ROBUST OBSERVER BASED FAULT DIAGNOSIS FOR NONLINEAR SYSTEMS

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For my family, who offered me unconditional love and support throughout my life.

ABSTRACT

The field of observer based fault diagnosis for nonlinear systems has become an important topic of research in the control community over the last three decades. In this thesis, the issues of robust fault detection, isolation and estimation of actuator faults and sensor faults for Lipschitz nonlinear systems has been studied using sliding mode, adaptive and descriptor system approaches.

The problem of estimating actuator faults is initially discussed. The sliding mode observer (SMO) is constructed directly based on the uncertain nonlinear system. The fault is reconstructed using the concept of equivalent output injection. Sensor faults are treated as actuator faults by using integral observer based approach and then the problem of sensor fault diagnosis, including detection, isolation and estimation is studied. The proposed scheme has the ability of successfully diagnosing incipient sensor faults in the presence of system uncertainties. The results are then extended to simultaneously estimate actuator faults and sensor faults using SMOs, adaptive observers (AO) and descriptor system approaches. $\mathcal{H}_\infty$ filtering is integrated into the observers to ensure that the fault estimation error as well as the state estimation error are less than a prescribed performance level.

The existence of the proposed fault estimators and their stability analysis are carried out in terms of LMIs. It has been observed that when the Lipschitz constant is unknown or too large, it may fail to find feasible solutions for observers. In order to deal with this situation, adaptation laws are used to generate an additional control input to the nonlinear system. The additional control input can eliminate the effect of Lipschitz constant on the solvability of LMIs.

The effectiveness of various methods proposed in this research has been demonstrated using several numerical and practical examples. The simulation results demon-

strate that the proposed methods can achieve the prescribed performance requirements.

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Nomenclature

$\mathcal{R}^n$	A set of real vector of length n numbers
$\mathcal{R}^{n\times n}$	A set of real $n \times n$ matrices
$\forall$	for all
$\in$	is an element of
$A > 0$	A symmetric positive-definite matrix
A^T	Transpose of A
A^{-1}	Inverse of A
$\lambda(A)$	Eigenvalue of A
$\lambda_{max}(A)$	Maximum eigenvalue of A
$\lambda_{min}(A)$	Minimum eigenvalue of A
$\sigma_{max}(A) = \sqrt{\lambda_{max}(A^TA)}$	Maximum singular value of A
$\Vert x\Vert = \sqrt{x^Tx}$	Euclidean norm of vector x
$\Vert A\Vert = \sigma_{max}(A)$	Matrix norm of A
I_n	An $n \times n$ identity matrix
$\Vert x\Vert^2_{\mathcal{L}_2} = \int_0^T x^T(t)x(t)dt$	L^2 norm of vector x

Chapter 1

Introduction

A fault can generally be defined as an unexpected deviation of at least one characteristic property, called the feature of the system, from the normal condition which tends to degrade the overall performance of a system and leads to undesirable but still tolerable behavior of the system [1].

The increased productivity requirements and stringent performance specifications have led to more demanding operating conditions in many modern engineering systems such as aircrafts, automotive vehicles, high-speed railways and power systems. Such conditions increase the possibility of faults which will result in off-specification production, increased operating costs, detrimental environmental impacts and even catastrophic disasters that claim both property and human life [2, 3]. Therefore, in order to satisfy the demands for reliability, availability, safety and maintainability of an industrial process, it is vital to promptly detect faults and diagnose the source and severity of each malfunction so that appropriate actions can be scheduled.

1.1 Basic concept of fault diagnosis

Faults can be of several types which may arise due to different conditions such as malfunctions in actuators and sensors, abnormal parameter variations of the process and hard failures in equipments due to structural changes, etc. Typical examples of faults are:

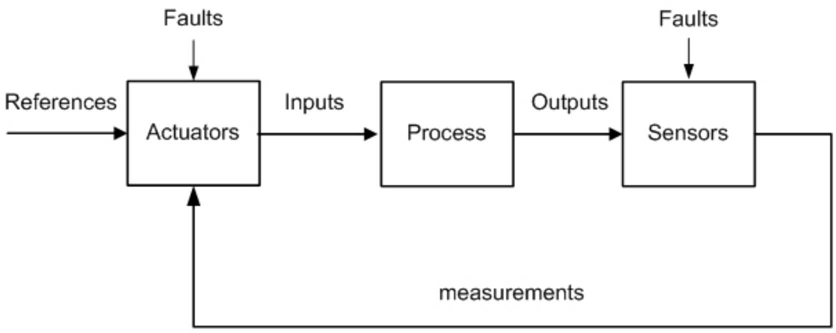

Figure 1.1: A faulty system which are subject to actuator faults and sensor faults

- Actuator faults, such as damages in the bearings, deficiencies in force and momentum, defects in the gears, aging effects and stuck faults. Actuators are used to generate the desired inputs to control the process to behave normally. When actuator faults occur, the faulty actuators are no longer to generate the desired control inputs.

- Sensor faults, such as scaling errors, drifts, dead zones, short cuts and contact failures. sensors are used to provide measurements that are needed for monitoring the system and computing the desired inputs. When sensor faults occur, the faulty sensors are no longer to provide accurate measurements which are needed to generate control inputs.

- Abnormal parameter variations in the system. When some components of the plant are faulty, the original process has changed into a different process so that the controller designed for the original process is no longer able to achieve the expected system performance.

- Construction defects such as cracks, ruptures, fractures, leaks and loose parts etc.

- External obstacles such as collisions and clogging of outflows.

Throughout this thesis, a fault is defined as an additive change appearing in an actuator or sensor and a faulty system is shown in Fig-1.1.

The growing demands for reliability, availability, safety and maintainability of modern control systems call for the research of fault diagnosis. This research field has attracted consideration worldwide in both theory and application in last two decades [4, 5, 6, 7, 8, 9, 10]. The role of a fault diagnosis is to detect faults and to identify their locations and significance in a system of interest. Such a system normally consists of three major tasks: fault detection, fault isolation and fault estimation [11].

Fault detection is the first step of fault diagnosis which is used to make a decision regarding the system working conditions, namely whether or not the system is working under normal conditions. This can be achieved from either the direct observation of system inputs and outputs, or the use of certain types of redundant relations. After a fault is being detected, the next step is fault isolation. In this step the locations of the faults are determined, i.e., amongst several sensors and actuators the faulty sensor or actuator is identified. Most practical diagnosis systems contain only fault detection and isolation (FDI). However, FDI cannot tell the magnitude of the fault. More comprehensive information of the fault such as magnitude, location and nature can be provided by the step of fault estimation, which is often referred to as fault identification [1]. This procedure can be regarded as an extension to FDI, since accurate fault estimation alternately implies the occurrence and location of the fault. Moreover, this step plays an important role in fault tolerant control (FTC), which stabilizes the closed-loop system and guarantees a prescribed performance level after the occurrence of any fault [12, 13].

1.2 Fault diagnosis methodologies

A traditional approach for fault diagnosis is hardware-based, where a particular variable is measured using multiple sensors, actuators and computers. Outputs from identical components are compared for consistency. This method is costly and sometimes may cause complex problems when incorporated with other redundant devices [7].

Another approach for fault diagnosis is the analytical redundancy method, which utilizes analytical models of the plant in equation to track the changes in the plant dynamics. The main idea behind this method is to generate directional residuals by failure detection filters. Different fault effects can be mapped into different directions or planes in the residual vector space so that fault isolation can be achieved [6]. The existing analytical redundancy fault diagnosis approaches can be broadly divided into knowledge-based FDI methods, signal-based FDI methods and model-based FDI methods:

1. Knowledge-based FDI methods

The knowledge-based methods of fault diagnosis are essentially being developed from the heuristic symptoms. These are obtained either from expert human operators, or from a qualitative model. In some of the knowledge-based methods, the model is built up by expert reasoning [14], fuzzy reasoning [15] and neural networks [16, 17], for mapping the inputs and outputs of the unknown system. In many other knowledge-based methods, measurement data is mapped to a known pattern which includes different normal and abnormal operating conditions directly, so that the system condition can be identified. One of the major advantages of the knowledge-based methods is that they do not require explicit mathematical model of the monitored system. However, they require in advance the knowledge of the monitored plants, such as the training data which contains faults and the corresponding symptoms, under different faulty conditions.

2. Signal-based FDI methods

Many signal-based FDI methods have been developed and can be divided into two categories: spectral analysis [18] (time-frequency, time-scale analysis, etc) and statistical methods [19] (signal classification, pattern recognition, etc). These methods extract from the system proper signals or symptoms such as spectral power densities, correlation coefficients and covariances, for the analysis of faults. Although the signal-based techniques do not require a complete analytical model, as the knowledge based methods, their efficiency is particularly limited for early fault detection and for the detection of faults which occur during transient operation.

3. Model-based FDI methods

With the development of digital computers and the system identification techniques, model-based FDI methods have received considerable attention in recent years [7, 20, 21, 22]. The model-based FDI methods comprise of two principal steps: residual generation and residual evaluation. The difference between the actual system's behavior and the predicted or estimated behavior forms the residual signal. This signal is supposed to be zero when the system is free from fault and nonzero during the occurrence of fault. After generating the residual, the next step is to evaluate the residuals and make a decision on whether or not a fault has occured. This is carried out by comparing the residual signal with a threshold. An alarm is triggered when the residual exceeds the threshold. The three most popular model-based FDI methods are [23]:

1) Parity equation approach [1, 24]. This approach is based on the test of consistency of parity equations by using measurements and inputs. Any inconsistency of the parity equations can be used to detect the faults.

2) Parameter estimation approach [16, 25, 26]. This approach is based on the assumptions that the occurrence of any fault will change the values of the physical system parameters such as mass, friction, resistance, etc. The parameters of the actual process can be repeatedly estimated using online parameter estimation methods. A fault can be declared if there exists discrepancies between the true values of the system parameters and their estimated values.

3) Observer-based approach [27, 28, 29, 30, 31]. The observer-based FDI approaches generally compare the actual system's measurements with the predicted or estimated outputs by employing observers. The output estimation errors are taken as residuals. The most commonly used observers include Luenberger observers [32] in a deterministic setting, Kalman filters [33] in a stochastic setting, sliding mode observers [8] and unknown-input observers (UIO) [34].

1.3 Complexities in Model-based fault diagnosis

In the context of fault diagnosis, nonlinearities and uncertainties are two main types of complexities.

1. Nonlinearities

As most plants are inherently nonlinear and the faults may often amplify the nonlinearities by driving the plants from a relatively linear operating point into a more nonlinear operating region, the study of fault diagnosis for nonlinear systems has become a very active research topic for both theoretical and practical reasons. The presence of nonlinearities in control systems constitutes a major challenge to model-based fault diagnosis.

During the last decade, productive results have been reported on fault diagnosis for linear time-invariant (LTI) systems, such as pseudo-inverse approach [35], adaptive approach [36], descriptor approach [37] and sliding mode-based approach [38]. However, there is no universal design method available for general nonlinear systems and the research of fault diagnosis can only be carried out systematically for some special classes of nonlinear systems such as bilinear systems [39, 40], linearizable systems [41] and other special types of nonlinear systems [42, 43].

Amongst these systems, Lipschitz nonlinear systems have received special attention in recent years [44, 45, 46, 47, 48]. Many observer-based FDI approaches have been reported for this class of nonlinear systems, such as UIOs [49], adaptive techniques [50], descriptor system approaches [51] and high-gain observers [52].

Consider the following continuous-time nonlinear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x, u, t) \\ y(t) &= Cx(t) \end{aligned} \tag{1.1}$$

where $x \in \mathcal{R}^n$ are the state variables, $u \in \mathcal{R}^m$ are the inputs and $y \in \mathcal{R}^p$ are the outputs. $A \in \mathcal{R}^{n\times n}$ and $C \in \mathcal{R}^{p\times n}$ are known constant matrices with C being of full rank. Note that any system of the form $\dot{x}(t) = \psi(x, u, t)$ can be expressed in the form of (1.3) as long as $\psi(x, u, t)$ is continuously differentiable with respect to x. Since some nonlinearities can be treated as unknown inputs, (1.3) can represent a broader class of nonlinear systems than it first appears.

For the nonlinear term $f(x, u, t)$, if there exists a positive scalar $\mathcal{L}_f$ which is independent of x, u and t, such that

$$\|f(x_1, u, t) - f(x_2, u, t)\| \leq \mathcal{L}_f \|x_1 - x_2\| \tag{1.2}$$

for all x_1 and x_2. Then inequality (1.2) is the well-known Lipschitz condition [] and (1.3) represents a Lipschtiz nonlinear system. Many practical systems satisfy the Lipschtiz condition, at least locally. For example, trigonometric nonlinearities occurring in robotic applications and the nonlinearities which are square or cubic in nature, can be assumed to be Lipschitz. The main advantage of Lischitz nonlinear systems is that the observer design can be carried out in a systematic way by simply extending linear observer design technique.

In this thesis, we will only focus on this type of nonlinear systems.

2. Uncertainties

Due to the high dependence of the model-based FDI to the corresponding mathematical models, a major downside of this approach is that this approach requires an accurate mathematical model of the system which is difficult to obtain in many practical situations. Moreover, the system parameters often vary during the process and the characteristics of disturbances are unknown. The existence of system uncertainties and disturbances can cause a misleading alarm and therefore make the model-based fault diagnosis system ineffective [53]. Consequently, it is vital to take the issue of robustness into account when designing a model-based fault diagnosis system. A robust fault diagnosis system should have the ability to be sensitive to the fault signals and insensitive to other unknown signals.

Uncertainties can be broadly divided into two classes: non-parametric uncertainties [8, 54], which include modelling errors and disturbances, and parametric uncertainties [55, 56], which are characterized in terms of unknown parameters. In this thesis, only non-parametric uncertainties will be studied.

A nonlinear state-space system with non-parametric uncertainties can be formulated as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x, u, t) + \eta(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1.3}$$

where the unknown term $\eta(t)$ represents the uncertainty.

In the fault diagnosis literature, efforts to boost the robustness of FDI schemes can be made either by decoupling the fault from the uncertainty, or by minimizing the effects of the uncertainty on the state and fault estimation. In the first method, it is often assume that the uncertainty is structured, i.e., the uncertainty has the form $\eta(t) = E\Delta\psi(t)$, where E is known (or approximately known) and represents the uncertainty distribution matrix, $\Delta\psi(t)$ is an unknown function of time. This structured uncertainty allows the use of linear and nonlinear state transformations to exactly decouple faults from uncertainties [8, 46, 57, 58]. However, the distribution of uncertainties is normally unknown and the construction of the state transformation is not an easy task. In such a case, decoupling faults from uncertainties is no longer possible and this can be solved by integrating $\mathcal{H}_\infty$ filtering into observers [45, 48, 59] or using the adaptive threshold approach [50, 60].

The fault diagnosis of nonlinear systems with structured uncertainties or unstructured uncertainties will both be studied in this thesis.

1.4 Sliding mode observer based fault diagnosis: an overview

During past decade, several robust fault diagnosis methods have been developed, such as observer-based robust FDI [61, 62, 63, 64], unknown input observer (UIO) based approach [34, 65, 66] and eigenstructure assignment approach [67, 68]. Amongst these approaches, the observer-based robust FDI is deemed most promising due to several advantages, such as it can be implemented using only the on-line measurements and offers more design freedom, compared with other approaches.

One of the particular interesting techniques among all observer-based FDI is the sliding mode observer (SMO)-based approach. In recent years, the SMO has been widely studied and considerable success has been achieved in many areas [8, 53, 61, 69, 70, 71, 72, 73, 74]. The main characteristic of the SMO is that, despite system uncertainties and disturbances, the output estimation errors between the system and the SMO can be forced to and maintained at zero during sliding.

Early work of applying the SMO for fault diagnosis was shown in [75] where a SMO is designed with the assumption that the states of the system are available. In [70, 71], the authors attempted to design a SMO for systems with uncertainties. When a fault occurs, the sliding motion will be destroyed and the residual will deviate from zero. On the contrary, the SMO proposed in [8], which is similar to that of [76], can maintain the sliding mode even after the presence of faults by selecting an appropriate gain. Therefore, the constant actuator faults and sensor faults can be reconstructed by the so-called equivalent output injection under certain conditions. This result was extended to a more general case in [77] where the derivative of the sensor fault is nonzero. However, the requirement of a complicated coordinate transformation and that the system is accurately known limits its application. The assumption of open loop stability in [77] was later relaxed in [72] to achieve robust sensor fault estimation using a linear matrix inequality (LMI) formulation. In [78], a nonlinear diffeomorphism was introduced to explore the system structure and the sensor fault was transformed into a pseudo-actuator fault scenario using a filter. A SMO was then designed to estimate the sensor fault based on the filtered system. In [79], the authors proposed a scheme to estimate incipient sensor faults for both open-loop stable and unstable systems. The problem of early detection of incipient faults using SMOs is also studied in [58]. A high-order SMO was designed in [73] to estimate sensor faults. It has been shown in [46, 80] that if certain conditions on fault and uncertainty distribution matrices are met, then the system uncertainties can be perfectly decoupled from faults and the reconstruction of actuator faults can be achieved. If this condition is not satisfied, the approach must settle for minimizing the effect of the uncertainties on the fault estimation. In [72], a FDI scheme which can minimize the $\mathcal{L}_2$ gain between the uncertainty and the fault reconstruction signal was proposed for a class of linear systems with uncertainties.

In this thesis, the issues of robust fault detection, isolation and estimation of actuator faults and sensor faults for Lipschitz nonlinear systems will be studied mainly based on this technique. Several ideas presented in above references are applied, such as the concept of equivalent output injection introduced in [8], LMI technique used in [72] and coordinate transformations used in [58].

1.5 Objectives of the thesis

Motivated by the success of observer based methods of fault diagnosis in various systems, the present study focus on developing effective and robust solutions for fault detection, isolation and estimation for a class of nonlinear systems (Lipschitz nonlinear systems).

The thesis sets the following main objectives:

1. Develop SMO based method to reconstruct actuator faults;
2. Develop effective algorithms to detect and isolate sensor faults;
3. Develop novel schemes based on SMOs and adaptive technique to estimate sensor faults.
4. Develop robust schemes based on SMOs and adaptive technique to estimate coinstantaneous actuator faults and sensor faults.

5. Simultaneously estimate the actuator and sensor faults using descriptor system approach.

1.6 Thesis outline

The thesis is organized as follows:

The thesis starts with an introduction of actuator fault detection and isolation in chapter 2. A sliding mode observer is proposed based on a constrained Lyapunov equation and the sufficient conditions for the design of such an observer have been derived. The fault signal is estimated by employing the concept of equivalent output error injection. The estimated fault signal provides information about the occurrence of a fault as well as about the size and severity of the fault. Under certain conditions, the actuator fault can be precisely reconstructed. The example of a single-link flexible joint robot system is tested to show the effectiveness of the proposed scheme.

Compared with actuator fault detection and isolation, very little research has been carried out on the sensor fault diagnosis, especially for the incipient sensor fault diagnosis. Therefore, a robust sensor fault detection and isolation scheme is proposed in chapter 3. The essential idea behind the proposed scheme is to employ a coordinate transformation such that the sensor faults can be separated from system uncertainties. More specifically, after the coordinate transformation of the original system, one of the subsystems is only subject to sensor faults, but without modelling uncertainties, which only appears in another subsystem. Based on the transformed system and the integral observer based approach, multiple sensor faults are detected by using the classical Luenberger observer and then isolated by using a bank of sliding mode observers.

It follows in chapter 4 that after the detection and isolation of multiple sensor faults, they are further estimated by using two methods. In the first method, the sensor fault is estimated by using sliding mode observers while in the second method, the sensor fault is estimated based on adaptive technique. The most attractive feature of the schemes proposed in chapters 3 and 4 is that by using a coordinate transformation, sensor faults are completely decoupled from modelling uncertainties and therefore the diagnosis of small sized sensor faults can be successfully carried out.

Chapters 2, 3 and 4 deal separately with the actuator faults and sensor faults. In other words, it is assumed that there occur only actuator faults or sensor faults at a particular time. In chapter 5, the situation when both actuator faults and sensor faults happen in a process simultaneously is studied. It is worth noting that uncertainties considered in previous chapters are structured and the uncertainty distribution matrix is known. In this chapter, the case that the uncertainty is unstructured is studied. By integrating $\mathcal{H}_\infty$ criteria, the proposed observer is not only capable of estimating the states and faults, but also minimizes the $\mathcal{H}_\infty$ gain between the estimation error and system uncertainties. Since the integrated $\mathcal{H}_\infty$ filtering ensures more accurate state estimations, the fault estimation is much more robust against the system uncertainties, compared with the fault estimation obtained from observers without $\mathcal{H}_\infty$ filtering. An illustrative example is given to show that the proposed method can preserve the shape of the actuator fault and sensor fault effectively.

A descriptor system approach is introduced to investigate the simultaneous estimation of actuator fault and sensor fault in chapter 6. The descriptor system is constructed by taking sensor faults as auxiliary states. An observer together with adaptive technique is designed for the augmented system to estimate faults. Asymptotic estimates of the original system states and the sensor faults can be directly obtained from the descriptor observer. Meanwhile, the actuator faults are obtained by using adaptive observer-based approach. The difference between the method developed in this chapter and those proposed in chapters 2,3,4 and 5 is that, this approach does not require any coordinate transformation which may increase the design difficulty. Besides, the assumption that the unstructured uncertainty is bounded in chapter 5 is removed in this chapter.

Concluding remarks and suggestions for future research are discussed in chapter 7.

Chapter 2

Reconstruction of actuator faults for uncertain nonlinear systems

In this chapter, a scheme based on SMOs is proposed for the reconstruction of actuator faults for Lipschitz nonlinear systems with matched non-parametric uncertainties.

2.1 Introduction

The demand for a safer and more reliable automatic control system stimulates the research on fault diagnosis and this topic has received considerable attention during the last two decades. The related literature can be found in [4, 5, 20, 21, 22, 23, 81] and the references there in.

The approaches of FDI developed in the past can essentially be grouped into three main categories: knowledge-based FDI methods [14], signal-based FDI [18] and model-based FDI [7]. In knowledge-based FDI, both the dynamic behaviors of the process and heuristic symptoms are characterized by a small number of symbols, or by qualitative values. Signal-based FDI approaches employ statistical operations on the measurements or train some artificial network to extract the information regarding faults. The model-based FDI approaches generally compare the actual system's behavior with the predicted or estimated behavior based on its mathematical model. The difference of these behaviors, referred to as residuals, are very sensitive to any faults and therefore being used for fault detection. An alarm is triggered when the actual process behavior deviates from its expected behavior; more precisely, when the residuals exceed some predefined thresholds. However, due to the high dependence of the residual generation FDI to the corresponding mathematical model, any uncertainties existing in the model can cause a misleading alarm, which often make the FDI ineffective.

One of the methods to deal with the uncertainty such as system deviation, disturbances and unknown nonlinearities is to use the idea of sliding mode techniques. Several authors have reported sliding mode observer design methods in an FDI context. In [61], a discontinuous observer strategy has been used where the error between the estimated and measured outputs is forced to exhibit a sliding mode and the effects of measurement noises are reduced. In [76] a Lyapunov-based approach has been used to formulate an observer design where asymptotic stability can be obtained under certain assumptions in the presence of bounded nonlinearities or uncertainties. Early work of applying the SMO for FDI was shown in [75] where a sliding mode observer approach is considered with the assumption that the states of the system are available. In [71] Hermans and Zarrop attempted to design an observer such that in the presence of a fault the sliding motion was destroyed. However, the observer proposed in [8], which is similar to that of [76] can maintain the sliding mode even after the presence of faults. The actuator fault can therefore be reconstructed by the so-called equivalent output injection under certain conditions. Later it was extended to sensor fault reconstruction in [77]. Notice that the precise fault reconstruction shown in [8] and [77] was only for linear systems without uncertainties. When there are uncertainties, [82] provides a method to reconstruct faults for linear systems. It should be emphasized that the above work only considers linear systems. For nonlinear systems, the synthesis and computation of the switching gain of the SMO are much more difficult. [83] designed an adaptive method to update the sliding mode observer gain for counteracting uncertainty, so the upper bound of the uncertainty was not needed. In [84], a bank of observers were designed to isolate actuator faults for both linear and nonlinear systems. LMI techniques were used in [46] to design the SMO for a class of nonlinear systems with uncertainties.

In general, the SMO-based FDI techniques can be classified into two categories. The first category uses SMOs to generate residuals which are sensitive to faults, but insensitive to uncertainties [21, 75]. The second category uses SMOs to reconstruct or estimate the faults [8, 54, 77, 85]. The main content of this chapter falls into the second category and the objective is to propose an SMO-based fault estimation scheme for actuator fault reconstruction for Lipschitz nonlinear systems with matched non-parametric uncertainties. The developed method is inspired by the work presented in [80], in which a robust SMO is established in the presence of uncertainties and actuator faults based on a constrained Lyapunov equation. However, [80] does not explicitly give a systematic solution for computing the parameters of the observer. This problem is solved in this chapter by transforming the sufficient condition of the stability of the observer into a convex optimization problem and expressing in LMI form. Moreover, Unlike in above mentioned papers where the bound of the system uncertainty is a constant and independent of states, the bound in this chapter is assumed to have a more general form. Under certain geometric conditions, the fault can be completely decoupled from system uncertainties and an accurate estimation can be achieved.

The remainder of the chapter is organized as follows: Section-2.2 briefly describes the mathematical preliminaries required for designing SMO. Section-2.3 describes the design procedure of the proposed SMO and derives the sufficient condition for the existence and stability of the SMO, and expresses the condition in LMI form. Section-2.4 explores under what conditions the fault can be reconstructed with arbitrary accuracy in the presence of uncertainty. The results of simulation considering the example of single-link flexible joint robot system is shown in section-2.5 with conclusions in section-2.6.

2.2 Problem Formulation

Consider a nonlinear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x,t) + Bu(t) + E\Delta\psi(x,t) + Df_a(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.1}$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$ and $y \in \mathcal{R}^p$ denote respectively the state variables, inputs and outputs ; $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{p\times n}$, $D \in \mathcal{R}^{n\times q}$ and $E \in \mathcal{R}^{n\times r}(q \leq p < n)$ are known constant matrices with C and D both being of full rank and the nonlinear term $f(x,t)$ is assumed to be known. The unknown nonlinear term $\Delta\psi(x,t)$ models lumped uncertainties and disturbances experienced by the system and $f_a(t)$

represents actuator faults. Note that the assumption that the matrix D has full column rank q is not a restriction and can always be met by redefining the fault vector [86].

The objective is to design an asymptotic observer from the measurement $y(t)$ to estimate both the state $x(t)$ and the actuator fault $f_a(t)$. The following assumptions are made throughout the chapter:

Assumption 2.1 The matrix pair (A, C) is detectable.
It follows the assumption that there exists a matrix $L \in \mathcal{R}^{n\times p}$ such that $A - LC$ is stable, and thus for any $Q > 0$ the Lyapunov equation

$$(A - LC)^T P + P(A - LC) = -Q \tag{2.2}$$

has an unique solution $P > 0$ [80].

Assume that $P \in \mathcal{R}^{n\times n}, Q \in \mathcal{R}^{n\times n}$ are in the form:

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \tag{2.3}$$

It is obvious that $P_1 \in \mathcal{R}^{(n-p)\times(n-p)} > 0$, $P_3 \in \mathcal{R}^{p\times p} > 0$, $Q_1 \in \mathcal{R}^{(n-p)\times(n-p)} > 0$ and $Q_3 \in \mathcal{R}^{p\times p} > 0$ if $P > 0$ and $Q > 0$.

Assumption 2.2 rank($C[D\ E]$)=rank(D)+rank(E).

This assumption implies that rank $[D\ E] \leq p$. A necessary condition for this assumption to hold is that the number of measurements is greater than or equal to the sum of the number of unknown inputs (system uncertainties and disturbances) and the number of actuator faults.

Assumption 2.3 The nonlinear term $f(x, t)$ is assumed to be known and Lipschitz about x uniformly, i.e., $\forall x, \hat{x} \in \mathcal{X}$,

$$\|f(x, t) - f(\hat{x}, t)\| \leq \mathcal{L}_f \|x - \hat{x}\| \tag{2.4}$$

where $\mathcal{L}_f$ is the known Lipschitz constant. Many nonlinearities can be assumed to be Lipschitz, at least locally.

Assumption 2.4 The function $\Delta\psi(x, t)$ representing the structured uncertainty is unknown but bounded, and it satisfies

$$\|\Delta\psi(x,t)\| \leq \xi(x,t) \tag{2.5}$$

where the bounding function $\xi(x,t)$ is known and Lipschitz about x uniformly, i.e., $\|\xi(x,t) - \xi(\hat{x},t)\| \leq \mathcal{L}_\xi \|x - \hat{x}\|$. Also the actuator fault f_a is bounded by a known function: $\|f_a(t)\| \leq \rho(t)$. This assumption is quite general when the actuator fault is constant or time-varying at a limited rate.

Without loss of generality, it is assumed that the output matrix C has the form:

$$C = \begin{bmatrix} 0 & I_p \end{bmatrix} \tag{2.6}$$

If C does not have such a structure, we can always find a nonsingular transformation matrix T_c such that $CT_c^{-1} = [0 \;\; I_p]$ since C has full row rank [87].

Assume that the triple (A, E, D) has the following structure:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \; E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \; D = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \tag{2.7}$$

where $A_1 \in \mathcal{R}^{(n-p)\times(n-p)}$, $A_4 \in \mathcal{R}^{p\times p}$, $E_1 \in \mathcal{R}^{(n-p)\times r}$, $E_2 \in \mathcal{R}^{p\times r}$, $D_1 \in \mathcal{R}^{(n-p)\times q}$ and $D_2 \in \mathcal{R}^{p\times q}$. Then system (2.1) can be rewritten as:

$$\begin{aligned} \dot{x}_1 &= A_1x_1 + A_2x_2 + f_1(x,t) + B_1u(t) + E_1\Delta\psi + D_1f_a \\ \dot{x}_2 &= A_3x_1 + A_4x_2 + f_2(x,t) + B_2u(t) + E_2\Delta\psi + D_2f_a \\ y &= x_2 \end{aligned} \tag{2.8}$$

where $x = col(x_1, x_2)$ with $x_1 \in \mathcal{R}^{n-p}$, $f_1(x,t) \in \mathcal{R}^{n-p}$ is the first $n-p$ rows of $f(x,t)$ and $f_2(x,t) \in \mathcal{R}^p$ represents the remaining rows.

In order to introduce SMO design in the next section, two important lemmas are given as follows:

Assumption 2.5 There exist arbitrary matrices $F_1 \in \mathcal{R}^{r\times p}$ and $F_2 \in \mathcal{R}^{q\times p}$ such that:

$$\begin{bmatrix} E^T \\ D^T \end{bmatrix} P = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} C \tag{2.9}$$

Lemma 2.1 There exists a solution $P = P^T > 0$ such that (2.9) holds if and only if Assumption 2.2 is satisfied.

Proof. See Appendix-A.

Lemma 2.2 If P and Q have been partitioned as in (2.3), then the following two conclusions are obvious :

1. $P_1^{-1}P_2E_2 + E_1 = 0$ and $P_1^{-1}P_2D_2 + D_1 = 0$ if (2.9) is satisfied;

2. The matrix $A_1 + P_1^{-1}P_2A_3$ is stable if Lyapunov equation (2.2) is satisfied.

Proof. See Appendix-B and [80].

Remark 2.1 The equation (2.9) in the Assumption 2.5 is commonly called as the *matching condition*. This assumption seems to be restrictive, but fortunately, for many practical control systems, particularly mechanical systems, it is often satisfied. The satisfaction of this assumption allows to decouple the dynamics of the observer error from the system uncertainties and faults.

2.3 Sliding mode observer design

The design of sliding mode observer begins by introducing a new linear change of coordinates $z = Tx$ so as to impose specific structures on the uncertainty and fault distribution matrices, where

$$T := \begin{bmatrix} I_{n-p} & P_1^{-1}P_2 \\ 0 & I_p \end{bmatrix} \tag{2.10}$$

Then the system (2.1) can be transformed into the new coordinate system z as:

$$\begin{aligned} \dot{z}(t) &= TAT^{-1}z(t) + Tf(T^{-1}z, t) + TBu(t) + TE\Delta\psi(T^{-1}z, t) + TDf_a(t) \\ y(t) &= CT^{-1}z(t) \end{aligned} \tag{2.11}$$

where

$$
\begin{aligned}
TAT^{-1} &= \begin{bmatrix} \bar{A}_1 & \bar{A}_2 \\ \bar{A}_3 & \bar{A}_4 \end{bmatrix} \\
&= \begin{bmatrix} A_1 + P_1^{-1}P_2A_3 & A_2 - A_1P_1^{-1}P_2 + P_1^{-1}P_2(A_4 - A_3P_1^{-1}P_2) \\ A_3 & A_4 - A_3P_1^{-1}P_2 \end{bmatrix} \\
TB &= \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} = \begin{bmatrix} B_1 + P_1^{-1}P_2B_2 \\ B_2 \end{bmatrix} \\
Tf(T^{-1}z,t) &= \begin{bmatrix} \begin{bmatrix} I_{n-p} & P_1^{-1}P_2 \end{bmatrix} f(T^{-1}z,t) \\ f_2(T^{-1}z,t) \end{bmatrix} \\
CT^{-1} &= \bar{C} = \begin{bmatrix} 0 & I_p \end{bmatrix}
\end{aligned}
$$

and $f_2(T^{-1}z,t)$ is the last p rows of $f(T^{-1}z,t)$.

Using the conclusion (1) of Lemma 2.1, we can further get that

$$
\bar{E} = TE = \begin{bmatrix} 0 \\ E_2 \end{bmatrix}, \quad \bar{D} = TD = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}
$$

Therefore the system (2.11) can be rewritten as:

$$
\begin{aligned}
\dot{z}_1 &= \bar{A}_1z_1 + \bar{A}_2z_2 + \bar{B}_1u(t) + \begin{bmatrix} I_{n-p} & P_1^{-1}P_2 \end{bmatrix} f(T^{-1}z,t) \\
\dot{z}_2 &= \bar{A}_3z_1 + \bar{A}_4z_2 + \bar{B}_2u(t) + f_2(T^{-1}z,t) + E_2\Delta\psi(T^{-1}z,t) + D_2f_a \\
y &= \bar{C}z
\end{aligned}
\tag{2.12}
$$

where $z = col(z_1, z_2)$ with $z_1 \in \mathcal{R}^{n-p}$ and $z_2 \in \mathcal{R}^p$.

It is interesting to note that in the new coordinate, the Lyapunov matrix P which is in the form of (2.3), can be proved to have the following quadratic form:

$$
\bar{P} = (T^T)^{-1}PT^{-1} = \begin{bmatrix} P_1 & 0 \\ 0 & P_0 \end{bmatrix} \tag{2.13}
$$

where $P_0 = -P_2^TP_1^{-T}P_2 + P_3$.

Substituting $P = T^T\bar{P}T$, $C = \bar{C}T$, $E = T^{-1}\bar{E}$ and $D = T^{-1}\bar{D}$ into the matching condition (2.9) yields

$$\begin{bmatrix} \bar{E}^T \\ \bar{D}^T \end{bmatrix} \bar{P} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \bar{C} \tag{2.14}$$

From the structure of $\bar{D}$, $\bar{E}$, $\bar{P}$ and $\bar{C}$, it can be obtained that $F_1 = E_2^T P_0$ and $F_2 = D_2^T P_0$, which provides an easy way to determine the value of F_1 and F_2 in Assumption 2.5.

Based on the transformed system (2.12),the sliding mode observer is designed as :

$$\begin{aligned} \dot{\hat{z}}_1 &= \bar{A}_1 \hat{z}_1 + \bar{A}_2 z_2 + \bar{B}_1 u(t) + \begin{bmatrix} I_{n-p} & P_1^{-1} P_2 \end{bmatrix} f(T^{-1}\hat{z}, t) \\ \dot{\hat{z}}_2 &= \bar{A}_3 \hat{z}_1 + \bar{A}_4 \hat{z}_2 + \bar{B}_2 u(t) + f_2(T^{-1}\hat{z}, t) + (\bar{A}_4 - A_0)(y - \hat{y}) + \nu \\ \hat{y} &= \bar{C}\hat{z} \end{aligned} \tag{2.15}$$

where $\hat{z}_1$, $\hat{z}_2$ and $\hat{y}$ denote respectively the estimated states and output, $A_0 \in \mathcal{R}^{p\times p}$ is a stable design matrix and plays the role of a Luenberger observer gain, and the discontinuous vector ν is defined by

$$\nu = \begin{cases} k(t,y,u)\frac{P_0(y-\hat{y})}{\|P_0(y-\hat{y})\|} & \text{if } y - \hat{y} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.16}$$

where $k(t, y, u)$ is the gain to be determined and $P_0 \in \mathcal{R}^{p\times p}$ is the symmetric definite Lyapunov matrix for A_0. Note that a similar form of the discontinuous vector ν has been used in linear systems [8]. The result has been extended to nonlinear systems in this study.

If the state estimation errors are defined as $e_1 = z_1 - \hat{z}_1$ and $e_y = e_2 = z_2 - \hat{z}_2$, then the state estimation error dynamics can be obtained as:

$$\dot{e}_1 = \bar{A}_1 e_1 + \begin{bmatrix} I_{n-p} & P_1^{-1} P_2 \end{bmatrix} \big(f(T^{-1}z, t) - f(T^{-1}\hat{z}, t)\big) \tag{2.17}$$

$$\begin{aligned} \dot{e}_y &= \bar{A}_3 e_1 + A_0 e_y + f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t) \\ &\quad + E_2 \Delta\psi(T^{-1}z, t) + D_2 f_a - \nu \end{aligned} \tag{2.18}$$

The sliding surface is defined as:

$$\mathcal{S} = \{(e_1, e_y) | e_y = 0\} \tag{2.19}$$

Proposition 2.1 *Under the Assumption 2.1-2.5, the system (2.17)-(2.18) is asymptotically stable if there exist matrices $P_0 > 0$, $P_1 > 0$, A_0, P_2 and a positive scalar α satisfying*

$$\begin{bmatrix} \bar{A}_1^T P_1 + P_1 \bar{A}_1 + \frac{1}{\alpha} \bar{P}_1 \bar{P}_1^T + \alpha (L_f)^2 I & A_3^T P_0 \\ P_0 A_3 & A_0^T P_0 + P_0 A_0 \end{bmatrix} < 0 \tag{2.20}$$

where $\bar{P}_1 = P_1 \begin{bmatrix} I_{n-p} & P_1^{-1} P_2 \end{bmatrix}$.

Proof. Consider $V(e_1, e_y) = V_1(e_1) + V_2(e_y)$ as the Lyapunov candidate, where $V_1(e_1) = e_1^T P_1 e_1$ and $V_2(e_y) = e_y^T P_0 e_y$.

The time derivative of V_1 along the trajectories of system (2.17) can be shown to be equal to:

$$\begin{aligned} \dot{V}_1 &= \dot{e}_1^T P_1 e_1 + e_1^T P_1 \dot{e}_1 \\ &= e_1^T (\bar{A}_1^T P_1 + P_1 \bar{A}_1) e_1 + 2 e_1^T \bar{P}_1 \left(f(T^{-1} z, t) - f(T^{-1} \hat{z}, t) \right) \end{aligned} \tag{2.21}$$

Since the inequality $2X^T Y \leq \frac{1}{\alpha} X^T X + \alpha Y^T Y$ is true for any scalar $\alpha > 0$ [46], then

$$\begin{aligned} \dot{V}_1 \leq\ & e_1^T (\bar{A}_1^T P_1 + P_1 \bar{A}_1) e_1 + \frac{1}{\alpha} e_1^T \bar{P}_1 \bar{P}_1^T e_1 + \alpha \left(f(T^{-1} z, t) - f(T^{-1} \hat{z}, t) \right)^T \\ & \cdot \left(f(T^{-1} z, t) - f(T^{-1} \hat{z}, t) \right) \\ =\ & e_1^T (\bar{A}_1^T P_1 + P_1 \bar{A}_1) e_1 + \frac{1}{\alpha} e_1^T \bar{P}_1 \bar{P}_1^T e_1 + \alpha \| f(T^{-1} z, t) - f(T^{-1} \hat{z}, t) \|^2 \end{aligned} \tag{2.22}$$

From the fact that $\hat{z} := col(\hat{z}_1, y)$, we have:

$$\begin{aligned} \left\| T^{-1} z - T^{-1} \hat{z} \right\| &= \left\| T^{-1} \begin{bmatrix} e_1 \\ 0 \end{bmatrix} \right\| = \| e_1 \| \\ \left\| f\left(T^{-1} z, t\right) - f\left(T^{-1} \hat{z}, t\right) \right\| &\leq \mathcal{L}_f \| e_1 \| \\ \left\| f_2\left(T^{-1} z, t\right) - f_2\left(T^{-1} \hat{z}, t\right) \right\| &\leq \mathcal{L}_{f_2} \| e_1 \| \end{aligned} \tag{2.23}$$

therefore

$$\begin{aligned} \dot{V}_1 &\leq e_1^T (\bar{A}_1^T P_1 + P_1 \bar{A}_1) e_1 + \frac{1}{\alpha} e_1^T \bar{P}_1 \bar{P}_1^T e_1 + \alpha (\mathcal{L}_f)^2 \| e_1 \|^2 \\ &= e_1^T \left(\bar{A}_1^T P_1 + P_1 \bar{A}_1 + \frac{1}{\alpha} \bar{P}_1 \bar{P}_1^T + \alpha (\mathcal{L}_f)^2 I_{n-p} \right) e_1 \end{aligned} \tag{2.24}$$

The time derivative of V_2 along the trajectories of system (2.18) can be obtained as:

$$\begin{aligned}\dot{V}_2 &= e_y^T(A_0^TP_0 + P_0A_0)e_y + 2e_y^TP_0\bar{A}_3e_1 + 2e_y^TP_0\left(f_2(T^{-1}z,t) - f_2(T^{-1}\hat{z},t)\right)\\ &\quad + 2e_y^TP_0E_2\Delta\psi(T^{-1}z,t) + 2e_y^TP_0D_2f_a - 2e_y^TP_0\nu \end{aligned} \tag{2.25}$$

From the definition of ν, it can be obtained as:

$$2e_y^TP_0\nu = 2k\|P_0e_y\| \tag{2.26}$$

then

$$\begin{aligned}\dot{V}_2 &= e_y^T(A_0^TP_0 + P_0A_0)e_y + 2e_y^TP_0\bar{A}_3e_1 + 2e_y^TP_0\left(f_2(T^{-1}z,t) - f_2(T^{-1}\hat{z},t)\right)\\ &\quad + 2e_y^TP_0E_2\Delta\psi(T^{-1}z,t) + 2e_y^TP_0D_2f_a - 2k\|P_0e_y\| \end{aligned} \tag{2.27}$$

From the Cauchy-Schwartz inequality, we can impose a bound on the last four terms of (2.27):

$$\begin{aligned}&2e_y^TP_0\left(f_2(T^{-1}z,t) - f_2(T^{-1}\hat{z},t)\right) + 2e_y^TP_0E_2\Delta\psi(T^{-1}z,t) + 2e_y^TP_0D_2f_a - 2k\|P_0e_y\|\\ &\leq 2\,\|P_0e_y\|\left(\|f_2(T^{-1}z,t) - f_2(T^{-1}\hat{z},t)\| + \|E_2\|\|\Delta\psi(T^{-1}z,t)\| + \|D_2\|\|f_a\| - k\right)\\ &\leq 2\,\|P_0e_y\|\left(\mathcal{L}_{f_2}\,\|e_1\| + \|E_2\|\xi(T^{-1}z,t) + \|D_2\|\rho - k\right) \end{aligned} \tag{2.28}$$

It follows from $\|\xi(x,t) - \xi(\hat{x},t)\| \leq \mathcal{L}_\xi\|x - \hat{x}\|$ that

$$\|\xi(T^{-1}z,t) - \xi(T^{-1}\hat{z},t)\| \leq \mathcal{L}_\xi\|T^{-1}z - T^{-1}\hat{z}\| = \mathcal{L}_\xi\|e_1\| \tag{2.29}$$

then $\xi(T^{-1}z,t) \leq \xi(T^{-1}\hat{z},t) + \mathcal{L}_\xi\|e_1\|$.

If k is chosen to satisfy:

$$k \geq \|E_2\|\xi(T^{-1}\hat{z},t) + \mathcal{L}_\xi\|E_2\|\|e_1\| + \|D_2\|\rho + \mathcal{L}_{f_2}\|e_1\| + \eta \tag{2.30}$$

where η is a positive constant which needs to be determined to ensure that the state error dynamics can be driven to the pre-defined sliding surface (2.19) in finite time. Then it follows from (2.27) that

$$\begin{aligned}\dot{V}_2 &\leq e_y^T(A_0^TP_0 + P_0A_0)e_y + 2e_y^TP_0\bar{A}_3e_1 + 2\|P_0e_y\|(\mathcal{L}_{f_2}\|e_1\|\\ &\quad + \|E_2\|\xi(T^{-1}\hat{z},t) + \mathcal{L}_\xi\|E_2\|\|e_1\| + \|D_2\|\rho - k)\\ &\leq e_y^T(A_0^TP_0 + P_0A_0)e_y + 2e_y^TP_0\bar{A}_3e_1 - 2\eta\|P_0e_y\| \end{aligned}$$

$$\leq e_y^T(A_0^T P_0 + P_0 A_0)e_y + 2e_y^T P_0 \bar{A}_3 e_1 \tag{2.31}$$

Combining (2.24) and (2.31) yields

$$\begin{aligned}
\dot{V} &= \dot{V}_1 + \dot{V}_2 \\
&\leq e_1^T \left(\bar{A}_1^T P_1 + P_1 \bar{A}_1 + \frac{1}{\alpha} \bar{P}_1 \bar{P}_1^T + \alpha (\mathcal{L}_f)^2 I_{n-p} \right) e_1 + e_1^T A_3^T P_0 e_y + e_y^T P_0 A_3 e_1 \\
&\quad + e_y^T (A_0^T P_0 + P_0 A_0) e_y \\
&= \begin{bmatrix} e_1 \\ e_y \end{bmatrix}^T \begin{bmatrix} \bar{A}_1^T P_1 + P_1 \bar{A}_1 + \frac{1}{\alpha} \bar{P}_1 \bar{P}_1^T + \alpha (L_f)^2 I & A_3^T P_0 \\ P_0 A_3 & A_0^T P_0 + P_0 A_0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_y \end{bmatrix} \\
&< 0
\end{aligned} \tag{2.32}$$

If (2.20) holds, then $e \to 0$ exponentially, namely, the error dynamical systems (2.17)-(2.18) are asymptotically stable.

This completes the proof.

Remark 2.2 The inequality (2.20) can be transformed into the following LMI problem: the state estimation error dynamics (2.17)-(2.18) are asymptotically stable if there exist matrices $P_0 > 0$, $P_1 > 0$, P_2, Y and a positive scalar α such that:

$$\begin{bmatrix} \Pi + \alpha(\mathcal{L}_f)^2 I_{n-p} & P_1 & P_2 & A_3^T P_0 \\ P_1^T & -\alpha I_{n-p} & 0 & 0 \\ P_2^T & 0 & -\alpha I_p & 0 \\ P_0 A_3 & 0 & 0 & Y + Y^T \end{bmatrix} < 0 \tag{2.33}$$

has a solution. Where $\Pi = A_1^T P_1 + P_1 A_1 + A_3^T P_2^T + P_2 A_3$ and $Y = P_0 A_0$. If $\mathcal{L}_f$ is known, then the problem of finding P_0, P_1, P_2, Y to satisfy (2.33) is a standard LMI feasibility problem. Alternatively, if the value of $\mathcal{L}_f$ is unknown, then the admissible Lipschitz constant of the nonlinear function can be maximized through LMI optimization.

Remark 2.3 From Assumption 2.4 and the conclusion of Lemma 2.1, it can be obtained that

$$\begin{aligned}
E_1^T P_1 + E_2^T P_2^T &= 0 \\
D_1^T P_1 + D_2^T P_2^T &= 0
\end{aligned} \tag{2.34}$$

The problem of finding matrix $P > 0$ to satisfy both (2.9) and (2.20) simultaneously can be transformed into the following optimization problem:

$$
\begin{aligned}
& min(\gamma) \\
& s.t. \\
& (2.33)\ and \\
& \begin{bmatrix} \gamma I & \begin{bmatrix} E_1^T P_1 + E_2^T P_2^T \\ D_1^T P_1 + D_2^T P_2^T \end{bmatrix} \\ \begin{bmatrix} E_1^T P_1 + E_2^T P_2^T \\ D_1^T P_1 + D_2^T P_2^T \end{bmatrix}^T & \gamma I \end{bmatrix} > 0
\end{aligned} \tag{2.35}
$$

Lemma 2.3 (Bellman-Gronwall inequality [88]) Let t_0 be a given time instant, γ_0, γ_1, γ_2 and a nonnegative constants and $g(t)$ a nonnegative piecewise continuous function of time. If $f(t)$ satisfies

$$f(t) \leq \gamma_0 + \gamma_1 e^{-a(t-t_0)} + \gamma_2 \int_{t_0}^{t} e^{-a(t-\tau)} g(\tau) f(\tau) d\tau,\ \forall t \geq t_0 \tag{2.36}$$

then

$$f(t) \leq (\gamma_0 + \gamma_1) e^{-a(t-t_0)} e^{\gamma_2 \int_{t_0}^{t} g(s)ds} + \gamma_0 a \int_{t_0}^{t} e^{-a(t-\tau)} e^{\gamma_2 \int_{\tau}^{t} g(\tau)} d\tau,\ \forall t \geq t_0 \tag{2.37}$$

Lemma 2.4 Consider the system descried in (2.12) and the observer described in (2.15). Let a_0 and c_0 be positive constants such that $\|e^{\bar{A}_1 t}\| \leq c_0 e^{-a_0 t}$. If $a_0 \geq c_0 \mathcal{L}_f \| [\ I_{n-p} \quad P_1^{-1} P_2\ ] \|$, then the bound of the state estimation error $e_1(t)$ is independent of the system input and output and satisfies:

$$\|e_1(t)\| \leq c_0 \|e_1(0)\| \exp\{(c_0 \mathcal{L}_f \| [\ I_{n-p} \quad P_1^{-1} P_2\ ] \| - a_0) t\} \tag{2.38}$$

Proof. According to conclusion (2) of Lemma 2.1, $\bar{A}_1$ is stable. Therefore there always exist constants a_0 and c_0 such that $\|e^{\bar{A}_1 t}\| \leq c_0 e^{-a_0 t}$ [88].

From (2.17), we can obtain:

$$e_1(t) = e^{\bar{A}_1 t} e_1(0) + \int_0^t e^{\bar{A}_1 (t-\tau)} \begin{bmatrix} I_{n-p} & P_1^{-1} P_2 \end{bmatrix}$$

$$\cdot \left(f\left(T^{-1}z,\tau\right) - f\left(T^{-1}\hat{z},\tau\right)\right) d\tau \tag{2.39}$$

From the triangle inequality, for any $t > 0$

$$\begin{aligned}
\|e_1(t)\| &\leq \|e^{\bar{A}_1 t}\|\|e_1(0)\| + \int_0^t \|e^{\bar{A}_1(t-\tau)}\| \left\| \begin{bmatrix} I_{n-p} & P_1^{-1}P_2 \end{bmatrix} \right\| \\
&\quad \cdot \| \left(f\left(T^{-1}z,\tau\right) - f\left(T^{-1}\hat{z},\tau\right)\right) \| d\tau \\
&\leq c_0 e^{-a_0 t} \|e_1(0)\| + \int_0^t c_0 e^{-a_0(t-\tau)} \left\| \begin{bmatrix} I_{n-p} & P_1^{-1}P_2 \end{bmatrix} \right\| \\
&\quad \cdot L_f \|e_1(\tau)\| \, d\tau
\end{aligned} \tag{2.40}$$

Applying Gronwall-Bellman inequality to (2.40) with $t_0 = 0$, $\gamma_0 = 0$, $\gamma_1 = c_0\|e_1(0)\|$, $\gamma_2 = c_0\mathcal{L}_f\|[\ I_{n-p} \quad P_1^{-1}P_2\ ]\|$ and $g(t) = 1$, the bound of e_1 (2.38) can be obtained immediately as (2.38).

This completes the proof.

In general, the design of a sliding mode observer involves two steps. The first step is to determine the sliding surface that represents the desired stable dynamics, and the second step is to choose the control gain that ensures the trajectories be driven to the sliding surface and maintained on it thereafter. After getting the sufficient condition for the error system (2.17)-(2.18) to be asymptotically stable, the next objective is to choose the observer switching gain to satisfy the reaching condition [61]. This switching gain not only forces the output errors e_y to be zero, but keeps the output errors at zero as well.

Proposition 2.2 *Under the Assumptions 2.1-2.5, the error dynamics (2.17)-(2.18) are driven to the sliding surface (2.19) in finite time if η in (2.30) is chosen to satisfy*

$$\eta \geq \|A_3\|\delta + \sigma \tag{2.41}$$

where σ is a positive scalar.

Proof. Consider the Lyapunov candidate function $V_2(e_y) = e_y^T P_0 e_y$.

Then the time derivative of V_2 can be obtained as

$$\dot{V}_2 = e_1^T A_3^T P_0 e_y + e_y^T P_0 A_3 e_1 + e_y^T (A_0^T P_0 + P_0 A_0) e_y$$

$$
\begin{aligned}
&+ 2e_y^T P_0 \left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t) \right) \\
&+ 2e_y^T P_0 E_2 \Delta\psi(T^{-1}z, t) + 2e_y^T P_0 D_2 f_a - 2e_y^T P_0 \nu
\end{aligned} \tag{2.42}
$$

Since A_0 is a stable matrix by design, therefore

$$
A_0^T P_0 + P_0 A_0 < 0 \tag{2.43}
$$

It follows from (2.23), (2.16) and (2.43) that

$$
\begin{aligned}
\dot{V}_2 &\leq e_1^T A_3^T P_0 e_y + e_y^T P_0 A_3 e_1 + 2e_y^T P_0 \left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t) \right) \\
&\quad + 2e_y^T P_0 E_2 \Delta\psi(T^{-1}z, t) + 2e_y^T P_0 D_2 f_a - 2e_y^T P_0 \nu \\
&\leq 2 \left\| P_0 e_y \right\| \left(\|A_3\| \|e_1\| + \mathcal{L}_{f_2} \left\| e_1 \right\| + \left\| E_2 \right\| \xi(T^{-1}z, t) \right. \\
&\quad \left. + \left\| D_2 \right\| \rho - k \right) \\
&\leq 2 \left\| P_0 e_y \right\| \left(\|A_3\| \|e_1\| + \mathcal{L}_{f_2} \left\| e_1 \right\| + \left\| E_2 \right\| \xi(T^{-1}\hat{z}, t) \right. \\
&\quad \left. \|E_2\| \mathcal{L}_{\xi} \|e_1\| + \left\| D_2 \right\| \rho - k \right)
\end{aligned} \tag{2.44}
$$

If the condition (2.41) holds, then

$$
\begin{aligned}
\dot{V}_2 &\leq -2\sigma \left\| P_0 e_y \right\| \\
&\leq -2\sigma \sqrt{\lambda_{min}(P_0)} V_2^{1/2}
\end{aligned} \tag{2.45}
$$

where $\lambda_{min}(P_0)$ is the smallest eigenvalue of P_0. This shows that the reachability condition [61] is satisfied. As a consequence, an ideal sliding motion will take place on the surface $\mathcal{S}$ and after some finite time,

$$
e_y = \dot{e}_y = 0, \quad \forall t > t_s \tag{2.46}
$$

This completes the proof.

2.4 Estimation of actuator fault

Given a sliding mode observer which satisfies (2.20) and (2.41), the task in this section is to reconstruct the actuator fault using the so-called equivalent output injection [8]. Firstly, the following assumption is needed to completely decouple the actuator fault and the system uncertainty.

Assumption 2.6 There exists a nonsingular matrix $G \in \mathcal{R}^{p \times p}$ such that

$$G \begin{bmatrix} E_2 & D_2 \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix} \tag{2.47}$$

where $H_1 \in \mathcal{R}^{(p-q) \times r}$ and $H_3 \in \mathcal{R}^{q \times q}$ is nonsingular.

From Proposition 2.2, it is known that if the gain $k(.)$ is chosen to satisfy (2.41), the state estimation error dynamics (2.17)-(2.18) will be driven to the sliding surface defined by (2.19) and a sliding motion will be maintained thereafter. Since during the sliding motion $e_y = 0$ and $\dot{e}_y = 0$, then

$$\begin{aligned} 0 = A_3 e_1 + &\left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)\right) \\ &+ \begin{bmatrix} E_2 & D_2 \end{bmatrix} \begin{bmatrix} \Delta\psi(T^{-1}z, t) \\ f_a \end{bmatrix} - \nu_{eq} \end{aligned} \tag{2.48}$$

where ν_{eq} is the equivalent output error injection signal representing the average behavior of the discontinuous function ν and it can be approximated to any degree of accuracy by

$$\nu_{eq} = k(t, y, u) \frac{P_0 e_y}{\|P_0 e_y\| + \delta} \tag{2.49}$$

where δ is a small positive scalar to reduce the chattering effect.

Case-1. If Assumption 2.6 is satisfied, the following equation can be obtained after multiplying G on both sides of (2.48):

$$\begin{aligned} 0 = G A_3 e_1 + G &\left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)\right) \\ &+ \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix} \begin{bmatrix} \Delta\psi(T^{-1}z, t) \\ f_a \end{bmatrix} - G\nu_{eq} \end{aligned} \tag{2.50}$$

Since $\lim_{t \to \infty} e_1 = 0$, $f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)$ will also tends to zero. This implies from (2.50) that

$$f_a(t) \to H_3^{-1} G_2 v_{eq} \quad \text{as } t \to \infty \tag{2.51}$$

where G_2 represents the last q rows of G.

Therefore the reconstruction of the actuator fault can be obtained from

$$\hat{f}_a(t) = k(t, y, u) H_3^{-1} G_2 \frac{P_0 e_y}{\|P_0 e_y\| + \delta} \tag{2.52}$$

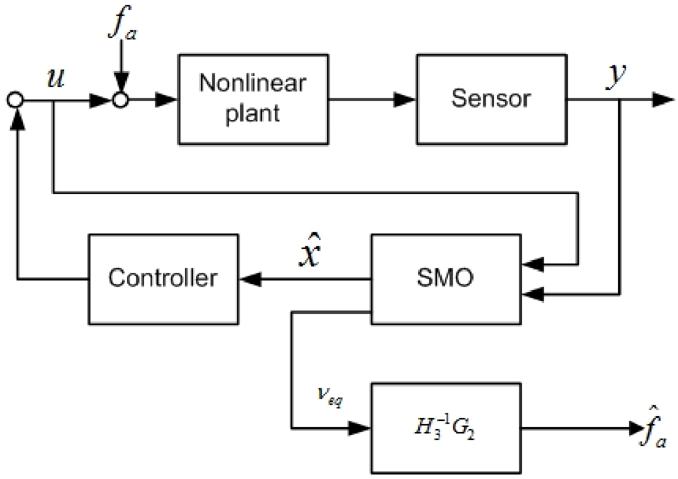

Figure 2.1: Schematic of the actuator fault reconstruction using (2.52)

The proposed actuator reconstruction scheme is shown in Fig-2.1.

Case-2. Alternatively, if Assumption 2.6 is not satisfied, we can approximately estimate the actuator fault $f_a(t)$ as

$$\hat{f}_a(t) \approx k(t, y, u) D_2^+ \frac{P_0 e_y}{\|P_0 e_y\| + \delta} \tag{2.53}$$

where D_2^+ is the left pseudo-inverse of D_2. Since D is of full column rank, such an inverse matrix always exists. D_2^+ is given by $D_2^+ = (D_2^T D_2)^{-1} D_2^T$.

It follows from (2.48) that

$$f_a(t) - D_2^+ \nu_{eq} = -D_2^+ A_3 e_1 - D_2^+ \left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t) \right) - D_2^+ E_2 \Delta\psi(T^{-1}z, t) \tag{2.54}$$

From the definition of $\hat{f}_a(t)$ in (2.53) and substitute $\hat{f}_a(t)$ for $D_2^+ \nu_{eq}$ in (2.54), we can obtain that

$$\begin{aligned} \|f_a(t) - \hat{f}_a(t)\| &= \| - D_2^+ A_3 e_1 - D_2^+ \left(f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t) \right) \\ &\quad - D_2^+ E_2 \Delta\psi(T^{-1}z, t) \| \\ &\leq (\|D_2^+ A_3\| + \mathcal{L}_{f_2} \|D_2^+\|) \|e_1\| + \|D_2^+ E_2\| \xi(x, t) \end{aligned} \tag{2.55}$$

and

$$\lim_{t \to \infty} \|f_a(t) - \hat{f}_a(t)\| \leq \|D_2^+ E_2\| \xi(x, t) \tag{2.56}$$

Remark 2.4 The existence of the matrix G in Assumption 2.6 guarantees that the actuator fault can be distinguished from the system uncertainty, which results in a precise reconstruction of the actuator fault. Otherwise the actuator can only be approximated and the estimation error satisfies (2.55) and (2.56). Since $\hat{f}_a(t)$ is only dependent on on-line measruements, therefore the this FDI scheme is practical for real implementation.

Remark 2.5 If the Assumption 2.6 is not satisfied, it follows from (2.56) that the actuator fault estimation error will mainly depend on the bound $\xi(x,t)$. One way to reduce the effect of the uncertainty on the fault estimation is to choose an appropriate matrix D_2^+ such that $\|D_2^+E_2\|$ is minimized [46].

Remark 2.6 The good feature of the proposed observer is that it not only enables fault detection, but also provides the amplitudes of faults, which is very useful for fault accommodation [89].

2.5 Simulation Results

The example of a single-link flexible joint robot system has been considered to demonstrate the effectiveness of the proposed SMO in reconstructing actuator faults. A dynamical model for the robot can be described by ([46, 90])

$$\begin{aligned}
\dot{\theta}_1 &= \omega_1 \\
\dot{\omega}_1 &= \frac{1}{J_1}(k_1(\theta_2-\theta_1)+k_2(\theta_2-\theta_1)^3) - \frac{B_v}{J_1}\omega_1 + \frac{K_\tau}{J_1}u \\
\dot{\theta}_2 &= \omega_2 \\
\dot{\omega}_2 &= -\frac{1}{J_2}(k_1(\theta_2-\theta_1)+k_2(\theta_2-\theta_1)^3) - \frac{mgh}{J_2}sin\theta_2 \\
&\quad + \psi(\theta_1,\omega_1,\theta_2,\omega_2,t)
\end{aligned} \tag{2.57}$$

where θ_1 and ω_1 are the motor position and velocity, respectively; θ_2 and ω_2 are the link position and velocity; J_1 is the inertia of the DC motor, J_2 is the inertia of the link, $2h$ is the length of the link while m_l represents its mass, B_v is the viscous friction, k_1 and k_2 are positive constants and K_τ is the amplifier gain. It is assumed that the motor position, motor velocity and the sum of link velocity and link position can be measured. The values of the parameters used in this simulation are: $J_1 = 3.7 \times$

$10^{-3}kg\cdot m^2$, $J_2 = 9.3\times10^{-3}kg\cdot m^2$, $h = 1.5\times10^{-1}m$, $m = 0.21kg$, $B_v = 4.6\times10^{-2}m$, $k_1 = k_2 = 1.8 \times 10^{-1}Nm/rad$ and $K_\tau = 8 \times 10^{-2}Nm/V$.

To illustrate the effectiveness of the prosed SMO and to reconstruct the actuator faults, a nonlinear uncertainty $\Delta\psi$ is added to the system, which satisfies the bound $\|\Delta\psi\| \leq 0.023(sin\theta_2)^2$. The uncertainty distribution matrix E is assumed to be $E = [0\ 1\ 0\ 0]^T$. For the illustration purpose, a linear state feedback controller $u = [-14.1 - 25.6 - 16.2 - 12.1]z$ has been utilized to stabilize the system. Suppose that a fault f_a occurs in the input channel, namely, the fault distribution matrix D is equal to the input matrix. f_a is given as:

$$f_a = \begin{cases} 0.05t & , \quad t \leq 2s \\ 0.1 + 0.5sin(2\pi t) & , \quad t > 2s \end{cases}$$

By reordering the system variables and defining $x = col(\theta_2, \omega_2, \theta_1, \omega_1)$, the output distribution matrix C becomes:

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Notice that C does not have the form in (2.6). A nonsingular transformation matrix

$$T_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is therefore introduced to obtain $CT_c^{-1} = [0\ \ I_p]$. This gives

$$A = \begin{bmatrix} -1.0000 & 1.0000 & 0 & 0 \\ -20.3548 & 1.0000 & 19.3548 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 48.6486 & 0 & -48.6486 & -12.4324 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} 0 \\ -19.3548(x_1 - x_3)^3 - 33.1935sinx_1 \\ 0 \\ 48.6486(x_1 - x_3)^3 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & I_3 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 21.6216 \end{bmatrix}$$

Imposing the stability constraint to the transformed system, as described in (2.20), and formulating the problem in an LMI framework gives the following solutions:

$$\begin{aligned}
\alpha &= 0.9328 \\
P_1 &= 0.7071 \\
P_2 &= 10^{-6} \times \begin{bmatrix} 0.0000 & -0.9657 & 0.0025 \end{bmatrix} \\
P_0 &= \begin{bmatrix} 0.7072 & 0.0001 & 0.2933 \\ 0.0001 & 0.8300 & 0.0001 \\ 0.2933 & 0.0001 & 0.1289 \end{bmatrix} \\
A_0 &= \begin{bmatrix} -23.6365 & 0.0277 & 38.3086 \\ -0.0001 & -0.5002 & 0.0003 \\ 55.4219 & -0.0665 & -92.3582 \end{bmatrix} \\
F_1 &= \begin{bmatrix} 0.7072 & 0.0001 & 0.2933 \end{bmatrix} \\
F_2 &= \begin{bmatrix} 6.3426 & 0.0014 & 2.7861 \end{bmatrix}
\end{aligned}$$

It is verified that the conclusion of Lemma 2.1 and Proposition 2.1 are all satisfied. The transformation matrix T is determined and the system is transformed into a new coordinate z and all the parameters of the proposed SMO (2.15) are obtained.

Fig 2.2-2.5 show the trajectories of the true states and their estimates. It can be seen from the figures that the proposed SMO can estimate the states very accurately, before and after the occurrence of a actuator fault. The result of fault estimation is depicted in Fig-2.6. It shows that despite the presence of system uncertainties $\Delta\psi$, the actuator fault can be reconstructed.

In addition, the proposed method can also work properly when considering measurement noise. This is shown in Fig-2.7 and Fig-2.8, where two noise environments are considered respectively, i.e., 30dB noise and 40dB noise. The measurement noise is added to the output signal so that the measured signal which is used by the SMO is corrupted. Arbitrarily large values of η would be needed to sustain a sliding motion, since the noise constitutes a large disturbance. From the figures, it can be seen that

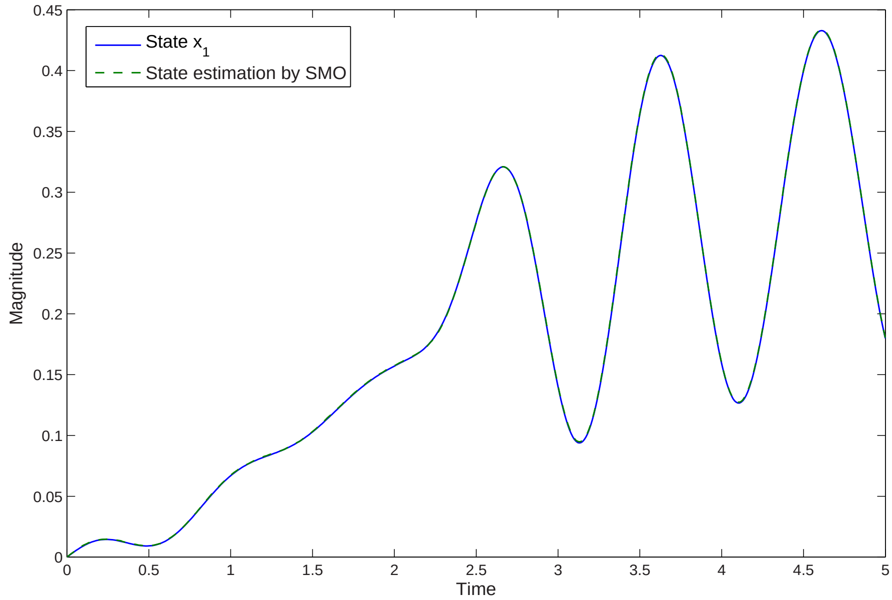

Figure 2.2: State x_1 and its estimated value $\hat{x}_1$

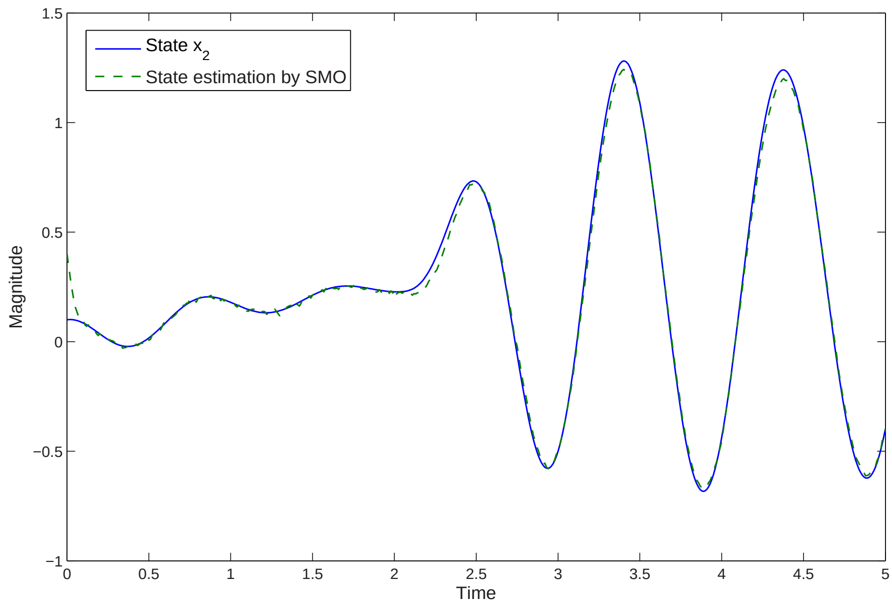

Figure 2.3: State x_2 and its estimated value $\hat{x}_2$

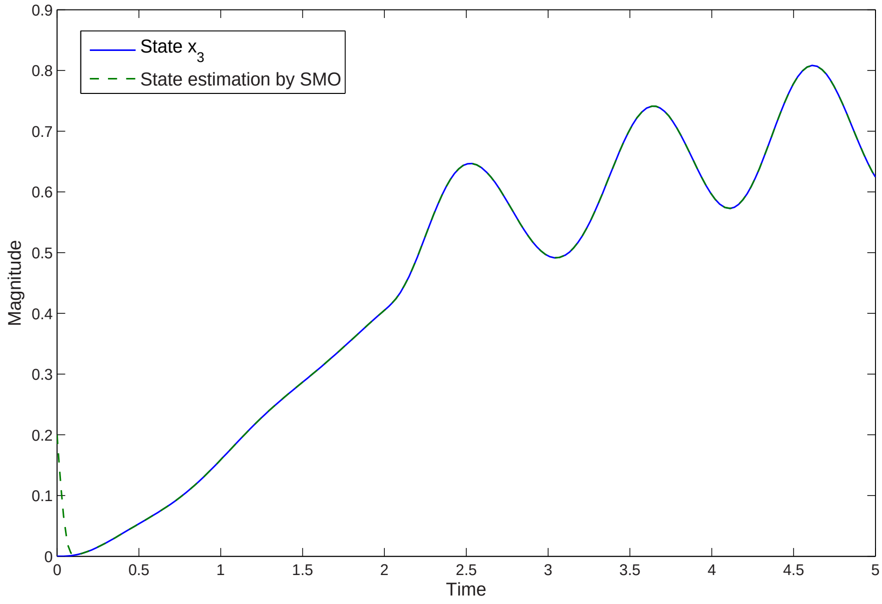

Figure 2.4: State x_3 and its estimated value $\hat{x}_3$

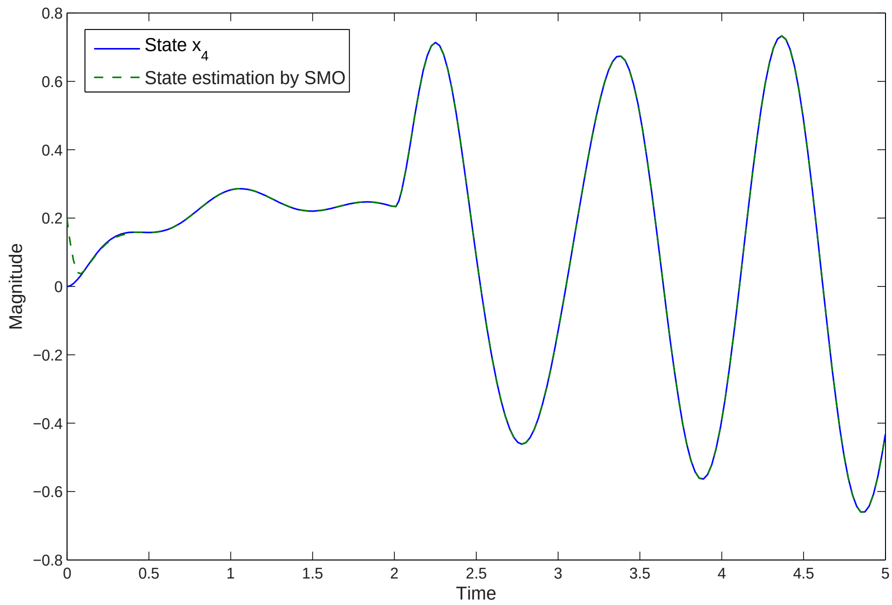

Figure 2.5: State x_4 and its estimated value $\hat{x}_4$

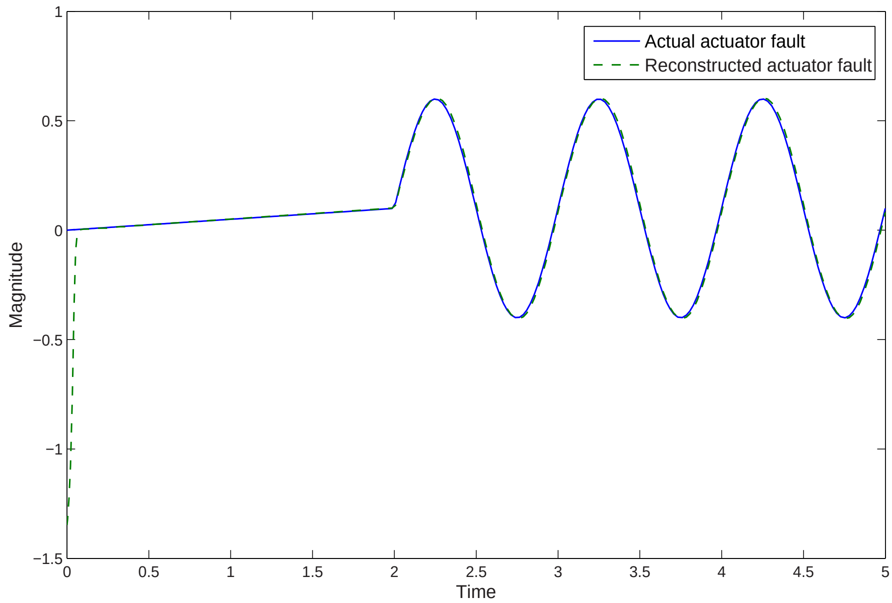

Figure 2.6: Actuator fault f_a and its estimated value $\hat{f}_a$

the proposed SMO can still preserve the shape of the actuator fault at different noise level.

2.6 Conclusions

A robust actuator fault estimation scheme for a class of Lipschitz nonlinear systems with matched non-parametric uncertainties has been put forward in this chapter. The proposed scheme is based on SMO technique and use the concept of equivalent output injection to estimate the actuator fault. The chapter reveals that under what conditions, the fault can be reconstructed with arbitrary accuracy in the presence of system uncertainty. The stability and reachability condition of the proposed SMO has been studied. The LMI based sufficient condition enables the designers to use Matlab's LMI toolbox, which makes the observer design applicable. The design parameters of the observer are obtained by LMI techniques. The effectiveness of the SMO has been demonstrated considering the example of a single-link flexible joint robot system.

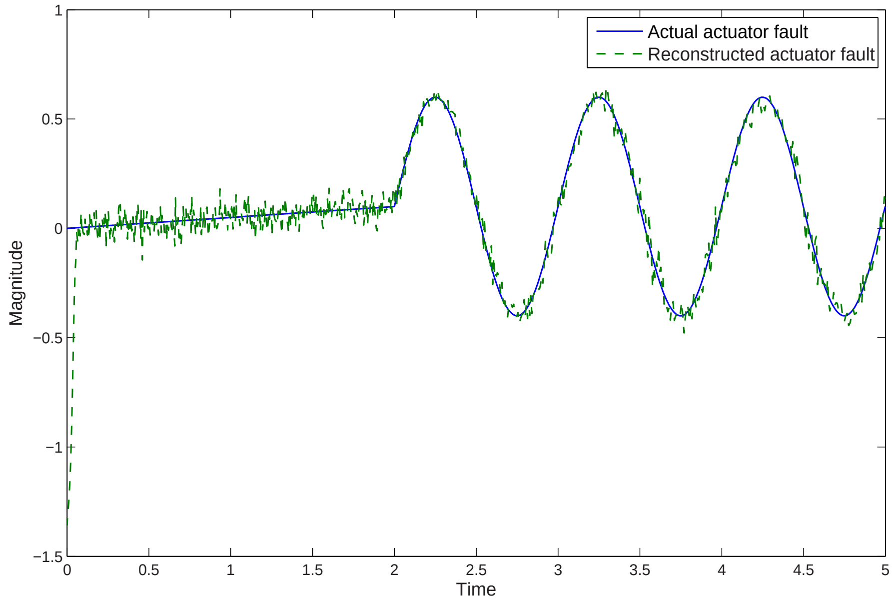

Figure 2.7: Reconstructed fault signal with sensor noise of 30dB

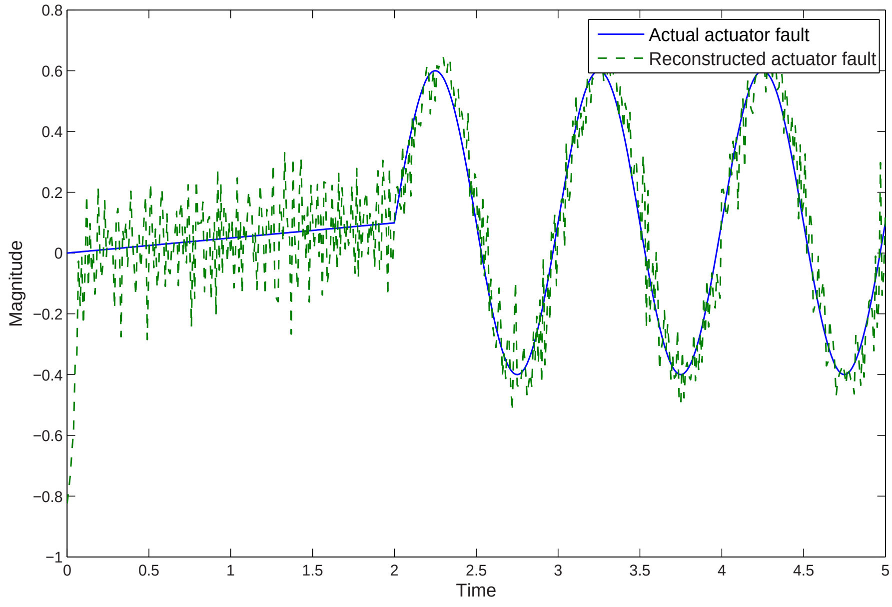

Figure 2.8: Reconstructed fault signal with sensor noise of 40dB

Chapter 3

Detection and isolation of incipient sensor faults for uncertain nonlinear systems

In this chapter, a sensor FDI scheme based on SMOs is proposed for the same class of nonlinear systems considered in Chapter 2. The research is carried out to solve not only sensor fault detection, but also sensor fault isolation problems by employing SMOs design.

3.1 Introduction

With the development of modern technology, autonomous systems are more and more dependent on sensors to acquire system information and signals from sensors often carry the most important information in automated/feedback control systems. A sensor fault may lead to poor regulation or tracking performance, or even affect the stability of the control system. Therefore the study of sensor FDI is becoming increasingly important. However, compared with the study of actuator FDI using SMOs, the research on sensor FDI is less studied in this realm.

Almost all the SMO-based approaches develpoed in the past mainly focus on relatively large-sized faults [8, 12, 72, 77, 78, 91]. The research on the detection and isolation of incipient faults has been less studied and still remains a challenge to model-based FDI techniques because they are almost unnoticeable during their initial stage and their effects to residuals are most likely to be concealed by system uncertainties. Incipient faults can cause very serious problems although they may develop slowly and be tolerable when they first appear. Hence it is necessary to detect and isolate incipient faults as early as possible to maintain the reliability of the system and this motivates the research reported in this chapter.

The proposed idea is inspired by the work presented in [58]. In [58], two coordinate transformations are introduced such that the original system can be decomposed into two subsystems. Based on the transformed systems, an SMO and a Luenberger observer are designed to eliminate the effects of disturbances and to detect incipient actuator faults, respectively. However, the proposed method is only applicable for linear systems and the problem of fault isolation still remains unsolved. In this chapter, the result in [58] for actuator fault detection for LTI systems is extended to sensor fault detection and isolation for Lipschitz nonlinear systems. The proposed method essentially transforms the original system into two subsystems (subsystem-1 and 2) where subsystem-1 includes the effects of system uncertainties but is free from sensor faults and subsystem-2 has sensor faults but without any uncertainties. Sensor faults in subsystem-2 are treated as actuator faults by using integral observer based approach [92]. For the purpose of fault detection, a traditional Luenberger observer is designed for this subsystem. The sensor fault is detected by considering the output estimation error of subsystem-2 as the residual. When this residual goes over a predefined threshold, a fault is detected. The most distinct feature of the proposed FDI scheme is that, by imposing a coordinate transformation to the original system, the effects of system uncertainties to the residual of subsystem-2 are completely de-coupled, which makes the scheme sensitive to incipient faults while robust to modelling uncertainty. Thus, early detection can be achieved and a false alarm caused by modelling uncertainties can be totally avoided.

After a fault is being detected, the next step is to determine the location of the fault, namely fault isolation. In principle the use of one single observer may permit the isolation of faults if their effect has independent projections onto the residual space. However if the system has significant nonlinearities, it is difficult to assure this independence. Therefore, a bank of observers is needed to isolate faults if they occur on different sensors. There are two schemes for fault isolation. The first one is called dedicated observer scheme [93]. In this scheme, N observers are designed to generate N residuals and the ith residual is expected to be only sensitive to the ith fault but insensitive to others. The other scheme is called generalized observer scheme [5], where N observers are also designed to produce N residuals. However, the difference is that the ith residual is sensitive to all possible faults except the ith one. In this chapter, the sensor fault isolation is carried out using the modified dedicated observer scheme to subsystem-2. Multiple observers, one for each possible sensor fault, are used to generate the estimated output vector. The estimated output vector is then compared with the actual output vector in order to determine which sensor is affected by the fault.

The rest of the chapter is organized as follows: Following the introduction, section-3.2 briefly describes the mathematical preliminaries required for designing observers. Section-3.3 proposes a sensor fault detection scheme and derives the stability condition of the proposed observers based on Lyapunov approach. The scheme of isolating multiple sensor faults is presented in section-3.4. The results of simulation are shown in section-3.5 with conclusions in section-3.6.

3.2 Problem Formulation

Consider a nonlinear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x,t) + Bu(t) + E\Delta\psi(t) \\ y(t) &= Cx(t) + Df_s(t) \end{aligned} \tag{3.1}$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$ and $y \in \mathcal{R}^p$ denote respectively the state variables, inputs and outputs. $f_s \in \mathcal{R}^q$ denotes the sensor fault. $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{p\times n}$, $D \in \mathcal{R}^{p\times q}$ and $E \in \mathcal{R}^{n\times r}$ ($p \geq q + r$) are known constant matrices with C and D both being of full column rank. The nonlinear continuous term $f(x,t)$ is assumed to be known. The unknown nonlinear term $\Delta\psi(t)$ models the lumped uncertainties and disturbances experienced by the system.

For the objective of achieving sensor fault detection and isolation, the following assumptions are made throughout:

Assumption 3.1 $rank(CE) = rank(E)$.

Lemma 3.1 Under Assumption 3.1, there exist state and output transformations

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{3.2}$$

such that in the new coordinate, the system matrices become:

$$\begin{aligned} TAT^{-1} &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad TE = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}, \\ SCT^{-1} &= \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix}, \quad SD = \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \end{aligned} \tag{3.3}$$

where $T \in \mathcal{R}^{n\times n}$, $S \in \mathcal{R}^{p\times p}$, $z_1 \in \mathcal{R}^{r}$, $w_1 \in \mathcal{R}^{r}$, $A_1 \in \mathcal{R}^{r\times r}$, $A_4 \in \mathcal{R}^{(n-r)\times(n-r)}$, $B_1 \in \mathcal{R}^{r\times m}$, $E_1 \in \mathcal{R}^{r\times r}$, $C_1 \in \mathcal{R}^{r\times r}$, $C_4 \in \mathcal{R}^{(p-r)\times(n-r)}$ and $D_2 \in \mathcal{R}^{(p-r)\times q}$. E_1 and C_1 are invertible and D_2 has the structure $D_2 = \begin{bmatrix} 0 \\ \bar{D}_2 \end{bmatrix}$, $\bar{D}_2 \in \mathcal{R}^{q\times q}$

Proof. See Appendix-C. Similar transformation procedure can be found in [91], in which the input and uncertainty distribution matrices after the transformation become $TB = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$ and $TE = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$.

Assumption 3.2 For every complex number s with nonnegative real part:

$$rank \begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix} = n + rank(E) \tag{3.4}$$

This assumption is known as the minimum phase condition.

Lemma 3.2 If there exist nonsingular matrices T and S such that the equation (3.3) holds, then the pair (A_4, C_4) is detectable if and only if Assumption 3.2 holds.

Proof. See [94, 95].

Assumption 3.3 The nonlinear term $f(x,t)$ is assumed to be known and Lipschitz about x uniformly, i.e., $\forall x, \hat{x} \in \mathcal{X}$,

$$\|f(x,t) - f(\hat{x},t)\| \leq \mathcal{L}_f \|x - \hat{x}\| \tag{3.5}$$

where $\mathcal{L}_f$ is the known Lipschitz constant.

Assumption 3.4 The function $\Delta\psi(t)$ representing the structured modeling uncertainty is unknown but bounded, and it satisfies $\|\Delta\psi(t)\| \leq \xi$. Also the unknown sensor fault is norm bounded, i.e., $\|f_s(t)\| \leq \rho$.

After introducing the state and output transformations (3.2), the original system (3.1) is converted into two subsystems:

$$\begin{aligned}
\dot{z}_1 &= A_1 z_1 + A_2 z_2 + f_1(T^{-1}z, t) + B_1 u + E_1 \Delta\psi \\
w_1 &= C_1 z_1 \qquad (3.6) \\
\dot{z}_2 &= A_3 z_1 + A_4 z_2 + f_2(T^{-1}z, t) + B_2 u \\
w_2 &= C_4 z_2 + D_2 f_s \qquad (3.7)
\end{aligned}$$

Partition T and S as:

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \tag{3.8}$$

where $T_1 \in \mathcal{R}^{r\times n}$, $S_1 \in \mathcal{R}^{r\times p}$. Then $f_1(T^{-1}z, t) = T_1 f(T^{-1}z, t)$, $f_2(T^{-1}z, t) = T_2 f(T^{-1}z, t)$ and the state z_1 can be obtained by the measured output y as:

$$z_1 = C_1^{-1} S_1 y \tag{3.9}$$

For subsystem (3.7), define a new state $z_3 = \int_0^t w_2(\tau)d\tau$ so that $\dot{z}_3(t) = C_4 z_2 + D_2 f_s$, and the augmented system with the new state z_3 is given as:

$$\begin{aligned}
\begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} &= \begin{bmatrix} A_4 & 0 \\ C_4 & 0 \end{bmatrix} \begin{bmatrix} z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} A_3 \\ 0 \end{bmatrix} z_1 + \begin{bmatrix} f_2(T^{-1}z, t) \\ 0 \end{bmatrix} \\
&\quad + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ D_2 \end{bmatrix} f_s \\
w_3 &= z_3 \qquad (3.10)
\end{aligned}$$

The augmented system (3.10) can then be rewritten in a more compact form as:

$$\begin{aligned}
\dot{z}_0 &= A_0 z_0 + A_{01} z_1 + F(z_0) + B_0 u + D_0 f_s \\
w_3 &= C_0 z_0 \qquad (3.11)
\end{aligned}$$

where $z_0 = \begin{bmatrix} z_2 \\ z_3 \end{bmatrix} \in \mathcal{R}^{n+p-2r}$, $w_3 \in \mathcal{R}^{p-r}$, $A_0 = \begin{bmatrix} A_4 & 0 \\ C_4 & 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times(n+p-2r)}$,

$A_{01} = \begin{bmatrix} A_3 \\ 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times r}$, $B_0 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times m}$, $D_0 = \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times q}$, $C_0 = \begin{bmatrix} 0 & I_{p-r} \end{bmatrix} \in \mathcal{R}^{(p-r)\times(n+p-2r)}$, $F(z_0) = \begin{bmatrix} f_2(T^{-1}z,t) \\ 0 \end{bmatrix}$.

Accordingly, subsystem (3.6) is rewritten as:

$$\begin{aligned} \dot{z}_1 &= A_1 z_1 + \bar{A}_2 z_0 + f_1(T^{-1}z,t) + B_1 u + E_1 \Delta\psi \\ w_1 &= C_1 z_1 \end{aligned} \tag{3.12}$$

where $\bar{A}_2 = \begin{bmatrix} A_2 \; 0_{r\times(p-r)} \end{bmatrix}$.

Lemma 3.3 The pair (A_0, C_0) is observable if Assumption 3.2 holds.

Proof. From the Popov-Belevitch-Hautus (PBH) test, the pair (A_0, C_0) is observable if and only if

$$rank \begin{bmatrix} sI - A_0 \\ C_0 \end{bmatrix} = rank \begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} = n + p - 2r \tag{3.13}$$

for all $s \in \mathcal{C}$. If $s = 0$, it is obvious that

$$rank \begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} = rank \begin{bmatrix} -A_4 \\ -C_4 \end{bmatrix} + p - r \tag{3.14}$$

If Assumption 3.2 holds, it follows that (A_4, C_4) is observable and thus

$$rank \begin{bmatrix} sI - A_4 \\ -C_4 \end{bmatrix} = n - r \;\; for \; all \; s \in \mathcal{C} \tag{3.15}$$

It follows that the rank test (3.13) holds when $s = 0$.

Moreover, since (A_4, C_4) is observable, if $s \neq 0$,

$$\begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \tag{3.16}$$

It means that the columns of $\begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix}$ are linearly independent and its rank is $n + p - 2r$. This completes the proof.

It follows from Lemma 3.3 that there exists a matrix $L_0 \in \mathcal{R}^{(n+p-2r)\times(p-r)}$ such that $A_0 - L_0C_0$ is stable, and thus for any $Q_0 > 0$, the Lyapunov equation

$$(A_0 - L_0C_0)^T P_0 + P_0(A_0 - L_0C_0) = -Q_0 \tag{3.17}$$

has a unique solution $P_0 > 0$ [80].

Assumption 3.5 There exists an arbitrary matrix $F_0 \in \mathcal{R}^{q\times(p-r)}$ satisfying:

$$D_0^T P_0 = F_0C_0 \tag{3.18}$$

Remark 3.1 The robustness of the SMO to bounded uncertainties inevitably makes it also robust to incipient faults. If Assumption 3.1 holds, then there exist coordinate transformations T and S such that sensor faults can be completely decoupled from uncertainties in the new coordinate. After the transformation, subsystem-1 formulated in (3.12) is free from sensor faults but subject to system uncertainties, and subsystem-2 formulated in (3.11) is prone to sensor faults but free from system uncertainties. Assumption 3.3 implies that there exists an asymptotic estimator for the system (3.11). In Assumption 3.4, the bound on uncertainty is assumed to be a constant, while in Chapter 2 the bound takes a nonlinear form. Assumption 3.5 together with the Lyapunov equation (3.17) is a passivity condition for system $((A_0 - L_0C_0), D_0, C_0)$ [80]. It follows form Lemma 2.1 that the sufficient and necessary condition for the existence of F_0 satisfying (3.18) is $rank(C_0D_0) = rank(D_0)$. From the structure of C_0 and D_0, it is easy to see that $C_0D_0 = D_2$. Therefore $rank(C_0D_0) = rank(D_2) = rank(D_0)$ is always satisfied since D is of full column rank.

3.3 Sensor fault detection scheme

Fault detection is the first step of fault diagnosis to determine whether a fault has occurred or not. The decision on the occurrence of a fault can be made if a significant residual change is observed. If we design sliding mode obervers directly for the original system, then the effect of incipient sensor faults on state estimation errors could be attenuated or even eliminated by the variable structure term [58] (the magnitude of the residual obtained will be within the chattering amplitude or smaller than the predefined threshold for a certain length of time if the gain $k(.)$ in (3.20) is chosen too large). The early detection, even detection of incipient sensor faults therefore becomes difficult. Observing the structure of subsystem-2 in (3.11), it is found that the state z_0 is neither subject to system uncertainties nor faults before the occurrence of any sensor fault. If we can design an observer for this particular subsystem and take the output estimation error $w_3 - \hat{w}_3$ ($\hat{w}_3$ is the estimation of w_3) as the residual, then the problem caused by designing conventional sliding mode observers for the original system can be solved. This intuition inspires the proposed fault detection scheme which is described in this section.

For subsystem-1, the proposed sliding mode observer has the form:

$$\begin{aligned} \dot{\hat{z}}_1 &= A_1\hat{z}_1 + \bar{A}_2\hat{z}_0 + f_1(T^{-1}\hat{z}, t) + B_1u + (A_1 - A_1^s)C_1^{-1}(w_1 - \hat{w}_1) + \nu_1 \\ \hat{w}_1 &= C_1\hat{z}_1 \end{aligned} \tag{3.19}$$

where $A_1^s \in \mathcal{R}^{r\times r}$ is a stable matrix which needs to be determined. For any $Q_1 > 0$ the Lyapunov equation $A_1^{sT}P_1 + P_1A_1^s = -Q_1$ has a unique solution $P_1 > 0$. The discontinuous output error injection term ν_1, that is used to eliminate the effects of uncertainties, is defined by

$$\nu_1 = \begin{cases} k_1\frac{P_1(C_1^{-1}S_1y-\hat{z}_1)}{\|P_1(C_1^{-1}S_1y-\hat{z}_1)\|} & \text{if } C_1^{-1}S_1y - \hat{z}_1 \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.20}$$

where k_1 is a positive scalar. It is worth noting that the state z_1 can be obtained by the measured output y as $z_1 = C_1^{-1}S_1y$.

For subsystem-2, a Luenberger observer with the following form is proposed:

$$\begin{aligned} \dot{\hat{z}}_0 &= A_0\hat{z}_0 + A_{01}C_1^{-1}w_1 + F(\hat{z}_0) + B_0u + L_0(w_3 - \hat{w}_3) \\ \hat{w}_3 &= C_0\hat{z}_0 \end{aligned} \tag{3.21}$$

where $L_0 \in \mathcal{R}^{(n+p-2r)\times(p-r)}$ is the gain of a traditional Luenberger observer which is used to make $(A_0 - L_0C_0)$ stable. If the state estimation errors are defined as

$e_1 = z_1 - \hat{z}_1$ and $e_0 = z_0 - \hat{z}_0$, then the state estimation error dynamics before the occurrence of sensor faults can be obtained as:

$$\dot{e}_1 = A_1^s e_1 + \bar{A}_2 e_0 + \big(f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\big) + E_1\Delta\psi - \nu_1 \tag{3.22}$$

$$\dot{e}_0 = (A_0 - L_0C_0)e_0 + (F(z_0) - F(\hat{z}_0)) \tag{3.23}$$

Define the sliding mode surface as:

$$\mathcal{S} = \{(e_1, e_0)|e_1 = 0\} \tag{3.24}$$

The condition of stability and reachability of the state estimation error dynamics (3.22) and (3.23) associated with the sliding motion (3.24) will be studied in the next.

Proposition 3.1 *Under the Assumptions 3.1-3.4 and that the system is free of faults, the error dynamical system (3.22) and (3.23) are asymptotically stable if there exist matrices* $A_1^s < 0$, L_0, $P_1 > 0$, $P_0 > 0$ *such that:*

$$\underbrace{\begin{bmatrix} -Q_1 + \frac{1}{\alpha_1}P_1P_1 & P_1\bar{A}_2 \\ \bar{A}_2^T P_1 & -Q_0 + \frac{1}{\alpha_0}P_0P_0 + aI \end{bmatrix}}_{-\tilde{Q}} < 0 \tag{3.25}$$

where $-Q_0 = (A_0 - L_0C_0)^T P_0 + P_0(A_0 - L_0C_0)$, $-Q_1 = A_1^{sT}P_1 + P_1A_1^s$, $a_1 = \mathcal{L}_{f_1}^2\|T^{-1}\|^2$, $a_0 = \mathcal{L}_{f_2}^2\|T^{-1}\|^2$, α_0 *and* α_1 *are two positive scalars,* $a = a_1\alpha_1 + a_0\alpha_0$.

Proof. Assume $V_1(e_1) = e_1^T P_1 e_1$, $V_0(e_0) = e_0^T P_0 e_0$ and consider $V(e_1, e_0) = V_1(e_1) + V_0(e_0)$ as a Lyapunov candidate. The time derivative of V_1, V_0 along the trajectories of system (3.22)-(3.23) can be shown to be:

$$\begin{aligned}\dot{V}_1 = {} & e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^T P_1\bar{A}_2 e_0 + 2e_1^T P_1 E_1\Delta\psi \\ & + 2e_1^T P_1\big(f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\big) - 2e_1^T P_1\nu_1\end{aligned}$$

Since the inequality $2X^TY \leq \frac{1}{\alpha}X^TX + \alpha Y^TY$ is true for any scalar $\alpha > 0$ [46], then

$$\begin{aligned}\dot{V}_1 \leq {} & e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^T P_1\bar{A}_2 e_0 + 2e_1^T P_1 E_1\Delta\psi \\ & + \frac{1}{\alpha_1}e_1^T P_1 P_1^T e_1 + \alpha_1\big(f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\big)^T \\ & \cdot\big(f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\big) - 2e_1^T P_1\nu_1\end{aligned} \tag{3.26}$$

Note that $\hat{z} := [(C_1^{-1}S_1y)^T, (\hat{z}_0)^T]^T$, then before the occurrence of sensor faults we have

$$z - \hat{z} = \begin{bmatrix} 0 \\ e_2 \end{bmatrix} \tag{3.27}$$

It is easy to see that

$$\begin{aligned} \|T^{-1}z - T^{-1}\hat{z}\| = \left\| T^{-1} \begin{bmatrix} 0 \\ e_2 \end{bmatrix} \right\| = \|T^{-1}e_2\| \leq \|T^{-1}e_0\| \\ \|f(T^{-1}z, t) - f(T^{-1}\hat{z}, t)\| \leq \mathcal{L}_f \|T^{-1}\| \|e_0\| \\ \|f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t)\| \leq \mathcal{L}_{f_1} \|T^{-1}\| \|e_0\| \\ \|f_2(T^{-1}z, t) - f_2(T^{-1}\hat{z}, t)\| \leq \mathcal{L}_{f_2} \|T^{-1}\| \|e_0\| \end{aligned} \tag{3.28}$$

where $\mathcal{L}_{f_1} = \|T_1\|\mathcal{L}_f$ and $\mathcal{L}_{f_2} = \|T_2\|\mathcal{L}_f$.

Moreover from the definition of ν_1, it can be obtained that

$$e_1^T P_1 \nu_1 = k_1 \|P_1 e_1\| \tag{3.29}$$

Then equation (3.26) can be simplified as:

$$\begin{aligned} \dot{V}_1 \leq & -e_1^T Q_1 e_1 + 2e_1^T P_1 \bar{A}_2 e_0 + \frac{1}{\alpha_1} e_1^T P_1 P_1 e_1 + \alpha_1 \mathcal{L}_{f_1}^2 \|T^{-1}\|^2 \|e_0\|^2 \\ & + 2\|P_1 e_1\| \|E_1\| \xi - 2k_1 \|P_1 e_1\| \end{aligned} \tag{3.30}$$

If the positive scalar gain k_1 is chosen to satisfy:

$$k_1 \geq \|E_1\|\xi + \eta_1 \tag{3.31}$$

where η_1 is a positive constant which needs to be determined to ensure that the state error dynamics (3.22) can be driven to the sliding surface (3.24).

Then it follows from (3.30) that

$$\begin{aligned} \dot{V}_1 & \leq -e_1^T Q_1 e_1 + 2e_1^T P_1 \bar{A}_2 e_0 + \frac{1}{\alpha_1} e_1^T P_1 P_1 e_1 + \alpha_1 \mathcal{L}_{f_1}^2 \|T^{-1}\|^2 \|e_0\|^2 - 2\eta_1 \|P_1 e_1\| \\ & \leq -e_1^T Q_1 e_1 + 2e_1^T P_1 \bar{A}_2 e_0 + \frac{1}{\alpha_1} e_1^T P_1 P_1 e_1 + \alpha_1 \mathcal{L}_{f_1}^2 \|T^{-1}\|^2 \|e_0\|^2 \end{aligned} \tag{3.32}$$

Similarly, the derivative of V_0 can be obtained as:

$$\begin{aligned}\dot{V}_0 &= -e_0^T Q_0 e_0 + 2e_0^T P_0 \left(F(z_0) - F(\hat{z}_0)\right) \\ &\leq -e_0^T Q_0 e_0 + \frac{1}{\alpha_0} e_0^T P_0 P_0 e_0 + \alpha_0 \mathcal{L}_{f_2}^2 \|T^{-1}\|^2 \|e_0\|^2 \end{aligned} \tag{3.33}$$

Combining (3.32) and (3.33) yields

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_0 \leq -e^T \tilde{Q} e \\ &\leq -\lambda_{min}(\tilde{Q}) \|e\|^2 = -\omega(t)\end{aligned} \tag{3.34}$$

where $\lambda_{min}(\tilde{Q})$ denotes the minimum eigenvalue of the matrix $\tilde{Q}$, $e = \left[e_1^T \; e_0^T\right]^T$. Integrating (3.34) from 0 to t yields

$$V(0) \geq V(t) + \int_0^t \omega(\tau) d\tau \geq \int_0^t \omega(\tau) d\tau \tag{3.35}$$

Therefore $0 < \int_0^t \omega(\tau) d\tau \leq V(0)$. Since $V(0)$ is positive and finite, $\lim\limits_{t\to\infty} \int_0^t \omega(\tau) d\tau$ exists and is finite. According to the Barbalat Lemma [96], then

$$\lim_{t\to\infty} \omega(t) = \lambda_{min}(\tilde{Q}) \lim_{t\to\infty} \|e\|^2 = 0 \tag{3.36}$$

which implies that $\lim\limits_{t\to\infty} e(t) = 0$, namely, the error dynamical systems (3.22) and (3.23) are asymptotically stable. This completes the proof.

Remark 3.2 The inequality (3.25) can be transformed into the following LMI feasibility problem: there exist matrices X, Y_0, $P_1 > 0$, $P_0 > 0$ and positive scalars α_0, α_1 such that:

$$\begin{bmatrix} \Theta_1 & P_1 & P_1 \bar{A}_2 & 0 \\ P_1 & -\alpha_1 I & 0 & 0 \\ \bar{A}_2^T P_1 & 0 & \Theta_0 - C_0^T Y_0^T - Y_0 C_0 + aI & P_0 \\ 0 & 0 & P_0 & -\alpha_0 I \end{bmatrix} < 0 \tag{3.37}$$

where $\Theta_1 = X + X^T$, $X = P_1 A_1^s$, $\Theta_0 = A_0^T P_0 + P_0 A_0$, $Y_0 = P_0 L_0$, $a = a_1 \alpha_1 + a_0 \alpha_0$.

Remark 3.3 Assume that P_0 in the new coordinate has the diagonal structure like:

$$P_0 = \begin{bmatrix} P_{01} & 0 \\ 0 & P_{02} \end{bmatrix} \tag{3.38}$$

where $P_{01} \in \mathcal{R}^{(n-r)\times(n-r)}$ and $P_{02} \in \mathcal{R}^{(p-r)\times(p-r)}$ are symmetric positive definite matrix. Then the inequality (3.25) can be transformed into the following LMI feasibility problem: there exist matrices X, Y_{02}, $P_1 > 0$, $P_{01} > 0$, $P_{02} > 0$ and positive scalars α_0, α_1 such that:

$$\begin{bmatrix} \Theta_1 & P_1 & P_1 A_2 & 0 & 0 & 0 \\ P_1 & -\alpha_1 I & 0 & 0 & 0 & 0 \\ A_2^T P_1 & 0 & A_4^T P_{01} + P_{01} A_4 + aI & C_4^T P_{02} & P_{01} & 0 \\ 0 & 0 & P_{02} C_4 & -Y_{02}^T - Y_{02} + aI & 0 & P_{02} \\ 0 & 0 & P_{01} & 0 & -\alpha_0 I & 0 \\ 0 & 0 & 0 & P_{02} & 0 & -\alpha_0 I \end{bmatrix} < 0 \tag{3.39}$$

where $Y_{02} = P_{02} L_{02}$, $L_0 = \begin{bmatrix} 0 \\ L_{02} \end{bmatrix}$, $L_{02} \in \mathcal{R}^{(p-r)\times(p-r)}$.

Substituting diagonal block matrix P_0 into (3.18) yields $F_0^T = P_{02} D_2$, which can be used to facilitate the design of sliding mode observers for subsystem-2 in Section-3.4.

Remark 3.4 If P_0 in the new coordinate does not have the diagonal structure in (3.38), then the problem of finding matrices P_0, F_0 to simultaneously satisfy both (3.25) and (3.18) can be transformed into the following LMI optimization problem: Minimize γ subject to $P_0 > 0$, $P_0 > 0$, (3.37) and

$$\begin{bmatrix} \gamma I & D_0^T P_0 - F_0 C_0 \\ (D_0^T P_0 - F_0 C_0)^T & \gamma I \end{bmatrix} > 0 \tag{3.40}$$

Proposition 3.1 implies that the error dynamical system (3.22)-(3.23) associated with the sliding surface (3.24) is asymptotically stable. The objective now is to choose the observer gain $k_1(.)$ in (3.20) such that the error dynamical system of (3.22) can be driven to the sliding surface and a sliding motion can be maintained thereafter. The following conclusion is presented.

Proposition 3.2 *Under the Assumption 3.1-3.4, the error dynamics (3.22) is driven to the sliding surface given by (3.24) in finite time if the gain η_1 is chosen to satisfy*

$$\eta_1 \geq \left(\|A_1^s\| + \|\bar{A}_2\| + \mathcal{L}_{f_1}\|T^{-1}\|\right)\|e\| + \eta_0, \quad \eta_0 > 0 \tag{3.41}$$

Proof. Consider a Lyapunov candidate function $V_1 = e_1^T P_1 e_1$. Then its time derivative can be obtained as:

$$\begin{aligned}\dot{V}_1 &\leq 2e_1^T P_1 A_1^s e_1 + 2e_1^T P_1 \bar{A}_2 e_0 - 2\|P_1 e_1\|\eta_1 + 2e_1^T P_1 \left(f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t)\right) \\ &\leq 2\|P_1 e_1\| \left((\|A_1^s\| + \|\bar{A}_2\| + \mathcal{L}_{f_1}\|T^{-1}\|)\|e\| - \eta_1\right)\end{aligned}$$

It follows from (3.41) that

$$\dot{V}_1 \leq -2\eta_0\|P_1 e_1\| \leq -2\eta_0\sqrt{\lambda_{min}(P_1)}\sqrt{V_1} \tag{3.42}$$

This shows that the reachability condition [61] is satisfied and a sliding motion is achieved and maintained after some finite time $t_s > 0$.

This completes the proof.

Lemma 3.4 Consider the system described by (3.11) and the observer described by (3.21). Let a_0 and c_0 be positive constants such that $\|e^{(A_0 - L_0 C_0)t}\| \leq c_0 e^{-a_0 t}$. If $a_0 \geq c_0 \mathcal{L}_{f_2}\|T^{-1}\|$, then the state estimation error $e_0(t)$ is bounded by

$$\|e_0(t)\| \leq c_0\|e_0(0)\|e^{(c_0\mathcal{L}_{f_2}\|T^{-1}\| - a_0)t} \tag{3.43}$$

Proof. From (3.23), the solution of $e_0(t)$ can be obtained as:

$$e_0(t) = e^{(A_0 - L_0 C_0)t}e_0(0) + \int_0^t e^{(A_0 - L_0 C_0)(t-\tau)} \cdot (F(z_0) - F(\hat{z}_0))\, d\tau \tag{3.44}$$

Applying the triangle inequality to (3.44), we can obtain

$$\|e_0(t)\| \leq c_0 e^{-a_0 t}\|e_0(0)\| + c_0\mathcal{L}_{f_2}\|T^{-1}\| \int_0^t e^{-a_0(t-\tau)}\|e_0(\tau)\| d\tau \tag{3.45}$$

where positive constants a_0 and c_0 are chosen such that $\|e^{(A_0 - L_0 C_0)t}\| \leq c_0 e^{-a_0 t}$. Such a_0 and c_0 can always be found since $A_0 - L_0 C_0$ is Hurwitz [88]. Applying Gronwall-Bellman inequality [88] to (3.45) yields:

$$\|e_0(t)\| \leq c_0\|e_0(0)\|e^{(c_0\mathcal{L}_{f_2}\|T^{-1}\| - a_0)t} \tag{3.46}$$

This completes the proof.

After any sensor fault occurs at time instant t_f, the state estimation error dynamics

(3.22) and (3.23) become:

$$\dot{e}_1 = A_1^s e_1 + \bar{A}_2 e_0 + \big(f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\big) + E_1\Delta\psi - \nu_1 \tag{3.47}$$

$$\dot{e}_0 = (A_0 - L_0C_0)e_0 + (F(z_0) - F(\hat{z}_0)) + D_0 f_s \tag{3.48}$$

Observing (3.48), one can find out that e_0 is only affected by sensor faults, but not subject to system uncertainties $\Delta\psi$ as well as the error injection term. Note that the sensor fault distribution matrix $D_0 = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}$. Thus the sensor fault f_s affects the last $p-r$ components of e_0, namely e_{z_3} directly. More specifically, if there occurs a fault, e_{z_3} will definitely change. The situation that the sensor fault only affects the first n-r components of e_0 does not exist. From Lemma 3.4, e_0 will approach to zero if there is no sensor fault. After the occurrence of any sensor fault, the last $p-r$ components of e_0 will deviate from zero. Therefore $\|e_{w3}\| = \|C_0e_0\| = \|e_{z_3}\|$ provides a good choice as the residual to detect the occurrence of sensor faults. Accordingly, the sensor fault detection scheme can be devised as follows:

Sensor fault detection scheme: Sensor faults can be detected if the residual $\|e_{w_3}\|$ exceeds a predefined threshold ς. Otherwise the system is healthy within the considered time. The detection time t_d $(t_d \geq t_f)$ is defined as the first time instant such that $\|e_{w_3}\|$ is observed greater than ς.

Remark 3.5 From (3.46), it is easy to see that before the occurrence of any sensor fault, the norm bound of the state estimation error $e_0(t)$ depends on the bound of the unknown initial condition $e_0(0)$. Since $\|e_0(0)\|$ is multiplied by $e^{(c_0\mathcal{L}_{f_2}\|T^{-1}\|-a_0)t}$, the effect of this bound will decrease exponentially and e_0 will approach to zero. It implies that a small threshold ς can be selected and the value does not significantly affect the performance of the fault detection scheme.

3.4 Sensor fault isolation scheme

After detecting a sensor fault, the next objective is to determine their locations if the system suffers from multiple faults instantaneously, which is referred to as fault isolation. Notice that $f_s = [f_s^{1T}, f_s^{2T}, \ldots, f_s^{qT}]^T$, if we can decide whether or not $f_s^i = 0,\ i = 1,2,\ldots,q$, then the sensor fault isolation can be achieved according to the known structure of D which describes the location of sensor faults. The objective of this section is to design a scheme that is capable of isolating multiple sensor faults even if they occur at the same time.

In the present work, the dedicated observer scheme which can only be used to detect and isolate one single fault has been modified and adopted. More specifically, to isolate one fault ($f_s^i \neq 0$) among q possible faults ($f_s \in \mathcal{R}^q$), a total number of $2q$ sliding mode observers is designed to generate q residuals (the original system is transformed into two subsystems under the new coordinate, that is why two sliding mode observers are needed). The SMOs are designed such that the ith residual is only sensitive to f_s^i but is insensitive to all other faults.

For subsystem-1, the proposed isolation observer designed for $f_s^i,\ i = 1, 2, \ldots, q$ has the following form:

$$
\begin{aligned}
\dot{\hat{z}}_1^i &= A_1\hat{z}_1^i + \bar{A}_2\hat{z}_0^i + f_1(T^{-1}\hat{z}^i, t) + B_1u + (A_1 - A_1^s)C_1^{-1}(w_1^i - \hat{w}_1^i) + \nu_1^i \\
\hat{w}_1^i &= C_1\hat{z}_1^i
\end{aligned}
\tag{3.49}
$$

where $\hat{z}^i$ and $\hat{w}^i$ denote respectively the estimated state and output obtained by the ith isolation estimator. The output error injection term ν_1^i is defined as:

$$
\nu_1^i = \begin{cases} k_1\frac{P_1(C_1^{-1}S_1y - \hat{z}_1^i)}{\|P_1(C_1^{-1}S_1y - \hat{z}_1^i)\|} & \text{if } C_1^{-1}S_1y - \hat{z}_1^i \neq 0 \\ 0 & \text{otherwise} \end{cases}
\tag{3.50}
$$

For subsystem-2, the proposed sliding mode observer designed for $f_s^i,\ i = 1, 2, \ldots, q$ has the following form:

$$
\begin{aligned}
\dot{\hat{z}}_0^i &= A_0\hat{z}_0^i + A_{01}C_1^{-1}w_1^i + F(\hat{z}_0^i) + B_0u + L_0(w_3^i - \hat{w}_3^i) + \bar{D}_0^i\nu_2^i \\
\hat{w}_3^i &= C_0\hat{z}_0^i
\end{aligned}
\tag{3.51}
$$

Partition D_0 and F_0 into $D_0 = (D_0^1, \ldots, D_0^q)$ and $F_0 = (F_0^{1^T}, \ldots, F_0^{q^T})^T$. D_0^i represents the ith column of D_0 and the rest columns are denoted as $\bar{D}_0^i$. Similarly, F_0^i is the ith row of F_0 and $\bar{F}_0^i$ consists of all other rows. The discontinuous output error injection term ν_2^i is defined by

$$
\nu_2^i = \begin{cases} \rho\frac{\bar{F}_0^i e_{w_3}^i}{\|\bar{F}_0^i e_{w_3}^i\|} & \text{if } e_{w_3} \neq 0 \\ 0 & \text{otherwise} \end{cases}
\tag{3.52}
$$

where $e^i_{w_3} = w^i_3 - \hat{w}^i_3$.

Note that for subsystem-2, a sliding mode observer is proposed instead of a normal Luenberger observer that being used in section-3.3 only for fault detection. The application of a bank of sliding mode observers associated with the proposed isolation scheme can successfully isolate multiple sensor faults even if they occur at the same time, which will be shown in the following. The property of the proposed sliding mode observer is characterised by the following result:

Proposition 3.3 *Under the Assumptions 3.1-3.5 and if there exist matrices A^s_1, L_0, $P_1 > 0$, $P_0 > 0$ such that:*

$$\begin{bmatrix} -Q_1 + \frac{1}{\alpha_1}P_1P_1 & P_1\bar{A}_2 \\ \bar{A}_2^T P_1 & -Q_0 + \frac{1}{\alpha_0}P_0P_0 + aI \end{bmatrix} < 0 \tag{3.53}$$

where P_1, P_0, Q_0, Q_1 are the same as that in Proposition 3.1, then the state estimation error of the ith isolator, e^i_0 will exponentially tend to zero if $f^i_s = 0$; otherwise e^i_0 satisfies $\dot{e}^i_0 = (A_0 - L_0C_0)e^i_0 + (F(z_0) - F(\hat{z}^i_0)) + D^i_0 f^i_s + \bar{D}^i_0(\bar{f}_s{}^i - \nu^i_2)$ if $f^i_s \neq 0$.

Proof. By using the ith isolation observer which is designed for f^i_s, the state estimation error dynamics after the occurrence of sensor faults can be obtained as:

$$\dot{e}^i_1 = A^s_1 e^i_1 + \bar{A}_2 e^i_0 + \big(f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}^i, t)\big) + E_1\Delta\psi - \nu^i_1 \tag{3.54}$$

$$\begin{aligned} \dot{e}^i_0 &= (A_0 - L_0C_0)e^i_0 + \big(F(z_0) - F(\hat{z}^i_0)\big) + D_0 f_s - \bar{D}^i_0\nu^i_2 \\ &= (A_0 - L_0C_0)e^i_0 + \big(F(z_0) - F(\hat{z}^i_0)\big) + D^i_0 f^i_s + \bar{D}^i_0(\bar{f}_s{}^i - \nu^i_2) \end{aligned} \tag{3.55}$$

Assume $V^i_1 = e^i_1{}^T P_1 e^i_1$ and $V^i_0 = e^i_0{}^T P_0 e^i_0$. Consider $V^i = V^i_1 + V^i_0$ as a Lyapunov candidate. The time derivative of V^i_1, V^i_0 along the trajectories of system (3.54)-(3.55) can be shown to be:

$$\begin{aligned} \dot{V}^i_1 &= e^i_1{}^T(A^s_1{}^T P_1 + P_1A^s_1)e^i_1 + 2e^i_1{}^T P_1\bar{A}_2 e^i_0 + 2e^i_1{}^T P_1E_1\Delta\psi \\ &\quad + 2e^i_1{}^T P_1\big(f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}^i, t)\big) - 2e^i_1{}^T P_1\nu_1 \\ &\leq -e^i_1{}^T Q_1 e^i_1 + 2e^i_1{}^T P_1\bar{A}_2 e^i_0 + \frac{1}{\alpha_1}e^i_1{}^T P_1P_1e^i_1 + \alpha_1\mathcal{L}^2_{f_1}\|T^{-1}\|^2\|e^i_0\|^2 \end{aligned} \tag{3.56}$$

If $f^i_s = 0$, the error dynamics of e^i_0 of (3.55) can be rewritten as:

$$\dot{e}^i_0 = (A_0 - L_0C_0)e^i_0 + \big(F(z_0) - F(\hat{z}^i_0)\big) + \bar{D}^i_0(\bar{f}_s{}^i - \nu^i_2) \tag{3.57}$$

The derivative of V_0^i becomes

$$
\begin{aligned}
\dot{V}_0^i &= -{e_0^i}^T Q_0 e_0^i + 2{e_0^i}^T P_0 \left(F(z_0) - F(\hat{z}_0^i)\right) + 2{e_0^i}^T P_0 \bar{D}_0^i(\bar{f_s}^i - \nu_2^i) \\
&\leq -{e_0^i}^T Q_0 e_0^i + 2{e_0^i}^T P_0 \left(F(z_0) - F(\hat{z}_0^i)\right) + 2\|\bar{F}_0^i e_{w3}^i\|(\|\bar{f_s}^i\| - \rho) \\
&\leq -{e_0^i}^T Q_0 e_0^i + 2{e_0^i}^T P_0 \left(F(z_0) - F(\hat{z}_0^i)\right) \\
&\leq -{e_0^i}^T Q_0 e_0^i + \frac{1}{\alpha_0}{e_0^i}^T P_0 P_0 e_0^i + \alpha_0 \mathcal{L}_{f2}^2 \|T^{-1}\|^2 \|e_0^i\|^2
\end{aligned} \tag{3.58}
$$

Combining (3.56) and (3.58) yields

$$
\dot{V}^i = \dot{V}_1^i + \dot{V}_0^i < 0
$$

This concludes that e^i tends to zero exponentially if $f_s^i = 0$ even after the occurrence of sensor faults $f_s^j \neq 0,\ j \in \{1, 2, \ldots, q\} \backslash \{i\}$. On the other hand, if $f_s^i \neq 0$, the term $D_0^i f_s^i$ in (3.55) can not be attenuated by $\bar{D}_0^i(\bar{f_s}^i - \nu_2^i)$, because D_0 is of full column rank. Therefore we can conclude that $\lim_{t\to\infty} e_0^i \neq 0$ if $f_s^i \neq 0$.

This completes the proof.

Proposition 3.3 characterises the property of the proposed sliding mode observer and also forms the intuitive principle of the fault isolation scheme as follows: the decision on which sensor is faulty can be made by jointly determining the value of $f_s^i,\ i = 1, 2, \ldots, q$ and the fault distribution matrix D (Note that the sensor fault vector is denoted as $f_s = [{f_s^1}^T, {f_s^2}^T, \ldots, {f_s^q}^T]^T$). Since D is known, the objective of finding faulty sensors moves to find which f_s^i is not equal to zero. After the system is detected to be faulty at some time instant t_d, a bank of $2q$ sliding mode observers is designed according to the possible faulty model. More specifically, for each $f_s^i,\ i = 1, 2, \ldots, q$, two observers given by (3.49) and (3.51) are designed to estimate states and outputs. It can be seen from the proof of Proposition 3.3 that the effect of $f_s^j,\ j \in \{1, 2, \ldots, q\} \backslash \{i\}$ is attenuated by the output error injection term ν_2^i, which can not eliminate the effect of f_s^i to the residual, then the state estimation error e_0^i obtained by observers which are designated for f_s^i will converge to zero if $f_s^i = 0$. On the other hand, the state estimation error e_0^i will go beyond a predefined threshold for some finite time $t_i > t_d$ if $f_s^i \neq 0$. As a result, choosing $\|e_{w3}^i\| = \|C_0 e_0^i\|$ as the residual and comparing it with the corresponding threshold ς_i which is associated with the observers designed for f_s^i, the location of sensor faults can be concluded.

In order to clearly illustrate the above strategy, an example of a system with three possible faulty models is given:

1. All $\|e_{w3}^i\|,\ i = 1, 2, \ldots, q$ are below the corresponding threshold ς_i during the experiment time, in which case it implies that all sensors are healthy;
2. There exist some finite time $t_i^1 > t_d$ and $t_i^3 > t_d$, such that $\|e_{w3}^1\| > \varsigma_1$ and $\|e_{w3}^3\| > \varsigma_3$ respectively, whereas others do not, in which case it implies that f_s^1 and f_s^3 do not equal to zero while others do;
3. All $\|e_{w3}^i\| > \varsigma_i,\ i = 1, 2, \ldots, q$ after some finite time, in which case it implies that all f_s^i do not equal to zero.

Based on this intuitive idea, the sensor fault isolation scheme can be summarized as follows:

Sensor fault isolation scheme: Multiple sensor faults can be isolated by comparing the residual $\|e_{w3}^i\|,\ (i = 1, 2, \ldots, q)$ with a predefined threshold ς_i, if $\|e_{w3}^i\|$ goes over the threshold for some finite time $t_i > t_d$, then it is concluded that $f_s^i \neq 0$. Otherwise if $\|e_{w3}^i\|$ is always below the threshold ς_i during the time studied, then $f_s^i = 0$. Considering the structure of D_0, the decision on which sensor is faulty can then be made.

Remark 3.6 The selection of the isolation threshold ς_i is similar to the selection of the detection threshold ς. Since the residual $\|e_{w3}^i\|$ obtained from the observer, which is designed to isolate f_s^i, is close to zero only if $f_s^i = 0$, the small value of ς_i can chosen.

The complete sensor FDI scheme is shown in Fig-3.1. The occurrence of fault can be declared if the residual $\|e_{w3}\|$ exceeds the pre-defined threshold ς at time instant t_d. After detecting a fault, its location can be isolated by designing a bank of SMOs. By comparing the residual $\|e_{w3}^i\|$ obtained from each SMO with the pre-defined threshold ς_i, f_s^1 and f_s^3 are isolated at time t_1 and t_3 respectively.

3.5 Simulation results

In this section, the effectiveness of the proposed scheme in detecting and isolating sensor faults has been demonstrated considering an example of a single-link robotic

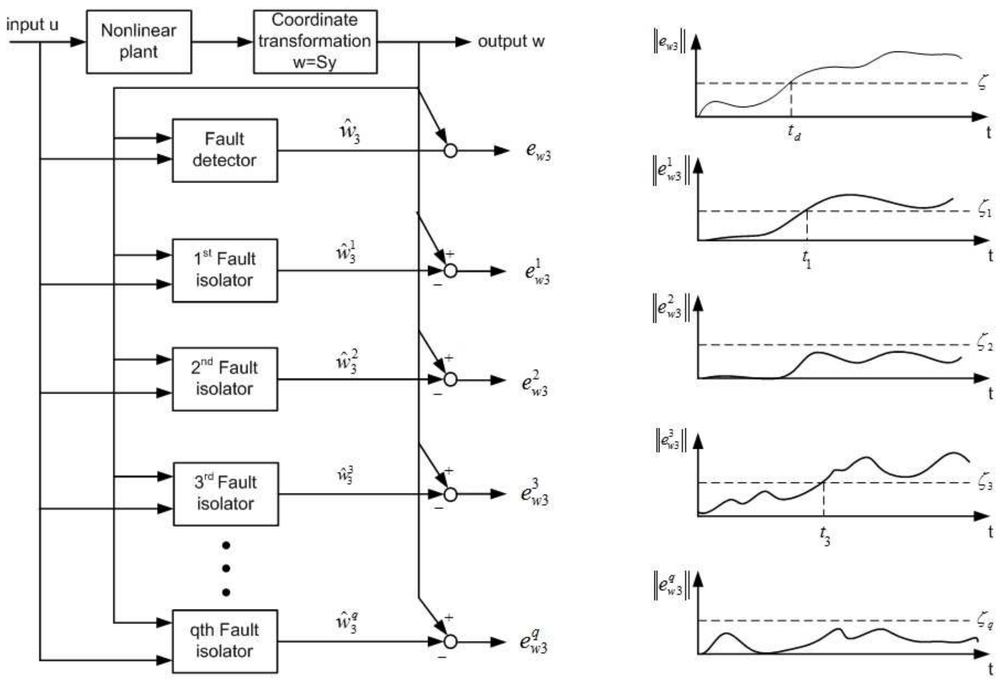

Figure 3.1: Sensor fault detection and isolation scheme

arm with a revolute elastic joint (see Fig-3.2). The dynamics is described by

$$\begin{aligned} J_l\ddot{q}_1 + F_l\dot{q}_1 + k(q_1 - q_2) + mglsinq_1 = 0 \\ J_m\ddot{q}_2 + F_m\dot{q}_2 - k(q_1 - q_2) = u \end{aligned} \tag{3.59}$$

where q_1 and q_2 denote the link position and the rotor position, respectively; u is the torque delivered by the motor; m is the link mass, l is the center of mass, J_m is the link inertia, J_l is the motor rotor inertia, F_m is the viscous friction coefficient, F_l is the viscous friction coefficient, k is the elastic constant and g is the gravity constant. In the simulation, the values of these parameters are chosen as: $m = 4$, $l = 0.5$, $J_m = 1$, $J_l = 2$, $F_m = 1$, $F_l = 0.5$, $k = 2$ and $g = 9.8$ (all in SI units).

Choosing $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$ and assuming that the link position, the link velocity and the rotor position can be measured, the dynamics (3.59) can be represented in state-space form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{J_l} & \frac{-F_l}{J_l} & \frac{k}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & \frac{-k}{J_m} & \frac{-F_m}{J_m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-mgl}{J_l} sinx_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix} (u + f_a)$$

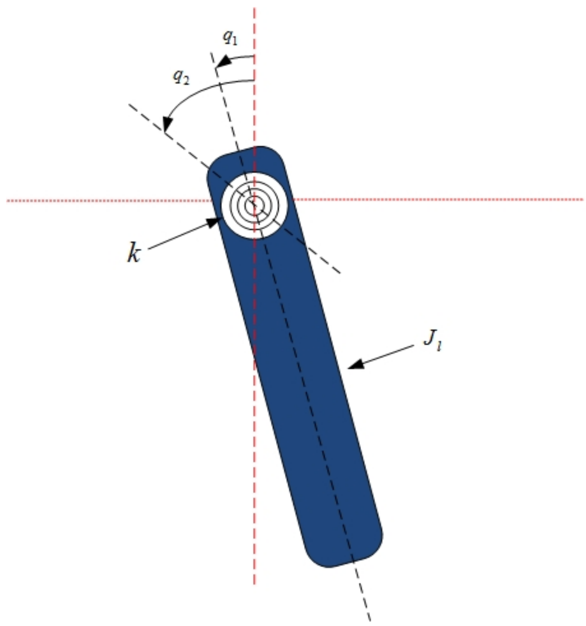

Figure 3.2: Single-link robotic arm with a revolute elastic joint, rotating in a vertical plane.

$$+\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\Delta\psi$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} f_s \tag{3.60}$$

Clearly, (3.60) is in the form of (3.1). The nonlinear term $f(x,t) = \frac{-mgl}{J_l} sinx_1$ has a Lipschitz constant of $\frac{mgl}{J_l}$. $\Delta\psi(t)$ denotes the system uncertainty and $f_s = [f_{s_1}^T, f_{s_2}^T]^T$ represents the sensor fault. These two external signals are included in the original system to testify the effectiveness of the proposed observers in detecting and isolating incipient sensor faults for a nonlinear system with uncertainties. In the simulation, the system uncertainty $\Delta\psi$ is assumed to be $-0.045\frac{mgl}{J_l} sin(t)$. It is easy to see that this uncertainty is bounded.

Two transformations $z = Tx$ and $w = Sy$ with

$$T = \begin{bmatrix} 1.0000 & 0 & -0.5000 & 0 \\ -1.0000 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$S = \begin{bmatrix} 1.0000 & -0.5000 & 0 \\ 0 & 1.0000 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

are introduced such that under the new coordinate, the system matrices become:

$$TAT^{-1} = \left[\begin{array}{c|ccc} 1 & 1 & 0.5 & -0.5 \\ \hline -2.25 & -1.25 & -0.125 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -1 & -1 \end{array}\right]$$

$$SCT^{-1} = \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$TB = \left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ 1 \end{array}\right], \quad TE = \left[\begin{array}{c} 1 \\ \hline 0 \\ 0 \\ 0 \end{array}\right], \quad SD = \left[\begin{array}{cc} 0 & 0 \\ \hline 2 & 0 \\ 0 & 1 \end{array}\right]$$

Imposing the stability constraint described in (3.25) to the transformed system and formulating the problem in an LMI framework gives the following solutions:

$$P_1 = 0.2930$$

$$A_1^s = -2.3063$$

$$P_{01} = \begin{bmatrix} 0.5620 & 0.0347 & 0.0251 \\ 0.0347 & 0.6723 & 0.3164 \\ -0.0251 & 0.3164 & 0.7064 \end{bmatrix}$$

$$P_{02} = \begin{bmatrix} 0.2254 & 0.0402 \\ 0.0402 & 0.2814 \end{bmatrix}$$

$$L_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 3.4774 & -0.4917 \\ -0.5056 & 2.7565 \end{bmatrix}$$
$$F_0 = \begin{bmatrix} 0.4508 & 0.0804 \\ 0.0402 & 0.2814 \end{bmatrix}$$

Case-1 In this case the sensor faults are given as:

$$\begin{aligned} f_{s_1} &= \begin{cases} 0 & , \quad t \leq 18s \\ 0.05\exp(0.01t) & , \quad t \geq 18s \end{cases} \\ f_{s_2} &= 0, \ \forall t \end{aligned}$$

The residual for fault detection is defined as the norm of the output estimation error e_{w3}, namely, $\|e_{w3}\|$. Lemma 3.4 states that the residual $\|e_{w3}\|$ will approach zero if there is no sensor fault. This result is shown in Fig-3.3 and the residual is very close to zero after about 10s. After the occurrence of any sensor faults, the residual will deviate from zero. Since the threshold used to detect the occurrence of a fault depends on its magnitude and the fault considered in this simulation is small in size, we choose the threshold to be 0.02 in the simulation. From Fig-3.3, a significant change of the residual can be observed at about 18s. The residual exceeds the threshold 0.02 at around $t = 18.25$s (the fault occurs at 18s), which implies that at least one of the sensors is faulty.

After the detection of a fault, the next stage is to determine which sensor amongst the various sensors is faulty. For this specific example, this is achieved by designing two isolation observers and determining whether or not $f_s^i = 0, \ i = 1, 2$. Results of the simulation are shown in Fig-3.4 and 3.5. The residual generated by the first isolation observer, which is designed for f_s^1 is compared with the threshold that is set to 0.02 in Fig-3.4. From the figure it is observed that the residual exceeds the threshold at about 18.23s, which implies that f_s^1 can be concluded to be nonzero after approximately 18.25s. Further, from Fig-3.5 it is seen that the residual generated by the second isolation observer always remains at zero, thus the possibility of $f_s^2 = 0$ can be excluded. It can therefore be concluded that the first and second sensors are faulty after some time instant, while the third sensor is healthy according to the fault distribution D.

Case-2 In this case the sensor faults are given as:

$$
\begin{aligned}
f_{s_1} &= \begin{cases} 0 & , \quad t \leq 18s \\ 0.05\exp(0.01t) & , \quad t \geq 18s \end{cases} \\
f_{s_2} &= \begin{cases} 0 & , \quad t \leq 25s \\ 0.07\exp(0.03t) & , \quad t \geq 25s \end{cases}
\end{aligned}
$$

The detectability of the proposed scheme is shown in Fig-3.6. Significant changes of the residual can be observed at about 18s and 25s. The residual firstly exceeds the threshold 0.02 at around $t = 18.25$s (the fault occurs at 18s), which implies that an incipient sensor has occurred to the system at around $t = 18.25$s.

The simulation result obtained by the isolation observer designed for f_s^1 is shown in Fig-3.7. It is observed from the figure that the residual exceeds the threshold 0.02 at about 18.25s, which implies that $f_s^1 \neq 0$ after about 18.25s. Fig-3.8 shows the simulation result when the isolation observer designed for f_s^2 is used. The residual obtained by the this observer exceeds the threshold (0.02) at approximately 25.17s, which denotes that f_s^2 can be found to be nonzero at about 25.17s. Thus, it can be concluded that sensor faults appear in the first and second sensors after about 18.25s and in the third sensor after about 25.17s.

3.6 Conclusions

A new residual-based scheme for robustly detecting and isolating incipient sensor faults for Lipschitz nonlinear systems is proposed in this chapter. The proposed FDI scheme essentially transforms the original system into two subsystems where subsystem-1 includes system uncertainties but is free from sensor faults and subsystem-2 has sensor faults but without uncertainties. By using the integral observer based approach, sensor faults in subsystem-2 are transformed into actuator faults and detected by designing a Luenberger observer for this subsystem. After being detected, multiple transformed sensor faults are then isolated based on the modified dedicated observer scheme using a bank of sliding mode observers. The sufficient condition of stability of the proposed FDI scheme has been studied and represented in the

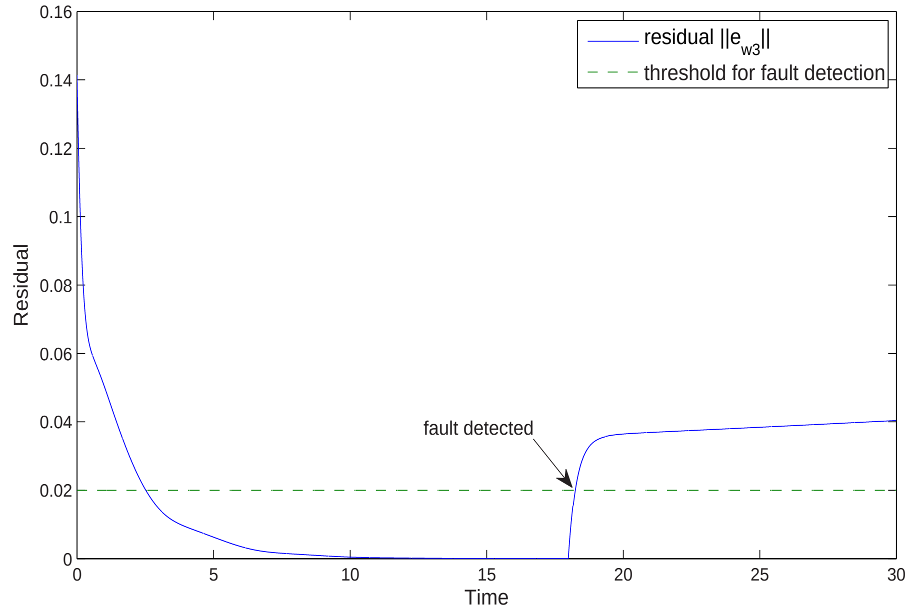

Figure 3.3: Detection of an incipient sensor fault using $||e_{w3}||$ as the residual. Threshold=0.02. An incipient sensor fault is introduced at 18.0s and is detected at about 18.25s

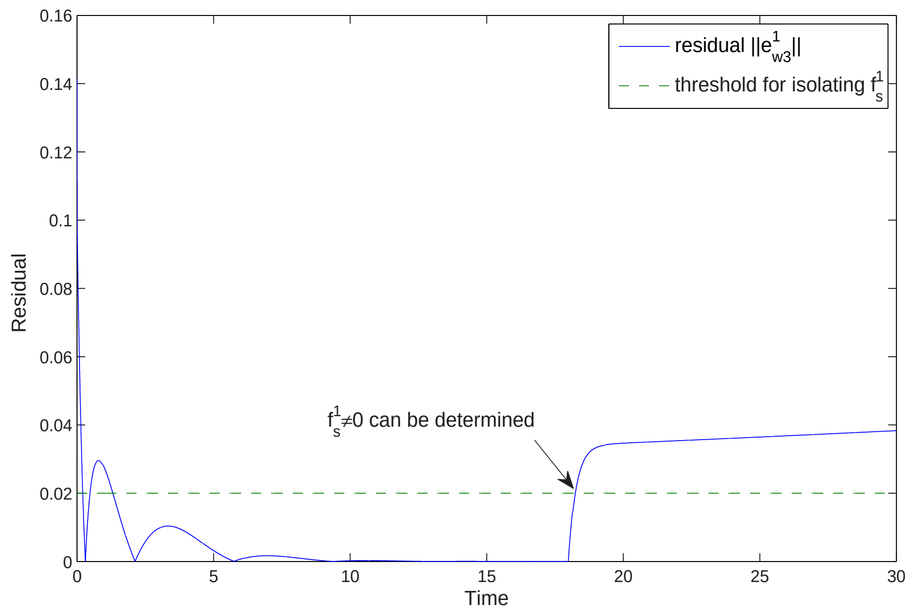

Figure 3.4: Residual generated by the first isolation observer which is designed for f_s^1. $||e_{w3}^1||$ is selected as the residual and the corresponding threshold is chosen to be 0.02. $||e_{w3}^1||$ exceeds the threshold at about 18.23s

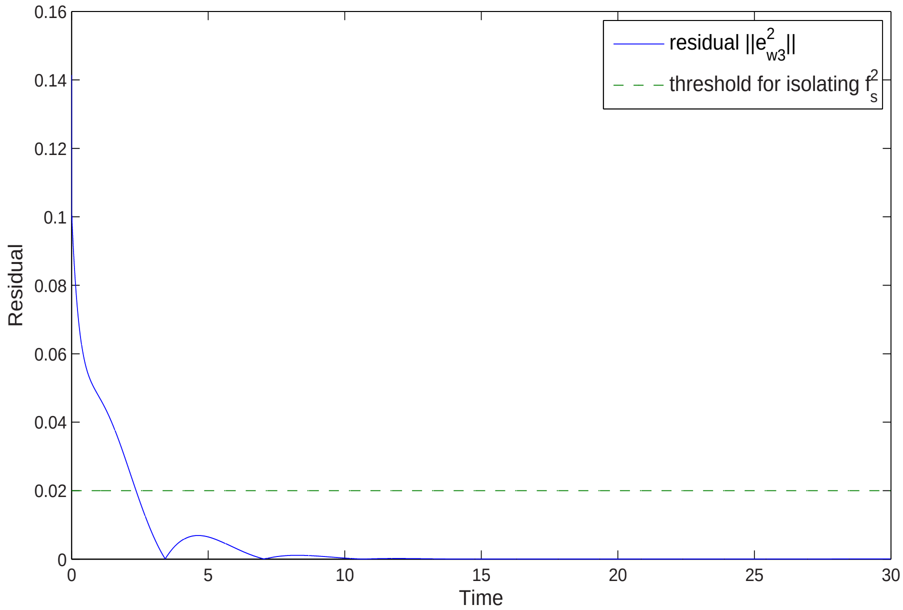

Figure 3.5: Residual generated by the second isolation observer which is designed for f_s^2. $\|e_{w3}^2\|$ is selected as the residual and the corresponding threshold is chosen to be 0.02. $\|e_{w3}^2\|$ remains under the threshold

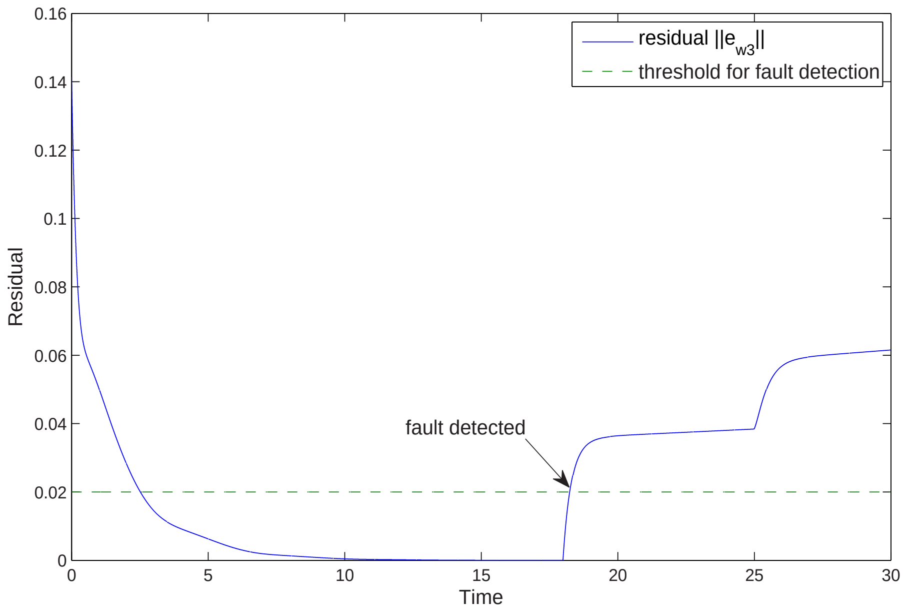

Figure 3.6: Detection of an incipient sensor fault using $\|e_{w3}\|$ as the residual. Threshold=0.02. An incipient sensor fault is introduced at 18.0s and is detected at about 18.25s

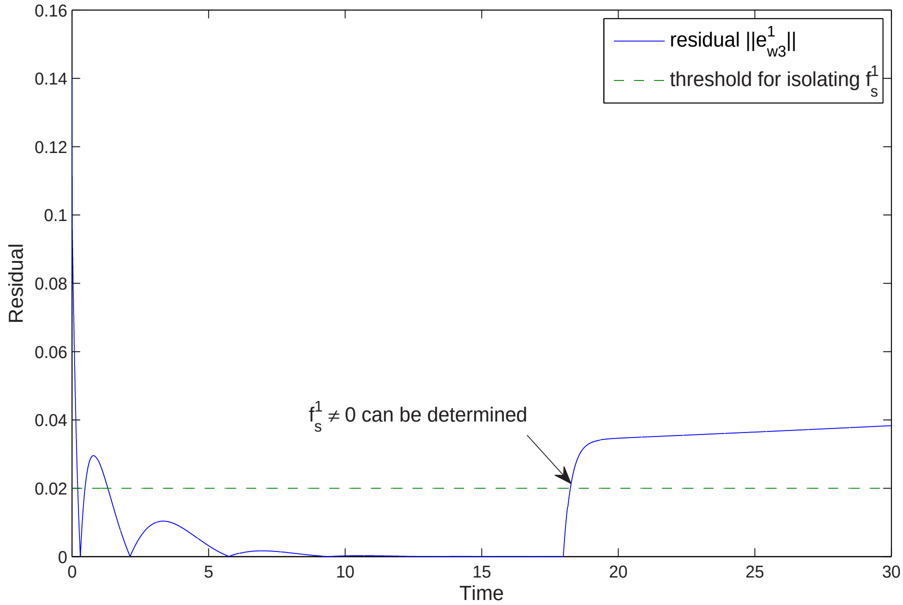

Figure 3.7: Residual generated by the first isolation observer which is designed for f_s^1. $\|e_{w3}^1\|$ is selected as the residual and the corresponding threshold is chosen to be 0.02. $\|e_{w3}^1\|$ exceeds the threshold at about 18.25s

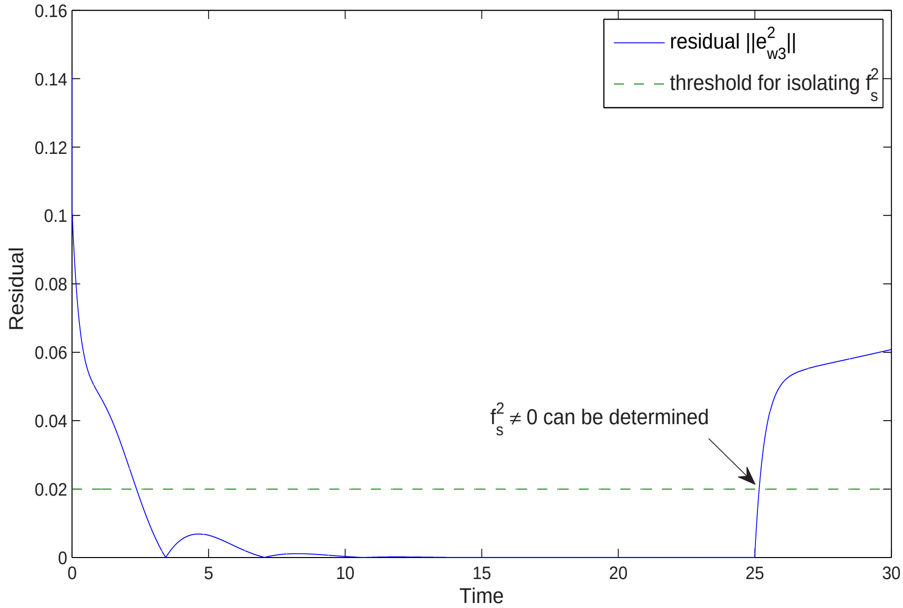

Figure 3.8: Residual generated by the second isolation observer which is designed for f_s^2. $\|e_{w3}^2\|$ is selected as the residual and the corresponding threshold is chosen to be 0.02. $\|e_{w3}^2\|$ exceeds the threshold at about 25.17s

form of LMI. Its effectiveness has been demonstrated considering the example of a single-link robotic arm with a revolute elastic joint. Simulation results confirm that the proposed method can effectively detect and isolate incipient sensor faults in the presence of system uncertainties.

Chapter 4

Estimation of sensor faults for uncertain nonlinear systems

The problem of incipient sensor FDI using residual-based observers has been discussed in Chapter 3. Specifically, we use a Luenberger observer to detect the occurrence of sensor faults and then design a bank of SMOs to exactly indicate which sensor is faulty. However, the developed FDI scheme cannot tell the amplitude, duration and shape of the fault, which are important to fault accommodation in the later process. Also, the use of multi-diagnostic observers will make the whole FDI system more complex. This problem can be addressed by fault estimation (FE) and will be studied in this chapter.

4.1 Introduction

FE is different from the majority of FDI methods in the sense that it not only detects and isolates the fault, but also provides details of the fault. Since FE directly gives an estimate of the location, size and duration of the fault, it is very useful for incipient faults and slow drifts, which are very difficult to detect. Also this approach is very useful for fault tolerant control systems. After estimating the sensor fault, the sensor compensation can be carried out by simply subtracting the estimated sensor fault from the measurement outputs. If the accuracy of the sensor fault estimation is satisfactory, then compensated measurement outputs will track the original measurements. Therefore the plant can still function normally without the need for controller reconfiguration after the occurrence of sensor faults.

Considerable research results have been reported on sensor fault estimation, see [97, 51, 98] and the references therein. In [97], an online estimation approach based on adaptive observer technique was adopted to reconstruct the sensor fault with an incipient time profile. A descriptor system approach was introduced to investigate sensor fault diagnosis for nonlinear systems in [51]. The proposed method is applicable for sensor faults of any forms. In [98], a sensor fault tolerant control scheme was presented for a crane system.

Among these approaches developed in the past, the SMO-based methods is the one that has been most extensively studied and considerable success has been achieved: [77] introduced two methods to estimate sensor faults for systems without uncertainties. In both methods, two SMOs were used in cascade. The second approach of [77] was later improved to achieve robust sensor fault estimation in [72] using an LMI formulation. The open loop stability required in [77] is no longer a necessary condition. The upper bound on the effect of the uncertainty on the reconstructed fault signals is minimized by using $\mathcal{H}_\infty$ concepts. However, for open loop unstable systems with certain classes of faults, the proposed method may not be applicable. This restriction was addressed in [79], where a new method for sensor fault reconstruction was proposed. The proposed observer designs are applicable for both open-loop stable and unstable systems. A high-order SMO was designed to reconstruct sensor faults in [73], where both disturbances and uncertainties were considered. For descriptor systems with actuator faults and sensor faults, the FE was carried out by using SMOs with feedforward signals in [99]. In [78], a nonlinear diffeomorphism was introduced to explore the system structure and a simple filter was presented to transform the sensor fault into a pseudo-actuator fault scenario. A SMO was designed to reconstruct the sensor fault precisely if the system does not experience any uncertainty, and to estimate the sensor fault when uncertainty exists.

In this chapter two sensor fault estimation schemes are developed. The proposed methods essentially transform the original system into two subsystems (subsystem 1 and 2), where subsystem-1 includes the effects of system uncertainties, but is free from sensor faults and subsystem-2 has sensor faults but without any uncertainties. Sensor faults in subsystem-2 are treated as actuator faults by using integral observer based approach. The effects of system uncertainties in subsystem-1 can be completely eliminated using a SMO. In the first scheme, the sensor faults present in subsystem-2 are reconstructed with arbitrary accuracy by a SMO. In the second scheme, the sensor faults are estimated by an adaptive observer (AO) instead of by a SMO. The most distinct feature of the proposed FDI schemes is that by imposing a coordinate transformation to the original system, the effects of system uncertainties are completely separated from subsystem-2. This makes the scheme only sensitive to faults while robust to modelling uncertainty. The method can not only reconstruct a time-varying sensor fault with relatively high amplitude, as well as can accurately estimate a sensor fault with trivial amplitude which is difficult to be detected using residual approaches.

The rest of the chapter is organized as follows: Section-4.2 briefly describes the mathematical preliminaries required for designing observers. Section-4.3 proposes a scheme using sliding mode observer to estimate sensor faults. The stability condition of the proposed observers based on Lyapunov approach is derived. In section-4.4, a scheme based on an adaptive estimator is proposed to estimate the fault. The results of simulation are shown in section-4.5 with conclusions in section-4.6.

4.2 Problem Formulation

Consider a nonlinear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Wf(x,t) + Bu(t) + E\Delta\psi(t) \\ y(t) &= Cx(t) + Df_s(t) \end{aligned} \tag{4.1}$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$ and $y \in \mathcal{R}^p$ denote respectively the state variables, inputs and outputs. $f_s \in \mathcal{R}^q$ is the sensor fault vector. $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{p\times n}$, $D \in \mathcal{R}^{p\times q}$, $E \in \mathcal{R}^{n\times r}$ ($p \geq q + r$) and $W \in \mathcal{R}^{n\times j}$ are known constant matrices with C and D both being of full rank. The nonlinear continuous term $f(x,t) \in \mathcal{R}^j$ is assumed to be known. The unknown nonlinear term $\Delta\psi(t)$ represents the uncertainty in system dynamics that includes parameter perturbations, external disturbances, unmodeled dynamics, etc.

Assumption 4.1 $rank(CE) = rank(E)$.

Lemma 4.1 Under Assumption 4.1, there exist state and output transformations

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{4.2}$$

such that in the new coordinate, the system matrices become:

$$\begin{aligned} TAT^{-1} &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad TE = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}, \\ SCT^{-1} &= \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix}, \quad SD = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}, \quad TW = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \end{aligned} \tag{4.3}$$

where $T \in \mathcal{R}^{n\times n}$, $S \in \mathcal{R}^{p\times p}$, $z_1 \in \mathcal{R}^r$, $w_1 \in \mathcal{R}^r$, $A_1 \in \mathcal{R}^{r\times r}$, $A_4 \in \mathcal{R}^{(n-r)\times(n-r)}$, $B_1 \in \mathcal{R}^{r\times m}$, $E_1 \in \mathcal{R}^{r\times r}$, $C_1 \in \mathcal{R}^{r\times r}$, $C_4 \in \mathcal{R}^{(p-r)\times(n-r)}$, $D_2 \in \mathcal{R}^{(p-r)\times q}$ and $W_1 \in \mathcal{R}^{r\times j}$. C_1 is invertible.

Proof. See Appendix-B.

Assumption 4.2 For every complex number s with nonnegative real part:

$$rank \begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix} = n + rank(E) \tag{4.4}$$

This assumption is known as the minimum phase condition.

Lemma 4.2 The pair (A_4, C_4) is detectable if and only if Assumption 4.2 holds.

Proof. See [94, 95].

Assumption 4.3 The nonlinear term $f(x,t)$ is assumed to be Lipschitz about x uniformly, i.e., $\forall x, \hat{x} \in \mathcal{X}$,

$$\|f(x,t) - f(\hat{x},t)\| \le \mathcal{L}_f \|x - \hat{x}\| \tag{4.5}$$

where $\mathcal{L}_f$ is the Lipschitz constant and assumed unknown in the present work.

Assumption 4.4 The function $\Delta\psi(x,t)$ representing the structured modeling uncertainty is unknown but bounded by $\|\Delta\psi(t)\| \le \xi$. Also the unknown sensor fault satisfies $\|f_s(t)\| \le \rho_s$ and its time derivative $\|\dot{f}_s(t)\| \le \rho_{ss}$.

After introducing the state and output transformations $T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$, $S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$ as that have been defined by (4.2), the original system is converted into two subsystems:

$$\begin{aligned} \dot{z}_1 &= A_1 z_1 + A_2 z_2 + f_1(T^{-1}z, t) + B_1 u + E_1 \Delta\psi \\ w_1 &= C_1 z_1 \end{aligned} \tag{4.6}$$

$$\begin{aligned} \dot{z}_2 &= A_3 z_1 + A_4 z_2 + f_2(T^{-1}z, t) + B_2 u \\ w_2 &= C_4 z_2 + D_2 f_s \end{aligned} \tag{4.7}$$

where $T_1 \in \mathcal{R}^{r\times n}$, $S_1 \in \mathcal{R}^{r\times p}$, $f_1(T^{-1}z, t) = W_1 f(T^{-1}z, t)$ and $f_2(T^{-1}z, t) = W_2 f(T^{-1}z, t)$.

For subsystem (4.7), define a new state $z_3 = \int_0^t w_2(\tau) d\tau$ so that $\dot{z}_3(t) = C_4 z_2 + D_2 f_s$, and the augmented system with the new state z_3 is given as:

$$\begin{aligned} \begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} &= \begin{bmatrix} A_4 & 0 \\ C_4 & 0 \end{bmatrix} \begin{bmatrix} z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} A_3 \\ 0 \end{bmatrix} z_1 + \begin{bmatrix} f_2(T^{-1}z, t) \\ 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ D_2 \end{bmatrix} f_s \\ w_3 &= z_3 \end{aligned} \tag{4.8}$$

The augmented system (4.8) can then be rewritten in a more compact form as:

$$\begin{aligned} \dot{z}_0 &= A_0 z_0 + \bar{A}_3 z_1 + \bar{W}_2 f(T^{-1}z, t) + B_0 u + D_0 f_s \\ w_3 &= C_0 z_0 \end{aligned} \tag{4.9}$$

where $z_0 \in \mathcal{R}^{n+p-2r}$, $w_3 \in \mathcal{R}^{p-r}$, $A_0 = \begin{bmatrix} A_4 & 0 \\ C_4 & 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times(n+p-2r)}$, $\bar{A}_3 = \begin{bmatrix} A_3 \\ 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times r}$, $B_0 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times m}$, $D_0 = \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times q}$, $C_0 = \begin{bmatrix} 0 & I_{p-r} \end{bmatrix} \in \mathcal{R}^{(p-r)\times(n+p-2r)}$, $\bar{W}_2 = \begin{bmatrix} W_2 \\ 0 \end{bmatrix}$.

Accordingly, subsystem (4.6) is rewritten as:

$$\begin{aligned} \dot{z}_1 &= A_1 z_1 + \bar{A}_2 z_0 + f_1(T^{-1}z, t) + B_1 u + E_1 \Delta\psi \\ w_1 &= C_1 z_1 \end{aligned} \tag{4.10}$$

where $\bar{A}_2 = \left[A_2 \; 0_{r\times(p-r)}\right]$.

Lemma 4.3 The pair (A_0, C_0) is observable if Assumption 4.2 holds.

Proof. See Lemma 3.3.

Assumption 4.5 There exist arbitrary matrices $F_0 \in \mathcal{R}^{q\times(p-r)}$ and $H_0 \in \mathcal{R}^{j\times(p-r)}$ such that:

$$D_0^T P_0 = F_0 C_0 \tag{4.11}$$

$$\bar{W}_2^T P_0 = H_0 C_0 \tag{4.12}$$

Remark 4.1 Assumption 4.1 ensures the existence of coordinate transformations T and S, such that in the new coordinate sensor faults can be completely decoupled from uncertainties. After the transformation, subsystem-1 which is formulated in (4.10) is free from sensor faults but subject to system uncertainties, and subsystem-2 which is formulated in (4.9) is prone to sensor faults but free from system uncertainties. The satisfaction of this assumption makes the accurate fault estimation (i.e., fault reconstruction) become possible, which is the most distinct feature of the proposed schemes. Assumption 4.2 implies that an asymptotic estimator can be designed for the system (4.9). In Assumption 4.4, the sensor fault is assumed to be non-zero and differentiable after its occurrence. This assumption is quite general either for constant faults or time-varying faults at limited rates [7, 100, 101]. The assumption of $\|f_s(t)\| \leq \rho_s$ is used in section-4.3 when designing SMOs, while the assumption $\|\dot{f}_s(t)\| \leq \rho_{ss}$ is used in section-4.4 when designing an AO. In Assumption 4.5, if an arbitrary matrix H_0 satisfying (4.12) can be found, then an adaptation law can be integrated into the estimator such that the situation that the Lipschitz constant is unknown or too large can be solved. This adds another important feature of the proposed schemes.

4.3 Sensor fault reconstruction using sliding mode observers with adaption laws

In this section, two sliding mode observers are designed for subsystem-1 (4.10) and subsystem-2 (4.9) respectively. One of the SMOs is being designed to eliminate the effects of system uncertainties on the state estimation while the other one is used to reconstruct the sensor fault. The sufficient condition of stability of the proposed scheme and the selection of design parameters of the observers will be discussed in this section.

For subsystem-1, the proposed sliding mode observer has the form:

$$\begin{aligned}\dot{\hat{z}}_1 &= A_1\hat{z}_1 + \bar{A}_2\hat{z}_0 + f_1(T^{-1}\hat{z},t) + B_1u + \frac{1}{2}\hat{k}_1P_1C_1^{-1}(S_1y_1 - \hat{w}_1) \\ &\quad + (A_1 - A_1^s)C_1^{-1}(w_1 - \hat{w}_1) + \nu_1 \qquad (4.13)\\ \hat{w}_1 &= C_1\hat{z}_1 \qquad (4.14)\end{aligned}$$

where $A_1^s \in \mathcal{R}^{r\times r}$ is a stable matrix which needs to be determined. For any $Q_1 > 0$, the Lyapunov equation $A_1^{sT}P_1 + P_1A_1^s = -Q_1$ has a unique solution $P_1 > 0$. The discontinuous output error injection term ν_1 that is being used to eliminate the effects of uncertainties is defined by

$$\nu_1 = \begin{cases}(\|E_1\|\xi + l_1)\frac{P_1(C_1^{-1}S_1y-\hat{z}_1)}{\|P_1(C_1^{-1}S_1y-\hat{z}_1)\|} & \text{if } C_1^{-1}S_1y - \hat{z}_1 \neq 0\\ 0 & \text{otherwise}\end{cases} \qquad (4.15)$$

where l_1 is a positive scalar to be determined later. It is worth noting that the state z_1 can be obtained by the measured output y as $z_1 = C_1^{-1}S_1y$.

$\hat{k}_1$ satisfies an adaption law as follows:

$$\dot{\hat{k}}_1 = l_{k_1}\|P_1(C_1^{-1}S_1y - \hat{z}_1)\|^2 \qquad (4.16)$$

where l_{k_1} is a positive constant.

For subsystem-2, the designed sliding mode observer has the form:

$$\begin{aligned}\dot{\hat{z}}_0 &= A_0\hat{z}_0 + \bar{A}_3C_1^{-1}w_1 + \bar{W}_2f(T^{-1}\hat{z},t) + B_0u + L_0(w_3 - \hat{w}_3) \\ &\quad + \frac{1}{2}\hat{k}_2\bar{W}_2H_0(w_3 - \hat{w}_3) + D_0\nu_2 \qquad (4.17)\\ \hat{w}_3 &= C_0\hat{z}_0 \qquad (4.18)\end{aligned}$$

where $L_0 = \begin{bmatrix} L_{01} \\ L_{02}\end{bmatrix} \in \mathcal{R}^{(n+p-2r)\times(p-r)}$ is a traditional Luenberger observer gain used to make $(A_0 - L_0C_0)$ stable. The sliding mode control law ν_2 which is being used to estimate the sensor fault is defined by

$$\nu_2 = \begin{cases} (\rho_s + l_2)\frac{F_0 e_{w_3}}{\|F_0 e_{w_3}\|} & \text{if } e_{w_3} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.19}$$

where $e_{w_3} = w_3 - \hat{w}_3$ and l_2 is a positive scalar to be determined later.

$\hat{k}_2$ also satisfies an adaption law as:

$$\dot{\hat{k}}_2 = l_{k_2}\|H_0 e_{w_3}\|^2 \tag{4.20}$$

where l_{k_2} is a positive constant.

If the state estimation errors are defined as $e_1 = z_1 - \hat{z}_1 = C_1^{-1}S_1 y - \hat{z}_1$ and $e_0 = z_0 - \hat{z}_0$, then the state estimation error dynamics after the occurrence of sensor faults can be obtained from:

$$\begin{aligned} \dot{e}_1 = {} & A_1^s e_1 + \bar{A}_2 e_0 + \big(f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\big) - \frac{1}{2}\hat{k}_1 P_1 e_1 \\ & + E_1\Delta\psi - \nu_1 \end{aligned} \tag{4.21}$$

$$\begin{aligned} \dot{e}_0 = {} & (A_0 - L_0 C_0)e_0 + \bar{W}_2(f(T^{-1}z,t) - f(T^{-1}\hat{z},t)) - \frac{1}{2}\hat{k}_2\bar{W}_2 H_0 C_0 e_0 \\ & + D_0(f_s - \nu_2) \end{aligned} \tag{4.22}$$

Note that $\hat{z} := [(C_1^{-1}S_1 y)^T, (\hat{z}_0)^T]^T$, it is easy to see that

$$\begin{aligned} \|f(T^{-1}z,t) - f(T^{-1}\hat{z},t)\| &\leq \mathcal{L}_f\|T^{-1}\|\|e_0\| \\ \|f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\| &\leq \mathcal{L}_{f_1}\|T^{-1}\|\|e_0\| \\ \|f_2(T^{-1}z,t) - f_2(T^{-1}\hat{z},t)\| &\leq \mathcal{L}_{f_2}\|T^{-1}\|\|e_0\| \end{aligned} \tag{4.23}$$

where $\mathcal{L}_{f_1} = \|W_1\|\mathcal{L}_f$ and $\mathcal{L}_{f_2} = \|W_2\|\mathcal{L}_f$.

With proper choice of positive constants l_1 and l_2, the error dynamics (4.21)-(4.22) can be driven to the sliding surface defined as:

$$\mathcal{S}_1 = \{(e_1, C_0 e_0) | e_1 = 0, C_0 e_0 = 0\} \tag{4.24}$$

The condition of stability and reachability of the state estimation error dynamics (4.21) and (4.22) associated with the sliding motion (4.24) will be studied next.

Proposition 4.1 *Under the Assumptions 4.1-4.5, the error dynamical system (4.21) and (4.22) are asymptotically stable if there exist matrices $A_1^s < 0$, L_0, $P_1 > 0$, $P_0 > 0$ such that:*

$$\begin{bmatrix} -Q_1 & P_1\bar{A}_2 \\ \bar{A}_2^T P_1 & -Q_0 + 2I \end{bmatrix} < 0 \tag{4.25}$$

where $-Q_1 = A_1^{sT} P_1 + P_1 A_1^s$, $-Q_0 = (A_0 - L_0 C_0)^T P_0 + P_0(A_0 - L_0 C_0)$.

Proof. Consider the Lyapunov function as:

$$V = V_1(e_1) + V_2(e_0) + V_3(e_{k_1}) + V_4(e_{k_2}) \tag{4.26}$$

where $V_1(e_1) = e_1^T P_1 e_1$, $V_2(e_0) = e_0^T P_0 e_0$, $V_3(e_{k_1}) = l_{k_1}^{-1} e_{k_1}^2/2$, $V_4(e_{k_2}) = l_{k_2}^{-1} e_{k_2}^2/2$, $e_{k_1} = k_1 - \hat{k}_1$ and $e_{k_2} = k_2 - \hat{k}_2$. k_1 and k_2 are two constants which can be determined by (4.34). The time derivative of V_1 and V_2 along the trajectories of system (4.21)-(4.22) can be shown to be:

$$\begin{aligned} \dot{V}_1 &= e_1^T (A_1^{sT} P_1 + P_1 A_1^s) e_1 + 2e_1^T P_1 \bar{A}_2 e_0 + 2e_1^T P_1 E_1 \Delta\psi \\ &\quad + 2e_1^T P_1 \left(f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t) \right) - 2e_1^T P_1 \nu_1 - \hat{k}_1 \|P_1 e_1\|^2 \end{aligned}$$

For any scalar $\alpha > 0$, since the inequality $2X^T Y \leq \frac{1}{\alpha} X^T X + \alpha Y^T Y$ holds ([46]), then

$$\begin{aligned} \dot{V}_1 &\leq e_1^T (A_1^{sT} P_1 + P_1 A_1^s) e_1 + 2e_1^T P_1 \bar{A}_2 e_0 + 2e_1^T P_1 E_1 \Delta\psi \\ &\quad + \frac{1}{\alpha_1} e_1^T P_1 P_1^T e_1 + \alpha_1 \left(f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t) \right)^T \\ &\quad \cdot \left(f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t) \right) - 2e_1^T P_1 \nu_1 - \hat{k}_1 \|P_1 e_1\|^2 \\ &\leq -e_1^T Q_1 e_1 + 2e_1^T P_1 \bar{A}_2 e_0 + \frac{1}{\alpha_1} \|P_1 e_1\|^2 + \alpha_1 \mathcal{L}_{f_1}^2 \|T^{-1}\|^2 \|e_0\|^2 \\ &\quad - \hat{k}_1 \|P_1 e_1\|^2 + 2e_1^T P_1 E_1 \Delta\psi - 2e_1^T P_1 \nu_1 \end{aligned} \tag{4.27}$$

Using (4.15), the last two terms of (4.27) can be expressed as :

$$2e_1^T P_1 E_1 \Delta\psi - 2e_1^T P_1 \nu_1 = -2l_1 \|P_1 e_1\| \leq 0 \tag{4.28}$$

Letting $\alpha_1 = 1/\mathcal{L}_{f_1}^2 \|T^{-1}\|^2$ and substituting (4.28) into (4.27) yields

$$\dot{V}_1 \leq -e_1^T Q_1 e_1 + 2e_1^T P_1 \bar{A}_2 e_0 + \|e_0\|^2 + (\mathcal{L}_{f_1}^2 \|T^{-1}\|^2 - \hat{k}_1) \|P_1 e_1\|^2 \tag{4.29}$$

Similarly, the derivative of V_2 can be obtained as:

$$\begin{aligned}\dot{V}_2 &\leq -e_0^T Q_0 e_0 + 2e_0 P_0 \bar{W}_2 (f(T^{-1}z,t) - f(T^{-1}\hat{z},t)) \\ &\quad - \hat{k}_2 \|H_0 C_0 e_0\|^2 + 2e_0^T P_0 D_0 (f_s - \nu_2) \\ &\leq -e_0^T Q_0 e_0 + \frac{1}{\alpha_0}\|H_0 C_0 e_0\|^2 + \alpha_0 \mathcal{L}_{f_2}^2 \|T^{-1}\|^2 \|e_0\|^2 \\ &\quad - \hat{k}_2 \|H_0 C_0 e_0\|^2 + 2e_0^T P_0 D_0 (f_s - \nu_2) \end{aligned} \tag{4.30}$$

From (4.19), it is easy to see

$$e_0^T P_0 D_0 (f_s - \nu_2) \leq 0$$

Let $\alpha_0 = 1/\mathcal{L}_{f_2}^2 \|T^{-1}\|^2$, it follows that

$$\dot{V}_2 \leq -e_0^T Q_0 e_0 + (\mathcal{L}_{f_2}^2 \|T^{-1}\|^2 - \hat{k}_2)\|H_0 C_0 e_0\|^2 + \|e_0\|^2 \tag{4.31}$$

Moreover, the time derivatives of V_3 and V_4 can be computed as:

$$\dot{V}_3 = l_{k_1}^{-1} e_{k_1} \dot{e}_{k_1} = -e_{k_1} \|P_1 e_1\|^2 \tag{4.32}$$

$$\dot{V}_4 = l_{k_2}^{-1} e_{k_2} \dot{e}_{k_2} = -e_{k_2} \|H_0 C_0 e_0\|^2 \tag{4.33}$$

Setting

$$k_1 = \mathcal{L}_{f_1}^2 \|T^{-1}\|^2 \text{ and } k_2 = \mathcal{L}_{f_2}^2 \|T^{-1}\|^2 \tag{4.34}$$

From (4.29), (4.31), (4.32) and (4.33), the time derivative of V can then be obtained from:

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 \\ &\leq -e_1^T Q_1 e_1 + 2e_1^T P_1 \bar{A}_2 e_0 - e_0^T Q_0 e_0 + 2\|e_0\|^2 \\ &= \begin{bmatrix} e_1 \\ e_0 \end{bmatrix}^T \begin{bmatrix} -Q_1 & P_1 \bar{A}_2 \\ \bar{A}_2^T P_1 & -Q_0 + 2I \end{bmatrix} \begin{bmatrix} e_1 \\ e_0 \end{bmatrix} \end{aligned} \tag{4.35}$$

It follows from (4.25) that $\dot{V} < 0$, which implies that the observer error dynamics (4.21) and (4.22) is asymptotically stable.

This completes the proof.

Remark 4.2 The problem of finding matrices $P_1 = P_1^T > 0$, $P_0 = P_0^T > 0$, L, $A_1^s < 0$ to satisfy both the inequality (4.25) and the matching condition (4.11) can be transformed into the following LMI optimization problem:

Minimize $\gamma_1 + \gamma_2$ subject to $P_1 > 0$, $P_0 > 0$, $X < 0$ and

$$\begin{bmatrix} X + X^T & P_1\bar{A}_2 \\ \bar{A}_2^T P_1 & P_0A_0 + A_0^T P_0 - Y_0C_0 - C_0^T Y_0^T + 2I \end{bmatrix} < 0,$$
$$\begin{bmatrix} \gamma_1 I & D_0^T P_0 - F_0C_0 \\ (D_0^T P_0 - F_0C_0)^T & \gamma_1 I \end{bmatrix} > 0,$$
$$\begin{bmatrix} \gamma_2 I & \bar{W}_2^T P_0 - H_0C_0 \\ (\bar{W}_2^T P_0 - H_0C_0)^T & \gamma_2 I \end{bmatrix} > 0 \tag{4.36}$$

where $X = P_1A_1^s$ and $Y_0 = P_0L_0$.

Remark 4.3 In practice, the Lipschitz constant in (4.5) is difficult to be obtained precisely. And if this parameter is too large, the LMIs which represent the stability of the observer may not be feasible. In the present work, the Lipschitz constant is assumed to be unknown and an adaptive sliding mode observer is proposed to deal with this situation. Specifically, the Lipschitz constants $\mathcal{L}_{f_1}$ and $\mathcal{L}_{f_2}$ are injected into the constants k_1 and k_2 which can be adjusted by the adaptation law (4.16) and (4.20). The asymptotical estimation of states can be guaranteed even if the estimate of k_1 and k_2 do not approach to their real values. It is worth noting that Lipschitz constant terms are not appearing in (4.36), which makes the above LMI optimization problem more solvable.

Proposition 4.1 implies that the error dynamical system (4.21)-(4.22) associated with the sliding surface (4.24) is asymptotically stable. The objective now is to choose the constants l_1 and l_2 such that the error dynamics can be driven to the sliding surface and a sliding motion can be maintained thereafter. The following conclusion is presented.

Proposition 4.2 *Under the Assumptions 4.1-4.5, the error dynamics (4.21)-(4.22) is driven to the sliding surface given by (4.24) in finite time if the gain l_1 and l_2 satisfy*

$$l_1 \geq \left(\|\bar{A}_2\| + \mathcal{L}_{f_1}\|T^{-1}\|\right)\|e_0\| + \eta_1 \tag{4.37}$$

$$l_2 \geq \frac{\mathcal{L}_{f_2}\|T^{-1}\|\|e_0\|}{\|D_0\|} + \eta_2 \tag{4.38}$$

where η_1 and η_2 are two positive scalars.

Proof. Consider the Lyapunov function as

$$V(t) = V_1(t) + V_2(t) \tag{4.39}$$

where $V_1 = e_1^T P_1 e_1$ and $V_2 = e_0^T P_0 e_0$.

Then

$$\begin{aligned}\dot{V}_1 \leq\ & e_1^T(P_1 A_1^s + A_1^{sT} P_1)e_1 + 2e_1^T P_1(\bar{A}_2 e_0 + f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t) \\ & + E_1 \Delta\psi - \nu_1)\end{aligned} \tag{4.40}$$

Since by design, A_1^s is a stable matrix which implies that $P_1 A_1^s + A_1^{sT} P_1 < 0$, therefore

$$\begin{aligned}\dot{V}_1 &\leq 2e_1^T P_1(\bar{A}_2 e_0 + f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t) + E_1 \Delta\psi - \nu_1) \\ &\leq 2\|P_1 e_1\| \left(\|\bar{A}_2\|\|e_0\| + \mathcal{L}_{f_1}\|T^{-1}\|\|e_0\| - l_1\right)\end{aligned}$$

It follows from (4.37) that

$$\dot{V}_1 \leq -2\eta_1\|P_1 e_1\| \leq -2\eta_1\sqrt{\lambda_{min}(P_1)}V_1^{\frac{1}{2}} \tag{4.41}$$

Similarly, if (4.38) is satisfied, then we have

$$\dot{V}_2 \leq -2\eta_2\|P_0 e_0\| \leq -2\eta_2\sqrt{\lambda_{min}(P_0)}V_2^{\frac{1}{2}} \tag{4.42}$$

This shows that the reachability condition ([61]) is satisfied and a sliding motion is achieved and maintained after some finite time $t_s > 0$.

This completes the proof.

After reaching the sliding surface, the sliding motion will be maintained even after the occurrence of any fault, i.e., $C_0 e_0 = 0$ and $C_0 \dot{e}_0 = 0$. Therefore it is clear to see from (4.22) that

$$0 = C_0 A_0 e_0 + C_0\left(F(z_0, t) - F(\hat{z}_0, t)\right) + D_2(f_s - \nu_{2_{eq}}) \tag{4.43}$$

where $\nu_{2_{eq}}$ is the equivalent output error injection signal representing the average behavior of the discontinuous function ν_2.

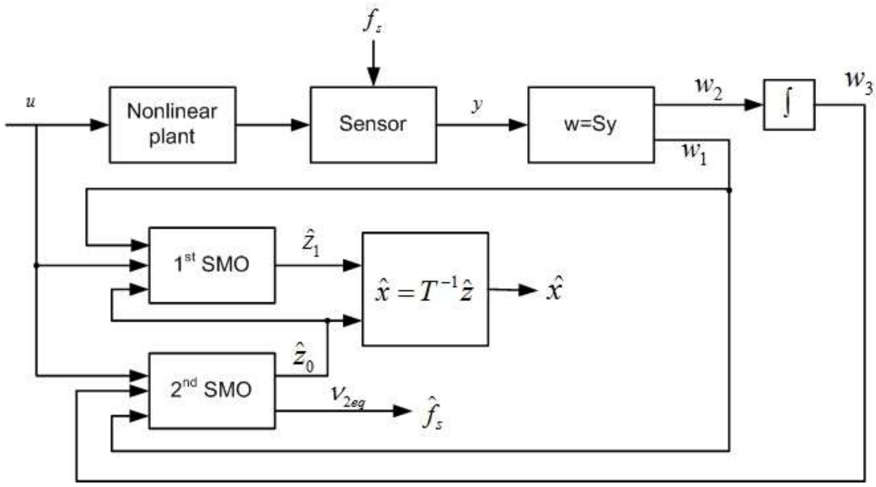

Figure 4.1: Schematic of the sensor fault estimation using (4.46)

According to Proposition 1, $\lim_{t\to\infty} e_0 = 0$ and also $\lim_{t\to\infty}(F(z_0,t) - F(\hat{z}_0,t)) = 0$. This implies (from (4.43)) that

$$f_s \to \nu_{2_{eq}} \quad \text{as } t \to \infty \tag{4.44}$$

The equivalent output error injection signal $\nu_{2_{eq}}$ can be approximated as:

$$\nu_{2_{eq}} = (l_2 + \rho_s)\frac{F_0 e_{w_3}}{\|F_0 e_{w_3}\| + \delta} \tag{4.45}$$

where δ is a small positive scalar to reduce the chattering effect. It can be shown that $\nu_{2_{eq}}$ can be approximated to any degree of accuracy by (4.45) ([61]). Therefore the sensor fault can be approximated as

$$\hat{f}_s \approx (l_2 + \rho_s)\frac{F_0 e_{w_3}}{\|F_0 e_{w_3}\| + \delta} \tag{4.46}$$

The complete sensor fault estimation scheme is shown in Fig-4.1.

4.4 Sensor fault estimation using adaptive observer

In section-4.3, a sensor fault estimation scheme using two sliding observers is proposed. One of which is used to eliminate the effect of the system uncertainty on the state estimation error, while the other one is being used to estimate the sensor fault. An adaption law is integrated in both sliding observers to tackle the situation when the Lipschitz constant is unknown. In this section, an alternative method which is based on using an adaptive observer (AO) technique [101] is proposed. More specifically, the proposed scheme combines one sliding mode observer and one adaptive observer, in which the sliding mode observer is used for subsystem-1 to remove the effect of the system uncertainty on the state estimation and the adaptive observer is used to directly estimate the sensor fault.

For subsystem-1, the sliding mode observer has the similar form as that being used in Section-4.3, i.e.,

$$\begin{aligned}\dot{\hat{z}}_1 &= A_1\hat{z}_1 + \bar{A}_2\hat{z}_0 + f_1(T^{-1}\hat{z}, t) + B_1 u + \frac{1}{2}n_1 P_1 C_1^{-1}(w_1 - \hat{w}_1) \\ &\quad + (A_1 - A_1^s)C_1^{-1}(w_1 - \hat{w}_1) + \nu \end{aligned} \tag{4.47}$$

$$\hat{w}_1 = C_1\hat{z}_1 \tag{4.48}$$

where $A_1^s \in \mathcal{R}^{r\times r}$ is a stable matrix. n_1 is a positive constant which satisfies $n_1 \geq \mathcal{L}_{f_1}^2\|T^{-1}\|^2$. The discontinuous output error injection term ν which is used to eliminate the effects of uncertainties is defined by

$$\nu = \begin{cases} l(.)\frac{C_1^{-1}S_1 y - \hat{z}_1}{\|C_1^{-1}S_1 y - \hat{z}_1\|} & \text{if } e_1 \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.49}$$

where $l(.)$ is a positive scalar gain to be determined.

The sliding surface is redefined as:

$$\mathcal{S}_2 = \{e_1 | e_1 = 0\} \tag{4.50}$$

For subsystem-2, the proposed adaptive observer has the form:

$$\begin{aligned}\dot{\hat{z}}_0 &= A_0\hat{z}_0 + \bar{A}_3 C_1^{-1} w_1 + \bar{W}_2 f(T^{-1}\hat{z}, t) + B_0 u + L_0(w_3 - \hat{w}_3) \\ &\quad + \frac{1}{2}n_2\bar{W}_2 H_0(w_3 - \hat{w}_3) + D_0\hat{f}_s \end{aligned} \tag{4.51}$$

$$\hat{w}_3 = C_0\hat{z}_0 \tag{4.52}$$

where n_2 is a positive constant which satisfies $n_2 \geq \mathcal{L}_{f_2}^2\|T^{-1}\|^2$ and $\hat{f}_s$ is the sensor fault estimation whose dynamics is defined as:

$$\dot{\hat{f}}_s = \Gamma F_0 e_{w_3} - \sigma\Gamma\hat{f}_s \tag{4.53}$$

where $\Gamma \in \mathcal{R}^{q\times q}$ is a symmetric positive definite matrix representing the learning rate and σ is a positive scalar.

Denote $e_f = f_s - \hat{f}_s$, then the state estimation error dynamics after the occurrence of faults can be obtained from:

$$\dot{e}_1 = A_1^s e_1 + \bar{A}_2 e_0 + \big(f_1(T^{-1}z,t) - f_1(T^{-1}\hat{z},t)\big) - \frac{1}{2}n_1 P_1 e_1 \\ + E_1\Delta\psi - \nu \tag{4.54}$$

$$\dot{e}_0 = (A_0 - L_0C_0)e_0 + \bar{W}_2\big(f(T^{-1}z,t) - f(T^{-1}\hat{z},t)\big) \\ - \frac{1}{2}n_2\bar{W}_2H_0C_0e_0 + D_0e_f \tag{4.55}$$

Define

$$\begin{aligned}
\Omega_1 &= \Big\{(e_{w_1}, e_{w_3}, \hat{f}_s) | \frac{\lambda_{min}(P_1)}{\|C_1\|^2}\|e_{w_1}\|^2 + \frac{\lambda_{min}(P_0)}{\|C_0\|^2}\|e_{w_3}\|^2 + \frac{\lambda_{min}(\Gamma^{-1})}{2}\|\hat{f}_s\|^2 \\
&\quad \leq \lambda_{min}(\Gamma^{-1})\rho_s^2 + \frac{\mu_4}{\mu_7}\Big\} \\
\Omega_2 &= \Big\{(e_{w_1}, e_{w_3}, \hat{f}_s) | \frac{\lambda_{min}(P_1)}{\|C_1\|^2}\|e_{w_1}\|^2 + \frac{\lambda_{min}(P_0)}{\|C_0\|^2}\|e_{w_3}\|^2 + \frac{\lambda_{min}(\Gamma^{-1})}{2}\|\hat{f}_s\|^2 \\
&\quad > \lambda_{min}(\Gamma^{-1})\rho_s^2 + \frac{\mu_4}{\mu_7}\Big\} \\
\mu_1 &= \lambda_{min}(-A_1^{sT}P_1 - P_1A_1^s) \\
\mu_2 &= \lambda_{min}(-(A_0 - L_0C_0)^TP_0 - P_0(A_0 - L_0C_0) - 2I) > 0 \\
\mu_3 &= \lambda_{min}(\sigma I - G) > 0 \\
\mu_4 &= \rho_{ss}^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}) + \sigma\rho_s^2 \\
\mu_5 &= min(\mu_1, \mu_2, \mu_3) \\
\mu_6 &= max(\lambda_{max}(P_1), \lambda_{max}(P_0), \lambda_{max}(\Gamma^{-1})) \\
\mu_7 &= \mu_5/\mu_6
\end{aligned}$$

where $G \in \mathcal{R}^{q\times q}$ is a symmetric positive definite matrix.

Proposition 4.3 *Under the Assumptions 4.1-4.5, if there exist matrices $A_1^s < 0$, L_0, $P_1 > 0$, $P_0 > 0$ such that:*

$$\begin{bmatrix} -Q_1 & P_1\bar{A}_2 \\ \bar{A}_2^TP_1 & -Q_0 + 2I \end{bmatrix} < 0 \tag{4.56}$$

where $-Q_1 = A_1^{sT} P_1 + P_1 A_1^s$, $-Q_0 = (A_0 - L_0 C_0)^T P_0 + P_0 (A_0 - L_0 C_0)$. *Then for a given matrix* Γ *and a positive scalar* σ, *the error dynamics (4.54)-(4.55) are uniformly bounded and* $(e_{w_1}, e_{w_3}, \hat{f}_s)$ *converges to* Ω_1 *at a rate greater than* $e^{-\mu_7 t}$.

Proof. Consider the Lyapunov function as:

$$V = e_1^T P_1 e_1 + e_0^T P_0 e_0 + e_f^T \Gamma^{-1} e_f \tag{4.57}$$

Its time derivative along the trajectories of system (4.55) and (4.53) can be shown to be:

$$\begin{aligned}
\dot{V} \leq & -e_1^T Q_1 e_1 + 2 e_1^T P_1 \bar{A}_2 e_0 + \|e_0\|^2 + (\mathcal{L}_{f_1}^2 \|T^{-1}\|^2 - n_1) \|P_1 e_1\|^2 \\
& - e_0^T Q_0 e_0 + \|e_0\|^2 + (\mathcal{L}_{f_2}^2 \|T^{-1}\|^2 - n_2) \|H_0 C_0 e_0\|^2 \\
& + 2 e_f^T \Gamma^{-1} \dot{f}_s + 2\sigma e_f^T f_s - 2\sigma e_f^T e_f \\
\leq & -e_1^T Q_1 e_1 + e_0^T (-Q_0 + 2I) e_0 + 2 e_1^T P_1 \bar{A}_2 e_0 + 2 e_f^T \Gamma^{-1} \dot{f}_s \\
& + 2\sigma e_f^T f_s - 2\sigma e_f^T e_f
\end{aligned} \tag{4.58}$$

Since $2X^T Y \leq \frac{1}{\alpha} X^T G X + \alpha Y^T G^{-1} Y$ holds for any scalar $\alpha > 0$ and symmetric positive definite matrix G [102], therefore

$$\begin{aligned}
2 e_f^T \Gamma^{-1} \dot{f}_s &\leq e_f^T G e_f + \dot{f}_s^T \Gamma^{-1} G^{-1} \Gamma^{-1} \dot{f}_s \\
&\leq e_f^T G e_f + \rho_{ss} \lambda_{max} (\Gamma^{-1} G^{-1} \Gamma^{-1})
\end{aligned} \tag{4.59}$$

Moreover,

$$2\sigma e_f^T f_s \leq \sigma \|e_f\|^2 + \sigma \rho_s^2 \tag{4.60}$$

Substituting (4.59) and (4.60) into (4.58) gives

$$\begin{aligned}
\dot{V} \leq & -e_1^T Q_1 e_1 - e_0^T (Q_0 - 2I) e_0 + e_f^T (G - \sigma I) e_f \\
& + \rho_{ss} \lambda_{max} (\Gamma^{-1} G^{-1} \Gamma^{-1}) + \sigma \rho_s^2 \\
\leq & -\mu_1 \|e_1\|^2 - \mu_2 \|e_0\|^2 - \mu_3 \|e_f\|^2 + \mu_4 \\
\leq & -\mu_5 (\|e_1\|^2 + \|e_0\|^2 + \|e_f\|^2) + \mu_4
\end{aligned} \tag{4.61}$$

Further from (4.57)

$$V \leq \lambda_{max}(P_1) \|e_1\|^2 + \lambda_{max}(P_0) \|e_0\|^2 + \lambda_{max}(\Gamma^{-1}) \|e_f\|^2$$

$$\leq max(\lambda_{max}(P_1), \lambda_{max}(P_0), \lambda_{max}(\Gamma^{-1}))(\|e_1\|^2 + \|e_0\|^2 + \|e_f\|^2)$$
$$= \mu_6(\|e_1\|^2 + \|e_0\|^2 + \|e_f\|^2) \tag{4.62}$$

Then

$$\dot{V} \leq -\mu_7 V + \mu_4 \tag{4.63}$$

On the other hand

$$V \geq \lambda_{min}(P_1)\|e_1\|^2 + \lambda_{min}(P_0)\|e_0\|^2 + \lambda_{min}(\Gamma^{-1})\|e_f\|^2$$
$$\geq \frac{\lambda_{min}(P_1)}{\|C_1\|^2}\|e_{w_1}\|^2 + \frac{\lambda_{min}(P_0)}{\|C_0\|^2}\|e_{w_3}\|^2 + \lambda_{min}(\Gamma^{-1})(\frac{\|\hat{f}_s\|^2}{2} - \rho_s^2) \tag{4.64}$$

If $(e_{w_1}, e_{w_3}, \hat{f}_s) \in \Omega_2$, then $V > \mu_4/\mu_7$ and consequently $\dot{V} < 0$. Therefore it can be concluded that $(e_{w_1}, e_{w_3}, \hat{f}_s)$ is uniformly bounded and converges to Ω_1 exponentially at a rate greater than $e^{-\mu_7 t}$.

This completes the proof.

Proposition 4.4 *Under the Assumptions 4.1-4.5, the error dynamics (4.54) is driven to the sliding surface given by (4.50) in finite time if the gain $l(.)$ satisfies*

$$l(.) \geq (\|\bar{A}_2\| + \mathcal{L}_{f_1}\|T^{-1}\|)\|e_0\| + \|E_1\|\xi + \eta_3 \tag{4.65}$$

where η_3 is a positive scalar.

Proof. Consider the Lyapunov candidate function $V_1 = e_1^T e_1$, then

$$\dot{V}_1 = e_1^T(A_1^s + A_1^{sT})e_1 + 2e_1^T(\bar{A}_2 e_0 + f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t) + E_1\Delta\psi - \nu)$$
$$\leq 2e_1^T(\bar{A}_2 e_0 + f_1(T^{-1}z, t) - f_1(T^{-1}\hat{z}, t) + E_1\Delta\psi - \nu)$$
$$\leq 2\|e_1\|\left(\|\bar{A}_2\|\|e_0\| + \mathcal{L}_{f_1}\|T^{-1}\|\|e_0\| + \|E_1\|\xi - l(.)\right) \tag{4.66}$$

It follows from (4.65) that

$$\dot{V}_1 \leq -2\eta_3\|e_1\| \leq -2\eta_3\sqrt{V_1} \tag{4.67}$$

This shows that the reachability condition is satisfied and a sliding motion is achieved and maintained after some finite time $t_s > 0$.

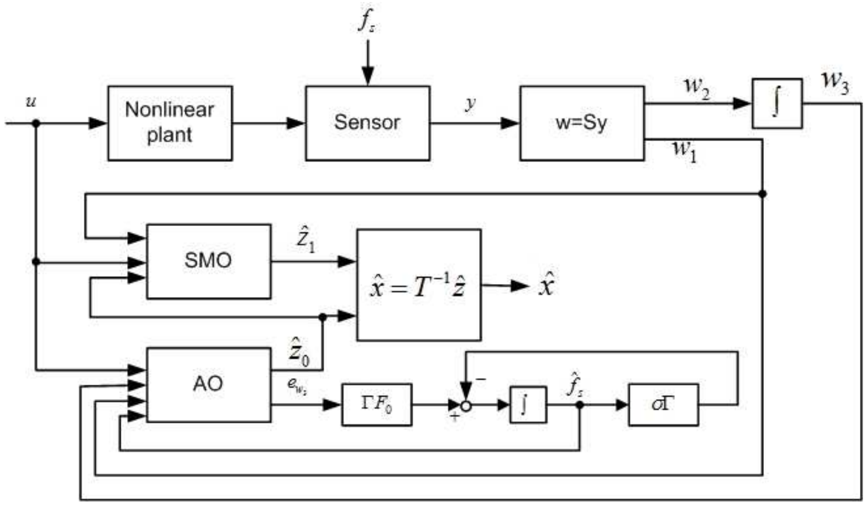

Figure 4.2: Schematic of the sensor fault estimation using (4.53)

This completes the proof.

The second sensor fault estimation scheme is shown in Fig-4.2.

4.5 Simulation results

Example-1. In this section, The effectiveness of the proposed schemes in estimating sensor faults has firstly been demonstrated by an example of a modified seventh-order aircraft model used in [103, 104], in which the states are defined as:

$$x_1 = \phi - bank\ angle(rad)$$
$$x_2 = r - yaw\ rate(rad/s)$$
$$x_3 = p - roll\ rate(rad/s)$$
$$x_4 = \delta - sideslip\ angle(rad)$$
$$x_5 = x_7 - washout\ filter\ state$$
$$x_6 = \delta_r - rudder\ deflection(rad)$$
$$x_7 = \delta_a - aileon\ deflection(rad)$$

the inputs are:

$$u_1 = \delta_{rc} - rudder\ command(rad)$$

$$u_2 = \delta_{ac} - aileon\ command(rad)$$

and outputs are:

$$y_1 = r_a - roll\ acceleration(rad/s)$$
$$y_2 = p_a - yaw\ acceleration(rad/s)$$
$$y_3 = \phi - bank\ angle(rad)$$
$$y_4 = x_7 - washout\ filter\ state$$

The system is in the form of (4.1) with

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -0.154 & -0.04 & 1.54 & 0 & -0.744 & -0.032 \\ 0 & 0.249 & -1 & -5.2 & 0 & 0.337 & -1.12 \\ 0.0386 & -0.996 & 0 & -2.117 & 0 & 0.02 & 0 \\ 0 & 0.5 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20.000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -25 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix}^T$$
$$C = \begin{bmatrix} 0 & -0.154 & -0.04 & 1.54 & 0 & -0.744 & -0.032 \\ 0 & 0.249 & -1 & -5.2 & 0 & 0.337 & -1.12 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$
$$D = \begin{bmatrix} 0.83 & 0 \\ 0 & 1.2 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}$$
$$E = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T$$
$$f(x,t) = \begin{bmatrix} \sin x_3 & \sin x_3 & 0 & 0 & \sin x_3 & 0 & 0 \end{bmatrix}^T$$
$$\Delta\psi = 2\sin t$$

Notice that in [103], the original model is linear with the nonlinear function $f(x,t) = 0$ and the system uncertainty $\Delta\psi = 0$. These two terms are added in the simulation example to show the effectiveness of the proposed methods on the sensor fault reconstruction of uncertain nonlinear systems.

The sensor fault $f_s = \begin{bmatrix} f_{s_1}^T & f_{s_2}^T \end{bmatrix}^T$ is applied to the system and defined as:

$$
\begin{aligned}
f_{s_1} &= \begin{cases} 0 & , \quad t \leq 15s \\ sin(0.5t) + 0.2sin(2t) & , \quad 15s < t < 30s \\ 0 & , \quad t \geq 30s \end{cases} \\
f_{s_2} &= \begin{cases} 0 & , \quad t \leq 20s \\ 0.6sin(2t) & , \quad t > 20s \end{cases}
\end{aligned}
$$

The nonsingular transformation matrices T and S are chosen as follows:

$$
T = \begin{bmatrix}
0.8440 & 0.1560 & 0.0405 & -1.5598 & 0 & 0.7535 & 0.0324 \\
-1.0000 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\
-1.0000 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.0000
\end{bmatrix}
$$

$$
S = \begin{bmatrix}
1.0000 & 0 & -0.8333 & 0 \\
-1.4359 & 1.0000 & -0.4701 & 0 \\
1.0128 & 0 & 0.1560 & 0 \\
1.0128 & 0 & -0.8440 & 1.0000
\end{bmatrix}
$$

The system matrices under the new coordinate become:

$$
TAT^{-1} = \left[\begin{array}{c|cccccc}
1.4794 & 1.3088 & 0.7373 & 5.6393 & 0 & -16.3183 & -0.9083 \\
\hline
-0.1540 & -0.1300 & -1.0338 & 1.2998 & 0 & -0.6280 & -0.0270 \\
0.2490 & 0.2102 & -1.0101 & -4.8116 & 0 & 0.1494 & -1.1281 \\
-0.9574 & -0.8466 & 0.0388 & -3.6104 & 0 & 0.7414 & 0.0310 \\
-3.5000 & 1.0460 & -0.8583 & -5.4593 & -4.0000 & 2.6372 & 0.1134 \\
0 & 0 & 0 & 0 & 0 & -20.0000 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -25.0000
\end{array}\right]
$$

$$
SCT^{-1} = \left[\begin{array}{c|cccccc}
-0.9873 & 0.0000 & -0.0000 & 0.0000 & 0 & -0.0001 & -0.0000 \\
\hline
0.0000 & 0.4701 & -0.9426 & -7.4112 & 0 & 1.4053 & -1.0741 \\
0.0000 & -0.1560 & -0.0405 & 1.5598 & 0 & -0.7535 & -0.0324 \\
0.0000 & -0.1560 & -0.0405 & 1.5598 & 1.0000 & -0.7535 & -0.0324
\end{array}\right]
$$

$$TB = \begin{bmatrix} 15.0700 & 0.8100 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20.0000 & 0 \\ 0 & 25.0000 \end{bmatrix}, \quad TE = \begin{bmatrix} 1 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad SD = \begin{bmatrix} -0.0033 & 0 \\ \hline -1.6619 & 1.2000 \\ 0.9966 & 0 \\ -0.0034 & 2.0000 \end{bmatrix}$$

Imposing the stability constraint to the transformed system, as described in (4.36), and formulating the problem in an LMI framework gives the following solutions for the SMO:

$$P_1 = 0.0239$$

$$A_1^s = -14.5082$$

$$F_0 = \begin{bmatrix} -0.7885 & 0.4797 & 0.0026 \\ 0.5646 & 0.0005 & 0.9426 \end{bmatrix}$$

$$H_0 = 1e - 15 \times \begin{bmatrix} 0.0002 & 0.0918 & -0.2619 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 2.1546 & -0.2331 & 0.9203 & 0.1433 & -0.0337 & 0.0306 \\ -0.2331 & 1.4204 & -1.5329 & -0.0615 & -0.0472 & -0.0612 \\ 0.9203 & -1.5329 & 2.8120 & -0.0828 & 0.0414 & 0.0726 \\ 0.1433 & -0.0615 & -0.0828 & 0.3196 & 0.0253 & 0.0074 \\ -0.0337 & -0.0472 & 0.0414 & 0.0253 & 0.0754 & 0.0022 \\ 0.0306 & -0.0612 & 0.0726 & 0.0074 & 0.0022 & 0.0544 \\ -0.5318 & -0.0796 & 0.0188 & -0.1035 & 0.0104 & -0.0048 \\ -0.8857 & -0.1326 & 0.0313 & -0.1725 & 0.0174 & -0.0079 \\ 0.3191 & 0.0478 & -0.0113 & 0.0621 & -0.0063 & 0.0029 \end{bmatrix}$$

$$\begin{bmatrix} -0.5318 & -0.8857 & 0.3191 \\ -0.0796 & -0.1326 & 0.0478 \\ 0.0188 & 0.0313 & -0.0113 \\ -0.1035 & -0.1725 & 0.0621 \\ 0.0104 & 0.0174 & -0.0063 \\ -0.0048 & -0.0079 & 0.0029 \\ 0.6355 & 0.2683 & -0.0990 \\ 0.2683 & 0.9281 & -0.1607 \\ -0.0990 & -0.1607 & 0.5307 \end{bmatrix}$$

$$L_0 = \begin{bmatrix} 3.8300 & 0.8530 & -1.2908 \\ -5.0129 & 1.9586 & 0.9128 \\ -5.5068 & 0.7652 & 1.5223 \\ -1.6720 & 4.2462 & 1.8103 \\ 9.7810 & -6.9812 & -5.5098 \\ -8.8265 & 1.6490 & -1.0835 \\ 3.4895 & 0.3753 & -0.1203 \\ 1.4576 & 3.1855 & -0.2212 \\ -0.5177 & -0.2265 & 2.6942 \end{bmatrix}$$

For the sake of comparison, the same design parameters are assigned to the adaptive observer. Select

$$\sigma = 0.02, \quad G = 0.01 I_8$$
$$\Gamma = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$$

to complete the adaptive observer design.

Fig-4.3-4.8 show the trajectories of the true states and their estimates provided by SMO-based method and AO-based method, respectively. It can be seen from the figures that both schemes can estimate the states accurately, before and after the occurrence of sensor faults. The results of sensor fault estimation are depicted in Fig-4.10 and Fig-4.11. It shows that the two proposed methods have similar performance in estimating sensor faults f_{s_1} and f_{s_2}, therefore we only analyze the results of the SMO-based method in the following. During the time period $0s - 15s$, there is no sensor fault and both estimations $\hat{f}_{s_1}$ and $\hat{f}_{s_2}$ approximate zero after an initial reaching phase. At time instant $15s$, sensor fault f_{s_1} occurs abruptly. The estimation of f_{s_1} converges very quickly to the actual value and the estimation of f_{s_2} oscillates between $15s - 17s$ and finally goes back to zero after $17s$. After $20s$, sensor fault f_{s_2} also happens. Between the time period $20s - 30s$, the estimation accuracy of f_{s_1} becomes a little lower and the estimate of f_{s_2} approaches to its actual value quickly after an initial transient. At time instant 30s, f_{s_1} becomes zero immediately and this causes a transient deviation of $\hat{f}_{s_2}$ from its actual value. The estimation of f_{s_1} preserves a sinusoidal curve since f_{s_2} still exists after $30s$.

Example-2. Consider the model of a single-link robotic arm with a revolute elastic joint [105] to further test the effectiveness of the proposed schemes. The dynamics

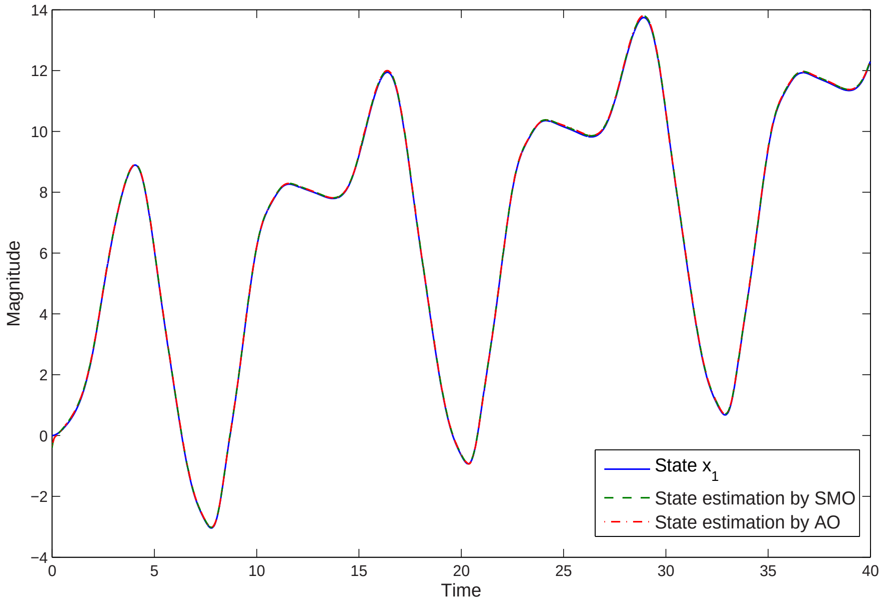

Figure 4.3: State x_1 and its estimated value $\hat{x}_1$

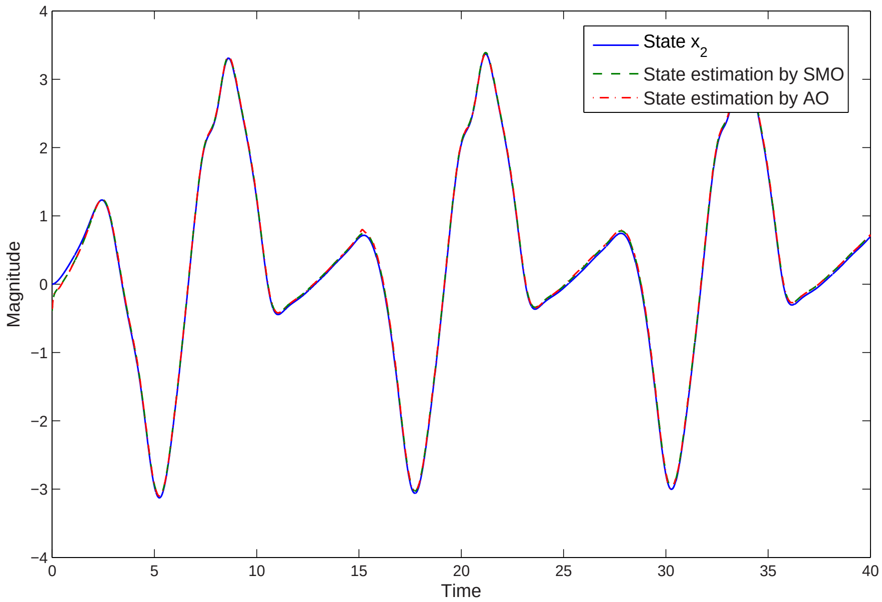

Figure 4.4: State x_2 and its estimated value $\hat{x}_2$

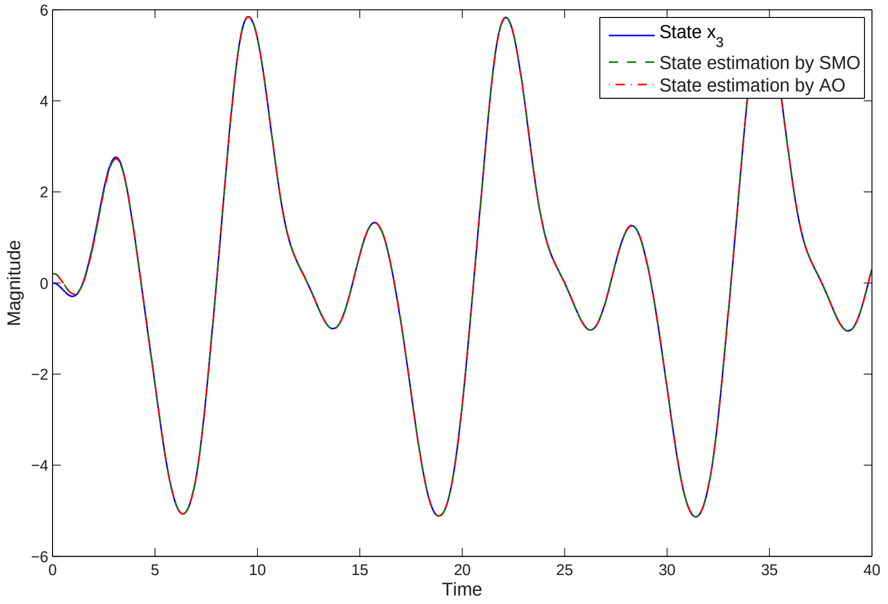

Figure 4.5: State x_3 and its estimated value $\hat{x}_3$

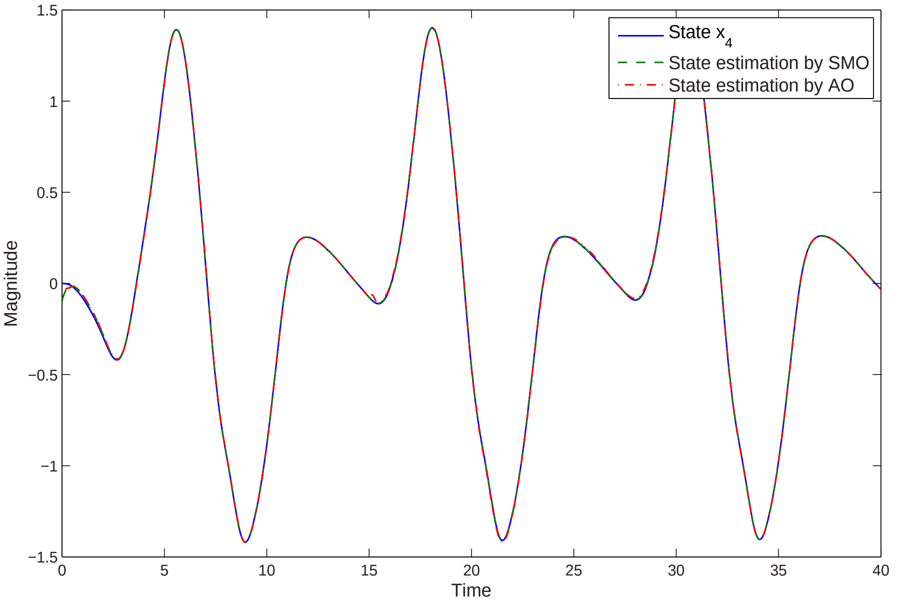

Figure 4.6: State x_4 and its estimated value $\hat{x}_4$

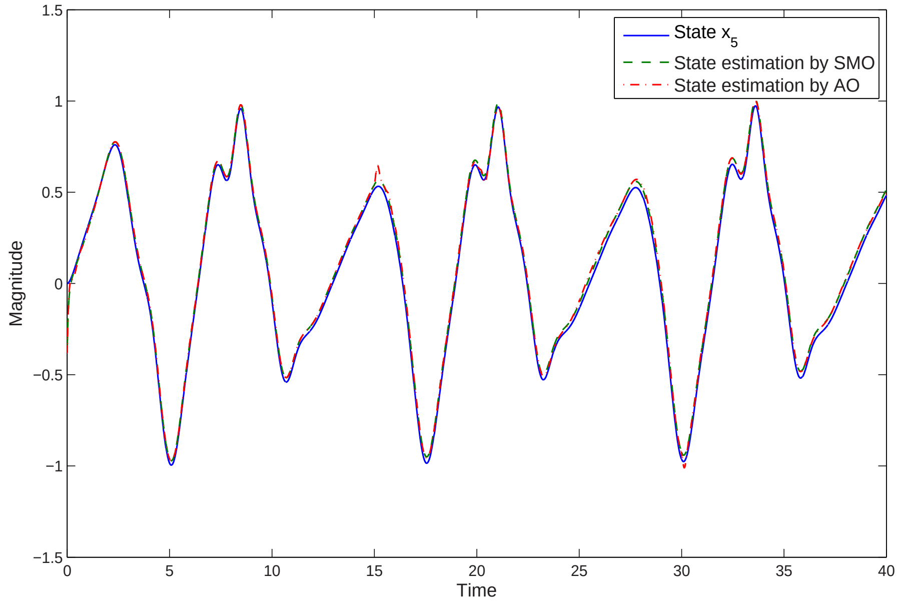

Figure 4.7: State x_5 and its estimated value $\hat{x}_5$

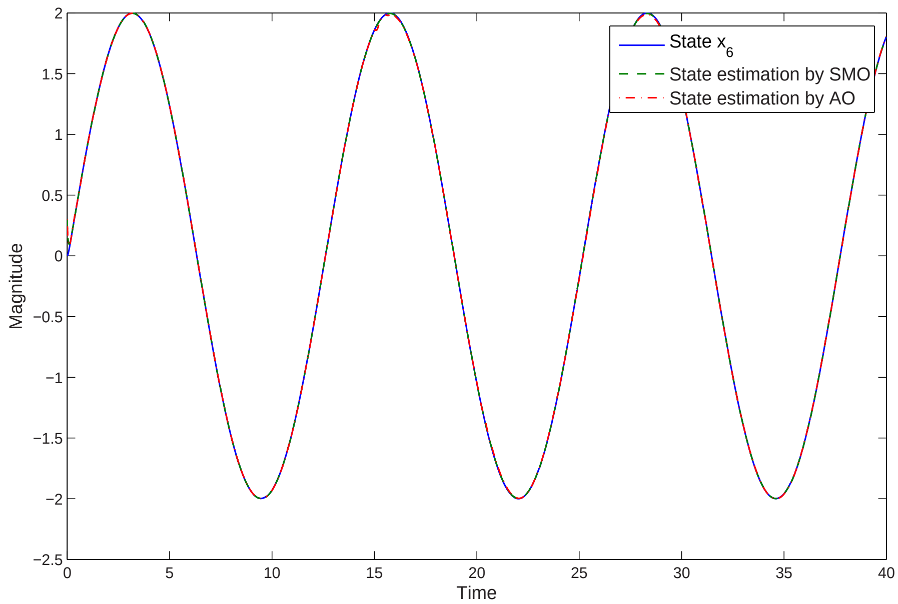

Figure 4.8: State x_6 and its estimated value $\hat{x}_6$

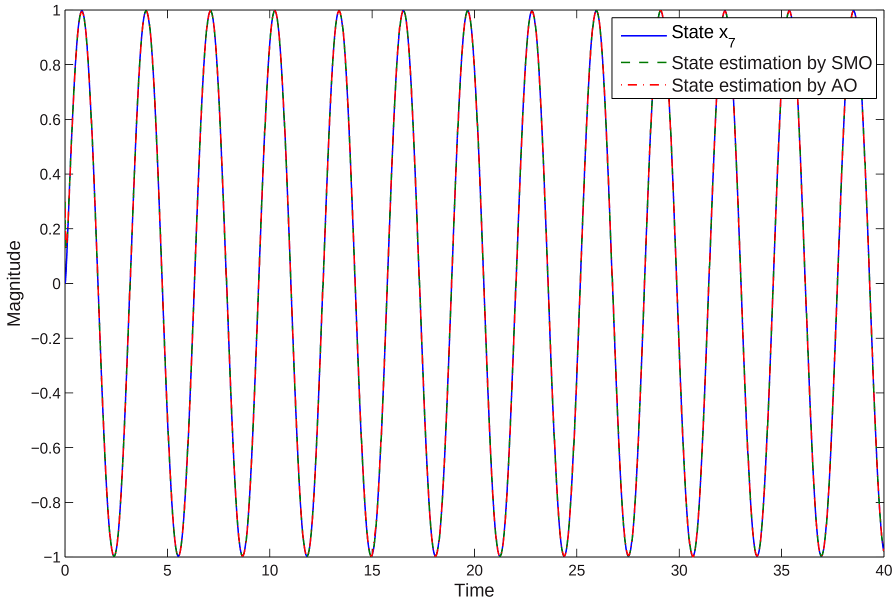

Figure 4.9: State x_7 and its estimated value $\hat{x}_7$

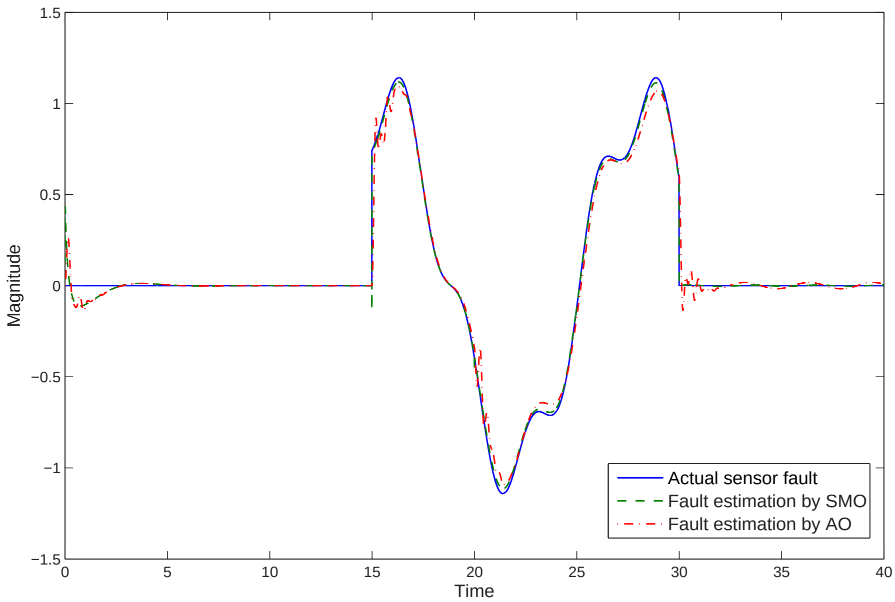

Figure 4.10: Sensor fault f_{s_1} and its estimated value $\hat{f}_{s_1}$

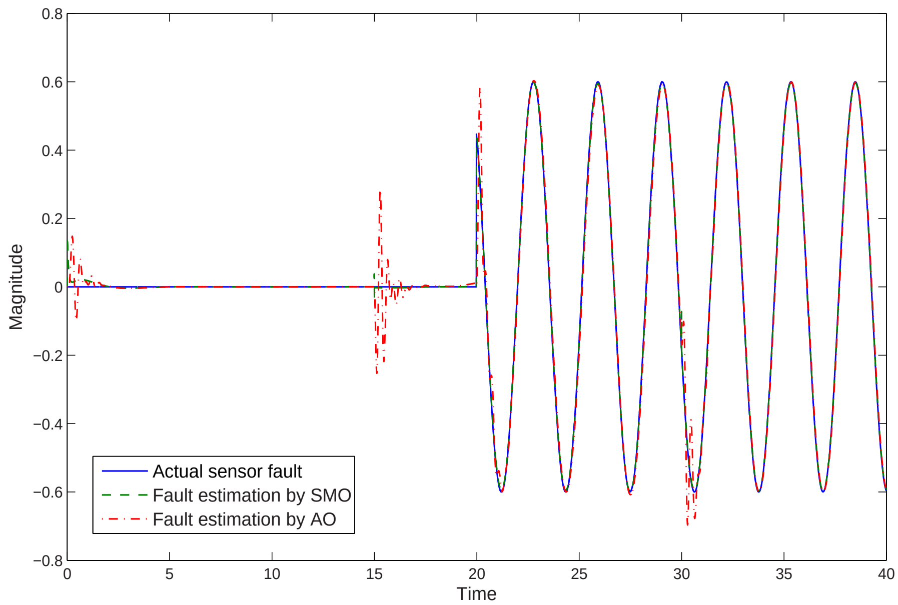

Figure 4.11: Sensor fault f_{s_2} and its estimated value $\hat{f}_{s_2}$

for this system can be described by

$$\begin{aligned} J_l\ddot{q}_1 + F_l\dot{q}_1 + k(q_1 - q_2) + mglsinq_1 = 0 \\ J_m\ddot{q}_2 + F_m\dot{q}_2 - k(q_1 - q_2) = u \end{aligned} \tag{4.68}$$

This example has been used in Chapter 3 and the same values are assigned to the parameters.

Choosing $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$ and assuming that the link position, the rotor position and the rotor velocity can be measured, the dynamics (4.69) can be represented in state-space form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{J_l} & \frac{-F_l}{J_l} & \frac{k}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & \frac{-k}{J_m} & \frac{-F_m}{J_m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix} u$$
$$+ \begin{bmatrix} 0 \\ \frac{-mgl}{J_l} sinx_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \Delta\psi$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} f_s \tag{4.69}$$

where $\Delta\psi = \frac{mgl}{J_l} sin(t)$ is the uncertainty. $f_s = [f_{s_1}^T \ f_{s_2}^T]^T$ denotes the sensor fault:

$$f_{s_1} = \begin{cases} 0 & , \quad t \leq 18s \\ 0.05\exp(0.01t) & , \quad 18s < t < 35s \\ 0 & , \quad t \geq 35s \end{cases}$$

$$f_{s_2} = \begin{cases} 0 & , \quad t \leq 15s \\ sin(0.5t) + 0.2sin(2t) & , \quad 15s < t < 30s \\ 0 & , \quad t \geq 30s \end{cases}$$

It should be emphasized that f_{s_1} is an incipient fault and the system uncertainty is relatively large. The input to the system is given by $u = 4sin(t/3)$.

The nonsingular transformations T and S are chosen as:

$$T = \begin{bmatrix} 1 & 0 & -0.5 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From the LMI synthesis, the design parameters can be obtained as:

$$P_1 = 0.0061$$

$$A_1^s = -5.9559 \times 10^4$$

$$P_0 = 10^3 \times \begin{bmatrix} 0.0008 & -0.0000 & -0.0000 & 0.0000 & -0.0000 \\ -0.0000 & 5.1446 & 2.5070 & 0.0000 & -0.0000 \\ -0.0000 & 2.5070 & 2.7057 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.6363 & -0.0000 \\ -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.6362 \end{bmatrix}$$

$$F_0 = 10^3 \times \begin{bmatrix} 1.2726 & -0.0001 \\ -0.0000 & 0.6362 \end{bmatrix}$$

$$H_0 = 10^{-4} \times \begin{bmatrix} 0.1376 & -0.0920 \end{bmatrix}$$

$$L_0 = \begin{bmatrix} -0.0000 & 0.0000 \\ 0.2255 & -0.2089 \\ -0.2090 & 0.4287 \\ 0.5014 & 0.0000 \\ 0.0000 & 0.5015 \end{bmatrix}$$

It can be shown that both the conditions of Proposition 4.1 and 4.3 are satisfied.

The parameters of the adaptive observer are selected as:

$$\sigma = 0.02, \ G = 0.01 I_5$$
$$\Gamma = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$$

It can be seen from Fig-4.12, 4.13, 4.14 and 4.15 that both schemes can estimate the states accurately. It is worth noting that incipient sensor faults are considered in this simulation. The fault estimation results are depicted in Fig-4.16 and 4.17. It shows that both proposed methods produce similar results. In the following, only the results obtained by SMOs are discussed. During the time period $0s - 15s$ both estimations $\hat{f}_{s_1}$ and $\hat{f}_{s_2}$ approximate zero after an initial reaching phase. After f_{s_1} occurs at $15s$ and f_{s_2} occurs at $18s$, both estimations converge very quickly to their actual values after a short-time oscillation. The sensor fault f_{s_2} vanishes at time instant $35s$. An initial transient appears before the estimation finally reaches its actual value.

4.6 Conclusions

Two schemes for estimating sensor faults for a class of nonlinear Lipschitz systems is proposed in this chapter. The proposed schemes essentially transform the original system into two subsystems where subsystem-1 includes system uncertainties but is free from sensor faults and subsystem-2 has sensor faults but without uncertainties. By using the integral observer based approach, sensor faults in subsystem-2 are transformed into actuator faults. In the first scheme, two sliding mode observers are designed. One of which is to eliminate the effect of system uncertainties while the other one is to estimate sensor faults using the concept of equivalent output injection. In the second scheme, an adaptive observer is adopted combined with a sliding mode

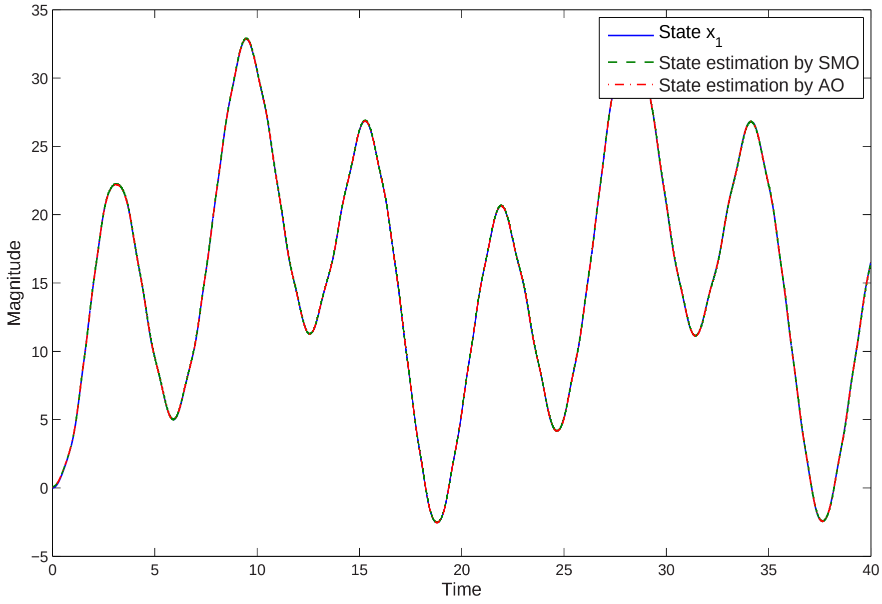

Figure 4.12: State x_1 and its estimated value $\hat{x}_1$

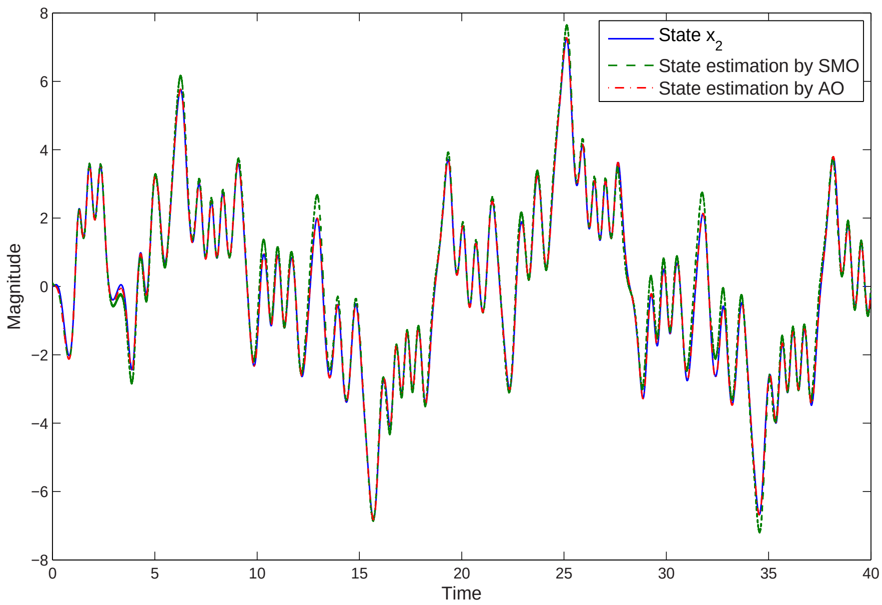

Figure 4.13: State x_2 and its estimated value $\hat{x}_2$

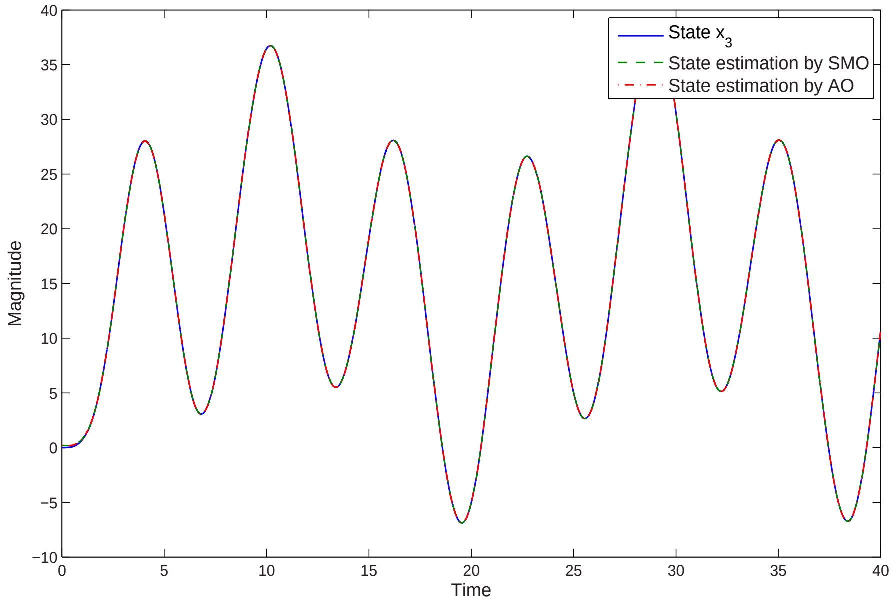

Figure 4.14: State x_3 and its estimated value $\hat{x}_3$

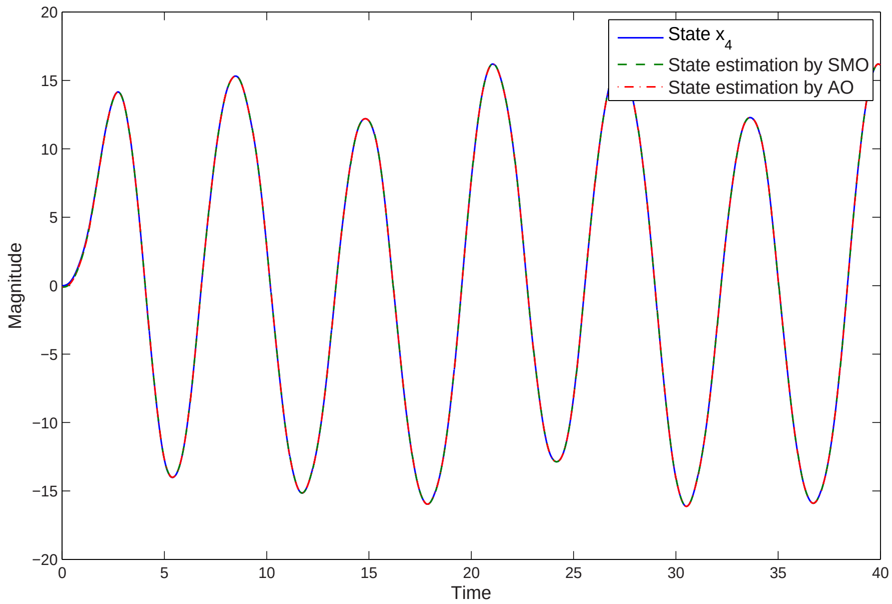

Figure 4.15: State x_4 and its estimated value $\hat{x}_4$

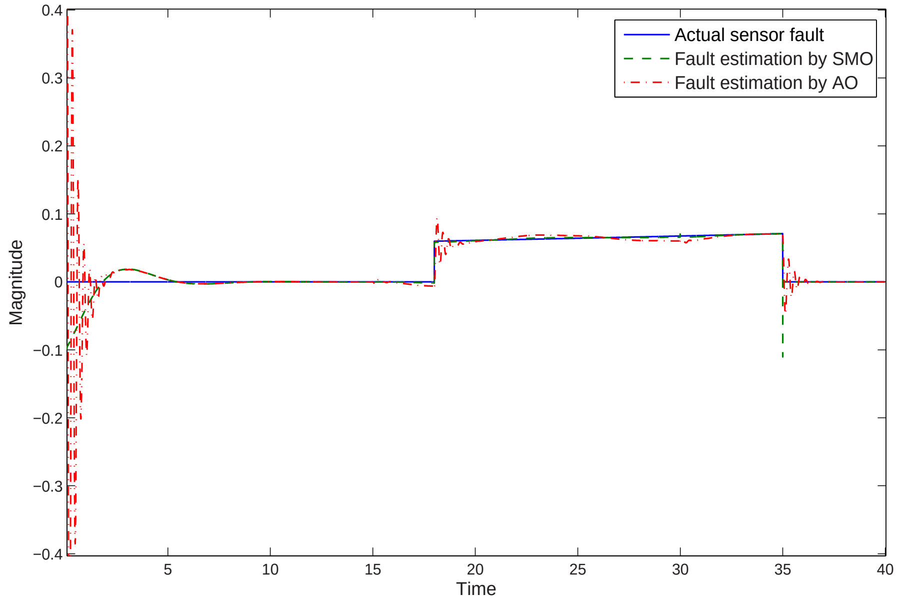

Figure 4.16: Sensor fault f_{s_1} and its estimated value $\hat{f}_{s_1}$

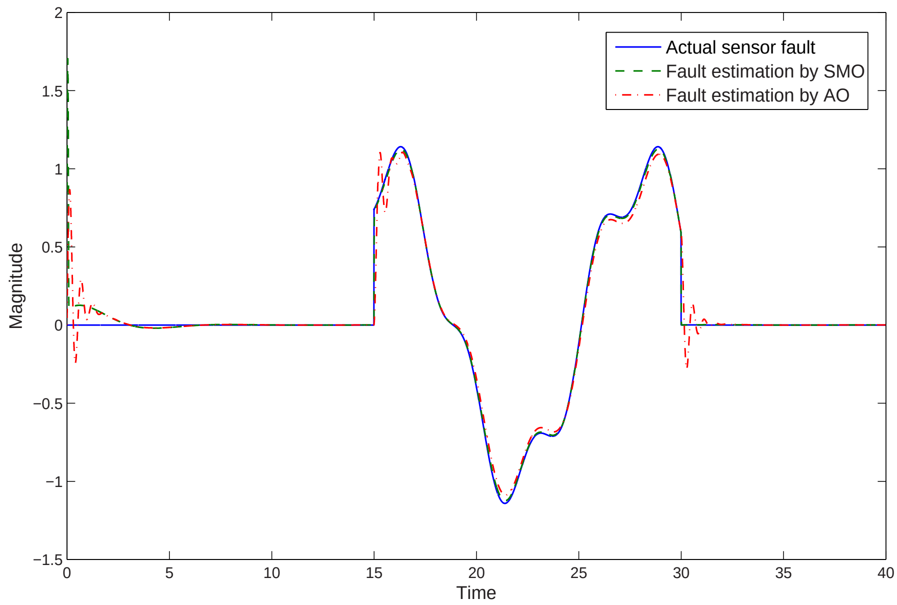

Figure 4.17: Sensor fault f_{s_2} and its estimated value $\hat{f}_{s_2}$

observer that is also used to reduce the effect of the system uncertainty. The sensor fault is directly estimated by the adaptive observer instead of using a sliding mode observer. Adaptation laws are integrated into both schemes to deal with the situation when the Lipschitz constant is unknown or too large, which may cause the failure of solving LMIs to find design parameters for observers. Two examples have been used to demonstrate the effectiveness of the proposed sensor fault estimation schemes. Simulation results confirm that both methods can accurately estimate sensor faults ,including the incipient faults, in the presence of large system uncertainties.

Chapter 5

Robust $\mathcal{H}_\infty$ filtering for uncertain nonlinear systems with fault estimation synthesis

The problem of actuator fault and sensor fault diagnosis has been discussed in previous chapters. However, it is assumed that there is no sensor fault when dealing actuator fault (chapter 2), or there is no actuator fault when dealing with sensor validation (chapters 3 and 4). In this chapter, the simultaneous estimation of actuator and sensor faults is studied.

5.1 Introduction

Almost all the approaches proposed in the past deal separately with actuator faults and sensor faults [19, 51, 56, 77, 82, 104]. In other words, it is usually assumed that there is no actuator fault when dealing with sensor validation, or sensors are healthy when faults occur in actuators. However, in many practical systems, both actuators and sensors are simultaneously prone to faults. Misinterpretation of actuator faults and sensor faults may cause a high rate of false alarm and unnecessary maintenance. Therefore, it would be desirable to consider actuator faults and sensor faults under one unified framework, and coincidentally detect and isolate them. This motivates the present study of estimating actuator faults and sensor faults which occur at the same time.

In this chapter, two schemes are developed for a class of uncertain nonlinear systems. The proposed methods essentially transform the original system into two subsystems (subsystem-1 and 2). Subsystem-1 includes the effects of actuator faults but is free from sensor faults and subsystem-2 only has sensor faults. Sensor faults in subsystem-2 take the appearance of actuator faults by using integral observer based approach [92]. The augmented subsystem-2 is further transformed by a linear coordinate transformation such that a specific structure can be imposed to the sensor fault distribution matrix. The first scheme is based on the matching condition and two sliding mode observers (SMOs) are designed to estimate actuator faults and sensor faults, respectively. However, this assumption is restrictive and sometimes it is difficult to find such matrices to satisfy both the Lyapunov equation and matching condition. In order to reduce this conservativeness, we remove the assumption of matching condition in the second scheme and use an adaptive observer (AO) to estimate the sensor fault.

It is worth emphasizing that unlike in chapters 2,3 and 4 where the uncertainty under consideration is assumed to be structured, the uncertainty considered in this chapter is assumed to be unstructured and can be a high-frequency noise or a slow-varying signal. For the case of structured uncertainty, certain rank conditions of the uncertainty distribution matrix should be satisfied such that the uncertainty can be completely decoupled from the fault. However, this additional assumption often limits the application of FDI schemes. For the case of unstructured uncertainty, the complete decoupling uncertainties from faults is not possible, but fortunately its effects on the estimation errors of states and faults can still be minimized. This can be done by integrating a prescribed $\mathcal{H}_\infty$ disturbance attenuation level into fault estimators. The $\mathcal{H}_\infty$ control problem is able to address the issue of system uncertainties, and also be applied to the typical problem of disturbance input control. It was initially formulated in [106] where the $\mathcal{H}_\infty$ norm from the norm-bounded exogenous disturbance signals to the observer error is guaranteed to be below a prescribed level. In this chapter, the sufficient condition for the stability of both proposed fault estimation observers is presented in an LMI form. By solving the LMI optimization problem, the $\mathcal{L}_2$ gain of the transfer from system uncertainties to the estimation errors can be minimized and observer parameters can be obtained.

The chapter is organized as follows: section-5.2 briefly describes the mathematical preliminaries required for designing observers. Section-5.3 proposes the first fault estimation scheme based on the matching condition. The stability condition of the proposed observers are derived and expressed in LMIs. If the matching condition is not satisfied, the second fault estimation scheme is given in section-5.4. The results of simulation are shown in section-5.5 with conclusions in section-5.6.

5.2 Problem Formulation

Consider a nonlinear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x,t) + B(u(t) + f_a(t)) + \Delta\psi(t) \\ y(t) &= Cx(t) + Df_s(t) \end{aligned} \tag{5.1}$$

where $x \in \mathcal{R}^n$ are the state variables, $u \in \mathcal{R}^m$ the inputs and $y \in \mathcal{R}^p$ the outputs. $f_a \in \mathcal{R}^m$ and $f_s \in \mathcal{R}^q$ denote the actuator fault and sensor fault, respectively. $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{p\times n}$ and $D \in \mathcal{R}^{p\times q}$ ($p \geq q$) are known constant matrices with C and D both being of full rank. The nonlinear continuous term $f(x,t) \in \mathcal{R}^n$ is assumed to be known. Note that nonlinear term $\Delta\psi(t)$ which represents the modelling errors or external disturbances is unstructured.

For the objective of achieving simultaneous fault estimation, the following assumptions are made throughout:

Assumption 5.1 $rank(CB) = rank(B)$.

This assumption implies that the number of measurements is at least equal to the number of effective inputs.

Lemma 5.1 Under Assumption 5.1, there exist state and output transformations

$$h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{5.2}$$

such that in the new coordinate, the system matrices become:

$$TAT^{-1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad TB = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad SCT^{-1} = \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix}, \quad SD = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}$$

where $T \in \mathcal{R}^{n\times n}$, $S \in \mathcal{R}^{p\times p}$, $h_1 \in \mathcal{R}^m$, $w_1 \in \mathcal{R}^m$, $A_1 \in \mathcal{R}^{m\times m}$, $A_4 \in \mathcal{R}^{(n-m)\times(n-m)}$, $B_1 \in \mathcal{R}^{m\times m}$, $E_1 \in \mathcal{R}^{m\times r}$, $C_1 \in \mathcal{R}^{m\times m}$, $C_4 \in \mathcal{R}^{(p-m)\times(n-m)}$ and $D_2 \in \mathcal{R}^{(p-m)\times q}$. B_1 and C_1 are invertible.

Proof. See [91, 94, 107].

Remark 5.1 Notice that in Chapter 4 a similar coordinate transformation has been used. The transformation is chosen such that the uncertainty distribution matrix after the transformation becomes $TE = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}$. While in this chapter, the transformation is chosen to make $TB = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$.

Assumption 5.2 For every complex number s with nonnegative real part:

$$rank \begin{bmatrix} sI_n - A & B \\ C & 0 \end{bmatrix} = n + m \tag{5.3}$$

This assumption is known as the minimum phase condition.

Lemma 5.2 The pair (A_4, C_4) is detectable if and only if Assumption 5.2 holds.

Proof. See [94, 95].

Assumption 5.3 The nonlinear function $f(x,t)$ is Lipschitz about x uniformly, that is, $\forall x, \hat{x} \in \mathcal{X}$,

$$\|f(x,t) - f(\hat{x},t)\| \leq \mathcal{L}_f \|x - \hat{x}\| \tag{5.4}$$

where $\mathcal{L}_f$ is the Lipschitz constant.

Assumption 5.4 The function $\Delta\psi(x,t)$,which represents the modeling uncertainty, is unknown but bounded, and it satisfies $\|\Delta\psi\| \leq \xi$. Also the unknown actuator fault f_a, sensor fault f_s and the derivative of f_s with respect to time are norm bounded, i.e., $\|f_a(t)\| \leq \rho_a$, $\|f_s(t)\| \leq \rho_s$, $\|\dot{f}_s(t)\| \leq \rho_{ss}$.

After introducing the state and output transformations $T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$ and $S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$, the original system is converted into two subsystems:

$$\dot{h}_1 = A_1 h_1 + A_2 h_2 + f_1(T^{-1}h, t) + B_1(u + f_a) + \Delta\psi_1$$

$$
\begin{aligned}
w_1 &= C_1 h_1 && (5.5)\\
\dot{h}_2 &= A_3 h_1 + A_4 h_2 + f_2(T^{-1}h,t) + \Delta\psi_2 \\
w_2 &= C_4 h_2 + D_2 f_s && (5.6)
\end{aligned}
$$

where (5.5) is referred to as subsystem-1 and (5.6) is referred to as subsystem-2, $T_1 \in \mathcal{R}^{m\times n}$, $S_1 \in \mathcal{R}^{m\times p}$, $f_1(T^{-1}h,t) = T_1 f(T^{-1}h,t)$, $f_2(T^{-1}h,t) = T_2 f(T^{-1}h,t)$, $\Delta\psi_1 = T_1\Delta\psi$ and $\Delta\psi_2 = T_2\Delta\psi$.

For subsystem-2, define a new state $h_3 = \int_0^t w_2(\tau)d\tau$ so that $\dot{h}_3(t) = C_4 h_2 + D_2 f_s$, then the augmented system with the new state h_3 is given as:

$$
\begin{aligned}
\dot{h}_0 &= A_0 h_0 + \bar{A}_3 h_1 + \bar{T}_2 f(T^{-1}h,t) + D_0 f_s + \bar{T}_2\Delta\psi \\
w_3 &= C_0 h_0 && (5.7)
\end{aligned}
$$

where $h_0 = \begin{bmatrix} h_2 \\ h_3 \end{bmatrix} \in \mathcal{R}^{n+p-2m}$, $w_3 \in \mathcal{R}^{p-m}$, $A_0 = \begin{bmatrix} A_4 & 0 \\ C_4 & 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2m)\times(n+p-2m)}$, $\bar{A}_3 = \begin{bmatrix} A_3 \\ 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2m)\times m}$, $\bar{T}_2 = \begin{bmatrix} T_2 \\ 0 \end{bmatrix} \in \mathcal{R}^{(n+p-2m)\times n}$, $D_0 = \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \in \mathcal{R}^{(n+p-2m)\times q}$ and $C_0 = \begin{bmatrix} 0 & I_{p-m} \end{bmatrix} \in \mathcal{R}^{(p-m)\times(n+p-2m)}$.

Lemma 5.3 The pair (A_0, C_0) is observable if Assumption 5.2 holds.

Proof. From the Popov-Belevitch-Hautus (PBH) test, the pair (A_0, C_0) is observable if and only if

$$
rank \begin{bmatrix} sI - A_0 \\ C_0 \end{bmatrix} = rank \begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} = n + p - 2m \tag{5.8}
$$

for all $s \in \mathcal{C}$. If $s = 0$, it is obvious that

$$
rank \begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} = rank \begin{bmatrix} -A_4 \\ -C_4 \end{bmatrix} + p - m \tag{5.9}
$$

If Assumption 2 holds, then

$$
rank \begin{bmatrix} sI - A_4 \\ -C_4 \end{bmatrix} = n - m \;\; for \; all \; s \in \mathcal{C} \tag{5.10}
$$

It follows that the rank test (5.8) holds when $s = 0$.

When $s \neq 0$

$$\begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \tag{5.11}$$

since (A_4, C_4) is observable, which implies that the columns of $\begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix}$ are linearly independent and its rank is $n + p - 2m$.

This completes the proof.

It follows from Lemma 5.3 that there exists a matrix $L_0 \in \mathcal{R}^{(n+p-2m)\times(p-m)}$ such that $A_0 - L_0C_0$ is stable, and thus for any $Q_0 > 0$, the Lyapunov equation

$$(A_0 - L_0C_0)^T P_0 + P_0(A_0 - L_0C_0) = -Q_0 \tag{5.12}$$

has an unique solution $P_0 > 0$ [80].

Partition $P_0 \in \mathcal{R}^{(n+p-2m)\times(n+p-2m)}$ and $Q_0 \in \mathcal{R}^{(n+p-2m)\times(n+p-2m)}$ as:

$$P_0 = \begin{bmatrix} P_{01} & P_{02} \\ P_{02}^T & P_{03} \end{bmatrix}, \quad Q_0 = \begin{bmatrix} Q_{01} & Q_{02} \\ Q_{02}^T & Q_{03} \end{bmatrix}, \tag{5.13}$$

It follows from $P_0 > 0$ and $Q_0 > 0$ that $P_{01} \in \mathcal{R}^{(n-m)\times(n-m)} > 0$, $P_{03} \in \mathcal{R}^{(p-m)\times(p-m)} > 0$, $Q_{01} \in \mathcal{R}^{(n-m)\times(n-m)} > 0$ and $Q_{03} \in \mathcal{R}^{(p-m)\times(p-m)} > 0$. If P_0 and Q_0 have the structure as shown in (5.13), then the following conclusion is obvious:

Lemma 5.4 The matrix $A_4 + P_{01}^{-1}P_{02}C_4$ is stable if Lyapunov equation (5.12) is satisfied.

Proof. According to the structure of A_0, C_0, P_0 and Q_0, it is easy to see that the first $n - m$ columns of $A_0 - L_0C_0$ are independent of L_0. After the block multiplication to (5.12), the following equation can be obtained as

$$A_4^T P_{01} + C_4^T P_{02}^T + P_{01}A_3 + P_{02}C_4 = -Q_{01}$$

This equation can be rewritten as

$$(A_4 + P_{01}^{-1}P_{02}C_4)^T P_{01} + P_{01}(A_4 + P_{01}^{-1}P_{02}C_4) = -Q_{01}$$

Since $P_{01} > 0$ and $Q_{01} > 0$, it follows that $A_4 + P_{01}^{-1}P_{02}C_4$ is stable from the Lyapunov theory.

In the next two sections 5.3 and 5.4, two schemes will be proposed to simultaneously estimate actuator faults and sensor faults. The design of fault estimation schemes begins by employing the nonsingular state transformation $z := [z_1^T \; z_2^T]^T = T_0 h_0$ to subsystem (5.7). T_0 is defined as:

$$T_0 = \begin{bmatrix} I_{n-m} & P_{01}^{-1}P_{02} \\ 0 & I_{p-m} \end{bmatrix} \tag{5.14}$$

After the transformation $z = T_0 h_0$, the subsystem (5.7) becomes

$$\begin{aligned} \dot{z} &= T_0 A_0 T_0^{-1} z + T_0 \bar{A}_3 h_1 + T_0 \bar{T}_2 f(T^{-1}h, t) + T_0 D_0 f_s + T_0 \bar{T}_2 \Delta\psi \\ w_3 &= C_0 T_0^{-1} z \end{aligned} \tag{5.15}$$

5.3 Fault estimation using SMOs

In this section, two sliding mode observers are developed to simultaneously estimate multiple actuator faults and sensor faults for the system described by (5.1).

5.3.1 Design of SMOs

Prior to the design of sliding mode observers, the following assumption is made.

Assumption 5.5 There exists an arbitrary matrix $F_0 \in \mathcal{R}^{q\times(p-m)}$ such that:

$$D_0^T P_0 \quad = F_0 C_0 \tag{5.16}$$

Lemma 5.5 If Assumption 5.5 holds, then $P_{01}^{-1}P_{02}D_2 = 0$.

Proof. See *Lemma 2.1*.

It is worth noting that a specific structure on the sensor fault distribution matrix T_0D_0 can be imposed in the new coordinate. If Assumption 5.5 is satisfied, then we can have

$$D_z = T_0D_0 = \begin{bmatrix} P_{01}^{-1}P_{02}D_2 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}$$

The system (5.15) can be rewritten as

$$\begin{aligned} \dot{z}_1 &= \bar{A}_1z_1 + \bar{A}_2z_2 + A_3h_1 + f_2(T^{-1}h, t) + \Delta\psi_2 \\ \dot{z}_2 &= \bar{A}_3z_1 + \bar{A}_4z_2 + D_2f_s \\ w_3 &= C_zz \end{aligned} \tag{5.17}$$

where

$$\begin{aligned} \bar{A}_1 &= A_4 + P_{01}^{-1}P_{02}C_4 \\ \bar{A}_2 &= -A_4P_{01}^{-1}P_{02} - P_{01}^{-1}P_{02}C_4P_{01}^{-1}P_{02} \\ \bar{A}_3 &= C_4 \\ \bar{A}_4 &= -C_4P_{01}^{-1}P_{02} \\ C_z &= C_0T_0^{-1} = \begin{bmatrix} 0 & I \end{bmatrix} \end{aligned} \tag{5.18}$$

The Lyapunov matrix P_0 in the new coordinate can be proved to have the following quadratic form:

$$P_z = (T_0^T)^{-1}P_0{T_0}^{-1} = \begin{bmatrix} P_{01} & 0 \\ 0 & \bar{P}_{03} \end{bmatrix} \tag{5.19}$$

where $\bar{P}_{03} = -P_{02}^TP_{01}^{-T}P_{02} + P_{03}$.

Substituting $P_0 = T_0^TP_zT_0$, $C_0 = C_zT_0$ and $D_0 = T_0^{-1}D_z$ into the matching condition (5.16) yields

$$D_z^TP_z = F_0C_z \tag{5.20}$$

From the structure of D_z, C_z and P_z, it is easy to obtain that $D_2^T\bar{P}_{03} = F_0$.

Since

$$\begin{bmatrix} h_2 \\ h_3 \end{bmatrix} = T_0^{-1} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} z_1 - P_{01}^{-1} P_{02} z_2 \\ z_2 \end{bmatrix} \tag{5.21}$$

then subsystem (5.5) can be rewritten as

$$\begin{aligned} \dot{h}_1 &= A_1 h_1 + A_2 h_2 + f_1(T^{-1}h, t) + B_1(u + f_a) + \Delta\psi_1 \\ &= A_1 h_1 + A_2 z_1 - A_2 P_{01}^{-1} P_{02} z_2 + f_1(T^{-1}h, t) \\ &\quad + B_1(u + f_a) + \Delta\psi_1 \end{aligned} \tag{5.22}$$

For subsystem (5.22), the proposed sliding mode observer has the form:

$$\begin{aligned} \dot{\hat{h}}_1 &= A_1 \hat{h}_1 + A_2 \hat{z}_1 - A_2 P_{01}^{-1} P_{02} w_3 + f_1(T^{-1}\hat{h}, t) + B_1(u + \nu_1) \\ &\quad + (A_1 - A_1^s) C_1^{-1} (w_1 - \hat{w}_1) \\ \hat{w}_1 &= C_1 \hat{h}_1 \end{aligned} \tag{5.23}$$

where $A_1^s \in \mathcal{R}^{m \times m}$ is a stable matrix which needs to be determined.

The discontinuous output error injection term ν_1 which is used to estimate the actuator fault is defined as

$$\nu_1 = \begin{cases} (\rho_a + l_1) \frac{B_1^T P_1 (C_1^{-1} S_1 y - \hat{h}_1)}{\| B_1^T P_1 (C_1^{-1} S_1 y - \hat{h}_1) \|} & \text{if } C_1^{-1} S_1 y - \hat{h}_1 \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.24}$$

where $P_1 \in \mathcal{R}^{m \times m}$ is the symmetric definite Lyapunov matrix for A_1^s and l_1 is a positive scalar. It is worth noting that the state h_1 can be computed from the measurement y as $h_1 = C_1^{-1} S_1 y$.

For subsystem (5.17), the proposed sliding mode observer has the form:

$$\begin{aligned} \dot{\hat{z}}_1 &= \bar{A}_1 \hat{z}_1 + \bar{A}_2 w_3 + A_3 C_1^{-1} w_1 + f_2(T^{-1}\hat{h}, t) \\ \dot{\hat{z}}_2 &= \bar{A}_3 \hat{z}_1 + \bar{A}_4 \hat{z}_2 + (\bar{A}_4 - L)(w_3 - \hat{w}_3) + D_2 \nu_2 \\ \hat{w}_3 &= \hat{z}_2 \end{aligned} \tag{5.25}$$

where $L \in \mathcal{R}^{(p-m) \times (p-m)}$ is the observer gain. The discontinuous output error injection term ν_2 is used to estimate the sensor fault and is defined as:

$$\nu_2 = \begin{cases} (\rho_s + l_2)\frac{D_2^T \bar{P}_{03}(w_3 - \hat{w}_3)}{\|D_2^T \bar{P}_{03}(w_3 - \hat{w}_3)\|} & \text{if } w_3 - \hat{w}_3 \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.26}$$

where l_2 is a positive constant which needs to be determined.

If the state estimation errors are defined as $e_1 = h_1 - \hat{h}_1$, $e_2 = z_1 - \hat{z}_1$ and $e_3 = z_2 - \hat{z}_2$, then their dynamics after the occurrence of faults can be obtained as:

$$\begin{aligned} \dot{e}_1 &= A_1^s e_1 + A_2 e_2 + \Big(f_1(T^{-1}h, t) - f_1(T^{-1}\hat{h}, t)\Big) + B_1(f_a - \nu_1) \\ &\quad + T_1 \Delta\psi && (5.27) \\ \dot{e}_2 &= \bar{A}_1 e_2 + \Big(f_2(T^{-1}h, t) - f_2(T^{-1}\hat{h}, t)\Big) + T_2 \Delta\psi && (5.28) \\ \dot{e}_3 &= \bar{A}_3 e_2 + L e_3 + D_2(f_s - \nu_2) \\ e_3 &= C_z e_0 && (5.29) \end{aligned}$$

Define

$$r(t) = H \begin{bmatrix} h_1(t) \\ z_1(t) \\ z_2(t) \end{bmatrix}, \quad and \;\; \bar{r} = He = H \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{5.30}$$

where $r(t) \in \mathcal{R}^{n+p-m}$ is a linear combination of state variables to be estimated over the horizon $[0,\ T]$. H is a pre-specified weight matrix and assumed to have full rank:

$$H := \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_2 & 0 \\ 0 & 0 & H_3 \end{bmatrix} \tag{5.31}$$

Consider the following worst-case performance measure:

$$J := \sup_{\|\Delta\psi\|_{\mathcal{L}_2} \neq 0} \frac{\|\bar{r}\|^2_{\mathcal{L}_2}}{\|\Delta\psi\|^2_{\mathcal{L}_2}} \tag{5.32}$$

The objective of this section is to design sliding mode observers in the form of (5.23) and (5.25) such that the observer error dynamics (5.27)-(5.29) is asymptotically stable and the performance measure satisfies $J \leq \mu$, where μ is a small positive constant. In other words, the $\mathcal{H}_\infty$ gain of the transfer function from the system uncertainty

$\Delta\psi$ to the state estimation error $\bar{r}$ is bounded by $\sqrt{\mu}$.

In the next, the stability of the proposed fault estimators is studied and the sufficient condition is summarized as follows:

Proposition 5.1 *Under the Assumptions 5.1-5.5, the observer error dynamics is asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level $\sqrt{\mu} > 0$ subject to $\|\bar{r}\|_{\mathcal{L}_2} \leq \sqrt{\mu}\|\Delta\psi\|_{\mathcal{L}_2}$ if there exist matrices A_1^s, L, $P_{01} > 0$, P_{02} and $\bar{P}_{03} > 0$ such that:*

$$\begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1}P_1P_1 & P_1A_2 & 0 & P_1T_1 \\ A_2^TP_1 & \Pi_2 + \frac{1}{\alpha_2}P_{01}P_{01} & C_4^T\bar{P}_{03} & P_{01}T_2 \\ 0 & \bar{P}_{03}C_4 & \Pi_3 & 0 \\ T_1^TP_1 & T_2^TP_{01} & 0 & -\mu I_r \end{bmatrix} < 0 \tag{5.33}$$

where $\Pi_1 = A_1^{sT}P_1 + P_1A_1^s + H_1^TH_1$, $\Pi_2 = A_4^TP_{01} + P_{01}A_4 + P_{02}C_4 + C_4^TP_{02}^T + \alpha_1\mathcal{L}_{f_1}^2\|T^{-1}\|^2I + \alpha_2\mathcal{L}_{f_2}^2\|T^{-1}\|^2I + H_2^TH_2$, $\Pi_3 = \bar{P}_{03}L + L^T\bar{P}_{03} + H_3^TH_3$, $\mathcal{L}_{f_1} = \|T_1\|\mathcal{L}_f$ and $\mathcal{L}_{f_2} = \|T_2\|\mathcal{L}_f$.

Proof. Based on the quadratic form of P_z, we consider the Lyapunov function as:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{5.34}$$

where $V_1(t) = e_1^TP_1e_1$, $V_2(t) = e_2^TP_{01}e_2$ and $V_3(t) = e_3^T\bar{P}_{03}e_3$.

The time derivative of $V_1(t)$ along the trajectories of state estimation error dynamics (5.27) can be shown to be:

$$\begin{aligned} \dot{V}_1(t) &= e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^TP_1A_2e_2 + 2e_1^TP_1T_1\Delta\psi \\ &\quad + 2e_1^TP_1\left(f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)\right) + 2e_1^TP_1B_1(f_a - \nu_1) \end{aligned}$$

Since for any scalar $\alpha > 0$, the inequality $2X^TY \leq \frac{1}{\alpha}X^TX + \alpha Y^TY$ [46], then

$$\begin{aligned} \dot{V}_1(t) &\leq e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^TP_1A_2e_2 + 2e_1^TP_1T_1\Delta\psi + \frac{1}{\alpha_1}\|P_1e_1\|^2 \\ &\quad + \alpha_1\|f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)\|^2 + 2e_1^TP_1B_1(f_a - \nu_1) \\ &\leq e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^TP_1A_2e_2 + 2e_1^TP_1T_1\Delta\psi + \frac{1}{\alpha_1}\|P_1e_1\|^2 \\ &\quad + \alpha_1\mathcal{L}_{f_1}^2\|T^{-1}\|^2\|e_2\|^2 + 2e_1^TP_1B_1(f_a - \nu_1) \end{aligned} \tag{5.35}$$

It follows from (5.24) the last term of (5.35) can be calculated as:

$$\begin{aligned} e_1^T P_1 B_1 (f_a - \nu_1) &= e_1^T P_1 B_1 f_a - (\rho_a + \eta_1) \frac{\|B_1^T P_1 e_1\|^2}{\|B_1^T P_1 e_1\|} \\ &\leq -l_1 \|B_1^T P_1 e_1\| < 0 \end{aligned} \tag{5.36}$$

Therefore

$$\begin{aligned} \dot{V}_1(t) \leq\ & e_1^T (A_1^{sT} P_1 + P_1 A_1^s + \frac{1}{\alpha_1} P_1 P_1) e_1 + 2 e_1^T P_1 A_2 e_2 \\ & + 2 e_1^T P_1 T_1 \Delta\psi + \alpha_1 \mathcal{L}_{f_1}^2 \|T^{-1}\|^2 \|e_2\|^2 \end{aligned} \tag{5.37}$$

Similarly, the derivatives of $V_2(t)$ and $V_3(t)$ with respect to time can be obtained as:

$$\begin{aligned} \dot{V}_2(t) =\ & e_2^T \left(\bar{A}_1^T P_{01} + P_{01} \bar{A}_1\right) e_2 + 2 e_2^T P_{01} \left(f_2(T^{-1}h, t) - f_2(T^{-1}\hat{h}, t)\right) \\ & + 2 e_2^T P_{01} T_2 \Delta\psi \\ \leq\ & e_2^T (A_4^T P_{01} + P_{01} A_4 + P_{02} C_4 + C_4^T P_{02}^T + \frac{1}{\alpha_2} P_{01} P_{01} + \alpha_2 \mathcal{L}_{f_2}^2 \|T^{-1}\|^2 I) e_2 \\ & + 2 e_2^T P_{01} T_2 \Delta\psi \end{aligned} \tag{5.38}$$

$$\begin{aligned} \dot{V}_3(t) =\ & e_3^T (\bar{P}_{03} L + L^T \bar{P}_{03}) e_3 + 2 e_3^T \bar{P}_{03} C_4 e_2 + 2 e_3^T \bar{P}_{03} D_2 (f_s - \nu_2) \\ \leq\ & e_3^T (\bar{P}_{03} L + L^T \bar{P}_{03}) e_3 + 2 e_3^T \bar{P}_{03} C_4 e_2 - 2 l_2 \|D_2^T \bar{P}_{03} e_3\| \\ \leq\ & e_3^T (\bar{P}_{03} L + L^T \bar{P}_{03}) e_3 + 2 e_3^T \bar{P}_{03} C_4 e_2 \end{aligned} \tag{5.39}$$

Therefore, the derivative of $V(t)$ can be obtained from:

$$\begin{aligned} \dot{V}(t) =\ & \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \\ \leq\ & e_1^T (A_1^{sT} P_1 + P_1 A_1^s + \frac{1}{\alpha_1} P_1 P_1) e_1 + e_2^T (A_4^T P_{01} + P_{01} A_4 + P_{02} C_4 + C_4^T P_{02}^T \\ & + \frac{1}{\alpha_2} P_{01} P_{01} + \alpha_1 \mathcal{L}_{f_1}^2 \|T^{-1}\|^2 I + \alpha_2 \mathcal{L}_{f_2}^2 \|T^{-1}\|^2 I) e_2 + e_3^T (\bar{P}_{03} L + L^T \bar{P}_{03}) e_3 \\ & + 2 e_1^T P_1 A_2 e_2 + 2 e_3^T \bar{P}_{03} C_4 e_2 + 2 e_1^T P_1 T_1 \Delta\psi + 2 e_2^T P_{01} T_2 \Delta\psi \end{aligned} \tag{5.40}$$

To attain the robustness of the proposed observer to the uncertainty $\Delta\psi$ in $\mathcal{H}_\infty$ sense, the following stability constraint is imposed instead of $\dot{V} < 0$:

$$\dot{V} + \bar{r}^T \bar{r} - \mu \Delta\psi^T \Delta\psi \leq 0 \tag{5.41}$$

If this constraint holds, then the state estimation error dynamics is stable and the $\mathcal{H}_\infty$ gain of the transfer function from $\Delta\psi$ to $\bar{r}$ is norm bounded by $\sqrt{\mu}$ [108, 107]. In other words, (5.41) minimizes the worst case effect of the system uncertainty $\Delta\psi$ on the state estimation error $\bar{r}$.

$$\dot{V} + \bar{r}^T\bar{r} - \mu\Delta\psi^T\Delta\psi \leq \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \Delta\psi \end{bmatrix}^T \begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1}P_1P_1 & P_1A_2 & 0 & P_1T_1 \\ A_2^TP_1 & \Pi_2 + \frac{1}{\alpha_2}P_{01}P_{01} & C_4^T\bar{P}_{03} & P_{01}T_2 \\ 0 & \bar{P}_{03}C_4 & \Pi_3 & 0 \\ T_1^TP_1 & T_2^TP_{01} & 0 & -\mu I_r \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \Delta\psi \end{bmatrix} < 0 \tag{5.42}$$

Under zero initial conditions, it follows that

$$\begin{aligned} \int_0^T (\bar{r}^T\bar{r} - \mu\Delta\psi^T\Delta\psi)dt &\leq \int_0^T (\bar{r}^T\bar{r} - \mu\Delta\psi^T\Delta\psi)dt + V \\ &= \int_0^T (\bar{r}^T\bar{r} - \mu\Delta\psi^T\Delta\psi + \dot{V})dt \\ &\leq 0 \end{aligned} \tag{5.43}$$

which implies that

$$\int_0^T (\bar{r}^T\bar{r})dt \leq \mu\int_0^T (\Delta\psi^T\Delta\psi)dt \tag{5.44}$$

namely,

$$\|\bar{r}\|_{\mathcal{L}_2} \leq \sqrt{\mu}\|\Delta\psi\|_{\mathcal{L}_2} \tag{5.45}$$

This completes the proof.

Remark 5.2 The effect of the system uncertainty on the state estimation error is decided by the value of μ. The smaller value of μ is, the more accurate fault estimation can be obtained. The minimization of μ can be found by solving the following LMI optimization problem:

$$\begin{aligned} &min(\mu) \\ &subject \;\; to \\ &X < 0, \; P_1 > 0, \; P_{01} > 0, \bar{P}_{03} > 0 \end{aligned}$$

$$and$$

$$\begin{bmatrix} \Pi_1 & P_1A_2 & 0 & P_1T_1 & P_1 & 0 \\ A_2^TP_1 & \Pi_2 & C_4^T\bar{P}_{03} & P_{01}T_2 & 0 & P_{01} \\ 0 & \bar{P}_{03}C_4 & \Pi_3 & 0 & 0 & 0 \\ T_1^TP_1 & T_2^TP_{01} & 0 & -\mu I & 0 & 0 \\ P_1 & 0 & 0 & 0 & -\alpha_1 I & 0 \\ 0 & P_{01} & 0 & 0 & 0 & -\alpha_2 I \end{bmatrix} < 0 \tag{5.46}$$

where $\Pi_1 = X+X^T+H_1^TH_1$, $\Pi_2 = A_4^TP_{01}+P_{01}A_4+P_{02}C_4+C_4^TP_{02}^T+\alpha_1\mathcal{L}_{f_1}^2\|T^{-1}\|^2I+\alpha_2\mathcal{L}_{f_2}^2\|T^{-1}\|^2I+H_2^TH_2$, $\Pi_3 = Y+Y^T+H_3^TH_3$, $X = P_1A_1^s$, $Y = \bar{P}_{03}L$. Once the LMIs are solved, A_1^s and L can be obtained from $A_1^s = P_1^{-1}X$ and $L = \bar{P}_{03}^{-1}Y$, respectively.

Remark 5.3 The convergence speed of the observer can be imposed by using a more restrictive condition $\dot{V} \leq -2\beta V$, instead of $\dot{V} < 0$. This implies that the state estimation error e will have a decay rate of at least β. It follows that there exists a positive scalar

$$\kappa := \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}} \tag{5.47}$$

where

$$P = \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_{01} & 0 \\ 0 & 0 & \bar{P}_{03} \end{bmatrix}$$

such that $\|e(t)\| \leq \kappa\|e(0)\|\exp(-\beta t)$. Following the same procedure of the proof as that has been carried out for Proposition 5.1, it can easily be proved that the observer error dynamics is asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level $\sqrt{\mu}$ and a prescribed decay rate of β if there exist matrices X, Y, P_1, P_{01}, P_{02} and $\bar{P}_{03}$ such that the following LMI optimization problem is solvable:

$$\begin{aligned} & min(\mu) \\ & s.t. \\ & X < 0, \;\; P_1 > 0, \;\; P_{01} > 0, \;\; \bar{P}_{03} > 0, \\ & and \end{aligned}$$

$$\begin{bmatrix} \Pi_1 + 2\beta P_1 & P_1 A_2 & 0 & P_1 T_1 & P_1 & 0 \\ A_2^T P_1 & \Pi_2 + 2\beta P_{01} & C_4^T \bar{P}_{03} & P_{01} T_2 & 0 & P_{01} \\ 0 & \bar{P}_{03} C_4 & \Pi_3 + 2\beta \bar{P}_{03} & 0 & 0 & 0 \\ T_1^T P_1 & T_2^T P_{01} & 0 & -\mu I & 0 & 0 \\ P_1 & 0 & 0 & 0 & -\alpha_1 I & 0 \\ 0 & P_{01} & 0 & 0 & 0 & -\alpha_2 I \end{bmatrix} < 0 \tag{5.48}$$

Remark 5.4 Without integrating the $\mathcal{H}_\infty$ filtering feature into the proposed observers, it can be proved that the error dynamics (5.27)-(5.29) is ultimately bounded if there exist matrices X, Y, P_1, P_{01}, P_{02} and $\bar{P}_{03}$ such that

$$\underbrace{\begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1} P_1 P_1 & P_1 A_2 & 0 \\ A_2^T P_1 & \Pi_2 + \frac{1}{\alpha_2} P_{01} P_{01} & C_4^T \bar{P}_{03} \\ 0 & \bar{P}_{03} C_4 & \Pi_3 \end{bmatrix}}_{-\Sigma} < 0 \tag{5.49}$$

The magnitude of the state estimation error is ultimately bounded with respect to the set

$$\Omega = \{e: \ \|e\| < \frac{2(\|P_1 T_1\| + \|P_{01} T_2\|)}{\lambda_{min}(\Sigma)} \xi + \eta_0\} \tag{5.50}$$

where η_0 is an arbitrarily small positive scalar.

Proposition 5.1 proves that the error dynamics is asymptotically stable. The objective now is to determine the constant gain l_1 in (5.24) and l_2 in (5.26) such that the trajectories (5.27)-(5.29) can be directed toward to the sliding surface which is defined as $\mathcal{S}_1 = \{(e_1, e_2, e_3) | e_1 = 0, \ e_3 = 0\}$.

Proposition 5.2 *Under the Assumptions 5.1-5.5, an ideal sliding motion will take place after some finite time on the hyperplane $\mathcal{S}_1$ if*

$$l_1 \geq \|A_2\| \|e\| + \mathcal{L}_{f_1} \|T^{-1}\| \|e\| + \|T_1\| \xi + \eta_1 \tag{5.51}$$

$$l_2 \geq \frac{\|C_4\| \|e\|}{\|D_2^T \bar{P}_{03} e_3\|} + \eta_2 \tag{5.52}$$

where η_1 and η_2 are positive scalars.

Proof. Consider the Lyapunov candidate functions $V_1(t) = e_1^T P_1 e_1$ and $V_3(t) =$

$e_3^T \bar{P}_{03} e_3$. The time derivative can be obtained as:

$$\dot{V}_1(t) = e_1^T({A_1^s}^T P_1 + P_1 A_1^s)e_1 + 2e_1^T P_1 A_2 e_2 + 2e_1^T P_1 T_1 \Delta\psi \\ + 2e_1^T P_1 \left(f_1(T^{-1}h, t) - f_1(T^{-1}\hat{h}, t)\right) + 2e_1^T P_1 B_1 (f_a - \nu_1)$$

It is easy to see that ${A_1^s}^T P_1 + P_1 A_1^s < 0$ since $P_1 A_1^s$ is symmetric negative definite by design. Furthermore, from the Cauchy-Schwartz inequality and (5.24), we obtain

$$\begin{aligned}\dot{V}_1(t) &< 2e_1^T P_1 A_2 e_2 + 2e_1^T P_1 T_1 \Delta\psi + 2e_1^T P_1 \left(f_1(T^{-1}h, t) - f_1(T^{-1}\hat{h}, t)\right) \\ &\quad + 2e_1^T P_1 B_1 (f_a - \nu_1) \\ &\le 2\|P_1 e_1\|(\|A_2\|\|e\| + \mathcal{L}_{f_1}\|T^{-1}\|\|e\| + \|T_1\|\xi) - 2l_1\|B_1^T P_1 e_1\| \\ &\le 2\|B_1^T P_1 e_1\|\|B_1^{-T}\|(\|A_2\|\|e\| + \mathcal{L}_{f_1}\|T^{-1}\|\|e\| + \|T_1\|\xi - l_1) \end{aligned} \quad (5.53)$$

It follows from (5.51) that

$$\dot{V}_1 \le -2\eta_1 \|P_1 e_1\| \le -2\eta_1 \sqrt{\lambda_{min}(P_1)} V_1^{\frac{1}{2}}$$

Similarly, it can be verified that if (5.52) is satisfied, then

$$\dot{V}_3 \le -2\eta_2 \|\bar{P}_{03} e_0\| \le -2\eta_2 \sqrt{\lambda_{min}(\bar{P}_{03})} V_3^{\frac{1}{2}}$$

This shows that the reachability condition [61] is satisfied and a sliding motion is achieved and maintained after some finite time $t_s > 0$.

This completes the proof.

5.3.2 Estimation of actuator and sensor faults using SMOs

Given observers which are in the form of (5.23) -(5.26), the objective in this subsection is to estimate the actuator faults and sensor faults simultaneously.

From Proposition 5.2, we know that an ideal sliding motion $\mathcal{S}_1$ will take place after some finite time t_s if the conditions (5.51)-(5.52) are satisfied. Therefore during the sliding motion,

$$e_1 = \dot{e}_1 = 0, \quad \forall t > t_s \quad (5.54)$$

and the error dynamics (5.27) becomes

$$0 = 0 + A_2e_2 + (f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)) + B_1(f_a - \nu_{1eq}) + T_1\Delta\psi \tag{5.55}$$

where ν_{1eq} denotes the equivalent output error injection signal to maintain the sliding motion [8]. We further rewrite (5.55) as:

$$f_a - \nu_{1eq} = -B_1^{-1}\left(A_2e_2 + (f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)) + T_1\Delta\psi\right) \tag{5.56}$$

Computing the $\mathcal{L}_2$ norm of(5.56) yields

$$\begin{aligned}\|f_a - \nu_{1eq}\|_{\mathcal{L}_2} &= \|B_1^{-1}(A_2e_2 + (f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)) + T_1\Delta\psi)\|_{\mathcal{L}_2} \\ &\leq (\sigma_{max}(B_1^{-1}A_2) + \sigma_{max}(B_1^{-1})\mathcal{L}_{f_1}\|T^{-1}\|)\|e_2\|_{\mathcal{L}_2} + \sigma_{max}(B_1^{-1}T_1)\|\Delta\psi\|_{\mathcal{L}_2} \\ &\leq (\sigma_{max}(B_1^{-1}A_2) + \sigma_{max}(B_1^{-1})\mathcal{L}_{f_1}\|T^{-1}\|)\|e\|_{\mathcal{L}_2} + \sigma_{max}(B_1^{-1}T_1)\|\Delta\psi\|_{\mathcal{L}_2}\end{aligned} \tag{5.57}$$

Since $\|e\|_{\mathcal{L}_2} \leq \sigma_{max}(H^{-1})\sqrt{\mu}\|\Delta\psi\|_{\mathcal{L}_2}$, we can obtain

$$\begin{aligned}\|f_a - \nu_{1eq}\|_{\mathcal{L}_2} \leq (&\sqrt{\mu}(\sigma_{max}(B_1^{-1}A_2) + \sigma_{max}(B_1^{-1})\mathcal{L}_{f_1}\|T^{-1}\|)\sigma_{max}(H^{-1}) \\ &+ \sigma_{max}(B_1^{-1}T_1))\|\Delta\psi\|_{\mathcal{L}_2}\end{aligned} \tag{5.58}$$

It follows that

$$\sup_{\|\Delta\psi\|_{\mathcal{L}_2}\neq 0} \frac{\|f_a - \nu_{1eq}\|_{\mathcal{L}_2}}{\|\Delta\psi\|_{\mathcal{L}_2}} = \sqrt{\mu}\beta_1 + \beta_2 \tag{5.59}$$

where $\beta_1 = (\sigma_{max}(B_1^{-1}A_2)+\sigma_{max}(B_1^{-1})\mathcal{L}_{f_1}\|T^{-1}\|)\sigma_{max}(H^{-1})$ and $\beta_2 = \sigma_{max}(B_1^{-1}T_1)$. Thus for a small $(\sqrt{\mu}\beta_1 + \beta_2)\|\Delta\psi\|_{\mathcal{L}_2}$, the actuator fault can be approximated as

$$\hat{f}_a(t) \approx \nu_{1eq} \tag{5.60}$$

The equivalent output error injection signal can be approximated as:

$$\nu_{1eq} \approx (\rho_a + l_1)\frac{B_1^TP_1(C_1^{-1}S_1y - \hat{h}_1)}{\|B_1^TP_1(C_1^{-1}S_1y - \hat{h}_1)\| + \gamma_1} \tag{5.61}$$

where γ_1 is a small positive scalar to reduce the chattering effect. It can be shown that ν_{1eq} can be approximated to any degree of accuracy by (5.61) for a small enough choice of γ_1 [53]. Therefore the actuator fault can be estimates as:

$$\hat{f}_a(t) \approx (\rho_a + l_1)\frac{B_1^T P_1(C_1^{-1}S_1 y - \hat{h}_1)}{\|B_1^T P_1(C_1^{-1}S_1 y - \hat{h}_1)\| + \gamma_1} \tag{5.62}$$

Similarly,

$$\sup_{\|\Delta\psi\|_{\mathcal{L}_2}\neq 0} \frac{\|f_s - \nu_{2eq}\|_{\mathcal{L}_2}}{\|\Delta\psi\|_{\mathcal{L}_2}} = \sqrt{\mu}\sigma_{max}(H^{-1})\sigma_{max}(D_2^+ \bar{A}_3) \tag{5.63}$$

where D_2^+ is the left pseudo-inverse of D_2. Such a matrix always exists because D_2 is of full column rank. ν_{2eq} is the equivalent output error injection signal and can be approximated as

$$\nu_{2eq} \approx (\rho_s + l_2)\frac{D_2^T \bar{P}_{03}(w_3 - \hat{w}_3)}{\|D_2^T \bar{P}_{03}(w_3 - \hat{w}_3)\| + \gamma_2} \tag{5.64}$$

where γ_2 is a small positive scalar to reduce the chattering effect.

Therefore for small $\sqrt{\mu}\sigma_{max}(H^{-1})\sigma_{max}(D_2^+ \bar{A}_3)\|\Delta\psi\|_{\mathcal{L}_2}$, the sensor fault can be estimated as

$$\hat{f}_s(t) \approx (\rho_s + l_2)\frac{D_2^T \bar{P}_{03}(w_3 - \hat{w}_3)}{\|D_2^T \bar{P}_{03}(w_3 - \hat{w}_3)\| + \gamma_2} \tag{5.65}$$

The proposed actuator and sensor fault estimation scheme based on SMOs is shown in Fig-5.1.

5.4 Fault estimation using SMO and AO

Most of the existing robust FDI methods based on sliding mode techniques [80, 84, 104, 109] and adaptive techniques [36, 92, 102, 110, 111] have the assumption that the fault distribution matrix is matched. The solvability of Lyapunov equation together with the matching condition is called Constrained Lyapunov Problem (CLP). Necessary and sufficient conditions for solving CLP can be found in [112]. However, it is difficult to find matrices to satisfy both the Lyapunov equation and the matching condition. Therefore Assumption 5.5 is quite restrictive. In this section, an alternative method which does not need this assumption is developed. More specifically, the ac-

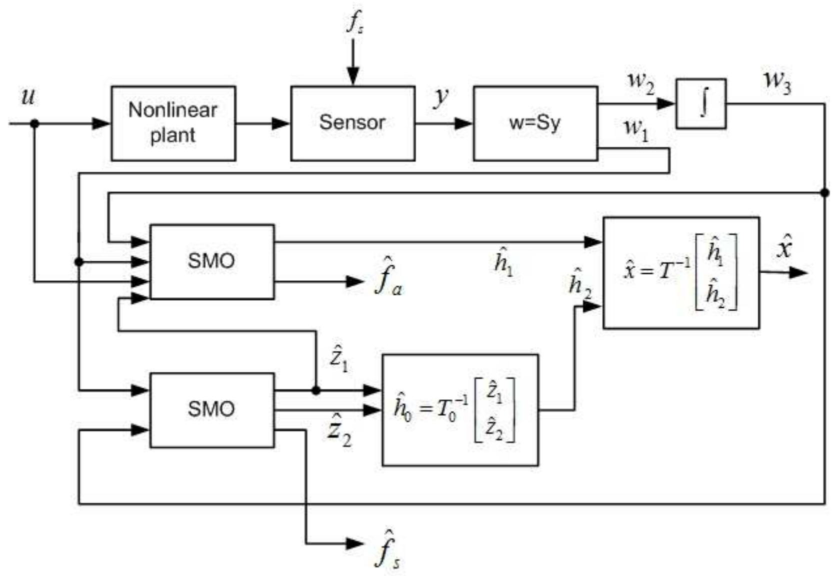

Figure 5.1: Schematic of the fault estimation using SMOs

tuator faults in subsystem (5.5) are estimated by a sliding mode observer, while the estimates of sensor faults in subsystem (5.7) is obtained by designing an adaptive observer.

5.4.1 Design of observers

The subsystems (5.15) and (5.5) in the new coordinate $z := [z_1^T \ z_2^T]^T = T_0 h_0$ become:

$$\begin{aligned} \dot{h}_1 &= A_1 h_1 + A_2 z_1 - A_2 P_{01}^{-1} P_{02} z_2 + f_1(T^{-1}h, t) + B_1(u + f_a) + T_1 \Delta\psi \\ w_1 &= C_1 h_1 \end{aligned} \tag{5.66}$$

$$\begin{aligned} \dot{z}_1 &= \bar{A}_1 z_1 + \bar{A}_2 z_2 + A_3 h_1 + f_2(T^{-1}h, t) + T_2 \Delta\psi + P_{01}^{-1} P_{02} D_2 f_s \\ \dot{z}_2 &= \bar{A}_3 z_1 + \bar{A}_4 z_2 + D_2 f_s \\ w_3 &= z_2 \end{aligned} \tag{5.67}$$

where $\bar{A}_1$, $\bar{A}_2$, $\bar{A}_3$ and $\bar{A}_4$ are defined in (5.18).

For subsystem (5.66), the proposed sliding mode observer has the form:

$$\begin{aligned} \dot{\hat{h}}_1 = {} & A_1 \hat{h}_1 + A_2 \hat{z}_1 - A_2 P_{01}^{-1} P_{02} w_3 + f_1(T^{-1}\hat{h}, t) + B_1(u + \nu) \\ & + (A_1 - A_1^s) C_1^{-1}(w_1 - \hat{w}_1) \end{aligned}$$

$$\hat{w}_1 = C_1 \hat{h}_1 \tag{5.68}$$

where $A_1^s \in \mathcal{R}^{m\times m}$ is a stable matrix, the discontinuous output error injection term ν is defined by

$$\nu = \begin{cases} (\rho_a + l)\frac{B_1^T P_1 (C_1^{-1} S_1 y - \hat{z}_1)}{\|B_1^T P_1 (C_1^{-1} S_1 y - \hat{z}_1)\|} & \text{if } C_1^{-1} S_1 y - \hat{z}_1 \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.69}$$

where $P_1 \in \mathcal{R}^{m\times m}$ is the Lyapunov matrix for A_1^s and l is a positive scalar.

For subsystem (5.67), the proposed adaptive observer has the form:

$$\begin{aligned} \dot{\hat{z}}_1 &= \bar{A}_1 \hat{z}_1 + \bar{A}_2 w_3 + A_3 C_1^{-1} w_1 + f_2(T^{-1}\hat{h}, t) + P_{01}^{-1} P_{02} D_2 \hat{f}_s \\ \dot{\hat{z}}_2 &= \bar{A}_3 \hat{z}_1 + \bar{A}_4 \hat{z}_2 + (\bar{A}_4 - L)(w_3 - \hat{w}_3) + D_2 \hat{f}_s \\ \hat{w}_3 &= \hat{z}_2 \end{aligned} \tag{5.70}$$

where $L \in \mathcal{R}^{(p-m)\times(p-m)}$ is a traditional Luenberger observer gain. $\hat{f}_s$ is the sensor fault estimates with the dynamics:

$$\dot{\hat{f}}_s = \Gamma D_2^T \bar{P}_{03} (e_3 + \dot{e}_3) \tag{5.71}$$

If the state estimation errors are defined as $e_1 = h_1 - \hat{h}_1$, $e_2 = z_1 - \hat{z}_1$, $e_3 = z_2 - \hat{z}_2$ and $e_f = f_s - \hat{f}_s$, then the state estimation error dynamics after the occurrence of faults can be obtained as:

$$\dot{e}_1 = A_1^s e_1 + A_2 e_2 + \Big(f_1(T^{-1}h, t) - f_1(T^{-1}\hat{h}, t)\Big) + B_1(f_a - \nu) + T_1 \Delta\psi \tag{5.72}$$

$$\dot{e}_2 = \tilde{A}_1 e_1 + \Big(f_2(T^{-1}h, t) - f_2(T^{-1}\hat{h}, t)\Big) + T_2 \Delta\psi + P_{01}^{-1} P_{02} D_2 e_f \tag{5.73}$$

$$\dot{e}_3 = \bar{A}_3 e_2 + L e_3 + D_2 e_f \tag{5.74}$$

Define

$$r(t) = H \begin{bmatrix} h_1(t) \\ z_1(t) \\ z_2(t) \\ f_s(t) \end{bmatrix}, \quad and \;\; \tilde{r} = H \begin{bmatrix} h_1 - \hat{h}_1 \\ z_1 - \hat{z}_1 \\ z_2 - \hat{z}_2 \\ f_s - \hat{f}_s \end{bmatrix} \tag{5.75}$$

where $r(t) \in \mathcal{R}^{n+p+q-m}$ is a linear combination of state variables to be estimated over the horizon $[0,\ T]$. H is a full rank known matrix and assumed to have the structure:

$$H := \begin{bmatrix} H_1 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 \\ 0 & 0 & H_3 & 0 \\ 0 & 0 & 0 & H_4 \end{bmatrix} \tag{5.76}$$

We now present Proposition 5.3 which gives the sufficient condition for the existence of the proposed observers with a prescribed $\mathcal{H}_\infty$ performance.

Proposition 5.3 *Under the Assumptions 5.1-5.4, the observer error dynamics is ultimately bounded with an $\mathcal{H}_\infty$ disturbance attenuation level $\sqrt{\mu} > 0$ subject to $\|\bar{r}\|_{\mathcal{L}_2} \leq \sqrt{\mu}\|\Delta\psi\|_{\mathcal{L}_2}$ if there exist matrices X, Y, P_{01}, P_{02}, $\bar{P}_{03}$, G and positive scalars α_1 and α_2 such that the following LMI optimization problem has a solution:*

$$min(\mu)$$
$$s.t.$$
$$X < 0,\ P_1 > 0,\ P_{01} > 0,\ \bar{P}_{03} > 0,\ G > 0\ \ and$$

$$\begin{bmatrix} \Pi_1 & P_1A_2 & 0 & 0 & P_1T_1 & P_1 & 0 \\ A_2^TP_1 & \Pi_2 & C_4^T\bar{P}_{03} & P_{02}D_2 - C_4^T\bar{P}_{03}D_2 & P_{01}T_2 & 0 & P_{01} \\ 0 & \bar{P}_{03}C_4 & \Pi_3 & -Y^TD_2 & 0 & 0 & 0 \\ 0 & D_2^TP_{02}^T - D_2^T\bar{P}_{03}C_4 & -D_2^TY & \Pi_4 & 0 & 0 & 0 \\ T_1^TP_1 & T_2^TP_{01} & 0 & 0 & -\mu I & 0 & 0 \\ P_1 & 0 & 0 & 0 & 0 & -\alpha_1 I & 0 \\ 0 & P_{01} & 0 & 0 & 0 & 0 & -\alpha_2 I \end{bmatrix}$$
$$< 0 \tag{5.77}$$

where $\Pi_1 = X + X^T + H_1^TH_1$, $\Pi_2 = A_4^TP_{01} + P_{01}A_4 + P_{02}C_4 + C_4^TP_{02}^T + \alpha_1\mathcal{L}_{f_1}^2\|T^{-1}\|^2I + \alpha_2\mathcal{L}_{f_2}^2\|T^{-1}\|^2I + H_2^TH_2$, $\Pi_3 = Y + Y^T + H_3^TH_3$, $\Pi_4 = G - 2D_2^T\bar{P}_{03}D_2 + H_4^TH_4$, $X = P_1A_1^s$ and $Y = \bar{P}_{03}L$. Once the problem is solved, A_1^s and L can be obtained from $A_1^s = P_1^{-1}X$ and $L = \bar{P}_{03}^{-1}Y$, respectively.

Proof. Consider the Lyapunov function as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \tag{5.78}$$

where $V_1(t) = e_1^TP_1e_1$, $V_2(t) = e_2^TP_{01}e_2$, $V_3(t) = e_3^T\bar{P}_{03}e_3$ and $V_4(t) = e_f^T\Gamma^{-1}e_f$.

The time derivative of $V_1(t)$, $V_2(t)$, $V_3(t)$ and $V_4(t)$ can be shown to be:

$$\begin{aligned}
\dot{V}_1(t) &= e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^TP_1A_2e_2 + 2e_1^TP_1T_1\Delta\psi \\
&\quad + 2e_1^TP_1\left(f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)\right) + 2e_1^TP_1B_1(f_a - \nu) \\
&\leq e_1^T(A_1^{sT}P_1 + P_1A_1^s + \frac{1}{\alpha_1}P_1P_1)e_1 + 2e_1^TP_1A_2e_2 \\
&\quad + 2e_1^TP_1T_1\Delta\psi + \alpha_1\mathcal{L}_{f_1}^2\|T^{-1}\|^2\|e_2\|^2 \qquad (5.79) \\
\dot{V}_2(t) &= e_2^T\left(\bar{A}_1^TP_{01} + P_{01}\bar{A}_1\right)e_2 + 2e_2^TP_{01}\left(f_2(T^{-1}h,t) - f_2(T^{-1}\hat{h},t)\right) \\
&\quad + 2e_2^TP_{01}T_2\Delta\psi + 2e_2^TP_{02}D_2e_f \\
&\leq e_2^T(A_4^TP_{01} + P_{01}A_4 + P_{02}C_4 + C_4^TP_{02}^T + \frac{1}{\alpha_2}P_{01}P_{01} + \alpha_2\mathcal{L}_{f_2}^2\|T^{-1}\|^2I)e_2 \\
&\quad + 2e_2^TP_{01}T_2\Delta\psi + 2e_2^TP_{02}D_2e_f \qquad (5.80) \\
\dot{V}_3(t) &= e_3^T(\bar{P}_{03}L + L^T\bar{P}_{03})e_3 + 2e_3^T\bar{P}_{03}C_4e_2 + 2e_3^T\bar{P}_{03}D_2e_f \qquad (5.81) \\
\dot{V}_4(t) &= 2e_f^T\Gamma^{-1}\dot{f}_s - 2e_f^T\Gamma^{-1}\dot{\hat{f}}_s \\
&= 2e_f^T\Gamma^{-1}\dot{f}_s - 2e_f^TD_2^T\bar{P}_{03}e_3 - 2e_f^TD_2^T\bar{P}_{03}(C_4e_2 + Le_3 + D_2e_f) \qquad (5.82)
\end{aligned}$$

For a positive definite matrix G it follows that

$$\begin{aligned}
2e_f^T\Gamma^{-1}\dot{f}_s &\leq e_f^TGe_f + \dot{f}_s^T\Gamma^{-1}G^{-1}\Gamma^{-1}\dot{f}_s \\
&\leq e_f^TGe_f + \rho_{ss}^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}) \qquad (5.83)
\end{aligned}$$

Substituting (5.83) into (5.82) yields

$$\begin{aligned}
\dot{V}_4(t) &\leq e_f^T(G - 2D_2^T\bar{P}_{03}D_2)e_f - 2e_f^TD_2^T\bar{P}_{03}e_3 - 2e_f^TD_2^T\bar{P}_{03}C_4e_2 \\
&\quad - 2e_f^TD_2^T\bar{P}_{03}Le_3 + \rho_{ss}^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}) \qquad (5.84)
\end{aligned}$$

Therefore, the derivative of $V(t)$ can be obtained as:

$$\begin{aligned}
\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) \\
&\leq e_1^T(A_1^{sT}P_1 + P_1A_1^s + \frac{1}{\alpha_1}P_1P_1)e_1 + e_2^T(A_4^TP_{01} + P_{01}A_4 + P_{02}C_4 + C_4^TP_{02}^T \\
&\quad + \frac{1}{\alpha_2}P_{01}P_{01} + \alpha_1\mathcal{L}_{f_1}^2\|T^{-1}\|^2I + \alpha_2\mathcal{L}_{f_2}^2\|T^{-1}\|^2I)e_2 + e_3^T(\bar{P}_{03}L + L^T\bar{P}_{03})e_3 \\
&\quad + e_f^T(G - 2D_2^T\bar{P}_{03}D_2)e_f + 2e_1^TP_1A_2e_2 + 2e_1^TP_1T_1\Delta\psi + 2e_2^TP_{01}T_2\Delta\psi \\
&\quad + 2e_2^TP_{02}D_2e_f + 2e_3^T\bar{P}_{03}C_4e_2 - 2e_f^TD_2^T\bar{P}_{03}C_4e_2 - 2e_f^TD_2^T\bar{P}_{03}Le_3 \\
&\quad + \rho_{ss}^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}) \qquad (5.85)
\end{aligned}$$

To attain the robustness of the proposed observers to the disturbances $\Delta\psi$ in $\mathcal{L}_2$ sense, we impose the following constraint on the stability criteria:

$$\dot{V} + \tilde{r}^T\tilde{r} - \mu\Delta\psi^T\Delta\psi < 0 \tag{5.86}$$

Substituting (5.85) and (5.75) into the constraint (5.86) yields

$$\begin{aligned}
\dot{V} + \tilde{r}^T\tilde{r} - \mu\Delta\psi^T\Delta\psi \leq & \begin{bmatrix} e_{h_1} \\ e_{z_1} \\ e_{z_2} \\ e_f \\ \Delta\psi \end{bmatrix}^T \begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1}P_1P_1 & P_1A_2 & 0 \\ A_2^TP_1 & \Pi_2 + \frac{1}{\alpha_2}P_{01}P_{01} & C_4^T\bar{P}_{03} \\ 0 & \bar{P}_{03}C_4 & \Pi_3 \\ 0 & D_2^TP_{02}^T - D_2^T\bar{P}_{03}C_4 & -D_2^T\bar{P}_{03}L \\ T_1^TP_1 & T_2^TP_{01} & 0 \end{bmatrix} \\
& \begin{matrix} \\ \\ \\ \\ \\ \end{matrix}\begin{bmatrix} 0 & P_1T_1 \\ P_{02}D_2 - C_4^T\bar{P}_{03}D_2 & P_{01}T_2 \\ -L^T\bar{P}_{03}D_2 & 0 \\ \Pi_4 & 0 \\ 0 & -\mu I_r \end{bmatrix} \begin{bmatrix} e_{h_1} \\ e_{z_1} \\ e_{z_2} \\ e_f \\ \Delta\psi \end{bmatrix} + \rho_{ss}^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1}) \\
& = -w^T\Phi w + \epsilon \\
& \leq 0
\end{aligned} \tag{5.87}$$

where $w = [e_1^T,\ e_2^T,\ e_3^T,\ e_f^T,\ \Delta\psi]^T$, $\epsilon = \rho_{ss}^2\lambda_{max}(\Gamma^{-1}G^{-1}\Gamma^{-1})$ and $-\Phi$ is the inner matrix.

When $-\Phi < 0$, one can obtain that $\dot{V}(t) \leq -\lambda_{min}(\Phi)\|w\|^2 + \epsilon - \tilde{r}^T\tilde{r} + \mu\Delta\psi^T\Delta\psi$. It follows that $\dot{V}(t) < 0$ if

$$\|w\| > \sqrt{\frac{\epsilon - \tilde{r}^T\tilde{r} + \mu\Delta\psi^T\Delta\psi}{\lambda_{min}(Q)}} \tag{5.88}$$

which implies that (e_1, e_2, e_3, e_f) will converge to a small set according to Lyapunov stability theory. Using Schur complement, $-\Phi < 0$ can be expressed in the LMI form by (5.77).

This completes the proof.

Define the sliding mode surface as $\mathcal{S}_2 = \{(e_1, e_2, e_3)|e_1 = 0\}$, the objective now is to determine the observer gain l in (5.69) such that the error dynamical system (5.72) can be driven to this the sliding surface.

Proposition 5.4 *Under the Assumptions 5.1-5.4, an ideal sliding motion will take place after some finite time on the hyperplane $\mathcal{S}_2$ if*

$$l_1 \geq \|A_2\|\|e\| + \mathcal{L}_{f_1}\|T^{-1}\|\|e\| + \|T_1\|\xi + \eta \tag{5.89}$$

where η is a positive scalar and $e = [e_1^T,\ e_2^T,\ e_3^T,\ e_f^T]^T$.

Proof. Consider the Lyapunov candidate function $V(t) = e_1^T P_1 e_1$. Its time derivative can be obtained as:

$$\begin{aligned}
\dot{V}(t) &= e_1^T(A_1^{sT}P_1 + P_1A_1^s)e_1 + 2e_1^TP_1A_2e_2 + 2e_1^TP_1T_1\Delta\psi \\
&\quad + 2e_1^TP_1\left(f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)\right) + 2e_1^TP_1B_1(f_a - \nu) \\
&< 2e_1^TP_1A_2e_2 + 2e_1^TP_1T_1\Delta\psi + 2e_1^TP_1\left(f_1(T^{-1}h,t) - f_1(T^{-1}\hat{h},t)\right) \\
&\quad + 2e_1^TP_1B_1(f_a - \nu) \\
&\leq 2\|P_1e_1\|(\|A_2\|\|e\| + \mathcal{L}_{f_1}\|T^{-1}\|\|e\| + \|T_1\|\xi) - 2l\|B_1^TP_1e_1\| \\
&\leq 2\|B_1^TP_1e_1\|\|B_1^{-T}\|(\|A_2\|\|e\| + \mathcal{L}_{f_1}\|T^{-1}\|\|e\| + \|T_1\|\xi - l)
\end{aligned} \tag{5.90}$$

It follows from (5.89) that

$$\dot{V} \leq -2\eta\|P_1e_1\| \leq -2\eta\sqrt{\lambda_{min}(P_1)}V_1^{\frac{1}{2}}$$

This shows that the reachability condition is satisfied.

The proof completes.

5.4.2 Estimation of actuator and sensor faults using SMO and AO

Following the same procedure as that is given in subsection-5.3.2, one can get that for small values of $(\sqrt{\mu}\beta_1 + \beta_2)\|\Delta\psi\|_{\mathcal{L}_2}$ and γ, where $\beta_1 = (\sigma_{max}(B_1^{-1}A_2) + \sigma_{max}(B_1^{-1})\mathcal{L}_{f_1}\|T^{-1}\|)\sigma_{max}(H^{-1})$ and $\beta_2 = \sigma_{max}(B_1^{-1}T_1)$, the actuator faults can be estimated as:

$$\hat{f}_a(t) \approx (\rho_a + l)\frac{B_1^TP_1(C_1^{-1}S_1y - \hat{h}_1)}{\|B_1^TP_1(C_1^{-1}S_1y - \hat{h}_1)\| + \gamma} \tag{5.91}$$

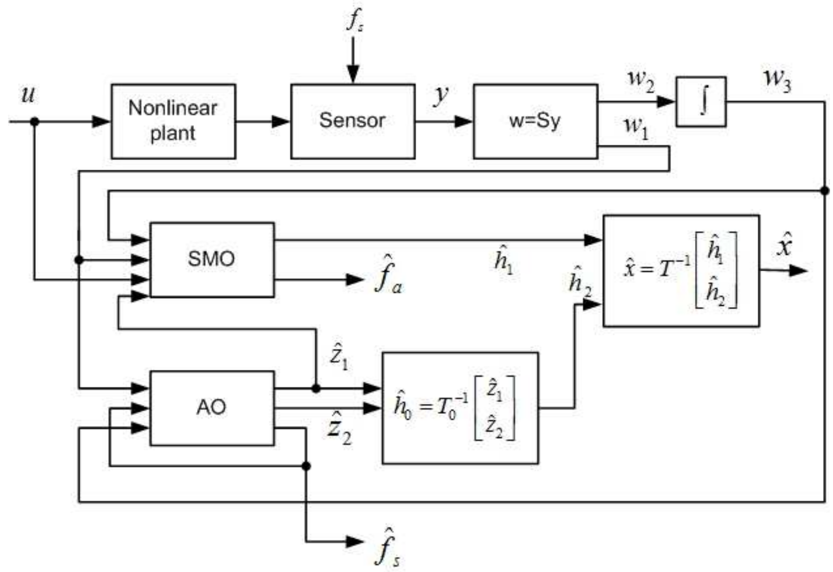

Figure 5.2: Schematic of the fault estimation using SMO and AO

From (5.71), the sensor faults can be estimated as:

$$\hat{f}_s(t) \approx \Gamma D_2^T \bar{P}_{03} e_3(t) + \Gamma D_2^T \bar{P}_{03} \int_{t_f}^{t} e_3(\tau) d\tau \tag{5.92}$$

where t_f is the time instant when a sensor fault occurs.

The proposed fault estimation scheme using SMO and AO is shown in Fig-5.2.

5.5 Simulation results

The effectiveness of the two proposed fault estimation schemes is illustrated by a numerical example in this section. The considered nonlinear system is in the form of (5.1) with

$$A = \begin{bmatrix} -5 & -1 & 0 & 1 & -2 \\ -1 & -4 & 1 & -1 & 1 \\ 0 & -1 & -2 & -2 & 1 \\ -1 & -2 & 1 & -2 & -1 \\ -2 & -1 & 2 & -1 & -3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 3 \\ 2 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$$

$$f(x,t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 2\sin x_1 \end{bmatrix}^T$$

$$\Delta\psi = 0.2\sin t$$

The actuator fault f_a and sensor fault f_s are chosen as:

$$f_a = \begin{cases} 0 & , \quad t \leq 10s \\ 0.8 & , \quad 10s < t < 20s \\ -1 & , \quad 20s \leq t < 30s \\ 0 & , \quad t \geq 30s \end{cases}$$

$$f_s = \begin{cases} 0 & , \quad t \leq 15s \\ 0.2t - 3 & , \quad 15s < t < 20s \\ -0.2t + 5 & , \quad 20s \leq t < 25s \\ 0 & , \quad t \geq 25s \end{cases}$$

Nonsingular transformations T and S are given as:

$$T = \begin{bmatrix} 1.3333 & 0 & 0 & -0.4444 & 0 \\ -0.7500 & 1.0000 & 0 & 0 & 0 \\ -0.2500 & 0 & 1.0000 & 0 & 0 \\ -0.7500 & 0 & 0 & 1.0000 & 0 \\ -0.5000 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$S = \begin{bmatrix} 1.0000 & 0 & 0 & -0.3333 \\ -0.3333 & 1.0000 & 0 & -0.5556 \\ -0.3333 & 0 & 1.0000 & 0.1111 \\ -1.0000 & 0 & 0 & 1.3333 \end{bmatrix}$$

such that in the new coordinate

$$TAT^{-1} = \left[\begin{array}{c|cccc} -6.1111 & -0.4445 & -0.4444 & -0.4937 & -2.2222 \\ \hline 0.5000 & -3.2500 & 1.0000 & -1.5278 & 2.5000 \\ -0.7500 & -0.7500 & -2.0000 & -2.5833 & 1.5000 \\ 0.2500 & -1.2500 & 1.0000 & -2.6389 & 0.5000 \\ -1.5000 & -0.5000 & 2.0000 & -2.1666 & -2.0000 \end{array}\right]$$

$$SCT^{-1} = \left[\begin{array}{c|cccc} 0.7500 & 0 & 0 & 0.0000 & 0 \\ \hline 0 & 1.0000 & 0 & -0.5556 & 0 \\ 0.0000 & 0 & 1.0000 & 0.1111 & 0 \\ -0.0000 & 0 & 0 & 1.3333 & 0 \end{array}\right]$$

$$TB = \left[\begin{array}{c} 4.0000 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array}\right], \quad TE = \left[\begin{array}{c} 0 \\ \hline 1 \\ 0 \\ 0 \\ 1 \end{array}\right], \quad SD = \left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ 2.9999 \end{array}\right]$$

Select $\mu = 1e - 4$, then from the LMI synthesis, the design parameters of the first fault estimation scheme can be obtained as:

$$P_1 = 0.2159$$
$$A_1^s = -2.3458$$
$$P_{01} = \begin{bmatrix} 0.0812 & -0.0069 & -0.0353 & -0.0779 \\ -0.0069 & 0.1362 & -0.0123 & 0.0070 \\ -0.0353 & -0.0123 & 0.1294 & 0.0346 \\ -0.0779 & 0.0070 & 0.0346 & 0.0804 \end{bmatrix}$$
$$P_{02} = \begin{bmatrix} -0.0761 & -0.5994 & -0.0000 \\ 0.7510 & 0.0422 & 0.0000 \\ -0.0246 & 0.5459 & -0.0000 \\ -0.5174 & -0.3473 & -0.0000 \end{bmatrix}$$
$$\bar{P}_{03} = \begin{bmatrix} 0.1251 & 0.0009 & 0.0203 \\ 0.0009 & 0.1320 & -0.0039 \\ 0.0203 & -0.0039 & 0.1169 \end{bmatrix}$$

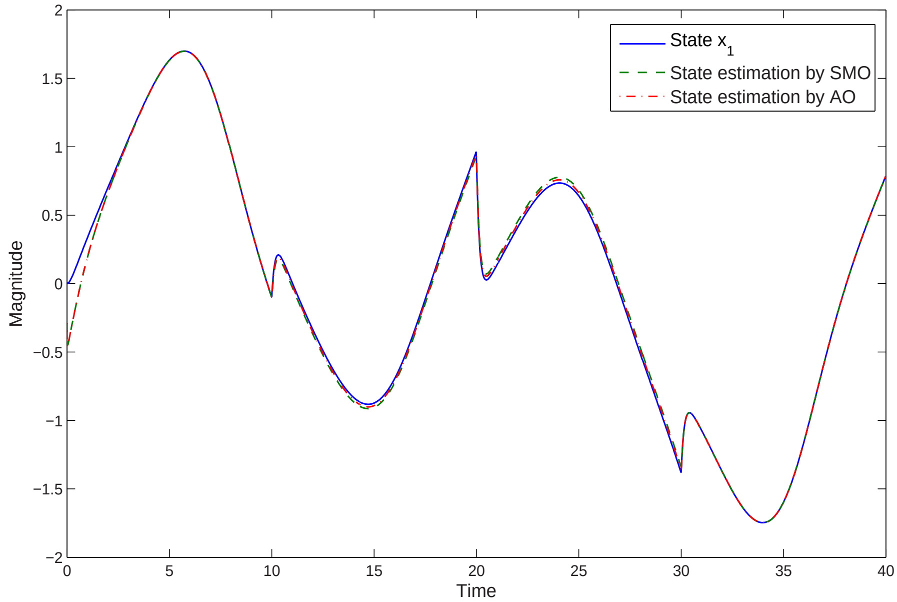

Figure 5.3: State x_1 and its estimated value $\hat{x}_1$

$$L = \begin{bmatrix} -1.4530 & 0.0189 & 0.2502 \\ 0.0236 & -1.2920 & -0.0466 \\ 0.3489 & -0.0595 & -1.5372 \end{bmatrix}$$

$$F_0 = \begin{bmatrix} 0.0610 & -0.0116 & 0.3505 \end{bmatrix}$$

For the sake of comparison, we select the same values for the parameters of the second scheme and also select

$$\Gamma = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \tag{5.93}$$

to complete the adaptive observer design.

Fig 5.3-5.7 show the true states and their estimates using SMO-based method and AO-based method, respectively. The results of fault estimation are depicted in Fig-5.8 and 5.9. It shows that despite the presence of system uncertainties $\Delta\Psi$, both methods can estimate the actuator fault and sensor fault with high accuracy.

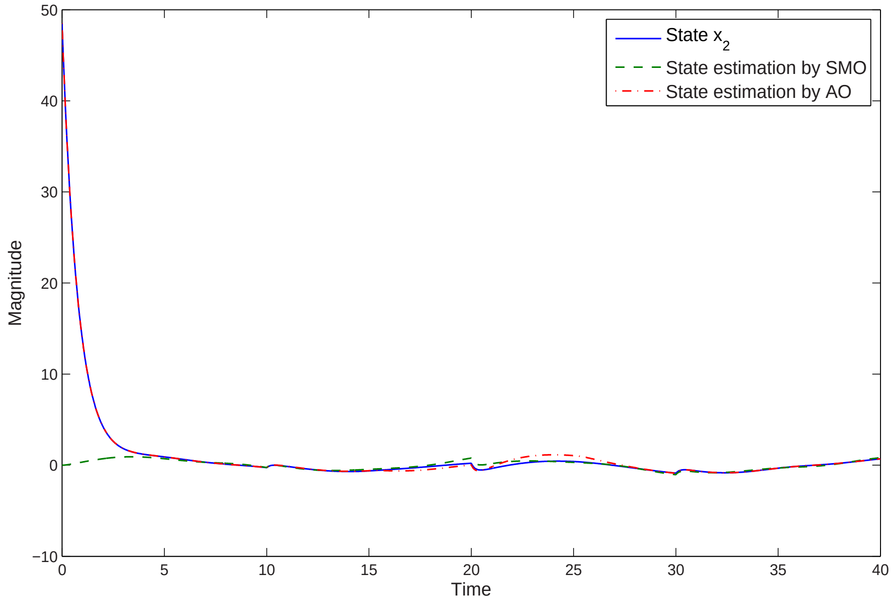

Figure 5.4: State x_2 and its estimated value $\hat{x}_2$

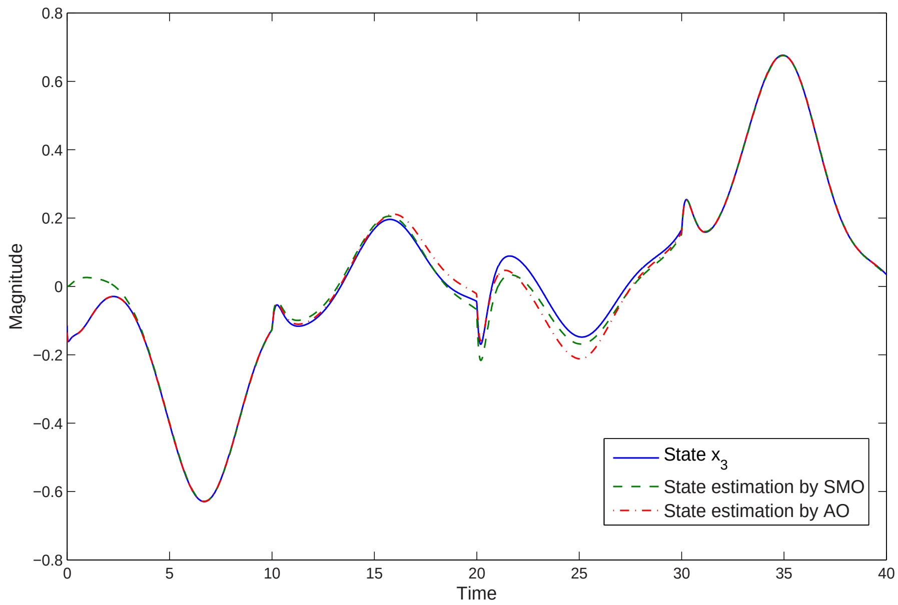

Figure 5.5: State x_3 and its estimated value $\hat{x}_3$

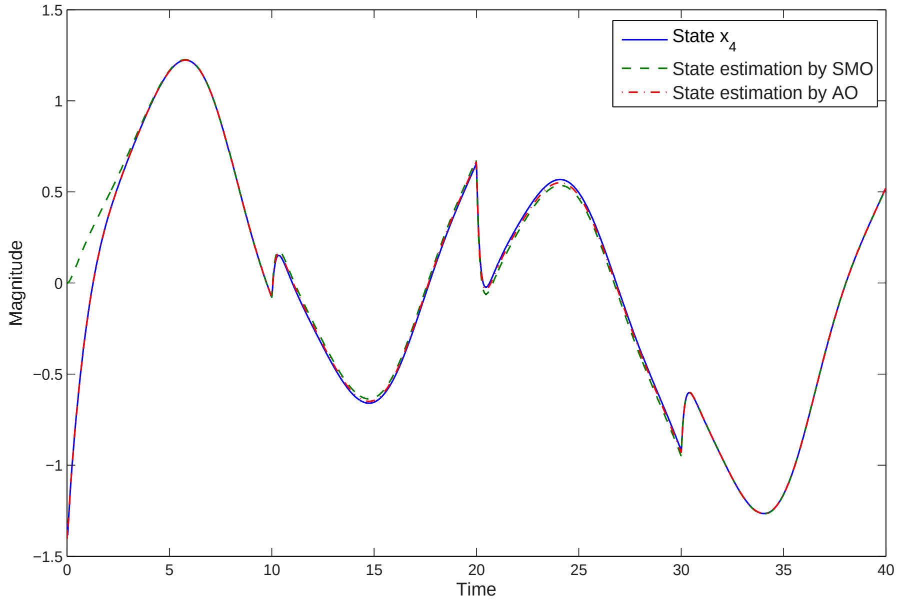

Figure 5.6: State x_4 and its estimated value $\hat{x}_4$

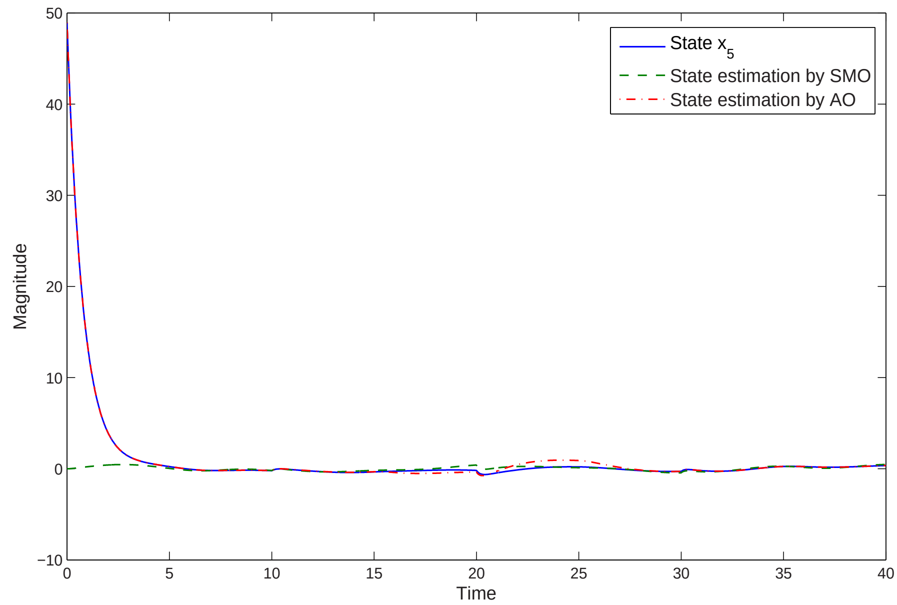

Figure 5.7: State x_5 and its estimated value $\hat{x}_5$

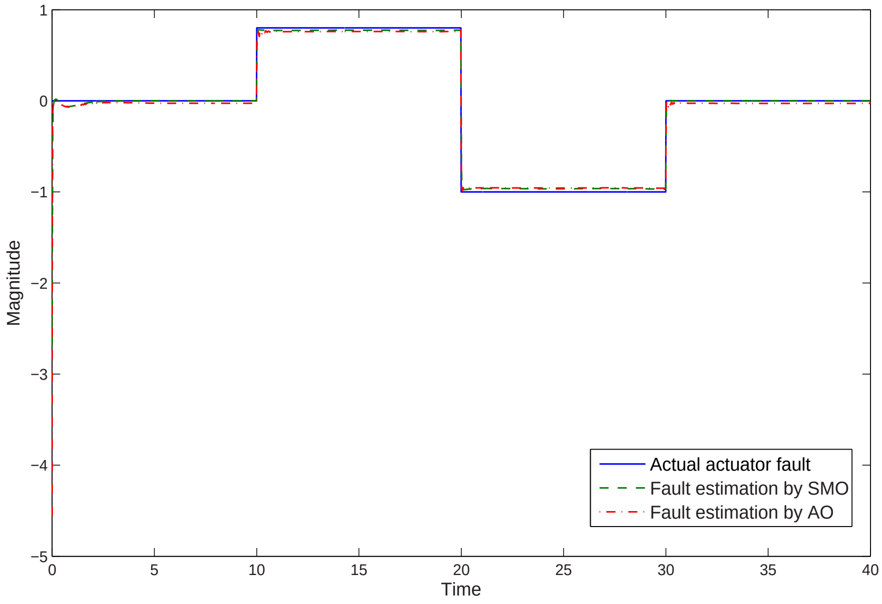

Figure 5.8: Actuator fault f_a and its estimated value $\hat{f}_a$

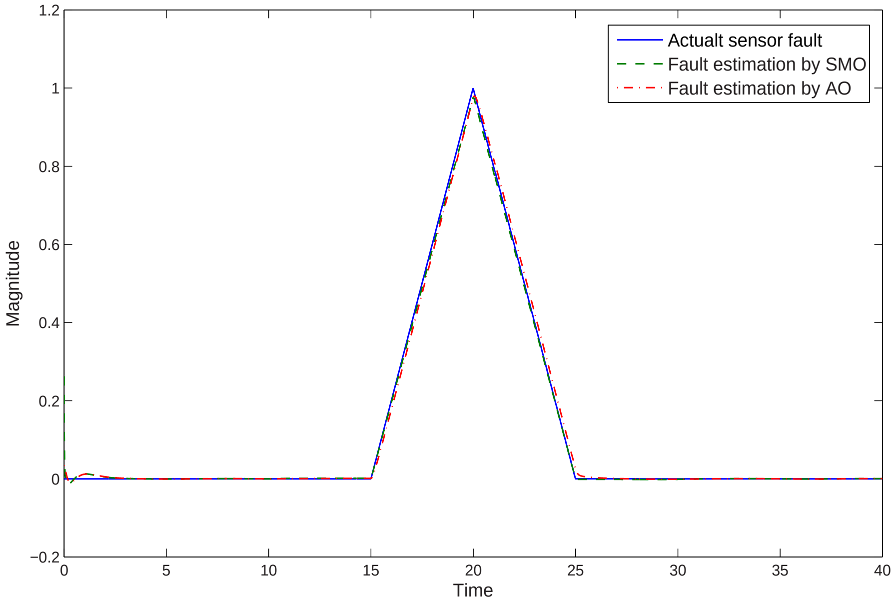

Figure 5.9: Sensor fault f_s and its estimated value $\hat{f}_s$

5.6 Conclusions

In this chapter, two schemes for simultaneously estimating actuator fault and sensor fault for uncertain Lipschitz nonlinear systems are proposed. The first scheme is based on the matching condition and consists of two SMOs. Each of them is used to estimate the actuator fault and sensor fault respectively. However, the matching condition is very restrictive and sometimes it is difficult to find such matrices to satisfy both the Lyapunov equation and the matching condition. The second scheme relaxes the constraint of the first method and employs an adaptive observer to estimate the sensor fault. Moreover, $\mathcal{H}_\infty$ is integrated into both schemes to attenuate the effects of the system uncertainties on state estimation and fault estimation. The effectiveness of the proposed methods is illustrated by considering a numerical example. Simulation results show that both methods can accurately estimate the actuator fault and sensor fault simultaneously.

It is worth noting that the uncertainty considered in this chapter is assumed to be unstructured. This type of uncertainty is usually modelled as norm-bounded perturbations and refers to the aspects of system uncertainty associated with unmodelled system dynamics, truncation of high frequency models, nonlinearities and the effects of linearization, etc. In practice, it is quite common that the uncertainties are given as a disturbance system whose elements in a disturbance matrix are not known exactly. Therefore, the methods developed in this chapter have a wide range of applications, such as aircrafts, high-speed railways and power systems.

Chapter 6

Estimation of actuator and sensor faults for uncertain nonlinear systems using a descriptor system approach

In previous chapters, sensor fault is transformed into the form of actuator fault. In this chapter, it is treated as an auxiliary state and an augmented descriptor system is constructed. Based on this system, an estimator is designed and the sensor fault can be obtained directly.

6.1 Introduction

Methods that are introduced in Chapter-4 and 5 all use an integral observer to transform sensor faults into the form of actuator faults and the design of observers is based on the transformed system. In this chapter, the sensor fault is treated in a different way. More specifically, sensor faults are taken as auxiliary states and an augmented descriptor system is therefore constructed. An estimator which is based on the descriptor system approach [51, 85, 113, 114, 115, 116, 117, 118] is designed for the augmented system, so that the simultaneous estimation of the system state and the sensor fault can directly be obtained.

In [119], a descriptor system approach has been introduced to investigate fault diagnosis for linear multi-variable systems with measurement noises. For Lipschitz nonlinear descriptor systems, the result of sensor fault estimation is shown in [116] and the work in [118] focuses on actuator fault signals. Note that the approaches proposed in [116, 118, 119] are for descriptor systems and only deal with one kind of faults at a time (either actuator fault or sensor fault). This motivates the present study to develop a novel fault estimation approach for a class of nonlinear state-space systems when faults occur at both sensors and actuators coincidentally. To cope with the actuator faults, the result of using adaptive observer-based approach to estimate actuator faults in Chapter-5 is extended in this chapter to yield the actuator fault estimation. The effects of the system uncertainties on the estimation errors of states and faults are reduced by integrating a prescribed $\mathcal{H}_\infty$ disturbance attenuation level into the observer. As a consequence of $\mathcal{H}_\infty$ filtering integrated into the descriptor observer, the estimation of states and faults is robust against uncertainties and can preserve the shape of fault signal effectively.

The main contributions of the present work are the following: 1. The uncertainty considered is unstructured and not necessarily bounded; 2. The proposed method is not only applicable for nonlinear state-space systems but also for nonlinear descriptor systems; 3. Not only simultaneous estimation of states, actuator faults and sensor faults can be obtained, but also the $\mathcal{L}_2$ gain minimization can be guaranteed at the same time; 4. Different types of faults such as unbounded faults, incipient faults and abrupt faults can be successfully estimated by the proposed method.

The rest of the chapter is organized as follows: section-6.2 introduces the problem and some mathematical preliminaries required for designing observers. In section-6.3, the observer is proposed and design parameters are formulated in LMI form. The stability condition of the proposed observers based on Lyapunov approach is derived. The results of simulation are shown in section-6.4 with conclusions in section-6.5.

6.2 Problem Formulation

Consider a nonlinear system described by

$$\dot{x}(t) = Ax(t) + f(x,t) + B(u(t) + f_a(t)) + \Delta\psi(t)$$

$$y(t) = Cx(t) + Df_s(t) \tag{6.1}$$

where $x \in \mathcal{R}^n$ are the state variables, $u \in \mathcal{R}^m$ are the inputs and $y \in \mathcal{R}^p$ are the outputs. $f_a \in \mathcal{R}^m$ and $f_s \in \mathcal{R}^q$ denote the actuator fault and sensor fault, respectively. $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{p\times n}$ and $D \in \mathcal{R}^{p\times q}$ ($p \geq q$) are known constant matrices with C and D both being of full rank. The nonlinear continuous term $f(x,t)$ is assumed to be known. The unstructured uncertainty $\Delta\psi(t) \in \mathcal{R}^n$ is more general than structured uncertainty that have been considered for fault diagnosis of Lipschitz nonlinear systems in literature (e.g.,[85, 117]).

Throughout the paper, the following assumptions are made:

Assumption 6.1 The matrix pair (A, C) is detectable.

Assumption 6.2 The nonlinear term $f(x,t)$ is assumed to be known and Lipschitz about x uniformly, i.e., $\forall x, \hat{x} \in \mathcal{X}$,

$$\|f(x,t) - f(\hat{x},t)\| \leq \mathcal{L}_f \|x - \hat{x}\| \tag{6.2}$$

where $\mathcal{L}_f$ is the known Lipschitz constant. Many nonlinearities can be assumed to be Lipschitz, at least locally.

Suppose that the sensor fault $f_s(t)$ is smooth and assume $\sigma := \dot{f}_s(t)$, then an augmented nonlinear system with states $z := \begin{bmatrix} x \\ f_s \end{bmatrix} \in \mathcal{R}^{n+q}$ can be constructed as follows

$$\begin{aligned} \bar{E}\dot{z} &= \bar{A}z + \bar{f}(x,t) + \bar{B}(u + f_a) + \bar{G}\bar{\Delta}\psi \\ y &= \bar{C}z \end{aligned} \tag{6.3}$$

where

$$\bar{E} = \begin{bmatrix} I_{n\times n} & 0_{n\times q} \\ 0_{q\times n} & I_{q\times q} \end{bmatrix} \in \mathcal{R}^{(n+q)\times(n+q)}$$

$$\bar{A} = \begin{bmatrix} A & 0_{n\times q} \\ 0_{q\times n} & 0_{q\times q} \end{bmatrix} \in \mathcal{R}^{(n+q)\times(n+q)}$$

$$\bar{B} = \begin{bmatrix} B \\ 0_{q\times m} \end{bmatrix} \in \mathcal{R}^{(n+q)\times m}$$

$$\bar{G} = \begin{bmatrix} I_{n\times n} & 0_{n\times q} \\ 0_{q\times n} & I_{q\times q} \end{bmatrix} \in \mathcal{R}^{(n+q)\times(n+q)}$$
$$\bar{C} = \begin{bmatrix} C & D \end{bmatrix} \in \mathcal{R}^{p\times(n+q)}$$
$$\bar{f}(x,t) = \begin{bmatrix} f(x,t) \\ 0_{q\times 1} \end{bmatrix} \in \mathcal{R}^{(n+q)\times 1}$$
$$\bar{\Delta\psi} = \begin{bmatrix} \Delta\psi \\ \sigma \end{bmatrix} \in \mathcal{R}^{(q+r)\times 1}$$

6.3 Design of the fault estimation observer

For the augmented system formulated in (6.3), the objective of the section is to design an observer to simultaneously estimate the states, actuator faults and sensor faults. The developed observer has the following form:

$$\dot{w} = (\bar{A} - L_1\bar{C})\hat{z} + \bar{B}(u + \hat{f}_a) + L_1 y + \bar{f}(\hat{x}, t) \tag{6.4}$$
$$\hat{z} = (\bar{E} + L_2\bar{C})^{-1}(w + L_2 y) \tag{6.5}$$
$$\hat{y} = \bar{C}\hat{x} \tag{6.6}$$

where $\hat{z} \in \mathcal{R}^{n+q}$ represents the estimated states of the augmented system, $L_1 \in \mathcal{R}^{(n+q)\times p}$ and $L_2 \in \mathcal{R}^{(n+q)\times p}$ are two observer gains. L_2 is selected to make $\bar{S} = \bar{E} + L_2\bar{C}$ nonsingular, $\hat{f}_a$ is the actuator fault estimation with the dynamics:

$$\dot{\hat{f}}_a = \Gamma F(e_y + \dot{e}_y) \tag{6.7}$$

where $\Gamma \in \mathcal{R}^{m\times m}$ is a symmetric positive definite matrix representing the learning rate, $F \in \mathcal{R}^{m\times p}$ is a design matrix which needs to be determined and $e_y = y - \hat{y}$ is the output estimation error.

It follows from (6.5) that the derivative of $\hat{z}$ can be obtained as:

$$\dot{\hat{z}} = \bar{S}^{-1}(\dot{w} + L_2\dot{y}) \tag{6.8}$$

Substituting (6.4) into (6.8) yields:

$$\dot{\hat{z}} = \bar{S}^{-1}((\bar{A} - L_1\bar{C})\hat{z} + \bar{B}(u + \hat{f}_a) + L_1 y + L_2\dot{y} + \bar{f}(\hat{x}, t)) \tag{6.9}$$

Note that system (6.3) can be rewritten as:

$$\dot{z} = \bar{S}^{-1}((\bar{A} - L_1\bar{C})z + \bar{B}(u + f_a) + L_1 y + L_2\dot{y} + \bar{f}(x,t) + \bar{G}\bar{\Delta}\psi) \tag{6.10}$$

Define the state estimation error and actuator fault estimation error as $e = z - \hat{z}$ and $e_f = f_a - \hat{f}_a$, respectively. Then after the occurrence of any faults, the dynamics of the state estimation error $e = z - \hat{z}$ can be obtained by comparing (6.9) and (6.10):

$$\dot{e} = \bar{S}^{-1}((\bar{A} - L_1\bar{C})e + (\bar{f}(x,t) - \bar{f}(\hat{x},t)) + \bar{B}e_f + \bar{G}\bar{\Delta}\psi) \tag{6.11}$$

Lemma 6.1 There exists an asymptotic estimator in the form of (6.4)-(6.5) for the system (6.3) if and only if Assumption 6.1 holds.

Proof. From the Popov-Belevitch-Hautus (PBH) test for observability, the pair $(\bar{S}^{-1}\bar{A}, \bar{C})$ is detectable if and only if

$$rank \begin{bmatrix} sI_{n+q} - \bar{S}^{-1}\bar{A} \\ \bar{C} \end{bmatrix} = n + q \tag{6.12}$$

for all $s \in \mathcal{C}$.

From the definition of $\bar{S}$, $\bar{A}$ and $\bar{C}$,

$$\begin{aligned}
& rank \begin{bmatrix} sI_{n+q} - \bar{S}^{-1}\bar{A} \\ \bar{C} \end{bmatrix} \\
&= rank \left\{ \begin{bmatrix} \bar{S} & 0_{(n+q)\times p} \\ 0_{p\times(n+q)} & I_p \end{bmatrix} \begin{bmatrix} sI_{n+q} - \bar{S}^{-1}\bar{A} \\ \bar{C} \end{bmatrix} \right\} \\
&= rank \begin{bmatrix} s\bar{S} - \bar{A} \\ \bar{C} \end{bmatrix} \\
&= rank \begin{bmatrix} s\bar{E} + sL_2\bar{C} - \bar{A} \\ \bar{C} \end{bmatrix} \\
&= rank \left\{ \begin{bmatrix} I_{n+q} & sL_2 \\ 0_{p\times(n+q)} & I_p \end{bmatrix} \begin{bmatrix} s\bar{E} - \bar{A} \\ \bar{C} \end{bmatrix} \right\} \\
&= rank \begin{bmatrix} s\bar{E} - \bar{A} \\ \bar{C} \end{bmatrix} = rank \begin{bmatrix} sI_{n+q} - A & 0_{(n+q)\times q} \\ 0_{q\times(n+q)} & sI_q \\ C & D \end{bmatrix}
\end{aligned}$$

$$= rank \begin{bmatrix} sI_n - A \\ C \end{bmatrix} + q \tag{6.13}$$

It follows that if Assumption 6.1 is satisfied, then

$$rank \begin{bmatrix} sI_n - A \\ C \end{bmatrix} = n \Rightarrow rank \begin{bmatrix} sI_{n+q} - \bar{S}^{-1}\bar{A} \\ \bar{C} \end{bmatrix} = n + q \tag{6.14}$$

which implies that the pair $(\bar{S}^{-1}\bar{A}, \bar{C})$ is detectable and the gain L can be chosen as $L = \bar{S}^{-1}L_1$ such that the error dynamics (6.11) is stable.

This completes the proof.

Define that

$$r(t) = H \begin{bmatrix} z(t) \\ f_a(t) \end{bmatrix}, \quad \hat{r}(t) = H \begin{bmatrix} \hat{z}(t) \\ \hat{f}_a(t) \end{bmatrix}, \quad and \ \bar{r} = H \begin{bmatrix} e \\ e_f \end{bmatrix} \tag{6.15}$$

where $r(t) \in \mathcal{R}^{n+q+m}$ is a linear combination of the augmented state variables and actuator faults, $\hat{r}(t)$ is the estimation and $\bar{r}$ is the estimation error, H is the pre-specified weight matrix which is assumed to have the form:

$$H := \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \tag{6.16}$$

where $H_1 \in \mathcal{R}^{(n+q)\times(n+q)}$ and $H_2 \in \mathcal{R}^{m\times m}$.

The objective of this section is to design an observer in the form of (6.4)-(6.5) such that the observer error dynamics is asymptotically stable and $\bar{r}$ satisfies the following $\mathcal{H}_\infty$ tracking performance:

$$J = \int_0^T \|\bar{r}\|^2 dt \leq \mu \int_0^T (\|\dot{f}_a\|^2 + \|\bar{\Delta\psi}\|^2) dt \tag{6.17}$$

The sufficient condition of stability of the proposed fault estimator is as follows:

Proposition 6.1 *Under the Assumptions 6.1-6.2, the observer error dynamics (6.11) and $\dot{e}_f$ are asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level $\sqrt{\mu} > 0$ subject to $\|\bar{r}\|_{\mathcal{L}_2} \leq \sqrt{\mu}(\|\bar{\Delta\psi}\|_{\mathcal{L}_2} + \|\dot{f}_a\|_{\mathcal{L}_2})$, if there exist matrices $P \in \mathcal{R}^{(n+q)\times(n+q)} >$*

0, $Y \in \mathcal{R}^{(n+q)\times p}$, $F \in \mathcal{R}^{m\times p}$ *and positive scalars* α_1 *and* α_2 *such that:*

$$\Omega := \begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1} P\bar{S}^{-1}\bar{S}^{-T}P & \Pi_2 + \bar{C}^T Y^T P^{-1}\bar{C}^T F^T & 0 & P\bar{S}^{-1}\bar{G} \\ \Pi_2^T + F\bar{C}P^{-1}Y\bar{C} & \Pi_3 + \frac{1}{\alpha_2} F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T & \Gamma^{-1} & -F\bar{C}\bar{S}^{-1}\bar{G} \\ 0 & \Gamma^{-T} & -\mu I_m & 0 \\ \bar{G}^T\bar{S}^{-T}P & -\bar{G}^T\bar{S}^{-T}\bar{C}^T F^T & 0 & -\mu I_{n+q} \end{bmatrix} < 0 \tag{6.18}$$

where $\Pi_1 = P\bar{S}^{-1}\bar{A} + \bar{A}^T\bar{S}^{-T}P - Y\bar{C} - \bar{C}^T Y^T + \alpha_1\mathcal{L}_f^2 I_{n+q} + \alpha_2\mathcal{L}_f^2 I_{n+q} + H_1^T H_1$, $\Pi_2 = P\bar{S}^{-1}\bar{B} - \bar{C}^T F^T - \bar{A}^T\bar{S}^{-T}\bar{C}^T F^T$, $\Pi_3 = -2F\bar{C}\bar{S}^{-1}\bar{B} + H_2^T H_2$. *The matrix gain* L_1 *can be obtained as* $L_1 = \bar{S}P^{-1}Y$.

Proof. Consider the Lyapunov function as

$$V(t) = V_1(t) + V_2(t) \tag{6.19}$$

where $V_1(t) = e^T P e$ and $V_2(t) = e_f^T \Gamma e_f$.

The time derivative of $V_1(t)$ along the trajectories of state estimation error dynamics (6.11) can be shown to be:

$$\begin{aligned} \dot{V}_1 &= e^T P\dot{e} + \dot{e}^T P e \\ &= e^T P((\bar{S}^{-1}\bar{A} - \bar{S}^{-1}L_1\bar{C})e + \bar{S}^{-1}(\bar{f}(x,t) - \bar{f}(\hat{x},t)) + \bar{S}^{-1}\bar{B}e_f + \bar{S}^{-1}\bar{G}\bar{\Delta\psi}) \\ &\quad + ((\bar{S}^{-1}\bar{A} - \bar{S}^{-1}L_1\bar{C})e + \bar{S}^{-1}(\bar{f}(x,t) - \bar{f}(\hat{x},t)) + \bar{S}^{-1}\bar{B}e_f + \bar{S}^{-1}\bar{G}\bar{\Delta\psi})^T P e \\ &= e^T(P\bar{S}^{-1}\bar{A} + \bar{A}^T\bar{S}^{-T}P - Y\bar{C} - \bar{C}^T Y^T)e + 2e^T P\bar{S}^{-1}(\bar{f}(x,t) - \bar{f}(\hat{x},t)) \\ &\quad + 2e^T P\bar{S}^{-1}\bar{B}e_f + 2e^T P\bar{S}^{-1}\bar{G}\bar{\Delta\psi} \qquad (6.20) \\ &\quad \le e^T(P\bar{S}^{-1}\bar{A} + \bar{A}^T\bar{S}^{-T}P - Y\bar{C} - \bar{C}^T Y^T)e + \frac{1}{\alpha_1} e^T P\bar{S}^{-1}\bar{S}^{-T}Pe \\ &\quad + \alpha_1\mathcal{L}_f^2\|e\|^2 + 2e^T P\bar{S}^{-1}\bar{B}e_f + 2e^T P\bar{S}^{-1}\bar{G}\bar{\Delta\psi} \\ &= e^T(P\bar{S}^{-1}\bar{A} + \bar{A}^T\bar{S}^{-T}P - Y\bar{C} - \bar{C}^T Y^T + \frac{1}{\alpha_1}P\bar{S}^{-1}\bar{S}^{-T}P + \alpha_1\mathcal{L}_f^2 I_{n+q})e \\ &\quad + 2e^T P\bar{S}^{-1}\bar{B}e_f + 2e^T P\bar{S}^{-1}\bar{G}\bar{\Delta\psi} \qquad (6.21) \end{aligned}$$

The time derivative of $V_2(t)$ can be shown to be:

$$\begin{aligned} \dot{V}_2 &= 2e_f^T\Gamma^{-1}\dot{e}_f \\ &= 2e_f^T\Gamma^{-1}\dot{f}_a - 2e_f^T\Gamma^{-1}\dot{\hat{f}}_a \end{aligned}$$

$$
\begin{aligned}
&= 2e_f^T\Gamma^{-1}\dot{f}_a - 2e_f^T F\bar{C}e - 2e_f^T F\bar{C}\dot{e} \\
&= 2e_f^T\Gamma^{-1}\dot{f}_a - 2e_f^T F\bar{C}e - 2e_f^T F\bar{C}(\bar{S}^{-1}\bar{A} - \bar{S}^{-1}L_1\bar{C})e - 2e_f^T F\bar{C}\bar{S}^{-1}(\bar{f}(x,t) \\
&\quad - \bar{f}(\hat{x},t)) - 2e_f^T F\bar{C}\bar{S}^{-1}\bar{B}e_f - 2e_f^T F\bar{C}\bar{S}^{-1}\bar{G}\bar{\Delta}\psi \\
&\leq 2e_f^T\Gamma^{-1}\dot{f}_a - 2e_f^T F\bar{C}e - 2e_f^T F\bar{C}(\bar{S}^{-1}\bar{A} - \bar{S}^{-1}L_1\bar{C})e + \frac{1}{\alpha_2}e_f^T F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T e_f \\
&\quad + \alpha_2\mathcal{L}_f^2\|e\|^2 - 2e_f^T F\bar{C}\bar{S}^{-1}\bar{B}e_f - 2e_f^T F\bar{C}\bar{S}^{-1}\bar{G}\bar{\Delta}\psi \\
&= e_f^T(-2F\bar{C}\bar{S}^{-1}\bar{B} + \frac{1}{\alpha_2}F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T)e_f + 2e_f^T(-F\bar{C} - F\bar{C}\bar{S}^{-1}\bar{A} \\
&\quad + F\bar{C}\bar{S}^{-1}L_1\bar{C})e + \alpha_2\mathcal{L}_f^2\|e\|^2 + 2e_f^T\Gamma^{-1}\dot{f}_a - 2e_f^T F\bar{C}\bar{S}^{-1}\bar{G}\bar{\Delta}\psi
\end{aligned} \tag{6.22}
$$

Therefore

$$
\begin{aligned}
\dot{V} &\leq e^T(\Pi_1 + \frac{1}{\alpha_1}P\bar{S}^{-1}\bar{S}^{-T}P)e + e_f^T(-2F\bar{C}\bar{S}^{-1}\bar{B} + \frac{1}{\alpha_2}F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T)e_f \\
&\quad + 2e^T(P\bar{S}^{-1}\bar{B} - \bar{C}^T F^T - \bar{A}^T\bar{S}^{-T}\bar{C}^T F^T + \bar{C}^T L_1^T\bar{S}^{-T}\bar{C}^T F^T)e_f \\
&\quad + 2e^T P\bar{S}^{-1}\bar{G}\bar{\Delta}\psi + 2e_f^T\Gamma^{-1}\dot{f}_a - 2e_f^T F\bar{C}\bar{S}^{-1}\bar{G}\bar{\Delta}\psi
\end{aligned} \tag{6.23}
$$

Let

$$
V_0 = \dot{V} + \bar{r}^T\bar{r} - \mu\varpi^T\varpi \tag{6.24}
$$

where $\varpi = \begin{bmatrix} \dot{f}_a \\ \bar{\Delta}\psi \end{bmatrix}$.

Substituting (6.23) and (6.15) into (6.24) yields

$$
V_0 \leq \begin{bmatrix} e \\ e_f \\ \dot{f}_a \\ \bar{\Delta}\psi \end{bmatrix}^T \begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1}P\bar{S}^{-1}\bar{S}^{-T}P & \Pi_2 + \bar{C}^T Y^T P^{-1}\bar{C}^T F^T & 0 & P\bar{S}^{-1}\bar{G} \\ \Pi_2^T + F\bar{C}P^{-1}Y\bar{C} & \Pi_3 + \frac{1}{\alpha_2}F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T & \Gamma^{-1} & -F\bar{C}\bar{S}^{-1}\bar{G} \\ 0 & \Gamma^{-T} & -\mu I_m & 0 \\ \bar{G}^T\bar{S}^{-T}P & -\bar{G}^T\bar{S}^{-T}\bar{C}^T F^T & 0 & -\mu I_{q+r} \end{bmatrix} \begin{bmatrix} e \\ e_f \\ \dot{f}_a \\ \bar{\Delta}\psi \end{bmatrix} \tag{6.25}
$$

It follows from (6.18)that

$$
V_0 = \dot{V} + \bar{r}^T\bar{r} - \mu\varpi^T\varpi < 0 \tag{6.26}
$$

Under zero initial conditions, it is easy to see that

$$\begin{aligned}\int_0^T (\bar{r}^T\bar{r} - \mu\varpi^T\varpi)dt &\leq \int_0^T (\bar{r}^T\bar{r} - \mu\varpi^T\varpi)dt + V \\ &= \int_0^T (\bar{r}^T\bar{r} - \mu\varpi^T\varpi + \dot{V})dt \\ &\leq 0 \end{aligned} \tag{6.27}$$

which implies that $\int_0^T \|\bar{r}\|^2 dt \leq \mu \int_0^T (\|\dot{f}_a\|^2 + \|\Delta\bar{\psi}\|^2)dt$. This function minimizes the worst case effect of the disturbance ϖ on the state estimation error $\bar{r}$. If this constraint holds, then the state estimation error dynamics is stable and the $\mathcal{H}_\infty$ gain of the transfer function from ϖ to $\bar{r}$ is norm bounded by $\sqrt{\mu}$ [107, 108]. It is clear that the smaller the μ is, the more robust the observer becomes.

This completes the proof.

To obtain matrices P, Y, F and positive scalars α_1, α_2 and ϵ in (6.18), we firstly recall the following lemma.

Lemma 6.2 (See [120, 121]) Given matrices $Q = Q^T$, F, M and N of appropriate dimensions, then

$$Q + MFN + N^T F^T M^T < 0 \tag{6.28}$$

for all F satisfying $F^T F \leq I$ if and only if there exists a scalar $\epsilon > 0$ such that

$$Q + \epsilon M M^T + \epsilon^{-1} N^T N < 0 \tag{6.29}$$

It is easy to show that the matrix inequality $\Omega < 0$ in (6.18) can be rewritten as

$$\begin{aligned}&\begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1} P\bar{S}^{-1}\bar{S}^{-T}P & \Pi_2 & 0 & P\bar{S}^{-1}\bar{G} \\ \Pi_2^T + F\bar{C}P^{-1}Y\bar{C} & \Pi_3 + \frac{1}{\alpha_2} F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T & \Gamma^{-1} & -F\bar{C}\bar{S}^{-1}\bar{G} \\ 0 & \Gamma^{-T} & -\mu I_m & 0 \\ \bar{G}^T\bar{S}^{-T}P & -\bar{G}^T\bar{S}^{-T}\bar{C}^T F^T & 0 & -\mu I_{q+r} \end{bmatrix} \\ &+ \begin{bmatrix} \bar{C}^T Y^T \\ 0 \\ 0 \\ 0 \end{bmatrix} P^{-1} \begin{bmatrix} 0 & \bar{C}^T F^T & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ F\bar{C} \\ 0 \\ 0 \end{bmatrix} P^{-1} \begin{bmatrix} Y\bar{C} & 0 & 0 & 0 \end{bmatrix} < 0 \end{aligned} \tag{6.30}$$

Applying Lemma 6.2 to (6.30) with $M = \begin{bmatrix} \bar{C}^T Y^T \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $N = \begin{bmatrix} 0 & \bar{C}^T F^T & 0 & 0 \end{bmatrix}$ and $F = P^{-1}$ gives that the satisfaction of (6.30) is equivalent to the following two inequalities:

$$P^{-2} < I_{n+q}, \tag{6.31}$$

$$\begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1} P\bar{S}^{-1}\bar{S}^{-T}P & \Pi_2 & 0 & P\bar{S}^{-1}\bar{G} \\ \Pi_2^T + F\bar{C}P^{-1}Y\bar{C} & \Pi_3 + \frac{1}{\alpha_2} F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T & \Gamma^{-1} & -F\bar{C}\bar{S}^{-1}\bar{G} \\ 0 & \Gamma^{-T} & -\mu I_m & 0 \\ \bar{G}^T\bar{S}^{-T}P & -\bar{G}^T\bar{S}^{-T}\bar{C}^T F^T & 0 & -\mu I_{q+r} \end{bmatrix}$$
$$+\epsilon \begin{bmatrix} \bar{C}^T Y^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & Y\bar{C} & 0 & 0 & 0 \end{bmatrix} + \epsilon^{-1} \begin{bmatrix} 0 \\ F\bar{C} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & \bar{C}^T F^T 0 & 0 \end{bmatrix} < 0 \tag{6.32}$$

where ϵ is a positive scalar.

Using Schur complement, (6.32) can further be written as

$$\begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1} P\bar{S}^{-1}\bar{S}^{-T}P & \Pi_2 & 0 & P\bar{S}^{-1}\bar{G} & 0 & \bar{C}^T Y^T \\ \Pi_2^T & \Pi_3 + \frac{1}{\alpha_2} F\bar{C}\bar{S}^{-1}\bar{S}^{-T}\bar{C}^T F^T & \Gamma^{-1} & -F\bar{C}\bar{S}^{-1}\bar{G} & F\bar{C} & 0 \\ 0 & \Gamma^{-T} & -\mu I_m & 0 & 0 & 0 \\ \bar{G}^T\bar{S}^{-T}P & -\bar{G}^T\bar{S}^{-T}\bar{C}^T F^T & 0 & -\mu I_{q+r} & 0 & 0 \\ 0 & \bar{C}^T F^T & 0 & 0 & -\epsilon I_{n+q} & 0 \\ Y\bar{C} & 0 & 0 & 0 & 0 & -\epsilon^{-1} I_{n+q} \end{bmatrix} < 0 \tag{6.33}$$

It is worth noting that (6.31) and (6.33) are non-convex feasibility problems. The problem of finding $P > 0$, Y, F and positive scalars α_1, α_2, ϵ, μ to satisfy both (6.31) and (6.33) can be converted into the following nonlinear minimization problem using cone complementary linearization (CCL) algorithm [122]. This method is easy to implement, however, the performance extremely depends on initial point and the specific problem to be solved so that the method often fails to converge in practice.

$$\begin{aligned} & min\ \ trace(P\bar{P} + \epsilon\bar{\epsilon}I_{n+q} + \mu I_{n+q}) \\ & s.t. \\ & P > 0,\ \ \bar{P} > 0,\ \ \alpha_1 > 0,\ \ \alpha_2 > 0,\ \ \mu > 0,\ \ \epsilon > 0,\ \ \bar{\epsilon} > 0, \end{aligned} \tag{6.34}$$

$$\begin{bmatrix} \Pi_1 & \Pi_2 & 0 & P\bar{S}^{-1}\bar{G} & 0 & \bar{C}^TY^T & P\bar{S}^{-1} & 0 \\ \Pi_2^T & \Pi_3 & \Gamma^{-1} & -F\bar{C}\bar{S}^{-1}\bar{G} & F\bar{C} & 0 & 0 & F\bar{C}\bar{S}^{-1} \\ 0 & \Gamma^{-T} & -\mu I_m & 0 & 0 & 0 & 0 & 0 \\ \bar{G}^T\bar{S}^{-T}P & -\bar{G}^T\bar{S}^{-T}\bar{C}^TF^T & 0 & -\mu I_{q+r} & 0 & 0 & 0 & 0 \\ 0 & \bar{C}^TF^T & 0 & 0 & -\epsilon I & 0 & 0 & 0 \\ Y\bar{C} & 0 & 0 & 0 & 0 & -\bar{\epsilon}I & 0 & 0 \\ \bar{S}^{-T}P & 0 & 0 & 0 & 0 & 0 & -\alpha_1 I & 0 \\ 0 & \bar{S}^{-T}\bar{C}^TF^T & 0 & 0 & 0 & 0 & 0 & -\alpha_2 I \end{bmatrix} < 0, \tag{6.35}$$

$$\begin{bmatrix} P & I \\ I & \bar{P} \end{bmatrix} \geq 0\ \ and\ \ \begin{bmatrix} \epsilon & I \\ I & \bar{\epsilon} \end{bmatrix} \geq 0 \tag{6.36}$$

where $\bar{P}$ and $\bar{\epsilon}$ are two new variables which are defined as $\bar{P} = P^{-1}$ and $\bar{\epsilon} = \epsilon^{-1}$.

The LMI optimization problem derived here seeks two objectives. The first one is to obtain the matrices P, Y and F from the LMIs and compute the observer gain L_1 from $L_1 = \bar{S}P^{-1}Y$; while the second objective is to boost the robustness of the observer against uncertainties ϖ by minimizing the $\mathcal{H}_\infty$ gain between the controlled estimation error $\bar{r}$ and ϖ. To solve this LMI optimization problem, the following iterative algorithm is being used:

- Step 1. Set i=0 and solve (6.34)-(6.36) to obtain the initial solutions $(P^0, \bar{P}^0, Y^0, F^0, \alpha_1^0, \alpha_2^0, \epsilon^0, \bar{\epsilon}^0, \mu^0)$

- Step 2. Solve the minimization problem: Minimize $trace(P\bar{P}^i + \bar{P}P^i + \epsilon\bar{\epsilon}^i I + \bar{\epsilon}\epsilon^i + \mu^i)$ subject to (6.34)-(6.36). The obtained solutions are denoted as $(P^{i+1}, \bar{P}^{i+1}, Y^{i+1}, F^{i+1}, \alpha_1^{i+1}, \alpha_2^{i+1}, \epsilon^{i+1}, \bar{\epsilon}^{i+1}, \mu^{i+1})$.

- Step 3. Check if the obtained solutions satisfy (6.31) and (6.33). If they do, then $L_1^{i+1} = \bar{S}P^{i+1^{-1}}Y^{i+1}$ is the desired observer gain and EXIT. Otherwise, set i=i+1 and return to step 2.

Remark 6.2 If there exists an estimator in the form of (6.4)-(6.5) for the system (6.3), then the estimation of the original system state x and sensor fault f_s can be obtained simultaneously. More specifically, the state estimation $\hat{x}$ can be obtained as $\hat{x} = \begin{bmatrix} I_n & 0_{n\times q} \end{bmatrix} \hat{z}$ and the sensor fault estimation $\hat{f}_s$ can be obtained as $\hat{f}_s = \begin{bmatrix} 0_{q\times n} & I_q \end{bmatrix} \hat{z}$. From (6.7), the actuator fault can be calculated as

$$\hat{f}_a(t) = \Gamma F \left(e_y(t) + \int_{t_f}^{T} e_y(t)dt \right) \tag{6.37}$$

where t_f is the time when an actuator fault occurs.

Remark 6.3 The result of the Proposition 6.1 for a state-space system in the form of (6.1) can easily be extended to a descriptor system with the following form

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + f(x,t) + B(u(t) + f_a(t)) + \Delta\psi(t) \\ y(t) &= Cx(t) + Df_s(t) \end{aligned} \tag{6.38}$$

where $E \in \mathcal{R}^{n\times n}$ is singular or nonsingular. Accordingly, the matrix $\bar{E}$ of the augmented system (6.3) changes to

$$\bar{E} = \begin{bmatrix} E & 0_{n\times q} \\ 0_{q\times n} & I_{q\times q} \end{bmatrix} \in \mathcal{R}^{(n+q)\times(n+q)} \tag{6.39}$$

6.4 Simulation results

Case-1. The effectiveness of the proposed fault estimation method for a class of nonlinear state-space system is illustrated by considering the following system

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}-1 & 0\\ 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 0.5\sin x_1\end{bmatrix} + \begin{bmatrix}1\\ 0.2\end{bmatrix}(u+f_a)+\Delta\psi\\ y &= \begin{bmatrix}2 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2\\ 4\end{bmatrix}f_s \end{aligned} \tag{6.40}$$

For illustration purpose, the input to the system is given by $u = 4sin(t/3)$. $\Delta\psi = \begin{bmatrix}0.02\sin(20t)\\ 0.05\cos(10t)\end{bmatrix}$ denotes the high-frequency unstructured uncertainty. The actuator fault f_a and sensor fault f_s have the form

$$\begin{aligned} f_a &= \begin{cases} 0 & , \quad t \le 20s\\ \sin(0.5t) + 0.2\sin(5t) & , \quad 20s < t < 30s\\ 0 & , \quad t \ge 30s \end{cases}\\ f_s &= \begin{cases} 0 & , \quad t \le 15s\\ 0.2(t-15) + 0.1\sin(5t) & , \quad t > 15s \end{cases} \end{aligned}$$

Choosing

$$L_2 = \begin{bmatrix}1.2 & 0.4\\ 0.2 & 2.5\\ 0 & 5\end{bmatrix}, \quad H_1 = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}, \quad H_2 = 0.2, \quad \Gamma = 100$$

and solving the LMI optimization problem described in (6.34)-(6.36) gives

$$\begin{aligned} \mu &= 0.0335\\ P &= \begin{bmatrix}1.4393 & -0.4951 & -0.3172\\ -0.4951 & 2.2044 & 1.0691\\ -0.3172 & 1.0691 & 2.0125\end{bmatrix}\\ \bar{P} &= \begin{bmatrix}1.2685 & -0.5183 & -0.5237\\ -0.5183 & 2.1502 & 1.2908\\ -0.5237 & 1.2908 & 2.5450\end{bmatrix} \end{aligned}$$

$$Y = \begin{bmatrix} 0.4213 & -0.2107 \\ 0.4378 & -0.2273 \\ -0.1135 & 0.3243 \end{bmatrix}$$

$$F = \begin{bmatrix} 0.0943 & -0.0412 \end{bmatrix}$$

$$\epsilon = 0.9493, \quad \bar{\epsilon} = 1.0534$$

$$\alpha_1 = 16.0370, \quad \alpha_2 = 13.0040$$

It can be verified that $P\bar{P} = I_3$ and $\epsilon\bar{\epsilon} = 1$, and the guaranteed disturbance attenuation level is $\|\mathcal{H}\|_\infty \leq \sqrt{\mu} = 0.1831$.

By using $L_1 = \bar{S}P^{-1}Y$, the observer gain L_1 can be obtained as

$$L_1 = \begin{bmatrix} 1.4570 & -0.2004 \\ -0.0729 & 1.0460 \\ -1.4924 & 2.9949 \end{bmatrix}$$

The state estimation results are shown in Fig-6.1 and 6.2. The state estimations can track their true values before and after the occurrence of any fault. The results of sensor fault estimation and actuator fault estimation are shown in Fig-6.3 and 6.4. It is worth noting that the sensor fault considered in the simulation is unbounded, which is often assumed in many fault diagnosis methods that the fault is bounded and the upper bound is known (e.g.,[46, 123]). It can be seen from the figures that despite the presence of disturbances ϖ, the proposed scheme can still estimate sensor faults and actuator faults accurately.

Case-2. The effectiveness of the proposed observer for estimating actuator faults and sensor faults for a class of nonlinear state-space system has been illustrated in *Case-1*. In this case, we will further test effectiveness of the proposed observer for descriptor systems. Consider the following plant

$$\begin{aligned} \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5\sin x_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0.2 \end{bmatrix} (u + f_a) \\ &\quad + \Delta\psi \\ y &= \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} f_s \end{aligned} \tag{6.41}$$

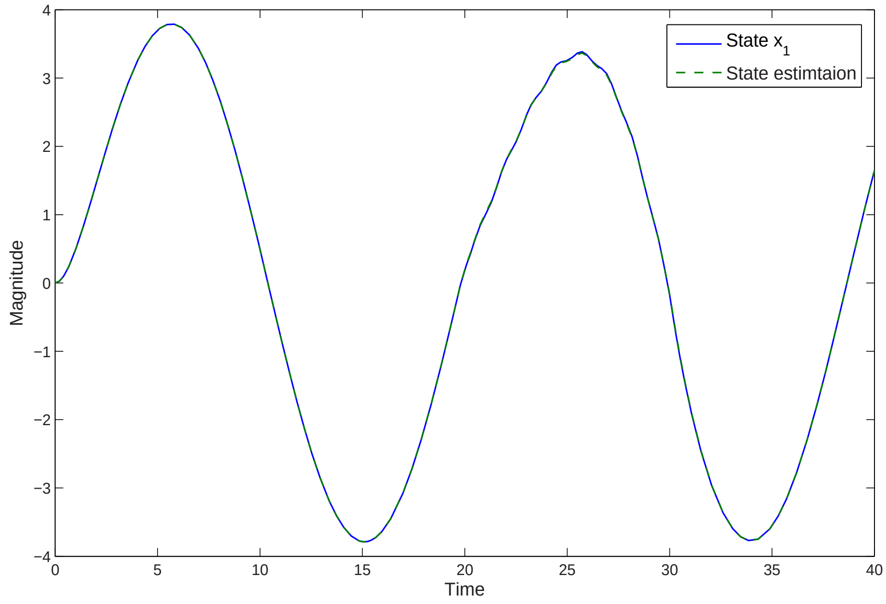

Figure 6.1: State x_1 and its estimated value $\hat{x}_1$

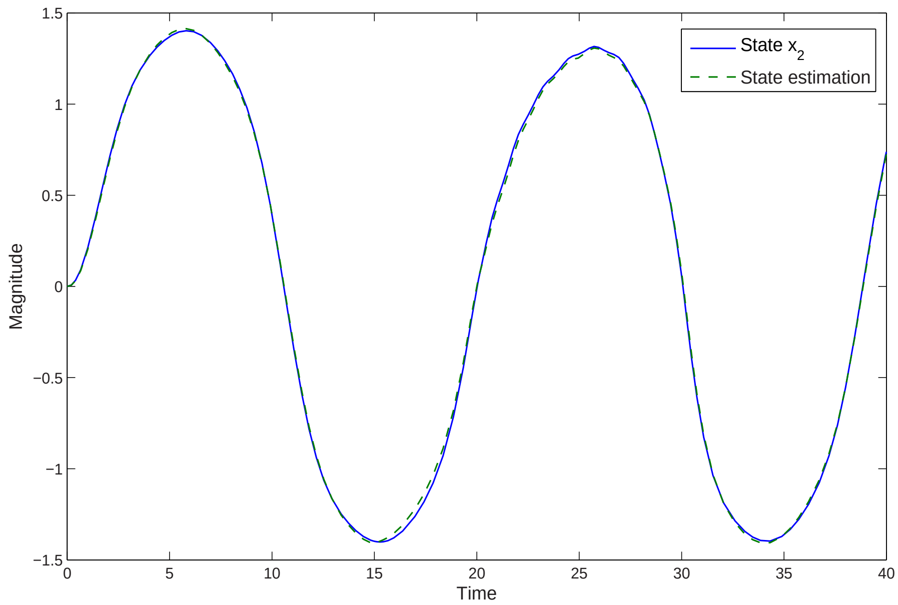

Figure 6.2: State x_2 and its estimated value $\hat{x}_2$

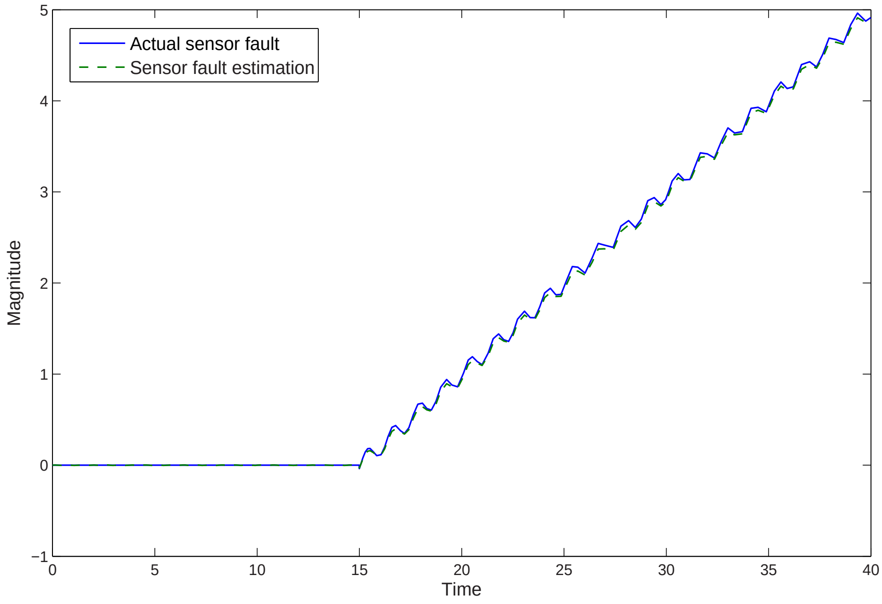

Figure 6.3: Sensor fault f_s and its estimated value $\hat{f}_s$

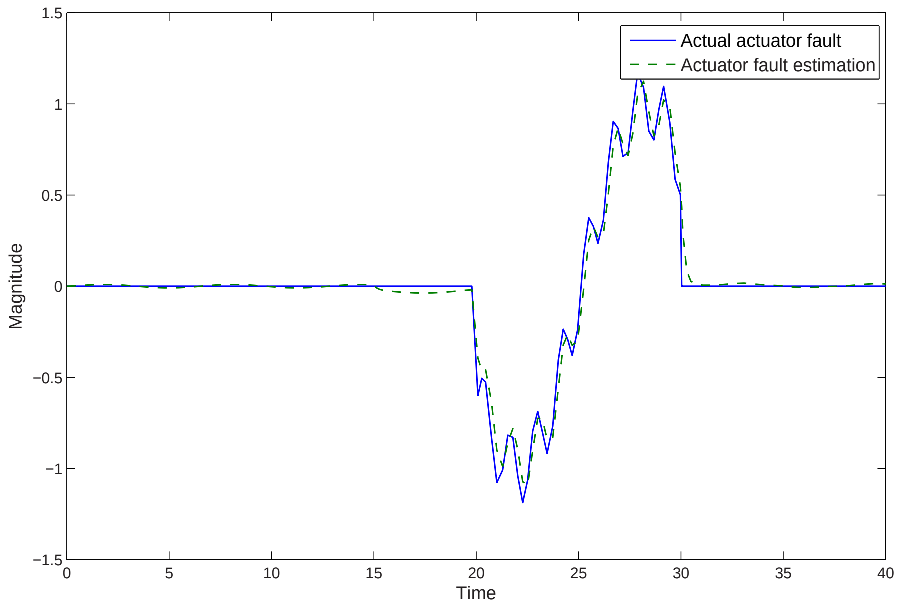

Figure 6.4: Actuator fault f_a and its estimated value $\hat{f}_a$

where $\Delta\psi = \begin{bmatrix} 0.02\sin(t) \\ 0.05\cos(t) \end{bmatrix}$ denotes the low-frequency uncertainty. This system is in the form of (6.38) with $E = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$. The actuator fault f_a and sensor fault f_s are represented as

$$\begin{aligned} f_a &= \begin{cases} 0 & , \quad t \leq 15s \\ \exp(0.01t) & , \quad t > 15s \end{cases} \\ f_s &= \begin{cases} 0 & , \quad t \leq 20s \\ 1 - \exp(-0.04(t-20)) & , \quad t > 20s \end{cases} \end{aligned}$$

It should be emphasized that both actuator fault and sensor fault are incipient. This kind of faults are difficult to detect because their sizes are small during the initial phase.

Choosing

$$L_2 = \begin{bmatrix} 1.2 & 0.4 \\ 0.2 & 2.5 \\ 0 & 5 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H_2 = 0.2, \quad \Gamma = 200$$

and formulating the LMIs described in (6.34)-(6.36) with $\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}$ gives the following solutions:

$$\begin{aligned} \mu &= 0.0094 \\ P &= \begin{bmatrix} 1.4988 & -0.0870 & -0.1187 \\ -0.0870 & 1.4693 & 0.4727 \\ -0.1187 & 0.4727 & 1.4957 \end{bmatrix} \\ \bar{P} &= \begin{bmatrix} 0.6723 & 0.0252 & 0.0454 \\ 0.0252 & 0.7586 & -0.2377 \\ 0.0454 & -0.2377 & 0.7473 \end{bmatrix} \\ Y &= \begin{bmatrix} 0.4724 & -0.2362 \\ 0.1768 & -0.1026 \\ 0.0297 & 0.2875 \end{bmatrix} \\ F &= \begin{bmatrix} 0.1128 & -0.0491 \end{bmatrix} \\ \epsilon &= 0.8468, \quad \bar{\epsilon} = 1.1810 \end{aligned}$$

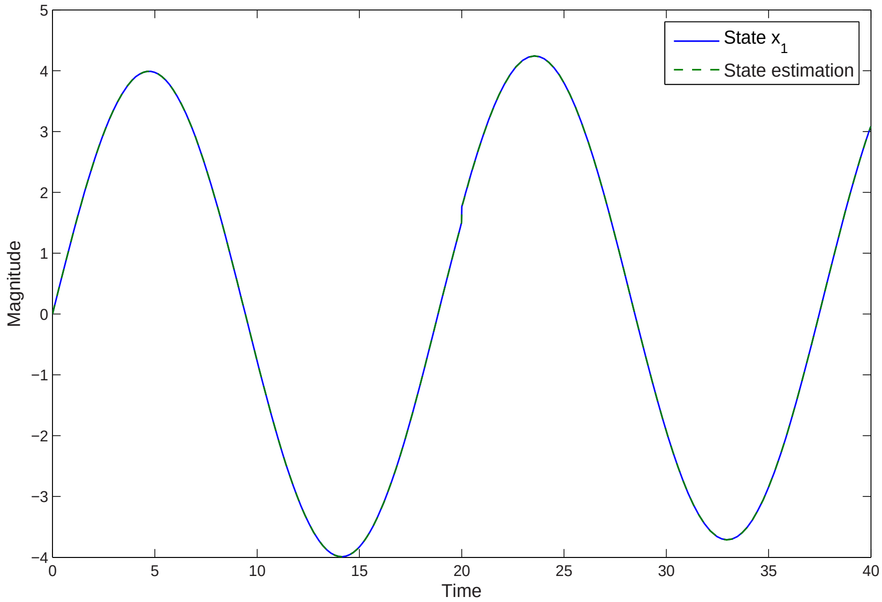

Figure 6.5: State x_1 and its estimated value $\hat{x}_1$

$$\alpha_1 = 11.7127, \quad \alpha_2 = 10.3196$$

The observer gain L_1 can be computed as

$$L_1 = \begin{bmatrix} 0.9310 & 0.3132 \\ 0.3380 & 1.5928 \\ -0.1939 & 4.0195 \end{bmatrix}$$

Fig-6.5-6.8 exhibit the states, sensor faults and actuator faults and their estimated trajectories, respectively. It can be seen from the figures that the estimates of sensor faults and actuator faults can still preserve the fault signal shape even in the presence of comparatively large system uncertainties.

6.5 Conclusion

In this chapter, a fault estimation scheme based on a descriptor system approach has been presented to simultaneously estimate system states, actuator faults and sensor faults of Lipschitz nonlinear systems. Specifically, the sensor faults are taken as auxiliary states and the original state-space system is transformed into a descriptor

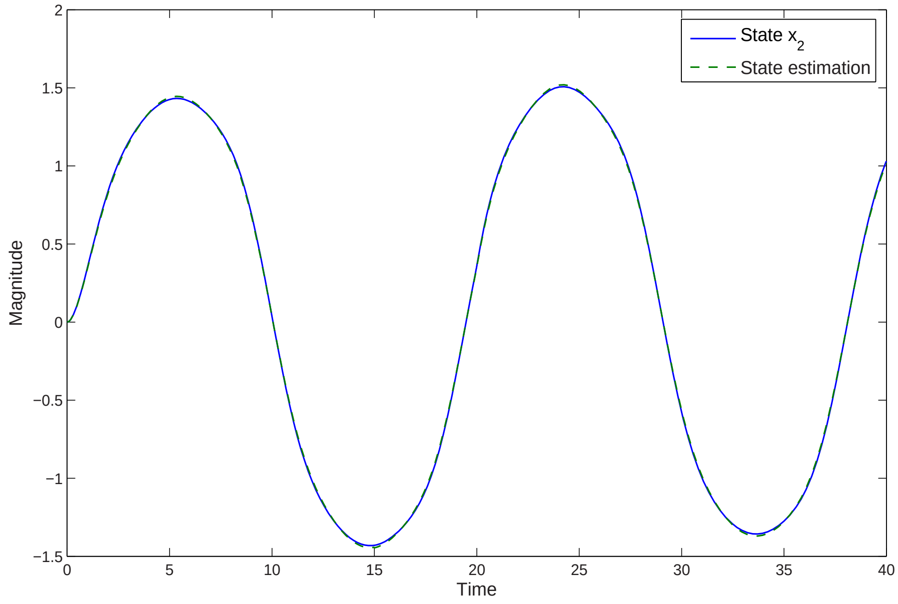

Figure 6.6: State x_2 and its estimated value $\hat{x}_2$

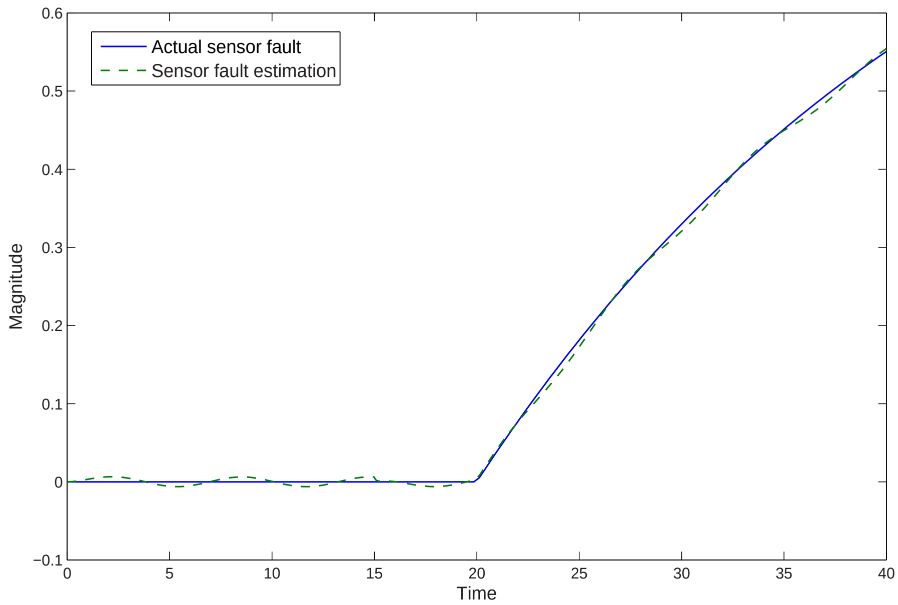

Figure 6.7: Sensor fault f_s and its estimated value $\hat{f}_s$

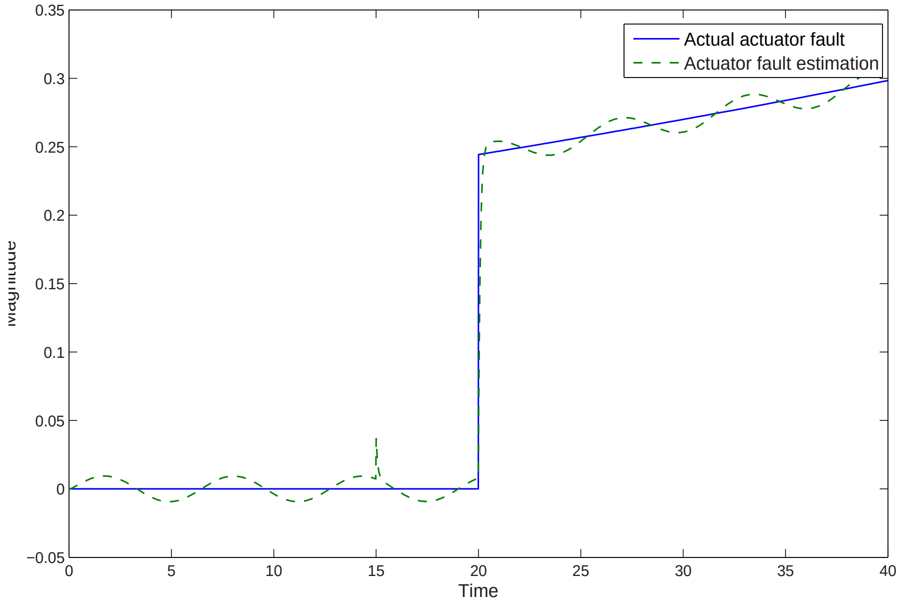

Figure 6.8: Actuator fault f_a and its estimated value $\hat{f}_a$

system accordingly. The estimation of sensor faults are obtained from the observer which is designed for this descriptor system, as a part of the augmented state vector. While the estimation of actuator faults is obtained using an adaptive observer. The design procedure has been presented by using the LMI approach. The efficiency of the proposed fault estimation scheme has been illustrated by considering two numerical examples. It shows from the results, the proposed method is not only able to successfully estimate the states and faults for nonlinear state-space systems, but also for nonlinear descriptor systems. The tracking performance of our approach is satisfactory for different types of faults, such as unbounded faults, incipient faults and abrupt faults.

Chapter 7

Conclusions and future work

7.1 Conclusions

This thesis proposes novel observer-based methods to diagnose faults for a class of nonlinear systems. Detection, isolation and estimation of actuator faults and sensor faults are three main topics which have been investigated in this study.

The nonlinear systems under consideration are assumed to be Lipschitz about the state uniformly, and are contaminated by modelling discrepancies and external disturbances, which are lumped as additive uncertainties. These unknown inputs may also cause changes in residuals such that the variations caused by real faults are concealed, and therefore make the model-based FDI ineffective. In order to deal with various uncertainties encountered in the problem of fault detection, isolation and estimation, robust observer based fault diagnosis schemes have been proposed by using sliding mode observers, adaptive observers and descriptor system approaches.

Initially, the estimation of actuator fault is studied in Chapter-2 and the result forms the basis of Chapter-5. FDI of incipient sensor faults is studied in Chapters-3 where a traditional Luenberger observer is designed to detect the occurrence of a fault and a bank of SMOs is used to diagnose the location. The problem of sensor fault estimation is addressed by two methods in Chapter-4. One method uses a SMO to estimate the sensor fault while the other one uses an adaptive observer to get the sensor fault estimation. Adaptation laws are imposed into both the methods to cope with the situation that the Lipschitz constant is unknown or too large, which may result in a failure of finding the observer design from LMIs. In Chapter-5, two methods are developed to explore the problem of simultaneous actuator and sensor fault estimation. The proposed methods are not only capable of estimating sensor faults, but also actuator faults at the same time. In the first method, it is assumed that the matching condition holds. Based on this assumption, two SMOs can be designed and the actuator fault and sensor fault can be estimated separately using the equivalent output injection term. In the second method, the assumption that the matching condition needs to be satisfied is removed. Instead of using a SMO to estimate the sensor fault in the first method, an adaptive observer is employed here. While the estimation of the actuator fault is still obtained by using a SMO. In both schemes, the $\mathcal{H}_\infty$ filtering is integrated to minimize the effects of uncertainties on the fault estimation. In Chapter-6, the problem of simultaneous estimation of actuator and sensor fault is further explored using the descriptor system approach. Based on this approach, the sensor faults can be directly estimated as auxiliary states rather than being transformed into actuator faults. In each chapter, the sufficient condition for the existence of observers is derived based on the Lyapunov method. The effectiveness of the proposed methodologies are verified by practical and numerical examples in each chapter. Simulation results show that the methods developed in this thesis can successfully detect, isolate and estimate the fault signals, and can achieve the prescribed performance.

The main contributions of this study are as follows:

1. Fault diagnosis for Lipschitz nonlinear systems with *structured non-parametric* uncertainties

- For a class of Lipschitz nonlinear systems with *matched* non-parametric uncertainties, the scheme of actuator fault estimation is proposed. Based on the matching condition, a state transformation is introduced to impose specific structures on the uncertainty and fault distribution matrices. The proposed scheme can only estimate actuator faults, but also reconstruct them under certain geometric conditions.

- For a class of Lipschitz nonlinear systems with *unmatched* non-parametric uncertainties, the incipient sensor FDI scheme based on SMOs are developed. Incipient faults are almost unnoticeable and their effects to residuals are most likely to be concealed by system uncertainties. Using the proposed method, sensor faults can be completely separated from uncertainties and easily detected and isolated.

- For the same class of uncertain Lipschitz nonlinear systems, two sensor fault estimation schemes, based on SMOs and AO respectively, are proposed. Adaptation laws are integrated into the design of observers, which makes the proposed schemes more applicable for the situation when the Lipschitz constant is large or unknown. Without the adaptation laws, the LMI solver may not provide a feasible solution when Lipschitz constant is too large.

2. Fault diagnosis for Lipschitz nonlinear systems with *unstructured non-parametric* uncertainties

- For a class of Lipschitz nonlinear systems with *unstructured* non-parametric uncertainties, nonsingular coordinate transformations are first introduced to split the original system into two subsystems. One subsystem only has actuator faults while the other only has sensor faults. Based on the transformed systems, several fault estimation schemes, based on SMOs, AO and descriptor system approaches respectively, are developed.

- By integrating $\mathcal{H}_\infty$ filtering into the design of fault estimators, fault estimation errors as well as the state estimation errors can be guaranteed to be less than a prescribed performance level, irrespective of uncertainties. It is shown that by by adjusting a single design parameter, it becomes possible to trade off between fault reconstruction performance and robustness to unknown inputs.

3. Sufficient conditions for the existence and stability of the proposed fault estimators are expressed in the form of LMIs. The problem of finding matrices to satisfy both Lyapunov equation and matching condition is modelled as a convex optimization problem and an LMI design procedure, which is solvable using commercially available software package, is presented.

In summary, the research in this thesis demonstrates that the proposed fault diagnosis schemes based on sliding mode observers, adaptive techniques and descriptor system theory are effective in dealing with fault detection, isolation and estimation for uncertain Lipschitz nonlinear systems.

7.2 Future work

The robust observer-based fault diagnosis has been studied extensively over the last two decades. However, this research area still remains open. The following are some of the directions that could be pursued in the future research:

- The thesis assumes that the system uncertainties are additive unknown terms. This is questionable because the model uncertainty describes internal property of the dynamic systems. The future work will extend the results of the proposed schemes in the thesis to systems with parametric uncertainties and other types of uncertainties.

- Networked control systems (NCSs) have advantageous over traditional systems in many aspects such as efficiency, practicality, energy consumption, installation , etc. However, one of the major problems of NCSs is the channel time delay and quantization error due to the limited communication capacity. The network-induced delay, including sensor-to-controller delay and controller-to-actuator delay, will deteriorate the system performance as well as stability. Therefore it is desirable to develop fault diagnosis schemes for networked control systems and for nonlinear systems with fixed or varying time delay in the states, outputs. The schemes proposed in this thesis are believed to have the potential to be extended to such systems.

- In this thesis, only fault detection, isolation and estimation were studied. However, successful fault diagnosis is not the ultimate goal for real applications. Fault-tolerant control (FTC) is needed to preserve the stability and reliability of the system when it is subject to a set of possible faults. The existing strategies of fault compensation control are based on adding an additional control input to the original control input in order to reduce or compensate the effects of faults, so that the controlled system can still continue to operate according to its original specifications. The additional input signal can be obtained from fault estimation and therefore the fault estimation schemes proposed in this thesis forms the foundation for FTC systems.

- As engineering plants grow in size and complexity, and the popularity of distributed systems, FDI for large scale nonlinear systems becomes increasingly important. In general, a fault that occurs in one subsystem will not only affect the behavior of this system, but it will also affect the behavior of the neighboring subsystems. It is believed that the results of this thesis can be extended to FDI for large scale nonlinear systems by taking interactions between subsystems into account.

- This thesis assumes that the output equation is linear. More complicated systems with both state dynamics and output dynamics being nonlinear can be studied in the future.

Appendix A

Proof of lemma 2.1

Lemma 2.1 There exist arbitrary matrices $F_1 \in \mathcal{R}^{r\times p}$ and $F_2 \in \mathcal{R}^{q\times p}$ such that:

$$\begin{bmatrix} E^T \\ D^T \end{bmatrix} P = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} C \tag{A.1}$$

if and only if Assumption 2.2 holds.

Proof. 1. Proof of sufficiency

It is shown in [8] that under Assumption 2.2, there exists a state transformation matrix T such that in the new coordinate, the transformed system matrices become $\bar{A} = TAT^{-1}$, $\bar{B} = TB$, $\bar{C} = CT^{-1}$, $\bar{D} = TD$ and $\bar{E} = TE$, where $\bar{C}$, $\bar{E}$ and $\bar{D}$ have the structure:

$$\bar{C} = \begin{bmatrix} 0 & I_p \end{bmatrix}, \quad \bar{D} = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} 0 \\ E_2 \end{bmatrix} \tag{A.2}$$

Therefore $A = T^{-1}\bar{A}T$, $B = T^{-1}\bar{B}$, $C = \bar{C}T$, $D = T^{-1}\bar{D}$ and $E = T^{-1}\bar{E}$.

If we select $P = T^T\bar{P}T$, $F_1 = \bar{F}_1$ and $F_2 = \bar{F}_2$, then substituting C, E and D into (A.1) yields

$$\begin{bmatrix} \bar{E}^T \\ \bar{D}^T \end{bmatrix} \bar{P} = \begin{bmatrix} \bar{F}_1 \\ \bar{F}_2 \end{bmatrix} \bar{C} \tag{A.3}$$

Letting $\bar{P} = \begin{bmatrix} \bar{P}_1 & 0 \\ 0 & \bar{P}_2 \end{bmatrix}$, $\bar{F}_1 = E_2^T \bar{P}_2$ and $\bar{F}_2 = D_2^T \bar{P}_2$, then it is clear that (A.3) always hold.

2. Proof of necessity

It follows from that P is positive definite, the matrices E and $E^T PE$ have the same null space and therefore $rank(E) = rank(E^T PE)$. From (2.9) it is easy to see that $E^T PE = F_1 CE$. Thus $rank(E^T PE) = rank(F_1 CE) \leq rank(CE) \leq rank(E)$ and hence $rank(CE) = rank(E)$. Similarly, it can be concluded that $rank(CD) = rank(D)$.

This completes the proof.

Appendix B

Proof of lemma 2.2

Lemma 2.2 If P and Q have been partitioned as in (2.3), then the following two conclusions are obvious :

1. $P_1^{-1}P_2E_2 + E_1 = 0$ and $P_1^{-1}P_2D_2 + D_1 = 0$ if (2.9) is satisfied;
2. The matrix $A_1 + P_1^{-1}P_2A_3$ is stable if Lyapunov equation (2.2) is satisfied.

Proof. 1. From the matrix partitions, it follows that

$$\begin{aligned}
E^TP &= \begin{bmatrix} E_1^T & E_2^T \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} \\
&= \begin{bmatrix} E_1^TP_1 + E_2^TP_2^T & E_1^TP_2 + E_2^TP_3 \end{bmatrix} \\
&= \begin{bmatrix} (P_1(E_1 + P_1^{-1}P_2E_2))^T & E_1^TP_2 + E_2^TP_3 \end{bmatrix} \\
D^TP &= \begin{bmatrix} (P_1(D_1 + P_1^{-1}P_2D_2))^T & D_1^TP_2 + D_2^TP_3 \end{bmatrix} &\text{(B.1)} \\
FC &= \begin{bmatrix} 0 & F_1 \\ 0 & F_2 \end{bmatrix} &\text{(B.2)}
\end{aligned}$$

By comparing (B.1) and (B.2), conclusion-1 can be obtained.

2. Applying block matrix multiplication to (2.2) yields

$$A_1^TP_1 + P_1A_1 + A_3^TP_2^T + P_2A_3 = -Q_1 \tag{B.3}$$

This implies that

$$(A_1 + P_1^{-1} P_2 A_3)^T P_1 + P_1 (A_1 + P_1^{-1} P_2 A_3) = -Q_1 \tag{B.4}$$

Therefore conclusion-2 can be obtained from the fact that $Q_1 > 0$ and $P_1 > 0$.

Appendix C

Proof of lemma 3.1

Lemma 3.1 Under Assumption 3.1, there exist state and output transformations

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{C.1}$$

such that in the new coordinate, the system matrices become:

$$\begin{aligned} TAT^{-1} &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad TE = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}, \\ SCT^{-1} &= \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix}, \quad SD = \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \end{aligned} \tag{C.2}$$

where $T \in \mathcal{R}^{n\times n}$, $S \in \mathcal{R}^{p\times p}$, $z_1 \in \mathcal{R}^r$, $w_1 \in \mathcal{R}^r$, $A_1 \in \mathcal{R}^{r\times r}$, $A_4 \in \mathcal{R}^{(n-r)\times(n-r)}$, $B_1 \in \mathcal{R}^{r\times m}$, $E_1 \in \mathcal{R}^{r\times r}$, $C_1 \in \mathcal{R}^{r\times r}$, $C_4 \in \mathcal{R}^{(p-r)\times(n-r)}$ and $D_2 \in \mathcal{R}^{(p-r)\times q}$. E_1 and C_1 are invertible.

Proof. Partition D as

$$D = \begin{bmatrix} \bar{D}_1 \\ \bar{D}_2 \end{bmatrix} \tag{C.3}$$

where $\bar{D}_2 \in \mathcal{R}^{q\times q}$.

Introduce a nonsingular transformation

$$S_0 = \begin{bmatrix} I_{p-q} & -\bar{D}_1\bar{D}_2^{-1} \\ 0 & I_q \end{bmatrix} \tag{C.4}$$

such that $S_0 D = \begin{bmatrix} 0 \\ D_2 \end{bmatrix}$, where $D_2 = \begin{bmatrix} 0 \\ \bar{D}_2 \end{bmatrix}$.

Introducing a nonsingular transformation T_0, we can obtain that

$$T_0 E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{C.5}$$

where $E_1 \in \mathcal{R}^{r\times r}$ and is invertible.

Introduce a nonsingular coordinate transform T_1 as:

$$T_1 = \begin{bmatrix} I_r & 0 \\ -E_2E_1^{-1} & I_{n-r} \end{bmatrix} \tag{C.6}$$

then

$$T_1T_0E = \begin{bmatrix} I_r & 0 \\ -E_2E_1^{-1} & I_{n-r} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \end{bmatrix} \tag{C.7}$$

and $CT_0^{-1}T_1^{-1}$ can be partitioned as $[\bar{C}_1 \ \ \bar{C}_4]$. Therefore

$$CE = CT_0^{-1}T_1^{-1}T_1T_0E = \begin{bmatrix} \bar{C}_1 & \bar{C}_4 \end{bmatrix} \begin{bmatrix} E_1 \\ 0 \end{bmatrix} = \bar{C}_1E_1 \tag{C.8}$$

It follows from $rank(CE) = rank(E)$ that

$$rank(\bar{C}_1E_1) = rank(CE) = rank(E_1) \tag{C.9}$$

then it can be conluded that

$$rank(\bar{C}_1) = rank(E_1) = r \tag{C.10}$$

Denote that

$$S_0\bar{C}_1 = \begin{bmatrix} C_{11} \\ C_{21} \end{bmatrix} \tag{C.11}$$

where $C_{11} \in \mathcal{R}^{r\times r}$ with $rank(C_{11}) = r$, $C_{21} \in \mathcal{R}^{(p-r)\times r}$.

Let

$$S_1 = \begin{bmatrix} I_r & 0 \\ -C_{21}C_{11}^{-1} & I_{p-r} \end{bmatrix} \tag{C.12}$$

then

$$S_1S_0\bar{C}_1 = \begin{bmatrix} I_r & 0 \\ -C_{21}C_{11}^{-1} & I_{p-r} \end{bmatrix} \begin{bmatrix} C_{11} \\ C_{21} \end{bmatrix} = \begin{bmatrix} C_{11} \\ 0 \end{bmatrix} \tag{C.13}$$

Let $S = S_1S_0$, then it can be obtained that

$$SCT_0^{-1}T_1^{-1} = \begin{bmatrix} S\bar{C}_1 & S\bar{C}_4 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix} \tag{C.14}$$

Let

$$T_2 = \begin{bmatrix} I_r & C_{11}^{-1}C_{12} \\ 0 & I_{n-r} \end{bmatrix} \tag{C.15}$$

and $T = T_2T_1T_0$, then

$$\begin{aligned} SCT^{-1} &= \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \\ TE &= \begin{bmatrix} E_1 \\ 0 \end{bmatrix} \\ SD &= \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \end{aligned} \tag{C.16}$$

Under the new coordinate, the matrices A and B can be transformed as in (3.3). This completes the proof.

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2. Jian Zhang, Akshya Kumar Swain and Sing Kiong Nguang, ”Reconstruction of sensor faults for a class of uncertain nonlinear systems”, The Journal of the Franklin Institute, (under revision)

3. Jian Zhang, Akshya Kumar Swain and Sing Kiong Nguang, ”Reconstruction of actuator fault for a class of nonlinear systems using sliding mode observer”, (Submitted)

4. Jian Zhang, Akshya Kumar Swain and Sing Kiong Nguang, ”Simultaneous estimation of actuator and sensor faults for a class of uncertain nonlinear systems”, (Submitted)

5. Jian Zhang, Akshya Kumar Swain and Sing Kiong Nguang, ”Estimation of actuator and sensor faults for uncertain nonlinear systems using a descriptor system approach”, IET Control Theory Applications, IET (under revision)

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