An improved method for conveying earthquake loss data utilising Monte Carlo simulations

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ABSTRACT:

A simple and intuitive method for estimating the probability distribution of cumulative direct earthquake losses is presented. The method is a Monte Carlo simulation which treats the occurrence of earthquake events as a random Poisson process. Examples carried out using this method show that the probabilistic skew in a cumulative loss distribution is greatest for short considered time periods and for regions with lower overall seismic hazard. Disaggregation and sample loss functions in time are used to examine which intensity level events contributed most to the overall cumulative loss estimates and how often these events occurred. Errors and limitations of the model are briefly discussed, with the conclusion that the presented method is potentially a valuable means of conveying earthquake loss information, as an addition to an expected loss term and/or loss hazard curve.

1 INTRODUCTION

Expected or mean direct losses is often the key output variable for loss estimation and cost-benefit studies. Whether the loss estimation method is the PEER performance based-design framework (see Porter, 2003), a method based on capacity and demand spectrum as in HAZUS (see Kircher, Whitman, & Holmes, 2006), a method based on the Modified Mercalli Intensity (MMI) scale (see Smith & Cousins, 2002), or any other methods, the following can be generally stated:

- For a loss estimation study, results are commonly presented as an expected loss, perhaps by way of a loss hazard curve (for example Bradley, Cubrinovski, Dhakal, & MacRae, 2010; Mander, Dhakal, Mashiko, & Solberg, 2007; Rahnama, Seneviratna, Morrow, & Rodriguez, 2004 and others)
- For a cost-benefit study, results are commonly presented as an expected cost-benefit, often in the form of an expected net present value (for example FEMA, 1994; Hopkins & Stuart, 2003 and others)

The loss-hazard curve plots the annual expected number of seismic events producing losses at or exceeding a given loss level, a direct extension of the seismic hazard curve. It provides information to the user about how frequently different loss level events are likely to occur. The area under the loss hazard curve represents the Expected Annual Loss (EAL). However, the limitation of the loss hazard curve is that while it informs users the annualised occurrence probability of different loss level events over a design period, it does not provide any information on how the actual event may likely vary from the expected amount. Disaggreations of expected losses are useful in allowing the user to see what were the most significant factors contributing to the expected losses, but they are still expected or mean terms.

Expected losses, or expected cost-benefits, is undoubtedly a valuable parameter in earthquake risk management. However, it stands as self-evident that at the end of a structure's design life, there can only be one outcome, which may be radically different than the 'expected' outcome. In this light, it would seem that a probability distribution of cumulative losses would be a very useful addition to expected value data, with the proviso that this can be attained with a reasonable accuracy and in a reasonable time.

Smith (2003a) has argued this in a journal paper in which he suggests the expected cost-benefit is, by its name, the mean or expected value of a probability distribution of many possible cost-benefit outcomes and in the context of decision-making, what is really needed is the full probability distribution.

Smith presented a Monte Carlo technique for predicting loss distributions based on knowing the Guttenberg-Richter magnitude-frequency relationships of nearby sources of seismicity, an appropriate strong motion attenuation model in terms of MMI and the vulnerability of the buildings in question (Smith, 2003b; Smith & Cousins, 2002). Application of this technique to a cost-benefit study showed that a probability distribution of cost-benefit may be highly skewed, and hence conventional expected cost-benefit may be a poor decision-making tool when compared to other available options (Smith, 2003a; Smith & Vignaux, 2006).

More recently, Pei and Lindt (2009) used a Monte Carlo method to calculate a probability distribution of long-term cumulative losses for a given time period. In the analysis, the occurrence of earthquake events was modelled as a random Poisson process (Anagnos & Kiremidjian, 1988; Benjamin & Cornell, 1970, pp. 236-249; Der Kiureghian, 2005). The analysis first established the Poisson rate of 'notable' earthquake shaking occurrences at a site and the probability distribution of shaking intensity for each 'notable' event (in terms of spectral acceleration), these were then combined with the loss intensity vulnerability model for the building in question, derived using PEER performance based design framework type methods. A key advantage of this technique was that conditional probability was implemented at all stages of the simulation through Bayesian updating. This therefore enabled the calibration of the results with any known data throughout the steps. Pei and Lindt applied this methodology in an example to calculate the probability distribution for cumulative loss for time periods of 1, 10, 20, 50 and 75 years for a woodframe building. Other notable studies using Monte Carlo simulation methods to calculate life cycle costs include those by Ergonul (2005, 2006) and Goda et al. (2010).

One downside of Monte Carlo methods mentioned above is that they may be difficult to implement from the point of view of a structural earthquake engineering researcher. For example, Smith's method required a model for sources of seismicity and their Guttenberg-Richter a, b and M_0 values. Pei's method required a thorough background in Bayesian updating. Furthermore, Bayesian updating required much supplementary data.

2 SIMPLIFIED METHOD FOR ESTIMATING PROBABILISTIC DISTRIBUTION OF CUMULATIVE LOSSES

2.1 Overview

The proposed overall framework for estimating the probability distribution of cumulative losses, modified from that presented by Pei and Lindt (2009), is illustrated in Figure 1. The earthquake occurrence models used in this framework are derived from a seismic hazard curve assuming a random Poisson process. The probabilistic loss given intensity model must be derived for the structure of interest using an appropriate method, such as the PEER performance based design framework. All of these are discussed in details in the following sections.

2.2 Probability distribution for the number of earthquakes

For each trial, the analysis randomly samples the number of earthquakes to occur in the considered time period. This requires a probability distribution for the number of earthquake events to occur. Assuming the occurrence of earthquakes at a site is adequately modelled by a random Poisson process, the probability of n earthquake events occurring of an intensity greater than equal to some intensity level, im, is given by Equation 1.

$$P(n, IM \geq im) = \frac{[\lambda(im)t]^n e^{-[\lambda(im)t]}}{n!} \quad (1)$$

where $\lambda(im)$ is the mean annual number of events producing an intensity of im or greater, which can be obtained directly from a seismic hazard curve and t is the time period considered.

In order to create a practicable probability distribution for the total number of events to occur in the time period, it is necessary to place a lower limit on earthquake intensity, im_{min}, so that a pool of 'important' events can be counted. Intuitively, this lower limit should be selected such that the occurrences of loss from earthquake events with shaking intensities less than the lower limit produce a negligible contribution to the overall losses. This can be achieved by disaggregating the expected annual loss by intensity and ensuring that the area under the resulting curve from earthquake intensities less than the lower limit is small.

With a lower bound now placed on the earthquake intensity measure, substituting $im = im_{min}$ into Equation 1 yields a probability distribution for the 'total' number of earthquake events to occur. For clarity, this is reproduced as Equation 2.

$$P(n, IM \geq im_{min}) = \frac{[\lambda(im_{min})t]^n e^{-[\lambda(im_{min})t]}}{n!} \qquad (2)$$

Users can create a cumulative probability distribution from Equation 2, from which different numbers of earthquake events can be randomly sampled for each trial of the Monte Carlo simulation. It is noted that as n becomes large, it may be necessary to use a Normal approximation to the Poisson distribution.

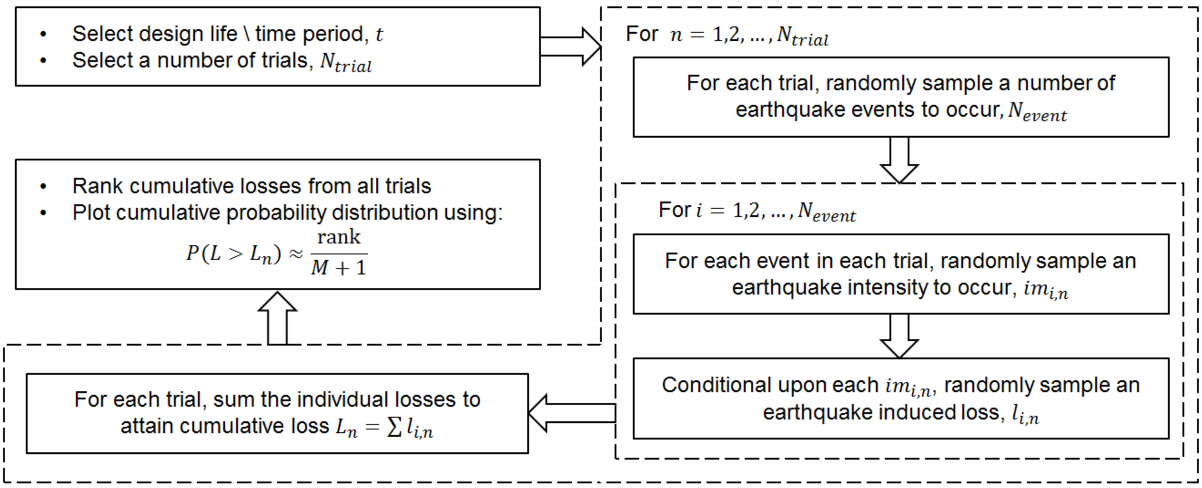

Figure 1 - Flowchart for the Monte Carlo simulation framework

2.3 Probability distribution for the intensity of each earthquake

A probability distribution of intensity is required in order to randomly sample an intensity for each event that occurs. Equation 3 gives the simple formula for this distribution. $\lambda(im)$ and $\lambda(im_{min})$ terms are the number of events expected to occur annually with intensity exceeding im and im_{min}, respectively. They can be read off a seismic hazard curve. This result has been previously reported by Der Kiureghian (2005, p. 1645).

$$G_{IM}(im) = \frac{\lambda(im)}{\lambda(im_{min})} \qquad (3)$$

where $G_{IM}(im)$ is the Complementary Cumulative Distribution Function (CCDF) of the intensity of each 'important' event (i.e. for intensities greater than im_{min})

2.4 Probability distribution for loss conditional upon intensity

For each intensity selected, a loss must then be selected conditional upon that intensity. This loss can be attained by random sampling from a CCDF of loss as a function of intensity, $G_{L|IM}(l, im)$. This distribution can be calculated using the PEER performance-based design framework as in Equation 4 (Der Kiureghian, 2005; Porter, 2003).

$$G_{L|IM}(l, im) = \int\int G_{L|DM}(l, dm)\left|dG_{DM|EDP}(dm, edp)\right|\left|dG_{EDP|IM}(edp, im)\right| \quad (4)$$

where $G_{L|IM}(l, im)$ is the CCDF of loss conditional upon intensity; $G_{EDP|IM}(edp, im)$ is the CCDF of demand conditional upon intensity that may be derived using incremental dynamic analysis; $G_{DM|EDP}(dm, edp)$ is the CCDF of damage conditional upon demand that may be derived by a damage analysis; and $G_{L|DM}(l, dm)$ is the CCDF of loss conditional upon damage that may be derived by a loss analysis.

2.5 A simplified approach

Including a probabilistic distribution of loss given intensity, $G_{L|IM}(l, im)$, in the Monte Carlo simulation adds a significant amount of set up and running time. This can be simplified by selecting the loss for each intensity measure as the expected or mean value of loss for that intensity measure, $E[L|IM]$. If this simplification is made, the random sampling of a loss for each intensity is not required because the expected or mean loss is a one-to-one, monotonic function of intensity measure and as such the simulation procedure can be shortened. Instead of using Equation 2 to randomly sample a number of events, Equation 3 to randomly sample an intensity for each event and Equation 4 to randomly sample a loss for each intensity, it is possible to use Equation 5 to randomly sample a number of events with intensities greater than im_{min}, and Equation 6 to randomly sample a loss for each event. As the $\lambda(l)$ and $\lambda(l_{min})$ terms in Equations 5 and 6 can be obtained directly from a loss hazard curve, this simplified method will result in a significant time reduction in the set up and simulation process.

$$P(n) = \frac{[\lambda(l_{min})t]^n e^{-[\lambda(l_{min})t]}}{n!} \quad (5)$$

$$G_L(l) = \frac{\lambda(l)e^{-\lambda(l)}}{\lambda(l_{min})e^{-\lambda(l_{min})}} \quad (6)$$

where l is the loss incurred from an earthquake; l_{min} is the lower bound loss level event (the expected loss resulting from the minimum intensity event im_{min}); $\lambda(l)$ is the mean annual number of events producing a loss of l or greater, which can be obtained directly from a loss hazard curve; and $G_L(l)$ is the CCDF of the loss of each 'important' event.

2.6 Example: Comparing cumulative loss distributions from different New Zealand regions

Consider the ten-storey reinforced concrete frame structure detailed in the New Zealand 'Red Book' (Bull & Brunsdon, 1998). The expected (mean) loss given intensity relationship for this structure was calculated by Bradley et al. (2009) using Incremental Dynamic Analysis and the PEER performance based framework. This expected loss given intensity relationship is reproduced as Figure 2(a). The building has a fundamental period of 1.5 s (Bradley, et al., 2009, p. 3) and is assumed to be situated on shallow (Class C) soil. Spectral acceleration hazard curves (T=1.5 s, ξ=5%) for Wellington, Christchurch and Otira were approximated by multiplying the peak ground accelerations given by Stirling et al. (2002, p. 1894) by 0.88, the appropriate spectral shape factor from the New Zealand loadings code (Standards New Zealand, 2004). The resulting hazard curves are given in Figure 2(b).

Cumulative loss distributions were then calculated for time periods of 1 year, 20 years, 50 years and 100 years. In all cases, a lower bound intensity of 0.05 g was used. The results for Wellington, Christchurch and Otira are shown in Figures 3 and 4 respectively.

In each location, as the simulation time period increased, the expected losses increased and the loss distributions became more normally distributed. Conversely, as the simulation time period decreased, the expected losses decreased and the loss distribution became more skewed towards zero losses, and producing a large divergence of mean and the median loss values.

Comparing between the three regions, for the same time periods, loss distributions for Otira have higher cumulative losses than for Wellington, which in turn have higher cumulative losses than for Christchurch (based on preconceived seismic hazard prior to 2010). This might be expected from Figure 2(b), where the annual rate of exceedances for large earthquakes (greater than 0.1g) is higher for Otira than for Wellington and higher for Wellington than for Christchurch. For example, for a design life of 50 years, the expected cumulative loss for Christchurch is $0.53 million, the expected cumulative loss for Wellington is $2.19 million and the expected cumulative loss for Otira is $7.54 million. Also, with increasing time period, greater reductions in skew can be observed for Otira than for Wellington, and the same for Wellington over Christchurch. For example, between 1 year and 100 years, the percentile value of the mean cumulative loss for Otira dropped 32.0% (87.4% to 55.4%), the percentile value of the mean cumulative loss for Wellington dropped 25.5% (88.8% to 63.3%) and the percentile value of the mean cumulative loss for Christchurch dropped 15.4% (89.3% to 73.9%).

Summarising the results in practical terms, when the considered time period is short (roughly in the order equating to the designed working life of a typical building), the earthquake loss distribution is bimodal in the extremes. Mean expected loss value is significantly over represented by small occurrences of large loss events, and in most instances one can expect no or minimum losses. The skewness of the loss distribution is a function of the seismic hazard curve in additional to the considered time period.

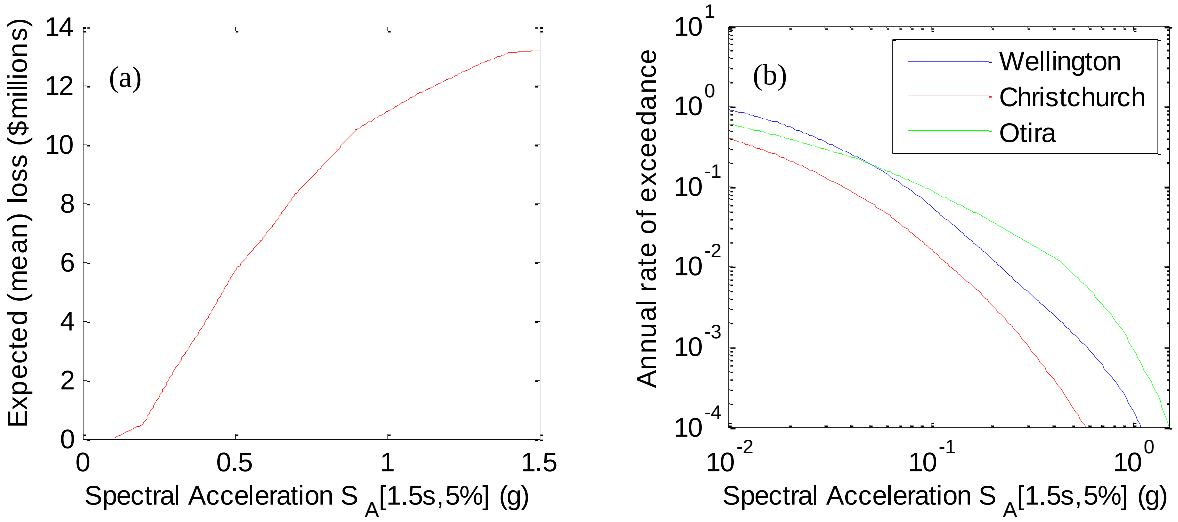

Figure 2 – (a) Expected loss given intensity relationship for the building, (b) Seismic hazard curves for Wellington, Christchurch and Otira

One of the disadvantages of cumulative distribution function plots for earthquake loss such as that shown in Figures 3 and 4 is that they do not convey information on how which intensity level or loss level events contributed to the overall cumulative losses, and how frequently these events occurred. These information are important for risk management and for checking the validity of the loss model. For this purpose, two alternate representations are helpful and are provided for the current simulation with the example Wellington building.

In the first method, EAL was disaggregated by intensity level or loss level, the output of these processes are presented in Figures 5(a) and 5(b) respectively (Bradley, et al., 2009, p. 18; Smith, 2008). In both figures, the Design Level Earthquake (DLE) is defined as the earthquake with 10% exceedance probability in 50 years and the Maximum Considered Earthquake (MCE) is defined as the earthquake with a 2% exceedance probability in 50 years. The second method plots the cumulative loss for individual Monte Carlo trial (loss sample function) against the simulation time., where the time of occurrence of an event was randomly and independently sampled from within the time period (Benjamin & Cornell, 1970, p. 236). Twenty loss sample functions for the Wellington example building over a 50 year time period are shown in Figure 6 for illustration.

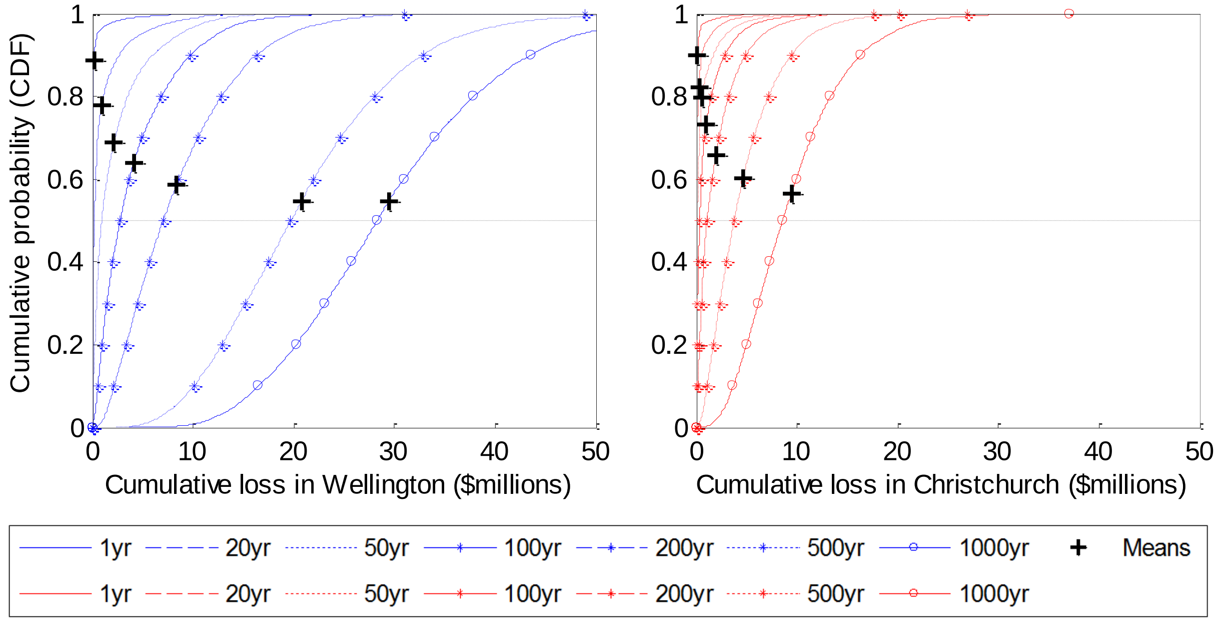

Figure 3 – Cumulative loss distributions for Wellington and Christchurch example structures

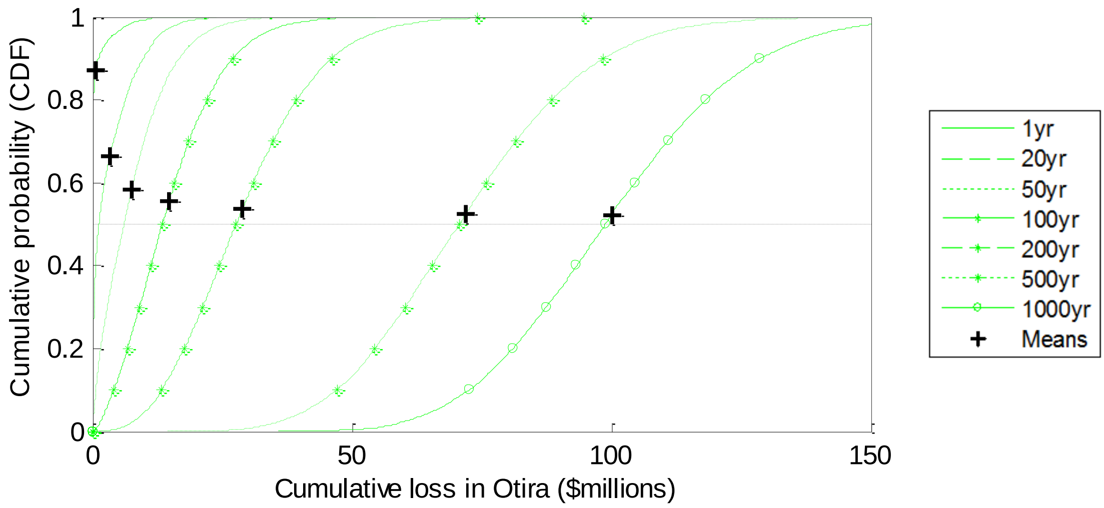

Figure 4 – Cumulative loss distributions for the Otira example structure

Figure 5(a) shows that events less than the DLE, with spectral accelerations between 0.1g to 0.4g, made up the majority of events contributing to the overall losses. Using Figure 2(b) it can be calculated that the return period for events in this acceleration range is about 20 years. Events larger than the MCE occurred rarely but has a noticeable contribution to the overall losses. The return period of an event exceeding the MCE is 2475 years. Figure 5(b) shows similar information to Figure 5(a), except in the context of the loss caused by each event. It shows that most likely events have losses ranging from small loss levels (about $10,000) up to loss levels of about $3 million.

In contrast, Figure 6 does not show an accurate breakdown of loss level events contributing to the overall losses, it instead has the advantage of providing a snapshot of 'typical' cumulative loss outcomes in the time domain based on the loss model used. The skew in the cumulative loss distribution is highlighted in that only 6 of the 20 sample trials shown reached a cumulative loss over the expected amount. The distribution in the contribution of each loss level event appears to fit with the distribution given in Figure 5(b).

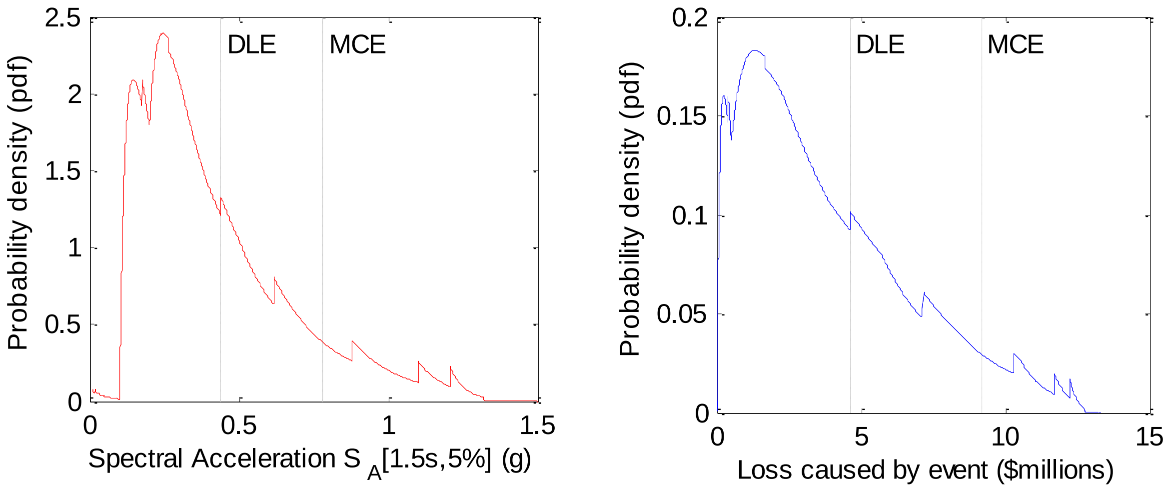

Figure 5 – Expected loss probability density function for the Wellington example building over a 50 year design life disaggregated by (a) intensity level, and (b) loss level

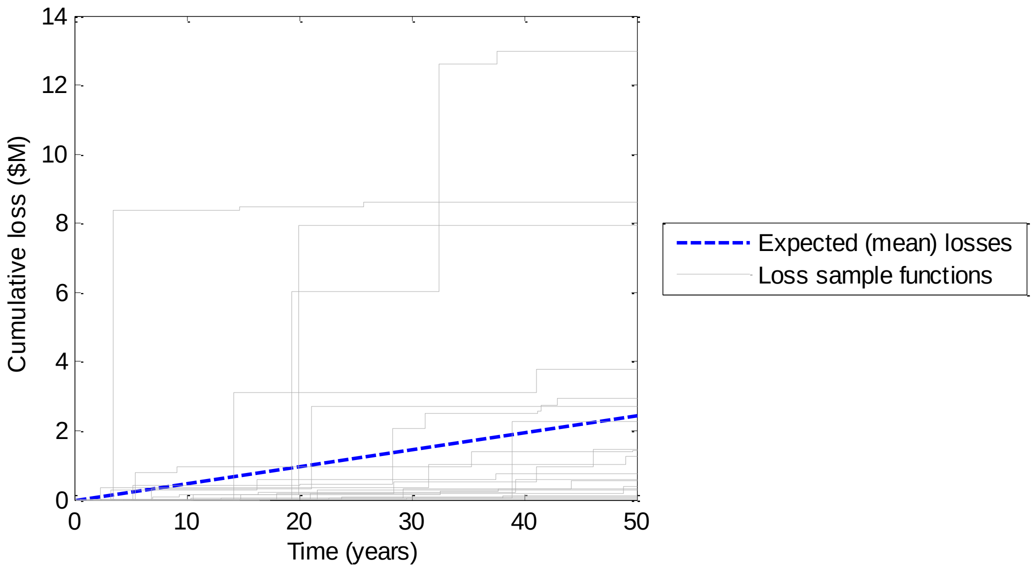

Figure 6 – Twenty sample functions of cumulative loss for the example building in Wellington

3 LIMITATIONS

The Monte Carlo method presented in this paper is subject to several sources of error due to the assumptions made in the modelling process. An outline of the major sources of error is given below.

- The annual expected number of events are assumed to be exactly as given by a seismic hazard curve. This seismic hazard curve is the output of a Probabilistic Seismic Hazard Analysis which is subject to significant epistemic uncertainty (Bradley, 2009), at least in part due to a limited number of accurate strong motions recordings. Uncertainty at this stage might be reduced by using Bayesian updating (Pei & Lindt, 2009).
- The ordinates on the seismic hazard curve are treated as Poisson means, in other words, earthquake events are assumed to occur randomly and independently according to a Poisson process. This model is mathematically convenient but over-simplified in terms of the actual geological processes that cause earthquakes, and is therefore likely to lead to some error. It is of course possible that more realistic models for earthquake occurrence could be used, for example, Markov or semi-Markov models or models incorporating a treatment of aftershocks (Anagnos & Kiremidjian, 1988). However, their implementation is likely to be considerably more complicated than the Poisson model, and hence they are avoided in this study.

- The relationship between financial loss and intensity level will typically be calculated by an analytical method such as the PEER performance base design framework. The choice of ground motions used in dynamic analysis, structural modelling assumptions, repair cost assumptions and, in particular, the treatment of component fragility functions will all have a significant effect on the results of such a study (Porter, 2003). Case studies into the magnitude of epistemic uncertainties in component fragility functions have been undertaken by Bradley (Bradley, 2010).
- As the method presented in this paper investigates cumulative losses, it is neatly assumed that after any earthquake event, the structure is repaired instantaneously and exactly to its pre-damage state. This may be acceptable for small events, but is likely to be untrue for larger events. In reality, the structure may be altered or even demolished.
- The method considered only direct losses, it does not consider other sources of loss including business downtime losses, relocation costs or economic ramifications of loss of life or injury.

One technique to minimise these errors would be to update the modelling relationships with physical data using Bayesian updating.

4 CONCLUSIONS

This paper presented a simple method for estimating the probability distribution of cumulative direct earthquake losses and net present values. The method treated the occurrence of earthquake events at a site as a random Poisson process and combined it with an appropriate loss intensity relationship to simplified loss estimation. Numerical examples showed that as the considered time period became shorter, the probability distribution for cumulative losses became progressively more skewed. Similarly, regions of lower overall seismicity had more skewed distributions of cumulative losses. In addition to disaggregation, a method based on Monte Carlo simulation was presented for investigating which intensity level or loss level events contributed most to the overall cumulative losses and how frequently these events occurred. Based on the examples undertaken and a brief examination of uncertainties the Monte Carlo method presented in this paper is potentially a valuable tool for conveying earthquake loss information, as an addition to an expected loss term and loss hazard curve.

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