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Quantum Interactions of Light and Atoms: Squeezing of Light by Atoms and Cooling of Atoms by Light

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Abstract

Light and atoms couple together via the electromagnetic interaction. The simplest form of this interaction is the electric dipole interaction, and in its quantised form it provides a useful starting point for the investigation of quantum effects in optics. Two examples of quantum noise manipulation of the light field due to interaction with atoms will be presented, as well as an analysis of a simple model for laser cooling of trapped atoms.

The first example of quantum noise manipulation is the investigation of a quantum non-demolition measurement scheme based on a three-level atomic system in the ladder configuration. An effective two-level model of the atomic system is used, which enables the inclusion of spontaneous emission noise from the upper atomic level. The system is found to perform well, when detuned far from resonance.

The second example is the treatment of squeezing in the intensity difference between two modes coupled by a three-level atomic system in the ladder configuration. The noise correlations are similar to those occurring in the optical parametric oscillator, and give rise to good squeezing when the system is well detuned from the intermediate level.

The simple model of laser cooling consists of a single two-level atom with quantised centre-of-mass motion constrained to move in a one-dimensional harmonic potential while interacting with a single-mode classical travelling light field. It is shown that there is an analogy between this model and the Jaynes-Cummings model. This gives rise to interesting coherent effects including quantum collapses and revivals in the atomic inversion.

Sideband cooling occurs for this model when the light field is tuned to the atom's first lower vibrational sideband. The strong sideband and Lamb-Dicke perturbation regimes are defined. Analytic results have previously been obtained for the latter regime, but we carry out a numerical investigation of the steady state and time evolution behaviour in the former regime. Differences in the behaviour in the two regimes are discussed. Finally the possibility of observing quantum jumps between trap levels is discussed.

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Contents

Abstract iii

Acknowledgements v

1 Introduction 1

1.1 Quantum mechanics 2

1.2 Squeezing 3

1.3 Cooling 4

I Concepts and methods in quantum optics 7

2 Quantum mechanics and the electromagnetic field 9

2.1 A brief introduction to quantum mechanics 9

2.1.1 The state vector 10

2.1.2 Observables 11

2.1.3 Eigenvectors and eigenstates of hermitian operators 11

2.1.4 Measurement 12

2.1.5 Time evolution 14

2.1.6 An example: Quantisation of the free electromagnetic field 15

2.2 Squeezed states 17

2.3 Quadrature amplitudes of the electromagnetic field 18

3 The matter-field Hamiltonian 21

3.1 Quantising the internal atomic states 21

3.2 Quantising the interaction energy 22

3.2.1 Interaction with a single mode: Jaynes-Cummings model 24

3.2.2 Interacting with many modes: coupling to a reservoir 25

3.2.3 Interaction with a single classical field 26

4 Modelling dissipative processes **27**

4.1 The density operator 28

4.2 The master equation 29

4.3 The master equation for cavity damping 32

II Quantum non-demolition measurements in a three-level atomic system **35**

5 An introduction to QND measurements **37**

5.1 What are QND measurements? 37

5.2 Some QND history 40

5.3 Two photon transitions and squeezing 42

5.4 Non-ideal QND measurement schemes 44

5.5 Quadrature operators 45

5.6 The correlation coefficients in frequency space 46

5.7 Input-output formalism 47

5.8 Linearised fluctuation analysis 49

6 The atomic three-level QND system **51**

6.1 The simple dispersive model 51

6.1.1 Semi-classical input-output treatment 51

6.1.2 Quantum input-output treatment 54

6.2 The effective two-level model 60

6.2.1 Linearised fluctuation analysis 62

6.2.2 Evaluating the correlation coefficients 65

6.3 Conclusion 71

III Intensity Squeezing in a three-level atomic system **75**

7 Noise reduction in non-linear optics **77**

7.1 Introduction 77

7.2 Nonlinear processes and squeezing 78

7.3 More about parametric amplification 80

7.4 Quantum correlations in three-level atomic systems 82
7.5 The master equation and its representations 83
7.5.1 The generalised P-representation 84
7.5.2 Stochastic differential equations 85
7.5.3 The noise spectrum 86
7.5.4 The output spectrum for the intensity 86

8 Intensity fluctuations in a three-level system **89**
8.1 The model 89
8.2 The derivation of the master equation 92
8.3 Spectrum of intensity fluctuations 97
8.4 Results 100
8.5 Conclusion 106

IV Coherent and laser cooling effects for a trapped ion **109**

9 Some background to laser cooling **111**
9.1 Introduction 111
9.2 Cooling of free particles 112
9.3 Cooling of trapped atoms 115
9.4 The model of laser cooling in a trap 117
9.4.1 Quantum effects in traps 118
9.4.2 The Lamb-Dicke regime 118
9.4.3 Overview of previous work 120
9.4.4 The system Hamiltonian 121
9.4.5 Equations of motion for the probability amplitudes 123

10 Coherent effects in a trap **127**
10.1 A comparison with the Jaynes-Cummings model 127
10.2 Solutions to the equations of motion 129
10.3 Quantum collapses and revivals in a trap 131
10.3.1 Collapse and revival times 131
10.3.2 Results 132
10.4 The Q-function for the trapped ion 134
10.5 Experimental realisation 137

10.6 Conclusion 139

11 The master equation for a trapped ion **141**

12 Cooling in the strong sideband regime **145**

12.1 The Equations of Motion 146

12.2 The Strong Sideband Regime 147

12.2.1 Perturbation Expansion in the Resolved Sideband Limit 150

12.3 The Steady State 151

12.3.1 The LDP regime 151

12.3.2 The SSB regime 153

12.4 Time Evolution 160

12.5 Conclusion 164

13 Quantum jumps in a trap **167**

13.1 Introduction 167

13.2 Monte-Carlo wavefunction simulations 168

13.3 Conclusion 171

14 Conclusion **173**

A The matrix elements $u_{n,m}$ **175**

B The coherent state **177**

C The Jaynes-Cummings model **179**

C.1 Quantum collapses and revivals 180

D Equations of motion in matrix form **183**

Bibliography **185**

List of Figures

2.1 The state vector for two simple systems 10
2.2 Uncertainties for a squeezed state 18

4.1 A typical reservoir correlation function 32

5.1 General model for measurement 38
5.2 General model for a QND measurement scheme 40

6.1 The three-level QND measurement scheme 52
6.2 Correlation functions for the simple dispersive model 59
6.3 The signal degradation correlation coefficient for the atomic model 68
6.4 The measurement correlation coefficient for the atomic model 69
6.5 The state preparation conditional variance for the atomic model 70
6.6 (a) A comparison of the atomic model with the beam splitter and (b) an optimisation of the three QND criteria. 72

7.1 Degenerate parametric amplifier 79
7.2 Degenerate parametric oscillator 80
7.3 Squeezing of the fluctuations in the phase quadrature of a coherent field by a degenerate parametric amplifier 82

8.1 Energy level diagram for the atomic three-level system 90
8.2 The squeezing spectrum for fluctuations in the intensity difference as a function of the detuning. 101
8.3 The squeezing spectrum for fluctuations in the intensity difference with unequal cavity decay rates. 102
8.4 The zero frequency component of the squeezing spectrum as a function of the ratio of the cavity decay rates. 103
8.5 The squeezing spectrum as a function of the ratio of the coupling constants. 104

8.6 The zero frequency component of the squeezing spectrum as a function of the ratio of the coupling constants. 105
8.7 The squeezing spectrum for the case of a resonant driving field. 107

9.1 Momentum transfer in an atom-light interaction 113
9.2 Laser cooling of a free atom in a standing wave 114
9.3 The absorption spectrum of a single ion bound in a harmonic potential . 116
9.4 Vibrational energy levels for a trapped ion 119
9.5 Energy level structure for a trapped ion 123

10.1 The dependence of the coupling $u_{n,n+1}$ on the trap quantum number n. . 133
10.2 The time behaviour of the atomic inversion. 135
10.3 The time behaviour of the atomic inversion for transitions involving the exchange of two trap quanta. 136
10.4 The time evolution of the Q-function. 138

12.1 The absorption spectrum for a trapped ion 149
12.2 The dominant processes considered in the determination of the steady state average quantum number in the LDP regime. 153
12.3 The steady state average trap number as a function of the scaled detuning 154
12.4 The steady state average trap number as a function of the spontaneous emission rate for various values of the trap frequency 155
12.5 The steady state average trap number as a function of the trap frequency for various values of the spontaneous emission 156
12.6 The steady state average trap number as a function of the Lamb-Dicke parameter for various values of the spontaneous emission rate. 157
12.7 The steady state average trap number as a function of the Lamb-Dicke parameter for various values of the trap frequency. 158
12.8 The steady state average trap number as a function of the spontaneous emission for various values of the Lamb- Dicke parameter 159
12.9 The time evolution of the average trap number for various values of the spontaneous emission. 161
12.10 The time evolution of the average trap number for various values of the Lamb-Dicke parameter. 162
12.11 The time evolution of the average trap number showing quantum collapses and revivals. 163

12.12 The time evolution of the average trap number for various values of the spontaneous emission on a logarithmic plot. 164