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THE AP INTEGRAL

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A Thesis submitted for the degree of
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University of Auckland, New Zealand

Department of Mathematics and Statistics

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To

The memory of J.C. Burkill

The originator of the AP Integral

The AP Integral

Abstract

This study attempts to develop the theory surrounding a controlled convergence theorem in the setting of AP integration. First, after a brief review, we spend some time on developing concepts and definitions. We are able to use these concepts to successfully present an initial control convergent theorem. Then we study our conditions more profoundly and find a number of equivalences and inequivalences between them. We are able to weaken some of the standard hypotheses significantly.

The Riesz type approach to our integral has been included in our theory.

Appropriately enough, we are able to use some of our approaches to extend the theory of the Henstock-Kurweil integral.

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35 years ago, a widowed mother dedicated her youngest son to serving the education of the national minorities in one of the most remote provinces of China. After 35 years, she rejoices in her son having the opportunity to study mathematics again, at the University of Auckland.

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THE AP INTEGRAL

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