A Glimpse of Reality – what mathematical modelling at secondary school could look like

By Kerri Spooner

Department of Mathematics
University of Auckland

Supervised by Professor Bill Barton

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The purpose of this thesis was to explore authentic mathematical modelling and use that experience to develop and research a genuine mathematical modelling experience for secondary school students. A classroom activity was developed and trialled with a group of New Zealand year 12 average ability students at a decile 10 school. The focus of the unit was the process of mathematical modelling. Data was collected on the classroom learning activities and what parts of the mathematical modelling process was remembered. The three data sources were student diaries, classroom assessment and student interviews. The results showed that an authentic modelling process is achievable within the restricted classroom environment. With the prompts provided in the classroom activity students coped well identifying the essential aspects of the situation being modelled, and there was good recall by all students for strategies to identify the essential aspects of the situation. Students did not do as well forming a model once they had identified the essential aspects. Further work is recommended in developing strategies to help students with the model formation stage. To allow for a full experience of the process of mathematical modelling more time is needed than was given to this activity.

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1. Introduction

I have always been interested in how we use mathematics in the real world and in teaching mathematics in ways that are useful and applicable for future employment and everyday life. In 2000 I carried out a study for my postgraduate dissertation investigating effective dynamics for mathematical teams in industry and mathematical groups in education. My interest in mathematical modelling fits with this philosophy. It is a team based mathematical activity used in industry as well as for research purposes.

Mathematical modelling is the process by which we represent a situation in useful mathematical terms (Dym, 2004; Wake, 1997). It is a real world mathematical activity and is used to gain insight to understand real situations. If we were to effectively be able to teach mathematical modelling at secondary school, these basic skills would give a student a tool box to tackle unfamiliar problems they may encounter later in life, and would also prepare them for future study. In life we are not given the appropriate model or answers to solve a problem. We need to seek it out, think through and formulate our approach, trailing it and modifying it as we go until we reach a satisfactory solution. This process is known as mathematical modelling (Treilibs et al, 1980).

Why would we want students to learn mathematical modelling skills at school? First, as previously mentioned “to be able later to use mathematics in tackling problems of concern to them” (Treilibs et al, 1980, pp 4) and to prepare students for further study and work as mathematical modellers. By including mathematical modelling in the curriculum we are helping to develop mathematically equipped citizens. Second, as a context for mathematical understanding. Using concrete examples reinforces the basic understanding of the mathematical concepts involved (Julie, 2002; Dindyal & Kaur, 2010). Third, modelling using realistic problems provides extra motivation for the study of mathematics, providing “a situation where a student needs to draw from his full range of mathematical knowledge” (Treilibs et al, 1980, pp 4). Modelling provides a context that is both practical and mathematical, developing higher level investigatory and creative skills (Treilibs et al, 1980). Mathematical modelling develops the skills needed to solve problems both within and outside of mathematics and promotes behaviours needed for working in teams.

The move for modelling to be included in school curriculums internationally began in the 1970s (Kaiser, Lederich & Rau 2010, Dindyal & Kaur, 2010). In the 1980s the International Community of Teachers of Mathematical Modelling and Applications was formed and the Shell Centre, University of Nottingham, produced material for the teaching of modelling and its assessment (Burkhardt, 2006). Despite this there was still a need for political pressure for clear curriculum statements to be written into curriculums (Burkhardt, 2006). It is only recently that good clear curriculum statements with support material are starting to be provided (Tam, 2011).

In 2011 I spent 6 months working with experienced modellers developing mathematical models for real time problems based in Industry. I have taken this experience, along with research into the particular problem of mathematical models for a rugby conversion kick, and brought these experiences into the classroom. In doing so I exposed students to the profession of mathematical modelling, to mathematical modelling as a valid form of mathematical activity and also how to build a tool box for approaching new problems they may meet later in life.

In this thesis, I will first describe my experience working with a real world mathematical modelling team and use this to identify the behaviours present in their work. I will then take a rugby conversion kick, look at the existing models, and critique them in terms of improvements to the models and the process of mathematical modelling. Next I look at reasons why modelling should be in the curriculum, the history of modelling and research into the learning and teaching of mathematical modelling to get an indication of where mathematical modelling and education is internationally. I introduce the theory of Realistic Mathematics Education (RME) and briefly describe the RME's activity developmental research theory. This theory develops learning activities from Hypothetical Learning Trajectories (HLT, which will be explained in section 4.4). I describe the process of developing an HLT for my study. This involved an in-depth examination of what the process of mathematical modelling might look like for a senior secondary school student. The HLT involved writing a mathematical goal for the classroom activities, designing the classroom activities and identifying conjectured learning processes. I then describe how these activities were trialled and data was collected in terms of what parts of the process had been experienced and could be recalled. The data was used to evaluate the teaching activity. These results are then discussed and conclusions drawn from the experience.

The purpose of this thesis is to explore authentic mathematical modelling and use this experience to develop a realistic mathematical modelling experience for secondary school students. A consequence of the research for this thesis has been the exposure in schools of mathematical modelling as a profession, acquisition of behaviours that are beneficial for future mathematical modelling and skills and thinking that can be applied to real life problems. The outcome of this research is better understanding of the issues, potential and problems of introducing modelling at senior secondary level.

2. What is authentic modelling: two experiences

In this section I discuss two experiences of mathematical modelling. The first is my experience working with a real world mathematical modelling team developing models for milk vats. The second is my experience researching and critiquing existing models for a rugby conversion kick.

2.1 Milk Vats

I had the pleasure of working on a mathematical modelling team through the Centre in Maths in Industry, Massey University in 2011.

I am going to describe the process we went through when working on one of the problems on which the centre had been employed. I am unable to go into detail of the actual problem because of commercial confidentiality but will describe the process we went through when working together.

The project was to develop mathematical models to describe the passage of powder through powder bins known as hoppers. The project consisted of different stages and models to be developed. A model was wanted to determine if core flow or mass flow was predominant through the hoppers for the powder concerned. A model was wanted to approximate the mixing that occurs in the conical section of a mass flow hopper. A model was wanted to approximate the dimensions of the powder inverted cone at the top of a core flow hopper. Mathematical assistance to incorporate models into the company's real time monitoring tools was in the brief and also modification to the models once validation testing had been completed. Written progress reports were to be provided throughout the project with project objectives stated and a detailed summary of accomplishments in accordance with the objectives to date. A final report was to be written encompassing the entire project and recommendation of any further action if appropriate. The mathematical team would be using existing materials and theoretical models in the public domain on which to base new models. An attempt to validate any new models would be carried out. The problem brief was given in six parts, with two of the parts being concerned with implementing the solutions. In summary the brief was to create mathematical models, apply models to reality then modify models if need be, provide written progress reports and a final report.

The parties involved in the contract were the client and a mathematical modelling team. The mathematical modelling team consisted of experts in the areas of contextual background, mathematical modelling, research and computer programming. It comprised one powder flow specialist, one applied mathematician, one research assistant, one matlab expert and one computer programmer. My role in the team was as a research assistant. I was another brain to bounce things off and test ideas, to find background contextual information, to research and carry out literature reviews, to find facts about how things behave so we can use and apply this information to our thinking.

The problem was presented from the client to two members of the mathematical modelling team with a meeting to clarify the problem and to establish context. Next a project plan was formulated by the mathematical modelling team with tentative deadlines for each stage of the project. This was done at a meeting while discussing and brainstorming possible strategies to use to attack the problem.

The next stage involved individual background reading and research to familiarise ourselves with context while at the same time thinking about generating ideas that would be helpful for formulating the models for the problem. Throughout this stage team meetings were held to bring together ideas and background information. All individuals worked on the problem individually from their area of expertise, then met to share knowledge. Face to face meetings were about bringing together information and thinking through the problem as a team, brainstorming, discussing, accepting and rebutting ideas, deciding on common ground and moving forward. Phone meetings also took place. Team members were allocated tasks at these meetings and after meetings each team member would go away and work on their individual tasks. Telephone meetings were a substitute when it was not practical to meet face to face. This process was repeated as ideas were discussed, formulated and tested.

A summary of what was actually done throughout the process in reference to the problem brief is as follows:

Part one of the contract was relatively straight forward to solve. It involved going to the literature to find the solution. So that the solution was user friendly for the client, the team, with the help of our computer programmer, developed computer software to generate an interactive form of the solution to be integrated into the client's computer system. C+ was used to produce the interactive programme. The other way of presenting the solution would have been to present charts found in the literature with instructions on how they could be interpreted.

Part two involved the generation of new ideas. There was a lot of brainstorming, critiquing ideas, finding relevant literature. To keep our ideas based in reality videos of the powder flow through a hopper were watched and ideas formulated and critiqued. A model was formed by looking at the video and finding literature to support what appeared to be happening in reality. The model was used and modified making assumptions based on literature and using existing models to apply to the situation.

Once we had what we thought was a working model, a MatLab computer simulation was designed to test the model. We then had to ask the question, is our model holding true to the situation? The model and our ideas were continually modified until we were satisfied that what we had produced was representative of the situation and useable to gain information about the problem. With the development of new ideas, during the work for part two, the question of intellectual property was raised.

Part three and four became modifications to apply in different situations of the work done in part two. That is, going back through the process of modifying the work done on part two to fit the situations for part three and four.

Part five and six were not completed when the final report was written. The models were given to the client with the final report for the client to test and then modifications made as necessary.

Progress reports were given to the client throughout the process, both verbal and written reports as well as a final written report containing the findings.

Overall the project would not have been possible without collaboration from all parties involved. It involved working as a team, working individually, participating in face to face and phone meetings.

In summary the parts of the process of mathematical modelling that was experienced and observed by me as a team member are:

- clarification of the problem
- formulation of a plan including deadlines and task allocations
- familiarising yourself with the context of the problem by reading background material and research and talking to context experts
- discussing, brainstorming, generating and critiquing new ideas and strategies
- using literature to find already known solutions
- working both individually and as a team
- presenting models as a computer programme
- using literature to support solutions and assumptions
- modifying already known solutions and new solutions
- testing models using computer simulation and critical thinking
- identifying relevant facts and relationships for the problem
- meeting to share and discuss progress
- challenging and discussing ideas to make sure ideas stand up to scrutiny
- working individually and as a group member
- report writing
- communicating with client both written and verbally.

2.2 Modelling the rugby kick

"Ahh….. damn it, I missed!" Have you ever wondered what it takes to make a successful conversion? I have. These curiosities led me to look into mathematical models of a rugby kick, in particular models predicting positions for successful conversion kicks. First I researched the different ways of modelling a rugby kick that had already been done, at the same time critically thinking about these models for possible improvements and limitations. I focussed on one model and set about planning to investigate if this model fitted reality and if it did not then to formulate a proposal for improvement. I started to carry out this plan. Unfortunately this investigation was not completed due to time restrictions. This is now future work. In this section I will describe the problem of the rugby conversion kick, give summaries of the already developed models, focus in on one model discussing the plan for testing the model to see how well it fits reality and the possible improvements and limitations for this model.

We can describe the problem as follows with reference to Figure 1. "Suppose a try is scored at T, then the kicker can kick to the posts from anywhere along line TF perpendicular to the tryline PQ. Where is the best place to kick from?" (De Villers, 1999, pp 64).

Figure 1 (De Villers, 1999, pp64)

or simply “Your rugby team has just scored a try. You are responsible for taking the conversion. Where on the field will you place the ball for the kick?”

In literature there are two different types of models for the rugby conversion kick, geometry based or statistical based. I will first look at the different geometry based models.

The simplest, most common model for the best position for a conversion kick is the rectangular hyperbola first mentioned by Hughes (1978). Wornsop (1989) extends Hughes work by making an attempt at justifying the best practical position to be the asymptote to the hyperbola.
De Villers (1999) and Polster and Ross (2010) pick up from Hughes work showing new ways of generating the rectangular hyperbola. Both De Villers (1999) and Polster and Ross (2010) make an attempt at modifying Hughes (1978) model to better fit reality.

Hughes (1978) was the earliest model I came across in literature. He states the locus of points giving the best position to take a conversion kick as a rectangular hyperbola. His model assumes distance is no object, kicker kicks in a straight line, there is no wind factor and we are only concerned with maximising the angle subtended by the posts. Hughes proposes that the point on the conversion line that gives the maximum angle is the intersection of a tangent to the circle which passes through the goal posts.

He demonstrates the tangent intersecting circle theory differently than later papers by demonstrating how to find it practically. He talks about finding the position of the centre of the circle by having a piece of rope where one player holds the end of the rope at goal post A, another player holds the other end at goal post B and a third player is between the ends of the rope standing on the try line corresponding to the position the try was scored. The rope is then pulled taunt between the three players and the third player walks the rope out until he is directly in front of the posts on the perpendicular bisector of line AB (figure 2). Now take the distance between player three and either goal post A or B (depending on which side try has been scored) and swing this distance out so that it is parallel with the try line. This is the position to take the kick. In theory the position player three is in is the centre for the circle to be drawn that goes through A and B, and which the conversion line will be tangent to. This position gives the maximum angle between the goal posts.

Figure 2 (Hughes, 1978, pp 293)

Analytically, with reference to figure 2 this distance position can be found by saying the distance between A and B is *d* and the distance from the midpoint of AB to where the try is scored *r* then "the ball should be brought back a distance of $(r^2 - \frac{1}{4}d^2)^{1/2}$" up the conversion line "and the angle subtended will be $sin^{-1}(\frac{d}{2r})$" (Hughes, 1978, pp 293).

A disadvantage of Hughes model is that it does not take into account the need to clear the crossbar. This means the model is only good for tries scored away from the goal posts.

In 1989 Worsnop suggested that instead of using the hyperbola for the best position, the asymptote of the hyperbola was a far more practical way of determining the best locus of positions (figure 7). The asymptote is a good approximation of the hyperbola for far away kicks and is a straight line. Being a straight line makes it a lot easier and more practical to find than the hyperbola. As this asymptote model is an extension of the hyperbola model it does not take into account the cross bar for in close kicks and is a good approximation for kicks away from goal posts.

Figure 3 (Worsnop, 1989, pp 226)

De Villers (1999) continues to work on the idea that the maximum angle between the goal posts is where the conversion line touches the circle which passes through the goal posts as its tangent (Hughes, 1978) and attempts to incorporate the crossbar. He develops Hughes model by giving the reader a choice of either using dynamic software like sketchpad or drawing expanding circles by hand. He first starts by drawing a circle using the goal posts as a chord and using the conversion line as tangent to this circle. He then explores different positions on this conversion line to show that the intersection of the tangent and the circle give the maximum angle given subtended by the goal posts.

Once De Villers has established that this is the best position for a particular try he then explores expanding and diminishing circles confirming this theory holds true for all try positions, plotting the locus of points. He provides theoretical evidence that the best position is a rectangular hyperbola, providing we are not in close to the goal posts.

De Villers further develops Hughes model by making an attempt to incorporate the cross bar. He does this by suggesting the “required tangent circles lie on a series of planes running through the crossbar with the optimal positions located where these planes cut the ground” (De Villers, 1999) (figure 4). This theory needs more explanation by De Villers.

Figure 4 (De Villers, 1999, pp 66)

Polster and Ross (2010) further look at producing a model that incorporates the cross bar and thus better reflecting reality. At the same time they propose an alternative proof to Hughes (1978) and De Villers (1993) work. Polster and Ross try to take reality into account far more than the previous models. They propose that the angle used by Hughes (1978) is wrong as it does not take into account that the ball must clear the crossbar. The angle should be some raised angle containing a vertical angle and a horizontal angle (figure 5).

Figure 5 (Polster & Ross, 2010, pp 12)

Polster and Ross make an attempt at optimising the positions this new angle, made up of two components, gives and call the path this takes a ‘rugby hyperloid”.

Now that we have looked at some of the geometry models another type of model used to investigate the best position of a rugby conversion kick is a statistical model. Wilensky (1996) proposed to look at the problem as a discrete probability problem. He used a computer software programme called StarLogo to produce a probability distribution for successful conversion kicks taken along a conversion line. The programme had several rugby players along a conversion line, randomly kicking thousands of balls while the programme keeps track of how many are converted. These results were displayed as a histogram thus giving a numerical solution instead of a general formula.

Figure 6 "Thousands of kicked balls. To the left of the line is a histogram of the successful kicks" (Wilnesky, 1996, pp 3)

The programme was written assuming each player had the same kicking ability. This first simulation of the model ignored factors such as grass friction, wind, player's ability and kicking difficulty. Working the solution using Starlogo meant that the next stage of developing the model was to introduce a wind of some velocity, then experiment with patches of grass and kicking ability. This implies that using software means it was possible to easily incorporate new factors into the model using very little time to develop a fuller model.

Whilst reading the different models in literature it became apparent to me that the geometry model was oversimplified and I started to question how well it fitted reality. I did not think it had effectively taken into account a player's kicking range. I decided to take the rectangular hyperbola model (Hughes, 1978; De Villers, 1999) and make a plan for improving the model. I wanted to see if it fitted reality and if not how we could improve it, in particular to incorporate kicking range of a player. For testing the model Curtis, a proficient conversion kicker, offered his services. I placed myself under the goal posts asking if a try was scored here please position yourself where you would take your conversion kick? I moved out two metres at a time along the try line asking the same question. Each time Curtis positioned himself and confirmed the position by taking a successful kick. I mapped Curtis' locus for where he had taken the conversion kicks and compared it with the geometry model above.

What we found was Curtis' locus under the goal posts was further out than the rectangular hyperbola. As we moved along the try line away from the posts Curtis' locus gently curves onto 'rectangular hyperbola locus' (rhlocus). Curtis' locus did not stay on the rhlocus for long. It then moved further in than rhlocus for kicks further away from the goal posts. Curtis' path was determined by his kicking distance or kicking range. If his kicking range was bigger I predict that he would have followed the rhlocus longer as we got further away from the goal posts.

The fact that Curtis' model did not match the rhm locus in close to the goal posts was not a surprise. I already knew that in theory the best position in close to the posts is a combination of angle width and angle of elevation (De Villers, 1999; Polster & Ross, 2010). What also contributes to the position close into the goal posts is charge down from the opposition, meaning there is a minimum distance away from the try line that the conversion kick must be taken to avoid being taken out by the opposition. In reality this distance for most players is between seven and ten metres (Reid, C., conversation 2011). For our purposes we assumed that the minimum distance is 10 metres for most players.

What we found was that, yes the geometry model is a good model but it did not fully fit reality. A players' kicking range determines the interval that the rectangular hyperbola model can be said to be a good model. In reality "when a try is scored directly in front of the goals, players choose to kick a minimum of about 10 metres from the goals" (Polster & Ross, 2010, pp6).
"Professional players seem to follow the Hughes hyperbola quite closely when near the goals, but then deviate to be nearer the goal line when the conversion line is further away"(Polster & Ross, 2010, pp14).

Now I wanted to try the model with other players. I wanted to record the positions they took their conversion kicks and draw these up on a scale drawing, compare these with the rhlocus, and then compare individual players locus' with each other to see when each players' individual kicking range starts having an effect. Finally asking the question can we write a generic model for real locus' that takes into account kicking range. I started collecting this data for other players but unfortunately ran out of time. Future work would involve collecting data and fitting a new model that incorporates a players kicking range. My proposal is to collect data of players and propose a realistic model based on distance as player kicking range is the biggest issue for conversion kicks (Polster & Ross, 2010).

Questions for further research include:

- What effect does a players' kicking range have on position for taking a conversion kick? (Polster & Ross, 2010).
- What is the probability distribution of a real kicker?
- What function could be used to model the decay of a balls' velocity if kicking strength of a player is assumed to be constant ?(Wilensky, 1996).
- How much of an effect does charge down from the opposition have on kicking position for goals close to the posts?
- How do we include flight path issues of the ball like deviation of flight path including curling away from the original direction into a model?

In summary the parts of the process I experienced by researching and critiquing existing models for the rugby kick are:

- clarification of the problem
- familiarising myself with the context of the problem by discussing with and reading background material and research by context experts
- discussing, brainstorming, generating and critiquing new ideas and strategies
- using literature to support new ideas and assumptions
- generating ideas to modifying already known solutions
- identifying relevant facts and relationships for the problem
- experiencing different ways of modelling the same situation
- critiquing models to test if they fit reality

I will draw on the two experiences described to develop the “mathematical goal for the senior secondary school student” (section 5) and the “hypothetical learning trajectory” (section 6a).

Now I will look at what the theory says about mathematical modelling.

3. Mathematical modelling

3.1 What it is mathematical modelling?

What is mathematical modelling? A mathematical model is a representation in mathematical terms of the behaviour of real devices, objects and situations (Dym, 2004; Bender, 2000; Wake, 1997). We draw upon models to aid in our understanding of, and our dealings with, the world around us (Barnes & Fulford, 2002; Svobodny, 1998). The process of developing a mathematical model is termed 'mathematical modelling'. In other words the process of mathematical modelling is the series of steps and actions taken to participate in a mathematical modelling experience (Wake, 2011).

Mathematical processes are described as the how of mathematics opposed to the content of the mathematics (Ministry of Education, 1995). Modelling is a mathematical process. The process of mathematical modelling is what mathematical modellers do (Begg, 1994a; Begg, 1994b).

What do mathematical modellers do? Mathematical modellers formulate models, work solutions from models, interpret solutions of models, validate solutions from models, improve models and report on the process of modelling (Wake, 2011; Bender, 2000; Treilibs, Hugh & Low, 1980). Traditionally in an applied mathematics course, a student is given the appropriate model for the situation and asked to solve it (Treilibs et al, 1980). In industry the role of the mathematical modeller includes formulating the model. I wish to focus on the process of model formulation, its key ideas and key experiences.

Model formulation can be either creating a new model for a situation or using a suitable model from what is already known. Model formulation involves first defining the task, then performing a stock take of what we know and what we want to know about the variables important to the task, then using this information to a develop the model or to choose a suitable model. Once the model has been formulated it needs to be tested, improved, validated and interpreted (Tam , 2011; Dym, 2004; Treilibs et al, 1980).

The key ideas and experiences of model formulation are: (Dym, 2004; Svobodny, 1998; Treilibs et al, 1980)

1. Forming a modelling group.
2. Observe the contextual background for the problem situation.
3. Define the task. Establishing why the model is wanted or needed. What is the model to be used for? Part of this is to identify the specific questions crucial to the typically ill-defined realistic problem.
4. Recognise and identify some of the situation's essential aspects. Identify the known and unknown variables we are interested in. What things can we measure (what do we know) and what do we need to find? What assumptions need to be made? Distinguishing the relative importance of variables is important in the building of a good model.
5. Use the essential aspects form or choose a suitable model. This can involve creating a new model drawing from what you know or using an already established model, modifying it if needed. Can we write what we want in terms of what we have got? This process involves

writing down any applicable relations between variables. Are there any physical laws relating the variables. Is there a conservable quantity? This is the equation formation stage

6. Test the model. This can be using the model to generate a solution and comparing with reality. Does the model match reality?
7. Improve the model. This is done by starting the process again. An iterative process is essential if we are to develop correct validated models.

The above are all an important part of the process for the formulation of models. A study conducted by Treilibs et al (1980) showed that there was strong evidence that generating variables, identifying questions that needed to be asked, generating relationships (model construction) and selecting relationships (model application) are all important modelling skills necessary for model formulation.

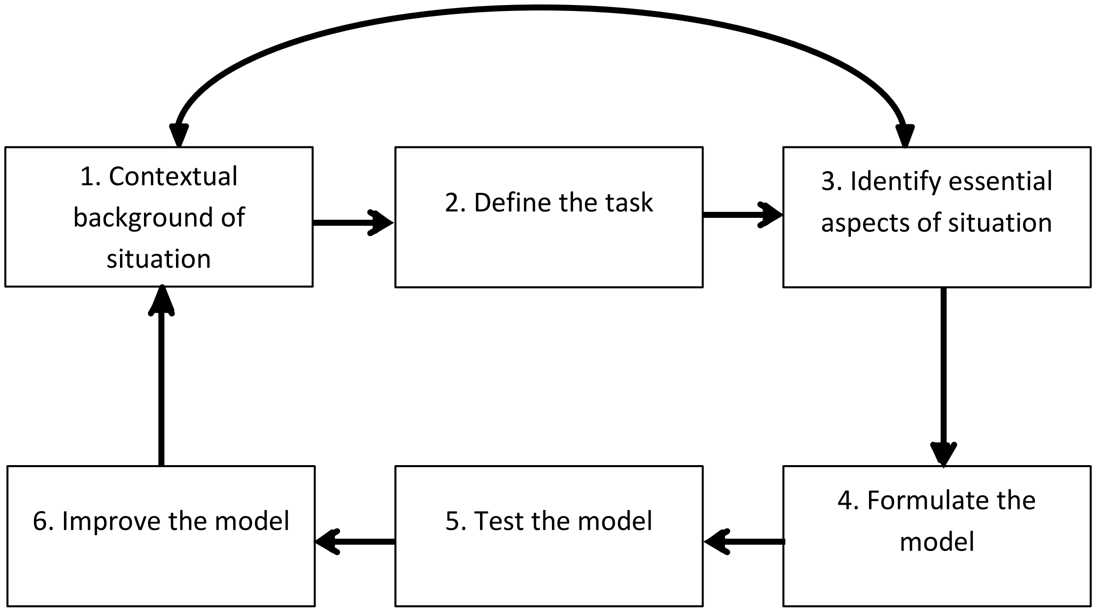

Figure 7
The process of mathematical modelling

It is widely acknowledged in the literature that a good understanding of the context of the problem is very important to devising a superior solution (Alsina, 2007; Pollak, 2007; Treilibs et al, 1980). Modelling uses initiative and creativity where the overall picture is just as important as the detail requiring modellers to draw from a broad range of mathematical knowledge (Bassanezi, 1994; Treilibs et al, 1980). As you model you begin to build a toolbox of ideas and different approaches to apply to realistic problems. An ability to be open to the experience, not to get fixated on detail and to think outside of the box is needed. It is essential to be able to see the big picture, make assumptions to simplify the detail and trying make a model as simple as possible (160.319 Robert McKibbon & Graeme Wake, conversation Massey University 2011). All of these things contribute to the success of mathematical modelling.

4. Mathematical modelling in the classroom

4.1 Why Should Modelling be in the Curriculum?

Should mathematical modelling be more prevalent in the school curriculum? In the past the mathematics of school has not reflected the mathematics of practice. For example practising engineers use reasoning that is very different from school mathematics (Julie, 2002; Tam, 2011). Pure mathematics as opposed to applied mathematics has continued to play a dominant role in curriculum (Tam, 2011). Should this change and if so why?

What are the reasons for applied mathematics, in particular mathematical modelling, to be included in the curriculum? First to introduce students to skills needed for the process of mathematical modelling for those who wish to pursue this field as an academic or professional career- that is to prepare students in using mathematics actively for their livelihood (Robitaille & Dirks, 1982). Second, for all students to broaden their development as mathematically equipped citizens. Every child can benefit from modellings' power of application (Tam, 2011). It provides a way for students to understand the world and is of prime value for solving problems in life. Mathematical practices in real life and the work place are context specific and applied to real situations (Julie, 2002; Robitaille & Dirks, 1982). Third modelling develops an alternative way to build mathematical concepts, learn to problem solve and develop higher order thinking. Fourth, modelling uses real world contexts which can provide contexts for motivation to explain and learn mathematics. Modelling also provides experience to work in a team. I will discuss these themes in this chapter. The reasons are not limited to the above mentioned.

Preparing students for further study and work as mathematical modellers
The inclusion of modelling in the curriculum provides experience of modelling before students are exposed to it in tertiary education. This experience offers the beginnings for a base of modelling competencies to be later developed on. In a study conducted by Galbraith and Stillman (2006), involving 14 and 15 year olds having their first experience of modelling, they noted potential difficulties for students in the areas of identifying essential aspects of the situation and model formation. At tertiary level it has been identified that one of the hardest stages of the modelling process and the one done least well is identifying the essential aspects of the situation (Alona Ben-Tal & Robert Mckinnon, conversation Massey University, 2011). An experience of modelling at school should focus in on these areas.

Developing Mathematically equipped citizens
If one of our curriculum objectives is to build competent mathematical citizens then the inclusion of mathematical modelling as a topic strand is an essential component of any curriculum (Kaiser, Blomhoj, Sriraman, 2006; MaaB, 2005; Treilibs et al, 1980). Mathematical modelling prepares students for everyday life by helping them to understand and make sense of the known world and by showing that mathematics is useful (Blum, 2011; Boaler, 2011; Dindyal & Kaur, 2010; Buckhardt, 2006; Lingefjard, 2006; Julie, 2002; Blum & Niss, 1991; Robitaille & Dirk, 1982). An example is how an aircraft stays in the sky. Modelling also gives students an effective tool for tackling problems for which they have no immediate solution. An example is finding your way through a foreign city. Burkhardt (2006) believes that "two kinds of student learning is essential for mathematics to be

functional in everyday life", that of "learning illustrative applications" of "standard models" and that of active modelling, where "students use mathematics to tackle problems that are unfamiliar to them" (Burkhardt, 2006, pp 178).

Illustrative examples of modelling are found in the commercial, economic and political world. They are used to develop successful aircraft designs and computer programmes, to predict global warming and the greenhouse effect, to establish inflation rates, predict future trends in derivatives and stock markets, and to understand likely future economic outcomes. "The world of today cannot go on without mathematical modelling" (Lingefjard, 2006, pp96). With models being used everywhere to deal with the world it is vital that students are exposed to illustrative applications of models. It is important that students are able to understand the models that have already been used and begin to develop these models further at the same time as developing their own modelling concepts (Buckhardt, 2006; Lingefjard, 2006).

Modelling is also used to confront situations which are unfamiliar and where there is no apparent solution. When a problem occurs in real life we are not given the appropriate model to solve it. We need to seek out and formulate a solution, weighing up different approaches, selecting appropriate strategies, reflecting on and improving the result, modifying it as we go, that is, using the skills of mathematical modelling. A range of mathematical modelling skills is beneficial if students are to be competent problem solvers in life (Lingefard, 2006; Robitaille & Dirks, 1982). We want students to be able to use mathematics in tackling problems of concern for them, both in their everyday and professional lives (Lingefjard, 2006; Blum & Niss, 1991; Treilibs et al, 1980). As we prepare students to use modelling as private citizens or as professionals, both now and in the future, we are giving students resources to solve problems that will confront them in all walks of life (Lingefjard, 2006). By providing experiences of mathematical modelling we are equipping individuals to deal with situations outside of mathematics (Lingefjard, 2006).

As we are using mathematics to understand the world and solve its problems, we are developing the eyes of a mathematical citizen, that is, to use the power of mathematics to describe, explain, predict and control real phenomena. Mathematics is a way of knowing about the world in which we live (Stillman, 2010). It is important that students start to develop this awareness and way of looking at the world early in life. Stillman (2010) and MaaB (2005) believe this awareness of mathematics and the development of equivalent mathematical beliefs should start early in one's education. Stillman (2010) believes exposure to modelling should start as early as childhood with exposure at primary and secondary school and thus sees modelling as an essential component of the school curriculum (Stillman, 2010; Lingefjard, 2006; MaaB, 2005). Modelling "is a way of equipping students with the power to exercise a fundamental duty as a citizen to appreciate, critique and use the models and modelling that permeate and format our modern world" (Stillman, 2010, pp301). Developing the eyes of mathematics through modelling is believed to be an essential component of the curriculum for the survival of commerce, industry and science (Lingefjard, 2006).

Mathematical understanding

Modelling takes contexts and views them through the world of mathematics (Stillman, 2010; Stillman, 2007). That is, modelling looks at the world and uses mathematics to describe it. Context based applications allow students to encounter the way abstract concepts are connected to practical examples and allows mathematics to becomes real, providing a hands on experience for concepts to become tangible mathematical ideas (Dindyal & Kaur, 2010; Julie, 2002). This is not to say that mathematical ideas removed from context are intangible. Modelling is about making connections between the real world and mathematics, bringing mathematics into the student's world (Dindyal & Kaur, 2010; Julie, 2002). This all contributes to understanding the concepts of mathematics relevant to the problem and provide experience of using mathematics to explain things (Treilibs et al, 1980). Armstrong and Bajpai (1988) believe that without sufficient practical experience pupils are unable to relate abstract mathematical concepts to any form of reality. Modelling provides one way of doing this. "Applications and modelling activities should be seen as embedded within mathematics (and thus the curriculum) and then together the two become an effective lens for describing and analysing the real world" (Stillman, 2007, pp466).

Modelling also helps to develop problem solving and higher order thinking. It uses context that is both practical and mathematical, developing higher level investigatory creative skills and fostering problem solving attitudes and competencies (Lingefjard, 2006; Treilibs et al, 1980). It does this by providing situations where students need to draw from appropriate mathematical ideas as well as real-world knowledge, using a logical sequence of steps to explore the situation. This is in line with the use of mathematics for practical situations in the real world (Blum, 2011; Julie, 2002; Treilibs et al, 1980). "Context-based mathematical modelling provides ideal settings to blend content and process to produce flexible mathematical competence" (Stillman, 2010, pp301). By using real applications to make connections between mathematical concepts and the real world we are developing mathematically versatile citizens.

Motivational Reasons

Treilibs et al (1980) believe that mathematical modelling using realistic problems provides extra motivation for studying and acquiring mathematical skills. By studying real life problems that students are familiar with, we can provide for some students more motivation for engagement compared to presenting abstract skills out of context (Kaiser et al, 2010; Treilibs et al, 1980). That is not to say that abstract mathematics cannot be taught in motivating ways. In Germany there are some educational groups which have implemented genuine modelling. Results from one such group found that over "the course ……. it was found that students who were initially sceptical about the problem and mathematics in general abandoned their existing notions and prejudices and became significantly involved in the solution of the problem"(Kaiser et al, 2010, pp236-237). As we develop models there is increasing student engagement with mathematics as its need becomes apparent. The search to find the mathematics to model and solve the problem provides the motivation to learn further mathematics (Lingefjard, 2006; Rowland & Jovanoski, 2004; Smith, 1996) with the situation itself providing motivation to learn the mathematics necessary to understand and solve it. Particular contexts can provide motivation for the study of some mathematical disciplines, for example modelling a parachute jump can provide the context for learning kinematics (Lingefjard, 2006).

While real life context can provide motivation for some students there is evidence that this is not the case for all students. While there is research showing familiar contexts that are part of students life experiences can have a positive effect on engagement (Kaiser & Schwarz, 2006) other research suggests over-familiar contexts having a negative effect (Busse, 2011). A study conducted by Busse (2011) showed contexts were interpreted very individually. This is supported also by Treilibs et al (1980). A study by Kaiser and Schwarz (2006) also showed that for some students the absence of a unique solution, common in real world problems, and the need to include common sense is seen as an additional barrier having a negative effect on engagement (Busse, 2011).

Developing future members of working teams
Mathematical modelling in the real world takes place in teams. In the workforce many things are not done in isolation. We are all called to work with one another and need the skills to be able to competently do so. By bringing modelling into the school curriculum we provide a context for group work and thus provide opportunities for students to develop the skills needed to work collaboratively in teams (Julie, 2002), again preparing our students for life as competent (mathematical) citizens and/or to take up professions such as engineering, architecture, and mathematical scientists.

Opposition to modelling in the curriculum
I have not found anyone in the literature that has strongly spoken out against mathematical modelling in the curriculum. What I have found is statements of things that make mathematical modelling difficult to implement in the curriculum. Mathematical beliefs (MaaB, 2005), teaching styles and peer group attitudes (Burkhardt, 2006), teacher perception that modelling is too difficult for the classroom (Armstrong & Bajpai, 1988), lack of teacher knowledge about mathematical modelling (Julie, 2002) and lack of suitable resources for successful implementation (Armstrong & Bajpai, 1988) are some of the reasons discussed in literature. Clathworthy and Galbraith go as far as to state "at best modelling curriculum statements are given lipservice and refuge is taken in the secure ideals of educability of structured applications" (Clathworthy & Galbraith, 1991, pp6-7).
I will take a look in more detail at the barriers to teaching mathematical modelling in section 4c.

Conclusion
We want a curriculum that prepares students to do further mathematics and mathematical modelling while simultaneously building competent mathematical citizens. Students need to experience how mathematics is used in everyday life, both on a personal and a professional level (Dindyal & Kaur, 2010). The inclusion of mathematical modelling at secondary school will help achieve this. Modelling provides opportunities to explain and see the world through mathematics as well as offering potential motivation and engagement in the subject. Mathematical modelling will help students to learn to reflect on situations and see how they are open to a mathematical analysis, developing the ability to see the world through the eyes of mathematics and the advantages of doing so (Julie, 2002). Using real applications helps students to understand the abstract concepts, giving concrete examples to explain things through. Our job is to expose students to current models, help them understand them and equip them with the skills to build on and further improve these models. We also need to teach them to evaluate and critique these models so they know the limitations of models with respect to the real world. To enable this we need to give students a toolkit of strategies from illustrative examples to hands on experiences of applying the process of mathematical modelling to unfamiliar situations (Kaiser et al, 2010; Burkhart, 2006).The importance of building competent mathematical citizens is imperative in the survival of the commercial world (Lingefjard, 2006) . To my mind, this makes mathematical modelling an essential part of the curriculum.

4.2 History of modelling and the curriculum

If modelling is the use of mathematics to make sense of the world, we can say that modelling has been in practice since the beginning of mathematical activity. Examples through the history of mathematics include Archimedes (c.287 BC – c.212 BC). He used mathematics to make sense of simple machines like levers and screws contributing to great developments in mechanics (Chrondros, 2010). Also, in the late 19th century, Klein's work had a great influence on the development of mathematics and modelling, using line geometry to make sense of mechanics and launching the idea of an encyclopaedia of mathematics and its applications, which became known as the Encyklopadie der mathematischen Wissenschaftn (Wikipedia, 2012).

Even though modelling has been used and informally taught for centuries, only recently in history has a move to formalise it in education occurred. The beginnings of recognising the need for its formalisation occurred in the 20th century, with the call to formalise it in education occurring in the 1970's (Kaiser et al, 2010). Let us look at the development of curriculum documentation in support of the teaching of modelling beginning in the 1970's to now.

1970's

The call for modelling to be in classrooms began in the seventies (Dindyal & Kaur, 2010; Kaiser et al, 2010) where it became "clear that applications, in particular, needed a stronger foothold in the school curriculum". (Dindyal & Kaur, 2010, pp330). Up until then curriculum reforms had minimal focus on applications and modelling being influenced by pure mathematicians, with focus being on content instead of applications and processes (Dindyal & Kaur, 2010). At this stage most early work with mathematical modelling was "individual rather than a coherent program and, though instructive, was not published" (Burkhardt, 2006, pp 183). What was needed was a coherent programme and political movement to formalise modelling in the curriculum.

One of the first major breakthroughs was in 1978 when the *Journal for Mathematical Modelling* for teachers was started in the UK by David Burghes (Houston, Galbraith, & Kaiser, 2012), thus starting a publication focused only on mathematical modelling.

1980's

The International Community of Teachers of Mathematical Modelling and Applications (ICTMA) was established and started to hold conferences in the 1980s. ICTMA provided the beginnings of a unified and international movement for the teaching of modelling. The community produced support material for the teaching and assessment of modelling and began publication of journals focused on the teaching of mathematical models (Burkhardt, 2006; Houston et al, 2012). These biannual journals were launched by David Burghes and continued to grow the international community of innovative modelling teachers. At the same time ICTMA conferences were being established, modelling was becoming a major theme at other conferences around the world, in particular Undergraduate Mathematics Teaching Conferences (UMTC). Thus "the teaching of modelling was established and steadily gained momentum" (Burkhardt, 2006, pp 186).

During this time the Shell Centre was formed as part of the University of Nottingham with one of its aims to develop and produce supportive material for the teaching of modelling and assessment material (Burkhardt, 2006). Around the same time the Spode Group and the Centre for Innovations in Mathematics Teaching at the University of Exeter, directed by David Burghes, were involved in several projects producing modelling exemplars suitable for secondary school. Examples of these can be found in Hobbs & Burghes (1989). At the same time the journal launched by David Burghes in 1978 was gaining momentum and became the *Journal of Teaching Mathematics and its Applications*. This journal is still being published today (Houston et al, 2012).

As the international movement was established, the move to get mathematical modelling in the curriculum was happening alongside. The UK Council for National Academic Awards (CNAA) had a major influence during this time (the 1980's) when they decided that "all polytechnic undergraduate courses in mathematics must have a modelling component in all three years" (Burkhardt 2006, pp 186), thus establishing modelling as a common goal for the polytechnics. People in these polytechnics during this time made major contributions to the development of the teaching of modelling (Burkhardt, 2006). The polytechnics were the first place to establish modelling as part of their curriculum. There is more freedom around curriculum in a polytechnic, more than universities and school (Houston et al, 2012).

Meanwhile the UK General Certificate of Secondary Education (GSCE) National Criteria aims had been written closely connected with mathematical modelling. These aims of the GSCE were not being translated into the classroom. Despite many of the aims of GSCE being connected closely with the process of mathematical modelling, very little of it was being carried out in schools. "Modelling processes rarely form part of the learning in classrooms" (Armstrong & Bajpai, 1988, pp 122). Discussions with teachers of mathematics suggest that the aims of the GSCE were too ambitious and a number of the aims were impossible to assess. Aims of GSCE where mathematical modelling could be applied lacked clarity and were hard to assess, meaning teachers put these in the "too hard" box. On top of this teachers of mathematics were inexperienced with mathematical models, did not comprehend the modelling process and tended to avoid using the real world in their teaching (Armstrong & Bajpai, 1988). Textbooks, syllabuses and assessment programmes ignored the potentials of the mathematical model and the learning opportunities it provides. "If students had some simple concept of what a mathematical model was a rudimentary understanding of the process of modelling many of the aims" of the GSCE "would be more readily attained" (Armstrong & Bajpai, 1988, pp 122).

In 1989 the American National Council of Teachers of Mathematics (NCTM) had one of its important goals for teaching mathematics as "Mathematically literate workers" (pp 3). Even though this goal implies mathematical modelling and its applications, again because the NCTM 1989 standards did not explicitly address applications and modelling consequently applications and modelling have not been a focus in school *curriculum* (Dindyal & Kaur,2010).

After reviewing the attempts to implement modelling in the 1980's there was clearly a need, as we moved into the 1990s, for curriculum documents to be written with clarity and specific examples.

1990's

Moving into the 1990's modelling was "no longer an upstart on the stage of mathematics education, fighting for attention and recognition" (Blum & Niss 1991, pp 45). Aspects of teaching modelling had now been on the agendas of quite a few conferences and in many publications (Blum & Niss, 1991). In the 1990's, ICTMA conferences became conferences of international significance (Houston et al, 2012). Globally modelling and its applications were appearing more frequently in textbooks for school and university. In Denmark and the Netherlands, attempts were being made to include applications of modelling in assessments (Blum & Niss, 1991).

During this time the need for specific curriculum statements was being recognised and there was an increase worldwide in including mathematical modelling in the school curriculum. "During the latter part of the 1990's, curriculum around the world started to acknowledge the importance and presence of mathematical modelling in comprehensive education" (Lingefjard, 2006, pp 96). Statements were appearing in curriculums and reports supporting the inclusion of modelling were being written. Syllabuses were starting to be written so mathematical modelling could be justified (Clathworthy & Galbraith, 1991). Even so, according to Tam (2011), these curriculum statements and support material were still not specific enough, but at least the need for more clarity and curriculum support was being acknowledged.

Even with all the support at the beginning of the 1990's most modelling courses were still about models and not the process of modelling (Clathworthy & Galbraith, 1991). Historically "the teaching of active modelling was more-or-less overwhelmed by the teaching of models, applications that the students were asked to learn rather than formulate themselves" (Burkhardt, 2006, pp 183). A move was needed to focus on the process of modelling as opposed to just the teaching of models.

2000's

As we moved into the 2000's the situation was still modest at best (Burkhardt, 2006). Curriculum statements were still too general and the need for political pressure still apparent (Burkhardt, 2006; Tam, 2011). Tam (2011) discusses how models and modelling are mentioned in the NCTM standards (2000) and New York State mathematics core curriculum (2005). Tams' (2011) conclusion is the descriptions in both were still too broad and needed to be more clearly defined, with further descriptions and more support material needed in the documents as to how models and modelling could be implemented.

In 2003 political pressure came from the large international comparative OECD study PISA which " emphasises developing the capacity of students to use mathematics in their present and future lives as goal of mathematics education……meaning that students should understand the relevance of mathematics in everyday life, in our environment, and for the sciences" (Kaiser et al, 2010, pp 219-220). This was an opening for mathematical modelling as a means to reality-orientated mathematics education (Kaiser et al, 2010).

Another significant political move came from the 2010 Common Core State Standards (CCSS) document in the States (Tam, 2011). Modelling was given a conceptual category and its own standard of mathematical practice. Being given a conceptual category meant it was able to be integrated throughout the standards and through all levels. As a conceptual category, the process of modelling is treated as a goal of the curriculum. Throughout the document there is an attempt to describe what modelling is (Tam, 2011).

Supportive statements in the document included a note that content must be understood in contexts, although contexts were not specified, making this goal obscure. There are comments supporting models to represent and describe, and using parts of the process of mathematical modelling. The whole process of modelling is not referred to, but parts of the process of modelling and necessary skills are noted. For example “choose and using appropriate …”; “construct a function …” (Tam, 2011, pp 31). There is a move in the document for more clarity instead of vague curriculum statements open to many interpretations as has been the case in the past. Tam (2011) believes that the “crucial step towards institutionalising mathematics curricula with a major emphasis on modelling and applications had not been made” until 2010 with the common core standards (Tam, 2011, pp 28).

From personal experience it has been observed that the development of modelling in the New Zealand curriculum has been following a similar pattern. Up until 2007 the New Zealand mathematics curriculum mentions modelling and modelling processes in mathematical processes as

- “develop flexibility and creativity in applying mathematical ideas and techniques to unfamiliar problems arising in everyday life, and develop the ability to reflect critically on the methods they have chosen” (Ministry of Education, 1992, pp 23)
- “become effective participants in problem-solving teams, learning to express ideas, and to listen and respond to the ideas of others” (Ministry of Education, 1992, pp23)
- “find, and use with justification, a mathematical model as a problem-solving strategy”(Ministry of Education, 1992, pp 24)
- “devise and use with justification a mathematical model as a problem-solving strategy” (Ministry of Education, 1992, pp 24)
- “using and justifying mathematical models” (Ministry of Education, 1992, pp 25)
- “organising and interpreting data, using diagrams, graphs, and models” (Ministry of Education, 1992, pp 27)

The new mathematics achievement standards, based on the New Zealand 2007 curriculum state “forming and using a model” and “relating findings to a context” are evidence for relational thinking in all achievement standards. The achievement standards define problems to be used in assessments as “problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or mathematical contexts” (NZQA, 2013).

Conclusion:

What is apparent after reviewing the literature is the need for political pressure for the inclusion of modelling in the curriculum. There is a need for good curriculum documentation and support for developing resources for modelling. Without these modelling will continue to be haphazard at best (Burkhardt, 2006). It is not until now that good clear curriculum statements with support material are starting to be provided (Tam, 2011). It is argued by Burkhardt (2006) and Lingefjard (2006) that for students to build their ability to use mathematics when dealing with the real world they need modelling experience, instruction on strategies for modelling and analytical approaches to a problem, and they need to reflect on the process invoked. To be useful in the real world for work and personal life we want students to learn applications of models and to be able to formulate their own models (Blum & Niss, 1991; Burkhardt, 2006).The question is how to effectively put these into a curriculum.

4.3 Research into modelling and teaching

Learning Modelling

Research on mathematical modelling is far from complete. The beginnings of work has been done on the impact of authenticity of tasks on mathematical modelling development (Palm, 2007), influence on learning from different styles of teaching modelling (Lege, 2007), competencies involved in the modelling process (MaaB, 2006), blockages students experience working with modelling (MaaB, 2006) and related challenges for teaching of mathematics modelling (Kaiser et al, 2006; Ikeda, 2007). The following are brief summaries of the findings of these studies.

Palm (2007) carried out a study to look at what part authenticity of a task plays in a student's response to that task and their mathematical modelling development. Previous studies showed that students were failing to use their common sense when dealing with real world problems. This was thought to be because of lack of exposure in school to realistic problems. Results of Palm's (2007) study showed that, even in a classroom setting, the more authentic a task was the more students drew on their real-world knowledge and the better the students developed their modelling abilities. The authenticity of the task also helped show the importance of mathematics. It was concluded that authentic problems are important in the development of modelling skills and significant of mathematics in the real world (Galbraith, 1995; Palm, 2007).

Lege (2007) conducted a study comparing two different styles of learning mathematical modelling. It was found that students who actively learned modelling by "doing it" outperformed those in a modelling assessment that had learnt it by looking at examples and questioning what had happened. The groups sat two different assessments with separate assessments to reflect both styles of learning. The active modelling group outperformed the questioning group in both of the assessments.

Jo Boaler (2001) conducted a 3 year study that compared two different teaching styles of mathematics. One group of students were taught through repeating skills in a standard format while another group of students were taught through open ended projects where skills were taught as the need for them was apparent. The results showed "that a modelling approach encouraged the development of a range of important practices, in addition to knowledge, that were useful in real world situations" (Boaler, 2001, pp 121). The group taught via open ended projects, learnt to choose and adapt different methods and hold mathematical discussions. This group outperformed the skill taught group in the national GSCE examination, although they were unfamiliar with the format of the assessment. The result of being taught by an open-ended modelling approach meant these students did not perceive any difference between the mathematics of school and the real world. This group drew on the mathematics they did in school and made use of it outside the classroom. The study demonstrated that modelling developed practices that are useful in real life. Mathematical modelling is an "opportunity to engage in important mathematical practices that have value beyond the mathematics classroom"(Boaler, 2001, pp 126).

Maab (2006) carried out a review of the empirical research into modelling competencies. The studies hinted that "mathematical skills, knowledge about the modelling process, a sense of direction and working in groups seem to have a positive impact on the development of modelling competencies whereas the context can motivate but also distract" (Maab, 2006, pp 119-120). Maab (2006) also carried out his own studies into how far students could carry out the modelling processes on their own and what competencies where being used or missing. The study showed that 13 year olds are able to develop modelling competencies. The study also showed that the mistakes that were made were assumptions that were either over-simplified reality or wrong. There was difficulty assigning mathematical symbols to factors and some students lost track of their own workings. The results clearly showed that more competencies are needed for doing mathematical modelling than standard textbook tasks and that "modelling competencies are more than just running through the steps of the modelling process" (Maab, 2006, pp 139).

From my look at past research the important things are: a task should be as authentic as possible (Palm ,2007; Alsina, 1998; Galbraith, 1995; and Kaiser-Messmer, 1993), students need to be actively modelling, that is learning by doing (Lege, 2007; Graeme Wake, conversation Massey University, 2011) and to be aware of student difficulties with the modelling process, in particular with making correct assumptions, assigning symbols and keeping track of the process (Maab, 2006).

Teaching Modelling

Ikeda (2007) carried out case studies with eight different countries, MaaB (2005) carried out a study with lower secondary school students in Germany, with Bukhardt (1984) and Armstrong and Bajpai (1988) writing papers, to help identify obstacles to be surmounted for the effective implementation and teaching of mathematical modelling. The following is a summary of obstacles identified from the different studies and written discussions.

The biggest issue for the successful implementation of mathematical modelling is teacher education and development of teacher attitudes conducive to modelling (Ikeda, 2007; Armstrong & Bajpai, 1988; Burkhardt, 1984). Burkhardt (1984) considered teaching styles and peer group attitudes to be the most serious challenge in effective integration of teaching modelling and its processes, with modelling considered too difficult and irrelevant for classrooms by certain educators (Armstrong & Bajpai, 1988). Ikeda's (2007) case studies in England, Japan, Netherlands and Canada all identified teacher attitudes and mathematical beliefs as an obstacle for the implementation of modelling with more professional development and experience of modelling needed for teachers in mathematical modelling(Armstrong & Bajpai, 1988). Canada and Japan both have modelling established in the curriculum but applications are being used as examples to teach mathematical concepts instead of 'doing' modelling. This could be because of "teacher's understandings of mathematical modelling, their view of mathematics and inexperience of doing mathematical modelling themselves" (Ikeda, 2007, pp 460) thus affecting resources used and developed to teach modelling. Lack of resources including adequate assessment resources where deficiencies are believed to be present in Czech Republic, Netherlands and Japan, with Czech republic talking about students viewing the language of mathematical modelling as new and hence having difficulties interpreting it. In the lesser developed countries like Mozambique, curriculums dominated by pure mathematics make it difficult for the teaching of mathematical modelling to gain any momentum. Student mathematical beliefs were also identified as a barrier in Germany with results from Maab's (2005) study supporting this. Generally "applications and modelling are not perceived as an important part of the mathematics curriculum" (Ikeda, 2007, pp 46) and hence there is a lack of teacher education, both pre- service and professional development, effecting teachers perceptions and understanding of mathematics and the development of adequate resources and assessment materials.

5. Framework for this study

5.1 Realistic Mathematics Education and Hypothetical Learning Trajectories

The framework for this study developed out of the theory of Realistic Mathematics Education (RME). RME is about giving students a realistic mathematical experience based on applications that use real contexts (De Lange, 1996). It is concerned with using real life examples that are experientially real for students and allowing students to find and create the mathematics themselves. The mathematics comes out of the student's experience (Gravemeijer, 1999). Modelling fits with the philosophy of RME. It takes a real situation, explores the situation with the goal being to represent the situation mathematically. That is, to create a mathematical representation of the situation, thus mathematics comes out of the experience.
Out of RME a theory known as 'developmental research' has emerged (Gravemeijer, 1999; Gravemeijer, 1994). This is concerned with the development of instructional classroom activities and involves the creation, trailing, reflection and redesigning of activities. The activities and process are not fixed and are continually reformed and reconsidered. The results and observations are not collected in a traditional test format but by looking at classroom experiences.

The classroom activities are developed based on a hypothetical learning trajectory (HLT). A HLT consists of a set of learning goals for students, planned instructional activities and conjectured learning process (Gravemeijer, 1999; Simon, 1995). The HLT is not fixed and open to adjustment day to day by the teacher. Cobb, Zhao and Visnovska (2008) found it useful while working in classrooms to view a hypothetical learning trajectory as being made of conjectures about the collective mathematical development of the classroom community opposed to hypothetical learning trajectories that focus on individual student's mathematical reasoning. I will take this approach for my research. That is, for this study I will use activities with real contexts that are real for the students. These activities will have collective classroom learning goals and conjectured learning processes. The data collected from the activities will be about the experience of the activities.

To help me develop my HLT, I started with a broad mathematical goal, namely the process of mathematical modelling for a senior secondary student. From this broad goal I developed mathematical goals for the HLT. I then looked at several different rich realistic activities and investigated them to see which ones might be suitable to develop into a classroom activity. I chose to develop three situations for the classroom, dropping a mobile phone, player positioning for a rugby conversion kick, and how far out to see can you see the light from a lighthouse. I then trialled these activities, collected data on students experiences of the activities, and reviewed the activities based on the learning goals and conjectured learning processes of the HLT.

5.2 The broad mathematical goal - mathematical modelling for a senior secondary school student

The first step of developing the RME research development statement in the form of a HLT is to explore our broad goal of the process of mathematical modelling for a senior secondary school student. That is, what would mathematical modelling for a secondary school student look like? This will be used as the foundation to develop our HLT.

Mathematical Goal for Senior Secondary Mathematics

Our broad mathematical goal is the process of mathematical modelling.

Our mathematical goal for senior secondary students' is to understand the process of mathematical modelling at an appropriate level. This comprises the following.

a) Observations and contextual background for the problem situation.
Students will participate in group discussions and brainstorming to establish a shared understanding of the problem situation.

Students will carry out a literature research that looks at the nature of the problem and will involve collecting background information for the problem situation. Background information includes previously collected observations and contextual information. Are there any physical laws relating to the situation? At the end of the literature research findings should be presented in a consolidated form.

b) Defining the task.
Students will contribute to class and group discussions to produce a set of goals for the model. These goals will be formulated by asking questions to identify the important aspects of the situations. Questions will establish why the model may be needed, what it will be used for, what things are already known about the situation, and what further information is needed

These questions will help define the task and set the direction for the model. For example, is the model to be used to gain a better understanding of what is happening, to make predictions, to answer a specific question? Is it to be used for decision making?

c) Recognise and identify the essential aspects of the situation.
Students will participate in class and group discussions to identify the essential aspects of the situation. Discussions will involve brainstorming all the known factors that may affect the situation. These factors will be classified by students into known (things that are given) and unknown factors (things we would like to know) and recorded as lists. Students will then rank the factors in order of importance. Part of this process will include deciding which things to assume will remain constant and which things we will ignore for the situation. Examples of the type of questions to ask are as follows. What things (variables) are we interested in? What information is missing? What things are given and what do we need to know? The goal of this step is to identify factors, rank them in order of importance and to describe and make assumptions for a particular situation. Part of the goal is to begin to establish the language of variables, constants and parameters that will be used later in their education.

d) Using the essential aspects form or choose a suitable model.
Students will participate in brainstorming any physical laws that relate to the essential aspects of the situation, choosing an approach giving reasons for their choices and attempting to form equations for the model. This process will draw from what students already know.

A suitable approach will be chosen by having a selection of tools and models available to choose from. This could be using appropriate software and or aids to help find relationships. For example geogebra, excel, matlab, graphics calculator, using tables or a selection of already known models. Examples of questions the students will be asking are as follows. What mathematical tools will be appropriate for the situation? Which mathematical approach will give the best result?

To form the equations students will have already participated in identifying the physical properties of the situation. These and any physical laws relating the factors will form the basis for the equations. The students will need to look particularly at what interactions are happening and decide how and if these can be written as equations.

e) Testing the model.
Once a model has been formed the student will test the model by applying critical thinking to it. This is done by asking whether the model makes sense? Does known data or results from physical experiments fit the predicted outcomes of the model? Do computer simulations of the model fit with what you would expect in reality? The students will assess the model by asking questions as follows. Does the model hold true with what we know? How do the results obtained from the model compare with reality?

f) Improving the model.
Students will review their models to see if they can make improvements. This is done by starting the modelling process again from the beginning with the underlying thinking being prompted by questions as follows. How can we better make the model fit reality? Why doesn’t the model give the expected results? What are the shortfalls of the model? Can we add in a new variable into the model? How will this affect the model? What factors could be used to make a ‘better’ model?

6. Implementing the Hypothetical Learning Trajectory

6.4 The Plan

6.1.1 Mathematical Goal

<u>*Statement of the mathematical goal*</u>
Our mathematical goal is for students to experience, remember, and be able to recreate, in basic form, the processes involved in the mathematical modelling of a real situation, with particular focus on identifying the essential aspects for model formation.

The broad stages of the modelling process are: forming a modelling group; establishing a shared understanding of the problem; undertaking necessary research of the context and what is known; setting a mathematical direction for the model (ensuring it is well-defined); identifying the essential aspects of the situation; mathematically interpreting the essential aspects of the situation for the model; constructing equations or other mathematical representations of the model; critiquing the model mathematically and otherwise; modifying the model mathematically and otherwise; applying the model; refining the model; repeating the cycle to improve the model; reporting on the model (Wake, 2011; Tam , 2011; Dym, 2004; Bender, 2000; Treilibs et al, 1980).

The level of each of these stages that comprise the mathematical goal for Yr 12 students are as follows.

Forming a Modelling Group
Modelling is not done in isolation but is a team effort with each member bringing their own strengths and expertise, drawing from their whole skill base, not only their mathematical one (Wake 2011). The modelling group compromises those who will work together on the problem situation, participating in discussions, researching, making decisions about their model, creating and refining one model, and reporting on their work.

The student modelling group (SMG) will differ from a real world mathematical modellers group (RMG) because a RMG will contain a group of experts from different fields, including mathematical experts and those expert on the situation to be modelled. The SMG are not yet professionals and will be experiencing the process of modelling for the first time. The SMG will still behave as a modelling group. The forming of a SMG will include developing an understanding of the difference between themselves and a RMG. The process will also include pooling the information they have together, identifying where the expertise in their group lies, and how they will access expertise and information they may need but not have amongst themselves. Finally, the process includes identifying the different tasks of the group and organising how these tasks will be accomplished.

Establishing a shared understanding
This is done through discussion of known information by group members. Further investigations may require access to further information. These investigations could be literature searches on the context and physical laws relating to the situation or sourcing any previously collected observations. Establishing shared understanding will differ for the SMG compared to the RMG as it will be teacher led. In the RMG the leading of the discussion and investigations comes from within the group and is shared by the members.

Defining the problem

The group needs to discuss why the model is needed and what it will be used for. From the discussion a set of goals will be written for the problem. These goals will set the direction for the model. Defining the problem will differ for the SMG compared to the RMG again as it will be teacher led. The teacher will ask leading questions whereas in the RMG there will be at least one person or a guide with underlying knowledge of the process to guide the group. This role is taken on by the teacher in the student activity.

Identify the essential aspects of the situation

This is done by brainstorming all the things that are connected to the situation, writing these in a list, splitting the list into what is known and what is unknown and ranking these factors in order of importance. Order of importance is sorted by how much of an affect a factor has on the situation or by how much variability a factor has. This is further sorted considering important factors that remain constant throughout the situation and factors we can ignore for the time being as they have minimal effect on the situation. The list is then reviewed again to simplify the situation. This stage will differ for the SMG compared to the RMG in levels of experience. This will most likely be the first time students have experienced identifying variables, classifying them in order of importance and simplifying a situation, the modellers having more experience at this. This step has been identified as one of the hardest stages of the modelling process and the one done least well at tertiary level (Alona Ben-Tal and Robert Mckinnon discussion Massey University 2011). Again the teacher plays a vital role in guiding the student and giving them effective tools to work through this stage.

Model Formation

Model formation is finding a way of relating the essential aspects of what we know with what we want to find out and involves using any known physical laws and equations and/or forming new equations. This is done through discussion and exploration of the list of essential aspects. Discussion is carried out, a plan for exploration is formulated and exploration begins. Exploration can take the form of using a software programme to mimic the situation, carrying out a physical experiment, using previously collected data and a package like Excel to find any patterns, or testing previously formed models. Discussion is initially teacher led with the teacher taking the role of the guide, prompting questions and providing knowledge of any physical known laws relating to the situation. Levels of experience in exploration will be different from the RMG with the RMGs having more exposure to forming connections between the essential aspects, knowledge of physical laws relating to the situation, access to more sophisticated computer programmes or programmers and the possibility of having come across similar problems before.

Testing the model

Testing the model is applying critical thinking to the model to see if it makes sense and fits reality. This involves questioning results produced by the model to see if they fit reality and asking why the model holds true or not. Results of models can be produced by running a simulation of the model using a computer or otherwise conducting a physical experiment to collect data. Testing the model will differ for the SMG due to time constrictions and limited access to equipment. The SMG will spend time discussing whether or not the model fits with reality discussions led by the teacher and in groups. In practice there will not be enough time or equipment to carry out tests to prove or disprove the formed model. In saying this the RMG also will most likely have a deadline set by the client and they too will be working to time constrictions. These time constraints will have been allowed for in the whole process instead of being determined by a school curriculum. Again the RMG's level of critical thought will be more in depth and they will have easier access to sophisticated equipment.

Improving the model
This stage of the modelling process is a review of the model. This is done by asking questions concerning whether it can be improved to better fit reality and discussing if it is practical to add things to the model to make it more realistic. This becomes an iterative process going through the cycle again brainstorming any changes and their effects on the model, all the time reviewing the modelling process from the beginning with the underlying aim of improving model and make it 'like' reality.

This will differ for the SMG again due to time restrictions. The students will enter into a teacher led discussion about how to improve the model and the cycling back process for improvement. The iterative process will not be possible due to time restrictions of school curricula. Students will discuss the obvious things to be added and tried for next time generating a list of possible factors to be included in the model to make it more realistic.

Reporting on the model
This is a clear concise report to a general audience. The SMG will present a 2 – 5 minute verbal report on the process they went through. The RMG would provide verbal and written reports throughout the process with a formal written and verbal report given at the end of a project. Academic literature reviews would be required in the RMG reports providing support to ideas, solutions and assumptions.

In summary the student will experience discussion, brainstorming and exploration in groups. This discussion, brainstorming and exploration will differ from a RMG in terms of the level of depth of material and expertise available to draw on. Nevertheless the student will still be able to go through the process of modelling and develop a simple model. The length of the experience will differ for the student and it is most likely there will not be time to fully develop a model, test it and go through the iteration cycle for improvement. These areas are most likely experienced through discussion to get a feel of what would happen if there was enough time. It is hoped out of this experience to see that there are multiple different ways of modelling a situation.

Due to curriculum restraints a 5-lesson model only is available for the teaching of the process of mathematical modelling. This will limit the students in how far through the process they will get. It is anticipated that, at a minimum, the students will get to experience exploring the formation of the model.

6.1.2 Planned instructional activities

Recommended Features
When teaching mathematical modelling the problems used to teach model formulation need to be simple and easy to understand (Wake, 1997; Treilibs et al, 1980). This is so that the formulation and interpretation aspect of the modelling process becomes accessible to students (Wake, 1997). The students must be able to acquire a good understanding of the situation being studied and understand the problem contextually.

Problems must also be real and need to be a new situation for the student, that is, a situation where the student has not thought about the problem analytically before. This is necessary to effectively enable the student to experience the process of mathematical modelling as opposed to an artificial or textbook experience of modelling (Treilibs et al, 1980).

The mathematics required to competently form a model of a problem needs to be learnt two to three years previously by the student (Clathworthy & Galbraith, 1991; Treilibs et al, 1980). This is in order for the student to be able to experience the process of modelling and not get bogged down in the mathematics. The activity should be of a low level demand using mathematical techniques that are well assimilated by the student for useful progress (Clathworthy & Galbraith 1991). Even if this is not 100% true it is safer to assume this for providing an experience where success can be obtained (Treilibs et al, 1980).

Taking into account all of the above, the activities need to be based on the principles of modelling as outlined in section 3, the above and processes further highlighted by our mathematical goal.

Planned instructional activities:
For these activities the students will be working in groups of three where two members are self-selected and the third member teacher selected. The groups will be consistent throughout the planned instructional activities.

For the last five minutes of each scheduled classroom lesson the students will be asked to keep a diary and write what they have done each day. This will be done so there is a record of what they experienced, and in order to undertake reflection on what they have done. Reflection at the end of a learning session makes it more likely to retain learning and make use of the experience at a later date (Mason, J. (2011) Plenary Talk. Volcanic DELTA Conference, Rotorua).

The teacher's role is to facilitate the discussions by asking leading questions to make up for the lack of knowledge behind the process from the students.

Planned Learning Activity 1: (Appendix A)
Goals:

- to introduce the process of mathematical modelling;
- to setup SMGs for the sequence of planned learning activities;
- to establish how these groups will operate.

The problem situation:

"You've dropped your phone. Arrgghhh….. Will it break?" The context of the problem was chosen as a relevant topic of concern for the students and therefore engaging. It was anticipated to spend one lesson using this topic to introduce the process of mathematical modelling, with the focus being on the process and not on producing a specific solution to the problem.

Lesson plans:

Use the Powerpoint “Mathematical Modelling. What is it?” to talk for two to three minutes about my experience of working on a mathematical modelling team at Maths in Industry Study Group 2004 and 2011 and with the Centre for Mathematics in Industry, Massey University 2011.

Introduce the problem by dramatically dropping my phone on the floor. “Arrgghhh Ive dropped my phone….. has it broken?”

Use PowerPoint Slide 3 to set up the discussion cycle to establish the background and context of the problem. A discussion cycle is first a teacher led whole class student discussion, then discussion in groups (3-4 minutes), followed by each group reporting to the class with further class discussion. Have a discussion cycle using the following questions as prompts to establish a shared class understanding of the background and context of the problem.

- What do we know about phones and dropping them?
- Are some phones more resistant to being dropped than others?
- What features of a phone might make it less (or more) susceptible to damage?

Use Slide 4 to go through the discussion cycle to establish exactly what we want to know and therefore form a shared class definition of the problem. Use the following questions as prompts

- What do we want to know?
- Why do we want to know it?
- What could the model be used for?

Use Slide 5 to go through the discussion cycle to identify the essential aspects of the situation. The discussion cycle will also consist of brainstorming everything that affects the situation and listing these on the board, then splitting the list into what things we know and don’t know. Go through the list asking how important each factor is. How big an effect that factor has on the situation? Is the factor constant or variable? Can we assume the factor is constant? Can we assume the factor has minimal effect and ignore it? The focus of this stage is to identify the important factors to use to form the model.
Use the following questions as prompts:

- What do we know about the situation?
- How can we define what we know?
- What do we want to find out?
- How can we define what we want to find out?
- What can we control?
- What things are we going to make assumptions about?
- What things can we assume are constant?
- What things are we going to ignore?

Use Slide 6 to go through the discussion cycle to explore an approach to formulate a model. The model may involve (but is not limited to) collecting data, forming an equation or physically building a model. Use the following questions as prompts:

Are there any physical laws that relate what we know with what we want to find out?
What is the physical relationship between a falling phone and the ground?
Can we write this relationship as an equation?
Can we write what we want to know in terms of what we know?
What is the simplest model possible?

Use Slide 7 as a teacher led class discussion to discuss ideas for how to test the model and models in general.

Use Slide 8 as a teacher led class discussion to discuss the iterative cycle of modelling. That is, how you go back and repeat the cycle to improve the model and improving models in general.

Planned Learning Activity 2: (Appendix B)
Goals:

- to mimic as closely as possible the experience of being a member of a mathematical modelling team.
- to experience the process of attempting to develop a mathematical model (solution).
- to experience and gain knowledge of some of the tools of mathematical modelling, in particular, but not limited to, software, tables, physical laws and known mathematics.

The problem situation:

"Your rugby team has just scored a try. You are responsible for taking the conversion. Where on the field will you place the ball for the kick?"
The context was chosen as I was already familiar with different models and their development for the best position to take the kick. This placed me in a good position to be able to provide guidance for this context. The aim was to go through the discussion cycle for all stages of the process. Groups would also work independently between discussions with the teacher providing guidance on the process (not the solution) where appropriate. Again the focus is on the process not on the actual solution. It was planned to spend three lessons on this situation.

Lesson plans:

Use Slide 10 to introduce the problem.

Use Slide 11 to go through the discussion cycle to establish the background and context of the situation discussing 'what do we already know about taking a conversion kick?'

Use Slide 12 to go through the discussion cycle to establish exactly what we want to know and therefore define the task. Use the following questions as prompts

- What is it exactly that we want to know?
- How do we define this?
- Why do we want to know it?

Use Slide 13 to go through the discussion cycle identifying the situation's essential aspects. The discussion cycle will be more student lead than in Activity 1. The discussion cycle will also consist of brainstorming everything that affects the situation and listing these on the board, with the teacher taking the role of the recorder, splitting the list into what things we know and don't know, going through the list asking how important each factor is. How big an effect that factor has on the situation? Is the factor constant or variable? Can we assume the factor is constant? Can we assume the factor has minimal effect and ignore it? The focus of this stage is to identify the important factors to use to form the model. Use the following questions as prompts:

- What do we already know about a rugby field?
- What do we already know about kicking?
- What are we going to assume/ignore for a simple model
- What factors are important?
- What things affect the position a kick is taken from?

Use Slide 14 to go through the discussion cycle brainstorming ideas to explore formulating the model. Out of the brainstorming ask students to write a plan for model exploration, giving individual group members roles where appropriate, all the time keeping in mind the aim of finding a generic model for the situation. Use the following questions as prompts.

- Write list of all important factors
- What do we know?
- What do we want to find out?
- Can we write what we know in terms of what we want?
- Are there any rules that relate what we know with what we want? (brainstorm)

Use Slide 15 to go through the discussion cycle to find ideas for testing the model. How are we going to test the model to see if it holds true? Test the model if time allows.

Use Slide 16 as a teacher led class discussion to discuss the iterative cycle of modelling. How you go back and repeat the cycle to improve the model and improving models in general.

Planned Learning Activity 3: (Appendix C)
Goals:

- to determine how much of the process of mathematical modelling the students picked up.
- to understand what they found easy.
- to understand what they found difficult.
- to find out whether they can identify the underlying ideas about the process of mathematical modelling?

Activity 3 is an assessment. The problem situation is "It is dusk and you are doing a night sail to Great Barrier and have just passed Tiri-tiri Matangi island. Tiri-tiri Matangi is home to what was the last manned light house in Auckland's Hauraki Gulf.
http://en.wikipedia.org/wiki/Tiritiri_Matangi_Lighthouse. The light house was built in 1851 and manned up to 1984 when it became automated. The lighthouse is 21 metres in height with its base

standing 91 metres above sea level. As you pass you wonder how far out to sea the light from Tiri-tiri Matangi will stay in sight?”

The assessment asks how you would go about forming a model to provide a solution for the problem. The assessment then presents different parts of models that could be used to form a solution and asks students to critique the models, thinking about what assumptions have been made to form the model so far, and how the model might work and what might you do next in the process of forming the model. The last part of the assessment asks students to talk about what things they liked and did not like about the modelling activities.

The context of sailing into the distance was used as it was hoped it was sufficiently interesting. It was based on an old problem of sighting a lighthouse when coming over the horizon but was thought this was too old fashioned and was not longer relevant with technology today and so was modified. The original problem had been quoted in literature as being a rich but achievable mathematical modelling experience (Kaiser et al, 2010) with the mathematics being year 11 right angled triangles. The assessment was to determine how much of the ‘process’ of mathematical modelling had the students picked up and also to see what solutions the students would come up with.

6.1.3 Conjectured Learning Process

<u>Thinking and learning students might engage in</u>

Forming a group

- Working in a group discussing ideas
- Working in a group carrying out individualised tasks that benefit the group

Establishing a shared understanding

- Discussing observations and contextual background for the problem situation eg rugby players sharing what they already know about kicking a conversion kick.
- Researching observations and contextual background for the problem situation eg looking at data already collected previously for the situation.
- Looking at what research has already been done towards a model eg use internet to see what and if any models have already been created

Defining the task

- Asking questions to define task eg what do we want to know?
- Defining the task for the problem situation

Recognise and identify the essential aspects of the situation

- Identifying what we know about a situation
- Identifying what we want to find out about a situation
- Recognising and identifying important factors (essential aspects) of a situation
- Use lists to identify important factors of a situation
- Identifying and classifying factors as important and unimportant to a situation
- Recognise what an assumption is eg assume all rugby players have the same kicking ability

Forming the model

- Using essential aspects try to formulate or find a model to describe the situation
- Looking for relationships amongst essential aspects.
- Use tables or other tools to look for relationships amongst important factors of a situation eg Excel
- Get an idea that there are physical laws that can be used to define relationships and form mathematical models eg Newton's Laws.
- Use computer software or other tools to help formulate a model eg Geogebra, Excel.

Testing the model

- Generating ideas for testing the model eg physical experiments, testing equations.

Improving the model

- Generating ideas for improving the model eg adding another variable to the model.

Overall process

- Experience the process of mathematical modeling

6.2 The Trial

6.2.1 Actual Classroom Activity

Activity 1:
This activity was trialed first by myself with a group of three 11 and 12 year old students. (See Appendix D). The thing that I learnt from the trial was in order for the activity to be successful, the first priority was to establish ground rules for respectful discussion techniques. The trial of the activity was successful for moving through the process of mathematical modelling, showing that this was possible even for younger students. We did not spend much time on improving the model but plenty of time on exploring how to find the model and testing the model through physical experiments.

In the actual classroom activity we first established groups of three students by asking students to get into pairs and splitting every second pair so that one member went with the pair in front and the remaining member went with the pair behind. A class discussion was had on group guidelines, respectful discussion and roles within a working group. Also established during the session was for the last 10 minutes of each lesson to be spent writing diaries reflecting on what they had done in each lesson.

Using the Powerpoint "Mathematical Modelling. What is it?" I introduced mathematical modelling by talking about my personal experiences of modelling at tertiary level and with industry. The problem was introduced by dropping my phone on the floor and pretending to stamp on it. "Arrgghh I've dropped my phone ……has it broken?"

Using the Slides 3,4,5,6 we discussed stages 2,3,4,5. For each stage of the process a class discussion was held, then an individual group discussion, then feedback to the class was carried out. This cycle of discussion was performed to establish a common understanding of the problem, to define the problem, to identify essential aspects, and to explore ideas for formulating the model.

For stage 4 the discussion cycle consisted of brainstorming as a class, writing a list of factors on the board which was then sorted into two lists, one for what we already knew and what we wanted to find out. The factors in both lists were then ranked into order of importance.

For stage 5 we had to find which factors would give us information to make the physical law f=ma useable, in particular which factors could realistically be used as estimates for force. We decided to use height as a measure of force assuming that the higher the phone was dropped the more force would be involved.

We didn't fully develop the model and therefore did not test it or improve it, though there was a lot of discussion early on about testing the model. The students were naturally curious about when a phone may actually break and were consequently engaged in the problem.

In summary, we did establish effective working groups and I got to model how to use lists to find essential aspects of the problem and how to simplify and transfer information into useable information for this particular situation.

We spent most of our time in class discussion. We spent two periods on this instead of one and students were engaged and motivated to find a solution. Overall the purpose of Activity 1 was meet, that is, establishing operative working groups and to introduce the process of mathematical modelling.

Activity 2:
I did not trial this with my small group of students, so it was untested.

The problem was introduced using Slide 10. The questions on Slide 11 were used as prompts to establish stage 2. Part of the contextual background for stage 2 was that we had looked at this problem earlier in the year when we had studied trigonometry. At this time we had physically gone out and measured distances and used these to calculate angles to find the biggest angle. This work was drawn on for stage 2 and 3 making defining the problem reasonably straight forward. As the problem had previously been done through physical exploration we defined the problem as the best position being the one that gave the biggest angle, this time our focus was on finding a formulae to generalize the situation with an equation that would work for different conversion positions.

Slide 12 was then used for prompts to brainstorm all the factors that could affect the position you would take a conversion kick, make a list of the factors separated into what we knew and what we didn't know and what we wanted to find out. Things we did not know we assumed were constant, for example the kicking ability of player.

The next day the focus was on taking the important factors and forming the model. Students discussed and explored ideas and then decided which approach they wanted to take. They made plans consisting of jobs to do for exploring and forming the model. I demonstrated Geogebra as one tool they would have available to them and discussed using Excel. Most groups used Geogebra to explore different positions and Excel to record findings and look for patterns. One group used the internet to research previously found general models.

I was away for the next lesson. The students carried on exploring without me, demonstrating that the activity was motivating and engaging enough to carry on without a teacher. No diaries were written this day.

In the final lesson each group verbally reported back on what they had found so far, as they would for a real client.

The discussion cycle established in Activity 1 was the discussion process used for all stages. This time it was more student led than teacher led. Feedback from the students was that discussion groups at school were harder than real life as people tend to 'self-select' for team work in the work environment. At school groups were not fully self-selected and not everyone wanted to participate in a group.

In summary the process of mathematical modelling was experienced except for stages 6 and 7, testing and improving the model. The main focus of this activity was to experience the process behind formulating a model which was experienced by all present.

6.2.2 Data Collection

Data was collected from three sources: student diaries, student assessment, and student interviews.

i) *Diaries*

Students were asked to keep a diary at the end of each lesson, writing between a paragraph to half a page per lesson. The last five minutes of each lesson was dedicated to writing diaries individually and in silence. There was one lesson when diaries were not written due to my absence. Seventeen out of the eighteen students handed in their dairies for evaluation. The reason one student choose not to hand in their diary is unknown.

The diaries are a record both of what they remembered about what they were taught in the classroom learning activities, and of their reflections of each lesson. The diaries were written straight after the teaching to provide a first person account of what occurred in the lessons (Burgess, 1981). These accounts provide insights into the activities. By looking at them collectively I can build a group account of what occurred. While diaries are a written record of what has happened each diary may vary in depth and detail, meaning that follow up in an interview may be useful (Burgess, 1981).

Diaries also provide a written recall of language used in lessons and evidence of students building their vocabulary for mathematical modelling.

ii) *Assessment*

Students sat an individual assessment at the end of the classroom learning activities. Fifteen out of the eighteen students did the assessment with two students absent and one student having left school. One student could not answer the questions in the assessment. Assessments varied in length from one page to six pages. In general all questions were attempted with question 5 and 6 being the most poorly answered. One student produced an outstanding assessment demonstrating a depth of understanding and potential as a mathematical modeller.

The assessment consisted of a series of open-ended questions on the process of mathematical modelling, what they had learnt about mathematical modelling and what they liked and didn't like about the modelling experience. For the process of mathematical modelling there were three different approaches to modelling a single problem, each presented at varying stages of formation. The students were asked to comment on the approach, discuss what they would do next and how they would explore using this approach to find a workable model. There was no opportunity to discuss the assessment with other people and it was done under test conditions. The assessment provided information on students' recall of the process and application of mathematical modelling and aspects they did and did not enjoy.

Like diaries assessments are a written record of students work and will vary in depth and detail. From the student point of view for an assessment they are trying to maximize their marks and hence trying to write what they think the teacher wants to see rather than what they really think. They also occur in a particular time interval which may be better for some students than others. They involve a certain amount of pressure which some students relate to better than others. Follow up interviews may be useful in some cases.

iii) *Interview*

Students participated in a 10 – 15 minute interview. The interviews were conducted between four and six weeks after the completion of the classroom learning activities. They took place at school during lunchtime. I was the interviewer. Eleven out of the eighteen students took part in the interviews. Two students had left school, one did not want to participate due to not wanting to be audio recorded and we ran out of time before the school year finished to interview the four students. The interviews provided information on what was learnt and could be recalled at a later date from the classroom learning activities.

The interviews were a series of open ended questions about modelling designed to transport the student back to their experience of mathematical modelling and see what they could recall of the process. All questions (see Appendix E) were asked of all students with some students being asked extra questions to probe for more in-depth information and in some cases to help students remember the experience.

The interviews were useful for confirmation and exploration of the experience and allowed for further clarification of students' thinking and experience that was not possible from individual written responses (Tashakkori, 2003). Disadvantages of the interviews were that they were time consuming and there was the potential for possible reactive and investigator effects (Tashakkori, 2003). As the interviewer I had to be as objective as possible in interpreting the students' answers so that my response to their answers and my extra questions was interpretative of what they had said and not leading in any way. I had to be conscious of not being subjective and placing words in their mouths. It was the same when analyzing the interview statements. I had to be as objective as possible when interpreting their statements as reflections of the different stages of the process and being conscious of not hearing what I wanted to hear.

Possible errors that could affect the validity of interviews are students telling me what they thought they wanted me to hear instead of their honest thoughts, me failing to be objective, decreased engagement by myself due to the amount of time it takes to conduct a set of interviews. I found as the interviews went on and time became a pressure I got tired and I rushed and did not engage in one interview as well as the others.

Summary of Data
The number references in this section and in 6biii refer to the student number in the summary of diaries (found in Appendix F), summary of assessments (found in Appendix G) and summary of interviews (found in Appendix H). 1 refers to student 1, d1 refers to student 1 in the summary of diaries, a1 refers to student 1 in the summary of assessments and i1 refers to student 1 in the summary of interviews.

i) *Diaries*
From the diaries what was remembered or felt worth mentioning from the classroom activities can be summarized as follows:

Parts of identifying the essential aspects of the situation was mentioned by nearly all of the students (fourteen of the seventeen). Most of the students mentioned planning for exploring how to develop the model (eleven students) and use of software and the internet as part of their investigation (twelve students). Experiencing the discussion cycle was mentioned by ten students. Using physical laws, making assumptions and formulating equations and formulae was mentioned by approximately a third of the students. Working in groups, discussing the use of the model, researching contextual background, delegating tasks, using diagrams and graphs and testing the model was mentioned by approximately a quarter of the students. Defining the problem, improvements to the model and using previous mathematics was mentioned the least (one student).

No mention was made in the diaries of my personal experience of modelling though this was mentioned by one student in the interviews. All aspects of the modelling process were mentioned at least once.

Overall, identifying the situations essential aspects, planning for developing the models, using software and involvement in discussion groups were the parts of the process that were most experienced by students and mentioned in the diaries.

A summary of individual diaries can be found in Appendix F.

ii) *Assessment*

From the assessment the two biggest areas remembered and commented on by students was the need to do research to know the background context of the problem, and identifying the essential aspects of the situation. Recognising and identifying the essential aspects was mentioned as brainstorming all the factors that affect the situation, sorting them into what was already known and what we wanted to find out and finally into order of importance. Factors that affect the situation was mentioned by nearly all students with over half the students discussing sorting factors into what we already know, what we wanted to find out and what we could ignore. Making assumptions was also discussed as part of this process.

Question 2 used a diagram to model the situation. There was strong evidence from almost all students of the recognition that to be useful diagrams needed to be to scale and the developed model needs to be realistic. Supporting comments supported the idea that for the model to be useful it needs to be true and fit reality (a2,a3,a4,a5,a6,a9,a10,a11,a13,a16,a18).

Making and recognising assumptions and testing the model was mentioned by half of the students. Testing of the model was discussed as using results from exploration, physical experiments or predicted measurements to verify any equations formed.

Using modelling tools such as physical laws, diagrams, scale drawings, tables, grids, known mathematics and formula, software and physical models was discussed by a few students when exploring developing a model further, with some students proposing a new and different models to the ones presented.

Making a plan for developing a model was the least mentioned by students.

All aspects of the process were mentioned by the collective group.

For the question on what things had you learnt about mathematical modelling the following comments were made:

- "I have learnt that mathematical modelling consists of a lot of trying and failing" (a2)
- I learnt "how to brainstorm ideas, use computer software, think deeply about a problem and with perseverance almost anything can be solved" (a4)
- "I learnt the process helps eliminate and simplify the model solution and software is used to aid the solution" (a5)
- "I learnt you need to work in a team" (a9)
- "It was hard but when we had group discussions the process was a lot clearer" (a10)
- "I leant how to get along in a team"(a12)

There were comments providing evidence of seeing the usefulness of modelling and its openness such as:

- "Mathematical modelling helps to solve things you never thought were solvable" (a6).
- "I learnt that there is more than one right answer in modelling ……it is used for a purpose that would be useful and save time" (a11).

Over half of students when answering what they liked about mathematical modelling communicated enjoyment of working in groups with supporting comments being:

- "Working in groups can sometimes be helpful as different people have different strengths and ideas" (a3).

- “It was hard but when we had group discussions the process was a lot clearer” (a10).
- “I liked working in a team” (a12).

A summary of individual assessments can be found in Appendix G.

iii) *Interviews*
From the interviews the following parts of the process of mathematical modelling, including the detail, have been experienced and retained from the teaching sequence:

Identifying the essential aspects of the situation, working in groups and forming the model were the parts of the mathematical modelling process remembered well and retained by students. I will discuss these first.

Identifying the essential aspects of the situation was experienced and retained by all students. Identifying the important aspects of the situation was described by all and identified by most as the first step in the process of mathematical modelling. The initial step in identifying the important aspects of the situation was through discussion of all the factors and variables that could affect the situation. This discussion involved brainstorming knowledge within the group, asking questions and research. The factors identified were listed and then sorted through discussion in two ways. First into what we knew and what we wanted to find out, then into order of importance. Order of importance was established by discussing which factors were consistent and had the biggest effect on the situation. Important variables were the ones found across the general situation. Factors that were ignored were ones that had a minimal effect or least common across the situation. Factors that were assumed to be constant were ones that contributed to the most variability for a general situation, producing inconsistency between reality and result. Assumptions were made for this group of factors. Factors that weren’t measurable were also ignored or assumed constant. Factors needed to be measureable in order to be included in the model. One student mentioned they explored how different variables affected the situation in establishing order of importance (i9). As students were identifying the situation’s essential aspects they were also simplifying the situation. Ranking variables into order of importance was part of this process. Simplifying was also done by grouping things related to each other together and having one variable to express that group.

Students talked about working in groups throughout the interviews. All students enjoyed working in groups, finding it beneficial and useful for bouncing ideas off each other and drawing from each other’s knowledge.

Forming the model was discussed by most students with the first step being planning, through discussion and brainstorming, how to explore forming the model. Some groups gave group members roles where they worked in roles separately then came back together and discussed findings. Exploration of forming the model was done both individually and as a group using known mathematics, formulas from physics, graphs, scale diagrams, tables, computer software, practical investigations (collected data, put into table, formed equation), discussing ideas and research. This stage was described as “figuring out the calculation” (i6), “using and thinking about calculations and putting the model together” (i9), “looking at making a generic model suitable for a wide variety of situations” (9), “a general formula being needed for the important variables” (i11) This could be done by exploring a specific situation then looking at the general situation (i1). Students felt they did not have enough time to complete this stage (i3,i9) and would have liked more guidance with this stage (i3).

Research for the model, planning for developing the model, using modelling tools and testing the model was discussed by approximately half the students. Research for the model was not identified as the first stage of the process but as a complementary part of other stages, particularly for identifying the important aspects of the situation and for forming a model. Research was mentioned to help establish and identify relevant variables (i2) and for finding solutions for the model (i13). Planning for the model was mentioned as brainstorming and discussion to develop a model plan (i1,i3,i4,i9,i11,i13).The most mentioned modelling tool was software (i1,i3,i4,i6,i9,i11,i18). Using known mathematics, scale diagrams, graphs and tables was mentioned by a few students.

Testing the model was also mentioned by approximately half of the students with some students discussing using the newly formed model to make a calculation and seeing if the result fitted reality (i6,i9). Some students discussed testing the model physically either through a physical experiment, previously collected data or previously formed model and seeing if model fitted with this reality (i10,i11,i13,i19).

Defining the problem was mentioned by a few students as looking at the question (i4,i6) and establishing what we want to find out (i9).

Improving the model and delegating roles were the least mentioned part of the process. When improvements were mentioned they were discussed as choosing one variable that had been assumed to be constant and add it into the model as a new variable (i13). One other student discussed improvement as an iterative process to use if the model did not work (i3).

Out of the interviews different themes came through. These themes were context of problem, difficulty of forming a model, lack of time to complete the process and enjoyment of the unit.

With reference to context of the problem a lot of the students liked the phone problem best as they could see the relevance and usefulness to themselves, though there were students who enjoyed the rugby problem best as they were more familiar and confident with the context (i1) and another student enjoyed the lighthouse problem best as this problem was the most unfamiliar to him and liked the fact that the solution was very unknown (i2). Overall the three problems covered or captured the audience, and had contexts that the audience could relate to. Nearly all students mentioned they would have liked to develop the model for the phone fully.

The most difficult part of the process talked about was coming up with the solution (i3,i4,i10,i11). This was described as “being able to figure out all the ingredients that go into the cake just not knowing how to put them together to make the cake” (i3).

Lack of time to complete the process of modelling was apparent from the interviews. Most students mentioned running out of time and not being able to fully develop model.

Overall the unit was enjoyed with reasons being modelling gave the opportunity for independent thought (i10, i13) they were actively involved instead of teacher led (i2,i13), worked in groups (i5,i9,i19) and it was practical work (i9,i10). The unit was described as “different and a breath of fresh air” by one student (i13).

In summary all aspects of the process were mentioned by the collective groups. Identifying the essential aspects of the situation, working in a group and forming the model were the parts of the mathematical modelling process experienced and retained most by the students. The parts recalled least were defining the problem, delegating roles and improving the model.

A summary of individual interviews can be found in Appendix H.

See Appendix I for a table of evidence of mathematical processes experienced collected from the diaries, assessments and interviews.

6.2.3 Discussion of Student Learning

1. Forming a group
Summary results: Discussing and brainstorming information, ideas, identifying factors, classifying factors and planning for model formation was experienced as a group. Ideas for testing the model and improvements to the model were also discussed by some groups.

Those groups that delegated jobs (2,3,12,and 13) experienced carrying out individualised tasks and coming back and sharing findings within the group (13).

Supporting evidence: Working in groups pooling ideas was beneficial and made the process easier (3). Working in groups helps to develop ideas (11). Modelling was perceived and experienced as something that would be very hard in isolation and needed the support of a group and the knowledge and resources a group provided (4,10,19).
“I like doing it in groups as it made it a bit easier as everyone has different ideas and also made the process go faster” (3).

Analysis: Techniques for effectively working in a group was experienced by all students. It was learnt that modelling is not done in isolation and the effectiveness of working in a group, through support and drawing on individual strengths within the group, was experienced by all students. It was found that discussing testing and improving the model was not formally taught though naturally occurred in some groups. This could be because of our natural curiosity to have things make sense. Behaviors of real world modelling group was experienced through delegation of roles and coming back to share and discuss together.

2. Establishing a shared understanding
Summary of results: The need to know the background context of an unfamiliar problem came through as one of the most important steps. Establishing a shared understanding was experienced through research and discussion of context of the situations, including details of the situation. This stage was not done as an isolated stage but occurred throughout the process whenever more

information was needed, in particular when establishing the situation's essential aspects. This was mentioned in the data by approximately a third of the students.

Supporting evidence
"Research and discussion of your model is a very important part as it gives you the foundations of your model" (2).
One group researched already formed models and critiqued for their own use (13).

Analysis:
It is not surprising that needing to know the background context of an unfamiliar problem came through as important. Without having an understanding of the problem it is very hard to begin to form an effective model. Students learnt that research and background information is important and it is ok to go and investigate if that knowledge is not already present in the group. They learnt that it was ok not to know everything . It is not a negative thing to source out additional information, rather a vital behaviour for the formation of a successful model. Information and ideas in the public domain can be used and built on.

3. Defining the task
Summary of results:
Defining the task was not strongly recalled by students. It was mentioned by only four students in the interviews. A diary recorded defining the problem in terms of usefulness and purpose (5). In the interviews defining the problem was mentioned as looking at the question (4,6) and then asking questions to clarify the question by defining what it is we want to find out (9). For example will the phone break? What does break mean? Is it chipped, or is it the battery falls out or the phone won't work again?

Supporting evidence:
There was only one diary recording evidence for defining the problem (5) and only 3 students mentioning it in their interviews (4,6,9).

Analysis:
The fact that defining the problem was not strongly recalled or recorded in the diaries could be as this part of the process was teacher led and the definition accepted as a group during the classroom activities. The problem that needed the most defining was the phone demonstration problem where this stage was all teacher led. This could mean the students may not feel much ownership of the process and therefore was not valued. It could be because it was not reflected on straight away in the diaries making future recall of this stage low. The rugby kick problem was very quickly defined as a class due to students having already had experience of it earlier in the year.

4. Recognise and identify the situations essential aspects
Summary of results:
There was evidence in all three forms of data collection that this step was experienced and retained by all students. There was strong evidence that sorting factors into what we know, what we wanted to find out and then ranking in order of importance was learnt as a technique for doing this. Lists

were mentioned throughout as an effective tool for sorting factors. One student identified this as a way of simplifying the process (4).The need to make assumptions was recognised by most students.

The brainstorming and discussion during this stage involved discussing knowledge within the group, asking questions about the situation, deciding if any more research needed to be done, discussing classifying factors and unconsciously simplifying the situation. Brainstorming and discussing was identified as the first step of this stage (1,2,3,4,5,6,7,9,10,11,13,19). Identifying the situations essential aspects was thought to be the first stage of the mathematical process by the majority of students.

There was mention of students enjoyment when classifying the factors of the situation and making decisions about the factors (3,4).

Supporting evidence:
"There is a lot of variables to be considered"(9)
"Most of the work done was on variables" (19)

Analysis:
The teaching sequence including questioning procedure and discussion cycle was successful for identifying the situations essential aspects. They were learning the concept of classification and simplifying when classifying (sorting) and ranking variables then moving one step further by finding one variable to represent a group of variables (4,10,11). It naturally occurred out of the experience that students had reached their own understanding that for a variable to be important it will have a big effect on the situation (6,19).

Whilst working in groups the students experienced a sense of what safety or unsafely felt like for sharing ideas and consequently learnt a safe group was needed to effectively express and develop ideas.

Brainstorming for this stage was identified by the majority of students as the first stage of the process of mathematical modeling. Subconsciously this may have been my focus having identified it as the part of the process that tertiary students struggle with the most (Alona Ben-Tal &Robert Mckibbin, conversation Massey University, 2011). My focus was to find a way to help give students tools for their future use and at the same time experiencing mathematical modelling and this would have influenced the delivery of the unit. Another reason could be because the contexts used for the problems students were already familiar with, so we could focus on building skills for identifying the important factors and variables.

5. Forming the model
Summary of results:
There was strong evidence that the first step of this stage was planning (1,2,6,8,9,11,12,13,14,15,17) with delegation of jobs being part of this planning (2,9,12,13). Computer software was used and acknowledged as a tool for mathematical modelling by all students with drawings, tables, previously known formulas and mathematics being acknowledged as other modelling tools. Carrying out practical investigations was mentioned in the interviews (4,10,19). Discussion of using physical laws

to help form model was noted in most of the diaries but only mentioned later in two interviews. Three students expressed finding forming model difficult (3,4,10). Lack of time for this part of the process was apparent.

Supporting evidence:
"Was like figuring out all the ingredients to make the cake just not knowing how to put them all together to actually make the cake" (3).
"Learnt techniques to help understand and identify patterns in data and use these patterns to devise a formula" (5).
"I have learnt that mathematical modeling consists of a lot of trying and failing" (2)
"You use various methods to find a solution not just one formula" (2)
Ran out of time and needed more guidance with the model formation stage (3)

Analysis:
Student groups experienced planning an investigation to develop a solution to a problem that is originally unknown to them. Within this group, they experienced how to have individual roles and come back together to share. This is in line with real world modeling.

They learnt that a broad range of tools are drawn from to put models together and that there is no specific one way to form a model or set path (2).

Physical laws were discussed in the classroom activities for forming the model for the phone but we did not follow this through till the end. The groups were left more on their own to explore and plan the model formation for the rugby kick. These reasons might explain why there was little mention of using physical laws, other than already known mathematics, in the interviews.

Though students enjoyed the fact that they were exploring they found it difficult to put together a model without help (3,4,10). Overall more guidance was needed for this stage.

Limited time was clearly a factor in how involved students got with this stage.

6. Testing the model
Summary of results:
Testing of the model was experienced by the students through discussion. No actual testing took place in the classroom activity due to lack of time. Even so 7/11 of the students discussed testing and critiquing the model in their interviews with ideas ranging from physical experiments to using the model to make predictions. Testing the model for the phone was talked about during the classroom activity very early on as the most exciting thing about the process and provided motivation for developing the model. Ideas for testing models came through in the assessment even though this was only experienced through discussion. Comments were made during the interviews to indicate that they would have liked to have developed fully and tested the phone model.

Supporting evidence:
Test calculations to see if it fits reality (6)
Make estimate to test model against (9)

Test model physically (10)
Critique model with data and discussion (13)

Analysis:
Here the students are learning about critical thinking "if this is true will this also hold" and seeing how to apply reality to a situation that has been mathematised. Ideas for testing the model, even though we did not actually test the model, could have been present in all the data because of our natural curiosity. We want to know whether or not something will work, and see this as part of the relevance of doing something. Perhaps ideas for testing come naturally. Testing also provided motivation for developing the model.

7. Improving the model
Summary of results, supporting evidence and analysis:
Improving the model was not experienced by the students. There were small discussions on improvements during the classroom activities. This had not had a big effect on the learning of the students with only two students mentioning improvements, one student in their diary and the other in their interview.

An interesting point came out of the trial of activity 1 with the group of three 11 and 12 year old students. As the activity was introduced and we were discussing the contextual background one student struggled to see the connection between mathematics. This could be because her mathematical experiences so far had been skill based or based on pattern finding and she had not had much experience with mathematical situations in context. This is something for primary and intermediate curriculum writers and teachers to be mindful of. Is there a need for more real applications of mathematics for years 1 to 8?

Conclusion:
Thinking and learning that the students engaged in that went well was forming a group, establishing a shared understanding, recognising and identifying the situations essential aspects and testing the model. Students worked in and developed skills for effective group work for mathematical modeling. They learnt that background and contextual information for the situation was important and "that you need a lot of background information to make your model accurate" (2). Identifying the situations essential aspects was experienced by all students and successfully taught. Even though testing the model was not formally taught it occurred logically through student's natural inquisitiveness, allowing for an experience of applying critical thinking in a modelling situation.

Defining the model, fully formulating the model and improving the model were the least experienced aspects of the process. Students did not get any real hands on experience on defining the model. The tools of planning, the idea of drawing on physical laws and known mathematics, using tables, lists and software was experienced. Formulation of a model was not experienced well by the students. This was due to lack of guidance and the need for more exploration time. Before the activities are done again work needs to be done to develop more strategies to look for relationships and using these to form a model. Extra time also needs to be added to allow this and for repeating the full process to improve the model.

The biggest focus on the process was on identifying the essential aspects of the situation and exploring formulating a model.

A large part of the process was about “using your own thinking to try and find the answer” (11) and gaining an experience that there is a flow on effect from one part of the process to the next and sometimes parts of the process intermingled.

No new mathematics came out of the experience other than the ‘process of mathematical modelling’. All other mathematical skills used were those previously learnt.

The underlying theme throughout the classroom activities was about effective working skills for group or team work. Modelling is not something done in isolation but an activity where everyone draws on each other’s skills (Wake, 2011)

There was strong evidence of enjoyment of the process and group work. All students enjoyed working in groups, finding it beneficial and useful for bouncing ideas off each other and drawing from each other’s knowledge.

6.3 The Next Cycle: Future Modifications

6.3.1 Recommendations for change and further research

General issues that came out of the trial
After trialing the unit it is apparent that the time required to more fully experience the process of mathematical modeling is greater than the time I allocated. If we want to teach modelling we need to find the time. This is something for curriculum writers to consider.

One of the issues that came out of the interviews was that students wanted more guidance with the model formation stage. This could have been because they wanted to be led through it and not think for themselves or it could be because the structure and prompts in the classroom activities were deficient. This brings us to a key issue of how to balance support that may interfere with being able to experience the process of mathematical modelling, which does involve being uncertain, and the need to independently explore different options to decide which option to take. If too much support is given then the whole point of modelling is lost.

Modelling needs to be real. Using real examples in the classroom creates an opportunity to revise mathematics students need to know. This can substitute many aspects of a formal review of past learnt material. Building models will also provide the incentive to learn new mathematical techniques and skills as modelling tools that can be applied to the present situation.

How much mathematics do students need to know to be able to model? As discussed in the literature, for students to properly experience the process of mathematical modelling the mathematics required should be a level or two below their current level. Students will develop a model regardless of their mathematical background. The less mathematically proficient the simpler the model will be that is formed and it may end up being oversimplified. The more mathematics the student knows the more complex the model may be. A complex model is not necessarily the best.

Either way a creative experience is entered into and models are formed drawing from a student's previous mathematics knowledge.

Was the experience motivating for the students? Most students communicated that they found the context relevant and engaging. The question is, did this motivate them to learn new mathematics or did it just keep them engaged longer when normally they would have been disinterested? The results from the interviews showed that overall they liked the modelling part of the course more than other parts. Most of the reasons offered were because they were actively involved and not just practicing skills and they could see the relevance.

Reviewing the actual activity

Another proposal for a modelling activity is to have an already developed simple model, and carry out discussion of its features and how it was formed. The students would then go through the modeling process to add a new variable to the model. This would help reduce some of the stages but still allow the student to experience the process of modelling. My concern with this plan is the engagement of the students through the process. If the exploration of the already formed model is too hands off there are opportunities for student disengagement and contradicting the principle of students learning by doing. Maybe this type of activity would be better suited as an assessment activity.

One thing that was missing in the powerpoint slides was forming the simplest model first. This comes into the model formation stage and is definitely one of the things that needs to be considered.

To gain richer data from the interview questions I would add the question "What do you see as the tools of mathematical modeling?" That is, what things do we use to help us make models?

Question 2 in the assessment is a good counter example, though when it was developed it was not intended to be. The model given was not realistic and if repeating the assessment again I would recommend developing it as a counterexample using questions like "Would this model work?" "Why or why not?" I would use this to build insight into what things would work and what will not.

If I was to repeat the classroom activities my recommendation would be to allow a minimum of 8 hours of classroom time. I would allow more time to fully develop the phone problem, develop more guidance prompts for stage 4 model formation, add reporting back as part of the process of modelling (by having students formally present their findings for their models and the processes they have gone through), and making the addition above to the powerpoint presentation. This would provide a richer experience and reporting on the model that is in line with a RMG.

7 Conclusion

7.1 Conclusions

It is important for mathematical modelling to be included in the curriculum because it prepares students to use mathematics as a problem solving tool in their daily lives, and it provides a foundation for students wanting to do further mathematics and mathematical modelling.

Main findings of research

The participants in this study were average ability year 12 students from a decile 10 school in New Zealand. They worked in groups as they took part in two authentic open ended teaching activities designed to experience the process of mathematical modelling. Data for research was collected in the form of diaries, assessments and interviews. The main findings of this study are:

- An authentic modelling process is achievable within the restricted secondary school classroom environment.
- More time is needed to allow for a full experience of the process of modelling. My recommendation is a minimum of 7 hours.
- Year 12 students were able to recognise and use strategies to identify the essential aspects of the situation.
- There were gaps in students' ability to form models.
- Students appreciated modelling because it allowed them to be involved in an activity.

We cannot extrapolate these findings to make conclusions for all students of diverse abilities, different backgrounds and internationally without further studies.

How could the research have been done better? By having an independent interviewer who was not their teacher to remove any pre-conceived ideas of the student understanding and any pressure felt by a student to say what they think their teacher wants to hear.

The classroom activity can be improved by having more prompts for stage 5, forming the model. This would provide scaffolding for students to experience, and develop skills, for forming a model. More time is needed to become familiar with this stage. Extra time is needed to experience the full process of mathematical modelling.

Relationship of my work to literature

The main points emerging from existing literature in regard to classroom activities is that the activity should be as authentic as possible (Palm, 2007), that students do better by 'doing' modelling opposed to being shown (Legge, 2007; Boaler, 2001), and working in groups helps develop modelling abilities (Maab, 2006). The style of activity for this research was chosen to provide a realistic situation to give students an opportunity to draw on all their knowledge, both mathematical and contextual. Having the lessons as a modelling experience allowed students to be involved and participate in the modelling. Having students working in groups meant they were working in the most effective way to develop modelling skills (Maab, 2006).

Maab (2006) showed that students as young as 13 could model which was consistent with our findings with the group used to trial the first activity. The results of Maab's (2006) study allowed us to have confidence in developing an activity for secondary school students.

Further research

Questions for further research for a better classroom experience of mathematical modelling include trialling activities with more able students, identifying mathematical modelling tools, and investigating the subjectivity of teacher knowledge for modelling success. New work is needed for stage 5, forming models.

Trialling activities with more academically able students will be beneficial to help identify effective prompts and tools for stage 5, forming the model. Research to identify mathematical modelling tools students use will be useful for developing resources for teaching a broad range of modelling techniques. Looking at the teacher's role in developing skills and behaviours for mathematical modelling will enable us to get a better idea for how much professional development will be needed for teachers for successful implementation of modelling into the curriculum. It was identified by Ikeda (2007) that the biggest barrier to mathematical modelling is teacher education and attitudes. Questions to be investigated include what sort of teacher knowledge is required to supervise modelling experiences? What type of involvement should a teacher have with mathematical modelling to effectively teach it?

A useful study would be to repeat the teaching activities with a larger time allocation in different decile schools and different ability students. It would be good to conduct the trial in a different country in order to solidify the tentative conclusions I have been able to draw.

Looking at the current literature on the best position for taking a rugby conversion kick, to further improve the developing model, a model that includes a players' kicking range needs to be explored.

The implication of my research for researchers and curriculum developers is that if we think mathematical modelling is important we need to write clear curriculum statements that are easy to implement, allocate sufficient time in teaching schemes and continue to develop and identify good authentic situations that can be used at secondary school. A good resource is one that has a real context that is relevant to the student, uses mathematics that is already known, and allows for the process of mathematical modelling to be experienced.

7.2 Limitations

Limitations of the study include scope of the participants, quantity and type of data collected, depth of the experience, time restrictions and subjectivity of the researcher and teacher.

The participants of the study were eighteen year 12 students of average ability. While it would have been interesting to see what a high achieving academic student would do with the experience, the result showed that modelling is possible for an average student. Trialling the activities with more academic students may have allowed us to develop effective tools for stage 5, formulation of the model, by establishing and analysing techniques and skills students use for this stage. As mentioned, investigating issues influencing a students' ability to form a model is a subject for further research.

Eighteen was an appropriate number of participants for the size of this investigative study and the scope and time limitations of a Masters thesis. Even so this number of participants is not large enough, along with the study being restricted to New Zealand average ability students from a decile 10 school, making it hard to make any substantial conclusions. We cannot make conclusions about other students in different situations. We can draw only tentative conclusions of what may or may not be possible for mathematical modelling and propose ideas for further research. A worthwhile succeeding study would investigate different decile schools (decile 1 and 5) or different ability classes (top academic class and bottom stream class). Gathering results from countries such as South Africa or Asia would allow us to strengthen the international picture. Extra studies are needed in order to supplement and consolidate the conclusions I have been able to draw.

The data was analysed by looking at the statements made by students and classifying them according to how they referred to different stages of the modelling cycle. Limitations of the data included the possibility that students wrote or told me what they thought they wanted me to hear instead of their honest thoughts. Even though my goal is always to provide an environment of free expression and good relationships with the students there is still the potential of my position to influence their responses to the research questions. Students are aware of the authority my position carries and my assessing their future work.

As a researcher, my limitations included the possibility of subjectivity in interpretation of student answers. Despite my best attempts my own engagement and commitment to the topic and any pre conceptions of individual students could have influenced my analysis of the results. I had to be very conscious that I was not putting a bias on the study and interpreting the answers as what was said and not what I wanted to hear.

The diaries, assessments and interviews gave a broad range of opportunities for recall of experience. Even so more in-depth questioning could have been used, with more structured questions in the assessment including good counter examples to provide richer data. As mentioned, a study focusing on mathematical modelling tools used by secondary school students would be a further useful study.

As mentioned in 6biii, a limitation of the teaching activity was my own personal focus on giving students tools for identifying the essential aspects of the model. This may have led to the main focus being on this stage of the process. Identifying the essential aspects of the situation was the part of the process that was experienced well by all students.

It would be helpful to repeat the trial with another teacher teaching. This would allow us to compare which stages of modelling were done well and provide further insight into why this might be the case. Questions for further research could include: Is student's modelling ability subjective to teacher knowledge? How much of an influence does teacher knowledge have on student modelling ability?

Despite best intentions, my commitment to modelling as a secondary school topic may have skewed my reporting towards the favourable, however it is my intention and resolve at all times that this report has been written without bias and with professional and academic integrity.

7.3 Personal reflections on my experience

My journey began by modelling the behaviour of a modeller so that I could experience and get a better understanding of the process. Instead of developing a model from scratch, I researched existing models and critiqued these for possible improvements.

The context I choose was the rugby conversion kick. I was interested in seeing if and how we could use mathematics to better understand sport. I evaluated existing models for enhancements, at the same time getting an idea of how a mathematical model was built. I focused in on one model to improve it to take into account a players' kicking range. I thought this variable would best better the model to make it more realistic and generic. I got as far as collecting data to be able to come back to at a later date.

The possible alternative approach to a modelling activity of looking at an already formed model and adding a variable to it, mentioned in 6c, is similar to the behaviour I first experienced as a modeller. The possible difference between myself and the students is that I picked my topic and consequently was highly motivated. This may not be the same for students in a classroom situation where choice is not always a possibility. In the classroom support would need to be given for the whole process. When you add a variable you are still going through the whole process of mathematical modelling, just not having to formulate the model from scratch.

Next I had the opportunity to work with the Centre for Mathematics in Industry and do a 300-level mathematical modelling paper. As a guest at the centre I participated as a member of a commercial mathematical modelling team and learned how such a team operates. This included carrying out research, discussing and developing ideas and communicating with a client. I participated in, and observed people developing ideas. Out of this I developed the behaviours of a mathematical modeller and gained an appreciation of intellectual property. Doing a course on Modelling helped me develop a broader range of mathematical skills to be drawn on to form models.

While I was at the Centre my research topic started to become clear. It was during this time I choose to focus on developing a model activity that would provide the experience of modelling that would be suitable for a secondary school student. From conversations with staff at Massey University I had identified that students in mathematical modelling struggled to identify the essential aspects of a modelling situation. I wanted to develop an activity that would address this. My focus became very narrow. I started to develop activities focused on developing definitions and identifying variables, constants and parameters. As I worked on this I realised that these were far too restricted and missed the experience of the process of mathematical modelling. I decided to pull back and develop a HLT that encompassed the process of mathematical modelling as well as an experience of identifying the essential aspects of the situation. Out of this I learnt one of the important modelling behaviours, which is to be able to see the bigger picture and not get bogged down with the detail.

Next I started looking and critiquing activities for their usefulness as a genuine experience of the process of mathematical modelling. The activities needed to be real, not artificially constructed activities as appear in many textbooks. I explored several contexts before deciding on the three used for the trial of the classroom activity.

The next stage was to develop the contexts into workable activities for the classroom. I did this by developing some questions that could be used as teacher prompts for each stage of the modelling process. Examples are in section 5. There is no set way to do modelling. These questions were designed to help students identify the important parts of each stage and to guide them forward through the process of modelling.

Once I had chosen the context of the activity and developed it, I then had the new experience of teaching mathematical modelling. This drew on and developed questioning techniques and prompting techniques. I decided that it was not my job to direct the modelling but to provide guidance and different options to consider. I also had to be confident in the process and in my students.

My next learning curve was that of the researcher. I began to learn how to conduct research always questioning my interpretation of the data, and trying to be a neutral as possible.

I have found the experience highly rewarding, having had the opportunity to participate as a member of a mathematical team, develop and teach a unit on mathematical modelling and conduct research on the teaching and learning of mathematical modelling. I have developed skills to effectively contribute to a mathematical modelling team, to begin to teach a genuine experience of mathematical modelling, and to act as a researcher.

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Appendix A

Mathematical Modelling

What is it?

- Team work – my experience

You've dropped your phone. Arrgghhh..... Will it break?

- What do we know about phones and dropping them?
- Are some phones more resistant to being dropped than others ?
- What features of a phone might make it less (or more) susceptible to damage?

These questions are the background or context to our problem.

- What do we want to know?
- Why do we want to know it?
- What could the model be used for ?

These questions help us to define the task. What is exactly that we want to know.

- What do we know or can control? How can we define what we know ?
- What do we want to find out?
- How can we define what we want to find out?
- Simplifying model – what is the simplest model possible?
- What things can we assume are constant?
- What things are we going to ignore?

These questions help us to decide what things are essential and needed to develop the model. What do we know? What do we want to find out? What things are we going to assume? Remember to start simple.

- Is there any physical laws that relates what we know with what we want to find out?
- What is the physical relationship between a falling phone and the ground?
- Can we write this relationship as an equation?

These questions help form the model. It may involve collecting data, forming an equation, physically building a model ....

- Does the model work?

Testing the model

- Improve the model.
- Does it fit reality?
- What things can we add to the model to make it more realistic?

These questions ask us to think about improvements that can be made to the model.

Appendix B

Best position to take a rugby conversion kick

You are responsible for taking the conversion. Where will you take it?

- **What do we already know about taking a conversion kick?**

These questions are the background or context to our problem.

- **What is it exactly we are wanting to know?**
- **How do we define this?**
- **Why do we want to know it?**

These questions help us to define the task. What is exactly that we want to know.

- What do we already know about a rugby field?
- What do we already know about kicking?
- What are we going to assume /ignore for a simple model
- What factors are important?
- What things affect the position a kick is taken from?

These questions help us to decide what things are essential and needed to develop the model? What do we know? What do we want to find out? What things are we going to assume? Remember to start simple

- Write list of all important factors
- What do we know?
- What do we want to find out?
- Can we write what we know in terms of what we want?
- Is there any rules that relate what we know with what we want? (brainstorm)
- Adjust for generic questions

These questions help form the model. It may involve collecting data, forming an equation, physically building a model ....

- How are you going to test model?
- Ideas?

Testing the model

- What could be added to model to improve it?
- How would you go about it?

These questions ask us to think about improvements that can be made to the model.

Appendix C

Classroom activity 3: Assessment

It is dusk and you are doing a night sail to Great Barrier Island and have just passed Tiri-tiri Matangi island. Tiri-tiri Matangi is home to what was the last manned light-house in Auckland's Hauraki Gulf.
http://en.wikipedia.org/wiki/Tiritiri_Matangi_Lighthouse
The light-house was built in 1851 and manned until1984 after which it became automated. The lighthouse is 21 metres in height with its base standing 91 metres above sea level.
As you pass you wonder how far out to sea the light from Tiri-tiri Matangi will stay in sight?

You decide to attempt to build a mathematical model to determine where you will be when you lose sight of the light from Tiri-tiri Matangi

Qu 1.

Describe in your own words how you would go about forming the mathematical model (you do not have to actually form the model, just describe the process you will go through to form the model).

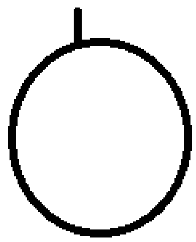
When you get home you find out that another sailing friend, Sam, had also been working on the above problem. He decided to build a real model of the problem, consisting of a sphere to represent the earth, and a small stick to represent the lighthouse.

Qu 2.

Do you think Sam's idea is a good idea?
Why or why not it is a good idea?

What assumptions are Sam making to use this model?

Instead of physically building the model Sam draws a circle to represent the earth and adds the lighthouse to the drawing.



Qu 3.

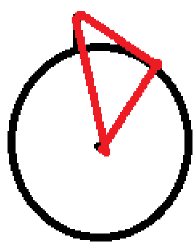
How could Sam use this model drawing to find a solution?

What background information would he need to know?

What maths (and physical) laws could he use to develop a model from the drawing?

What assumptions would he be making?

Sam added a right angle triangle to the drawing.



Qu 4.

How would this help in forming the model?

What things would he be assuming did not change?

Qu 5.

What would be the next stage in forming this model (you do not have to do the next stage, only write what you would do next)?

How would he test this model?

Another solution Sam came up with was to watch as lower items disappear below the horizon at various distances and then use this to estimate when taller objects would disappear.

Qu 6.

Explain how this model might work to find a solution.

Qu 7.

In your own words write 5 sentences (5 different things) that you have learned about mathematical modelling.

Qu 8.

What did you like about the modelling experience?
What did you not like about the modelling experience?

Appendix D

Trial of classroom activity 1 with the Fibonacci club

The Fibonacci club consists of three students aged 11 and 12. The first thing necessary was to establish respectful discussion techniques for the group. That is, talking in turn and how effective group work operates. We spent one 1 ¼ hour session on discussing the contextual background of the situation including everything we already knew about dropping phones, what features we thought affected the durability of a phone (material it is made of; make of phone; phone cover or no cover; possibility of relationship between desirability and durability of a phone). The group defined the problem as "Will the phone break by dropping it 1 metre above the ground" and defined break as "screen cracks" or "plastic chipped" or "battery comes out". The discussion identified what the group perceived to be the important factors that would influence whether phone would break or not as height phone dropped from; force phone hits ground; surface phone dropped on to. The assumptions made were to look at one type of phone only, keep weight constant, gravity is constant and the phone screen hitting surface face down when dropped.

The background information I contributed was newtons 2nd law of motion f=mxa. As a group, including myself, we brainstormed to see if we could find a relationship between any of our identified factors and newtons 2nd law of motion. We thought height would influence the force the phone would hit the ground, and force is determined weight (mass x gravity) and gravity. We decided to test our model by dropping phone at different heights and measuring force with a pair of padded kitchen scales, recording the results in a table.

At the next session we tested the model. We brainstormed, drew up a table for results, made a model involving kitchen scales to estimate force to test model our model. We then tested and found limitations with model, adjusted model and re tested.

We found our first test wasn't holding true to the equation we formed. We had to critique the situation to see if equation model wasn't holding true or if there were limitations with the kitchen scales used in the test model or some other factor. It was concluded that there were physical model limitations to our test model. It did not hold true over a height of 70cm. This could be due to the mechanics of the scale we were using to drop the weight into.

As the activity was introduced, by discussing the contextual background, there was one student who struggled to see the connection to mathematics. One possible explanation for this could be that a lot of her mathematical experiences so far have been skill based or based on pattern finding and the student had not had much experience with mathematical situations in context.

Appendix E

Interview Questions —Kerri Spooner—Authentic Mathematical Modelling in the Classroom

Do you remember the unit we did on mathematical modelling?

We looked at three different situations. What do you remember about them?

Which one did you find the most interesting?

Why was it interesting?

Did you learn any mathematics from it?

Was there anything you found difficult about it?

How did you know where to start?

Tell me how far you got with the mathematics of the model.

How did you decide what was important about the situation?

How did you know what to ignore about the situation?

Is there anything specific you remember about the other two situations?

Was there anything you found difficult about the other two situations?

Were they easy to understand?

Tell me what you remember about how far you got with those models.

Do you remember the framework we used to work through the situations?

Tell me what parts you remember?

Did you enjoy the unit as a whole?

Did you enjoy the modelling unit more than other maths taught this year?

Why or why not?

Do you remember the written assessment?

Did you find it easy or difficult?

Tell me about it.

Would you like to do more mathematical modelling?

Have you thought about modelling since we did the unit?

If so what?

Can you think of a mathematical modelling example now?

Appendix F

Summary of individual diaries

Student 1

She has mentioned the situation has different variables (aspects) and that some of these will need to be controlled for developing a model. She has mentioned using known equations that relate to the situation. Identifying important variables, which things could be ignored till later and planning for exploration for developing the model was experienced.

Student 2

Research; narrowed down causes for problem; factors that might help problem (looking at all factors that affect the situation); forming an equation; measuring factors (variables); made predictions; planning formulation of model; delegating different tasks to group members; used computer programme to develop model (geogebra).

Student 3

Looking at context of problem; working in groups; discussion in groups and as a class to identify key things factors that affect situation (what we know, what to find out and what we want to control). Finding a formula; factors that go into a formula. Measuring factors. Use of software including excel and geogebra. Using diagrams.

Student 4

Worked in groups of 3; brainstorming what things affect the situation; brainstormed assumptions; discussion and what we know and what we want to find out (classifying things that affect the situation); discussion and exploration of physical laws relating to situation; relating things that affect the situation; discussion of the important factors of the situation.

Student 5

Discussion of things that affect the situation; discussion on usefulness of model and what it would be used for; discussion on important factors that affect the situation; what things can be controlled (assumptions); looked ahead to improvements; discussion of physics (physical laws) and how they can be used to create an equation for the model; discussed and defined the problem; using software (geogebra); learnt techniques to help understand and identify patterns in data and use these patterns to devise a formula.

Student 6

Learnt how physics comes into the equation (using physical laws to write equations for a situation); forming a plan to create a model.

Student 7

Discussion of situation and factors that affect situation; investigated variables that we could control (assumptions); what we know and what we want to find out (classifying factors); learnt about physics and how this relates to forming an equation.

Student 8

Decided on a brand (made an assumption); thought about what we already knew about the situation and what we wanted to find out; formulated a plan to work through exploring to develop a model; use online resource (geobegra) and a table in excel.

Student 9

Different variables are involved in modeling; discussed all the variables; modeling used to make predictions for complex situations; discussed context in detail; listed all the factors that affect the situation; make prediction of what reasonable solution would be; delegated people jobs; made plan for formulating model; learnt about software (geogebra); used excel for data collected.

Student 10

Discussed variables of the situation (what we can control and what we can't); learnt equations in relation to variables; discussed what we knew and what we wanted to find out and what we wanted to ignore; used online software (geogebra); internet and excel software for tables to record findings; used cosine rule (drawing on previous mathematical knowledge).

Student 11

Working in groups helps to develop ideas; modeling can be used to improve items of everyday life; controlled and uncontrolled variables; what we know and what we want to find out; using physics formulae to get variables we need; how to measure variables; use evidence to prove models; plan development of model; use drawings; using software (geogebra); ignoring factors (making assumptions).

Student 12

Brainstormed what variables are present (affect the situation) and how to find out the variables; discussed all the factors that could affect the situation; made estimate for possible solution; allocated jobs to group members; planned development of model; used software (geogebra).

Student 13

Discussion on background and context and how to find an equation for the situation; discussed the physics of the situation which helped to understand the situation better. Delegated jobs to group members; made plan for developing model; used software (geogebra).

Student 14

Experiment to see what happens; made list of what we already knew about the situation and what we wanted to find out; made a plan for developing the model; used online programme (geogebra); made table in excel; draw a graph of the situation.

Student 15

Worked in groups; talked; discussed; brainstormed; used software (geogebra).

Student 16

Talked about different outcomes and influences different outcomes; talked about what we already know and what we want to find out; used computer (excel); talked and planned formation of our model; working as a group; looking at results.

Student 17

What things we know about the situation and what things we want to find out about the situation; planning how to develop model; used scale mode drawings.

Student 18

This student did not hand in any diaries.

Summary of diaries

From the diaries these things (parts of the process of mathematical modeling) have been experienced from the teaching sequence:

The students worked in groups of 3. A large part of the process involved discussion and talking with each other about the context of the situations, the purpose and usefulness of the model to be developed, defining the problem for the situation, identifying the important aspects of the situation, making assumptions, exploring physical laws that relate to the situation, planning exploration for developing the model. The context of the situations was looked at in detail through discussion and research. The different aspects of the situation was brainstormed and classified into what we already know, what we want to find out and what we can control. These aspects were then classified into their importance and things that could be ignored or assumed were constant. When looking at what things could be ignored discussion was also had at looking ahead to making improvements to the model once it had been formed. Predications were also made for realistic solutions. In planning for the development of the model group members were delegated jobs. Computer software, tables, scale drawings and diagrams was used to explore and develop the model as well as previous mathematical knowledge. Evidence and results were used to accept or reject models.

One thing missing is talking about focusing on forming the simplest model first.

Appendix G

Summary of individual assessments

Student 1

This student was absent for the assessment.

Student 2

Qu 1.

Find out what factors affect the situation, make assumptions, make a reasonable prediction and use that to inform the model. Using scale drawing hard, identifying factors need to know.
Research to see if other models have been done and compared with mine. Do a physical experiment.

Qu 2.

Discusses need for realistic scale, model that fits reality. Need for accurate scale drawings.
Identified the need to know radius of earth, information about how light travels. Identifying the need to know the background context of the problem.

Qu 4.

Saw could use trig, identified what things needed to know and what things would be ignored and or assumed.

Qu 5.

Discussed exploring the model further and had a plan

Qu 6.

Stated assumption that would make this model work and could see how it would work.

Qu 7.

"I have learnt that mathematical modelling consists of a lot of trying and failing"
"to find a solution you must sometimes assume a lot of factors that may affect your results" (make model simple)
"you use various methods to find a solution not just one formula"
"that research and discussion of your model is a very important part as it gives you the foundations of your model."
"that you need a lot of background information to make your model accurate".

Student 3

Qu 1.

List what already know about the problem, then what we really want to know about the problem, then classify factors into what really matter and what don't. Then make a plan, do some research, use software.

Qu 2.
Discussion of the importance of model being realistic of situation.

Qu 3.
Proposed a different model from one in assessment. Looked at things wanted to know, the factors affecting the situation and made assumptions.

Qu 5.
Form a model using a formula or diagram, using software and the internet.

Qu 7 .
Research, use software, work in groups “working in groups can sometimes be helpful as different people have different strengths and ideas”.

“I liked doing it in groups as it made it a bit easier as everyone has different ideas and also made the process go faster”.

<u>Student 4</u>

Qu 1.
In groups brainstorm what you know about the situation, asking and answering questions. Use software to form a solution.

Qu2.
Model needs to be realistic (scale).

Qu 3.
Identified background information.

Qu 4.
Made assumptions.

Qu 5.
Research into background information.

Qu 7.
I learnt how to:
Brainstorm ideas; use computer software, think deeply about a problem, with perseverance almost anything can be solved.
Liked working in group and not alone.

Student 5

Qu1.
Work in group of 3. Find the problem, investigate what you want to find out al l things that can affect the problem and try to eliminate them so your test is accurate.

Qu 2.
Good, shows situation simply. Again discusses need for model to be realistic (earth round or flat ?).

Qu 3.
Identifies background information needed.
Do physical experiment as a model.
Diagram needs to be to scale.

Qu 5, 6.
If he knows the factors of the situation he may be able to find a way to calculate the distance.

Qu 7.
The process helps eliminate and simplify the model solution. Software is used to aid solution. “some mathematical models are impossible to solve.”

Student 6

Qu 1.
Research. Factors that effect the situation.

Qu 2.
Good idea if was to scale but thought this would be difficult (model needs to be realistic).

Qu 3.
Drawing needs to be to scale.

Qu 7.
Mathematical modeling is useful in life “it can be used to make money” “it can give you the greater edge in sport” “basically it helps to solve things you never thought were solvable”

Student 7
This had left school by the time of the assessment.

Student 8
This student was absent for the assessment.

Student 9

Qu1.
Consider all possible variables that could affect the situation then identify the important variable's.

Qu 2.
Assumptions had been made. Proposed new, more realistic model (surrounding area opposed to world).

Qu 3.
Could see how model could be used.
Identified needed information.
Made assumptions.

Qu 4.
Made assumptions to use model.

Qu 5.
Made a physical model of the situation to scale.

Qu 7.
"There is a lot of variables to be considered."
"Helps if you have experienced team members."
"Need to work with a team."
Liked working in groups.

Student 10

Qu 1.
This student identified what she/he knew about the problem and what she/he wanted to find out about the problem. An assumption was made to ignore one of the factors that could have an effect on the problem. A table and grid would be used to explore the situation. Results from exploration would be tested to verify an equation or predicted measurements are correct.

Qu 2.
Discussed if going to make a model needed to be realistic.

Qu 4.

Qu 7.
"It was hard but when we had group discussions the process was a lot clearer."
Interesting and useful in real life situations.
Perceived to be something you could contribute to even if you were not good at mathematics.
Needed more clarified guidance.

Student 11

Qu1.
Do physical experiment, draw on known mathematics, use factors already known.

Qu 2.
Need for realistic model (not drawn to scale or 'true').

Qu 3.
Discussion of unrealistic model.

Qu 5.
Proposed new model in relation to immediate surroundings not in relation to earth.

Qu 7.
"I leant that there is more than one right answer in modeling ……it is used for a purpose that would be useful and save time".
Using scale drawings to check answers.
Using known maths esp trigonometry.

Liked working in groups not independently as in assessment.

Student 12

Qu 2.
Made assumption to be able to use model.

Qu 3.
Discussed using trig to use model (applying known maths to problem).

Qu 4.
Made assumptions to be able to use the model.

Qu 5.
Use software.

Qu 7.
Team work, research, using software, how to get along in a team.
"I liked working in a team."

Student 13

Qu 1.
Context, what we know, what we want to find out.

Qu 2.
Model needs to be realistic eg to scale.

Qu 3, 4, 5, 6.
Can't read writing.

Qu 7.
Important to understand context, what you know and what you want to find out, to work as a group member. Liked it was challenging and difficult. Didn't like how there wasn't enough time to complete the model.

<u>Student 14</u>

Allie couldn't start the assessment, her question was describe your modeling experience? Did you enjoy it? Did you learn any maths from it?

Still couldn't answer

<u>Student 15</u>

Qu 7.

Talking and writing and answering questions in our groups.

<u>Student 16</u>

Qu 1.
Disscussed factors that could affect the situation.

Qu2.
Discussed need for realistic model (not to scale).

Qu 7.
" I learnt there is a lot of things you have to consider like fair testing."
" I like discussing….."

<u>Student 17</u>
This student was absent for the assessment.

<u>Student 18</u>

Qu 1.
Extra study (background information), form a design of the model by looking at all the variables that affect the situation, then sort the variables, form the model and test the model.

Qu2.
Discussion of realistic model (not to scale). Discussed assumptions made for this model to be usable.

Qu 3.
Explore situation by using diagram and drawing on, adding to as explore. Need to know variables of importance, made assumption, ‘if use mathematical formula…may get a near guess to the answer.”

Qu4.
Discussed assumptions need to be made to use this model.

Qu 5.
Use software.

Qu6.
Discussed assumptions.

Qu 7.
Team work, software, how to work in a team, how to research.

Summary of assessments

From the assessment the following parts of the mathematical process were identified.

It was recognised that there was a need to do research and know the background context of the problem. Recognising and identifying the essential aspects was done through brainstorming and asking questions. Brainstorming all the factors that affect the situation and sorting into what we already knew, what we wanted to find out and into order of importance. During this process assumptions were made and decision made to ignore factors. Exploration for forming a model was done through forming a plan for exploration and using modelling tools such as using diagrams, scale drawings, tables, grids, known mathematics and formula, software and physical models. Testing of the model could be done by using results from exploration, physical experiments or predicted measurements to verify any equations formed.

Students enjoyed working in groups with supporting comments being “working in groups can sometimes be helpful as different people have different strengths and ideas” (3). “It was hard but when we had group discussions the process was a lot clearer” (10). “I liked working in a team” (12). There was evidence of the usefulness of modelling and its openness. “Mathematical modelling helps to solve things you never thought were solvable” (6). “I leant that there is more than one right answer in modelling ……it is used for a purpose that would be useful and save time” (11).

Appendix H

Summary of individual interviews

1. Forming a modelling group
2. Establishing a shared understanding
3. Defining the problem
4. Identifying the essential aspects of the situation
5. Model formation
6. Testing the model
7. Improving the model

Student 1

Lost half the video and only discussed one situation. This student was absent a lot.

4. Identified variables needed to find.
5. Planned for developing model.
Used known mathematics, scale diagrams, computer software to explore forming the model.
Explored for a specific situation then thought about looking at the 'general' situation.

Rugby kick was easier for this student as was more familiar and confident with the context of rugby. Thought rugby context was more interesting and applicable to life.

Student 2

Lost the last part of his interview. Was up to assessment questions.

4. List factors, identify relevant factors. Eliminate and ignore factors that won't affect the result or had too much variation producing inconsistency between reality and result. Find out about known factors. Made assumptions. Establish what things we already know about the situation, find out information about known factors (eg gravity if needed).
5. Used graphs

Enjoyed lighthouse (assessment) the most as for this student this was the problem where the solution was the most unknown. Enjoyed finding out something you didn't already know the solution for.

Student 3

4. Looked at all variables that can affect the situation. Made a list of factors. Classified factors into what we knew, what we didn't know and what we wanted to find out. Ignored factors and made assumptions. Made list, discussed what was important and what to ignore.
5. As a group discussed, brainstormed and planned how to form model. Used software, table and known mathematics (cosine) to explore forming the model. Ran out of time and needed more guidance to complete forming model.

6. Discussed testing model
7. Discussed repeating process (iterative process). If the model didn't work repeat the process again.

Context of problem was important to the student. This student liked the phone problem as could see the relevance and usefulness to the student. Found the process easy but finding the solution difficult. The process was teacher led at first. Student felt process was open enough that they could add to it and modify it if they wanted. Found working in a group pooling ideas beneficial. Ran out of time and needed more guidance with the formation stage. Found going from important variables to model formation difficult. Student thought identifying the situations' important aspects was the first stage of the process. Had thought about modelling in reference to dropping their phone. Could think about applying the modelling process to shooting a netball goal. Enjoyed working in groups and finding out what they already knew and what they wanted to find out. Didn't like not being able to find model. Was like figuring out all the ingredients to make the cake just not knowing how to put them all together to actually make the cake.

Student 4

2. Brainstormed what already knew
3. Looked at the question.
4. Brainstormed all the factors that affect the situation. If things related to each other grouped together and have one variable to express that group (simplifying variables). List factors and sort to what we knew, what we wanted to know, what we wanted to find out.
5. Used known mathematics, practical investigations, software, diagrams, scale drawings to explore model. Designed model (planning?).

Cell phone was the most interesting as liked the process and discussion of all the factors. Liked the complexity of the phone situation. First stage of process was identified as brainstorming for 4. The most difficult part was coming up with a solution. Needed more than one way of exploring the situation. Enjoyed and found unit interesting. Did not enjoy it more than other mathematics, preferred more structured mathematics and less writing. Would do more mathematical modelling. Enjoyed the process and deciding what factors were important and which were not (making decisions).Which factors related to situation. Found finding solution hard, did not enjoy this part. Rather do modelling in a group than by self.

Student 5

Lost first part of interview. Only have answers from "have you thought about modelling since?"

Could see the possibility of how to create a mathematical model for a tennis serve. Enjoyed working in groups, the discussion and work on computer. Didn't like it when didn't understand, then it felt like a waste of time.

Student 6

2. Researched solution on internet

3. Start out with a question.
4. Make list of variables/ factors by asking questions, talking, discussing. List sorted into relevance/ irrelevance to the situation. Discussed how to rank variables in order of importance. (For it to be important it will have the biggest effect on situation, ignore it if it has minimal effect on situation or has too much variation (too many different effects) on situation meaning too complex).
5. Discussed ideas, used graphs, computer software to explore forming the model. This stage is about figuring out calculation.
6. Test calculation to see if it fits reality.

Phone was most interesting as could see relevance to himself. Enjoyed unit. Did not view mathematical modelling as working as hard as normal, was more relaxed (did not view discussion, group work, exploration as work). Liked other units better. Did not want to do anymore modelling.

Student 9

2. Discussed research when looking at relevant variables. Research was not identified as 1st stage of process but as part of 3.
3. Define what we want to find out.
4. Lots of variables involved in both situations. Variables identified by brainstorming, known variables, what we wanted to find out, how this could be measured. Identify relevance of variables decided through brainstorming, which variables to focus on and which to ignore. Explored how different variables affect the situation. Relevant variables found through research.
5. Plan was discussed and formed for exploring how to form model. Used software, graphs. Looking at making a generic model suitable for a wide variety of situations. About using and thinking about calculations and putting the model together. Viewed model as equation. Ran out of time with this stage.
6. Made estimate to test model against.

Didn't feel supported in her group. Found phone the most interesting as it is relevant to the student. Found the rugby problem more straightforward compared to the phone (due to complexity of phone). Saw usefulness of models. Made predictions and calculations. Improvement for next time to take phone problem through to completion. Identified stage 3 as first stage of mathematical modelling process. Enjoyed unit as it is more practical work and was fun. Had to think for the assessment. Would like to do more mathematical modelling. Had thought about modelling since unit. Enjoyed that it was group work, practical, interesting and different though confusing at times.

Student 10

2. Context is important to understand. It is easier to start the process when familiar with the situation.
4. Looked at causes of the problem. Brainstormed for a list of variables. Simplify the problem by sorting list of things that affect the problem. Important variables were the ones found across the general situation.

5. Tables and diagrams were tools to help form a model. Collect data, put in table and form equation.
6. Test model physically.

This student was only here for the phone. Found phone interesting as could see relevance to how it was a problem for students. Would have liked to have fully formed model and tested. Need context for problem that most of the audience can relate to. Equation forming part difficult. Liked fact that it was practical making it easier to understand. Liked working in group better than individually. Would like to do more mathematical modelling. Enjoyed learning about everyday stuff (saw the relevance of modeling to everyday stuff).

<u>Student 11</u>

4. Discussed and identified variables affecting the situation. Simplified the situation by identifying important variables and recognised the need to use variables that were measurable. Ignored variables that weren't measurable or assumed they were constant.
5. Developed a plan by thinking about how to formulate the model. A general formula is needed for the important variables. Used formulas from physics. Used diagrams, scale drawings and software to help formulate model.
6. Discussed and planned how they would test the model. Could carry out a physical experiment. Was not enough time to test the model.

The process was about "using your own thinking to try and find the answer" (independent thought). There was flow on from one part of the process to the next. The phone was the most interesting because the student was already familiar with and saw the relevance of modelling the situation. The difficult part was testing the model by trying to come up with an example that would work with the formula. There was not enough time to fully test the model. An improvement for next time was to develop the model for the phone fully (most students were engaged in this activity). This could be due to relevance of the situation and this was first case of mathematical modeling students had experienced (so interest high). It was important for this student to have gone through the process and known whether it had worked or not. The variables were easier to identify in the rugby kick situation opposed to the phone.

<u>Student 13</u>

2. Research for solution
3. Defined problem – what the problem was. Background information given for assessment gave contextual background for situation.
4. Looked at variables for the situation. Variables identified from discussion and knowledge within the working group. Wrote down everything they knew about the situation, identify important variables, made assumptions.
5. Group was split and given roles, used software, tried to come up with formula and then used research to solve. Worked separately and came back together and discussed. Used research to find other models.
6. Critiqued model with data and discussion. Made plan to test model for phone.

7. Discussed improvements – take variables had assumed to be constant, choose one and add to model making it variable.

Improvement for next time – allow more time to follow the process right the way through. Saw practical application of modelling. Liked the situation where one could simplify it and form a general model (situation needs to be real and doable). Saw benefits of group work. Applied mathematics which was already known to situations opposed to learning new mathematics. Assumptions made based on time restraints. Found it difficult that there was lack of time and no time to physically test model.
Enjoyed unit, enjoyed more than other units as “different and a breath of fresh air.” “nice to be challenged in a different way.” Enjoyed being “actively involved instead of teacher led.” “More independent thought.” Would like to do more mathematical modelling in the future. Assessment was structured.

<u>Student 18</u>

2. Drawn on previous experience (from practical experience at beginning of year).
4. Lots of variables. First step is discussion, talking, to identify all the variables that could affect the situation. List all the variables then sorted into most important to least important. Sorted into which factors would have the biggest effect (most important) and left the rest alone (the ones that had a minimal effect or least common factor across the situation or contributed to the most variability for a general situation). Made assumptions eg no wind.
5. Used software, practical experiment, formula.
6. Critiqued a previously found model.

Liked phone best as liked complexity (all the different things that could affect it) and the relevance of the problem. Felt that once they understood the mathematics to apply to the situation, the task was easy. Did not learn any new mathematics as such just mathematics vocab eg variable – modelling was good for developing mathematics language. Most of the work done was on variables. Ran out of time. #4 was identified as first stage of process (then #2 came in as part of 3). Enjoyed the unit. Liked working in groups compared to individually. Found assessment difficult. Wanted the group to draw on and bounce ideas off to be able to start. Open to doing mathematical modelling again in a group, not individually. Could see other practical applications of modelling. Liked working in a group and thought would be different if not in a group.

Summary of interviews

From the interviews, identifying the essential aspects of the situation and forming the model were the parts of the mathematical modelling process experienced and retained most by the students. Students talked about working in groups throughout the interviews.

The following parts of the process of mathematical modelling, including the detail, have been experienced and retained from the teaching sequence:

Identifying the important aspects of the situation was described by all students and identified by most as the first step in the process of mathematical modelling. The first step in identifying the important aspects of the situation was through discussion of all the factors and variables that could affect the situation. This discussion involved brainstorming knowledge within the group, asking questions and research. The factors identified were listed and then sorted through discussion in two ways. First into what we knew and what we wanted to find out, then into order of importance. Order of importance was established by discussing which factors were consistent and have the biggest effect on the situation. Important variables were the ones found across the general situation. Factors that were ignored were ones that had a minimal effect or least common factor across the situation. Factors that were assumed to be constant were ones that contributed to the most variability for a general situation, producing inconsistency between reality and result. Assumptions were made for this group of factors. Factors that were not measurable were also ignored or assumed constant. Factors needed to be measureable in order to be included in the model. One student mentioned that they explored how different variables affect the situation in establishing order of importance (9). As students were identifying the situations' essential aspects they were also simplifying the situation. Ranking variables into order of importance was part of this process. Simplifying was also done by grouping things related to each other together and have one variable to express that group.

Forming the model was discussed by most students with the first step being planning, through discussion and brainstorming, how to explore forming the model. Some groups gave group members roles where they worked in roles separately then came back together and discussed findings. Exploration of forming the model was done individually and as a group using known mathematics, formulas from physics, graphs, scale diagrams, tables, computer software, practical investigations (collected data, put into table, formed equation), discussing ideas and research. This stage was described as figuring out the calculation (6), using and thinking about calculations and putting the model together (9), looking at making a generic model suitable for a wide variety of situations (9), a general formula being needed for the important variables (22) This could be done by exploring a specific situation then looking at the general situation (2). Students felt they did not have enough time to complete this stage (3,9) and would have liked more guidance with this stage (3).

Most of the interviews discussed testing the model. They discussed testing the model by using the newly formed model to make a calculation and seeing if the result fitted reality (6,9). Some students discussed testing the model physically either through a physical experiment or previously collected data or previously formed model and seeing if model fitted with this reality (20,22,23,29).

Research was not identified as the first stage of the process but as a complementary part of other stages, particularly for identifying the important aspects of the situation and for forming a model. Research was mentioned for carrying out to help establish and identify relevant variables (2) and for finding solution for the model (23).

Defining the problem was mentioned as looking at the question (4, 6) and establishing what we want to find out (9).

Improving the model was the least mentioned part of the process. When it was mentioned improvements were discussed as choosing one variable that had been assumed to be constant and add it into the model as a new variable (23). One other student discussed improvement as an iterative process to use if the model did not work (3).

Many of the students liked the phone problem best as they could see the relevance and usefulness to themselves, though there were students who enjoyed the rugby problem best as they were more familiar and confident with the context (2) and another student enjoyed the lighthouse problem best as this problem was the most unfamiliar to him and he liked the fact that the solution was very unknown (2). Overall all 3 problems covered or captured the audience and had contexts that the audience could relate to. Nearly all students mentioned that they would like to develop model for the phone fully.

All students except one enjoyed working in groups, finding it beneficial and useful for bouncing ideas off each other and drawing from each others' knowledge.
The most difficult part of the process was coming up with the solution (3,4,20,22) being described as "being able to figure out all the ingredients that go into the cake just not knowing how to put them together to make the cake" (3).

Most students mentioned running out of time and not being able to fully develop model.

Modelling gave the opportunity for independent thought (20, 23) and they were actively involved instead of teacher led (23).

Interviews were more specific with how to carry out the process whereas dairies where more a record of what was done on the day.

Appendix I	Diary	Assessment	Interview
1. Forming a group	Worked in a group (3,4,11,15,16) Discussion in groups (3,4) Delegation of tasks to group members(2,11,13)	Working in groups beneficial (3,9,10) Enjoyed working in groups (4,9,11,12) Brainstorm in groups (4,5) Learnt how to work in a team (18)	Working in groups beneficial (3,4,11) Enjoyed working in groups (3,5,9,10,18) Brainstorm in groups (3) Delegation of tasks to group members(13)
2. Observations and contextual background	Research in to context of situation (2,3,13) Discussion of context of situations in detail (9)	Do research (3,4,6,18)	Brainstormed what already knew about situation (4 Research (on internet (6, 13)) (9) Context important to understand (10) Drawn on previous experience (19)
3. Defining the task	Discussion on purpose and usefulness of the model (5) Discussed on defined problem (5)		Looked at question(4,6) Define what we want to find out (9) Defined what the problem was (13)
4. Recognise and identify situation's essential aspects	Discussion and looking at all of the factors that affect the situation (1,2,3,4,5,7,9,10) Classifying things that affect the situation (what we already know about a situation and what we want to find out– what we know, what we want to find out and what we can control)(3,4,7,8,10,14,17) Discussion and Identification of important variables (1,4,5) Brainstorming what things affect the situation (4) Measure factors, measuring factors (2,3,11) Brainstorming and making assumptions (1,4,5,7,8,11) Controlling certain aspects of the situation, what things can be controlled (making assumptions) (5,7,11) Ignoring certain aspects	Brainstorm, asking questions(4) Find out what factors affect the situation (2,3, 4,5,6, 9,16,18) Factors already know about situation (3,10,11,13) Factors want to find out (10,13) make assumptions (2,3,12,18) identifying factors need to know(2,9) identify important factors (9,18) what things would be ignored and or assumed.(2,5,10) Stated and made assumptions(2,4,9,10)	Brainstormed, discussion to identify all factors that (3,4,6,9,10,11,13,19) Identify relevant factors (2,6,9) Identify variables need to find (1,4) Ignore and eliminate factors (2,3,11) What things do we already know about situation (2 What things do we need to find (2,3,4,9) How to measure variables (9) Classified/sorting factors (3,10,19) Rank variables in order of importance (6,13,19) Made assumptions (2,3,13,19) Simplified variables (4) Simplified situation (4,10,11)

	(making assumptions) (1)		
5. Using the essential aspects to form or choose a suitable model	Discussion and exploration of physical laws and equations relating to situation and how they can relate to the situation (1,4,5,6,7,11,13) Forming equations, finding a formula and the factors that go into a formula (2,3,10,13) Planning for exploration for the formulation of the model, forming a plan for developing a model (1,2,6,8,9,11,12,13,14,15, 17) Delegation of jobs to group members (2,9,12,13) Using computer software (Geogebra, Excel and Internet) (2,3,5,8,9,10,11,12,13,14, 15) Using previous mathematical knowledge (10) Using diagrams, tables and scale drawings (3,8,9,10,11,14,17) Using graphs (14)	Using scale drawing, diagrams (2,3,18) Using known maths (2,11,12) Discussed exploring the model further (2) and had a plan(2, 3) Use software (3,5,12,18) Using formula (3) Proposed new model (9,11) Made physical model (9) Used table and grid (10)	Group members given roles (13) Discussed and brainstormed (3,6) Used known mathematics (1,3,4) Scale diagrams and drawings (1,4,10,11) Computer software (1,3,4,6,9,11,13,19) Develop general solution (1,9,11) Used formulas from physics (11,19) Used graphs (2,6,9) Used table (3,10) Practical investigations (4,10,19)
6. Testing the model	Make predictions, estimates for possible solutions (2,9,12) Performed experiment to see what happens(14) Looking at results (16) Using evidence to prove models (11)	Research to see if other models have been done and compare with mine.(2) Do a physical experiment(2,5,11) Results from exploration or predicted measurements tested to verify equations (10)	Discussed testing and critiquing model (3,6,9,10,11
7. Improving the model	Looked ahead to improvements (5)		Discussed improvements (13)

Table 1: Results from diary, assessment and interviews