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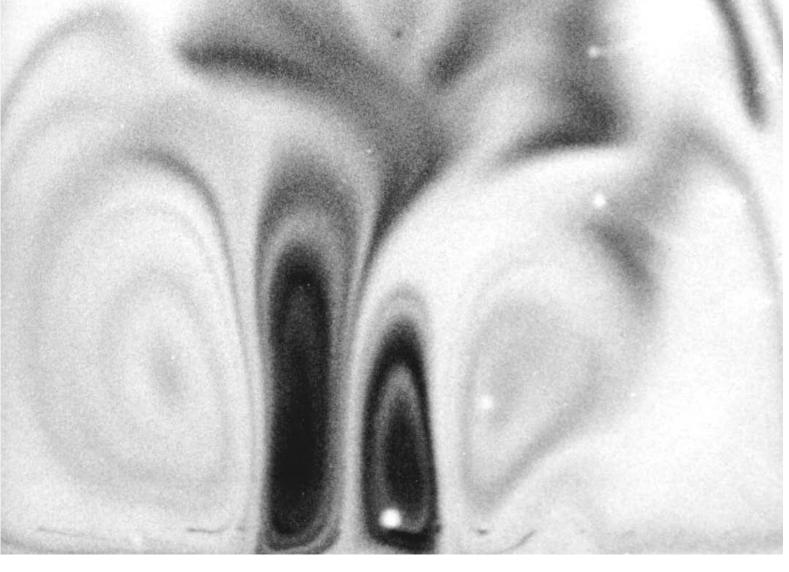
TRANSIENT EFFECTS IN GEOTHERMAL CONVECTIVE SYSTEMS

A thesis submitted in partial fulfilment of the requirements of the degree of Doctor of Philosophy at the University of Auckland

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ABSTRACT

This work is a detailed analysis of the transient behaviour of geothermal convective systems. The flow in these systems is found to be fluctuating or regular oscillatory in a simplified two-dimensional model and these unsteady effects persist when the model is refined to include the concepts of temperature dependent viscosity and fluid withdrawal and recharge. The analysis is extended into three dimensions to verify this behaviour. The supplementary exploration of added salinity gradients indicates transient effects of a different kind in this case. The examination of the porous insulator problem confirms the results of previous authors and verifies the viability of the numerical methods that are used throughout the investigation.

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The Hele-Shaw apparatus used for the experimental work was borrowed from the Department of Civil Engineering and I am grateful for the assistance of technicians Mr C. Raymond and Mr D. Browne, who helped set it up. I am also indebted to Dr John Meikle and the Auckland Industrial Development Division of the D.S.I.R. for the use of their AGA Thermovision camera. The many photographs taken of these experiments were processed by the Photographic departments of the University of Auckland and the A.I.D.D.

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Parts of the work have appeared previously in Horne and O'Sullivan (1974 a and b) and I am indebted to Ms Kitty Young for the preparation of these two papers.

I would also like to thank Professor C.M. Segedin of the Department of Theoretical and Applied Mechanics, Associate Professor M.P. Hochstein of the Department of Geology, University of Auckland, and Dr Ian Donaldson and Dr Robin Wooding of the D.S.I.R., Wellington for the various helpful discussions I have had with each of them.

The text has been typed by Ms Mamie Long who was fortunately persuaded out of her retirement from such tasks.

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NOTATION

All variables and operators used are defined when they first appear in the text, however the commonly used ones are summarised here.

n

Dimensional Variables

```
bl, b2, b {}^{\bullet} Coefficients of variation of viscosity v with temperature T.
            - The gravitational acceleration
            - The permeability of the medium
            - The porosity of the medium
            - The dynamic pressure
            - The mean flux velocity
            - Time
            - Spatial dimensions

    Concentration of dissolved mineral salts

            - Minimum and maximum values of C
c_0, c_1
            - Buoyancy force
Ę
            ■ Thermal dispersion tensor
K<sub>ii</sub>
            - Temperature
            - Maximum and minimum values of T
            - Thermal expansion coefficient
            - Linear and quadratic thermal expansion coefficient
\alpha_1, \alpha_2
            - Solutal expansion coefficient
\beta_1, \beta_2, \beta_3 - Viscosity variation coefficients
            - Thermal. diffusivity
            - Solutal diffusivity
            - Ratio of volumetric heat capacities
            - Dynamic viscosity of fluid
            - Kinematic viscosity of fluid
            - Low temperature value of {\bf v}
            - Density of fluid
            - Low temperature value of \rho
```

Non-Dimensional Variables

Fraction of lower boundary heated
Strength of fluid sink
Velocity due to flow into sink
Velocity not due to flow into sink
Concentration

Nu Nusselt number

P Pressure

R Rayleigh number

S Solutal Rayleigh number

U Fluid velocity

X - Spatial dimensions

Y Buoyancy ratio

AX, ΔY Spatial increments

Δτ Time increment

v · Viscosity

ρ' Density

e Temperature

 ψ - Stream function

e - Vector potential

T Time

Operators

δ Jacobian

∇² - Laplacian