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# **TRANSIENT EFFECTS IN GEOTHERMAL CONVECTIVE SYSTEMS**

**A thesis submitted in partial fulfilment of  
the requirements of the degree of Doctor of Philosophy  
at the University of Auckland**

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## ABSTRACT

This work is a detailed analysis of the transient behaviour of geothermal convective systems. The flow in these systems is found to be fluctuating or regular oscillatory in a simplified two-dimensional model and these unsteady effects persist when the model is refined to include the concepts of temperature dependent viscosity and fluid withdrawal and recharge. The analysis is extended into three dimensions to verify this behaviour. The supplementary exploration of added salinity gradients indicates transient effects of a different kind in this case. The examination of the porous insulator problem confirms the results of previous authors and verifies the viability of the numerical methods that are used throughout the investigation.

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# CONTENTS

Notation	ii
<u>CHAPTER 1 - INTRODUCTION</u>	
1.1 The Precept	1
1.2 The Wairakei Geothermal Region	2
1.3 Past Work	4
1.4 The Scope of this Work	6
<u>CHAPTER 2 - MATHEMATICAL FORMULATION</u>	
2.1 The Equations of Motion	8
2.2 Boundary Conditions	14
2.3 Two-Dimensional Regions	16
2.4 Three-Dimensional Regions	20
2.5 Fluid Sinks and Sources	22
<u>CHAPTER 3 - EXPERIMENTAL SOLUTION IN TWO-DIMENSIONAL REGIONS</u>	26
<u>CHAPTER 4 - NUMERICAL METHODS</u>	
4.1 Past Numerical Solutions	35
4.2 Finite Difference Methods	35
4.3 Variational Techniques	44
<u>CHAPTER 5 - RESULTS FOR THE TWO-DIMENSIONAL PROBLEMS</u>	
5.1 The Uniformly Heated Model - $f = 1$	47
5.2 The Non-Uniformly Heated Model - $f < 1$	52
5.3 Review of Constant Viscosity Solutions	58
5.4 The Recharge - Discharge Solutions	61
5.5 The Variable Viscosity Model	71
<u>CHAPTER 6 - THREE-DIMENSIONAL TRANSIENT FLOW</u>	
6.1 Introduction	76
6.2 The Range of Solutions	76
6.3 The Numerical Results	77
<u>CHAPTER 7 - THERMOHALINE CONVECTION IN POROUS MEDIA</u>	
7.1 Introduction	81
7.2 The Equation of Motion	82
7.3 The Numerical Results	84
<u>CHAPTER 8 - THE POROUS INSULATOR PROBLEM</u>	
8.1 Introduction	87
8.2 The Numerical Solutions	88

<u>CHAPTER 9 - SUMMARY AND CONCLUSIONS</u>	
9.1 Evaluation of the Results	91
9.2 The Thermal Boundary Layer	91
9.3 The Preferred Solution	92
9.4 Disturbance Interactions	92
9.5 Temperature/Salinity Effects	93
9.6 The Presence <i>of</i> Boundaries	93
9.7 The Physical Implications	93
9.8 Exploitation	94
9.9 Future Modelling of Geothermal Fields	94
 <u>APPENDICES</u>	
A - The Arakawa Differencing Schemes	96
B - <i>An</i> Extension of the Buneman Algorithm to Fourth-Order Accuracy	97
C - The Nusselt Number	101
D - <i>An</i> Extension of the Buneman Algorithm to Neumann-type Boundary Conditions in Two and Three Dimensions	103
E - Spectral Representation of Equations of Motion	105
F - Evolution of the Thermal Boundary Layer	107
G - Streamline Representation in Three-Dimensional Flows	108
 <u>REFERENCES</u>	110

## NOTATION

All variables and operators used are defined when they first appear in the text, however the commonly used ones are summarised here.

### Dimensional Variables

$b_1, b_2, b$	- Coefficients of variation of viscosity $\nu$ with temperature $T$ .
$g$	- The gravitational acceleration
$k$	- The permeability of the medium
$m$	- The porosity of the medium
$P$	- The dynamic pressure
$S$	- The mean flux velocity
$t$	- Time
$x$	- Spatial dimensions
$C$	- Concentration of dissolved mineral salts
$C_0, C_1$	- Minimum and maximum values of $C$
$F$	- Buoyancy force
$K_{ij}$	- Thermal dispersion tensor
$T$	- Temperature
$T_0, T_1$	- Maximum and minimum values of $T$
$\alpha$	- Thermal expansion coefficient
$\alpha_1, \alpha_2$	- Linear and quadratic thermal expansion coefficient
$\alpha'$	- Solutal expansion coefficient
$\beta_1, \beta_2, \beta_3$	- Viscosity variation coefficients
$\kappa$	- Thermal diffusivity
$\kappa'$	- Solutal diffusivity
$X$	- Ratio of volumetric heat capacities
$\mu$	- Dynamic viscosity of fluid
$\nu$	- Kinematic viscosity of fluid
$\nu_0$	- Low temperature value of $\nu$
$\rho$	- Density of fluid
$\rho_0$	- Low temperature value of $\rho$

### Non-Dimensional Variables

$f$	- Fraction of lower boundary heated
$q$	- Strength of fluid sink
$u$	- Velocity due to flow into sink
$u'$	- Velocity not due to flow into sink
$C$	- Concentration



$Nu$	- Nusselt number
$P$	- Pressure
$R$	- Rayleigh number
$S$	- Solutal Rayleigh number
$U$	- Fluid velocity
$X$	- Spatial dimensions
$\gamma$	- Buoyancy ratio
$\Delta X, \Delta Y$	- Spatial increments
$\Delta \tau$	- Time increment
$\nu$	- Viscosity
$\rho$	- Density
$e$	- Temperature
$\psi$	- Stream function
$\phi$	- Vector potential
$T$	- Time

#### Operators

$\delta$	- Jacobian
$\nabla^2$	- Laplacian