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NONLINEAR STRUCTURAL ANALYSIS USING STRUT-AND-TIE MODELS

A thesis submitted in partial fulfilment of the
requirements for the degree of Doctor of Philosophy
in Civil Engineering at the University of Auckland

– by –

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ABSTRACT

Increasing popularity of the strut-and-tie methodology among research communities and practising engineers is due to its rational analytical approach and its superiority, compared to the conventionally employed empirical methods for analysing disturbed regions in structural systems. Nevertheless, this analysis methodology is not used as a routine procedure in design offices, primarily because of the perceived ambiguity and complexity involved in appropriate model formulation. In addition, until recently application of the strut-and-tie methodology has been limited to the prediction of strength, with utilisation of this modelling technique to capture nonlinear structural deformation being rather minimal [ACI Bibliography (1997)].

The research project reported herein represents an original contribution to the development of the strut-and-tie methodology by providing a systematic approach for applying this modelling technique to nonlinear structural concrete analyses. The study proposes a orginally developed computer-based strut-and-tie model formulation procedure that permits prediction of the nonlinear monotonic and cyclic response of structural systems with distinct reinforcement details. The procedure being presented in this thesis is a refined version of that reported previously [To *et al.* (2001 & 2002b)] and the accuracy of the analytical modelling is verified using experimental data.

Several issues pertaining to model formulation are thoroughly investigated. These issues include the strategy of model formulation for Bernoulli (or beam) and disturbed regions of structural systems, the satisfactory positioning of model elements, the appropriate stress-strain material models for concrete and reinforcing steel, the suitable effective strength of model elements, the inclined angle of diagonal concrete struts in beam and column members, and the concrete tension carrying capacity and associated tension stiffening effect.

In addition, the seismic response of various prototype structures when subjected to the experimentally employed cyclic forces and the time-history earthquake loadings was predicted using the originally developed cyclic strut-and-tie models. A summary encapsulating the findings of this project and recommendations for future research work in the area of nonlinear strut-and-tie modelling is also presented.

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LIST OF SYMBOLS

a = development length of ultimate bond stress

A_g = gross section area

A_p = total prestressed reinforcement area

A_s = flexural tension reinforcement area

A'_s = flexural compression reinforcement area

A_{cs} = area of concrete struts in B-regions

A_{ct} = area of concrete ties in B-regions

A_{rs} = area of rebar struts in B-regions

A_{rt} = area of rebar ties in B-regions

A_{st} = total area of longitudinal reinforcement in column sections

A_{s-t} = area of rebar strut-tie for cyclic strut-and-tie models

A_v = area of transverse rebar ties

A_{vs} = total area of transverse reinforcement in a single layer parallel to the applied shear

A_{ve} = effective seaction area for carrying shear

b_o = concrete core width measured from centreline to centreline of longitudinal rebars

b_w = total section width

c = neutral axis depth measuring from extreme compression edge

c_c = concrete coverage

$C_{c(max)}$ = maximum concrete flexural compression

C_s = total reinforcement compression at yielding

d_b = flexural rebar diameter

d_v = effective section depth

d_{vs} = transverse rebar diameter

D' = diameter of circular concrete core measuring from centre to centre of peripheral hoops

D_c = total diameter of the circular sections

D_o = depth of concrete core measured from centreline to centreline of longitudinal rebars

D_r = total depth of the rectangular column sections

E_c = concrete elastic modulus

E_cA_e = effective section stiffness

E_cA_g = gross section stiffness

E_cI_e = effective flexural stiffness

E_cI_g = gross flexural stiffness

E_s = reinforcing steel elastic modulus

f_2 = compressive stress in diagonal concrete struts

f_c = concrete compressive stress

f_{cont} = contact stress developed across concrete cracks

f_{cy} = effective strength of rebar struts in structural B-regions

f_c' = concrete cylinder strength

f_{cc}' = confined concrete compressive strength

f_{cr} = concrete cracking strength

f_{ct} = concrete tensile stress in a prism member

f_{cts} = average concrete tensile stress in the member sections

f_d = compressive strength of concrete struts in structural B-regions

f_{dt} = tensile strength of concrete ties in structural B-regions

f_p = stress in prestressed reinforcement

f_s = stress in reinforcement

f_{sy} = yield strength of rebar ties in structural B-regions (for monotonic models)

f_{s-t} = yield strength of rebar ties in structural B-regions (for cyclic models)

f_t' = plain concrete tensile strength

f_{ts} = average value of cracked concrete tension carrying capacity (for cyclic models)

f_{ult} = reinforcement ultimate tensile strength

f_y = measured yield strength of flexural reinforcement

f_v = shear stress in the member sections

f_{vy} = measured yield strength of transverse reinforcement

h_p = parpendicular distance between diagonal concrete struts in structural B-regions

ℓ_c = rebars development length

ℓ_{pj} = length of joint-links

ℓ_s = lap splice length of rebars

ℓ_t = length required to develop full bond stress between rebars and the surrounding concrete

ℓ' = half length of concrete ties

M_y^{1st} = moment measured at the serviceability limit state

n = ratio of E_s / E_c

N = externally applied column axial load

P = externally applied tension

$P_{\ell p}$ = lap splice capacity

p'' = volumetric ratio of transverse reinforcement

p_ℓ = cross-sectional length of rupture surface between the lap spliced rebars

r_o = radius of circular concrete core measuring from section centre to the centreline of longitudinal rebars

s = pitch distance between transverse reinforcement

s_L = surface area of reinforcement per unit volumn of concrete

s_R = flexural reinforcement spacing

T_s = maximum tension in reinforcement before yielding develops in flexural members

t = thickness of the imaginary flexural reinforcement tube

u_m = bond stress between reinforcment and concrete

u_{ult} = ultimate bond stress between reinforcement and concrete

v = total shear stress resisted by concrete and transverse reinforcement

V_n = Member shear strength

V_s = transverse reinforcement shear contribution

V_c = concrete shear contribution

V_p = shear contribution from axial force component

x_c = position of flexural compression centroid, measuring from the extreme compression edge

x_t = position of flexural tension centroid, measuing from the extreme compression edge

α_N = angle between member longitudinal axis and the line of externally applied axial action

β_t = empirical factor dicting the slope of descending branch of the tension stiffening model

ε_1 = average principal tensile strain

ε_2 = average principal compressive strain

ε_c = concrete compressive strain

ε'_c = concrete strain at f'_c

ε'_{cc} = ultimate concrete compressive strain

ε_{ct} = concrete compressive strain at f_{ct}

ε_{50} = concrete compressive strain at $0.5f_d$

ε_{dt} = concrete tensilestrain at f_{dt}

ε_s = reinforcement tensile strain

ε_{sh} = reinforcement tensile strain at the begining of strain hardening

ε_t = average member strain in transverse direction

ε_u = reinforcement tensile strain at f_{ult}

ε_x = average member strain in longitudinal direction

ε_y = reinforcement yield strain

γ = Poisson ratio

θ = angle between diagonal concrete strut and member longitudinal axis;

ρ = ratio of A_{rt}/A_{ct}

ρ_l = ratio of A_{st}/A_g

ρ_w = ratio of A_s/A_g

σ_{ct} = peak stress in concrete being transferred from the rebars through bonding

$\bar{\sigma}_{ct}$ = average stress in concrete being transferred from the rebars through bonding

ϕ = half angle of the fan shaped compression sector, measured to the circular section edge

ϕ' = half angle of the fan shaped compression sector, measured to the centre line of imaginary flexural reinforcement tube

ϕ_c = 0.85, efficiency factor for evaluating the concrete compressive strength under cyclic loading

ϕ_o = 4/3, over strength factor for evaluating the effective strength of flexural rebar ties in circular columns for monotonic models

ϕ_r = 3/4, reduction factor for evaluating the effective area of flexural rebar strut-tie