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NONLINEAR STRUCTURAL ANALYSIS USING STRUT-AND-TIE MODELS

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Civil Engineering at the University of Auckland

— by —

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March 2005
ABSTRACT

Increasing popularity of the strut-and-tie methodology among research communities and practising engineers is due to its rational analytical approach and its superiority, compared to the conventionally employed empirical methods for analysing disturbed regions in structural systems. Nevertheless, this analysis methodology is not used as a routine procedure in design offices, primarily because of the perceived ambiguity and complexity involved in appropriate model formulation. In addition, until recently application of the strut-and-tie methodology has been limited to the prediction of strength, with utilisation of this modelling technique to capture nonlinear structural deformation being rather minimal [ACI Bibliography (1997)].

The research project reported herein represents an original contribution to the development of the strut-and-tie methodology by providing a systematic approach for applying this modelling technique to nonlinear structural concrete analyses. The study proposes a originally developed computer-based strut-and-tie model formulation procedure that permits prediction of the nonlinear monotonic and cyclic response of structural systems with distinct reinforcement details. The procedure being presented in this thesis is a refined version of that reported previously [To et al. (2001 & 2002b)] and the accuracy of the analytical modelling is verified using experimental data.

Several issues pertaining to model formulation are thoroughly investigated. These issues include the strategy of model formulation for Bernoulli (or beam) and disturbed regions of structural systems, the satisfactory positioning of model elements, the appropriate stress-strain material models for concrete and reinforcing steel, the suitable effective strength of model elements, the inclined angle of diagonal concrete struts in beam and column members, and the concrete tension carrying capacity and associated tension stiffening effect.
In addition, the seismic response of various prototype structures when subjected to the experimentally employed cyclic forces and the time-history earthquake loadings was predicted using the originally developed cyclic strut-and-tie models. A summary encapsulating the findings of this project and recommendations for future research work in the area of nonlinear strut-and-tie modelling is also presented.
ACKNOWLEDGEMENTS

For the course of this research project I feel a deep sense of gratitude:

to my supervisors Dr. J. Ingham, Dr. S. Sritharan and Dr. B. J. Davidson for their much valued input and guidance;

to my friends for many enjoyable moments we have had during my years at University;

to my parents who have provided me an unlimited amount of support and encouragement;

and to Emeritus Prof. R. Park for his invaluable comments provided as an external examiner. His recent pass away is a big loss to the concrete and earthquake strengthening research community.
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LIST OF SYMBOLS

\( a \) = development length of ultimate bond stress
\( A_g \) = gross section area
\( A_p \) = total prestressed reinforcement area
\( A_s \) = flexural tension reinforcement area
\( A_s' \) = flexural compression reinforcement area
\( A_{cs} \) = area of concrete struts in B-regions
\( A_{ct} \) = area of concrete ties in B-regions
\( A_{rs} \) = area of rebar struts in B-regions
\( A_{rt} \) = area of rebar ties in B-regions
\( A_{st} \) = total area of longitudinal reinforcement in column sections
\( A_{s-t} \) = area of rebar strut-tie for cyclic strut-and-tie models
\( A_v \) = area of transverse rebar ties
\( A_{rs} \) = total area of transverse reinforcement in a single layer parallel to the applied shear
\( A_{ve} \) = effective section area for carrying shear
\( b_o \) = concrete core width measured from centreline to centreline of longitudinal rebars
\( b_w \) = total section width
\( c \) = neutral axis depth measuring from extreme compression edge
\( c_c \) = concrete coverage
\( C_{c(max)} \) = maximum concrete flexural compression
\( C_s \) = total reinforcement compression at yielding
\( d_b \) = flexural rebar diameter
\( d_v \) = effective section depth
\( d_{vs} \) = transverse rebar diameter
\( D' \) = diameter of circular concrete core measuring from centre to centre of peripheral hoops
$D_c =$ total diameter of the circular sections

$D_o =$ depth of concrete core measured from centreline to centreline of longitudinal rebars

$D_r =$ total depth of the rectangular column sections

$E_c =$ concrete elastic modulus

$E_c A_e =$ effective section stiffness

$E_c A_g =$ gross section stiffness

$E_c I_e =$ effective flexural stiffness

$E_c I_g =$ gross flexural stiffness

$E_s =$ reinforcing steel elastic modulus

$f_2 =$ compressive stress in diagonal concrete struts

$f_c =$ concrete compressive stress

$f_{cont} =$ contact stress developed across concrete cracks

$f_{cy} =$ effective strength of rebar struts in structural B-regions

$f'_c =$ concrete cylinder strength

$f'^e_c =$ confined concrete compressive strength

$f_{cr} =$ concrete cracking strength

$f_{ct} =$ concrete tensile stress in a prism member

$f_{cts} =$ average concrete tensile stress in the member sections

$f_d =$ compressive strength of concrete struts in structural B-regions

$f_{dt} =$ tensile strength of concrete ties in structural B-regions

$f_p =$ stress in prestressed reinforcement

$f_s =$ stress in reinforcement

$f_{sy} =$ yield strength of rebar ties in structural B-regions (for monotonic models)

$f_{sy-\tau} =$ yield strength of rebar ties in structural B-regions (for cyclic models)

$f'_t =$ plain concrete tensile strength

$f_{ts} =$ average value of cracked concrete tension carrying capacity (for cyclic models)

$f_{ult} =$ reinforcement ultimate tensile strength

$f_y =$ measured yield strength of flexural reinforcement

$f_b =$ shear stress in the member sections

$f_{vy} =$ measured yield strength of transverse reinforcement

$h_p =$ perpendicular distance between diagonal concrete struts in structural B-regions
$\ell_c$ = rebar development length
$\ell_{pj}$ = length of joint-links
$\ell_s$ = lap splice length of rebar
$\ell_l$ = length required to develop full bond stress between rebar and the surrounding concrete
$\ell'$ = half length of concrete ties
$M_{y,1}^{st}$ = moment measured at the serviceability limit state
$n$ = ratio of $E_s/E_c$
$N$ = externally applied column axial load
$P$ = externally applied tension
$P_{lp}$ = lap splice capacity
$p''$ = volumetric ratio of transverse reinforcement
$p_c$ = cross-sectional length of rupture surface between the lap spliced rebars
$r_o$ = radius of circular concrete core measuring from section centre to the centreline of longitudinal rebars
$s$ = pitch distance between transverse reinforcement
$s_L$ = surface area of reinforcement per unit volume of concrete
$s_R$ = flexural reinforcement spacing
$T_s$ = maximum tension in reinforcement before yielding develops in flexural members
$t$ = thickness of the imaginary flexural reinforcement tube
$u_m$ = bond stress between reinforcement and concrete
$u_{ult}$ = ultimate bond stress between reinforcement and concrete
$v$ = total shear stress resisted by concrete and transverse reinforcement
$V_n$ = Member shear strength
$V_s$ = transverse reinforcement shear contribution
$V_c$ = concrete shear contribution
$V_p$ = shear contribution from axial force component
$x_c$ = position of flexural compression centroid, measuring from the extreme compression edge
$x_t$ = position of flexural tension centroid, measuring from the extreme compression edge
$\alpha_N$ = angle between member longitudinal axis and the line of externally applied axial
action

\( \beta_t = \) empirical factor dicting the slope of descending branch of the tension stiffening model

\( \varepsilon_1 = \) average principal tensile strain

\( \varepsilon_2 = \) average principal compressive strain

\( \varepsilon_c = \) concrete compressive strain

\( \varepsilon'_c = \) concrete strain at \( f'_c \)

\( \varepsilon'_{cc} = \) ultimate concrete compressive strain

\( \varepsilon_{ct} = \) concrete compressive strain at \( f_{ct} \)

\( \varepsilon_{50} = \) concrete compressive strain at \( 0.5f_d \)

\( \varepsilon_{dt} = \) concrete tensile strain at \( f_{dt} \)

\( \varepsilon_s = \) reinforcement tensile strain

\( \varepsilon_{sh} = \) reinforcement tensile strain at the beginning of strain hardening

\( \varepsilon_t = \) average member strain in transverse direction

\( \varepsilon_u = \) reinforcement tensile strain at \( f_{ult} \)

\( \varepsilon_x = \) average member strain in longitudinal direction

\( \varepsilon_y = \) reinforcement yield strain

\( \gamma = \) Poisson ratio

\( \theta = \) angle between diagonal concrete strut and member longitudinal axis;

\( \rho = \) ratio of \( A_{rt}/A_{ct} \)

\( \rho_l = \) ratio of \( A_{st}/A_g \)

\( \rho_w = \) ratio of \( A_s/A_g \)

\( \sigma_{ct} = \) peak stress in concrete being transferred from the rebars through bonding

\( \bar{\sigma}_{ct} = \) average stress in concrete being transferred from the rebars through bonding

\( \phi = \) half angle of the fan shaped compression sector, measured to the circular section edge

\( \phi' = \) half angle of the fan shaped compression sector, measured to the centre line of imaginary flexural reinforcement tube

\( \phi_c = 0.85, \) efficiency factor for evaluating the concrete compressive strength under cyclic loading

\( \phi_0 = 4/3, \) over strength factor for evaluating the effective strength of flexural rebar ties in circular columns for monotonic models

\( \phi_r = 3/4, \) reduction factor for evaluating the effective area of flexural rebar strut-tie
in cyclic model

\( \phi_{y}^{1st} \) = section curvature measured at the serviceability limit state

\( \omega \) = ratio of \( A'_{s} / A_{s} \)

\( \zeta \) = compressive strength degradation factor for concrete under orthogonal tensile strain

\( \Sigma_{o} \) = total perimeter of flexural rebar located in flexural tension zone
CHAPTER 1 - INTRODUCTION

1.1 Research Objective

The objective of this research project was to develop an alternative methodology to the mainstream engineering analysis and design practice for reinforced concrete structures by using the strut-and-tie modelling technique. This was a conceptual study, which placed particular emphasis on the pursuit of technical validity using the proposed strut-and-tie modelling procedure.

1.2 General Information on Strut-and-Tie Model

The conventional method of designing a structure requires differentiation between the Bernoulli or beam (B-) regions and disturbed (D-) regions in structural systems. As the force transfer mechanism in these two types of region are significantly different, each type is traditionally analysed and designed using different approaches.

For the B-regions of the structural members, the internal distributions of stress and strain can be assumed to be regular, and therefore their structural behaviour when subjected to external actions can be analysed with high precision using long-established solid mechanics relationships. However, in D-regions of structural members, due to high irregularity of the internal stress and strain distributions, it is not possible to evaluate structural performance using these solid mechanics relationships. These structural regions are therefore typically analysed using empirical or “rule of thumb” methods which could potentially lead to incorrect design solutions and premature structural failure. Sophisticated computer analytical approaches, such as finite element analysis, are commonly used in predicting the elastic response of D-regions, but are nevertheless ineffective in understanding the structural behaviour of these regions after concrete cracks. This is because of inadequate material models for concrete and reinforcing steel, and most importantly, the lack of an accurate bond slip model in finite element packages. Accordingly, a relatively simple analytical procedure based upon the strut-and-tie modelling technique to characterise D-region structural behaviour is investigated in this research.
Illustrated in Fig. 1.1c is an example of the strut-and-tie model depicting the force transfer mechanism that developed inside a reinforced concrete bridge portal frame when subjected to a left to right lateral force. The model assumes that compression is carried only by struts that represent concrete and tension is carried exclusively by ties that represent rebars. These two model elements are adjoined by nodes in which the curvature of actual force stems is concentrated and force equilibrium is achieved. Reader is referred to section 2.5 for further theoretical background of strut-and-tie model.

1.3 Design Procedure for Structural Concrete

The conventional procedure for designing reinforced concrete structures can be divided into three stages: (a) selecting member dimensions, (b) determining the quantity, location and anchorage details of reinforcement to ensure ultimate strength criteria are satisfied, and (c) satisfying criteria including member deformation under service load conditions.

As the example of the portal frame shows in Fig. 1.1a, a planar frame model as illustrated in Fig. 1.1b is typically formulated for the second and the third design phases identified above, to ensure satisfactory member strength at the ultimate limit state and structural deformation at the serviceability limit state, respectively. Note that the structural performance of the joint panel regions is not investigated in a planar frame model, and they are represented by rigid elements. Strut-and-tie models (STMs) have been commonly used in conjunction with the planar frame model to examine force equilibrium conditions of the structural D-regions, including the joint panel regions for the portal frame example, shown in Fig. 1.1c. Also depicted in Fig. 1.1c is a strut-and-tie modelling solution to the entire structural system, indicating that this modelling technique is theoretically capable of combining the three previously described design phases using a single modelling solution. This may ultimately result in a consistent analytical standard to facilitate effective dimensioning and reinforcement detailing of structural systems.

1.4 Scope of Research

The aim of this research was to use the strut-and-tie modelling technique to develop a consistent analytical procedure for reinforced concrete structural members situated in B- and D-regions. The proposed STM analysis is capable of:

• assessing the demand on various structural components within a structural system, and to provide insight into the member failure sequence;
Figure 1.1: Different structural models for a portal frame supporting bridge superstructure.
• predicting the monotonic force-displacement response envelope and hysteretic behaviour of an entire structural system;
• facilitating a unified analytical approach to all structural members within a single reinforced concrete structural system;
• providing an effective analytical solution by allowing the performance of the complete structural system to be examined simultaneously.

The primary objective of the current research was to identify if the strut-and-tie method of analysis could be significantly more practical through the provision of a systematic model formulation procedure for structural concrete analyses. It is noteworthy that the research project reported in this thesis made no attempt to capture structural behaviour in any great detail at the microscopic level, nor was it intended to predict all possible structural failure modes using the STM. Instead, attention was given to investigating the development of a robust procedure to generate computer-based STMs, allowing rapid implementation by practising engineers, primarily for structural concrete analyses.

1.5 Outline of Dissertation

This thesis is organised into five subsequent chapters. The first of these is Chapter 2, which provides a comprehensive literature review addressing historic and recent development of the strut-and-tie method, sometimes referred to as truss models. The review of important features pertaining to the advancement of modelling methodology, including the plasticity truss model, the Mohr’s compatibility truss model and the strut-and-tie model are presented. Explanation of the compression field theory, the modified compression field theory and the softened truss theory are given. Furthermore, several commercial and academic computer packages available to the author are examined, to identify the computer programme most suitable for the research detailed in this thesis.

In Chapter 3, the model formulation procedure that was employed for monotonic and cyclic strut-and-tie model analyses is described in detail. This modelling procedure was originally developed by the author for the research reported herein, and is a refined version of that previously proposed by To et al. (2001 & 2002b). In addition, described in the following Chapters 4 and 5 are the application of the strut-and-tie models that were formulated using this originally developed procedure, to perform nonlinear analysis of various reinforced concrete structures.

In Chapter 4, the structural response of various reinforced concrete structural systems are estimated using strut-and-tie models. Push-over analysis is performed using monotonic strut-and-tie models, and the cyclic loading history employed in laboratory tests is adopted for
cyclic strut-and-tie analyses. Analytical results are compared with experimental data, and the discrepancies and similarities are discussed.

In Chapter 5, the dynamic behaviour of various full-scale prototype structures under several time-history earthquake records are examined using the cyclic strut-and-tie models. Strut-and-tie analytical results are then compared with that obtained from the conventional planar frame models. In addition, a hybrid modelling solution that integrates strut-and-tie and planar frame modelling techniques is also proposed in this study to examined if computation time required for analysing large structures could be minimised.

It must be emphasised that all the strut-and-tie dynamic analyses described in Chapter 5 were originally conducted for this research to determine if the strut-and-tie method presents a significant advantage over the conventional planar frame modelling method.

Chapter 6 summarises the findings of the current research project and recommends future research directions on the related topic. Also, the new idea of integrated design procedure using strut-and-tie method is also delineated.
CHAPTER 2 - THEORETICAL BACKGROUND OF STRUT-AND-TIE METHODOLOGY

2.1 Introduction

The truss model concept to comprehend the shear behaviour of reinforced concrete has existed for more than a century. However, it was only in a relatively recent time that this modelling technique received renewed attention, and subsequently, much research has been conducted in the past few decades to refine and expand the theoretical basis of the truss model. Nevertheless, much research effort is still needed to derive a rational model formulation procedure to allow ease of application in practice. In this chapter, an overview with much emphases were given to the theoretical background at different advancement stages which form the bases to development of truss modelling techniques. Other theoretical details pertaining to the evaluation of suitable strut-and-tie model element properties are given in Chapter 3.

2.2 Historic and Current Developments in Structural Concrete Design

Flexure and shear are two actions that must be considered in the analysis of structural concrete. Flexural theory that originated from Hooke's Law, based upon the linear elastic stress method, was originally the predominant approach employed in analyses for flexure. However, the deficiency of Hooke's Law in accounting for the energy-absorbing capability of structural systems through plasticity resulted in this analysis methodology being replaced at around the 60s in the last century by Bernoulli's Theorem. Instead of assuming a linear sectional stress distribution, Bernoulli's Theorem was developed based upon the assumption of a linear strain distribution as depicted in Fig. 2.1. Bernoulli's Theorem assumes: 1) strain distribution along the section remains plane before and after bending, see Fig. 2.1b; 2) there is no slippage between the reinforcement steel bars (rebars) and the surrounding concrete; and 3) concrete in the tension zone of a section is neglected and all the tension is carried only
by the tension rebars, see Fig. 2.1c. Also depicted in Fig. 2.1c are the concrete stress envelope and the rebar stress level corresponding to the strain value at the ultimate load state. Bernoulli’s Theorem has been widely employed in practice as a standard procedure for flexural analyses and designs because it is very accurate yet simple to perform.

Contrary to bending, the analysis and design of structural concrete to resist shear has always been based on a semi-empirical procedure. Design provisions for shear strength of a member that are employed in the major design standards [ACI 318 (2002), CEB-FIP (1978), CSA (1994) Eurocode 2 (1992) and NZS 3101 (1995)] are derived partly from a rational truss model and partly from an empirical approach based on test results.

The first analytical models that were used to examine the rebar contribution to shear strength of structural concrete were proposed by Ritter (1889) and Mörsch (1912) at the turn of last century. The truss models, shown in Fig. 2.2, were developed according to the observation that a conventionally reinforced concrete beam subjected to shear stress would develop diagonal cracks inclined at approximately 45° to the member longitudinal axis. These models assume that after concrete cracking, the reinforced concrete beam behaves analogous to a truss, with the top longitudinal concrete chords in compression, the bottom longitudinal rebar chords and the transverse rebar ties in tension, and the diagonal concrete struts in compression.

Following Ritter and Mörsch, much research along this line of study has been conducted, predominately in the past three decades, aiming at developing a rational model that is capable of describing the shear behaviour of structural concrete. The development of these analytical models based upon a truss-like mechanism can be categorised into three groups, namely the plasticity truss model, the Mohr’s compatibility truss model and the strut-and-tie model.

Theoretical Background of Strut-and-Tie Methodology: Historic and Current Developments in Structural
2.3 Plasticity Truss Model

The plasticity truss model was developed according to the theory of plasticity, to examine the load-carrying capacity of a structural member when subjected to bending, shear or combined bending and shear. A solution generated using this model will ensure that the internal stress level is lower than the designated material strength, and that force equilibrium, together with the static boundary conditions for externally applied actions, are all satisfied. Furthermore, instead of using a 45° inclination for diagonal concrete struts, the major advancement of this plasticity truss model from Ritter and Mörsch’s truss model was that the inclined angle of diagonal concrete struts could be rationally evaluated, based on the assumed failure mechanism and the volumetric ratio of flexural to shear reinforcing steel provided inside the structural members.

Summaries of applying the plastic truss model in the analysis of various structural types were given by Thurlimann et al. (1983) and Nielsen (1984). The model assumes rigid-plastic structural behaviour, with no deformations occurring until the applied stress reaches the yield strength of a structural member. Subsequent to yielding, an arbitrarily large deformation is possible without additional applied load, indicating that the structural system has reached its
load-carrying capacity, and has collapsed. Although it is apparent that structural concrete does not behave analogously to a rigid-plastic material, this analytical approach is satisfactory when employed in the design of under-reinforced concrete structures, for which the strength is essentially governed by yielding of the tension reinforcement. See the representative force-displacement diagram in Fig. 2.3a. In addition, for overly-reinforced concrete structures, a suitable design solution could also be derived using the plasticity theory if an appropriate effective concrete compressive strength is taken into account, see Fig. 2.3b.
2.4 Mohr’s Compatibility Truss Model

Mohr’s compatibility truss model is applicable to the analysis and design of a structural member when subjected to shear. Contrary to a plastic truss model, a Mohr’s compatibility truss model considers not only stress equilibrium, but also ensures that the average strain condition of the model complies with the Mohr’s compatibility requirement. Furthermore, as this modelling technique considers both stress and strain conditions, it is capable of predicting the complete shear force-shear displacement response envelope of a structural member when applied in conjunction with the realistic material stress-strain characteristics of concrete and reinforcing steel.

Several theories were developed for applying this modelling technique, namely compression field theory, modified compression field theory and softened truss theory. These theories are briefly described in the following sections.

2.4.1 Compression Field Theory

The Mohr’s compatibility truss model was first established using the compression field theory, which assumes that concrete carries no tension after cracking, and that shear is transferred through diagonal concrete struts in the beam web, see Fig. 2.4. Furthermore, the angle of inclined diagonal concrete strut, $\theta$, was assumed to coincide with the orientation of average principal compressive strain. According to the strain compatibility condition, the relationship between the strut angle, $\theta$, and the strain values in three different directions could be described as:

$$\tan^2 \theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_t - \varepsilon_2}$$  \hspace{1cm} (Eq. 2.1)

where $\varepsilon_x$ is the average longitudinal strain in the structural member, see Fig. 2.5b; $\varepsilon_t$ is the average transverse strain in the structural member; and $\varepsilon_2$ is the average principal compressive strain in the diagonal concrete struts.

For a structural member subjected to a given level of shear, $V_n$, there are four unknowns: 1) the stress in the flexural rebars, $f_s$; 2) the stress in the transverse rebars, $f_v$; 3) the compressive stress in diagonal concrete struts, $f_2$; and the concrete strut inclination, $\theta$. These four unknowns can be found by simultaneously solving the three force equilibrium equations, see Fig. 2.5a, in conjunction with the compatibility relationship described in Eq.
Figure 2.4: A reinforced concrete beam subjected to a point load.

2.1, see Fig. 2.5b. Furthermore, with knowledge of the constitutive stress-strain response of concrete and rebar as illustrated in Fig. 2.5c, a complete shear force-shear displacement response envelope of a structural member can be evaluated [Mitchell and Collins (1974) and Collins (1978)].

2.4.2 Modified Compression Field Theory

At a later stage, the compression field theory was modified by Vecchio and Collins (1986) to incorporate both the concrete strength degrading behaviour and cracked concrete tension-
Figure 2.5: Compression field theory proposed for a reinforced concrete beam. [Collins and Mitchell (1997)]

carrying capability in the constitutive stress-strain relationship. In their study, Vecchio and Collins realised that the concrete stress-strain relationship adopted from the standard concrete cylinder tests does not represent the correct average stress-strain behaviour of the cracked concrete in a structural member. The different concrete material response illustrated in Fig. 2.6 suggests that the concrete in a cylinder is subjected to only small tensile strain resulting from Poisson effect when compared with the cracked concrete in a structural member. As a result of this intense orthogonal tensile strain, the compressive strength of the cracked concrete in a structural member is weaker than that exhibited in standard cylinder tests.

The average stress-average strain relationship of the cracked concrete was evaluated by Vecchio and Collins (1986) through the testing of 30 reinforced concrete panels, with a variety of well-defined uniform biaxial stress conditions, including the results from pure shear. Test results indicated that concrete compressive strength is not only dictated by the compressive strain in the direction of applied stress, but is also dependent on the orthogonal
co-existing compressive or tensile strain. By taking account of the orthogonal strain state, Vecchio and Collins proposed the material stress-strain relationship for diagonal concrete struts to be calculated as:

\[ f_c = \eta \cdot f_c' \cdot \left[ 2 \cdot \frac{\varepsilon_2}{\varepsilon_c'} - \left( \frac{\varepsilon_2}{\varepsilon_c'} \right)^2 \right] \]  

(Eq. 2.2a)

and \( \eta = \frac{1}{0.8 + 170\varepsilon_1} \leq 1.0 \)  

(Eq. 2.2b)
where $f_c$ is the concrete stress; 
$f_c'$ is the concrete cylinder strength; 
$\varepsilon_c'$ is the concrete strain at $f_c'$; 
$\varepsilon_1$ is the average principal tensile strain of concrete; 
$\zeta$ is the concrete compressive strength reduction factor.

According to Mohr’s stress circle, the average concrete tensile stress, $f_{cts}$, that is orthogonal to the orientation of diagonal concrete struts is given as:

$$f_{cts} = (\tan \theta + \cot \theta) v - f_c$$  \hspace{1cm} (Eq. 2.3)

where $v$ is the member shear stress.

It follows that the total shear strength of a member, $V_n$, is computed as:

$$V_n = f_{ct} b_w d_v \cot \theta + \frac{A_{vy} f_{yy}}{s} d_v \cot \theta$$  \hspace{1cm} (Eq. 2.4)

where $A_{vs}$ is the total area of transverse reinforcement in a single layer parallel to the applied shear; 
$b_w$ is the total section width; 
$d_v$ is the effective section depth; 
$f_{vy}$ is the yield strength of transverse reinforcement; and 
$s$ is the pitch distance between transverse reinforcement.

The mathematical expression for the member shear strength described in Eq. 2.4 is the combined contributions of concrete and shear reinforcement. The treatment of member shear strength using the Modified Compression Field Theory was also adopted in the 1994 edition of the Canadian concrete design standard [CSA (1994)].

2.4.3 Softened Truss Theory

During the time when the Modified Compression Field Theory was proposed, studies on degrading concrete compressive strength were also conducted by two groups of Japanese researchers [Shirai and Noguchi (1989) and Mikame et al. (1991)]. According to their studies, the identified concrete strength degrading factors were significantly different from those recommended by Vecchio and Collins (1986). The first group [Miyahara et al. (1987) and Izumo et al. (1991)], at the University of Tokyo, used hollow reinforced concrete...
cylinders, which had an outer diameter of 330 mm and a wall thickness of 38 mm. The test specimens were subjected to internal pressure followed by longitudinal loads. The proposed degrading factor, $\zeta$, was:

$$\zeta = 1.0 \quad \text{for } \varepsilon_1 \leq 0.0012 \quad \text{(Eq. 2.5a)}$$

$$\zeta = 1.15 - 125 \varepsilon_1 \quad \text{for } 0.0012 \leq \varepsilon_1 \leq 0.0044 \quad \text{(Eq. 2.5b)}$$

$$\zeta = 0.6 \quad \text{for } \varepsilon_1 \geq 0.004 \quad \text{(Eq. 2.5c)}$$

Following the testing of two reinforced concrete panels, the second group of researchers, at Chia and Nihon University, proposed the following degrading factor:

$$\zeta = \frac{1}{0.27 + 2.71 \cdot \varepsilon_1^{0.167}} \quad \text{(Eq. 2.6)}$$

The plots of Eqs. 2.2b, 2.5 and 2.6, are compared in Fig. 2.7 to illustrate the discrepancies between the concrete strength reduction factors. Significant difference between the proposed degrading factors is noticed even when the orthogonal tensile strain, $\varepsilon_1$, is as low as 0.01.

Realising the discrepancy between the proposed concrete strength degrading factors by different researchers, Abdeldjilil and Hsu (1995) conducted a detailed experimental study on 22 full-sized reinforced concrete panels to examine the stress and strain softening effect of reinforced concrete. From the test results, they concluded that concrete suffers not only strength degradation, but also experiences strain softening, when loaded in a biaxial stress state. This physical phenomenon of concrete is primarily governed by the coexisting orthogonal tensile strain and the loading sequence, while other factors such as quantity and spacing of tension reinforcement were found to be insignificant. According to their investigation, the proposed reduction factor, which is common to the stress degradation and strain softening effect, is given as:

$$\zeta = \frac{0.9}{\sqrt{1 + 400 \varepsilon_1}} \quad \text{(Eq. 2.7)}$$

The plot of Eq. 2.7 is also depicted in Fig. 2.7 for comparison.
Figure 2.7: Concrete strength degradation factors to account for orthogonal tension effect.

The refined concrete softened stress-softened strain relationship, as shown in Eq. 2.8, was integrated in the Mohr's compatibility truss model to evaluate the shear load-shear displacement of a reinforced concrete member.

\[
f_c = \zeta \cdot f_c' \cdot \left[ 2 \cdot \left( \frac{e_c}{\zeta e_c'} \right) - \left( \frac{e_c}{\zeta e_c'} \right)^2 \right] \quad \text{for } e_c < \zeta e_c' \quad (\text{Eq. 2.8a})
\]

\[
f_c = \zeta \cdot f_c' \cdot \left[ 1 - \left( \frac{e_c}{\zeta e_c'} - 1 \right)^2 \right] \quad \text{for } e_c > \zeta e_c' \quad (\text{Eq. 2.8b})
\]

Shown in Fig. 2.8 are the plots of the stress-strain relationship of reinforced concrete derived from Eqs. 2.2 and 2.8 compared to a typical cylinder concrete stress-strain curve.

Based on the framework of Mohr's compatibility model that was derived from the softened truss theory, Pang and Hsu (1995) have taken a step further to include the concrete shear resistance that resulted from aggregate and shear-key interlocking actions, when evaluating the shear behaviour of a structural member. The aggregate interlocking action is attributable
to the protruding concrete particles being trapped in the cracks, while shear-key interlocking is induced between concrete struts being developed along the meandering crack patterns.

The theory that considers concrete shear resistance has been successfully incorporated in the Mohr’s compatibility truss model to analyse the shear behaviour of reinforced concrete panels [Hsu and Zhang (1997)]. However, analytical solutions to treat shear in other structural types are yet to be established.

2.5 Strut-and-Tie Model

Although the plasticity truss model and the Mohr’s truss model are promising treatments for shear, their development was based on the assumed regular stress and regular strain distributions, and therefore, their application is limited to Bernoulli (B-) regions of structural systems. As the internal stress and strain distributions of D-regions are highly irregular, due to the disturbance of discontinuous externally applied actions or physical geometry, structural D-regions are particularly difficult to analyse and design. Numerical computer analysis such as the finite element method may be used for D-regions, but are seldom employed as a routine procedure in design practice due to its complexity, time-consumption.
Figure 2.9: B- and D-regions in a multi-column reinforced concrete frame and a strut-and-tie model representation.

and other disadvantages previously mentioned in Section 1.2. Consequently, the common analysis procedure for D-regions has been based on the empirical or “rule of thumb” approach. However, this common analysis approach may potentially lead to an incorrect design solution and premature structural failure. An example of distinguishing between the B- and D-regions in a structural system is illustrated in Fig. 2.9 using a multicolumn reinforced concrete frame. Also depicted in this figure is a typical strut-and-tie modelling solution for this structural system corresponding to a lateral action applied from left to right.

The strut-and-tie modelling procedure was established according to the assumption that compression is only carried by concrete, and tension is transferred exclusively through reinforcement. This modelling technique is particularly useful in depicting the force path being developed inside a structure by means of struts and ties respectively, representing the compression and tension force paths. The model elements are adjoined by nodes in which the curvature of actual force stems is concentrated and force equilibrium is achieved.

The strut-and-tie modelling technique has received renewed interest especially in the past three decades, as it was found to be the only rational method which was reasonably simple to perform, for analyzing and designing structural D-regions. It is generally agreed that this model is particularly useful not only in member dimensioning, but also in deriving satisfactory critical reinforcement details. In addition, this modelling technique is applicable for treating shear in structural B-regions. Shown in Fig. 2.10 are several examples of conceptual strut-and-tie modelling solutions for various structures proposed by Schlaich et al. (1987). In their work, they demonstrated that a strut-and-tie model can simultaneously

Theoretical Background of Strut-and-Tie Methodology: Strut-and-Tie Model
analyse the load-carrying capacity of a whole structural system and hence, a consistent level of analysis accuracy to all structural components can be achieved.

Much research was conducted in the past few decades to provide examples of suitable strut-and-tie models for different structural systems including deep beams, dapped end beams, shear wall coupling beams and corbels [Marti (1985a, 1985b & 1999), Rogowsky and MacGregor (1986) and Schlaich and Schafer (1991)]. However, the developed models are frequently related strictly to a particular load configuration and cannot be used for another

Theoretical Background of Strut-and-Tie Methodology: Strut-and-Tie Model
load case without modification. The disadvantage of a unique model is not peculiar to the strut-and-tie modelling technique, but is inherent to the non-homogeneous material property of the cracked reinforced concrete.

2.6 Computer Packages

A brief review of several commercial and academic computer packages is provided in this section. The appropriateness of this software for the current research project is identified.

2.6.1 Space Gass 8.0 & Sap 2000

Space Gass 8.0 and Sap 2000 are commercial packages for structural analysis and design. Both packages are similar in the available functions for structural analysis. They both have a user-friendly graphic interface, allowing users to set up the structural problems on the monitor or through internally provided spread sheets.

Both computer packages are capable of performing static and dynamic analyses. Space Gass 8.0 is more capable in handling design-orientated problems as the programme has a selection of different member sections to choose from. It has also incorporated several design Standards in the package to ensure that structural forces comply with code requirements. Contrary to Space Gass 8.0, Sap 2000 is more powerful in performing dynamic analyses, especially for earthquake time-history analyses. As both computer programmes were developed primarily for the design office environment, they employ a limited number of material stress-strain models. This is very common to most commercial packages and was considered disadvantageous and inadequate for the research described in this thesis.

2.6.2 Ruaumoko & Drain-2DX

Ruaumoko and Drain-2DX are both computer software developed primarily for academic research purposes. Both programmes were written using Fortran computer language. Ruaumoko was developed by Carr (1998), while Drain-2DX was developed by Allahabadi and Powell (1988). Ruaumoko is very powerful in performing dynamic analysis, and has a variety of model element types, including beam, spring and shear panel. Also, it has a wide range of material stress-strain relationships, ranging from the simple bilinear curve to the sophisticated curvilinear models for analysis. In addition, a graphic interface at the analytical stage is provided, allowing users to examine the layout of the input structural model. However, the data input procedure is less straightforward in Ruaumoko.

Contrary to Ruaumoko, Drain-2DX is more appropriate for performing static analysis, especially when analysing a test unit using a displacement history as employed in the
Although it has a limited number of material stress-strain models, the programme is relatively stable which makes it suitable for conducting strut-and-tie model (STM) analyses in this research.

2.6.3 Computer Aid Strut-and-Tie (CAST)

Computer Aid Strut-and-Tie (CAST) is a finite element computer programme which was developed by Tjin and Kuchma (2002). This programme is equipped with a graphical design interface, allowing designers to formulate STMs graphically, and to quickly optimise design solutions based on the analytical results that are illustrated along the model elements. Depicted in Fig. 2.11 is an analytical example of a corbel structure obtained from CAST. This computer programme is capable of evaluating model element forces for both statically determinate and indeterminate analytical cases, and accordingly, the maximum stress level in the model elements and stress distribution inside the nodal zones could be predicted. CAST could also perform a nonlinear pushover analysis however, the available material model does not allow stress degradation to occur. Furthermore, the dynamic analysis option is not supported by CAST and thus this programme was not employed in this research project.
2.6.4 NL-STM

NL-STM was developed by Yun (2000) for designing reinforced concrete members and predicting the corresponding structural behaviour based upon strut-and-tie methodology. It has a pre-processor to perform finite-element analysis of reinforced concrete structures. Based on the obtained analysis results, including internal stress levels and principal stress directions, users can formulate appropriate STMs and facilitate member-size checking with the aid of an integrated graphic interface. Again, this programme was not used in this research project because it does not support dynamic analysis.
CHAPTER 3 - NONLINEAR STRUT-AND-TIE MODEL FORMULATION PROCEDURE

3.1 Introduction

The strut-and-tie method of analysis is a modelling technique used to discretely represent the force paths that are established inside a structural system when subjected to external actions. This modelling technique is especially useful in examining the load-carrying capacity of structural D-regions, because it allows designers to envisage the internal force flow and permit the critical load-carrying elements to be identified.

Although the strut-and-tie modelling technique is simple to comprehend, it is a surprisingly complex task to apply in structural analysis. This is mainly due to successful application of this modelling technique often pre-requiring sufficient knowledge of the internal force path that is dictated by the reinforcement arrangement and the support conditions. Furthermore, the position of the force path may vary significantly for different types of structural failure mechanisms, imposing an additional challenge in identifying a suitable analytical model.

In this chapter, a strut-and-tie model (STM) formulation procedure that is capable of assessing the nonlinear behaviour of an entire structural system is presented. This modelling procedure is a refined version of that previously reported by To et al. (2001 & 2002b).

3.2 Strategy for Model Formulation Procedure

When formulating a STM for assessing the appropriate reinforcement details of D-regions in structural systems, Schlaich et al. (1987) recommended that an established model based on the uncracked elastic load-path could be used for a simple and consistent modelling solution. Their work was also supported by Marti (1999), who proposed that in conjunction with identifying the uncracked elastic load-path, some modification according to the redistribution of internal forces and moments could also be incorporated when formulating...
a suitable STM for structural D-regions. From a design viewpoint, this modelling approach is considered suitable because it guarantees a conservative design solution. However, some critics [MacGregor (1988) and Sritharan (1998)] argued that it is inadequate when used for analysing the ultimate structural strength when significant inelastic strain and progressive damage is expected to develop in structural D-regions. This is primarily because a STM formulated using elastic theory is insufficient in capturing the redistribution of forces associated with the inelastic response of structures. MacGregor (1988) further suggested that the stress trajectories should not be expected to match the shear crack patterns observed in structural components if flexural cracks develop first. This implies that the suitability of using an elastic theory to determine the force path in structural D-regions is dependent on the combination of flexural, shear and axial load conditions. The influence of combined loading conditions on crack patterns was confirmed by Bhide and Collins (1989) through a series of tests on reinforced concrete panels loaded with pure shear and combined shear and tension. They reported that the orientation of the crack pattern that developed at the final stage of testing may vary as much as 38° from the orientation of the initially observed crack pattern. This implies a significant change of force path.

As per previous discussions, it is impossible to achieve a highly accurate representation of the internal force transfer mechanism being established at different load states using a single model. For a STM formulation procedure which is sufficiently robust to be adopted as a routine design exercise by practising engineers, a suitable model formulation procedure to be developed in this study required satisfactory analytical accuracy, while aspects necessary for simple implementation were also considered.

The force paths occurring in structural D-regions, such as beam-column joints, can be very different at various load states, but this is not usually the case for structural B-regions. D-regions exhibit comparatively little elastic deformation and if improperly designed, potential brittle failure associated with drastic strength degradation would be expected. In contrast, B-regions often have ductile response and are mostly responsible for the nonlinear inelastic behaviour of an entire structural system. As only a single model can be used in the computer analyses, the following two objectives were prioritised when developing a strut-and-tie model formulation procedure in this research:

- To assess the load-carrying capacity of structural D-regions so that brittle failure can be avoided;
- To evaluate the nonlinear behaviour of B-regions so that the force-displacement response of an entire structural system can be satisfactorily calculated.

Accordingly, the eventual strategy employed for the model formulation procedure was to adopt the force path that occurred at the ultimate limit state for the structural D-regions, and
the force path that developed at the serviceability limit state for the structural B-regions. Structural components are expected to suffer significant inelastic strain and progressive damage at the ultimate limit state, but to exhibit cracked elastic behaviour at the serviceability limit state.

3.3 Section Force Analysis

When formulating a STM for the structural components located in B-regions, predominately beams and columns, a comprehensive section force analysis was performed using a computer programme SEFAP written in FORTRAN. This section force analytical programme was developed based on the Bernoulli compatibility condition that plane sections remain plane after bending. Within this programme, realistic nonlinear stress-strain characteristics of confined and unconfined concrete were accounted for, using the material model proposed by Mander *et al.* (1988), see Fig. 3.1a. In addition, the nonlinear stress-strain characteristic of rebars shown in Fig. 3.1b was also considered [Dodd and Restrepo (1995)], as were aspects such as non-regular section geometry and the exact location of the flexural reinforcement.

3.4 Strut-and-Tie Modelling Procedures

Two modelling procedures were proposed in this research. The first, a monotonic model formulation procedure as described in Section 3.5, was developed to perform push-over analyses and evaluate the monotonic force-displacement response envelope. This modelling procedure is particularly useful in providing an insight into suitable load paths and to identify the failure sequence of the load-carrying components in a structural system.

The second, a cyclic model formulation procedure as described in Section 3.6, shares great similarity with the first procedure, except that it was developed primarily for predicting the hysteretic response of a structural system and the structural dynamic performance under time-history earthquake loading.

3.5 Monotonic Model Formulation Procedure

The proposed model formulation procedure for analysing the monotonic force-displacement response envelope of a structural system consists of six different model element types, as illustrated in Fig. 3.2a. The type 1 model element is employed in the STM to represent flexural compression zones in B-regions. This model element is composed of a concrete strut and a rebar strut arranged in tandem. The type 2 model element consists of a concrete tie and...
Figure 3.1: Material stress-strain characteristics.
a rebar tie also arranged in tandem, to simulate the structural behaviour of the flexural tension zone in structural B-regions. The type 3 and 4 model elements are respectively composed of a concrete strut and a rebar tie to represent the diagonal concrete mass and the transverse reinforcement located in structural B-regions. Type 5 and 6 model elements are respectively constructed using a concrete strut and a rebar tie to simulate the concrete mass and the reinforcement located in structural D-regions.

3.5.1 Model Element Positions

As STMs are used to discretely represent the force path developed inside a reinforced concrete structure, it is logical that the “struts” and the “ties” are located at the centre of the respective force path. Although the concept is simple, the major obstacle arises from the fact that the position of the force path may shift along the length of a structural member, and also may vary at a different load state.

For the monotonic model formulation procedure, the type 1 element representing flexural struts and the type 2 element representing flexural ties in the B-region structural members are located at the corresponding compression centroid, \( x_c \), and tension centroid, \( x_t \), measured at the serviceability limit state, see Fig. 3.2a. The serviceability limit state was defined by the commencement of rebar yielding, or the extreme concrete fibre reaching a compressive strain of 0.002, whichever occurred first. The serviceability limit state was chosen because this effectively defines the cracked-section elastic stiffness, after which plastic behaviour commences. See Fig. 3.3 for the moment-curvature characteristics of the beam and column members employed in large scale beam-column joint test units [Ingham et al. (1994)], which are the examples considered in Chapter 4. Also, the correct computation of elastic stiffness is essential in ductile design in order to accurately determine the yield displacement, which subsequently dictates the displacement ductility capacity of the structures.

It is noted that the position of the flexural force path inside the B-region structural members, measured at the serviceability limit state and at the ultimate limit state, is different. As only one of these two positions can be used, the change in structural response and material stress-strain characteristics as a result of alternation in force path positions at a different load state can be accounted for by assigning suitable post-yielding properties to struts and ties in type 1 and type 2 elements.

The type 3 element and the type 4 element are positioned in the model to ensure an angle between the diagonal concrete struts and the member longitudinal axis between 31° and 59°, as recommended by CEB-FIP (1978). A sophisticated energy minimisation approach based on the virtual work theory in assessing the appropriate angle of the diagonal concrete strut is
Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
Prakhya and Morley's model

Trilinear representation

\( f_{ct} \)

\( f_{dt} \)

\( E_c \)

\( \varepsilon_{ct} \)

\( \varepsilon_{dt} \)

c) Concrete ties subjected to tension.

d) Reinforcement ties and struts subjected to tension and compression.

Figure 3.2: Continued.

b. Moment-curvature relationship of knee-joint beam section for the redesigned unit in section 4.4.

c. Moment-curvature relationship of knee-joint column section for all test units in section 4.4.

Figure 3.3: Moment-curvature relationships for beam and column sections.
available [Kim and Mander (1999)]. However, that approach was not considered in this research because the correct evaluation of a diagonal concrete strut angle is essential in determining the shear strength of an unreinforced concrete member, but is less important for concrete members with adequate shear reinforcement. As all test units considered in Chapter 4 were sufficiently reinforced against shear action in the structural B-regions, strut orientation selected according to the CEB-FIP range was considered appropriate.

The force path that develops within a D-region is essentially influenced by the employed reinforcement detail, the support conditions and the types of external loads. Thus, it is not possible to establish a definite guide for locating the type 5 model elements, representing concrete struts, and the type 6 model elements, representing rebar ties. For all the test unit examples considered in Chapter 4, the positions of type 5 and 6 model elements were located at the corresponding centroids of force paths developed at the ultimate limit state that have been previously identified in literature [ACI Bibliography (1997)]. The ultimate limit state is defined when a structural component reaches its load-carrying capacity and suffers significant inelastic strain and structural damage, and when drastic strength degradation is imminent.

3.5.2 Type 1 Element

A quadlinear curve replicating the curvilinear model of concrete stress-strain characteristics that was proposed by Kent and Park (1971) was employed to represent the material response of the concrete strut in the type 1 element. Kent and Park’s material model shown in Fig. 3.2b is composed of a second degree parabolic curve as the ascending branch, and a bilinear curve as the descending branch. The ascending branch reaches its full strength at a strain of 0.002 for both confined and unconfined concrete, and the gradient of the descending branch is dictated by the strain $\varepsilon_{50}$, at which the material model loses 50% of its maximum compressive strength. The model also assumes that concrete has a residual strength identical to 20% of the concrete cylinder strength, $f'_{c}$. The strain $\varepsilon_{50}$ is calculated using Eq. 3.1 as:

$$\varepsilon_{50} = \frac{3 + 0.2f'_{c}}{145f'_{c} - 1000} + \frac{3}{4} p'' \frac{b_{o}}{s} \left(f'_{c} \text{ in MPa}\right) \quad \text{(Eq. 3.1a)}$$

and

$$p'' = \frac{2(b_{o} + D_{o})A_{ns}}{b_{o} D_{o} s} \quad \text{(Eq. 3.1b)}$$

where $\varepsilon_{50}$ is the strain value at 0.5$f_{d}$;
$p''$ is the volumetric ratio of transverse reinforcement;
$s$ is the pitch distance between transverse reinforcement;
\( b_o \) is the width of concrete core measured from centreline to centreline of the longitudinal rebar, as defined in Figs. 3.4b and 3.4c; and

\( D_o \) is the depth of concrete core measured from centreline to centreline of the longitudinal rebar, as defined in Figs. 3.4b and 3.4c.

The quadlinear material response employed for concrete struts in the type 1 element is essentially identical to Kent and Park's model except for the ascending branch, see Fig. 3.2b. For the ascending branch of the quadlinear curve between points A and B, the curve ascends with a gradient equal to the elastic modulus of concrete, \( E_c \), calculated using the suggested equation in Eurocode 2 (1992):

\[
E_c = 9500 \cdot (f'_c + 8)^{1/3} \text{ (MPa)} \quad \text{(Eq. 3.2)}
\]

where \( f'_c \) is the unconfined concrete compressive strength.

This equation was preferred to that recommended in NZS 3101 (1995) because it provides a comparable estimation to the actual value of \( E_c \), rather than a suitable design solution. The curve changes the ascending gradient at point B, at which the stress level is equal to 0.75 \( f_d \), and connects with point C, where the curve reaches the full strength of concrete struts, \( f_d \), at a strain equal to 0.002. The computation of concrete strut strength, \( f_d \), is described in Section 3.5.2.2. Note that the change of ascending gradient at the stress level of 0.75 \( f_d \) was adopted in the quadlinear curve to allow satisfactory comparison with the second degree parabolic ascending curve employed in Kent and Park's model. The descending branch of quadlinear curve, between points C and D, is computed using Eq. 3.1 and the residual compressive strength from point D to infinite strain is equal to 0.2 \( f'_c \).

A bilinear curve was also employed to represent the stress-strain characteristic of the rebar struts utilised in the type 1 model element, see the compression branch in Fig. 3.2d. The commonly assumed value of \( E_s = 200 \text{ GPa} \) was employed as the gradient of the elastic branch. The strain hardening ratio (s.h.r) of rebar struts employed in type 1 elements was assessed by comparing the moment-curvature relationship of the B-region structural members, evaluated using section force analysis, see Fig. 3.3, with that calculated using the corresponding STMs. It was found that s.h.r = 5% is suitable for rebar struts representing flexural compression reinforcement in beams, and that s.h.r = 2.5% is appropriate for rebar struts replicating the flexural compression reinforcement located in columns and beams with significant side reinforcement.
3.5.2.1 Effective Area of Flexural Concrete Struts

The cross-sectional area of struts which represent the concrete flexural compression zone, $A_{cs}$, is defined by the area between the neutral axis position and the extreme compression edge of the section:

$$A_{cs} = \frac{\phi D_c^2}{4} - \frac{D_c^2}{4} \times \sin\phi \times \cos\phi$$

for a circular section \( \text{(Eq. 3.3a)} \)

$$= \frac{D_c^2 \cdot (\phi - \sin\phi \cos\phi)}{4}$$

and \( \phi = \cos^{-1}\left(\frac{0.5D_c - c}{0.5D_c}\right) \)

$$A_{cs} = c \cdot b_w$$

for a rectangular section \( \text{(Eq. 3.3b)} \)

where \( c \) is the neutral axis depth, measured from the extreme compression edge;

\( b_w \) is the section width;

\( D_c \) is the diameter of a circular section; and

\( \phi \) is half angle of the fan shaped compression sector, measured to the circular section edge, see Fig. 3.4b.

3.5.2.2 Effective Compressive Strength of Flexural Concrete Struts

The maximum compression, $C_{c(max)}$, which can be supported by concrete in the flexural zone of B-region structural members is first assessed using section force analysis. The magnitude of this parameter is dependent upon the longitudinal reinforcement quantity and the applied load level. Subsequently, the effective strut strength, $f_d$, is derived using Eq. 3.4:

$$f_d = \frac{C_{c(max)}}{A_{cs}}$$

\( \text{(Eq. 3.4)} \)

3.5.2.3 Effective Area of Flexural Rebar Struts

Columns usually have the flexural rebars uniformly distributed adjacent to the perimeter of the section. For simplicity of calculating the effective area of ties representing the flexural
tension reinforcement, an imaginary reinforcement tube is assumed to be located along the centre line of longitudinal rebars, see Fig. 3.4.

For a circular column section shown in Fig. 3.4a, the half angle of the flexural compression sector, $\phi'$, measuring to the centreline of the reinforcement tube and the thickness of flexural reinforcement tube, $t$, are calculated using Eqs. 3.5 and 3.6, respectively.

$$\phi' = \cos^{-1}\left(\frac{0.5D_o - c}{r_o}\right)$$  \hspace{1cm} (Eq. 3.5)

$$t = \frac{A_{st}}{2\pi r_o}$$  \hspace{1cm} (Eq. 3.6)

$$r_o = \frac{D' - d_b - d_{vs}}{2}$$

where $D'$ is the circular concrete core diameter measured from centre to centre of peripheral hoop;

$d_b$ is the diameter of longitudinal rebars;

$d_{vs}$ is the diameter of transverse rebars;

$A_{st}$ is the total area of longitudinal rebars; and

$r_o$ is the radius of circular concrete core measuring from the section centre to the centreline of the longitudinal rebar, see Fig. 3.4a.

For a rectangular column that is transversely reinforced with interlocking spirals as shown in Fig. 3.4b, it was found appropriate to use a rectangular imaginary reinforcement tube for replicating the actual distribution of the flexural reinforcement. The adoption of a rectangular imaginary reinforcement tube for a rectangular column section, such as that shown in Fig. 3.4c, is more obvious, and the tube thickness, $t$, is calculated as:

$$t = \frac{A_{st}}{2 \cdot (b_o + D_o)}$$  \hspace{1cm} (Eq. 3.7)

Accordingly, the effective area for rebar struts, $A_{rs}$, representing the flexural compression reinforcement in the circular and rectangular columns, is given as:

Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
a, A typical circular column section transversely reinforced using circular hoops.

b, A typical rectangular column section transversely reinforced using interlocking spirals.

c, Linear stress profile.

d, A typical rectangular column section transversely reinforced with rectangular hoops.

Figure 3.4: Circular and rectangular column sections.
For a circular imaginary reinforcement tube:

\[ A_{rs} = 2 \cdot \phi' \cdot t \cdot r_o \]  
(Eq. 3.8a)

For a rectangular imaginary reinforcement tube:

\[ A_{rs} = t \cdot (b_o + 2c + D_o - D_r) \]  
(Eq. 3.8b)

where \( D_r \) is the total sectional depth of rectangular column.

For beams designed to resist reversing seismic moments, the flexural reinforcement is commonly located adjacent to the tension and compression edges of the section. If the side reinforcement quantity is comparatively small, it may be neglected as it has minimal influence on the modelling results. Therefore, the effective area of rebar struts representing the flexural compression reinforcement is assigned as identical to the actual rebar area.

\[ A_{rs} = A'_s \]  
(Eq. 3.9)

where \( A'_s \) is the area of flexural compression reinforcement.

For beam sections with significant side reinforcement, the imaginary reinforcement tube approach described above can be used to compute the effective area of the rebar struts.

3.5.2.4 Effective Yield Strength of Flexural Rebar Struts

Rebar struts employed in the type 1 element start yielding when the compressive stress being supported by the rebar struts reaches an effective yield strength, \( f_{cy} \), evaluated using Eq. 3.10. Beyond compression yielding, the compressive strength of rebar struts increases following the post-yielding branch of the bilinear curve shown in Fig. 3.2d, at a strain-hardening ratio described in Section 3.5.2.

\[ f_{cy} = \frac{C_s}{A_{rs}} \]  
(Eq. 3.10)

The \( C_s \) term employed in Eq. 3.10 is the maximum compression that can be supported by the reinforcement that is located in the structural B-regions before compression yielding develops. This evaluation of different structural member types is described in the following two sections.
(a) **Rebar Struts in Beams**

For beam sections with minimal side reinforcement, the maximum compression, $C_s$, that can be supported by reinforcement before yielding is the product of the compression rebar area and measured material yield strength, $C_s = A_s' f_y$. The effective compressive strength of flexural rebar strut, $f_{cy}$, is therefore given as:

$$f_{cy} = f_y$$  \hspace{1cm} (Eq.3.11)

(b) **Rebar Struts in Columns**

For column sections, an imaginary reinforcement tube is considered again to compute the maximum compression that can be supported by reinforcement before yielding. This assumes that linearity is valid in describing the stress profile across the column sections. When stress in the extreme compression fibre of the imaginary reinforcement tube reaches the measured reinforcement yield strength, $f_y$, then the total compression, $C_s$, carried by a circular imaginary reinforcement tube, see Fig. 3.4a, is calculated as:

$$C_s = 2 \cdot \int_0^{\phi'} \sigma_s \, dA_{rs}$$

$$= 2 f_y \cdot t \cdot r_o \cdot \int_0^{\phi'} \left[ \frac{\cos \phi - \cos \phi'}{1 - \cos \phi'} \right] d\phi$$

$$= 2 f_y \cdot t \cdot r_o \cdot \frac{\sin \phi' - \phi' \cos \phi'}{1 - \cos \phi'}$$  \hspace{1cm} (Eq. 3.12a)

where $\sigma_s$ is the linear stress function across the imaginary longitudinal steel tube; and

$d\phi$ is the differential angle of $\phi$, defined in Fig. 3.4a.

Subsequently, by incorporating $A_{rs}$ derived from Eq. 3.8a for the circular imaginary reinforcement tube, $f_{cy}$ is given as:

$$f_{cy} = f_y \cdot \frac{\sin \phi' - \phi' \cos \phi'}{\phi' \cdot (1 - \cos \phi')}$$  \hspace{1cm} (Eq. 3.12b)

For a rectangular imaginary steel tube, $C_s$ is calculated as:

$$C_s = f_y \cdot t \cdot (b_o + c + 0.5D_o - 0.5D_i)$$  \hspace{1cm} (Eq. 3.13a)

Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
Again, by substituting $A_{rs}$, computed from Eq. 3.8b for the rectangular imaginary reinforcement tube, $f_{cy}$ is evaluated as:

$$f_{cy} = f_y \cdot \frac{b_o + c + 0.5D_o - 0.5D_r}{b_o + 2c + D_o - D_r}$$

(Eq. 3.13b)

For beams with significant side reinforcement, Eq. 3.13b can be used to compute the effective strength of flexural rebar struts.

### 3.5.3 Type 2 Element

The type 2 element is composed of a concrete tie and a rebar tie arranged in tandem. The material response of the concrete ties employed in this element is represented by a trilinear curve illustrated in Fig. 3.2c. The gradient of the elastic branch, $E_c$, was also calculated using the suggested equation in Eurocode 2 (1992), described in Eq. 3.2. The gradient of the descending branch and the residual tensile strength were established by tracing the Prakhyta and Morley (1990) tension stiffening model described in the following Section 3.5.3.1.

The material response of the rebar ties is represented by the tension branch of the bilinear curve shown in Fig. 3.2d. Again, the commonly assumed value of $E_s = 200$ GPa was used as the gradient of the elastic branch. The strain-hardening ratio of the bilinear stress-strain characteristic of the rebar ties was also evaluated by comparing the moment-curvature responses generated, using both section force analysis and STM. It was found suitable to use 5% when representing the flexural tension reinforcement in the beams, and 2.5% when representing either columns or beams with significant side reinforcement.

#### 3.5.3.1 Tension Stiffening Effect

When a reinforced concrete member cracks, the intact concrete between cracks tends to rebound to the unstressed state but is restrained by the reinforcement, resulting in tensile stress being developed in the concrete. This phenomenon is known as tension stiffening, which effectively reduces the tension force carried by the reinforcement and thus indirectly increases the reinforcement material stiffness.

The tension-carrying capacity of cracked concrete is often neglected in design as it does not generally influence structural ultimate strength. However, the tension-stiffening effect should be considered when attempting to accurately predict the force-displacement response of structural concrete, particularly in the elastic regime.

Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
The tension-stiffening model suggested by Prakhya and Morley (1990) and expressed by Eq. 3.14 and illustrated in Fig. 3.2c, was used to model the stress-strain characteristic of concrete ties representing the average tensile response of cracked concrete in structural B-regions. This model was derived from experimental studies of reinforced concrete beams and slabs that were subjected to short-term loads [Clark and Speirs (1978) and Clark and Cranston (1979)].

Parameters that are employed in the model to govern the strength degradation branch include specific surface, (i.e. the surface area of reinforcement per unit volume of concrete, $s_L$), concrete cover to the reinforcement, $c_c$, reinforcement spacing, $s_R$ and rebar diameter, $d_R$. This model is preferred because it considers the effect of reinforcement arrangement on the degrading characteristic of concrete tensile strength and thus the associated response of the structural member. Furthermore, the simplicity and versatility of the model allows important features of material response to be sufficiently replicated using a trilinear curve.

$$f_{ct} = \frac{\beta_t \cdot f_{dt} \cdot \left( \frac{\varepsilon_{ct}}{\varepsilon_{dt}} \right)}{\beta_t - 1 + \left( \frac{\varepsilon_{ct}}{\varepsilon_{dt}} \right)^{\beta_t}} \quad \text{for} \quad \varepsilon_{ct} > \varepsilon_{dt} \quad (\text{Eq. 3.14a})$$

$$\beta_t = \left( \frac{100 A_{ct}}{A_g - A_{ct}} \right)^{0.366} \cdot \left( \frac{1}{s_L \cdot c_c} \right) \cdot \left( \frac{c_c}{s_R} \right)^{0.146} \geq 1 \quad (\text{Eq. 3.14b})$$

where $f_{ct}$ is the concrete tensile stress;

$\varepsilon_{ct}$ is the concrete strain at $f_{ct}$;

$\varepsilon_{dt}$ is the concrete strain at $f_{dt}$;

$\beta_t$ is an empirical factor dictating the slope of descending branch; and

$A_g$ is the gross section area of structural members.

3.5.3.2 Effective Area of Flexural Concrete Ties

The effective area of concrete ties, $A_{ct}$, employed in the type 2 element to represent the flexural tension being carried by concrete is computed as:

$$A_{ct} = \frac{A_g - A_{ct}}{2} \quad \text{for all section geometries} \quad (\text{Eq. 3.15})$$

Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
Notably, the concrete tie area given in Eq. 3.15 corresponds to half of the concrete area in the flexural tension zone. This equation was selected because it provides satisfactory comparison with the experimentally recorded uncracked member stiffness, when used in the examples of STM analyses, described in Chapter 4.

### 3.5.3.3 Effective Strength of Flexural Concrete Ties

It was observed by Blackman et al. (1958) and Somayaji and Shah (1981) that the cracking strength of reinforced concrete members is different from the tensile strength of plain concrete, due to the non-uniform concrete stress distribution induced by rebars orientated in both the longitudinal and transverse directions. Thus, the plain concrete tensile strength is not suitable to be adopted as the effective tensile strength of concrete ties.

It is assumed that the concrete ties located in the structural B-region, as illustrated in Fig. 3.5 a(i), are analogous to the reinforced concrete prism that is depicted in Fig. 3.5 a(ii). Accordingly, the procedure first proposed by Somayaji and Shah (1981) and later modified by Chan et al. (1992) was adopted to evaluate the effective strength of concrete ties. This procedure was developed based on the assumed reinforcement-concrete bond stress distribution that was associated with tension being carried in the concrete component immediately before cracking occurs. This procedure begins by guessing the tension-carrying capacity, \( P \), of the uncracked concrete component and then calculating the transfer length, \( \ell_t \), using Eqs. 3.16 and 3.17. The transfer length is the distance required for transferring the applied tension through the reinforcement to the surrounding concrete, see Fig. 3.5a(iii).

\[
\ell_t = k_p \cdot \frac{P}{(1 + np) \cdot \Sigma_o} \quad \text{[Somayaji and Shah (1981)]} \quad (\text{Eq. 3.16})
\]

where:
- \( n \) is the ratio of \( E_s / E_c \);
- \( \rho \) is the area ratio of \( A_{rt} / A_{ct} \); and
- \( \Sigma_o \) is the total perimeter of longitudinal rebar located in the flexural tension zone.

\( k_p \) is a constant which was determined by Chan et al. (1992) as:

\[
k_p = \frac{1}{u_{ult}} \quad \text{(Eq. 3.17a)}
\]

\[
u_{ult} = 0.07467 \cdot \left( \frac{f'_c}{2.5} \right)^{0.3} \cdot \frac{f'_c}{d_b} \quad \text{(Eq. 3.17b)}
\]
a (i), Tension zone in a B-region.

a (ii), A tie member representing the tension zone in B-regions.

a (iii), Bond stress distribution when subjected to small tension forces.

a (iv), Bond stress distribution when subjected to large tension forces.

b, Algorithm for determining the concrete tie effective tensile strength.

**Figure 3.5: Determining the effective strength of concrete ties.**

where $u_{ult}$ is the ultimate bond stress between reinforcement and concrete;

$c_c$ is the concrete coverage to reinforcement.

An algorithm to describe the subsequent steps is illustrated in Fig. 3.5b, in conjunction with application of Eqs. 3.16 to 3.21. These equations were developed by Chan et al. (1992)
assuming a nonlinear piecewise function of the reinforcement-concrete bond stress distribution, assuming no slippage of the reinforcement.

Expressions to calculate the peak and average concrete stress corresponding to the different tension force level in the concrete component are provided in Eqs. 3.18 and 3.19 as:

For \( \ell_t < \ell' \):

\[
\begin{align*}
    u_m &= \frac{P}{0.4736 \cdot \Sigma_o \cdot \ell_t \cdot (1 + np)} \quad \text{(Eq. 3.18a)} \\
    \sigma_{ct} &= \frac{0.4736 \cdot \Sigma_o \cdot u_m \cdot \ell_t}{A_{ct}} \quad \text{(Eq. 3.18b)} \\
    \bar{\sigma}_{ct} &= \frac{(\ell' - 0.3408 \ell_t) \cdot P}{(1 + np) \cdot A_{ct} \cdot \ell'} \quad \text{(Eq. 3.18c)}
\end{align*}
\]

For \( \ell_t > \ell' \):

\[
\begin{align*}
    a &= 1.513 \ell' - \frac{1.6565P}{\Sigma_o \cdot u_{ult} \cdot (1 + np)} \quad \text{(Eq. 3.19a)} \\
    \sigma_{ct} &= \frac{0.9134 \cdot \Sigma_o \cdot u_{ult} \cdot \ell'}{A_{ct}} - \frac{0.6037 \cdot \Sigma_o \cdot u_{ult} \cdot a}{A_{ct}} \quad \text{(Eq. 3.19b)} \\
    \bar{\sigma}_{ct} &= \frac{0.4192 \cdot \Sigma_o \cdot u_{ult} \cdot \ell'}{A_{ct}} - \frac{0.2014 \cdot \Sigma_o \cdot u_{ult} \cdot a^2}{A_{ct} \cdot \ell'} \quad \text{(Eq. 3.19c)}
\end{align*}
\]

where \( \ell' \) is half of the tie member length, defined in Fig. 3.5a(ii);

\( u_m \) is the maximum bond stress corresponding to the tension force level in concrete component, \( P \), being considered;

\( a \) is the development length of ultimate bond stress, see Fig. 3.5a(iv);

\( \sigma_{ct} \) is the peak stress in concrete component; and

\( \bar{\sigma}_{ct} \) is the average stress in concrete component.
The cracking strength of the concrete component in a reinforced concrete prism, $f_{cr}$, for both tension load levels is then calculated as:

$$f_{cr} = 0.92f'_t \cdot \left(\frac{\sigma_{cl}}{\sigma_{ct}}\right)^{0.8}$$  \hspace{1cm} (Eq. 3.20)

where $f'_t$ is the plain concrete tensile strength, $=0.6\sqrt{f'_c}$ [NZS 3101 (1995)] for concrete subjected to flexural tension and $=0.5\sqrt{f'_c}$ [Priestley (1996)] for concrete subjected to direct tension.

Finally, the effective strength of concrete ties, $f_{dt}$, was assessed as being identical to the average tensile stress in the concrete component of a reinforced concrete prism:

$$f_{dt} = \bar{\sigma}_{ct}$$  \hspace{1cm} (Eq. 3.21)

Note that the suggested procedure to calculate the effective tensile strength can be easily implemented using a spreadsheet.

To assist with determining the effective strength of concrete ties conveniently, the design charts shown in Appendix A were formulated for reinforcement sizes commonly used in New Zealand. The effective strength of concrete ties, $f_{dt}$, given in these charts are normalised with respect to the plain concrete tensile strength, $f'_t$, and are plotted for practical $\rho$ values as a function of $\ell'/d_b$. Since $0.5 \leq c_c/d_b \leq 3$, where $c_c$ is the concrete cover to the longitudinal reinforcement and $d_b$ is the rebar diameter, has minimal influence on the computed value of the effective strength of concrete ties, a practical value of $c_c = 35$ mm is adopted in establishing the charts. Moreover, for a section detailed with a uniform rebar size, the total perimeter of longitudinal reinforcement located in the flexural tension zone, $\Sigma_o$, is directly proportional to the area of rebar ties representing the longitudinal tension reinforcement, $A_{rt}$, and is effectively considered when evaluating the $\rho$ value. Therefore, computation of $\Sigma_o$ is not necessary when using these charts.

It is noted that the procedure described above for calculating the effective strength of concrete ties suggests no concrete cracking for the domain of $\ell'/d_b$ smaller than that covered by the charts. This is because Chan’s model would suggest insufficient transfer of tensile stress from the embedded reinforcement to the surrounding concrete through bonding. For the length of concrete ties shorter than that accounted for in the charts, cracking may still occur due to compatibility. In such cases, when located in structural B-regions, a linearly-inclined stress distribution profile, analogous to that shown in Fig. 3.4c, with an

Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
extreme concrete tension fibre stress equal to $f'_t$ is assumed. The effective tensile strength is then evaluated by dividing the total concrete tension force by the concrete tie area, $A_{ct}$, to give:

\[
\begin{align*}
    f_{dt} &= \frac{D_c^2 \cdot f'_t}{6 \cdot A_{ct}} & \text{for circular sections} & \quad \text{(Eq. 3.22a)} \\
    f_{dt} &= \frac{D_r \cdot b_w \cdot f'_t}{4 \cdot A_{ct}} & \text{for rectangular sections} & \quad \text{(Eq. 3.22b)}
\end{align*}
\]

where $f'_t = 0.6\sqrt{f'_c}$ [NZS 3101 (1995)].

The significance of including the concrete ties in the type 2 element for simulating the tension-stiffening effect is illustrated in Fig. 3.6 using the strut-and-tie analytical results of circular columns that are subjected to double bending, see Fig. 3.6a [To et al. (2002b)]. It is demonstrated in Figs. 3.6b and 3.6c that the STMs equipped with concrete ties to simulate the tension-stiffening effect have predicted the column elastic stiffness and unit yield strength with higher satisfaction.

### 3.5.3.4 Effective Area of Flexural Rebar Ties

When computing the effective area of rebar ties employed in the type 2 element to represent flexural tension reinforcement in columns, the imaginary reinforcement tube shown in Fig. 3.4 is considered again.

\[
\begin{align*}
    A_{rt} &= 2(\pi - \Phi')t r_o & \text{for circular imaginary reinforcement tube} & \quad \text{(Eq. 3.23a)} \\
    A_{rt} &= t \cdot (b_o + D_o + D_r - 2c) & \text{for rectangular imaginary reinforcement tube} & \quad \text{(Eq. 3.23b)}
\end{align*}
\]

For the rebar ties employed to represent the flexural tension reinforcement in beams with minimal side reinforcement, the actual tension reinforcement quantity was adopted as the effective tie area:

\[
A_{rt} = A_s
\]

where $A_s$ is the flexural tension reinforcement area in a beam.

For beam sections with significant side reinforcement, Eq. 3.23b derived from the imaginary reinforcement tube method can be used to calculate the effective area of flexural rebar ties.
a) Column physical configurations and STMs

b) Force-displacement response for column (I) subjected to double bending.

**Figure 3.6: Force-displacement response comparison diagrams. [To et al. (2002b)]**
3.5.3.5 Effective Yield Strength of Flexural Rebar Ties

Similar to the method for computing the effective yield strength of flexural rebar struts described in Section 3.5.2.4, the effective yield strength of flexural rebar ties, $f_{sy}$, is computed by dividing the maximum tension that could be carried by reinforcement before yielding, $T_s$, by the effective tie area, see Eq. 3.25. Once the tensile stress inside rebar ties reaches its effective yield strength, the strength of rebar ties increases following the post-yielding branch of a bilinear curve, shown in Fig. 3.2d, at a strain-hardening ratio described in section 3.5.3.

$$f_{sy} = \frac{T_s}{A_{rt}}$$  \hspace{1cm} (Eq. 3.25)

The maximum tension supported by reinforcement inside the B-region structural members before yielding, $T_s$, is computed for different structural member types in the following two sections.
(a) Rebar Ties in Beams

For beam sections with minimal side reinforcement, \( T_s \) is the product of the reinforcement area and measured material yield strength, \( T_s = A_s f_y \). Hence the effective yield strength of rebar ties is given as:

\[
f_{sy} = f_y
\]  
(Eq. 3.26)

(b) Rebar Ties in Columns

For column sections it was assumed that the imaginary reinforcement tube has a linear stress profile with the stress value at the extreme tension fibre equal to the measured reinforcement yield strength, see Fig. 3.4a. Accordingly, \( T_s \) is calculated as:

\[
T_s = 2 \cdot \int_0^{\pi/2} \sigma_s dA_{rt}
\]

\[
= \frac{f_y t (D' - d_b - d_{vs})}{\cos(\phi') \cdot \sin\left(\frac{\pi + \phi'}{2}\right)} \cdot \int_0^{\pi/2} \cos\left(\frac{\pi - \phi}{2}\right) \cdot \sin\left(\phi' + \frac{\phi}{2}\right) d\phi
\]

\[
= f_y t r_o \cdot \left[ \frac{\sin\phi' - (\phi' - \pi) \cos\phi'}{\cos(\phi') \cdot \sin\left(\frac{\pi + \phi'}{2}\right)} \right]
\]  
(Eq. 3.27a)

By incorporating \( A_{rt} \), calculated from Eq. 3.23a for the circular imaginary reinforcement tube into Eq. 3.25, \( f_{sy} \) is calculated as:

\[
f_{sy} = \phi_o \cdot f_y \cdot \frac{\sin\phi' - (\phi' - \pi) \cos\phi'}{2(\pi - \phi') \cos\left(\frac{\phi'}{2}\right) \cdot \sin\left(\frac{\pi + \phi'}{2}\right)}
\]  
(Eq. 3.27b)

where \( \phi_o = 4/3 \) is an effective overstrength factor only applied for circular column sections.

For the rectangular imaginary reinforcement tube, \( T_s \) is computed as:

\[
T_s = f_y t \cdot (b_o + 0.5D_o + 0.5D_r - c)
\]  
(Eq. 3.28a)
Again, by incorporating $A_{ct}$ derived from Eq. 3.23b for the rectangular reinforcement tubes into Eq. 3.25, $f_{sy}$ is given as:

$$f_{sy} = f_y \cdot \frac{b_o + 0.5D_o + 0.5D_r - c}{b_o + D_o + D_r - 2c}$$  \hspace{1cm} (Eq. 3.28b)

Note that this equation can also be used for calculating the effective strength of flexural rebar ties representing beam sections with significant side reinforcement.

### 3.5.4 Analytical Charts for Type 1 and Type 2 Elements

It has been shown that formulation of the type 1 and type 2 elements rely heavily upon the section force analysis. For the case when a comprehensive section force analysis is not available, or if the scope of the analytical exercise is such that a high degree of accuracy is not required, the properties of type 1 and type 2 elements can be derived using the figures shown in Appendix B [To et al. (2001) & (2002b)] for two concrete cylinder strengths, 30 MPa and 40 MPa, as a function of flexural reinforcement quantity. Note that the analytical data plotted on these figures are correlated with trend lines for ease of application. The neutral axis positions and the force centroid positions given in the figures were normalised with respect to the total section depth for rectangular sections, and to the section diameter for circular sections, see Figs. B1(i-iv), B2(ii-vii) & B3(ii-vii). Also, illustrated in Figs. B1(vi), B2(viii-ix) & B3(viii-ix) is the effective strength of flexural concrete struts, the strength values those derived using equations described in Section 3.5.2.1 were normalised to the corresponding concrete cylinder strength.

Grade 60 rebar, with respective nominal yield and ultimate strength of 413 MPa and 600 MPa, was employed in the computation of the analytical charts. These charts can be satisfactorily used for grade 430 steel, which was commonly used in New Zealand prior to 2004, as the discrepancy observed in analytical results that derived from these two reinforcement grades is insignificant.

### 3.5.5 Type 3 Element

The type 3 model element is composed of a diagonal concrete strut representing the diagonal concrete mass located in beams and columns. The quadlinear curve depicted in Fig. 3.2b is also employed to describe the stress-strain characteristic of the concrete strut used in this model element type.
3.5.5.1 Effective Area of Diagonal Concrete Struts

The effective area of struts representing concrete between diagonal cracks in beams and columns is computed by multiplying the perpendicular distance between the strut member, \( h_p \), see Fig. 3.2a and the effective web width. In this study, the effective web width was taken as the width of confined concrete core equal to \( 1.6r_o \) for circular columns, and \( b_o \) for rectangular columns and beams, see Fig. 3.4.

3.5.5.2 Effective Strength of Diagonal Concrete Struts

Based on the plasticity theory, the equation proposed by Nielsen et al. (1978) and that recommended by Eurocode (1992), as shown in Eq. 3.29, were employed for evaluating the effective strength of diagonal concrete struts, \( f_d \), as:

\[
f_d = \psi f'_c
\]

and \( \psi = 0.7 - \frac{f'_c}{200} \geq 0.5 \) \(( f'_c \text{ in MPa})\) \hspace{1cm} \text{(Eq. 3.29)}

3.5.6 Type 4 Element

The type 4 model element consists of a transverse rebar tie, replicating the transverse reinforcement in beams and columns. The material response of the transverse rebar ties employed for this model element type was represented by the tension branch of the bilinear curve shown in Fig. 3.4d.

3.5.6.1 Effective Area of Transverse Rebar Ties

The effective area of the transverse rebar ties was calculated by dividing the nominal shear strength of a member, \( V_n \), by the measured yield strength of the transverse reinforcement, \( f_{vy} \), to give:

\[
A_v = \frac{V_n}{f_{vy}} \hspace{1cm} \text{(Eq. 3.30)}
\]

3.5.6.2 Effective Strength of Transverse Rebar Ties

The effective shear strength of a reinforced concrete member, \( V_n \), is the combined contribution of transverse reinforcement, \( V_s \), the concrete tensile strength before cracking or the concrete aggregate interlocking action after cracking, \( V_c \), and the force component in the direction of applied shear arising from the prestressing or externally applied action, \( V_p \) [Rahal and Collins (1999)]. The expression for nominal member shear resistance is thus written as:

Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
\[ V_n = V_s + V_c + V_p \]

(Eq. 3.31)

(a) \textbf{Reinforcement Contribution}

To compute the shear strength capacity attributed to transverse reinforcement, \( V_s \), Eqs. 3.32a [Ang \textit{et al.} (1989)] and 3.32b were employed for transverse reinforcement in the form of circular spirals or hoops and rectangular ties, in conjunction with consideration of the inclined angle of diagonal concrete struts, \( \theta \).

\[ V_s = \frac{\pi D'}{4 s \tan \theta} A_{vs} f_{vy} \quad \text{for circular spirals or hoops} \quad \text{(Eq. 3.32a)} \]

\[ V_s = \frac{d_v}{s \tan \theta} A_{vs} f_{vy} \quad \text{for rectangular ties} \quad \text{(Eq. 3.32b)} \]

where \( A_{vs} \) is the total area of transverse reinforcement in a layer in the direction of the shear force; and

\( d_v \) is the effective section depth.

(b) \textbf{Concrete Contribution}

The member shear strength attributed to the tension-carrying capacity of cracked concrete was computed using the modified compression field theory (MCFT) [Vecchio and Collins (1986)] that was briefly addressed in Section 2.4.2, and is recommended by the Canadian Concrete Design Standard [CSA A23.3 (1994)]. By taking account of the assumed diagonal crack angle, \( \theta \), and the maximum longitudinal reinforcement tensile strain, \( \varepsilon_s \), the concrete shear strength, \( V_c \), was given by:

\[ V_c = \beta \sqrt{f'_{c} A_{ve}} \]

(Eq. 3.33a)

and \[ \beta = \frac{0.33 \cot \theta}{1 + \sqrt{500 \varepsilon_1}} \leq \frac{0.18}{0.3 + \frac{7200 \varepsilon_1}{35}} \];

(Eq. 3.33b)

\[ \varepsilon_1 = \varepsilon_s + [\varepsilon_s + 0.002 (1 - \sqrt{1 - \nu (\tan \theta + \cot \theta (0.8 + 170 \varepsilon_1)))})] \cot \theta^2 ; \]

(Eq. 3.33c)

\[ \nu = \frac{V_s + V_c - V_p}{A_{ve} f'_{c}} \]

(Eq. 3.33d)
\[ A_{ve} = 0.628D^2 \quad \text{for circular section [Priestley et al. (1996)]} \quad (\text{Eq. 3.33e}) \]

\[ A_{ve} = 0.8A_g \quad \text{for rectangular section} \quad (\text{Eq. 3.33f}) \]

where \( A_{ve} \) is the effective shear area;
\( \varepsilon_1 \) is the average principal tensile strain in diagonal concrete.
\( \nu \) is the total shear stress resisted by concrete and transverse reinforcement.

A trial-and-error computation procedure for determining concrete shear strength using MCFT is described in the following. As the procedure pre-requires an assumption of the anticipated maximum longitudinal reinforcement tensile strain, \( \varepsilon_s \), this value was selected between the practical upper and lower limits of \( 0 \leq \varepsilon_s \leq 0.002 \), which were recommended by Rahal and Collins (1999). While the lower limit of \( \varepsilon_s \) is applicable to a member subjected to high compression, such as a partially or fully prestressed section, the upper limit of \( \varepsilon_s \) is suitable for a member that is not subjected to axial compression.

A rigorous equation to compute \( \varepsilon_s \) is documented in recent literature [Collins and Mitchell (1997), CSA A23.3 (1994) and Rahal and Collins (1999)]. However, as a simple approach for practical use, \( \varepsilon_s \) is defined in Table 3.1 in this research according to the practical range of applied axial load level.

Notably, the MCFT does not examine the possible shear strength degradation in the potential plastic hinge zones. However, it was considered suitable to be adopted in this research project primarily because it accounts for the orientation of diagonal concrete struts when computing the concrete shear strength. Furthermore, all the STM analytical examples considered in Chapter 4 did not exhibit premature shear failure in the structural B-regions.

The trial-and-error procedure for computing \( \beta \) is given as follows:

**Step 1:** Determine the appropriate inclined angle of diagonal concrete strut, \( \theta \).

**Step 2:** Calculate \( V_s \) using Eq. 3.32.

**Step 3:** Select an \( \varepsilon_s \) value from Table 3.1 according to the applied axial load level.

**Step 4:** Guess the value of \( \nu \), the ratio of shear strength (excluding \( V_p \)) to concrete cylinder strength.

Nonlinear Strut-and-Tie Model Formulation Procedure: Monotonic Model Formulation Procedure
Step 5: Solve the quadratic equation 3.33c to find $\varepsilon_1$.

Step 6: Compute $\beta$ using Eq. 3.33b.

Step 7: Calculate $V_c$ using Eq. 3.33a.

Step 8: Calculate $v$ using Eq. 3.33d.

Step 9: Check if $v$ (derived in step 8) = $v$ (guess at step 4); Return to step 4 if necessary, otherwise obtain $V_c$.

Table 3.1 Appropriate $\varepsilon_s$ values. [Rahal and Collins (1999)]

<table>
<thead>
<tr>
<th>$\varepsilon_s$</th>
<th>Axial load level, $\frac{N}{A_g f'_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>high, $N/A_g f'_c &gt; 0.35$</td>
</tr>
<tr>
<td>0.001</td>
<td>moderate, $0.35 \geq N/A_g f'_c \geq 0.1$</td>
</tr>
<tr>
<td>0.002</td>
<td>low, $0.1 &gt; N/A_g f'_c$</td>
</tr>
</tbody>
</table>

The procedure described above for computing the $\beta$ value usually converges rapidly when using an initial trial value of $v = 0.1$. Furthermore, to assist in determining the $\beta$ value without solving the quadratic Eq. 3.33c, the design charts shown in Appendix C are plotted for different $v$ values as a function of $\theta$.

(c) Externally Applied Load Contribution

The force component of the externally applied load orientated parallel to the applied shear action is considered as an independent contribution of member shear strength. Externally applied loads such as those illustrated in Fig. 3.7, include prestressing in beams and axial loads in columns. The force component is calculated as:

$$V_p = N \tan \alpha_N$$

(Eq. 3.34)

where $N$ is the externally applied axial load; and

$\alpha_N$ is the angle between the member longitudinal axis and the line of the applied axial action.
3.5.7 Type 5 Element

The type 5 model element is composed of a concrete strut to represent the compressed concrete mass of structural components located in the structural D-region. Again, the material stress-strain response of concrete struts employed in this model element type is represented by a quadlinear curve as depicted in Fig. 3.2b.

3.5.7.1 Effective Area of Concrete Struts

In the STM analysis, the effective area of the concrete struts was calculated by multiplying the effective strut depth parallel to the plane of the model layout, by the strut width orthogonal to the plane of the model layout. The strut width was measured by the average distance between adjacent concrete struts, and the effective strut depth was taken as the

![Diagram](image)

Figure 3.7: Axial load contribution to member shear strength.
concrete core width, which was approximately assessed as \(0.8b_w\), where \(b_w\) is the total width of a structural component.

### 3.5.7.2 Effective Strength of Concrete Struts

The compressive strength of mass concrete within a reinforced concrete structure is dependent upon the confining effects attributed to the structural D-region boundary conditions, the quantity of transverse reinforcement, the extent of cumulative damage to the concrete, the multiaxial stress state and the presence of transverse tensile strains.

The effective concrete strength employed in the type 5 model element could be determined using Eq. 3.35 as:

\[
f_d = \phi_e f'_c
\]

(Eq. 3.35)

where \(\phi_e\) is the strength efficiency factor.

Following a study of nonflexural structural members, Marti (1985b) proposed \(\phi_e = 0.6\) for general application, with this value varying depending on the presence of lateral confinement. Rogowsky and MacGregor (1986) also agreed that \(\phi_e = 0.6\) should be employed in design practice. They further suggested that a value as high as \(\phi_e = 0.85\) can be used if concrete struts are orientated within \(\pm15^\circ\) of the uncracked elastic force paths. Following an experimental study on the effect of transverse splitting stress on concrete cylinders, Adebar and Zhou (1993) also proposed a similar value of \(\phi_e = 0.6\) for concrete struts that have no transverse reinforcement. Other strength degrading factors were also proposed by Alshegeir and Ramirez (1992), \(\phi_e = 0.2, 0.5\) and \(0.8\), and Schlaich et al. (1987), \(\phi_e = 0.34, 0.51\) and \(0.68\), for different strut conditions.

In conjunction with \(\phi_e\) for considering the stress field conditions being experienced by the concrete struts, MacGregor (1997) introduced an additional factor, \(\phi_2\), in his more recent work to account for the concrete brittle effect with the increase of cylinder strength as:

\[
f_d = \phi_e \phi_2 f'_c
\]

(Eq. 3.36)

and

\[
\phi_2 = 0.55 + \frac{1.25}{\sqrt{f'_c}}
\]

It is clear that no general consensus is concluded amongst the research community regarding the suitable efficiency factors to be used in STMs. However, those factors proposed by Schlaich et al. (1987) were justified to be appropriate when adopted in evaluating a suitable
design solution for a beam-column joint under seismic loading [Sritharan (1998)]. These efficiency factors are listed in Table 3.2 corresponding to the anticipated strut condition at the ultimate limit state.

For analysis orientated problems such as those considered in this study, application of the effective strength factors listed in Table 3.2, would significantly underestimate the structural strength particularly at the low to mid displacement ductility levels. For all analytical examples considered in Chapter 4, it was discovered that the maximum structural strength at all displacement ductility levels could be satisfactorily captured using the modified Kent and Park’s concrete stress-strain characteristic model, as shown in the quadlinear curve in Fig. 3.2b, when \( f_d = 0.51 f'_c \) was adopted as the effective strength of concrete struts located in the structural D-regions.

<table>
<thead>
<tr>
<th>Effective strut strength, ( f_d )</th>
<th>Expected concrete strut conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.68 ( f'_c )</td>
<td>This value can be adopted for concrete struts locating in regions where minor cracking is expected. An example of this application would be the strut residing in a fully prestressed structural member.</td>
</tr>
<tr>
<td>0.51 ( f'_c )</td>
<td>This value is appropriate for concrete struts when the neighbouring rebar is not expected to have significant strain hardening. A suggested value of maximum rebar strain is ( \varepsilon_s &lt; 0.01 ).</td>
</tr>
<tr>
<td>0.34 ( f'_c )</td>
<td>This is the maximum permissible design stress for concrete struts when there is potential development of significant inelastic strain, such as ( \varepsilon_s &gt; 0.02 ) in the neighbouring reinforcement. This value can be used in concrete struts with no effective confinement.</td>
</tr>
</tbody>
</table>

### 3.5.8 Type 6 Model Element

The type 6 model element is composed of a rebar tie to represent the reinforcement in D-regions of a structural system. The tension branch of the bilinear curve shown in Fig. 3.4d is used as the stress-strain characteristic of the rebar ties. A strain hardening ratio equal to 2.5%
was adopted for the bilinear material response to account for the post-yielding strength enhancement at the material plastic regime. The actual rebar area and measured material yield strength were employed as the effective area and effective yield strength of rebar ties in the type 6 element, respectively.

3.5.9 Special structural features in D-regions

Special structural features encountered in the analytical examples considered in the next chapter are addressed in this section.

3.5.9.1 Reinforcement Lap Splicing

An assessment procedure for lap spliced rebars that have little or no transverse reinforcement has been previously developed [Ingham et al. (1994) and Priestley et al. (1996)] and is applied to the as-built bridge knee-joint test unit considered in Chapter 4 as one of the STM analysis examples. A failure mechanism is presumed to develop between each of the tension lapped bars and around the inside perimeter of the bar group as shown in Fig. 3.8. Assuming the concrete tensile strength across the rupture surface to be $0.29 \sqrt{f_c}$, the lap splice capacity, $T_{lp}$ is established as:

$$T_{lp} = 0.29 \sqrt{f_c} \ell_s p_\ell$$

(Eq. 3.37)

where $\ell_s$ is the lap splice length of rebars; and $p_\ell$ is the cross section length of rupture surface, see Fig. 3.8.
The lap splicing between rebar ties is replicated by a translational spring with zero length, see the inset in Fig. 3.9. The tension-slip response of lap splicing that is depicted in the same figure assumes no slipping until the applied tension, $T$, is equal to the lap splice capacity. Thereafter, the lap splice capacity decreases until it reaches the residual capacity attributable to friction between lug and surrounding concrete. Since no appropriate values for the gradient of the degrading branch and residual lap splice capacity were found in literature, these two parameters were assessed by comparing the experimental data with analytical STM results that were generated for the as-built knee-joint test unit considered in Section 4.4.1, with the incorporation of a reinforcement lap splicing model. It was found appropriate to assume the residual lap splice capacity becomes effective at the slip, defined as $s$ in Fig. 3.9, identical to 5 times the elongation of the lap spliced rebar ties when the rebar tension reaches the lap splice capacity. Also, a residual lap splice capacity equal to 40% of the maximum value was assessed as suitable.

3.5.9.2 Reinforcement Clamping Length

The capacity of reinforcement anchorage in concrete requires careful examination because it affects correct positioning of the nodal zones, which are generally located at the point where full tensile strength of the reinforcement may be developed.

This can be illustrated by examining the force-transfer mechanism developed inside a knee-joint subjected to joint-opening moment. When concrete starts to crack in the joint panel...
region, the associated crack pattern when subjected to joint-opening moment does not allow a uniform bond stress along the embedded length of tension rebar. If bond stress along the effective anchorage zone of the column reinforcement is equated to the maximum experimentally observed value of \(2.5\sqrt{f'_c}\) MPa [Ingham et al. (1997)], the effective clamping length, which is referred to the minimum length measured from the rebar end to allow the reinforcement to develop tensile yield strength, \(\ell_c\), can be calculated as:

\[
\ell_c = \frac{d_b f_y}{10\sqrt{f'_c}} \quad \text{(Eq. 3.38)}
\]

Note that the rebar overstrength factor was not included in Eq. 3.38, to predict the actual clamping length rather than providing a suitable design solution.

For the effective clamping length of reinforcement outside the joint panel region, the equation recommended by Priestley et al. (1996) shown in Eq. 3.39 was used.

\[
\ell_c = \frac{d_b f_y}{3.0\sqrt{f'_c}} \quad \text{(Eq. 3.39)}
\]

### 3.6 Cyclic Model Formulation Procedure

A STM formulation procedure that allows hysteretic response to be predicted, and time-history earthquake analyses to be conducted for various structural systems is proposed in this section. As adjustments in the model element positions are necessary to allow for symmetrical model layout, it must be emphasised that the proposed procedure was not aiming at replicating the exact force path that develops within a structural system under cyclic actions. Instead, it is a diagnostic tool which is sufficiently simple and robust to be potentially adopted as a routine procedure for seismic-resistant analysis of complicated structural problems.

#### 3.6.1 Model element types

The proposed cyclic STM is formulated using five different element types, as shown in Fig. 3.10. The type 1 element that represented the flexural zone in beams and columns, was composed of a uniaxial fibre model described in Section 3.6.2.1. The type 2 element consisted of a concrete strut and a concrete tie arranged in tandem, to represent the diagonal concrete mass in beams and columns. The type 3 element was composed of a rebar tie to replicate the transverse reinforcement in beams and columns. The mass concrete located in a D-region was represented by the type 4 element, which consisted of a concrete strut and a
concrete tie arranged in tandem. Finally, reinforcement located in the D-region was replicated by the type 5 element, which was composed of a rebar strut-tie.

### 3.6.2 Type 1 element property

The uniaxial fibre model employed in the Type 1 element is developed based on the experimental study by Tjokrodimuljo (1985). This model element type is used in the cyclic STMs to replicate behaviour of the flexural zone in beams and columns. The computation procedure for the fibre model properties, such as effective area and effective strength, were established using the section force analysis described in Section 3.3. Model elements were located at the force centroid of the tension reinforcement, $x_t$, see Fig. 3.9, measured at the first yield states for each flexural action direction.

#### 3.6.2.1 Uniaxial Fibre Model

Seventeen reinforced concrete prisms were tested by Tjokrodimuljo (1985) with reversed cyclic axial loads, to investigate the combined hysterestic response of concrete and reinforcement in the flexural zone of reinforced concrete beams and columns. Based upon

Figure 3.10: Cyclic STM of a portal frame and the model element types.
his findings, a uniaxial fibre model as shown in Fig. 3.11, was developed in this research project to replicate the cyclic material behaviour in flexural zones. This uniaxial fibre model consists of three elements arranged in parallel, namely a concrete tie, a concrete strut and a rebar strut-tie. A typical form of the stress-strain characteristics of concrete in reinforced concrete members found by Tjokrodimuljo (1985) is shown in Fig. 3.12a. In this figure the "contact stress effect" is illustrated by the non-zero compressive stress at zero strain on the loading path. This effect was due to the wedging type action by dislocated aggregate particles in the concrete cracks, and was partially responsible for the beam elongation under cyclic loading [Fenwick et al. (1996)]. Furthermore, Tjokrodimuljo's test results showed that the phenomenon of "tension stiffening", that is attributed to the tension-carrying capability of
a, Typical response of concrete component in reinforced concrete member [Tijkrodimuljo (1985)].

b, Concrete tie.

c, Concrete strut.

d, Combined analytical response of concrete tie and concrete strut.

e, Rebar strut-tie.

f, Uniaxial fibre model.

Figure 3.12: Model member stress-strain characteristic.
concrete, continues even after concrete cracks. To replicate the behaviour of the “contact stress” and “tension stiffening” effects, a concrete tie with the stress-strain characteristic represented by a bilinear elastic-perfectly plastic curve, as illustrated in Fig. 3.12b, was adopted in the analysis.

The elastic stiffness of concrete ties was assessed in this study to be 0.1\(E_c\) as this was found comparable to Tjokrodimuljo’s test data. The compressive strength for the “contact stress effect”, \(f_{\text{cont}} = 0.05f_d\), was selected according to the recommendation by Douglas (1996), where \(f_d\) is the effective compressive strength of concrete struts discussed in Section 3.5.2.2. Also, the tension-carrying capacity of concrete ties, \(f_{ts} = 0.5f_d\), was evaluated in this study to replicate the average value of effective concrete tensile strength, \(f_{dt}\), that can be calculated using the procedure established in Section 3.5.3.3.

The stress-strain characteristic of concrete struts was replicated using the cyclic curve shown in Fig. 3.12c. The monotonic profile of the curve was established according to the modified Kent and Park’s model, described in Section 3.5.2. The degradation of unloading and reloading stiffness was defined automatically by the internal function in Drain-2DX according to the monotonic response envelope and concrete strut axial displacement.

Illustrated in Fig. 3.12d is the combined stress-strain characteristic of the concrete strut and concrete tie generated using Drain-2DX, which is the analytical replication of typical experimental results depicted in Fig. 3.12a. A bilinear elasto-plastic curve shown in Fig. 3.12e was used to represent the material response of reinforcement, having the commonly assumed elastic stiffness, \(E_s = 200\text{ GPa}\). The strain hardening ratio of the bilinear curve was equal to either 2.5% or 5%, as suggested in Sections 3.5.2 and 3.5.3, chosen depending on the type of B-region structural members being modelled. In Fig. 3.11f, the analytical axial load-axial displacement traces generated using an idealised uniaxial fibre model are shown to compare satisfactorily with the typical experimental results obtained by Tjokrodimuljo (1985). Discrepancies between the two sets of data, in particular to the unloading and reloading stiffness, is due to the analytical material model employed for rebar ties and does not consider the Bauschinger effect. This model deficiency is further discussed in Section 3.7.

3.6.2.2 Effective Area and Strength of Flexural Concrete Struts

The effective area of concrete struts, \(A_{ce}\), employed in the type 1 element to represent concrete in flexural compression zone was calculated as the area between the neutral axis position and the extreme compression edge of the section. Equations for evaluating the area of concrete struts were identical to those employed in the monotonic STMs.
\[ A_{cs} = \frac{D_c^2 \cdot (\phi - \sin \phi \cos \phi)}{4} \] for circular sections \quad \text{(from Eq. 3.3a)}

and \[ \phi = \cos^{-1} \left( \frac{0.5D_c - c}{0.5D_c} \right) \]

\[ A_{cs} = c \cdot b_w \] for rectangular sections \quad \text{(from Eq. 3.3b)}

Experimental results from Tjokrodimuljo (1985) showed that a 15% reduction of concrete compressive strength occurred in reinforced concrete prisms if the rebar was previously subjected to inelastic tensile strain. Consequently, to take account for this possible material damage, the effective strength of concrete struts, \( f_d \), was calculated as:

\[ f_d = \frac{0.85 \cdot C_{c(max)}}{A_{cs}} \] \quad \text{(Eq. 3.40)}

where \( C_{c(max)} \) is the maximum compression that could be supported by concrete in the flexural zone, see Section 3.5.2.2.

### 3.6.2.3 Effective Area and Strength of Concrete Ties

The effective area of concrete ties utilised in the type 1 element for representing concrete in the flexural tension zone is equal to the concrete strut area, \( A_{cs} \), as:

\[ A_{ct} = A_{cs} \] \quad \text{(Eq. 3.41)}

The effective compressive strength of concrete ties, \( f_{cont} \), for replicating the "contact stress effect" and the effective tensile strength, \( f_{ts} \), for replicating the cracked concrete tension-carrying capacity is calculated as:

\[ f_{cont} = 0.05f_d \] \quad \text{(Eq. 3.42)}

\[ f_{ts} = 0.5f_{dt} \] \quad \text{(Eq. 3.43)}

Nonlinear Strut-and-Tie Model Formulation Procedure: Cyclic Model Formulation Procedure
where \( f_{dt} \) is the effective tensile strength of concrete evaluated using the procedure described in Section 3.5.3.3.

### 3.6.2.4 Effective Area and Yield Strength of Flexural Rebar Strut-Tie

For the rebar strut-tie employed in the type 1 element to represent the flexural reinforcement in beams, the effective area, \( A_{s-t} \), and the effect yield strength, \( f_{s-t} \), were computed using Eqs. 3.44 and 3.45, respectively.

\[
A_{s-t} = \phi_r A_s \quad \text{for the tension reinforcement} \quad (\text{Eq. 3.44a})
\]

\[
A_{s-t} = \phi_r A'_s \quad \text{for the compression reinforcement} \quad (\text{Eq. 3.44b})
\]

\[
f_{s-t} = f_y \quad (\text{Eq. 3.45})
\]

where \( \phi_r = 3/4 \) is the reduction factor, based on satisfactory comparison between analysis results and experimental data for the test units considered in Chapter 4.

For the effective properties of rebar strut-ties representing flexural reinforcement in columns, the imaginary reinforcement tube described in Section 3.5.4.2 was employed again. The effective area, \( A_{s-t} \), and effective strength, \( f_{s-t} \), were calculated using Eqs. 3.46 and 3.47 as:

\[
A_{s-t} = 2(\pi - \phi') tr_o \quad \text{for circular reinforcement layout} \quad (\text{Eq. 3.46a})
\]

\[
A_{s-t} = \phi_r t \cdot (b_o + D_o + D_r - 2c) \quad \text{for rectangular reinforcement layout} \quad (\text{Eq. 3.46b})
\]

\[
f_{s-t} = f_y \frac{\sin \phi' - (\phi' - \pi) \cos \phi'}{2(\pi - \phi') \cos \left(\frac{\phi'}{2}\right) \sin \left(\frac{\pi + \phi'}{2}\right)} \quad \text{for circular reinforcement tube} \quad (\text{Eq. 3.47a})
\]
\[ f_{s-t} = f_y \frac{b_o + 0.5D_o + 0.5D_r - c}{b_o + D_o + D_r - 2c} \]  

for rectangular reinforcement tube (Eq. 3.47b)

where \( b_o \) is the width of concrete core defined in Figs. 3.4b and 3.4c;  
\( D_o \) is the depth of concrete core defined in Figs. 3.4b and 3.4c; and  
\( D_r \) is the total section depth of rectangular column.

### 3.6.3 Type 2 element property

The type 2 element for representing diagonal mass concrete in beams and columns is composed of a concrete strut and a concrete tie. Within this element, a concrete strut was used to replicate the compression-carrying behaviour of concrete, and a concrete tie was used to simulate the effects of “contact stress” and “tension stiffening” exhibited in concrete when subjected to cyclic loading. The inclined angle of the diagonal concrete struts was selected between 31° and 59° as recommended by CEB-FIP (1978).

#### 3.6.3.1 Effective Area and Strength of Diagonal Concrete Struts

The diagonal concrete struts located in structural B-regions are used to carry shear force that developed inside the structural members. This is in contrast to the concrete struts discussed in section 3.6.2.2, of which the primary function is to carry flexural compression force. The effective area of diagonal concrete struts is computed by multiplying the perpendicular distance between the strut members, \( h_o \), see Figs. 3.2a and 3.10, by the core concrete width taken as 1.6\( r_o \) for circular columns, and \( b_o \) for rectangular columns and beams.

The effective strength of diagonal concrete struts for cyclic STMs was calculated using the Eurocode (1992) equation shown in Eq. 3.48. An additional efficiency factor, 0.85, was also included in the equation to correspond with the 15% strength reduction when struts are subjected to repeated cyclic action [Tjokrodimuljo (1985)].

\[ f_d = 0.85 \cdot f'_c \]  

(Eq. 3.48)

and  
\[ \nu = 0.7 - \frac{f'_c}{200} \geq 0.5 \quad (f'_c \text{ in MPa}) \]
3.6.4 Type 3 element property

The type 3 element is composed of a rebar tie representing the transverse reinforcement in beams and columns. A bilinear curve of material response, shown in Fig. 3.2d, was adopted as the stress-strain characteristic of the rebar tie in this element type. The effective area, $A_y$, and the effective strength, $V_n$, established for rebar ties in the type 3 element were identical to those proposed for monotonic STMs detailed in Section 3.5.5.1 and 3.5.5.2, respectively.

$$A_y = \frac{V_n}{f_{vy}} \quad \text{(from Eq. 3.30)}$$

$$V_n = V_s + V_c + V_p \quad \text{(from Eq. 3.31)}$$

where $f_{vy}$ is the measured yield strength of the transverse reinforcement; $V_s$ is the shear contribution of transverse reinforcement; $V_c$ is the shear contribution of concrete tensile strength; and $V_p$ is the force component of the externally applied load that is parallel to the applied shear.

3.6.5 Type 4 element property

The type 4 element consists of a concrete strut and a concrete tie arranged in tandem to represent the mobilised mass concrete in structural D-regions. The nonlinear curves illustrated in Figs. 3.12c and 3.12b are employed in this model element type to respectively represent the stress-strain characteristic of concrete struts and concrete ties.

The concrete struts and concrete ties in this element had the same effective area. This was calculated by multiplying the strut width, measured as the average distance between the adjacent concrete struts, by the effective strut depth, approximately calculated as $0.8 b_w$, where $b_w$ is the total width of a structural component.

Again, the effective strength of concrete struts, $f_d = 0.51 f'_c$, was found to be suitable in most of the analytical examples considered in Chapter 4. The effective compressive strength of a concrete tie for modelling the “contact stress effect” was evaluated as $f_{cont} = 0.05 f_d$, and the effective tensile strength for representing the “tension stiffening effect” of cracked concrete was computed as $f_{ts} = 0.5 f_{dt}$. 

Nonlinear Strut-and-Tie Model Formulation Procedure: Cyclic Model Formulation Procedure
3.6.6 Type 5 element property

The type 5 model element consists of a rebar tie to represent the reinforcement in structural D-regions. Again, the bilinear curve shown in Fig. 3.12e was used to represent the stress-strain characteristic of the rebar ties employed in this element. The actual reinforcement area and the measured material yield strength were adopted as the effective area and effective strength of the rebar tie, respectively.

3.6.7 Reinforcement lap splice property

Cyclic lap splice behaviour was also developed in this research project for conducting an STM analysis of the as-built bridge knee-joint test unit considered in Section 4.4.1. Special effort was made to establish the most appropriate cyclic lap splice analytical model using the limited stress-strain material models available in Drain-2DX. The cyclic tension-slip characteristic of reinforcement lap splicing, as illustrated in Fig. 3.13, was proposed based on the monotonic profile described in Fig. 3.9. The tension-slip cyclic response curve has no compression slip branch as this was not possible in the as-built knee-joint test unit. Additionally, no tension slip as assumed until the lap splice capacity was attained. Thereafter, the tension being transferred across the lap spliced rebars decreases to a residual strength, $0.4T_{tp}$ at the tension slip, $s$, which was previously evaluated in Section 3.5.9.1.
3.7 Material models deficiencies

Although much effort was exercised to utilise the analytical material models available in Drain-2DX to establish the realistic stress-strain characteristics of concrete and rebars, noteworthy discrepancies remain between the actual material cyclic responses and their corresponding analytical representations.

Illustrated in Fig. 3.14a is a typical stress-strain response of concrete when subjected to axial cyclic loading, and the corresponding stress-strain representation described previously for the cyclic STM formulation procedure. According to this figure, the analytical model is not capable of predicting the actual concrete strength beyond the first load cycle, when cyclically loaded at the same displacement ductility level. Subsequently, this may result in the STMs being unable to satisfactorily predict the progressive strength degradation due to concrete damage when the structural components were under cyclic loading at the same displacement ductility level.

The strength prediction is less difficult when using a bilinear curve to replicate the stress-strain characteristics of rebars, see Fig. 3.14b. However, the inability of the curve to capture the unloading and reloading stiffness softening due to the Bauschinger effect that is exhibited in the typical stress-strain characteristic of rebars when loaded cyclically, is expected to be detrimental to the accuracy of cyclic analysis results. Also, for the same reason, it explains the inconsistency exhibited between the analytically derived and experimentally measured hysteretic behaviour of the uniaxial fibre model shown in Fig. 3.12f.

Although critical features of the tension-slip behaviour, including the peak and residual lap splice strengths, could be satisfactorily replicated by the cyclic tension-slip model described in Fig. 3.13, substantial differences remain between the analytical representation and the actual behaviour of reinforcement tension-slip. Illustrated in Fig. 3.15 is the nonlinear cyclic reinforcement bond-slip response model that was proposed by Yankelevsky et al. (1992). Yankelevsky’s model suggested that the residual lap splice capacity and the rate of lap splice strength degradation are dependent on the number of load cycles and the maximum slip amplitude previously experienced by the reinforcement.

Rectification of all the identified discrepancies between the actual and analytical material stress-strain characteristics was not possible in this research project due to restrictions on the material models available in Drain-2DX. Furthermore, the incorporation of sophisticated material models in STM analysis requires substantial efforts in computer programming,
Figure 3.14: Actual and analytical cyclic stress-strain characteristics.

Nonlinear Strut-and-Tie Model Formulation Procedure: Material models deficiencies
which was considered beyond the scope of the current research project and hence, this was not pursued further.

Figure 3.15: Typical cyclic bond-slip behaviour of reinforcement in well-confined concrete [Yankelevsky et al. (1992)].
CHAPTER 4 - NONLINEAR STRUT-AND-TIE MODEL ANALYTICAL EXAMPLES

4.1 Introduction

First illustrated in this chapter are comprehensive worked examples to assess the monotonic and cyclic structural response of a beam and a column using STMs that are formulated using the originally developed procedures described in Chapter 3. Subsequently, the force-displacement response envelopes and the hysteretic structural behaviour of various reinforced concrete structural test units are evaluated using monotonic and cyclic STMs, respectively. The test units those are considered here include cantilever beams, columns subjected to double curvature action, bridge knee- and tee- joints, multicoloum bridge bents, interior and exterior building frames and an experimentally scaled building frame system. Similarities and discrepancies between the analysis results and experimental data are presented.

4.2 Reinforced Concrete Cantilever Beams

To demonstrate the STM formulation procedure using the section analysis charts listed in Appendix B1, consider the doubly reinforced cantilever beam depicted in Fig. 4.1a. [Liddell et al. (2000)]. This beam was designed to demonstrate a ductile response by providing adequate shear reinforcement, as recommended by the New Zealand Concrete Design Standard [NZS 3101 (1995)]. The cantilever beam had an unconfined concrete compressive strength, $f'_c = 30$ MPa, compression to tension reinforcement area ratio, $\omega = 1.0$ and the ratio of flexural tension reinforcement area to gross section area, $\rho_w = 0.005$. The elastic moduli of the STM model elements representing concrete and reinforcement were $E_c = 31.9$ GPa and $E_s = 200$ GPa, respectively, where $E_c$ was calculated using Eq. 3.1 and $E_s$ was based on the commonly assumed value.
As identified by arrow lines in Figs. B1(i), B1(iii) and B1(v) in Appendix B1, the neutral axis depth, compression centroid and effective concrete compressive strength that were measured at first yield state were $0.215D_r$, $0.075D_r$ and $0.4f'_c$, respectively.

Two models with different angles for the diagonal concrete struts were formulated for both monotonic and cyclic STMs. This was done in order to illustrate the minimal discrepancy in

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Cantilever Beams
the analytical results due to different inclined strut angles, conforming to the CEB-FIP (1978) recommendations previously mentioned in Sections 3.5.1 and 3.6.2.

4.2.1 Monotonic STM

*Type 1 element - Flexural Concrete Struts*

\[
A_{cs} = c b_w \quad \text{from Eq. 3.3b}
\]

\[
= 129 \times 270
\]

\[
= 34830 \text{ mm}^2
\]

\[
f_d = \frac{C_c^{(max)}}{A_{cs}} \quad \text{from Eq. 3.4 or from design chart B1(v) in Appendix B}
\]

\[
= 0.4 f'_c
\]

Definition of the stress-strain curve of a concrete strut located in the highly reinforced section:

\[
p'' = \frac{2(b_o + D_o)A_y}{b_o D_o s} \quad \text{from Eq. 3.2b}
\]

\[
= \frac{2790 \times 28.3}{220 \times 550 \times 50}
\]

\[
= 0.013
\]

\[
\varepsilon_{50} = \frac{3 + 0.2 f'_c}{145 f'_c - 1000} + \frac{3}{4} p''^{\frac{b_o}{s}} \quad \text{from Eq. 3.2a}
\]

\[
= \frac{3 + 0.2 \times 30}{145 \times 30 - 1000} + \frac{3}{4} \times 0.013 \times \frac{220}{50}
\]

\[
= 0.023
\]
Definition of the stress-strain curve of a concrete strut located in the lightly reinforced section:

\[ p'' = \frac{2(b_o + D_o)A_{xs}}{b_o D_o s} \]

\[ \frac{1540 \times 28.3}{220 \times 550 \times 155} \]

= 0.0023

\[ \varepsilon_{50} = \frac{3 + 0.2f'_c}{145f'_c - 1000} + \frac{3}{4}p'' \frac{b}{s} \]

\[ = \frac{3 + 0.2 \times 30}{145 \times 30 - 1000} + \frac{3}{4} \times 0.0023 \times \frac{220}{\sqrt[4]{155}} \]

= 0.0048

*Type 1 element - Flexural Rebar Struts*

\[ A_{rs} = A'_s \]

= 804.3 mm²

\[ f_{cy} = f_y \]

= 430 MPa

*Type 2 element - Flexural Rebar Ties*

\[ A_{rt} = A_s \]

= 804.3 mm²

\[ f_{sy} = f_y \]

= 430 MPa

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Cantilever Beams
Type 2 element - Flexural Concrete Ties

\[ A_{ct} = \frac{A_g - A_{cs}}{2} \]

from Eq. 3.15

\[ = \frac{162000 - 34830}{2} \]

\[ = 63585 \text{ mm}^2 \]

For the effective strength of concrete ties,

\[ \ell'/d_b = 215/16 \]

\[ = 13.5 \]

\[ \rho = \frac{A_{rt}}{A_{ct}} \]

\[ = \frac{804.3}{63585} \]

\[ = 0.013 \]

According to the design chart shown in Fig. A1(iii) in Appendix A, no solution is provided by the algorithm illustrated in Fig. 3.5. Therefore, the effective strength of concrete ties is calculated using Eq. 3.22b as:

\[ f_{dt} = \frac{D_r \cdot b_w \cdot f_i'}{4 \cdot A_{ct}} \]

\[ = \frac{600 \times 270 \times 3.3}{4 \times 63585} \]

\[ = 2.1 \text{ MPa} \]

\[ = 0.64 f'_{i} \text{ for } f'_{i} = 0.6 \sqrt{f'} \]
Figure 4.2: Tensile stress-strain characteristic of concrete in the cantilever beam example.

Shown in Fig. 4.2 is the stress-strain curve derived from Prakhy and Morley’s model (1990) accompanied by a trilinear trace adopted in the STM analysis. The residual tensile strength is estimated from this figure to be 0.42 MPa commencing at a strain value of 0.0005.

Type 3 model element - Diagonal Concrete Ties

\[ A_{cs} = h_p \times b_o \]
\[ = 325 \times 192 \]

for model 1, \[ = 62400 \text{ mm}^2 \]
\[ = 365 \times 192 \]

for model 2, \[ = 70080 \text{ mm}^2 \]

\[ f_d = v f_c' \]
and

\[ \nu = 0.7 - \frac{f'_c}{200} \geq 0.5 \]

\[ \nu = 0.7 - \frac{30}{200} \]

\[ \nu = 0.55 \]

\[ 0.55f'_c \]

**Type 4 element - Transverse Rebar Ties (for the heavily reinforced section)**

\[ V_s = \frac{d_v}{s \tan \theta} A_{vs} f_{vy} \quad \text{from Eq. 3.32b} \]

\[ = \frac{561}{50 \times \tan 49} \times 113.1 \times 300 \times \frac{1}{1000} \]

for model 1, \( V = 330.9 \text{ kN} \)

\[ = \frac{561}{50 \times \tan 42.5} \times 113.1 \times 300 \times \frac{1}{1000} \]

for model 2, \( V = 415.6 \text{ kN} \)

Assuming the maximum flexural reinforcement tensile strain, \( \varepsilon_s = 0.002 \), for no axial load being applied to the beam, and by using the trial-and-error procedure described in Section 3.5.6.2-ii, in conjunction with the design chart shown in Fig. C(ii) in Appendix C, it can be calculated that \( \nu = 0.1 \) and \( \beta = 0.122 \) for model 1, and \( \nu = 0.131 \) and \( \beta = 0.135 \) for model 2. Therefore:

\[ V_c = \beta \sqrt{f'_c A_{ve}} \quad \text{from Eq. 3.33a} \]

for model 1, \( V_c = 0.122 \times \sqrt{30} \times 129600 \times 1000 \)

\[ = 86.6 \text{ kN} \]

for model 2, \( V_c = 0.135 \times \sqrt{30} \times 129600 \times 1000 \)

\[ = 95.8 \text{ kN} \]

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Cantilever Beams
and \[ V_n = V_s + V_c + V_p \] from Eq. 3.31

for model 1, \[ = 330.9 + 86.6 + 0 \]
\[ = 417.5 \text{ kN} \]

for model 2, \[ = 415.6 + 95.8 + 0 \]
\[ = 511.4 \text{ kN} \]

The area of transverse rebar ties as:

\[ A_v = \frac{V_n}{f_{vy}} \]

for model 1, \[ = \frac{417.5 \times 1000}{300} \]
\[ = 1392 \text{ mm}^2 \]

for model 2, \[ = \frac{511.4 \times 1000}{300} \]
\[ = 1705 \text{ mm}^2 \]

Type 4 element - Transverse Rebar Ties (for the lightly reinforced section)

\[ V_s = \frac{d_v}{s \tan \theta} A_{vz} f_{vy} \] from Eq. 3.32b

for model 1, \[ = \frac{561}{133 \times \tan 49} \times 56.5 \times 300 \times \frac{1}{1000} \]
\[ = 62.2 \text{ kN} \]

for model 2, \[ = \frac{561}{133 \times \tan 42.5} \times 56.5 \times 300 \times \frac{1}{1000} \]
\[ = 78.0 \text{ kN} \]
Using the trial-and-error procedure described in Section 3.5.6.2-ii, in conjunction with the design chart of Fig. C(iii) in Appendix C for the maximum tensile strain of flexural reinforcement, $\varepsilon_s = 0.002$, it was established that $\nu = 0.038$ and $\beta = 0.123$ for model 1, and $\nu = 0.046$ and $\beta = 0.143$ for model 2. The concrete shear resistance is then calculated as:

$$V_c = \beta \sqrt{f'_c} A_{ve}$$

from Eq. 3.33a

for model 1, $V_c = 0.123 \times \sqrt[3]{30} \times 129600 / 1000$

$= 87.3$ kN

for model 2, $V_c = 0.143 \times \sqrt[3]{30} \times 129600 / 1000$

$= 101.5$ kN

and

$V_n = V_s + V_c + V_p$

from Eq. 3.31

for model 1, $V_n = 62.2 + 87.3 + 0$

$= 149.5$ kN

for model 2, $V_n = 78.0 + 101.5 + 0$

$= 179.5$ kN

The area of transverse rebar ties as:

$$A_v = \frac{V_n}{f_{vy}}$$

for model 1, $A_v = \frac{149.5 \times 1000}{300}$

$= 498$ mm$^2$

for model 2, $A_v = \frac{179.5 \times 1000}{300}$

$= 598$ mm$^2$
The type 5 and type 6 elements are not included in this example as the D-regions in a cantilever beam are assumed to be structural insignificant. A summary of model properties is given in Table 4.1 and the layouts of model 1 and model 2 are shown in Figs. 4.1b and 4.1c, respectively. The analytical force-displacement response envelopes generated using the two STMs are illustrated in Fig. 4.3 in the positive displacement quadrant, accompanied by the experimental data. These analytical response envelopes are also transposed to the negative displacement quadrant for comparison. Both models have satisfactorily captured the initial elastic stiffness and unit yield strength. Analytical results suggest that the cantilever beam suffered flexural failure with no concrete crushing in the diagonal struts, which is consistent with the experimental observation. More importantly, both models essentially provide an identical prediction, suggesting that the different inclined angle of diagonal concrete strut selected between the range recommended by CEB-FIP (1978), predicts similar structural response if the structure has sufficient shear reinforcement.

Table 4.1 Monotonic STM properties for the cantilever beam example.

<table>
<thead>
<tr>
<th>Model elements</th>
<th>Effective area</th>
<th>Effective strength</th>
<th>Strain hardening ratio, s.h.r.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1 - Flexural concrete struts</td>
<td>34830 mm²</td>
<td>0.4f_c</td>
<td>n/a</td>
</tr>
<tr>
<td>Type 1 - Flexural rebar struts</td>
<td>804.2 mm²</td>
<td>430 MPa</td>
<td>5%</td>
</tr>
<tr>
<td>Type 2 - Flexural concrete ties</td>
<td>63585 mm²</td>
<td>0.64f_t</td>
<td>n/a</td>
</tr>
<tr>
<td>Type 2 - Flexural rebar ties</td>
<td>804.2 mm²</td>
<td>430 MPa</td>
<td>5%</td>
</tr>
<tr>
<td>Type 3 - Diagonal concrete struts</td>
<td>model 1 - 62400 mm²</td>
<td>0.55f_c</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>model 2 - 70080 mm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 4 - Transverse rebar ties (heavily reinforced section)</td>
<td>model 1 - 1392 mm²</td>
<td>300 MPa</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>model 2 - 1705 mm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 4 - Transverse rebar ties (slightly reinforced section)</td>
<td>model 1 - 498 mm²</td>
<td>300 MPa</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>model 2 - 598 mm²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.2.2 Cyclic STM

Type 1 element - flexural concrete struts

\[ A_{cs} = c b_w \]

from Eq. 3.38b

\[ = 129 \times 270 \]

\[ = 34830 \text{ mm}^2 \]

\[ f_d = \frac{0.85 C_{c(max)}}{A_{cs}} \]

from Eq. 3.39 or from design chart B1(v) in Appendix B

\[ = 0.34 f'_c \]

Type 1 element - flexural concrete ties

\[ A_{ct} = A_{cs} \]

from Eq. 3.40

![Graph showing vertical displacement and force for a cantilever beam example.](image)

**Figure 4.3:** Monotonic analytical results of the cantilever beam example.
\[ f_{\text{cont}} = 0.05f_d \]

from Eq. 3.41

\[ = 0.017f' \]

\[ f_{ls} = 0.5f_{dt} \]

from Eq. 3.42

\[ = 0.32f' \]

Type 1 element - flexural rebar struts-ties

\[ A_{s-t} = \phi_r A_s \]

from Eq. 3.43

\[ = 603.2 \text{ mm}^2 \]

\[ f_{o-t} = f_y \]

from Eq. 3.44

\[ = 430 \text{ MPa} \]

Type 2 element - diagonal concrete struts

\[ A_{cs} = h_p \times b_o \]

for model 1, \[ = 315 \times 192 \]

\[ = 60480 \text{ mm}^2 \]

for model 2, \[ = 357 \times 192 \]

\[ = 68544 \text{ mm}^2 \]

\[ f_d = \phi_c v f'_c \]

from Eq. 3.47
0.47f'_c

Type 3 element - transverse rebar ties (for the heavily reinforced section)

\[ V_s = \frac{d_v}{s \tan \theta} A_{vs} f_{vy} \]

from Eq. 3.32b

for model 1,

\[ = \frac{561}{50 \times \tan 51.9} \times 113.1 \times 300 \times \frac{1}{1000} \]

= 298.5 kN

for model 2,

\[ = \frac{561}{50 \times \tan 45.5} \times 113.1 \times 300 \times \frac{1}{1000} \]

= 374.1 kN

Using \( \varepsilon_y = 0.002 \) and the chart depicted in Fig. C(ii) in Appendix C, it is calculated that \( \nu = 0.1 \) and \( \beta = 0.113 \) for model 1, and \( \nu = 0.12 \) and \( \beta = 0.131 \) for model 2.

\[ V_c = \beta_s \sqrt{f'_c} A_{ve} \]

from Eq. 3.33a

for model 1, \( V_c = 0.113 \times \sqrt{30} \times 129600 / 1000 \)

= 80.2 kN

for model 2, \( V_c = 0.131 \times \sqrt{30} \times 129600 / 1000 \)

= 93.0 kN

and

\[ V_n = V_s + V_c + V_p \]

from Eq. 3.31

for model 1, \( V_n = 298.5 + 80.2 + 0 \)

= 378.7 kN

for model 2, \( V_n = 374.1 + 93.0 + 0 \)

= 467.1 kN

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Cantilever Beams
\[ A_v = \frac{V_n}{f_{vy}} \]

for model 1,
\[ = \frac{378.7 \times 1000}{300} \]
\[ = 1262 \text{ mm}^2 \]

for model 2,
\[ = \frac{467.1 \times 1000}{300} \]
\[ = 1557 \text{ mm}^2 \]

Type 3 element - Transverse Rebar Ties (for the lightly reinforced section)

\[ V_s = \frac{d_v}{s \tan \theta} A_{vs} f_{vy} \]

from Eq. 3.32b

for model 1,
\[ = \frac{561}{133 \times \tan 51.9} \times 56.5 \times 300 \times \frac{1}{1000} \]
\[ = 56.1 \text{ kN} \]

for model 2,
\[ = \frac{561}{133 \times \tan 45.5} \times 56.5 \times 300 \times \frac{1}{1000} \]
\[ = 70.3 \text{ kN} \]

Again, using \( \varepsilon_s = 0.002 \) and the chart shown in Fig. C(ii) in Appendix C, it is established that \( v = 0.035 \) and \( \beta = 0.114 \) for model 1, and \( v = 0.042 \) and \( \beta = 0.134 \) for model 2. The concrete shear resistance is then calculated as:

\[ V_c = \beta \sqrt{f'_c} A_{ve} \]

from Eq. 3.33a

\[ V_c = 0.114 \times \sqrt{30} \times 129600 / 1000 \]

for model 1,
\[ = 80.9 \text{ kN} \]

\[ V_c = 0.134 \times \sqrt{30} \times 129600 / 1000 \]

for model 2,
\[ = 95.1 \text{ kN} \]
and

\[ V_n = V_s + V_c + V_p \]

from Eq. 3.31

for model 1, \( V_n = 56.1 + 80.9 + 0 \)

\( = 137 \text{ kN} \)

for model 2, \( V_n = 70.3 + 95.1 + 0 \)

\( = 165.4 \text{ kN} \)

\[ A_v = \frac{V_n}{f_{vy}} \]

for model 1, \( A_v = \frac{137 \times 1000}{300} \)

\( = 457 \text{ mm}^2 \)

Figure 4.4: Cyclic analytical results of the cantilever beam example.
Table 4.2 Cyclic STM properties for the cantilever beam example.

<table>
<thead>
<tr>
<th>Model elements</th>
<th>Effective area</th>
<th>Effective strength</th>
<th>Strain hardening ratio, s.h.r.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1 - Flexural concrete struts</td>
<td>34830 mm²</td>
<td>0.34f'_c</td>
<td>n/a</td>
</tr>
<tr>
<td>Type 1 - Flexural concrete ties</td>
<td>34830 mm²</td>
<td>f_{com} = 0.017f'_c</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>f_{ls} = 0.32f'_l</td>
<td></td>
</tr>
<tr>
<td>Type 1 - Flexural rebar struts-ties</td>
<td>603.2 mm²</td>
<td>430 MPa</td>
<td>5 %</td>
</tr>
<tr>
<td>Type 2 - Diagonal concrete struts</td>
<td>model 1 - 60480 mm²</td>
<td>0.47f'_c</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>model 2 - 68544 mm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 3 - Transverse rebar ties (heavily reinforced section)</td>
<td>model 1 - 1262 mm²</td>
<td>300 MPa</td>
<td>5 %</td>
</tr>
<tr>
<td></td>
<td>model 2 - 1557 mm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 3 - Transverse rebar ties (slightly reinforced section)</td>
<td>model 1 - 457 mm²</td>
<td>300 MPa</td>
<td>5 %</td>
</tr>
<tr>
<td></td>
<td>model 2 - 551 mm²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{for model 2, } \quad \frac{165.4 \times 1000}{300} = 551 \text{ mm}^2
\]

Again, type 4 and type 5 elements were not included in this example as the D-region was assumed to have no structural significance in a cantilever beam. A summary of the cyclic model properties is provided in Table 4.2. The formulated cyclic STMs for different diagonal strut angle are depicted in Figs. 4.1d and 4.1e.

The two sets of cyclic STM analytical results illustrated in Fig. 4.4 indicate close proximity to each other. Analyses had accurately predicted the initial elastic stiffness and the yield strength of the test unit. However, the estimated unloading and reloading stiffness were less precise, due to the Bauschinger effect not being considered in the material stress-strain response of rebar strut-ties, see Section 3.7. Since the strain energy stored in the reinforcement was not examined by the model, the reinforcement rupture that occurred in the 2nd load cycle at displacement ductility \( \mu = 10 \), corresponding to the beam-tip displacement +85 mm, was not captured.
4.3 Reinforced Concrete Columns

An example is provided in this section to illustrate the analysis of a reinforced concrete column using STM. The circular column considered here was tested with double curvature bending action and was identified as Column 1 in a series of tests conducted by Priestley et al. (1994) to evaluate the suitable quantity of shear reinforcement under seismic loading.

The cross section of this column is depicted in Fig. 4.5a, which had a section diameter, \( D_c = 609.6 \, \text{mm} \), unconfined concrete strength, \( f'_c = 30 \, \text{MPa} \), ratio of flexural reinforcement area to gross section area, \( \rho_f = 0.0052 \) and axial load ratio, \( N/f'_c A_g = 0.057 \), where \( N \) is the axial force and \( A_g \) is the gross section area. According to the section force analysis, the neutral axis
depth at the first-yield state, \( c = 0.3 \, D_c \), the compression centroid position, \( x_c = 0.13 \, D_c \), the tension centroid position, \( x_t = 0.81 \, D_c \), and the effective strength of flexural concrete struts was evaluated as \( f_c' = 0.43 f_c \). The concrete core diameter, \( D' = 563.3 \text{ mm} \), and the flexural and transverse rebar diameter were \( d_h = 12.7 \text{ mm} \) and \( d_{vs} = 6.35 \text{ mm} \), respectively. The elastic moduli of concrete, \( E_c = 31.9 \text{ GPa} \), was calculated using Eq. 3.1, and \( E_s = 200 \text{ GPa} \) was adopted for reinforcement. The properties for different element types are computed below.

4.3.1 Monotonic STM

Type 1 element - Flexural Concrete Struts

\[
\phi = \cos^{-1}\left(\frac{0.5D_c - c}{0.5D_c}\right) \quad \text{from Eq. 3.3a}
\]

\[
= \cos^{-1}\left(\frac{0.5 \times 609.6 - 0.3 \times 609.6}{0.5 \times 609.6}\right)
\]

\[
= 1.16 \text{ rad.}
\]

\[
A_{cs} = \frac{D_c^2 \cdot (\phi - \sin \phi \cos \phi)}{4}
\]

\[
= \frac{609.6^2 \times (1.16 - \sin 1.65 \times \cos 1.65)}{4}
\]

\[
= 70800 \text{ mm}^2
\]

\[
f_d = \frac{C_{c(max)}}{A_{cs}}
\]

\[
= 0.43 f_c'
\quad \text{from design chart C3(viii) in Appendix C}
\]

Type 1 element - Flexural Rebar Struts

\[
r_o = \frac{D' - d_h - d_{vs}}{2}
\]

\[
= \frac{563.3 - 12.7 - 6.35}{2}
\]

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Columns
\[
\phi' = \cos^{-1}\left(\frac{0.5D_c - c}{r_o}\right) \quad \text{from Eq. 3.5}
\]
\[
= \cos^{-1}\left(\frac{0.5 \times 609.6 - 0.3 \times 609.6}{272}\right)
\]
\[
= 1.1 \text{ rad}
\]
\[
t = \frac{A_{st}}{2\pi r_o} \quad \text{from Eq. 3.6}
\]
\[
= \frac{1520}{2 \times \pi \times 272}
\]
\[
= 0.9 \text{ mm}
\]
\[
A_{rs} = 2 \phi' t r_o \quad \text{from E. 3.8a}
\]
\[
= 2 \times 1.1 \times 0.9 \times 272
\]
\[
= 540 \text{ mm}^2
\]
\[
f_{cy} = f_y \cdot \frac{\sin\phi' - \phi' \cos\phi'}{\phi' \cdot (1 - \cos\phi')} \quad \text{from Eq. 3.12b}
\]
\[
= f_y \times \frac{\sin 1.1 - 1.1 \times \cos 1.1}{1.1 \times (1 - \cos 1.1)}
\]
\[
= 0.65f'_y
\]

**Type 2 element - Flexural Rebar Ties**

\[
A_{rt} = 2(\pi - \phi') t r_o
\]
\[
= 2 \times (\pi - 1.1) \times 0.9 \times 272
\]
\[
= 1000 \text{ mm}^2
\]

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Columns
\[ f_{sy} = \phi_y \cdot f_y \cdot \frac{\sin \phi' - (\phi' - \pi) \cos \phi'}{2(\pi - \phi') \cos \left(\frac{\phi'}{2}\right) \cdot \sin \left(\frac{\pi + \phi'}{2}\right)} \]

\[ = f_y \times 4 \times \frac{\sin 1.1 - (1.1 - \pi) \times \cos 1.1}{2 \times (\pi - 1.1) \times \cos \left(\frac{1.1}{2}\right) \times \sin \left(\frac{\pi + 1.1}{2}\right)} \]

\[ = 0.82f_y \]

Type 2 element - Flexural Concrete Ties

\[ A_{ct} = \frac{A_e - A_{cs}}{2} \]

\[ = \frac{291860 - 73600}{2} \]

\[ = 109100 \text{ mm}^2 \]

For the effective strength of concrete ties,

\[ \ell' / d_b = 183 / 12.7 \]

\[ = 14.4 \]

\[ \rho = A_{rt} / A_{ct} \]

\[ = 1000 / 109100 \]

\[ = 0.0092 \]

According to Section 3.5.3.3, no solution was suggested by the algorithm illustrated in Fig. 3.5, and Eq. 3.22a was adopted to calculate the effective strength of concrete ties.

\[ f_{dt} = \frac{D_c^2 \cdot f_t'}{6 \cdot A_{ct}} \quad \text{and} \quad f_t' = 0.6 \sqrt{f_c'} \]

\[ = \frac{609.6^2 \times 0.6 \sqrt{f_c'}}{6 \times 109100} \]

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Columns
Shown in Fig. 4.6 is the stress-strain curve derived from Prakhya and Morley's model (1990) accompanied by a trilinear approximation that was used in the STM analysis. The residual tensile strength was determined from the figure to be 0.34 MPa commencing at a strain value of 0.00045.

**Type 3 element - Diagonal Concrete Struts**

\[ A_{cs} = h_p \times 1.6r_o \]

\[ = 261 \times 1.6 \times 272 \]

\[ = 113600 \text{ mm}^2 \]

\[ v = 0.7 - \frac{30}{200} \geq 0.5 \quad \text{from Eq. 3.29} \]
Hence, \( f_d = 0.55 f'_c \)

**Type 4 element - Transverse Rebar Ties**

\[
V_s = \frac{\pi D'}{4s \tan \theta} A_{vs} f_{vy} \quad \text{from Eq. 3.32a}
\]

\[
= \frac{\pi \times 563.3}{4 \times 76 \times \tan 48.5^\circ \times 1000} \times 63.3 \times 275
\]

\[= 89.6 \text{ kN} \]

When calculating concrete shear strength, the trial-and-error procedure described in Section 3.5.6.2-ii was used in conjunction with the design chart shown in Fig. C(ii) in Appendix C. For \( N/f'_c A_g = 0.057 \) and \( \varepsilon_s = 0.002 \) that are determined from Table 3.1, \( \nu = 0.034 \) and \( \beta = 0.124 \) are read off from Fig. C(ii). Hence the concrete shear resistance is calculated as:

\[
V_c = \beta \sqrt{f'_c A_{ve}} \quad \text{from Eq. 3.33a}
\]

\[
= 0.124 \times \sqrt{30 \times 233372} / 1000
\]

\[= 158.5 \text{ kN} \]

\[
V_p = 503 \times \tan 14^\circ \quad \text{from Eq. 3.34}
\]

\[= 125 \text{ kN} \]

\[
V_n = V_s + V_c + V_p
\]

\[= 89.6 + 158.5 + 125
\]

\[= 373 \text{ kN} \]

\[
A_v = \frac{V_n}{f_{vy}}
\]
Again, type 5 and type 6 elements were not included in this example as D-regions in the column have no structural significance. A summary of model properties is provided in Table 4.3 and the layout of the model is shown in Figs. 4.5b. The analytical force-displacement response envelope generated using the monotonic STM is illustrated in Fig. 4.7 in the positive displacement quadrant, accompanied by the experimental data. The analytical response envelope was also transposed to the negative displacement quadrant for comparison with experimental data. Analytical results indicated that the initial elastic stiffness and the yield strength of the column were accurately predicted. It should be noted that the column lateral strength at the strain hardening excursion was slightly under-estimated by the STM because the reinforcement strain hardening characteristic was not accurately represented by the assumed strain hardening ratio (s.h.r) equal to 2.5%, see Table 4.3, that was employed.
for rebar struts and rebar ties. However, s.h.r. = 2.5% was deemed appropriate to be used in column STMs for general analysis as suggested by To et al. (2002b).

Table 4.3 Monotonic STM properties for the column example.

<table>
<thead>
<tr>
<th>Model elements</th>
<th>Effective area</th>
<th>Effective strength</th>
<th>Strain hardening ratio, s.h.r.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1 - Flexural concrete struts</td>
<td>73600 mm²</td>
<td>0.43$f'_c$</td>
<td>n/a</td>
</tr>
<tr>
<td>Type 1 - Flexural rebar struts</td>
<td>540 mm²</td>
<td>0.65$f_y$</td>
<td>2.5 %</td>
</tr>
<tr>
<td>Type 2 - Flexural concrete ties</td>
<td>109100 mm²</td>
<td>0.062$f'_c$</td>
<td>n/a</td>
</tr>
<tr>
<td>Type 2 - Flexural rebar ties</td>
<td>1000 mm²</td>
<td>0.82$f_y$</td>
<td>2.5 %</td>
</tr>
<tr>
<td>Type 3 - Diagonal concrete struts</td>
<td>113600 mm²</td>
<td>0.55$f'_c$</td>
<td>n/a</td>
</tr>
<tr>
<td>Type 4 - Transverse rebar ties</td>
<td>1360 mm²</td>
<td>275 MPa</td>
<td>5 %</td>
</tr>
</tbody>
</table>

4.3.2 Cyclic STM

Type 1 element - Flexural Concrete Struts

\[ A_{cs} = 73600 \text{ mm}^2 \quad \text{from Eq. 3.3a} \]
\[ f_d = 0.37f'_c \quad \text{from Eq. 3.40} \]

Type 1 element - Flexural Concrete Ties

\[ A_{ct} = A_{cs} = 73600 \text{ mm}^2 \quad \text{from Eq. 3.41} \]
\[ f_{cont} = 0.05f_d = 0.019f'_c \quad \text{from Eq. 3.42} \]
\[ f_{ts} = 0.5f_{dt} \quad \text{from Eq. 3.43} \]

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Columns
= 0.031 f'c

Type 1 element - Flexural Rebar Struts-Ties

\[ A_{s,t} = 2(\pi - \phi') tr_o \quad \text{from Eq. 3.46a} \]

= 1000 mm²

\[ f_{s,t} = \phi_o \cdot f_y \cdot \frac{\sin \phi' - (\phi' - \pi) \cos \phi'}{2(\pi - \phi') \cos \left(\frac{\phi'}{2}\right) \sin \left(\frac{\pi + \phi'}{2}\right)} \quad \text{from Eq. 3.47a} \]

= 0.82 f'c

Type 2 element - Diagonal Concrete Struts

\[ A_{cs} = 113600 \text{ mm}² \]

\[ f_d = 0.47 f'c \quad \text{from Eq. 3.48} \]

Type 3 element - Transverse Rebar Ties

\[ V_s = \frac{\pi \times 563.3}{4 \times 76 \times \tan 45.3° \times 1000} \times 63.3 \times 275 \]

= 100.3 kN

When calculating concrete shear strength, the trial-and-error procedure described in Section 3.5.6.2-ii was used in conjunction with the design chart shown in Fig. C(ii) in Appendix C. For \( N/f'c A_c = 0.057 \) and \( \varepsilon_s = 0.002 \) as determined from Table 3.1, \( v = 0.039 \) and \( \beta = 0.135 \) are read off from Fig. C(ii). Hence the concrete shear resistance is calculated as:

\[ V_c = \beta \sqrt{f'c A_{ve}} \quad \text{from Eq. 3.33a} \]

\[ = 0.135 \times \sqrt[3]{30 \times 233372 / 1000} \]

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Columns
\[ V_p = 503 \times \tan 14^\circ \quad \text{from Eq. 3.34} \]

\[ V_p = 172.6 \text{ kN} \]

\[ V_n = V_s + V_c + V_p \]

\[ V_n = 100.3 + 172.6 + 125 \]

\[ V_n = 398 \text{ kN} \]

\[ A_v = \frac{V_n}{f_{vy}} \]

\[ A_v = \frac{398 \times 1000}{275} \]

\[ A_v = 1450 \text{ mm}^2 \]

Table 4.4 Cyclic STM properties for the column example.

<table>
<thead>
<tr>
<th>Model elements</th>
<th>Effective area</th>
<th>Effective strength</th>
<th>Strain hardening ratio, s.h.r.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type 1 - Flexural concrete struts</strong></td>
<td>73600 mm²</td>
<td>0.37f'_c</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>Type 1 - Flexural concrete ties</strong></td>
<td>73600 mm²</td>
<td>0.019f'_c</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>Type 1 - Flexural rebar struts-ties</strong></td>
<td>1000 mm²</td>
<td>0.82f_y</td>
<td>2.5 %</td>
</tr>
<tr>
<td><strong>Type 2 - Diagonal concrete struts</strong></td>
<td>113600 mm²</td>
<td>0.47f'_c</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>Type 3 - Transverse rebar ties</strong></td>
<td>1450 mm²</td>
<td>275 MPa</td>
<td>5 %</td>
</tr>
</tbody>
</table>
Again, type 4 and type 5 elements were not applicable in this example as D-regions had no structural significance. The formulated cyclic STM for the column is illustrated in Fig. 4.5c, while a summary of the cyclic model properties is given in Table 4.4 and the analytically obtained hysteretic response is illustrated in Fig. 4.8. The analysis had satisfactorily predicted the initial elastic stiffness and lateral yield strength of the column. The general shape of the experimentally recorded hysteretic loops was captured by the model, but the unloading and reloading branches were insufficiently predicted due to the Bauschinger effect being not considered in the material stress-strain curves employed for rebar strut-ties, see Section 3.7. Also, the column lateral strength degradation that was experimentally observed when cyclically loaded at the same displacement ductility level was not achieved due to the limitations of the material stress-strain curve employed for the concrete struts. This model deficiency was previously discussed in Section 3.7.

4.4 Beam-column bridge knee-joints

Four large-scale bridge knee-joint test units are analysed in this section using STMs. An earlier version of STM analytical results of these knee-joint test units was reported by To et
al. (2002b and 2003a). The newer models presented in this section were refined using the model formulation procedure described in Chapter 3.

The STMs developed for the structural component located in the structural D-regions, i.e. the joint panel of the test units, were based on the force path occurring at the ultimate limit state, previously recommended by Ingham et al. (1997). All knee-joint test units were constructed to replicate details of Bent 38 of the I-980 southbound connector in Oakland, California. Dimensions of the prototype structure and the configuration of test units are illustrated in Figs. 4.9a and 4.9b respectively [Ingham et al. (1994)].

All test specimens were built using a common column reinforcement detail and the appropriate joint force-transfer mechanisms were incorporated in the joint design. The test unit reinforcement details replicated an as-built, a repaired, an externally post-tensioned retrofitted and a redesigned joint. Comprehensive work examples showing calculation of the modelling properties are not provided here. However, pertinent modelling features of the test units are given below.

![Diagram](image)

**Figure 4.9:** Prototype structure and corresponding test unit configuration.

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
4.4.1 As-built knee-joint

The column and beam reinforcement details of the as-built knee-joint test unit are illustrated in Figs. 4.10a and 4.10b, respectively. The column section had a rectangular shape and was transversely reinforced with interlocking spirals. The beam section was doubly reinforced to resist cyclic flexural action.

4.4.1.1 Monotonic Knee-joint STM

The knee-joint reinforcement details of the as-built test unit are illustrated in Fig. 4.11a. The column flexural reinforcement was not fully extended to the top of the joint region, and was prematurely terminated with straight bar extension, as seen in the circled region in Fig. 4.11a, implying the absence of an effective end anchorage mechanism. As a consequence, the flexural compression force in the cap beam, see Fig. 4.12a, was diverted in the vicinity of the joint region from the original compression path, to connect with the column flexural reinforcement at a distance $\ell_c = 85$ mm from the rebar end, being one-half of the clamping length evaluated using Eq. 3.38. Also, note that the beam flexural strength of the joint-opening STM was reduced due to shortening of the internal lever arm, attributable to the compression force diversion.

The effective strength $f_d = 0.51f'_c$ was adopted for the joint strut, as discussed in Section 3.5.6.2, to achieve satisfactory structural strength prediction, especially at the low to mid displacement ductility level. The width of joint strut depicted in Fig. 4.15a was assessed as: $a = 312.5$ mm.

When formulating the joint-closing STM, shown in Fig. 4.12b, the lap splice mechanism that is discussed in Section 3.5.9.1 was considered when replicating the behaviour of lap spliced rebars located at the back of the joint panel. Also discussed in the same section, the residual lap splice capacity was evaluated from the experimental data to be 40% of the maximum value. Although minimal joint damage resulting from the joint-closing moment was anticipated due to premature lap splice failure, the concrete compressive strength could have been weakened by the joint-opening moment. Accordingly, a conservative value, $f_d = 0.51f'_c$, was deemed appropriate to be adopted as the effective strength of joint struts for the joint-closing STM. The area of rebar ties “a” and “b” as identified in Fig. 4.12b, was $3420$ mm$^2$ and $2300$ mm$^2$, respectively. The width of the joint strut illustrated in the same figure was assessed as: $c = 375$ mm.
Figure 4.10: Section reinforcement details of the knee-joint test units

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
The cyclic knee-joint STM depicted in Fig. 4.13 was formulated based on the procedure described in Section 3.6. This model does not replicate exactly the actual force path. In particular, the beam flexural compression force was not diverted when entering the joint panel region as described for the monotonic joint-opening STM. Model elements for the beam and the column were located at the force centroid of tension reinforcement, $x_t$, evaluated from the respective flexural direction. The force transfer path developed in the joint panel when under cyclic loading was simplified, being represented by two diagonal joint struts. It must be emphasised that the layout of cyclic STM was not aiming at replicating...
Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints

Figure 4.12: Monotonic STM's of the as-built knee-joint test unit.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.14a.)
the exact force transfer path that occurred inside the test unit. Instead, it was primarily intended to provide a simplified modelling solution for examining hysteretic structural performance. Again, the behaviour of lap splicing when subjected to cyclic loading was considered and replicated, using the analytical model discussed in Section 3.6.7. Also, \( f_d = 0.51f'_c \) was adopted as the effective strength of joint struts, which was consistent with that employed for the monotonic model.

### 4.4.1.2 Analytical Results

The monotonic force-displacement response envelopes derived from the joint-opening and joint-closing STMs are illustrated in the positive and negative displacement quadrants of Fig. 4.14a, respectively. Also captured in the analysis was the reinforcement yielding sequence at different applied load levels, which is depicted on the respective response envelope using triangles for the joint-opening direction, and diamonds for the joint-closing direction. Both analytical response envelopes indicated satisfactory prediction of initial elastic stiffness and unit strength. Furthermore, joint-moment carrying capacity in the joint-closing direction was also satisfactorily estimated.
Figure 4.14: As-built knee-joint force-beam tip displacement responses.

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
For the joint-opening direction, the analytical events 1-5 as shown in Fig. 4.12a suggested that unit strength was dictated by extensive yielding in the beam flexural reinforcement, which was consistent with experimentally recorded strain gauge readings [Ingham et al. (1994)]. For the joint-closing direction, event 1 as shown in Fig. 4.12b estimated lap splice failure prior to the test unit reaching its yield strength, and at a load level comparatively similar to that observed in the test. No concrete crushing, resulting in significant unit strength degradation, was suggested by either model.

The hysteretic response generated using the cyclic STM is illustrated in Fig. 4.14b, using the solid lines accompanied by the experimental data. Although overall comparison of the structural responses was satisfactory, apparent discrepancies occurred between the analytical and experimental data, including the unloading and reloading stiffness, and strength degradation when subjected to repeated cyclic loading at the same displacement ductility level. These analytical inaccuracies were due to deficiencies in the material models employed for concrete struts and rebar ties, as previously discussed in Section 3.7. Rectification of these analytical deficiencies was unfortunately not possible in this study due to limitations of the material stress-strain curves available in Drain-2DX. STM analytical results would be expected to improve if more sophisticated material stress-strain models were utilised for the concrete and rebar.

4.4.2 Repaired knee-joint

The knee-joint reinforcement details of the repaired unit are shown in Fig. 4.11b. This test unit replicated the repair details implemented by the California Department of Transportation to bent 38 of the I-980 freeway connector following the 1989 Loma Prieta earthquake.

In the laboratory, the as-built test unit discussed in the previous section was repaired to establish this test unit. Consequently, this test unit shared the same column and beam members as the as-built unit, but the joint panel region was strengthened by incorporating a joint haunch and extra transverse reinforcement to improve joint integrity.

4.4.2.1 Knee-joint STM

The monotonic STMs replicating the force transfer path that developed within the test unit when subjected to joint-opening and joint-closing moments are depicted in Figs. 4.15a and 4.15b, respectively. Noteworthy features of these models are the extra struts and ties being introduced in the joint panel to account for the additional reinforcement and improved anchorage of the column longitudinal reinforcement due to enlarged dimension of the joint.
Figure 4.15: Monotonic STM of the repaired knee-joint test unit.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.17a)
panel. Additionally, a concrete tie was introduced in the joint-opening STM to account for the tension carried by concrete across the joint haunch.

Again, \( f'_d = 0.51 f'_c \) was adopted as the effective strength of concrete struts located in the joint panel regions. For the monotonic joint-opening model shown in Fig. 4.15a, the width of joint struts was assessed as: \( a = 125 \text{ mm} \); \( b = 95 \text{ mm} \); \( c, d, e, \& f = 95 \text{ mm} \); \( g = 325 \text{ mm} \); \( h = 125 \text{ mm} \) and \( i = 62.5 \text{ mm} \). For the monotonic joint-closing model shown in Fig. 4.15b, the areas of rebar ties were: \( a = 3420 \text{ mm}^2 \) and \( b = 2850 \text{ mm}^2 \). The width of joint struts was assessed as: \( c \& d = 125 \text{ mm} \) and \( e = 187.5 \text{ mm} \).

The cyclic STM formulated for the repaired knee-joint test unit is depicted in Fig. 4.16. The model constructed for the joint panel region was a combined modelling solution to account for both the joint-opening and joint-closing moments effects. The properties of the model elements were computed according to the procedure described in Section 3.6. Also, the area of model elements located in the structural D-regions were identical to the corresponding model elements employed in the monotonic models.

![Figure 4.16: Cyclic STM of the repaired knee-joint test unit.](image-url)
4.4.2.2 Analytical Results

The monotonic STM analytical results obtained for the repaired unit in both joint loading directions are shown in Fig. 4.17a using solid lines. Excellent estimation of the experimentally recorded stiffness was obtained for both joint loading directions. The unit strength was accurately predicted in the joint-opening direction, but was slightly underestimated in the joint-closing direction. The underestimated unit strength in the joint-closing direction in the inelastic displacement region was due to yielding of the top beam flexural reinforcement being inaccurately predicted by the STM at the beam displacement of -125 mm.

The reinforcement yielding sequence captured by the joint-opening STM at different applied load levels is illustrated using triangles on the positive displacement response envelope as shown in Fig. 4.17a. Similar to the as-built unit considered in the previous section, the unit strength of the repaired knee-joint in the joint-opening direction was dictated by extensive yielding in the bottom beam flexural reinforcement, see events 1-2 & 4-5 in Fig. 4.16a. Also suggested by event 3 in the same figure was reinforcement yielding in the joint panel region. The extensive yielding in the beam bottom flexural reinforcement predicted by the STM was consistent with experimental strain gauge measurements [Ingham et al. (1994)].

The reinforcement yielding sequence captured by the joint-closing STM is depicted using diamonds on the negative displacement response envelope in Fig. 4.17a. Consistent with the experimental observation, analysis results identified that yielding occurred in both the beam and column flexural reinforcement, see events 1-4 in Fig. 4.15b. Analysis also suggested that no significant concrete crushing developed in the joint panel region. The unit reached its maximum strength at a displacement of approximately -125 mm, corresponding to a displacement ductility level \( \mu = 2.2 \), as a result of significant flexural reinforcement yielding in both of the beam and column members.

The analytical hysteretic response generated using the cyclic STM is included in Fig. 4.17b, accompanied by the experimental data. The two data sets compared well with each other except for the discrepancies that occurred in the unloading and reloading stiffness due to the Bauschinger effect of the rebar not being included in the material stress-strain response model, see Section 3.7. Also, note that the stepwise reloading and unloading analytical curves were due to stiffening of the concrete struts, as a result of crack closing. The cyclic model does not suggest any concrete crushing in the joint panel region which was comparable with experimental observations.
vertical displacement (in)

vertical displacement (m)

a, Monotonic analytical results.

vertical displacement (in)

vertical displacement (m)

b, Cyclic analytical results.

Figure 4.17: Repaired knee-joint force-beam tip displacement responses.

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
In general, the analysis captured the knee-joint structural response with sufficient precision. Also, it suggested that the adoption of effective strength of joint struts, \( f_d = 0.51 f'_e \) was appropriate.

### 4.4.3 Retrofit Knee-Joint

The knee-joint test unit considered in this section represented a retrofit solution to the as-built joint. The reinforcement details of the retrofitted unit, as illustrated in Fig. 4.11c, were identical to those of the as-built unit, except for the external prestressing anchored to a concrete bolster at the back of the joint. This externally applied prestressing was intended to provide better joint performance against seismic action. Also, the flexural and shear strength of the cap beam was enhanced to ensure that plastic hinges would develop in the column for both joint-opening and joint-closing directions.

#### 4.4.3.1 Knee-Joint STMs

The corresponding monotonic STMs for both joint displacement directions, that are shown in Figs. 4.18a and 4.18b, resemble those of the as-built unit, except that extra concrete struts were mobilised at both ends of the cap-beam to account for the prestressing. Also, the reinforcement clamping length described in Section 3.5.8.2, was considered when assessing the appropriate anchor position of the column longitudinal rebar tie in the joint panel region for the joint-opening model. Again, \( f_d = 0.51 f'_e \) was adopted as the effective strength of concrete struts located in the joint panel. The width of joint struts for the joint-opening model was assessed as: \( a = 375 \text{ mm} \); \( b = 140 \text{ mm} \) and \( c = 250 \text{ mm} \), and for the joint-closing model was evaluated: \( a = 187.5 \text{ mm} \); \( b = 95 \text{ mm} \) and \( c = 187.5 \text{ mm} \).

The cyclic STM formulated for the retrofitted knee-joint is shown in Fig. 4.19. This cyclic model also resembled that of the as-built knee-joint, but with additional concrete struts at both ends of the cap-beam being mobilised by prestressing.

#### 4.4.3.2 Analytical Results

The force-displacement response envelopes generated using the two monotonic STMs are illustrated in Fig. 4.20a using solid lines. The experimental results suggest that the test unit suffered strength degradation primarily due to concrete crushing in the joint panel region for the joint-opening direction of loading, and subsequently weakened the compression-carrying capacity of concrete struts when loaded in the joint-closing direction. When the beam-tip displacement reached \( \pm 180 \text{ mm} \), corresponding to a displacement ductility level, \( \mu = 3 \),

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
Figure 4.18: Monotonic STMs of the retrofitted knee-joint test unit.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.20a)

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
joint shear failure occurred as a result of significant concrete damage, leading to continuous joint integrity deterioration. The shear failure resulted in pinching being exhibited in the experimentally recorded hysteretic loop, see Fig. 4.20a. As unit strength degradation was due to the cyclic loading effect, it was not therefore captured by either of the monotonic STMs. However, the elastic unit stiffness and maximum unit strength were accurately predicted.

The reinforcement yielding sequence captured by both monotonic STMs at different applied load levels are identified on the response envelopes in Fig. 4.21a, using triangles for the joint-opening direction, and diamonds for the joint-closing direction. The joint-opening model suggested that yielding occurred in the column flexural reinforcement at the column-joint interface and in the beam flexural reinforcement embedded in the joint panel, see events 1-2 in Fig. 4.18a. Similarly, the joint-closing model predicted that yielding first developed in the column flexural reinforcement embedded in the joint panel, see event 1 in Fig. 4.18b, and then extended to the mid-column height as indicated by events 2-4 in the same figure. Further suggested by event 5 was yielding of the beam reinforcement at the beam-joint interface. Except for the joint shear failure that was attributable to the cyclic loading effect but was not captured, analytical events suggested a close comparison with experimental observations.

The hysteretic response computed using the cyclic STM is illustrated in Fig. 4.20b, together with the experimental data. In general, the analytical results satisfactorily predicted the actual

Figure 4.19: Cyclic STM of the retrofitted knee-joint test unit.
Figure 4.20: Retrofitted knee-joint force-displacement responses.

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
structural behaviour of the retrofitted knee-joint test unit. However, the unloading and reloading stiffness and strength degradation at the beam-tip displacement ±180 mm, corresponding to the displacement ductility level, μ = 3, were not satisfactorily captured due to deficiencies in the material models employed for concrete and rebar, as earlier discussed in Section 3.7. Note that the stepwise reloading and unloading curves were due to the closing and opening of concrete struts that represented cracked concrete. The adoption of $f_d = 0.51f'_c$ as the effective strength of concrete struts located in the joint panel appears appropriate.

### 4.4.4 Redesigned Knee-Joint

The redesigned knee-joint unit had an identical column to the as-built unit, but was constructed with different beam and joint reinforcement details, see Fig. 4.11d. The beam member was adequately reinforced in this test unit to ensure that plasticity occurred only in the column. The capacity design philosophy that was employed in the joint design was based on the joint reinforcement approaching its yield strength when the column had developed its flexural strength. In addition, the embedded column flexural reinforcement was bent within the joint panel, to provide effective anchorage and allow a superior force transfer mechanism to be established.

#### 4.4.4.1 Knee-Joint STM

The monotonic STMs formulated for the redesigned knee-joint test unit are depicted in Figs. 4.21a and 4.21b for the joint-opening and joint-closing directions, respectively. The effective strength value $f_d = 0.51f'_c$ was adopted for the concrete struts located in the joint panel region. For the joint-opening model, the effective area of rebar ties are “a” = 2850 mm² and “b” = 3420 mm². The width of concrete struts were assessed as: c = 275 mm and d = 312.5 mm. For the joint-closing model, the effective area of rebar ties “a” = 6400 mm² and “b” = 4850 mm². The width of concrete struts was evaluated as: c = 270 mm; d = 162 mm; e = 150 mm and f = 125 mm.

The cyclic STM of the redesigned knee-joint that was formulated according to the stress path depicted by the monotonic STMs is illustrated in Fig. 4.22. This cyclic model resembled those of other knee-joint test units discussed previously, except for the model layout in the joint-panel which was constructed differently, according to the specified reinforcement details. Notably, the strut-and-tie representation of the joint panel is common to the sufficiently reinforced joint panel in other structural types, see Sections 4.5.1, 4.8 and 4.10.
Figure 4.21: Monotonic STMs of the redesigned knee-joint test unit.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.23a)
4.4.4.2 Analytical Results

The force-displacement response envelopes obtained from the monotonic STMs for both joint-opening and joint-closing directions are illustrated in Fig. 4.23a, in the positive and negative displacement quadrants, respectively. Analytical results provided satisfactory comparison with the experimental records. The unit strength degradation exhibited in the recorded data was due to the rupture of transverse reinforcement and buckling of flexural column reinforcement in the plastic hinge zone immediately below the column-joint interface. As the model predicted only the average transverse tension force rather that the localised value, local transverse reinforcement rupture was not suggested by the analysis. Furthermore, reinforcement buckling was not considered in the model formulation procedure, and the associated strength degradation that was observed in the testing was not predicted by the model.

The reinforcement yielding sequence captured by the monotonic STMs at different applied load levels is depicted on the response envelopes shown in Fig. 4.23a, using triangles for the joint-opening direction and diamonds for the joint-closing direction. For the joint-opening

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
Figure 4.23: Redesigned knee-joint force-displacement responses.

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column bridge knee-joints
direction, analysis suggests that extensive yielding developed in the column flexural reinforcement, see events 1-5 in Fig. 4.21a. This prediction is consistent with experimental strain gauge readings, but yielding in the beam bottom flexural reinforcement, observed in the testing, is not suggested in the analysis. For the joint-closing direction, the model predicted that yielding occurred in both the beam and column flexural reinforcement, see events 1-4 in Fig. 4.21b. Again, analytical results were comparable with the experimental records.

The hysteretic response which was computed using the cyclic STM is illustrated in Fig. 4.23b, accompanied by the experimental data. Satisfactory prediction was obtained except that strength degradation of the test unit resulting from buckling of the column flexural reinforcement was not captured at high ductility levels.

In general, the analytical results obtained from both monotonic and cyclic models had satisfactorily captured the critical aspects of the experimentally measured structural response, including elastic stiffness and unit yield strength.

4.4.5 Analysis Remarks

The STM analysis conducted for the large-scale knee-joint test units has, in general, captured the critical aspects of the experimentally observed structural performance. The effective strength $f_d = 0.51f'_c$ was applicable to the joint struts in all cases. The cyclic STMs were, in general, capable of predicting the initial elastic stiffness and unit yield strength. However, the strength degradation due to concrete damage resulting from cyclic loading was not estimated by the cyclic models. This was due to the deficiencies of the concrete material model that was discussed previously in Section 3.7.

4.5 Beam-column Tee-Joints

The three large-scale bridge tee-joint test units analysed in this section were adopted from testing conducted by Sritharan et al. (1996 and 1999). STM analytical results of these tee-joints were previously documented in To et al. (2003b), and the results reported in this section were regenerated using the refined model formulation procedure described in Chapter 3.

The cap-beam of the test units had conventionally reinforced, partially prestressed and fully prestressed details. These test units represented redesigns of the as-built interior joint from three-column Bent 793+57 of the Santa Monica Viaduct in Los Angeles, California, see Fig.
4.24. The as-built joint was a typical joint constructed according to the pre-1960 standard, with no shear reinforcement, and was found to provide poor seismic performance [MacRae, et al. (1994)]. All test units were constructed at half-scale and tested in an inverted position. The soffit and deck slab were not constructed in the test units to simplify the testing procedure and data interpretation.

These tee-joint tests, together with the knee-joint testing described previously, were sponsored by the California Department of Transportation, and represented part of an extensive experimental research project carried out over the past decade, aiming at improving the seismic design of bridge structures subsequent to widespread damage in the 1989 Loma Prieta and 1994 Northridge earthquakes.

4.5.1 Conventionally Reinforced Tee-Joint, IC1

The overall dimensions and reinforcement details of tee-joint IC1 are illustrated in Fig. 4.25. The test unit was composed of a circular column and rectangular beam, and was detailed with conventional reinforcement in the joint panel region. Notably, additional cap-beam bottom longitudinal rebars (i.e. the top layer of longitudinal reinforcement in the inverted position), and an extra amount of stirrups in the vicinity of the joint region were provided to facilitate effective joint force transfer by mobilising an external joint strut, see the STMs in Fig. 4.26.

![Diagram of Bridge Joint](image-url)
Horizontal joint reinforcement was provided by #3 \((d_b = 9.5 \text{ mm})\) spirals at 57 mm spacing and 2 sets of 4-legged #3 hairpin-type stirrups were employed as the vertical joint reinforcement. In order to allow the column flexural rebars to be effectively anchored into the joint strut, the column rebars were extended to be close to the top of the cap-beam (i.e. the bottom layer as depicted in Fig. 4.26 in the inverted position), terminating at a distance of 63.5 mm away from the beam edge.

4.5.1.1 Tee-joint IC1 STM

The formulated monotonic STMs replicating the stress path developed inside the tee-joint IC1, when subjected to push (+ve displacement) and pull (-ve displacement) actions, are depicted in Fig. 4.26. As the stress paths that occurred inside the test unit were similar for both action directions, the pull direction STM was assumed to mirror the push direction STM. For the push STM, shown in Fig. 4.26a, the area of rebar ties was “a” = 1550 mm\(^2\) and “b” = 2330 mm\(^2\). The width of the concrete struts located within, and adjacent to the joint panel region was evaluated as: \(c = 200 \text{ mm; } d = 160 \text{ mm; } e = 95 \text{ mm; } f = 150 \text{ mm; } g = 100 \text{ mm; } h = 150 \text{ mm and } i = 100 \text{ mm.}\) For the pull direction STM depicted in Fig. 4.27b, the area of the rebar ties was “a” = 2330 mm\(^2\) and “b” = 1550 mm\(^2\). The width of the concrete struts identified in the same figure was assessed as: \(c = 150 \text{ mm; } d = 100 \text{ mm; } e = 150 \text{ mm; } f = 100 \text{ mm; } g = 160 \text{ mm; } h = 200 \text{ mm and } i = 100 \text{ mm.}\)

![Figure 4.25: Reinforcement detail of the conventionally reinforced tee-joint, IC1.](image)
Figure 4.26: Monotonic STMs of the tee-joint test unit, IC1.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.28a)

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column Tee-Joints
The cyclic STM formulated for tee-joint IC1 is illustrated in Fig. 4.28. The model layout for the joint panel region was simplified from the discrete stress path employed in the monotonic STMs to allow symmetric layout, yet the actual stress path was sufficiently retained.

4.5.1.2 Tee-joint IC1 Analytical Results

The derived force-displacement response envelopes of tee-joint IC1 using both push (+ve displacement) and pull (-ve displacement) monotonic STMs are illustrated in Fig. 4.28a. Satisfactory estimation was obtained for the unit elastic stiffness and yield strength.

The reinforcement yielding sequence obtained from both pull and push STMs at different applied load levels is illustrated in the corresponding response envelope in Fig. 4.28a. Both models suggested that extensive yielding developed in the column flexural reinforcement, see events 1-3 & 5 for the push direction model in Fig. 4.26a, and events 1-2 & 4 for the pull direction model in Fig. 4.26b. Also suggested by events 4 and 3 in the respective push and pull models, was the yielding of transverse reinforcement in the column plastic hinge zone. Analytical results were consistent with the experimentally recorded data. Notably, yielding of the beam flexural reinforcement embedded inside the joint panel region was observed in the testing at high displacement ductility levels, $\mu = 6 (\pm 100 \text{ mm})$, but was not suggested in

Nonlinear Strut-and-Tie Model Analytical Examples: Beam-column Tee-Joints
Figure 4.28: STM analysis results for the tee-joint IC1.
the analysis. As the beam flexural reinforcement that was located outside the joint panel remained elastic throughout the test, yielding that was experimentally observed in the beam reinforcement inside the joint may have been due to high local tensile stress required for the development of effective joint force transfer mechanism.

Experimental observations suggested that the performance of the tee-joint IC1 was satisfactory until the displacement ductility level, $\mu = 6.0$, corresponding to a column displacement of 100 mm, when gradual strength deterioration was recorded at this ductility level due to the progressive tension yielding of joint reinforcement, causing extensive concrete spalling in the joint panel region and in the column plastic hinge zone. Also contributing to unit strength degradation was buckling of the column flexural reinforcement in the plastic hinge zone. The analysis was not able to predict joint strut crushing in the joint panel, as the adoption of $f_d = 0.51f'_c$ appears to be higher than the actual concrete strength. This effective strength value was nevertheless regarded as suitable, because the adoption of a lower strength value in this STM analysis, $f_d = 0.34f'_c$, see Table 3.2, would predict premature strut crushing prior to plasticity being developed in the column, leading to a significant under-estimation of the maximum unit lateral strength by the STMs.

The hysteretic structural response generated using the cyclic STM is illustrated in Fig. 4.28b. The analysis provided a reasonable insight into critical aspects of the measured cyclic structural performance, including the elastic stiffness and unit yield strength. Again, structural strength degradation resulting from concrete damage when subjected to repeated cyclic action at the same displacement ductility level was not predicted, due to limitations of the employed material models, see Section 3.7. Notably, the stepwise reloading and unloading curves were due to closing and opening of concrete struts that represented cracked concrete.

4.5.2 Partially Prestressed Tee-Joint, IC2

This tee-joint test unit was comparable to tee-joint IC1 discussed above, but was detailed with partial prestressing equal to 0.1$f'_c$. Because of the applied prestressing, it was anticipated that all of the column flexural rebar could be directly anchored into the joint diagonal concrete strut. Therefore, no extra stirrups were provided in the cap beam adjacent to the joint region, see Fig. 4.29. The column flexural rebar was also extended to the top of the cap beam (i.e. bottom layer of the flexural reinforcement in the inverted position). Additional bottom beam reinforcement required by the joint design model was also provided in this unit to allow the development of external struts that utilised the reserve capacity of the beam shear reinforcement. The horizontal joint reinforcement was provided by #3 ($d_b = 9.5$ mm) spirals at 121 mm spacing, and 2 sets of 4-legged hairpin type reinforcement were
utilised as the vertical joint reinforcement, which was identical to that provided in IC1. The employed horizontal and vertical joint reinforcement were 32% and 18% of that required, based on the maximum horizontal and vertical shear forces, respectively. The cap beam prestressing was applied using straight bars, and the total applied prestressing force was 1737 kN.

4.5.2.1 Tee-Joint IC2 STMs

The monotonic STMs of the tee-joint test unit IC2 are depicted in Fig. 4.30. Noteworthy is that the additional concrete struts that run along the length of the cap-beam were incorporated into the STMs to represent the concrete being mobilised by prestressing. These concrete struts were not effective when the test unit was subjected to a lateral load corresponding to the column ultimate flexural strength, but they were required here to provide analytical stability for the application of axial load at the top of the column.

For the push direction STM, shown in Fig. 4.31a, additional rebar ties were required in the joint and beam to transfer tension to the reaction point on the right. Accordingly, the push direction STM was slightly different from the pull direction STM, shown in Fig. 4.31b. As the cap-beam was partially prestressed, a broader diagonal concrete strut was mobilised in the joint panel region to transfer larger shear forces across the joint. Also, this broader diagonal concrete strut was beneficial to the development of an effective force-transfer mechanism as it was less reliant upon joint reinforcement. Again, $f_d = 0.51f'_c$ was adopted as the effective strength of joint struts. For the push model, shown in Fig. 4.31a, the area of

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**Figure 4.29:** Reinforcement details of the tee-joint with partially prestressed cap beam, IC2.
Figure 4.30: Monotonic STMs of the tee-joint test unit, IC2.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.32a)
rebar ties was evaluated as “a” = 1550 mm² and “b” = 2330 mm². The width of concrete struts identified in the same figure was assessed as: c, d, e & f = 100 mm; g = 235 mm; h = 100 mm and i = 195 mm. For the pull model, shown in Fig. 4.31b, the area of rebar ties was evaluated as “a” = 2330 mm² and “b” = 1550 mm². Also, the width of concrete struts identified in the same figure was evaluated as: c = 150 mm; d = 250 mm; e = 62.5 mm and f = 250 mm.

The cyclic STM formulated for the tee-joint IC2 is shown in Fig. 4.32. This model was constructed according to the discrete stress path identified in both monotonic STMs.

4.5.2.2 Tee-joint IC2 Analytical Results

When tested, the unit was initially subjected to an unintentional impulsive horizontal load of 388 kN in the pull (-ve displacement) direction, corresponding to a displacement ductility, μ = 3. Unfortunately, no test data were recorded during the mishap. The test unit was then unloaded and the horizontal displacement of the column was brought back to zero displacement, requiring an actuator force of 158 kN in the push direction. From this point, the test was started again.

Due to the erroneous testing procedure, the experimentally recorded elastic stiffness in the pull (-ve displacement) direction action was substantially lower than that predicted by the

![Figure 4.31: Cyclic STM of the partially prestressed tee-joint test unit, IC2.](image)
monotonic STM, see Fig. 4.32a. However, as the elastic stiffness was accurately calculated for the push direction displacement, it was assumed that the pull model had predicted the elastic response satisfactorily. Also suggested by the analytical response envelopes in both displacement directions is that the unit strength was accurately analysed.

The force-displacement response envelope and the structural hysteretic behaviour generated while using the monotonic models and the cyclic model are illustrated in Fig. 4.32a and 4.32b respectively. Analytical results compared satisfactorily with the experimentally recorded data, especially to the initial elastic stiffness and unit yield strength. Lateral strength degradation that was observed at high displacement ductility levels was due to significant concrete damage in the column plastic hinge zone under cyclic loading. This was not captured by the cyclic model, due to limitations of the material model employed for concrete struts, see Section 3.7.

The reinforcement yielding sequence captured by both monotonic STMs at different applied load levels is illustrated on the respective response envelopes in Fig. 4.32 using solid circles. Both models suggested extensive yielding in the column flexural reinforcement, but minimal concrete damage in the joint panel region, see events 1-4 for both models in Fig. 4.30. The analysis results were consistent with experimental observations.

Again, the analytical results obtained from the STMs had, in general, satisfactorily predicted the critical aspects of experimentally measured structural response of the tee-joint IC2 test unit, including the elastic stiffness, lateral unit yield strength and the location of reinforcement yielding.

4.5.3 Fully Prestressed Tee-Joint, IC3

The fully prestressed tee-joint test unit considered here had a prestress force equal to $0.2f'_c$. The unit reinforcement details of this test unit are illustrated Fig. 4.33. The cap-beam of this unit was constructed with precast units to examine the feasibility of precast construction of concrete multicolunn bents and their associated behaviour when subjected to simulated seismic loads. The column flexural reinforcement detail of test unit IC1 was duplicated in test unit IC3 for strength comparison. No continuous flexural mild steel reinforcement was provided in the cap-beam; however, 4 #4 ($d_b = 12.7$ mm) top and bottom rebars were provided in each precast beam segment, primarily to support the beam transverse rebars during construction. A layer of epoxy with a thickness of 3.2 mm was used between the precast elements to improve the grouting procedure. The joint reinforcement detail of this test unit was identical to that of test unit IC2, except that the joint vertical stirrups were provided as closed ties. Due to flexural reinforcement buckling that was observed in the
Figure 4.32: STM analysis results for the tee-joint IC2.
column plastic hinge zones when testing the previous two test units, resulting in the development of pronounced column shear cracks, the volumetric ratio of column spiral reinforcement in IC3 was increased by 50%. Again, cap beam prestressing was applied using straight bar with zero eccentricity, and the total prestressing force was 3002 kN.

4.5.3.1 Tee-Joint IC3 STMs

The corresponding STMs of test unit IC3, when subjected to both loading directions, are illustrated in Fig. 4.34. A noteworthy modelling feature is that the column flexural reinforcement tie was directly anchored into the joint concrete strut and the diagonal concrete strut that extended from the test unit supports. Similar to IC2, the STMs for IC3 for both action directions had additional concrete struts in the cap-beam to represent concrete being mobilised by the prestressing. As the force transfer models suggested that similar force paths occurred in the joint for both loading directions, the pull direction STM was assumed to be the mirror image of the push direction STM. Again, $f_{d} = 0.51f'_{c}$ was adopted as the effective strength of the main diagonal joint struts. The rebar ties identified in both the pull and push direction models had the same area of $a = 3420 \text{ mm}^2$. The width of the main diagonal joint strut in both models was assessed as $b = 325 \text{ mm}$.

The cyclic STM formulated for tee-joint IC3 is illustrated in Fig. 4.35. This model was formulated to approximately replicate the stress path depicted in both monotonic STMs.

![Figure 4.33: Reinforcement detail of the fully prestressed tee-joint, IC3.](image-url)
Figure 4.34: Monotonic STMs of the tee-joint test unit, IC3.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.36a)
4.5.3.2 Tee-joint IC3 Analytical Results

The force-displacement response envelopes derived from both monotonic push and pull models are shown in Fig. 4.36a, accompanied by the experimental data. As illustrated, the elastic stiffness of the test unit in both displacement directions was accurately captured, and the unit lateral yield strength was predicted satisfactorily. Furthermore, the hysteretic response envelope derived using the cyclic model, shown in Fig. 4.37b, resembled that recorded in the experiment. Since the cyclic STM could not examine the strain energy being absorbed by the rebars, fracture of column longitudinal reinforcement which occurred during the 2\textsuperscript{nd} and 3\textsuperscript{rd} load cycle at the pull direction displacement of -150 mm, corresponding to the displacement ductility of $\mu = 10$, was not predicted. Again, stepwise unloading and reloading curves were due to the opening and closing of concrete struts that represented cracked concrete.

The reinforcement yielding sequence that was captured by both monotonic models at different applied load levels is also illustrated on the respective response envelope shown in Fig. 4.36a, using solid circles. The analysis suggested that inelastic response of the test unit in both push and pull direction actions was attributable to the formation of a column plastic hinge immediately above the column-joint interface. Moreover, no joint strut crushing was

![Diagram](image)

**Figure 4.35: Cyclic STMs of the tee-joint test unit, IC3.**
Figure 4.36: STM analyses for the tee-joint IC3.
predicted by the analysis. Analytical predictions were, in general, consistent with the experimental observations except that the fracture and buckling of column flexural reinforcement at a very high displacement ductility level, \( \mu = 10 \ (\pm 150 \text{ mm}) \), were not captured. Cyclic STM analytical results could be improved by incorporating a more sophisticated rebar stress-strain model that is capable of accounting for the strain energy absorption and buckling behaviour.

### 4.5.4 Analysis Remarks

Three large-scale tee-joint test units are analysed in this section using STMs. Analytical results sufficiently predicted the structural response of the test units. The tee-joint IC1 STM demonstrated that \( f_d = 0.51f'_c \) was suitable to be adopted as the effective strength of lightly confined joint struts. However, caution must be exercised by the designer when interpreting the analytical results, as they are likely to over-estimate the unit strength at high displacement ductility levels, at which significant concrete damage is likely. The analysis conducted for tee-joint IC3 demonstrated that a structure with predominant flexural response may be analysed by the proposed STM with sufficient precision.

### 4.6 Bridge Portal Frame

A full-scale reinforced concrete bridge portal frame tested by Innamorato et al. (1996) was analysed using STM. The key reinforcement details and the geometric configuration of this test unit are illustrated in Fig. 4.37. This test unit consisted of two circular columns and a rectangular beam, which were constructed in accordance with the as-built details of a multicolumn bridge bent of the 110 Santa Monica Viaduct in Los Angeles, California.

#### 4.6.1 Bridge Portal Frame STM

The monotonic STM formulated for the bridge portal frame is illustrated in Fig. 4.38a. As the joint panels of this test specimen were effectively unreinforced, a simple joint force-transfer mechanism, similar to that employed for the as-built knee-joint test unit and described in Section 4.4.1, was utilised in formulating this model. Additionally, the column flexural reinforcement was terminated into the joint panels with straight extension, implying that zero tension could be developed at the top end of rebars. Consequently, the beam compression entering the joint-opening panel was diverted, connecting with the column flexural reinforcement at a distance away from the rebar ends equal to half of the effective clamping length, \( \ell_c \), calculated using Eq. 3.38. Only the model formulated for the push direction action was examined in the monotonic analysis, as the model for the pull direction...
action was the mirror image reflection of that for the push direction action. Similar to the STM analyses considered in the previous sections, was adopted as the effective strength of the joint struts. As indicated in Fig. 4.38a, the area of rebar ties representing the flexural reinforcement embedded in the joint panel was: \(a = 5750 \text{ mm}^2\) and \(b = 1920 \text{ mm}^2\). The width of joint struts identified in the same figure was evaluated as: \(c = 590; d = 380 \text{ mm};\) and \(e = 610 \text{ mm}\).

The cyclic STM formulated for the bridge portal frame is shown in Fig. 4.38b. This model was formulated according to the discrete stress-path described in the monotonic model. Notably, diversion of the beam flexural compressive stress when entering into the joint

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**Figure 4.37:** Configuration and reinforcement details of bridge portal frame.

Nonlinear Strut-and-Tie Model Analytical Examples: Bridge Portal Frame
Figure 4.38: STMs for the bridge portal frame.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.39a)

Nonlinear Strut-and-Tie Model Analytical Examples: Bridge Portal Frame
panels was not replicated, due to the restriction of a symmetrical model layout. This resulted in a slight over-estimation of the beam flexural strength.

4.6.2 Analytical Results for the Bridge Portal Frame

The force-displacement response envelope generated using the monotonic STM for the push direction action is illustrated in the positive displacement quadrant of Fig. 4.39a. This response envelope was also transposed to the negative displacement quadrant for comparison. Test results indicated that the first signs of significant shear cracking in both joint panels occurred at displacement ductility $\mu = 1$ (±42 mm). Also at this displacement ductility level, plastic hinges developed at the top of both circular columns. Further concrete cracking occurred in both joint panel regions, attributable to yield penetration of the column flexural reinforcement at displacement ductility $\mu = 2$ (±84 mm). Moreover, anchorage failure in the bottom flexural rebars of the cap-beam that were embedded inside the joint panel was observed, resulting in less stable hysteretic response beyond $\mu = 2$. The test was terminated after three load cycles at displacement ductility $\mu = 4$, prior to the development of significant strength degradation, in order to preserve the integrity of the structure for the second phase of testing, which examined a retrofit solution.

The analytical force-displacement response envelope indicated satisfactory comparison with the experimental record except for slight under-estimation of the unit yield strength. Consistent with experimental observation, the STMs predicted that the lateral strength of the test unit was governed by the development of plasticity in both circular columns. Analytical events, illustrated in Fig. 4.38a, suggested that yielding first occurred in the column bases, see events 1-2, and subsequently in the column flexural reinforcement, both located below the joint-opening panel, see event 3, and that embedded inside the joint-closing panel, see event 4. Further suggested by the analysis and confirmed by experiment was that no crushing developed in the joint struts.

The hysteretic response of the bridge portal frame computed using the cyclic STM is illustrated in Fig. 4.40b. The predicted initial elastic stiffness and unit yield strength were satisfactory. Furthermore, the general shape of the hysteretic loop was sufficiently captured by the cyclic model. As plastic hinges were not suggested to develop in the cap-beam, neglecting diversion of the beam flexural compression when entering the joint-opening panel by the cyclic STM, as mentioned in Section 4.6.1, did not affect the accuracy of the predicted unit lateral strength.
Figure 4.39: STM analysis results for the bridge portal frame.
4.6.3 Analysis Remarks

As the test specimen was loaded up to moderate displacement ductility levels, it has predominantly exhibited ductile structural behaviour. Therefore, as expected, the STM captured critical aspects of the experimentally observed structural response with satisfactory precision.

4.7 Multicolumn Bridge Bents

STM analytical results of multicolumn bridge bents were previously reported by To et al. (2003b), but the modelling results presented in this section are from reanalysis, using the refined model formulation procedure presented in Chapter 3. The first unit, MCB1, of the two multicolumn bridge bents analysed in this section, was designed with a precast fully-prestressed cap-beam, while the second unit, MCB2, was designed with a mix of conventional and headed reinforcement, and with mechanical couplers [Sriharian et al. (1997, 2001)]. Notably, these two multicolumn bridge bents were also designed according to the force transfer models proposed by Priestley (1993), Ingham (1995) and Sriharian (1998), resulting in substantially reduced amounts of joint reinforcement, which improved constructability. In conjunction with the design concept based on force transfer models, reinforcement quantities in the joint region were further decreased with the utilisation of prestressing in the cap-beam.

4.7.1 Multicolumn Bridge Bent, MCB1

The reinforcement detail of the precast fully-prestressed multicolumn test unit, MCB1, is illustrated in Fig. 4.41a. A longitudinal reinforcement quantity of 4% was provided in the interior column in order to ensure development of the maximum practicable joint shear, while a flexural reinforcement content of 3% was provided in the exterior column in order to avoid excessive cap-beam seismic moment. Also illustrated in Fig. 4.40a is the reinforcement detail of the pin connection at the column bases. This pin connection was achieved by terminating all the column flexural reinforcement just above the footing and by reducing the gross area of the column section at the column-footing interface by 50%. Adequate reinforcement was provided at the centre of the column to transfer axial tension and to provide lateral connectivity.

Prestressing was applied to the bent cap with zero eccentricity using straight bars, and there was no continuous flexural reinforcement provided in the cap-beam in order to allow
Figure 4.40: Key reinforcement details of multicolumn bridge bents: (a) MBC1, (b) MCB2, and (c) Cap beam reinforcement details of MCB2. [Sritharan et al. (2001)]
construction of the test unit using precast modules, see Fig. 4.40a. However, 6 #4 \((d_b = 12.7 \text{ mm})\) top and bottom longitudinal rebars were provided in each precast segment of the capbeam, primarily to support the beam transverse rebars during construction.

A nominal quantity of horizontal spirals and vertical rectangular ties were provided to both tee- and knee-joints. In order to increase the effectiveness of rectangular ties in the out-of-plane direction, two j-hooks were also employed as crossties with each stirrup in the tee-joint. Minimal reinforcement was also provided within the knee-joint. The horizontal spirals of this joint were identical to that of the tee-joint, while only 3 sets of 6 legged #3 \((d_b = 9.5 \text{ mm})\) stirrups were placed as vertical joint reinforcement. Moreover, for the column flexural rebars to be effectively anchored in the diagonal joint strut, these rebars were extended into the joint as close to the top beam reinforcement as possible.

4.7.1.1 MCB1 STM

The monotonic STMs formulated for MCB1, when subjected to push (+ve displacement) and pull (-ve displacement) direction loadings, are illustrated in Fig. 4.41a and 4.41b respectively. The monotonic model formulated for the test unit when subjected to the push and pull direction loadings are similar due to the comparable discrete force paths. Also noteworthy is that the rebars connecting the column and footing were assumed to be anchored at a distance of \(\ell_c/2\) away from the bar end, where \(\ell_c\) is the reinforcement anchorage length calculated using Eq. 3.39. Furthermore, \(f_d = 0.51f_c'\) was adopted as the effective strength of joint struts in both monotonic models. The area of rebar ties representing the column flexural reinforcement embedded in the tee-joint and the knee-joint was: \(a = 6820 \text{ mm}^2\) and \(b = 5520 \text{ mm}^2\), respectively. The width of joint struts identified in Figs. 4.42 was assessed as \(c = 315 \text{ mm}\) and \(d = 315 \text{ mm}\). The MCB1 cyclic STM is illustrated in Fig. 4.42. Notably, the pin connection detail at the column bases was also modelled with an internal lever arm identical to that employed in the monotonic model pin base to provide comparable flexural strength.

4.7.1.2 Analytical Results

The force-displacement envelope generated using the monotonic STMs is illustrated in Fig. 4.43a. Both push (+ve displacement) and pull (-ve displacement) models satisfactorily predicted the initial elastic stiffness and unit yield strength. According to the experimental results, plastic response of the test unit was due to column plastic hinges that developed below both of the tee- and knee-joints. Significant structural damage was experimentally observed in the plastic hinge zones including concrete crushing, flexural rebar buckling and
Figure 4.41: Monotonic STMs of multicolumn bridge bent, MCB1.

(Note: Numbers refer to nonlinear event history shown in Fig. 4.43a)

Nonlinear Strut-and-Tie Model Analytical Examples: Multicolumn Bridge Bents
flexural rebar fracture at the final stage of testing. Only fine cracks were detected in both joint panels.

The analytical events captured by both models are depicted on the respective force-displacement response envelope, shown in Fig. 4.43, using solid circles. The reinforcement yielding sequences predicted by both models were similar. Analytical events 1-2 shown in Figs. 4.41a and 4.41b, suggested that yielding first occurred in the rebars connecting the column bases and the foundation. Subsequently, yielding developed in the column flexural reinforcement below the joint panel regions, implying the formation of column plastic hinges, see events 3-5 in Fig. 4.41a for the push model, and events 3-6 in Fig. 4.41b for the pull model. Event 7 in Fig. 4.41a indicated rebar yielding in compression, suggesting a high possibility of rebar buckling. In addition, concrete crushing inside the column plastic hinge zones was predicted by both models. This is demonstrated by strength reduction in the push direction (-ve displacement) response envelope and gradual stiffness softening in the inelastic regime of the push direction (+ve displacement) response envelope.

The analytical hysteretic response derived using the cyclic STM is illustrated in Fig. 4.42. The analysis satisfactorily predicted the initial elastic stiffness and unit yield strength predicted by the analysis was the unit strength degradation in the push direction. Concrete crushing in the column plastic hinge zone being developed at a lateral displacement of +260 mm, corresponding to displacement ductility level, $\mu = 8$. This analysis was consistent with experimental observation except that buckling and fractu

Nonlinear Strut-and-Tie Model Analytical Examples: Multicolumn Bridge Bents
Figure 4.43: STM analysis results for the multicolumn bridge bent, MCB1.

Nonlinear Strut-and-Tie Model Analytical Examples: Multicolumn Bridge Bents
column flexural reinforcement that developed subsequently and did significant concrete damage in the column plastic hinge zone, were not captured by the cyclic STM. A sophisticated stress-strain model of reinforcement that accounts for strain energy absorption and buckling behaviour is required for improving analytical accuracy.

4.7.2 Multicolumn Bridge Bent, MCB2

The multicolumn bridge bent test unit, MCB2, had the same column reinforcement detail as MCB1, but the cap-beam was detailed with a mix of conventional and headed reinforcement, mechanical couplers and zero prestressing, see Fig. 4.41b.

To simplify construction of the bent cap, short u-shape headed reinforcement was fed through the knee-joint and was connected to the headed flexural beam reinforcement adjacent to the column face using mechanical couplers. In addition, mechanical couplers were also provided in the low bending moment region of the bent cap to connect rebars with different diameters over zero lap length. For the reinforcement detail of the tee-joint, 5 sets of 6-legged #3 \((d_b = 9.5 \text{ mm})\) stirrups were placed within a 381 mm distance from each face of the interior column, see Fig. 4.40c(i), in order to support an effective joint force-transfer mechanism. Three of the same stirrup sets and #3 \((d_b = 9.5 \text{ mm})\) stirrups at 57 mm spacing were provided in the joint as the vertical and horizontal reinforcement, see Fig. 4.40c(iii). Furthermore, the flexural reinforcement was extended into the joint as close to the top beam reinforcement as possible.

As the exterior column had less flexural reinforcement content than the interior column, a reduced amount of knee-joint reinforcement was required. In order to support an effective joint force-transfer mechanism, 4 sets of 6-legged #3 \((d_b = 9.5 \text{ mm})\) stirrups were placed within a 381 mm distance adjacent to the exterior column face, see Fig. 4.40c (ii).

4.7.2.1 MCB2 STMs

The formulated STMs for MCB2, when subjected to the externally applied push and pull actions, are illustrated in Fig. 4.44. As depicted in the pull model, additional reinforcement ties in both tee- and knee-joint regions were mobilised as a result of tension demand developed in the cap-beam due to the externally applied action. Again, the reinforcement connecting columns and footings was assumed to be anchored at half of the distance calculated by Eq. 3.38, away from the rebars end. As the beam member of this unit was not prestressed, only narrow concrete struts were expected to be mobilised to transfer shear across the joint regions. Again, \(f_d = 0.51f'_c\) was adopted as the effective strength of the joint strut. For the push direction model shown in Fig. 4.44a, the area of rebar tie representing
Figure 4.44: Monotonic STMs of multicolumn bridge bent, MCB2.

(Note: Numbers refer to nonlinear event history shown in Fig. 4.46a)

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the column flexural reinforcement embedded in the joint panels was: $a = 3850 \text{ mm}^2$; $b = 3850 \text{ mm}^2$ and $l = 540 \text{ mm}^2$. The width of concrete strut identified in the same figure was assessed as: $c = 195 \text{ mm}$; $d = 425 \text{ mm}$; $e = 195 \text{ mm}$; $f = 130 \text{ mm}$; $g = 180 \text{ mm}$; $h = 320 \text{ mm}$; $i = 195 \text{ mm}$; $j = 65 \text{ mm}$; $k = 230 \text{ mm}$; $m = 215 \text{ mm}$; $n = 425 \text{ mm}$; $o = 165 \text{ mm}$; $p = 130 \text{ mm}$; $q = 265 \text{ mm}$; $r = 265 \text{ mm}$; and $s = 130 \text{ mm}$. For the pull direction model, shown in Fig. 4.44b, the area of rebar ties representing the reinforcement embedded in the joint panels was: $a = 3850 \text{ mm}^2$; $b = 3850 \text{ mm}^2$; $m = 3850 \text{ mm}^2$; and $l = 2570 \text{ mm}^2$. The width of joint struts identified in the same figure was evaluated as: $c = 195 \text{ mm}$; $d = 325 \text{ mm}$; $e = 180 \text{ mm}$; $f = 130 \text{ mm}$; $g = 130 \text{ mm}$; $h = 225 \text{ mm}$; $i = 165 \text{ mm}$; $j = 260 \text{ mm}$; $k = 130 \text{ mm}$; $n = 195 \text{ mm}$; $o = 325 \text{ mm}$; $p = 165 \text{ mm}$; $q = 195 \text{ mm}$; $r = 115 \text{ mm}$; $s = 195 \text{ mm}$; $t = 130 \text{ mm}$; and $u = 225 \text{ mm}$.

The cyclic STM that approximately replicated the stress-path that developed inside the test unit MCB2 under the lateral cyclic actions is illustrated in Fig. 4.45. Again, the pin connection at column bases was included in the model to provide a flexural resistance comparable to the actual situation.

4.7.2.2 Analytical Results

The calculated force-displacement response envelopes using the monotonic STMs are illustrated in Fig. 4.46a, accompanied by the experimental data. Unit strength and initial elastic stiffness in both push (+ve displacement) and pull (-ve displacement) direction
Figure 4.46: STM analyses for the multicolumn bridge bent, MCB2.
loadings were predicted satisfactorily. The elastic response was accurately predicted in the push direction but was less precisely matched in the pull direction, especially after concrete cracking, as indicated by the softened elastic stiffness at a lateral load level of approximately 300 kN. In addition, joint strut crushing in the tee-joint panel was predicted, resulting in the lateral strength degradation demonstrated in the response envelope depicted in the positive displacement quadrant of Fig. 4.46a.

The reinforcement yielding sequence captured by both STMs is depicted on the respective force-displacement response envelope, shown in Fig. 4.47a, using solid circles. For the push model, yielding first occurred in the rebars connecting the column bases and the foundation, see events 1-2, 4-5 & 9 in Fig. 4.44a. This was followed by yielding developing in the column flexural reinforcement below the knee-joint panel, see events 3 & 6, then in the beam flexural reinforcement, see event 7, and subsequently in the knee-joint transverse reinforcement, see event 8. A similar reinforcement yielding sequence was captured by the pull model. Events 1-2 & 9-10, shown in Fig. 4.44b, predicted that yielding developed in the rebars connecting the column bases and the foundation. Subsequently, yielding occurred in the column flexural reinforcement below the joint panels, see events 3-5 & 7, and in the beam flexural reinforcement located inside and in the vicinity of the joint panels, see events 6 & 8.

The analytical hysteretic response computed using the cyclic STM is illustrated in Fig. 4.46b, accompanied by the experimental data. The prediction of unit yield strengths, the initial elastic stiffness and the unloading and reloading stiffness were captured with sufficient accuracy. Furthermore, unit strength degradation resulting from the crushing of tee-joint concrete struts was captured by the model at a lateral displacement of -270 mm corresponding to displacement ductility level, $\mu = 6$, in the pull loading direction. However, the cyclic STM did not predict the unit strength degradation that was attributable to progressive damage of the tee-joint when loaded cyclically at the same displacement ductility level. Again, this was due to the deficiency of the material stress-strain models employed for the concrete struts, as earlier discussed in Section 3.7.

4.7.3 Analysis Remarks

In general, the analytical results obtained from the STMs were consistent with the experimental observations. STM analysis for both test units predicted joint stresses resulting in unit strength degradation. However, due to the deficiency of the strain models employed for the concrete struts and rebar ties, the predicted degradation of test units and reinforcement buckling and fracture was not
4.8 Building Frame Interior Joint Systems

The four 70% scale beam-column joint subassemblies considered in this section were tested by Lin (2000), representing subassemblies of one-way precast reinforced concrete ductile perimeter frames. All test units were designed to comply with the ductile frame requirements recommended in the New Zealand Concrete Design Standard (NZS 3101 (1995)), which follows capacity design principles and ensures satisfactory lateral load resistance for the frames applicable to high-rise multistorey buildings. The beams and columns in all test units were precast and connected at the joint region through grouted steel ducts. Grade 500 and grade 300 steel were used as the flexural reinforcement and transverse reinforcement, respectively.

4.8.1 Interior joint system, Units 1 and 2

The physical dimensions and key reinforcement details of test units 1 and 2 are illustrated in Fig. 4.47. These two interior joint subassemblies were designed to reconcile the discrepancy between the theoretical design model developed by Lin (2000) and the design recommendations given in NZS 3101 (1995), for joints subjected to a combined high column compression, $0.4A_gf'_c$, and high joint shear stress, $0.19A_gf'_c$. All test units had identical beam and column sections, but different transverse reinforcement quantities were employed in the joint panel region. The flexural reinforcement in beams and columns was provided by 16-HD12 ($d_b = 12$ mm) and 8-HD12 ($d_b = 12$ mm) + 4-HD16 ($d_b = 16$ mm), respectively. Six sets of transverse reinforcement were provided in the joint panel for unit 1, and nine sets of transverse reinforcement were located in the joint panel for unit 2.

4.8.1.1 Unit 1 and 2 STMs

As test units 1 and 2 were essentially identical, they were assumed to develop the same stress path inside the structure. The monotonic STM illustrated in Fig. 4.48a, which is a discrete representation of this stress path, was formulated using the proposed procedure described in Chapter 3. A strength value of $f_{d} = 0.51f'_c$ was adopted as the effective strength of the joint struts. Also illustrated in Fig. 4.48a, the area of the rebar ties employed for the STM to represent the joint transverse reinforcement, denoted as "a" in Fig. 4.48a, provided in test unit 1 and unit 2 was $1280\text{mm}^2$ and $1920\text{mm}^2$, respectively. The width of the concrete struts applicable to both test units was assessed as: $b = 45$ mm; $c = 245$ mm and $d = 45$ mm. As the STMs for the test units were identical in both lateral action directions, the analysis was limited to the push (left to right) action direction.
The cyclic STM used to analyse the hysteretic structural response of test units 1 and 2 is illustrated in Fig. 4.48b. Again, the modelling solution for both test units was identical, except for the area of the transverse rebar located in the joint panel region.

Notably, a very strong steel cast extension was connected to the column base to generate a suitable shear demand in the joint panel when testing. For a simple modelling solution, this steel cast fitting was assumed undeformable and was therefore represented by rigid model elements in both monotonic and cyclic STMs, see Fig. 4.48.

4.8.1.2 Analytical Results for Unit 1

The force-displacement response envelope generated using the monotonic STM for test unit 1, when subjected to a push direction action (left to right), is illustrated in the positive displacement quadrant of Fig. 4.49a. This response envelope was also transposed to the
Figure 4.48: STMs for interior joint Units 1 and 2.
(Note: Numbers refer to nonlinear event history shown in Figs. 4.49a and 4.50a; Bold elements assumed undeformable)
Figure 4.49: STM analyses for the Interior joint Unit 1.
negative displacement quadrant for comparison. Analytical results demonstrated slight under-estimation of the unit initial elastic stiffness, as well as the unit yield strength, possibly due to the distorted internal force distribution arising from the application of column axial load as a result of asymmetrical layout of the monotonic STMs. However, the response envelope has successfully predicted concrete crushing in the joint panel, resulting in strength reduction commencing at a column lateral displacement of approximately $\pm 75$ mm. This prediction was consistent with experimental observations which indicated that the lateral strength of the test specimen started to deteriorate due to concrete crushing in the joint panel at a lateral displacement of $\pm 90$ mm, corresponding to the displacement ductility level, $\mu = 4$.

The captured reinforcement yielding sequence, illustrated in Fig. 4.48a, suggested yielding occurred only in the beam flexural reinforcement, implying the formation of plastic hinges at both sides of the joint panel. The column flexural reinforcement was predicted to remain elastic, identical to that suggested by the experimental data.

The hysteretic structural response derived from the cyclic STM is illustrated in Fig. 4.49b. The cyclic model had a better prediction of the initial elastic stiffness than that obtained from the monotonic model. This was possibly due to the symmetrical model layout allowing better prediction of the internal force distribution under the initially applied column axial load. In general, the critical aspects of structural response including initial elastic stiffness, unit yield strength and joint strut crushing were satisfactorily captured by the cyclic STM. Again, the stepwise unloading and reloading curves were due to closing and opening of the concrete struts representing cracked concrete.

### 4.8.1.3 Analytical Results for Unit 2

The force-displacement response envelope for test unit 2, generated using the monotonic STM, is illustrated in Fig. 4.50a. Analyses again slightly under-estimated the initial elastic stiffness and unit yield strength, as expected when using the same STM as for unit 1. Experimental observation suggested that bond failure in the top beam flexural reinforcement embedded in the joint panel occurred at the first load cycle of displacement ductility $\mu = 6$, resulting in an acute pinching behaviour exhibited in the experimentally recorded hysteretic loop. Also at this displacement ductility level, concrete damage in the joint panel was observed. As the bond-slip behaviour for Grade 500 reinforcement was not previously established through experimentation, it was not incorporated in this STM, and hence not captured in the analysis. Nevertheless, the joint strut crushing that was experimentally observed in the test unit was predicted by the model.
Figure 4.50: STM analyses for the Interior joint Unit 2.
The reinforcement yielding sequence captured by the unit 2 STM was identical to that obtained for unit 1, see Fig. 4.48a, and also comparable to experimental observations. The analysis suggested that plastic hinges developed in the beams on both sides of the joint panel, and that the column remained elastic.

The hysteretic behaviour of test unit 2 computed using the cyclic STM is shown in Fig. 4.51b. Key structural response including initial elastic stiffness, unit yield strength, joint strut crushing and unloading and reloading stiffness were all satisfactorily predicted.

### 4.8.2 Interior joint system, Units 3 and 4

Interior joint unit 3 and unit 4 were designed for a joint shear stress of $0.19f'_c$ and an axial column compression of $0.1A_g f'_c$. The reinforcement details employed in both test units are depicted in Fig. 4.51. Both test units shared an identical column section but had different beam section reinforcement details. Flexural reinforcement provided in the column was 8-HD12 ($d_b = 12$ mm) + 4-HD16 ($d_b = 16$ mm). Both test units had the same reinforcement quantity provided in the beams but the top:bottom flexural reinforcement quantity ratio were 1:1 in unit 3 and 2:1 in unit 4, see Fig. 4.51.

#### 4.8.2.1 Unit 3 STMs

The monotonic STM formulated for the interior joint system unit 3 is shown in Fig. 4.52a. Again, only the push (left to right) model was examined here, as the pull model was assumed to be a mirror image of the push direction model. The area of the transverse rebar tie in the joint panel reinforcement was $a = 855 \text{ mm}^2$. The width of the concrete struts identified in Fig. 4.52a was assessed as: $b = 45$ mm; $c = 245$ mm and $d = 45$ mm. The cyclic STM was also formulated for unit 3 to replicate the cyclic force path developed inside a structure, see Fig. 4.52b. Again, rigid model elements were employed in both monotonic and cyclic STMs to represent the strong steel cast extension connected to the column base.

#### 4.8.2.2 Unit 4 STMs

The monotonic and cyclic STMs constructed for the interior joint system unit 4 are shown in Figs. 4.53a and 4.53b, respectively. These two models were essentially identical to those formulated for unit 3. The area of the rebar tie representing the transverse joint reinforcement was: $a = 1050 \text{ mm}^2$, and the width of the concrete struts identified in Fig. 4.53a was assessed as: $b = 45$ mm; $c = 245$ mm and $d = 45$ mm.
Figure 4.51: Reinforcement details of interior-joint unit 3 and unit 4. [Lin(2002)]
Figure 4.52: STMs for interior joint Unit 3.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.54a;
Bold elements assumed undeformable)
Figure 4.53: STMs for interior joint Unit 4.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.55a);
Bold elements assumed undeformable)
4.8.2.3 Analytical Results for Unit 3 and Unit 4

The analytical force-displacement response envelopes derived for unit 3 and unit 4 using monotonic STMs are shown in Figs. 4.54a and 4.55a respectively. As expected, the analytical response envelopes were very similar for the two units due to similar reinforcement details, see Fig. 4.51. Experimental data indicated that plastic hinges developed in the beam members adjacent to both sides of the joint panel. In addition, reinforcement-concrete bond failure was observed in the embedded top flexural beam reinforcement inside the joint when at a column lateral displacement of ±125 mm, corresponding to a displacement ductility \( \mu = 6 \). This resulted in a reduction of unit strength to more than 20% of the peak value, with significant pinching exhibited in the experimentally recorded hysteresis loops. Both STMs for unit 3 and unit 4 suggested the development of plastic hinges next to the beam-joint interfaces, see analytical events 1-2 & 9-10 in Fig. 4.52a for unit 3, and analytical events 1-4 in Fig. 4.53a for unit 4. As bond failure was not suggested by STMs because this feature was not included in the model formulation procedure, the associated unit strength degradation was not captured in the analysis. Also for the same reason, STMs for both test units had inaccurately predicted the formation of column plastic hinges, see events 3-8 in Fig. 4.52a for unit 3, and events 5-8 in Fig. 4.53a for unit 4.

The hysteretic response generated using cyclic STMs for both test units 3 and 4 is illustrated in Fig. 4.54b and 4.55b. In general, both models captured the critical aspects of measured structural response including the initial elastic stiffness and unit yield strength. However, as bond failure was not predicted, the associated pinching demonstrated in the experimentally recorded data, and the associated strength degradation at displacement ductility level \( \mu = 6 \) were not replicated by the analytical hysteresis loops.

4.8.3 Analysis Remarks

In general, the STM analytical results obtained for the interior joint system demonstrated a satisfactory comparison with the experimental data. In particular, concrete crushing in the joint panel was correctly estimated. Again, the inability of STM to predict reinforcement-concrete bond failure led to an inaccurate prediction of structural response at high displacement ductility levels.
Figure 4.54: STM analyses for the Interior joint Unit 3.

Nonlinear Strut-and-Tie Model Analytical Examples: Building Frame Interior Joint Systems
Figure 4.55: STM analyses for the Interior joint Unit 4.
4.9 Building Frame Exterior Joint Systems

Two full-scale building frame exterior joint systems that were tested by Chen (2000) were analysed using STMs and are considered in this section. Both test specimens were designed conforming to the New Zealand Concrete Design Standard [NZS 3101 (1995)], featuring the provision of effective rebar anchorage through bending of the beam and column flexural reinforcement around the back of the joint panel. Notably, the exterior joint units were tested at 90° orientation with the column lying horizontally. All STMs considered in this section were established based on the discrete force path expected to develop inside the exterior joint test units at the ultimate limit state that was proposed by Chan (2000).

4.9.1 Exterior Joint System, Unit-A

The overall dimensions and the key reinforcement details of the exterior joint system unit-A are illustrated in Fig. 4.56. The flexural reinforcement provided in the beam and column were 5-HD28 ($d_b = 28$ mm) and 6-HD28 + 2-HD20 ($d_b = 20$ mm) respectively. Six sets of R10 ($d_b = 10$ mm) reinforcement ties were used as the joint transverse reinforcement.

4.9.1.1 Unit-A STM

The monotonic STMs replicating the stress-path that developed inside unit-A when subjected to joint-opening (+ve displacement) and joint-closing (-ve displacement) moments are illustrated in Figs. 4.57a and 4.57b respectively. Notably, the diagonal concrete struts and the transverse rebar ties provided in the columns of both STMs were redundant because of zero shear force being supported by the columns. However, these elements were provided to ensure analytical stability. Again, $f_d = 0.51 f'_c$ was adopted as the effective strength of joint struts in both models. The area of model elements used in both models was identical, with the area of rebar ties representing joint transverse reinforcement being: $a = 630 \text{ mm}^2$, and the area of rebar ties representing the column flexural reinforcement embedded in the joint panel being: $b = 1850 \text{ mm}^2$ and $c = 950 \text{ mm}^2$. The width of the concrete struts identified in both STMs, shown in Fig. 4.57, was assessed as: $d = 125$ mm; $e = 125$ mm; $f = 125$ mm and $g = 50$ mm.

The cyclic STMs replicating the stress-path that developed inside the exterior joint system unit-A when subjected to cyclic actions is illustrated in Fig. 4.58. Again, the diagonal concrete struts and transverse rebar ties employed in the column model were only employed to provide analytical stability.
a, Unit-A

Figure 4.56: Key reinforcement details of the exterior joint systems. [Chen (2000)]
Figure 4.56: Continued.
**Figure 4.57:** Monotonic STM's for the building frame exterior joint system unit-A.

(Note: Numbers refer to nonlinear event history shown in Fig. 4.59a)

Nonlinear Strut-and-Tie Model Analytical Examples: Building Frame Exterior Joint Systems
4.9.1.2 Analytical Results for Unit-A

The force-displacement response envelopes generated using the monotonic STMs are depicted in Fig. 4.59a, accompanied by the experimental data. Experimental observations indicated the formation of a plastic hinge in the beam next to the beam-joint interface. Also, concrete crushing was observed in the joint panel, resulting in unit strength degradation and anchor failure of beam flexural reinforcement inside the joint panel when the beam-tip force-displacement reached +180 mm in the joint-opening direction and -240 mm in the joint-closing direction, corresponding to a displacement ductility $\mu = 6.0$. Both force-displacement response envelopes that were generated using STMs captured the initial elastic stiffness sufficiently. Analysis results of the joint-opening model suggested that the unit developed a ductile failure mode, while the response envelope derived from the joint-closing model predicted a brittle failure due to joint strut crushing.

The reinforcement yielding sequence captured by both monotonic models is depicted on the respective response envelope in Fig. 4.59a, using solid circles. For the joint-opening model,
Figure 4.59: STM analyses for the exterior joint system unit-A.

Nonlinear Strut-and-Tie Model Analytical Examples: Building Frame Exterior Joint Systems
yielding was predicted to occur in the beam and column flexural reinforcement, see events 1 & 3-5 in Fig. 4.57a, and in the joint transverse reinforcement, see event 2 in Fig. 4.57a. For the joint-closing model, yielding was predicted to occur only in the joint transverse reinforcement, see event 1 in Fig. 4.57b, and subsequently the joint struts started crushing.

The hysteretic response of the exterior joint system unit-A computed using cyclic STM is illustrated in Fig. 4.59b, accompanied by the experimental results. Critical aspects of measured structural response, including elastic stiffness and unit yield strength, were sufficiently predicted. However, unit strength degradation occurred at a beam-tip displacement ductility $\mu = 6.0$ due to joint damage, and the associated anchorage failure of beam flexural reinforcement was not predicted by the cyclic STM. This was due to deficiencies of the material model employed for concrete struts, as previously discussed in Section 3.7.

4.9.2 Exterior Joint System unit-B

The reinforcement details provided in the exterior joint system unit-B are illustrated in Fig. 4.56b. Reinforcement details employed in unit-B were essentially identical to those employed in unit-A, except for the additional diagonal rebars that were provided at the re-entrant corner of the joint panel in unit-B. The longitudinal reinforcement provided in the beam and the column was 5-HD28 ($d_b = 28$ mm) and 6-HD28 + 6-HD12 ($d_b = 12$ mm), respectively.

4.9.2.1 Unit-B STM

The monotonic STMs replicating the stress-path expected to be developed inside the exterior joint system unit-B for both joint-opening (+ve displacement) and joint-closing (-ve displacement) moments are depicted in Fig. 4.60a and 4.60b respectively. Both monotonic models resembled those employed for unit-A because of the similar reinforcement details. The diagonal rebars located at the re-entrant corner of the joint panel were not included in the STM because experimental evidence suggested that they were inactive in transferring shear across the joint panel [Chen (2000)]. As no shear forces were supported by the column, diagonal concrete struts and transverse rebar ties in the column model were provided to ensure analytical stability only. An effective strength of $f_d = 0.51f'_c$ was adopted for joint struts in the unit-B STM. The area of rebar ties that were identified in both monotonic models, shown in Figs. 4.60a and 4.60b was: $a = 950$ mm$^2$; $b = 1850$ mm$^2$ and $c = 680$ mm$^2$. The width of the concrete struts identified in the same figure was evaluated as: $d = 100$ mm; $e = 175$ mm; $f = 100$ mm and $g = 50$ mm.
Figure 4.60: Monotonic STMs for the building frame exterior joint system unit-B.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.62a)

Nonlinear Strut-and-Tie Model Analytical Examples: Building Frame Exterior Joint Systems
The cyclic STM formulated for unit-B is illustrated in Fig. 4.61. Again, concrete struts and transverse rebar ties were provided only to ensure analytical stability.

4.9.2.2 Analytical Results for unit-B

The analytical force-displacement response envelopes generated using both monotonic STMs are illustrated in Fig. 4.62a, accompanied by experimental data. Experimental results suggested that unit strength degradation started from the second load cycle at +115 mm in the joint-opening direction, and -180 mm in the joint-closing direction, corresponding to displacement ductility $\mu = 4.0$, as a result of significant joint damage and reinforcement anchorage failure being developed in the beam flexural reinforcement that was embedded inside the joint panel. Notably, the substantial joint damage was primarily due to the actual concrete strength at testing being lower than the design value, leading to the maximum joint shear stress being 16% higher than that permitted by NZS 3101 (1995).
Figure 4.62: STM analyses for the exterior joint system unit-B.

Nonlinear Strut-and-Tie Model Analytical Examples: Building Frame Exterior Joint Systems
Both analytical force-displacement envelopes satisfactorily predicted the elastic stiffness of the test unit. Joint strut crushing was captured by the analysis, but at an earlier stage than that observed experimentally. The residual unit strength at high displacement ductility levels was also sufficiently predicted for both joint loading directions. No reinforcement yielding was suggested by the joint-opening model however, yielding of the transverse rebar was predicted by the joint-closing model, see event 1 in Fig. 4.60b, which was comparable to experimental observations.

The hysteretic response of unit-B was computed using the cyclic STM shown in Fig. 4.62b. Analysis has sufficiently predicted the elastic stiffness and unit yield strength of the test unit. However, the reinforcement-concrete bond failure was not predicted, and the associated pinching exhibited in the experimentally recorded hysteretic loop was not accurately captured by the analysis.

4.9.3 Analysis Remarks

According to the STM analysis performed for the exterior joint systems described in this section, it was discovered that the formulated STMs, based on the proposed modelling procedure, resulted in certain limitations in the computation accuracy. Analytical inaccuracy frequently occurs when conducting STM analysis for structural systems that develop brittle failure, resulting in drastic loss of strength and energy-absorbing capability. It is expected that the utilisation of more sophisticated material stress-strain models for concrete and rebar would improve the precision of cyclic STM analysis.

4.10 Reinforced Concrete Building Frame

A one-third scale 3-bay, 2-storey building frame tested by Wuu (1996) was analysed using STM. The overall dimensions and the key reinforcement details of the building frame system are depicted in Fig. 4.63. Seismic actions were simulated using actuators positioned horizontally at the top of each column. The ratio of horizontal column forces applied to interior versus exterior columns was 2:1. No external axial load was applied to the columns of the test unit. This building frame test unit was designed conforming to NZS:3101 (1995) for ductile structural behaviour. Joint damages due to extensive yield penetration of reinforcement were prevented by fillet welding R6 ($d_b = 6$ mm) rebars to the portion of the beam flexural reinforcement embedded within the interior-joint and exterior-joint panel regions. A similar arrangement was used for the column reinforcement that was embedded in the foundation beam. Four R6 U-shaped bars were each fillet welded to the two outermost
Figure 4.63: Key reinforcement details of the building frame system.
column flexural rebars that were embedded in the foundation beam. Furthermore, effective anchorage of the flexural reinforcement was provided in the exterior-joints by extending the rebars to the beam stubs with a 90° hook. The flexural reinforcement provided in the beam section was 10-D10 \((d_b = 10 \text{ mm})\), and in the exterior and interior column sections was 12-D10 and 12-D12 \((d_b = 12 \text{ mm})\) respectively. Notably, the foundation beams were heavily reinforced to avoid any structural damage during testing.

STM analytical results of the building frame were previously reported by To et al. (2002a), but the newer model presented in this section was refined using the procedure established in Chapter 3.

### 4.10.1 STM of the building frame

The monotonic STM of the reinforced concrete building frame system, when subjected to a push direction (left to right) action, is illustrated in Fig. 4.64. Only the push direction STM was examined as the pull direction STM was expected to be the mirror image of the push direction STM. Model layouts in the interior-joint and the exterior-joint panel regions were adapted from those established for the building joint test units considered in Sections 4.7 and 4.8. Due to the foundation beams being heavily reinforced, and no deformation or damage due to strain penetration being experimentally observed, they were not considered in the analysis. Furthermore, \( f_d = 0.51 f'_c \) was adopted as the effective strength of joint struts. The area of rebar ties in the monotonic STM shown in Fig. 4.65 to represent the transverse reinforcement provided in the exterior- and interior-joints was: \( a = 340 \text{ mm}^2 \) and \( b = 565 \text{ mm}^2 \), respectively. The width of joint struts identified in the same figure was measured as: \( c = 22.5 \text{ mm} \), \( d = 135 \text{ mm} \) and \( e = 22.5 \text{ mm} \).

The cyclic STM for the building frame system is illustrated in Fig. 4.65. Again, the foundation beams were not considered in this model as they had no structural significance. The cyclic STMs for the interior-joint and exterior-joint panels were formulated to resemble those established for the interior and exterior joint systems considered in Sections 4.7 and 4.8, respectively.

### 4.10.2 Analytical Results for the Building frame system

The analytical force-displacement response envelope generated using the monotonic STM is shown in the positive displacement quadrant of Fig. 4.66a. This envelope was also transposed to the negative displacement quadrant for comparison with experimental data. The analytical response envelopes satisfactorily captured the initial elastic stiffness and the lateral yield strength of the structural system for both load directions. This was expected, as
Figure 4.64: Monotonic STM for the building frame system.
(Note: Numbers refer to nonlinear event history shown in Fig. 4.66a)
Figure 4.65: Cyclic STM for the building frame system.
Figure 4.66: STM analysis results for the building frame system.

Nonlinear Strut-and-Tie Model Analytical Examples: Reinforced Concrete Building Frame
the building frame system had exhibited superior ductile response when tested. The captured reinforcement yielding sequence illustrated in Fig. 4.64 suggested that plastic hinges had developed, primarily at the column bases and in the beam sections next to the joint panel regions. This beam sway failure model was consistent with experimental observations. STM analysis also suggested no serious joint strut crushing.

Hysteretic response of the building frame system generated using the cyclic STM is illustrated in Fig. 4.66b. Again, the cyclic STM predicted the initial elastic stiffness, the unit yield strength and the unloading and reloading stiffness with sufficient accuracy. However, the model was not capable of capturing unit strength degradation due to concrete damage in the plastic hinge zones when the building frame system was under cyclic action at the same displacement ductility level. This deficiency was attributed to the limitation of the material model currently employed for concrete struts.

4.10.3 Beam Elongation

Elongation exhibited in the flexural members when subjected to cyclic loading was partly due to a tension lag effect that was associated with diagonal concrete cracks, and also due to concrete particles being trapped inside the cracks developing wedging-type action. These phenomena are particularly noticeable in beam members where there is no applied axial force to impede length growth.

The beam elongation of the building frame system captured by the STM is illustrated in Fig. 4.67, accompanied by experimental data. This figure indicates that the calculated beam elongation for both level 1 and level 2 at the post-yielding state was approximately two-thirds of the experimentally measured values. This discrepancy is believed to be due to the STM being only capable of calculating beam elongation attributed to the tension lag effect, but not to the wedging-type action established by the loose concrete particles being trapped inside the cracks. Also for the same reason, the analysis indicated that no uni-directional plastic hinges were developed in the external columns at level 1, as the beams did not elongate sufficiently.

4.10.4 Analysis Remarks

The STM analysis performed for the building frame system described in this section have, in general, predicted the recorded structural behaviour satisfactorily. This was primarily because the test unit had a dominant ductile response, and brittle failure as a result of concrete damage in the joint panels or reinforcement anchor failure was not observed.
Figure 4.67: Beam elongation of the building frame system.
CHAPTER 5 - DYNAMIC ANALYSES USING STRUT-AND-TIE MODELS

5.1 Introduction

As an extension of the nonlinear analytical examples for various structural types described in Chapter 4, the dynamic response of three prototype structures, namely a bridge portal frame system, a multicolumn bridge bent and a building frame, are demonstrated in this chapter using cyclic STMs. In addition, the dynamic response evaluated from the STM analyses are compared with those obtained from planar frame model analyses.

5.2 Earthquake Records

Dynamic analyses were performed using four earthquake records. These were scaled to force all STMs of the prototype structures, except for the as-built bridge portal frame, to develop nonlinear response with displacement ductility reaching $\mu = 6$. As the as-built bridge portal frame system suffered lap-splice failure between the beam and column flexural reinforcement located at the back of the joint panel, the structural yield displacement could not be identified due to structural brittle failure. Consequently, the unscaled earthquake records were employed when conducting dynamic analysis for the as-built bridge portal frame system.

The measured ground acceleration of the four earthquake records are shown in Fig. 5.1. Relevant details of the four earthquakes and the corresponding scale factors adopted for the dynamic analyses are listed in Table 5.1. Only one component of the ground acceleration was
Figure 5.1: Earthquake records chosen for the time-history analyses of STMs.
Table 5.1 Selected Earthquake records and the scale factors for the time-history analyses of STMs.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Kobe - JR-Takator EW</th>
<th>El-Centro - Imperial Valley EW</th>
<th>Northridge - Santa Monica City Hall EW</th>
<th>Loma Prieta - Oakland outer harbour wharf EW</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA (g)</td>
<td>0.68</td>
<td>0.35</td>
<td>0.88</td>
<td>0.28</td>
</tr>
<tr>
<td>Digitisation interval (sec)</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Duration (sec)</td>
<td>35.0</td>
<td>53.74</td>
<td>60.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Scaled for as-built bridge portal frame</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Scaled for redesigned bridge portal frame</td>
<td>2.7</td>
<td>12.5</td>
<td>6.5</td>
<td>13.5</td>
</tr>
<tr>
<td>Scaled for Multicolumn bridge bents</td>
<td>1.5</td>
<td>7.0</td>
<td>4.0</td>
<td>7.25</td>
</tr>
<tr>
<td>Scaled for Building frame system</td>
<td>2.5</td>
<td>9.0</td>
<td>5.5</td>
<td>8.5</td>
</tr>
</tbody>
</table>

applied in the horizontal direction. All earthquake records had a digitisation interval of 0.02 seconds, except for the Kobe earthquake record, for which the digitisation interval was equal to 0.01 seconds. No initial ground displacement and velocity was assumed when performing the dynamic analyses.

5.3 Calibration for viscous damping

Drain-2DX allows the viscous damping matrix, \( C \), to be calculated proportional to the nodal masses and the element stiffness in the form of \( C = a_o M + a_1 K \), where \( a_o \) and \( a_1 \) are the Rayleigh damping factors that can be evaluated using the solution of a pair of simultaneous equations, shown in Eq. 5.1, if the damping ratios \( \xi_a \) and \( \xi_b \) that are associated with the two specific natural frequencies of the structure \( \omega_a \) and \( \omega_b \) are known [Clough and Penzien (1993)].

\[
\begin{bmatrix}
a_o \\
a_1
\end{bmatrix} = 2 \cdot \frac{\omega_a \cdot \omega_b}{\omega_b^2 - \omega_a^2} \begin{bmatrix}
\omega_b - \omega_a \\
\omega_b \\
\omega_a
\end{bmatrix} \cdot \begin{bmatrix}
\xi_a \\
\xi_b
\end{bmatrix}
\]

Eq. 5.1

Dynamic Analyses using Strut-and-Tie Models - Calibration for viscous damping
For the dynamic analyses conducted in this study, 5% of the critical damping, i.e. \( \xi_a = \xi_b = 0.05 \), was specified for the first and the tenth modes when computing the suitable Rayleigh damping factors, \( a_0 \) and \( a_1 \), of the viscous damping matrix, \( C \).

### 5.4 Planar frame model

The planar frame models of the prototype structures were formulated along the member centrelines. A bilinear stress-strain curve, similar to that illustrated in Fig. 5.2, was adopted for the flexural elements that replicated the beam and column members. The effective flexural stiffness, \( E_c I_e \), of the flexural elements that are illustrated in Figs. 5.4d and 5.4e for the respective as-built and repaired portal frame systems, was evaluated using Eq. 5.2:

\[
E_c I_e = \frac{M_{1st}^y}{\phi_{1st}^y}
\]

Eq. 5.2

where \( M_{1st}^y \) is the moment measured at the first yield state; 
\( \phi_{1st}^y \) is the section curvature measured at the first yield state.

---

**Figure 5.2:** Bilinear force-displacement response of planar frame models.
The effective axial stiffness, $E_cA_e$, of the flexural elements as defined in Eq. 5.3, was reduced from the gross section stiffness, $E_cA_g$, in proportion to the effective flexural stiffness [Priestley et al. (1996)] to reflect the influence of axial load, flexural cracking and reinforcement ratios.

$$E_cA_e = E_cA_g \frac{E_c I_e}{E_c I_g}$$  \hspace{1cm} \text{Eq. 5.3}

where $E_c I_e$ is the gross flexural stiffness.

According to discussion previously presented in Section 3.5.2, the strain hardening ratio of the bilinear stress-strain curve was selected as 5% for flexural elements replicating beams, and 2.5% for flexural elements replicating columns and beams with significant side reinforcement. All flexural elements were connected by rigid-links that are located inside joint panels, and joint-links situated at the beam- and column-joint interfaces, see Figs. 5.4c and 5.4d. Nodes were typically located at the member-joint interfaces to predict the maximum flexural demand on members. Rigid-links were employed to provide geometric connectivity and compatibility between model elements at the end of joint-links, and joint-links were used to represent yield penetration into the joint panel region by the flexural reinforcement.

The joint-links were assigned the effective member properties calculated using Eqs. 5.2 and 5.3. Also, adequate flexural strength was assigned to these joint-links to avoid plastic hinges being formed inside the joint panels. The length of joint-links, $\ell_{pj}$, was calculated using Eq. 5.4 [Priestley et al. (1996)] as:

$$\ell_{pj} = 0.022 f_y d_b$$ \hspace{1cm} \text{Eq. 5.4}

where $f_y$ is the flexural strength of reinforcing steel bars;

$d_b$ is the diameter of flexural reinforcing steel bars.

All prototype structures were expected to have sufficient shear strength to prevent shear failure. Hence, the shear component was neglected when formulating the planar frame models, as shear deformation contributes only a very small magnitude of elastic deformation.
5.5 Bridge portal frame

Two full-scale prototype bridge portal frame systems, namely an as-built unit and a redesigned unit, are considered in this section. The physical dimensions of the prototype structures are depicted in Fig. 5.3. The joint and the member reinforcement details of these two bridge frame systems were identical to the corresponding knee-joint test units examined in Sections 4.4.1 and 4.4.4, except that the reinforcement area was increased by nine times to convert the details of the one-third scaled model to the prototype scale. Reinforcement details employed for the prototype units were identical in the columns, but were different in the joint panels and cap-beams. Sufficient joint reinforcement was provided to the redesigned unit to avoid brittle joint failure. This is in contrast to the effectively unreinforced joint panels of the as-built unit. Additional flexural reinforcement was employed in the cap-beam of the redesigned unit rather than in the cap-beam of the as-built unit to ensure elastic beam behaviour.

The prototype bridge portal frames were connected to the foundation using a pin-type connection, with the section depth of column bases being reduced by one-half to 1067 mm, see the inset of Fig. 5.3. The detail of the pin-connection was achieved by connecting the column centre to the foundation, with the area of longitudinal rebar being 25% of that provided to the column flexural reinforcement. As the pin connections between the column bases and the foundation were not entirely moment resistance-free, they were modelled to have flexural strength comparable to the actual situation. The integrated cap-beams were not expected to be deformable, therefore they were modelled using rigid members, as depicted by the bold lines in Fig. 5.4. The STMs and the planar frame models of the bridge frame systems formulated according to the different reinforcement details are illustrated in Fig. 5.4. A summary of the model properties is presented in Table 5.2.

Table 5.2 Model properties of the bridge portal frame system

<table>
<thead>
<tr>
<th>Model properties</th>
<th>As-built unit - STM</th>
<th>As-built unit - Planar frame model</th>
<th>Redesigned unit - STM</th>
<th>Redesigned unit - Planar frame model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode period</td>
<td>2.0</td>
<td>1.7</td>
<td>0.75</td>
<td>0.8</td>
</tr>
<tr>
<td>10th mode period</td>
<td>$7.3 \times 10^{-5}$</td>
<td>$1.9 \times 10^{-5}$</td>
<td>$2.7 \times 10^{-5}$ s</td>
<td>$1.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>Dead load</td>
<td>560 kN/m</td>
<td>560 kN/m</td>
<td>560 kN/m</td>
<td>560 kN/m</td>
</tr>
<tr>
<td>$a_o$</td>
<td>0.44</td>
<td>0.36</td>
<td>0.84</td>
<td>0.79</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$1.16 \times 10^{-6}$</td>
<td>$2.94 \times 10^{-7}$</td>
<td>$4.34 \times 10^{-7}$</td>
<td>$2.94 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Dynamic Analyses using Strut-and-Tie Models - Bridge portal frame
The monotonic force-displacement response envelopes generated using the two modelling techniques for both portal frame systems are shown in Fig. 5.5. Notably, the elastic stiffness obtained from the planar frame model was derived based on the effective member properties that assumed a cracked member section. This is in contrast to the elastic stiffness that was evaluated using STM, which used gross member properties until the flexural demand exceeded the crack bending moment. Accordingly, the 1\textsuperscript{st} mode period that was obtained from the two modelling techniques was expected to be different, and it was considered that in general the evaluation made using STM and the planar frame model were the respective lower limit and upper limit of the actual 1\textsuperscript{st} mode period of the structural system. Also for the same reason, the monotonic response envelopes generated using STMs could capture the
progressive stiffness softening at the elastic regime and were, in general, expected to match the actual structural behaviour better than the planar frame models.

For the as-built portal frame system, the STM suggested that lap-splice failure between the longitudinal rebars of the cap-beam and column, that are located at the back of the joint panel, would develop when the applied lateral load reached approximately 2150 kN, see Fig. 5.5a. This resulted in a drastic loss of lateral strength and brittle structural failure. As the lap-splice failure was not captured by the planar frame model, it was incorrectly predicted that the structure would develop a plastic failure mechanism.

![Diagram of analytical models of the bridge portal frame.](image)

**Figure 5.4: analytical models of the bridge portal frame.**

(Note: Bold members identify the undeformable rigid elements.)

Dynamic Analyses using Strut-and-Tie Models - Bridge portal frame
Figure 5.5: Force-displacement response envelopes for the bridge portal frame system.

Dynamic Analyses using Strut-and-Tie Models - Bridge portal frame
According to the analytical results obtained for the redesigned knee-joint test unit considered in 4.4.4, the redesigned portal frame system is expected to develop a plastic failure mechanism due to the adequate provision of joint reinforcement, as confirmed by the monotonic force-displacement response envelopes generated using both STM and the planar frame model. Both sets of response envelopes are considered to satisfactorily match actual structural behaviour. However, the response envelopes obtained from STM are considered superior as they estimated progressive stiffness softening due to concrete cracking in beams and columns in the elastic regime. The yield displacement and yield strength of the redesigned portal frame were identified as approximately 0.175 m and 5000 kN, respectively.

Notably, the elastic stiffness of the redesigned unit is higher than the as-built unit due to more flexural reinforcement being provided in the cap beam. Therefore, its 1st mode period is comparatively shorter, see Table 5.2.

5.5.1 Dynamic Analysis Results for the As-built Bridge Portal Frame

The dynamic performance predicted for the as-built portal frame when subjected to the Kobe earthquake is shown in Fig. 5.6a. Analysis results obtained from both modelling techniques suggested excessive residual displacement at the end of earthquake. Displacement demand on the as-built portal frame was predicted as 550 mm from the STM, and 650 mm from the planar frame model. At these displacement levels, lap-splice failure would have been developed, causing significant structural damage. Accordingly, it is expected that the bridge would be inoperable after the earthquake.

It is surprising that both modelling techniques did not suggest significant differences in the general nature of the dynamic analysis results. Both estimated reasonably similar maximum lateral drift and large residual lateral displacement. This is probably due to the fact that both analytical models predicted structural failure for the applied level of seismic shaking intensity.

According to the cyclic STM analysis results of the as-built knee-joint test unit previously discussed in Section 4.4, it is concluded that the dynamic response estimated by the STM is assessed as being closer to the actual structural behaviour than that suggested by the planar frame model.

For the Northridge earthquake analysis results, shown in Fig. 5.6c, both modelling techniques predicted similar displacement demand and little residual displacement at the end of the earthquake, suggesting minimal structural damage and predicting that the bridge
Figure 5.6: Time-history displacement response of the as-built prototype portal frame system.
should be operable after the earthquake. Notably, the STM exhibited a greater oscillating displacement amplitude at the end of the earthquake, which is presumably due to the elastic stiffness softening as a result of extensive concrete cracking in the cap-beam and columns, see Fig. 5.4a.

Dynamic analysis results for the El-Centro and Loma Prieta earthquakes are shown in Figs. 5.6b and 5.6d, respectively. Results obtained from both modelling techniques suggested that low displacement and no residual lateral displacement occurred in the as-built bridge frame. This implied that the structural system would remain elastic with minimal structural damage when subjected to this intensity of seismic excitation, similar to that of the El-Centro and Loma Prieta earthquakes.

5.5.2 Dynamic Analysis Results for the Redesigned Bridge Portal Frame

Since the redesigned bridge portal frame was expected to develop plastic failure mechanism, both modelling techniques suggested similar structural dynamic performance for each earthquake record, see Fig. 5.7. Nevertheless, discrepancy between the two sets of analytical results started to occur after the model reached its maximum displacement ductility, $\mu = 6.0$. This analytical discrepancy was due to different stress-strain material models being employed by the two modelling techniques. According to the cyclic STM analysis results generated for the redesigned knee-joint test unit discussed in Section 4.4.4, the dynamic response of the redesigned bridge portal frame predicted using STMs is expected to match the actual behaviour more closely than that estimated using the planar frame models, as the planar frame model exhibits pure bilinear cyclic response.

5.6 Multicolumn bridge bent

An imaginary multicolumn bridge bent system that supports a four-lane traffic superstructure is considered in this section. The physical configuration and reinforcement details of this prototype bridge bent are illustrated in Fig. 5.8. This bridge bent system has a rectangular cap-beam and circular columns. The section dimensions and reinforcement details of the cap-beam, columns, knee- and tee-joints and column base pin connections were identical to those employed in the test unit MCB2, described in Section 4.7.2., except that the section and reinforcement areas were increased by four times to convert the details of the half-scaled model to the prototype scale. The superstructure dead loads were lumped at the corresponding cap-beam nodes in the STM and were uniformly distributed along the cap-beam in the planar frame model. A summary of model properties is presented in Table 5.3.

Dynamic Analyses using Strut-and-Tie Models - Multicolumn bridge bent
Figure 5.7: Time-history displacement responses of the redesigned prototype portal frame system.
The STM and the planar frame model formulated for the bridge bent system are illustrated in Fig. 5.9a and 5.9b, respectively.

The monotonic force-displacement response envelopes of the prototype multicolumn bridge bent are depicted in Fig. 5.10. Both modelling techniques predicted the bridge bent system
Figure 5.9: Analytical models of the multicolumn bridge bent.
to exhibit plastic failure mechanism with minimal joint damages. Based on the STM analysis results generated for the test unit MCB2 described in Section 4.7.2.2, the STM response envelope is considered to match closely with the actual structural behaviour, in particular to the progressive stiffness softening demonstrated in the elastic regime, due to concrete cracking in beam and column members. It is expected that the response envelope generated
from the planar frame model predicted only the effective elastic stiffness of the bridge bent system, and thus was less accurate than that evaluated using STM. The yield displacement and yield strength of the multicolour bridge bent were identified as 0.1 m and 2100 kN, respectively.

The dynamic response of the multicolour bridge bent derived using both modelling techniques are illustrated in Fig. 5.11. As both modelling types suggested that the multicolour bridge bent would develop a plastic failure mechanism with minimal joint damage, both sets of dynamic analysis results were similar. Notably, the planar frame model, in general, predicted a slightly larger lateral drift, possibly due to its comparative lower initial elastic stiffness, as illustrated by the corresponding monotonic response envelope in Fig. 5.10.

5.7 Building frame system

The key reinforcement details and physical dimensions of the prototype building frame system are illustrated in Figs. 5.12 and 5.13, respectively. This building frame system has no corner columns and was adopted from the Red Book [C & CA (1998)], representing a typical perimeter frame widely employed in recent years for resisting seismic force in buildings.

The cyclic STM and the planar frame model formulated for the building frame system are illustrated in Figs. 5.13a and 5.13b, respectively. The dead loads and live loads that were established according to NZS 4203 (1992) for earthquake loading criteria, were lumped to the corresponding beam nodes in the STM and were uniformly applied to the beam members in the planar frame model.

Notably, computation instability is likely to occur when processing dynamic analyses for STMs which have a large number of model elements, such as the example shown in Fig. 5.13a. This problem could be mitigated at the expense of computation time by choosing very small analytical steps, but would result in the strut-and-tie modelling technique being uneconomical and unfeasible to be adopted in routine design practice. In order to minimise the lengthy computation time required for large STMs, while maintaining the advantages of using this modelling technique in estimating the critical aspects of D-region structural performance, a hybrid modelling solution that is depicted in Fig. 5.14 is proposed. This model amalgamates strut-and-tie and planar frame modelling techniques for representing the D- and B-regions, respectively. Relevant modelling properties for all three analytical models are provided in Table 5.4.
Figure 5.11: Time-history displacement responses of the multicolumn bridge bent system.
a) Beam reinforcement layout (excluding joint reinforcement).

b) Joint reinforcement.

c) Column section.

Figure 5.12: Key reinforcement details of the building frame system [C & CA (1998)].
Figure 5.13: Physical dimensions and analytical models of the building frame system.
Table 5.4 Model properties of the building frame system

<table>
<thead>
<tr>
<th>Model properties</th>
<th>STM</th>
<th>Planar frame model</th>
<th>Hybrid model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; mode period</td>
<td>0.84 s</td>
<td>1.63 s</td>
<td>1.21 s</td>
</tr>
<tr>
<td>10&lt;sup&gt;th&lt;/sup&gt; mode period</td>
<td>0.04 s</td>
<td>0.07 s</td>
<td>0.06 s</td>
</tr>
<tr>
<td>Dead load at each floor</td>
<td>37 kN/m</td>
<td>37 kN/m</td>
<td>37 kN/m</td>
</tr>
<tr>
<td>$a_o$</td>
<td>0.71</td>
<td>0.37</td>
<td>0.53</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$6.8 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-3}$</td>
<td>$8.7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Monotonic force-displacement response envelopes of the building frame system generated using all analytical models are shown in Fig. 5.15. All models suggested that the building frame would develop a plastic failure mechanism. The yield displacement and yield strength were evaluated as 0.17 m and 1600 kN, respectively.
Again, the initial elastic stiffness suggested by the planar frame model was calculated basing on the effective member properties, contrary to that obtained from the STM, which was evaluated using gross member properties until the flexural demand exceeded the crack bending moment. Therefore, the 1st mode period obtained from the two modelling techniques is different. Evaluations using the STM and the planar frame model are considered to be the respective lower limits and upper limits of the actual 1st mode period of the building frame system. Also, the differences between the 1st mode period predicted by the STM and the planar frame model is believed to be amplified by the number of model elements involved in the modelling solution.

The predicted top-story displacement time-history response of the building frame system under the four scaled earthquake records is illustrated in Fig. 5.16. The planar frame model had, in general, predicted the maximum lateral drifts to be larger than the other two analytical models, due to the lower elastic stiffness as suggested by the monotonic response envelope in Fig. 5.15. Also, due to the different stress-strain material characteristics employed for the model elements in the STM and the planar frame model, the corresponding estimated dynamic responses started to vary once the lateral drift of the building frame system exceeded its yield displacement. Since the hybrid model is a combined analytical solution of
Figure 5.16: Time-history displacement response of building frame system.

Dynamic Analyses using Strut-and-Tie Models - Building frame system
the STM and the planar frame model, the calculated lateral drifts of the building frame are, in general, between the values suggested by the other two analytical models.

Presented in Table 5.5 is the comparison of the model complexity of all three analytical models, and the approximate computation time required for performing the dynamic analysis using a computer equipped with an Intel 3 (1.6 GHz) central processing unit. Notably, the computation time required by the hybrid model is only approximately one-third of that necessary for the STM.

<table>
<thead>
<tr>
<th>Model properties</th>
<th>STM</th>
<th>Planar frame model</th>
<th>Hybrid model</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of nodes</td>
<td>1016</td>
<td>376</td>
<td>732</td>
</tr>
<tr>
<td>number of elements</td>
<td>5942</td>
<td>402</td>
<td>3232</td>
</tr>
<tr>
<td>approximate computation time</td>
<td>3 hrs</td>
<td>10 mins</td>
<td>1 hr</td>
</tr>
</tbody>
</table>

5.8 Analysis Remarks

The dynamic response of various full-scale prototype structures was evaluated using both STMs and planar frame models. It was demonstrated that both modelling techniques predicted similar dynamic response for structural systems that exhibit predominant ductile behaviour, and for those that suffer brittle failure.

Dynamic analysis results obtained from STMs were considered to have matched the actual structural behaviour more closely than the results evaluated from the planar frame models. However, not all analytical results demonstrated in this chapter suggested sufficient evidence to conclude that STMs are superior to planar frame models when used for performing earthquake time-history dynamic analyses. Additionally, the lengthy computation time required for large STMs would make the strut-and-tie modelling technique unattractive and uneconomical when adopted for dynamic analyses in routine design practice.

The computation time required by the hybrid modelling solution remains very time-consuming in comparison with that necessary for the planar frame model, when used to conduct dynamic analysis for large scale structural systems.
CHAPTER 6 - CONCLUSION AND RECOMMENDATIONS

This chapter summarises the findings of the current research. Conclusions regarding strut-and-tie modelling capability and deficiencies are discussed and future improvements to this modelling technique are proposed. Also, the next generation of design procedures using STMs is outlined.

6.1 STM Formulation Procedures

The critical aspects of monotonic and cyclic STM formulation procedures are briefly addressed in the following:

- Monotonic and cyclic STM formulation procedures were proposed in Chapter 3. They were respectively constructed with six and five different types of model elements to replicate the behaviour of various structural components;
- A quadilinear curve with a degrading branch was used to represent the material response of concrete struts in both monotonic and cyclic STMs;
- A trilinear curve with a degrading branch was adopted for the concrete ties to represent the concrete tension-carrying capacity in the monotonic STMs;
- A uniaxial fibre model was developed for the cyclic STMs to replicate the cyclic material response of the flexural zone in beam and column members;
- For monotonic STMs representing B-region structural members, the model elements were situated at the respective force centroids measured at the first yield state;
- For monotonic STMs representing D-region structural members, the model elements were positioned according to the force path expected to develop at the ultimate limit state, as previously identified in the literature;
- For cyclic STMs representing B-region structural members, model elements were located at the flexural tension centroids measured at the first yield state for the respective flexural action direction;
- For cyclic STMs representing the D-region of structural systems, model elements were positioned to approximately replicate the cyclic force path expected to develop at the ultimate limit state, as previously suggested in the literature. Some adjustments were
made to simplify the models when necessary, to allow a symmetrical layout for the cyclic and dynamic analyses;

- The effective strength and effective area of model elements located in B-regions were calculated according to section force analysis results measured at the first yield state;
- For the cyclic loading case, a factor of 0.85 was adopted when evaluating the effective strength of concrete struts that were located in structural B-regions, to simulate the reduction of maximum compression capacity in concrete if first subjected to a large tensile strain;
- The effective strength of concrete struts in D-regions was $f_d = 0.51 f'_c$, and the effective area was calculated by multiplying the average distance between the neighbouring struts and the effective width, which was taken as the concrete core width approximating to 80% of the total width of structural components;
- The measured yield strength of reinforcement was adopted as the effective strength of rebar ties located in D-regions, with the effective area being identical to the total rebar area of the reinforcement group that the rebar ties represented;
- The lap-splicing response curve employed in this study assumed no slipping before the tension carried by the lap-spliced rebars reached the lap-splice capacity. Thereafter, the lap-splice strength decreased to 40% of its maximum value; and
- A hybrid modelling solution that integrates STM and planar frame models to respectively represent the D- and B-region structural members was proposed in Chapter 5 in order to provide a more time-effective modelling solution for dynamic analytical problems.

### 6.2 Analytical Results

The proposed monotonic and cyclic STM formulation procedures were extensively examined using test data for various types of structural systems. Obtained analysis results are summarised below:

- In general, analytical results that were generated using the STMs satisfactorily captured the elastic stiffness and yield strength of all structural units considered in Chapter 4. The maximum strength of the concrete struts that were located in the joint panel region was sufficiently predicted using the effective strength value of $0.51 f'_c$;
- The maximum tension transferring capacity between lap-spliced reinforcement was satisfactorily captured by both the monotonic and cyclic STMs;
- Monotonic STMs are very effective in predicting the monotonic force-displacement response envelopes of various structural systems and are very useful in identifying the failure sequence of different structural components;
- Hysteretic loops that were derived using cyclic STMs adequately predicted the cyclic response of structural systems that exhibit predominant ductile response and brittle failure due to concrete damage, but failed to satisfactorily estimate the cyclic behaviour...
of structural systems that suffer failure types not considered in the model formulation procedures;

- The dynamic response of various full-scale prototype structural systems was computed using both STMs and planar frame models. It was demonstrated that both modelling techniques predicted similar dynamic response for all structural systems considered in Chapter 5;

- The dynamic response of a building frame system that was generated using the hybrid model was similar to that obtained from the STM and planar frame model. It is a more time-effective modelling solution than the STM; and

- Although dynamic analysis results obtained from STMs were considered to match the actual structural behaviour more closely than those calculated using the planar frame model, it was comparatively very time-consuming. The marginal advantage of analytical accuracy does not support a preference for adoption of STM over the planar frame model for use in dynamic analyses.

### 6.3 Model Deficiencies

Inaccuracy of analysis results was attributed to various model deficiencies that are listed below:

- Progressive concrete strength degradation when loaded under cyclic action at the same ductility level was not captured by the existing cyclic STMs. Also, different types of reinforcement failure modes including buckling, fracture and loss of anchorage in concrete could not be predicted by the existing model formulation procedure;

- The Bauschinger effect could not be included in the bilinear stress-strain material model currently employed for rebars;

- The lengthy computation time required when performing dynamic analysis for large STMs makes this modelling technique unattractive and uneconomical when adopted in routine design practice;

- As the hybrid modelling solution remains very time-consuming when used in conducting dynamic analysis for large scale structural systems, this modelling technique has insufficient advantages over conventional planar frame models when used for dynamic analysis;

- When evaluating the general dynamic behaviour of large-scale structural systems, it is recommended that planar frame models be used in conjunction with the STMs, to verify the integrity of the D-region structural members; and

- For small-scale structural system, dynamic behaviour could be examined directly using STMs as the computation time is less of an issue.
6.4 Model Improvements in Future Research

As discussed previously, the analysis results obtained from monotonic STMs, in general, compared satisfactorily with the test data, so it is assessed that further model improvements are not necessary. In contrast, the cyclic STMs have major weaknesses in accurately predicting the hysteretic behaviour of structural systems that suffer brittle failure. These model deficiencies are mainly attributable to the inherent weaknesses of the analytical cyclic stress-strain material models of concrete and rebars that are available in Drain-2DX. As rectification to this problem required substantial effort in computer programming, this task was considered beyond the scope of the current research and therefore was not pursued. Nevertheless, the following listed suggestions are considered critical to future improvement in analytical accuracy of the cyclic STMs.

- The material model of concrete struts should be capable of predicting the gradual compressive strength degradation under repeated cyclic actions at the same displacement ductility level;
- The incremental effect of wedge action due to an increasing number of dislocated concrete particles being trapped inside concrete cracks during sequential load cycles should also be included in the material model of concrete, in order to improve the estimation of beam elongation;
- The material model of rebar ties should examine the buckling effect when reinforcement experiences excessive compression;
- The material model of rebars should be capable of assessing the strain energy being absorbed by reinforcement in order to predict reinforcement fracture; and
- The material model of rebars should consider the reinforcement-concrete bond-slip behaviour to allow pinching response in the analytical hysteretic loops when anchorage failure occurs.

6.5 Integrated Design Approach using Strut-and-Tie Models

The research documented here demonstrates that a single STM is technically capable of predicting monotonic and cyclic response, as well as dynamic behaviour of an entire structural system. With the incorporation of sophisticated stress-strain material models for concrete and rebars, the main advantage of applying the strut-and-tie modelling technique over conventional planar frame model analysis is its capability to predict the realistic nonlinear response of an entire structural system simultaneously. Furthermore, it provides a highly transparent insight to the force demands of every structural component, in particular those located in structural D-regions, allowing possible premature structural failure to be detected.
As previously discussed, a new structural design procedure, shown in Fig. 6.1, that permits synergistic linkage between analysis and design phases could be developed based on the strut-and-tie modelling technique. This design procedure has a major advantage of encouraging designers to consider the force transfer mechanism that develops inside an entire structural system, i.e. both the D- and B-regions, and the force interaction between different structural components at various load states. This procedure could potentially lead to a consistent design standard for all components of a structural system. Moreover, the

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**Figure 6.1: Algorithm to the design procedure using strut-and-tie methodology.**
procedure could deliver speedy design solutions when an interactive computer programme that allows automatic model formulation can be established.
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APPENDICES

Appendix A - Effective Strength of Flexural Concrete Ties

Presented in Appendix A are design charts established based on the procedure described in section 3.5.3.3 to evaluate the effective strength of flexural concrete ties employed in the type 1 element of monotonic STMs. Note that $\rho = \frac{\text{area of rebar tie divided by area of concrete tie}}{\text{concrete tie}} = \frac{A_{rt}}{A_{ct}}$.

![Graph showing effective strength of concrete ties.](image)

A(i), Effective strength of concrete ties for rebar size, $d_b = 10$ mm.
A(ii), Effective strength of concrete ties for rebar size, \( d_b = 12 \text{ mm} \).

\[
\frac{f_{db}}{f_i} \quad \rho = 0.005 \text{, } 0.01 \text{, } 0.02 \text{, } 0.03
\]

\( \ell' \) \( (d_b = 12 \text{ mm}) \)

A(ii), Effective strength of concrete ties for rebar size, \( d_b = 16 \text{ mm} \).

\[
\frac{f_{db}}{f_i} \quad \rho = 0.005 \text{, } 0.01 \text{, } 0.02 \text{, } 0.03
\]

\( \ell' \) \( (d_b = 16 \text{ mm}) \)

A(iii), Effective strength of concrete ties for rebar size, \( d_b = 20 \text{ mm} \).

\[
\frac{f_{db}}{f_i} \quad \rho = 0.005 \text{, } 0.01 \text{, } 0.02 \text{, } 0.03
\]

\( \ell' \) \( (d_b = 20 \text{ mm}) \)
A(iv), Effective strength of concrete ties for rebar size, \( d_b = 25 \text{ mm} \).

A(v), Effective strength of concrete ties for rebar size, \( d_b = 32 \text{ mm} \).
Appendix B - Section Force Analytical Charts

Illustrated in Appendix B are the plots of modelling parameters for different section geometries evaluated using section force analysis. Analytical data obtained from these plots are used in formulating STM elements simulating the structural flexural components, see section 3.5.2, 3.5.3 and 3.6.2. Note that $\omega$ is the compression rebar area divided by the tension rebar area, $A'_c / A'_s$.

B1 Beam Members

B1(i), Neutral axis position at the first yield state for beam sections, $f'_c = 30$ MPa.

B1(iii), Compression centroid position at the first yield state for beam sections, $f'_c = 30$ MPa.
B1(ii), Neutral axis position at the first yield state for beam sections, $f'_c = 40$ MPa.

B1(iv), Compression centroid position at the first yield state for beam sections, $f'_c = 40$ MPa.
B1(v), Effective strength of flexural concrete struts in beam sections, $f'_c = 30$ MPa.

B1(vi), Effective strength of flexural concrete struts in beam sections, $f'_c = 40$ MPa.
B2 Rectangular Column Members

B2(i), A rectangular column section and the STM representation.

B2(ii), Column section neutral axis position at the first yield state, $f'_c = 30$ MPa.
B2(iii), Column section neutral axis position at the first yield state, \( f'_c = 40 \) MPa.

B2(iv), Column section compression centroid position at the first yield state, \( f'_c = 30 \) MPa.

B2(v), Column section compression centroid position at the first yield state, \( f'_c = 40 \) MPa.
B2(vi), Column section tension centroid position at the first yield state, $f'_c = 30 \text{ MPa}$.

B2(vii), Column section tension centroid position at the first yield state, $f'_c = 40 \text{ MPa}$.

B2(viii), Effective strength of flexural concrete struts in rectangular column sections, $f'_c = 30 \text{ MPa}$.
B2(ix), Effective strength of flexural concrete struts in rectangular column sections, 
\[ f_c' = 40 \text{ MPa} \]
B3 Circular Column Members

B3(i), A circular column section and the STM representation.

B3(ii), Circular column neutral axis position at the first yield state, $f_c = 30$ MPa.
B3(iii), Circular column neutral axis position at the first yield state, $f'_c = 40$ MPa.

B3(iv), Circular column compression centroid position at the first yield state, $f'_c = 30$ MPa.

B3(v), Circular column compression centroid position at the first yield state, $f'_c = 40$ MPa.
B3(vi), Circular column tension centroid position at the first yield state, $f'_c = 30 \text{ MPa}$.

B3(vii), Circular column tension centroid position at the first yield state, $f'_c = 40 \text{ MPa}$.

B3(viii), Effective strength of concrete strut in circular column section, $f'_c = 30 \text{ MPa}$.
B3(ix), Effective strength of concrete strut in circular column section, $f'_c = 40$ MPa.
Appendix C - Charts of $\beta$ Values

Presented in Appendix C is the plots of $\beta$ value calculated using the quadratic Eq. 3.33c for assessing the concrete contribution to the member shear strength, see section 3.5.6.2 (b).

C(i), $\beta$ value for $\varepsilon_2 = 0.0$
C(ii), $\beta$ value for $\varepsilon_s = 0.001$

C(iii), $\beta$ value for $\varepsilon_s = 0.002$