sign of the quantity at the diagonal. The DNS data shows this change
is much smoother in reality.

More insight can be gained by examining the Lumley triangles and the barycentric
maps for the flow presented in Figures 7.8, 7.9 and 7.10. A Lumley triangle is a
commonly used diagram that shows the state of the turbulence in terms of the two
independent invariants of the anisotropy tensor (Pope, 2000). Barycentric maps are
an alternative way to plot the state of the turbulence by using its eigenvalues and
also prove a useful tool for understanding the flow (Banerjee et al., 2007). Plot (a)
in Figures 7.8, 7.9 and 7.10 shows a path plotted in the solution domain with the red
and the green indicating its start and it finish. The adjacent plots show the Lumley
triangle and the barycentric maps for this path. Plots (b) and (c) corresponds to the
DNS data while plots (d) and (e) show the predictions of the ASBM closure.

Figure 7.8 shows that mid-stream the predictions of the ASBM closure agree well
with the DNS data, moving from the center of the flow to the wall. The barycentric
Figure 7.5: Comparison of DNS results for (a) $u' u'$, (b) $v' v'$ and (c) $w' w'$ with results calculated using the ASBM closure for (d) $u' u'$, (e) $v' v'$ and (f) $w' w'$. All plots in $m^2/s^2$ using contour levels indicated for each component.
Figure 7.6: Comparison of DNS results for (a) $\overline{u'v'}$, (b) $\overline{u'w'}$ and (c) $\overline{w'v'}$ with results calculated using the ASBM closure for (d) $\overline{u'v'}$, (e) $\overline{u'w'}$ and (f) $\overline{w'v'}$. All plots in m$^2$/s$^2$ using contour levels indicated for each component.
map shows that along this path the turbulence changes from 3-component towards 1-component until very close to the wall where the DNS data shows it goes to 2-component in the limit of the wall. The ASBM closure predicts this correctly except very near the wall where instead of 2-component it returns to 3-component turbulence. For the path running near to the wall in Figure 7.9 the agreement between the ASBM closure and the DNS is also quite good with both remaining predominantly 1-component. However after crossing the diagonal (indicated by the magenta point) and nearing the corner the results from the ASBM closure rapidly move to a 3-component state again. The poorest agreement is seen in Figure 7.10 where the path moves from the center to a corner, along the diagonal. Here the DNS data clearly shows the turbulence moves from 3-component in the center to a 1-component state in the corner. Whereas the ASBM closure predicts a return to 3-component turbulence at the corner after approaching towards 1-component turbulence along the path.
Figure 7.8: Lumley triangles and barycentric maps comparing the state of the turbulence along the mid-stream path shown in (a) between DNS data (b) and (c), and calculated using the ASBM closure (d) and (e).
Figure 7.9: Lumley triangles and barycentric maps comparing the state of the turbulence adjacent to the wall along the path shown in (a) between DNS data (b) and (c), and calculated using the ASBM closure (d) and (e).
7.3. SIMULATIONS OF A SQUARE DUCT

Figure 7.10: Lumley triangles and barycentric maps comparing the state of the turbulence along the diagonal path shown in (a) between DNS data (b) and (c), and calculated using the ASBM closure (d) and (e).
7.4 Discussion

To validate the ASBM closure against accurate data for more complex three-dimensional flows the Stanford asymmetric diffuser was selected as a test case problem. The asymmetry in the diffuser leads to a truly complex but steady three-dimensional flow that standard eddy-viscosity closures have difficulty predicting. The availability of good quality experimental and LES data also make the asymmetric diffuser a good benchmark problem.

Unfortunately simulations using a blending of more than 60% ASBM closure resulted in unsteady behaviour near the corners of the diffuser that prevented the solution converging. This was particularly frustrating because the instantaneous results showed that the mean flow away from the corners had achieved a very good agreement with the experimental data. Investigations revealed that the unsteady behaviour in the corners occurred well upstream of the diffuser and so the focus was turned to flow through a square duct.

The problem of flow through a square duct is also a useful benchmark problem as standard eddy-viscosity closures are not capable of predicting the secondary flows which are a result of Reynolds stress imbalance. As expected, simulations of the square duct also developed unsteady behaviour near the corners when the blending of the ASBM closure was increased above 60%. Using the ASBM closure to post-process DNS data allowed direct comparisons to be made. This revealed that the ASBM closure did not predict the Reynolds stress tensor well near the corners of the flow. The Reynolds stress imbalance was too great near the diagonal which caused too much secondary motion. Using Lumley triangles and barycentric maps showed that near the corners of the flow the ASBM closure predicts the state of the turbulence incorrectly. Strategies to address these issues are currently being investigated as part of an ongoing research project and are discussed in Chapter 9.

Despite the difficulties posed by these two test cases, the results discussed in this chapter are very positive. Away from the corners the ASBM closure performed very well at estimating truly complex three-dimensional flows. The issues caused by corners are unlikely to affect wind flow over terrain except in the most extreme
circumstances. Regardless of this fact, due to the insight gained through the work discussed in this chapter it is anticipated that improvements to the ASBM closure will address the issue of unsteady behaviour in the corners of flows in the near future.
CHAPTER 7. CURRENT BENCHMARK SIMULATIONS
Chapter 8

Conclusion

The study of wind flow over complex terrain and its accurate prediction has become increasingly important. However, numerical methods still often rely on using linearised models beyond their range of applicability or CFD approaches with standard turbulence closures that cannot accurately predict complex flows. In particular, standard CFD approaches give poor estimates of separation and are not capable of representing the anisotropy of turbulence. The objective of this thesis was to develop an approach for modelling wind flow over complex terrain that is capable of capturing the dynamics of separation and accurately estimating the anisotropy of turbulence.

There are several different CFD frameworks currently available for simulating atmospheric flows, each with advantages and disadvantages. For the work in this project a RANS approach was selected for several reasons. The most important were its ability to simulate wind farm sized domains at a reasonable computational cost, its position as the most commonly used CFD approach in industry and the potential for improvements to flow through to hybrid and LES approaches. The turbulence closures selected to use with the RANS approach were the $v^2f$ closure and the ASBM closure because of key properties they possess. The $v^2f$ closure models the near-wall anisotropy and has been shown to predict separation well. The ASBM closure is based on a family of closures that take the structure of turbulence into account. It calculates the full Reynolds stress tensor and does not require the solution of additional transport equations. The simulations in this project are the first known
applications of the ASBM closure to atmospheric flows.

One of the necessities of simulating wind flow over terrain is that a model is required for the interaction between the flow and the ground. Standard wall functions provide an adequate model by assuming that the mean flow is parallel to the ground and the log law applies. To apply the $v^2f$ and ASBM closures to simulations of wind flow over complex terrain estimates of the wall-normal fluctuations and the Reynolds stress components adjacent to the ground were required. Using a combination of fundamental physics and previously published results new wall functions were developed that provided these estimates for both smooth surfaces and rough terrain. The same scalings were applied for key parameters as are used in the standard wall functions to ensure that new wall functions are consistent. The new wall functions developed for the ASBM closure are general models that can be used with other closures that calculate the full Reynolds stress tensor. The results of the simulations carried out during this project demonstrate the robustness and accuracy of these new wall functions.

Many turbulence closures which calculate the full Reynolds stress tensor have issues with stability. The ASBM closure is no exception and in previous work its highly nonlinear nature led to problems achieving stable solutions. As part of this project a new algorithm was developed that blends the ASBM closure with an eddy-viscosity closure. Using this algorithm to gradually move towards solution using only the ASBM closure greatly improves its stability. This algorithm could also be applied to other closures that calculate the full Reynolds stress tensor.

A method for generating inflow boundary conditions that are consistent with the ground boundary conditions and the top boundary conditions were described. By using this approach a horizontally homogeneous boundary layer free of streamwise gradients was simulated. This successful simulation enabled a quantitative investigation of the impact of using inconsistent boundary conditions. The results showed that using inflow boundary conditions that are inconsistent with the boundary conditions at the top of the domain causes systematic errors throughout the domain and particularly near the ground. Analytic inflow boundary conditions cause relatively small errors if the top boundary conditions that are used are consistent with them. These results were found to also apply when obstacles were introduced into the flow.
An application was developed which extended the standard RANS approach to include a global pressure gradient and Coriolis effects. It was used to generate realistic Ekman profiles which can be used as inflow profiles for simulations with large domains where the log law does not apply throughout. For simulations of moderate sized hills this is often the case and simulations showed that errors were introduced if log law inflow profiles were used rather than the Ekman profiles.

A number of simulations of flows over a two-dimensional hill were carried out to validate the $\nu^2 f$ and ASBM closures and their new wall functions. While the flows selected have relatively low $Re$ they include significant separation which provided a good test. Accurate and detailed experimental data was available for both smooth and rough hills. For the smooth hill comparisons were made between the results obtained by simulations in this work, RANS simulations carried out by other researchers and the experimental data. The simulation using the ASBM closure outperformed all of the other simulations and accurately represented the flow. The separated region was captured well and the anisotropy of the turbulence was predicted well. Of the closures that did not resolve the full Reynolds stress tensor, the $\nu^2 f$ closure achieved the best agreement with the experimental data. A number of higher $Re$ simulations of a smooth hill were carried out to test the agreement between the solutions obtained using the new wall functions and wall-resolved boundary conditions. The results showed a good agreement for both of the new wall functions though like standard wall functions they are not as accurate in the separated region. The simulations of the rough hill reinforced the results of the smooth hill simulations. The simulation using the ASBM closure outperformed those using other closures and it accurately captured the separated region and represented the turbulence anisotropy.

Simulations of fully three-dimensional flows were carried out to validate the ASBM closure and new wall functions further. Preliminary three-dimensional simulations showed that the method developed during this project could be applied successfully to large-scale, high $Re$, real three-dimensional flows over complex terrain. Comparisons were then made with two well-known wind flow experiments, one at the laboratory-scale and one full-scale. The first was the wind-tunnel experiments of Kettles hill for which detailed profiles of mean wind velocity and turbulence are available.
issues surrounding the aerodynamic roughness the results showed that simulations using the ASBM closure were able to accurately estimate both the mean wind velocity and the turbulence. The second experiment was the full-scale study of Askervein hill. A method for achieving good agreement with the reference site wind data was presented and the results for a simulation using the ASBM closure were compared with the field data and previously published results. It was found that apart from a region on the lee side of the hill where intermittent separation is thought to occur, the simulation using the ASBM closure accurately predicts the experimental data. When compared with the other simulations it was found to perform as well as the LES and hybrid RANS/LES solutions in most aspects. In some aspects, such as estimating the shear stress it was the most accurate closure considered. This indicates the effectiveness of the new wall functions which explicitly models the shear stress.

Two three-dimensional test cases were investigated involving complex flows that are difficult to simulate, and impossible to simulate using isotropic turbulence closures. While the flows have not yet been simulated successfully due to unsteady behaviour near the corners of the flows, a number of important insights were gained and possible improvements to the ASBM closure identified.

In summary, an effective and robust approach for modelling wind flow over complex terrain has been developed using the ASBM turbulence closure and new wall functions developed as part of this work. The method has been validated against a number of representative two-dimensional and three-dimensional flows and also against a well-know full-scale three-dimensional flow. Accurate information is predicted for both the mean wind velocity and the full Reynolds stress tensor. Separation and re-attachment are also captured well. Because of the success of this project is envisaged that this approach will be adopted more widely and further validation of the ASBM closure and the new wall functions will occur.
Chapter 9

Future Work

The results of the two-dimensional simulations carried out in Chapter 5 and the three-dimensional simulations in Chapter 6 showed that ASBM closure incorrectly predicts positive values for the shear stress component of the Reynolds stress tensor near the top of a hill. It is suspected that this occurs because the mean strain rate that is generated by the curvature of the hill is not interpreted correctly by the ASBM closure. While this did not have a major impact on the results presented, work could be carried out using DNS data of flow over a hill (Laval & Marquillie, 2010) that is now available to understand and correct the problem.

Similarly, errors were observed in the region near the ground at the top of the hills when wall functions were used. This was the case for both the standard wall functions and the new wall functions developed in this work. It is likely this is because the speed-up that occurs due to the curvature is incorrectly interpreted by the wall functions as an increased friction velocity. The DNS data from Laval & Marquillie (2010) could also be used to assess the wall functions response to curvature and to develop corrections.

The results presented in Chapter 7 demonstrated the need to develop a method for incorporating the effect of corners in the ASBM closure. The current wall blocking model described in Chapter 2 is based on the elliptic relaxation model proposed by Durbin (1991) but uses only a single parameter, $\phi$ to calculate the influence of the wall. In the presence of multiple walls, such as at a corner, each wall must affect the flow
and will play a part in determining the state of the turbulence as highlighted by the barycentric maps in Chapter 7. By improving the wall blocking to include the effect of multiple walls it is anticipated that the ASBM will be able to better represent the physical behaviour in the corners of the test case flows and hence converged, accurate solutions may be obtained.

Finally the approach described in this report should be tested against more complex and difficult validation data as it becomes available. In particular, a simulation over the steep, three-dimensional hill studied in the Bolund project (Berg et al., 2011) could be carried out. The Bolund experiment is the perfect next validation test case as it specifically is designed to provide a new dataset for validating models of flow in complex terrain. Also, additional simulations of the potential wind farm ridge near Palmerston North could be carried out using the ASBM closure for a number of wind directions and the results compared with field data as it is acquired.
Publications and Presentations

Papers accepted for publication


Presentations


- O’Sullivan, J. P., Pecnik, R., & Iaccarino, G. 2010, Applying the $\nu^2 f$ and the algebraic structure-based Reynolds stress closures to wind flow over complex terrain, *APS Division of Fluid Dynamics 63rd Annual Meeting*, Long Beach, USA.

Appendix A

Derivation of Equation 3.5

Beginning with the steady momentum equation (2.11):

\[
\frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial u'_i u'_j}{\partial x_j} \tag{A.1}
\]

For the simulations in this project the density is constant so it may be absorbed into the mean pressure \(\bar{p} = \bar{p}/\rho\):

\[
\frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial u'_i u'_j}{\partial x_j} \tag{A.2}
\]

Using the Boussinesq approximation equation (2.13):

\[
\frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( 2\nu T S_{ij} - \frac{2}{3} k \delta_{ij} \right) \tag{A.3}
\]

\[
\frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( 2\nu T S_{ij} \right) - \frac{2}{3} \frac{\partial k}{\partial x_i} \tag{A.4}
\]

\[
\frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial}{\partial x_i} \left( \bar{p} + \frac{2}{3} k \right) + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( 2\nu T S_{ij} \right) \tag{A.5}
\]
Defining the modified pressure as in equation (2.17) and using the definition of the strain rate equation 2.14.

\[
\frac{u_j}{\partial u_i} \frac{\partial u_i}{\partial x_j} = - \frac{\partial \hat{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left[ \nu_T \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]
\]  
(A.6)

\[
\frac{u_j}{\partial u_i} \frac{\partial u_i}{\partial x_j} = - \frac{\partial \hat{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu + \nu_T \right) \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial u_j}{\partial x_i} \right)
\]  
(A.7)

where

\[
\frac{\partial R_{ij}^*}{\partial x_j} = - \frac{\partial}{\partial x_j} \left( \nu + \nu_T \right) \frac{\partial u_i}{\partial x_j} - \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial u_j}{\partial x_i} \right)
\]  
(A.9)
Appendix B

Derivation of Equation 3.8

Beginning again with the steady momentum equation (2.11):

\[ \frac{u_j}{u_i} \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial u'_i u'_j}{\partial x_j} \]  \hspace{1cm} (B.1)

Using a blend of the Boussinesq approximation equation (2.13) and solving the full Reynolds stress tensor:

\[ \frac{u_j}{u_i} \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) + (1 - \alpha) \left[ \frac{\partial}{\partial x_j} \left( 2\nu T_{S ij} - \frac{2}{3} k \delta_{ij} \right) \right] + \alpha \left[ -\frac{\partial u'_i u'_j}{\partial x_j} \right] \]  \hspace{1cm} (B.2)

Splitting the viscous term between the two turbulence terms:

\[ \frac{u_j}{u_i} \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x_i} + (1 - \alpha) \left[ \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( 2\nu T_{S ij} - \frac{2}{3} k \delta_{ij} \right) \right] + \alpha \left[ \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial u'_i u'_j}{\partial x_j} \right] \]  \hspace{1cm} (B.3)
Using the definition of the strain rate equation (2.14).

\[
\frac{u_j}{u_i} \frac{\partial u_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + (1 - \alpha) \left[ \frac{\partial}{\partial x_j} \left( \nu + \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{2}{3} \frac{\partial k}{\partial x_i} \right] \\
+ \alpha \left[ \frac{\partial}{\partial x_j} \nu \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right] \quad (B.4)
\]

\[
\frac{u_j}{u_i} \frac{\partial \bar{u}_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + (1 - \alpha) \left[ \frac{\partial}{\partial x_j} \left( \nu + \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{2}{3} \frac{\partial k}{\partial x_i} \right] \\
+ \alpha \left[ \nu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] \quad (B.5)
\]

\[
\frac{u_j}{u_i} \frac{\partial \bar{u}_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu + \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) + (1 - \alpha) \left[ \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{2}{3} \frac{\partial k}{\partial x_i} \right] \\
+ \alpha \left[ \nu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] \quad (B.6)
\]

\[
\frac{u_j}{u_i} \frac{\partial \bar{u}_i}{\partial x_j} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu + \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) + (1 - \alpha) \left[ \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] \\
- \frac{2}{3} \frac{\partial k}{\partial x_i} + \frac{2}{3} \alpha \frac{\partial k}{\partial x_i} + \alpha \left[ \nu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] \quad (B.7)
\]

Using the definition of the modified pressure as in equation (2.17):

\[
\frac{u_j}{u_i} \frac{\partial \bar{u}_i}{\partial x_j} = - \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu + \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) + (1 - \alpha) \left[ \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] \\
+ \alpha \left[ \frac{2}{3} \frac{\partial k}{\partial x_i} - \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{u}_i}{\partial x_j} \right) \right] \quad (B.8)
\]
\[
\frac{u_j \partial u_i}{\partial x_j} = -\frac{\partial \hat{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( (\nu + \nu_T) \frac{\partial u_i}{\partial x_j} \right) \\
+ \frac{\partial}{\partial x_j} \left[ (1 - \alpha) \nu_T \frac{\partial u_i}{\partial x_j} - \alpha \left( \nu_T \frac{\partial u_i}{\partial x_j} + u_i' u'_j - \frac{2}{3} k \right) \right] \quad (B.9)
\]

Giving the final form of equation (3.8):

\[
\frac{u_j \partial u_i}{\partial x_j} + \frac{\partial R^*_{ij}}{\partial x_j} = -\frac{\partial \hat{p}}{\partial x_i} \quad (B.10)
\]

where

\[
\frac{\partial R^*_{ij}}{\partial x_j} = -\frac{\partial}{\partial x_j} \left( (\nu + \nu_T) \frac{\partial u_i}{\partial x_j} \right) \\
- \frac{\partial}{\partial x_j} \left[ (1 - \alpha) \nu_T \frac{\partial u_i}{\partial x_j} - \alpha \left( \nu_T \frac{\partial u_i}{\partial x_j} + u_i' u'_j - \frac{2}{3} k \right) \right] \quad (B.11)
\]
APPENDIX B. DERIVATION OF EQUATION 3.8
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