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Picosecond Pulse Generation and Propagation in Erbium Doped Optical Fibres

by

Paul Bollond

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Physics at the University of Auckland

The University of Auckland 1997
Abstract

This thesis is concerned with the generation of picosecond pulses and their propagation through both resonant and non-resonant media. This was achieved by constructing a passively modelocked Erbium doped fibre laser (EDFL) which was used to study pulse propagation through sections of standard communications grade optical fibre, dispersion shifted optical fibre, and also through an Erbium doped fibre amplifier (EDFA) module.

The EDFL produced a train of ~2 psec pulses at 4 MHz, tunable over the erbium gain band (1520 - 1570 nm). The laser was constructed from commercially available components and had the property of stability combined with low pump power requirements to produce ~50 Watt peak power pulses. The laser cavity geometry included a nonlinear optical loop mirror, which has the property of efficiently switching high peak power pulses, and allowed pulsed operation without the aid of any high-speed electronics.

An EDFA module of identical geometry to that used in the laser was also constructed, and this was probed using the pulses from the EDFL. The traditional temporal and spectral measurements were found to be inadequate to allow a complete description of the pulse amplification process to be developed. To overcome this problem the technique of frequency resolved optical gating (FROG) was applied for the first time to optical fibre research, and allowed an indirect measurement of the electric field of the pulse. This complete description of the pulse was used in a numerical model to describe pulse propagation in an optical fibre. Fundamental propagation terms in the model were treated as free parameters in a minimisation scheme, which could be determined for a fibre under examination. This technique was shown to be accurate when used to examine pulse propagation through both standard and dispersion shifted optical fibre.

A comprehensive numerical model was developed for the EDFA, and it was apparent from this model that a pulse propagating through an optimised EDFA encounters an atomic inversion distribution which is a strong function of distance along the amplifying fibre. It was also shown from the experimental results that the EDFA exhibited resonant dispersion, which is characteristic for propagation through an atomic medium on resonance.
Acknowledgments

The work described in this thesis was conducted over a five year period, and was made possible by the contributions from many individuals and organisations. I would like to take this opportunity to thank them.

I would like to thank my supervisor Professor John Harvey for the assistance he has given me over this period, which has allowed the experimental laser physics research group at the University of Auckland to establish a research program in optical fibre technology for telecommunications. I would also like to thank Dr Graham Town of the University of Sydney for the opportunity to work in his lab during 1995. I also thank the research group at the Optical Fibre Technology Centre, The University of Sydney for manufacturing the Erbium doped fibre which was used in this thesis, and also for answering my many questions about it.

This work was initiated with financial support from Telecom New Zealand Ltd, who awarded me the 1992 Telecom New Zealand Fellowship in Telecommunications Engineering. I would like thank Telecom New Zealand Ltd for this award, and for their support of research conducted in New Zealand.

I also thank the Physics Department and the Applied Optics Centre at the University of Auckland for financial support over the latter part of this research. I also would like to thank the University of Auckland for allowing me to attend the Optical Fiber Communication (OFC) conference in Dallas Texas during February 1997 to present some results of the thesis, by awarding me a grant from the Graduate Research Fund. I also thank FibreNet New Zealand Ltd for sponsoring my trip to the Australian Conference on Optical Fibre Technology (ACOFT) in Surfers Paradise, Queensland, during December 1996, where I also presented some results. Thanks also to the Physico-Chemistry Department, the University of Auckland Medical School, for sponsoring me to travel to the OFC and ACOFT conferences. I thank the Applied Optics Centre at the University of Auckland for sponsoring my trip to the International Conference on Quantum Electronics (IQEC) in Sydney during July 1996, where I also gave a presentation and presented a poster, and also numerous other conferences. I also would like to thank Apple Computers for sponsoring me to give a presentation at the Apple University Consortium Conference in Perth during July 1995.
I would also like to thank Dr Rick Trebino for his useful discussions on the subject of Frequency Resolved Optical Gating (FROG), which was initiated from a meeting at the IQEC conference in July 1996. I would also like to thank Dr John Dudley and Dr Liam Barry for their assistance with the experiment and also with interpreting the electrical field measurements from the FROG results. Thanks also to Dr Rainer Leonhardt for his support and help with the many experiments conducted over the years. Finally, I thank my family for their support during my time at University.
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<tr>
<td>APC</td>
<td>Angled Polished Connector</td>
</tr>
<tr>
<td>ASE</td>
<td>Amplified Spontaneous Emission</td>
</tr>
<tr>
<td>BBO</td>
<td>$\beta$-Barium Borate</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous Wave</td>
</tr>
<tr>
<td>DS</td>
<td>Dispersion Shifted</td>
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<tr>
<td>EDF</td>
<td>Erbium Doped Fibre</td>
</tr>
<tr>
<td>EDFA</td>
<td>Erbium Doped Fibre Amplifier</td>
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<tr>
<td>EDFL</td>
<td>Erbium Doped Fibre Laser</td>
</tr>
<tr>
<td>ESA</td>
<td>Excited State Absorption</td>
</tr>
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<td>F8L</td>
<td>Figure of Eight Laser</td>
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<td>FROG</td>
<td>Frequency Resolved Optical Gate</td>
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<td>FWHM</td>
<td>Full Width at Half Maximum intensity</td>
</tr>
<tr>
<td>GSA</td>
<td>Ground State Absorption</td>
</tr>
<tr>
<td>GVD</td>
<td>Group Velocity Dispersion</td>
</tr>
<tr>
<td>MFD</td>
<td>Mode Field Diameter</td>
</tr>
<tr>
<td>MI</td>
<td>Modulational Instability</td>
</tr>
<tr>
<td>NA</td>
<td>Numerical Aperture</td>
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<tr>
<td>NALM</td>
<td>Nonlinear Amplifying Loop Mirror</td>
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<td>NLSE</td>
<td>Nonlinear Schrödinger Equation</td>
</tr>
<tr>
<td>NOLM</td>
<td>Nonlinear Optical Loop Mirror</td>
</tr>
<tr>
<td>OMA</td>
<td>Optical Multichannel Analyser</td>
</tr>
<tr>
<td>PC</td>
<td>Polarisation Controller</td>
</tr>
<tr>
<td>SAM</td>
<td>Self Amplitude Modulation</td>
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<tr>
<td>SHG</td>
<td>Second Harmonic Generation</td>
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<td>SMF</td>
<td>Single Mode Fibre</td>
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<td>SPM</td>
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<td>WDM</td>
<td>Wavelength Division Multiplexer</td>
</tr>
<tr>
<td>WPS</td>
<td>Weak Pulse Shaping</td>
</tr>
<tr>
<td>ZDW</td>
<td>Zero Dispersion Wavelength</td>
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Chapter 1

Introduction

1.1 Historical Perspective

The telecommunications industry has undergone a revolution over the past 25 years by using glass optical fibres for the transmission of information encoded as pulses of light. Although light has been used since antiquity as a vehicle to carry information across lines of sight, optical fibre links today have created networks on a planetary wide scale which form the backbone of our information age.

This technology began with Alexander Graham Bell, when he transmitted a telephone signal over a distance of greater than 200 metres using light as the carrier signal in 1880. However, his photophone was abandon because of the lack of a reliable light source, and the lack of a dependable transmission medium, and he chose to work on the electrical telephone.

The laser was invented in 1960 and gave a new impetus to the idea of lightwave communications, and much research followed on the necessary optical devices and transmission media. The first demonstration of laser action in semiconductors was reported in 1962, but it was in the 1970s that semiconductor light sources became practical for use in fibre communications systems. Since then device characteristics such as efficiency, modulation bandwidth, and reliability have been continuously improved. It was predicted in early work that laser beams would transmit through the air at high bit rates between distant stations. However the same problems as had previously been encountered by Bell soon lead to the conclusion that guided paths offer the only practical means of optical transmission over long distances.

The transmission medium remained a significant problem until Kao and Hockham proposed in 1966 that glass optical fibres with a core surrounded by a lower refractive index cladding could be used for transmitting light over long distances. At that time available glasses had losses of more than 1000 dB per
km, and could not realistically have been used for long distance telecommunications. In 1970 Robert Maurer and others of Corning Glass Works were able to produce an optical fibre from fused silica with a loss of 20 dB/km [Kapron 1970]. The revolutionary concept introduced by Corning was primarily a materials one - high purity fused silica glass, with very small index changes between the core and cladding layer could be used successfully to guide light for many tens of kilometers before the detection limit was encountered. The first large scale application appeared 7 years later when two telephone exchanges in Chicago were connected with optical fibre with a loss of 3 dB/km at 800 nm. At that time digital technology could transmit 45 MBit/s over 10 km of optical fibre, before electronic repeaters were required to regenerate the signal. Today the optical losses have decreased to ~ 0.19 dB/km when operating at 1550 nm which is limited mainly by the fundamental process of Rayleigh scattering, to the point where signal regeneration is only required every ~70 - 100 km. Commercially available systems today provide links with 10 Gbit/s over much greater distances.

Hasegwa and Tappert first showed in 1973 that the slowly varying field approximation in a single mode optical fibre obeys the Nonlinear Schrodinger Equation (NLSE), and that soliton solutions are possible in the anomalous and normal dispersion regimes. This observation gave birth to the field of nonlinear fibre optics. In 1980 solitons were experimentally created by Mollenauer [Mollenauer 1980]. A soliton pulse retains its shape during propagation by balancing dispersion against the nonlinear effect of Self Phase Modulation (SPM). Nonlinear propagation effects can occur because the light inside a single mode fibre is confined to an area of ~ 5x10^{-11}m^2, and has peak powers ranging from 1 mW in telecommunications systems to much greater values in research, producing an extremely high intensity over this small area. Self Phase Modulation occurs when the refractive index is increased by an amount proportional to the intensity of light (via the optical Kerr effect) which leads to the phase of the field becoming self modulating.

The method of signal regeneration has traditionally been to use electronics to detect the optical signal, recover the digital signal, and then retransmit using another laser diode. In 1985 optical communications changed radically when a research group at the University of Southampton demonstrated gain in optical fibres at 1550 nm, which were doped with the rare earth Erbium. In the following few years the design of the Erbium doped fibre was optimised. In 1989 a practical semiconductor laser diode laser became available to invert the Erbium, and the first optical fibre amplifiers appeared. Advantages of
Erbium Doped Fibre Amplifiers (EDFA) are a wide gain bandwidth across the minimum loss region of silica fibres, polarisation insensitivity, temperature stability, quantum limited noise figure, and immunity to interchannel crosstalk. System breakthroughs occurred at an unprecedented rate once the intrinsic propagation losses had been counter balanced by the introduction of EDFAs. By 1993 as many as 274 Erbium doped optical fibre amplifiers had been used in a link spanning 9,000 km carrying data at 10 Gbit/s. This distance is close to the symbolic 13,000 km across the Pacific ocean, the longest link necessary on Earth. As the spans become longer it becomes more important to reduce the levels of amplified spontaneous emission noise from the EDFAs in the network, and to optimise the amplifier design to have the largest gain with the smallest noise figure.

In a sense, the technologies of communication have become out of balance; large increases in the speed of electronics are becoming harder to achieve, while the capacity of fibre optic links remains largely untapped. Potential electronics have a potential bandwidth of ~50 GHz, which is only a few times greater than the performance of current devices. However each optical fibre has three transmission bands in the near infrared (850, 1310, and 1550 nm) defined by local attenuation minima, and there is ~ 4 THz capacity in each of these bands. Today multiple signal wavelengths are commonly multiplexed (so called Wavelength Division Multiplexing - WDM) on to each fibre, each optical channel transmitting ~ 5 to 10 Gbit/s and separated by several nanometres within the Erbium gain band. The immunity to interchannel crosstalk between each channel as it is amplified in the EDFA allows each channel to be treated almost as an independent system.

Fibre optics will be the method of choice for many communications applications in the future, with its unparalleled information carrying capacity. Reduction of single mode fibre losses, progress in high sensitivity optical receivers, development of high speed semiconductor laser diodes, and the advent of optical amplifiers have led to a steady increase in transmission capacity measured by the highest bit rate over the longest unrepeatered distance, and to a consequent reduction in the cost of transmission per bit of data.

Questions still remain, however, about network and component design, for example whether solitons or WDM systems will be used for very high bit rate systems. This thesis is concerned with the design of the optical amplifier, and with subsequent pulse propagation. Optical amplifiers are a crucial technology since they will be used in many wide band optical links. This study is aided by indirect measurements of
the electrical field from the fibre devices for the first time, using the technique of frequency resolved optical gating. Complete information about the propagation of the pulses can be calculated by solving the NLSE once the electrical field is known. In this thesis the dispersion and nonlinearity of the EDFA is inferred from propagation experiments, which is different from that of standard single mode fibres since the inverted EDFA is an atomic system close to resonance. A complete understanding of these amplifiers is essential given their pivotal role in future communications systems.

1.2 Thesis Overview

The work described in this thesis is the generation picosecond pulses and a study of their propagation through both resonant and non-resonant media. The pulses were generated in a passively modelocked Erbium Fibre laser, and were tunable across the 1517 - 1570 nm gain band of this fibre.

The second chapter describes general properties of single mode optical fibres and phenomena affecting pulse propagation. The discussion includes the nature of weakly birefringent fibres, the origin of important dispersive and nonlinear propagation effects, and formation of optical solitons through an analytic solution to the propagation equation.

The following chapter describes the apparatus and experimental techniques used in the experiments conducted in this thesis. The construction of optical fibre based wave plates is explained. It is shown how a variety of mirrors and switches (NALMs) can be designed and constructed from 2x2 port directional couplers. The remaining part of the chapter describes instruments used in the work which include an infrared autocorrelator, spectrometer, and a frequency resolved optical gate.

The Erbium doped fibre amplifier is discussed in detail in chapter 4. An extensive numerical model was developed following previous work [Pederson et al 1991] based on rate equations to describe the atomic system and includes wide band Amplified Spontaneous Emission (ASE). The model was refined so that it accurately represents the performance of the particular Erbium doped fibre used in this thesis.

The Erbium doped fibre has a smaller core size than standard single mode fibre and splicing between the two presents problems. In chapter 5 a numerical model was used to relate the splice loss to the slope efficiency of a continuous wave fibre laser, by extending the EDFA model. A continuous wave fibre laser was constructed and the splice loss was determined from several experiments where the laser output coupler had 4 % reflectivity.
Chapter 6 describes a passively modelocked Erbium fibre laser. This was based on the Figure of Eight design and uses a Nonlinear Amplifying Loop Mirror (NALM) to switch high power pulses within the cavity. The laser is tunable over the Erbium gain band (1517 - 1570 nm) and the pulse width could be adjusted over the range ~ 1.2 to 3.0 psec. The electrical field of the pulses was measured using the frequency resolved optical gate, and a numerical model was used to determine the field within the laser cavity. Optimal switching of the NALM was shown to occur when a low order soliton propagates into the NALM. This method of analysis provided direct experimental confirmation of the previous models which have been developed to describe the operation of the laser.

Chapter 7 presents experimental results for pulse propagation through an EDFA, which was almost identical to the amplifier used within the laser cavity. Frequency resolved optical gating has been used to determine the electrical field before and after the EDFA. By using a numerical model to propagate the experimental fields through the EDFA module, the dispersion and nonlinearity have been determined from a minimisation scheme, and resonant dispersion has been shown to occur in the Erbium doped fibre.

The final results chapter is concerned with the pulse propagation through dispersion shifted optical fibre. This is an important area of study since many new telecommunications systems are using this low dispersion fibre in the 1550 nm band together with EDFAs to counteract propagation loss. The dispersion parameters for this fibre are known from measurements from the manufacturer. The laser was tuned both below and above the zero dispersion wavelength to study normal and anomalous dispersion propagation. Numerical simulations were used to confirm the presence of deep modulation on the pulse when propagating in the normal dispersion regime. The modulation occurs from a type of four wave mixing from interaction between SPM and third–order dispersion in this regime, and has previously been studied at 1300 nm, near the zero dispersion wavelength for standard single mode fibre [Stern et al 1992] [Yanovsky and Wise 1994] [Beaud et al 1987] [Gouveia–Neto et al 1988].

Chapter 9 is the conclusion of the thesis, where the main results are summarised. The experimental investigations presented in this thesis are by no means complete, but have highlighted some difficulties when measuring the amplification of pulses in resonant atomic media. It is hoped that this will form the basis for further study.
Chapter 2

The Propagation of Light in Optical Fibres

In this chapter a review of fibre characteristics and nonlinear propagation effects is presented. The first section discusses the geometrical construction of optical fibres. This is followed by an introduction to the attenuation, chromatic dispersion, birefringence and polarisation mode dispersion of the waveguide and its particular importance to the propagation of ultrashort pulses. The nonlinear propagation effects resulting from a nonlinear refractive index are treated in the next section. The chapter concludes with a brief numerical investigation of the generalised propagation equation.

2.1 Introduction to Optical Fibres

A glass optical fibre consists of a central core surrounded by a cladding layer with a lower refractive index [Agrawal 1989]. Such fibres are known as step-index fibres to distinguish them from graded-index fibres, in which the refractive index of the core decreases smoothly (e.g. quadratically) from the centre to the core boundary. Figure 2.1 shows schematically the cross section and the refractive-index profile of a step-index fibre.

![Cross section and the refractive index profile of a step index fibre.](image)

Two parameters which characterise the fibre are the relative core-cladding index difference \( \Delta n \),

\[
\Delta n = \frac{n_2 - n_1}{n_1}
\]
and the normalised frequency $V$,

$$V = k_o a \sqrt{n_2^2 - n_1^2}$$  \hspace{1cm} 2.2

where $k_o = \frac{2\pi}{\lambda}$ is the wave number, $a$ is the core radius, and $\lambda$ is the wavelength of light [Yariv 1985].

The $V$ parameter determines the number of modes supported by the fibre. A step-index fibre will support only one mode if $V < 2.405$. The main difference between the single-mode and multimode fibres is in the core size. A core radius of $a = 25-30 \mu m$ is typical for multimode fibres. However, single-mode fibres with a typical $\Delta n \approx 3 \times 10^{-3}$ require $a$ to be in the range $2-4 \mu m$. The numerical value of the outer radius $b$ is less critical as long as it is large enough to confine the fibre mode entirely. Typically $b = 50-60 \mu m$ for both single-mode and multimode fibres.

### 2.2 Fibre Characteristics

#### 2.2.1 Intrinsic Fibre Losses

The material of choice for low-loss optical fibres is silica glass, which is formed by fusing pure $SiO_2$. The refractive-index difference between the core and the cladding is realised by the selective use of dopants during the fabrication process. Dopants such as $GeO_2$ and $P_2O_5$ increase the refractive index of the pure silica core, while boron and fluorine are primarily used for the cladding because they decrease the refractive index of silica.

An important fibre parameter is a measure of the power loss transmission of optical signals inside the fibre. If $P_0$ is the power at the input of a fibre of length $L$, the transmitted power $P_T$ is given by

$$P_T = P_0 \exp(-\alpha L)$$  \hspace{1cm} 2.3

where $\alpha$ is the attenuation constant, usually quoted in units of dB/km.

Due to power loss the effective interaction length of the fibre is

$$L_{Att} = \frac{1}{\alpha_p} [1 - \exp(-\alpha_p L)]$$  \hspace{1cm} 2.4
where $\alpha_p$ is the attenuation at the pump frequency, $L$ is the length of the fibre.

There are several factors contributing to the attenuation. Pure silica absorbs predominantly in the ultraviolet and the mid infrared. However, even a small amount of impurities can lead to significant absorption in range 0.3-2 $\mu$m, the most important impurity being hydroxyl ions, which have characteristic absorption peaks at 1.37 and 1.23 $\mu$m. At shorter wavelengths, Rayleigh scattering is the dominant loss mechanism. This arises from random density fluctuations frozen into the fused silica during manufacture and sets the ultimate limit on the fibre loss. The attenuation constant due to Rayleigh scattering can be written as

$$\alpha_R = C \lambda^{-4}$$  \hspace{1cm} (2.5)

where the constant $C = 0.63$ dB $\mu$m$^4$/km for fused silica. The total attenuation for a typical fused silica single mode fibre has a minimum loss of approximately 0.2 dB/km at 1.55 $\mu$m (figure 2.2).

![Figure 2.2](image)

**Figure 2.2** Typical loss profile of a single mode optical fibre [Corning SMF-28].

The effect of attenuation on the evolution of a slowly varying electric field $A(z,t)$ can be described by

$$\frac{\partial A}{\partial z} = -\frac{1}{2} \alpha A$$  \hspace{1cm} (2.6)

where $\alpha$ is the total optical power attenuation and $z$ is the propagation distance.
2.2.2 Chromatic Dispersion

When an electromagnetic wave interacts with the bound electrons of a dielectric material, the medium response depends on the optical frequency $\omega$. This property, referred to as chromatic dispersion, manifests itself through the frequency dependence of the refractive index $n(\omega)$. On a fundamental level, it is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of the bound electrons. Far from the medium resonances, the refractive index is well approximated by the Sellmeier equation [Born & Wolf 1985]

$$n^2(\omega) = 1 + \sum_{i=1}^{m} \frac{B_i \omega_i^2}{\omega_i^2 - \omega^2}$$

where $\omega_i$ is the resonance frequency and $B_i$ is the strength of the resonance.

Fibre dispersion plays a critical role in the propagation of short optical pulses since different spectral components of the pulse travel at different speeds given by $\frac{c}{n(\omega)}$. The fibre dispersion is the sum of the material dispersion and the waveguide dispersion. The material dispersion arises from the dispersion of the core material and the lower refractive index cladding (as some of optical mode is in the cladding). The waveguide dispersion occurs because of dielectric waveguiding of the optical mode, where the effective mode index is slightly lower than the material index. The total dispersion can be engineered by the use of multiple cladding layers, where, for example, the zero dispersion wavelength can be shifted from 1.3 $\mu$m to 1.55 $\mu$m by the use of a second cladding layer (chapter 8).

The effects of fibre dispersion are accounted for by expanding the mode propagation constant $\beta(\omega)$ in a Taylor series about the central frequency $\omega_0$,

$$\beta(\omega) = n(\omega) k(\omega) = \frac{\omega n(\omega)}{c}$$

$$\beta(\omega) = \beta(\omega_0) + \left( \frac{d\beta(\omega)}{d\omega} \right)_{\omega_0} (\omega - \omega_0) + \frac{1}{2} \left( \frac{d^2\beta(\omega)}{d\omega^2} \right)_{\omega_0} (\omega - \omega_0)^2 + \frac{1}{6} \left( \frac{d^3\beta(\omega)}{d\omega^3} \right)_{\omega_0} (\omega - \omega_0)^3 + ...$$

The first term in the expansion is the group delay of the pulse envelope. This is proportional to the accumulated phase shift $\Phi(\omega)$ that the pulse envelope suffers after propagation through a length $L$ of material. The net phase shift for a spectral component of frequency $\omega_0$ as a function of $L$ can be written as [Siegman 1986]
\[ \Phi(\omega_0) = \beta(\omega_0) L \quad 2.10 \]

The second term \( \beta_1 \) is related to the group velocity, \( v_g \) as the pulse envelope travels in a medium with a modified refractive index (equation 2.11). The phase shift suffered by the group velocity is proportional to \( (\omega - \omega_0) \), and is termed a linear frequency shift.

\[
\beta_1 = \left( \frac{d\beta(\omega)}{d\omega} \right)_{\omega_0} = \frac{1}{v_g} = \frac{1}{c} \left[ n(\omega_0) + \omega_0 \left( \frac{dn}{d\omega} \right)_{\omega_0} \right] \quad 2.11
\]

The third term \( \beta_2 \) is the group velocity dispersion (GVD) and is responsible for pulse broadening due to the different frequency components travelling at slightly different velocities in the pulse. The phase shift suffered by the group velocity dispersion is proportional to \( (\omega - \omega_0)^2 \), and is termed a quadratic frequency shift. The GVD is written as

\[
\beta_2 = \left( \frac{d^2\beta(\omega)}{d\omega^2} \right)_{\omega_0} = \frac{1}{c} \left[ 2 \left( \frac{dn}{d\omega} \right)_{\omega_0} + \omega_0 \left( \frac{d^2n}{d\omega^2} \right)_{\omega_0} \right] \approx \frac{\omega_0}{c} \left( \frac{d^2n}{d\omega^2} \right)_{\omega_0} = \frac{\lambda_0^3 \left( \frac{d^2n}{d\lambda^2} \right)}{2\pi c^2} \quad 2.12
\]

The fourth order term \( \beta_3 \) becomes important only when the pulse wavelength coincides with the zero dispersion wavelength \( \beta_2 = 0 \), or when the pulse width is less than 0.1 ps (ie when \( \Delta \omega \approx \omega_0 \)). For most practical situations the expansion is truncated at \( \beta_2 \).

Another useful parameter is the dispersion parameter \( D \), which is commonly used instead of \( \beta_2 \). It is related to the GVD by

\[
D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2 \quad 2.13
\]

where \( D \) has units of ps/km.nm.

The sign of \( \beta_2 \) is positive in the visible part of the spectrum (which is also referred to as the normal dispersion regime), changes sign at 1.27 \( \mu \)m for bulk fused silica and is negative at longer wavelengths. The zero dispersion wavelength in a single mode fused silica fibre is typically slightly higher but can be shifted to 1.55 \( \mu \)m by suitably adjusting the design parameters [Agrawal 1989].
The effect of chromatic dispersion on the evolution of a slowly varying electric field \( A(z,T) \) is given by

\[
\frac{\partial A}{\partial z} = -\frac{i}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} - \frac{i}{24} \beta_4 \frac{\partial^4 A}{\partial T^4}.
\]

where \( A \) is the pulse amplitude, and \( T = t - \beta_z z \) is a frame of reference moving with the pulse at the group velocity \( v_g \).

### 2.2.3 Birefringence and Polarisation Mode Dispersion

**Birefringent Single Mode Fibres**

Single mode fibres with nominally circular symmetry about the fibre axis are actually bimodal since they can propagate two modes with orthogonal polarisation, the \( HE_{11}^i \) and \( HE_{11}^j \) modes. The principal axes, \( x \) and \( y \) are determined by the symmetry of the cross section. The larger the anisotropy of the cross section the greater the difference in the propagation constants \( \beta_x \) and \( \beta_y \) for the two polarisation modes. If the fibre cross section is independent of the fibre length then the fibre behaves like a birefringent medium with a modal birefringence given by

\[
B = \frac{\lambda (\beta_x - \beta_y)}{2\pi} = |n_x - n_y|
\]

where \( \lambda \) is the optical wavelength, and \( n_x \) & \( n_y \) are the refractive indices of the principal axes. Light polarised along one of the principle axes will retain its polarisation along all the propagation distance. However, light polarised at an angle with respect to an axis will pass through various states of elliptic polarisation, returning back to the initial state of polarisation after a distance

\[
L_p = \frac{\lambda}{B} = \frac{2\pi}{(\beta_x - \beta_y)}
\]

known as the beat length. The beat length can be observed directly by means of scattering from the fibre. Since the radiation pattern of a dipole has a null along the dipole axis and a maximum normal to the axis, a fibre viewed along the direction of the incident polarisation will exhibit a series of dark and light bands, with a period equal to the beat length.
A large amount of modal birefringence can be intentionally introduced into a single mode fibre with either the use of an elliptical core, or by using a stress induced asymmetry. In a high birefringence single mode fibre the beat length is typically several millimetres. However, in a standard single mode fibre with a nominally circular core, the inherent birefringence is typically four orders of magnitude smaller.

Polarisation Mode Dispersion in Single Mode Fibres

Nominally circular fibres do not maintain the state of polarisation present at the input for more than a few meters, because of polarisation coupling perturbations that are randomly distributed along the length of the fibre. The perturbations may be variations in geometry, composition, or strain, and can couple energy from one normal polarisation mode to the other. The perturbations of the fibre birefringence can be described as the vector sum of a length independent component, and randomly fluctuating length dependent component,

\[ \bar{B}(\omega, z) = B_0(\omega) + B(z). \]  \hspace{2cm} (2.17)

The length independent birefringence, \( B_0(\omega) \) is assumed to be equal to the linear birefringence (\( \beta_x - \beta_y \)), and the randomly fluctuating length dependent component, \( B(z) \) is assumed to be white Gaussian noise, with mean zero, and variance \( \sigma^2 = 2h \) [Foschini and Poole 1991].

Strong coupling occurs when the random perturbations are spatially distributed with a period equal to the beat length of the fibre. When this phase matching condition is satisfied substantial coupling will occur. In practice, a large number of perturbing birefringences exist and are distributed in a pseudo random fashion. The cross coupling then becomes an average over all of these perturbations and is then described by the polarisation mode coupling coefficient \( h \). Typical coupling coefficients in good polarisation maintaining fibres are in the range \( h \approx 3 - 5 \times 10^{-6} m^{-1} \), while for standard single mode fibres \( h \approx 3 - 5 \times 10^{-2} m^{-1} \) [Newport].

An expression for the powers in each orthogonal polarisation state \( P_x \) and \( P_y \) as a function of fibre length can be derived from mode coupling theory [Kaminow 1981]. If a normal mode with polarisation \( x \) is excited at the input of a fibre of length \( L \), and the output powers in the two normal modes are \( P_x \) and \( P_y \), then the extinction coefficient can be defined as
The powers $P_x$ and $P_y$ as a function of fibre length due to the random perturbations that couple the two polarisation are

$$\langle P_x \rangle = e^{-hz} \cosh(hz) \quad \text{and} \quad \langle P_y \rangle = e^{-hz} \sinh(hz), \quad \text{where} \quad \langle \eta \rangle = \tanh(hz)$$

for $P_x(0) = 1$ and $P_y(0) = 0$. The powers in each polarisation mode shown in figure 2.3. For a high birefringent fibre the coupling distance, $h^{-1}$ is usually much greater than the propagation distance, and hence very little power transfer takes place. For a standard single mode fibre, the coupling distance is approximately $h^{-1} \approx 20 - 30m$, and the polarisation mode quickly degrades for typical propagation lengths.

**Figure 2.3** Polarisation mode dispersion for a typical single mode optical fibre.

The polarisation dispersion is shown in figure 2.3. For standard single mode fibres the polarisation mode may be preserved over 20 to 40 metres before degrading to a completely random state. However, this value varies from fibre to fibre because of the underlying geometric and stress irregularities, which can also be introduced into the fibre with ambient temperature.

Polarisation mode dispersion can be a problem in communications systems because when a pulse enters a single mode fibre that has its spectral content confined to a single polarisation state, the output
pulse typically has its polarisation state spread over many states (as described by the telegraphers equation [Gisin et al 1991]) and which can lead to broadening in time. In this thesis a polariser was placed inside a fibre laser cavity, and it was important that the cavity length was not excessive so that the polarisation state was not degraded excessively after one roundtrip.

2.3 Propagation Nonlinearities

The response of any dielectric medium to light becomes nonlinear for intense electromagnetic fields, and optical fibres are no exception. The origin of the nonlinear response is related to the anharmonic motion of bound electrons under the influence of an applied field. As a result, the induced polarisation $P$ from the electric dipoles is not linear in the electric field $E$, but satisfies a more general relation

$$P = \varepsilon_0 \left( \chi^{(1)} \cdot E + \chi^{(2)} \cdot E \cdot E + \chi^{(3)} \cdot E \cdot E \cdot E + \ldots \right)$$ 2.20

where $\varepsilon_0$ is the vacuum permittivity and $\chi^{(j)} (j = 1, 2, \ldots)$ is the jth order susceptibility. To account for the light polarisation effects, $\chi^{(j)}$ is a tensor of rank $j+1$. The linear susceptibility $\chi^{(1)}$ represents the dominant contribution to $P$, its effects are responsible for the refractive index and the attenuation coefficient. The second order term is nonzero only for media which lack an inversion symmetry at the molecular level. Since SiO$_2$ is a symmetric molecule, $\chi^{(2)}$ vanishes for silica glasses.

The lowest-order nonlinear effects in optical fibres originate from the third order susceptibility which is responsible for phenomena such as third harmonic generation, four wave mixing and nonlinear refraction. Terms of order higher than third have sufficiently small susceptibility coefficients that they can be neglected. Successive components of the nonlinear susceptibility diminish in accordance with the approximate rule

$$\chi^{(n+2)} / \chi^{(n)} \approx |E_0|^{-2}$$ 2.21

where the electric field $E_0$ is characteristic of the nonlinear medium [Loudon 1988]. The electrical fields used in the experiments in this thesis are in the order of $10^{12}$ Vm$^{-1}$, therefore the susceptibility expansion can be truncated at the third order term.

Unless special efforts are made to achieve phase matching, the nonlinear processes responsible for the generation of new frequencies are not efficient in optical fibres. Most of the nonlinear effects in
optical fibres therefore originate from nonlinear refraction, the intensity dependence of the refractive index resulting from the contribution of $\chi^{(3)}$, which is also known as the Kerr effect. The refractive index of the fibre becomes

$$n(\omega,E) = n(\omega) + n_2 |E|^2$$

where $n(\omega)$ is linear refractive index given in equation 2.6, $|E|^2$ is the optical intensity inside the fibre, and $n_2$ is the nonlinear refractive index related to the third order susceptibility by

$$n_2 = \frac{3}{8n(\omega)} \chi^{(3)}$$

The value of $n_2$ for standard single mode optical fibres is approximately $n_2 = 2.5 \times 10^{-20}$ m$^2$/W [Barry et al. 1997]. The intensity dependent refractive index leads to a number of interesting nonlinear optical effects including self phase modulation and self steepening. These will now be discussed in turn, with the other nonlinear effects comprehensively reviewed by Agrawal [1989].

### 2.3.1 Self Phase Modulation

When a light pulse with the envelope $|E|^2$ travels in a nonlinear medium different parts of the pulse undergo various nonlinear perturbations due to the intensity dependence of the refractive index. From equation 2.22 the phase of the optical field changes by

$$\phi(\omega) = n(\omega,E(t)) k(\omega) L = (n(\omega) + n_2 |E(t)|^2) k(\omega) L$$

where $k(\omega) = \frac{\omega}{c}$ and $L$ is the fibre length. This intensity dependent nonlinear phase shift is called self phase modulation (SPM). For $n_2 > 0$, the leading edge of the pulse will be down-shifted in frequency, and the trailing edge of the pulse will be up-shifted in frequency. The phase shift is related to a frequency shift of the frequency components by

$$\delta\omega(t) = -\frac{1}{2\pi} \frac{\partial \Phi(t)}{\partial t}$$
This frequency shift is responsible for spectral broadening of the pulse. The instantaneous frequency shift for a Gaussian pulse is shown in figure 2.4 (b) and is nonlinear over the entire pulse.

![Figure 2.4](image)

**Figure 2.4** (a) Temporal intensity profile of a Sech pulse, and (b) the corresponding frequency shift due to SPM.

The effect of self phase modulation in the normal dispersion regime is to generate a linear frequency shift across the central region of the pulse. This is a consequence of the lower frequencies in the leading edge of the pulse travelling faster than the peak, and the trailing edge travelling slower, and therefore the pulse becomes broader.

If a pulse with a frequency shift generated by SPM passes through an optical fibre with anomalous GVD, the trailing half of the pulse containing the blue frequencies is advanced, while the leading half containing the red frequencies is retarded, and the pulse will be initially compressed (eg. figure 6.42), while in the normal dispersion regime the opposite applies.

The interaction length required for SPM to approximately double the spectral width of a pulse (in the absence of any GVD) is known as the nonlinear length \( L_{NL} \)

\[
L_{NL} = (\gamma P_o)^{-1}
\]

where \( P_o \) is the peak power. \( \gamma \) is the nonlinearity coefficient and it is defined as

\[
\gamma = \frac{n_2 \omega_o}{c A_{eff}}
\]
where $A_{\text{eff}}$ is the effective mode area, which is approximated by the core area in the case when $V \sim 2.4$ [Stolen 1980].

2.3.2 Self Steepening

Self steepening (SS) of an optical pulse results from the intensity dependence of the group velocity, and leads to an asymmetry in the SPM broadened spectra. As the pulse propagates inside the fibre, it becomes asymmetric with its peak (with a higher refractive index) shifted toward the trailing edge. As a result the trailing edge of the pulse becomes steeper.

Self steepening of the trailing edge of the pulse eventually creates an optical shock analogous to the development of an acoustical shock on the leading edge of a sound wave. The critical distance corresponding to the shock formation for a hyperbolic secant pulse is

$$L_{\text{SS}} = 1.32 L_{\text{NL}} \frac{T_o}{T_{\text{opt}}}$$

where $T_o$ is the pulse width, $T_{\text{opt}}$ is the optical period ($= 2\pi/\omega_o$) and $L_{\text{NL}}$ is the nonlinear length (equation 2.26). A notable feature is that SS induced spectral broadening becomes larger on the blue frequency side of the peak, than the red side, leading to a spectral asymmetry. Self steepening becomes important when the peak power is greater than $\sim 1$ kW, and the pulse duration $\ll 0.1$ psec, hence this effect can be neglected for most studies conducted in this thesis [Agrawal 1989].

2.3.3 Stimulated Scattering Processes

The nonlinear effects governed by the third order susceptibility are elastic since no energy is transferred from the electromagnetic field to the dielectric medium. A second class of nonlinear effects results from stimulated inelastic scattering in which the optical field transfers part of its energy to the nonlinear medium. Two important nonlinear effects in optical fibres fall into this category; both related to the vibrational modes of silica, known as stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS). In SRS phonons of optical frequencies participate in the scattering, whereas in SBS the phonons are of acoustic frequencies. A fundamental difference is that SBS in optical fibres occurs predominantly in the backward direction (due to a phase matching requirement) whereas SRS dominates
in the forward direction. The experiments conducted in this thesis were below the threshold for stimulated scattering processes and hence these effects were not observed. For a comprehensive review of these nonlinear effects in optical fibres, the reader is referred to the review by Agrawal (1989).

2.4 The Propagation Equation

The evolution of an optical pulse in a single-mode fibre under the influence of attenuation, dispersion and Self Phase Modulation, is described by the dimensionless nonlinear Schrödinger equation (NLSE) [Tomlinson et al. 1984]

\[
\frac{\partial A}{\partial z} - \frac{1}{2} \alpha_s A + \frac{i}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} + \frac{i}{24} \beta_4 \frac{\partial^4 A}{\partial T^4} = -i |A|^2 A
\]

where \( A \) is the normalised pulse amplitude. The reference frame is taken to be moving with the pulse at the group velocity \( v_g \). The second term is from energy loss due to attenuation, the next three terms account for group velocity dispersion and the final term represents the contribution from self phase modulation. The NLSE is solved in general by using the Split Step Fourier method, which gives numerical solutions for the field as a function of distance along the fibre [Agrawal 1989]. There are also several analytical solutions which may be obtained when only a small number of nonlinear effects are considered.

There are two length scales that can be introduced to determine the relative importance of the terms in the NLSE. The dispersion length \( L_D \) and the nonlinear length \( L_{NL} \) provide length scales over which the effects of GVD and nonlinearity (SPM) become important.

\[
L_D = \frac{T_0^2}{|\beta_2|} \quad L_{NL} = \frac{1}{P_0}
\]

If the length scale is shorter than the propagation length then that process is important to the evolution of the pulse. When the nonlinear length \( L_{NL} \) is short compared to the length of the fibre, self phase modulation becomes an important effect. The spectral width increases from the self-phase shifts introduced on the various frequency components which make up the pulse. In the normal group velocity dispersion regime the pulse width widens due to GVD. This acts to linearise the frequency chirp of the
output pulse. The interaction lengths will be used to identify the dominant mechanisms in the experiments discussed in chapters 6, 7 & 8.

In the anomalous dispersion regime the GVD frequency chirp is in the opposite direction and can balance the frequency chirp due to SPM, to create a pulse which will propagate unchanged. Solutions of equation 2.29 indicate that if a hyperbolic secant pulse, with $P_o$ and $T_o$ suitably chosen, is launched inside an ideal lossless fibre it will propagate undistorted for an arbitrary long distance. This type of pulse is known as a soliton. The peak power required to support the fundamental soliton is given by

$$P_1 = \frac{\beta_2}{gT_o^2} = \frac{3.11\beta_2}{gT_{FWHM}^2}$$  \hspace{1cm} 2.32$$

where $T_{FWHM} = 1.76 \ T_o$ for a hyperbolic secant pulse. Higher order solitons are also described using equation 2.31, and have a peak power $P_N = N^2 P$, which evolve during propagation with a period equal to $\pi/2$ times the dispersion length.
Numerical solutions of the NLSE for a second order soliton over two soliton periods are shown in figure 2.5. The periodic evolution is characterised by pulse compression and then broadening, to recover the original shape at the end of one soliton period. However, when the initial pulse shape is not matched to an integer order soliton, the pulse may adjust itself as it propagates by dispersing energy to eventually evolve into an integer order soliton [Agrawal 1989].
Chapter 3

Apparatus and Experimental Techniques

This chapter describes the measurement equipment and techniques used in the experiments conducted for this thesis. It begins with a description of a technique for introducing a controlled amount of propagation loss. Then it is shown how optical fibre based waveplates may be constructed to control polarisation transformations. Later sections describe optical fibre based components which are available commercially, and it is shown how optical loop mirrors and switches may be constructed from these components. The next sections give details of the measurement equipment, including a frequency resolved optical gate which is used to recover the electrical field envelope of the laser pulses. The chapter concludes with a discussion of chronocyclic representations, which allow an intuitive visualisation of the experimental laser pulses in the time-frequency domain.

3.1 Introduction of Excess Attenuation

1. Splice Loss
When two fibres which have different mode field diameters are connected together, some of the light will be lost because of the mode field mismatch. This loss can be estimated by

$$\alpha_s = 20 \log_{10} \left( \frac{2 \omega_1 \omega_2}{\omega_1^2 + \omega_2^2} \right)$$

3.1

where $\omega_1$ and $\omega_2$ are the mode field diameters of the two fibres [Marcuse 1977]. For similar fibres the splice loss was typically less than 0.05 dB for experiments in this thesis when fusion splicing was used. However, since Erbium doped fibres can have a core which is almost half the diameter of standard single mode fibres, the mode field mismatch in this case will be significant.
2. Curvature Losses

The loss caused by a constant curvature has been studied in some detail by earlier workers, and an empirical relationship has developed [Jeunhomme 1982]. The curvature loss can be estimated from

\[ \alpha_c = 3.0 \times 10^7 (\Delta n)^{1/4} (\lambda R)^{-1/2} \left( \frac{\lambda_c}{\lambda} \right)^{3/2} \exp \left[ -7.1 \times 10^5 \frac{R}{\lambda} (\Delta n)^{2/3} \left( 2.743 - 0.996 \frac{\lambda}{\lambda_c} \right)^3 \right] \]

in dB per km, where \( \Delta n \) is the core cladding refractive index difference, \( \lambda \) is the operating wavelength in microns, \( \lambda_c \) is the cut-off wavelength and \( R \) is the radius of curvature in meters. The expression is accurate to approximately 10 percent for \( 1 \leq \lambda/\lambda_c \leq 2 \). Figure 3.1 shows the curvature loss for various values of the cut off wavelengths and index differences when 1.55 \( \mu \)m is the operating wavelength.

![Curvature Loss for several values of the cut-off wavelength and index difference, \( \lambda = 1.55 \mu \)m.](image)

When the cut off wavelength is close to the operating wavelength the curvature losses are only significant when the bend radius is small, less than 5 cm for typical single mode fibres. However, when the fibre is guiding light which is far from the cut off wavelength the curvature losses can be the dominant loss mechanism. This can arise in an Erbium doped optical fibre which is designed to support both the 0.98 \( \mu \)m pump and 1.55 \( \mu \)m signal light in a single mode. It is desirable in this instance to wind the fibre on a spool with a diameter larger than the typical commercial fibre spool (\( R = 7.7 \) cm). Standard single
3.2 The Control of Polarisation

Birefringence in single mode fibre is known to result (1) from deviations of the core geometry from the ideal straight circular cylinder and (2) from mechanical stress through the elasto-optic effect. A perfectly azimuthally symmetric single mode fibre would not have any birefringence, i.e. the group index of the propagating HE\textsubscript{11} mode would be independent of the plane in the fibre. In practice all single mode fibres have some birefringence due to azimuthal asymmetry [Kaminow 1981].

1. Fibre Birefringence due to Bending

Birefringence in single mode optical fibres arises from both a geometrical deformation of the mode fields and through an elasto-optic effect. The geometrical birefringence has been shown to be approximately 3 orders of magnitude smaller than the stress induced birefringence [Stone 1988], and hence it is neglected in this analysis.

When a fibre experiences a pure bending stress, there is a component of induced birefringence in the direction of the fibre radius, which varies across the fibre in the direction of the bend radius. From the center of the fibre towards bend radius pivot the stress component is compressive, but it is a tensile stress in the outer radial direction of the bend. The stress is an odd function, with the optical mode being an even function, and it has been shown that this cannot produce birefringence in the HE\textsubscript{11} mode [Ulrich et al 1980]. There is a second order effect, in the direction of fibre bend radius which does produce birefringence. The change in refractive index can be calculated from this second order effect and is given by

\[ \delta n_{\text{bend}} = \frac{n^3 (p_{11} - p_{12}) (1 + \nu) (r/R)^2}{4} \]

where \( n \) is the refractive index, \( p_{11} - p_{12} \) are the strain optical coefficients, \( \nu \) is Poisson's ratio, \( r \) is the outer fibre diameter, and \( R \) is the bend radius [Lefevre 1980]. For fused silica the typical material constants are \( n = 1.46 \), \( p_{11} - p_{12} = -0.15 \), \( \nu = 0.17 \). For silica based optical fibres equation 3.3 reduces to
\[ \delta n_{\text{bend}} = a \left( \frac{r}{R} \right)^2 \]

where \( a = 0.1365 \), with the fast axis coinciding with the radius of curvature. When the light has a polarisation state with an axis between the fast and slow axes of the fibre the power between the two orthogonal polarisation modes is periodically exchanged with a period given by the beat length \( L_b \) defined by

\[ L_b = \frac{\lambda}{\delta n} = \frac{\lambda}{a} \left( \frac{R}{r} \right)^2 \]

where \( \lambda \) is the wavelength of the light.

Using equation 3.5 the beat length due to the bend induced birefringence can be calculated. For typical single mode fibres the diameter \( 2r = 125 \, \mu\text{m} \), a typical commercial fibre spool has diameter of \( R = 15.4 \, \text{cm} \), at \( 1.55 \, \mu\text{m} \) the beat length is approximately 17.2 m. The beat length as a function of bend radius, using these parameters is plotted in figure 3.2.

![Figure 3.2](image)

Figure 3.2 Beat length due to bend induced birefringence, fibre diameter \( 2r = 125 \, \mu\text{m}, \lambda = 1.55 \, \mu\text{m} \).

The bending induced birefringence produces a small change in refractive indices between the two axes in the fibre. It is possible to construct an all fibre device where the total phase shift between the two axes is an integral multiple of the wavelength of the light, ie a waveplate. The condition for the fractional waveplate is
where \( N \) is the number of turns on the coil of radius \( R \), and \( m \) is fractional order of the waveplate. If the birefringence is only from the bending of the fibre, the radius of the waveplate can be calculated to be

\[
R = \frac{2\pi r^2}{\lambda} Nm
\]

To control the state of polarisation completely it is necessary to have a sequence of a quarter wave plate, half wave plate and then another quarter wave plate. This can be achieved using \( Nm = 4 \) as a minimum requirement, but for a fibre of diameter \( 2r = 2 \times 125 \mu m \), the radius of curvature is 8.6 mm. To increase the radius of curvature (decreasing losses) \( Nm = 8 \) can be used, giving a radius of curvature \( R = 1.73 \) cm, and a total length of the wave plate sequence of 87 cm. Therefore the fibre optic quarter wave plate would be constructed using 2 turns, and the half wave plate with 4 turns around the coil of radius \( R = 1.73 \) cm.

2. Other Methods for Inducing a Change in Birefringence

There are several other methods for inducing a change in birefringence, and all involve subjecting the fibre to mechanical stress. These methods include twisting [Ulrich and Simon 1989], or applying a transverse clamping stress [Stone 1988]. Since these methods are not used in this thesis they will not be discussed any further.

3.3 Fibre based Isolators and Polarisers

The performance of many optical components (eg. laser diodes and optical amplifiers) is very sensitive to light reflected back into them, from splices, connectors, filters and even backwards scattered light from Rayleigh scattering in the fibre itself. In many communication networks and laser configurations it is also necessary to enforce unidirectional propagation. To achieve unidirectional propagation optical fibre based isolators are widely used.
The isolators used in fibre optic communications systems all use the nonreciprocal nature of Faraday rotation, which is conveniently achieved in bulk materials by coupling the light from the optical fibre with a graded index lens. The fibre end faces are typically polished at 8 degrees, and all interfaces are antireflection coated to reduce back reflections. There are two types of isolators available, polarisation sensitive and polarisation insensitive, and both were used in this thesis.

The single stage polarisation sensitive isolator is shown in figure 3.3.a, and is comprised of two graded index lenses (G), two polarisers (P) and a rod of Faraday material (F) in a magnetic field. The two polarisers are oriented at 45 degrees to one another. The Faraday material, usually a Yttrium Iron Garnet (Bi-YIG) film of < 500 μm thickness, rotates the polarisation by 45 degrees in one direction, to pass through the second polariser with minimal losses. Counter-propagating light is blocked by the polariser on the right, since the 45 degree rotation will bring the polarisation state to be at 90 degrees to that polariser.

The most widely used isolators are those which are insensitive to the input polarisation state. This is necessary since the state of polarisation is quickly degraded to a random state after propagation through standard fibre (section 2.2.3). The polarisation insensitive isolator is shown in figure 3.3.b, where two Faraday rotators (F) each provide 45 degree rotation [Chang and Sorin, 1990]. Birefringent TiO₂ (ie Rutile; R) crystals provide spatial walk-off for the extraordinary polarisation (e ray) travelling through the isolator. Considering propagation from right to left, the first crystal R provides walk-off for the e ray, the o ray (ordinary polarisation) is not deflected. After a rotation of 45 degrees at the first Fraday rotator, the centre crystal R provides a walk-off for the o ray only. After the second 45 degrees rotation, the e ray only walks off to recombine with o ray, and is efficiently coupled into the output fibre. Propagation in the other direction is strongly attenuated since the walk-off in central crystal R (of 90 degree different polarisation) deflects the rays in the incorrect direction, so are not collimated at the right hand side fibre.

The isolators used in this thesis were of commercial manufacture, and exhibited high performance. The polarisation sensitive isolator had an isolation of 44 dB, an insertion loss of 0.54 dB (12 % loss), and
a reflectivity of smaller than -60 dB. The polarisation insensitive isolator had an isolation of greater than 68 dB, an insertion loss of 0.68 dB, and a reflectivity of smaller than -60 dB. Insertion losses of 0.2 - 0.3 dB are commercially available, but have a smaller isolation (25 - 40 dB) [Etek].

### 3.4 Directional Couplers and Wavelength Division Multiplexing

Directional couplers are the fibre optic equivalent of a beam splitter. They have found widespread application in fibre amplifiers and lasers because they are in fibre form and have low insertion loss, low excess loss (~<0.04 dB), are relatively low cost, and were used extensively in the experimental work performed in this thesis.

The fused fibre coupler is formed by bringing two single mode fibres into close proximity, twisting them together slightly to ensure good contact, and heating until the glass softens and fuses. During the fusion process, the fibre ends are pulled apart so that the glass in the vicinity of the heat is tapered down to a diameter that is a fraction of the diameter of one of the fibres. The reduced core diameter allows most of the optical field to reside outside of the core and coupling occurs when the evanescent field from one waveguide couples to the other. Light is launched into one of the ports while it is being made, and the output from the two ports at the opposite sides is monitored to determine the coupling ratio. The tension is released and the heat is no longer applied when the desired coupling is reached.

![Schematic of a fused fibre coupler](image)

**Figure 3.4** Schematic of a fused fibre coupler

A variety of useful optical components can be made that exploit the fact that the power coupled from one fibre to another can be varied by changing $Z$, the length of the coupling region over which the fields interact, $a$ the core radius in the coupling region, and $\Delta a$, the difference in core radii in the coupling region. The equation that governs the power coupling coefficient from one fibre to the other is
\[ K^2 = F^2 \sin^2 \left( \frac{CZ}{F} \right) \]

where \( F \) expresses the effect of the core-diameter difference, and

\[ F^2 = \left[ 1 + \left( \frac{234 a^2}{\lambda^3} \right) \left( \frac{\Delta a}{a} \right)^2 \right]^{-1} \quad \text{and} \quad C = 21 \lambda^{5/2} / a^{7/2} \]

expresses the coupling between the fields of the two fibres [Tekippe 1990]. Since \( K^2 \) is a sinusoid, there are many opportunities for trading \( Z, L, a \) and \( \Delta a \) against one another. For example, a coupler could be made that delivers almost all of the energy around 1550 nm to one output and almost all the energy around 980 nm to the other. This is done by making a peak of \( K^2 \) lie at 980 nm and a trough at 1550 nm. By conservation of energy, this situation corresponds to a trough at 980 nm and a peak at 1550 nm for the other output port. These wavelength division multiplexing (WDM) couplers are also used in the experiments described later in this thesis.

### 3.5 Optical Fibre Loop Reflectors

In this section the performance of fibre loop reflectors, made from directional couplers, are examined. The loop mirror theory [Mortimore 1988] [Urquart 1993] is generalised to describe nonlinear and amplifying effects, which are discussed in the next three sections.

The loop mirror is made by joining together two arms of a directional coupler to form a Sagnac interferometer, and since it is a non resonant interferometer its performance must be explained by the coherent superposition of fields. The complex amplitude components of the wavefronts are subscripted 1 to 4, corresponding to the four ports indicated in figure 3.5.
The direction of propagation is indicated by the superscript $t$, or $a$, for towards or away from the directional coupler. The amplitude of the wavefront incident on the directional coupler from port 1 is $E'_1$.

Wavefront division occurs at the coupler to produce $E'_2$ and $E'_3$, given by

$$E'_2 = j\sqrt{g} \sqrt{k} E'_1 \quad \text{and} \quad E'_3 = \sqrt{1-k} E'_1,$$

where $k$ is the coupling ratio, and $j$ represents the $\pi/2$ phase shift between ports 2 and 3 [HP Fiber Optics Handbook 1988]. Having entered the loop the two wavefronts propagate in opposite directions until incident on the coupler. The optical path lengths of the two oppositely propagating waves are identical, and in the absence of nonlinear effects the waves experience the same total phase shift. The amplitude of the wavefronts incident on the directional coupler after propagating around the loop is

$$E'_2 = E'_3 \sqrt{g} e^{(-a'/2+j\beta'_t)L} \quad \text{and} \quad E'_3 = E'_2 e^{(-a'/2+j\beta'_t)L}$$

where $a$ is the power loss coefficient, $\beta$ is the propagation constant of the light in the fibre, and $g$ is the gain of a linear gain element (of infinitesimal length). At the coupler the two wavefront components coherently combine to produce wavefront components propagating away from the coupler, $E'_4$ and $E'_5$, given by

$$E'_4 = j\sqrt{k} E'_2 + \sqrt{1-k} E'_3 \quad \text{and} \quad E'_5 = j\sqrt{k} E'_3 + \sqrt{1-k} E'_2$$

The transmission and reflection of the loop mirror can be calculated from the equations describing the wavefront amplitudes. The transmitted amplitude and power are given by

$$t = E'_4 / E'_1 = \sqrt{g}[(1-k)e^{j(1-k)n_z|r|2 \pi L / \lambda} - ke^{igk_n_z|r|2 \pi L / \lambda}]e^{-at'/2}$$

and

$$T = t \ast t = g[(1-k)^2 + k^2 - 2k(1-k)[1 + \cos((1-k(1+g))n_z|r|^2 \pi L / \lambda)]e^{-at'}$$

respectively. Similar equations can be derived for the reflected amplitude and power

$$r = E'_5 / E'_1 = j\sqrt{g} \sqrt{1-k} [e^{j(1-k)n_z|r|2 \pi L / \lambda} + ke^{igk_n_z|r|2 \pi L / \lambda}]e^{-at'/2}$$

29
and \[ R = r^* r = 2gk(1-k)[1 + \cos[(1-k(1+g))n2|E_1|^2 2\pi L/\lambda]e^{-aL}. \] 3.17

3.5.1 Linear Optical Loop Mirror

When the loop reflector is constructed without a gain element \((g = 1)\), and the length \(L\) is comparatively short, so that the nonlinear phase shift and attenuation can be neglected, the device behaves as a Sagnac interferometer. The equations for the transmitted and reflected power reduce to simple parabolic forms,

\[ T = (1-2k)^2 \quad \text{and} \quad R = 4k(1-k) \] 3.18

which are only functions of the coupling ratio \(k\).

The transmitted and reflected intensities are shown in figure 3.6. The loop mirror behaves as a perfect reflector when the coupling ratio is exactly 50:50. However, when there is a slightly asymmetric splitting ratio the device still operates as an excellent mirror, transmitting only 1\% when the splitting ratio reaches \(k = 0.45\).

![Figure 3.6 Fibre Loop Mirror Performance, Transmitted and Reflected Power](image)

The phase changes associated with transmission and reflection can be determined from the equations describing the transmitted and reflected amplitude. When operated in reflection the amplitude undergoes a \(\pi/2\) phase shift, as indicated by the \(j\) in equation 3.16. On transmission, the amplitude is negative when
the coupling ratio is less than \( k = 0.5 \), indicating a \( \pi \) phase shift. There is no net phase shift on transmission when the coupling ratio is greater than \( k = 0.5 \), since the amplitude is positive.

### 3.5.2 Nonlinear Optical Loop Mirror (NOLM)

In the previous section the performance of fibre loop reflectors, made from directional couplers, was studied. The transmission and reflection coefficients were found to be dependent only on the coupling of the directional coupler. This is a consequence of the equal path lengths experienced by the oppositely propagating wavefronts. In this section the loop reflector is studied with an additional nonlinear phase shift, causing the oppositely propagating wavefronts to travel slightly different distances. Although both wavefronts are contained in the same physical medium, when the optical intensities are large the refractive index of the medium becomes slightly intensity dependent due to SPM, and this is the basis of nonlinear fibre based devices for ultra fast switching and logic [Andrekson et al. 1992].

![Figure 3.7](image)

**Figure 3.7** Transmission of a NOLM, using coupling \( k = 0.1 \) and \( k = 0.4 \).

The transmitted power for the nonlinear optical loop reflector can be found by using equation 3.15, and making the gain element transparent \( (g = 1) \) [Doran and Wood 1988]. The phase shift acquired by the field propagating a distance \( L \) is proportional to the product of the length and the power. Figure 3.7 shows the transmission performance for coupling \( k = 0.1 \), and \( k = 0.4 \), using \( \lambda =1.55 \) \( \mu \)m and an effective area of \( 30 \mu m^2 \). The best switching ratio (ie. the contrast between the linear off and the higher intensity on) occurs for a coupling closest to 0.5, but the switching energy increases correspondingly. When the product of the
power and length is small the results for the linear optical loop mirror are adequate, and equation 3.18 can be used to find the minimum transmission of the NOLM.

The power at which the NOLM becomes transparent can be found from equation 3.15, and is when the cosine term equals zero. The NOLM transmits all incident power the when the relation

\[ n_2 |E_{11}|^2 2\pi L/\lambda = m \frac{\pi}{1-2k} \quad m \text{ odd} \]  

is satisfied, \( k \neq 0.5 \). Figure 3.8 shows the minimum \( P*L \) (power*length) requirement for a nonlinear optical loop to reach transparency, as a function of the coupling \( k \). The curve is calculated using the same parameters used in figure 3.7. Although a highly asymmetric coupling provides the lowest switching energy, the contrast is degraded. For example using \( k = 0.1 \), \( P*W = 908 \) W m, with \( T_{\text{min}} = 64 \% \), compared to using \( k = 0.4 \), \( P*W = 3.63 \) W km, with \( T_{\text{min}} = 4.0 \% \). For this reason 40:60 couplers are often used in NOLM's.

3.5.3 Nonlinear Amplifying Loop Mirror (NALM)

An improvement to the Nonlinear Optical Loop Mirror can be made by further exploiting the nonlinearities by using an asymmetrically located gain element in the fibre loop [Fermann et al 1990]. The gain element is commonly a section of Erbium doped optical fibre for amplification in the 1550 nm telecommunications band. The power at which the NALM becomes transparent is
Figure 3.9 Minimum switching power * length condition for a NALM operating at 1.55 μm.

Figure 3.9 shows the minimum P*L (power*length) requirement for a NALM to reach transparency, as a function of the coupling k. The curve is calculated using the same parameters used in figure 3.7.

Figure 3.10 Minimum switching power * length condition for a k=0.5 NALM operating at 1.55 μm.

Figure 3.10 shows the minimum P*L (power*length) requirement for a NALM with a 50:50 coupler to reach transparency, as a function of the amplifier gain. For a modest gain of 100 transparency is reached with approximately 20 metres in the phase shifting length for a 1 Watt peak power pulse, which can be easily realised experimentally.
The switching power in the previous equations is for a CW light field, but has been equated to the fundamental soliton power (section 2.4) as the input power to the NALM when analysing modelocked fibre lasers [Duling 1991]. However, theoretical studies have shown that in the pulsed situation complete switching can occur when the pulse is a fundamental soliton after the splitter [Pearson et al 1993], and therefore this theory can only be taken as approximate when considering switched pulses, since the effect of dispersion was not considered in this theory.

3.6 Data Acquisition and Analysis

All the measurement techniques, which are described in the following sections of this chapter, involved transferring a raw electrical signal to a personal computer for further analysis. This was mostly done using a Macintosh Quadra 840AV computer equipped with an analogue to digital (A/D) conversion card, and a GPIB card. The A/D card operated at 10 kHz and had 12 bits of digitisation, providing 4096 discrete voltage levels. There was also an option of an additional programmable gain up to 100, which allowed spectral measurements in particular to have an effective dynamic range of 5 orders of magnitude. However, at this level the spectrometer performance was usually limited by internal scattering inside the instrument. The GPIB card was used for communicating with digital oscilloscopes and RF spectrum analysers.

The software package which remotely controlled the measurement instruments and processed the data was LabVIEW (National Instruments). Programs written in this package are called Virtual Instruments (VI's) and these were specifically designed for data transfer, instrument remote control and raw data processing. In the course of this thesis many VI's were written to control all the instruments described in the next sections.

The data from the LabVIEW VI's was stored as ASCII text files in vector format, and further analysis was performed using dedicated mathematical analysis software. MATLAB is a programming environment specifically designed for working with matrices, and it was used primarily for the analysis of the raw data. Presentation of the results in this thesis was done using Igor from WaveMetrics (graphs), MacDraw Pro from Claris (diagrams) and Microsoft Word (text).
3.7 Time Domain Diagnostics

3.7.1 Intensity Photo-detection

The simplest way to observe the pulse train from a modelocked laser is to measure the photo-current from a photodiode on an oscilloscope. The two photodiodes used had a response of 25 psec and 3.5 nsec. Since the modelocked pulses from the fibre laser had a duration less than 2 psec, this technique was only suitable for monitoring the average performance or stability.

The 25 psec photodetector was used with a digital oscilloscope which had a response of approximately 15 psec. The digitisation of the signal was a problem when the timebase was relatively long compared to pulse width; occasionally the signal would not be detected by the oscilloscope because of the sampling time of the digital oscilloscope, and averaging was needed. In this case analogue oscilloscopes were most valuable. Figure 3.11 shows the response from the 3.5 nsec photodetector with an analogue oscilloscope [Tektronix Model 1904]. Laser stability was observed by using the analogue oscilloscope on a relatively long timebase and viewing the pulse envelope. The laser stability was considered acceptable if intensity fluctuations were less than a few percent.

![Image](image.png)

Figure 3.11 4.2 MHz pulse train detected by the monitor photodiode.

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3.7.2 SHG Autocorrelation

The problem of measuring the duration of mode-locked ultra short optical pulses, which are much less than the response time of the fastest optical detectors, is commonly solved by the technique of autocorrelation. The spatial distribution of a pulse of duration of $t_0 = 10^{-12}$ sec to the speed of light is 0.3 mm and it is a relatively simple task to measure the spatial extent of the pulse.

Each individual pulse is split into two equal intensity pulses at a beamsplitter. The two pulses travel different paths in a Michaelson interferometer and recombine in a nonlinear crystal to generate a second harmonic pulse. The second harmonic pulse is a self-convolution as the two pulses overlap in the nonlinear crystal, for the particular amount of delay. By varying the delay in one of the interferometer arms, the entire autocorrelation function is reconstructed from the second harmonic pulse energy. The detector usually has a response time which is very slow compared to the rate of incident pulses, and typically a large number of pulses (~$10^6$) are averaged at each delay.

The normalised second order autocorrelation function is

$$G^2(\tau) = \frac{< I(t)I(t-\tau) >}{< I(t)^2 >} \tag{3.21}$$

where $I(t)$ is the optical intensity of one of the pulses, and the brackets indicate a time average [Yariv]. Although the intensity distribution cannot be uniquely determined solely from the second order autocorrelation function, the assumption of the temporal distribution function allows an analytic form of $G^2(\tau)$ to be calculated. The full width at half maximum of $G^2(\tau)$, $\Delta \tau_G$ can then be related to the pulse duration, $\Delta t_p$, as shown in table 3.1 [Sala et al 1980].

<table>
<thead>
<tr>
<th>$l(t)$</th>
<th>$\Delta t_p$</th>
<th>$G^2(\tau)$</th>
<th>$\Delta t_p / \Delta \tau_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>1; $</td>
<td>t</td>
<td>&lt; \Delta t_p / 2$</td>
</tr>
<tr>
<td></td>
<td>0; $</td>
<td>t</td>
<td>&gt; \Delta t_p / 2$</td>
</tr>
<tr>
<td>Gaussian</td>
<td>$\exp \left[ -\frac{4 \ln 2 \tau^2}{\Delta t_p^2} \right]$</td>
<td>0.3148</td>
<td>0.6482</td>
</tr>
<tr>
<td>Hyperbolic</td>
<td>$\text{sech}^2 \left[ \frac{1.7627 \tau}{\Delta t_p} \right]$</td>
<td>$\exp \left[ \frac{2 \ln 2 \tau^2}{\Delta t_p^2} \right]$</td>
<td></td>
</tr>
<tr>
<td>Secant</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1 Second Order Autocorrelation Functions and Time-Bandwidth Products for several pulse shape models.
The autocorrelator used in the analysis of the pulses in this thesis was based on a standard design and was constructed locally (figure 3.12). Real-time observations of the autocorrelation function were achieved by a loud speaker containing a retro-reflecting mirror, to provide a variable delay in the interferometer.

The other arm of the interferometer was mounted on a stepper motor driven translation stage, which could driven by a personal computer. The translation stage had resolution of 1.0 μm allowing a time base increment of 6.67 fsec per step. In this configuration the computer was used to move the translation stage, and the voltage from the SHG photomultiplier tube was sampled using a lock-in amplifier before digitisation at the computer. This technique allowed autocorrelation functions to be retrieved in the presence of noise, and stored in the computer.

![Figure 3.12 Schematic of a Background Free Autocorrelator](image)

Second harmonic generation was performed in a nonlinear crystal (BBO) with phase matching achieved by adjusting the inclination of the optic axis with a micrometer. The large phase matching bandwidth of BBO at 1550 nm ensured that all frequency components of spectra measured in this thesis were phased matched for SHG. The second harmonic signal was generated in the near infrared, and was detected using a photomultiplier tube with a sensitivity of approximately 3.5 % at 770 nm (Hamamatsu R928).
3.8 Frequency Domain Diagnostics

3.8.1 An Infrared Spectrometer

![Diagram of an Infrared Spectrometer]

Figure 3.13 Schematic of the Infrared Spectrometer

The infrared spectral data was collected using a modified Jarrell-Ash MonSpec 18 monochromator (figure 3.13). The basic instrument measured 16.5 x 16.5 cm by 12.5 cm high, and had a focal length of 156 mm. Operation in the infrared was achieved by using a diffraction grating blazed for use at 1000 nm (600 lines per mm), providing a useful range from 900 nm to 1800 nm (figure 3.15).

To speed the collection of data, the spectrometer was driven by a stepper motor (taken from a computer disk drive) mounted on a locally made micrometer assembly. The stepper motor rotated with 1.8 degrees per step, and the lead screw in the micrometer had 600 turns per inch, which gave a linear translation of 1.5625 μm per step. The dispersion of the diffraction grating (9.0 nm/mm) allowed 0.125 nm of the spectrum to be scanned per step. The optical resolution of the instrument was limited by the input and output slits of the monochromator (25 μm wide), and was measured to be approximately 0.3 nm when measuring the spectral line of a laser.
The infrared sensitive detector used was a Germanium photodiode (Hamamatsu B2538-02). The peak responsivity occurred at 1520 nm, and was approximately 0.8 A/W (figure 3.14). This detector had a single stage thermoelectrical Peltier cooler which allowed low noise operation at -10 degrees C. The spectral efficiency of the spectrometer can be roughly approximated by the product of the diffraction grating efficiency and the detector sensitivity, and this is shown in figure 3.15. The peak efficiency occurs at approximately 1500 nm, and decreases by about 20 percent at 1600 nm.

The absolute spectral calibration was done using the third diffracted order of an Argon ion laser line (514.5 nm x 3 = 1543.5 nm), since this was conveniently located in the middle of our measurement range.
The spectra were all measured as the spectrometer scanned towards larger wavelengths, as slight mechanical imperfections gave measurable differences between forward and backward scanning spectra.

![Figure 3.16 Spectral Linearity Curve comparison to Frequency Double Fibre Laser Measurement](image)

The imperfect mechanical system of the monochromator became apparent when the spectral linearity was measured (figure 3.16). This was done by measuring the location of a spectral peak from a tunable modelocked fibre laser (chapter 6) and comparing its frequency doubled spectral location with a high resolution visible spectrometer measurement. The cyclic error of approximately 2 nm, over 25 nm scan, was corrected for in the spectral measurements by using a fitted correction curve (figure 3.17).

![Figure 3.17 Spectral Linearity Correction Curve.](image)
3.9 The Time-Frequency Domain

3.9.1 SHG Frequency Resolved Optical Gating

Ultrashort pulse characterisation is traditionally carried out by measuring the intensity autocorrelation function and the optical power spectrum, but it is well known that these techniques do not provide complete intensity and phase information about the pulse. Recently the technique of frequency resolved optical gating (FROG) has been developed which overcomes the limitations of these techniques by allowing the retrieval of the complete electrical field [Kane and Trebino 1990]. Once the electrical field is known the spectrum and autocorrelation function can be computed to provide an independent check against the traditional measurements.

The FROG measurement technique has two parts; the experimental apparatus and the electrical field retrieval algorithm. The apparatus spectrally resolves a nonlinearly mixed signal from two replicas of the pulse, at various time delays between the two pulses. This allows the measurement of a time-frequency spectrogram - the FROG trace - which the algorithm is able to deconvolve into the electrical field as a function of time.

In this work Second Harmonic Generation (SHG) was the nonlinear optical process used, and experimentally this involved measuring the SHG spectrum from the autocorrelator. Third order nonlinear optical processes (eg. Third Harmonic Generation, Polarisation Gate, or Self Diffraction) may also be used in the FROG technique. However, these higher order processes require significantly larger peak power pulses to generate a detectable nonlinear signal, and for this reason SHG is the only practical FROG technique for examining pulses from fibre optics experiments.
Figure 3.18 Schematic of the Infrared Frequency Resolved Optical Gate

The experimental apparatus is shown in figure 3.18, and used a visible spectrometer to detect the SHG spectrum from the existing autocorrelator. The envelope of the SHG FROG field has the form

$$E_{\text{sig}}(t,\tau) = E(t)E(t-\tau)$$

3.22

where $E_{\text{sig}}(t,\tau)$ is the complex envelope of the pulse, and $\tau$ is the delay between the two arms in the autocorrelator. The signal field of equation 3.22 is symmetric about $\tau$, which leads to a temporal ambiguity in the retrieved electrical field. $E(t)$ yields the same FROG spectrogram as $E^*(-t)$, This implies that the retrieved intensity may be replaced by $I(t)\leftrightarrow I(-t)$, similarly the electrical field phase (and frequency chirp) $\phi(t)\leftrightarrow -\phi(-t)$. The time direction must be inferred from additional information, such as by propagating in a dispersive medium to create a known distortion of the phase.

The spectrometer detects the Fourier transform of this signal on its CCD array,

$$I_{\text{FROG}} = \left| \int_{-\infty}^{\infty} E_{\text{sig}}(t,\tau)\exp(i\omega t)dt \right|^2 = \left| E_{\text{sig}}(w,\tau) \right|^2.$$

3.23
The FROG spectrogram is built up from many SHG spectra taken at different time delays between the two pulses. This experimentally measured trace is then used as an input to the numerical algorithm which determines the full complex electrical field.

The deconvolution algorithm is shown in figure 3.19 [DeLong 1994 b]. An initial guess for the electrical field $E(t)$ is Fourier transformed using an FFT algorithm, then the theoretical FROG spectrogram is computed. This is weighted with the experimental data (first constraint) using

$$
E_{\text{sig}}'(\omega, \tau) = \frac{E_{\text{sig}}(\omega, \tau)}{|E_{\text{sig}}(\omega, \tau)|} \sqrt{I_{\text{FROG}}(\omega, \tau)}.
$$

An inverse FFT is done to get back to $E_{\text{sig}}'(t, \tau)$, and the time delay is integrated out. Convergence of the algorithm to fit the experimental data is achieved by using a second constraint (eq 3.22). This involves the minimisation of the metric

$$
Z = \frac{1}{N^2} \sum_{i,j=1}^{N} \left| E_{\text{sig}}(t_i, \tau_j) - E(t_i)E(t_i - \tau_j) \right|.
$$

Fortunately only single dimensional minimisation is needed along the gradient of $Z$ for the algorithm to converge rapidly [DeLong 1994 c]. Typically an experimental FROG spectrogram contains 128 x 128 points, the deconvolution algorithm attempts to find a global minimum for the electrical field in 128 x 2 dimensions. In practice the minimum is always located in a time significantly less than the actual data.
acquisition time, with the help of fast personal computers. The redundancy of the FROG data (N×N data points when 2×N electrical field points are required) helps to ensure rapid convergence of the algorithm.

A measurement of the fitting error is the value of $Z_{err}$:

$$Z_{err} = \frac{1}{N^2} \sum_{i,j=1}^{N} \left| I_{\text{exp}}(\omega_i, \tau_j) - I_{\text{FROG}}(\omega_i, \tau_j) \right|$$  \hspace{1cm} (3.26)

with an acceptable level taken to be $Z_{err} < 0.005 \ (0.5 \%)$.

Experimentally, pulses examined in this thesis had duration of approximately 2 picoseconds in the 1550 nm spectral region. In the frequency domain, a transform limited hyperbolic secant has a spectral bandwidth (FWHM) of approximately 1.4 nm, but experimental pulses usually had much wider spectra due to SPM. The spectrometer had a resolution of approximately $d\lambda = 0.14 \ \text{nm}$ per pixel at $\lambda = 775 \ \text{nm}$, which conveniently enabled the SHG spectra to be captured with $N = 128$ points. The time delays were determined by a Fourier relation

$$d\tau = \frac{\lambda^2}{300 \cdot d\lambda \cdot N},$$  \hspace{1cm} (3.27)

and for a standard measurement this was approximately $d\tau = 106 \ \text{fsec}$. Therefore the FROG measurement technique retrieves the electrical field at 20 - 40 temporal locations across the pulse.

Figure 3.20 Electrical field intensity versus time for a 1.9 psec 25 W soliton pulse
3.9.2 Chronocyclic Representations

In the previous sections several pulse detection methods were discussed. A complete characterisation involves using all these measurement techniques to create chronocyclic representations of the pulse. Chronocyclic refers to a representation in both the time and frequency domains, and has the advantage over traditional methods in that any chirp is easily identified.

To illustrate this technique a soliton-like pulse, consistent with the experiments conducted in this thesis, will be examined theoretically. Figure 3.20 shows the field intensity in time for a 1.9 picosecond pulse. The soliton pulse profile is hyperbolic secant, and has a peak power of 25 Watts in this case. The pulse is initially unchirped since the phase of the electrical field is zero.

![Autocorrelation Delay (psec) vs. SHG Intensity](image)

**Figure 3.21** Background free autocorrelation function versus time for a 1.9 psec soliton pulse

The autocorrelation function of the hyperbolic secant pulse has a similar profile to the intensity graph (figure 3.21), the analytical form is given in table 3.1. The pulse width can be determined from the autocorrelation width using the relation $\Delta \tau_c = \Delta t_p / 0.6482$ (table 3.1). However, this technique requires that the experimental data closely fits the theoretical autocorrelation function for this particular pulse profile.
The optical spectrum of the unchirped pulse is shown in figure 3.22. The spectral profile in the frequency domain is a hyperbolic secant, since this follows from the Fourier transformation. The spectral bandwidth is approximately 1.4 nm centred around 1550 nm, which is determined by the time-bandwidth product $\Delta \nu \Delta t_p = 0.3148$ (table 3.1).

The FROG spectrogram is shown in figure 3.23. The energy is concentrated in a narrow frequency band which extends over the temporal duration of the pulse. The grid is taken from a typical experimental
setup, which is determined by the diffraction grating in the spectrometer (0.14 nm per pixel around 775 nm). A typical FROG measurement uses 128 x 128 points, hence using equation 3.27 the temporal resolution is 106 fsec.

Figure 3.24  Wigner function for a 1.9 psec 1550 nm unchirped soliton pulse

The SHG FROG spectrogram has the disadvantage that it is always symmetric in time, and it is not immediately obvious how the frequency components are distributed in time. An alternative chronocyclic representation is the Wigner distribution function [Paye 1992], which for a light pulse is

$$ W(t, \omega) = \int_{-\infty}^{\infty} E(t + \frac{\tau}{2}) E^*(t - \frac{\tau}{2}) \exp[-i\omega\tau] d\tau $$  \hspace{1cm} (3.28) $$

This is a real function defined in the time-frequency domain, which can have negative values. Integration over the $\omega$ axis gives the temporal intensity, and similarly integration over $\tau$ axis gives the optical spectrum. The function $(E.E^*)$ has a quadratic nature, and this leads to artificial peaks in the Wigner function which are not physically present because of a computed cross term. The difficulty of interpreting the negative regions of the Wigner function is also a disadvantage of this method. However, frequency chirp is easily identified from the Wigner function.
Figure 3.24 shows the computed Wigner function for the unchirped hyperbolic secant pulse. Since there is no frequency chirp, the energy is concentrated symmetrically in both the time and frequency domains.

Figure 3.25 Phase after the 1.9 psec 25 W solit on propagates through 100 meters of optical fibre.

Frequency chirp can be generated after the pulse propagates through a medium with a nonlinear refractive index. To show this effect the nonlinear Schrödinger equation was numerically solved for the 25 W pulse propagating through 100 meters of optical fibre. Neglecting dispersion, and taking the nonlinearity to be 2.6 /W.km, the pulse develops a maximum phase shift of approximately 2π radians (figure 3.25). The intensity is the same as figure 3.20, since SPM alone acts only in the frequency domain.

Figure 3.26 Frequency chirp of the pulse
The frequency chirp of the pulse can be computed using

$$\delta \omega(t) = -\frac{1}{2\pi} \frac{\partial \phi(t)}{\partial t}$$

and has an approximately linear gradient over the central region of the pulse.

![Optical spectrum of the 1.9 psec pulse](image)

**Figure 3.27** Optical spectrum of the 1.9 psec pulse

The effect of SPM broadening is clearly seen in the frequency domain. The origin of the oscillatory structure can be understood from the chirp of the field (figure 3.26). When the same chirp occurs for different times, these same frequency components will either add constructively or destructively depending on their relative phase difference, to create an oscillatory frequency structure [Agrawal 1989].
Figure 3.28  SHG FROG Spectrogram for a 1.9 psec 1550 nm chirped pulse

The FROG spectrogram of the chirped pulse does not allow an intuitive identification of the frequency structure. The main disadvantage of the SHG FROG is that because of the temporal symmetry (equation 3.22), the traces are identical for positive and negative frequency chirps.

The computed Wigner function shows clearly signature of frequency chirp on the pulse (figure 3.29). The leading edge of the pulse (negative times) has longer wavelength components, which agrees with the graph of frequency chirp versus time (figure 3.26). Integrating the Wigner function over the frequency axis yields the intensity versus time (figure 3.20), and similarly integrating over the time axis gives the optical spectrum (figure 3.27).
In practice the FROG is measured, and then the electrical field envelope is retrieved from the experimental FROG spectrogram. The autocorrelation and optical spectrum are also measured, and then compared to the computed autocorrelation function and spectrum. The temporal ambiguity of the retrieved electrical field is resolved using propagation models. Usually the pulse develops an SPM chirp through anomalous propagation inside the fibre laser, and the direction of time can be set by comparing the retrieved chirp to figure 3.26.
3.9.3 Minimisation technique for determining Dispersion and Nonlinearity

In section 3.9.1 the SHG-FROG technique was shown to allow the measurement of the electric field of an optical pulse. With accurate measurement of the average optical power, and using a known repetition rate for the laser, the pulse envelope can be scaled so that the energy contained within the pulse equals the product of the average power and laser repetition rate (e.g. figure 6.27).

The evolution of an optical pulse in a single-mode fibre under the influence of second order dispersion and Self Phase Modulation, is described by the nonlinear Schrödinger equation (NLSE):

\[
\frac{\partial A}{\partial z} = -\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i \gamma |A|^2 A
\]

where \( A(z,t) \) is the pulse envelope in a comoving frame, and the nonlinearity and group velocity dispersion parameters are \( \gamma \) and \( \beta_2 \), respectively. To determine \( \gamma \) and \( \beta_2 \) for an arbitrary section of single mode fibre an algorithm was developed which simply takes the fully characterised input field, propagates it through the NLSE for an initial choice of \( \gamma \) and \( \beta_2 \), and compares the computed optical intensity and spectrum of this propagated input field with that from the field which was experimentally measured at the fibre output. The algorithm determines the optimum values of \( \gamma \) and \( \beta_2 \) to minimise the rms-error \( \sigma \) between these two optical intensity and spectral plots, and the algorithm continues until this error has reached an acceptably low value.

Tests of this algorithm with numerically generated data reveal that rms errors of \( \sigma < 0.5\% \) determine \( \gamma \) and \( \beta_2 \) to an accuracy of less than 5\%, even in the presence of simulated random noise typical of experimental measurements. Alternatively, the algorithm can compare the FROG trace which was directly measured at the fibre output with that which was computed by propagating the initial field through the fibre via the NLSE, however in practice there was negligible difference between the results from these two approaches.

This technique was used throughout this thesis to study pulse propagation through standard single mode fibre (Section 7.2.1), Erbium doped optical fibre (Chapter 7), dispersion shifted fibre (Chapter 8), and also to examine nonlinear switching within the source of these optical pulses (Section 6.3.3). In all cases excellent agreement was observed with previously published physical models for these processes.
Chapter 4

The Erbium Doped Fibre Amplifier

4.1 Rare Earth Elements and Ions

The Rare Earths or Lanthanides, are a set of 15 elements ranging from Lanthanum (La) with an atomic number of 57, to Lutetium (Lu), with an atomic number of 71. All of the rare earth atoms have the same outer electronic structure of 5s^25p^66s^2, which are filled shells. The number of electrons occupying the inner 4f shell dictate their optical characteristics, and range from zero (La) to fourteen (Lu).

Ionisation of the rare earths usually takes place to form a trivalent state, where the two 6s electrons and one of the 4f electrons are removed, but the outer shells are left intact. Consequently, the remaining 4f electrons are partially shielded from perturbations of external fields. The fluorescence and absorption wavelengths are therefore less dependent on external electric fields than, for example, the ions of the transition elements, which do not experience similar electronic shielding. For this reason rare earth ions have been the only ions that have exhibited laser oscillation in glass, where as several transition elements will lase in crystalline media (eg. Ti : Sapphire). All of the trivalent Lanthanides have been reported to exhibit stimulated emission on the 4f transitions in either a crystalline or glass host. Restricting the discussion to glass optical fibres, laser oscillation has been observed for nine Lanthanides, all of which were triply ionised. This includes Pr^{3+}, Nd^{3+}, Pm^{3+}, Sm^{3+}, Tb^{3+}, Ho^{3+}, Er^{3+}, Tm^{3+} and Yb^{3+} [Handbook of Laser Science and Technology 1995].

The Er^{3+} impurity has been made to lase on at least 13 transitions, spanning a spectral range from 0.55 to 4.75 μm [Handbook of Laser Science and Technology 1982]. The reason for a large number of
transitions can be understood with reference to figure 4.1, where it can be seen that the energy levels tend to be widely and evenly spaced (figures 4.2). This reduces the rate of non radiative decay, and gives rise to many metastable levels. The metastable nature of the levels also allows for numerous types of up conversion schemes to be devised.

4.2 Properties of Erbium in Glass

Glasses are commonly made up of network formers, eg SiO$_2$, which form a covalently bonded structure and network modifiers, eg. Na$^+$ which form ionic bonds to the SiO$^-$ groups and break up the covalent structure, thus allowing more modest processing temperatures. The disordered matrix has a wide range of bond lengths and bond angles, order is only displayed over short ranges at the local level. In doped glasses, the rare earth ions take the place of the network modifier. In glasses where network modifiers are absent, such as pure fused silica, a very rigid structure exists and there is a lack of non bridging groups.

![Figure 4.2](image)

Figure 4.2 Measured visible to near infrared fluorescence spectrum of a 12m Er Fibre; 250 mW Pump at 980 nm.

The absorption and emission spectra of the Er$^{3+}$ ion are signatures of the energy states of its 4f inner electrons (figure 4.3). In a glass or crystal host, these energy states are modified by local electric fields that cause Stark splitting and by dynamical perturbation, ie. homogenous broadening. Inhomogeneous broadening results from the structural disorder of the glass that causes differences in the electric fields at various sites. Rare earth ions in silicate glasses are too large to occupy sites between the network formers and are more easily incorporated into the glass structure by adding a network modifier such as Al, to
produce unbridged oxygen to which the rare earth attaches. This changes the Stark splitting and enhances the inhomogeneity of the gain medium.

![Graph](image)

**Figure 4.3** Typical emission and absorption Cross sections of Erbium and Aluminium co-doped optical Fibre. [Pederson et al 1991].

![Graph](image)

**Figure 4.4** Emission minus Absorption Cross sections of Erbium and Aluminium co-doped optical Fibre.

The Erbium doped optical fibre used in this thesis was co doped with Aluminium, and was pumped at 980 nm. Another pump scheme is possible at 1.48 μm (figure 4.4), but this was not investigated since a pump laser was not available at this wavelength.
4.3 Theory of Erbium Doped Optical Amplifiers

The EDFA is described in a \((r, \phi, z)\) cylindrical coordinate system with the \(z\) axis along the fibre. Considering only the \(LP_{01}\) modes and the circular symmetry for the EDFA, the steady state population concentrations, \(N_1(r,z)\) and \(N_2(r,z)\), in the ground and excited state, respectively, are evaluated from the transition rates of the pump, signal and spontaneous emission.

Assuming a three level amplification process only, the population concentrations in the ground \(N_1\) and excited states \(N_2\) can be related to the total concentration of active atoms \(\rho_{\text{Er}}(r)\), and is given by

\[
N_1 = \rho_{\text{Er}}(r) - N_2 \tag{4.1}
\]

[Pederson et al 1991]. The rate of absorption is proportional to the population in the ground state, and similarly, the rate of emission is proportional to the population in the upper state, as described by

\[
R_a \propto N_1 \quad \& \quad R_e \propto N_2. \tag{4.2}
\]

The rate of absorption is the sum of the pump \((R_{pa})\) and signal \((W_{sa})\) absorption coefficients. The rate of emission is the sum of the pump \((R_{pe})\), signal \((W_{se})\) and spontaneous emission \((A_e)\) rates. The spontaneous emission rate \(A_e\) equals the inverse of the spontaneous emission lifetime \(\tau_e\), for Erbium the excited state lifetime is approximately 10 milliseconds. The absorption and emission coefficients are in practice determined experimentally, and vary from fibre to fibre, since they are strongly dependant on the spectroscopy of the particular fibre.

\[
R_a = R_{pa} + W_{sa} \quad \quad \quad R_e = R_{pe} + W_{se} + A_e \tag{4.3}
\]

Once the population concentrations have reached equilibrium, the number of transitions from the ground state to excited state equals the number of transitions from the excited state to the ground state. The steady state can be described by equating the products of the transition rates and population concentration (equation 4.4).

\[
R_a \cdot N_1 = R_e \cdot N_2 \tag{4.4}
\]

Combining equations 4.1 - 4.4 leads to the excited state population in the steady state (equation 4.5). Equation 4.1 is used to determine the ground state population from the excited state population and the erbium concentration \((\rho_{\text{Er}}(r)\) is an initial condition).
\[ N_2(r,z) = \rho_{p_e}(r) \frac{R_{pe}(r,z) + W_{sa}(r,z) + A_c}{R_{pe}(r,z) + R_{po}(r,z) + W_{sa}(r,z) + W_{so}(r,z) + A_c} \]

The growth of the signal and pump fields as they propagate down the fibre can be calculated once the atomic populations are known. The differential equations describing the growth of the signal and pump are given by

\[ \frac{dP_s(z)}{dz} = \left[ \gamma_e(v_s,z) - \gamma_a(v_s,z) \right] P_s(z) \]
\[ \frac{dP_p(z)}{dz} = \left[ \gamma_e(v_p,z) - \gamma_a(v_p,z) \right] P_p(z) \]

where \( \gamma_{e,a}(v_{s,p},z) \) are emission and absorption factors describing the overlap between the optical fields and distribution of active atoms. The emission, \( \gamma_e(v,z) \) and absorption \( \gamma_a(v,z) \) factors are determined from the emission and absorption cross sections (Figure 4.3 [Pederson et al 1991]), and the overlap integral (equation 4.7). The overlap integral describes the coupling between the optical mode and population, and is extended to the Erbium doping radius \( \alpha_d \) (\( \alpha_d < \) core radius).

\[ \gamma_e(v,z) = \sigma_e(v) 2\pi \int_0^{\alpha_d} N_2(r,z) I_p^{01}(r) dr \]
\[ \gamma_a(v,z) = \sigma_a(v) 2\pi \int_0^{\alpha_d} N_1(r,z) I_p^{01}(r) dr \]

The absorption and emission rates are now described in more detail. The pump absorption rate \( R_{po} \) is given by

\[ R_{po}(r,z) = \sigma_{po} \frac{P_p(z)}{h \nu_p} I_p^{01}(r) \]

where \( \sigma_{po} \) is the absorption cross section at the pump wavelength, \( P_p(z) \) is the pump power at position \( z \) along the fibre, \( h \) is Planck's constant, \( \nu_p \) is the pump frequency, and \( I_p^{01}(r) \) is the normalised optical mode.

The pump emission rate \( R_{pe} \) is considered only when pumping at 1.48 \( \mu m \), as the emission cross section from the excited state is negligible at 980 nm.

\[ R_{pe}(r,z) = \sigma_{pe} \frac{P_p(z)}{h \nu_p} I_p^{01}(r) \]
The signal emission and absorption rates, $W_{se}$ and $W_{sa}$, are calculated from the signal and the amplified spontaneous emission (ASE), where $\sigma_s$ and $\sigma_a$ are the emission and absorption cross sections, respectively, at the frequency $\nu$. The broadband nature of the absorption and emission cross sections in the 1550 nm band necessitates a broadband treatment of the ASE, requiring many frequency slots, with a typical resolution of at least 1 nm over 1440 - 1620 nm.

$$W_{sa}(r,z) = \left[ \frac{\sigma_a(\nu)}{h\nu_s} P_s(z) + \int_0^\infty \frac{\sigma_a(\nu)}{h\nu_s} S_{ASE}(\nu,z) d\nu \right] f_s^0(r)$$  \hspace{1cm} 4.10

$$W_{se}(r,z) = \left[ \frac{\sigma_e(\nu)}{h\nu_s} P_s(z) + \int_0^\infty \frac{\sigma_e(\nu)}{h\nu_s} S_{ASE}(\nu,z) d\nu \right] f_s^0(r)$$  \hspace{1cm} 4.11

The spontaneous emission is amplified in both the forward and backward directions along the fibre. This means that the total amplified spontaneous emission spectrum $S_{ASE}$ has to be determined from the forward $S_{ASE}(\nu,z)^+$ and backward $S_{ASE}(\nu,z)^-$ travelling components

$$S_{ASE}(\nu,z) = S_{ASE}(\nu,z)^+ + S_{ASE}(\nu,z)^-$$  \hspace{1cm} 4.12

The growth of the spontaneous emission occurs in a similar manner as the pump and signal fields (equation 4.6), with the addition of the two orthogonal polarisation state vacuum photons, which are emitted from the excited state. The ASE initiates from zero power at each end of the fibre, with the vacuum photons providing the seed for the growth of this broadband noise.

$$\frac{dS_{ASE}(\nu,z)^-}{dz} = -[2h\nu \gamma_e(\nu,z) + \gamma_e(\nu,z) - \gamma_a(\nu,z)] S_{ASE}(\nu,z)^-$$  \hspace{1cm} 4.13

$$\frac{dS_{ASE}(\nu,z)^+}{dz} = +[2h\nu \gamma_e(\nu,z) + \gamma_e(\nu,z) - \gamma_a(\nu,z)] S_{ASE}(\nu,z)^+$$  \hspace{1cm} 4.14

Equations 4.5 - 4.14 describe the amplification and absorption processes which occur during a single pass through an optical fibre amplifier. From the growth of the signal field the amplifier gain $G$ can be calculated. An additional quantity useful in describing amplifier performance is the noise figure $F$, and is given in equation 4.15.
The noise figure is defined as the degradation in signal to noise ratio of the initial signal entering the amplifier, and can be interpreted as a measure of the spectral density of the ASE around the signal frequency. From equation 4.15 it can be seen that the more rapid the gain increases in the beginning of EDFA the lower the noise figure.

### 4.4 Numerical Solution of the Rate Equations

The equations of the previous section have been solved numerically on a personal computer, as an general analytical solution is known to be intractable. The intractability arises from the complexity of interdependence of the equations, in particular, division of rates to find the populations in equation 4.5, and the inhomogeneous nature of the differential equations 4.13 - 4.14. The results were used initially to compare with those measured by the manufacturer of the Erbium doped optical fibre which was used in this work. Once reasonable agreement was achieved, the model was used to study amplifier performance under variation of physical parameters. These results are presented in the later sections of this chapter.

The procedure for reaching the steady state within the amplifier is shown as a flow chart in figure 4.5. The procedure is iterative, using repeated substitution of the previous solution as an estimate for the optical power distributions and population concentrations, until equilibrium is reached. On the first iteration, an estimate for the optical power distributions is made, then using the initial conditions the rate equations are solved (equations 4.8 - 4.11). The energy level populations are calculated, then the overlap integrals, and finally the optical power distributions.

A measure of convergence is the absolute difference between two successive iterations' calculated signal gain. The iterations are stopped once the product of two of these successive gain differences is less than 10^{-10} dB. Stable convergence usually occurs after 20 - 30 iterations, with the signal gain numerical accuracy of approximately 10^{-5} dB. Reducing the number of required iterations is identified as an important area for speeding the total calculation time. The best initial estimate for the optical power distributions was made by using the final distributions of a previous calculation (which was saved); this usually only saved a few iterations. In addition several techniques were used estimate the final solution, given the nature of convergence, and these were implemented (if needed) after four or five iterations.

The technique of momentum weighting (eg. Atikens improvement formula, [Vetterling et al 1992]) was found to be invaluable and was extensively used. This technique improves the convergence by smoothing out oscillations of the solution, when the signal would oscillate between two values from
iteration to iteration. This was a common problem, simply countered by estimating the final solution by averaging two previous population distributions, after four or five iterations (if oscillations were detected).

**Figure 4.5** Flow chart for reaching the steady state solution to the EDFA equations
In some cases convergence was almost linear (the change in gain error decreased as a constant ratio), particularly when relatively small optical fields were studied. In this case the convergence is almost like a geometric series and Aitken's improvement formula was used to further speed the rate of convergence. However, in practice this proved to be less effective than momentum weighting, as convergence was never reached in a deterministic manner. Consequently no other convergence acceleration techniques could be developed, although considerable effort was expended.

The numerical grid size which provided solutions which no longer displayed grid size dependence were initially located. The distance along the fibre was divided into 40 segments, the radial distance was divided into 80 segments, and 80 optical frequencies were used to simulate the broad band ASE. Each optical power distribution along the fibre is determined from the solution of a differential equation, solved using the Runge Kutta Fehlberg method (fifth order ordinary differential equation solution). The grid size could have be reduced by a factor of two for many problems, but was retained to solve the wide range of problems encountered in this thesis.

The algorithm also allowed the pump and signal fields to propagate in either direction or both directions, to allow for the possibility of co propagating or counter propagating signals and pump fields. This also allows for the study of bi-directional pumping schemes, although this was not investigated. Counter propagating pump and signal fields were not extensively investigated because convergence only occurred with relatively small signals. The main reason for the generality of propagation direction was the ease in which to convert the amplifier into a laser calculation, by simply redefining the initial conditions to match laser reflectivities (considered in the next chapter).

## 4.5 Refinement of the Numerical Model

The primary goal of the numerical model was to predict the performance of experimental amplifier and laser configurations. Considerable importance was placed on determining the length of Erbium doped fibre needed to construct these devices, as only fifty metres was available and cutting the fibre is a destructive process. The Erbium doped optical fibre used was manufactured at the Optical Fibre Technology Centre, University of Sydney, who provided optical amplifier measurements, as well as a list of fibre specifications. The numerical model was tested using the physical parameters provided, and best estimates from the literature where these were absent, and compared to the manufacturers amplifier measurements.
Table 4.1 EDF-2 Erbium co-doped optical fibre parameters.

The physical parameters for the Erbium doped optical fibre given by the manufacturer are listed in table 4.1. In addition, several other physical parameters are needed for the numerical model and these were derived from the literature which discussed a spectroscopically similar fibre. The additional parameters are listed in table 4.2.

Table 4.2 Erbium doped, Alumino-germano-silicate core Optical Fibre parameters.

Table 4.3 shows the measured signal gain provided by the manufacturer. The fibre length was stated as 10 meters, and the pump power 40 mW at 980 nm. In this work only 980 nm pumping is considered, as this pumping scheme was used exclusively in latter experiments. The amplifier is assumed to be pumped in a co-propagating configuration, where the pump and signal propagate in the same direction, as this is the most common pumping scheme, giving higher signal gains and lower noise figures.

Table 4.3 Measured EDF-2 fibre in various amplifier configurations;
10 m length: 40 mW pump at 980 nm & signal at 1533 nm; 50 mW pump at 1480 nm & signal at 1550 nm

An important consideration is the level of uncertainty associated with the physical parameters. To determine if the numerical model was adequate certain physical parameters were varied slightly, and the
calculated signal gain was recorded. Table 4.4 shows the calculated gain when the amplifier length is varied from 9 to 11 meters. The middle row is the calculation using all the manufacturers parameters; pump power was 40 mW at 980 nm, signal was at 1533 nm.

<table>
<thead>
<tr>
<th>Amplifier Length</th>
<th>Small Signal (36 dB)</th>
<th>In-Line (26 dB)</th>
<th>Power (18 dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 m</td>
<td>37.76 dB</td>
<td>28.63 dB</td>
<td>20.25 dB</td>
</tr>
<tr>
<td>10 m</td>
<td>37.08 dB</td>
<td>28.46 dB</td>
<td>20.16 dB</td>
</tr>
<tr>
<td>11 m</td>
<td>36.34 dB</td>
<td>28.28 dB</td>
<td>20.06 dB</td>
</tr>
</tbody>
</table>

Table 4.4 Calculated EDF-2 amplification fibre in various amplifier configurations; Fibre length variation

The calculated signal gain, using the manufacturers data, is 1.08 dB, 2.46 and 2.16 dB too large for the small signal, in-line and power amplifier configurations, respectively. It is difficult to account for this large disagreement between the theory and experiment under amplifier length variation.

<table>
<thead>
<tr>
<th>Erbium conc (ppm)</th>
<th>Small Signal (36 dB)</th>
<th>In-Line (26 dB)</th>
<th>Power (18 dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>38.36 dB</td>
<td>28.76 dB</td>
<td>20.33 dB</td>
</tr>
<tr>
<td>500</td>
<td>37.08 dB</td>
<td>28.46 dB</td>
<td>20.16 dB</td>
</tr>
<tr>
<td>600</td>
<td>35.51 dB</td>
<td>28.08 dB</td>
<td>19.96 dB</td>
</tr>
</tbody>
</table>

Table 4.5 Calculated EDF-2 fibre amplification in various amplifier configurations; Erbium doping concentration variation

Table 4.5 shows the calculated signal gain when the concentration of Erbium is varied in the core, assuming all other parameters are the manufacturers data. The concentration of Erbium is known to be a difficult measurement to make, however, from the table it is evident that the actual concentration is likely to be close to 500 ppm, as specified.

<table>
<thead>
<tr>
<th>Pump Power (mW)</th>
<th>Small Signal (36 dB)</th>
<th>In-Line (26 dB)</th>
<th>Power (18 dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>37.08 dB</td>
<td>28.46 dB</td>
<td>20.16 dB</td>
</tr>
<tr>
<td>35</td>
<td>36.21 dB</td>
<td>27.73 dB</td>
<td>19.46 dB</td>
</tr>
<tr>
<td>30</td>
<td>35.15 dB</td>
<td>26.84 dB</td>
<td>18.63 dB</td>
</tr>
</tbody>
</table>

Table 4.6 Calculated EDF-2 fibre amplification in various amplifier configurations; Pump power level variation

Most of the other physical parameters were studied under similar variation, and the similar large discrepancies were encountered at relatively large signal powers (in-line and power). However, it is known that there are coupling difficulties when pumping Erbium doped fibre. The difficulties of coupling into the
single mode fibre are compounded with the problem of the mode field mis-match between standard single fibre, and the smaller core Erbium doped fibre. Table 4.6 shows the calculated signal gain when there is reduced pump power actually inside the Erbium doped fibre. Taken together with slight uncertainties in the measurement of the physical parameters and signal gains, the model is representative of the measurements when inefficient pump coupling is considered.

4.6 Optical Amplifier Design

The numerical model has been shown to predict the measured amplifier performance, to within a reasonable level of experimental uncertainty. The model is now used to examine optical amplifier design, and to identify optimal design strategies for possible experimental configurations. The study is in two parts, the first is concerned with an analysis of the three amplifier designs, as implemented by the manufacturer. The final section takes a more global view, and considers optical amplifier performance when practical parameters, such as signal wavelength, amplifier length, etc, are changed. Other workers have previously used this technique to optimise both the erbium doped fibre and amplifier design [Pederson et al 1991, 1992] [Giles and Desurvire, 1991] [Desurvire et al, 1990].

Figure 4.6 shows the steady state solution for the small signal Erbium doped fibre amplifier using the parameters in Table 4.1 - 4.2. The fibre length is 10 meters, with 40 mW of pump light propagating in the forward direction. The small signal input to the amplifier is -31 dBm (0.774 µW average power) at 1533 nm, and also travels in the forward direction (co propagating).

![Figure 4.6 Steady state solution for a co-propagating small signal EDFA : Pump and Signal Powers](image)

The pump field is quickly absorbed by the Erbium ions which are present in the fibre core. The signal encounters an inverted medium and becomes amplified, almost exponentially. However, as the pump is depleted it can no longer fully invert the medium, and signal amplification becomes less efficient. After approximately 8.2 meters the signal reaches its maximum level, acquiring 37.29 dB of amplification.
Upon further propagation in the now absorbing medium, the signal loses energy, to emerge from the amplifier with 37.08 dB of optical gain.

\[ \eta = \frac{\lambda_s P_{\text{out}}}{\lambda_s P_{\text{in}} + \lambda_s P_{\text{pump}}} \]  

4.16

The quantum conversion efficiency \( \eta \) of the pump to signal photons can be calculated using equation 4.16. Virtually unity quantum conversion efficiency can be achieved in optimised EDFA's. However, for the small signal EDFA the efficiency is calculated to be 15.9%.

![Figure 4.7](image)

**Figure 4.7** Steady state solution for a co-propagating small signal EDFA; Amplified Spontaneous Emission

The pump field is also degraded by the generation of amplified spontaneous emission (ASE), which propagates in both directions. The ASE initiates from quantum noise inside the fibre, hence has zero power at the end it originates from (on the graph the first point is made equal to the second point, so that it can be seen on a logarithmic scale). Figure 4.7 shows the total power of the forward and backward travelling ASE. The power is broad band, with a similar structure to the emission cross section (figure 4.4), and hence the power at any particular optical frequency is considerably less than the signal frequency. The calculated optical spectrum emerging from the amplifier shows a signal that is approximately 20 dB above the ASE noise floor. Using equation 4.15(a) the noise figure of the amplifier is calculated to be 4.67 dB, i.e. the signal to noise ratio is degraded by 4.67 dB after propagation through the amplifier.

The second configuration investigated was the in-line amplifier. The parameters used were the same as for the small signal amplifier, with the exception that the input signal level is now -16 dBm (25.1 \( \mu \)W). This configuration is used in a communications link where the optical amplifier is located midway between the source and detector, for example. The pump and signal powers for the in-line amplifier are
shown in figure 4.8. The amplifier can be divided into three sections, whose approximate boundaries are indicated by the changes in of the signal gain and pump power. From the graph, the pump is relatively slowly depleted over the first 2 meters. In this region the pump and the backward propagating ASE are strong, and this combination leads to significant recycling of the Er ions through stimulated emission and pump absorption. Consequently, there is a significant transfer of power from the pump to the backward propagating ASE.

In the middle section both the forward and backward ASE are low and the inversion is high since the only de-excitation process is spontaneous emission. In this region the gain increases most rapidly and the pump attenuation is the highest. In the final section the forward propagating ASE has reached a level where stimulated emission has again become important, but the pump intensity is not sufficient to maintain inversion and the gain per unit length falls off.

![Figure 4.8](image)

**Figure 4.8** Steady state solution for a co-propagating In-line amplifier; Pump and Signal Powers.

The maximum signal level occurs after approximately 7.2 meters, with a gain of 28.62 dB. The signal emerging from the amplifier is slightly absorbed in the last 3 meters, having a gain of 28.46 dB. The calculated noise figure is 3.482 dB, which is near the theoretical minimum of 3.01 dB. The pump power exiting from the amplifier is calculated to be 55 μW, and the exiting total ASE powers 0.52 mW and 2.9 mW in the forward and backward directions respectively. The backward travelling ASE power is always larger than the forward ASE power when the pump field co-propagates with signal in the forward direction, because the high pump power in the first section of the fibre. The quantum conversion efficiency is calculated to be 68.9 %.

The last configuration studied was the power amplifier, where a relatively large signal is amplified. Using the parameters in the previous sections, and a signal power of -7 dBm (0.20 mW) the steady state
solution can be calculated. Figure 4.9 shows the pump and signal powers as they both propagate in the fibre amplifier. The signal is now much larger than the ASE in the previous cases, and experiences an almost exponential gain over the first 3 metres.

![Figure 4.9 Steady state solution for a co-propagating Power amplifier: Pump and Signal Powers.](image)

The maximum signal occurs after 6.4 metres, with the signal exiting from the amplifier with a gain of 20.16 dB. The noise figure is calculated to be 3.69 dB. The forward travelling ASE total power emerging from the fibre is calculated to be 0.16 mW, and in the backwards direction 0.68 mW. The conversion efficiency of the amplifier is calculated to be 80.4%.

It is important to note that if the fibre length was made shorter, to be 6.4 metres for example, the conversion efficiency would increase, but the signal level may still reach a larger level in the interior of the fibre. This is because the steady state solution depends on the ASE as well as the pump and signal powers. To optimise the design of the amplifier, the calculation must be repeated with the physical parameter (e.g., the length) changed slightly each time. In the final section of this chapter the performance will be studied when these physical parameters are varied.

### 4.6.1 Amplifier length variation

The calculated performance of the amplifier is shown in figure 4.10 when the length is changed from 3 to 15 meters. This would be a destructive measurement in practice, therefore it is useful to identify the magnitude of the penalties when the amplifier is not at the optimum length using the numerical model. For the parameters used (those from the previous sections) the optimum length occurs at approximately 6.0 meters for the small signal amplifier, to 6.3 meters for the power amplifier. These lengths are slightly shorter than the peak signal distance, derived from the 10 metre amplifier graphs of the previous section. This is to be expected, as the generation of ASE is reduced in the shorter length amplifiers.
The small signal amplifier is most sensitive to the length, since when the signal is relatively small it has the most difficulty competing with the ASE for pump photons.

The noise figure is shown in figure 4.11. As the amplifier length increases, the ASE power generated increases and hence there is an increase in the noise figure. The lowest noise figure occurs with the in-line amplifier, when the signal level into the fibre is between small signal and power amplifier cases, and is a consequence of a favourable high gain and lower ASE power combination (equation 4.15.a). Although the small signal amplifier has the highest gain the relatively large level of ASE generated serves to degrade the noise figure of the amplifier, compared to the in-line case.

Figure 4.10 Gain for the three types of EDFAs, using a 40 mW co-propagating pump at 980 nm, signal 1533 nm.

Figure 4.11 Noise figure for the three types of EDFAs, using a signal at 1533 nm.
4.6.2 Signal level variation

Amplifier performance has been shown in the previous sections to be dependent on the input signal power level. Figure 4.12 shows the calculated signal gain for 10 meter amplifier when the signal is at 1533 nm, and also when the signal is at 1550 nm, co-propagating with the pump. The graph distinctly shows the amplifier operating in the unsaturated regime when the signal level is less than approximately 1 \( \mu \)W, and saturated performance above this value. The amplifier is only able to provide a maximum gain of 38 dB and 30 dB for the 1533 nm and 1550 nm signals respectively. The maximum possible gain is related to emission cross section (figure 4.3), and for this particular spectroscopy the largest emission cross section occurs at 1533 nm.

![Graph showing signal gain for 10 m EDFA using a 40 mW co-propagating pump at 980 nm](image)

**Figure 4.12** Signal gain for a 10 m EDFA using a 40 mW co-propagating pump at 980 nm

When the product of the input signal and maximum gain is large compared to the saturation power, the amplifier is operating in saturation and the gain is reduced.

![Graph showing signal noise figure for 10 m EDFA using a 40 mW co-propagating pump at 980 nm](image)

**Figure 4.13** Signal noise figure for a 10 m EDFA using a 40 mW co-propagating pump at 980 nm
The noise figure for the 10 metre amplifier is shown in figure 4.13. The noise figure is calculated from equation 4.14 using the ASE power at the signal wavelength emerging from the amplifier, and the signal gain. The graph shows that there is a minimum noise figure at 50 µW input signal power for 1533 nm, and 140 µW for 1550 nm. Although the amplifier is not providing the largest gain with these input signal levels, it is the favourable combination of the gain and ASE power that gives the lowest noise figure.

![Figure 4.14 Gain for a 10 m small signal (-31 dBm) EDFA, using a 40 mW co-propagating pump at 980 nm](image)

**4.6.3 Signal wavelength variation**

The EDFA is a broad band device, capable of large amplification over the entire 1550 nm telecommunications band. This broad band performance is shown in figure 4.14, where the small signal gain is calculated from 1510 nm to 1590 nm. The performance is strongly dependent on the actual spectroscopy of the doped fibre, and in this case the data in figure 4.3 was used, and hence may vary slightly in practice.

The gain spectrum displays the characteristic peak at 1533 nm, and a broad near flat gain region between 1540 nm to 1560 nm. The 1533 nm peak is associated with a stark splitting level. The flat region is intentionally engineered by using the Aluminium co-doping in the core, and facilitates multiple optical channel amplification (wavelength division multiplexing).
The noise figure is shown in figure 4.15 as the signal wavelength is varied over the 1510 nm to 1590 nm region. The trend is one of a decreasing noise figure towards longer wavelengths, which is not obvious when studying the gain characteristics. The noise figure is calculated using the ASE power emerging from the amplifier, at the signal wavelength, and it is apparent that the total ASE power is reduced when the signal is at longer wavelengths.

### 4.6.4 Pump power variation

The final section is the study of fibre amplifiers when the pump power is increased. Current technology provides fibre pigtailed - semiconductor laser diodes with an output power up to 100 mW, at 980 nm. To investigate whether an improved amplifier could be constructed with a larger pump power, the numerical model was used with a small signal input power and amplifier lengths ranging from 5 to 20 meters. Figure 4.16 shows the calculated signal gain when the co-propagating pump is increased from 5 mW to 250 mW, using either 5, 10 or 20 meters amplifier fibre lengths.
Figure 4.16 Signal gain for a small signal EDFA, using a -31 dBm signal at 1533 nm and various fibre lengths.

Although there is an increase in the small signal gain as the pump power is increased, the pumping efficiency is degraded. The pump efficiency is defined to be the maximum signal gain (dB) divided by the launched pump power (mW), and has a maximum of 4 dB/mW for the 5 meter amplifier using 5 mW pumping. The pumping efficiency is degraded as an increased amount of ASE power is generated. This is a particular problem for long amplifiers, where it is difficult to invert the medium along the entire length.

Figure 4.17 Noise figure for a small signal EDFA.

The noise figure is calculated for the three different length amplifiers in figure 4.17. From the section which investigated the length variation, the 5 meter amplifier is close to the optimum length when the pumping is approximately 40 mW. Therefore this short length is expected to have the lowest noise figure. Furthermore, as the pump is increased there is a more rapid the gain in the beginning of the amplifier, which helps to lower the noise figure further (equation 4.15.b). The longer length amplifiers serve to generate excessive amounts of ASE, and therefore have a large noise figure.
4.7 Conclusion

The numerical model developed has been used extensively to analyse the operation of Erbium doped fibre amplifiers. Initially the results were used to check the theory against some amplifier measurements made by the manufacturer of the Erbium doped fibre (used in the experimental parts of this thesis). The manufacturer had made signal gain measurements for a small signal, in-line and power amplifier configurations, and these were shown to agree with numerical model, to within a reasonable degree of experimental uncertainty.

The model was then used to study the effect on amplifier performance when physical parameters are changed. The study investigated the signal gain and noise figure when the length, signal power, signal wavelength and pump power are changed. This analysis showed the limitations of optical fibre amplifier design in a practical manner, and sets the way for the experimental work now that the amplifier construction is understood. The results of further calculations, which relate specifically to later chapters, are presented in Appendix 1.
Chapter 5

The Continuous Wave Erbium Doped Fibre Laser

The previous chapter was concerned with the amplification of an electromagnetic wave propagating in a doped optical fibre when the population is inverted and the signal wavelength falls within the Erbium transition lineshape. In this chapter the amplification medium is placed inside an optical resonator to provide optical feedback, and the physical characteristics of the resulting laser are examined.

The analysis begins with a discussion of lasers whose amplification medium has a relatively large small signal gain. The theory of high gain lasers allows the optimum output coupling requirement to be calculated. The next section uses an analytical model to calculate the fibre laser threshold power, slope efficiency, and also the minimum and optimum fibre length. Further theory is used to predict the fibre laser oscillation wavelength.

After considering the analytical theory, a numerical model is used to study practical fibre lasers. The model is based on the numerical solution to the Erbium amplifier equations used in the previous chapter, with modified boundary conditions for laser oscillation. Based on the design recommendations from the theory several Erbium doped fibre lasers have been constructed. The experimental results are presented in the following section. The experimental laser slope efficiency was used with the model to estimate the intracavity losses non-destructively. The source of the intracavity losses is identified, and guidelines are presented for the optimisation of Erbium fibre laser cavity design.

5.1 The Physics of High Gain Lasers

In the previous chapter the Erbium doped optical fibre amplifier was studied, and it was found that small signal gains in excess of 30 dB are possible. The analysis in this chapter considers the situation when the fibre amplifier has optical feedback to form a fibre laser. Initially the problem of finding the optimum reflectivity of the mirrors at each end of the laser to give the maximum output power is studied. Since there is a large single pass small-signal amplification factor, elements from the theory of high gain lasers are used to find the optimum output coupling.

For a cavity with an output coupler transmission of \( T_2 \), and a high reflector transmission of \( T_b \), the problem of interest is to determine the value of the transmission coefficient \( T_2 \) which yields the maximum output intensity. If \( T_2 \) is too large, the losses exceed the gain and no lasing is possible. If \( T_2 \) is too small
the power may be large inside the cavity, but too little is coupled out. Clearly there is an optimum
coupling for maximum power output. The output power \( I_{out} \) for a laser can be written as

\[
I_{out} = I_s \left[ \frac{T_0 T_z (g_0 - \alpha_l l_g)}{1 - e^{-\alpha_l l_g}} \right]
\]

5.1

where \( I_s \) is the saturation intensity \((I_s = h\nu / \sigma z)_s, g_0 \) is the single pass gain, \( \alpha_l \) is the total internal losses, and \( l_g \) is the length of the gain medium [Verdeyen 1989]. The total cavity loss \( \alpha_l l_g \) is the sum of the
internal losses \( L \), and the output coupling loss \( T_{out} \).

In the low loss laser configuration the integrated loss is small \((\alpha_l l_g \sim 2)\), and the exponential term in
equation 5.1 can be approximated by a two term expansion of the Taylor series

\[
1 - e^{-\alpha_l l_g} \approx \alpha_l l_g
\]

5.2

To find the optimum output coupling equation 5.1 is differentiated with respect to \( T_z \) to find the
maximum output power. The optimum output coupling transmission is calculated to be

\[
T_{out}^{Max} = \sqrt{g_0} L - L.
\]

5.3

[Verdeyen 1989]. Equation 5.1 can be used to find the output intensity of the laser with optimum coupling

\[
I_{out}^{Max} = I_s T_z \left( \sqrt{g_0} - \sqrt{L} \right)
\]

5.4
Our purposed Coupling Transmission trpeiecng

Figure 5.1 Calculated Output Power for a laser with an unsaturated single pass gain $g_0 = 12\%$.

The output power is shown in figure 5.1 for a laser with low losses and a low gain (12 % per pass), which accurately represents the operation of a He-Ne laser at 632.8 nm [Yariv 1985].

When the integrated loss, $\alpha_l l_k$ exceeds approximately 2, the exponential term in the denominator of equation 5.1 can be neglected. Following a similar procedure to the low gain case, the optimum output coupling $T_2$ for the high gain laser is the solution of the transcendental equation

\[ \frac{T_2}{1 - T_2} + \ln \left( \frac{1}{1 - T_2} \right) = \left( g_0 - \alpha_{\text{int}} \right) l_k \]

where $T_h = 1$ [Verdeyen 1989].

Figure 5.2 Calculated Output Power for a laser with an unsaturated single pass gain =20, 30, and 40.
The output power for several high gain lasers is shown in figure 5.2, and the optimum output coupling is shown in figure 5.3 as a function of the single pass gain. For a single pass gain of 27 the optical output reflectivity is 4 percent, which can be achieved simply from the reflection from a glass-air interface (e.g., a cleaved fibre end). For single pass gain (minus losses) of greater than 27, the optimal output coupling has a reflectivity less than 4 percent, but the penalty for not choosing 4 percent is small (less than 4 percent). Therefore a good choice for the output coupler for an Erbium doped fibre laser is one with 4 percent reflectivity.

5.2 The Analytical Model for Erbium Doped Fibre Lasers

The output power for fibre lasers has been analytically studied for linear and ring cavity configurations [Barnard et al 1994]. The analytical model describes the output power in terms of the attenuation coefficients and saturation powers. The linear laser is assumed to have two counter propagating waves at the laser wavelength confined in a cavity by a mirror at each end of the amplifying fibre.

Figure 5.3 Optimal Reflectivity for a laser as a function of the unsaturated single pass gain.

Figure 5.4 Schematic Diagram of Erbium Doped Fibre Laser with Intracavity Loss Elements
The linear fibre laser configuration has discrete loss elements, with transmissions \( \varepsilon_1 \) and \( \varepsilon_2 \), and mirror reflectivities \( R_1 \) and \( R_2 \). The slope efficiency has been calculated to be

\[
\eta = \frac{n_d \varepsilon_2}{T_{\text{eff}}} \left[ \frac{1 - R_2}{P_{\text{x}}^{\text{fs}}} \right] \frac{P_{\text{x}}^{\text{fs}}}{P_{\text{x}}^{\text{fs}}} \left( G_{\text{max}} \varepsilon R \right)^{\delta}
\]

and the threshold power

\[
P_{p}^{th} = \frac{h \nu P_{\text{x}}^{\text{fs}} \left[ \alpha, L - \ln(\varepsilon R) \right]}{1 - \left( G_{\text{max}} \varepsilon R \right)^{\delta}}
\]

where \( \varepsilon = \varepsilon_1 \varepsilon_2 \), \( R = \sqrt{R_1 R_2} \), so that the power out of the laser cavity is

\[
P_{\text{Laser}} = \eta \left( P_{p}^{in} - P_{p}^{th} \right)
\]

where \( P_{p}^{in} \) is the 980 nm power into the cavity, \( P_{x}^{fs} \) is the cross saturation power for the signal, \( P_{x}^{fs} \) is the intrinsic saturation power for the signal, \( \delta \) is saturation power ratio, \( G_{\text{max}} \) is the maximum gain, and \( T_{\text{eff}} \) is the effective output transmission [Barnard et al 1994].

![Figure 5.5 Slope Efficiency as a function of the Transmission of intracavity Loss Elements.](image_url)
Figure 5.6 Threshold Power as a function of the Transmission of intracavity Loss Elements.

The equations 5.6 - 5.7 were used to calculate the slope efficiency and threshold power of 6 and 12 meter Erbium doped fibre lasers (figures 5.5 - 5.6). The reflectivity of the output coupler is $R_1 = 4\%$ and the high reflector $R_2 = 100\%$. The intracavity losses can be easily determined by constructing a fibre laser and measuring the laser slope efficiency. One disadvantage is that the equations 5.6 - 5.8 have parameters which are difficult to measure experimentally, hence it will be difficult to accurately determine the intracavity losses. Another disadvantage is that the inversion properties of the Erbium ions cannot be determined from this model.

The laser oscillation wavelength can be predicted if the inversion properties of the Erbium doped fibre are known [Nielsen et al 1991]. The roundtrip gain $G = \exp(g)$ at the laser wavelength $\lambda_\text{c}$, and the single pass loss at the pump wavelength can be expressed as

$$g = 2\Gamma_p \rho_{E_r} \left[ \langle x_0 \rangle \sigma_e (\lambda_\text{c}) - (1 - \langle x_0 \rangle) \sigma_a (\lambda_\text{c}) \right] - \left[ \log_e (R_1) + \log_e (R_2) \right]$$

$$\alpha = \Gamma_p L \rho_{E_r} \left[ \langle x_0 \rangle \sigma_e (\lambda_p) - (1 - \langle x_0 \rangle) \sigma_a (\lambda_p) \right]$$

where $L$ is the fibre length, $\rho_{E_r}$ the Er concentration and $R_1$ and $R_2$ are reflectivities of the endfaces. $\sigma_e (\lambda)$ and $\sigma_a (\lambda)$ are the emission and absorption cross sections, $\langle x_0 \rangle$ is the average atomic inversion along the fibre length, and $\Gamma_e$ is the modal confinement factor. In the steady state $g = 0$, and the lasing wavelength $\lambda_\text{c}$ is determined by the global maximum of the gain cross section.
\[ \sigma_x = (x_0)\sigma_e(\lambda) - (1 - (x_0))\sigma_a(\lambda). \]

For a given fibre length, Erbium concentration and reflectivities this gives both the laser wavelength and average inversion along the length.

\[ \text{Figure 5.7} \quad \text{Laser Wavelength as a function of the average inversion along the fibre.} \]

The predicted laser wavelength is shown in figure 5.7 as a function of the average atomic inversion along the fibre. This is the wavelength at which the gain cross section is a maximum for the emission and absorption cross sections (and other parameters) of the previous chapter. There is a discontinuity at \(<X_0> = 0.58\) since there are two peaks in \(\sigma_e(\lambda)\), and for higher average inversions equation 5.9 predicts that the laser will oscillate at shorter wavelengths. However since from the previous chapter (and Appendix 1) the atomic inversion is known to be strongly dependent upon the 980 nm pumping, these analytical models can only be of little value when describing actual experimental results.

5.3 The Numerical Model for Erbium Doped Fibre Lasers

The numerical model describing the Erbium Doped Fibre Laser is based on the amplifier model, described in the previous chapter, with modified boundary conditions. The amplifier model has only a single pass through the amplifying medium of the pump and signal radiation. The boundary conditions allow no reflection of the pump, signal, or the forward or backward propagating Amplified Spontaneous Emission (ASE). The laser cavity has boundary conditions which allow the reflection of (at least) the signal from both ends of the active medium, when in a linear laser configuration.

The numerical model also includes loss elements before and after the active medium. This is necessary since in practice excess losses due to splicing etc. can introduce significant losses.
The linear fibre laser configuration is shown in figure 5.8. The boundary conditions provide feedback of the pump and signal, before being attenuated by the loss element, and then being amplified by the Erbium doped fibre.

5.3.1 Numerical Solution of Erbium Doped Fibre Laser Equations.

The equations describing the propagation through the Erbium doped fibre have been described in the previous chapter. These have been solved numerically on a personal computer, with the modified boundary conditions. The procedure for reaching the steady state within the laser is shown as an algorithm in table 5.1. Similar to the amplifier solution, the procedure is iterative, using repeated substitution of the previous solution as an estimate for the optical power distributions and population concentrations, until equilibrium is reached. Since for a laser in the steady state, the round trip gain must equal the losses, this additional information can be used to speed convergence to the steady state.
On the first iteration to find the steady state solutions, an estimate for the optical power distributions is made, then using these initial conditions the rate equations are solved (equations 4.8 - 4.11). The laser signal in practice grows from optical noise present in the cavity. For the first iteration a signal is injected to represent this noise fluctuation (there are no other injected fluctuations after the first iteration). When the process of modelling began the first iteration injected signal was small, at least 30 dB less than the pump power, but as more was understood about the system of equations, an estimate for the final signal power was used instead to speed the modeling.

A measure of convergence is the absolute difference between two successive iterations' calculated signal power level. The iterations are stopped once the product of two of these successive power level differences is less than $10^{-5}$ (as a proportion). A measure of the solutions convergence to the steady state is the difference of the actual numerical to the theoretical round trip gain. Stable convergence usually occurs after 60 - 100 iterations, with the signal power level numerical accuracy of approximately 0.05 %. There is no guarantee that the solution will converge to a physically correct steady state, and the difference between the calculated and theoretical round trip gain is used to test for this departure. This fractional departure is usually less than $10^{-5}$ for convergent solutions. Unstable numerical simulations are aborted if convergence is not reached within 400 iterations.

Similar to the amplifier situation, reducing the number of required iterations is an important area for speeding the total calculation time. The best initial estimate for the signal inside the laser came from a numerical model, which was built up from the results of many simulations, and is described later on in this section. The best initial estimate for the optical power distributions was made by using the final distributions of a previous calculation; this usually only saved a few iterations. In addition several
techniques were used to estimate the final solution, given the nature of convergence, and these were implemented (if needed) after four or five iterations, as described in the previous chapter.

1. Load Pump and Signal Power vectors.
2. z=0 boundary reflectivity and Intracavity loss conditions.
3. Forward Propagation; Solve the Rate Equations from existing Pump and Signal Power vectors.
4. z=L boundary reflectivity and Intracavity loss conditions.
6. Calculate Round trip gain.
7. Force new Pump and Signal Power vectors to fit physical constraints.
8. Do averaging to force convergence, if necessary.
9. Stop iteration when the round trip gain and output signal power settles down.
10. Calculate output power and difference between round trip and theory gain.

Table 5.1 Algorithm for Solving the Erbium Doped Fibre Laser Rate Equations

The procedure for solving the fibre laser rate equations is shown in table 5.1. To improve the convergence of the equations it was necessary to force the Pump and Signal fields to fit all known physical constraints after each iteration. The most obvious constraint is the boundary conditions imposed by the reflectivity of the end mirrors. There are three other conditions which are apparent from the physics of the laser, and these are that 1) the roundtrip gain equals the losses, 2) the gain in the forward direction equals that in the backward direction, and 3) conservation of photon number between the pump and signal fields.

The first condition is that in the steady state, the product of the saturated double pass laser gain and the total intracavity losses equals unity. This can be described by equation 5.10, where \( g \) is the saturated single pass gain of the laser, \( R_1 \) and \( R_2 \) are the mirror reflectivities at \( z=0 \) and \( z=L \) respectively, and \( T_1 \) and \( T_2 \) is the transmission of the lumped loss elements at each end. The propagation losses due to Rayleigh scattering can be disregarded since they are significantly less than the lumped loss, for fibre lengths considered here.

\[
g = \frac{1}{R_1 R_2 [T_1 T_2]} \tag{5.10}
\]

The saturated single pass gain experienced during the forward propagation equals that for the backward propagation, for optical fields of the same wavelength.

The final physical constraint used was the conservation of photon number. Since the laser is approximated as a three level system, there cannot be more signal photons generated than the number of pump photons supplied to the cavity. Equation 5.11 is true for any position along the fibre, where \( P_s F(z) \) and \( P_s B(z) \) is the signal power at position \( z \), propagating in the forward and backward directions along the fibre, respectively.
\[
PsF + PsB \leq \frac{\Delta P}{\lambda s}(PpF + PpB)
\]

From the second constraint, the gain in the forward direction is equal to that in the backward direction, the optical power can be evaluated at \(PsF(z=0), PsF(z=L), PsB(z=0)\) and \(PsB(z=L)\) in a closed form. The equal gains experienced by the forward and backward propagating fields is described by Equation 5.12.

\[
\frac{PsF(0)}{PsF(L)} = \frac{PsB(L)}{PsB(0)}
\]

The boundary conditions describing the mirror reflectivity’s and lumped loss elements are shown in equations 5.13.a and 5.13.b.

\[
PsF(0) = PsB(0) \cdot \left[ T_1^2 R_1 \right] \quad PsB(L) = PsF(L) \cdot \left[ T_2^2 R_2 \right]
\]

Substituting Equations 5.13 into 5.12 gives the forward propagating signal at \(z=0\), as a function of the forward propagating signal at \(z=L\), the mirror reflectivity’s and the intracavity losses (equation 5.14.a). A similar equation relating the backward propagating signal at \(z=L\) to \(z=0\) is shown in equation 5.14.b. Using these four equations (5.13 and 5.14) the signal power at \(z=0\) and \(z=L\) can be found for the forward and backward propagating fields, if it is known at any one of the positions.

\[
PsF(0) = PsF(L) \cdot \left[ T_1 T_2 \sqrt{R_1 R_2} \right] \quad PsB(L) = PsF(0) \cdot \left[ T_1 T_2 \sqrt{R_1 R_2} \right]
\]

The numerical solution seeks to find the optical power for the both the signal and pump fields, propagating in the forward and backward directions, at many points along the optical fibre of the laser cavity. In practice it is the most intense power which has the largest effect on the solution to the equations. If the \(R_1\) is taken to be the output coupler (a partially transmitting mirror) and \(R_2\) is the high reflector of the fibre laser (100% reflecting mirror), it is apparent that the largest signal power occurs at \(z=0\) propagating in the backwards direction, ie \(PsB(z=0)\).

The numerical solution algorithm (table 5.1) assumes that \(PsB(z=0)\) is correct, and then calculates \(PsF(z=0), PsF(z=L), PsB(z=L)\) using equations 5.13 and 5.14. Then the numerical solutions for \(PsF(z)\) and \(PsB(z)\) are scaled linearly, to force agreement with the known physical constraints. If \(PsB(z=0)\) is incorrect the next numerical iteration would change it slightly. This procedure of forcing the numerical solution to fit the physical constraints was used after each iteration and improved both the stability of the equations, as well as the rate of convergence to a solution. The physically correct solution is identified by comparing the theoretical roundtrip laser gain (equation 5.10) to the numerical one (before it is forced to
fit the physical constraints). From the simulations, there was found to be a certain range of parameters where the solution did not converge to the physically correct solution.

5.3.2 Results of the Numerical Simulations of Erbium Doped Fibre Laser.

The method for numerically solving the Erbium doped fibre laser equations has been outlined in the previous section. In this section the simulation results are presented, which complement the experimental results in the next section. The main problem, as in the amplifier chapter, is that of design optimisation. This problem is discussed in detail, as the numerical model is used to simulate fibre lasers with varying pump power, output coupling, intra-cavity loss, as well as Erbium fibre length.

The active medium of the fibre laser is assumed to have the same physical parameters as in the optical amplifier chapter, and these are used in the model. The basic physical parameters of the Erbium fibre are summarised in table 5.2.

<table>
<thead>
<tr>
<th>Fibre Type</th>
<th>Erbium doped, Alumino-germano-silicate core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erbium concentration $\rho_{\text{Er}}(r)$</td>
<td>approximately 500 ppm</td>
</tr>
<tr>
<td>Numerical Aperture</td>
<td>0.22 ± 0.03</td>
</tr>
<tr>
<td>Erbium confinement factor ($R_{\text{Er}}/R_{\text{core}}$)</td>
<td>0.80</td>
</tr>
<tr>
<td>$LP_{11}$ Cut Off Wavelength</td>
<td>875 nm</td>
</tr>
<tr>
<td>Fibre attenuation</td>
<td>6 dB/km @ 1100nm</td>
</tr>
<tr>
<td>Outside Diameter</td>
<td>125 ± 1.5 μm</td>
</tr>
<tr>
<td>Coating Diameter</td>
<td>250 μm</td>
</tr>
<tr>
<td>Pump Absorption Cross section $\sigma_{\text{pa}}$</td>
<td>$2.1 \times 10^{-25} \text{ m}^2$ (@ 980 nm)</td>
</tr>
<tr>
<td>Signal Band Absorption Cross sections $\sigma_{\text{sa}}$</td>
<td>see graph 4.4</td>
</tr>
<tr>
<td>Signal Band Emission Cross sections $\sigma_{\text{se}}$</td>
<td>see graph 4.4</td>
</tr>
<tr>
<td>Excited State Lifetime $\tau_{\text{e}}$</td>
<td>10.2 msec</td>
</tr>
</tbody>
</table>

Table 5.1 EDF-2 Erbium co-doped optical fibre parameters.

The physical design of the laser is shown in the previous section (figure 5.9). The laser cavity configuration is a linear design (as opposed to a ring cavity), which is end pumped through the first input coupler. The pumping scheme is always from first end only in these calculations ($R_1(\lambda_{\text{pump}}) = 4\%$), with the other end always containing a high reflector ($R_2(\lambda_{\text{pump}}) = R_2(\lambda_{\text{signal}}) = 100\%$). The two intra-cavity loss elements are always assumed to be equal at both ends of the cavity ($T_1 = T_2$), and introduce equal power losses for the pump and signal.
A typical steady state solution for the signal to the numerical model is shown in figure 5.11. In this case the linear fibre laser cavity has 6 meters of EDF-2 type fibre, with 85% transmission of the loss elements at each end of the Erbium fibre. The output coupler is has a 4 percent reflectivity at the signal wavelength \( R_s = 4\% : \lambda_s = 1533\,\text{nm} \).

The forward propagating signal has initially 0.3708 mW of signal power at the output coupler end of the fibre laser. As it travels along the Erbium doped fibre, which is pumped from the output coupler end, it is amplified, and reaches a maximum power of approximately 2.606 mW after 5.078 meters. The power level has dropped slightly to 2.566 mW after 6.0 meters. The single pass gain is 6.920, but undergoes a 15 percent loss, before encountering the high reflector \( R_2 = 100\% \). After the high reflector, the signal is now propagating in the backwards direction, and undergoes another 15 percent loss before entering the Erbium fibre.

The backward travelling signal at \( z=6 \) meters is initially 1.854 mW \( (= 2.566 \times 0.85 \times 1.00 \times 0.85) \). The backward propagating signal is again amplified by a factor of 6.920, to have 12.8236 mW at the output coupler end of the Erbium fibre. The power emerges from the fibre laser after the backwards propagating signal passes through the 15 percent loss element, and the 4 percent output coupler. Therefore the output power in this case is 10.4641 mW \( (12.8236\,\text{mW} \times 0.85 \times 0.96) \).
The forward and backward propagating pumping field is shown in figure 5.12 as determined from the steady state solution. The previous signal field coexists with this pump field in the steady state. The laser cavity is pumped with 30 mW, however, this must pass through the pump output coupler ($R_1(\lambda_p) = 4\%$), and then the loss element ($T_1 = 85\%$), before reaching the Erbium fibre. There is initially 24.48 mW (30 mW x 0.96 x 0.85) of pump power at the output coupler end of the fibre laser. As it travels along the Erbium doped fibre most of the power is absorbed by Erbium to invert the medium, with 0.2660 mW at the high reflector end of the Erbium fibre. The pump absorption by a factor of approximately 92 (-19.64 dB) on the first single pass ensures that the laser is reasonably efficient, even though the intracavity losses are high.

The backwards propagating pump begins with a 192.179 $\mu$W (0.2660 mW x 0.85 x 1.00 x 0.85) at the Erbium fibre at the high reflector end. The pump undergoes another absorption by a factor of -19.64 dB, before reaching the output coupler end with 2.088 $\mu$W of pump. The residual pump exiting the laser has a power of approximately 1.704 $\mu$W (2.088 $\mu$W x 0.85 x 0.96) which may interfere with the pump laser source. The pump absorption is an almost exponential process as the pump propagates in the forward direction, although in other cases there is a slightly weaker rate of absorption in the first few meters of the fibre when the signal level is large.
The efficiency of this laser, of converting the pump photons into signal photons, can be calculated from the pump and signal powers of the steady state solution. For the case studied in this instance, the signal power out of the laser cavity is 10.4641 mW at 1533 nm. The laser is pumped with 30 mW of 980 nm, however the Erbium fibre has only 24.48 mW coupled into it. The laser efficiency is therefore \( \frac{10.4641}{24.48} = 42.75 \% \). However, since the pump photons have significantly more energy than the signal photons, it is more useful to define a quantum efficiency of the laser, by dividing the laser efficiency by the quantum efficiency of the pumping scheme. Therefore the quantum efficiency of this laser is \( \frac{42.75 \%}{(980 \text{nm}/1533 \text{nm})} = 66.87 \% \). Therefore on average almost 67% percent of the pump photons are converted to signal photons in this laser.

\[
P_{\lambda_{\text{signal}}} (z = 0_{\text{Out of Cavity}}) = \frac{\eta_{\text{Slope Eff.}}}{\eta_{\text{Quantum Eff.}}} \left[ P_{\lambda_{\text{pump}}} (z = 0_{\text{Inside Cavity}}) - \text{Threshold} \right]
\]

\[
\frac{\eta_{\text{Quantum Eff.}}}{\lambda_{\text{pump}}} \frac{\lambda_{\text{pump}}}{\lambda_{\text{signal}}}
\]

Equation 5.14 defines the slope efficiency and threshold for the laser. The slope efficiency \( \eta_{\text{Slope Eff.}} \) is the determined from the gradient of a signal power versus pump power graph. Equation 5.15 divides the slope efficiency by the quantum efficiency \( \eta_{\text{Quantum Eff.}} \) so that slope efficiency has a range from 0 to 1. A slope efficiency equal to 1 is when every pump photon is converted to a signal photon. The laser threshold is the signal power versus pump power graphs intercept with the x axis, and is the pump power which is coupled into the Erbium fibre for which laser oscillation begins.

![Figure 5.13](image)

Figure 5.13 Erbium Fibre Laser output power as the Pump Power is increased:
6 meter 500ppm Erbium Doped Fibre, 980 nm Pump, 1533 nm Signal, R1=4% R2=100%, Losses=15%

To determine the slope efficiency and threshold for this fibre laser configuration a signal power versus pump power graph is calculated. This is shown in figure 5.13, and uses the same parameters of the
previous case, except that the pump power is increased. The graph shows the output power of the fibre laser as the pump power is increased from 30 to 100 mW, as calculated using the numerical model at 40 successive pump power levels. The computation is numerically intensive, and yields a slope which is linear to the precision of the individual signal power levels (relative error of ~$10^{-6}$). The pump power on the x axis is that which is directed at the laser cavity, as discussed before. 0.96 x 0.85 of this is actually coupled into the Erbium fibre (which is used in the efficiency calculation).

The slope efficiency is 79.24%, and the laser threshold is 3.85 mW. This means that almost 80% of the pump photons above threshold (which are coupled into the cavity) are converted into signal photons which are coupled out of the cavity.

![Graph showing laser slope efficiency vs. transmission of intracavity loss elements.](image1)

**Figure 5.14** Erbium Fibre Laser Slope Efficiency as the Transmission of loss element is increased:
6 meter 500 ppm Erbium Doped Fibre, 980 nm Pump, 1533 nm Signal, $R1=4\%$, $R2=100\%$.

![Graph showing pump threshold power vs. transmission of intracavity loss elements.](image2)

**Figure 5.15** Erbium Fibre Laser Pump Threshold as the Transmission of loss element is increased:
6 meter 500 ppm Erbium Doped Fibre, 980 nm Pump, 1533 nm Signal, $R1=4\%$, $R2=100\%$. 

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The primary reason for developing the numerical model is to identify optimal fibre laser cavity designs. The calculation of the slope efficiency and laser threshold is important, but for design optimisation it is necessary to observe how these parameters that define the laser performance change when certain physical parameters of the laser cavity are varied. The physical parameters which were varied in this study are 1) the intracavity loss, 2) the output coupler reflectivity, and 3) the Erbium fibre length. With each of these three parameters held constant, the input pump power is changed from 30 mW to 100 mW to determine the slope efficiency and laser threshold, from a graph of signal power versus pump power.

![Graph showing the relationship between transmission of intracavity loss elements and laser slope efficiency.](image)

**Figure 5.16** Erbium Fibre Laser Slope Efficiency as the Transmission of loss element is increased.

6 and 12 meter 500 ppm Erbium Doped Fibre, 980 nm Pump, 1533 nm Signal, R1=4,10 and 50% R2=100%.

Figures 5.15 to 5.18 show the slope efficiency and threshold for 6 meter Erbium fibre lasers, as a function of the intracavity loss element transmission. The results are significantly different from the analytical model [section 5.2], because of the difficulty of accurately obtaining the parameters used in the analytical model.
Figure 5.17 Erbium Fibre Laser Slope Efficiency as the Transmission of loss element is increased:
6 and 12 meter 500 ppm Erbium Doped Fibre, 980 nm Pump, 1533 nm Signal, R1 = 90% and 96% R2 = 100%.

Figure 5.18 Erbium Fibre Laser Pump Threshold as the Transmission of loss element is increased:
6 meter 500 ppm Erbium Doped Fibre, 980 nm Pump, 1533 nm Signal, R1 = 4%, 10%, 50%, 90% and 96% R2 = 100%.
The results of the numerical simulations are shown in figures 5.15 to 5.19. The laser output power can be calculated using equations 5.16 and 5.17. The laser slope efficiency is taken from figure 5.15 - 5.16 and shows quantum limited conversion efficiency when there are no intracavity losses. There is a strong decline in efficiency as the intracavity losses increase, and to a less extent as the output coupling reflectivity is increased. The optimal output coupler was found to be as low as possible, which is consistent with the theory of high gain - high loss lasers [section 5.1]. However, the simulations did not always converge, particularly when the losses were small, or the output coupling reflectivity was large.

\[
P_{\text{output}} = \frac{\eta_{\text{slope eff.}}}{\eta_{\text{Quantum Eff.}}} \cdot \left[ P_{\text{input}}^{\lambda_{\text{pump}}} - P_{\text{threshold}}^{\lambda_{\text{pump}}} \right]
\]

\[
\eta_{\text{Quantum Eff.}} = \frac{P_{\text{output}}^{\lambda_{\text{signal}}}}{P_{\text{input}}^{\lambda_{\text{pump}}}}
\]

The pump threshold power is the pump power required to initiate laser oscillation, after the first loss element inside the Erbium fibre, and was between 3.4 and 4.2 mW for the range of parameters studied. Simulations performed using 12 meters of Erbium fibre, instead of 6, have given almost identical slope efficiencies, but an increased threshold (between 6.8 and 7.2 mW). From the analytical theory, the threshold power is directly proportional to the Erbium fibre length (equation 5.7) [Nielsen et al 1991].
The model gave results which were identical, to within the numerical convergence precision, when forward and backward propagating amplified spontaneous emission (ASE) was included. However, the model became unstable when the ASE was allowed to be reflected by the boundary conditions. All results presented here exclude the ASE from the model, since it was found to have a negligible effect on the output power.

5.4 Experimental Results for CW Erbium Doped Fibre Lasers

The experimental design used was the linear fibre laser (figure 5.20). The 980 nm pump was coupled into the Erbium fibre using a fused fibre wavelength division multiplexer (WDM), with the second WDM port used as the output coupler for the laser in the 1550 nm band. The high reflector consisted of a loop mirror fusion spliced onto to other end of the Erbium fibre. The loop mirror is constructed using a 2x2 port splitter, with low excess loss and a low loss fusion splice to join the 2 ports together.

The reflectivity of the output coupler was approximately 4% when bare fibre endface. to 96% by inserting a calibrated variable reflecting mirror after the output coupler fibre. When the fibre endface was cleaved at an angle the laser threshold increased beyond 150 mW of pump power, giving instead of a narrow band laser spectrum, a characteristic wide band amplified spontaneous emission spectrum.

Experimental results are shown in figure 5.21 for a 6 meter linear Erbium fibre laser. A microscope objective was used to couple the pump into the 980 nm port of the WDM, and a loss of 40% was measured. There was a further loss of pump power as it is coupled into the Erbium fibre, through a fusion splice.
Figure 5.21  Measured Output Power and Results of Numerical Model
6 meter 500 ppm Erbium Doped Fibre, 980 nm Pump, 1533 nm Signal, R1=4% R2=100%

Using the results of the simulations the transmission of the Erbium fibre fusion splice could be calculated (figure 5.16). This was estimated to be approximately 59% transmission for the output coupler investigated. Further studies, with different fusion splices and either 6 or 12 meters of Erbium fibre, have shown the same consistent estimates of the intracavity loss.

Figure 5.22  Fusion Splice Duration versus Splice Loss as Estimated by Fusion Splicer for Er Fibre.

During the construction of the laser the intracavity losses are usually difficult to estimate, and although low loss fusion splices were used there was a large intracavity loss. Using the intracavity losses as the single unknown quantity in the numerical model has lead to consistent predictions for the intracavity losses. The predicted intracavity losses was at a reasonable level when compared to the
maximum transmission due to the mode field mismatch splice loss. However, during the fusion splicing process there is some diffusion of core material, which can lead to a slightly tapered splice region. This tapered splice can reduce the splice losses to insignificant levels (~0.05 dB = 1%) [Zheng et al 1994]. This technique was found to only increase the intracavity losses in our case, and might be attributed to incompatible diffusion rates between the fibres used in these experiments (figures 5.22 - 5.23). The splice loss as estimated by the fusion splicer (figures 5.22 - 5.23) is proportional to the actual splice loss, and can be made to be similar to the actual splice loss by adjusting the parameters within the fusion splicer algorithm, if a large number of splice losses have been accurately measured using cutback techniques. In figures 5.22 - 5.23 the estimated splice loss is only proportional to the actual splice loss, since the equipment for cutback techniques was not available at the time.

![Graph](image)

**Figure 5.23** Erbium Laser Power versus Splice Loss as Estimated by Fusion Splicer for Er Fibre.

The major source for the intracavity losses is the coupling mismatch due to different core radii of the Erbium fibre and standard single mode fibre, as the optimal geometry of the doped fibre is when the core is smaller than usual [Pederson et al 1991]. This serves to increase the intensity of the pump mode, and provides an optimal overlap with the signal mode. The inherent splice losses between two different fibres can be estimated by the following formula, using the butt-joint approximation [Marcuse 1977]:

$$\text{Loss(dB)} = 20 \cdot \log_{10} \left[ \frac{2 \omega_1 \omega_2}{\omega_1^2 + \omega_2^2} \right]$$

where $\omega$ is the modefield radius. Using equation 5.18 and typical parameters for the standard fibre ($\omega_1 = 6.0 \, \mu m$) and the Erbium fibre ($\omega_2 = 1.688 \, \mu m$) the maximum transmission is 76 %. Considering that the
light encounters four of these splices per round-trip, it is apparent that intracavity losses will significantly degrade the fibre laser performance.

### 5.5 Conclusion

A numerical model of the linear Erbium fibre laser with intracavity loss elements has been described. The results of simulations have allowed the intracavity loss to be predicted for experimental results. The output coupler reflectivity was 4%, and the slope efficiency and threshold was measured for several erbium fibre lengths. The agreement between the model and the experiment is reached when the intracavity losses (the Erbium fusion splices) is approximately 41%. The model can be used to optimise the design of the linear fibre laser, once the intracavity losses are estimated from experimental data. The elimination of intracavity losses has been attempted by modifying the fusion splicer parameters, but this was not successful. Careful choice of fibres to allow low loss fusion splices or the use of tapered fibre sections is necessary to ensure higher performance Erbium fibre lasers.
Chapter 6

A Passively Modelocked Erbium Doped Fibre Laser

6.1 Introduction

Passive modelocking is a powerful technique for subpicosecond pulse generation in many types of laser systems. In this chapter an Erbium Fibre laser that is passively modelocked using Self Phase Modulation (SPM) within a Nonlinear Amplifying Loop Mirror (NALM) is described. SPM has been shown to provide a Self Amplitude Modulation (SAM) for certain cavity configurations, which is necessary to provide stability for the passively modelocked laser operation [Haus et al 1991]. In contrast to active modelocking the amplitude modulation mechanism coexists on the time scale of the oscillating pulse, which ensures pulse stability even in a highly nonlinear cavity.

Modelocked laser operation may be considered analytically in the steady state if various pulse shaping mechanisms produce only small change of the electric field envelope per pass. This assumption is called the Weak Pulse Shaping (WPS) approximation [Krausz et al 1992], and in the context of this work is analogous to the propagation of a fundamental soliton in an optical fibre having anomalous dispersion. Formally the WPS approximation is valid for solitary lasers if the cavity length is much smaller than the soliton period. When this condition is satisfied an average soliton model may be used to describe the pulse formation. The average soliton model contains no information on the possible pulse shape variations within the cavity, and therefore is only valid for a completely homogenous cavity; where the pulse shaping elements are continuously distributed along its length. Although this is impractical for actual cavities, this average soliton model provides an insight to qualitative behavior.

![Figure 6.1](image.png)  
**Figure 6.1** A schematic of a ring laser with the various pulse shaping elements
6.2 The Average Soliton Model

The main pulse shaping mechanisms in a unidirectional passively modelocked solid state laser system are shown in figure 6.1 [Haus et al 1991]. A common feature of these systems is that the Self Phase Modulation (SPM) is substantially larger than the effect of Self Amplitude Modulation (SAM). The pulse shortening mechanisms are primarily dominated by the effects of SPM and Group Velocity Dispersion (GVD), with SAM providing pulse stabilisation.

A convenient technique to describe pulse shaping after propagation through cavity elements is by using transfer operators [Krausz et al 1992]. The complex envelope of the electrical field experiences pulse shaping given by

\[ \frac{\partial A(t,z)}{\partial z} = \hat{P} A(t,z) \]  

where \( A(t,z) \) is the pulse envelope, \( t \) is measured in a frame of reference moving with the pulse at the group velocity, \( z \) is the propagation distance, and \( \hat{P} \) is the propagation operator describing the cavity element.

The transfer operator which describes a full round trip is given by \( \hat{T} \), which is the sum of the pulse shaping processes describing the separate cavity elements.

\[ \hat{T} = \hat{L} + \hat{G} + \hat{D} + \hat{N} \]  

where the \( \hat{L} \) operator describes the effect of linear loss (the output coupling, etc.) and phase shift. The gain medium is described by the \( \hat{G} \) operator. The effect of the Group Velocity Dispersion is given by \( \hat{D} \). The combined nonlinear effects of Self Phase Modulation (SPM) and Self Amplitude Modulation (SAM) are represented by \( \hat{N} \).

After one complete cavity propagation the electrical field envelope has evolved to

\[ A_{n+1}(t,z) = e^{i\theta} A_n(t,z) \]  

where the index \( n \) counts the number of round trips. Using the WPS assumption of the average soliton model, the individual cavity operators become commuting and therefore equation 6.3 can be decomposed to
and hence equation 6.1 becomes

\[ A_{n-1}(t, z) = e^{i} e^{\hat{L}} e^{\hat{G}} \hat{N} A_n(t, z) \]  

6.4

\[ \frac{\partial A(t, z)}{\partial z} = \left[ \hat{L} + \hat{G} + \hat{D} + \hat{N} \right] A(t, z). \]  

6.5

In the steady state all changes add to zero, hence the average soliton model is given by equating the right hand side of equation 6.5 to zero.

In the presence of large dispersive perturbations in the cavity, a better description of the pulse formation may be obtained by using higher order terms of the non-commuting operators in the cavity transfer operator, referred to as the solitary laser model. This implies that the steady state pulse is characterised by different parameters at different positions in the resonator. Under practical operating conditions numerical simulations may be required to provide accurate parameters [Ippen 1994], as the operator approach violates the WPS assumption.

6.2.1 Analytic solution of the Average Soliton Model

Although the average soliton model is impractical for actual laser cavities, it is useful to provide qualitative behaviour of the pulse in the steady state. In particular, parameters describing the pulse amplitude, width, and frequency chirp need to be evaluated for certain cavity configurations. It is also useful to gain an insight into the overall stability of the modelocked laser system, by calculating a stability criterion [Haus et al 1991].

The average soliton model of equation 6.5 requires calculating the transfer operators for each pulse shaping process. The linear loss and phase shift operator is

\[ \hat{L} = -(l + jx)A \]  

6.6

where \( l \) is the linear loss per pass, and \( x \) is the phase shift per pass. The power loss per pass is \( 2|A(t, z)|^2 \).

The gain medium is described by the transfer operator

\[ \hat{G} = g \left( 1 + \frac{1}{\Omega_g^2} \frac{d^2}{dt^2} \right) \]  

6.7
where $\Omega_g$ is the bandwidth of the gain profile, approximated to be parabolic.

The group velocity dispersion modifies the pulse envelope according to the dispersive transfer operator

$$\hat{D} = j \beta_2 \frac{d^2}{dt^2}$$  \hspace{1cm} (6.8)

where $\beta_2$ is the second order group velocity dispersion for the optical fibre, typically $\sim -23$ ps$^2$/km at 1550 nm in standard single mode optical fibre.

The nonlinear propagation transfer operator $\hat{N}$ has a Self Phase and a Self Amplitude Modulation terms

$$\hat{N} = (\delta - j \gamma) |A(t, z)|^2$$  \hspace{1cm} (6.9)

where $\gamma$ is the Self Phase Modulation coefficient and $\delta$ is the Self Amplitude Modulation coefficient. The Self Amplitude Modulation coefficient is inversely proportional to the saturation intensity and must be positive so that the loss is reduced with increasing intensity, and is discussed in more detail in section 6.3. The Self Phase Modulation coefficient is

$$\gamma = \frac{\omega_0 n_z d}{c A_{eff}}$$  \hspace{1cm} (6.10)

where $A_{eff}$ is the effective area of the optical mode, $\omega_0$ is the optical frequency, $n_z$ is the nonlinear refractive index, $d$ is the length of optical fibre and $c$ is light speed.

In the steady state all temporal changes must add to zero. Using equation 6.5, a master equation of state can be derived by substituting equations 6.6 - 6.9 into 6.5

$$\left[ -(1 + j \chi) + g \left( 1 - \frac{1}{\Omega_g^2} \frac{d^2}{dt^2} \right) + j \beta_2 \frac{d^2}{dt^2} + (\delta - j \gamma) |A(t)|^2 \right] A(t) = 0$$  \hspace{1cm} (6.11)

A solution of this master equation is a hyperbolic secant profile for the pulse envelope

$$A(t) = A_0 \text{sech}(t / \tau)^{1 - \delta / \beta_2}.$$  \hspace{1cm} (6.12)
The pulse is described by three parameters, the peak amplitude \( A_0 \), the pulse width \( \tau \) and the linear chirp \( \beta \). These three parameters can be obtained by substituting equation 6.12 into the master equation. The second derivative of equation 6.12 is required by the master equation, and upon substituting into the master equation gives constant terms and terms involving a secant multiplier. Each of these two types of terms can be grouped together and set to zero individually, to give

\[
g - l - jx + \frac{(l + j\beta)^2}{\tau^2} \left( \frac{g}{\Omega_p} + j\beta_2 \right) = 0, \tag{6.13}
\]

\[
\frac{1}{\tau^2} \left( \frac{g}{\Omega_p} + j\beta_2 \right) \left( 2 + 3j\beta - \beta^2 \right) - (\delta - j\gamma) A^2 = 0. \tag{6.14}
\]

The two complex equations (6.13 & 6.14) can be solved for the four unknowns. The amplitude \( A_0 \) can first be solved by using the properties of the optical soliton. The fundamental soliton has a peak power \( P_1 \) which must satisfy

\[
P_1 = \frac{|\beta_2|}{\gamma \tau^2} = \frac{|D| \lambda^2}{2 \pi c \gamma \tau^2}, \tag{6.15}
\]

for propagation through a fibre with a negative dispersion of \( D \) (or \( \beta_2 \) [ps/\text{km}] ). The average soliton model is based on an averaged dispersion, as well as a distributed intracavity loss & gain, hence equation 6.15 is appropriate in this case.

The laser is assumed to have one pulse circulating in the cavity at a time. This allows the pulse energy \( W \) to be equated to the soliton power \( P_1 \) and the cavity round trip time \( T_R \). The pulse energy is

\[
W = 2 A^2 \tau = P_1 T_R. \tag{6.16}
\]

To lowest order the gain \( g \) balances the loss \( l \). Using this information the pulse width \( \tau \) and chirp parameter \( \beta \) can be obtained by substituting equation 6.12 into the master equation. A solution for the chirp parameter is

\[
\beta = \chi \pm \sqrt{\chi^2 + 2} \tag{6.17}
\]

where \( \chi \) is
\[ \chi = \frac{2(\gamma + \delta D_n)}{3(\gamma D_n - \delta)}. \]

Only one of the two signs \( \pm \) is an acceptable solution of equation 6.14, which arises from the requirement that the real and imaginary parts of the derivative of the master equation must balance. The sign of \( \pm \) is taken to be the sign of \( \delta D_n - \gamma \) [Haus et al 1991].

The normalised dispersion and pulse width of \( \chi \) are

\[ D_n = \frac{\Omega^2}{g} \beta, \quad \tau_n = \frac{W_{\Omega^2}}{2g} \tau. \]

The solution for the normalised pulse width is

\[ \tau_n = \frac{2 - 3\beta D_n - \beta^2}{\delta} = \frac{-2D_n - 3\beta + D_n \beta^2}{\gamma}. \]

The normalised bandwidth of the pulse is given by the pulse width and chirp

\[ \omega_n = \frac{\sqrt{1 + \beta^2}}{\tau_n}. \]

Finally, it is useful to evaluate the overall stability of the passively modelocked laser system, by calculating a stability criterion. This is done by examining the gain slightly before and after the pulse. If the gain \( g \) is less than the loss \( l \) in this region then small noise perturbations will not have a chance to grow and to destroy the pulse. Using equation 6.13, the gain is calculated in the presence of a pulse \( (\tau \neq 0) \), and also in the absence of a pulse \( (l \tau^2 \neq 0) \). This gives a stability criterion

\[ 1 - \beta^2 - 2\beta D_n > 0. \]

### 6.2.2 Numerical solutions of the Analytical Average Soliton Model

In this section a typical fibre laser configuration will be qualitatively analysed using the previously described equations. For these calculations the laser is assumed to have a cavity length of 50 meters, containing an Erbium doped fibre amplifier to provide gain, and be supporting a single pulse in the cavity.
The gain of the amplifying section of the laser will be taken to be a factor of 5 (hence g=0.1 m⁻¹), to counter intracavity losses and the output coupler. The cavity is assumed to be constructed from typical standard telecommunications single mode fibre, which has a core diameter of 8 µm and a nonlinear refractive index of approximately n₂ = 2.5 x 10⁻²⁰ m²/W, hence the Self Phase Modulation parameter γ = 1.2 x 10⁻³ /W.m.

![Diagram](image-url)

**Figure 6.2** The normalised pulsewidth (τₚ) as a function of the Self Amplitude Modulation parameter (δ)

The modelocked laser is assumed to support a single pulse in the cavity, which (in the anomalous dispersion regime of the fibre) can be assumed to be a fundamental soliton. For a pulse width of 1 psec, it has a peak power of ~ 8 W when the dispersion β₂ = -23 ps²/km. The cavity length of 50 meters implies a repetition rate of approximately 4 MHz, and hence a pulse energy of ~ 8 pJ. The gain bandwidth of the laser required to support these pulses only needs to extend 2.5 nm around the gain peak, hence Ω₉ = 0.32 THz. Using these parameters, the normalised pulsewidth and dispersion relate to the physical values by

\[ \beta₂ = Dₙ \text{ psec}^2 / m \quad \tau = \frac{\tau_p}{4} \text{ psec}. \]  

6.20
The normalised pulsewidth is plotted in figure 6.2, and the chirp parameter is plotted in figure 6.3 versus the Self Amplitude Modulation parameter (δ), for three values of the normalised dispersion (D_n). The SAM parameter is the dependent variable in these figures, since in an actual laser this is the easiest parameter to be make adjustable, as described in the next section. The pulsewidth is a maximum with no SAM, and decreases almost symmetrically with SAM. The chirp parameter is almost constant with SAM, but has the same sign as the SAM.

The stability criterion is shown in figure 6.4. It is clear that stable modelocked operation is only possible for positive values of SAM. In the complete absence of SAM there are no additional losses to any optical fluctuations which may destabilise the main pulse, hence this configuration may also be considered
as unstable. This reveals that a pure soliton fibre laser is always unstable, and even with SAM a sufficient amount of GVD is required for stability.

6.3 Sources of Self Amplitude Modulation

SAM has been shown to provide an indispensable mechanism for stabilising a passively modelocked fibre laser. The physical sources of SAM in an actual laser are related to fast saturable absorber theory, and are described in terms of Additive Pulse Modelocking (APM).

The SAM arises from coherent interference of a pulse from the main cavity, with a second synchronised pulse from a nonlinear cavity. The resulting SAM action may be thought of as similar to a mirror with a nonlinear reflectivity or a fast saturable absorber, with transmission of only the central portion of the pulse a possible configuration. These two cavities can be the same physical component, but the two pulses may have orthogonal polarisations or counterpropagating propagation directions. The two techniques are a convenient way of providing SAM for fibre lasers since they are easily implemented and provide a stabilised form of APM in the same cavity.

![Figure 6.5](#)  
APM action through Nonlinear Polarisation Rotation, Implementation in a Fibre Laser.

Intensity dependent polarisation rotation provides a mechanism for artificial saturable absorption, and can be a stabilised form of APM since the coherent additions of polarisation can occur in the same cavity. If the two polarisation components undergo different nonlinear phase shifts, then upon recombination a nonlinear polarisation rotation will occur. With a polariser, to select the coherent addition of polarisation states, this is converted into a self amplitude modulation (figure 6.5). This APM mechanism has been used to passively modelock Erbium doped fibre ring lasers [Matsas et al 1993] [Haus et al 1994].
6.4 The Figure of Eight Laser Configuration

The remaining sections of this thesis describe experiments involving pulse generation and propagation. This was achieved by constructing a compact source of pulses, and then performing pulse characterisation measurements of the laser pulses, and also of the pulses in external propagation experiments. The average soliton model is based on a ring configuration, but a modelocking mechanism is not explicitly described in this model. However as mentioned in chapter 3, optical fibre based loop mirrors have switching properties when propagating high power pulses, and this is exploited by the Figure of Eight design.

![Figure 6.6: The Figure of Eight Laser Design.](image)

The Figure of Eight laser configuration is composed of two rings, the passive ring, and the switching ring. The passive ring has an optical fibre based isolator to ensure unidirectional propagation into the other ring. The switching ring is either a NOLM or a NALM. At low powers, the loop acts to reflect the light back towards the isolator, resulting in a large loss for CW operation. The switching ring becomes transparent when the clockwise propagating field has an additional \( \pi \) phase shift relative to the counter clockwise propagating field when combined back at the central coupler. The additional phase shift is caused by SPM on the pulse which is amplified before propagation clockwise around the NALM. This design favours modelocked operation for pulses of a critical peak power.

Assuming that solitons are transmitted through the NALM with the least loss, the fundamental soliton power will balance the optimum switching power. The generated pulse width in a F8L is then given by

\[
\tau^2 = \left( \frac{0.776\lambda^2}{\pi^2 c} \right) D L (g - 1) \tag{6.21}
\]

where \( \lambda \) is the signal wavelength, \( D \) is the fibre dispersion, \( L \) is the loop length and \( g \) is the gain [Duling 1991].
6.4.1 Initial Design and General Performance

The initial Figure of Eight laser configuration is shown in figure 6.7. The passive ring contained the output coupler where 70 percent of the power was coupled out of the cavity. The polarisation sensitive isolator was used to ensure unidirectional propagation in the passive ring. The polarisation state after leaving the isolator evolves while propagating through the Erbium doped fibre (5.5 meters, ~500 ppm doping) and phase shifting length of the NALM (25 meters).

![Figure 6.7](image)

The polarisation state is transformed using two sets of polarisation controllers (PC). The PC in the NALM maximises transmission of the high power pulse by providing a phase bias and prevents the transmission of a CW signal [Duling et al 1994]. The PC in the output loop is then set to minimise the propagation loss at the polarisation sensitive isolator for the high peak power pulses, since the nonlinear birefringence of the fibre provides an additional polarisation rotation. An external cavity polarisation insensitive isolator was necessary to eliminate destabilising reflections from returning into the laser cavity.
The Erbium doped fibre was pumped using a commercial 980 nm laser diode, which could provide up to 90 mW of 980 nm pump from its fibre pigtail. A second laser diode was also purchased to pump an Erbium fibre amplifier, which was current limited by the power supply, to provide only 68 mW (figure 6.8) and was used for the experiments in the next chapter.

Figure 6.8 980 nm Laser Diode Performance.

The CW lasing threshold is approximately 7 mW for the Erbium fibre laser under optimum conditions. With correct adjustment of the polarisation controllers, and by introducing a mechanical perturbation, stable single pulse operation was observed with pump powers in the range of 8-10 mW. Modelocked operation of the fibre laser could also be spontaneously initiated by increasing the pump power to 40 mW or more. In this case, a comb of multiple closely spaced pulses are spontaneously generated and single pulse operation may be obtained by gradually reducing the pump power to 8 mW.

Figure 6.9 Optical Spectrum of the fundamental pulse, centred at 1558.4 nm with a width of 2.5 nm.
The repetition rate of the pulse train is 4.15 MHz, which is fixed by the round-trip time of the cavity (240 ns). The pulse spectrum is centred at 1558.4 nm (FWHM = 2.5 nm), and the average power at 9 mW pumping (32 mA 980 nm laser diode current, at 2.6 Volts) is 0.25 mW (figure 6.9). The autocorrelation trace shows a full width at half maximum of 1.6 psec (figure 6.10), hence the pulse width is approximately 1.0 psec assuming a hyperbolic secant profile. It thus remains close to the time-bandwidth product for transform limited sech pulses (0.32). The energy per pulse is 60 pJ, with a peak power is in excess of 60 W. The peak to peak amplitude fluctuations were better than two percent over many hours.

![Figure 6.10](image)

**Figure 6.10** Non-collinear autocorrelation trace, showing a FWHM of 1.6 psec.

The polarisation sensitive isolator was chosen to provide additional Self Amplitude Modulation within the cavity, and hence provide stability to the modelocked operation. To verify this, it was replaced by a polarisation insensitive isolator [Noske and Taylor 1993]. The threshold for passive modelocking then rose to about 40 mW. Single pulse operation could also be achieved in this laser by reducing the pump power to 8 mW after modelocking had been initiated, however the resulting pulses did not have the same long term stability associated with the pulses generated when the polarisation sensitive isolator was used.
As the pump laser diode power is increased, additional pulses are formed after the fundamental soliton [Guy et al. 1993]. At approximately 14 mW pump power an identical second pulse was observed to follow the first by about 60 psec. Additional trailing pulses occurred with each 3 mW increase in laser diode power, until at 90 mW when there were 20 pulses in the modelocked train (each separated by 82 psec, i.e. 12 GHz). The energy quantisation occurs because of the switching condition of the NALM which has to be satisfied for the solitons within the cavity [Grudinin et al. 1992]. The inter-pulse spacing was observed to be highly dependent on the PC settings as the interaction between these adjacent solitons depends on their relative phase [Agrawal 1989]. Increasing the pump power beyond 90 mW only contributed to a CW power component in the spectrum.
The cavity roundtrip time could be changed by adding or cutting fibre from the phase shifting length in the NALM. Experiments were conducted with the repetition rate of the laser from 3.3 to 8.5 MHz, but at 4.15 MHz provided the lowest self starting threshold, as described earlier. The total cavity length in this case represents a favourable trade off between a sufficient phase shifting length, and a low polarisation mode dispersion due to a short cavity length.

The repetition rate was found to be slightly temperature dependent (figure 6.14) by -10.5 Hz per degree C, which is consistent with an thermal expansion coefficient of the 50 meter glass fibre cavity of 1.75x10^-6 per degree C.
Figure 6.15  Fibre laser frequency dependence on ambient temperature, with air conditioning

Figure 6.16  Laboratory ambient temperature, with air conditioning
The temperature dependence of the repetition frequency is more dramatically shown when the laboratory air conditioning heaters switch on at night, to maintain a minimum lab temperature (figures 6.15 - 16).

The average power from the modelocked fibre laser over time is shown in figure 6.17 with a single pulse propagating in the cavity. The ambient temperature during this time period shown in figure 6.13, which cools the components in the laser power supply, to produce a slightly different electrical loading on the 980 nm laser diode over time (figure 6.18). The 980 nm laser diode itself was temperature stabilised, and normally operated at 13 degrees C, with a measured variation of 0.004 degrees C over 16 hours (but no correlation with the average power). The power supply was designed and constructed locally, and with further improvements the temperature dependence may be eliminated, to give improved fibre laser performance.
The combination of a temperature dependent diode laser output power, and the expansion and birefringence of the glass fibre cavity creates a weak time dependence in the output power. However, as typical experiments conducted in this thesis had durations of 1-5 minutes, the time dependent performance of the laser was negligible in practice.

### 6.4.2 Tunable Operation

In order to generate wavelength tunable pulses, an optical filter was placed in the linear loop of the F8L after the polarisation sensitive isolator. With the additional losses introduced into the cavity by the filter single pulse modelocking self starting from noise was not possible. The output coupler was then repositioned after the fibre amplifier in the nonlinear loop to increase the power of the pulses out of the cavity [Margulis et al 1995]. The threshold for modelocking with the generation of multiple pulses per round trip was about then 25 mW and by subsequently reducing the pump power to about 8 mW, stable, single pulse per round trip operation was again obtained.

![Figure 6.19](image)

Figure 6.19 Tunable Figure of Eight Laser Configuration
The tunable bandpass filter used (JDS Fitel 1570) had a bandwidth of approximately $3.0 \pm 0.5$ nm over the range 1520 to 1568 nm, which coincided with the gain band of the Erbium doped fibre. This allowed pulses to be generated ranging from 1517 nm to 1570 nm by adjusting the passband of the filter. In practice modelocked operation had to be reinitiated at each new wavelength location, making the laser not continuously tunable while modelocked.

The spectrum of the generated optical pulses at 5 sample wavelengths is shown in figure 6.20. The spectrum remained symmetric as the output wavelength was tuned, although the lasing bandwidth was slightly wavelength dependent as can be expected from the gain variation.

From figure 6.21 the time-bandwidth product of the generated pulse varies from 0.31 to 0.36 over the range of tuneability, when pumping with minimum power. It therefore remained close to the time-
bandwidth product for transform limited sech pulses (0.32). At the output wavelength of 1559 nm the average output power was approximately 0.5 mW, which corresponds to a peak pulse power of about 80 Watts.

Figure 6.22 Pulse width and Peak power tunability

The settings of the polarisation controllers provided many degrees of freedom for pulse formation in the laser. When the pump laser diode was operated at a constant current, the pulse width could be varied by adjusting the polarisation controller before the polarisation sensitive isolator (since this changes the intracavity losses). Increased 980 nm pumping also could lead to larger energy pulses. Figure 6.22 shows single pulse modelocked operation at 1548 nm, using 9 - 12 mW of 980 nm pump power and different settings of the polarisation controller. The pulse width could be tuned by a factor of 2, and peak powers increasing by a factor of 6.4.

Figure 6.23 Optical Spectra for modelocked laser operating at 1517 nm, and 1570 nm.
The useful tuning range of the modelocked laser was less than the bandwidth provided by the tunable filter and Erbium doped optical fibre, as a large proportion of the energy emitted from the laser was ASE when tuned to the edge of the Erbium gain band. As shown in figure 6.23, when operating at 1517 or 1570 nm more than 50 percent of the energy emitted from the laser was in a continuous wave component centred on the peak of the ASE spectral peak at 1531 nm. In the region of useful amplification (1530 - 1550 nm) the ASE contribution was typical less than 5 percent, and only discernible on a logarithmic plot of the spectrum.

6.4.3 Analysis of Modelocked Operation

In the previous section the general performance of the laser was described. To understand the modelocked operation of the laser in the steady state, the previously described diagnostic techniques were employed to measure the pulses exiting the laser at several locations. The use of frequency resolved optical gating allows both the intensity and phase of the pulse to be measured. The experimentally determined electrical field is then used to calculate the field within the laser by a backwards propagation numerical simulation into the laser. This technique was limited by knowledge of pulse propagation in the amplifier fibre while in the laser configuration, and will be discussed more fully in the next chapter.

![Optical Spectra for modelocked laser operating at 1537 nm.](image)

For the purpose of laser characterisation, measurements of the optical spectrum, autocorrelation, FROG and power were made at 10 wavelengths from 1527 to 1562 nm. The method of analysis at one of these wavelengths is presented in detail, and then the remaining results are summarised at the end of this section.
Figure 6.25   Autocorrelation measurement for modelocked laser operating at 1537 nm.

In this analysis the results are taken from one experiment when the laser was operating at 1537.6 nm, producing 242 µW of average power when pumping with 11 mW of 980 nm light. The optical spectrum has a full width at half maximum intensity of 1.98 nm (figure 6.24). The autocorrelation has a FWHM = 2.844 psec which implies a pulse width of 1.835 psec assuming that the pulse has a hyperbolic secant intensity profile (figure 6.25). The time-bandwidth product of \( \Delta \nu \Delta \tau = 0.46 \) indicates that the pulse has some frequency chirp. The repetition rate was 4.08865 MHz, and this implies a peak power of approximately 31.8 W, assuming a square pulse profile ("Autocorrelation method" for determining the peak power).

Figure 6.26   FROG measurement of the modelocked laser operating at 1537 nm.
The implied existence of frequency chirp from the time bandwidth product was checked by measuring the FROG spectrogram (figure 6.26). The spectrogram was measured at 128 different time delays with 106.7 fsec resolution. The SHG spectrum was also measured at 128 wavelengths, with a resolution of 0.14 nm, using 3 seconds integration of the spectrum at each temporal delay.

![Figure 6.27 Retrieved electrical field Intensity and Phase from FROG spectrogram](image)

The electrical field envelope of the pulse was retrieved from the spectrogram data using an algorithm described in section 3.9.1. The fitting error $G = 0.22 \% (<0.5 \%)$ implies that the retrieved electrical fields computed FROG spectrogram closely resembled the experimental spectrogram, and a visual inspection between the two confirmed that this was the case. The electrical field intensity is slightly asymmetric, with a tail on the trailing edge, and the phase shows a small chirp due to SPM. The peak intensity was recomputed to be 27.4 W based on the numerically integrated pulse area, average power and repetition rate.

![Figure 6.28 Frequency chirp of the modelocked laser operating at 1537 nm.](image)
The frequency chirp is shown in figure 6.28, and has similar features to SPM chirp, and is approximately linear over the central 2 picoseconds of the pulse. There is a large measurement error with the data in the edges of the pulse, hence the frequency shift beyond +/- 3 psec can be regarded as noise in figure 6.28. The computed Wigner function clearly shows the linear chirp (figure 6.29), with the leading edge of the pulse containing longer wavelength frequency components.

The optical spectrum and autocorrelation computed from the retrieved electrical field are shown in figures 6.24 - 25, along with the measured spectrum and autocorrelation function, and there is excellent agreement between them.
The analysis of the internal operation of the modelocked laser in the steady state is based on a numerical simulation of measured electrical fields back into the laser cavity. The physical layout of the laser is shown in figure 6.30, and a description of the propagation lengths and losses is given in table 6.3.1.

![Physical Layout of the Figure of Eight Laser](image)

The repetition rate of the modelocked laser was 4.15 MHz, which gave a total cavity length was approximately 50.22 metres, using a refractive index of 1.46. All optical fibre in the cavity is assumed to be standard single mode fibre with a dispersion of $-23 \text{ ps}^2/\text{km}$, and a non-linearity parameter of $\gamma = 1.2 \times 10^{-3} \text{ W}^{-1}\text{m}^{-1}$, as measured in section 7.2.1.

The gain of the Erbium doped fibre amplifier was determined from measurements of the clockwise and counter-clockwise propagating powers. The power at the output coupler A was compared to A', and this was divided by the losses at C, D, F & G to determine gain of the doped fibre length.

$$A/A' = G$$

$$\text{Intensity}(F)/\text{Intensity}(G) = G/(0.95*0.85*0.8*0.8) = \exp(\gamma \text{Gain} \cdot L_{Er})$$  \hspace{1cm} 6.3.1

For the purpose of this analysis, the pulse is assumed to experience a constant exponential gain along the fibre amplifier length $L_{Er}$ (eg. an isotropic inversion)
\[
\frac{\partial A(t, z)}{\partial t} = \gamma_{Gain} A(t, z)
\]

where \(A(t, z)\) is the amplitude of the Electrical field envelope, and \(\gamma_{Gain}\) is the gain per unit length over the 5.44 meters of amplification fibre.

<table>
<thead>
<tr>
<th>Interface</th>
<th>Loss (%)</th>
<th>Distance (length) (m)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40</td>
<td>0</td>
<td>Microscope Objective Output Coupler</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>45</td>
<td>APC Output Fibre Connector</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>261 (216)</td>
<td>Polarisation Insensitive Isolator</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>3.71 (1.1)</td>
<td>980 / 1550nm Wavelength Division Multiplexer</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>4.60 (0.89)</td>
<td>70 % Output Coupler</td>
</tr>
<tr>
<td>F</td>
<td>20</td>
<td>6.60 (2.0)</td>
<td>Erbium Fibre Splice</td>
</tr>
<tr>
<td>G</td>
<td>20</td>
<td>12.04 (5.44)</td>
<td>Erbium Fibre Splice</td>
</tr>
<tr>
<td>H</td>
<td>5</td>
<td>12.82 (0.78)</td>
<td>980 / 1550nm Wavelength Division Multiplexer</td>
</tr>
<tr>
<td>I</td>
<td>50</td>
<td>14.42 (1.6)</td>
<td>50 % Output Coupler</td>
</tr>
<tr>
<td>J</td>
<td>30</td>
<td>17.42 (3.0)</td>
<td>Tunable Bandpass Filter</td>
</tr>
<tr>
<td>K</td>
<td>10</td>
<td>21.42 (4.0)</td>
<td>Polarisation Sensitive Isolator</td>
</tr>
<tr>
<td>L</td>
<td>50</td>
<td>24.42 (3.0)</td>
<td>50 % Output Coupler</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>50.22 (30.40)</td>
<td>70 % Output Coupler</td>
</tr>
</tbody>
</table>

Table 6.1 Propagation Lengths and Losses of the Figure of Eight Laser
The electrical field of the pulses from the laser was measured using the FROG technique and are shown in figure 6.31. The main output port is taken from pulses clockwise propagating around the NALM, after the EDFA and had an average power of 0.292 mW when pumping with 9 mW of 980 nm. By numerically integrating the intensity profile and using the repetition rate of 4.15 MHz the peak intensity was determined to be 46.1 W (70.4 pJ), the pulse width was 1.34 psec. The phase shift across the pulse was approximately 1.5 radians, which is due to Self Phase Modulation in the output fibre since the nonlinear length (8.3 m) is comparable to the output fibre length (4.60 m).

The electrical field from the weak output port was also measured using our FROG detection system. The average power was only 45.7 μW, which corresponded to a peak power of 6.0 W for the 11 pJ pulses. with a duration of 1.70 psec. The phase shift across these low energy pulses was less than 0.3 radians, as seen in figure 6.31 (top). The lower peak power of output $E_2$ arises since the pulse is coupled out of the cavity before amplification in the EDFA. As a consequence of the lower peak power, the pulse evolution
during propagation within the phase shifting length of the NALM and the output coupling fibre is governed primarily by dispersion, and the presence of a quadratic dispersive phase shift is clearly shown as the dashed line in figure 6.31 (top). The electrical field has a small amount of noise present since these pulses are close to the detection limit of the measurement system (~1 W peak power).

The accuracy of the FROG technique was checked by comparing the measured spectrum and autocorrelation functions with separate instruments, to that computed from the electrical fields. In all cases excellent agreement was observed. The laser was centred at 1559.0 nm with a spectral width of 2.36 nm for the main output port, and 1.56 nm for the weak output port.

Using the propagation distances, intensity losses, and amplifier gain, the measured electrical fields were propagated back into the laser cavity using the Nonlinear Schrödinger equation (section 2.4). The field intensity as function of time and distance is shown in figure 6.32.

The results of the numerical simulations of the field back into the laser cavity, at the central splitter, are shown in figure 6.32. The output pulse from the laser (figure 6.31.bottom) is shown at distance z = 0.
At 2.6 meters back into the cavity the polarisation insensitive isolator was encountered, with a loss of 15%. The output coupler was located at 4.6 meters, and has a loss of 30%, since 70% of the power is coupled out of this cavity. The Erbium doped fibre was located between 6.6 and 12.0 meters, and provides a gain of ~13 in this instance (when pumped with 9 mW of 980 nm). A 20% splice loss between the Erbium and standard fibre was assumed, which was consistent with measurements of the CW laser slope efficiency with these internal losses (chapter 5). The central 3 dB splitter was located at 14.4 meters from the output of the laser.

The weak output port \(E_1\) is accurately propagated back to the central 3 dB splitter, since the anticlockwise field encounters only standard single mode fibre during propagation in the laser cavity (the 27 m phase shifting length). For standard fibre propagation, attenuation was neglected and propagation parameters used were \(\gamma = 1.2 \times 10^{-3} \text{ W}^{-1}\text{m}^{-1}\) and \(\beta_2 = -23 \text{ ps}^2/\text{km}\) respectively [Barry et al 1997]. To determine the equivalent clockwise propagating field \(E'_c\) leaving the 50:50 coupler, it is necessary to accurately model pulse propagation within the EDFA.

The dispersion and nonlinearity parameters of the EDFA are not known as accurately as for standard fibre, however in the steady state the anticlockwise propagating field \(E'_A\) must be identical (within a constant phase shift of \(\pi/2\) from the 50:50 coupler) to the equivalent clockwise propagating field \(E'_C\). The parameters of the EDFA can therefore be determined accurately by backward–propagating the measured clockwise output \(E_1\) to obtain \(E'_C\) for an initial choice of EDFA dispersion and nonlinearity, and then using a minimisation algorithm to determine optimum values of dispersion and nonlinearity to minimise the rms error between the propagated field \(E'_C\) and the constraint field \(E'_A\) (figure 6.34). For a value of EDFA gain of \(g = 0.45 \text{ m}^{-1}\), the results of this minimisation procedure yielded EDFA parameters of \(\beta_2 = +48 \text{ ps}^2/\text{km}\) and \(\gamma = 6.0 \times 10^{-3} \text{ W}^{-1}\text{m}^{-1}\), in good agreement with reported measurements of similar erbium doped fibre [Deutsch and Pfeiffer 1992].
Figure 6.34  Calculated Intensity and Spectrum at the central 50:50 splitter using a minimisation algorithm

The intensity and phase of the optimised clockwise propagating field $E'_c$ are shown as the open circles in figure 6.35 and it is clear that there is very good agreement with the constraint field $E'_\lambda$. Note that the phase characteristics of the pulses $E'_\lambda$ and $E'_c$ leaving the coupler are very flat, with a variation of less than 0.1 radian over the pulse FWHM. Given this small phase variation, and using values of peak power and FWHM of 9.5 W and 1.70 psec respectively, the equivalent soliton order of $E_\lambda$ and $E_c$ was calculated to be $N \sim 0.7$. This implies that the pulse incident on the 50:50 coupler from the linear loop of the F8L is a fundamental soliton, in agreement with theoretical descriptions of F8L operation [Duling et al. 1994].
The clockwise propagating field has a slightly different profile to the anti-clockwise propagating field, which results from difficulties in modelling the amplifier realistically. This problem occurs because for an optimised EDFA the gain is maximised by having almost complete inversion at one end of the amplifier and ~ 50 \% inversion at the other, which leads to an exponential gain only in the first few meters [Appendix 1]. The non-isotropic inversion also implies a position dependent resonant dispersion within the Erbium doped fibre. The pulse was also assumed to experience equal amplification in both directions (equation 4.6).

With the parameters of all the F8L fibre segments accurately determined, the switching characteristics were determined by the forward-propagation of the fields \( E'_A \) and \( E'_C \) to yield the corresponding anti-clockwise and clockwise fields incident on the 50:50 coupler after propagation through the NALM. The intensity and phase of these fields \( E_A \) and \( E_C \) are shown in Figure 36 (top) and Figure 36 (bottom) respectively. The incident anticlockwise field \( E \) has peak-power and FWHM of 17.6 W and 1.70 psec, with a small phase variation of 0.2 radian across the pulse FWHM. Comparing figure 35 (solid) and 36 (top) shows that there is little evolution in the pulsewidth and phase of the initial field \( E_A \), and this arises since it propagates through the phase-shifting length of 27 m at low power before amplification in the EDFA, and is therefore not distorted significantly by nonlinear effects in the fibre.
By contrast, the anticlockwise field $E'_C$ undergoes significant nonlinear evolution because it is amplified in the EDFA immediately after leaving the 50:50 coupler, and propagates in the phase-shifting length at high-power. The resulting incident field $E_C$ has peak-power and FWHM of 38 W and 0.94 psec respectively, and comparing Figure 6.35 (circles) and Figure 6.36 (bottom) clearly shows the effect of pulse compression and nonlinear phase distortion. The pulse shows the presence of low intensity wings, and although there is a phase shift of around 1.3 radians between the peak of the pulse and the wings, there is less than 0.1 radian variation in the phase across the pulse FWHM. Note that this non-linear phase evolution has been theoretically predicted to occur in a NALM as a result of soliton-like pulse evolution [Duling et al 1994], but these results are the first experimental verification of this expected behaviour.

At the central 3 dB splitter the clockwise and anti-clockwise propagating pulses (figure 6.36) interfere constructively to efficiently switch the pulse out of the NALM towards the isolator in the unidirectional loop. The degree of interference between the two pulses at the central splitter is in practice
difficult to calculate because of the birefringence of the cavity [Mortimore 1988]. A birefringence of $2 \times 10^{-8}$ will provide a $\pi$ phase shift for a 1550 nm signal between the fast and slow polarisation states over a distance 50 metres. This level of birefringence can be obtained by winding the fibre on a spool with a radius of 15 cm [Lefevre 1980], which was done in this experiment. Furthermore, the inherent birefringence of standard single mode fibre has been measured to be $-2 \times 10^{-7}$ by other researchers [Stone 1988].

Figure 6.37 Calculated Electrical field intensity after interference at the central 3 dB splitter propagating towards the isolator (top) and towards the filter (bottom).

The exact switching of the NALM when these two pulses recombine in the central 50:50 coupler depends on the combination of the relative nonlinear phase shifts, the linear phase bias introduced by the PC, and the birefringence of the loop mirror. In the model it was assumed that the total phase bias set by the PC in the NALM was adjusted to maximise the transmission, ie, that the total relative phase shift
between the two pulses in the NALM is $\pi$ radians [Fermann et al 1990]. In the numerical simulation a small phase bias of $\sim 0.3$ radians was added to the clockwise propagating pulse to achieve optimal switching (figure 6.37). The interfered pulse has a peak power of 50 W and a duration of 1.2 psec, with a phase variation of 0.1 radians across the FWHM, which propagates towards the isolator in the uni-directional loop. This corresponds to a soliton order of $N$~1.2, and note that approximate modelling of the effects of the isolator and spectral filter indicates that this pulse evolves into a fundamental soliton as it propagates in the linear loop. Efficient switching is demonstrated since the field propagating towards the filter (and will be blocked by the isolator) has a computed intensity of less than 1.8 W, representing less than 5 percent of the energy incident at the coupler. During propagation around the unidirectional ring the pulse is broadened and attenuated by the polarisation sensitive isolator and filter, to have a peak power of 19 W and 1.70 psec and a duration of 1.70 psec (as shown in 6.35). The most interesting feature of these calculations is that efficient switching can occur with the interference of two pulses with significantly different durations. Note that the large phase variations of around $\pi$ radians in the reflected field in figure 6.37 (bottom) arise from the oscillatory structure of the multiple–peaked pulse.

![Figure 6.38](image)

Figure 6.38 Calculated Anti-clockwise (solid) and Clockwise (dashed) Peak Power propagating from the central 3 dB splitter around the NALM.

The large intensity variations during propagation in the NALM are more clearly shown in figure 6.38. During clockwise propagation the pulse initially has a peak power of 9.5 W, a pulse width of 1.70 psec (figure 6.39) and a soliton order of $N=0.77$ (figure 6.40).
Figure 6.39  Calculated Anti-clockwise (solid) and Clockwise (dashed) Pulse Width propagating from the central 3 dB splitter around the NALM.

At the central 50:50 coupler (distance \( z = 0 \) and \( z = 37.66 \) m in figure 6.38) the pulse from the linear loop is divided into two pulses which subsequently propagate in the clockwise (solid line) and anti-clockwise (dashed line) directions around the NALM. The pulse which travels anti-clockwise around the NALM first propagates through the phase shifting length. Initially the pulse resembles a soliton of order \( N = 0.54 \), and undergoes pulse compression through the phase shifting length. At the 70 % output coupler (27.2 m) the anti-clockwise travelling pulse has a soliton order of \( N = 0.69 \). The EDFA section is encountered between \( z = 29.2 \) m and 34.7 m, where the pulse is amplified to a peak power of \( \sim 21 \) W, before travelling back to the central splitter (as shown in figure 6.36.top).

Figure 6.40  Calculated and Anti-clockwise (solid) and Clockwise (dashed) Soliton Order propagating around the NALM.
The clockwise travelling pulse is also compressed during propagation, since it also propagates as a chirped pulse in the anomalous dispersion regime. From figure 6.40 the pulse is close to a soliton-like pulse of order $N=1.05$ before the central 50:50 splitter in the linear loop. The possibility of efficient switching in NALMs with a fundamental soliton before the splitter has been previously confirmed by other experimental workers [Duling et al. 1994], where they measured soliton orders at this point in their laser of between $N = 1.1 - 1.4$, and also theoretical studies which have shown the importance of a fundamental soliton at this location before the NALM to provide the most efficient switching [Pearson et al 1993].

The direction of time in this analysis has been taken such that the chirp is consistent as if it was generated by SPM, but there is a time ambiguity of the SHG-FROG measurement since $E(t)$ can be replaced with $E'(-t)$ (section 3.9.1). However, if then this was done in the analysis the simulation gave a pulse width which was less than 0.36 psec after the tunable bandpass filter ($z = 17.4$ m). The subpicosecond pulse has a bandwidth of 7.0 nm if it is transform limited at this point, and cannot be transmitted through the 3 nm filter. Therefore identifying the chirp to be generated by SPM at the output of the laser, by choosing either $E(t)$ or $E'(-t)$ was the correct method.

![Figure 6.41](image_url)  
**Figure 6.41** Theoretical Initial Chirp on a 2.5 W 2.2 psec Soliton.

The direction of time was also resolved by an external propagation experiment. Theoretically it is well known that if a pulse has negative chirp (figure 6.41) and then propagates through a length of dispersive fibre (neglecting SPM) then the pulse will initially compress, before broadening, whereas a pulse will a positive chirp will only broaden [Agrawal 1989]. Figure 6.42 shows the pulse width (FWHM) calculated by solving the NLSE over 120 metres for solitons with positive, negative and no frequency chirp.
Figure 6.42  Pulse Broadening as a function of distance for 3 Chirped 2.5 W 2.2 psec Soliton Pulses.

The experiment consisted of attenuating the pulse, using a variable bend-loss attenuator, to 2.5 W peak power so that the dispersive length was much smaller (and more important) than the nonlinear length for standard single mode fibre ($L_D = 78 < L_{NL} = 155$ m).

Figure 6.43  Measured Autocorrelation before and after 60 metres of Standard SMF-28 Fibre.

The measured autocorrelation after propagation through the 60 metres of standard SMF-28 single mode fibre is compared to the laser output only in figure 6.43. The compression of the pulses after propagation is a clear indication that the chirp of the laser pulses is negative ($C < 0$), and agrees with the previous analysis.
6.5 Conclusion

In conclusion, a polarisation based APM Erbium fibre laser based has been constructed which overcomes the modelocking problems of the classic figure of eight laser design. Very stable amplitude pulse trains were observed, with one pulse in the cavity at a time, with low pump powers. A single pulse in the cavity could be obtained by introducing a mechanical perturbation with pump powers as low as 8 mW. By placing a tunable filter in the cavity the pulses were generated which were wavelength tunable from 1517 to 1570 nm with pulse peak powers up to 80 W, and at a pump power of only 8 mW, thus this fibre laser is an extremely efficient and useful source of picosecond optical pulses.

Within the range of wavelength tunability the time-bandwidth product of the generated pulses remains close (0.31 - 0.36) to that of a transform limited sech shaped pulse. Evidence for a nonlinear polarisation rotation cannot be inferred from pulse measurements because the polarisation sensitive isolator transmission was determined by the polarisation controller settings (which were not known), however significantly improved modelocked stability was observed when a polarising element was in the cavity.

The use of Frequency Resolved Optical Gating enabled the pulse characteristics to be calculated within most of the fibre laser cavity. The pulse was calculated to be close to a fundamental soliton before the central 50:50 splitter, before entering the NALM. Theoretical studies have also shown the importance of a fundamental soliton at this location before the NALM to provide the most efficient switching [Pearson et al 1993], which is necessary for passively modelocked operation.

The dispersion and nonlinearity parameters of the EDF section in the NALM were determined to be +48 ps²/km, and γ = 6.0 x 10⁻³ W⁻¹m⁻¹. For the small core of the EDF (ω = 1.688 μm) this gave a nonlinear refractive index of n₂ = 2.65 x 10⁻²⁰ m²W⁻¹, which is similar to that for standard single mode fibres. The exact analysis of the operation of the NALM in the steady was found to be limited by the knowledge of, 1) the pulse propagation through the EDFA and 2) by the phase bias introduced by using weakly birefringent fiber in the cavity. However, these problems may be overcome in the future by using this back-propagation of experimental fields technique in conjunction with more measurements at several other sampling points (ie. weak output couplers) in the cavity.
Chapter 7

Picosecond Pulse Propagation through an Erbium Doped Fibre Amplifier

Picosecond pulse propagation through an Erbium doped fibre amplifier is studied both theoretically and experimentally in this chapter. An extended nonlinear Schrodinger equation has been used to include the effect of the gain on the pulse propagation. The effects of dispersion and refractive index of the Erbium doped optical fibre is also discussed for the special case when the incident light field interacts with an atomic resonance.

Preliminarily experimental results indicated a wavelength dependent compression or broadening consistent with a wavelength dependent dispersion. In subsequent more detailed experiments, the electrical field was measured before and after the EDFA using Frequency Resolved Optical Gating. The extended nonlinear Schrodinger equation was then numerically solved to find the dispersion and nonlinearity of the EDFA for these electrical fields. The technique allowed the dispersion and nonlinearity to be determined consistently at each wavelength and EDFA pump power investigated. The total second order dispersion for the Erbium Doped Fibre section was determined to be positive, except around 1543 nm, and was strongly wavelength dependent which is consistent with resonant dispersion.

Experiments were conducted with the signal and pump fields either co-propagating or counter-propagating through the EDFA. When the signal and pump power remained constant, the dispersion and nonlinearity of the EDF section was measurably different for the two pumping schemes. It is noted that between the two pumping schemes different atomic inversion distributions exist within the EDF, which may lead to a different resonant contribution the total dispersion of the EDA module.

7.1 Theory of Pulse Propagation in Erbium Doped Optical Fibres

The evolution of an optical pulse in a single-mode fibre under the influence of attenuation, second and third order dispersion and Self Phase Modulation, can be described by the nonlinear Schrödinger equation
\[
\frac{\partial A}{\partial z} - \frac{1}{2} \alpha_g A + \frac{i}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} = i \gamma |A|^2 A
\]

where \( A \) is the pulse amplitude, and \( T = t - \beta_1 z \) is a frame of reference moving with the pulse at the group velocity \( v_g \). The second term accounts for propagation amplification or loss \( \alpha_g \) [Desurvire 1994]. This description of pulse propagation in an amplifying fibre assumes that \( \alpha_g \) does not depend on distance along the amplifier, and that the dispersion and nonlinearity are also constant.

Equation 7.1 is valid for pulsewidths \( T > 1 \text{ psec} \). If the pulse width is comparable or shorter than the dipole relaxation time for Erbium (~100 fsec) a set of Maxwell-Bloch equations must solved. This leads to a modified nonlinear Schrödinger equation, known as the Ginzburg-Landau equation, which is valid for subpicosecond pulse amplification [Agrawal 1995]. For pulse amplification experiments studied in this thesis equation 7.1 is a sufficient description, provided that the gain, nonlinearity and dispersion are known for the Erbium doped fibre.

The dispersion of the medium is the variation of refractive index with wavelength, and is caused by the chromatic response of the mediums bound electrons to the electromagnetic excitation. Far from the medium resonances, the refractive index \( n \) is approximated by the Sellmeier equation,

\[
n^2(\omega) = 1 + \sum_{j=1}^{m} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2} = 1 + \chi(\omega)
\]

where \( \omega \) is the optical frequency, \( B_j \) is the strength of the jth resonance, and \( \chi(\omega) \) is the medium’s electric susceptibility [Hermansson et al 1983].

Atomic resonances of the medium will cause additional wavelength and power dependent changes of the refractive index, and therefore will cause additional dispersion and nonlinearity within the medium. As the frequency of the EM field approaches an atomic resonance the polarisation of the activator ions generates a complex atomic susceptibility, in addition to the host material’s susceptibility [Desurvire 1994].

\[
\chi = \chi^\text{re} - i \chi^\text{im}
\]
The real part of the atomic susceptibility causes a refractive index change relative to the host material, and the imaginary part causes amplification of the EM field. The change in refractive index of the host medium is

$$\delta n(\omega) = \Gamma_s \frac{1}{2\pi L} \int_0^L \chi^{re}(\omega, z) \, dz$$  \hspace{1cm} (7.4)$$

and the amplification of the EM field is

$$g(\omega) = \exp\left(-\Gamma_s \frac{\omega}{nc} \int_0^L \chi^{im}(\omega, z) \, dz\right)$$  \hspace{1cm} (7.5)$$

where $\Gamma_s$ is the fraction of power in the doped region of the fibre core. The integral averages over the position dependent susceptibility, since the inversion of the active ions changes along the amplifier as the pump field becomes depleted, as described in chapter 4. These equations neglect any changes in inversion in the radial direction.

The imaginary part of the susceptibility is related to emission and absorption cross sections, and the atomic population densities by

$$-\chi^{im}(\omega) = \frac{nc}{\omega} (\sigma_e(\omega)N_2 - \sigma_a(\omega)N_1)$$  \hspace{1cm} (7.6)$$

where $\sigma_e(\omega)$ and $\sigma_a(\omega)$ are the emission and absorption cross-sections of the Erbium doped fibre, and where $N_1$ and $N_2$ are the atomic population densities of the upper and lower states [Desurvire 1994]. The real and imaginary parts of the complex atomic susceptibility are related through the Kramers Kronig relation

$$-\chi^{re}(\omega) = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{\chi^{im}(\omega')}{\omega' - \omega} \, d\omega'$$  \hspace{1cm} (7.7)$$

where PV is the principal value of the integral.

The emission and absorption cross-sections $\sigma_{e,a}^{im}$ are taken to be a sum of N Lorentzian line shapes, chosen to fit experimental data, which represent cross sections for transitions between the Stark split upper and lower atomic states,
Emission and absorption cross-section data has been published for a typical aluminosilicate Erbium doped fibre [Barnes et al. 1990] [Pederson et al. 1991], but the cross-section data for each Lorentzian line was not available for the particular fibre studied in this thesis.

The Kramers Kronig transform of the Lorentzian line shape function is

\[
-\frac{1}{\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{L_i(\omega')}{\omega - \omega'} d\omega' = 2 \frac{\omega - \omega_i}{\Delta \omega_i} L_i
\]

It is useful to define real emission and absorption cross sections

\[
\sigma_{e,a}^{re}(\omega) = \sum_i 2 a_i \frac{\omega - \omega_i}{\Delta \omega_i} L_i(\omega)
\]

The real part of the atomic susceptibility can now be calculated from equation 7.6 and 7.9 using the Kramers Kronig relation to give

\[
\chi'(\omega) = \frac{n c}{\omega} \left[ N_2 \sigma_{e}^{re}(\omega) - N_1 \sigma_{a}^{re}(\omega) \right]
\]

The refractive index change \( \delta n(\omega) \) can calculated from equations 7.4 and 7.11 to give [Desurvire 1994]

\[
\delta n(\omega) = \frac{\Gamma_s c}{2 \omega L} \left[ \sigma_{e}^{re}(\omega) \int dz N_2 - \sigma_{a}^{re}(\omega) \int dz N_1 \right]
\]

If the position dependence of the atomic inversion along the amplifying fibre is neglected, the amplification of the EM field becomes

\[
g(\omega) = \exp \left[ \Gamma_s \left[ \sigma_{e}^{im}(\omega)(N_2) - \sigma_{a}^{im}(\omega)(N_1) \right] \right]
\]
where \( \langle N_1 \rangle \) and \( \langle N_2 \rangle \) are the average populations in the lower and upper atomic states over the amplifier length. Similarly, the refractive index change is calculated to be

\[
\delta n(\omega) = \frac{\Gamma_c}{2\omega} \left[ \sigma_v^\text{in}(\omega)\langle N_2 \rangle - \sigma_v^\text{ou}(\omega)\langle N_1 \rangle \right]
\]  

7.14

The total dispersion of the medium is therefore

\[
\beta(\omega) = \beta_{\text{host}}(\omega) + \frac{\Gamma_c}{2} \left[ \sigma_v^\text{in}(\omega)\langle N_2 \rangle - \sigma_v^\text{ou}(\omega)\langle N_1 \rangle \right]
\]  

7.15

where \( \beta_{\text{host}}(\omega) = n_{\text{host}}\omega / c \) is the host contribution to the dispersion [Hoffman and Buck 1996]. The propagation constant can be expanded in a Taylor series to give the higher order dispersive terms

\[
\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \beta_2(\omega - \omega_0)^2 / 2 + \beta_3(\omega - \omega_0)^3 / 6 + \ldots
\]  

7.16

where \( \beta_i = \frac{\partial^n \beta}{\partial \omega^n} \bigg|_{\omega=\omega_0} \). Equation 7.15 can be used to numerically compute the dispersion of the Erbium doped fibre across the atomic resonances, if the individual cross sections are accurately known.

The contribution to the host fibre refractive index is given in equation 7.15. It is strongly dependent upon the atomic population densities of the upper and lower states, which is primarily related to the pump EM field (980 nm) along the fibre, and to a lesser extent the signal field (1550 nm). This refractive index change has been measured using interferometric techniques, and shown to be of the order of \( 10^{-7} \) for modest 980 nm pumping [Fleming and Whitley 1996] [Betts et al. 1991].

Measurements of the pump induced refractive index change (Equation 7.14) and the gain of a broadband signal (equation 7.13) have allowed the complex susceptibility to be determined for several EDF samples [Hoffman and Buck 1996] [Matera et al. 1991] [Desurvire. 1994]. An analytical expression for the susceptibility can then be obtained by fitting the experimental results to the sum of many Lorentzian lineshape functions. Once the lineshapes are determined equation 7.15 can be used to calculate the resonant contribution to the EDF dispersion. Using previously published data for the lineshapes [Matera et al. 1991], the gain, refractive index change and dispersion were computed for a fully inverted medium (Figure 7.1).
It can be seen that significant pulse shaping due to resonant dispersion in the EDF may occur across the Erbium gain band. Note that since the amplifier gain (dB/m) is directly proportional to the dopant concentration the units of resonant dispersion are often quoted as fs/nm/dB instead of ps/nm/km [Fleming and Whitley 1996].

![Theoretical EDF Resonant Dispersion](image)

**Figure 7.1** Theoretical EDF Resonant Dispersion, inset shows the (sign reversed) imaginary (solid, gain) and real (dashed, dn) parts of the excited ion susceptibility.

The nonlinearity $\gamma$ in the propagation equation 7.1 arises from the nonlinear refractive index $n_2$, which is related to the third order atomic susceptibility $\chi^{(3)}$ by

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}} \quad n(\omega,|E|^2) = n(\omega) + n_2 |E|^2 \quad n_2 = \frac{3}{8n} \chi^{(3)}$$

7.17

The propagation nonlinearity is therefore expected to be different from that for a passive silica single mode fibre, but not significantly different. The smaller effective area of the Erbium doped fibre (chapter 3) will increase the nonlinearity $\gamma$ by $\sim 3$ times compared to $\gamma$ for standard single mode fibre, and this may be the dominant contribution since $\delta n(\omega)$ is small (equation 7.14).
7.2 Experimental Results

The experimental configuration used to study picosecond pulse amplification is shown in figure 7.2. The amplifying fibre was from the same spool as the fibre used in the modelocked Erbium fibre laser described in the previous chapter. The wavelength division multiplexer had an insertion loss of 0.2 dB for the signal and 0.07 dB for the pump (980 nm) allowing efficient coupling of the pump power into the amplifier fibre. The pump light was provided by a fibre coupled laser diode, similar to the one which pumped the modelocked fibre laser (figure 6.8).

![Figure 7.2 Experimental Configuration for the Erbium Fibre Amplifier](image)

The amplifier was enclosed in a metal box along with the pump laser diode, with the Angle Patchcord Connector (APC) fibres coming out of the case. The use of APCs allowed quick connection to the modelocked laser and various other fibre lengths, with low back-reflection from the angled cleaved connector. Connection between two APCs a loss of typically less than 0.5 dB.

The results of the amplifying experiments will first be presented with the pump co-propagating with the signal, to minimise the propagation distance through the standard single mode fibre after the EDFA. This allowed the high power pulses to be studied with minimal SPM generated on the detected pulses.

The characteristics of the pulses used for the tunable amplification experiments are shown in table 7.1. The modelocked laser was tuned to 9 different wavelengths across the Erbium gain band, and adjusted for approximately 1.9 psec pulses. The peak power ranged from approximately 30 to 50 Watts for the co-propagation experiments. In the next section, the power was attenuated to be ~ 25 W for comparison between co-propagation and counter propagation of the ~ 1.9 psec pulses.
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<th>dTdv</th>
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<td>52.9</td>
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</tbody>
</table>

**Table 7.1**  
Pulse Characteristics for Co-propagating Tunable Amplification Experiments

![Graph](image)

**Figure 7.3**  
Amplification of the 1.9 psec pulses as a function of pump power

The power amplification of the ~1.9 psec pulses is shown in figure 7.3. The amplifier had no net gain or loss at all wavelengths when pumping with ~4 mW of 980 pump power, and typically gave 10 dB of gain at 14-18 mW of pump power. Signals which are closer to the Erbium gain peak at ~1532 nm, experience more gain. Spontaneous emission is generated strongly at the gain peak wavelength, and this process becomes more efficient as the signal wavelength is further from this peak. The amplifier is operating as a power amplifier when the input signals are comparable to the saturation power (chapter 4), and this regime is appropriate to study since it allows an insight into the modelocked laser operation. As the pump power was increased beyond 20 mW there was a risk of the amplifier oscillating as a laser, due to spurious reflections.
Preliminary amplification experiments consisted of measuring the amplified pulse's spectrum and autocorrelation function, as a function of laser wavelength and amplification (980 nm pump power to the fibre amplifier). The spectrum was observed to broaden due to self phase modulation as the higher power pulse propagated along the output connector fibres.

The autocorrelation function of the laser pulses closely fitted a hyperbolic secant profile, but typically developed some structure after amplification. The Full Width at Half Maximum (FWHM) intensity of the autocorrelation function after amplification can either be greater or less than the original pulses, depending on the wavelength (figure 7.4). This pulse compression or broadening effect upon amplification has been investigated theoretically by other researchers, and is dependent upon a wavelength dependent resonant dispersion in the Erbium doped fibre amplifier [Hoffman and Buck 1996]. In that study they used the analysis of the previous section to calculate the resonant dispersion, and concluded that pulse broadening or compression occurs for subpicosecond pulses ($T < \sim 0.5$ psec).

The measurements of the spectrum and autocorrelation function provide only incomplete information about the amplification process, since the electrical field cannot be recovered from these measurements alone. Frequency Resolved Optical Gating was used to measure the electrical field, and provided this additional information in an independent way. The remaining results presented in this chapter use all three measurement techniques to give a complete characterisation of the amplification process.

Figure 7.4 Ratio of amplified pulse autocorrelation width to that of the laser pulses
7.2.1 Pulse Propagation through Standard Single Mode Fibre

The Erbium doped fibre amplifier module was constructed from two APC patch-cord connectors, a wavelength division multiplexer and some Erbium doped optical fibre, as shown in figure 7.2. This chapter is concerned with the characterisation of the pulse propagation through Erbium doped optical fibre, however, since the EDFA module has a significant amount of standard single mode fibre, it is important to characterise pulse propagation through standard single mode fibre.

The characterisation of pulse propagation through standard single mode fibre was done using the FROG technique to measure the electrical field before and after a 20 metre standard single mode fibre (Corning SMF-28) patchcord [Barry et al 1997]. With accurate characterisation of the input pulse power, it is possible to relate the input and output fields via the nonlinear Schrödinger equation (NLSE):

$$\frac{\partial A}{\partial z} = -i\beta_2 \frac{\partial^2 A}{\partial T^2} + i\gamma |A|^2 A$$  \hspace{1cm} (7.18)

Here, $A(z,T)$ is the pulse envelope in a comoving frame, and the nonlinearity and group velocity dispersion parameters are $\gamma$ and $\beta_2$ respectively. With this measurement technique, $\gamma$ and $\beta_2$ are treated as unknown variables whose values are to be determined by a numerical minimisation algorithm. An algorithm was developed which simply takes the fully characterised input field, propagates it through the NLSE for an initial choice of $\gamma$ and $\beta_2$, and compares the FROG trace of this propagated input field with the FROG trace which was directly experimentally measured at the fibre output. The algorithm determines the optimum values of $\gamma$ and $\beta_2$ to minimise the rms-error $\sigma$ between these two FROG traces, and the algorithm continues until this error has reached an acceptably low value. Tests of this algorithm with numerically generated data reveal that rms errors of $\sigma < 0.5\%$ determine $\gamma$ and $\beta_2$ to an accuracy of less than 5\%, even in the presence of simulated random noise typical of experimental measurements.

Figure 7.5 shows the results of this algorithm for input pulses at 1542 nm. Here, the optimum values of $\beta_2$ and $\gamma$ were found to be $-21.4$ ps$^2$/km and $1.10 \times 10^{-3}$ W/m respectively, with $\sigma = 0.4\%$. A check on the accuracy of this technique can be made by comparing the intensity, phase and spectrum of the propagated input pulse (using the optimised values of $\beta_2$ and $\gamma$) with the intensity, phase and spectrum obtained from the direct experimental measurements. These results are shown as the circles in figure 7.5, and it is clear the agreement is excellent.
Figure 7.5  Intensity, phase, and spectrum of pulses after fibre propagation from experimentally measured FROG (solid lines), and from the numerical algorithm (circles).
Using $\gamma = n_2 \omega_0/cA_{\text{eff}}$, where $A_{\text{eff}}$ is calculated from the given mode field diameter (MFD) of the fibre ($\text{MFD}_{\text{SMF-28}} = 9.3$ \(\mu\text{m} \) @ 1300 nm), and $D = (2\pi/c^2)\beta_2$, Table 7.2 tabulates all the measurements using this technique, including the numerical error $\sigma$ from the algorithm. Figure 7.6 plots these results against wavelength, with appropriate error bars, to account for errors in optical power measurements and in the MFD. The average value obtained for the nonlinear refractive index was $n_2 = (2.53 \pm 0.09) \times 10^{-20}$ m$^2$/W, in good agreement with recent measurements [Artiglia et al. 1995]. This average is shown as the dashed line in the figure 7.6.

<table>
<thead>
<tr>
<th>$\lambda$ (nm)</th>
<th>$D$ (ps/km.nm)</th>
<th>$n_2 \times 10^{-20}$ m$^2$/W</th>
<th>$\sigma$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1534</td>
<td>16.4</td>
<td>2.54</td>
<td>0.3</td>
</tr>
<tr>
<td>1542</td>
<td>16.9</td>
<td>2.34</td>
<td>0.5</td>
</tr>
<tr>
<td>1549</td>
<td>17.3</td>
<td>2.51</td>
<td>0.6</td>
</tr>
<tr>
<td>1555</td>
<td>17.6</td>
<td>2.63</td>
<td>0.3</td>
</tr>
<tr>
<td>1559</td>
<td>17.9</td>
<td>2.54</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Table 7.2** Summary of Dispersion and Nonlinear Refractive Index results for standard single mode fibre.

For the dispersion results in figure 7.6, the least-squares fit to the data is also plotted (dashed line) and the dispersion curve obtained from the manufacturers specifications (solid line), and there is excellent agreement within experimental error.
These results show that the FROG-based technique provides a convenient and accurate simultaneous measurement of fibre dispersion and nonlinearity which may find wide application in optical fibre research. This technique is used throughout the remainder of this thesis to characterise pulse propagation through an Erbium doped fibre amplifier, and also through dispersion shifted fibre.
7.2.2 Copropagating Pulse Amplification at 1562 nm

The first series of amplification results and analysis were with the modelocked laser operating at 1562 nm, and the pulses were amplified in the co-propagating configuration. This is the easiest amplifier configuration to understand since the output fibre length after the EDFA is minimised, and also since the signal wavelength was detuned the furthest from the main Erbium transition (~1532 nm) thus minimising effects due to resonant dispersion.

![Measured Optical Spectrum of the Modelocked Fibre Laser Pulses at 1562 nm](image)

**Figure 7.6** Measured Optical Spectrum of the Modelocked Fibre Laser Pulses at 1562 nm

![Measured Autocorrelation function of the Modelocked Fibre Laser Pulses at 1562 nm](image)

**Figure 7.8** Measured Autocorrelation function of the Modelocked Fibre Laser Pulses at 1562 nm
The passively modelocked fibre laser was tuned to 1562.6 nm, and the spectrum and autocorrelation function were measured (figures 7.7 - 7.8). The spectral FWHM is \( \sim 1.55 \) nm, and the autocorrelation width FWHM = 3.01 psec, implying a pulse width of \( \sim 1.94 \) psec assuming a sech profile, and a time-bandwidth product of 0.37. The FROG spectrogram was also measured, and the electrical field was retrieved with an fitting error of 0.27% (acceptable).

Figure 7.9  Measured FROG of the Modelocked Fibre Laser Pulses at 1562 nm

Figure 7.10  Retrieved Intensity and Phase of the Modelocked Fibre Laser Pulses at 1562 nm
The average power from the fibre laser was measured to be 0.273 mW, with a repetition rate of 4.088 MHz. The output microscope objective had a measured transmission of approximately 60% (it was anti-reflection coated for use in the visible), and hence the average power from the output fibre of the laser was approximately 0.42 mW. The electrical field intensity is scaled so that the integrated area multiplied by the repetition rate equals 0.42 mW, this implies a peak power of 41.3 W. This was less than 53 W computed from the autocorrelation function method (table 7.1), but is more accurate since it uses the actual intensity profile, not the assumed hyperbolic secant.

![Graph showing frequency chirp](image1)

**Figure 7.11** Chirp of the Modelocked Fibre Laser Pulses at 1562 nm

![Graph showing computed Wigner function](image2)

**Figure 7.12** Computed Wigner Function of the Modelocked Fibre Laser Pulses at 1562 nm
The optical spectrum and autocorrelation function are computed from the retrieved electrical field, and are shown along with the measured data (figures 7.7 - 7.8). The agreement is excellent in both cases. The Wigner function was also computed from the electrical field, the small amount of chirp on the pulse is immediately evident by the tilt of the pulse in the time-frequency domain.

The pulses were then amplified using an Erbium doped optical fibre amplifier (figure 7.2). The signal pulses were traveling in the same direction as the 980 nm pump (copropagating). With the 980 nm laser diode producing approximately 4 mW, the average power from the EDFA was the same as the input signal. The next series of results (figures 7.13-7.18) show the amplifier operating in transparency.

![Graph showing measured optical spectrum and retrieved spectrum](image)

**Figure 7.13** Measured Optical Spectrum of the Amplified Pulses at 1562 nm, 4 mW 980 nm Pump
Figure 7.14  Measured Autocorrelation of the Amplified Pulses at 1562 nm, 4 mW 980 nm Pump

The spectrum and autocorrelation function of the pulses passing transparently through the amplifier is shown in figures 7.13 - 7.14. The spectrum shows SPM on the pulse, which is expected, since although the amplifier is providing no net gain when pumped with 4 mW of 980 nm, the nonlinear length of the input laser pulses is $\sim 11.5$ m (assuming fibre is SMF), which was comparable to the amplifier length, and acts to increase the frequency chirp on the pulses. The autocorrelation function has increased from $\sim 3.01$ psec for the laser, to $\sim 3.68$ psec after the amplifier, which is evidence of a positive dispersion section within the amplifier acting to broaden the pulses.

Figure 7.15  Measured FROG of the Amplified Pulses at 1562 nm, 4 mW 980 nm Pump
The FROG spectrogram was measured, and the electrical field was retrieved (with an error of 0.177 %). The phase of the electrical field changes by less than 2 radians over the central region of the pulse, which is approximately twice the phase change of the original laser pulses. The frequency chirp was computed from the gradient of the phase, and is more linear than that of the original laser pulses. The gradient of the chirp is 0.272 THz/psec for the amplified pulses, compared to 0.151 THz/psec for the laser.
The optical spectrum, and autocorrelation function were computed from the retrieved electrical field, and are plotted along with the measured data (figures 7.13 -7.14). Agreement in the optical spectrum is excellent, any slight differences may be attributed to inaccuracies with the infrared spectrometer. The autocorrelation function fit was good over the central region of the pulse, but tends to be worse at the wings of the pulse.

The Wigner function was computed from the electrical field and is shown in figure 7.18. The larger tilt on the central energy packet, compared to the original laser pulse, is indicative of the lasers linear chirp.

The pulses were further amplified using an Erbium doped optical fibre amplifier, by increasing the 980 nm pump power. The next series of results show the amplified pulses when the pump power was at 10 mW. 10 mW pump is significant since this was a typical pumping scheme for the fibre laser.
The measured optical spectrum is shown in figure 7.19. The SPM on the pulse has increased to cause a more pronounced spectral splitting and broadening. The measured autocorrelation function (figure 7.20) has increased slightly from the case when the amplifier was pumped with 4 mW, and has a FWHM = 3.8 psec.
Figure 7.21  Measured FROG of the Amplified Pulses at 1562 nm, 10 mW 980 nm Pump

Figure 7.22  Retrieved Intensity and Phase of the Amplified Pulses at 1562 nm, 10 mW 980 nm Pump

The retrieved electrical field from the FROG spectrogram (figure 7.22) shows a pulse intensity profile which has more small scale structure. This may be an artefact, since the retrieval error in this case was 0.46 % (marginal), indicating that there was some noise in the FROG data.

The phase closely resembles a parabola (figure 7.22), and the frequency chirp is further linearised, compared to 4 mW case. The chirp gradient (0.28 THz/psec) is also slightly larger than the 4 mW amplifier case (0.27 THz/psec).
Figure 7.23  Chirp of the Amplified Pulses at 1562 nm, 10 mW 980 nm Pump

The computed Wigner function shows the characteristic linear frequency chirp of SPM in figure 7.24. The chirp is seen from the tilt of the function. By integrating over the time domain of the Wigner function the spectrum is recovered (figure 7.19). The negative regions of the Wigner function contribute to the spectral splitting due to SPM (by decreasing the spectral intensity). Integrating the Wigner function over the frequency domain gives the temporal intensity (figure 7.22).
The amplifier was studied at a total of six 980 nm pump power levels, with the laser at 1562.6 nm. The pump powers considered were 4, 6, 8, 10, 14, and 18 mW, with the previously described measurement techniques. The most interesting observation, aside from the power amplification (figure 7.3), was the linearising of the frequency chirp. At 18 mW pump the chirp had gradient of 0.375 THz/psec, more than double that of the laser pulses, and was very linear (figure 7.25).

The results from these measurements can be used to provide an insight into the physical processes occurring in the fibre amplifier. The propagation of the pulses through the fibre amplifier module can be described by the Nonlinear Schrodinger Equation (equation 7.1), which is well known for propagation...
through the standard single mode fibre sections (section 7.2.1). In this thesis, the electrical field is measured at the input and output of the amplifier, and the results were used in conjunction with a numerical model based on the NLSE to estimate the dispersive and nonlinear terms in the NLSE for the Erbium doped fibre.

The physical layout of the amplifier module is shown in figure 7.26. In the co-propagating configuration the signal pulses from the fibre laser propagate through approximately 3.5 meters of standard single mode fibre (of the Wavelength Division Multiplexer) before encountering the Erbium Doped Fibre (EDF) section. It was assumed that the input coupling losses were 40 % at each end of the EDF, due to the mode field mismatch at the EDF-SMF splice (chapter 5). The short SMF fibre after the amplifying EDF had a total length of approximately 0.80 meters, which consisted of a 0.35 metre patch cord and APC connector after the EDF, and a 0.45 metre patch cord and APC connector to the output microscope objective stage. The microscope objective stage collimated the beam for propagation through free space, towards the detectors, and had a total transmission of approximately 40 %. In the counter propagating configuration, the input fibre length was therefore 0.35 metres before the EDF, and the output fibre was $3.5 + 0.45 = 3.95$ meters after the EDF section.
Figure 7.27 Simulation of the Amplified Pulses at 1562 nm, 10 mW 980 nm Pump

The measured electrical field of the laser (figure 7.10) was propagated forwards through the input 3.5 meters of SMF (at the EDF boundary), by numerically solving the NLSE for the standard SMF. Similarly, the measured electrical field emitted from the EDFA module (figure 7.22) was propagated backwards through the 0.80 meters of SMF after the EDF section, by numerically solving the NLSE. The field is therefore known at each end of the EDF section.

The amplifier was assumed in this analysis to exponentially amplify the signal along the EDF length. In practice this implies that SPM develops to a lesser extent for the amplified pulse propagating through the 0.8 meters of SMF, than if the pulse was amplified rapidly in the first few meters of the EDF where the pulse would have a longer distance to develop SPM. The EDFA amplification process was examined in chapter 4. Pulse temporal broadening is expected to be affected by the exponential amplification assumption to a smaller extent, since the nonlinear length in these experiments was always significantly less than the dispersion length.
The amplifier field was then numerically propagated backwards, through the EDF, and compared to the forward propagated laser field. The dispersive ($\beta_2$ & $\beta_3$) and nonlinear ($\gamma$) terms are unknown for the EDF, which has a non-standard core geometry and possible resonant dispersion. A minimisation routine was written in the computer program, which solved the NLSE, to locate the optimal choice of the dispersive and nonlinear terms (similar to section 7.2.1), so that the forward propagating laser field matched the backward propagating amplifier field, at the input to the EDF section. The function to be minimised was the sum of squares error between the normalised temporal intensity profile and the normalised spectrum in the frequency domain. This was necessary since the dispersion acts mainly to distort the temporal profile, and the nonlinearity acts mainly to distort the spectrum in the frequency domain. Normalising both profiles ensured that the minimisation process was equally weighted for the dispersive and nonlinear terms.
The simulation for the input laser field (figure 7.10) and the output amplified laser field (figure 7.22) was found to agree, with minimum error, for \( \beta_2 = 35 \text{ psec}^2/\text{km} \), \( \beta_3 = 13 \text{ psec}^3/\text{km} \) and \( \gamma = 2.76 /\text{W.km} \). The fields are compared at the input to the EDF section (\( z = 3.95 \text{ m} \)), and are shown in the time domain (figure 7.28) and the frequency domain (figure 7.29). The agreement was good, error may be attributed to the incorrect isotropic inversion assumption, and to changes in the pulse characteristics over the between measurements.

The dispersion was determined to be positive for the amplifier at this wavelength, with this pumping scheme, compared to \( \beta_2 \approx -23 \text{ psec}^2/\text{km} \) for standard SMF. The nonlinear term is similar to SMF, which is taken to be 1.2 /W.km. The dispersion and nonlinearity are plotted as a function of the 980 nm pump power in figure 7.30. The dispersion is in the range 36 - 38 psec²/km for this range of pump powers. The consistent value of dispersion shows that this technique is robust over the long time frame required for these measurements (~ 1 hour). This was possible because of the stability of the modelocked fibre laser used in these experiments. The optical spectrum and autocorrelation function of the laser was compared to the original data (figure 7.5 - 7.6) after the measurements were completed (figure 7.30), and excellent agreement was found, confirming laser stability over this time period.
7.2.3 Copropagating Pulse Amplification at 1543 nm

The next series of results presented concern the modelocked laser operating at 1543 nm, which corresponds to the amplified pulse being compressed upon propagation through the amplifier module. The pulse width was measured to have the greatest compression at this wavelength (figure 7.4), which indicates a net anomalous dispersion of the EDFA at this wavelength.
The characteristics of the modelocked laser operating at this wavelength are shown in figures 7.31 to 7.32. The pulse profile is similar to the previous case (1562 nm), so the FROG and Wigner function are not shown for the 1543 nm data. The average power from the fibre laser was measured to be 0.193 mW, with a repetition rate of 4.088 MHz. The output coupling losses of the microscope objective (of 40 %) implied that the actual average power from the fibre laser was 0.321 mW. Using the measured autocorrelation function width of 2.95 psec, the pulse width was approximately 1.90 psec. The peak power was calculated to be ~ 40 W using the autocorrelation function method (Table 7.1).

![Figure 7.32 Measured Autocorrelation function of the Modelocked Fibre Laser Pulses at 1543 nm](Image)

![Figure 7.33 Retrieved Intensity and Phase of the Modelocked Fibre Laser Pulses at 1543 nm](Image)
The peak power was determined to be 30.4 Watts, using the retrieved pulse profile from the FROG spectrogram data. This method for determining the peak power is more accurate than the autocorrelation method, since it numerically integrates the retrieved pulse profile. The measured autocorrelation width was 2.95 psec at 1543 nm, compared to 3.01 psec at 1562 nm, and the frequency chirp was also similar as shown in Figure 7.34 (gradient of 0.157 THz/psec at 1543 nm, compared to 0.151 THz/psec at 1562 nm). The only significant difference between the two cases here was the peak power (30.4 W at 1543 nm, and 41.3 W at 1562 nm).

![Figure 7.34 Chirp of the Modelocked Fibre Laser Pulses at 1543 nm](image)

The 1543 nm pulses were propagated through the amplifier module, as in the previous case. The first measurement was when the EDFA provided no net gain (transparency), and this occurred with 4 mW of 980 nm pump to give the same average power out. The measured spectrum is shown in figure 7.35, and SPM is seen to broaden the spectrum, but significantly less than in the 1562 nm case (figure 7.13). This could be due to smaller peak power at 1543 nm, or to a different value of the nonlinearity in the NLSE (the numerical simulations at the end of this section have been used to resolve this question).
The measured autocorrelation function after the transparent EDFA module is shown in figure 7.36. The FWHM of the autocorrelation function was 2.28 psec, implying a pulse width of approximately 1.47 psec (if the pulse profile is assumed to be a hyperbolic secant). The pulse has clearly been compressed at this wavelength, compared to the 1562 nm case (FWHM = 3.8 psec).
The FROG spectrogram was measured for the amplified pulses with 4 mW of 980 nm pump, and the intensity and phase of the electrical field envelope was retrieved with an error of 0.156 % (excellent). The electrical field intensity shows that the trailing edge of the pulse (positive time) was eroded, and the pulse was therefore compressed (figure 7.37). The chirp was linear over the central 2 psec of the pulse (figure 7.38), and had a greater gradient than the 1562 nm case (figure 7.15). The combined propagation effects of SPM and normal GVD act to linearise the frequency chirp over the central region of the pulse [Agrawal 1989]. Linear chirp generation in the normal dispersion regime is commonly used in combination with a prism pair to make a pulse compressor, since the prism pair can provide anomalous dispersion.
The retrieved electrical field was used to compute the optical spectrum and autocorrelation function (figure 7.35 - 7.36). In both cases the agreement was good with the measured data. The main difference was for the optical spectrum, and may be attributed to the inaccuracies of the locally made infrared spectrometer (section 3.8.1).

![Figure 7.39 Measured Optical Spectrum of the Amplified Pulses at 1543 nm, 18 mW 980 nm Pump](image)

The pulses were again further amplified, by increasing the 980 nm pump power to the EDFA module. At 18 mW pump, the average power from the microscope objective / output coupler stage was 2.45 mW, an amplification factor 12.4 over the input laser power (0.198 mW). The pulses exiting the EDFA module, before the microscope objective (with its transmission of 60 %) had an average power of 4.10 mW, implying that the quantum efficiency of the amplifier was approximately 35 %. The amplifier efficiency could possibly be further improved by improving the EDF-SMF splice losses.
The measured optical spectrum is shown in figure 7.39, and shows that SPM is present on the amplified pulse. There was also a small peak at 1528 nm which is the ASE emitted from the laser, and amplifier. The measured autocorrelation function is shown in Figure 7.40 and has the same profile as in the 4 mW case (figure 7.36), the FWHM = 2.22 psec, (compared to 2.28 psec). As the pump power was increased from 4 mW to 18 mW, the autocorrelation width remained between 2.22 to 2.28 psec. This provides further evidence of a net anomalous GVD at this wavelength in the EDFA module.

The electrical field was retrieved from the FROG spectrogram with an error of 0.24 % (figure 7.41). The field has the same shape as the 4 mW amplification case, the trailing edge of the pulse has been eroded. The combination of pulse compression and large amplification boosts the peak power to 457 W (before the microscope objective - output coupler). The chirp was also linear in this case over the central 2 psec of the pulse (figure 7.42), and had a gradient of 0.621 THz/psec (compared to 0.351 THz/psec for the 4 mW case, and 0.151 THz/psec for the laser). The electrical field was used to compute the optical spectrum and autocorrelation function, and these were plotted over the measured data (figures 7.37 - 7.38), with good agreement.
The numerical simulation of the measured electrical fields through the EDFA module was again done, as described in the previous section. The minimisation scheme was again implemented for the unknown dispersion and nonlinearity in the EDF section. For comparative purposes to the 1563 nm result (figures 7.28 - 7.29), case of 10 mW pump into the EDFA at 1543 nm is shown in figures 7.43 - 7.44.

The minimisation scheme for the numerical simulation for the input laser field (figure 7.33) and the output amplified laser field (10 mW case, similar to figure 7.39) was found to agree, with the minimum error, for the EDF section of $\beta_2 = -5.5$ psec$^2$/km, $\beta_3 = 3.3$ psec$^3$/km and $\gamma = 2.1$ /W.km.
The fields were compared at the input to the EDF section \((z = 3.95 \text{ m})\), and are shown in the time domain (figure 7.43) and the frequency domain (figure 7.44). The agreement was good in the time domain, which implied that the dispersion is accurately estimated by this method. In the frequency domain there was less satisfactory agreement. The single unknown quantity, \(\gamma\) was inadequate to allow a better fitting. This could be due to the exponential amplification assumption, when in practice the pulse may be amplified to a maximum level before the end of the EDF section is reached, and propagate at a high power over a greater distance than assumed by this model. This numerical simulation technique could be further improved by using the results of chapter 4 to calculate the actual amplification versus distance.
7.2.4 Summary of Copropagation measurements

The numerical simulation technique was applied to all data sets, with laser wavelengths over the range 1527 to 1562 nm, and amplifier pump powers from 4 mW to 18 mW. The dispersion and nonlinearity of the EDF section was then determined from the minimisation scheme, as described previously.

Figure 7.45 Beta 2 versus Wavelength for EDF, from FROG measurements and Simulations. The uncertainty is estimated to be +/- 10 psec2/km.

Figure 7.46 Gamma versus Wavelength for EDF, from FROG measurements and Simulations. The uncertainty is estimated to be +/- 4 /W.km.
The technique allowed $\beta_2$ to be determined consistently at each wavelength investigated. The total second order dispersion for the Erbium Doped Fibre section was determined to be positive, except around 1543 nm. From this result, the host contribution to the total dispersion of the EDF is positive across the Erbium gain band, since $\beta_2$ has a positive dispersion offset. This is consistent with the zero dispersion wavelength greater than absorption wavelength for the EDF [Zhu et al. 1996], which agrees with a previously published host dispersion for an EDF of similar geometry [Deutsch and Pfeiffer 1992; Type 3 fibre]. The resonant contribution to the total dispersion is the oscillation across the Erbium gain band in figure 7.45.

The third order dispersion did not contribute significantly to the total dispersion, except around 1543 nm (figure 7.47). This is because in the expansion of the dispersion, $\beta_2$ is multiplied by $(\omega - \omega_0)^2$, and $\beta_3$ is multiplied by $(\omega - \omega_0)^3$, so that $\beta_1$ is only significant when $\beta_3$ is numerically greater than $\beta_2$. The nonlinearity was difficult to determine by this method, as previously described, but is estimated to be in the range 2 - 10/\text{W.km} (figure 7.46), in good agreement with section 6.4.3 (ie. the nonlinear refractive index $n_2$ is the same order of magnitude as for standard SMF). The effect of the pump power level was to show the uncertainty in this measurement technique, since for this range of pump powers the measurement uncertainty dominanted over the physical contributions to the propagation processes.
7.2.5 Comparison between Co-Propagating and Counter Propagating Pulse Amplification

The final series of results is a comparison between the signal and pump fields co-propagating and counter propagating along the Erbium doped optical fibre. The modelocked laser was operated at nine different wavelengths across the Erbium amplifier gain band (table 7.1). The pulses were tuned to be approximately 1.9 psec and then attenuated to be ~ 25 Watts peak power. The amplifier was pumped with 10 mW of 980 nm pump, and then the co-propagating and counter propagating amplified pulses were measured using the previously described techniques. This technique worked well at each wavelength, where the laser pulses and the amplifier power did not change during the measurement process, but some variation in the results between other wavelengths may be expected as the conditions were not exactly the same.

![Graph showing ratio of amplified pulse autocorrelation width to that of the laser pulses for the Counter Propagating Amplified Pulses at 1532 & 1550 nm](image)

**Figure 7.48** Ratio of amplified pulse autocorrelation width to that of the laser pulses for the Counter Propagating Amplified Pulses at 1532 & 1550 nm

Preliminarily studies between the two propagation directions was done by measuring the pulse broadening or compression using autocorrelation. The pulse was measured to compress the most at approximately 1543 nm in the copropagating configuration (the pulses from the laser had FWHM = 2.95 psec compared to 1.98 psec after amplification). At this wavelength the pulse broadened from having a FWHM of the autocorrelation function of 2.95 psec to 4.16 psec in the counter propagating configuration. Such a dramatic change in pulse characteristics is difficult to explain using a constant value for the
dispersion and nonlinearities, between the co-propagating and counter propagating configurations. The analysis on the data was done by solving the NLSE for unknown dispersion and nonlinearity (similar to the previous sections) to determine the experimental propagation parameters.

The difference between the two amplification configurations is illustrated by the results presented in the next few pages, with the laser tuned to 1562 nm, and then 1543 nm. The section concludes by using all the results for the nine wavelengths to determine the dispersion and nonlinearity for both amplification configurations.

![Figure 7.49](image1.png)  
**Figure 7.49** Measured Optical Spectrum of the Modelocked Fibre Laser Pulses at 1562 nm

![Figure 7.50](image2.png)  
**Figure 7.50** Measured Optical Spectrum of the Co-propagating Amplified Pulses at 1562 nm
The modelocked fibre laser was tuned to 1562 nm and the average power, optical spectrum (figure 7.49), autocorrelation, and FROG spectrogram was measured. The average power was 0.273 mW after the microscope objective output stage (40 % transmission losses) and the autocorrelation function had a FWHM = 3.01 psec (implying a sech pulse width ~ 1.94 psec). The peak power was estimated to be ~ 39.5 W using numerical integration of the electrical field method (figure 7.52).

![Figure 7.51](image)

**Figure 7.51** Measured Optical Spectrum of the Counter propagating Amplified Pulses at 1562 nm

The laser pulses were then attenuated to 0.140 mW average power (~20.2 W peak power) using a variable attenuator after the fibre laser output coupler (using a variable bend in the fibre - section 3.1). The laser pulses were then propagated through the EDFA module counter propagating, and then co-propagating with the same 980 nm pump. The use of APC connectors to connect the laser and EDFA and output coupler ensured that the was experiment done was over a short time interval (~ 20 minutes), as opposed to fusion splicing the connections.
The optical spectrum for the co-propagating and counter propagating amplified pulses is shown in figures 7.50 - 7.51. The electrical field was retrieved from the FROG spectrograms and is shown in figures 7.52 - 7.54. The optical spectrum was computed from the retrieved electrical field and is shown on the measured spectrum graphs, and in all cases the agreement was excellent.
The average power of the amplified pulses (using 10 mW of 980 nm pump power) was 1.10 mW in the copropagation configuration, and 1.03 mW in the counter-propagating direction. The difference may be in part by the power meter detected residual 980 nm as well as the 1562 nm signal in the copropagation configuration, but not entirely since co-propagating amplification is slightly more efficient. The 980 nm power detected by the power meter was measured to be insignificant since the power coming out of the EDFA was reflected off a 1550 nm dielectric mirror in a power meter, the dielectric mirror had a low reflectivity at 980 nm.

The electrical field intensity broadened during co-propagating amplification, since the EDF section had a large positive dispersion at this wavelength (section 7.2.2). The pulse compressed in the counter propagating configuration, which could be due to the relatively long external standard SMF fibre length (~3.5 m WDM - figure 7.48) in this configuration. Using the method of solving the NLSE for unknown dispersion and nonlinearity, the dispersion of the EDF section was determined to be $\beta_2 = \sim 40 \text{ psec}^2/\text{km}$ and $\gamma = \sim 3.5 \text{ /W.km}$ for both configurations.
The modelocked fibre laser was tuned to 1543 nm and similar measurements of the pulse characteristics were taken. The average power was 0.193 mW after the microscope objective output stage and the autocorrelation function had a FWHM = 2.95 psec (implying a sech pulse width ~ 1.90 psec). The peak power was estimated to be ~ 28.4 W using numerical integration of the electrical field method (figure 7.55). The average power of the laser was then attenuated 0.118 mW to give a peak power of approximately 17.3 W. The average power of the amplified pulses was 1.167 mW in the copropagation configuration, and 1.108 mW in the counter-propagating direction.
The electrical field intensity compressed during co-propagating amplification (figure 7.56), since the EDF section had a small positive dispersion at this wavelength. The pulse broadens significantly in the counter propagating configuration (figure 7.57). Using the method of solving the NLSE for unknown dispersion and nonlinearity, the dispersion of the EDF section was determined to be $\beta_2 = \sim 80 \text{ psec}^2/\text{km}$ for the counter propagating configuration and $\beta_2 = \sim 3 \text{ psec}^2/\text{km}$ for the co-propagating configuration. The nonlinearity was $\gamma = \sim 3.5 /\text{W.km}$ for both configurations.

The results of the method of solving the NLSE with all the experimental data are shown in figure 7.58. The second order dispersion is larger when the pump and signal fields are counter propagating, this could be due to the signal and pump both being large at the one end of the EDF, serving to induce a larger
resonant dispersion, compared to the co-propagating case when the pump is large at one end and the signal is large at the other. The 1543 nm result was midway between the main two Erbium transitions at 1532 nm and 1555 nm (figure 4.4), and pulse propagation was the most susceptible to the effects of resonant dispersion at this wavelength.

7.3 Conclusion

In this chapter experimental results have been presented for picosecond pulse propagation through an Erbium Doped Fibre Amplifier. The use of Frequency Resolved Optical Gating allowed the electrical field to be measured before and after amplification. The propagation through amplifier was modelled as an exponential gain, and this was achieved by numerically solving the NLSE. A minimisation process was used to determine the propagation parameters which produced the measured electrical field. It was apparent that the system could not be sensibly characterised by single dispersion and nonlinearity parameters. The dispersion was found to be strongly wavelength dependent, and to a lesser extent pump and signal power dependent, which is consistent with resonant dispersion. The second order dispersion was normal and approximately 2 to 4 times the dispersion of standard single mode fibre [SMF-28], and hence will not significantly contribute to the pulse distortion when the total propagation length is longer than several hundred metres, which is the case for commercial telecommunications networks. However, the contribution of the EDFA dispersion is significant in a fibre laser configuration, where the total cavity length is only a few tens of metres, such as the one studied in this thesis (chapter 6).

The theory of resonant dispersion is unable to compute the actual dispersion because the individual emission and absorption cross sections are not known for the particular EDF used in this thesis. The theory also assumes that the inversion is isotropic throughout the EDF, which from chapter 4 and Appendix 1, is known not to be the case. The results of Appendix 1 show that the atomic inversion is dependent upon the signal level within the amplifier, but is less sensitive to whether the signal is co-propagating or counter-propagating with the pump field. The experimental results of the dispersion show a measurable difference between the two pumping schemes, especially at 1543 nm. This could be due to the exponential amplification assumption used to model the amplifier, which is only true for an amplifier with an isotropic inversion. The nonlinear propagation constant (\(\gamma\)) was also determined from the experimental results, and was only ~ 2-4 times that for standard fibre. This could be explained by the smaller core size of the EDF, which has a higher optical power density, to increase \(\gamma\) for the same nonlinear refractive index. There was some evidence for pump power induced \(\gamma\), but the effect was small which is consistent with the theory developed in section 7.1.
Chapter 8

Picosecond Pulse Propagation near the Zero Dispersion Wavelength

This chapter describes the results of experiments when the picosecond pulses from the modelocked fibre laser propagate through dispersion shifted fibre. The dispersion shifted fibre had a group velocity dispersion zero at approximately 1547 nm, and by tuning the laser either below or above the zero dispersion wavelength normal or anomalous dispersion propagation could be studied. For the first time, the use of the Frequency Resolved Optical Gating measurement technique provided a measurement of the temporal and phase characteristics of the pulse, which can be severely distorted after long distance propagation near to the zero of the group velocity dispersion. The results are compared to numerical simulations using SPM with second and third order dispersive terms, and excellent agreement was obtained between theory and experiment.

8.1 Dispersion Shifted Fibre Characteristics

The studies on pulse propagation near the zero dispersion wavelength were achieved with 2.2 km of Dispersion Shifted Fiber™ [Corning SMF/DS], and by using the previously described passively modelocked fibre laser (Chapter 6). SMF/DS fibre is optimised for use in the 1550 nm wavelength region.

The segmented core design used by Corning achieves low dispersion, attenuation, and bend loss at the 1550 operating wavelength. In this core design the refractive index difference between the peak of the core and the cladding is 0.9%, and the difference between the peak of the ring and the cladding is 0.3% (figure 8.1).

<table>
<thead>
<tr>
<th>Attenuation</th>
<th>Cut off Wavelength</th>
<th>1550 nm M.F.D.</th>
<th>NA</th>
<th>Zero Dispersion Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20 dB/km</td>
<td>1220 nm</td>
<td>7.93 μm</td>
<td>0.17</td>
<td>~ 1547 nm</td>
</tr>
</tbody>
</table>

Table 8.1 Corning SMF/DS Fibre characteristics
The fibre characteristics are shown in table 8.1, and since the parameters are similar to standard single mode optical fibre, fusion splicing provided low loss splices between sections of standard and dispersion shifted single mode fibre.

The dispersion \( D \) of the SMF/DS was determined from measured data over 1300 to 1650 nm, by the manufacturer from the actual fibre spool (figure 8.2). A third order polynomial was fitted to this data to allow interpolation and to account for any measurement errors. The fitted dispersion curve was used to calculate the higher order terms in the expansion of the group velocity dispersion from
\[ \beta_2 = -\frac{\lambda^2 D}{2\pi c} \quad \beta_3 = \frac{d\beta_2}{d\omega} \quad \beta_4 = \frac{d\beta_3}{d\omega} \]  

where \( \lambda \) is the wavelength and \( \omega \) is the angular frequency. The higher order expansion of the dispersion was necessary for theoretical analysis since the second order dispersion term passed through zero in the wavelength region of interest.

![Graph showing calculated second and third order group velocity dispersion.](image)

**Figure 8.3** Calculated second and third order group velocity dispersion.

![Graph showing calculated fourth order group velocity dispersion.](image)

**Figure 8.4** Calculated fourth order group velocity dispersion.
The zero dispersion wavelength (ZDW) is estimated to be at 1544.1 nm using the fitted curve of figure 8.2 and equation 8.1 (1547.0 nm on the manufacturers data sheet). The third order dispersion is positive and the fourth order dispersion is negative over the wavelength range of interest.

The empirical relations in table 8.2 were obtained by fitting cubic polynomials to the data used in figures 8.3 - 8.4, and was used in the theoretical analysis later in this chapter.

\[
\begin{align*}
\beta_2(\lambda) &= -5.78433919e+19 \lambda^3 + 1.833361169e+14 \lambda^2 - 2.47930010e+08 \lambda + 1.58659877e+02 \\
\beta_3(\lambda) &= 7.81449945e+17 \lambda^3 - 2.974027667e+12 \lambda^2 + 3.967463633e+06 \lambda + 1.79166853e00 \\
\beta_4(\lambda) &= -8.42559876e+15 \lambda^3 + 3.27849972e+10 \lambda^2 - 4.32464070e+04 \lambda + 1.9159199e-02
\end{align*}
\]

**Table 8.2**  Empirical relations for dispersion of SMF/DS fibre (\(\lambda = \text{metres}\))

### 8.2 Theory of Pulse Propagation near the Zero Dispersion Wavelength

The propagation of picosecond pulses through single mode optical fibres, in the region of zero second order group velocity dispersion, can be described by an extended nonlinear Schrödinger equation

\[
\frac{\partial A}{\partial z} - \frac{1}{2} \alpha_k A + \frac{i}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} + \frac{i}{24} \beta_4 \frac{\partial^4 A}{\partial T^4} = i \gamma |A|^2 A \tag{8.2}
\]

where \(A\) is the pulse amplitude, and \(T = t - \beta_4 z\) is a frame of reference moving with the pulse at the group velocity \(v_g\). The nonlinearity term was taken to be \(\gamma = 2.5 \times 10^{-3} \text{W.m}\), since the core radius of DSF is smaller than for standard SMF. The dispersion is expanded to fourth order since the second order term vanishes at the zero dispersion wavelength.

A steady-state solution of equation 8.2 is \(A^{*w} = \sqrt{P_0} \exp(i\gamma P_0 z)\), where \(\gamma P_0 z\) is the nonlinear phase shift. It is interesting to consider if equation 8.2 is stable against small perturbations of the field. This is done by substituting

\[
A = (\sqrt{P_0} + a) \exp(i\gamma P_0 z), \quad \text{where } |a|^2 << P_0 \tag{8.3}
\]
into equation 8.2, to obtain

\[ a(z, T) = a_1 \cos(Kz - \Omega T) + ia_2 \sin(Kz - \Omega T). \]  

The dispersion relation given by

\[ K = \frac{1}{6} \beta_3 \Omega^3 \pm \sqrt{\frac{1}{2} \beta_2 \Omega^2 + \frac{1}{24} \beta_4 \Omega^4 + 2\gamma P_0 \sqrt{\frac{1}{2} \beta_2 \Omega^2 + \frac{1}{24} \beta_4 \Omega^4}} \]  

where \( K \) is the wave number and \( \Omega \) is the frequency of modulation [Abdullaev et al. 1994] [Cavalcanti et al. 1991]. When \( K \) has imaginary values the perturbation \( a(z, T) \) (equation 8.4) experiences exponential growth and Modulation Instability (M.I.) occurs.

From the dispersion relation (equation 8.5) it is apparent that a CW field is unstable against small perturbations in the anomalous dispersion regime \( (\beta_2 < 0) \) [Agrawal 1989]. The third order dispersion term does not influence whether the dispersion relation has imaginary terms, hence it does not contribute to M.I. gain. The fourth order dispersion term can provide M.I. gain in the normal dispersion regime \( (\beta_2 > 0) \), provided that

\[ -2\gamma P_0 < \frac{1}{2} \beta_2 \Omega^2 + \frac{1}{24} \beta_4 \Omega^4 < 0. \]  

For the SMF/DS fibre equation 8.6 can be satisfied in the normal dispersion regime because the fourth order dispersion term is negative (figure 8.4).

![Figure 8.5](image_url)  

**Figure 8.5** Dispersion relation term with SMF/DS data.
The dispersion relation term which contains $\beta_4$ (equation 8.6) is shown in figure 8.5 using the SMF/DS data of the previous section. When it takes negative values the dispersion relation (equation 8.5) has an imaginary terms, and M.I. can occur. From figure 8.5 this condition can be satisfied for MI frequencies beyond 60 THz for typical pulse propagation in the normal dispersion regime.

### 8.3 Pulse Propagation in the Normal Dispersion Regime

The possibility of observing scalar Modulational Instability in the normal dispersion regime is interesting, and provided the motivation for a series of experiments. In an initial experiment, the fibre laser was tuned to 1532.5 nm, and the pulses where propagated though 700 metres of SMF/DS.

![Figure 8.6](image6.png)  
**Figure 8.6** Measured Optical Spectrum of the Modelocked Fibre Laser Pulses at 1532.5 nm.

![Figure 8.7](image7.png)  
**Figure 8.7** Measured Autocorrelation function of the Modelocked Fibre Laser Pulses at 1532.5 nm.
The passively modelocked fibre laser spectrum and autocorrelation function was measured (figures 8.6 - 8.7) when the laser was tuned to 1532.5 nm. The pulse duration is 2.2 psec and at an average power of 0.200 mW the peak power was estimated to be 22 W.

![Graph](image)

**Figure 8.8** Retrieved Intensity and Phase of the Modelocked Fibre Laser Pulses at 1532.5 nm

The FROG spectrogram was also measured and the intensity and phase of the electrical field was retrieved with an error of 0.3 % (figure 8.8). The intensity has only small structure which may be attributed to small measurement errors. The phase shift is less than one radian across the central part of the pulse, consistent with SPM generated during propagation through the external fibre lengths to the laser (as discussed in chapter 6). The optical spectrum and autocorrelation function was calculated from this electrical field and excellent agreement was obtained with the measured data (figures 8.6 - 7).

![Graph](image)

**Figure 8.9** Measured Optical Spectrum after 700 metres SMF/DS propagation
After 700 meters of SMF/DS fibre the spectrum has broadened considerably, and a second small peak occurs at 1558 nm (figure 8.9). It is important to note that the second peak at 1558 nm has the same frequency shift from the zero dispersion wavelength as the input laser spectrum, indicating that a phase matched process may be occurring. The energy of the spectrum is calculated to be conserved about the input laser wavelength, to within the uncertainties of the infrared spectrometer.

![Autocorrelation function](image)

**Figure 8.10** Measured Autocorrelation function after 700 metres SMF/DS propagation.

The autocorrelation function had a triangular shape, with some modulation at the origin (figure 8.10). This implies an approximately square pulse shape, as characteristic of propagation in the normal dispersion regime, with some fast modulation on the envelope. The modulation frequency of \( \sim 300 \) fs agreed with the 3.3 THz frequency shift in the spectrum between the main peak and small peak at 1558 nm.

The FROG spectrogram was measured after 700 metres of SMF/DS propagation and is shown in figure 8.11 [Dudley et al 1997]. The trace clearly possesses much more temporal and spectral structure than a typical laser FROG trace, separated into three distinct spectral bands. The largest amplitude is at a wavelength of around 768 nm, corresponding to the portion of the broadened pulse spectrum with largest amplitude at a fundamental wavelength around 1534 nm. There was also a much smaller amplitude signal around 779 nm, corresponding to the small peak in the pulse spectrum at a fundamental wavelength of 1558 nm. Of particular interest is the large FROG signal around 773 nm, since there is no appreciable signal around the fundamental wavelength of 1546 nm in the pulse spectrum. It can be seen that this component of the trace is temporally modulated as a function of delay time, and it is clear that the origin
of this modulation is in the mixing of the two components of the pulse spectrum on either side of the ZDW (this sum-frequency mixing process is also phase-matched with the BBO crystal). Importantly, the physical origin of this temporal modulation can be seen from the FROG trace in a way which is not possible with isolated spectral or temporal measurements.

**Figure 8.11**  Measured FROG after 700 metres SMF/DS propagation.

**Figure 8.12**  Retrieved Intensity and Phase after 700 metres SMF/DS propagation.
The FROG measurement after propagation through 700 m of DSF was more difficult than that of the input pulse because of the relatively low output pulse peak power (~ 5 W) and because of the presence of significant structure in both the temporal and spectral domains. In addition, after propagation through 700 m of fibre, significant depolarisation of the originally linearly polarised light from the F8L occurred, and a linear polariser was used to ensure that the same polarisation state was input to the spectrometer, autocorrelator and the FROG set-up. In these experiments a 256x256 grid of points scanning over a SHG wavelength range of 35 nm with 0.14 nm resolution, and a temporal range of 25 ps with 100 fs resolution was used. Because of this limited temporal scan range, there was some minor temporal clipping at the edges of the trace. Consequently, the very low retrieval errors which were obtained for the input pulse, could not be obtained for this pulse. The retrieved intensity and phase shown in figure 8.12 was obtained with a retrieval error of 0.9 %.

![Graph](image)

**Figure 8.13** Measured and Retrieved Autocorrelation function after 700 metres SMF/DS propagation.

The broadened pulse with the rapid oscillations on the leading edge (figure 8.12) was used to compute the optical spectrum and autocorrelation function. Figure 8.13 shows that the experimentally observed modulation in the autocorrelation observed near zero–delay is in good quantitative agreement with the autocorrelation function from the retrieved pulse. Note that even with the limited temporal resolution in this experiment, the two–dimensional nature of the FROG measurement allows very good retrieval of complicated temporal structure on the pulse profile.
This propagation was modelled using a modified NLSE which included dispersive terms up to third order (equation 8.2). At a wavelength of 1532.5 nm, the fibre dispersion parameters were $\beta_2 = 1.09 \times 10^{-3}$ ps$^2$m$^{-1}$, $\beta_3 = 1.16 \times 10^{-4}$ ps$^3$m$^{-1}$ (table 8.2), and the non-linearity parameter was $\gamma = 2.5 \times 10^{-3}$ W$^{-1}$m$^{-1}$. The input pulse was based on an asymmetric sech$^2$-fit to the retrieved intensity and phase shown in figure 8.8, with a peak-power of 22 W, equivalent to that measured experimentally. The simulations were carried out including both fourth-order dispersion and Raman effects [P. D. Drummond 1996], but with the experimental parameters, these additional nonlinear effects were found to be unimportant.

Figure 8.14 Simulation of 22 W 1532.5 nm pulses through 700 metres SMF/DS fibre (time domain).

The simulation of the 2.2 psec pulses over 700 metres is shown in figure 8.14. The pulse broadens due to propagation in the normal dispersion regime, and develops rapid oscillations on the leading edge. These oscillations arise from the interference between components of the pulse spectrum, generated
through four-wave mixing around the ZDW, which propagate through the fibre with the same group

Figure 8.15  Calculated Intensity and Phase after 700 metres SMF/DS propagation.

Figure 8.16  Simulation of 22 W 1532.5 nm pulses through 700 metres SMF/DS fibre (frequency domain).
The nonlinear length was approximately 17.5 metres for the 22 W pulses in the SMF/DF fibre, and therefore significant SPM develops during the 700 metre propagation (figure 8.16). The energy transfer to the 1558 nm peak occurs after SPM has broadened the spectrum to this extent, then a stable phase-matched soliton-like pulse develops (since this was now in the anomalous dispersion regime) (figure 8.17).

Figure 8.17  Calculated Optical Spectrum after 700 metres SMF/DS propagation.

Figure 8.18  Calculated Autocorrelation function after 700 metres SMF/DS propagation.
Figure 8.19 Calculated FROG after 700 metres SMF/DS propagation.

The autocorrelation trace was computed from the electrical field after the 700 m simulation (figure 8.18) and had similar features to the measured experimental autocorrelation function (figure 8.10). The qualitative agreement was very good, particularly in view of the difficulty in obtaining the experimental parameters for the simulation.

The FROG spectrogram was also computed from the electrical field after the 700 m simulation (figure 8.19). It has the same qualitative features as the experimental FROG spectrogram (figure 8.11), since the measured and calculated electrical fields are similar. The FROG technique provided good agreement between the retrieved pulse from the experimental trace and numerical simulation for propagation at 1532.5 nm. In this next section the modelocked fibre laser was tuned to 1542 nm and the pulses were again propagated through 700 meters of SMF/DS fibre. Since the four wave mixing process becomes more efficient closer to the zero dispersion wavelength, the peak power from the laser was attenuated to ~ 10 Watts peak power to avoid very wide band spectral components being generated.
The spectrum from the modelocked fibre laser, and after 700 metres SMF/DS propagation is shown in figure 8.20. The pulses had a centre wavelength at 1542.0 nm, a duration of 1.91 psec, and the peak power was 10.2 Watts. After propagation the spectrum has broadened, to create peaks at 1562 nm (-2.5 THz), 1546.5 nm (-0.57 THz), 1534 nm (+1.0 THz), and 1521 nm (+2.68 THz). Note that the ZDW is at 1546.5 nm, and a stable soliton-pulse formed at 1562 nm. Four wave mixing between two original signal photons (1542 nm) and a 1562 nm photon creates a small peak at 1521 nm. The large peak at 1534 nm travels at the same group velocity as the 1562 nm peak and hence there is efficient energy transfer between two spectral components. These nonlinear processes have been previously observed using standard fibre at 1300 nm [Yanovsky et al 1994].

Figure 8.20  Measured Spectrum from F8L and after 700 metres SMF/DS propagation.
Figure 8.21 Simulation of 10 W 1542 nm pulses through 700 metres SMF/DS fibre (time domain).

The experiment was studied theoretically by numerically solving the NLSE (equation 8.2). At a wavelength of 1542.0 nm, the fibre dispersion parameters were \( \beta_2 = 0.1942 \times 10^{-3} \text{ ps}^2\text{m}^{-1} \), \( \beta_3 = 0.1198 \times 10^{-3} \text{ ps}^3\text{m}^{-1} \) (using the empirical relations table 8.2) and the non-linearity parameter was \( \gamma = 2.5 \times 10^{-3} \text{ W}^{-1}\text{m}^{-1} \). The numerical simulation in the time domain is shown figure 8.21, where after 300 meters modulation develops on the pulse envelope. After 700 metres the pulse has developed complete and deep modulation across almost the entire pulse envelope (figure 8.22). The frequency of modulation is 3.55 THz (0.28 fsec modulation) and is equal to the frequency difference between the 1534 nm and 1562 nm peaks (figure 8.20) which travel at the same group velocity along the fibre (and interfere with one another).
Figure 8.22  Calculated Intensity and Phase after 700 metres SMF/DS propagation

Figure 8.23  Simulation of 10 W 1542 nm pulses through 700 metres SMF/DS fibre (freq domain)
The numerical simulation in the frequency domain is shown figure 8.23. The original laser frequency components are soon broadened by SPM (the nonlinear length is \( \sim 44 \) meters), and a pulse is created in the anomalous dispersion regime \((\sim -2.55 \text{ THz})\). Through four wave mixing a smaller peak is created at \( \sim +2.55 \text{ THz} \), but since this peak is in the normal dispersion regime it broadens temporally and remains small.

![Figure 8.24](image)

**Figure 8.24** Comparison between Simulation and Experimental Spectrum.

The measured spectrum after 700 metres propagation was compared to the results of the numerical simulation (figure 8.24), and excellent quantitative agreement is seen. The differences may be attributed to the spectrometer and the estimates of the dispersion terms (since the ZDW was estimated to be 1544.1 nm from the empirical results, compared to 1547 nm given by the manufacturer). The spectrometer had a known wavelength calibration error, as well as a wavelength dependent detection efficiency (the spectrometer may be 20% less sensitive at 1600 nm, compared to 1500 nm, section 3.8).

From figure 8.23 it can be seen that there is a dependence on the frequency separation with the propagation distance. Since Tera-Hertz frequency generation is a topic of current interest, the tunability of the frequency modulation was investigated further by using a fixed length of SMF/DS fibre and varying the laser wavelength and pulse power. The measured spectrum from 2 km of SMF/DS fibre is shown in figure 8.25 as a function of the input 1533.8 nm pulse peak power. The modelocked fibre laser was tuned to 1533.8 nm and pumped with 7 mW 980 mW to produce 2.52 psec 15.8 Watt pulses at 4 MHz repetition rate. The soliton peak at \( \sim 1560 \) nm was observed to move out to higher wavelengths at larger pump powers, and spectrally splits (possibly due to SPM).
Figure 8.25 Spectrum as a function 1533 nm Pulse Peak Power into 2 km SMF/DS.

Figure 8.26 Typical Autocorrelation function after 2 km SMF/DS propagation.
Figure 8.29  Simulation of 27 W 1560 nm pulses through 80 metres SMF/DS fibre (time domain).

Figure 8.30  Calculated Intensity and Phase after 80 metres SMF/DS propagation.
The electrical field intensity and phase are shown in figure 8.30 after propagation through 80 metres of SMF/DS fibre. The phase had a maximum shift of \(\sim 8.2 = 2.7\) radians, which was in agreement with the spectrum. The optical spectrum is calculated from this field and excellent agreement was obtained with the measured spectrum (figure 8.28).

![Autocorrelation function](image_url)

**Figure 8.31** Measured Autocorrelation function after 80 metres SMF/DS propagation.

The autocorrelation function is shown in figure 8.31. The central peak at zero time delay results from the autocorrelation of the sub-picosecond peak in figure 8.30.

![Spectrum](image_url)

**Figure 8.32** Measured Spectrum for 33 W pulses after 80 metres SMF/DS propagation

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The pulses from the modelocked fibre laser were then amplified slightly to 33 Watts peak power, and again propagated through 80 m SMF/DS fibre. The further spectral broadening which resulted is shown in figure 8.32. The measured autocorrelation function was similar to figure 8.31, which is expected because of pulse compression.

![Graph showing measured spectrum for 72 W pulses after 80 metres SMF/DS propagation.]

The pulses were further amplified to approximately 72 W peak, before propagation through 80 m SMF/DS fibre. The spectrum broadened further and frequency components were generated in the normal dispersion regime. This spectrum has some features which appear similar to propagation in normal dispersion regime, however numerical simulations were not able to give agreement with the measured experimental results. This could be attributed to the NLSE not having the additional nonlinear terms required for the shorter time scales and higher peak powers (SS & SRS). Another problem with the numerical simulation strategy was that it was very sensitive to the initial pulse used, since from the previous chapter it is clear that the amplification process broadens and chirps the pulse, this can make agreement with numerical simulation difficult for these last two experiments (figures 8.32 - 8.33).

The final series of results presented in this chapter were obtained when the modelocked fibre laser was tuned closer to the ZDW of the SMF/DS fibre. Figure 8.34 shows the measured spectrum after 43 W 1550 nm pulses have propagated through 80 metres of SMF/DS fibre. In this case no external EDFA was used to amplify the pulse power, instead the peak power was attenuated down to 18.7 W by using the bendloss-variable attenuator on the output of the fibre laser. At 18.7 W peak power numerical simulation
gave good agreement with the measured spectrum and autocorrelation function, however at higher pulse powers this was not the case.

Figure 8.34  Measured Spectrum as a function of Peak Power.

The lack of agreement between the numerical simulation and experimental results which occurred for input peak powers greater than 18.7 W, corresponded to the situation where the pulses were significantly compressed (figure 8.35). From the autocorrelation function measurements, it was clear that at large peak powers the pulse had broken up into several short duration pulses. However, the measurements of the spectrum and autocorrelation alone do not allow the electrical field to be uniquely determined, and therefore unfortunately the extent and depth of the modulation on the pulse could be not determined from this data. The FROG technique was not employed to study the experiment because the
wide spectral extent and relatively low peak power of the pulses produced spectrograms with an intensity that was comparable to the spectrometer's detection limit.

![Graph showing measured autocorrelation function as a function of peak power.](image)

**Figure 8.35** Measured Autocorrelation function as a function of Peak Power.

### 8.5 Conclusion

The development of high-capacity, long-distance optical communication systems will require the use of ultrashort optical pulses propagating near the zero-dispersion wavelength (ZDW) of optical fibres. It is therefore very important that ultrashort pulse propagation in this regime be accurately characterised. In particular, it is well-known that near-ZDW propagation can lead to severe pulse distortion because of the combined effects of self-phase modulation (SPM) and higher-order dispersion [Agrawal 1989]. Numerical simulations [Agrawal et al 1986] [Boyer and Carlotti 1986] using the nonlinear Schrödinger
equation, and several experimental studies at 1300 nm [Stern et al 1992] [Yanovsky and Wise 1994] [Beaud et al 1987] [Gouveia–Neto et al 1988], have indicated that the interaction between SPM and third-order dispersion in this regime results in the development of rapid temporal oscillations on the pulse and the corresponding appearance of characteristic peaks in the pulse spectrum.

A combined experimental and theoretical study of the propagation of 3ps pulses at 1.3 μm measured the autocorrelation function and spectral characteristics of pulses after propagation through 1.3 km of standard silica fibre and showed that they were in good agreement with the predictions of numerical simulations using a modified NLSE including higher–order dispersive and Raman effects [Schütz et al 1993]. In these experiments, it was noted that while the spectral–splitting was quite apparent from the measurements of the pulse spectrum, the severe temporal distortion of the pulse predicted by the NLSE was not reflected in the measurements of the optical autocorrelation function. In this work the application of the FROG technique has enabled direct experimental confirmation of the predicted complex pulse shapes which result from long distance pulse propagation near the zero dispersion wavelength under the influence of dispersion and nonlinear interactions.

In conclusion, it is shown that the FROG technique is extremely valuable for characterising the intensity and phase of an optical pulse after transmission through an optical fibre. Whilst other techniques can provide useful insight into the shape of simpler pulses, no other technique can provide a direct measurement of the intensity and phase of severely distorted or rapidly modulated pulses such as those generated by propagation near the zero–dispersion wavelength of optical fibres.
Chapter 9

Conclusions

This thesis has studied the generation of picosecond pulses and their propagation through both resonant and non-resonant media. This was done by constructing an Erbium doped fibre laser (EDFL) which was passively modelocked to produce a stable train of ~2 psec pulses at 4 MHz, tunable over the Erbium gain band. This fibre laser had the unique property of stability combined with low pump power requirements, to produce 30 - 100 Watt peak power pulses. It was shown that the laser operated in the quasi-steady state by efficiently switching a fundamental soliton within the NALM section in the cavity.

A second EDFA, of identical construction to that used in the laser, was probed using the pulses from the EDFL, and by indirectly determining the electrical field before and after the EDFA module, the propagation parameters could be inferred from a numerical model. It was apparent that the EDFA exhibited resonant dispersion, which is characteristic for propagation through an atomic medium on resonance. The measurements combined with a numerical simulation showed that the atomic inversion distribution along the amplifying fibre is crucial for determining the dispersion for different amplification configurations. The overall dispersion was found to be normal and therefore ruled out the generation of Modulational Instability within this Erbium doped fibre [Agrawal 1992]. The normal dispersion could be explained from the host dispersion, and the resonant dispersion contribution of the inverted Erbium ions.

A comprehensive numerical model was developed for the EDFA, and was used extensively to analyse its operation. Initially the results were used to check the theory against some amplifier measurements made by the manufacturer of the Erbium doped fibre. The manufacturer had made signal gain measurements for a small signal, in-line and power amplifier configurations, and these were shown to agree with the numerical model, to within a reasonable degree of experimental uncertainty.
The model was then used to study the effect on amplifier performance when physical parameters are changed, which is of interest for the design optimisation of commercial EDFAs. The study investigated the signal gain and noise figure when the EDF length, signal power, signal wavelength and pump power were changed. The results of further calculations, which relate specifically to the atomic inversion of the Erbium ions, are presented in Appendix 1.

The numerical model of the EDFA was extended to be a linear Erbium fibre laser with intracavity loss elements. The results of simulations allowed the intracavity loss to be predicted from experimental results. The experiments had an output coupler reflectivity for the CW linear Erbium laser of 4% for several different Erbium fiber lengths, and measured the laser slope efficiency and threshold in each case. Using the experimental data the model gave consistent results for intracavity losses (the Erbium fusion splices), which were approximately 40%. This large splice loss is expected when splicing small core Erbium doped fibre to standard single mode fibre.

In chapter 8, experimental results were presented for picosecond pulse propagation through a section of dispersion shifted optical fibre. Propagation near the zero dispersion wavelength can lead to severe pulse distortion because of the combined effects of higher–order dispersion and self–phase modulation (SPM). Pulse propagation in the normal dispersion regime was demonstrated to generate a deep THz frequency modulation across the pulse envelope, which could be tuned from ~ 2 to 6 THz, by changing the pulse peak power or wavelength. The Frequency Resolved Optical Gating (FROG) measurement technique was shown to allow complete characterisation of a pulse propagating through an optical fibre, which had previously only been inferred from performing separate time and frequency domain measurements. This additional information can be used to completely characterise the fibre parameters, and therefore will lead to a better understanding of the processes involved in high speed communications.
Rate Equation Solutions for a 6 metre EDFA in the Steady-State

This appendix presents results of numerical solutions for the rate equations of a 6 metre Erbium Doped Fibre Amplifier. The parameters of the EDF are those used in chapter 4, which accurately represent the Erbium doped fibre used in the experimental work in this thesis. The core doping is taken to be 500 parts per million of Erbium ions in the glass fibre, with a doping radius of 80% of the fibre core radius, assuming a square doping profile.

The numerical model solves the rate equations for the pump, signal and wideband Amplified Spontaneous Emission (ASE) fields. The numerical solution is solved iteratively, since the propagation of each field depends on the others, as described in chapter 4.

This appendix presents numerical solutions for the signal gain, and average inversion for a 6 metre EDFA. The average atomic inversion is important since it in part determines the resonant dispersion, which effects the pulse propagation through the EDFA (chapter 7).

The results are presented in two sections. The first represents telecommunication applications, and has signal powers from 0.1 μW to 10 mW (in 12 logarithmic steps) with the EDF pumped with 10, 50 and 100 mW of 980 nm pump in either co-propagating or counter propagating with the signal field. The final section of results has the signal power from 50 μW to 1 mW) with the EDF pumped with 2 to 18 mW of 980 nm pump, which closely model the experiments described in chapter 7.
Figure A1.1 Inversion along an EDFA using 50 mW 980 nm Pump, and 0.1 mW Signal at 1532 nm.

The inversion of the Erbium doped fibre amplifier is shown as a function of distance in figure A1.1. The signal and pump are co-propagating along the amplifier, from left to right. The inversion reaches a maximum after ~20 cm, and then is depleted until the end of the EDF is reached. The numerical model includes amplified spontaneous emission and this depletes the inversion slightly at the beginning of the EDF. The inversion is slightly larger at the centre of the core (Radius = 0) than at the Erbium doping radius (towards the edge of the core), because the pump mode decreases more rapidly than the signal mode across the core and the Erbium ions experience relatively less pump radiation.

It is apparent from figure A1.1 that the inversion is not isotropic along the EDF length, and that amplification is not a purely exponential process. A more uniform inversion may be achieved by reducing the EDF length to less than 1 metre, however only a relatively small amplification can be expected from this configuration.
The signal gain for a 6 metre EDFA is calculated in figure A1.2 using 10 mW 980 nm. The signal power is from 0.1 μW to 10 mW in 12 logarithmic steps (0.1 μW, 0.28, 0.8, 2.3, 6.6, 18.7, 53 152, 432 μW, 1.23, 3.5, 10 mW). The maximum signal gain is experienced by the lowest signal at the peak of the emission cross section curve (figure 4.4). The minimum gain is for the 10 mW signal pump, which at 100 percent quantum efficiency for converting the 980 mW pump photons to ~1550 nm photons is a gain of ~1.58.
There is a small difference for the signal gain when the 980 nm pump is co-propagating, compared to the counter propagating case, and this shown in figure A1.3. The counter propagating configuration can provide up to approximately 5 percent more amplification. The two pumping schemes generate different amounts of ASE, which serve to deplete the atomic inversion and hence reduce the amplification.

![Graph](https://via.placeholder.com/150)

**Figure A1.4** Average Atomic Inversion versus Wavelength using 10 mW 980 nm Pump

The average inversion of the Erbium doped fibre amplifier is shown in figure A1.4. The counter propagating configuration has a slightly higher inversion. It is apparent that the 10 mW pump field alone produces an average inversion of ~ 0.74, and is depleted with as little as 0.2 μW input signal power.
Figure A1.5  Signal Gain versus Wavelength using 50 mW 980 nm Pump

Figure A1.6  Average Atomic Inversion versus Wavelength using 50 mW 980 nm Pump
The inversion for the 50 mW and 100 mW pumping schemes is shown in figures A1.6 and A1.7. The average inversion has increased to ~ 0.85 for the 100 mW case, but is still dramatically effected by the signal level power. This has implications for the resonant dispersion, since from these calculations it is dependent upon signal wavelength and power level. It can also be observed that it is difficult to fully invert the atomic medium for an EDFA which has an optimised length (to provide the maximum gain).
The signal gain for a 6 metre EDFA is calculated in figure A1.9 using 2 mW 980 nm. The signal power is from 50 µW to 1 mW in 9 steps (50 µW, 100, 200, 300, 400, 500, 600, 800, & 1000 µW). The amplifier is absorbing, since the pumping scheme is inadequate to provide significant inversion.
The average inversion is quite different depending upon whether the pump field is co-propagating or counter propagating with the signal field, when the pump is only 2 mW. The inversion is larger for the largest signal levels, since more photons can be absorbed into the upper atomic state. The higher atomic inversion provides less loss for the larger signal field.
The case of 4 mW 980 nm pumping is shown in figures A1.12 and A1.13. From the experimental results in chapter 7 this was when the amplifier was transparent (gain = 1). There is theoretical agreement if splicing losses between the Erbium and WDM fibres attenuate the pump to less than 4 mW. For many conditions, the average inversion is depleted to less than 0.5 after signal amplification, however it has to be greater than 0.5 prior to the signal field being incident to provide an net gain greater than 1.
Figure A1.14  Signal Gain versus Wavelength using 6 mW 980 nm Pump

Figure A1.15  Average Atomic Inversion versus Wavelength using 6 mW 980 nm Pump
Pumping the EDFA with more than 4 mW of 980 nm pump provides significant signal amplification (figures A1.14 and A1.16). The case of 10 mW 980 nm pump was studied in detail in chapter 7. The signal gain is almost identical between the two pumping schemes, with the average inversion only changing by 1-2 percent.
Figure A1.18  Signal Gain versus Wavelength using 18 mW 980 nm Pump

Figure A1.19  Average Atomic Inversion versus Wavelength using 18 mW 980 nm Pump

The final two figures are for pumping with 18 mW 980 nm pump. The largest amount of signal gain is experienced by the smallest signal level (50 μW). The average inversion is strongly dependent upon the signal wavelength and signal level, and to a smaller degree, upon the pumping scheme.


Corning SMF/DS http://www.usa.net/corning-fiber/products/pi1037.html

Corning SMF-28 http://www.usa.net/corning-fiber/products/pi1036.html


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