

Quantum Interactions
of
Light and Atoms:
Squeezing of Light by Atoms
and
Cooling of Atoms by Light

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Abstract

Light and atoms couple together via the electromagnetic interaction. The simplest form of this interaction is the electric dipole interaction, and in its quantised form it provides a useful starting point for the investigation of quantum effects in optics. Two examples of quantum noise manipulation of the light field due to interaction with atoms will be presented, as well as an analysis of a simple model for laser cooling of trapped atoms.

The first example of quantum noise manipulation is the investigation of a quantum non-demolition measurement scheme based on a three-level atomic system in the ladder configuration. An effective two-level model of the atomic system is used, which enables the inclusion of spontaneous emission noise from the upper atomic level. The system is found to perform well, when detuned far from resonance.

The second example is the treatment of squeezing in the intensity difference between two modes coupled by a three-level atomic system in the ladder configuration. The noise correlations are similar to those occurring in the optical parametric oscillator, and give rise to good squeezing when the system is well detuned from the intermediate level.

The simple model of laser cooling consists of a single two-level atom with quantised centre-of-mass motion constrained to move in a one-dimensional harmonic potential while interacting with a single-mode classical travelling light field. It is shown that there is an analogy between this model and the Jaynes-Cummings model. This gives rise to interesting coherent effects including quantum collapses and revivals in the atomic inversion.

Sideband cooling occurs for this model when the light field is tuned to the atom's first lower vibrational sideband. The strong sideband and Lamb-Dicke perturbation regimes are defined. Analytic results have previously been obtained for the latter regime, but we carry out a numerical investigation of the steady state and time evolution behaviour in the former regime. Differences in the behaviour in the two regimes are discussed. Finally the possibility of observing quantum jumps between trap levels is discussed.

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