Copyright Statement

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand).

This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author’s right to be identified as the author of this thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author’s permission before publishing any material from their thesis.

To request permissions please use the Feedback form on our webpage. [http://researchspace.auckland.ac.nz/feedback](http://researchspace.auckland.ac.nz/feedback)

General copyright and disclaimer

In addition to the above conditions, authors give their consent for the digital copy of their work to be used subject to the conditions specified on the Library Thesis Consent Form.
Quantum Interactions
of
Light and Atoms:
Squeezing of Light by Atoms and
Cooling of Atoms by Light

Craig Andrew Blockley

A thesis
submitted in partial fulfillment
of the requirements for the degree
of
Doctor of Philosophy
at the
University of Auckland

University of Auckland,
1993
Abstract

Light and atoms couple together via the electromagnetic interaction. The simplest form of this interaction is the electric dipole interaction, and in its quantised form it provides a useful starting point for the investigation of quantum effects in optics. Two examples of quantum noise manipulation of the light field due to interaction with atoms will be presented, as well as an analysis of a simple model for laser cooling of trapped atoms.

The first example of quantum noise manipulation is the investigation of a quantum non-demolition measurement scheme based on a three-level atomic system in the ladder configuration. An effective two-level model of the atomic system is used, which enables the inclusion of spontaneous emission noise from the upper atomic level. The system is found to perform well, when detuned far from resonance.

The second example is the treatment of squeezing in the intensity difference between two modes coupled by a three-level atomic system in the ladder configuration. The noise correlations are similar to those occurring in the optical parametric oscillator, and give rise to good squeezing when the system is well detuned from the intermediate level.

The simple model of laser cooling consists of a single two-level atom with quantised centre-of-mass motion constrained to move in a one-dimensional harmonic potential while interacting with a single-mode classical travelling light field. It is shown that there is an analogy between this model and the Jaynes-Cummings model. This gives rise to interesting coherent effects including quantum collapses and revivals in the atomic inversion.

Sideband cooling occurs for this model when the light field is tuned to the atom's first lower vibrational sideband. The strong sideband and Lamb-Dicke perturbation regimes are defined. Analytic results have previously been obtained for the latter regime, but we carry out a numerical investigation of the steady state and time evolution behaviour in the former regime. Differences in the behaviour in the two regimes are discussed. Finally the possibility of observing quantum jumps between trap levels is discussed.
I would like to thank my supervisors, Professor D. F. Walls and Dr. M. J. Collett, for guiding me through my research. It has been especially enjoyable being part of a large (by NZ standards) and active research group. One of the obvious advantages of this is the continual stream of students, post-docs and visitors, with whom one can discuss current problems. Special thanks goes to Drs. T. A. B. Kennedy, S. M. Tan and Z. Weiping, as well as to Professor H. Risken. I would also like to thank Dr P. Grangier, J. F. Roch and Professor C. N. Cohen-Tannoudji for useful discussions. Three years of study would be difficult indeed without the companionship of fellow students - my thanks to M. Holland, R. Levien, M. Dance, K. Gheri, E. P. Storey, A. Duhs, P. Smith and others.

I would like to thank the Vice Chancellors' Committee for financial support, and also the Royal Society of NZ for travel funds.

I would like to dedicate this thesis to my parents, who have dedicated so much of their time and effort to me over the years.
Contents

Abstract
Acknowledgements

1 Introduction
  1.1 Quantum mechanics ............................................. 2
  1.2 Squeezing .................................................. 3
  1.3 Cooling .................................................. 4

I Concepts and methods in quantum optics 7

2 Quantum mechanics and the electromagnetic field 9
  2.1 A brief introduction to quantum mechanics ................. 9
    2.1.1 The state vector .................................... 10
    2.1.2 Observables ........................................ 11
    2.1.3 Eigenvectors and eigenstates of hermitian operators .... 11
    2.1.4 Measurement ....................................... 12
    2.1.5 Time evolution .................................... 14
    2.1.6 An example: Quantisation of the free electromagnetic field ... 15
  2.2 Squeezed states ......................................... 17
  2.3 Quadrature amplitudes of the electromagnetic field ......... 18

3 The matter-field Hamiltonian 21
  3.1 Quantising the internal atomic states ...................... 21
  3.2 Quantising the interaction energy ......................... 22
    3.2.1 Interaction with a single mode: Jaynes-Cummings model ...... 24
    3.2.2 Interacting with many modes: coupling to a reservoir ....... 25
3.2.3 Interaction with a single classical field ........................................ 26

4 Modelling dissipative processes ...................................................... 27
  4.1 The density operator ....................................................................... 28
  4.2 The master equation ....................................................................... 29
  4.3 The master equation for cavity damping ......................................... 32

II Quantum non-demolition measurements in a three-level atomic system ... 35

5 An introduction to QND measurements ................................................. 37
  5.1 What are QND measurements? ...................................................... 37
  5.2 Some QND history ....................................................................... 40
  5.3 Two photon transitions and squeezing .......................................... 42
  5.4 Non-ideal QND measurement schemes ......................................... 44
  5.5 Quadrature operators .................................................................. 45
  5.6 The correlation coefficients in frequency space ......................... 46
  5.7 Input-output formalism ................................................................. 47
  5.8 Linearised fluctuation analysis ....................................................... 49

6 The atomic three-level QND system .................................................... 51
  6.1 The simple dispersive model ......................................................... 51
    6.1.1 Semi-classical input-output treatment .................................... 51
    6.1.2 Quantum input-output treatment .......................................... 54
  6.2 The effective two-level model ....................................................... 60
    6.2.1 Linearised fluctuation analysis ............................................. 62
    6.2.2 Evaluating the correlation coefficients ................................... 65
  6.3 Conclusion .................................................................................... 71

III Intensity Squeezing in a three-level atomic system ......................... 75

7 Noise reduction in non-linear optics .................................................. 77
  7.1 Introduction ................................................................................. 77
  7.2 Nonlinear processes and squeezing .............................................. 78
  7.3 More about parametric amplification ......................................... 80
7.4 Quantum correlations in three-level atomic systems ........................................ 82
7.5 The master equation and its representations ....................................................... 83
  7.5.1 The generalised P-representation ................................................................. 84
  7.5.2 Stochastic differential equations .................................................................... 85
  7.5.3 The noise spectrum ......................................................................................... 86
  7.5.4 The output spectrum for the intensity ............................................................ 86

8 Intensity fluctuations in a three-level system ......................................................... 89
  8.1 The model ........................................................................................................... 89
  8.2 The derivation of the master equation ............................................................... 92
  8.3 Spectrum of intensity fluctuations .................................................................... 97
  8.4 Results ............................................................................................................... 100
  8.5 Conclusion ......................................................................................................... 106

IV Coherent and laser cooling effects for a trapped ion ............................................ 109

9 Some background to laser cooling ........................................................................ 111
  9.1 Introduction ......................................................................................................... 111
  9.2 Cooling of free particles ................................................................................... 112
  9.3 Cooling of trapped atoms .................................................................................. 115
  9.4 The model of laser cooling in a trap ................................................................. 117
    9.4.1 Quantum effects in traps ............................................................................. 118
    9.4.2 The Lamb-Dicke regime ............................................................................. 118
    9.4.3 Overview of previous work ......................................................................... 120
    9.4.4 The system Hamiltonian ............................................................................. 121
    9.4.5 Equations of motion for the probability amplitudes .................................. 123

10 Coherent effects in a trap .................................................................................... 127
  10.1 A comparison with the Jaynes-Cummings model ............................................ 127
  10.2 Solutions to the equations of motion ............................................................... 129
  10.3 Quantum collapses and revivals in a trap ....................................................... 131
    10.3.1 Collapse and revival times ........................................................................ 131
    10.3.2 Results ......................................................................................................... 132
  10.4 The Q-function for the trapped ion ................................................................. 134
  10.5 Experimental realisation .................................................................................. 137
List of Figures

2.1 The state vector for two simple systems ........................................ 10
2.2 Uncertainties for a squeezed state ............................................. 18
4.1 A typical reservoir correlation function ....................................... 32
5.1 General model for measurement ..................................................... 38
5.2 General model for a QND measurement scheme ............................... 40
6.1 The three-level QND measurement scheme ...................................... 52
6.2 Correlation functions for the simple dispersive model ..................... 59
6.3 The signal degradation correlation coefficient for the atomic model .... 68
6.4 The measurement correlation coefficient for the atomic model .......... 69
6.5 The state preparation conditional variance for the atomic model ......... 70
6.6 (a) A comparison of the atomic model with the beam splitter and (b) an optimization of the three QND criteria. .......................... 72
7.1 Degenerate parametric amplifier .................................................. 79
7.2 Degenerate parametric oscillator .................................................. 80
7.3 Squeezing of the fluctuations in the phase quadrature of a coherent field by a degenerate parametric amplifier ............................. 82
8.1 Energy level diagram for the atomic three-level system .................... 90
8.2 The squeezing spectrum for fluctuations in the intensity difference as a function of the detuning. .............................................. 101
8.3 The squeezing spectrum for fluctuations in the intensity difference with unequal cavity decay rates. .............................................. 102
8.4 The zero frequency component of the squeezing spectrum as a function of the ratio of the cavity decay rates. ................................. 103
8.5 The squeezing spectrum as a function of the ratio of the coupling constants.104
8.6 The zero frequency component of the squeezing spectrum as a function of the ratio of the coupling constants. 105
8.7 The squeezing spectrum for the case of a resonant driving field. 107
9.1 Momentum transfer in an atom-light interaction 113
9.2 Laser cooling of a free atom in a standing wave 114
9.3 The absorption spectrum of a single ion bound in a harmonic potential 116
9.4 Vibrational energy levels for a trapped ion 119
9.5 Energy level structure for a trapped ion 123
10.1 The dependence of the coupling $\mu_{n,n+1}$ on the trap quantum number $n$. 133
10.2 The time behaviour of the atomic inversion. 135
10.3 The time behaviour of the atomic inversion for transitions involving the exchange of two trap quanta. 136
10.4 The time evolution of the Q-function. 138
12.1 The absorption spectrum for a trapped ion 149
12.2 The dominant processes considered in the determination of the steady state average quantum number in the LDP regime. 153
12.3 The steady state average trap number as a function of the scaled detuning 154
12.4 The steady state average trap number as a function of the spontaneous emission rate for various values of the trap frequency 155
12.5 The steady state average trap number as a function of the trap frequency for various values of the spontaneous emission 156
12.6 The steady state average trap number as a function of the Lamb-Dicke parameter for various values of the spontaneous emission rate. 157
12.7 The steady state average trap number as a function of the Lamb-Dicke parameter for various values of the trap frequency. 158
12.8 The steady state average trap number as a function of the spontaneous emission for various values of the Lamb- Dicke parameter 159
12.9 The time evolution of the average trap number for various values of the spontaneous emission. 161
12.10 The time evolution of the average trap number for various values of the Lamb-Dicke parameter. 162
12.11 The time evolution of the average trap number showing quantum collapses and revivals. 163
12.12 The time evolution of the average trap number for various values of the spontaneous emission on a logarithmic plot.  

164