Coronary Flow Mechanics

An Anatomically Based Mathematical Model of Coronary Blood Flow Coupled to Cardiac Contraction

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Abstract

Coronary blood flow through the ventricular contraction cycle has been investigated in this thesis using an anatomically accurate computational model.

Using Strahler ordered morphological data and an avoidance algorithm a three dimensional finite element model has been constructed of the largest six generations of the coronary arterial network within an anatomically accurate finite element model of the left and right ventricles. Segment radii, lengths and connectivity are consistent with the literature, local network branch angles are consistent with the principle of minimum shear stress at bifurcations, and an even spatial distribution of vessel segments throughout the myocardium has been achieved.

A finite difference collocation grid has been generated on the coronary finite element mesh. The Navier Stokes equations governing blood flow through elastic vessels have been reduced to one dimension and are solved on this finite difference grid using the two step Lax Wendroff method. Blood flows at bifurcations are calculated using an iterative method ensuring conservation of mass and momentum. The microcirculation networks are modelled using a lumped parameter model incorporating the nonlinear variation of resistance and compliance with pressure by fitting results from published anatomical data. The venous network is assumed to parallel the generated arterial model. The calculated blood flow through the network model demonstrates physiological pressure drops, flow rates and a regional distribution within the ventricular geometry consistent with experimental data.

The intramyocardial pressure (IMP) exerted on the coronary vasculature during contraction is calculated from quasi-static solutions of the equations of finite deformation applied to the ventricular model with a nonlinear anisotropic constitutive law. IMP is found to vary approximately linearly between ventricular pressure at the endocardium and atmospheric pressure at the epicardium.

IMP and vessel stretch are included in the transmural pressure radius relationship to model

the effect of myocardial deformation on coronary flow. The calculated coronary blood flow through the contraction cycle shows that arterial flow is predominantly diastolic while venous flow is significantly increased during systole. Calculated time varying velocity profiles in the large epicardial vessels compare well with published experimental results. Regionally averaged velocities in small vessels show that arterial inflow is most significantly impeded at the left ventricular endocardium. Furthermore, the large time constant associated with the capillary and venule networks limits the filling of these vessels during diastole particularly at the endocardium.

An increase in heart rate, modelled by reducing the time for diastole causes an increase in small vessel epicardial blood flow and a decrease in blood flow through small vessels within the myocardium. The decrease in flow is most pronounced at the left ventricular endocardium.

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List of Symbols

The following symbols (listed by chapter) are used in this thesis :

λ,μ,θ	Prolate Spheroidal coordinates
x, y, z	Rectangular Cartesian coordinates
$ER(n, m)$	the network connectivity for parent order n and daughter vessel m
$SN(n)$	the segment to element ratio of order n
$\boldsymbol{\xi}$ (ξ_1, ξ_2, ξ_3)	local element coordinate system
$\boldsymbol{z}$	data point position
$\hat{\boldsymbol{z}}$	data point orthogonal projection
ψ_i	finite element weighting function for node i
$\boldsymbol{p}$	the newton Raphson step in ξ space
U_n	the vessel avoidance source term
β_v	the vessel avoidance exponent
U_b	the boundary avoidance source term
β_b	the boundary avoidance exponent
U_l	the local junction avoidance source term
β_3	the local junction avoidance exponent
r, θ, x	Cylindrical coordinates
t	time
(v_r, v_θ, v_x)	radial, circumferential and axial velocity respectively

r^*, x^*, t^*	non dimensionalised radius, axial distance and time respectively
v_r^*, v_x^*	non dimensionalised radial and axial velocity respectivly
ρ	blood density
ν	blood viscosity
p	internal vessel pressure
R	inner vessel radius
V	average cross sectional velocity
α	vessel cross section velocity profile parameter
R_o	unstressed vessel radius
G_o	vessel pressure area parameter
β	vessel pressure area exponent
Δ_x	finite difference space step
Δ_t	finite difference time step
ω	finite difference scheme amplification factor
C_o	vessel wave speed
Q	vessel flowrate
S	vessel area
F_a, F_b, F_c	flow through the parent and two daughter segments of a bifurcation respectively
p_a, p_b, p_c	pressures of the parent and two daughter segments of a bifurcation respectively
p_o	bifurcation junction pressure
$\boldsymbol{s}$	the Newton Raphson step for bifurcation pressures
R_a, R_c, R_v	arterial, capillary and venule resistance lumped parameter resistances respectively
C_1, C_2	the lumped parameter capacitance values
c_i	microcirculation segment conductance values
K	microcirculation shape factor
Nu	microcirculation recruitment factor
p_t	microcirculation transmural pressure
N_{1-8}	coefficients for the numerator polynomial in the lumped parameter models
D_{1-8}	coefficients for the denominator polynomial in the lumped parameter models
F_a, F_c, F_v	arteriole, capillary, venule microcirculation flows

P_a, P_1, P_2, P_v	arteriole, distal capillary, proximal capillary, and venule pressures
μ_f, σ_f	mean and standard deviation of flow for a given myocardial chunk volume
$h(t)$	probability density function of transit times
(X_1, X_2, X_3)	material Cartesian coordinates
(x_1, x_2, x_3)	reference Cartesian coordinates
(ν_1, ν_2, ν_3)	micro structure material coordinates
(Y_1, Y_2, Y_3)	Cartesian material vessel coordinates
(y_1, y_2, y_3)	Cartesian reference vessel coordinates
$(\theta_1, \theta_2, \theta_3)$	curvilinear reference coordinates
$\boldsymbol{T}$	2nd Piola-Kirchoff stress tensor
$\boldsymbol{A}^{(\nu)}$	the covariant base vector for the undeformed ν coordinates
$\boldsymbol{\sigma}$	Cauchy stress tensor
$\boldsymbol{F}$	the deformation gradient tensor
J	the Jacobian for coordinate transformations
t_{wall}	average stress acting normal to the vessel direction
λ	vessel stretch
λ_o	residual vessel stretch
β_1, β_2	vessel stretch pressure radius exponents
F_o	vessel collapse parameter
κ	vessel collapse exponent
$\boldsymbol{C}$	Green's deformation tensor
I_1, I_2, I_3	principle strain invariants
W	strain energy function
$G_{oA}, \beta_A, \beta_{2A}$	arterial vessel pressure area parameters
$G_{oV}, \beta_V, \beta_{2V}, \kappa_V$	venous pressure area parameters

List of Acronyms

The following acronyms (listed in the order that they appear) are used in this thesis :

IMP	Intramyocardial Pressure
LV	Left Ventricle
RV	Right Ventricle
LAD	Left Anterior Descending Coronary Artery
RCA	Right Coronary Artery
CX	Circumflex Coronary Artery
RMS	Root Mean Squared Error
RD	Relative Dispersion
SPQR	Super Convergent Patch Recovery
LVP	Left Ventricular Pressure
RVP	Right Ventricular Pressure
D	diastole
IC	isovolumic contraction
E	ejection
IR	isovolumic relaxation
FTI	Flow Time Integral

Notation

- For variables at finite difference points the superscript represents the time step and the subscript represents spatial position. For example V_i^k is the quantity V at point i and time step k.
- In general, uppercase letters are used for material coordinates and lowercase letters are used for reference coordinates.
- Mathematical variables represented by bold lowercase letters refer to vector quantities, while bold uppercase letters refer to tensor quantities, except for some finite elasticity tensors where uppercase is used for the reference configuration and lowercase for the deformed configuration.
- The mechanics section of this thesis uses the Einstein summation convention, where repeated indices imply summation over the individual components. For example a vector dot product of two N component vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ may be written as :

$$a_i b_i = \boldsymbol{a} \cdot \boldsymbol{b} = \sum_{i=1}^{N} a_i b_i$$