# Probability Distributions of Cumulative Losses Caused by Earthquakes

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**SUMMARY:**
A simple and intuitive method for estimating the probability distribution of cumulative earthquake losses is presented. The method is a Monte Carlo simulation which treats the occurrence of earthquake events as a random Poisson process. Examples carried out using this method show that the probabilistic skew in cumulative loss is high for time periods corresponding to the design lives of typical buildings, and is therefore important in decision making. Deaggregation and sample loss functions in time are discussed as a way of investigating which intensity level events contributed most to the overall losses and how frequently these events occurred. Mathematical expressions for the coefficient of variation and skew in cumulative loss distributions are derived. Finally, limitations of the model are briefly discussed with the conclusion that the presented method is potentially a valuable means of conveying earthquake loss information, as an addition to an expected loss term and/or loss hazard curve.

*Keywords: Earthquake loss estimation, Monte Carlo simulation*

## 1. INTRODUCTION

The results of financial loss estimation studies in earthquake engineering are commonly presented in two forms: a 'loss hazard curve' and an Expected Annualised Loss (EAL) term. The loss hazard curve is a plot of the monetary loss caused by an event (on the $x$-axis) against the expected number of events each year to exceed that loss, $\lambda(l)$ (on the $y$-axis). In this context, 'loss' refers to the cost required to repair the structure to its undamaged, pre-earthquake state after an earthquake. This loss hazard curve is a direct extension of a seismic hazard curve, the same in every respect except that the $x$-axis of the seismic hazard curve measures a ground motion intensity measure as oppose to a financial loss. In the case of PEER framework loss estimation methods (Bradley, Dhakal, Cubrinovski, MacRae, & Lee, 2009; Porter, 2003), the EAL term and loss hazard function are calculated by Equations 1.1 and 1.2, respectively.

$$\text{EAL} = \int_0^\infty E[L|IM] \left|\frac{d\lambda(im)}{dim}\right| dim \tag{1.1}$$

$$\lambda(l) = \int_0^\infty G_{L|IM}(l, im) \left|\frac{d\lambda(im)}{dim}\right| dim \tag{1.2}$$

where EAL is the expected annualised loss; $\lambda(l)$ is the mean annual frequency of exceedance of loss level $l$; $im$ is the intensity measure, typically a spectral ordinate or peak ground motion parameter; $\left|\frac{d\lambda(im)}{dim}\right|$ is the absolute value of the slope of the seismic hazard curve at intensity measure $im$; $E[L|IM]$ is the expected loss conditional upon intensity; and $G_{L|IM}(l, im)$ is the complementary cumulative distribution function (ccdf) of loss conditional upon intensity.

Additionally, on knowing the EAL, the expected loss in an arbitrary time period, $t$, is calculated using Equation 1.3. This equation is showed graphically as Figure 1. The discount rate, $\alpha$, reduces the magnitude of future losses to account for, in very simple terms, the fact that a loss sustained in the future is not so bad as a loss sustained now, because that lost money could have been 'growing' through investment or earning interest, and also because people tend to value negative impacts in the short term more highly than those in the long term. After adjustment for inflation, the 'real' discount rate for decision making purposes in earthquake engineering is typically in the range of 2.5% - 6%, depending on the situation (FEMA, 1992; Zerbe & Falit-Baiamonte, 2001).

$$\mathrm{EL}(t) = \begin{cases} \mathrm{EAL}\left(\dfrac{1-e^{-\alpha t}}{\alpha}\right) & \alpha > 0 \\ \mathrm{EAL}\; t & \alpha = 0 \end{cases} \tag{1.3}$$

where $\mathrm{EL}(t)$ is the expected value of cumulative loss; and $\alpha$ is the continuously compounding, real discount rate (adjusted for inflation)

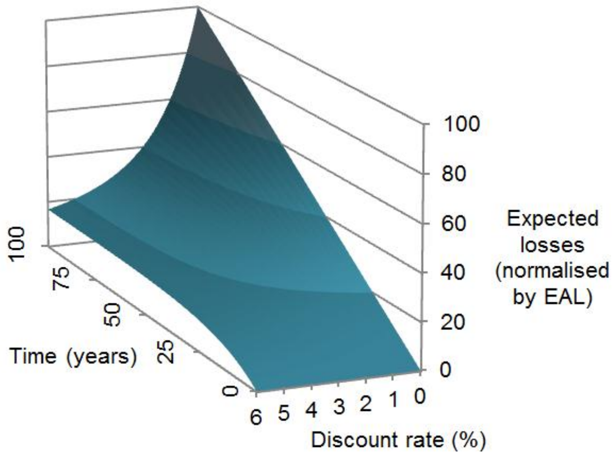


**Figure 1.** Normalised expected losses as a function of time and the discount rate

These expected loss estimates (and the annual loss exceedance frequencies from the loss hazard curve) are undoubtedly valuable parameters for decision-making. However, there is one obvious disadvantage in that both pieces of information are expected loss estimates: 'expected' in that they are based only on the mean annual frequencies of earthquakes of various intensities. This can be misleading in situations where there is a high probability of no significant seismic action (and hence no or minimal losses) and a small probability of a large earthquake (and so very high losses), because the expected loss outcome may not representative of a likely outcome(Smith, 2003). In other words, expected loss estimates can be misleading where the probability distribution of cumulative losses has a high right skew. In such situations, having the probability distribution of cumulative losses provides valuable information and, in conjunction with expected loss estimates, can help to better-inform decision making about seismic risk.

## 2. CALCULATING DISTRIBUTIONS OF CUMULATIVE LOSS

### 2.1. Background

Calculating probability distributions of cumulative loss requires modelling of earthquake occurrence as a stochastic process in time, rather than occurring at average rates. Anagnos & Kiremidjian (1988) reviewed different approaches to earthquake occurrence modelling and divided available approaches into five categories: Poisson models, Markov models, semi-Markov models, renewal models and trigger models. This paper investigates the probability distribution of cumulative losses based on a homogenous Poisson process of earthquake occurrence. Readers interested in loss estimation methods using earthquake occurrence models other than a random Poisson process are referred, for example, to the work of Takahashi, Der Kiureghian & Ang (2001) and Liek Yeo & Cornell (2005, Chapter 5).

Perhaps the easiest way to calculate a probability distribution of cumulative losses is Monte Carlo simulation, because of difficulties in deriving a closed form solution. Several such examples are available in literature. For example, Smith & Cousins (Smith & Cousins, 2002) discussed a Monte Carlo method based on randomly prompting earthquake occurrence at a fault based on Guttenburg Richter $a$ and $b$ values, attaining the Modified Mercalli Scale intensity at the site by an appropriate attenuation relationship and then using this intensity to model the loss. When applied to a cost-benefit study, this method showed that the probability distribution of benefits derived from a retrofit can be highly skewed (Smith, 2003). Pei & Van de Lindt (2009) described a loss estimation framework based on Monte Carlo simulation where the Poisson rate of earthquake occurrence was one of several parameters that were selected based on Bayesian updating from real data. This method was used to create probability distributions of cumulative loss for two woodframe buildings. Ergonul (2006) used Monte Carlo simulation to estimate the future worth of a shopping centre including life cycle costs due to earthquake occurrence. Goda, Lee & Hong (2010) used Monte Carlo simulation to approximate the life cycle costs of base isolated and non-base isolated structures.

However, one downside of Monte Carlo methods mentioned above is that they may be difficult to implement from the point of view of a structural earthquake engineering researcher. For example, Smith's method required a model for sources of seismicity and their Guttenberg-Richter $a$ and $b$ values. Pei's method required a thorough background in Bayesian updating. Furthermore, Bayesian updating required much supplementary data.

### 2.2. Proposed Monte Carlo method

The proposed easy-to-use method of calculating distributions of cumulative losses is shown as a flowchart in Figure 2. The method is very similar to that proposed by Pei &Van de Lindt (2009), except with the inclusion of discounting and without the use of Bayesian updating. It relies on treating earthquake occurrence as a random Poisson process and also assumes that any damage done is quickly repaired after an earthquake.

#### *2.2.1. Probability distribution for the number of events*

For each Monte Carlo trial, the analysis randomly samples the number of earthquakes to occur in the considered time period. This requires a probability distribution for the number of earthquake events to occur. Assuming the occurrence of earthquakes at a site is adequately modelled by a random Poisson process, the probability of $n$ earthquake events occurring of an intensity greater than equal to some intensity level, $im$, is given by a Poisson distribution as Equation 2.1.

$$P(n, IM \geq im) = \frac{[\lambda(im)t]^n e^{-[\lambda(im)t]}}{n!} \tag{2.1}$$

where $\lambda(im)$ is the mean annual number of events producing an intensity of $im$ or greater, which can be obtained directly from a seismic hazard curve; and $t$ is the time period considered.

In order to create a practicable probability distribution for the total number of events to occur in the time period, it is necessary to place a lower limit on earthquake intensity, $im_{min}$, so that a pool of 'important' events can be counted. Intuitively, this lower limit should be selected such that the occurrences of loss from earthquake events with shaking intensities less than the lower limit produce a negligible contribution to the overall losses. This might be achieved by disaggregating the expected annual loss by intensity and ensuring that the area under the resulting curve from earthquake intensities less than the lower limit is small.

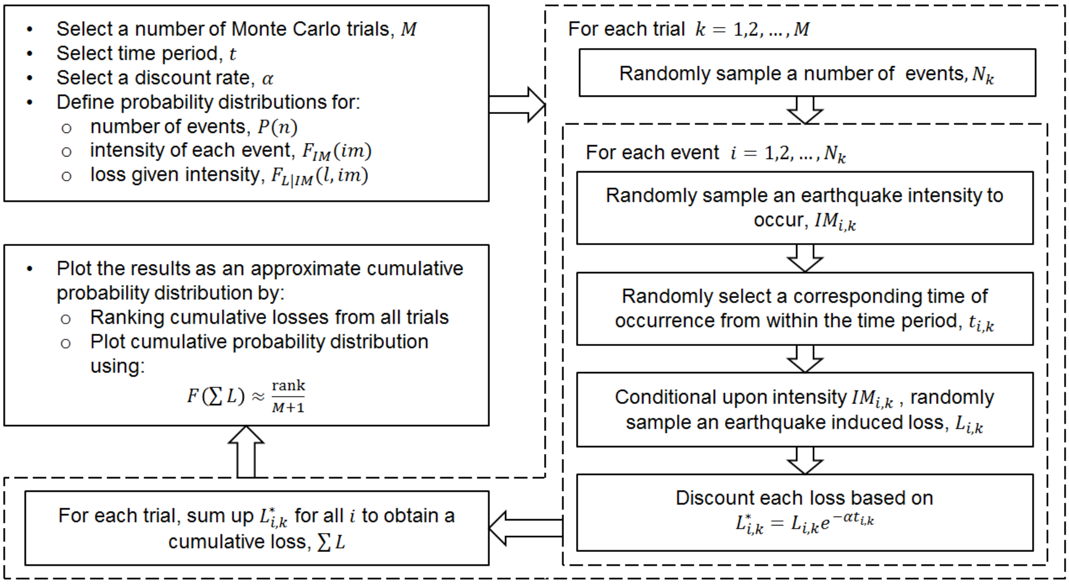


**Figure 2.** Flowchart for the Monte Carlo simulation framework

With a lower bound now placed on the earthquake intensity measure, substituting $im = im_{min}$ into Equation 2.1 yields a probability distribution for the total number of 'important' earthquake events to occur. For clarity, this is reproduced as Equation 2.2.

$$P(n, IM \geq im) = \frac{[\lambda(im_{min})t]^n e^{-[\lambda(im_{min})t]}}{n!} \tag{2.2}$$

Users can create a cumulative probability distribution from Equation 2.2, from which different numbers of earthquake events can be randomly sampled for each trial of the Monte Carlo simulation. It is noted that as $n$ becomes large, it may be necessary to use a Normal approximation to the Poisson distribution.

### *2.2.2. Probability distribution for intensity of each event*

A probability distribution of intensity is required in order to randomly sample an intensity for each event that occurs. Equation 2.3 gives the simple formula for this distribution. The $\lambda(im)$ and $\lambda(im_{min})$ terms are the number of events expected to occur annually with intensity exceeding $im$ and $im_{min}$, respectively. These can be read off a seismic hazard curve. This result has been previously reported by Der Kiureghian (2005, p. 1645).

$$G_{IM}(im) = \frac{\lambda(im)}{\lambda(im_{min})} \tag{2.3}$$

where $G_{IM}(im)$ is the complementary cumulative distribution function (ccdf) of the intensity of each 'important' event (i.e. for intensities greater than $im_{min}$)

### *2.2.3. Sampling a loss for each intensity*

For each intensity, a loss must then be selected conditional upon that intensity. This loss can be attained by random sampling from a probability distribution of loss conditional upon intensity, that in turn may be derived by a number of loss modelling approaches including MMI-based, Capacityspectrum and PEER framework methods. It is assumed that this conditional probability distribution is renewable or ergodic (Der Kiureghian, 2005).

*2.2.4. Selecting a time of occurrence*

In order to apply discounting, a time of occurrence of each event must be known. In keeping with the memoryless property of a random Poisson process, this time of occurrence can be randomly selected from within the time period of interest.

**2.3. Example**

The proposed Monte Carlo method was applied to the 10 storey, 1.5 s period, reinforced concrete structure detailed in the New Zealand Red Book (Bull & Brunsdon, 1998). The mean and variance of loss as a function of intensity, as well as the probability of collapse as a function of intensity, were calculated by Bradley et al. (2009) using PEER framework loss estimation methods. From this data, an approximate probabilistic relationship for loss conditional upon intensity was created, and this is shown in Figure 3a. The financial loss is represented as a damage ratio, the financial loss caused by an earthquake divided by the replacement cost of the building, which was taken as $NZ 14 million with no variability. The structure was assumed to be situated on rock in either of two locations: Wellington or Christchurch. Spectral acceleration hazard curves (T=1.5 s, ξ=5%) for these locations were approximated by multiplying the peak ground accelerations given by Stirling et al. (2002, p. 1894) by 0.88, the appropriate spectral shape factor from the New Zealand loadings code (SNZ, 2004). The resulting hazard curves are shown in Figure 3b. It is noted that the seismic hazard curve for Christchurch was based on the pre-conceived seismic hazard prior to the 2011 and 2012 Christchurch earthquakes.

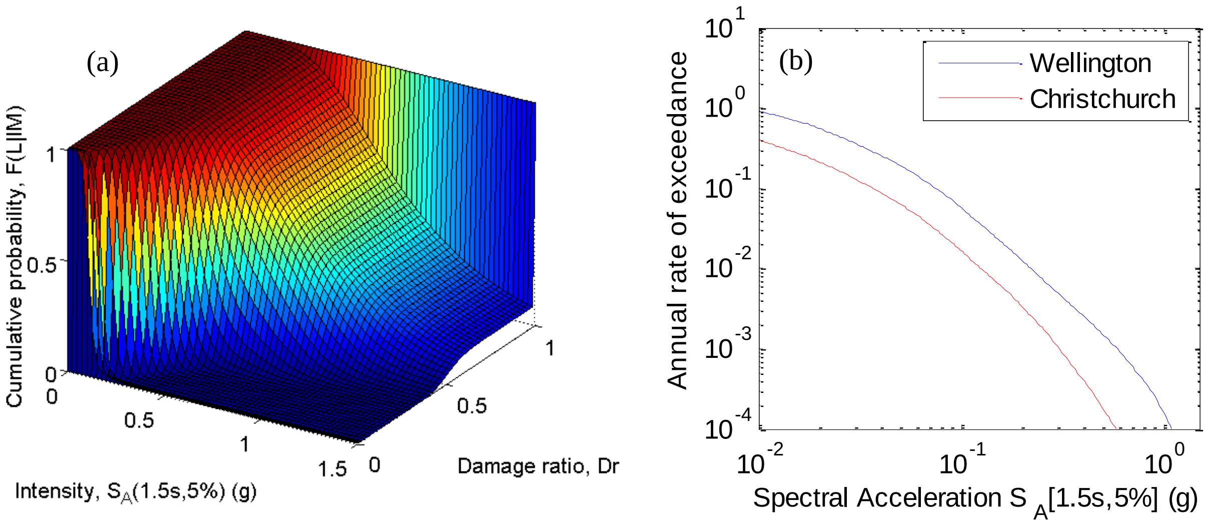


**Figure 3.** The loss given intensity relationship (a) and seismic hazard curve (b) for the example structure

Monte Carlo simulation was then applied to derive probability distributions of cumulative loss, for 1, 20, 50, 100, 200, 500 and 1000 year time periods. Simulations were run with no discounting as well as with a discount rate of 3%. The resulting cumulative loss distributions are shown in Figures 4 and 5. Each simulation involved 5000 trials and took about 30 seconds to run on a Dell T3500 personal computer. In viewing Figures 4 and 5, the percentile difference between the mean and the 50$^{th}$ percentile median may be thought of as a measure of the skew in the distributions.

For the undiscounted cases, Figure 4 shows that as the simulation time period increased, the expected cumulative losses increased and the cumulative loss distributions became progressively more normally distributed. Similarly, as the simulation time period decreased, the expected cumulative losses decreased and the loss distribution became more skewed towards zero losses, producing a large divergence of mean and the median loss values. It is noted that service lives of 200, 500 and 1000 years are unlikely to be achieved in reality. These are included in order to inform the reader as to the overall trends in the accumulation of earthquake induced losses with time.

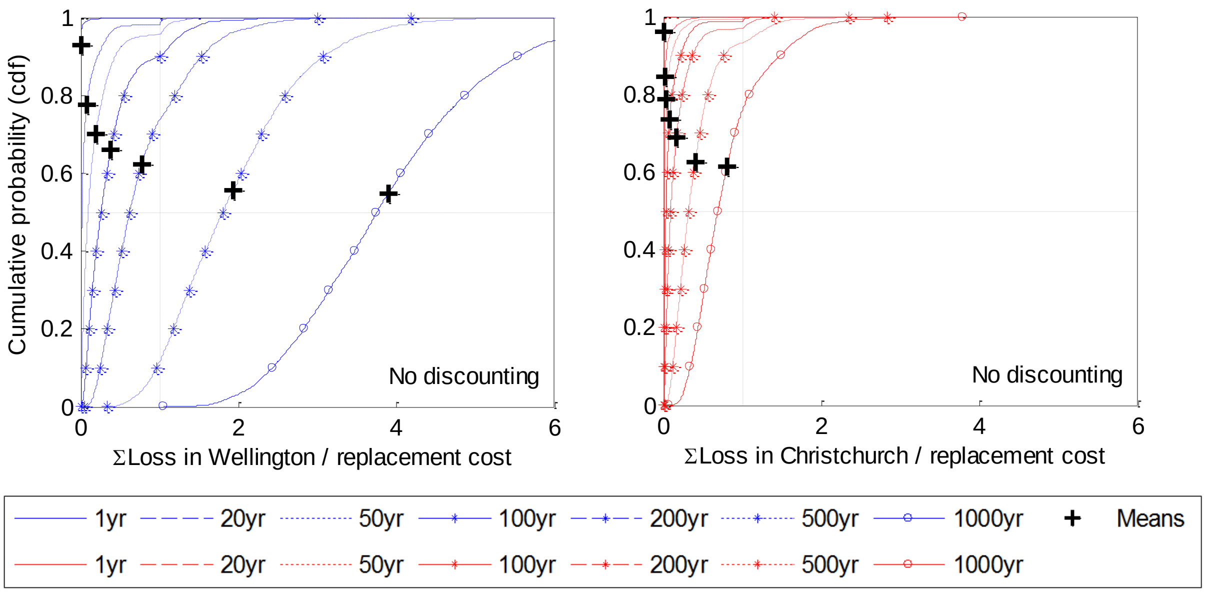


**Figure 4.** Cumulative loss distributions for the example structure when situated in Wellington and Christchurch, with no discounting

Figure 5 highlights the impact of discounting. As the time period becomes progressively greater, the probability distribution of discounted cumulative tends to a fixed shape which is approximately that given by the 1000 year distribution. It is interesting to note that when discounting is applied, the probability of discounted cumulative losses being greater than the replacement cost of the building is almost negligible. The reason is that discounting reduces the magnitude of future losses to the point that a structure would have to be heavily damaged and then collapse early in its design life in order for cumulative discounted losses to exceed the replacement value.

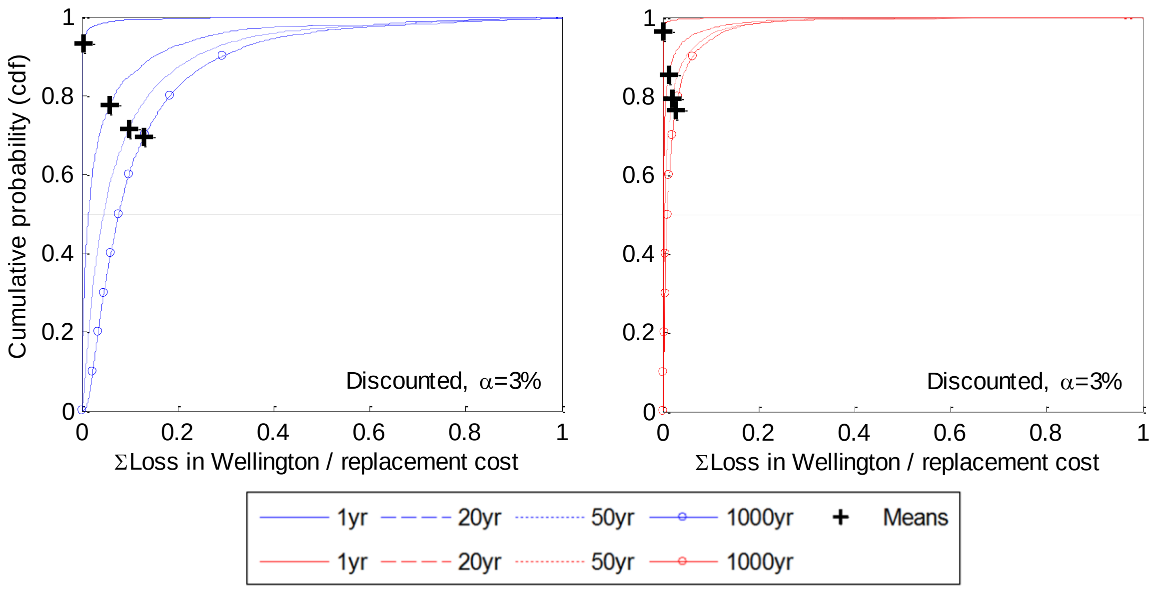


**Figure 5.** Cumulative loss distributions for the example structure when situated in Wellington and Christchurch, with a discount rate of 3%

Comparing between time periods, the loss distributions for Wellington consisted of higher losses than the corresponding distributions from Christchurch. This is simply due to the greater overall level of seismic hazard assumed for Wellington, as shown in Figure 3b.

In practical terms, Figures 4 and 5 show that when the considered time period is short (roughly in the order equating to the designed working life of a typical building), the earthquake loss distribution tends to be highly skewed, meaning there is a high probability of no (or minimal) losses and a small chance of very high losses. This has important implications for risk management, particularly so if a high discount rate is used. It suggests that decision makers should consider skew in the distribution of cumulative losses.

It is noted that the information in Figures 4 and 5 may be plotted in a variety of different ways. For example, it may be plotted as a histogram for which a probability density function may be fitted for each time period. Furthermore it may be plotted as a graph of cumulative loss against time with lines corresponding to the mean loss as a function of time, median loss as a function of time and also relevant percentile losses as a function of time.

## 3. EXTENSIONS

### 3.1. Contributions to expected losses

One of the disadvantages of probability distributions of loss, such as Figures 4 and 5, is that they do not convey information on which intensity level or loss level events contributed most to the overall cumulative losses, nor how frequently these events occurred. Such information is important for decision makers as well as researchers. For example, if the overall losses are dominated by small and frequent events, it is known that improvement strategies should pay particular attention to the protection of fragile non-structural components and contents under moderate earthquake shaking and that a seismic protection strategy that only strengthens the structural frame against major damage without reducing in-structure floor accelerations may provide little overall benefit to the structure. It is also known that the loss model will be highly sensitive to assumptions made about the fragility of building components at low demands.

One method of attaining information about which intensity level events contributed most to the overall cumulative losses is to deaggregate the EAL by intensity (Bradley, et al., 2009, p. 18). Deaggregation is simply breaking up a term that is derived by integration or summation (for example Equation 1.1) into incremental contributions of a given parameter at different values. A curve that is derived by deaggregation of EAL by intensity level will tend to zero as the intensity becomes small, say 0.05g peak ground acceleration, signifying a negligible contribution to the overall losses. This is because such earthquakes, although common, are unlikely to cause any significant damage to the structure. It will also tend to zero at high intensities, say 2g, because such earthquakes, although devastating, are extremely uncommon. In between, there will be a ‘hump’ centred around those intensities that contribute most to the overall losses.

Another method of viewing this information that is useful for the purposes of visualising and understanding the ways in which losses accumulate, is to plot out several sample functions of cumulative losses in time, where each sample path corresponds to a single Monte Carlo trial (Benjamin & Cornell, 1970, p. 236; Cutfield & Ma, 2012, p. 7). Where the cumulative losses tend to accumulate by small, repeated losses, this implies that the overall losses are dominated by small but frequent earthquakes. Where the losses tend to accumulate by infrequent but large ‘jumps’, this implies that the overall losses are dominated by infrequent but large earthquakes.

### 3.2. Coefficient of variation and coefficient of skew

Yeo and Cornell (2005, p. 88) derived the moment generating function for the probability distribution of cumulative losses based on a random Poisson process of earthquake occurrence and used it to derive expressions for the mean and variance as functions of time and discount rate. Drawing on and extending these results, it can readily be shown that the coefficient of variation, COV, and coefficient of skewness, $\gamma$, of a probability distribution of cumulative losses are as given by Equations 3.1 and 3.2, respectively. The coefficient of variation is a normalised measure of the variability in a distribution defined as the standard deviation divided by the mean. Likewise, the coefficient of skewness is a normalised measure of the skew in a distribution defined as the third central moment divided by the standard deviation, cubed. These are important measures of risk for decision makers. Higher coefficients of variation imply that there is greater likelihood that an actual outcome will differ from the expected loss outcome. Higher coefficients of skew imply that there is a greater risk of an unlikely 'extreme' loss outcome along with a higher probability of no or minimal losses.

$$\mathrm{COV}\big(\Sigma L(\alpha,t)\big) = \begin{cases} \mathrm{K}_1\left[\dfrac{\alpha\sqrt{1-e^{-2\alpha t}}}{\sqrt{2\alpha}(1-e^{-\alpha t})}\right] & \alpha > 0 \\ \mathrm{K}_1\dfrac{1}{\sqrt{t}} & \alpha = 0 \end{cases} \tag{3.1}$$

$$\gamma\big(\Sigma L(\alpha,t)\big) = \begin{cases} \mathrm{K}_2\left[\dfrac{\left(\sqrt{2\alpha}\right)^3(1-e^{-3\alpha t})}{(3\alpha)\left(\sqrt{1-e^{-2\alpha t}}\right)^3}\right] & \alpha > 0 \\ \mathrm{K}_2\dfrac{1}{\sqrt{t}} & \alpha = 0 \end{cases} \tag{3.2}$$

where $\mathrm{COV}\big(\Sigma L(\alpha,t)\big)$ is the coefficient of variation of cumulative losses; $\gamma\big(\Sigma L(\alpha,t)\big)$ is the coefficient of skew of cumulative losses; $\alpha$ is the discount rate; $t$ is the time period considered; $\mathrm{K}_1$ is a constant equal to $\sqrt{\frac{\mathrm{E}[L^2]}{\lambda(\mathrm{E}[L])^2}}$; $\mathrm{K}_2$ is a constant equal to $\sqrt{\frac{(\mathrm{E}[L^3])^2}{\lambda(\mathrm{E}[L^2])^3}}$; $\lambda$ is the Poisson rate of 'considerable' earthquakes; and $L$ is a random variable for the loss induced by each single 'considerable' earthquake.

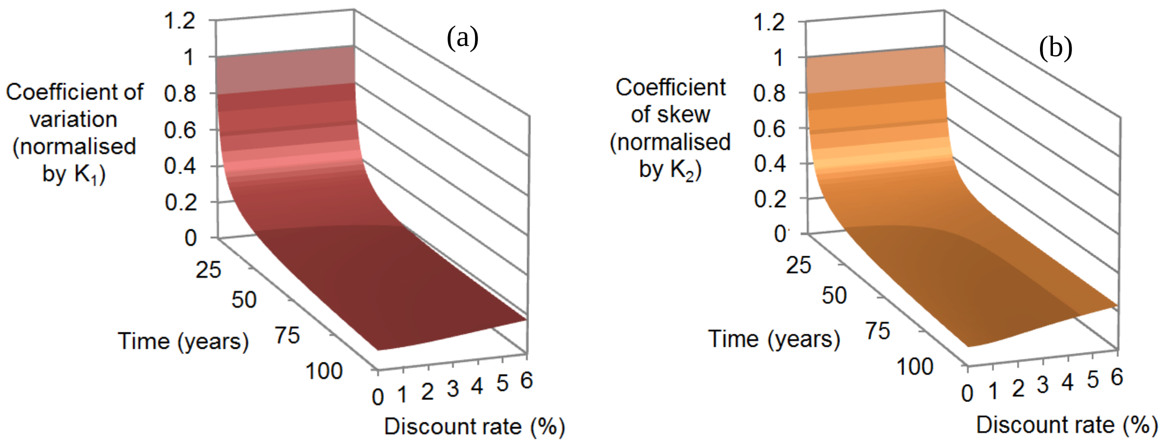


**Figure 6.** Plots of the (a) coefficient of variation and (b) coefficient of skew of the probability distribution of cumulative losses, as they depend on the time period and the discount rate

Figure 6 shows Equations 3.1 and 3.2 plotted as functions of time and discount rate, normalised by the constants $\mathrm{K}_1$ and $\mathrm{K}_2$, respectively. It can be seen that the trends observed for both the coefficient of variation and skew are very similar. For example, both coefficients tend to infinity as time tends to zero and are proportional to the inverse square of time when the discount rate is equal to zero (this fits with the trends seen in Figure 4). Importantly, after about 25 years, the both coefficients tend to 'plateau', with little more change occurring after this time. As such, after about 25 years, it is the values of the constants $K_1$ and $K_2$ that are most important for determinants of the variance and skew, and these will primarily be a function of the seismic hazard to which the structure is exposed.

For the purposes of calculating the constants $K_1$ and $K_2$, the Poisson rate of 'considerable' earthquakes, $\lambda$, and the probability distribution of the 'single loss' random variable, $L$, must be known. These might be selected to match observed data, or alternatively calculated by: (1) selecting $\lambda$ as the $\lambda(im_{min})$ term discussed in Section 2.2.1; (2) calculating a probability distribution for the intensity of each earthquake, $G_{IM}(im)$, by Equation 2.3; and (3) calculating the distribution of loss according to Equation 3.3 using closed form or numerical integration.

$$f_L(l) = \int_0^1 f_{L|IM}(im, l)\, dG_{IM}(im) \tag{3.3}$$

where $f_L(l)$ is the probability density function (pdf) of loss that can be used to calculate the constants $K_1$ and $K_2$; and $f_{L|IM}(im, l)$ is the pdf of the financial loss conditional upon the intensity, which must be calculated by a loss analysis of the structure in question.

## 4. LIMITATIONS

Any earthquake induced financial loss estimation will be subject to a great deal of aleotoric uncertainty (inherent randomness that is irreducible) and epistemic uncertainty (knowledge uncertainty that is reducible). These uncertainties are present throughout the simulation procedure, from defining expected seismic hazard and dynamic structural response to estimating damage and repair costs (Bradley, et al., 2009), and the credibility of loss estimation results will be limited by how well these uncertainties are represented and controlled. On top of this, the method presented in this paper has some further limitations in that:

- It assumes earthquakes occur according to a random Poisson process. This is not unreasonable for smaller events, but is likely to be unrealistic in locations where the seismic hazard is dominated by a 'characteristic' rupture of a known fault, because the inter-arrival times of these characteristic ruptures are may not well represented by the decaying exponential distribution assumed by a random Poisson process (Takahashi, et al., 2001).

- It assumes that earthquake induced damage is instantaneously repaired. Yeo and Cornell (2005, Chapter 5) showed that while this assumption is not unreasonable in the mainshock environment, it is not entirely appropriate in the aftershock environment. It also has obvious limitations in the case of structural collapse.

## 5. CONCLUSIONS

This paper presented a simple Monte Carlo method for estimating the probability distribution of cumulative direct earthquake losses. The method treated the occurrence of earthquake events at a site as a homogenous Poisson process. Numerical examples showed that for a typical design life of a building, the probability distribution of cumulative losses is likely to have a high skew, and hence that the proposed method (or similar methods) are useful tools for conveying earthquake loss information for decision making, because neither skew nor variability are not captured in an expected loss estimate.

Some related topics were then discussed. Firstly, the importance of using deaggregation or Monte Carlo sample functions to examine which intensity level events contributed most to the overall losses is emphasized. Secondly, formulae were developed to allow quick calculation of the coefficients of variation and skew.

Lastly, the limitations with the approach taken in this paper are briefly discussed. The two most important limiting assumptions are identified as (1) the assumption that earthquakes occur randomly according to a homogenous Poisson process and (2) the assumption that damage is instantaneously repaired.

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