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Wireless power for heart pumps: Control and energy measurement

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Supervised by: Dr. David Budgett and Dr. Aiguo Patrick Hu

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Bioengineering

The University of Auckland
New Zealand
April 2013
Abstract

The use of implantable heart pumps, such as Left Ventricular Assist Devices (LVADs), has been steadily increasing since gaining approval to be used for bridge-to-transplant therapy for heart disease patients. These pumps require 5 to 10W of electrical power, and this is supplied by a percutaneous cable which connects to an external energy source. However, this method of power delivery is prone to infections and is a major cause of serious adverse events related to the use of LVADs. This thesis focuses on the development of Inductive Power Transfer (IPT) as a wireless solution for providing power to implantable heart pumps. The use of oscillating magnetic fields to wirelessly transfer power into the body eliminates the risk of infection from a percutaneous cable. IPT has been successfully demonstrated; however, currently there is no commercial TET system available for use in LVADs.

In this thesis, frequency domain models are developed to analyse the steady-state characteristics of resonant IPT systems under the operating conditions appropriate for powering an implantable heart pump. A methodology for selecting the parameters and topology of an IPT system which minimises the power loss in the resonant circuit has been developed. An investigation to determine the power dissipation of resonant IPT components has been carried out to allow evaluation of the heating caused by the IPT system. Electrical methods of power loss determination may be unreliable for measuring the loss from resonant components during operation. More suitable thermal power measurement techniques are developed where the measurement is unaffected by electrical operating conditions. A novel Peltier device balance calorimeter is proposed to measure the power dissipation of the power transfer components. Primary side power flow control methods for an IPT system based on varying the operating frequency and energy injection have been proposed and thoroughly studied. The new energy injection control method improves control action over the secondary output and sustains the same efficiency throughout the control range. The power flow control methods have been implemented on all the basic resonant topologies. A standalone closed-loop IPT system that incorporates important aspects of implantation has been successfully developed. The system is capable of supplying 5W to 12W of power across a wide coupling range at an end to end efficiency of above 70%.

The research undertaken contributes to both theoretical development and commercialisation of inductive power technology for implantable heart pumps.
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# Nomenclature

## Acronyms

<table>
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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>ADC</td>
<td>Analogue to Digital Converter</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital to Analogue Converter</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DUT</td>
<td>Device Under Test</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic Interference</td>
</tr>
<tr>
<td>ESR</td>
<td>Equivalent Series Resistance</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
</tr>
<tr>
<td>IPT</td>
<td>Inductive Power Transfer</td>
</tr>
<tr>
<td>LC</td>
<td>Inductor-Capacitor connection</td>
</tr>
<tr>
<td>LVAD</td>
<td>Left Ventricular Assist Device</td>
</tr>
<tr>
<td>MET</td>
<td>Maximum Efficiency Tuning</td>
</tr>
<tr>
<td>MOSFET</td>
<td>Metal Oxide Semiconductor Field Effect Transistor</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
</tr>
<tr>
<td>PDM</td>
<td>Pulse Density Modulation</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulation</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RTD</td>
<td>Resistive Temperature Device</td>
</tr>
<tr>
<td>LTSPICE</td>
<td>PC Simulation Program with Integrated Circuit Emphasis</td>
</tr>
<tr>
<td>TAH</td>
<td>Total Artificial Heart</td>
</tr>
<tr>
<td>TET</td>
<td>Transcutaneous Energy Transfer</td>
</tr>
<tr>
<td>VA</td>
<td>Volt-Ampere</td>
</tr>
<tr>
<td>VAD</td>
<td>Ventricular Assist Device</td>
</tr>
<tr>
<td>ZCS</td>
<td>Zero Current Switching</td>
</tr>
<tr>
<td>ZPA</td>
<td>Zero Phase Angle</td>
</tr>
<tr>
<td>ZVS</td>
<td>Zero Voltage Switching</td>
</tr>
</tbody>
</table>
Symbols

\( C \) - Capacitance (Farads)
\( f \) - Frequency (Hz)
\( \omega \) - Angular frequency (Radians/s)
\( i \) - Instantaneous current (Amperes)
\( I \) - Current (Amperes)
\( j \) - Complex operator
\( k \) - Magnetic coupling coefficient
\( L \) - Self-inductance (Henrys)
\( M \) - Mutual inductance (Henrys)
\( Q \) - Quality factor
\( R \) - Resistance (\( \Omega \))
\( v \) - Instantaneous voltage (Volts)
\( V \) - Voltage (Volts)
\( t \) - Time (seconds)
\( T \) - Period (seconds)
\( \theta \) - Phase angle, switching angle (degrees)
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1.1 Implantable heart pump therapy

The gold standard for curing medically refractory congestive heart disease patients is heart transplantation. It is estimated by the Registry of the International Society for Heart and Lung Transplantation (ISHLT) that around 5000 heart transplantations are performed worldwide each year [1]. However, in the US alone there are 3000 candidates on any given day waiting for a heart transplant. Since hearts for transplantation can only come from deceased donors there will always be a lack of supply, which means that most of these patients will not receive a transplant.

During the last three decades left ventricular assist devices (LVADs), for assisting a failing heart; and total artificial hearts (TAHs), for complete replacement of the heart have been clinically trialled in patients [2, 3]. Due to recent developments in LVAD technology, current state-of-the-art heart pumps have shown to be mechanically durable over a time frame of several years and adverse events due to device related malfunctions have declined. With each new generation of heart assist devices, increasing survival rates have been reported; from 55% rate of survival after 12 months in 2001 with a 1st generation device, to 70% rate of survival after 12 months in 2009 with a 2nd generation device [4]. According to the latest report from the US Interagency Registry for Mechanically Assisted Circulatory Support (INTERMACS) [5], over 90% of all LVAD implantations were used for bridge-to-transplant therapy before the year 2010. However, in 2010 the number of LVAD implantations approved for destination therapy has rapidly increased and it currently surpasses the number of implantations for bridge-to-transplant therapy. LVAD therapy has also shown to be so
effective that it can help the native heart of a patient to fully recover, such that the use of the LVAD is no longer required [6].

1.1.1 Overview of current implantable heart pump systems

Below is a recent list of companies who are currently developing implantable heart pump systems and have a commercial medical product. This review outlines their latest pumps, their place in the market, the method of power delivery and any recent developments.

**Thoratec**

The Thoratec HeartMate II is an implantable axial flow LVAD and is the most dominant LVAD used in the US. Currently power for the LVAD is delivered through a percutaneous cable that exits through the abdomen. The HeartMate II was approved for Bridge-to-transplant therapy in the US in 2008 and later received approval by the FDA for destination therapy on Jan 2010. In May 2011 Thoratec entered into an agreement with the company Witricity, where Thoratec will support the development of a Fully Implantable Ventricular Assist System (FILVAS) based on the HeartMate II, which will incorporate the use of a wireless power system [7].

**Berlin Heart**

Berlin Heart currently develops EXCOR (external) and INCOR (implantable axial flow) heart pumps in Europe, while only EXCOR Pediatric has received FDA approval for use in North America. The EXCOR systems consist of an external pump which hangs from hoses that pass through the skin connecting to the heart, while INCOR systems are fully implantable with a percutaneous power cable. Berlin Heart has also recognised the need for a wireless power delivery system and have partnered with a company to develop the technology.

**HeartWare**

HeartWare Ventricular Assist System HVAD is an implantable centrifugal pump which has received the CE mark in the European Union, but has not gained approval for use in the US. Power to the HVAD is provided percutaneously via a cable through the abdomen. As part of their technology pipe line HeartWare is developing a miniaturised ventricular assist device (MVAD). In parallel with heart pump development, HeartWare is putting significant effort on
the development of a wireless power system. In 2012 they have agreed to fund a wireless power system development programme in collaboration with Dualis MedTech [8].

**Micromed**
The Micromed implantable LVAD the Heart Assist 5 is claimed to be the world’s smallest and lightest implantable LVAD [9]. Power to the pump is delivered percutaneously. It is currently only sold outside the US and has obtained approval to conduct clinical studies in the US on June 2012. There is no official information from Micromed about any recent developments; however, they have previously contacted Telemetry Research Ltd. regarding the development of a wireless power system.

**Abiomed**
Abiomed’s latest product is the Impella range of micro-axial heart pumps. These are catheter like pumps designed for temporal use and are inserted through the femoral artery then to the left ventricle providing support in critical situations. It has received the CE mark in the European Union in April 2012 for use of up to 5 days on patients when during cardiac surgery. Abiomed has an LVAD developed in 2004 called AbioCor. There are unofficial announcements of the development of a fully implantable heart pump called the AbioCor II together with a wireless power supply [10], however no information about this development is on their official website.

**Terumo Heart**
Terumo Heart is a division of the medical device company Terumo. They currently have a heart assist system called the Duraheart LVAS. The Duraheart is an implantable assist device with a percutaneous power cable and carries the CE mark available for sale in Europe and in Japan. Only minor clinical trials have been performed with the Duraheart. No signs of development of a wireless power system for their LVAS device were found at the time of writing.

**Jarvik Heart**
Jarvik Heart’s latest implantable LVAD is called the Jarvik 2000. Power is delivered to the device via a skull pedestal instead of an abdominal cable. The unique skull pedestal is a permanent plug that is mounted onto the skull of a patient, this type of percutaneous connection has been proved to be less prone to infection and reduces the risk of injury [11].
The Jarvik 2000 has earned CE Mark in the European Union for both bridge-to-transplant and destination therapy. The Jarvik 2000 supported the longest survivor of any heart assist device patient for a record of 7.5 years. In August 2012, Jarvik Heart announced that their Jarvik 2000 system has been granted full FDA approval for destination therapy trials. No development or interest of a wireless power system has been reported from Jarvik Heart.

1.1.2 Power sources for implantable heart pumps

The average power consumption for the implantable heart pumps mentioned in Section 1.1.1 were collected from several sources and are tabulated in Table 1-1. From Table 1-1 it can be seen that the average power consumption of a current implantable heart pumps is in the range of 5 to 10W. Compared to other implantable devices such as a pacemaker, which only consumes microwatts of power and can run on a battery for up to ten years [12], implantable heart pumps are considered as high power devices. Since the pump has to operate continuously, a stack of the four largest implantable lithium-ion batteries (700mAh) can only power a 5W pump for roughly an hour [13]. Due to the level of power consumption, running a heart pump off implantable batteries is not a feasible option; thus, the use of an external power source is absolutely necessary.

<table>
<thead>
<tr>
<th>Heart pump manufacturer – pump model</th>
<th>Pump type</th>
<th>Average power consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thoratec – HeartMate II</td>
<td>Axial flow LVAD</td>
<td>6W [14]</td>
</tr>
<tr>
<td>Berlin Heart – INCOR</td>
<td>Axial flow LVAD</td>
<td>3 – 4W [15]</td>
</tr>
<tr>
<td>HeartWare - MVAD</td>
<td>Axial flow LVAD</td>
<td>3.5 – 6W [16]</td>
</tr>
<tr>
<td>HeartWare - HVAD</td>
<td>Centrifugal LVAD</td>
<td>5W [17]</td>
</tr>
<tr>
<td>Micromed – Heart Assists 5</td>
<td>Axial flow LVAD</td>
<td></td>
</tr>
<tr>
<td>Terumo Heart – Duraheart</td>
<td>Centrifugal LVAD</td>
<td>8W [18]</td>
</tr>
<tr>
<td>Jarvik Heart – Jarvik 2000</td>
<td>Axial flow LVAD</td>
<td>12W [19]</td>
</tr>
</tbody>
</table>

Ever since the first implantable heart pump was made, power was supplied by a driveline cable that passes through a hole created on the skin. Most current LVAD systems pass this percutaneous lead through the skin in the abdominal area of the body [6]. However, these percutaneous leads are one of the major causes of serious adverse events for LVAD patients.
Examples of these adverse events are cable breakages, injuries caused by the cable and infections around the wound [2]. At present, the risk of driveline related infections is still very high [5]; and is one of the reasons that an LVAD cannot be approved for a patient unless they are critically ill, despite all the benefits the LVAD itself can provide. Heart pump company Jarvik Heart has tried to improve the percutaneous connection by creating a pedestal which is mounted on the skull to provide a secure and permanent connection which reduces the likelihood of infection [11]. This proposed pedestal is shown in Figure 1-1 and is similar to the power connection made by a cochlear implant. However, this method is rather aesthetically unpleasing and the instalment is also considered permanent since a hole is drilled into the skull [11].

1.1.3 Summary

Implantable heart pump therapy is currently the second best option for medically refractory heart disease patients when compared to heart transplantation. Although the amount of available hearts for transplantation is always lacking since it has to be obtained from a deceased donor, more implantable heart pumps can always be made. Within the last decade, several companies have developed effective heart pumps which have gone through clinical trials and have gained approval for use in patients. Currently, one downfall of these existing systems is the requirement of a percutaneous power delivery link.
1.2 Inductive power transfer for implantable heart pumps

The standard practice of delivering power with a percutaneous cable is high risk, since it is necessary to create a wound on the skin for the power delivery cable. This wound is unhygienic and acts as a path for bacteria to get into the body, making it prone to infection. This wound is also vulnerable to tear and the cable itself is susceptible to breakages. In addition, it causes inconvenience for activities such as bathing and other water related activity.

One way of improving the current situation is to deliver power wirelessly to the heart pump. In the medical field this is commonly referred to transcutaneous energy transfer (TET) where power is delivered through intact skin. Currently the most promising method of achieving this is with inductive power transfer (IPT) where power can be transferred across a gap between two inductive coils. There are other ways to accomplish TET based on different principles, such as using electromagnetic waves (radio, light) [21] and acoustics (mechanical vibration) [22]. However, these options are not as suitable for powering high power implantable devices when compared to IPT. These technologies currently cannot provide enough power on a small footprint or are not efficient enough for this level of power delivery [23].

![System block diagram of a general IPT system](image)

**Figure 1-2: System block diagram of a general IPT system**

The underlying principle behind IPT is electromagnetic induction; a primary coil is driven by a high frequency alternating current generating an oscillating magnetic field around the coil. When a secondary coil is placed near the vicinity of the primary coil, the oscillating magnetic fields induces current in the secondary coil, which then can be conditioned to drive a load. When this happens the two coils are magnetically coupled together; the amount of power transferred depends on the level of coupling between coils, the frequency of operation and the
strength of the magnetic field. A system block diagram of a general IPT system is illustrated in Figure 1-2.

IPT is not suitable for delivering power across ferromagnetic or metallic media, since these materials can absorb or alter the path of the magnetic flux. All other media are practically transparent to magnetic fields, which mean it is suitable for transferring power across the skin.

Successful application of IPT will mean that heart pumps can be made fully implantable with no open wound. This reduces the probability of adverse events and means that it can be offered to more patients at earlier stages of heart disease [6].

1.2.1 Operating environment of the IPT system

The operating environment and constraints of an IPT system for implantable heart pumps are very different to that of an industrial IPT system. In this application, the IPT system will be considered as a medical device. Thus, it has to comply with health and safety regulations and quality-of-life standards. Its reliability and performance must also be verified through extensive clinical trials, since a failure to deliver power could be fatal for the patient.

The IPT system will also be carried by the patient and operated continuously over a period of several months when used in bridge-to-transplant therapy, or over many years in destination therapy. In order to assure constant power delivery, an implanted backup battery should be incorporated into the IPT system. This implanted battery can also be used to provide a period of time where the primary side of the system can be taken off. This allows the patient to engage in activities such as bathing and showering, without any restrictions and improving the quality-of-life.

Since the implanted coil and circuits will be in full contact of internal tissue and blood, it needs to be enclosed in a biocompatible material. The conventional enclosure of long term implantable devices is a titanium case [24]. However, this is not suitable for the secondary pickup coil since titanium would act as a shield to the magnetic field generated by the primary power transfer coil. Hence, alternative encapsulation is required for the secondary pickup coil. The external coil may also be in contact with the skin, therefore both power transfer coils need to be encapsulated in biocompatible material.
1.2.2 Challenges associated with the IPT system

The level of magnetic coupling with the external and internal coil is a key factor in determining the amount of power that can be transferred and the efficiency at which it can be delivered. Unlike most industrial IPT applications where the coupling is more or less fixed [25, 26] the coupling variation in this application may vary during deployment and operation. Firstly, when the secondary coil is implanted underneath the skin of the patient, the coil separation would be affected by the thickness of the skin and fat layer which is different for every patient and may even vary over time. Since the coil is not fixed onto something rigid, variations in surgical placement will also affect the coil separation. Both of these factors will establish a different long term coupling offset. Secondly, the coils will experience fluctuations in coupling due to repetitive body movements such as respiration and from actions such as walking and posture changes. Hence, the system must be able to run continuously over a range of fixed coupling levels with disturbances which will be occurring all of the time.

Another challenge is to do with the heat generated by the power transfer coils and the implanted circuitry. Since these components are in contact with live tissue, heat dissipation must not be excessive such that it can damage the tissue. It has been reported by T.M. Seese et al. that chronic heating of muscle tissue above 45°C causes necrosis or cell death to occur in the tissue [27]. In another study, Liu et al. has shown in [28] that chronic heating of muscle tissue above 42°C causes increased perfusion rates in tissue due to angiogenesis, a response that is triggered when tissue requires healing. Several other publications have indicated that damage to human skin tissue occurs when the contact temperature exceeds 42°C [28-30]. Although there is no general consensus to the maximum rise in tissue temperature that is considered safe, a temperature rise of less than 2°C above core temperature seems to be widely accepted as a safe level of heating [29-31]. All things considered, the power dissipation of the power transfer coils must be kept at a minimum. This implies that a certain level of efficiency in the power transfer must be achieved.

Since half of the system needs to be implanted inside the body, there will also be size and weight constraints. The shape of the implanted pickup coil should aid implantation underneath the skin; this means that a flat coil with a larger surface area is more favourable than a bulky coil with the same volume. The primary coil can be of different shape and size, but it should also be low profile which makes it easier to hold in place. The weight of the
Chapter 1

Introduction

coils is also important as heavier coils are more difficult to secure in place and can cause fatigue for the patient. Due to this weight constraint, the common use of ferrite cores, which is used to increase the coupling between the power transfer coils is unfavourable, since ferrite is very dense.

1.2.3 Summary
In summary, an IPT system for powering implantable heart pumps should have good power flow control so that it can provide sufficient power (5-15W) to tolerate different levels of coupling and fluctuations in coupling, while maintaining good levels of efficiency as excessive power loss may cause damage to the skin tissue. In addition, the coils are constrained in terms of weight and size which lowers the maximum level of coupling that is possible between the power transfer coils.

1.3 Overview of the development of TET systems for implantable heart pumps

1.3.1 Historical overview of TET systems for implantable heart pumps
The concept of using IPT for powering heart pumps emerged in the mid 1960s [32-34]. Early IPT systems commonly consists of a split ferrite transformer [34, 35]. The systems were designed like traditional transformers, where the only difference was that there was a large air-gap between the ferrite cores. Since bulky ferrite cores were used, the coils were very sensitive to misalignments. The efficiency of these early TET systems ranged from 50 to 80% depending on the coupling and load. The first TET systems were designed to transfer up to 100W of power, since this was required by the first generation of artificial hearts [34]. Due to the high power level and high frequency operation of these early TET systems, the core losses in the ferrite core coils required cooling in order to prevent them from getting too hot [34].

A unique skin tunnel TET transformer was proposed by Carl F. Andren et al. in 1968, where a coil was implanted in a loop of skin created in surgery and an external coil is threaded through the loop, such that the coils are almost fully coupled and can function like a normal transformer [33]. Apart from being surgically difficult to implant, this system was very aesthetically unpleasing.
From the year 1990 onwards there was an obvious change as to how TET systems for implantable heart pumps were designed. Researchers began to move away from designs which consisted of using bulky ferrite power transfer coils to designs that were lighter and more planar in shape by designing flexible ferrite coils [36, 37] and air-cored coils [30, 38]. This change was also made possible due to advances in heart pump technology, since more efficient continuous flow pumps replaced the first generation power hungry pulsatile flow pumps. These second generation heart pumps are smaller and lighter, which make them more convenient to carry, improving the quality-of-life of the patients.

Since power transfer coils based on transformers began to phase out, to compensate for the reduced level of coupling between power transfer coils, resonant TET systems began to emerge [39]. Resonant TET systems works by introducing a capacitor in parallel or in series with the power transfer coil to create a resonant LC tank. Energy oscillates between the magnetic flux store in the coil inductance (L) and the electric charge stored in the resonant capacitance (C). Only a small amount of energy is required to build up energy in the LC tank, also an ideal LC tank can hold an infinite amount of energy. Hence, a resonant system requires less input power to create the same magnetic field strength than that is required in a non-resonant system. Presently, resonance is a key part of designing TET systems since it provides the advantages of higher power transfer capability, higher efficiency and allows the use of smaller coils to deliver the required power. Many resonant TET systems with over 80% efficiency have been published from 1990 onwards [40, 41] with compact resonant coils and circuits [42].

One part of the New Energy and Industrial Development Organisation (NEDO) artificial heart project which launched in 1995 [43], was to design a TET system for powering an implantable heart pump. The latest report on the TET side of the project was a system based on hybrid power transfer coils. This proposed system consisted of a secondary coil with a relatively large ferrite core for the purpose of creating a bump on the skin. The primary coil which is air-cored can then be easily fitted over the secondary coil, due to the bump made on the skin. The dimensions of the primary coil were relatively compact measuring 92mm in diameter and 5mm in thickness. An efficiency of over 80% was reported for this system when delivering more than 10W of power [40].

Another set of work carried out by T.D. Dissanayake in 2009 developed a closed-loop IPT system for implantable heart pumps [44]. The power transfer coils were air-cored and were
very compact measuring only 50mm in diameter and less than 10mm in thickness. A maximum efficiency of 88% was report while delivering 15W of power.

In 2007 a research group from the Massachusetts Institute of Technology, which later became Witricity, claimed to have found a new way to transfer power over large gaps by operating in a “strongly coupled” regime and commonly described using Coupled-mode theory [45]. They claim that this is different from traditional inductive power transfer. However, soon after their claims many have shown in [46, 47] that this is the same phenomenon as traditional IPT. Coupled-mode theory is a common representation used in science to describe resonant objects, whereas reflected impedance representation is commonly used by electrical engineers. Witricity has also started using four coils instead of two coils to boost efficiency over long separations and recently many other research groups are designing multiple coil systems to boost the range of the IPT system. The use of multiple resonant tanks is basically an impedance transformation to allow a practical power converter to drive a set of very high quality factor coils at its natural resonant frequency. This type of impedance transformation is not new as it is commonly used with transformers. Recently, a TET system for implantable devices using multiple resonant coils was designed. It comprises of two highly coupled primary coils transferring power to two highly coupled secondary coils where the primary and secondary coils are very loosely coupled ($k<0.05$) [48].

In summary, many researchers have demonstrated the capability to deliver sufficient power to heart pumps with wireless power systems. However, there is currently still no system on the market for powering an implantable heart pump. This is most likely due to the challenges associated clinical trials and the many regulatory requirements of medical devices which have not been sufficiently proved to be met by such wireless power systems.

1.3.2 Overview of power converters and control of IPT systems for implantable heart pumps

A resonant IPT system for powering an implantable heart pump consists of four main components: the primary and secondary resonant tank (the resonant circuit), the power converter to drive the resonant circuit, the power conditioning circuit on the secondary circuit, and a power flow controller to regulate the load power.

The purpose of the power converter is to convert a DC source into a high frequency source to drive the primary power transfer coil. For operating frequencies of less than 1 MHz, this is
usually achieved by implementing a non-linear switching network. The control of these switching networks can be classified under two main schemes: hard-switching and soft-switching. Hard-switching is when the converter switches without any regard for the voltage or current across the switch, this can generate substantial losses if switched during high voltage or current. The switching frequency is usually fixed for hard switching converters, which makes them simple to implement. Soft-switching converters are generally much more efficient, since the concept is to eliminate switching loss by switching when there is zero voltage or current across the switch (Class-D) or when there is zero voltage and current across the switch (Class-E). A resonant load makes this possible as there are zero voltage or current instances across the tank during resonance. Soft-switching converters require the detection of zero voltage or current crossings in the resonant tank making start up and control more difficult. Due to the amount of power to be delivered for powering a heart pump (5 to 15W) and the resonant frequency of the tank (usually above 100 kHz), hard-switching will be very inefficient and is almost never used in this application.

Class-D converters such as the soft-switched voltage-fed full-bridge or half-bridge and the soft-switched current-fed full-bridge or push-pull converters [49] are the most common methods of driving a resonant load for powering a heart pump. Many different power flow control strategies exist for each type of Class-D converter to control power flow. More recently many Class-E converters are appearing for powering implantable devices [41]. TET systems based on Class-E converters are usually used for low coupling and lower power systems, as variations in coupling can disrupt the Class-E switching conditions. Since a Class-E converter usually reduces the number of switching components in the circuit, the power flow control methods and its control range are limited.

Power flow control can either be done on the primary converter side or on the secondary implant side. Primary side control regulates the energy in the primary tank or alters resonant parameters to control the power at the load. One downside is that feedback from the load must be obtained and this also has to be done transcutaneously. The most common methods of receiving feedback are via a wireless communications link [50] or via another set of TET coils [51].

Secondary side power regulation requires the primary converter to deliver more power than is required making it slightly less efficient, since the controller can only dissipate the excess power as heat or de-couple the secondary pickup coil so that less or no power is received.
Secondary side power regulation also adds more components to the implant and increases its size making it undesirable for implantation. In addition, having a controller on the secondary side complicates the implanted circuitry which cannot be easily accessed in case of failure. Secondary side power regulation is most useful when one primary coil has to power multiple secondary pickups [52], which allows each pickup to regulate the amount of power it needs. Since the coupling and load may change significantly during operating when powering an implantable heart pump, secondary side regulation would not be very efficient. This will cause both the primary and secondary to dissipate more heat, which itself is problematic for this application. Thus, this thesis will only focus on primary side power flow control techniques.

1.4 Objective and scope

The objective of this research project is to contribute to the development and commercialisation of a TET system for implantable heart pumps. This research primarily seeks to improve the design of moderately coupled resonant circuits and the control range of the power flow controller. Emphasis is also placed on increasing efficiency and the determining the power dissipation of the TET system.

The following chapters of this thesis are organised as follows:

Chapter 2 is an analysis of inductively coupled resonant circuits. The analysis is focused around the likely resonant circuit characteristics of a TET system for heart pumps, such as the use of air-cored coils and operation at moderate levels of coupling (0.1<$k<$0.6). Frequency domain models of the coupled resonant circuits are established and numerical methods are used to determine its steady-state characteristics. From these models the resonant topology, parameters and operating frequency for maximum resonant efficiency can be obtained. The effect of changes in coupling and load are also analysed and system instability regions such as frequency bifurcation points are also discussed.

Chapter 3 of this thesis presents a study on the power dissipation of the TET system with particular focus on the power loss of the power transfer components. Different methods of determining power losses in components under high frequency excitation are discussed. A large part of this chapter focuses on calorimetric techniques of power measurement, since high levels of uncertainty exists with electrical power measurement methods. The design and
verification of a novel calorimeter to accurately measure resonant component losses is presented and experimental results are given.

Chapter 4 presents an analysis on primary side power flow control based on varying the resonant frequency of the resonant IPT system. Different methods of frequency control are analysed and its control performance is simulated. The design of practical power converters utilising frequency power flow control are presented on both parallel and series tuned primary resonant tanks. IPT systems based on primary side frequency power flow control are practically implemented and their performance is analysed.

Chapter 5 investigates a primary side power flow control strategy for a TET system based on the principle of energy injection. Different energy injection control strategies are analysed and a new control method based on pulse density modulation is presented. Energy injection power flow control was practically implemented on both parallel and series tuned primary resonant tanks, and their performance is analysed.

Chapter 6 presents the implementation and design of a standalone closed-loop TET system for powering implantable heart pumps. This TET system adheres to a specification that has been put together from a variety of sources, such that it will meet most people’s expectations. This chapter puts the resonant circuit design methodology and power flow control methods presented in previous chapters into perspective. The performance of the implemented closed-loop TET system is presented across a practical coupling and load range.

Chapter 7 provides a summary of the conclusions drawn in this thesis. A list of the contributions made in this thesis is presented, and direction for future work is also provided.
Inductive power transfer (IPT) operates by driving a high frequency current through a power transfer coil, to generate high frequency magnetic flux. High inductance power transfer coils are able to store more magnetic flux, however due to its higher impedance, high input VA is required to drive the coil. A sinusoidal drive current for the power transfer coils is also desired, since only magnetic fields oscillating at the frequency to which the pickup is tuned to will transfer power. Hence, resonant circuits are used in most IPT systems to improve the harmonic content of the magnetic flux and to improve the system efficiency [53].

The most basic resonant circuit is an inductor-capacitor (LC) circuit. It has an oscillation frequency known as the resonant frequency, at which energy naturally resonates between the inductor and capacitor. Adding a capacitor with the power transfer coil reduces the VA power requirements to drive a certain current through the coil, by cancelling out the reactance of the power transfer coil, when driven at the resonant frequency. At resonance, the greatest amplitude of the oscillating voltage and current can be achieved and only small amounts of energy is needed to sustain the resonating voltage and current.

A resonant circuit exhibits zero current flowing in the circuit, when energy in the capacitor is at its maximum; and zero voltage across the circuit, when energy in the inductor (coil) is at its maximum. This phenomenon allows for zero current switching (ZCS) or zero voltage switching (ZVS) of switching networks used to drive the resonant tank. ZCS and ZVS are soft-switching techniques and can be regarded as common practice for resonant converters, since their efficiency is significantly higher than that of hard-switched resonant converters [54].
The resonant circuit is a key part of an IPT system since it determines the characteristics of the system, such as its operating frequency, coupling and load sensitivity and its power transfer capability. Fine selection of these resonant parameters is vital for good efficiency and system stability. This chapter provides an analysis of the basic resonant topologies used in IPT systems and presents a methodology for selecting the most suitable resonant topology and parameters for an IPT system.

2.1 Resonant tank topology

The two basic resonant tank topologies are the series tuned and parallel tuned resonant tanks and are shown in Figure 2-1a) and b) respectively. The series tank has a current source characteristic and must be driven by a voltage source (voltage-fed) and the parallel tank has a voltage source characteristic and must be driven by a current source (current-fed) [54].

![Figure 2-1: Basic resonant tank topologies; a) Ideal series tuned tank, b) Ideal parallel tuned tank](image)

The steady-state characteristics of the basic resonant tanks can be obtained by analysing their input impedance expressed in the frequency domain. The impedance seen by the source for an ideal series tuned and parallel tuned tank is given by equation (2-1) and (2-2) respectively.

\[ Z_{\text{series}} = j\omega L + \frac{1}{j\omega C} \]  
\[ Z_{\text{parallel}} = \frac{j\omega L}{1 - \omega^2 LC} \]

where \( \omega = 2\pi f \) is the angular frequency and \( f \) is the frequency at which the circuit is driven. Note that the impedance of an inductor \( L \) is given as \( j\omega L \) and the impedance of a capacitor \( C \) is given as \( 1/j\omega C \).
The undamped natural resonant frequency $f_0$ is the frequency at which energy in the resonant tank will oscillate at when left unforced. The undamped natural resonant angular frequency $\omega_0$ is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$  \hspace{1cm} (2-3)

At $\omega_0$ the reactance of the series tuned and parallel tuned input impedances are cancelled out, such that $\text{Im}\{Z|_{\omega_0}\} = 0$. While the real part of the input impedance for a series tuned tank is zero, and for a parallel tuned tank it is infinite. However, damping is inevitably present in a practical tank due to imperfections in the reactive components. This damping can be represented as resistive losses $R_L$ and $R_C$ in series with the inductor and capacitor respectively. The equivalent circuits of the damped resonant tanks are shown in Figure 2-2.

![Figure 2-2: Resonant tank topologies with series component resistances; a) series tuned tank, b) parallel tuned tank](image)

The input impedance of a series tuned tank with series component resistances shown in Figure 2-2a) is given as

$$Z_{\text{series}} = j\omega L + \frac{1}{j\omega C} + R_L + R_C$$  \hspace{1cm} (2-4)

The impedance of a parallel tuned tank with series component resistances shown in Figure 2-2b) is given as

$$Z_{\text{parallel}} = R_L - \omega^2 L R_C + j\omega (L + CR_C R_L) \frac{1}{1 - \omega^2 LC + j\omega C (R_C + R_L)}$$  \hspace{1cm} (2-5)

The most commonly accepted resonant frequency of a damped resonant tank is the Zero Phase Angle (ZPA) frequency $f_r$ [54]. This is given by the angular frequency that cancels out
the imaginary part of the input impedance, such that a resonant tank running at its ZPA frequency draws only real power from the source. For the series tuned tank shown in Figure 2-2a), \( \omega_r \) is given as

\[
\omega_{r\text{-series}} = \omega_0
\]  

(2-6)

Note that \( \omega_r \) for a series tuned tank is that same as its undamped natural frequency \( \omega_0 \). However \( \omega_r \) for the parallel tuned tank shown in Figure 2-2b) is not the same as \( \omega_0 \) and is given as

\[
\omega_{r\text{-parallel}} = \omega_0 \sqrt{\frac{L - CR_L^2}{L - CR_C^2}}
\]  

(2-7)

Apart from the two basic resonant tank topologies, additional reactive components may be added to form higher order resonant circuits. Since each LC pair in a high order resonant tank has a unique resonant frequency, beating will occur when left unforced. High order resonant tanks are not part of this study.

### 2.2 Frequency domain modelling of the resonant system.

A resonant IPT system consists of a driven primary resonant tank and a secondary resonant tank that is inductively coupled to the primary coil. This coupling allows voltage/current to be induced in the secondary coil, which means that power is transferred to the secondary pickup. When the secondary coil is coupled to the primary coil, they share a mutual inductance \( M \) given by [55]

\[
M = k \sqrt{L_1 L_2}
\]  

(2-8)

where \( L_1 \) and \( L_2 \) are the self inductance of the primary and secondary coil respectively and \( k \) is the coupling co-efficient. The effect of the mutual inductance on the primary and secondary resonant circuit can be modelled as an induced current-dependent voltage source in series with the coil inductance (Thevenin’s equivalent), or as a voltage-dependent current source in parallel with the coil inductance (Norton’s equivalent). The Norton’s equivalent circuit due to mutual inductance \( M \) is shown in Figure 2-3a), and the Thevenin’s equivalent circuit due to mutual inductance \( M \) is shown in Figure 2-3b).
The expression for the primary and secondary induced voltage in series with the coil inductance is given as

\[
V_p = j\omega MI_2 \\
V_S = j\omega MI_1
\]  \hspace{1cm} (2-9)

where \(I_1\) is the current through \(L_1\) and \(I_2\) is the current through \(L_2\). The expression for the primary and secondary induced current in parallel with the coil inductance is given as

\[
I_p = \frac{M}{L_1}I_2 \\
I_S = \frac{M}{L_2}I_1
\]  \hspace{1cm} (2-10)

Since there are two basic resonant tanks, it is possible to make four basic resonant circuit topologies; they are Series-series (SS), Series-parallel (SP), Parallel-series (PS) and Parallel-parallel (PP). The equivalent circuits for the four basic resonant circuit topologies with their series component resistances are shown in Figure 2-4.

Since we now have a way of linking the coupled primary and secondary resonant tanks. The steady-state characteristics of the coupled resonant circuit can be obtained by defining its input impedance. This can be achieved by reflecting the secondary impedance onto the primary tank. The derivation for the input impedance seen by the source for a series-series resonant circuit is shown below and is depicted in Figure 2-5. Here the Thevenin’s equivalent circuits are used to analyse the effect of the mutual inductance, although the same outcome can be achieved with the Norton’s equivalent circuits.
Figure 2-4: Equivalent circuits of the four basic resonant topologies

Figure 2-5: Input impedance of a series-series tuned resonant system in terms of reflected impedance
The impedance $Z_{2S}$ as seen by the secondary induced voltage $V_S$ is given by

$$Z_{2S} = j\omega L_2 + \frac{1}{j\omega C_2} + R_2 + R_{L2} + R_{C2}$$

(2-11)

where resistor $R_2$ is the resistive load connected to the secondary pickup. The current flowing through the secondary pickup coil can generally be expressed as

$$I_2 = \frac{V_S}{Z_2} = \frac{j\omega M I_1}{Z_2}$$

(2-12)

The primary side reflected impedance $Z_R$ as seen by the reflected voltage $V_P$ is given by

$$Z_R = \frac{V_P}{I_1} = \frac{-j\omega M I_2}{I_1}$$

(2-13)

Combining equation (2-12) and (2-13) the primary side reflected impedance in terms of $Z_2$ is given as

$$Z_R = \frac{\omega^2 M^2}{Z_2}$$

(2-14)

Hence, the input impedance seen by the source for a series-series resonant circuit is given as

$$Z_{1SS} = \frac{1}{j\omega C_1} + j\omega L_1 + R_{C1} + R_{L1} + Z_{RS}$$

(2-15)

where $Z_{RS}$ is the primary side reflected impedance of a series tuned secondary circuit. A similar approach was taken to work out the input impedance expressions of the PP, SP and PS coupled resonant circuit topologies and the resulting impedances are summarised in Table 2-1.
Table 2-1: Frequency domain impedance equations of the four basic coupled resonant topologies

<table>
<thead>
<tr>
<th>Topology</th>
<th>Impedance Equation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{2P}$</td>
<td>$j\omega L_2 + R_{L_2} + \frac{R_2(j\omega C_2R_{C_2} + 1)}{j\omega C_2(R_2 + R_{C_2}) + 1}$</td>
<td>$Z_{RP} = \frac{\omega^2 M^2}{Z_{2P}}$</td>
</tr>
<tr>
<td>$Z_{2S}$</td>
<td>$j\omega L_2 + R_{L_2} + \frac{1}{j\omega C_2} + R_{C_2} + R_2$</td>
<td>$Z_{RS} = \frac{\omega^2 M^2}{Z_{2S}}$</td>
</tr>
<tr>
<td>$Z_{1PP}$</td>
<td>$\frac{1}{j\omega C_1} + R_{C_1} + \frac{(j\omega L_1 + R_{L_1} + Z_{RP})}{j\omega L_1 + R_{L_1} + \frac{1}{j\omega C_1} + R_{C_1} + Z_{RP}}$</td>
<td></td>
</tr>
<tr>
<td>$Z_{1SP}$</td>
<td>$\frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} + j\omega L_1 + R_{L_1} + Z_{RP}$</td>
<td></td>
</tr>
<tr>
<td>$Z_{1PS}$</td>
<td>$\frac{(j\omega L_1 + R_{L_1} + Z_{RS})}{j\omega L_1 + R_{L_1} + \frac{1}{j\omega C_1} + R_{C_1} + Z_{RS}}$</td>
<td></td>
</tr>
<tr>
<td>$Z_{1SS}$</td>
<td>$\frac{1}{j\omega C_1} + R_{C_1} + j\omega L_1 + R_{L_1} + Z_{RS}$</td>
<td></td>
</tr>
</tbody>
</table>

2.3 Effects of tuning on a resonant system

The simplest and most common method for tuning an IPT resonant system is to tune both individual primary and secondary resonant tanks to the same nominal undamped frequency $f_0$. If both the primary and secondary resonant tanks are tuned to the same nominal undamped natural frequency, such that $L_1C_1 = L_2C_2$. It can be worked out from the expressions given in Table 2-1, that only the SS topology has an input impedance $Z_I$ that is purely resistive at $\omega_0$. Note that $\omega_0$ is always a ZPA frequency for the SS topology, however it may also have other ZPA frequencies [56]. For all other topologies the ZPA frequency $\omega_r$ of the input impedance $Z_I$ is in fact different to the nominal frequency that the individual tanks are tuned to. This is due to the fact that, the secondary reflected impedance is reactive and its magnitude increases as coupling $k$ increases. An example of how much $\omega_r$ can deviate from $\omega_0$ for each of the four basic resonant topologies, when coupling increases is illustrated in Figure 2-6. For this plot, the secondary circuit quality factor $Q_S$ of each topology was set to 1 and both primary and secondary tanks are tuned to the same undamped natural frequency such that $L_1C_1 = L_2C_2$. 

---
When designing loosely coupled systems ($k<0.1$), most of the time it is safe to make the assumption that the system ZPA frequency is the same as the individual resonant frequency of the primary resonant tank. Since when loosely coupled the primary and secondary circuits practically operate independently from each other [57]. However, the system ZPA frequency can be significantly different, when higher levels of coupling are involved ($k>0.1$) as illustrated in Figure 2-6.

Moderately coupled IPT systems that have their individual primary and secondary tanks tuned to the same nominal frequency, will not be able to soft-switch at the nominal frequency, since soft-switching can only be achieved at a ZPA frequency of the system. Although driving the primary at the nominal frequency would provide maximum power transfer to the secondary load [58], switching losses will increase greatly [54].

In addition to the varying system ZPA frequency as coupling changes, the resonant circuit may also exhibit multiple ZPA frequencies. Once the primary and secondary circuits are coupled together the result is a system made up of four reactive components, this combined circuit has 4th order input impedance in terms of $\omega$. The magnitude of the high order terms is insignificant when the coupling is low. Thus, when loosely coupled the input impedance of the resonant circuit is essentially 2nd order. However, as coupling increases the 4th order terms of the input impedance become more and more significant. Due to the fact that high order expressions can have multiple roots, the high order input impedance may have multiple ZPA frequencies, a phenomenon known as frequency bifurcation [56]. A PP resonant system with
tuning $L_1C_1=L_2C_2$ and a secondary quality factor $Q_s=2$ is used to illustrate the effect of frequency bifurcation in Figure 2-7. The normalised ZPA frequency plotted against coupling of this system is shown in Figure 2-7a), and the phase of $Z_1$ plotted against normalised frequency is shown in and Figure 2-7b).

![Figure 2-7: Characteristic of PP configuration when $Q_{s_0}=2$ and $L_1C_1=L_2C_2$: a) Normalised ZPA frequency versus coupling, b) Phase of $Z_{1PP}$ versus normalised frequency.](image)

Figure 2-7a) indicates that frequency bifurcation is encountered once the coupling $k$ between the primary and secondary coils exceeds 0.43. Since three distinct ZPA frequencies now exist after that point. Figure 2-7b) shows how the phase or imaginary part of $Z_{1PP}$ changes with frequency at different levels of coupling, when $k>0.43$ ($k=0.5$ and $k=0.6$) three distinct zero phase crossing points can be seen on the graph. This means that the resonant circuit may be soft-switched at three different ZPA frequencies. The determination of precise frequency bifurcation points is an involved and cumbersome process and has been described by Covic et al. [56]. If the operating frequency of the converter is designed to track the ZPA frequency, then as coupling is increase passed the bifurcation point, the ZPA frequency at which the converter will operate at is dependent on the transient condition and the zero crossing detection. It is extremely challenging to determine the practical operating frequency of an IPT system once the impedance has bifurcated. Thus, it is usually avoided in most designs.

Resonant parameters for moderately-coupled IPT systems cannot be tuned with predictable characteristics, by merely tuning the individual primary and secondary tanks to the same nominal frequency. Since the primary and secondary circuits of a TET system for implantable heart pumps will not stay in the loosely coupled mode. Resonant parameters must be
carefully selected such that it is frequency stable and stays efficient throughout the coupling and load range of the TET system.

2.4 Resonant system design considerations

The design of a resonant IPT system is one with many variables that has to be taken into account. Depending on the application of the IPT system, some variables such as coupling, operating frequency and coil size will be restricted or even strictly defined. Due to the large number of factors that have to be taken into account, there is no one design method that will work for all IPT systems. Hence, there is the need for tools and techniques to analysis a certain resonant configuration. This section will attempt to highlight some design tradeoffs and compromises that exist in the design of the resonant parameters of an IPT system for the application of powering implantable heart pumps.

2.4.1 Power transfer coils

The design of power transfer coils is an important part of TET system design and is a standalone research topic by itself. Power transfer coil design involves determining physical dimensions of the coil, the magnetic flux pattern generated, the type of wire to be used and the coupling between power transfer coils.

For analysing the resonant circuit of an IPT system, it is just necessary to know the self inductance and self capacitance of the primary and secondary coils, the operating coupling range, and the equivalent series resistance (ESR) profile of the coil. However, since in this study the operating frequency (several hundreds of kHz) is very far from the self resonant frequency of the coils (several MHz), the self capacitance of the coils is negligible. Therefore, only the remaining three characteristics are required to fully define the electrical operating conditions and losses of the resonant power transfer coils. The other factors of coil design do not affect the electrical operation of the resonant system. For example, the physical dimensions and the magnetic flux pattern generated by the coil, play a part in determining the inductance of the coil, and the relationship between coupling and physical coil separation. Aspects such as physical coil dimensions and coil separation is usually constrained or dictated by the application. Although these aspects are important in the design of the IPT system, they however do not affect the electrical characteristics of the system.
The greatest constraint in the design of power transfer coils for implantable devices is its physical size and weight. Since the secondary pickup coil has to be implanted, it is ideal if the coil is as compact and as low profile as possible. Also due to the fact that the power transfer coils need to be carried on the body the weight of the coil also needs to be taken into consideration. Thus, the use of bulky and heavy ferrite cores, which are used to improve the coupling between the coils is usually avoided [44]. For the application of powering an implantable device, the operating coupling range of the power transfer coils is usually quite wide. Minimum coupling coefficient in the operating coupling range can be three to five times lower than the maximum coupling coefficient, which is a huge change for the resonant circuit. A large operating coupling range is required for this application, since the alignment and depth of the implanted coil will be different in every patient. Variation is introduced from both surgical placement and the different physical physique of the patient. In addition, the external primary cannot be clamped on the skin due to heating and discomfort. Hence, the alignment between the primary and secondary coil will constantly be changing.

Since the focus of this study is not on power transfer coil design, the coils used in this study are adopted from previous work by T.D Dissanayake [44]. High quality Litz wire supplied by PACK Feindrähte [59] was used to make all the power transfer coils. The coils used in this study are compact and thin, a pair of these power transfer coils is shown in Figure 2-8.

![Spiral pancake power transfer coils encapsulated in silicon](image)

**Figure 2-8: Spiral pancake power transfer coils encapsulated in silicon**

### 2.4.2 Resonant capacitors

Resonant capacitors unlike the power transfer coils are not designed and made for the IPT system. Rather they are just purchased off the shelf and are used with the power transfer coils to form a resonant tank. For analysing the resonant circuit of an IPT system, it is only necessary to know the capacitance and ESR profile of the capacitor.
Important aspects of the resonant capacitor for an IPT system include its physical size and electrical ratings. Since the capacitor will be used in a resonant tank, the voltage and current ratings have to be relatively high, since resonant voltages can be as high as several hundred volts and currents as high as several amps. In addition, the frequency of these voltage and currents will be oscillating at several hundred kHz. Capacitors not rated for these conditions will either fail or generate huge losses in the system. The aforementioned conditions are actually very heavy requirements for a capacitor. Thus, polypropylene film capacitors which are regarded as the best high frequency capacitors are often used to keep losses down. For the application of powering implantable heart pumps it is ideal to keep the physical size of the capacitor to a minimum. However, larger and bulkier capacitors have better voltage and current ratings which results in a lower ESR. When selecting resonant capacitors, a compromise has to be made between physical size and electrical performance. One way to reduce the physical size of the capacitor is to lower the tuning capacitance, since capacitors with lower capacitances are smaller for the same electrical ratings. However, lowering the capacitance will increase the resonant frequency which may increase losses in other parts of the system.

The resonant capacitors used throughout the practical implementations of this study were polypropylene film capacitors from Cornell Dubilier’s 940C range. The capacitors used had voltage ratings of 2000Vdc and 600Vac (50 Hz) or better. The range of capacitor values available in this series ranged from 10nF to 4.7µF.

### 2.4.3 Resonant topology

The resonant topologies that will be covered in this study are the four basic resonant topologies PP, PS, SP and SS as introduced in section 2.2. Theoretically, all four basic resonant topologies are the same, since they are just dual circuits of each other [54]. However, in practice the choice of adopting a resonant topology for the system depends on factors such as the impedance of the load and the VA limitations of the power supply. For instance, a high impedance load would result in a high secondary circuit quality factor $Q_S$ for a parallel tuned pickup; whereas a high impedance would make the circuit $Q$ low with a series tuned pickup. Another example is that, a parallel tuned primary would require more voltage than current to drive, since a parallel tank operating at its resonant frequency will exhibit very high input impedance. Compared to a series tuned primary which has a very low
input impedance when driven at its resonant frequency, would not need much voltage to drive, but tends to draw more current.

2.4.4 Primary converter
In a practical IPT system a power converter circuit is necessary to supply energy to the resonant circuit. Since soft-switching is a vital requirement for good efficiency for resonant systems, the voltage waveform supplied across the resonant circuit by the power converter can be assumed to be a sinusoidal source operating at the ZPA frequency of the resonant circuit. In practice the input voltage is not sinusoidal and consists of many harmonics. Since the harmonics do not contribute in building up the energy in the resonant circuit, they can be ignored when analysing the resonant system.

The choice of the power converter to be used on the system is dependent on the resonant topology of the primary tank and the power flow control method that is desired. Theoretically, all ideal power converters are the same as they just supply an AC voltage for the resonant circuit.

2.4.5 Expressions for output voltage and resonant currents
Using the impedance expressions given in Table 2-1 and given all the terms apart from $\omega$, the operating resonant frequency $\omega_r$ can be obtained by solving

$$\text{Im}\{Z_1(\omega_r)\} = 0 \quad (2-16)$$

Since Equation (2-16) is not a polynomial for all four basic resonant topologies, it requires extensive techniques to solve for the angular frequency analytically [56]. Hence, it is much simpler to solve Equation (2-16) numerically.

A script in MATLAB was written to find all the ZPA frequencies of any given impedance, this code can be found in Appendix A and is based on finding the zero crossings of an expression. Note that $\omega_r$ has to be re-calculated whenever a resonant parameter is altered or when coupling $k$ changes. Once the operating frequency $\omega_r$ has been obtained the resonant system is fully characterised for that coupling and load point.
The input voltage $V_{IN}$ required for sustaining output voltage $V_d$ across load $R_2$ at the ZPA frequency $\omega_r$ for each of the four basic coupled resonant circuit topologies are given by

\begin{align}
V_{INSS} &= V_d \cdot \frac{Z_{1SS}(\omega_R) Z_{2S}(\omega_R)}{j \omega_R MR_2} \\
V_{INPS} &= V_d \cdot \frac{Z_{2S}(\omega_R) (j \omega_R L_1 + R_{L1} + Z_{RS}(\omega_R))}{j \omega_R MR_2} \\
V_{INSP} &= V_d \cdot \frac{Z_{1SP}(\omega_R) (1 - j \omega_R L_2 + R_{L2})}{j \omega_R M} \\
V_{INPP} &= V_d \cdot \frac{(j \omega_R L_1 + R_{L1} + Z_{RP}(\omega_R))}{j \omega_R M (1 - j \omega_R L_2 + R_{L2})} \tag{2-17}
\end{align}

The magnitude of the resonant currents can be given in terms of $V_{IN}$. For the SS topology the resonant currents are given by

\begin{align}
I_{L1SS} &= I_{C1SS} = \frac{V_{INSS}}{Z_{1SS}(\omega_R)} = V_d \cdot \frac{Z_{2S}(\omega_R)}{j \omega_R MR_2} \\
I_{L2SS} &= I_{C2SS} = \frac{V_d}{R_2} \tag{2-18}
\end{align}

For the PS topology the resonant currents are given by

\begin{align}
I_{L1PS} &= \frac{V_{INPS}}{(j \omega_R L_1 + R_{L1} + Z_{RS}(\omega_R))} = V_d \cdot \frac{Z_{2S}(\omega_R)}{j \omega_R MR_2} \\
I_{C1PS} &= V_{INPS} \cdot j \omega_R C_1 = V_d \cdot \frac{C_1}{MR_2} Z_{2S}(\omega_R) (j \omega_R L_1 + R_{L1} + Z_{RS}(\omega_R)) \\
I_{L2PS} &= I_{C2PS} = \frac{V_d}{R_2} \tag{2-19}
\end{align}

For the SP topology the resonant currents are given by

\begin{align}
I_{L1SP} &= I_{C1SP} = \frac{V_{INSP}}{Z_{1SP}(\omega_R)} = V_d \cdot \frac{1}{j \omega_R M (1 - \frac{j \omega_R L_2 + R_{L2}}{Z_{2P}(\omega_R)})} \\
I_{L2SP} &= \frac{j \omega_R M I_{L1SP}}{Z_{2P}(\omega_R)} = V_d \cdot \frac{1}{Z_{2P}(\omega_R) (1 - \frac{j \omega_R L_2 + R_{L2}}{Z_{2P}(\omega_R)})} \\
I_{C2SP} &= V_d \cdot \frac{j \omega_R C_2}{1 + j \omega_R C_2 R_{C2}} \tag{2-20}
\end{align}
For the PP topology the resonant currents are given by

\[ I_{L1PP} = \frac{V_{INPP}}{j\omega RL_1 + R_{L1} + Z_{RP}(\omega_R)} = V_d \cdot \frac{1}{j\omega M \left( 1 - \frac{j\omega RL_2 + R_{L2}}{Z_{2P}(\omega_R)} \right)} \]

\[ I_{C1PP} = V_{INPP} \cdot j\omega R C_1 = V_d \cdot \frac{C_1(j\omega RL_1 + R_{L1} + Z_{RP}(\omega_R))}{M \left( 1 - \frac{j\omega RL_2 + R_{L2}}{Z_{2P}(\omega_R)} \right)} \]

\[ I_{L2PP} = \frac{j\omega M I_{L1PP}}{Z_{2P}(\omega_R)} = V_d \cdot \frac{1}{Z_{2P}(\omega_R) \left( 1 - \frac{j\omega RL_2 + R_{L2}}{Z_{2P}(\omega_R)} \right)} \]

\[ I_{C2PP} = V_d \cdot \frac{j\omega R C_2}{1 + j\omega R C_2 R_{C2}} \]  \hspace{1cm} (2-21)

Note that the terms \( R_{L1}, R_{L2}, R_{C1}, R_{C2} \) in Equations (2-17) through to (2-21) are frequency dependent ESR functions. Using a loss model based on the frequency dependent ESR of the resonant components, the total power loss in the resonant circuit can be given by

\[ P_T = I_{L1}^2 R_{L1}(\omega) + I_{C1}^2 R_{C1}(\omega) + I_{L2}^2 R_{L2}(\omega) + I_{C2}^2 R_{C2}(\omega) \]  \hspace{1cm} (2-22)

The resonant currents in Equation (2-22) are expressed in RMS quantities.

### 2.5 Selection of resonant topology and parameters

The resonant topology and tuning parameters are the two fundamental components of determining the maximum power capacity and efficiency of an IPT system. Due to the sensitive nature of the resonant system at moderate levels of coupling \( k>0.1 \), it is risky to make assumptions that simplify the complex input impedance expressions of the system. Thus, the system must be analysed across all operating coupling and load conditions in order to determine its performance and stability. A numerical analysis performed in the frequency domain is presented in this section and is used to determine the steady-state characteristic of the resonant circuit across its entire operating range. Several metrics were used to compare different resonant parameters and a selection procedure was developed to determine the resonant topology and tuning that gives minimum power loss.

#### 2.5.1 Objective function

In an IPT system, the power loss of the resonant coils and capacitors are the most difficult to reduce, since their power loss is related to their ESR, which is dependent on the quality of the component. Although the ESR of the components are difficult to control, other factors such
as the operating frequency and the magnitude of the resonant current, are determined by the resonant topology, tuning and load. This leads to the notion that there is a resonant circuit setup that produces minimum loss in the resonant circuit, given constraints on certain parameters. Thus, the objective function for the most efficient resonant setup is to minimise the average power loss in the resonant circuit given by Equation (2-22) across the operating coupling and load range.

Another objective function that could be used to determine the resonant topology and parameters is to maximise its power transfer capability. Maximum power transfer capability is defined as the ability to supply a certain power to the load with a minimum input voltage at the primary converter. However, this does not imply maximum efficiency system efficiency. A resonant setup that provides maximum power transfer for a certain load condition, is most useful when it is difficult to supply enough power to the load and when efficiency is not a primary requirement. Thus, given a load \( R_2 \) and desired output voltage \( V_d \), the objective function is to find the resonant setup that requires the smallest input voltage \( V_{IN} \).

2.5.2 Constraints
In order to minimise the loss in the resonant circuits, there must be constraints put in place for each parameter such that a unique solution can be obtained. The input impedance \( Z_l \) of the resonant topologies shown in Table 2-1 consists of eleven terms, they are: \( \omega, k, L_1, L_2, C_1, C_2, R_{L1}, R_{L2}, R_{C1}, R_{C2} \) and \( R_2 \). The terms \( L_1, L_2, R_{L1}, R_{L2} \) and the coupling \( k \) range are determined by the power transfer coils, and the terms \( C_1, C_2, R_{C1} \) and \( R_{C2} \) are determined by the tuning capacitors. The load resistance \( R_2 \) is specified by the application along with a desired output voltage \( V_d \). Finally, the angular frequency of the source \( \omega \) is not specified, rather it is set to be \( \omega_r \), the ZPA angular frequency of \( Z_l \), which can only be determined once all the other terms have been given.

Since \( L_1, L_2, R_{L1}, R_{L2} \) and the coupling \( k \) range are determined by the set of the power transfer coils, it is difficult to change one term without affecting the others. Also it may not be feasible to request for a certain coil inductance and coupling range, given that the physical dimensions of the coils are constrained by the application at hand. Since power transfer coil design is not a primary focus of this thesis, the terms of the power transfer coils will be obtained by practically measuring different coils which satisfy the physical constraints of the application. Hence, the loosely constrained variables of the procedure will be the resonant
topology and the tuning capacitances $C_1$ and $C_2$, since $R_{C1}$ and $R_{C2}$ will be obtained by measuring $C_1$ and $C_2$ respectively.

### 2.5.3 Maximum efficiency tuning procedure

A numerical procedure called the Maximum Efficiency Tuning (MET) procedure was written in MATLAB to find the resonant topology and tuning capacitors $C_1$ and $C_2$, which satisfies the objective function of maximum efficiency given in section 2.5.1. Since $C_1$ and $C_2$ is fixed across the operating coupling and load conditions, the desired output voltage $V_d$ is maintained by adjusting the magnitude of the source voltage. Thus, the resonant topology and tuning capacitors found in the procedure, is useful if input magnitude power flow control is used on the converter. For a given set of power transfer coils ($L_1$, $L_2$, $R_{L1}$ and $R_{L2}$), the operating coupling range ($k_{min}$ to $k_{max}$), the load range ($R_{2min}$ to $R_{2max}$) and the desired output voltage $V_d$. The MET procedure iterates $C_1$ and $C_2$ from $C_{min}$ to $C_{max}$ and locates the pair that provides the minimum average resonant tank loss for each resonant topology. The MET procedure operates as follows:

For each $C_1$ and $C_2$ pair (between $C_{min}$ to $C_{max} at 1nF steps$) the frequency dependent input impedance of each resonant topology is calculated for the current coupling $k$ and load $R_2$. From this the ZPA frequency $\omega_r$ is obtained by numerically solving Equation (2-16) using the code provided in Appendix A. If multiple ZPA frequencies exists then both the lower and upper ZPA frequencies are taken into account, as practical control circuits will only track on to these two frequencies since the center ZPA frequency is unstable [56].

After $\omega_r$ is obtained, the required input voltage $V_{IN}$ to generate $V_d$ across the load $R_2$ for each topology is calculated with the Equations in (2-17). The magnitudes of the resonant currents for each topology are then calculated with the respective equations from (2-18) to (2-21). Then the power loss of each resonant topology is obtained with equation (2-22) with the frequency dependent ESRs of each component.

Since each $C_1$ and $C_2$ pair needs to be compared, the average power loss $\overline{P_T}$ across the coupling and load conditions is calculated for each topology and pair of tuning capacitances. The $C_1$ and $C_2$ pair with the lowest $\overline{P_T}$ is the MET pair for the topology.
A flowchart for the MET procedure is shown in Figure 2-9 and the MATLAB code for the procedure is provided in Appendix B.

Figure 2-9: Flowchart for the Maximum Efficiency Tuning (MET) procedure
2.6 Comparison of resonant topology and parameter selection methods

To demonstrate the effectiveness of the proposed MET procedure, nominally tuned resonant circuits \((L_1C_1=L_2C_2)\) will be compared to resonant circuits tuned by the MET procedure. This will be done in a case study analysis where a hypothetical specification for an IPT system for powering implantable heart pumps is given. In addition, the design considerations of the resulting topologies and parameters will be discussed in regard to the specific application.

2.6.1 Case study: Specifications

A Transcutaneous Energy Transfer (TET) system for powering implantable heart pumps consists of identical primary and secondary power transfer coils. The outer diameter of the coil is 52mm and has a thickness of only 4mm; the inductance measured for these coils was 11.5\(\mu\)H. These coils are compact and flat, and should be suitable for implantation. The expected range of concentric coil separation, while the power transfer coils are operating on a patient is expected to be anywhere between 7mm and 20mm. The average power drawn by the heart pump is expected to be between 10W to 15W. Given that the operating voltage of the pump is 12V \((V_{dc}=12V)\), this would correspond to an output load range of 15Ω to 12Ω.

2.6.2 Case study: Modelling of resonant components

The power loss of reactive components can be modelled with a frequency dependent ESR, which can be empirically measured with a LCR meter. The power dissipated by the resistive component can then be calculated with \(I^2R\), where \(I\) is the RMS current flowing through the component and \(R\) is the ESR at the operating frequency of the current.

**Power transfer coils**

The ESR of the power transfer coils given in this case study were measured with a Fluke PM6306 LCR at several frequencies within the range of 0 to 500 kHz. Since the primary and secondary coils are identical, their ESR is also identical and its frequency dependency is illustrated in Figure 2-10. A second order curve was fitted to the empirical measurements, and an expression for the frequency dependent ESR of the coil can be given as

\[
R_L(f) = 7 \times 10^{-13} f^2 + 4 \times 10^{-9} f + 0.026
\]  

(2-23)
The coupling coefficient $k$ was also measured between the primary and secondary coil at concentric separations from 0 to 20 mm, and its relationship is shown in Figure 2-11. An exponential curve was fitted to obtain an expression for coupling $k$ versus concentric separation and is given as

$$k = 0.655e^{-0.073d} \tag{2-24}$$

where $d$ is the concentric separation measured in millimetres.

![Figure 2-10: ESR of power transfer coils versus frequency used in the case study](image)

![Figure 2-11: Coupling $k$ versus concentric separation for power transfer coils used in the case study](image)

The maximum coupling possible for the coils (when directly on top of each other) has been measured to be $k=0.66$. The expected concentric separation of the coils given in section 2.6.2
was 7mm to 20mm. This separation range corresponds to a coupling range of \( k = 0.15 \) to 0.4 for this set of power transfer coils.

**Resonant capacitors**

The ESR of the capacitors that will be used to tune the resonant tank, are polypropylene film capacitors from Cornell Dubilier’s 940C series. Upon measuring the ESR of these capacitors, it was found that the ESR of these capacitors was fairly constant within 100 kHz to 500 kHz. In fact the capacitance was found to decrease slightly with frequency [60]. However, the ESR of these capacitors was found to be dependent on the value of its capacitance. The ESR of capacitors from the 940C series, with values between 10nF to 220nF was measured with a Fluke PM6306 LCR meter. The ESR of the capacitors was measured at 200 kHz and at 500 kHz and the average of these two measurements was then plotted against its capacitance, as shown in Figure 2-12. An exponential approximation was fitted on Figure 2-12, and the capacitor’s ESR as a function of its own capacitance is approximated as

\[
R_C(C) = 2 \times 10^{-7} \cdot C^{-0.73} \tag{2-25}
\]

In practice several capacitors may be connected in parallel to lower the ESR of the resonant capacitance. The model given in Equation (2-25) assumes that a single capacitor is used.

![Figure 2-12: Capacitor ESR versus capacitance of capacitors used in the case study](image)

**2.6.3 Case study: Comparison of resonant circuits**

Given system parameters in section 2.6.1, the MET procedure was performed. The tuning resulting from the MET procedure was then compared to nominally tuned resonant circuits
whose individual primary and secondary tanks were tuned to 180 kHz. Operating frequencies around 200 kHz are commonly chosen for similar TET systems [31, 40], although no explicit reasons are given for choosing nominal frequencies around 200 kHz. It is commonly described as not too high for the operation of the power converter and not too low such that resonant circuit Q is low [40].

Table 2-2 provides a summary of the given power transfer coils and the specified operating coupling and load range. The capacitor pairs found in the MET procedure are listed, and the simulated efficiency of the resonant configurations is also compared in Table 2-2. Note that the tuning capacitances are given to the nearest nanofarad.

Table 2-2: Summary of given parameters and tuning outcomes

<table>
<thead>
<tr>
<th>Nominal 180 kHz tuning</th>
<th>MET</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PP</td>
</tr>
<tr>
<td>$L_1=11.5\mu F$</td>
<td></td>
</tr>
<tr>
<td>$L_2=11.5\mu F$</td>
<td></td>
</tr>
<tr>
<td>$R_{L1}=7 \times 10^{-13} f^2 + 4 \times 10^{-9} f + 0.026$</td>
<td></td>
</tr>
<tr>
<td>$R_{L2}=7 \times 10^{-13} f^2 + 4 \times 10^{-9} f + 0.026$</td>
<td></td>
</tr>
<tr>
<td>$R_{2\text{min}}=12\Omega$</td>
<td></td>
</tr>
<tr>
<td>$R_{2\text{max}}=15\Omega$</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{min}}=0.15$</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{max}}=0.4$</td>
<td></td>
</tr>
<tr>
<td>$V_d=12\text{V}_\text{rms}$</td>
<td></td>
</tr>
<tr>
<td>$R_C=2 \times 10^{-7} C^{-0.73}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-2 shows the maximum, minimum and average efficiency points between the nominal 180 kHz tuned circuits and the MET tuned circuits for all four resonant topologies. Note that the maximum and minimum efficiency points are at different coupling and load conditions for each topology. For this setup the maximum efficiency achieved across the coupling and load conditions by all the resonant configurations were quite similar, ranging from 93.8% to 96.8%, and occurs when coupling is at its maximum. However, the minimum efficiency
obtained across the coupling and load range was significantly higher with the MET tuned circuits.

In this case study, the minimum efficiency achieved with MET tuning is approximately 15% higher than the nominal 180 kHz tuning for the PS and SS topologies. A 2% improvement in the minimum efficiency the PP and SP topologies was seen with the MET tuning over the nominal tuning. Also the average efficiency of the PS and SS configurations are higher than the PP and SP configurations. This implies that this given set of power transfer coils are more suitable for resonant topologies with a series tuned secondary.

In terms of resonant circuit efficiency, it can clearly be seen that the MET tuned PS and SS configurations are superior. However, the decision to adopt the PS or SS topology is unclear as the resonant circuit efficiency is almost identical. Hence, the decision will be made by looking at the other characteristics of the system.

2.6.4 Case study: Discussion

It can be seen in Table 2-2 that the MET procedure tuned the PS topology to a nominal frequency of 428.4 kHz, while the SS topology was tuned to a nominal frequency of 396.6 kHz, such that \( L_1C_1 = L_2C_2 \). However, the efficiency of these resonant circuits is much better than their counterparts which are nominally tuned to 180 kHz.

From Equation (2-9) it can be seen that the voltage induced at the secondary (which is proportional to \( V_{out} \)), is directly proportional to \( \omega \) and \( I_1 \). This means that if the same output voltage is to be maintained, an increase in operating frequency will lead to the decrease of the resonant current. This relationship can be illustrated with the MET tuned and nominal 180 kHz tuned SS topologies. Shown in Figure 2-13 are two plots that show primary resonant current \( I_1 \) required for the MET SS configuration and the nominal 180 kHz tuned SS configuration to maintain \( V_d \) across \( R_2 \), over the operating coupling and load range. While Figure 2-14 shows two plots that show the system resonant frequency \( \omega_r \) of the MET SS configuration and the nominal 180 kHz tuned SS configuration across the operating coupling and load range for the SS topology. From Figure 2-13 it can be seen that the magnitude of the current \( I_1 \) required by the MET tuned configuration is less than half of the current required by the nominal 180 kHz tuned configuration, across the operating coupling and load range. While Figure 2-14 shows that the operating frequency is constant throughout the operating
range, and the frequency for the MET configuration is more than double the frequency of the nominal 180 kHz tuned configuration.

Since power dissipated in the power transfer coils is given by $I^2R$, where $I$ is inversely proportional to $\omega$, and $R$ is proportional to $\omega^2$ (as given in Table 2-2). There exists an operating frequency which would minimise the power dissipation of the power transfer coils. The MET procedure has shown that maximum resonant circuit efficiency is achieved with a nominal frequency of 396.6 kHz for the SS topology in this case study. This suggests that the MET procedure gives a means to justify the choice of the nominal frequency.

![Figure 2-13: Required $I_1$ for MET SS and nominal 180 kHz SS configurations across coupling and load range](image1)

![Figure 2-14: $\omega_r$ for MET SS and nominal 180 kHz SS configurations across coupling and load range](image2)
Both MET PS and SS configurations have almost identical performance in terms of resonant circuit efficiency. Hence, let us compare their other system characteristics. Figure 2-15 shows the operating frequency $\omega_r$, primary resonant current $I_1$, resonant circuit efficiency and the input voltage $V_{IN}$ across the operating coupling and load range for the MET PS and SS configurations.

Firstly, if we compare the operating frequency of the two configurations, it can be seen that $\omega_r$ does not change for the MET SS configuration across the coupling and load range. However, for the MET PS configuration, $\omega_r$ decreases as coupling increases or when the load resistance decreases. The operating frequency varies from 428 kHz to 387 kHz and no frequency bifurcation occurs. Since the MET SS configuration is frequency stable, it is slightly more favourable than the MET PS configuration. However, since the operating frequency of both configurations are quite similar, there is no clear advantage of one configuration over the other.

Secondly, if we compare the primary resonant current $I_1$, we can see that the magnitude of the current is similar for every coupling and load condition. Although, the MET SS configuration does have a slightly larger range between maximum and minimum current, there is not much difference between the two configurations in this aspect.

Lastly, if we compare the required input voltage $V_{IN}$ for the two configurations, one can see that there are some clear differences. The MET PS plot for $V_{IN}$ vs. $k$ vs. $R_2$ indicates that higher input voltage is required at low coupling and when load resistance is low. Higher input voltage is required at low coupling for a parallel tuned primary since the input impedance of the tank increases as coupling decreases. This is because the impedance of an uncoupled ideal parallel tank has infinite impedance (Equation (2-2)). Hence, more voltage is required to maintain the resonant current. The input voltage range for the MET PS configuration is 35V$_{rms}$ to 81.5V$_{rms}$ and decreases non-linearly with coupling. This input voltage range represents a 57% decrease from maximum voltage. The MET SS plot for $V_{IN}$ vs. $k$ vs. $R_2$ indicates that higher input voltage is required at high coupling and when load resistance is low. Higher input voltage is required at high coupling since the input impedance of the tank increases as coupling increases, due to the increasing magnitude of the reflected impedance. Hence, more voltage is required to maintain the resonant current. The input voltage range for the MET SS configuration is 4V$_{rms}$ to 11.6V$_{rms}$ and approximately increases linearly with coupling. This input voltage range represents a 65.5% decrease from maximum voltage.
The absolute input voltages mentioned here, refer to the magnitude of the AC voltage required across the input impedance \( Z_1 \). However, this does not correspond to the absolute DC voltage required for driving the power converter. The required DC voltage can be approximately obtained by using a conversion factor which is dependent on the power.
converter. Hence, the actual DC voltage required may be larger or smaller than the AC voltage. Nonetheless, the required DC voltage is directly proportional to the AC voltage obtained here. Thus, the percentage change between maximum and minimum input voltage will remain the same and is important since it indicates power flow control range required for the system. Therefore, since the MET SS configuration requires are 65.5% change from maximum input voltage in order to regulate $V_d$ across the coupling and load range, as compared to 57% with the MET PS configuration. The PS configuration is more favourable, since a smaller power flow control range is required.

### 2.6.4 Case study: Conclusion

The investigation suggests that either the MET PS of SS configuration should be implemented for the set of power transfer coils and operating conditions provided. Both configurations will give the same resonant circuit efficiency across the coupling and load range. The decision of whether to adopt the MET PS or SS configuration depends on the practical implementation of the primary converter. For instance; the MET SS configuration has the advantage of having a stable operating frequency across the operating coupling and load range. Also series tuned tank operates off simpler voltage-fed converters and do not need high DC voltages to operate. Whereas, the MET PS configuration has the advantage of requiring a narrower control range, since many controllers have a limited control range. Also power flow control is more intuitive for converters driving a parallel tuned circuit, since more input voltage is needed when coupling is low.

### 2.7 Summary

This chapter has illustrated the significance of the design of resonant topology and parameters for a resonant IPT system. A steady-state frequency domain analysis has been presented to analyse the characteristics of the resonant circuit across the operating coupling and load range.

This study has shown that nominal tuning methods do not work well for moderately coupled systems due to the complex nature of the resonant circuit impedance. Since the resonant parameters dictate the operating frequency, efficiency and power transfer capability of the TET system.
A selection procedure called the Maximum Efficiency Tuning procedure was presented to systematically choose the most suitable resonant topology and tuning capacitors for a given set of power transfer coils and operating conditions.

A comparison between nominally tuned configurations and MET tuned configurations showed that the efficiency of the system can improve by up to 15% by using the MET configuration. The methods presented in this chapter provide valuable techniques to analyse moderately coupled resonant systems and helps the design engineer to make knowledgeable decisions in IPT system design.
A TET system for high power implantable devices needs to have good efficiency in order to keep the heating of the system to an acceptable level for use as a medical device. Of greater importance is the physical distribution of the losses in the system. Since the secondary pickup coil is implanted inside the body, the heat dissipation of the pickup coil should not damage adjacent tissue. The primary power transfer coil which will be in close proximity of the skin must also not generate excessive heat that can damage the skin. Studies have shown that if internal tissue temperature is raised by several degrees above core temperature, damage will occur over chronic periods of time [28, 29]. Heating cause by other parts of the TET system, such as the primary power converter is not as critical, since it does not have to be close to the body and can have active cooling if required. Thus, it is very important to know exactly where the losses of the TET system are occurring and the exact level of power dissipation.

3.1 The importance of power loss determination of an implantable TET system

The physical power dissipation of a TET system can be divided into four parts: an external converter/controller circuit, a primary power transfer coil near the proximity of the skin, a secondary pickup coil implanted underneath the skin and a pickup circuit implanted inside the body. Since the TET system is a medical device, it is of high importance to know accurately how much power each component of the TET system is giving off in order to determine whether these losses will lead to tissue damage.
The four parts of the TET system which can be considered separately in terms of heat dissipation are shown in Figure 3-1 in their respective locations when implanted in a patient. Among the four parts, the power dissipation of the primary converter/controller is of least concern. This is because it does not need to be located close to the body and since its size is not strictly defined, cooling devices such as heat sinks and fans can be installed to help dissipate any concentrated heat. However, it is almost impossible to attach cooling devices to the power transfer coils, since the primary power transfer coil must be located near the surface of the skin and the secondary pickup coil is implanted beneath the skin. In addition, materials that are metallic or ferromagnetic must not be placed near the coils, as eddy currents will be generated in these materials resulting in even more losses.

The secondary pickup coil and circuit must have low power dissipation since they are implanted and are in direct contact with internal body tissue. Out of the implanted parts of the system, the pickup circuit may have a metallic enclosure to help spread any concentrated heat; however, the pickup coil may not due eddy current generation. When inside the body, both the secondary pickup coil and the secondary circuit experience temperature regulation via blood perfusion in the surrounding tissue [29], this increases convection and helps to spread concentrated heat energy. Nevertheless, it is of primary importance to know what the losses of the power transfer coils are as they are in direct contact with body tissue; and of
secondary importance to know the power dissipated by the implanted pickup circuit and the primary power converter.

Since a TET system consists of high frequency switching circuits and reactive components driven at high frequencies, measuring the power loss of these components during operation may become highly inaccurate with common electrical measurement techniques [61], making it a somewhat difficult task to accurately determine the power loss of each component. The following sections of this chapter investigates the different ways to determine power loss of TET system components; Section 3.2 will look at electrical methods of determining power and Section 3.3 will look at calorimetric methods, and discuss their advantages and disadvantages.

3.2 Electrical power loss measurement techniques

Electric power cannot be measured directly and can only be obtained indirectly by measuring two other quantities from which electric power can be calculated [62]. The instantaneous power drawn by a component at any time \( t \) is given by

\[
p(t) = v(t)i(t)
\]  

(3-1)

where \( v(t) \) is the instantaneous voltage across the component and \( i(t) \) is the instantaneous current flowing through the component. Instantaneous power \( p(t) \) is usually not very useful, since the instantaneous power may vary all the time. Of more interest is the average power dissipation \( P \). For steady-state DC voltage and current waveforms the average power dissipated is given by

\[
P = VI
\]  

(3-2)

where \( V \) is the DC voltage across the component and \( I \) is the DC current flowing through the component. For periodic voltage/current waveforms the steady-state power can be expressed by finding the RMS voltage/current of the waveform. The RMS of a time varying waveform \( x(t) \) of period \( T \) is given as

\[
x_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 \, dt}
\]  

(3-3)
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Determination of power loss in power transfer components

The steady-state average power dissipated across a component can then be given by

\[ P = V_{\text{rms}} I_{\text{rms}} \cos \phi \]  \hspace{1cm} (3-4)

where \( I_{\text{rms}} \) and \( V_{\text{rms}} \) are the RMS values of \( i(t) \) and \( v(t) \) respectively and \( \phi \) is the phase angle between \( i(t) \) and \( v(t) \). If the real part of the impedance or equivalent series resistance (ESR) \( R_{\text{ESR}} \) of the component is known, then instantaneous power dissipated can be given by

\[ p(t) = i(t)^2 R_{\text{ESR}} \]  \hspace{1cm} (3-5)

and the steady-state average power dissipated is given as

\[ P = I_{\text{RMS}}^2 R_{\text{ESR}} \]  \hspace{1cm} (3-6)

Note that the average power dissipated cannot be given by \( \frac{V_{\text{RMS}}^2}{R_{\text{ESR}}} \), since \( R_{\text{ESR}} \) is the series resistance not the parallel resistance and because \( v(t) \) may not be in phase with \( i(t) \). For components like an inductor whose ESR is frequency dependent, the power dissipated by the inductor must be calculated by with the ESR of the inductor measured at the operating frequency of the current.

In the following sections, the power losses of each of the four main components of a TET are broken down. A description of how the power losses can be practically measured with the electrical methods is given.

3.2.1 Power losses in the primary converter/controller

The losses of the primary converter/controller can be broken down into the following main categories: DC-DC regulator losses, control circuitry losses, MOSFET losses and inductor/transformer losses.

DC-DC regulator losses

DC-DC regulators are used in the primary circuit to provide auxiliary supply voltages for the control circuitry. The steady-state power \( P_{\text{reg}} \) dissipated across a DC-DC regulator can be measured by subtracting its output power from the input power given as

\[ P_{\text{reg}} = V_{\text{in}} I_{\text{in}} - V_{\text{aux}} I_{\text{aux}} \]  \hspace{1cm} (3-7)
where $V_{in}$ and $I_{in}$ are the DC voltage and current at the input of the regulator, $V_{aux}$ is the output DC auxiliary voltage and $I_{aux}$ is the DC current at the output of the regulator. Equation (3-7) can be applied for both linear and switch-mode DC-DC voltage regulators.

**Control circuitry losses**

The power consumed by the control circuitry can be calculated by summing up the output power of all the DC-DC regulators supplying auxiliary supply voltages. Hence, the steady-state power $P_{con}$ consumed by the control circuitry is given as

$$P_{con} = \sum_{n=1}^{n} V_{auxn} I_{auxn}$$  (3-8)

Equation (3-8) is only accurate when the $V_{aux}$ and $I_{aux}$ of each auxiliary supply rail are sustained at a constant level across the entire rail. This can be ensured by using a powerful regulator and the use of de-coupling capacitors across the supply rail.

**MOSFET losses**

The power losses of the MOSFETs in the converter can be determined via many loss mechanisms of a MOSFET [63]. For this analysis only the most significant loss mechanisms will be accounted for. An ideal switch has zero voltage across it when it is ON, zero current flowing through it when it is OFF and the switching between states is completed instantaneously. However, a MOSFET is not an ideal switch since it consists of conduction losses when it is ON, reverse diode conduction loss when OFF and losses due to overlapping voltage and current waveforms when switching. The steady-state power $P_{gate}$ consumed by the gate resistor at an operating frequency with period $T$ is given as

$$P_{gate} = \frac{1}{T} \int_{0}^{T} I_{g}^{2}(t) R_{g} \, dt$$  (3-9)

where $I_{g}(t)$ is the gate drive current waveform and $R_{g}$ is the gate resistor of the MOSFET.

The steady-state conduction loss $P_{con}$ when the MOSFET is ON with an on-resistance $R_{on}$ at an operating frequency with period $T$ is given by

$$P_{con} = \frac{1}{T} \int_{0}^{T} I_{ds}^{2}(t) R_{on} \, dt$$  (3-10)

where $I_{ds}$ is the drain source current flowing through the MOSFET.
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The steady-state reverse diode conduction loss $P_d$ when the MOSFET is OFF is determined by the diode’s forward voltage $V_{fw}$ and the conduction current $I_d$ at an operating frequency with period $T$ is given as

$$P_d = \frac{1}{T} \int_0^T I_d(t)V_{fw} \, dt$$  \hspace{1cm} (3-11)

The switching loss of a MOSFET is primarily due to the drain source voltage of the MOSFET $V_{ds}$ and current flowing through the MOSFET $I_{ds}$ overlapping at the switching instant. At an operating frequency with period $T$, the steady-state switching loss $P_s$ is given by

$$P_s = \frac{1}{T} \int_0^T I_{ds}(t)V_{ds}(t) \, dt$$  \hspace{1cm} (3-12)

which accounts for both turn on and turn off instants in one switching cycle.

Inductor/transformer losses

The losses of small signal transformers and cored inductors consist of conduction losses and core losses. The conduction loss $P_L$ of an inductor/transformer can be given by

$$P_L = I_{Lrms}^2 R_L$$  \hspace{1cm} (3-13)

where $R_L$ is the ESR of the inductor/transformer winding at the operating frequency and $I_{Lrms}$ is the RMS current flowing through the inductor/transformer. The core losses of an inductor/transformer cannot be determined with a generic formula and can only be estimated by equations given on the data sheet of the core, together with the current/voltage across the inductor/transformer and the frequency of operation.

Although the majority of the primary circuit losses have been accounted for there are many other loss mechanisms which are considered insignificant. In high frequency switching circuits, DC auxiliary supplies and small signal voltages may be affected by conducted harmonics or induced harmonics. These parasitic losses in the control circuitry are extremely difficult to take into account. Other losses which were considered negligible for this high level breakdown include losses such as, de-coupling capacitor discharge losses and small signal transistor switching losses.
3.2.2 Power losses in the primary and secondary resonant tank

The primary and secondary power transfer coils resonate with a capacitor to form a resonant tank. The resonant capacitor can be located near the power transfer coil to minimise the resonant current loop; however, it is usually located in the enclosure of the primary converter and in the secondary pickup enclosure so that the power transfer elements are less bulky. Thus, the power dissipated by the resonant capacitors can be considered to be part of the primary and secondary circuits.

Ideal inductors (power transfer coils) and capacitors are purely reactive components and do not dissipate any power, rather they only store energy in the form of magnetic flux and electric charge respectively. However, in practice, inductors have resistive and magnetic losses while capacitors have dielectric losses.

Resonant capacitor losses

The ESR measured in a capacitor is mainly the result of dielectric losses in the capacitor. When a capacitor is charged or discharged, electrons are displaced through the dielectric, as a result of this movement heat is produced [64]. In order to reduce this loss, the capacitor must have suitable ratings for the application. A resonant capacitor for a TET system must have a dielectric this is able to handle high voltages (several hundred volts) and high frequency currents (several hundred kHz). The most suitable type of capacitor that could be purchased at the time of writing was polypropylene film capacitors. The steady-state power loss of a resonant capacitor provided with its ESR $R_C$ at the frequency of operation is given by

$$P_C = I_{Crms}^2 R_C$$  \hspace{1cm} (3-14)

where $I_{Crms}$ is the RMS current flowing through the capacitor.

Resonant coil losses

Since the power transfer coils used in the TET systems presented in this thesis are all air-cored coils, core losses such as hysteresis loss and eddy current losses do not exist. Therefore, the power transfer coils only have resistive losses. It is well known that the resistance of a wire wound inductor increases from its DC resistance when the frequency of the current passing through it increases. This is due to phenomena known as the skin effect and the proximity effect.
The skin effect describes the phenomenon that; high frequency current only flows near the surface of a conductor. This reduces the effective cross-sectional area of the conductor and in turn increasing its effective resistance. The skin depth is a parameter that accounts for the effect of this phenomenon, and corresponds to the depth measured from the conductor surface to where electrons will flow. The smaller the skin depth, the smaller the effective cross-sectional area and the effective resistance increases. The skin depth $\delta$ is inversely proportional to the operating frequency and is given by

$$\delta = \frac{2\rho}{\omega\mu}$$  \hspace{1cm} (3-15)

where $\rho$ is the resistivity of the conductor, $\omega$ the angular frequency of the current and $\mu$ the permeability of the conductor.

The proximity effect describes the loss generated by eddy currents induced in the coil by the current flowing in the windings of the coil itself; this further changes the distribution of current in the wire and further increases the effective resistance. Like the skin effect the proximity effect increases with frequency.

In order to minimise the effect of the skin and proximity effect, Litz wire is commonly used to create the power transfer coils. Litz wire consists of hundreds of thin individually insulated wires which are twisted together to form a thicker wire. The thin wires are designed to be only a few skin depths thick so that the entire cross-sectional area of the wire will be used during conduction, overcoming the skin effect [65]. The thin wires are twisted in a way that minimises the proximity effect caused by adjacent wires [66]. Even with the use of Litz wire, the resulting power transfer coil is still not ideal and still has an ESR $R_{coil}$ which is frequency dependent due to the combined effect of the two phenomena. The power loss $P_{coil}$ of a power transfer coil is given as

$$P_{coil} = I_{coil}^2 R_{coil}$$  \hspace{1cm} (3-16)

where $I_{coil}$ is the RMS current flowing through the power transfer coil.

### 3.2.3 Power losses in the secondary pickup circuit

The purpose of the secondary pickup circuit is to rectify the induced oscillating voltage/current at the pickup coil into DC voltage/current. The most common and effective
rectifier circuit is the full-bridge diode rectifier followed with an inductor-capacitor filter. A DC to DC voltage regulator can also be placed after this stage for further conditioning. However, since the secondary circuit is implanted inside the body the circuit should be kept as simple and compact as possible. Thus, the power loss analysis of a full-bridge diode rectifier is given here.

The power consumed by the full-bridge rectifier comes primarily from the conduction losses of the rectification diodes. At any given time, provided that the rectifier is in continuous conduction, two of the four rectifier diodes can be assumed to be in conduction. Therefore, given that the forward voltage of the diode is $V_f$, the power $P_{rec}$ dissipated by the rectifier is given as

$$P_{rec} = 2I_{R2}V_f$$  \hspace{1cm} (3-17)

where $I_{R2}$ is the average current drawn by the secondary load. The power loss of the smoothing capacitor is considered negligible provided that the voltage is constant across the capacitor (the output voltage) is constant. While the conduction loss of the smoothing inductor (if present) can be given as

$$P_{Lrec} = I_{R2}^2R_{Lrec}$$  \hspace{1cm} (3-18)

where $R_{Lrec}$ represents the DC resistance of the smoothing inductor. Other losses which were considered insignificant in the loss breakdown of the secondary circuit include: the diode turn on and turn off losses and the losses due to the harmonics created by the full-bridge rectifier.

3.2.4 Practical considerations of electrical measurement techniques

Electrical power can only be derived by measuring electrical quantities of voltage, current and resistance [62]. Common instruments for measuring these quantities include multi-meters and oscilloscopes. For higher accuracy, instruments such as high speed digital acquisition systems, LCR meters and spectrum analysers are used. All these instruments can quickly provide measurements of voltage, current or resistance from which power can be derived. Also, the aforementioned instruments are all excellent at measuring constant DC voltages and currents. However, when high frequency waveforms, switching waveforms or high quality reactive components need to be measured, the measurements obtained must be verified and taken with caution.
Chapter 3  Determination of power loss in power transfer components

Firstly, the difficulty lies within the fact that electrical instruments are used to measure the characteristic of another electrical circuit. For the electrical instrument to make an accurate measurement, the maximum rate of change of voltage \((dv/dt)\) or the maximum frequency of the circuit being measured, must be significantly lower than the response or sample frequency of the instrument. Otherwise the changes in voltage would be misrepresented and the waveforms will be under sampled. Since the operating frequency of the TET systems presented here are only a few hundred kHz and resonant voltage peaks are only several hundred volts, the current state-of-the-art instruments are fully rated to measure such waveforms. However, this is only true for clean sinusoidal waveforms. Circuits which contain switching elements will most likely generate high frequency harmonic content. To date, there is still no instrument that can accurately sample the \(dv/dt\) of a hard-switched transient waveform [62].

Another challenge for electrical instruments is measurement delay matching. Instantaneous power, given off by a component is given as the product of the instantaneous voltage and current across the component. With even the slightest time delay in matching the voltage and current waveforms being measured, the resulting power calculation can be drastically misleading [67]. If the computed RMS values of the current and voltage waveforms are used, the need for delay matching is eliminated. However, the phase between the voltage and current waveforms which is needed for this calculation (Equation (3-4)) also depends on the matching up the current and voltage waveforms. In addition, any noise present in the signal can seriously affect the RMS value.

Using Equation (3-6), with the expression of resistance to work out the steady-state power may help alleviate the problem of response time and delay matching. Since only the current waveform is required, however it also has its own set of problems. This is due to the fact that resistance cannot be measured while the component is running in circuit; it must be disconnected from the circuit and measured independently.

DC resistance is measured by applying a small test voltage across the component and by measuring the current flowing through the device the resistance is then derived. One potential problem with this resistance measurement is that the resistance obtained for the component is only accurate under the test conditions, which is low voltage and low current. The resistance of the component may be significantly different while running at higher voltage/current, due to factors such as heating and voltage/current dependent characteristics [68].
AC resistance is measured by applying a small sinusoidal test voltage at the frequency of interest across the component, by analysing the magnitude of the resulting current and the phase shift between the current and the test voltage the impedance of the component may be determined. In addition to suffering the same problem of only characterising the component at low voltage/current levels, the detection of phase shift between the test voltage and current is difficult to acquire accurately, when measuring reactive components with high quality factors [69]. Since high quality inductors and capacitors have very low resistances, the phase shift between the test voltage and test current will be near 90°. When this is the case, it is very difficult for the instrument to determine the real part of its impedance, since this is calculated by measuring the deviation of the phase from 90°. The power transfer coils and resonant capacitors used in the TET system are designed to have the highest quality factor possible to increase efficiency, but at the same time it makes the determination of its power dissipation difficult.

An in-depth analysis of the power losses of a TET system using electrical measurement techniques was previously presented by H. Chen in [70]. Chen’s investigation shows that there can be a difference of up to 30% between the power consumption measured using similar methods given in this chapter, and the DC input power consumption which can be accurately determined from the DC inputs and outputs. Therefore, electrical methods of power loss only provide a rough estimation of the power losses when high frequency switching circuits and high quality factor reactive components are involved.

### 3.3 Calorimetric power loss measurement techniques

Power can be measured more directly with calorimetric techniques, since the quantity being measured is temperature, which is directly related to the heating caused by power dissipation [62]. Any component which is not 100% efficient consumes energy and that energy will eventually be dissipated as heat energy. This heat energy will either be conducted and/or radiated away from the component. It does not matter how the component is electrically characterised or excited, any power loss will be converted in to heat. Since calorimetric methods measure power by means of detecting heat, it can be used to accurately measure both DC and AC power losses.

The main principle behind calorimetric power measurement, suggests that if all the heat energy released is trapped and used to heat up a known substance, then the power dissipation
of the component may be calculated by observing the temperature rise of the substance. A chamber used to perform such measurements is known as a calorimeter.

The main drawback of using a calorimeter to measure power is that the measurement is much more time consuming to perform [71] and also instantaneous power cannot be measured. Also there are no standard calorimeters available for purchase; hence, they must be custom built, verified and calibrated. There are a few types of calorimeters that are suitable for measuring the power loss of electrical components, their principle of operation are described in the following sections.

### 3.3.1 Closed-type no-flow calorimeter

A diagram of a closed-type no-flow calorimeter is shown in Figure 3-2. It consists of a highly thermally insulated chamber such as a vacuum flask, filled with a coolant of certain mass and known thermal properties. The device under test (DUT) to be measured for its power dissipation is then placed in the chamber and left to run for a time $t$. The difference between starting and end temperatures or the rate of change of temperature of the coolant is used to infer the power dissipation of the DUT. The general principle of power loss determination for a closed-type no-flow calorimeter is given by

$$ P = \frac{c_p m \Delta T}{t} $$

(3-19)

where $\Delta T$ is the temperature rise of the coolant over the period of time $t$, $c_p$ is the specific heat of the coolant and $m$ is the mass of the coolant.

![Closed-type calorimeter](image)

**Figure 3-2: Closed-type calorimeter**

A calorimeter based on this principle was developed by G.S. Dimitrakakis [72] to measure the power dissipation of power converters in the range of 2 to 30W and an overall accuracy of ±2% was achieved for this range.
The main assumptions made for a closed-type no-flow calorimeter are:

- The leakage heat into or out of the chamber is negligible over the time of the experiment.
- The temperature rise of the coolant is small such that the rate of leakage heat into or out of the chamber remains constant.
- All the heat dissipated is released from the component and absorbed by the coolant.
- The temperature distribution of the coolant is constant. (this can be improved by stirring the coolant)

### 3.3.2 Closed-type flow calorimeter

A Closed-type flow calorimeter consists of a mechanism to remove the heat generate by the DUT from the calorimeter chamber. One way to achieve this is to pump the coolant across the DUT at a known flow rate and measure the temperature difference of the coolant at the inlet and an outlet of the chamber. A calorimeter based on this principle to measure the power loss of kilowatt transformers presented in [73] regulates the coolant such that its temperature is always the same at the inlet. A diagram of this calorimeter is shown in Figure 3-3. Another calorimeter based on the same principle was designed by A. J. Taberner [74]. This calorimeter was accurate enough to measure the heat production of cardiac trabeculae, which is in the order of microwatts.

Other types of closed-type flow calorimeters include the use of coolant in heat exchangers [75] to remove and measure the heat generated by the DUT. Many have published how this can be achieved with greater accuracy such as using a double jacket type calorimeter [76] to decrease the energy lost to the ambient environment. The design of such a calorimeter is shown in Figure 3-4. This type of calorimeter is usually used to measure the losses of power transformers [73, 77] and large electric machines [78], and is seldom seen for determining power dissipation of several watts, most likely because of the heavy equipment involved with heat exchangers.
The general principle of power loss determination of a closed-type flow calorimeter is given by

\[ P = c_p \dot{m}(T_{out} - T_{in}) \]  

(3-20)

where \( c_p \) is the specific heat of the coolant, \( \dot{m} \) is the mass flow rate of the coolant, \( T_{in} \) is the temperature of the coolant at the inlet and \( T_{out} \) is the temperature of the coolant at the outlet.
The main assumptions made for a closed-type flow calorimeter are:

- The leakage heat into or out of the chamber is negligible over the time of the experiment.
- The temperature rise of the coolant is small such that the rate of leakage heat into or out of the chamber remains constant.
- All the heat dissipated is released from the component and absorbed by the flowing coolant/heat exchanger.

### 3.3.3 Balance calorimeter

A balance calorimeter does not directly measure the heat energy released by the DUT, rather it determines its power loss by imitating the heating profile of the DUT (main test) with a heating element of known output power (balance test) [62]. If the heating profiles of the main test and balance test are matched, the power dissipation of the DUT is assumed to be equal to that of the heating element. The advantage of using a balance calorimeter is that the thermal properties of the coolant and the chamber do not need to be known, as long as the thermal environment is identical during the main test and the balance test. A balance calorimeter can consist of one chamber where the main test and balance test are performed sequentially. Or it can consist of two identical chambers such that the heating of the main test is balanced on the fly. Heating profiles used to balance the power loss includes matching the steady-state temperature such that

\[ T_{1\text{steady}} = T_{2\text{steady}} \]  

(3-21)

where \( T_{1\text{steady}} \) is the average temperature of the main test at steady-state and \( T_{2\text{steady}} \) is the average temperature of the balance test at steady-state. A power balance can also be achieved by matching the rate of the coolant temperature rise in the main test and in balance test over the same period of time such that

\[ \Delta T_1 = \Delta T_2 \]  

(3-22)

Given that the starting coolant temperatures of both chambers are identical.

Balance calorimeters have been developed by many to measure the losses of kilowatt devices [79] and devices dissipating under 50W of power [80]. Many of these balance calorimeters in literature involve the use of large chambers, heat exchanges, pumps and other heavy
hardware, an example schematic of such a calorimeter is shown in Figure 3-5. A dual chamber calorimeter was specifically designed to measure the losses of film capacitors in [81]; however, these were film capacitors used in traction motors and experience several hundred amps of current.

![Figure 3-5: Schematic of 50W Balance calorimeter [80]](image)

The main assumptions made for a balance calorimeter are:

- During both the main test and the balance test, the environment is thermally identical (including any heat leakage) and that the chamber used is identical in size and the coolant is identical in mass.
- The DUT and balance heating element are similar in size and their heat dissipation characteristics are similar.

### 3.3.4 Practical considerations of calorimetric measurement techniques

Creating a calorimeter to measure power dissipation requires a considerable amount of hardware, compared to simply using electrical instruments to take measurements. A chamber also has to be custom built to house the component to be measured, in order to obtain good accuracy and to minimise measurement time.

Since the rate of heat leakage into or out of the calorimeter largely depends on the temperature of the coolant and the ambient room temperature. Since the ambient temperature
usually depends on the time of the day, the accuracy of the calorimeter may vary. One method to overcome this is to use a double jacketed calorimeter [75]. This basically involves placing the calorimeter in another temperature controlled chamber (e.g. water bath) to create a stable ambient temperature for the calorimeter.

Another difficulty is resetting the coolant temperature after each test. Since the temperature of the coolant will increase after taking a measurement. It may take a long time for the coolant to cool back down to the ambient temperature especially if the chamber is well insulated. Installing heat exchangers to reset the temperature of the calorimeter is effective, however it is also costly [82].

**Temperature sensors**

In order for any calorimeter to function, a temperature sensor of some sort is required to detect the amount of heat dissipated. There are three common types of electrical temperature sensors, they are: thermocouples, resistance temperature detectors (RTDs) and thermistors.

A thermocouple is made of two different metal alloy strips joined together at one end (hot junction) and separated at the other (cold junction). When a strip of metal is subjected to a temperature gradient a voltage is formed across the metal, this phenomenon is known as the Seebeck effect. Hence, when there is a temperature difference between the hot and cold junction of a thermocouple a voltage is produced. In order to determine the temperature of the hot junction, the characteristic of the metals must be known and the cold junction must be kept at a reference temperature. Out of the three types of sensors, thermocouples have the widest temperature measuring range and can handle high temperatures of up to 2300°C [83]. Due to the requirement of a stable reference temperature at the cold junction (whose temperature must be maintained by another sensor), the accuracy of thermocouples are not as great as thermistors and RTDs.

A resistance temperature detector (RTD) operates on the principle of measuring the temperature dependent resistance of a well known pure metal. Pure metals such as platinum have very stable and well known temperature coefficients over a large temperature range. Because of this stability, RTDs are considered to have the highest accuracy amongst the three types of electrical temperature sensors. Hence, RTDs are commonly used to calibrate thermocouples and thermistors [31]. The main drawback of a RTD is its response time, which is slower than thermocouples and thermistors.
A Thermistor operates on the same principle as an RTD, but instead of measuring the temperature dependent resistance of pure metals, the temperature dependent resistance of a ceramic or polymer is measured. The temperature response of a thermistor can be very different to RTDs, since they can have either positive temperature coefficients (PTC) or negative temperature coefficients (NTC). Typically thermistors are only accurate across a narrow range of temperatures and must be calibrated over the temperature measurement range of interest. Despite this, thermistors are much more responsive than RTDs and can exhibit a higher precision within their limited measuring range.

A temperature sensor for a calorimeter typically does not need to measure a wide range of temperatures, rather accuracy and sensitivity is more important. Because of this thermocouples are not often used due to their low accuracy. RTDs are a good choice for calorimeters when steady-state temperatures need to be measured, due to their accuracy. Whereas thermistors are a better choice if the rate of change of temperature needs to be recorded, because of their fast response.

Since temperature sensors only measure temperature at a point, several sensors are usually placed at different locations in the calorimeter chamber in order to determine the average temperature of the coolant.

Several publications have reported that calorimetric techniques have provided more accurate power loss measurements than electrical measurement techniques [80, 84]. An investigation of designing a calorimeter, for measuring the power dissipation of power transfer components of a TET system for powering implantable devices is presented in the following sections.

### 3.4 Calorimeter for measuring the power loss of power transfer components

Many calorimeters have been designed to measure the loss of large kilowatt electric machines [85] and motors [86] or power converters that draw several tens of watts [72]. However, what is sought after here is a calorimeter for measuring the power loss of power transfer components of a 5 to 15W IPT system, whose power dissipation is estimated to be around 0.5W. Although there are highly sophisticated calorimeters designed to measure microwatts of power like the one by Taberner et al. to measure the power output of tiny muscles [74], the difference here is that the power dissipation is quite small compared to the physical size of
the power transfer components. This means that a relatively large calorimeter chamber and mass of coolant is required.

3.4.1 Calorimeter specifications

The purpose of this study is to develop a calorimeter that can give us an accurate power measurement of the power transfer components while they are operating. The calorimeter should be simple enough to build in the environment of a power electronic lab bench and at the same time be as cost effective as possible.

The calorimeter should also be able to hold different sized power transfer components. The power transfer coils used in the TET systems of interest are disc shaped and have leads running out the side of the disc. The maximum diameter of the coils is 70mm and the maximum thickness is 10mm. The resonant capacitors used in the TET system are cylindrical polypropylene film capacitors from the 940C series by Cornell Dubilier Electronics, they measure 35mm in length and have a maximum diameter of 20mm.

Ideally the calorimeter should be able to take the measurement within a time period of one hour; otherwise the experiment will be too time consuming to perform. Also the calorimeter should be able to be used again within ten minutes after each measurement in order for it to be effective.

The calorimeter should also not interfere with the operation of the TET system in any way. Since the power transfer coils generate a strong magnetic field around itself during operation. The calorimeter chamber should not be made of anything that is metallic or ferromagnetic; otherwise eddy currents will be induced in these materials creating additional sources of heating. Even the silver coating found on the walls of many thermal isolation chambers used to trap radiated heat, is prone to eddy current heating if the walls of the chamber are too close to the coils. This implies that commercially available vacuum flasks and Dewar vessels may not be appropriate for this calorimeter.

Three calorimeters based on different principles for measuring the power loss of power transfer components of a TET system are presented in the following sections. The operating principle is explained and a simple prototype was built for each calorimeter to test its feasibility.
3.4.2 Closed-type water calorimeter

**Operating principle**
The first calorimeter that will be tested will be a closed-type no-flow calorimeter whose operating principle is described in section 3.3.1. This type of calorimeter should be relatively easy to set up.

**Implementation**
Dewar vessels or other types of vacuum flasks are commonly used for a closed-type calorimeter as it provides the best thermal isolation. However, since these vessels usually contain a silver coating to trap radiated heat, it cannot be used with the power transfer coils due to eddy current heating. Therefore, a plastic container surrounded by insulating foam on all sides was used as the calorimeter chamber.

Six thermistors were used to determine the average temperature of the coolant, four were placed at the edge of the chamber and two were placed near the DUT at the center of the chamber. Each thermistor was calibrated using the Steinhart method, which requires three temperature/resistance pairs for calibrating each thermistor. The reference temperature points used for calibration was 20°C, 25°C and 30°C and a platinum RTD was used to obtain these reference temperatures. In the prototype, water at room temperature was used as the coolant. Water has a relatively high specific heat of 4.21kJ/kg°C, the higher the specific heat the more energy it takes to heat up a certain mass of the substance. Hence, in order to see a significant change in water temperature, the mass of the water should be kept to a minimum. A diagram illustrating the prototype setup is shown in Figure 3-6.

![Diagram of closed-type water calorimeter setup](image-url)
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*Initial testing*

The aim of the first test with the calorimeter was to investigate whether it could obtain a good estimate of the power loss of a DC resistor, whose power dissipation can be accurately measured electrically.

Several of these tests were performed at different starting coolant temperatures while the ambient temperature was uncontrolled. The tests revealed that while the resistor was dissipating a constant level of power, a significant temperature gradient existed in the water even at steady-state, such that the temperature detected at the edge were much lower than the temperature at the center of the chamber. Despite the temperature difference, the temperature rise measured by of the six thermistors over five minutes was averaged and Equation (3-19) was used to calculate the power loss. It was found that the estimated power loss varied greatly with starting temperature and the estimate was always significantly lower than the actual power consumption.

*Verdict*

A closed-type calorimeter must be highly thermal insulated in order to produce accurate measurements. The temperature measurements obtained in the initial tests showed a significant temperature gradient in the coolant at steady-state; this suggests that the coolant should be stirred in order to maintain an even temperature distribution in the coolant. The temperature gradient also suggested that there were significant amounts of heat leaking out of the chamber, most probably due to the very simplistic setup. The rate of leakage heat is dependent on the difference in temperature between the coolant and the ambient temperature, which makes factoring in the leakage heat extremely difficult since the ambient room temperature is not constant.

Calibrating the calorimeter to work for a certain starting and ambient temperature means that there has to be a means of adjusting the coolant and ambient temperature before starting a test, else the measurement will be inaccurate. Also, due to the low power dissipation and the comparably large mass of the coolant, the rate at which the DUT is dissipating heat to the coolant may even be lower than the rate at which the coolant loses heat to the ambient environment. Due to these uncertainties this method was quickly discarded, since it is not suitable to be used as a simple setup to measure the power loss of power transfer components.
Since it is difficult and costly to create a highly thermally insulated chamber for a power transfer coil, balance calorimetry is preferred since heat leakage does not affect accuracy given that the leakage is the same during the main test and the balance test [62].

3.4.3 Vacuum chamber balance calorimeter

**Operating principle**

The second concept for a calorimeter to measure the power loss of power transfer components was to build a coolant free vacuum calorimeter.

The principle of operation for the vacuum calorimeter is that, since there is no medium present for conduction or convection, all the heat that would have conducted out would remain on the DUT. This should result in a sudden temperature increase on the DUT and would be measured by the thermistors attached to the DUT. In order to determine the power dissipation, balance type calorimetry would be used. For instance, if the power dissipation of the power transfer coils were to be measured; the main test would consist of the driving the coil with the power converter and the balance test would consist of driving same coil with a DC current, over a certain period of time. Several balance tests would be performed until the temperature increase of the coil in the balance test matched the main test. Thus, the power dissipation of the coil in the main test is determined.

The key advantage of the vacuum chamber calorimeter over the closed-type water calorimeter is that the measurement time is greatly shortened as the heat energy would be concentrated on the coil, as the only means of conduction would be through the connecting wires of the coil. This also means that the measurement would not be very sensitive to different ambient temperatures as the mode of conduction is removed. It was also assumed that the radiated heat is insignificant considering our low power level and absolute temperature.

**Implementation**

An acrylic tube with an inside diameter of 12 cm was used to build the vacuum chamber and an illustration of the chamber is shown in Figure 3-7. The bottom acrylic plate of the chamber was attached via acrylic welding to provide an air tight seal. The top acrylic plate is removable; hence it was fitted with an O-ring which would keep the chamber air tight when the air is sucked out of the chamber due to suction. A pressure gauge was attached to the top
plate and rubber bungs with holes drilled in them were used as wire entry points for the chamber. The rubber bungs would also provide air tight seals due to suction when air is extracted from the chamber. The chamber implemented for this prototype was capable of holding a -85kPa vacuum for around 30 minutes, which should be sufficient for the measurement. Thermistors calibrated with the same method and range as the thermistors in section 3.4.2 was also used in this calorimeter.

![Diagram](image)

**Figure 3-7: Side view of the vacuum balance calorimeter setup**

**Initial testing**

The first test for the calorimeter was to compare the temperatures detected by the thermistors, between the cases when the DUT is dissipating power in air and when it is dissipating power in a vacuum.

Our initial hypothesis was that when in a vacuum, no temperature increase or very little temperature increase would be detected by the thermistors that are not touching the DUT; and a sharp temperature increase for the thermistors attached to the DUT since heat is trapped within the DUT. Conversely when in air, the temperature of the thermistors that are not touching the DUT should increase due to heat conduction and convection. A smaller temperature increase is expected for the thermistors attached to the coil, since heat is being conducted and convected away.
An experiment was set up to test the hypothesis mentioned above: the DUT in the chamber was a power transfer coil with a DC current passing through it dissipating 0.418W of power. Four thermistors were then placed in the chamber at different distances from the coil and a simple data logging script was written in MATLAB to log the temperature of the four sensors and the results are shown in Figure 3-8. Figure 3-8a) shows the temperature measured by the thermistors when the coil was dissipating power in a vacuum and Figure 3-8b) shows the temperature measured when the coil was dissipating power in air.

![Figure 3-8: Vacuum balance calorimeter initial testing; a) in vacuum, b) in air.](image)

The initial testing showed that the thermistors not touching the coil rose to a higher temperature when the coil was dissipating power in a vacuum than when in air, which was contrary to what was expected. After repeated experiments with similar outcomes, we realised that the amount of heat radiated was not insignificant after all. When there is no medium for heat conduction power dissipated greatly increases the temperature of the coil. this in turn increases the amount of radiated heat, since the amount of heat radiated from a body is proportional to the fourth power of the temperature of the body [87]. In addition, the larger temperature increase detected in the vacuum test was due to the fact that the radiated heat energy which would previously been absorbed by the air, is now only heating up the thermistors. It can be seen in Figure 3-8a) when in a vacuum, the temperature increase detected by the thermistors were inversely proportional to the distance it was from the coil, which is expected for an omni-directional radiating point source. However, when in air as shown in Figure 3-8b), this temperature difference was not as profound and was not inversely proportional to distance, since the convection of the air evens out the temperature distribution.
**Verdict**

Since our original predictions were not correct, this method was also discarded. Since the vacuum chamber did not provide the environmental conditions that we hoped to achieve. In addition, it is both difficult and time consuming to match temperature rise profiles between the main test and the balance test. Also it is difficult to set up a controller that automatically performs the measurement, since the tests are not done simultaneously. Furthermore, the starting DUT temperature of each test needs to be the same, since the temperature increase rate is dependent on initial temperature of the DUT. Lastly, the same procedure cannot be replicated for measuring the loss of a resonant capacitor, since a DC current cannot be used on a capacitor to create power loss.

**3.4.4 Peltier device balance calorimeter**

**Operating principle**

The operating principle of this balance calorimeter is based on the double chamber calorimeter where power balance is achieved on the fly. Power dissipation is considered balanced between the DUT and test heating element, when both chambers are at the same temperature and the rate of further temperature increase is the same. This implies that in order to achieve a power balance, the absolute temperature of the calorimeter does not need to be known. Since the controller only needs to know whether there is a temperature difference between the main and balance chamber. This further suggests that it is not necessary to use temperature sensors to achieve the power balance. Hence, the idea of using a thermoelectric device as the temperature balance sensor was conceived.

A thermoelectric device is a device that directly converts temperature to electric voltage and vice-versa. These devices are commonly used as heat pumps and active cooling devices, or can be used in reverse to generate electricity. A common thermoelectric device is a Peltier device; it is made up of an array of dissimilar metals sandwiched in between two ceramic surfaces. When a voltage is applied to the Peltier device heat energy from one surface will be transferred to the other. The phenomenon of heat transferral across the junction of dissimilar metals in the presence of a voltage across the junction is known as the Peltier effect. A similar effect also happens in reverse; when there is a temperature difference between the junctions of dissimilar metals, a voltage will be along the metals, and this phenomenon is known as the Seebeck effect.
A Peltier device when used as a sensor is similar to the operation of a thermocouple. In a thermocouple, the hot junction of the thermocouple is located at the temperature measurement point and the cold junction is kept at a reference temperature. Hence, the voltage generated by the Seebeck effect can infer the temperature at the hot junction. On a Peltier device, the cold and hot junctions are located on the two surfaces of the device. When the temperatures of the two surfaces are equal, no voltage will appear across the Peltier device terminals. However, when one side is hotter than the other, a voltage whose magnitude is proportional to the temperature difference will be generated. The voltage generated will either be positive or negative giving an indication as to which side is hotter. Since a Peltier device exhibits a self generating Seebeck effect, it requires no power supply to operate as a sensor.

Equal temperatures on the two surfaces of the Peltier device will always produce zero voltage at the Peltier terminals, since a voltage potential cannot be created from no energy. Thus, the temperature balance point of a Peltier device when used as a sensor does not need to be calibrated. In addition to its inherent ability to detect a temperature balance, it also serves as a better sensor for a balance calorimeter, since the temperature is sensed across the surface of the device. Whereas the electrical temperature sensors mentioned in section 3.3.4 only measure the temperature at one point in space. Hence, several sensors are required in the calorimeter to give the average temperature of the coolant.

**Implementation**

A Peltier balance calorimeter was built using two moulded acrylic boxes with inner dimensions 50mm by 70mm by 70mm, serving as the two chambers of the balance calorimeter. One side of each box was milled such that a standard 40x40mm Peltier device could be sandwiched between the boxes ensuring a stable and tight fit. A hole was then drilled in each box to allow the leads of the DUT and the balance heating element to be inserted into the chambers.

Water was chosen to be the coolant for the initial prototype; hence, the leads connected to the DUT and balance heating element had to be insulated with water tight heat shrink. Two thermistors were placed inside each chamber on the surface of the Peltier device for verification that the calorimeter is functioning as expected. A cross-sectional diagram of the prototype setup is shown in Figure 3-9.
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Figure 3-9: Cross-sectional view of the Peltier device balance calorimeter setup

**Initial testing**

The aim of the first test whether the Peltier device calorimeter would detect a balance (zero voltage across terminals) when both chambers contained the same DC load dissipating the same power. Another aim of this test was to observe the stability of the Peltier device voltage under steady-state balanced conditions.

![Diagram of Peltier device balance calorimeter setup](image)

*Figure 3-10: Peltier device balance calorimeter initial testing*

During the test equal amounts of power were supplied to the DC load in each chamber; at the start of the test and the load power was set to 2.3W, then at the 10 minute mark the power dissipation of both loads were decreased to 1.77W and finally the power was increased to 4.38W at the 15 minute mark. Figure 3-10a) shows the Peltier voltage recorded during this test, while Figure 3-10b) shows the temperature recorded from the four thermistors whose
locations are shown in Figure 3-9. Throughout the test the temperature of both chambers remained the same as indicated by temperature sensed by the thermistors in Figure 3-10b). In Figure 3-10a) it can be seen that the Peltier device voltage deviated to -0.0005V after 30 minutes of run time. This voltage deviation roughly corresponds to a temperature difference of 0.16°C between the Peltier device surfaces, since it was tested that the Peltier device had a sensitivity of roughly 3mV/°C.

This initial test verifies that the Peltier device voltage remains at zero when the temperature of both surfaces is the same, regardless of the absolute temperature. Also the accuracy of the Peltier device balance point shown in this test is approximately ±0.2°C. This actually implies that it has higher balance accuracy than the thermistors used, whose accuracy is only ±0.5°C after calibration.

**Verdict**

The Peltier device balance calorimeter removes the need for using temperature sensors in traditional balance calorimeters and uses a Peltier device as the temperature balance sensor. Initial testing of this concept confirmed our initial hypothesis for this calorimeter and also showed promising results, since the temperature balance accuracy was better than that of the thermistors used in the experiment.

Using a Peltier device as the balance sensor simplifies the calorimeter design. Since one Peltier device sensor can replace the multiple temperature sensors required, in order to get an average temperature, for traditional balance calorimeters. Moreover, calibration is not required for the temperature balance point of the Peltier device, given that the device is symmetrical and that the calorimeter chambers are identical. The temperature readings of un-calibrated temperature sensors can be significantly different, even between the same type and make of sensor. Because of this, temperature sensors must be calibrated for the temperature measurement range of interest.

**3.4.5 Summary**

In section 3.4 several conceptual calorimeters were built for the purpose of measuring the power dissipated in power transfer coils.

Firstly, the closed-type water calorimeter estimates the power loss of the DUT by observing the temperature rise of the water. However, due to varying ambient conditions and starting
temperatures the calorimeter was difficult to calibrate and experiments were difficult to repeat.

Next, the vacuum balance calorimeter attempted to use a vacuum to create a controlled environment by removing the conduction and convection modes of heat transfer. However, during initial testing it was identified that there is a significant amount of heat that is dissipated as radiated heat. Moreover, trying to match a temperature increase profile between the main test and balance test using temperature sensors was very difficult.

The next concept was to use a double chamber balance calorimeter, which utilised a Peltier device as the temperature balance sensor. Initial testing of this calorimeter showed that the temperature balance accuracy of the Peltier device was better than that of the thermistors. In addition, a Peltier device when used as a sensor does not need to be calibrated. Being a passive device it must have an inherent temperature balance point of 0V. Also since the Peltier device senses the temperature of the coolant over a surface, it provides more accurate detection of the average coolant temperature.

Out of the three calorimeters mentioned above, the Peltier device balance calorimeter was the most promising for measuring the power loss of the power transfer components. Hence, an improved balance calorimeter based on this principle was designed to measure to the power dissipation of power transfer components used in TET systems for powering implantable heart pumps.

3.5 Implementation of an improved Peltier device balance calorimeter

This section presents a carefully designed Peltier device balance calorimeter whose proof of concept was presented in section 3.4.4. The calorimeter presented adheres to the specifications given in section 3.4.1. The details of the design are first presented in section 3.5.1, then followed by its performance verification in 3.5.2 and finally the power loss of the power transfer components is determined in 3.5.3.
3.5.1 Calorimeter design

Shown in Figure 3-11 is a cross-sectional diagram of the improved Peltier device balance calorimeter. The design of each part of the calorimeter is presented below.

![Diagram of the improved Peltier device balance calorimeter](image)

**Figure 3-11: Cross-sectional view of the improved Peltier device balance calorimeter**

**Calorimeter Chamber**

Each chamber of this double chamber calorimeter is identical and is designed to house power transfer coils with a maximum diameter of 70mm and a maximum thickness of 10mm. Thus, the rectangular chambers were designed with inner dimensions of 40mm wide by 80mm long by 90mm high. The chambers were built using 3mm thick clear acrylic bonded together with di-chloromethane and sealed with varnish.

A 2mm deep 40mm by 40mm groove was milled out on each chamber such that a 40x40mm Peltier device fits tightly between the chambers and minimises the distance between the Peltier device surface and the coolant. Thermal paste was also applied in this groove to ensure good thermal contact between the chamber and the Peltier device.

The two chambers were then wrapped with a layer of 10mm high density polyurethane foam to provide some thermal isolation to the ambient environment. However, this chamber is not designed to be perfectly insulated, since this is not required for a balance calorimeter. As long as the two chambers are exposed to identical conditions, the calorimeter should balance correctly.
**Peltier device**

Since the Peltier device is not used as a heat pump or a generator, its power rating and efficiency is not very important when used as a sensor. This is because equal temperatures on both surfaces of any Peltier device must give zero voltage, since the Peltier device does not require power to operate as a sensor. Of greater importance when selecting the Peltier device is the surface area of the device, because the larger the surface area the greater the temperature averaging effect. Thus, a low cost 40mm by 40mm by 3.8mm Peltier device by CUI Inc. (CP85438) was used for the calorimeter. The relationship between the temperature difference of the two surfaces and the voltage across the Peltier device was measured to be approximately 3mV/°C.

**Stirrer**

A stirrer was added to each chamber of the calorimeter to ensure an even temperature distribution within the coolant. This is to ensure that the temperature detected by the Peltier device is the average coolant temperature. An ideal stirrer would allow for the heating component to be located at any position within the chamber, since the coolant is always thoroughly mixed. A plastic propeller mounted onto a long plastic shaft was attached to a low voltage DC motor to act as a stirrer for each chamber. The speed of the stirrer was set to approximately 2000 revolutions per minute such that the coolant was well mixed, but not turbulent.

**Balance heating element**

An 18.2Ω balance load resistor rated at 25W was used as the balance heating element. The physical dimension of the resistor is 28mm long, 15mm wide and 15mm high. It consists of a finned metallic casing which increases the rate of heat dissipation into the coolant. This in turn reduces the time taken for the Peltier device to sense a change of power dissipation in the balance heating element.

The balance load resistor was driven by a 100W power Op-amp (MP108FD) which has a maximum output of 7.5V on a ±15V supply. 7.5V corresponds to a maximum output power of 3.1W with the 18.2Ω load. This should be sufficient for balancing the loss of the power transfer components which have a maximum estimated loss of around 0.5W. A linear power Op-amp was used instead of a more efficient switched mode DC-DC converter, to ensure that there is no ripple on the DC voltage and current powering the heating element. Since the
power dissipation of the balance heating element has to be determined with electrical instruments.

**Temperature sensors**

Three thermistors were placed in each chamber of the calorimeter to capture the absolute temperature of the chambers. This was added to obtain the temperature change profile and to double check that a balance has been achieved. These thermistors were calibrated with the Steinhart method using a water bath at the temperatures 20°C, 25°C and 30°C. A platinum RTD was used to determine the temperature of the water bath.

**Controller**

A controller is needed to balance the heat coming from the DUT to the heat coming from the balancing heating element. The only source of feedback for the controller is the voltage across the Peltier device. Since it takes a few seconds for the heat energy from the DUT/balance heating element to be sensed by the Peltier device, a relatively slow controller is required. Since it is difficult to implement a slow analogue controller with large time constants, a digital PID controller was designed on LabView software to control the calorimeter.

A NI USB-6009 multifunction I/O card was used to obtain the Peltier device voltage for the input of the LabView PID controller and also to send control signals to the power Op-amp. Since the output voltage of the Peltier device is only a few milli-volts, the Peltier device voltage was first pre-amped by a precision Op-amp (AD8561) configured as a non-inverting amplifier with a gain of 1000; providing a sensitivity of approximately 3V/°C for the LabView PID controller. The sampling interval for the digital PID controller was set to 5 seconds per sample and the constants for proportional, integral and derivative control were set to 0.5, 0.01 and 1 respectively.

These constants were empirically determined such that it was possible to achieve a stable power balance within 45 minutes. A graphical data display window was created in LabView to monitor the calorimeter during each measurement; it graphs the real time voltage across the Peltier device, the PID values, the output voltage and the temperature measured by the thermistors in the calorimeter. Shown in Figure 3-12 is the graphical data display window showing the data at the end of a typical measurement. Note that a balance is considered to be achieved when the Peltier device voltage stays at zero and the output remains at a constant
value. The thermistors can be used to double check that a balance has been achieved by checking that the temperature detected by all the thermistors are rising at the same rate, that is checking that all the lines in the temperature plot are parallel to each other.

**Figure 3-12: LabView graphical display for Peltier device balance calorimeter**

Shown in Figure 3-13 is a photograph of the improved Peltier device balance calorimeter implemented in practice. The different parts of the calorimeter are also labelled in Figure 3-13.

**Figure 3-13: Photograph of the improved Peltier device balance calorimeter**
3.5.2 Performance verification

Before the proposed calorimeter can be used to determine the unknown power loss of the power transfer components, the power balance accuracy of the calorimeter has to be verified. A series of verification tests were carried out and are presented in this section. Each experiment was repeated several times to obtain a good average and to ensure repeatability. In all the tests the power dissipation of the DUT and the balance heating element was accurately known since they were driven by DC sources. The power dissipated by the DUT and the balance heating element was determined by measuring its voltage and current with a multi-meter.

**DC-DC balance test**

The balance accuracy of the improved Peltier device balance calorimeter was determined by testing it against a test load resistor driven by a DC voltage. The calorimeter was put under four different scenarios, in each scenario a 50Ω resistor was driven at a different DC voltage such that its power dissipation corresponded to 0.2W, 0.288W, 0.5W and 1W. These scenarios cover the power dissipation range of interest, since the maximum power loss each power transfer component is estimated to be around 0.5W. Each scenario was repeated six times over the period of one week to ensure that the calorimeter is consistent regardless of ambient temperature changes and to ensure repeatability.

![Figure 3-14: DC-DC balance test: absolute error vs. test load dissipation](image)

Shown in Figure 3-14 is a plot showing the absolute error between the power dissipation of the balance heating element and the test load at different levels of power dissipation. The DC-
DC balance testing indicates that the calorimeter starts losing accuracy when the power dissipation of the test load is smaller or equal to 0.2W. This is most likely due to the fact that there is significant amount of heat exchange between the calorimeter and the ambient environment, since the calorimeter is not fully insulated. If the rate of heat exchange with the ambient environment is similar to that of the power dissipated by the test load, it will become difficult for the Peltier device to detect any temperature differences. When this happens the power balance accuracy will be lowered. Figure 3-14 indicates that the improved Peltier device balance calorimeter has an absolute accuracy of ±0.036W and an average error of ±0.0125W (6 samples) when load power dissipation is between 0.2W and 1W. This is increased to an absolute accuracy of ±0.022W and an average error of ±0.0076W (6 samples) when load power dissipation is between 0.288W to 1W.

**Size and position DC-DC balance test**

Next the balance accuracy of the calorimeter was tested by changing the position of the test load in the chamber and by varying the physical size of the test load. Three different test loads were used in these tests: a 20 turn single layer spiral coil measuring 50mm in diameter made with 1mm diameter Litz wire (thin DC coil), a 6.8Ω load resistor measuring 28mm by 15mm by 15mm (large 6.8Ω) and the same 50Ω load resistor used in the previous DC-DC balance test measuring 15mm by 10mm by 10mm (small 50Ω).

A total of five test scenarios were created with the three test loads they are: thin DC coil at center, small 50Ω at center, small 50Ω at corner, large 6.8Ω at center and large 6.8Ω at corner. The regions designated as “center” or “corner” within the test load chamber is illustrated in Figure 3-15. For each of these scenarios the test load was set to dissipate 0.5W of power and each test scenario was repeated four times.

Shown in Figure 3-16 is a plot showing the absolute error between the power dissipation of the balance heating element and the test load for each test scenario. Here an absolute accuracy of ±0.042W and an average accuracy of ±0.0169W (4 samples) were observed across the different load positions and sizes. Since the accuracy of calorimeter from this set of experiments has only gone down slightly when compared to the previous DC-DC balance tests. This proves that the stirrer implemented does a good job in mixing the coolant and maintains a constant temperature distribution in the coolant. This experiment also shows that the DUT and the balance heating element do not have to be similar in shape or mass, in order to receive an accurate power balance with this calorimeter.
Chapter 3  
Determination of power loss in power transfer components

Figure 3-15: Diagram for Size and position DC-DC balance test (Top view)

Figure 3-16: Size and position DC-DC balance test: absolute error vs. test scenario (0.5W)
Starting temperature test

The DC-DC balance tests and the Size and position DC-DC balance tests mentioned above were performed at different starting coolant temperatures. Shown in Figure 3-17 is a plot of the absolute balance error versus the starting coolant temperature of each of the forty-four balance tests performed. Note that at the start of each test the coolant temperature of each chamber were approximately ±0.1°C of each other.

Figure 3-17 clearly indicates that there is no correlation between starting coolant temperatures and the power balance error for starting temperatures between 22°C and 27°C. For starting temperatures higher than 27°C no judgement can be made due to a lack of data. However, it is unlikely that the starting coolant temperature will be higher than 27°C and less than 22°C in an air conditioned lab environment, except during several minutes straight after a measurement. Hence, the implemented calorimeter is capable of maintaining an accuracy of ±0.042W between 22°C and 27°C. This result also verifies the fact that a balance calorimeter will give an accurate power measurement regardless of the leakage heat of the calorimeter given that the same thermal environment is experienced by the main test and the balance test.

Figure 3-17: Starting temperature test: absolute error vs. starting coolant temperature
3.5.3 Determination of power loss in power transfer components

The purpose of improved Peltier device balance calorimeter is to accurately determine the power dissipation of power transfer components in a TET system while it is in operation. But due to the physical constraints of the calorimeter, it is not possible to align the power transfer coils if one coil is in the calorimeter. Nonetheless, if the uncoupled power transfer components are driven with the same operating frequency and current as when they are coupled and transferring power, the power dissipation of these components will be the same. Hence, in this study a controllable soft-switched converter will be used to drive both the primary and secondary power transfer components at the determined operating conditions.

The resonant setup for this study consists of: a 10.9µH primary coil (52mm in diameter and 4mm thick), a 3.3µH secondary coil (52mm in diameter and 2mm thick), a 33nF primary capacitor and a 100nF secondary capacitor. The output load on the secondary is a 10Ω resistor and the output voltage is regulated at 12Vrms. For this set of power transfer coils the maximum concentric separation is 16mm and the minimum concentric separation is 5mm, this corresponds to a coupling range of $k=0.18$ to 0.45 respectively. Since the primary inductance is much larger than the secondary, a suitable resonant topology for these coils would be Parallel-parallel (Please refer to Chapter 2 for details). Inputting this setup into the frequency domain simulation presented in Chapter 2 predicts that the primary resonant current will have a maximum value of $3.7A_{\text{rms}}$ at $k=0.18$ and a minimum of $1.51A_{\text{rms}}$ at $k=0.45$, while the secondary resonant current will stay between 2.37 and 2.32$A_{\text{rms}}$ within the coupling range of $k=0.18$ to 0.45. The operating frequency of the system stays between 263 and 273 kHz for the parallel-parallel topology across the coupling range.

With this information the power dissipation of the power transfer components were measured in the improved Peltier device balance calorimeter at values in between maximum and minimum current. The power dissipation of the primary coil and capacitor were measured at a frequency of 265 kHz with a resonant current of $3.7A_{\text{rms}}$ and $2.4A_{\text{rms}}$. The power dissipation of the secondary coil and capacitor was measured at a frequency of 267 kHz with a resonant current of $2.5A_{\text{rms}}$. The power measurement for each component was repeated three times so that an average can be obtained.

The power measured for each component by the calorimeter is presented in Table 3-1. Note that the operating frequency was measured with an oscilloscope and the current was approximated by measuring the RMS voltage across the power transfer coil and dividing it by...
the reactance of the coil. This method provides sufficient accuracy since the quality factor of the coils are very high (coil Q>200), which means that the resistance of the coil is negligible compared to its reactance. A current probe was used to verify the accuracy of this method, but the current probe could not be used to measure the resonant current going through the DUT during operation, since it acts like a load to the resonant tank. Keep in mind that the accuracy of a single measurement for this calorimeter is ±0.042W, given that the power dissipation is above 0.2W.

<table>
<thead>
<tr>
<th>Device Under Test</th>
<th>Measurement 1</th>
<th>Measurement 2</th>
<th>Measurement 3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.9µH primary coil @ 265 kHz, 2.4A&lt;sub&gt;rms&lt;/sub&gt;</td>
<td>0.456W</td>
<td>0.412W</td>
<td>0.413W</td>
<td>0.427W</td>
</tr>
<tr>
<td>10.9µH primary coil @ 265 kHz, 3.7A&lt;sub&gt;rms&lt;/sub&gt;</td>
<td>0.972W</td>
<td>0.923W</td>
<td>0.936W</td>
<td>0.944W</td>
</tr>
<tr>
<td>33nF primary capacitor @ 265 kHz, 2.4A&lt;sub&gt;rms&lt;/sub&gt;</td>
<td>0.101W</td>
<td>0.156W</td>
<td>0.12W</td>
<td>0.126W</td>
</tr>
<tr>
<td>33nF primary capacitor @ 265 kHz, 3.7A&lt;sub&gt;rms&lt;/sub&gt;</td>
<td>0.295W</td>
<td>0.315W</td>
<td>0.346W</td>
<td>0.319W</td>
</tr>
<tr>
<td>3.3µH secondary coil @ 267 kHz, 2.5A&lt;sub&gt;rms&lt;/sub&gt;</td>
<td>0.180W</td>
<td>0.186W</td>
<td>0.174W</td>
<td>0.180W</td>
</tr>
<tr>
<td>100nF primary capacitor @ 267 kHz, 2.5A&lt;sub&gt;rms&lt;/sub&gt;</td>
<td>0.123W</td>
<td>0.152W</td>
<td>0.138W</td>
<td>0.137W</td>
</tr>
</tbody>
</table>

From these power measurement values it can be seen that the power dissipation of the primary power transfer coil and resonant capacitor may change by a factor of two, depending on the coupling conditions. However, since the output power is always regulated to the same power level, the secondary pickup coil and resonant capacitor losses are relatively constant across the different coupling conditions. When coupling is at its minimum (k=0.18) the power loss of the primary coil is at its maximum of 0.944W, taking the size of the coil into account this corresponds to a power loss density of 1.4mW/mm³. Similarly the 0.18W dissipated by the secondary coil corresponds to a power loss density of 0.55mW/mm³.

### 3.5.4 Comparison between electrical and calorimetric measurements

The steady-state power dissipation of each power transfer component measured by the calorimeter was compared against the power loss measured by electrical instruments using the method given by Equation (3-6) which is \( P = I_{\text{RMS}}^2 R_{\text{ESR}} \). The ESR of the power transfer
coils and resonant capacitors were measured by three different LCR meters at the frequency of 265 kHz. Shown in Table 3-2 is a list of the ESRs for each component obtain from a Hewlett Packard 4285A precision LCR meter (75kHz to 30MHz), a Fluke PM6306 LCR meter (DC to 1MHz) and an Agilent E4980A precision LCR meter (20Hz to 2MHz). It can be observed that the HP 4285A and the Agilent E4980A produced readings which were in good agreement with each other, whereas the Fluke PM6306 meter gave readings that were significantly different. The Fluke PM6306 ESR measurements of the power transfer coils were approximately 40% below and the resonant capacitor ESR measurements were three times as large as the measurements from the other two meters. The resulting steady-state power loss obtained by using Equation (3-6) and the different ESR measurements is compared against the calorimeter power loss measurements in Figure 3-18.

Firstly, it can be seen in Figure 3-18 that the power transfer coil power dissipation measured by the calorimeter and the electrical measurements determined by the HP 4285A and Agilent E4980A LCR meter were quite similar. The electrical measurement method only slightly over-estimated the power dissipation of the power transfer coils by roughly 5%. From this it can be seen that the power transfer coil ESR measurements made with the Fluke PM6306 are very inaccurate, since it under-estimates the power loss by over 40%.

Secondly, from Figure 3-18 it can be seen that the capacitor power dissipation determined with HP 4285A or Agilent E4980A LCR meter under-estimates the loss by about 30% when compared to the calorimeter measurement; whereas the Fluke PM6306 meter over-estimates the power loss by more than 100%. Again the Fluke PM6306 appears to be very inaccurate.

<table>
<thead>
<tr>
<th>Device under test</th>
<th>HP 4285A</th>
<th>Agilent E4980A</th>
<th>Fluke PM6306</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.9µH primary coil</td>
<td>79mΩ</td>
<td>75mΩ</td>
<td>44.7mΩ</td>
</tr>
<tr>
<td>33nF primary capacitor</td>
<td>15mΩ</td>
<td>14mΩ</td>
<td>54mΩ</td>
</tr>
<tr>
<td>3.3µH secondary coil</td>
<td>32mΩ</td>
<td>29mΩ</td>
<td>17.6mΩ</td>
</tr>
<tr>
<td>100nF primary capacitor</td>
<td>7.3mΩ</td>
<td>8mΩ</td>
<td>22.3mΩ</td>
</tr>
</tbody>
</table>
If we ignore the ESR measurements made by the Fluke PM6306 LCR meter, assuming its inaccuracy is due to its narrower frequency measurement range of DC to 1MHz. The ESR measurement of the HP 4285A and Agilent E4980A LCR meters were more accurate for the power transfer coils than it was for the resonant capacitors. This makes sense considering that the ESR measurements are taken under low voltage and current conditions.

The losses of an inductor consists of core loss and conduction losses. It is well known that core losses increase as voltage and operating frequency increases [89], and that conduction losses increase as operating frequency increases due to the skin and proximity effect as discussed in section 3.2.2. Since no ferrite core is used in the power transfer coils and the operating frequency is within the rating of the Litz wire; the ESR of the coils under high voltage and current excitation would be similar to the ESR measured at small signal levels. Hence, the electrical power loss measurement is quite similar to the loss measured by the calorimeter.

However, for the resonant capacitor, the dielectric stress caused by the large resonant voltage would be much higher than the stress caused at small signal levels. A capacitor under large
voltage excitation will lead to an increase in ESR [60]. Thus, the capacitor ESR during operation would be higher than the ESR value measured with an LCR meter and making the electrical measurement under-estimate the actual power loss.

This study shows that there are large variations associated with electrical power measurement of reactive components at high frequencies. Although the power loss of some high frequency components can be accurately determined by electrical methods (such as air-cored coils); it must first be verified with calorimetric techniques, before an electrical instrument can be assumed accurate for the job.

3.6 Summary

The accurate determination of power losses in the different parts of a TET system for powering implantable heart pumps is a necessary step, in order for it to be approved for medical use. Excessive heat from components implanted in the body or touching the surface of the body may lead to tissue damage, which can make the TET system do more harm than good.

Power loss is commonly determined using electrical instruments, since it is quick and convenient to perform. However, the accuracy of such measurements is questionable, when the electronic instruments are measuring high frequency waveforms, while other system characteristics such as resistance cannot be measured while the system is running. In addition, it has also been found that there may be large measurement variations between precision electronic measuring instruments.

However, power loss measurements via calorimetric methods can accurately determine the loss of electronic components while they are in operation. A novel balance calorimeter which uses a Peltier device as a balance sensor was proposed and implemented. This calorimeter is simple and cost efficient, making it a useful tool for determining high frequency power losses on a power electronics lab bench. The presented calorimeter was able to measure power dissipation from 0.2W onwards with an absolute accuracy of ±0.042W. The power loss measured with the calorimeter was compared to the loss measured with different electrical instruments and the results are discussed.
Chapter 4

Variable frequency power flow control for TET systems

The analysis in Chapter 2 presented a design methodology for selecting resonant topology and parameters of a TET system. In order to complete the system it is necessary to have a power flow control method that can regulate the output to a desired output voltage while tolerating the coupling and load changes during operation. Power flow control of an IPT system can be achieved through many ways. Power flow can be controlled on either the primary or secondary side of the system [41], and is achieved by changing the parameters of the resonant system [90-92] or by controlling the voltage and currents in the system [93-95]. Since only power flow control methods which are suitable for IPT systems for powering implantable heart pumps are analysed; only primary side power flow control methods which do not compromise soft-switching of the primary resonant tank will be considered in this thesis.

The focus of this chapter is to investigate methods by which power flow to the load can be varied by changing the resonant frequency of the system. The principle of frequency power flow control will first be introduced, followed by practical methods on how this can be achieved. The effect of frequency power flow control will be analysed for all four basic resonant topologies in this chapter.

4.1 Principle of frequency power flow control

Frequency power flow control is a method of changing the output voltage or power by changing the operating frequency of an IPT system. The operating frequency of a resonant
system is a vital parameter, since each resonant tank has a unique resonant frequency at which most energy can be stored in the tank. Any deviation from this resonant frequency will result in a dramatic change in energy and consequently changing the power delivered to the load [90].

### 4.1.1 Effect of frequency change on a secondary pickup

Regulating the voltage/power at the load of the secondary pickup is the primary purpose of frequency power flow control. Firstly, the effect of changing the source frequency used to drive the secondary resonant circuit is analysed. The effect of the primary converter can be expressed as a variable voltage source in series with the pickup inductance as explained in section 2.2, the equivalent circuits of the series and parallel tuned pickups are shown in Figure 4-1.

![Figure 4-1: Equivalent circuits of series tuned (Left) and parallel tuned (Right) resonant pickups](image)

The power transferred to a secondary resonant pickup is dependent on the voltage $V_S$ induced at the pickup coil. $V_S$ is determined by the strength and frequency of the magnetic field passing through the pickup coil and is given by

$$V_S = j\omega M I_1$$  \hspace{1cm} (4-1)

where $I_1$ is the current passing through the primary coil at angular frequency $\omega$ and $M$ is the mutual inductance between the primary and secondary coils. The voltage $V_{out}$ generated across a resistive load $R_2$ due to $V_S$ for a series tuned secondary is given by

$$V_{out} = V_S \frac{R_2}{\frac{1}{j\omega C_2} + j\omega L_2 + R_2}$$  \hspace{1cm} (4-2)

and the voltage generated across a resistive load $R_2$ due to $V_S$ for a parallel tuned secondary is given by
From the above expressions it can be seen that $V_{out}$ will be affected if the operating frequency changes. However, the size of this effect is also dependent on the circuit parameters. An effective method to characterise any RLC circuit is to determine its circuit quality factor $Q$. The $Q$ of an RLC circuit is given by the ratio of the energy stored in the circuit over the power dissipated in one cycle at a particular operating frequency and the circuit quality factor of a secondary pick will be denoted as $Q_S$. The $Q_S$ of a series tuned secondary circuit is given by

$$Q_S = \frac{\omega L_2}{R_2}$$  \hspace{1cm} (4-4)$$

and the $Q_S$ of a parallel tuned secondary is given by

$$Q_S = \omega C_2 R_2$$  \hspace{1cm} (4-5)$$

To see the effect that a change in the operating frequency of the primary converter has on $V_{out}$, let us assume that the magnitude of $V_S$ stays constant while its angular frequency $\omega$ is varied. Figure 4-2 and Figure 4-3 show the output voltage $V_{out}$ versus the normalised operating angular frequency ($\omega/\omega_0$) at different values of $Q_S$ for a series tuned and parallel tuned pickup respectively. $Q_{S0}$ is the quality factor of the secondary circuit at the undamped natural angular frequency $\omega_0 = \frac{1}{\sqrt{L_2 C_2}}$.

From Figure 4-2 and Figure 4-3 it can be seen that the effect of frequency change, affects the output voltage the most when $Q_S$ is high. It can be seen that the maximum output voltage for a series tuned pickup is achieved at $\omega_0$ and is also true for parallel tuned pickups when $Q_S>2$. When $Q_S<2$ the maximum voltage frequency starts decrease from $\omega_0$, this becomes really apparent when $Q_S<1$ and no boosting of $V_S$ occurs (maximum $V_{out}=V_S$, when $\omega=0$). However, at such a low Q the circuit is highly damped and is no longer self-resonant [96], making the parallel tuned pickup almost no different to an RL pickup circuit.
Figure 4-2: Output voltage of series tuned pickup versus normalised $\omega$

Figure 4-3: Output voltage of parallel tuned versus normalised $\omega$

It is also important to note that for a series tuned pickup the change in output voltage is symmetrical around $\omega_0$ (on a log scale), but is not symmetrical for a parallel tuned pickup. This means that as the operating frequency deviates from $\omega_0$ the effect on the output voltage
is different depending on whether frequency is increased or decreased. It is usually preferred that the frequency is decreased below $\omega_0$, since in practice losses tend to increase with increasing frequency.

The change in output voltage shown in Figure 4-2 and Figure 4-3 only apply when the secondary tank is driven by a variable frequency sinusoidal source of constant magnitude. However, for a soft-switched resonant converter the operating frequency cannot be changed by just changing the switching frequency. The operating frequency can only be changed by modifying the reactance of the resonant tank such that it’s natural resonant frequency changes.

### 4.2 Primary side frequency power flow control methods

For a soft-switched ZVS or ZCS resonant converter the only means to vary the operating frequency is to vary the reactance of the primary resonant tank. This means that either the inductance or the capacitance of the primary resonant tank has to be varied. Practical methods of varying the inductance and capacitance of a resonant tank are discussed in section 4.2.1 and section 4.2.2 respectively.

#### 4.2.1 Frequency power flow control by means of variable inductance

The inductance $L_1$ and $L_2$ shown in the resonant circuits presented here, come from the inductance of the power transfer coil and pickup coil respectively. There are primarily two ways to vary the inductance of coupled resonant circuits. The first method is to change the inductance of the power transfer coil, such that the additional inductance $L_V$ is also coupled to the secondary inductance as depicted by Figure 4-4a). The second method is to introduce an inductance $L_V$ to the primary tank such that it is not coupled to the secondary inductance as depicted by Figure 4-4b).

To introduce an inductance into a resonating tank a discrete inductance $L_V$ can be switched in and out of the tank. Due to the current source property of inductors, $L_V$ cannot be switched in with a series switch shunted across $L_1$. This will involve open circuiting $L_V$ which can lead to current arcing if $L_V$ is switched out with energy still stored in it. Rather $L_V$ can be switched in with a switch shunt across $L_V$ and placed in series with $L_1$. Closing the switch would allow the current in $L_V$ to be freewheel and removing it from the tank, while opening the switch would add the inductance $L_V$ to the tank. However, such a method is uncommon since an AC switch
will need to be used because of the oscillating resonant current, and also it is difficult to match the currents through \( L_v \) and \( L_1 \) during the switching instants else a current conflict would occur.

![Figure 4-4: Variable inductance methods; a) coupled \( L_1 \), b) uncoupled \( L_v \)](image)

To achieve variable inductance, it is more common to have the inductor \( L_v \) built into the circuit and its inductance is changed by varying the properties of its core. Chapter 1 previously mentions that it is undesirable for the power transfer coil of a TET system for implantable heart pumps to have a ferrite core, due to increased weight and heating from the ferrite. Thus, this eliminates the option of having \( L_v \) coupled to the secondary. However, if \( L_v \) is not coupled to the secondary then it is not a problem for it to have a ferrite core since it can be located on the primary converter.

A coil wound around a ferrite core increases its inductance by a factor that is proportional to the relative permeability \( \mu_r \) of the ferrite core. The inductance is determined by the magnetic flux density in the inductor for a given amount of current flowing through it. If there is a secondary bias winding on the same ferrite core, the inductance of the primary winding may be changed by applying a DC current to the bias winding. The DC bias current sets up a constant flux density in the ferrite core and changes the amount of amp-turns required to saturate the inductor. When the inductor core saturates the inductance of the inductor effectively decreases as it can no longer store additional energy. Hence, a secondary bias winding can be used to vary the inductance of the inductor. Hsu et al. [97] and Medini et al. [98] have implemented such a variable inductance (also known as a magnetic amplifier) to tune resonant circuits. Impressive variations in inductance (decreased by a factor of 5) can be achieved with this method as shown by Medini in [98]. Since the bias winding of the inductor is controlled by a continuous DC current the amount of inductance added can be controlled very smoothly without introducing switching harmonics to the resonant tank.
Since the variable inductor $L_V$ is in series with the power transfer coil $L_1$ it also experiences the full high frequency resonant current. This means that the core material must be suitable for high frequency operation and the wire used should also be Litz wire in order to decrease losses due to Skin and Proximity effects. Also the inductance of $L_V$ should be similar or larger than that of the primary power transfer coil, in order to obtain a good frequency control range. Taking all these factors into account, the result could be a rather bulky component on the primary converter. Other disadvantages include the continuous DC current that needs to be passed through the secondary winding which requires drive circuitry and consumes continuous power.

Furthermore, the variable inductor should not affect the secondary or any other object. This means that only closed ferrite cores should be used to eliminate any leakage flux.

### 4.2.2 Frequency power flow control by means of variable capacitance

The capacitance of the resonant circuit is mainly made up of discrete high frequency AC capacitors used to tune the resonant circuit and partly of parasitic capacitances. There exist several types of variable capacitors but are mostly unsuitable for power applications. There are mechanical parallel plate capacitors which physically change the gap of the capacitor plates, these are extremely bulky and are not electronically controllable. Another device known as the Varactor, is a diode like component whose capacitance changes with the DC bias voltage across it. Varactors are commonly used to tune high frequency radio circuits of several MHz or even GHz, so they only exhibit capacitances of several picofarads [99], which is not useful in this application.

The capacitance of a capacitor is determined by the separation, surface area and the permittivity of the dielectric in between two conductive surfaces. Discrete capacitors are available in different dielectrics and sizes to produce different capacitances for different applications. At the time of writing there are no electronic methods known to control the capacitance of a power capacitor, such that its permittivity can be varied.

A discrete capacitor $C_V$ can be used to introduce capacitance to a resonant tank by switching the capacitor in and out of the tank. Since capacitors have a voltage source property, it is possible to disconnect a charged capacitor from the circuit safely. This can be achieved by using a series switch and shunting $C_V$ across the primary resonant capacitor $C_1$, as shown in Figure 4-5. In order to vary the effective capacitance added to the circuit, $C_V$ must be
switched in and out of the tank with a switching scheme that controls the amount of time the capacitor is in circuit. Keep in mind that capacitances add when connected together in parallel.

![Variable capacitance method: switched capacitance \( C_V \)](image)

Since the \( C_V \) is switched in across \( C_1 \), one has to ensure that this only happens when the voltages across both capacitors are equal. Otherwise a voltage conflict would occur and destructive currents may be generated.

Disadvantages of having a switched capacitor in a resonant circuit include the addition of a power switch that also has to be controlled with timings related to the operating frequency of the converter. Since the additional capacitor \( C_V \) will also be part of the high frequency resonant tank when switched in, \( C_V \) has to have a similar rating to that of the main resonant capacitor \( C_1 \). Both the power switch and \( C_V \) would increase the physical size of the primary side circuit. Furthermore, an AC switch may be required to switch \( C_V \) in and out of the oscillating resonant tank.

### 4.2.3 Comparison of variable reactance methods

Discussed in this section are practical considerations for changing the resonant frequency of a system by changing the reactance of a resonating tank. For the application of an IPT system for powering implantable heart pumps, the size of the device and good efficiency (to minimise heating) are key considerations of the design. In terms of size both variable inductance and capacitance control require a significant and similar amount of physical space; a ferrite cored inductor with current control circuitry or an additional AC capacitor with its power switch, will required roughly the same PCB area to implement.

In terms of efficiency, while ignoring additional control losses; an extra ferrite cored inductor of similar inductance to \( L_I \) will generate a similar amount of loss as the power transfer coil,
since it experiences the same high frequency resonant current. Although an extra capacitor $C_V$ in parallel with the primary capacitor $C_1$, also has high frequency resonant current passing through it. The resonant current will be split between the two capacitors which lowers the total series resistance and the power loss due to ESR. Although there is an extra power switch in the path of the resonant current, given that it is soft-switched and the power dissipation of the switch may not be very significant. Hence, in terms of efficiency the variable capacitor method is theoretically superior.

In terms of the control over the variable reactance, variable inductance via the magnetic amplifier method would provide a much finer and accurate control over the added inductance since it can be controlled with a continuous bias current. Control of the variable capacitance via the switched capacitor method can also provide fine control through switch schemes; however, since the capacitor $C_V$ is either in or out of the circuit, the resonant frequency will actually be switching between two resonant frequencies. This will create non-sinusoidal waveforms and introduce harmonics into the circuit. Hence in terms of reactance control the variable inductance method is theoretically superior.

Since each reactance control method has its own pros and cons. Based on the fact that variable capacitance control is potentially more efficient than variable inductance control, and because of the fact that variable inductance actually changes the resonant topology. A switched capacitance method was chosen for analysis and implementation of this study on frequency power flow control.

### 4.3 Effect of primary capacitance change on coupled resonant circuits

The operating resonant frequency $\omega_r$ of a resonant IPT system may be changed by modifying the tuning capacitance in the system as discussed in the previous section. Since only primary side control is discussed in this thesis, this refers to varying the capacitance of the primary resonant tank. The effect of a variable primary capacitance on the secondary circuit and ultimately the power delivered to the load is dependent on the circuit topology of the IPT system. Analysis for the four basic coupled resonant topologies (PP, PS, SP and SS) will be presented here and analysis will be performed under loosely coupled and moderately coupled conditions.
4.3.1 Effect of primary capacitance change under loosely coupled conditions

Under loosely coupled conditions \((k<0.1)\), it can be assumed that the operating frequency of an IPT system is dominated by the primary resonant tank, since the effect of the secondary pickup can be considered to be negligible [57]. Assuming that the primary circuit has a high quality factor under loosely coupled conditions, the operating frequency of the system can be approximated as:

\[
f_0 = \frac{1}{2\pi\sqrt{L_1C_1}}
\]

From this it can be seen that any change to \(C_1\) will change the resonant frequency. However, when \(C_1\) is varied, the impedance of the primary tank will also change which would in turn affect the magnitude of the primary resonant current. In other words changing \(C_1\) will affect both the operating frequency and the resonant current.

Using the coupled resonant circuit models in Chapter 2 the effect of changing the primary capacitor \(C_1\) under loosely coupled conditions \((k<0.1)\) is analysed. In order to present a fair comparison; the parallel and series tuned secondary pickups that are compared in each case will have the same undamped resonant frequency \(\omega_{S0}\), secondary circuit quality factor \(Q_{S0}\) \((Q_S\text{ at }\omega_{S0})\) and load \(R_2\). In order to maintain these properties, the following relationships must be maintained:

\[
\begin{align*}
L_{2p} &= R_2^2C_{2S} \\
L_{2S} &= R_2^2C_{2P} \\
\frac{1}{L_{2p}C_{2P}} &= \frac{1}{L_{2S}C_{2S}} = \omega_{S0}
\end{align*}
\]

where \(L_{2p}\) and \(C_{2p}\) are the inductance and capacitance of the parallel tuned secondary, and where \(L_{2S}\) and \(C_{2S}\) are the inductance and capacitance of the series tuned secondary. If the above relations are satisfied then the \(Q_{S0}\) of the two parallel and series secondary tanks will be given by

\[
Q_{S0} = \frac{L_{2S}}{L_{2p}} = \frac{\omega_{S0}L_{2S}}{R_2} = \omega_{S0}C_{2P}R_2
\]

Normalised simulation results show that the effect of varying \(C_1\) always has the same effect on the output voltage and operating frequency given that the circuits have the same secondary quality factor \(Q_{S0}\). Regardless of what \(L_1\) and \(C_1\) are as long as \(L_1C_1 = L_2C_2\). The value of \(L_1\) and \(C_1\) only affect the absolute value of the output voltage and determines the required input voltage.
Given in Table 4-1 is a summary of the two coupled resonant circuits which gives a $Q_{S0}$ of 2 and satisfies the relationships given in Equation (4-6). Followed by the effect on the output voltage and operating frequency due to varying $C_j$ in Figure 4-6. Note that $C_{total}$ is the equivalent resonant capacitance, which is equal to $C_j + C_y$.

The same analysis was performed for two coupled resonant circuits with a $Q_{S0}$ of 1, the summary of the parameters are given in Table 4-2 and the effect on the output voltage and operating frequency due to varying $C_j$ is shown in Figure 4-7. Note that for both simulations, an input voltage source with an RMS of 1V operating at the ZPA frequency of the resonant system was used.

Table 4-1: Resonant setup for analysing the effect of primary capacitance change ($Q_{S0}=2$)

<table>
<thead>
<tr>
<th>$Q_{S0}$=2</th>
<th>$f_0$=265 kHz</th>
<th>$R_2$=10</th>
<th>$L_jC_j=1/4\pi^2f_0^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series secondary</td>
<td>$L_2=12\mu H$, $C_2=30nF$</td>
<td>Parallel secondary</td>
<td>$L_2=3\mu H$, $C_2=120nF$</td>
</tr>
<tr>
<td>Series secondary</td>
<td>$L_2=12\mu H$, $C_2=13.3nF$</td>
<td>Parallel secondary</td>
<td>$L_2=3\mu H$, $C_2=53.3nF$</td>
</tr>
</tbody>
</table>

Figure 4-6: Effect of primary capacitance change $Q_{S0}=2$, $k=0.1$
Table 4-2: Resonant parameters for analysing the effect of primary capacitance change (Q_{S0}=1)

<table>
<thead>
<tr>
<th>Q_{S0}=1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_0=212 kHz</td>
</tr>
<tr>
<td><img src="image1.png" alt="Image" /></td>
</tr>
<tr>
<td>R_2=16</td>
</tr>
<tr>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>L_2C_1= 1/4πf_0^2</td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>Series secondary</td>
</tr>
<tr>
<td>L_2= 12μH, C_2=47nF</td>
</tr>
<tr>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>Parallel secondary</td>
</tr>
<tr>
<td>L_2= 12μH, C_2=47nF</td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
</tr>
<tr>
<td>f_0=352 kHz</td>
</tr>
<tr>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>R_2=6.6</td>
</tr>
<tr>
<td><img src="image7.png" alt="Image" /></td>
</tr>
<tr>
<td>L_2C_1= 1/4πf_0^2</td>
</tr>
<tr>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td>Series secondary</td>
</tr>
<tr>
<td>L_2= 3μH, C_2=68nF</td>
</tr>
<tr>
<td><img src="image9.png" alt="Image" /></td>
</tr>
<tr>
<td>Parallel secondary</td>
</tr>
<tr>
<td>L_2= 3μH, C_2=68nF</td>
</tr>
<tr>
<td><img src="image10.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure 4-7: Effect of primary capacitance change Q_{S0}=1, k=0.1

Figure 4-6 and Figure 4-7 show that there is a more profound change in the output voltage V_{out} for the same change in C_1 if Q_{S0} is larger. The output voltage of the PP and PS configurations at Q_{S0}=1 and Q_{S0}=2 changes in a way that is very similar to the output voltage of individual series and parallel tuned pickups excited at different frequencies previously shown in Figure 4-2 and Figure 4-3. However, how the output voltage of the SP and SS configurations is this study are very different to Figure 4-2 and Figure 4-3.

The reason for the above observation is primarily due to the fact that all the circuit models presented in Chapter 2 and subsequently used here are driven by a voltage source, although parallel tuned resonant tanks should be current-fed. However, this is not a mistake in the
simulation rather it portrays a limitation in the practical implementation of resonant converters. A parallel primary resonant tank needs to be driven by a current source, however in practice there is no such thing as a true current source, as all current sources are quasi-current sources derived from a voltage source. The simplest way to make a voltage source act like a current source during steady-state operation is to add a series inductance to the voltage source.

Theoretically, if a parallel resonant tank is driven by a current source the resonant voltage across the tank is proportional to the Q of the tank. However, with quasi-current sources the magnitude of the resonant voltage is restricted by the input voltage $V_{in}$. On the contrary the resonant current of a voltage-fed series resonant tank is also proportional to Q, however the current magnitude is not restricted due to the fact that voltage sources do exist (although voltage sources do have current limits, this limit can be increased with a higher capacity supply). Since all converters are ultimately voltage driven (like the models given in Chapter 2), the primary resonant current for a parallel tuned primary and series tuned primary are given as

$$I_{1\text{parallel}} = \frac{V_1}{j\omega L_1 + R_{L1} + Z_R}$$

$$I_{1\text{series}} = \frac{V_1}{j\omega L_1 + \frac{1}{j\omega C_1} + R_{C1} + Z_R}$$

where $V_1$ is the AC input voltage source operating at an angular frequency of $\omega$, whose magnitude is dependent on the DC input voltage $V_{in}$ of the converter. Given that secondary is loosely coupled ($k<0.1$), it can be assumed that the primary resonant tank has a high Q such that $R_{L1}$ and $R_{C1}$ are negligible and the that $\omega = \frac{1}{\sqrt{L_1 C_1}}$, then the primary resonant current can be approximated by

$$I_{1\text{parallel}} \approx \frac{V_1}{j\omega L_1 + Z_R}$$

$$I_{1\text{series}} \approx \frac{V_1}{j\omega L_1 + \frac{1}{j\omega C_1} + Z_R}$$

When the system is loosely coupled, it can be assumed that the reflected impedance $Z_R$ is much smaller than the impedance of the primary coil $j\omega L_1$. This means that $I_{1\text{parallel}} \approx \frac{V_1}{j\omega L_1}$. Since the induced voltage at the secondary is given by $V_S = j\omega M I_1$ (Equation (4-1)),

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substituting in $I_{\text{parallel}}$ gives $V_S = jM V_I / L_1$. Therefore, a change in $C_I$ does not change $V_S$ but only changes the operating frequency, which is exactly the same condition set by the analysis in 4.1.1. Hence, the output voltage vs. $C_I$ in this study matches the curves in Figure 4-2 and Figure 4-3.

In the expression for $I_{\text{series}}$, the magnitude of the terms $j\omega L_1 + \frac{1}{j\omega C_1}$ exactly cancels out when $\omega = \frac{1}{\sqrt{L_1 C_1}}$. But since there is still a small amount of reflected impedance under loosely coupled conditions, the terms $j\omega L_1 + \frac{1}{j\omega C_1}$ do not exactly cancel out, resulting in a term whose magnitude is very similar to $Z_R$. This means that $I_{\text{series}}$ is very sensitive to small frequency deviations from the undamped natural resonant frequency of the series tuned tank. Hence, the analysis of SP and SS configurations cannot be simplified to the analysis performed in section 4.1.1.

4.3.2 Effect of primary capacitance change under moderately coupled conditions

If the primary and secondary are moderately coupled ($k > 0.1$) then the frequency of the resonant system is dictated by both the primary and secondary LC values. As discussed previously in Chapter 2, calculating the system’s resonant frequency analytically is not trivial; however, it can easily be obtained through numerical methods. Moderately coupled resonant circuits are essentially $4^{\text{th}}$ order circuits meaning that the operating frequency is no longer dominated by the primary resonant tank. It also means that the reflected impedance from the secondary pickup cannot be ignored and affects the Q of the primary resonant tank. In addition, frequency bifurcation phenomena may also be encountered, introducing multiple possible resonant frequencies which can have drastic effects on the power delivered to the load.

For analysing the effect of a change in the primary capacitance, the same resonant circuit parameters used in section 4.3.1 are used here. These parameters give series tuned and parallel tuned secondary circuits that have the same $R_2$, the same resonant frequency $\omega_{S0}$ and the same $Q_{S0}$. In addition, the individual undamped resonant frequencies of the primary and secondary are also made the same such that $L_1 C_1 = L_2 C_2$. Again, regardless of what $L_1$ and $C_I$ are, the normalised change in operating frequency and output voltage is always identical.
Using the parameters in Table 4-1 for a TET system with $Q_{S0}=2$, Figure 4-8 shows the effect of $C_1$ at a coupling of $k=0.3$ and Figure 4-9 shows the effect of $C_1$ at a coupling of $k=0.5$.

Using parameters in Table 4-2 for a system with $Q_{S0}=1$. Figure 4-10 shows the effect of $C_1$ at a coupling of $k=0.3$ and Figure 4-11 shows the effect of $C_1$ at a coupling of $k=0.5$.

At moderate levels of coupling a frequency bifurcation point may be encountered when the primary capacitance is being varied. This phenomenon is seen in Figure 4-9 for all four topologies system when $Q_{S0}=2$ and $k=0.5$. The determination of precise frequency bifurcation points is an involved process and is described by Covic et al. in [56]. In general, the chance of encountering frequency bifurcation increases as coupling $k$ increases, since this increases...
the significance of the reflected impedance. Frequency bifurcation is problematic for frequency control since the relationship between output voltage and operating frequency can change completely as shown in Figure 4-9. Generally speaking it is impossible to adopt frequency power flow control if frequency bifurcation exists within the operation range of interest. Although it is possible to use smart controllers to overcome the problem, it is easier to change the setup of the resonant circuit in order to avoid bifurcation. Decreasing $Q_{S0}$ reduces the chance of bifurcation at high levels of coupling, but at the same time the output voltage control range is decreased.

This study shows that in order to adopt primary side frequency power flow control for a moderately coupled IPT system the steady-state operating characteristics must be analysed.
across the entire coupling and load range. This is to ensure that both the control range is sufficient and that frequency bifurcation is not encountered. The methods and models presented in Chapter 2 is an appropriate tool for such an analysis.

4.3.3 Summary

If we observe the output voltage versus change in $C_L$ of the PP and PS systems across the coupling range of $k=0.1$ to 0.5 (Figure 4-6 to Figure 4-11) it can be seen that for the same $Q_{so}$, the effect of a varying primary capacitance is very similar given that the system does not bifurcate. The plots are also similar to Figure 4-2 and Figure 4-3. This shows that the assumption of $\omega L_1 \gg Z_R$ made in Equation (4-9) is still valid up to moderate levels of coupling for $Q_{so}=1$ or 2. This suggests that the output voltage can be approximately determined using a simple secondary pickup equivalent circuit.

Given that there is no frequency bifurcation this makes a primary side frequency controlled PP or PS system very stable and predictable, since it’s tolerance to variations/mismatches in system parameters is much higher than a primary side frequency controlled SP or SS system. Therefore, primary side frequency power flow control is much suited for TET systems with a parallel primary tank.

In the next section, primary side frequency power flow control is implemented for a parallel tuned primary converter and also for a series tuned primary converter. Practical data is obtained. In both converters a switched capacitor is used for controlling the resonant frequency of the primary tank.

4.4 Implementation of primary capacitance control on a parallel resonant primary tank

A current-fed half-bridge converter (or more commonly known as a push-pull converter) was chosen for the implementation of frequency power flow control on a parallel tuned resonant tank.

4.4.1 The parallel push-pull converter

Shown in Figure 4-12a) is the schematic diagram a general parallel-tuned push-pull converter. The converter consists of two current splitting inductors and two MOSFETs that
drive a parallel tuned LC resonant circuit. The two current splitting inductors $L_{S1}$ and $L_{S2}$ acts as a current-source which is necessary to drive the parallel tank, these two splitting inductors should be identical such that the current is evenly split. The two states of this converter is shown in Figure 4-12b), in each state one of the two ends of the parallel resonant tank is grounded. The state of the switches $S_1$ and $S_2$ are always mutually exclusive. Since the converter is driving a parallel resonant tank the MOSFETs can be soft-switched at the zero voltage crossings of the resonating voltage across the tank. Detailed analysis regarding the operation of a push-pull converter can be found in [54, 91].

![Diagram](image)

**Figure 4-12: a) Push-pull converter; b) Equivalent circuit states of a push-pull converter**

The main reason why the parallel resonant push-pull converter was chosen for the implementation of switched capacitor control is because that all voltages in the converter remain positive with respect to ground during operation. This unique characteristic allows a capacitor $C_V$ to be switched in and out of the resonant tank without the use of an AC switch. In both states of the converter, one side of the parallel resonant tank is shorted to the ground terminal. This means that instead of placing the capacitor $C_V$ with a series switch across $C_I$, it is possible to place $C_V$ with its switch across $S_1$ and $S_2$ to achieve the same effect. However, this also means that two sets of $C_V$ and its switch are needed, such that one is used for when
$V_{C1}$ is positive and the other when $V_{C1}$ is negative. This placement of $C_V$ allows the capacitor to be switched in and out of the resonant tank with a low side N-MOSFET, which simplifies the required gate drive.

4.4.2 The parallel push-pull converter with switched capacitor control

The schematic of the parallel tuned push-pull circuit with switched capacitors $C_V$ is shown in Figure 4-13a) and equivalent circuits states when the switched capacitor $C_V$ is in circuit are shown in Figure 4-13b).

![Diagram](image)

**Figure 4-13:** a) Switched capacitor push-pull converter; b) Equivalent circuit states of a switched capacitor push-pull converter

When $S_1$ is ON and $S_2$ is OFF, $S_{V2}$ can be turned on to effectively add $C_{V2}$ in parallel to $C_I$ and similarly when $S_2$ is ON and $S_1$ is OFF, $S_{V1}$ can be turned on to add $C_{V1}$ in parallel to $C_I$. This has the effect of increasing the resonant capacitance and decreases the resonant frequency.

To control the amount of effective capacitance added to the primary resonant tank, the switches $S_{V1}$ and $S_{V2}$ need to be modulated so that it is on for different periods of time within the period of half the resonant frequency. A method for achieving this while making sure that there will be no conflict between the voltage across $C_V$ and $C_I$ is presented by Si in [91]. This method has been adopted for this study and is briefly described below.
The switching instances of the switches $S_{V1}$ and $S_{V2}$ are determined by comparing the voltages $V_{SI}$ and $V_{S2}$ to a control voltage $V_{CON}$. $S_{V1}$ is kept ON whenever $V_{SI} < V_{CON}$, and this shunts $C_V$ across $C_I$. $S_{V1}$ is also kept on when $S_I$ is ON ($V_{SI} = 0$) to ensure that the capacitor is fully discharged at the beginning of the positive half-cycle of $V_{SI}$. Once $V_{SI}$ is larger than $V_{CON}$ then $S_{V1}$ is turned OFF, during this time $C_{VI}$ holds its voltage since is no discharge path. Since $C_{VI}$ is charged, the only time it is safe for $C_{VI}$ to be switched back into the circuit is at the instant when $V_{CV1} = V_{SI}$, this happens at the instant when $V_{SI}$ drops back to $V_{CON}$ after reaching its peak, at this point $S_{V1}$ is turned back ON. After this the cycle repeats itself. The same switching scheme applies for the switch $S_{V2}$, with its voltage $V_{S2}$ compared to the same $V_{CON}$. 

**Figure 4-14: Waveforms and timings of a switched capacitor push-pull converter**
The waveforms and the timings of a switched capacitor parallel tuned push-pull converter are shown in Figure 4-14. Note that the resonant tank voltage is non-sinusoidal when $C_{V1}/C_{V2}$ is only partially switched in during the half period of the resonant waveform.

### 4.4.3 Resonant voltage waveform of switched capacitor push-pull converter

Figure 4-14 shows that the resonant voltage of a switched capacitor push-pull converter when $C_{V}$ is partially switched in can be approximated by piecewise sinusoidal waveforms at two different frequencies. When the control voltage $V_{CON}=0V$ (0% frequency control) the switched capacitors are always disconnected from the tank, this is when the converter runs at its maximum resonant frequency $f_H$. For an unloaded converter, $f_H$ is equal to the undamped resonant frequency of the primary resonant tank given as

$$f_H = \frac{1}{2\pi \sqrt{L_1 C_1}} \tag{4-10}$$

When the control voltage $V_{CON} > \bar{V}_{C1}$ (100% frequency control) the switches $S_{V1}$ and $S_{V2}$ are constantly kept ON and the switched capacitors $C_{V1}$ and $C_{V2}$ are always connected to the resonant tank. This is when the converter runs at its minimum resonant frequency $f_L$. For an unloaded converter, $f_L$ can be given as

$$f_L = \frac{1}{2\pi \sqrt{L_1 (C_1 + C_{V})}} \tag{4-11}$$

For proper operation of the push-pull converter, the split inductors $L_{S1}$ and $L_{S2}$ should be identical in inductance and have an inductance much greater than $L_1$ [54]. At steady-state the split inductors act like a current source and supplies current to the parallel resonant tank. Since during one half of the resonant cycle, one end of the split inductors are shorted to the ground by the switches $S_{V1}$ and $S_{V2}$; and because $L_{S1}$ and $L_{S2}$ act like a center tapped transformer with $V_{IN}$ applied at the center tap, the average voltage at the other end of the split inductors must be equal to $2V_{IN}$ [54]. With this assumption, the peak of the resonant voltage $\bar{V}_{C1}$ can then be calculated by equating the steady-state half-cycle average voltage of $V_{S1}$ and $V_{S2}$ (equivalent to the half-cycle of $V_{C1}$) to $2V_{IN}$.

For the case when $V_{CON}=0$ and $V_{CON} \geq \bar{V}_{C1}$, the voltage across the parallel resonant tank can be assumed to be purely sinusoidal at frequencies $f_H$ and $f_L$ respectively. If the half-cycle
average of $V_{C1}$ is equated to $2V_{IN}$, the peak of the resonant voltage $\hat{V}_{C1}$ (identical to the peak of $\hat{V}_{S1}$ and $\hat{V}_{S2}$) can be obtained as $\pi V_{IN}$ [54].

However, for the case when $0 < V_{CON} < \hat{V}_{C1}$ the converter switches between two resonating states; one with $C_V$ in the resonant tank and one without. The voltage waveform $V_{C1}$ is no longer a pure sinusoid nor is its peak voltage equal to $\pi V_{IN}$. By applying the same assumption that during steady-state the average voltage of one half-cycle of $V_{C1}$ is equal to $2V_{IN}$, and assuming that $V_{C1}$ is made of piecewise sinusoids at frequencies $f_H$ and $f_L$; the relationship between the half-cycle average of $V_{C1}$ and $V_{IN}$ can be given by

$$\frac{2\theta_L}{\omega_L} + \frac{\pi - 2\theta_H}{\omega_H} \left( \int_{0}^{\theta_L/\omega_L} \hat{V}_L \sin(\omega_L t) \, dt + \int_{\theta_H/\omega_H}^{\pi - \theta_H/\omega_H} \hat{V}_H \sin(\omega_H t) \, dt \right) + \int_{\pi - \theta_L/\omega_L}^{\pi} \hat{V}_L \sin(\omega_L t) \, dt = 2V_{IN}$$

(4-12)

where $\omega_H = 2\pi f_H$ and $\omega_L = 2\pi f_L$, and $\hat{V}_H/\hat{V}_L$ are the peak voltages of the respective piecewise sinusoids. $\theta_H$ and $\theta_L$ are the switching angles of the two piecewise sinusoids and are given as

$$\theta_H = \sin^{-1}\frac{V_{CON}}{\hat{V}_H}$$

$$\theta_L = \sin^{-1}\frac{V_{CON}}{\hat{V}_L}$$

(4-13)

By resolving the integrals of Equation (4-12) the expression is simplified to

$$\frac{\hat{V}_L}{\omega_L} (1 - \cos \theta_L) + \frac{\hat{V}_H}{\omega_H} \cos \theta_H = V_{IN} \left( \frac{2\theta_L}{\omega_L} + \frac{\pi - 2\theta_H}{\omega_H} \right)$$

(4-14)

Substituting $\theta = 0$ into Equation (4-14) we obtain $\hat{V}_H = \pi V_{IN}$, and substituting $\theta = \pi$ into Equation (4-14) we obtain $\hat{V}_L = \pi V_{IN}$. Equating the previous equations we can see that $\hat{V}_L = \hat{V}_H$ and as a result $\theta_H$ will also be equal to $\theta_L$. With this the peak of the resonant voltage can be given as

$$\hat{V}_{C1} = \hat{V}_L = \hat{V}_H = V_{IN} \left( \frac{2\theta \omega_H + (\pi - 2\theta) \omega_L}{(1 - \cos \theta) \omega_H + \cos \theta \cdot \omega_L} \right)$$

(4-15)
In can be shown from Equation (4-15) that when $\theta = 0$ ($C_V$ completely disconnected) or when $\theta = \frac{\pi}{2}$ ($C_V$ always connected), the peak voltage is equal to $\pi V_{IN}$. Also when $\omega_H = \omega_L$ as in the case when there is only one resonant capacitor, Equation (4-15) simplifies down to $\hat{V}_{C_1} = \pi V_{IN}$. Shown in Figure 4-15 is a plot of $\hat{V}_{C_1}/\pi V_{IN}$ against $\theta$ for the case when $C_V = C_I$.

![Figure 4-15: Peak resonant voltage versus $\theta$ for switched capacitor push-pull converter when $C_V=C_I$](image)

### 4.4.4 Equivalent capacitance of a switched capacitor push-pull converter

The operating frequency of a switched capacitor push-pull converter can be found by obtaining the steady-state total equivalent capacitance $C_T$ seen by the converter. At switching angle of $\theta=0$, $C_T$ is equal to $C_I$ and when $\theta=\pi/2$, $C_T$ is equal to $C_I+C_V$. When the switching angle $\theta$ is in between 0 and $\pi/2$, the equivalent steady-state capacitance $C_{EQ}$ that is added to $C_I$ can be found by considering the charge stored in $C_V$ over half a resonant cycle.

The derivation of this equivalent capacitance is based on the voltage-time integral of the voltage across $C_V$, and has been adapted from the work by Si [100]. The original derivation by Si assumes that $C_V$ is smaller than $C_I$, such that the resonant voltage can be approximated as a sinusoid. This derivation has been extended by allowing $C_V$ to be larger than $C_I$ and considers the resonant voltage as a piecewise waveform made up of sinusoids at two different frequencies $f_H$ and $f_L$. 

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For a certain switching angle $\theta$ the equivalent capacitance added to the resonant circuit at steady-state may be approximated by, equating the charge stored in $C_V$ over one half period of the resonant cycle to an equivalent capacitor $C_{EQ}$ with sinusoidal voltage excitation with a peak voltage of $\pi V_{IN}$ over one half period. This integral equation can be expressed as

$$
\int_0^{\frac{2\theta}{\omega_L} + \frac{\pi - 2\theta}{\omega_H}} C_{EQ} \cdot \pi V_{IN} \sin(\omega_{EQ}t) \, dt
= \int_0^{\frac{\theta}{\omega_L}} C_V \mathcal{V}_{S_1} \sin(\omega_L t) \, dt + \int_{\frac{\theta}{\omega_H}}^{\frac{\pi - \theta}{\omega_H}} C_V V_{CON} \, dt + \int_{\frac{\pi - \theta}{\omega_L}}^{\frac{\pi}{\omega_L}} C_V \mathcal{V}_{S_1} \sin(\omega_L t) \, dt
$$

(4-16)

where $\omega_L$ and $\omega_H$ are as previously defined in section 4.4.3 and $\mathcal{V}_{S_1}$ is obtained from Equation (4-15) at switching angle $\theta$. $\omega_{EQ}$ is the angular frequency of the equivalent sinusoidal voltage with peak $\pi V_{IN}$ and is given by

$$
\omega_{EQ} = \frac{2\pi}{\frac{2\theta}{\omega_L} + \frac{\pi - 2\theta}{\omega_H}}
$$

(4-17)

Note that $\omega_{EQ}$ is not the same as the angular resonant frequency $\omega_r$, since this is only an approximation for working out the equivalent capacitance. By solving Equation (4-16) the equivalent capacitance due to $C_V$ at a switching angle $\theta$ can be expressed as

$$
C_{EQ} = \frac{\mathcal{V}_{S_1}}{\pi V_{IN}} \left( \frac{\omega_{EQ}}{\omega_L} C_V (1 - \cos \theta) + \frac{\omega_{EQ}}{2\omega_H} C_V \sin \theta (\pi - 2\theta) \right)
$$

(4-18)

where $\mathcal{V}_{S_1}$ is given by Equation (4-15) and $\omega_{EQ}$ by Equation (4-17). The total equivalent capacitance $C_T$ as seen by the converter is given by

$$
C_T = C_1 + C_{EQ}
$$

(4-19)

and the operating frequency $f_r$ of the converter with primary capacitance $C_T$ is given as

$$
f_r = \frac{1}{2\pi \sqrt{L_1 C_T}}
$$

(4-20)

For the case when $C_V = C_1$ the total equivalent capacitance $C_{EQ}$ plotted against $\theta$ is shown in Figure 4-16a), and the normalised operating frequency $f_r/f_H$ plotted against $\theta$ is shown in Figure 4-16b).
4.4.5 Practical implementation of switched capacitor push-pull converter

A control circuit was designed to implement a fully ZVS switched capacitor push-pull converter whose schematic was previously given in Figure 4-13a). The different parts of the control circuitry are described below.

**ZVS Gate drive and start up circuitry**

To soft-switch the push-pull converter switches $S_1$ and $S_2$ an internal feedback loop is created to detect the zero voltage crossings of the resonant voltage. A 50:1 step down transformer was placed across the resonant tank to create a small signal representation $V_{V_{ij}}$ of the resonant voltage. $V_{V_{ij}}$ is then fed into a non-inverting Schmitt trigger to generate a square wave with rising and falling edges at the zero voltage crossings of the resonant voltage $V_{C1}$. This square wave signal is then used as the input signal for the low side gate drive of the converter switches $S_1$ and $S_2$.

The ZVS push-pull converter encounters a problem during start up, since at start up the voltage across the tank is zero and no switching signal can be generated. Starting with one switch ON and one switch OFF does not guarantee start up, this is because steady-state operation of a push-pull converter requires that both splitting inductors $L_{S1}$ and $L_{S2}$ have equal current passing through them [54]. In order to ensure the converter starts up properly, a start up circuit was implemented which turns ON both $S_1$ and $S_2$ for a period of time determined by a RC circuit. Temporarily turning on both switches at start up establishes the required
current in the split inductors, afterwards turning off one of the switches will induce oscillation in the resonant tank. The ZVS gate drive circuitry with the start up circuit is shown in Figure 4-17.

![Figure 4-17: ZVS gate drive circuitry and start up circuit of a push-pull converter](image)

**Switched capacitor gate drive circuitry**

The circuitry implemented for controlling the switches $S_{V1}$ and $S_{V2}$ is shown in Figure 4-18. A resistor divider connected across $S_1$ and $S_2$ is used to reduce $V_{S1}/V_{S2}$ to $V_{S1d}/V_{S2d}$ which is used for comparison at the comparator, note that the signal $V_{V1}$ the step down transformer cannot be used since the signal is negative for one half of the resonant cycle. The switching signal for $S_{V1}$ and $S_{V2}$ are generated by comparing voltages $V_{S1d}/V_{S2d}$ with a control voltage $V_{CONd}$ which is the small signal representation of $V_{CON}$. The two comparators shown in Figure 4-18 are set up as inverting Schmitt triggers. If $V_{S1d}/V_{S2d}>V_{CONd}$ the output of the comparator is low, and if $V_{S1d}/V_{S2d}<V_{CONd}$ the output of the comparator is high. The output of the comparators is connected to a low side gate drive for switches $S_{V1}$ and $S_{V2}$.

High speed components were used in these control circuits to get switching as close to the zero voltage crossings of the resonant voltage as possible. The comparators used in this design had typical propagation times of 7ns and the gate drives had propagation times of less than 150ns. The total propagation time is less than 5% of the resonant period for a resonant frequency of 300 kHz.
Figure 4-18: Gate drive circuitry for the switched capacitors of a push-pull converter

Practical implementation

To demonstrate the operation of the switched capacitor push-pull circuit and the proposed control circuitry. A practical implementation of the converter was made and tested.

In this demonstration, two 1mH ferrite cored inductors were used for the split inductors $L_{S1}/L_{S2}$ and an input voltage of 18Vdc was used for the push-pull converter. The primary coil $L_1$ had an inductance of 12µH and the resonant capacitor $C_1$ had a capacitance of 36nF. The value of the switched capacitor $C_V$ was set to 47nF; $C_V$ was chosen to be larger than $C_1$ for greater frequency control and also accentuates the non-sinusoidal resonant voltage. This also demonstrates that this is an improved implementation of the switched capacitor push-pull as it was not previously possible to have $C_V>C_1$ [44, 91]. This is made possible by using faster opamps in the control circuit and faster MOSFETs for the switched capacitor switch. These resonant parameters give a theoretical $f_{th}$ of 242 kHz and $f_L$ of 159 kHz.

Shown in Figure 4-19a) to f) are six oscilloscope snap shots of the switched capacitor push-pull converter in action at increasing percentages of frequency control. The yellow trace in Figure 4-19 is the ZVS switching signal for $S_1$, the pink trace is the resonant voltage $V_{S1}$, the blue trace is the resistor divided signal $V_{S1d}$ and the green trace is the control voltage $V_{COND}$. Keep in mind that there is no secondary pickup present.
Figure 4-19: Practical waveforms of a switched capacitor push-pull parallel tuned converter at six different switching angles

At 0% and 100% frequency control shown in Figure 4-19a) and Figure 4-19f) respectively, it can be seen that one half cycle of the waveform $V_{S1}$ is sinusoidal. The experimentally measured $f_H$ was 239 kHz and $f_L$ was 162 kHz, this is within ±3 kHz of the theoretical $f_H$ and $f_L$. Also at 0% and 100% frequency control, $\bar{V}_{S1}$ was measured to be 60V and 58.4V which is slightly higher than the predicted peak of $\pi V_{IN}$ which is 56.5V. This could be due to the ZVS
being slightly off, which will appear as a phase shift in the switching and can boost the resonant voltage peak [54].

The resonant voltage peaks $\bar{V}_{C1}$ obtained from the practical implementation was plotted against the estimated practical switching angle $\theta$ and is shown in Figure 4-20. Figure 4-20 was also overlaid with the peak voltage that can be obtained from Equation (4-15). In this figure, it can be seen that there is a considerable gap between the calculated and practical values, where the maximum difference of the calculated value is approximately 7% below practical. This difference may be to do with the assumption made in Equation (4-12) that the resonant voltage $V_{C1}$ is a piecewise sinusoid which may not be the case. This difference can also be accounted by the fact that the practical voltage peak was observed to be boosted, since its peak was higher than $\pi V_{IN}$ at 0% and 100% frequency control. Also the accuracy of the measurement of the oscilloscope used to obtain the practical peak voltage is only ±0.5V.

![Figure 4-20: Practical vs. calculated voltage peak of a switched capacitor push-pull converter](image)

The operating frequency obtained from the practical implementation was also extracted and is plotted against the estimated practical switching angle $\theta$ and is shown in Figure 4-21. This figure was also overlaid with the operating frequency that can be calculated with Equation (4-20), from the estimated switching angle $\theta$. A good correlation is seen between the practical and calculated operating frequency.
4.5 Implementation of primary capacitance control on a series resonant primary tank

A modified voltage-fed half-bridge converter was chosen for the implementation of frequency power flow control on a series tuned tank.

4.5.1 The modified half-bridge converter

The modification includes the removal of the DC splitting capacitors of a traditional half-bridge converter. This allows the primary capacitance to be connected directly to ground, which makes the implementation of a switched capacitor possible without a floating AC switch. The schematic of this converter is shown in Figure 4-22a).

This converter is very simple and consists of two converter switches $S_1$ and $S_2$, whose states are always mutually exclusive to each other. The equivalent circuits of the two states of this converter are shown in Figure 4-22b), and its operation is similar to that of a synchronous buck converter. When $S_1$ is ON and $S_2$ is OFF the input voltage source is applied across the resonant tank, supplying energy to the tank. When $S_1$ is OFF and $S_2$ is ON, the series tuned tank is shorted together and allows energy to freely oscillate in the resonant tank between $L_1$ and $C_1$. To soft-switch this converter, $S_1$ and $S_2$ are zero current switched off the resonant current $I_1$; when $I_1$ is positive (flowing from source to tank) $S_1$ is ON and $S_2$ if OFF, when $I_1$ is negative $S_2$ is ON and $S_1$ is OFF. More information about this converter can be found in Chapter 5.

Figure 4-21: Practical operating frequency of a switched capacitor push-pull converter
The half-bridge converter was chosen over the full-bridge for the implementation of a switched capacitor, because it requires less power components which make it potentially more efficient. Furthermore, $C_I$ can be permanently grounded with the half-bridge converter which greatly simplifies the control circuitry required for the switched capacitor.

![Diagram](image)

**Figure 4-22:** a) Modified half-bridge series tuned converter, b) Equivalent circuit states of the modified half-bridge converter

### 4.5.2 The modified half-bridge converter with switched capacitor

The modified half-bridge converter with a switched capacitor $C_V$ added across $C_I$ is shown in Figure 4-23a). When $S_V$ is ON the capacitance $C_V$ is shunted across $C_I$ adding capacitance to the resonant tank. However, when $S_V$ is OFF this does not necessary disconnect $C_V$ from the resonant tank. This is because the MOSFETs are directional switches, when the voltage across $C_I$ is negative, the body diode of the MOSFET will start conducting effectively shunting $C_V$ across $C_I$. This means implies that $C_V$ will be connected to the resonant tank during every half cycle when the resonant current $I_I$ is negative. The equivalent circuit states of the switched capacitor circuit are given in Figure 4-23b).
It has been previously mentioned in section 4.4.2, that \( C_V \) can only be safely switched into the circuit when its voltage \( V_{CV} \) is equal to \( V_{C1} \) the voltage of the resonant capacitor \( C_1 \). The method for achieving this is basically the same as the method used for the switched capacitor push-pull converter. This method has been adopted from [91] and is described below.

Since the MOSFET has a body diode which will conduct when \( V_{C1} \) is negative, the switch \( S_V \) is kept ON during the negative half cycle of \( V_{C1} \) to direct the resonant current through the switch instead of flowing through the less efficient body diode. This means that \( C_V \) will be connected to the resonant tank for at least 50% of each resonant cycle.

When the resonant voltage \( V_{C1} \) increases above 0V, \( S_V \) can be switched off at any time before \( V_{C1} \) reaches its peak value. When \( S_V \) is switched OFF, \( C_V \) will hold its voltage at that instant and the switch can only be safely turned back ON when \( V_{C1} \) equals to \( V_{CV} \) again. This happens once \( V_{C1} \) is decreasing after reaching its peak value. To control the amount of effective capacitance added to the resonant tank, the switch \( S_V \) needs to be modulated so that it is on for different periods of time within the positive half cycle of \( V_{C1} \).

The switching of the switch \( S_V \) is determined by comparing the voltage \( V_{C1} \) to a reference voltage \( V_{CON} \). This allows \( C_V \) to be switched in and out of the resonant tank during the positive half-cycle of \( V_{C1} \). Whenever \( V_{C1} \) is smaller than \( V_{CON} \), \( S_V \) is kept ON and \( C_V \) is
shunted across $C_1$. When $V_{Cl}$ is larger than $V_{CON}$, $S_V$ is switched off disconnecting $C_V$ from $C_1$.

![Waveforms of a switched capacitor half-bridge series tuned converter](image)

**Figure 4-24: Waveforms of a switched capacitor half-bridge series tuned converter**

The waveforms and the timings of a switched capacitor half-bridge converter are shown in Figure 4-24. Note that the resonant voltage $V_{Cl}$ is always sinusoidal during its negative phase and is non-sinusoidal during its positive phase when $C_V$ is only partially switched in over the half cycle. Keep in mind that the main converter switches $S_1/S_2$ are zero current switched off $I_I$, while $S_V$ is switched off $V_{Cl}$. 

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4.5.3 Resonant voltage waveform of switched capacitor half-bridge converter

If $V_{CON}$ is greater than the peak of the resonant voltage $V_{C_{1}}$, then $S_V$ will be ON 100% of the time. This is the only time when the resonant voltage $V_{C1}$ is purely sinusoidal and is the converter is also operating at its minimum frequency $f_L$. When unloaded the operating frequency $f_L$ can be approximated by the undamped natural frequency of the tank, given as

$$f_L = \frac{1}{2\pi \sqrt{L_1 (C_1 + C_V)}}$$  \hspace{1cm} (4-21)

The steady-state peak of the resonant voltage or current for a voltage-fed series tuned tank is not limited by the input voltage source. The RMS of the resonant current $I_1$ is purely dependent on any impedance present in the primary series tuned tank when the operating frequency is at $\omega_r$.

The switched capacitor is fully switched in during the negative half cycle of the resonant voltage and partially switched in during the positive half cycle. The resonant voltage $V_{C1}$ is proportional to the primary circuit quality factor $Q$ times the equivalent input voltage. Since the quality factor of a series tuned circuit is given as $Q = \frac{\omega_r}{\sqrt{R}}$, the ratio between the positive voltage peak $V_{C_{1A}}$ and the negative voltage peak $V_{C_{1B}}$ is approximately equal to the ratio of the quality factor of the circuit during the positive half cycle $Q_a$ and negative half cycle $Q_b$. Hence, the ratio between the positive and negative peak of the resonant voltage $V_{C1}$ is approximated as

$$\frac{V_{C_{1A}}}{V_{C_{1B}}} = \frac{Q_a}{Q_b} = \frac{\omega_a}{\omega_b}$$  \hspace{1cm} (4-22)

where $Q_a = \frac{\omega_a}{\sqrt{R}}$ and $Q_b = \frac{\omega_b}{\sqrt{R}}$; and the operating angular frequency of the positive and negative half cycle of the resonant voltage is given by $\omega_a$ and $\omega_b$, respectively. The approximation given in Equation (4-22) can also be expressed in terms of the equivalent capacitance during each half cycle. Simple substitution shows that the ratio of the positive peak $V_{C_{1A}}$ to that of the negative peak $V_{C_{1B}}$ can be given as

$$\frac{V_{C_{1A}}}{V_{C_{1B}}} = \sqrt{\frac{C_1 + C_{EQ}}{C_1}}$$  \hspace{1cm} (4-23)
From this it can be seen that the peak of $V_{C1}$ is inversely proportional to the equivalent capacitance present during that half cycle. $C_{EQ}$ is the equivalent capacitance added in that half cycle due to $C_V$ and can be given by Equation (4-18). Note that this is not the same $C_{EQ}$ given in the following section which accounts of the entire resonant cycle.

### 4.5.4 Equivalent capacitance of a switched capacitor half-bridge converter

The new resonant frequency of the switched capacitor half-bridge converter can be found by obtaining the steady-state total equivalent capacitance $C_T$ seen by the converter. The control voltage $V_{CON}$ can be translated into a switching angle $\theta$ as previous given in Equation (4-13) for the push-pull converter. When $\theta=\pi/2$ the switched capacitance is switched in 100% of the time and the total equivalent capacitance $C_T$ is equal to $C_I+C_V$; this is also the only time when the resonant voltage is fully sinusoidal. When $\theta<\pi/2$ the equivalent steady-state capacitance $C_{EQ}$ that is added to $C_I$ can be found by considering the charge and discharge of the capacitor $C_V$ in one resonant cycle.

The derivation of the equivalent capacitance is based on the voltage-time integral of the voltage across $C_V$, this is similar to the derivation of $C_{EQ}$ for the switched capacitor push-pull in section 4.4.4. However, since $C_V$ can only be switched out during the positive cycle of the resonant voltage, the voltage-time integral must be taken across the whole resonant cycle. The derivation assumes that the resonant voltage is a piecewise waveform made up of sinusoids at the two frequencies $f_H$ and $f_L$ which are previously given by Equations (4-10) and (4-11).

For a certain switching angle $\theta$ the equivalent capacitance added to the resonant circuit at steady-state may be approximated by equating the charge stored in $C_V$ over one period of the resonant cycle to an equivalent capacitor $C_{EQ}$ with sinusoidal excitation over the same period. This integral equation can be expressed as

$$
C_{EQ} \cdot \pi V_{IN} \sin(\omega_{EQ} t) \, dt
$$

$$
= \int_0^{\theta} C_V \hat{y}_{S1} \sin(\omega_L t) \, dt + \int_{\pi-\theta}^{\pi} \frac{C_V}{\omega_H} V_{CON} \, dt + \int_{\pi-\theta}^{\pi} C_Y \hat{y}_{S1} \sin(\omega_L t) \, dt
$$

(4-24)
By solving Equation (4-24) the equivalent capacitance due to $C_V$ at a switching angle $\theta$ can be expressed as:

$$C_{EQ} = \frac{\hat{V}_{S1}}{\pi V_{IN}} \left( \frac{\omega_{EQ}}{2\omega_L} C_V (2 - \cos \theta) + \frac{\omega_{EQ}}{4\omega_H} C_V \sin \theta (\pi - 2\theta) \right) \tag{4-25}$$

where $\hat{V}_{S1}$ is given by Equation (4-15) and $\omega_{EQ}$ by Equation (4-17). Here it can be seen that when $\theta=\pi/2$, $C_{EQ} = C_V$, and when $\theta=0$, $C_{EQ} = \frac{C_V}{2}$. The total equivalent capacitance $C_T$ as seen by the converter is

$$C_T = C_1 + C_{EQ} \tag{4-26}$$

For the case when $C_V = C_1$ the total equivalent capacitance $C_{EQ}$ plotted against $\theta$ is shown in Figure 4-25a), and the normalised operating frequency $f_r/f_H$ plotted against $\theta$ is shown in Figure 4-25b).

![Figure 4-25: Switched capacitor half-bridge converter when $C_V=C_1$; a) $C_{EQ}/C_1$ versus $\theta$, b) $f_r/f_H$ versus $\theta$](image-url)
4.5.5 Practical implementation of switched capacitor series half-bridge converter

A control circuit was designed to implement the fully ZCS switched capacitor half-bridge series tuned converter whose schematic was previously given in Figure 4-22a. The different parts of the control circuitry are described below.

ZCS gate drive circuitry

A circuit to soft-switch the half-bridge converter switches $S_1$ and $S_2$ are shown in Figure 4-26. The switches are zero current switched off the resonant current $I_{L1}$. To detect the zero current crossings of $I_{L1}$, a current transformer with a turn ratio of 1:200 was placed on resonant tank to transform the resonant current into a small signal voltage. This voltage is then put through a non-inverting Schmitt trigger to generate a square wave with rising and falling edges at the zero crossing instances of the resonant current. This square wave signal can then be used to drive the half-bridge gate driver for the switches $S_1$ and $S_2$.

At start up the high side switch $S_1$ must be turned ON in order to start the oscillation of the series tuned resonant tank. This converter does not usually require a start up circuit, since when the circuit is turned on transient noise should trigger the zero crossing comparator, turning on the high side switch.

![Figure 4-26: ZCS gate drive circuitry of a half-bridge series tuned converter](image)

Switched capacitor gate drive circuitry

The circuitry for controlling the switch $S_V$ is shown in Figure 4-27. The switching signal for $S_V$ is generated by comparing $V_{C1d}$ (the small signal representation of $V_{C1}$) with the control voltage $V_{CONd}$ (the small signal representation of $V_{CON}$) in an inverting Schmitt trigger. The output of the comparator is connected to a low side gate drive, such that when $V_{C1d}<V_{CONd}$, $S_V$ is turned ON and when $V_{C1d}>V_{CONd}$, $S_V$ is turned OFF.
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Figure 4-27: Gate drive circuitry for the switched capacitor of a half-bridge converter

High speed components were used in these control circuits to get switching as close to the zero current/voltage crossings as possible. The comparators used in this design had typical propagation times of 7ns and the gate drives had propagation times of less than 150ns. This accounts for less than 5% of the resonant period for a resonant frequency of 300 kHz.

**Practical implementation**

To demonstrate the operation of the switched capacitor half-bridge circuit and the proposed control circuitry. A practical implementation of the converter was made and tested.

In this demonstration, an input of 2Vdc was used for the main converter and 12Vdc was used for the control circuitry. Separate input voltages were used, because a high quality factor series tuned tank does not require much voltage to drive. The primary coil \( L_1 \) had an inductance of \( 12 \mu \text{H} \) and the resonant capacitor \( C_1 \) had a capacitance of 22nF. The value of the switched capacitor \( C_V \) was made equal to \( C_1 \) with a capacitance of 22nF. Since only one half cycle of the resonant frequency has switched capacitor action, the harmonic content of the switched capacitor half-bridge converter is much higher than the switched capacitor push-pull converter. Because of this, it was difficult to set \( C_V \) larger than \( C_1 \) in this implementation. These resonant parameters give a theoretical \( f_{H1} \) of 253 kHz and \( f_L \) of 219 kHz.

Shown in Figure 4-28a) to f) are six oscilloscope snap shots of the switched capacitor half-bridge circuit in action at increasing percentages of frequency control. The yellow trace in Figure 4-28 is the ZVS switching signal for \( S_V \), the pink trace is the resonant voltage \( V_{C1} \), the blue trace is the ZCS switching signal for \( S_1/S_2 \) and the green trace is the resonant current \( I_{L1} \). Note that there is no secondary pickup present.
At 100% frequency control, it can be seen in Figure 4-28f) that $V_{CI}$ and $I_{LI}$ are purely sinusoidal. The experimentally measured $f_H$ was 254 kHz and $f_L$ was 220 kHz, which is within $\pm 1$ kHz of the theoretical $f_H$ and $f_L$. At 0% frequency control, it can be seen in Figure 4-28a) that the positive and negative peaks of the $V_{CI}$ were 66.4V and 47.2V respectively. This gives a ratio of 1.407 between positive and negative peaks; this closely matches the ratio
of 1.414 which can be calculated by Equation (4-23). Shown in Figure 4-29 is the ratio between positive and negative peaks plot against the estimated switching angle $\theta$, and is overlaid with the ratio given by Equation (4-23). Although the calculated and practical peak voltage ratios shown in Figure 4-29 match at $\theta=0$ and at $\theta=\pi/2$, there is a noticeable difference between calculated and practical points in between these two values of $\theta$. This is probably due to the assumptions made in section 4.5.3.

**Figure 4-29:** Practical vs. calculated peak voltage of a switched capacitor half-bridge converter

**Figure 4-30:** Practical operating voltage of a switched capacitor half-bridge converter

Shown in Figure 4-30 is the plot of the measured operating frequency vs. the estimated switching angle $\theta$ of the switched capacitor $C_V$. The operating frequency predicted with equivalent capacitance given in Equation (4-26) is also overlaid, and a good correlation is seen between the two.
4.6 Performance of switched capacitor converters

Here the presented switched capacitance power converters are compared experimentally. The main performance metric of interest is the power flow control range of the converter under different operating conditions. All four basic resonant topologies were compared and observations are discussed.

In order to evaluate these converters fairly, both the parallel tuned push-pull and the series tuned half-bridge were given the same power transfer coil $L_1$. The primary resonant capacitor $C_1$ and the switched capacitor $C_V$ were different for each primary topology, such that the $\omega_H$ of the push-pull converter is equal to the $\omega_L$ of the half-bridge converter. This is done so that the unloaded resonant frequency of the secondary is equal to that of the $\omega_H$ of the push-pull converter and $\omega_L$ of the half-bridge converter. This is to ensure that primary resonant waveform is purely sinusoidal at the frequency $\omega_0$ where the quality factor of the secondary circuit $Q_{S0}$ is measured. Since the half-bridge converter is only fully sinusoidal at $\omega_L$ or at 100% frequency control. Hence, this means that the frequency control of the push-pull converter decreases in frequency from $\omega_0$ and the half-bridge converter increases in frequency from $\omega_0$. For both primary tuned and series tuned converters $C_V$ was made equal to $C_1$. Therefore, the theoretically frequency control range for the parallel tuned push-pull converter will be from $f_0$ to $0.707f_0$ and for the series tuned half-bridge converter the frequency control range will be from frequency to $f_0$ to $1.155f_0$. All other parameters and test conditions used in this practical study are given in Table 4-3.

Two series tuned secondary circuits and two parallel tuned secondary circuits were designed to measure the performance of the switched capacitor converters. All four secondary circuits were made to have a natural resonant frequency of approximately 215 kHz. The secondary circuit parameters were designed so that there are two series tuned pickups, one with $Q_{S0}=1$ and the other $Q_{S0}=2.3$; and two parallel tuned pickups, one with $Q_{S0}=1$ and the other $Q_S=2.3$.

Practical and simulated data was collected for the eight experimental setups described above. Each figure from Figure 4-31 to Figure 4-38 consists of part a) and part b). Part a) of the figures plots the estimated percentage of $C_V$ against the operating frequency; where the percentage of $C_V$ is obtain with Equation (4-18) and (4-25) with the switching angle $\theta$ measured in practice. Part b) of the figures plots the output voltage $V_{out}$ against the operating frequency. In both part a) and b) the practical data is overlaid with simulated data using the
model presented in Chapter 2, using the same resonant parameters that are shown in Table 4-3.

**Table 4-3: Resonant parameters of experimental study on frequency power flow control converters**

<table>
<thead>
<tr>
<th>Parallel tuned push-pull primary converter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1=12\mu H$</td>
</tr>
<tr>
<td>$f_{hf}=212\ kHz$ (0% control)</td>
</tr>
<tr>
<td>Series secondary Q$_{S_0}=1$</td>
</tr>
<tr>
<td>$V_{IN}=25V,\ k=0.14$</td>
</tr>
<tr>
<td>$L_2$</td>
</tr>
<tr>
<td>$C_2$</td>
</tr>
<tr>
<td>$R_2$</td>
</tr>
<tr>
<td>$f_0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Series tuned half-bridge primary converter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1=12\mu H$</td>
</tr>
<tr>
<td>$f_{hf}=257\ kHz$ (0% control)</td>
</tr>
<tr>
<td>Series secondary Q$_{S_0}=1$</td>
</tr>
<tr>
<td>$V_{IN}=4V,\ k=0.14$</td>
</tr>
<tr>
<td>$L_2$</td>
</tr>
<tr>
<td>$C_2$</td>
</tr>
<tr>
<td>$R_2$</td>
</tr>
<tr>
<td>$f_0$</td>
</tr>
</tbody>
</table>

| Parallel secondary Q$_{S_0}=1$ | Parallel secondary Q$_{S_0}=2.35$ |
|-------------------------------------------|
| $V_{IN}=4V,\ k=0.14$ | $V_{IN}=6V,\ k=0.3$ | $V_{IN}=5V,\ k=0.14$ | $V_{IN}=7V,\ k=0.3$ |
| $L_2$ | 10.9$\mu H$ | $L_2$ | 3.3$\mu H$ |
| $C_2$ | 47$nF$ | $C_2$ | 183$nF$ |
| $R_2$ | 14.6 | $R_2$ | 9.9 |
| $f_0$ | 222 kHz | $f_0$ | 204 kHz |
Figure 4-31: Performance of PP frequency control when $Q_{s0}=2.35$

Figure 4-32: Performance of PP frequency control when $Q_{s0}=1$
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**Figure 4-33**: Performance of PS frequency control when $Q_{s0}=2.3$

**Figure 4-34**: Performance of PS frequency control when $Q_{s0}=1$
Figure 4-35: Performance of SP frequency control when $Q_{s0}=2.3$

Figure 4-36: Performance of SP frequency control when $Q_{s0}=1$
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Figure 4-37: Performance of SS frequency control when $Q_{s0}=2.3$

Figure 4-38: Performance of SS frequency control when $Q_{s0}=1$
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**Observations of PP frequency control experiment**

Firstly, the parallel tuned push-pull converter together with a parallel tuned secondary to form the PP topology was implemented and the results are shown in Figure 4-31 and Figure 4-32. In part a) of both figures the simulated operating frequencies matched very closely to the frequency obtain in practice. The practical output voltages shown in Figure 4-31b) and Figure 4-32b) also matched quite closely to the simulated results and have the same trend as the simulated results. The only exception was that there was a significant difference between simulated and actual output voltage when \( Q_{S0} = 1 \) at \( k = 0.3 \). This difference is most probably to do with parasitic values that were not modelled and also because the practical coupling factor \( k \) may not be exactly 0.3. With \( Q_{S0} = 2.3 \), the output voltage at 100% frequency control decreased to 70% of its output voltage at 0% frequency control. However, when \( Q_{S0} = 1 \), the output voltage increases by approximately 10% when frequency control is varied from 0 to 100%.

**Observations of PS frequency control experiment**

Shown in Figure 4-33 and Figure 4-34 are the plots comparing practical data with the simulated data for a parallel tuned push-pull converter driving a series tuned secondary to form the PS topology. A high level of agreement was seen between practical and simulated data in these two figures at coupling levels of \( k = 0.14 \) and 0.3 at both secondary \( Q_{S0} \) of 1 and 2.3. With \( Q_{S0} = 2.3 \), the output voltage at 100% frequency control was 48% of its value at 0% frequency control. When \( Q_{S0} = 1 \), this control range decreases and the output voltage at 100% frequency control only decreases to 75% of its value at 0% frequency control.

Keep in mind that for both the PP and PS systems \( C_v = C_i \), which means the operating frequency at 100% frequency control decreases to approximately 70.7% of its value at 0% frequency control. \( \omega_0 \) is obtained at 0% frequency control.

**Observations of SP frequency control experiment**

Shown in Figure 4-35 and Figure 4-36 are the plots comparing practical data with the simulated data for a series tuned half-bridge converter driving a parallel tuned secondary to form the SP topology. Here there was good correlation between the simulated operating frequency in part a) of the figures; however, the correlation of the simulated and practical output voltage was only good when \( k = 0.3 \), but not when \( k = 0.14 \). At \( k = 0.14 \) the practical output voltage actually diverges away from the simulated output voltage. This is primarily
due to the highly unpredictable primary resonant current, which is sensitive to the slightest amounts of reflected impedance and deviations in frequency. Hence, the series tuned primary is difficult to simulate accurately at low coupling levels. When \( Q_{S0} = 2.35 \) and \( k = 0.3 \), the output voltage at 100\% frequency control was 78\% of its value at 0\% frequency control; at \( k = 0.1 \) the simulation indicates that output voltage at 100\% frequency control drops to 85\% of its maximum, but in practice it only dropped to 92\%. In addition, like the PP configuration the output voltage stayed around the same value throughout the frequency control when \( Q_{S0} = 1 \) at both levels of coupling.

**Observations of SS frequency control experiment**

Shown in Figure 4-37 and Figure 4-38 are the plots comparing practical data with the simulated data for a series tuned half-bridge converter driving a series tuned secondary to form the SS topology. Here there was good correlation between the estimated \( C_V \) in circuit and the operating frequency in part a) of the figures, but the correlation of the simulated and practical output voltage was not consistent with \( Q_{S0} \) or coupling \( k \). Although the trend of the simulated and practical output voltages is the same, the difference in the absolute voltages is due to the high sensitivity of the primary resonant current of a series tuned primary. When \( Q_{S0} = 2.3 \), there is a slight increase of the output voltage of up to 10\%, when frequency control is varied from 100 to 0\%. When \( Q_{S0} = 1 \), the output voltage at 0\% frequency decreases down to 92\% of its value at 100\% frequency control.

Keep in mind that for both the SP and SS systems \( C_V = C_I \), which means the operating frequency at 0\% frequency control increases to approximately 1.15\% of its value at 100\% frequency control and \( \omega_0 \) is achieved at 100\% frequency control.

**Summary of observations**

In conclusion, this experimental study confirms that the PP and PS topologies are more suitable for primary side frequency power flow control due to its much more stable primary resonant current. However, the study shows that the control range of primary side frequency power flow control is not overly impressive, since the maximum change in output voltage seen in this study was only a 50\% decrease from maximum output. This is due to the fact that the change in frequency is only proportional to the square root of any change to the capacitance of the primary resonant tank. In order to increase the control range the ratio of \( C_V \) to \( C_I \) must be increased. However, it is difficult to make \( C_V \) much larger than \( C_I \) as this will
cause the resonant waveform to be highly distorted; the additional harmonics may cause problems for a ZVS/ZCS soft-switched converter and will increase the resonant tank losses. Moreover, since the range of output voltage control changes with different levels of coupling and load resistance at the secondary pickup, frequency power flow control range is highly unstable.

4.7 Summary

In this chapter power flow control based on primary side frequency control for all four basic resonant topologies were analysed with the frequency domain impedance analysis presented in Chapter 2. Discussion on the different ways to implement primary side frequency power flow control on a soft-switched converter is also presented and the method of using a switched capacitor for varying the frequency of a resonant tank was thoroughly analysed. The proposed switched capacitor controllers were practically implemented and good correlation between practical and simulation results is reported. Improvements were made to previous work on switched capacitor control by increasing the control range and extending the control to series tuned primary circuits.

In this study, primary frequency power flow control was found to work best on parallel tuned primaries due to its more stable primary resonant current which provides more predictable control characteristics. Finally, the output control range of a frequency power flow controlled system is dependent on the circuit quality factor $Q_S$ of the secondary pickup. The control range increases as $Q_S$ increases which is undesirable for IPT systems which have a wide load range. Hence, frequency power flow control is more suitable for powering constant loads.
Primary side power flow control of a soft-switched TET system may either be achieved by varying the resonant frequency as discussed in Chapter 4, or by varying the actual input power of the primary converter. A common form of input power control is known as input magnitude control [93], where simply the DC input voltage of the converter is varied to change the power at the output of the IPT system. This is usually achieved with a DC-DC converter stage before the primary resonant converter [41, 93]. However, such a system requires an additional converter stage which adds additional power components and control circuitry. Not only does this increase the size of the system, but it also significantly reduces the overall efficiency of the system due to the losses in the DC-DC converter. This chapter investigates the use of a type of input power control known as energy injection control [95] for the application of a TET system for implantable heart pumps.

5.1 Principle of energy injection control

Energy injection control is a control that can only be implemented on resonant systems. It relies on the system to be able to oscillate freely, thereby its operation can be self-sustained when no energy source is connected [95]. Energy injection power flow control works by injecting energy into a resonant system whenever more energy is required to power the load; and lets the resonant system free oscillate whenever there is sufficient energy in the system to sustain the output. During free oscillation, energy is consumed by the load and by any resistance present in the resonant system. For resonant converters, energy injection power flow control is potentially more efficient than input magnitude control as extra
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Power/switching components may not be required. Moreover, it is also potentially more efficient than frequency control as only the required amount of energy is injected into the system when needed.

To keep the converter soft-switched with energy injection control the switching network of the converter must only change between injection and free oscillation states during a zero voltage/current crossing of the resonant tank.

![Diagram of a voltage-fed series tuned resonant tank](image1)

Figure 5-1: Voltage-fed series tuned tank

Shown in Figure 5-1 is a diagram of a voltage-fed series tuned resonant tank together with its resonant waveforms, and Figure 5-2 is a diagram of a current-fed parallel tuned resonant tank together with its resonant waveforms. A voltage source is used to drive the series tuned tank, due to the current source nature of the series inductor; and a current source is used to drive the parallel tuned tank, due to the voltage source nature of the parallel capacitor [49]. In Figure 5-1 and Figure 5-2 the frequency of the source is operating at the ZPA frequency of...
resonant tank such that the voltage source $V_{AC}$ for the series tuned tank is in phase with the resonant current $I_{L1}$, while the current source $I_{AC}$ for the parallel tuned tank is in phase with the resonant voltage $V_{CI}$. Hence, in order to soft-switch these resonant tanks the series tank must be zero current switched and the parallel tank zero voltage switched.

**Figure 5-2: Current-fed parallel tuned tank**

In this thesis, the positive phase of the resonant tank shown in Figure 5-1 and Figure 5-2 refers to the period where a positive voltage/current source will inject energy into a series/parallel tuned tank. The negative phase refers to the period where a negative voltage/current source will inject energy into a series/parallel tuned tank. Energy injection can occur during the positive or negative phase of the resonant tank. However, a positive source will not inject energy during the negative phase and vice versa. A converter designed to inject energy during both phases will be called a full-wave energy injection converter; and a converter that can only inject energy during the positive phase it is called a half-wave
energy injection converter in this thesis. The resonant tank can be in free oscillation at any time, but the circuit should only enter or exit free oscillation during the zero voltage/current crossing of the resonant tank in order to maintain soft-switching.

### 5.1.1 Simulating the waveforms of a resonant tank under energy injection control

In order to design a resonant IPT system that uses energy injection control one must understand the build up of resonant energy and the damping of resonant energy in a resonant tank. Shown in Figure 5-3 are diagrams showing the equivalent circuits of series and parallel tuned resonant tanks, in their respective injection and free oscillation states, damped by a purely resistive load $R_1$. Note that during free oscillation both the series and parallel tuned tanks form the same shorted circuit, also note that a parallel tuned tank is already connected in its shorted form.

![Figure 5-3: Injection state and free oscillation states on primary resonant topologies](image)

In practice, the resonant tank would not be driven by an AC source; rather a switching network powered by a DC source will be used. Commonly, the input waveform across the resonant tank from a full-wave injection converter is represented by the $V_{DC}$ or $I_{DC}$ trace in
Figure 5-1 and Figure 5-2. The input from a half-wave injection converter would be the same $V_{DC}/I_{DC}$ trace with the negative phase of the waveform equal to zero. The equivalent AC input voltage can be approximated by taking the time domain Fourier transform of $V_{DC}$ and taking only the fundamental component. The high frequency harmonics of the input waveform can be considered negligible in building up the energy in the tank, since they are effectively filtered by the resonant tank [101]. However, the high frequency harmonics of a square wave input will generate additional AC losses in the resonant components. The RMS of the equivalent sinusoidal input waveform at the resonant frequency for a full-wave energy injection converter with input voltage $V_{DC}$ is approximated by

$$V_{AC} = \frac{2\sqrt{2}}{\pi} V_{DC}$$  \hspace{1cm} (5-1)

For the half-wave energy injection converter the RMS of the equivalent sinusoidal input is given as

$$V_{AC} = \frac{\sqrt{2}}{\pi} V_{DC}$$  \hspace{1cm} (5-2)

Note that $V_{AC}$ and $V_{DC}$ can be interchanged with $I_{AC}$ and $I_{DC}$ respectively, for obtaining the RMS of the equivalent sinusoidal input for a parallel tuned tank.

In order to analyse different energy injection control strategies it is necessary to know how the resonant waveforms change transiently during the injection and free oscillation states. Since time domain waveforms are of interest here, a set of state-space equations are used to obtain the resonant waveforms. Since there are only nodes in a resonant tank, two distinct differential equations are required to simulate the circuit. Assigning the current $I_{L1}$ and voltage $V_{C1}$ as the state variables, the state vector $x$ is expressed as

$$x = \begin{bmatrix} I_{L1} \\ V_{C1} \end{bmatrix}$$  \hspace{1cm} (5-3)

The state-space representation of the series tuned circuit shown in Figure 5-1 in the form of $A\dot{x} + Bx = C$ is given as

$$\begin{bmatrix} L_1 & 0 \\ 0 & C_1 \end{bmatrix} \dot{x} + \begin{bmatrix} R_1 & 1 \\ -1 & 0 \end{bmatrix} x = C$$  \hspace{1cm} (5-4)

and the state-space representation of a parallel tuned circuit shown in Figure 5-2 in the form of $A\dot{x} + Bx = C$ is given as
Since the circuit switches between two states the system is no longer linear. In order to simulate this behaviour a set of differential equations are used to represent each switching state. For this circuit only the output matrix $C$ changes between the different sets of equations and is dependent on whether the circuit is in the injection state or the free oscillation state. If an AC voltage source is to be used then the output matrices $C$ for the series and parallel tuned circuit are given as

$$
\begin{bmatrix}
0 & C_{inj} \\
-L_1 & 0
\end{bmatrix} \dot{x} + \begin{bmatrix}
1 & 0 \\
-R_1 & 1
\end{bmatrix} x = C
$$

(5-5)

where $C_{inj}$ is for the injection state and $C_{fre}$ is for the free oscillation state. $\omega_r$ is the ZPA frequency of the tank and for a series tuned tank it is obtained with Equation $(2-6)$ and for a parallel tuned tank with Equation $(2-7)$. To simulate the input source waveform due to a DC to AC switching network, the output matrices $C$ for the series and parallel tuned circuit are given as

$$
C_{inj} = \begin{bmatrix}
V_{AC} \sin \omega_r t \\
0
\end{bmatrix}, C_{fre} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

(5-6)

$$
C_{pos} = \begin{bmatrix}
V_{DC} \\
0
\end{bmatrix}, C_{neg} = \begin{bmatrix}
-V_{DC} \\
0
\end{bmatrix}, C_{fre} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

(5-7)

where $C_{pos}$ is for the positive phase injection state, $C_{neg}$ for the negative phase injection state, and $C_{fre}$ is for the free oscillation state. Note that for a half-wave energy injection converter $C_{neg}$ is not used.

The natural characteristics of the both series and parallel tuned circuits can be found by solving $x$ when the system is unforced. The solution $x_{CF}$ of the unforced complementary function for the above 2nd order differential equations for both series and parallel tuned tanks is given by

$$
x_{CF} = Ke^{\lambda t}
$$

(5-8)
where matrix $K$ is determined by initial conditions and $\lambda$ is the solution of the characteristic polynomial. For an LC resonant tank $\lambda$ is given as

$$\lambda = -\frac{\omega_r}{2Q} \pm \omega_r \sqrt{\frac{1}{4Q^2} - \frac{1}{4Q^2}}$$  \hspace{1cm} (5-9)$$

where $\omega_r$ is the resonant frequency and $Q$ is the quality factor of the primary resonant tank. Since the loading of the tank $R_1$ is modelled to be in series with $L_1$ the $Q$ of the resonant tank for both series and parallel tuned primary tanks is given by

$$Q = \frac{\omega L_1}{R_1}$$  \hspace{1cm} (5-10)$$

In order for the tank to be able to resonate, the discriminant of the characteristic polynomial must be less than zero. Hence the condition for resonance is given by

$$Q > 0.5$$  \hspace{1cm} (5-11)$$

In practice, the operating primary $Q$ is much larger than 0.5, as a circuit with low $Q$ will have damped sinusoidal waveforms. Given that $Q \gg 0.5$ the rising envelope $I_{1p}$ of the peak of $I_{L1}$ during injection instances can be approximated by [101]

$$I_{1p} = I_{1max} \left(1 - e^{-\frac{t}{\tau}}\right) + I_{10} e^{-\frac{t}{\tau}}$$  \hspace{1cm} (5-12)$$

where $I_{10}$ is the initial current in the tank and $I_{1max}$ is the asymptotic value of the peak of $I_{L1}$. When free oscillating, the decaying envelope $I_{1p}$ of the resonant current $I_{L1}$ is given by

$$I_{1p} = I_{10} e^{-\frac{t}{\tau}}$$  \hspace{1cm} (5-13)$$

where $I_{10}$ is the initial current in the tank. The time constant $\tau$ of the current envelope $I_{1p}$ is given as

$$\tau = \frac{2L_1}{R_1} = \frac{2Q}{\omega_r}$$  \hspace{1cm} (5-14)$$

Equation (5-14) implies that as primary $Q$ is increased the primary current takes longer to reach its peak asymptotic value and also takes longer to decay, while the vice versa is true when primary $Q$ is decreased. Since the circuit only changes state at the zero crossings of the resonant waveform in a soft-switched energy injection converter, it is more convenient to
measure time in number of resonant cycles elapsed. Time \( t \) can be replaced by the number of resonant cycles elapsed given by

\[
t_n = f_r t
\]  (5-15)

where \( f_r \) is the resonant frequency. Similarly the time constant \( \tau \) in terms of resonant cycles can also be expressed as

\[
\tau_n = f_r \tau = \frac{Q}{\pi}
\]  (5-16)

Hence, the envelope of \( I_{L1} \) given in Equations (5-12) and (5-13) can also be expressed in terms of resonant cycles elapsed. Thus, during injection instances the envelope of \( I_{L1} \) can be expressed as

\[
I_{1p} = I_{1max} \left( 1 - e^{-\frac{t_n}{\tau_n}} \right) + I_{10} e^{-\frac{t_n}{\tau_n}}
\]  (5-17)

and during free oscillation instances the envelope \( I_{1p} \) can be expressed as

\[
I_{1p} = I_{10} e^{-\frac{t_n}{\tau_n}}
\]  (5-18)

The asymptotic value of \( I_{L1} \) is dependent on the load present in the tank. For the series tuned primary tank \( I_{1max} \) is given as

\[
I_{1max} = \frac{\sqrt{2}V_{AC}}{R_1}
\]  (5-19)

and for the parallel tuned primary tank \( I_{1max} \) is given as

\[
I_{1max} = Q\sqrt{2}I_{AC}
\]  (5-20)

In all practical converters that detect the zero crossings of the resonant waveforms to achieve soft-switching, the magnitude of the resonant waveforms must stay above a certain level for it to be detectable by the controller. Thus, it is useful to determine the number of free oscillation instances before \( I_l \) reduces down to a level where zero current crossings can no longer be detected, or before the resonant current becomes discontinuous. Given that the minimum detectable limit for the peak of \( I_l \) is \( I_{l_{min}} \), in order to prevent the tank from collapsing the maximum allowable number of consecutive free oscillation cycles is given by

\[
t_n < -\ln \frac{I_{1min}}{I_{10}} \tau_n
\]  (5-21)
5.1.2 Relationship between resonant current, coupling and output voltage

The real part of the reflected impedance $Z_R$ can be used to estimate the resonant current required to maintain a certain secondary output voltage. By assuming that losses in the resonant components are negligible then the relationship between $Z_R$ and $Z_2$ is given as

$$I_1^2 Z_R = \frac{V_S^2}{Z_2} \tag{5-22}$$

where $Z_2$ is the impedance of the secondary tank seen by the induced voltage source $V_S$. Rearranging Equation (5-22) and substituting in Equation (2-14) for $Z_R$ we have

$$V_S = \omega I_1 k \sqrt{I_1 I_2} \tag{5-23}$$

From this we can see that the magnitude of the secondary induced voltage is dependent on the coupling $k$, the magnitude of $I_I$ and the operating frequency. Since the energy injection converter will be switched at the system resonant frequency and the reactance of the system is not altered, the operating frequency will be constant for a given coupling $k$. Since the secondary output voltage $V_{out}$ is shown to be directly proportional to $V_S$ in Equations (4-2) and (4-3). This implies that $V_{out}$ is also directly proportional to $I_I$ and $k$ given that operating frequency does not change. Hence, if the coupling decreases by 50% during operation then $I_I$ must increase by approximately 50% in order to maintain the same output voltage. Note that this is an approximation, since the operating frequency will also change with changes in $k$, which was previously illustrated in Figure 2-6.

In the following sections, energy injection control methods are explored for both series tuned and parallel tuned primary resonant tanks. The implementation of energy injection converters for both series and parallel tuned topologies are also presented.
5.2 Implementation of energy injection control on a series resonant primary tank

Common voltage-fed DC to AC converters can be converted into an energy injection converter for driving a series resonant tank without the need of additional power components. The most well known voltage-fed converters are the full-bridge and half-bridge converters [49]. Shown in Figure 5-4a) is the schematic of a full-wave energy injection converter and in Figure 5-4b) is the schematic of a half-wave energy injection converter. No changes are necessary to convert the full-bridge converter to a full-wave energy injection converter. However, the DC splitting capacitors in a conventional half-bridge converter are removed to allow free oscillation in the half-wave energy injection converter.

The full-wave energy injection converter consists of three switching states; positive injection state when \( S_1 \) and \( S_4 \) are ON and \( S_2 \) and \( S_3 \) are OFF, negative injection state when \( S_1 \) and \( S_4 \) are ON and \( S_2 \) and \( S_3 \) are OFF, and free oscillation and is achieved by turning \( S_2 \) and \( S_4 \) ON and turning \( S_1 \) and \( S_3 \) OFF. The free oscillation state shorts the series tuned tank allowing energy to oscillate between the coil and capacitor.

The operation of a half-wave energy injection converter consists of two switching states; positive injection state when \( S_1 \) is ON and \( S_2 \) is OFF, and free oscillation state when \( S_2 \) is ON and \( S_1 \) is OFF. Instead of applying half the AC voltage of a full-bridge converter across the tank like in a conventional half-bridge converter, the tank experiences the full DC input voltage during the positive injection state and must free oscillate during the negative phase of the resonant current \( I_{L1} \). The main difference between the two energy injection converters is that the full-wave can provide a larger equivalent \( V_{AC} \) for the same input voltage \( V_{DC} \) when compared to the half-wave, and the full-wave can inject energy during both positive and negative phases of \( I_{L1} \).

![Figure 5-4: Series tuned energy injection converter; a) Full-wave, b) half-wave](image-url)
For this study on series tuned energy injection control a half-wave energy injection converter was chosen due to its low component count and simpler switching scheme. This is more suitable for a TET system for powering implantable heart pumps as it makes the primary circuit more compact, there is also no need for the higher power which a full-wave converter can supply for the same input voltage. More details about a full-wave energy injection system can be found in [102].

5.2.1 Half-wave series tuned energy injection systems

The injection and free oscillation states of half-wave energy injection converter are shown in Figure 5-5. The injection state consists of $S_1$ being ON and $S_2$ being OFF as shown in Figure 5-5a), note that the converter should only be in injection state when the phase of $I_{L1}$ is positive. The free oscillation state consists of $S_1$ being OFF and $S_2$ being ON as shown in Figure 5-5b) and the converter can be in free oscillation regardless of the phase of $I_{L1}$.

![Equivalent circuit states of half-wave energy injection circuit](image)

**Figure 5-5: Equivalent circuit states of half-wave energy injection circuit, a) Injection state; b) free oscillation state**

When coupled together with a series or parallel tuned secondary pickup, the equivalent circuit topologies with the effect of coupling modelled as a current-dependent voltage source is shown in Figure 5-6. Note that the ESR of the resonant components is added to the equivalent circuit.
Figure 5-6: Half-wave series tuned energy injection system topologies, a) Series-series; b) Series-parallel

5.2.2 State-space simulation of half-wave series tuned energy injection converter

In order to simulate any energy injection control strategies, a two part state-space simulation of the half-wave energy injection controller together with the secondary pickup was established based on the circuits shown in Figure 5-6. The simulation is similar to the two part simulation presented in section 5.1.1, but is extended to incorporate the effects of coupling. Two more state variables $I_{L2}$ and $V_{C2}$ is added so that the circuit is fully defined. The switches $S_1$ and $S_2$ are modelled as ideal switches, such that they are modelled as a short circuit when ON and an open circuit when OFF. The state vector $x$ of the state-space simulation is given as
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\[ x = \begin{bmatrix} I_{L_1} \\ V_{C1} \\ I_{L_2} \\ V_{C2} \end{bmatrix} \quad (5-24) \]

The state-space representation of the half-wave series tuned energy injection converter with a series tuned pickup in the form of \( \dot{x} + Bx = C \) is given as

\[
\begin{bmatrix}
0 & -C_1 & 0 & 0 \\
L_1 & 0 & M & 0 \\
0 & 0 & 0 & -C_2 \\
M & 0 & L_2 & 0
\end{bmatrix} \dot{x} + \begin{bmatrix}
1 & 0 & 0 & 0 \\
(R_{L1} + R_{C1}) & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & (R_{L2} + R_{C2} + R_2) & 1
\end{bmatrix} x = C \quad (5-25)
\]

and the state-space representation of the half-wave series tuned energy injection converter with a parallel tuned pickup in the form of \( \dot{x} + Bx = C \) is given as

\[
\begin{bmatrix}
0 & -C_1 & 0 & 0 \\
L_1 & 0 & M & 0 \\
0 & 0 & 0 & C_2 \\
M & 0 & L_2 & -R_{C2}C_2
\end{bmatrix} \dot{x} + \begin{bmatrix}
1 & 0 & 0 & 0 \\
(R_{L1} + R_{C1}) & 1 & 0 & 0 \\
0 & 0 & (1 + \frac{R_{L2}}{R_2}) & 0 \\
0 & 0 & R_{L2} & -1
\end{bmatrix} x = C \quad (5-26)
\]

The output matrix \( C \) is different for the two parts of the simulation. The output matrices \( C \) for both SS and SP topologies are given as

\[
C_{INJ} = \begin{bmatrix} 0 \\ V_{DC} \\ 0 \end{bmatrix}, \quad C_{FRE} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5-27)
\]

where \( C_{INJ} \) is used for the energy injection state and \( C_{FRE} \) is used for the free oscillation state.

The output voltage \( V_{out} \) which is equivalent to the voltage across \( R_2 \) for the SS topology is given by

\[
V_{SSout} = [0 \quad 0 \quad R_2 \quad 0] x \quad (5-28)
\]

and the output voltage \( V_{out} \) for the SP topology can be approximated by the voltage \( V_{C2} \) assuming that \( I_{C2}R_{C2} \ll V_{C2} \) is given by

\[
V_{SPout} = [0 \quad 0 \quad 0 \quad 1] x \quad (5-29)
\]
5.2.3 Implementation of a ZCS half-wave series tuned energy injection converter

The control circuitry of the half-wave series tuned energy injection converter presented in this thesis consists of two main parts. The part described here is the zero current detection circuit for soft-switched operation. The other part consists of the power flow controller which determines when injection and free oscillation instances should occur in order to regulate the output voltage. Different control strategies are discussed and their practical implementations are given in Section 5.3.

ZCS gate drive circuitry

The zero current crossing gate drive circuitry and its signal labels are shown in Figure 5-7.

Figure 5-7: ZCS gate drive circuitry for half-wave series tuned energy injection circuit

To achieve soft switching a current transformer is used to detect the zero current crossings of the primary resonant current $I_{L1}$. The output of the current transformer is then fed into an inverting Schmitt trigger to generate a square wave signal $V_{ZCS}$ whose rising and falling edges are at the zero current crossings of $I_{L1}$. Before $V_{ZCS}$ used to drive $S_1$ and $S_2$ via the half-bridge gate driver, it is AND gated with the signal $V_{INJ}$ which governs whether an injection instance can occur or not. Whenever $V_{ZCS}$ and $V_{INJ}$ are high the circuit will be forced into its injection state, if either $V_{INJ}$ or $V_{ZCS}$ is low the circuit be forced into its free oscillation state. Since injection can only occur when the current is in its positive phase $V_{INJ}$ should only change during the negative phase of the resonant current. To achieve this $V_{INJ}$ is produced from the output of a positive-edge triggered D flip-flop $FF1$ whose input is the control signal $V_{CON}$ from the power flow controller and is clocked by $V_{ZCS}$. $FF1$ acts as a current phase detector, since the control signal $V_{CON}$ only propagates to $V_{INJ}$ at the start of the negative current phase preparing $V_{INJ}$ for the next positive current phase. This ensures that any commands from the
control signal is synchronised to the resonant frequency so that it does not compromise zero current switching.

5.3 Power flow control methods for half-wave series tuned energy injection converter

Although the concept of energy injection is simple there are many primary side power flow control strategies that can be implemented on the half-wave series tuned energy injection converter. The methods presented in this section are different to control strategies used in magnitude control such as Pulse Width Modulation (PWM) control and hysteresis control [103]. This is because many magnitude control methods are hard-switched or continuous and cannot be applied to a ZCS resonant converter. The control methods presented here are integer based controllers, since control action can only takes place in between resonant cycles such that it is in sync with the zero current switching.

5.3.1 Cycle-by-cycle output voltage control

The most straightforward way to regulate the voltage at the output with energy injection is to inject energy when the output voltage is below what is desired, and to free oscillate when output voltage is too high. A cycle-by-cycle output voltage controller means that a decision has to be made during each resonant cycle whether to inject energy or free oscillate. This implies that during the period of one resonant cycle the output voltage has to be sampled, sent back to the primary controller and compared to generate the control signal $V_{CON}$. A schematic of a cycle-by-cycle output voltage controller which uses wireless communications to receive feedback for the power flow controller is shown in Figure 5-8.
Since feedback from the load must be obtained wirelessly there is always going to be more delay than feedback from a direct wired connection. Depending on the operating frequency and speed of the wireless communications, this method may be difficult to implement on a high frequency TET system. If feedback is obtained at a rate slower than the resonant frequency, the controller may not be able to keep the output voltage steady, especially if damping in the primary tank is high. If feedback is slow or dropouts occur in the wireless link, the cycle-by-cycle controller can easily be saturated. This will result in extremely large output oscillations and most likely lead to the collapse of the primary resonant current. Thus, the cycle-by-cycle control is not very practical for TET systems as the output feedback delay is unavoidable. Nonetheless, the control resolution of a functioning cycle-by-cycle output voltage controller is unmatched by all other control methods, since control action is taken at every resonant cycle and is derived directly from the output voltage. The MATLAB code for simulating the time domain waveforms of a series tuned half-wave energy injection converter with cycle-by-cycle output voltage control is presented in Appendix C.

5.3.2 Cycle-by-cycle peak current control

Since a very high speed wireless link is required to control the output voltage directly cycle-by-cycle, this problem can be overcome by regulating the primary resonant current instead. Since the sampling of the primary current $I_1$ can be performed directly by the power flow controller feedback delay will not be an issue for this cycle-by-cycle controller. A sample of the primary current can be obtained from the same current transformer used for the zero current switching gate drive circuit. The voltage $V_{II}$ from the current transformer is compared to a current limit $V_{ref}$. If the peak of $V_{II}$ (corresponds to the peak of $I_1$) is greater than $V_{ref}$, then no energy injection will occur at the next positive current phase, rather the converter will continue to free oscillate. However, if the peak of $V_{II}$ is less than $V_{ref}$ then energy injection will occur at the next positive current phase. Since the magnitude of the primary resonant current does not directly dictate the output voltage, the current limit $V_{ref}$ will need to be determined by a PID controller. The PID controller compares the output voltage $V_{out}$ received wirelessly to the desired output voltage $V_d$. Since the feedback from the secondary now only determines the reference voltage $V_{ref}$ (current limit), feedback delay will only make the response of the controller slower rather than saturating the control. Proper design of the PID controller can also help improve the response of the controller by compensating for minor
delays in the feedback. Shown in Figure 5-9 are plots illustrating the resonant current and the timings of \( S_1 \) and \( S_2 \) in cycle-by-cycle peak current control.

![Diagram](image)

**Figure 5-9: Cycle-by-cycle primary peak current control waveforms**

To generate the control signal \( V_{CON} \) with this control two comparators (\( COMP1 \) and \( COMP2 \)) and a positive-edge triggered D flip-flop \( FF2 \) can be configured in a way shown in Figure 5-10. \( COMP1 \) compares the resonant current represented by \( V_{I1} \) to the current limit \( V_{ref} \) using an inverting Schmitt trigger with no hysteresis. The output of \( COMP1 \) is high when \( V_{ref} > V_{I1} \) and low when \( V_{ref} < V_{I1} \). Since the instantaneous value of \( V_{I1} \) will always drop below \( V_{ref} \) at the zero crossing of \( V_{I1} \); it can never trigger the free oscillation that should occur at the next positive current phase when \( V_{ref} < V_{I1} \). Hence, the flip-flop \( FF2 \) is used to hold the signal when the peak of \( V_{I1} \) is greater than \( V_{ref} \). This is achieved by tying the input of \( FF2 \) to +5V (logic “high”) and connecting the output of \( COMP1 \) to the clock input of \( FF2 \). This will produce the correct injection control signal \( V_{CON} \) at the inverted output of \( FF2 \) when free oscillation needs to occur at the next positive current phase. However, since the input of \( FF2 \) is always high, \( FF2 \) needs to be reset straight after the positive current phase, so that peak current comparison can be performed again. Hence, the comparator \( COMP2 \) is used to produce a square wave that resets \( FF2 \). The output of \( COMP2 \) has a rising edge at the zero crossing instant when \( I_{L1} \) enters the negative current phase; and has a falling edge delayed by the
hysteresis on COMP2, such that it is slightly after the zero crossing instant when $I_{L1}$ enters the positive current phase. This signal when fed to the active-low CLEAR pin of FF2, resets the output of FF2, making it ready to trip whenever the peak of $V_{Il}$ increases above $V_{ref}$.

![Diagram](image)

**Figure 5-10: Cycle-by-cycle peak current control circuitry**

The biggest drawback of the primary peak current controller is that its control resolution is dependent on the operating Q of the primary resonant tank. Equation (5-16) states that if Q is high the envelope of the resonant current takes longer to build up and longer to die down. This implies that the difference between consecutive current peaks is small, such that small changes in the current limit $V_{ref}$ would change the energy injection scheme resulting in control action. However, when primary circuit Q is low the difference between consecutive current peaks can be quite large. This implies that small changes in the current limit $V_{ref}$ may not produce any control action, as the energy injection scheme along with the resonant current $I_{L1}$ will stay the same. Therefore, the resolution of a peak current control decreases when primary circuit Q decreases. The MATLAB code for simulating the time domain waveforms of a series tuned half-wave energy injection converter with cycle-by-cycle peak current control is presented in Appendix D.
5.3.3 Pulse density modulation control

The two previous control techniques presented were both cycle-by-cycle techniques, which mean the decision to inject energy or to free oscillate is determined during every resonant cycle. Although this provides the advantage of making the controller react as fast as possible to any changes in the system, it may be practically difficult to implement due to issues with slow feedback for the output voltage control, and unstable control resolution for the peak current control. Instead of determining the control action cycle-by-cycle, a Pulse Density Modulation (PDM) pattern which consists of a pre-determined sequence of injections and free oscillations can be selected to drive the converter based on the feedback received. The concept of PDM is in some ways similar to that of PWM. PWM control comprises of a fixed control frequency with a varying duty cycle. Whereas a PDM pattern has a control frequency measured in number of cycles, so that ZCS is not compromised. This means that the period of the pattern is related by the resonant frequency. Each PDM pattern also has a certain “pulse density” or “injection ratio” which is analogous to the duty cycle of a PWM waveform. Theoretically the number of control steps possible with a PWM waveform is limitless, whereas in PDM pattern control the number of control steps is equal to the pattern length of the PDM pattern measured in number of resonant cycles. Shown in Figure 5-11 are the resonant current and gate drive waveforms for a half-wave energy injection converter driven by a PDM pattern. The PDM pattern shown in Figure 5-11 has a length of 5 cycles and an injection ratio of 3/5, which means that the pattern has a density of 3 energy injection instances within 5 resonant cycles. For the PDM control patterns described in this thesis a “1” will denote an energy injection instance and a “0” will denote a free oscillation instance for PDM patterns written in line. The PDM pattern shown in Figure 5-11 can be represented in line by “11100”.

High speed feedback is also not necessary for PDM pattern power flow control, the rate of the feedback only determines the rate of pattern changes which corresponds to the response speed of the control. PDM pattern control is also superior to peak current control as its resolution is not dependent on the Q of the primary circuit. Control action is guaranteed with every control step, since the injection ratio is directly modified. Since all the energy injection and free oscillation instances are pre-defined by the PDM pattern, provided that the PDM pattern has enough resolution, a slow feedback loop would not create large output oscillations.
Depending on the complexity of the PDM patterns and whether variable pattern lengths are required, generating $V_{\text{CON}}$ for PDM control may be difficult with just discrete electronics and logic. Discrete component implementation will also limit the ability of the controller to only one set of PDM patterns, whereas a digital controller such as a micro-controller or FPGA will provided much more flexibility. Since the algorithm used in a digital controller can easily
generate a wide range of PDM patterns of variable lengths. Shown in Figure 5-12 is a block diagram of a PDM controller, it consists of a pattern generator which is clocked by $V_{ZCS}$ so that the PDM is in sync with the ZCS frequency of the system.

The main disadvantage with a PDM controller is that its control circuitry requires the use of digital controllers which can output the patterns at the required frequency. Since the resonant frequency is several hundred kHz, common micro-controllers may not have the speed to do the job. Thus, more expensive controllers such as FPGAs may be required. Apart from this, the control performance of a PDM controller depends on the PDM patterns and the pattern selection algorithm.

PDM patterns of a certain pattern length and injection ratio does not define a distinct PDM pattern. In fact there may be several different patterns that will fit the same description. For example, given a pattern length of 8 and an injection ratio of 3/8, three distinct PDM patterns can be formed: “11100000”, “10101000” and “10010010”. Note that the patterns “10100010”, “10010100” and “10101000” are all identical due to the repeating nature of the pattern. PDM patterns that have the same injection ratio will apply an input voltage waveform (square wave pulses) of the same RMS voltage across the series resonant tank. However, different PDM patterns with the same injection ratio can greatly affect the waveform and RMS of the resonant current/voltage. To illustrate this, shown in Figure 5-13 is the simulated input voltage waveform and the primary resonant current $I_{LL}$ for a series tuned resonant circuit driven by the PDM patterns “11110000” and “10101010”. These two PDM patterns have the same pattern length of 8 and the same injection ratio of 4/8. However, the RMS value of $I_{LL}$ due to the “11110000” pattern is $0.226A_{\text{rms}}$ and due to the “10101010” pattern it is $0.217A_{\text{rms}}$. Whereas the RMS value of the input voltage waveform is fixed at $0.5V_{\text{rms}}$. Apart from differences in the RMS value of the resonant current due to the different patterns, the Crest factor or the peak-to-RMS ratio of the resonant current is also significantly different. A sine wave with constant amplitude has a Crest factor of $\sqrt{2}$ or approximately 1.414. The resulting Crest factor for the PDM pattern “11110000” was 1.77 and for the pattern “10101010” it was 1.42. A waveform with a low crest factor or a relatively constant envelope is highly desirable as it has lower peak values, which mean that less stress is placed on the components. The MATLAB code for producing the time domain waveforms shown in Figure 5-13 for a series tuned half-wave energy injection converter with PDM pattern control is presented in Appendix E.
5.3.4 Proposed PDM control pattern control

Proposed in this section is a PDM pattern generator that aims to produce PDM patterns that minimises the resulting Crest factor of the resonant current. It can be seen intuitively that the envelope ripple of the primary resonant current $I_{LI}$ will decrease, if the energy injection instances of a PDM pattern is spread evenly amongst the free oscillation instances. For a given injection ratio $n/N$, where $n$ is the number of injection instances and $N$ is the pattern length. The proposed algorithm suggests that if the injection instances are evenly spread for a given injection ratio, the maximum number of free oscillation instances in between injection instances is equal to the quotient of $N/n$. For the case when $n$ divides exactly into $N$ the maximum number of free oscillation instances in between injection instances is equal to $N/n-1$. Presented in Figure 5-14 is a diagram showing how this proposed PDM pattern, which minimises the number of free oscillation instances in between each injection instance, is generated given an injection ratio $n/N$. 
Given an energy injection ratio $\frac{n}{N}$

$N = \text{Pattern length}$

$n = \text{Number of injection instances/sub-patterns}$

If $n > \frac{N}{2}$ solve for $n = N - n$ and invert sub-pattern

Sub-pattern length $l = \text{quotient} \frac{N}{n}$

Number of long sub-patterns $n_L = \text{mod} \frac{N}{n}$

Number of short sub-patterns $n_S = n - n_L$

A simpler sub-optimal method of generating a PDM pattern of a certain length and injection ratio is to simply place all the energy injection instances together, followed by all the free oscillation instances. An example of what the Proposed PDM patterns look like compared to this sub-optimal method of generating a PDM pattern is shown in Table 5-1 for a pattern length of 16.

![Proposed PDM Pattern Structure](image)

**Figure 5-14: Proposed PDM pattern algorithm**
Table 5-1: Proposed PDM patterns versus sub-optimal PDM patterns, N=16

<table>
<thead>
<tr>
<th>Injection Ratio</th>
<th>Proposed PDM pattern</th>
<th>Sub-optimal PDM pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16</td>
<td>10000000 00000000</td>
<td>10000000 00000000</td>
</tr>
<tr>
<td>2/16</td>
<td>10000000 10000000</td>
<td>11000000 00000000</td>
</tr>
<tr>
<td>3/16</td>
<td>10000100 00100000</td>
<td>11100000 00000000</td>
</tr>
<tr>
<td>4/16</td>
<td>10001000 10001000</td>
<td>11110000 00000000</td>
</tr>
<tr>
<td>5/16</td>
<td>10010010 01001000</td>
<td>11111000 00000000</td>
</tr>
<tr>
<td>6/16</td>
<td>10101001 00100100</td>
<td>11111100 00000000</td>
</tr>
<tr>
<td>7/16</td>
<td>10101010 10101000</td>
<td>11111110 00000000</td>
</tr>
<tr>
<td>8/16</td>
<td>10101010 10101010</td>
<td>11111111 00000000</td>
</tr>
<tr>
<td>9/16</td>
<td>01010101 01011011</td>
<td>11111111 10000000</td>
</tr>
<tr>
<td>10/16</td>
<td>01010110 11011011</td>
<td>11111111 11000000</td>
</tr>
<tr>
<td>11/16</td>
<td>01101101 11011101</td>
<td>11111111 11100000</td>
</tr>
<tr>
<td>12/16</td>
<td>01110111 01110111</td>
<td>11111111 11110000</td>
</tr>
<tr>
<td>13/16</td>
<td>01111011 11011111</td>
<td>11111111 11111000</td>
</tr>
<tr>
<td>14/16</td>
<td>01111111 01111111</td>
<td>11111111 11111100</td>
</tr>
<tr>
<td>15/16</td>
<td>01111111 11111111</td>
<td>11111111 11111110</td>
</tr>
<tr>
<td>16/16</td>
<td>11111111 11111111</td>
<td>11111111 11111111</td>
</tr>
</tbody>
</table>

5.3.5 Comparison of half-wave series tuned energy injection control methods

Implementation

Each of the energy injection control methods except for the cycle-by-cycle output voltage control (which cannot be implemented open loop) was practically implemented on a series tuned half-wave energy injection converter with the control circuitry shown in their respective sections.

For the implementation of the PDM control a Xilinx Spartan 3E FPGA was used to generate the PDM patterns to drive the half-wave converter. A ring counter with parallel load was implemented in the FPGA so that any PDM pattern could be easily generated. The PDM pattern was kept in sync with the resonant cycle by using $V_{ZCS}$ as the clock input to the ring counter. The FPGA was programmed with a Hevday HLM-0503 Logic module and the implementation of an 8-bit PDM pattern generator is shown in Figure 5-15.
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Figure 5-15: 8-bit PDM pattern generator implemented in Hevday Interactive logic

Experimental setup and results

To compare the energy injection control methods, the open loop characteristics of cycle-by-cycle peak current control, 16-bit Sub-optimal PDM pattern control and 16-bit proposed PDM pattern control was compared. The control was swept from 0 to 100% and the resulting output voltage was recorded at each step.

A series tuned and a parallel tuned secondary circuit with load $R_2=10$ and secondary circuit $Q_S$ of approximately 1.8 was used for the comparison. The coupling between primary and secondary power transfer coils were fixed at $k=0.3$. A summary of the resonant parameters of the experiment is shown in Table 5-2.

The state-space representation of the series-tuned half-wave energy injection converter was also used to simulate the output voltage and Crest factor of the resonant waveforms. The same set of resonant parameters shown in Table 5-2 was used in the simulation.

Table 5-2: Resonant parameters of Series tuned energy injection systems

<table>
<thead>
<tr>
<th>Series tuned half-wave energy injection primary converter</th>
<th>$L_1=11.2\mu \text{H}$</th>
<th>$C_1=33\text{nF}$</th>
<th>$f_0=262 \text{ kHz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series secondary $Q_S=1.81$</td>
<td>$V_{IN}=14\text{V}$, $k=0.3$</td>
<td>$V_{IN}=14\text{V}$, $k=0.3$</td>
<td></td>
</tr>
<tr>
<td>$L_2$</td>
<td>10.9\mu H</td>
<td>$L_2$</td>
<td>3.3\mu H</td>
</tr>
<tr>
<td>$C_2$</td>
<td>33\text{nF}</td>
<td>$C_2$</td>
<td>110\text{nF}</td>
</tr>
<tr>
<td>$R_2$</td>
<td>10</td>
<td>$R_2$</td>
<td>10</td>
</tr>
<tr>
<td>$f_0$</td>
<td>265 kHz</td>
<td>$f_0$</td>
<td>264 kHz</td>
</tr>
</tbody>
</table>
The simulated RMS output voltage and the measured RMS voltage for the Series-series energy injection system is shown in Figure 5-16. While the same simulated and experimental results for the Series-parallel energy injection system is shown in Figure 5-17. In both figures the simulation and experimental results closely matched with each other. Since the coupling and the secondary circuit $Q_s$ was similar for both systems, the absolute output voltage was also very similar between the two setups. Shown in Figure 5-18 is the simulated Crest factor of the resonant current $I_{L1}$ across the control range for each control method.

![Figure 5-16: Output voltage versus control percentage for peak current control and PDM control on a SS energy injection circuit at $k=0.3$](image-url)
Figure 5-17: Output voltage versus control percentage for peak current control and PDM control on a SP energy injection circuit at $k=0.3$

Figure 5-18: Crest factor of primary current $I_{L1}$ versus output voltage for peak current control and PDM control on a SS energy injection circuit at $k=0.3$
Minimum control percentage

It can be seen in Figure 5-16 and Figure 5-17 that the practical results only exist for control percentages over 40% for the peak current control, over 50% for Sub-optimal PDM control and over 25% for the proposed PDM control. In practice, when the resonant current $I_{L1}$ decreases to a level where the ZCS controller can no longer detect the current, the resonant current in the tank will collapse. Hence, the point in the control when this happens is known as the minimum control percentage. This minimum control percentage is dependent on the control method and the operating Q of the primary circuit, since $I_{L1}$ dampens at a rate that is inversely proportional to Q.

It was practically determined that the peak current controller of the energy injection circuit used in this study cannot compare the peak of $V_{I1}$ (peak of $I_{L1}$) to $V_{CON}$ if the peak is less than 1A. This means that the minimum current limit allowed for the system is $I_{1\text{min}}=1$ (this was determined at the highest possible operating Q of the system, that is when there is no secondary). For the PDM controller used in this study, the zero current crossing signal $V_{ZCS}$ can no longer be generated when $I_{L1}$ drops below 0.05A, such that $I_{1\text{min}}=0.05$. The limit for the PDM controller is much lower than the peak current controller because $I_{L1}$ does not need to be compared with $V_{CON}$ in PDM pattern control.

To analytically determine what the injection ratio will be at the practical minimum control percentage for each control method, the peak of the primary current can be estimated using Equations (5-17) and (5-18) which are the equations for the envelope of the resonant current. Using the parameters in Table 5-2 the required information can be worked out. Firstly, the reflected impedance seen by the SS system is calculated to be $Z_{RSS}=2.97$ and the reflected impedance seen by the SP system calculated to be $Z_{RSP}=3-1.63i$. This corresponds to a primary circuit $Q=6.2$ and $\tau_{n}=1.97$ for the SS system, and $Q=5.65$ and $\tau_{n}=1.8$ for the SP system. The input voltage for the series tuned half-wave converter was $V_{DC}=14V$ and by using Equation (5-2) the equivalent RMS AC voltage operating at $\omega_r$ can be worked out to be $V_{AC}=8.91\text{V}_{\text{rms}}$. $I_{1\text{max}}$ for the system is obtained by Equation (5-19), which gives $I_{1\text{max}}=3$ for the SS system and $I_{1\text{max}}=2.97$ for the SP system.

Peak current control analysis

Since the next state of the converter is determined during the previous cycle for a cycle-by-cycle controller, the injection ratio at minimum control percentage can be obtained with envelope analysis at the time when the peak of $I_i$ is just below $I_{1\text{min}}$. For the peak current
controller $I_{1\text{min}}$ was previous found to be equal to 1A. Hence, the control action taken by the peak current controller for the SS system is as follows:

<table>
<thead>
<tr>
<th>Cycle no.</th>
<th>At start of cycle</th>
<th>At end of cycle</th>
<th>Control action for next cycle (compare $I_L$ to $I_{1\text{min}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$I_{10} = 1$</td>
<td>$I_{1p} = 3 \left(1 - e^{-\frac{1}{1.97}}\right) + I_{10}e^{-\frac{1}{1.97}} = 1.8$</td>
<td>Energy injection</td>
</tr>
<tr>
<td>1</td>
<td>$I_{10} = 1.8$</td>
<td>$I_{1p} = I_{10}e^{-\frac{1}{1.97}} = 1.08$</td>
<td>Free oscillate (must Inject)</td>
</tr>
<tr>
<td>2</td>
<td>$I_{10} = 1.08$</td>
<td>$I_{1p} = I_{10}e^{-\frac{1}{1.97}} = 0.65$</td>
<td>Collapsed ($I_I &lt; I_{1\text{min}}$)</td>
</tr>
</tbody>
</table>

For this experimental setup, if the initial peak of $I_{L1}$ is equal $I_{1\text{min}}$, two consecutive free oscillation instances will cause the peak of $I_{L1}$ to fall below $I_{1\text{min}}$ and collapsing the resonant current. Hence, the minimum control percentage for this setup is when the current limit causes the injection ratio of the converter to fall below $\frac{1}{2}$. From the analysis the control pattern at minimum control percentage should be “10”, this matches with what was obtained experimentally. The analysis for the SP system is shown below and is almost exactly the same as the SS system.

<table>
<thead>
<tr>
<th>Cycle no.</th>
<th>At start of cycle</th>
<th>At end of cycle</th>
<th>Control action for next cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$I_{10} = 1$</td>
<td>$I_{1p} = 1$</td>
<td>Inject</td>
</tr>
<tr>
<td>1</td>
<td>$I_{10} = 1$</td>
<td>$I_{1p} = 2.97 \left(1 - e^{-\frac{1}{1.8}}\right) + I_{10}e^{-\frac{1}{1.8}} = 1.84$</td>
<td>Free oscillate</td>
</tr>
<tr>
<td>2</td>
<td>$I_{10} = 1.84$</td>
<td>$I_{1p} = I_{10}e^{-\frac{1}{1.8}} = 1.06$</td>
<td>Free oscillate (must Inject)</td>
</tr>
<tr>
<td>3</td>
<td>$I_{10} = 1.06$</td>
<td>$I_{1p} = I_{10}e^{-\frac{1}{1.8}} = 0.61$</td>
<td>Collapsed ($I_I &lt; I_{1\text{min}}$)</td>
</tr>
</tbody>
</table>

**PDM pattern control analysis**

Envelope analysis can also be performed to determine the minimum control percentage of PDM pattern control. Since the patterns are fixed the minimum control percentage of an open loop system be calculated at start up, where $I_{10} = 0$. The envelope analysis for each of the 16-bit sub-optimal patterns is shown below for the SS system. Since the reflected impedance of the SP system is similar, the end results are the same and the analysis is not shown.
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<table>
<thead>
<tr>
<th>n/N</th>
<th>After injection instances</th>
<th>At end of pattern</th>
<th>$I_{lp}&gt;I_{imin}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{1}{1.97}}\right) = 1.194$</td>
<td>$I_{lp} = 1.194 e^{-\frac{15}{1.97}} = 0.0006$</td>
<td>No</td>
</tr>
<tr>
<td>2/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{2}{1.97}}\right) = 1.913$</td>
<td>$I_{lp} = 1.913 e^{-\frac{14}{1.97}} = 0.0010$</td>
<td>No</td>
</tr>
<tr>
<td>3/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{3}{1.97}}\right) = 2.346$</td>
<td>$I_{lp} = 2.346 e^{-\frac{13}{1.97}} = 0.003$</td>
<td>No</td>
</tr>
<tr>
<td>4/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{4}{1.97}}\right) = 2.606$</td>
<td>$I_{lp} = 2.606 e^{-\frac{12}{1.97}} = 0.006$</td>
<td>No</td>
</tr>
<tr>
<td>5/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{5}{1.97}}\right) = 2.763$</td>
<td>$I_{lp} = 2.763 e^{-\frac{11}{1.97}} = 0.01$</td>
<td>No</td>
</tr>
<tr>
<td>6/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{6}{1.97}}\right) = 2.857$</td>
<td>$I_{lp} = 2.857 e^{-\frac{10}{1.97}} = 0.018$</td>
<td>No</td>
</tr>
<tr>
<td>7/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{7}{1.97}}\right) = 2.914$</td>
<td>$I_{lp} = 2.914 e^{-\frac{9}{1.97}} = 0.03$</td>
<td>No</td>
</tr>
<tr>
<td>8/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{8}{1.97}}\right) = 2.948$</td>
<td>$I_{lp} = 2.948 e^{-\frac{8}{1.97}} = 0.05$</td>
<td>Borderline</td>
</tr>
<tr>
<td>9/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{9}{1.97}}\right) = 2.969$</td>
<td>$I_{lp} = 2.969 e^{-\frac{7}{1.97}} = 0.085$</td>
<td>Yes</td>
</tr>
<tr>
<td>10/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{10}{1.97}}\right) = 2.981$</td>
<td>$I_{lp} = 2.981 e^{-\frac{6}{1.97}} = 0.142$</td>
<td>Yes</td>
</tr>
<tr>
<td>11/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{11}{1.97}}\right) = 2.989$</td>
<td>$I_{lp} = 2.989 e^{-\frac{5}{1.97}} = 0.236$</td>
<td>Yes</td>
</tr>
<tr>
<td>12/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{12}{1.97}}\right) = 2.993$</td>
<td>$I_{lp} = 2.993 e^{-\frac{4}{1.97}} = 0.393$</td>
<td>Yes</td>
</tr>
<tr>
<td>13/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{13}{1.97}}\right) = 2.996$</td>
<td>$I_{lp} = 2.996 e^{-\frac{3}{1.97}} = 0.653$</td>
<td>Yes</td>
</tr>
<tr>
<td>14/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{14}{1.97}}\right) = 2.998$</td>
<td>$I_{lp} = 2.998 e^{-\frac{2}{1.97}} = 1.086$</td>
<td>Yes</td>
</tr>
<tr>
<td>15/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{15}{1.97}}\right) = 2.999$</td>
<td>$I_{lp} = 2.999 e^{-\frac{1}{1.97}} = 1.805$</td>
<td>Yes</td>
</tr>
<tr>
<td>16/16</td>
<td>$I_{lp} = 3 \left(1 - e^{-\frac{16}{1.97}}\right) = 2.999$</td>
<td>$I_{lp} = 2.999 e^{-\frac{0}{1.97}} = 2.999$</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The analysis shows that the PDM patterns should work for injection ratios above 8/16 which is exactly what was observed in practice. It also indicates that the peak of $I_1$ at the end of the 8/16 pattern is right on the current limit. Experimentally the sub-optimal PDM pattern 8/16 was achievable with the SS system but not with the SP system.

For the proposed PDM patterns the envelope analysis for each pattern is not as straightforward as the injection instances are spread out. However, since we are only interested in the minimum injection percentage the only the longest stretch of free oscillations in each pattern needs to be considered. Also many of the low injection ratio patterns are made up of
repetitions of shorter patterns. The envelope analysis for the 16-bit proposed PDM patterns for the SS system is shown below. Once again the analysis for the SP system is not shown since the end result is the same.

<table>
<thead>
<tr>
<th>n/N</th>
<th>After injection instances</th>
<th>At end of pattern</th>
<th>$I_{1f} &gt; I_{1\text{min}}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16</td>
<td>$I_{1p} = 3 \left( 1 - e^{-\frac{1}{1.97}} \right) = 1.194$</td>
<td>$I_{1p} = 1.194e^{-\frac{15}{1.97}} = 0.0006$</td>
<td>No</td>
</tr>
<tr>
<td>2/16 (1/8)</td>
<td>$I_{1p} = 3 \left( 1 - e^{-\frac{1}{1.97}} \right) = 1.194$</td>
<td>$I_{1p} = 1.194e^{-\frac{7}{1.97}} = 0.034$</td>
<td>No</td>
</tr>
<tr>
<td>3/16 (1/6)</td>
<td>$I_{1p} = 3 \left( 1 - e^{-\frac{1}{1.97}} \right) = 1.194$</td>
<td>$I_{1p} = 1.194e^{-\frac{5}{1.97}} = 0.09$</td>
<td>Yes (not in practice)</td>
</tr>
<tr>
<td>4/16 (1/4)</td>
<td>$I_{1p} = 3 \left( 1 - e^{-\frac{1}{1.97}} \right) = 1.194$</td>
<td>$I_{1p} = 1.194e^{-\frac{3}{1.97}} = 0.26$</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The analysis indicates that the proposed PDM patterns should be able to work down to an injection ratio of 3/16. However, experimentally the lowest injection ratio that could be stably sustained was 4/16 for both the SS and SP systems.

**Control percentage versus output voltage**

Figure 5-16 and Figure 5-17 show that the proposed PDM control patterns give a linear relationship between its control percentage and output voltage, whereas the sub-optimal PDM patterns exhibit a square root type relationship. The plots indicate that the sub-optimal patterns may generate larger output voltages than the proposed PDM patterns given the same injection ratio, seemingly implying that the sub-optimal patterns are more efficient. However, the reason why the sub-optimal patterns generate larger output voltages is because the patterns generate a higher RMS primary current and consequently more power is drawn from the supply. Although PDM patterns with the same injection ratio impose the same RMS voltage across the resonant tank the harmonic content of the input voltage is different. Keep in mind that only harmonic content of the input that is at the same frequency as the frequency of the resonant tank will contribute most significantly in building up the resonant current. The greatest difference between the output voltage due to the sub-optimal and proposed PDM patterns, was at injection ratio 8/16. With the sub-optimal PDM pattern “1111111000000000” the input voltage generated from the 8 consecutive injection instances has a fundamental frequency which is the same as the resonant voltage. Hence, $I_{L1}$ is built up rapidly reaching a peak close to its asymptotic value. With the proposed PDM pattern “1010101010101010” each injection instance is separated by a free oscillation instance.
Although the injection instances have a frequency that is the same as the resonant frequency, the overall fundament frequency of the pattern is half the resonant frequency. Another way to look at why the proposed PDM patterns generate lower primary current is because the Crest factor of the current waveform is kept at a minimum. This means that although the current waveform from the sub-optimal PDM pattern has the same current-time integral as the proposed PDM pattern, the higher peak current values caused by the sub-optimal PDM pattern would make the RMS value of the current higher. Thus, the sub-optimal patterns actually provide better power transfer capability at the center of its control range.

Regarding the control action of the peak current controller, it can be seen in Figure 5-16 and Figure 5-17 that there are control percentage intervals that have no effect on the output voltage or no control action. In this is particular setup, there are large intervals of no control action between the control percentage of 46% to 64% (when injection ratio is 1/2 with pattern “10”) and between the control percentage of 70% to 83% (when injection ratio is 2/3 and pattern is “110”). Intervals of no control action become more apparent when the primary circuit Q decreases, causing $I_{Li}$ to build up rapidly during injection instances and damp rapidly during free oscillation. Since it is the peak current that is being compared, large variations between consecutive peak values will decrease the resolution of the controller.

**Crest factor**

Shown in Figure 5-18 is a plot of the Crest factor of the simulated resonant current $I_{Li}$ against the simulated output voltage for the three control methods compared in this study. It can be seen that the proposed PDM patterns produces resonant waveforms with Crest factors much lower than the sub-optimal patterns. Note that the Crest factor is the same at both ends on the control as the PDM patterns are identical. Peak current control also performs well in this regard as the Crest factor of the resonant waveforms are almost identical to the PDM patterns at the same output voltage. However, note the output voltage increments are not even due to the non-linearity of peak current control as previously discussed. Resonant waveforms with lower Crest factors are beneficial as this reduces the peak current/voltage stresses on the switches and resonant components, allowing smaller and faster components to be used. In addition, decreasing the Crest factor also decreases the EMI generated by the converter as waveforms with lower peaks result in lower radiated power.
5.4 Implementation of energy injection control on a parallel resonant primary tank

A current-fed converter is required to drive a parallel tuned resonant tank. Since a current source does not exist naturally like a voltage source, a quasi-current source is often achieved by connecting a series inductor to a voltage source [54]. Shown in Figure 5-19 are quasi current-fed full-bridge and half-bridge (push-pull) converters where the input voltage source is connected to a series inductor $L_S$ which acts as the current source. However, it is not as trivial to apply energy injection control on a parallel tuned tank. When the converter enters into free oscillation, the current source cannot just be disconnected; rather it must also be freewheeled to keep the current source going. Thus, this may imply the use of extra power components to direct the flow of the current source elsewhere.

![Figure 5-19: Current-fed parallel resonant converters; a) full-wave, b) half-wave](image)

In order to put these converters into a free oscillation state there are actually two ways this can be achieved. When entering free oscillation the parallel tank, the first option is to disconnect $L_S$ from the resonant tank and freewheel $L_S$ and the resonant tank separately. The secondary option is to freewheel $L_S$ together with the resonant tank by disconnecting $L_S$ from the input voltage source and shunting the resonant tank across the current source $L_S$. The modified converters for freewheeling $L_S$ and the tank separately are illustrated in Figure 5-20a) and b), while the converters for freewheeling $L_S$ and the tank together are illustrated in Figure 5-20c) and d).
Figure 5-20: Energy injection converters for parallel tuned resonant tank

To put the converter shown in Figure 5-20a) into free oscillation, switches $S_1$ to $S_4$ are turned OFF while $S_F$ is turned ON; during injection instances $S_F$ is OFF while the switches $S_1$ to $S_4$ are zero voltage switched.

To put the converter shown in Figure 5-20b) into free oscillation, switches $S_1$, $S_2$, $S_{F2}$ and $S_{F4}$ are tuned OFF while $S_{F1}$ and $S_{F3}$ are turned ON; during injection instances $S_{F1}$ and $S_{F3}$ are OFF, $S_{F2}$ and $S_{F4}$ are ON, while $S_1$ and $S_2$ are zero voltage switched.

To put the converters shown in Figure 5-20c) and d) into free oscillation, $S_{F1}$ is turned OFF and $S_{F2}$ is turned ON, and for injection instances $S_{F1}$ is ON and $S_{F2}$ is OFF, while the switches $S_1$ to $S_4$ are constantly zero voltage switched regardless of the state.
Once again the main difference between the full-wave and half-wave converters is that double the power can be delivered with the full-wave converter with the same input voltage $V_{DC}$.

The parallel tuned energy injection converters which are separately freewheeled (Figure 5-20a and b)) are similar to the series tuned energy injection converters presented in section 5.2. Since the converter can only change states at the zero crossing instants of the resonant waveforms, in order to keep the converter soft-switched. This means that the same control methods presented in section 5.3 can be used with the parallel tuned energy injection converters that are separately freewheeled. The only difference is that an additional switch is required for the full-wave converter and four addition switches are required for the half-wave converter, in order to freewheel the current source inductors separately from the resonant tank. Four additional switches are required for the push-pull converter since it consists of two current source inductors, which need to be disconnected from the resonant tank during free oscillation. This amount of additional power switches makes the converter very inefficient as they may be switched at a rate as high as the resonant frequency. These additional switches are all high side switches, which are difficult to keep ON for extended periods of free oscillation, due to limitations on high side gate drives.

The parallel tuned energy injection converters which freewheels the entire circuit (Figure 5-20c and d)) are quite different to all the previously presented energy injection converters, since the converter may change states during anytime without being restricted to the zero crossing instants of the resonant waveform. This is because the additional switches $S_{F1}$ and $S_{F2}$ are before the current source inductors. Thus, the hard switching waveforms are buffered out by the current source inductors and do not affect the resonant tank. Since the inductors will not let high frequency waveforms pass, it has the effect of averaging out the input voltage, and effectively changes the magnitude of the constant current source. Technically speaking, the resonant tank is never in free oscillation; rather power flow control is achieved by varying the input current. Since the MOSFET $S_{F2}$ is only used to freewheel the converter, it can be replaced with a diode, making it bear a resemblance to a buck converter. However, using a MOSFET is preferred, since it lowers the conduction loss during free oscillation. Since $S_{F1}$ and $S_{F2}$ are always contrary to each other and are in a high side low side switch configuration, a half-bridge gate driver can be used to drive the switches.
For this study on parallel tuned energy injection control, a half-wave (push-pull) converter was chosen due to its lower component count and simpler switching scheme. Since the same control methods described in section 5.3 can be used on the parallel tuned energy injection converters, which freewheel the current source and resonant tank separately. The converter which freewheels the current source and resonant tank together is chosen for implementation in this study and different control strategies are investigated.

### 5.4.1 Push-pull parallel tuned energy injection converter

The injection and free oscillation states of the push-pull energy injection converter, which freewheels the current source and resonant tank together, are shown in Figure 5-21. The injection state consists of $S_{F1}$ turned ON and $S_{F2}$ turned OFF, as shown in Figure 5-21a), and the free oscillation state consists of $S_{F1}$ turned OFF and $S_{F2}$ turned ON, as shown in Figure 5-21b). There are no restrictions on when the converter can be in injection or free oscillation state and the main converter switches $S_1$ and $S_2$ are constantly zero voltage switched off the primary resonant voltage regardless of the energy injection state.

![Figure 5-21: Equivalent circuit states of a push-pull parallel tuned energy injection converter; a) injection states, b) free oscillation states](image-url)
When coupled together with a series or parallel tuned secondary pickup the equivalent circuit topologies with the effect of coupling modelled as a current dependent voltage source is shown in Figure 5-22. Note that the ESR of each resonant component is added to the equivalent circuit.

![Figure 5-22: Parallel tuned push-pull energy injection system topologies; a) Parallel-series, b) Parallel-parallel](image)

### 5.4.2 State-space simulation of a parallel tuned push-pull energy injection converter

In order to simulate the energy injection control strategies a state-space representation of the push-pull energy injection controller together with the secondary pickup was established based on the circuits shown in Figure 5-22. The simulation is similar to the simulation presented in section 5.2.2. However, two more state variables are added as there are two additional inductors $L_{S1}$ and $L_{S2}$. The switching network of this push-pull energy injection converter consists of two pairs of mutually exclusive switches, $S_i/S_2$ and $S_{F1}/S_{F2}$, creating a total of four different circuit topologies. Hence, a 4 part state-space representation is used to simulate the system. Note that the switches are modelled as ideal switches, such that they are
modelled as a short circuit when ON and an open circuit when OFF. The state vector $x$ is now given by

$$
x = \begin{bmatrix} I_{LS1} \\ I_{LS2} \\ I_{L1} \\ V_{C1} \\ I_{L2} \\ V_{C2} \end{bmatrix} \tag{5-30}
$$

The four states of the circuit are defined by the following: let state 1 be when $S_1$ is ON and $S_{F1}$ is ON (injection); state 2, when $S_1$ is ON and $S_{F1}$ is OFF (free oscillation); state 3, when $S_1$ is OFF and $S_{F1}$ is ON (injection); and state 4, when $S_1$ if OFF and $S_{F1}$ is OFF (free oscillation). The 4 part state-space representation of the parallel tuned push-pull energy injection converter with a series tuned pickup in the form of $A\dot{x} + Bx = C$ is given as

**State 1**

$$
\begin{bmatrix}
0 & L_{S2} & 0 & 0 & 0 & 0 \\
L_{S1} & 0 & 0 & C_1 R_{C1} & 0 & 0 \\
0 & 0 & -L_1 & C_1 R_{C1} & -M & 0 \\
0 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 0 & 0 & 0 & C_2 & 0 \\
0 & 0 & M + L_2 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
R_{S1} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -R_{L1} & 1 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
= \begin{bmatrix}
V_{DC} \\
0
\end{bmatrix}
$$

**State 2**

$$
\begin{bmatrix}
0 & L_{S2} & 0 & 0 & 0 & 0 \\
L_{S1} & 0 & 0 & C_1 R_{C1} & 0 & 0 \\
0 & 0 & -L_1 & C_1 R_{C1} & M & 0 \\
0 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 0 & 0 & 0 & C_2 & 0 \\
0 & 0 & M + L_2 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
R_{S1} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & R_{L1} & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

**State 3**

$$
\begin{bmatrix}
0 & L_{S2} & 0 & 0 & 0 & 0 \\
L_{S1} & 0 & 0 & -C_1 R_{C1} & 0 & 0 \\
0 & 0 & -L_1 & -C_1 R_{C1} & -M & 0 \\
0 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 0 & 0 & 0 & C_2 & 0 \\
0 & 0 & M + L_2 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
R_{S1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -R_{L1} & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
= \begin{bmatrix}
V_{DC} \\
0
\end{bmatrix}
$$

**State 4**

$$
\begin{bmatrix}
0 & L_{S2} & 0 & 0 & 0 & 0 \\
L_{S1} & 0 & 0 & -C_1 R_{C1} & 0 & 0 \\
0 & 0 & -L_1 & -C_1 R_{C1} & M & 0 \\
0 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 0 & 0 & 0 & C_2 & 0 \\
0 & 0 & M + L_2 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
R_{S2} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & R_{L1} & -1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
= \begin{bmatrix}
V_{DC} \\
0
\end{bmatrix}
$$

-172-
The 4 part state-space representation of the parallel tuned push-pull energy injection converter with a parallel tuned pickup in the form of $A\dot{x} + Bx = C$ is given as

State 1

$$
\begin{bmatrix}
0 & L_{S2} & 0 & 0 & 0 & 0 \\
L_{S1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -L_1 & C_1R_{C1} & -M & 0 \\
0 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 0 & M & 0 & L_2 & C_2R_{C2}
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\end{bmatrix}
= \begin{bmatrix}
0 & R_{S2} & 0 & 0 & 0 & 0 \\
R_{S1} & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -R_{L1} & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & -\frac{1}{R_2} & 1 \\
0 & 0 & 0 & 0 & R_{L2} & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix}
\begin{bmatrix}
V_{DC} \\
V_{DC}
\end{bmatrix}
$$

State 2

$$
\begin{bmatrix}
0 & 0 & C_1R_{C1} & 0 & 0 & 0 \\
0 & 0 & L_1 & -C_1R_{C1} & M & 0 \\
0 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 0 & 0 & 0 & -C_2(1 + \frac{R_{C2}}{R_2}) & 0 \\
0 & 1 & 0 & 0 & 0 & C_2R_{C2}
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\end{bmatrix}
= \begin{bmatrix}
R_{S1} & 0 & 0 & 1 & 0 & 0 \\
0 & R_{L2} & 0 & -1 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -\frac{1}{R_2} & 1 \\
0 & 0 & 0 & 0 & R_{L2} & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

State 3

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -L_1 & -C_1R_{C1} & -M & 0 \\
0 & 0 & 0 & -C_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix}
= \begin{bmatrix}
R_{S1} & 0 & 0 & 0 & 0 & 0 \\
0 & R_{L2} & 0 & -1 & 0 & 0 \\
0 & 0 & -R_{L2} & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & -\frac{1}{R_2} & 1 \\
0 & 0 & 0 & 0 & R_{L2} & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
\begin{bmatrix}
V_{DC} \\
V_{DC}
\end{bmatrix}
$$

State 4

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & L_1 & -C_1R_{C1} & M & 0 \\
0 & 0 & 0 & -L_1 & -C_1R_{C1} & -M \\
0 & 0 & 0 & 0 & -C_1 & 0 \\
0 & 0 & 0 & 0 & 0 & -C_2(1 + \frac{R_{C2}}{R_2}) \\
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix}
= \begin{bmatrix}
0 & R_{S2} & 0 & -1 & 0 & 0 \\
0 & 0 & R_{L2} & -1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -\frac{1}{R_2} & 1 \\
0 & 0 & 0 & 0 & R_{L2} & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

The output voltage which is equivalent to the voltage across $R_2$ for the PS energy injection system is given by

$$
V_{PSout} = [0 \ 0 \ 0 \ 0 \ R_2 \ 0]x
$$

(5-33)

The output voltage for the PP energy injection system can be approximated by the voltage $V_{C2}$ assuming that $I_{C2}R_{C2} << V_{C2}$, hence the output voltage is given by

$$
V_{PPout} = [0 \ 0 \ 0 \ 0 \ 0 \ 1]x
$$

(5-34)
5.4.3 Implementation of a ZVS parallel tuned push-pull energy injection converter

The control circuitry of a parallel tuned push-pull energy injection converter consists of two main parts. The part which will be described here is the zero voltage detection circuit for soft-switched operation. The other part consists of the power flow controller, which controls the energy injection in order to regulate the output voltage. Different control strategies are discussed and their practical implementations are given in Section 5.5.

**ZVS gate drive circuitry**

The ZVS gate drive circuitry used here is the same circuit that was used for the primary side frequency control push-pull converter that was presented in Chapter 4 in section 4.4.5. The diagram of the gate drive circuitry is reproduced below in Figure 5-23, for details please refer back to section 4.4.5.

![ZVS gate drive circuitry](image)

**Figure 5-23: ZVS gate drive circuitry for push-pull parallel tuned energy injection circuit**

5.5 Power flow control methods for parallel tuned push-pull energy injection converter

A primary side energy injection controller, which freewheels the entire push-pull converter, is no longer restricted to integer based energy injection control methods. Since only the magnitude of the input voltage is effectively being changed, the control action does not have to be synchronous with the zero voltage switching. Hence, continuous control methods such as PWM or other fixed frequency control methods may be used.
5.5.1 Fixed frequency PWM output voltage control

Since the output voltage is directly proportional to the input voltage as shown in Equation (2-17). The voltage at the secondary output can be controlled by effectively varying the input voltage of the push-pull energy injection converter. To achieve this, the energy injection switches $S_{F1}$ and $S_{F2}$ can be switched in a PWM fashion. A PWM signal consists of a fixed period $T_{PWM}$ (PWM frequency $f_{PWM}$), and a duty cycle $D$ which is a percentage that corresponds to the proportion of the period $T_{PWM}$ when the signal is high. When the switches $S_{F1}$ and $S_{F2}$ are switched with a PWM signal with duty cycle $D$, the average input voltage in between the split inductors of the converter is given by

$$V_{IN} = DV_{DC}$$  \hspace{1cm} (5-35)

Hence, the average peak voltage of the resonant voltage $V_{C1}$ is now given as

$$\bar{V}_{C1} = \pi DV_{DC}$$  \hspace{1cm} (5-36)

The split inductors $L_{S1}$ and $L_{S2}$ of a push-pull converter act as the quasi current source for the parallel tuned resonant tank. At steady-state the split inductors supply the real power dissipated by the parallel resonant tank. The current through the split inductors $I_{LS}$ is made up of a DC component that corresponds to the RMS real current drawn by the tank. On top of this DC components are AC components related to the frequency of the resonant tank and the frequency of the PWM signal. The ripple caused by the resonant tank is not analysed here, since it is not related to the PWM control. More information on the frequency and magnitude of this ripple can be found in [91].

To analyse the ripple in $I_{LS}$ due to the PWM input voltage at steady-state. The resonant tank is removed from the converter and is replaced by two voltage sources $V_{S1}$ and $V_{S2}$, which represent the steady-state voltage waveforms across $S_I$ and $S_2$. This equivalent circuit is shown in Figure 5-24. If only the ripple component of $I_{LS}$ due to the PWM control is desired, the circuit can be further simplified by approximating $V_{S1}$ and $V_{S2}$ with their average voltage across the resonant period which can be calculated to be $DV_{DC}$. The equivalent steady-state circuit for this transformation is also shown in Figure 5-24. Note that $L_{S1}$ and $L_{S2}$ are assumed to be identical. Shown in Figure 5-25 are the simplified input voltage and current waveforms for the equivalent circuit in shown Figure 5-24. Note that harmonics due to the resonant tank are neglected.
Figure 5-24: Equivalent circuit for analysing split inductor current of a PWM driven energy injection push-pull converter

\[
V_{DC} \quad S_{F1} \quad V_{IN} \quad L_{S1} \quad V_{S1} \quad V_{S2} \quad L_{S2} \quad + \quad -
\]

\[\varphi_{S1} = \varphi_{S2} = \pi DV_{DC}\]

Figure 5-25: Simplified split inductor current waveforms of an energy injection push-pull converter

With this model, the change in current \( I_{LS} \) during the ON period of the PWM input voltage \( (V_{IN}=V_{DC}) \) can be given as

\[
\Delta I_{LS} = \int_0^{DT_{PWM}} \frac{V_{LS}}{L_s} \, dt = \frac{V_{DC}(1-D)}{L_s} DT_{PWM}
\]

(5-37)

During the OFF period of the PWM \((V_{IN}=0)\) the change in current \( I_{LS} \) can be given as

\[
\Delta I_{LS} = \int_{DT_{PWM}}^{T_{PWM}} \frac{V_{LS}}{L_s} \, dt = \frac{-V_{DC}D}{L_s} (1-D)T_{PWM}
\]

(5-38)

At steady-state the energy stored in the split inductor \( L_s \) at the beginning of the PWM cycle is equal to the energy stored at the end of the PWM cycle. Hence, the current ripple given in Equations (5-37) and (5-38) are identical. The relationship between \( \Delta I_{LS} \) versus the duty cycle \( D \) is shown in Figure 5-26. A large \( \Delta I_{LS} \) is undesired, since this will increase the AC losses in the split inductors. For a given input voltage \( V_{DC} \) the current ripple of \( I_{LS} \) can be reduced by
increasing the split inductance or by increasing the PWM frequency. However, increasing the inductance will increase the physical size of the split inductors and increasing the PWM frequency would lead to higher switching losses in $S_{F1}$ and $S_{F2}$, which are both undesirable.

![Figure 5-26: $\Delta I_{LS}$ versus duty cycle $D$ for a PWM energy injection push-pull converter](image)

Using the 4 part state-space simulation presented in 5.4.2 combined with fixed frequency PWM, the instantaneous current and voltage waveforms of this converter can be obtained via simulation. The MATLAB code for simulating the time domain waveforms of a parallel tuned push-pull energy injection converter with fixed frequency control, based on the state-space representation given in section 5.4.2 is provided in Appendix F.

**PWM control circuitry**

Since the output voltage is not just dependent on the duty cycle of the PWM signal, but also dependent on coupling, a PID controller is necessary to determine the required duty cycle. A high level schematic of PWM output voltage controller is shown in Figure 5-27. The secondary output voltage $V_{out}$ is received by the primary side controller via a wireless radio link. This output voltage is then compared to the desired output voltage $V_d$ at the PID controller. The output of the PI controller $V_{ref}$ is then sent to the PWM generator, where $V_{ref}$ is compared to an internal reference which determines the duty cycle $D$ of the PWM. The PWM signal $V_{PWM}$ is then produced by the PWM generator at the frequency $f_{PWM}$ with duty cycle $D$. A PWM generator can be commonly found as discrete integrated circuits. The specific controller used in this study was a Texas instrument TL5001, whose PWM frequency $f_{PWM}$ can be selected from 20 kHz to 500 kHz.
Since the switching frequency of the energy injection switches \( S_{F1} \) and \( S_{F2} \) are not restricted to the operating resonant frequency \( f_r \) like \( S_1 \) and \( S_2 \). Since \( f_{PWM} \) can either be slower than \( f_r \) or faster than \( f_r \), the two cases will be analysed separately.

**PWM frequency lower than resonant frequency**

At steady-state the current through the split inductors will have a ripple \( \Delta I_{LS} \) due to the PWM input voltage. Since \( f_{PWM} \) is smaller than \( f_r \), the current will take several resonant cycles to ramp up and down, which effectively acts like a varying current source to the resonant tank. Because of this, although the average peak of the resonant voltage across the tank will still be given by Equation (5-36), a peak envelope with amplitude proportional to \( \Delta I_{LS} \) will be present on the resonant waveforms.

Using the state-space representation of the energy injection push-pull converter given in Appendix F, the envelope of resonant waveforms can be simulated. To illustrate the effect of \( f_{PWM}<f_r \), a primary converter with no secondary circuit is established. The loading due to the secondary circuit will be represented by the resistance \( R_1 \) in series with the resonant coil \( L_1 \). The parameters of this converter are given in Table 5-3.

**Table 5-3: Simulation parameters for a PWM energy injection push-pull converter**

<table>
<thead>
<tr>
<th>( L_{SI} )</th>
<th>1mH</th>
<th>( L_{S2} )</th>
<th>1mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 )</td>
<td>11.2( \mu )H</td>
<td>( C_1 )</td>
<td>33nF</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>2( \Omega )</td>
<td>( f_0 )</td>
<td>262 kHz</td>
</tr>
<tr>
<td>( V_{DC} )</td>
<td>20V</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Shown in Figure 5-28 and Figure 5-29 are the resonant waveforms and the current \( I_{LS} \) for a PWM signal with \( D=0.5 \) and \( f_{PWM} \) of 0.25\( f_0 \)=65.5 kHz and 0.125\( f_0 \)=32.75 kHz respectively. Subsequently, shown in Figure 5-30 and Figure 5-31 are the resonant waveforms and current \( I_{LS} \) for a PWM signal with \( D=0.2 \) and \( f_{PWM} \) of 0.25\( f_0 \)=65.5 kHz and 0.125\( f_0 \)=32.75 kHz.
respectively. From these figures it can clearly be seen that the resonant waveforms no longer have constant amplitude.

It can be obtained with Equation (5-37) that when $f_{PWM}=65.5$ kHz with $D=0.5$, $\Delta I_{LS}=0.076A$ (sim. 0.085A); when $f_{PWM}=32.75$ kHz with $D=0.5$, $\Delta I_{LS}=0.153A$ (sim. 0.173A); when $f_{PWM}=65.5$ kHz with $D=0.2$, $\Delta I_{LS}=0.049A$ (sim. 0.053A); and when $f_{PWM}=32.75$ kHz with $D=0.2$, $\Delta I_{LS}=0.0977A$ (sim. 0.11A). Comparing the values obtained from Equation (5-37) to the simulation, shows that the approximation is ±10% accurate.

At $D=0.5$, the average peak of the resonant current $I_{L1}$ was 1.7A, while the simulated amplitude of the current peak envelope was 0.129A and 0.64A for the case when $f_{PWM}=65.5$ kHz and $f_{PWM}=32.75$ kHz respectively.

At $D=0.2$, the average peak of the resonant current $I_{L1}$ was 0.682A, while the simulated amplitude of the current peak envelope was 0.099A and 0.37A for the case when $f_{PWM}=65.5$ kHz and $f_{PWM}=32.75$ kHz respectively.

Figure 5-28: PWM energy injection push-pull waveforms, $f_{PWM}=0.25f_0=65.5$ kHz, $D=0.5$
Figure 5-29: PWM energy injection push-pull waveforms, $f_{PWM}=0.125f_0=32.75$ kHz, $D=0.5$

Figure 5-30: PWM energy injection push-pull waveforms, $f_{PWM}=0.25f_0=65.5$ kHz, $D=0.2$
When $f_{PWM} > f_r$ the magnitude of the current ripple $\Delta I_{LS}$ due to the PWM input voltage will be reduced since $\Delta I_{LS}$ is proportional to $T_{PWM}$. Moreover, even if the same current ripple magnitude is present, the peak envelope of the resonant waveforms would still remain constant. Since $f_{PWM}$ is greater than equal to $f_r$, the ripple in the split inductors ramp up and back down all within a resonant cycle. Hence, the ripple is averaged out during the resonant cycle and the resonant waveform is unaffected.

To demonstrate this, an energy injection push-pull with parameters given in Table 5-3 was controlled by a PWM signal with $f_{PWM}=f_r$ and at $f_{PWM}=2f_r$. A duty cycle $D=0.5$ was used for both cases, since $\Delta I_{LS}$ is greatest at $D=0.5$. Shown in Figure 5-32 and Figure 5-33 are the resonant waveforms and current $I_{LS}$ of the respective PWM controlled energy injection push-pull converters. It can be seen in these two figures that the resonant waveforms are almost identical.

**Figure 5-31: PWM energy injection push-pull waveforms, $f_{PWM}=0.125f_r=32.75$ kHz, $D=0.2$**
Figure 5-32: PWM energy injection push-pull waveforms, $f_{PWM}=f_0=262$ kHz, $D=0.5$

Figure 5-33: PWM energy injection push-pull waveforms, $f_{PWM}=2f_0=524$ kHz, $D=0.5$
The main disadvantage of setting $f_{PWM}$ higher than $f_r$ is that the energy injection switches $S_{F1}$ and $S_{F2}$ will be switching more rapidly than the main converter switches. Since the switches $S_{F1}$ and $S_{F2}$ are hard-switched, significant losses may be generated by these switches which will lower the overall efficiency of the system.

### 5.5.2 Comparison of parallel tuned push-pull energy injection control methods

A PWM power flow controller was practically implemented based on the control circuitry depicted in Figure 5-27. The open loop performance of fixed frequency PWM control was compared for the case when $f_{PWM}$< $f_r$ and when $f_{PWM}$> $f_r$. The same secondary circuits that were used with the series tuned energy injection system in section 5.3.5 were used to test the parallel tuned energy injection system. The parallel tuned energy injection converter was used to drive and series tuned pickup and a parallel tuned pickup. The resonant parameters of the experimental implementation and of the MATLAB state-space simulation are shown in Table 5-4.

| Parallel tuned push-pull energy injection primary converter |
|-----------------------------|-----------------------------|
| $L_1$=11.2µH | $C_1$=33nF | $f_0$=262 kHz |
| $V_{IN}$=14V, $k$=0.3 | $V_{IN}$=14V, $k$=0.3 |
| Series secondary $Q_{S0}$=1.81 |
| Parallel secondary $Q_{S0}$=1.82 |
| $L_2$ | $C_2$ | $R_2$ |
| 10.9µH | 33nF | 10 |
| $L_2$ | $C_2$ | $R_2$ |
| 3.3µH | 110nF | 10 |
| $f_0$ | $f_0$ |
| 265 kHz | 264 kHz |

In this experiment, the natural resonant frequency of the tank $f_0$ was set to 262 kHz, and PWM signals with a frequency of 58 kHz ($f_{PWM}$< $f_r$) and 340 kHz ($f_{PWM}$> $f_r$) was used for the PWM power flow controller. At coupling $k$=0.3, the operating frequency $f_r$ was approximately 270 kHz. Thus, the slow PWM frequency is 4.6 times slower than $f_r$ and the fast PWM frequency is 1.26 times faster than $f_r$. 

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The duty cycle of the PWM signal was swept from 0 to 100% and was plotted against the resulting secondary output voltage. The results from both simulation and the experiment for the parallel-series system are shown in Figure 5-34, and the results for the parallel-parallel system are shown in Figure 5-35. In both figures, the simulation and experimental plots
showed that there is a linear relationship between output voltage and duty cycle. Also the results indicate that the RMS value of the output voltage is the same for a given duty cycle, regardless of the PWM frequency.

Shown in Figure 5-36a) and c) are oscilloscope snapshots of the low frequency PWM control in action, and shown in Figure 5-36b) and d) are snapshots of the high frequency PWM control in action. The yellow trace is the voltage waveform $V_{in}$ at the split inductors, and also represents the duty cycle $D$; the pink trace is the secondary output voltage; and the blue trace is the voltage $V_{S1}$ across the push-pull converter switch $S_1$. Resonant waveforms with oscillating amplitudes are clearly seen for the low frequency PWM control shown in Figure 5-36a) and c), whereas resonant waveforms with constant amplitude is observed for high frequency PWM control shown in Figure 5-36b) and d).

![Figure 5-36: Push-pull parallel tuned energy injection converter waveforms: a) Slow PWM control, $D=0.5$; b) Fast PWM control, $D=0.5$; c) Slow PWM control, $D=0.1$; d) Fast PWM control, $D=0.1$;](image-url)
The practical control range of a PWM energy injection control which freewheels the entire push-pull converter is superior to that of integer based ZVS/ZCS energy injection controllers which freewheels the resonant tank. A minimum control percentage exists for the integer based controllers since a minimum current or voltage needs to be sustained in the resonant tank for the power flow control to operate. However, since the energy injection switches of a PWM power flow controller are hard-switched, the minimum control percentage is determined by the lowest duty cycle the PWM controller can generate. This can be as low as 5% as shown in Figure 5-34 and Figure 5-35. Furthermore, since PWM control is very smooth, there are no fixed control steps and control action is always taken given a change in the duty cycle or control percentage.

5.7 Summary

In this chapter power flow control based on primary side energy injection control for all four basic resonant topologies of IPT systems was presented. The energy injection control is to regulate the secondary output voltage of an inductive power system, by injecting energy into the primary resonant tank when more output power is required; and when the output power is sufficient, the resonant tank is left to oscillate freely. The energy injection control methods presented allowed the resonant tank to remain fully soft-switched and converters were presented for both series and parallel tuned primaries. Time domain state-space simulations were provided for each converter implemented in this study, and good correlation between experimental and simulation results have been seen.

This study has shown that the concept of energy injection works well on all four of the basic resonant topologies and provides the wide control range of input voltage magnitude control, but without the extra conversion stage. Energy injection control is straightforward to implement on series tuned voltage-fed resonant converters. However, it is more cumbersome for it to be implemented on parallel-tuned current-fed converters, due to the need to freewheel the current source.

Energy injection control is relatively easy to implement and provides an extremely wide control range. Such a power flow control method is highly suitable for controlling IPT systems for powering implantable heart pumps, since it means that wide coupling variations can be tolerated.
The final chapter of this thesis presents the implementation and performance of a standalone closed-loop TET system. The resonant parameters of the TET system are determined using the procedure presented in Chapter 2 and incorporates the energy injection power flow controller presented in Chapter 5. This chapter illustrates how all the analysis presented in this thesis can be put into practical use. At the same time it demonstrates that an IPT system for implantable heart pumps can physically be implemented, with all the components compact enough for implantation, even when in the prototyping stage.

6.1 A standalone closed-loop TET system for implantable heart pumps

A standalone system refers to a fully independent system that is able to run off a single external source such as a power supply or battery, while a closed-loop system is a self controlled system that is able to regulate the output power.

In an IPT system, output power regulation can either be performed on the primary side or on the secondary side of the system. Since the secondary side of this TET system is to be implanted in the body; the secondary circuit should remain compact and complexity should be reduced when possible to minimise the probability of failure. Thus, primary side output regulation should be adopted.

In order to achieve primary side output regulation, feedback from the secondary circuit is required. Various attempts have been made to transmit feedback data over the same power transfer link [104, 105], since this eliminates the need of a wireless communications link.
However, due to low bandwidth and limited range, this technique is limited and unreliable. In the application of an implantable heart pump wireless communication over at least several metres is likely to be required to provide information about the status of the pump and the backup battery charge. Hence, the feedback for primary side output regulation can share a reasonably high bandwidth wireless communication link that is independent from the inductive link.

A system block diagram of a standalone closed-loop TET system is shown in Figure 6-1.

**Figure 6-1: Block diagram of a standalone closed-loop TET system for implantable heart pumps**

In a standalone closed-loop IPT system for implantable heart pumps, the primary circuit can be broken down into the following components:

- The primary resonant circuit including the power transfer coil
- The DC to AC converter
- The power flow controller
- A Wireless receiver

The secondary circuit can be broken down into the following components:

- The secondary resonant circuit including the power pickup coil
- An AC to DC converter
- A Wireless transmitter
The goal of this study is to develop an IPT system with the techniques presented in the previous parts of this thesis, such that the system is able to tolerate wide levels of coupling while maintaining good efficiency. Since the design of resonant circuits are presented in detail in Chapter 2 and the design of DC-AC converters with their power flow controllers are given in Chapters 4 and 5; this chapter will focus on the closed-loop aspects and performance of the TET system.

6.2 Specification and design of a standalone closed-loop TET system

6.2.1 Operating specifications

There is no public document specifying the user requirements of an IPT system for implantable heart pumps. However, from a range of published literature, personal communications with companies and medical practitioners, a user requirement can be derived that meets most people’s expectations.

Size and displacement of power transfer coils

A specification of high interest is the physical size of the power transfer coils, especially the implanted secondary pickup coil. Several publications have presented the design of power transfer coils for delivering power to implantable devices and others have specified the size of their power transfer coils used in their TET systems. The general consensus is that the thinner the coils the better it is for implantation and is more aesthetically appealing.

An IPT system design by T. Mussivand et al. in 1993 [30] consists of a air-cored primary coil measuring 58mm in diameter, 30mm in height and weighs 75g; and a air-cored secondary coil measuring 70mm in diameter, 20mm in height and weighs 125g. The system was able to cope with a maximum displacement of 20mm and had an efficiency of 80% at 10mm displacement while delivering 10 to 25W of power [30]. This early system reported good efficiency and tolerance to coupling changes but consisted of extremely bulky coils.

A more modern IPT system developed by the NEDO artificial heart project 2008 [40] used a set of “hybrid coils” where the primary was air-cored and the secondary was ferrite-cored, the ferrite on the secondary measured 38mm in diameter and 9mm in height and was wrapped with Litz wire forming an outer diameter of 53mm. The secondary coil was purposely made
to create a lump on the skin, so that the primary coil measuring 92mm in diameter could easily be fitted over the secondary providing stable coil alignment and coupling. The system delivered 16W of power at an efficiency of 80% [40].

Another recent IPT system by H. Miura et al. in 2005 [42] reported a system whose secondary circuit consisting of a synchronous rectifier was imbedded together with the secondary coil. The primary coil of the system measured 120mm in diameter and the secondary coil was 90mm in diameter, the thickness of both coils was less than 10mm. While a efficiency of 89% could be sustained at up to 10mm displacement [42].

Having been in contact with several heart pump companies, a good understanding of the preferred size of the power transfer coils and the required level of coupling tolerance has been obtained. From this the size constraints for the power transfer coils in this TET system implementation are as follows:

- Thickness of primary and secondary power transfer coil < 7mm
- Diameter of primary coil < 75mm
- Diameter of secondary coil < 55mm
- Full operation at up to 20mm displacement (in any direction) between power transfer coils

**Output power**

The power requirements of several state-of-the-art implantable heart pumps have been given in Table 1-1. From this the average power consumption of modern LVADs known to be in the range of 5 to 10W; however, very little information about their operating voltage is found in public documents or published literature.

Having previously obtained a used Micromed’s DeBakey HA5 LVAD and its pump controller, it was found that it operated off an input of 12Vdc and draws up to 1A of current. Other heart pump companies that we have been in contact with were also satisfied with the 12Vdc output which we incorporate on all our demonstration systems. Therefore, the output requirements for this TET system implementation are as follows:

- Average load voltage regulated above 12Vdc
- Controllable load power of 5W to 12W, this corresponds to a load range of 28Ω to 12Ω for a 12V output
6.2.2 Power flow controller selection

In this thesis, primary side frequency power flow control and energy injection control have been thoroughly analysed and the greatest advantage of energy injection control over frequency control is that its control range is much larger. In the specifications given above, the system must be able to maintain 12V at the output anywhere from 0 to 20mm of concentric coil displacement.

Since moderately coupled systems can have a coupling range of \( k = 0.1 \) to 0.6, a minimum coupling which is six times smaller than maximum coupling, the output voltage can change by a factor of six across this coupling range. It would be impractical to use frequency control to regulate such a large control range, since the maximum change in output voltage reported in Chapter 4 was less than a factor of two when the operating frequency changed by a factor of two. Hence, due to the large coupling range that has to be tolerated, energy injection control will be used.

The energy injection strategy to be implemented is dependent on the choice of the primary resonant tank topology. If a parallel tuned primary is chosen, the energy injection method that is most suitable will be the PWM control method presented in section 5.5.1. Since this is a control method with a very large practical control range and only requires two additional components. If a series tuned primary is chosen, the energy injection method that is most suitable will be the peak current control method presented in section 5.3.2. Since this is a control method with good control range and can be implemented without the use of complex controllers. Both of these control strategies are simple and robust, which are important aspects of the power flow controller for an implantable TET system.

6.2.3 Power transfer coil selection

T.D. Dissanayake has proposed a flat pancake coil design which meets the physical constraints in section 6.2. [44]. These coils were wound in a flat spiral such that there is an inner diameter and an outer diameter. Coils with large inner diameter maintain higher levels of coupling when the coils are displaced concentrically, but coupling drops off much faster when displaced off-axis; and the vice versa is true with a small inner diameter.

More layers with the same geometry can be added to increase the inductance of the coil at the cost of thickness. When the power transfer coils are used on a patient the secondary coil will be implanted under the skin, while the primary will be held over the skin on top of the
secondary coil. Mis-alignments of the power transfer coils will mostly likely be due to off-axis displacements, since the location of the implanted secondary coil may not be very clear. Whereas the concentric displacement of the coil is determined by the thickness of the coil’s encapsulation, the patient’s skin and the any clothing between the coils; hence, the concentric displacement is relatively constant for each patient. This implies that the coils should be designed with a small inner diameter, since this would allow the coils to maintain better coupling when displaced off-axis.

In this study 405 strands of 41 AWG Litz wire [59] with an overall diameter of 2mm were used for the design of the power transfer coils. This Litz wire was chosen since 41 AWG wire is recommended for an operating frequency of approximately 200 kHz [106]. Choosing the physical coil winding structure is a trade-off between size (and weight) and trying to maintain a minimum coupling coefficient at the limits of the physical displacement where effective power transfer is required. Without attempting to provide an optimal coil design, a number of examples were created to find a good design. Table 6-1 provides physical characteristics of eight coils which meet the user requirements given in section 6.2.1. Each coil in Table 6-1 is given an ID which is constructed by its inner diameter/outer diameter/number of layers.

Table 6-1: Selection of coils meeting design specifications: size versus inductance

<table>
<thead>
<tr>
<th>Coil ID</th>
<th>Inner dia.</th>
<th>Outer dia.</th>
<th>No. of layers</th>
<th>No. of turns</th>
<th>Weight</th>
<th>Inductance ±10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/50/1</td>
<td>10mm</td>
<td>50mm</td>
<td>1</td>
<td>11</td>
<td>20g</td>
<td>3µH</td>
</tr>
<tr>
<td>10/60/1</td>
<td>10mm</td>
<td>60mm</td>
<td>1</td>
<td>14</td>
<td>25g</td>
<td>5µH</td>
</tr>
<tr>
<td>10/70/1</td>
<td>10mm</td>
<td>70mm</td>
<td>1</td>
<td>16</td>
<td>30g</td>
<td>7µH</td>
</tr>
<tr>
<td>10/50/2</td>
<td>10mm</td>
<td>50mm</td>
<td>2</td>
<td>22</td>
<td>40g</td>
<td>11.5µH</td>
</tr>
<tr>
<td>10/60/2</td>
<td>10mm</td>
<td>60mm</td>
<td>2</td>
<td>28</td>
<td>50g</td>
<td>20µH</td>
</tr>
<tr>
<td>10/70/2</td>
<td>10mm</td>
<td>70mm</td>
<td>2</td>
<td>32</td>
<td>60g</td>
<td>28µH</td>
</tr>
<tr>
<td>25/60/1</td>
<td>25mm</td>
<td>60mm</td>
<td>1</td>
<td>9</td>
<td>22g</td>
<td>3.5µH</td>
</tr>
<tr>
<td>25/60/2</td>
<td>25mm</td>
<td>60mm</td>
<td>2</td>
<td>18</td>
<td>45g</td>
<td>14µH</td>
</tr>
</tbody>
</table>

Power transfer coils with large coil inductances improves the output voltage at the secondary pickup, since the voltage induced at the secondary is proportional to \( \sqrt{L_1L_2} \) (Equation (2-9)). Since \( L_2 \) affects the secondary circuit quality factor where \( L_1 \) does not, \( L_1 \) is usually chosen to
be as large as possible especially if the required output voltage is high. This implies that the primary coil should be a two layer coil.

Since the secondary coil has to be implanted under the skin, it is more favourable to make the coil as flat as possible. A flat coil eases implantation and minimises the protrusion created on the skin, which suggests the use of a single layer coil. The only single layer coil presented in Table 6-1 that falls within the specification of being less than 55mm in diameter for a secondary coil, is the coil 10/50/1.

To select the primary coil, the 10/50/1 secondary coil was paired up with the three two layer primary coils with small inner diameters. The coupling $k$ was measured at up to 20mm of concentric and off-axis displacement. Shown in Figure 6-2 is the coupling vs. concentric displacement for the coils 10/50/2, 10/60/2 and 10/70/2 paired up with the 10/50/1 secondary coil. The coupling and concentric displacement is almost identical between the three pairs of coils, with the 10/50/2 primary coil giving a coupling that is 0.02 lower for the same displacement.

Shown in Figure 6-3 is the off-axis displacement (with 0mm concentric displacement) for the coils 10/50/2, 10/60/2 and 10/70/2 paired up with the 10/50/1 secondary coil. Here we can see that naturally the larger diameter coils maintain a slightly higher coupling for the same off-axis displacement. In practice the coils will be encapsulated in a bio-compatible material and separated up a layer of skin and fat, assuming that the encapsulation is at least 1mm thick and the minimum thickness of the skin and fat is 3mm, the minimum concentric displacement between the power transfer coils would be approximately 5mm. Between 5mm to 20mm of concentric displacement the coupling variation between the secondary coil and the small 10/50-2 coil is $k=0.14$ to 0.44 giving a coupling change of 3.14 times minimum; the coupling variation with the large 10/70/2 coil is $k=0.16$ to 0.47 giving a coupling change of 2.9 times minimum. Since there is only a slight benefit in using the larger 10/70/2 coil, the smaller 10/50/2 coil was chosen instead since it is much more compact and light.

The chosen power transfer coil pair 10/50/2 and 10/50/1 was then measured with a HP4285A precision LCR meter, which was deemed accurate for ESR measurements of these air-cored coils in Chapter 3. The ESR vs. frequency profile of the primary and secondary coils is shown in Figure 6-4. A second order approximation was fitted on the ESR profiles of the power transfer coils shown in Figure 6-4. The ESR of the 10/50/2 primary coil as a function of frequency is approximated as $R_{L1}(f) = 6.48 \times 10^{-7}f^2 + 4.58 \times 10^{-5}f + 0.0265$, and
the ESR of the 10/50-1 secondary coil as a function of frequency is approximated as $R_{12}(f) = 9.15 \times 10^{-8} f^2 + 4.54 \times 10^{-5} f + 0.0144$.

![Figure 6-2: Coupling $k$ versus concentric displacement of power transfer coil pairs](image1)

![Figure 6-3: Coupling $k$ versus off-axis displacement of power transfer coil pairs](image2)
6.2.3 Topology and tuning selection

With the power transfer coils selected, the next step is to determine the resonant parameters and the resonant topology. The Maximum Efficiency Tuning (MET) procedure was used here to determine the tuning and the topology of the system. The following parameters were inserted into the procedure: \( L_1 = 11.2 \mu \text{H}, \ L_2 = 3.3 \mu \text{H}, \ R_{L1}(f) = 6.5 \times 10^{-13} f^2 + 4.6 \times 10^{-8} f + 0.026, \ R_{L2}(f) = 9.2 \times 10^{-14} f^2 + 4.5 \times 10^{-8} f + 0.0144, \ V_d = 12, \ k_{\text{min}} = 0.14, \ k_{\text{max}} = 0.44, \ R_{C1} = R_{C2} = 0.05 \) and \( R_2 = 12 \text{ to } 28 \Omega \).

The full MATLAB script to implement the MET procedure is found in Appendix B. The results for the most efficient tuning parameters for each of the four basic resonant topologies across the specified coupling and load range are shown in Table 6-2.

For this set of power transfer coils, the average efficiency figures from the MET procedure clearly show that a topology with a parallel tuned secondary is the preferred option. The average efficiency of the PP and SP topologies are over 90%, while average efficiency of the PS and SS topologies are below 80%. This is mainly because the inductance of the secondary coil \( L_2 \) is much smaller than \( L_1 \), and in order for the resonant frequency of both primary and secondary tanks to be similar the value of \( C_2 \) is pushed up. This increases the secondary quality factor \( Q_s = \frac{\sqrt{L}}{R} \) for parallel tuned pickups which improves efficiency. Since the theoretical efficiency of the PP and SP topologies are so similar, the practical efficiencies
would also be very similar. Although the loss elements are different in a parallel tuned converter and in a series tuned converter, the efficiency of both converters can be very comparable. This is because class D ZVS/ZCS converters have a theoretical efficiency of 100%, thus the practical efficiency is largely dependent on how well the converter is practically implemented.

Table 6-2: Results from MET procedure for the standalone IPT system

<table>
<thead>
<tr>
<th>Topology</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>Average efficiency</th>
<th>Operating frequency</th>
<th>$Q_s$</th>
<th>AC Input voltage $V_1$</th>
<th>Required $V_{DC}$ for half-wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel-parallel</td>
<td>30nF</td>
<td>90nF</td>
<td>92.15%</td>
<td>244-276 kHz</td>
<td>1.98-4.62</td>
<td>25.71 to 78.8V</td>
<td>35.5V</td>
</tr>
<tr>
<td>Parallel-series</td>
<td>5nF</td>
<td>20nF</td>
<td>78.93%</td>
<td>667-674 kHz</td>
<td>0.46-1.07</td>
<td>50 to 159V</td>
<td>71.6V</td>
</tr>
<tr>
<td>Series-parallel</td>
<td>40nF</td>
<td>125nF</td>
<td>92.29%</td>
<td>208-239 kHz</td>
<td>2.34-5.45</td>
<td>2.42 to 9.82V</td>
<td>21.8V</td>
</tr>
<tr>
<td>Series-series</td>
<td>5nF</td>
<td>20nF</td>
<td>78.95%</td>
<td>673-689 kHz</td>
<td>0.46-1.07</td>
<td>1.75 to 4.73V</td>
<td>10.5V</td>
</tr>
</tbody>
</table>

The required DC input voltage $V_{DC}$ of a half-wave converter corresponds to the maximum AC input voltage $V_1$ required in the simulation. The PP topology requires a maximum $V_1$ of 78.8V$_{RMS}$ when $k=0.14$ and $R_2=12\Omega$ on a half-wave push-pull converter, this corresponds to a DC input voltage of 35.5V [54]. The SP topology requires a maximum $V_1$ of 9.82V$_{RMS}$ when $k=0.44$ and $R_2=12\Omega$ on a half-wave converter, this corresponds to a DC input voltage of 21.8V (Equation (5-1)).

In a practical standalone IPT system, the input voltage can only be certain voltages determined by a combination of cell voltages or common DC adapter voltages. Common DC voltages include 5V, 12V, 18V, 24V, 36V and 48V. In this case, using an input voltage of 36V for the PP configuration may be a bit tight, especially if the input source is a battery, since battery voltages tend to sag as they drain. A 24V input can be used for the SP configuration, since the maximum required input is 21.8V there is sufficient head room for voltage sag, and since the DC input voltage is lower it is considered to be safer. Hence, in terms of input voltage the SP topology is more favourable.

The simulated operating frequency of the PP and SP across the coupling and load range is shown in Figure 6-5. The simulated frequency range for the PP system is 232 to 262 kHz and the frequency range for the SP system is 203 to 231 kHz, and no frequency bifurcation was
encountered for both topologies across this coupling and load range. The frequency range for both topologies is almost identical. However, since SP system has a slightly lower operating frequency, this is usually more favourable since the power losses in many components tend to increase with frequency. Note that there is usually very little margin above or below the coupling and load range before frequency bifurcation is encountered when the parameters are determined from the MET procedure.

Since the SP topology in this case has been shown to be superior in efficiency, required input voltage and operating frequency range over all the other basic resonant topologies. The SP topology with its tuning values determined with the MET procedure is used in this implementation of a TET system for powering implantable heart pumps.

Figure 6-5: Frequency vs. coupling and load range of MET selected configurations; a) Parallel-parallel, b) Series-parallel
6.3 Implementation

A summary of the parameters of the IPT system to be implemented is given in Table 6-3. Each of these parameters has been determined and justified in section 6.2. Note that the primary and secondary resonant capacitors $C_1$ and $C_2$ could not be tuned to the values found by the MET procedure due to the limited range of capacitor values available.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary coil $L_1$</strong></td>
<td>Series tuned</td>
<td>11.2 µH, dimensions: 10/50/2</td>
</tr>
<tr>
<td><strong>Primary cap. $C_1$</strong></td>
<td>42nF (closest practical equivalent to 40nF)</td>
<td></td>
</tr>
<tr>
<td><strong>Power flow control</strong></td>
<td>Peak current control</td>
<td></td>
</tr>
<tr>
<td><strong>Secondary coil $L_2$</strong></td>
<td>Parallel tuned</td>
<td>3.3 µH, dimensions: 10/50/1</td>
</tr>
<tr>
<td><strong>Secondary cap. $C_2$</strong></td>
<td>122nF (closest practical equivalent to 125nF)</td>
<td></td>
</tr>
<tr>
<td><strong>AC to DC converter</strong></td>
<td>Full-bridge rectifier with LC smoothing filter</td>
<td></td>
</tr>
</tbody>
</table>

6.3.1 Primary circuit

The schematic of the series tuned energy injection converter (Figure 5-5) with peak current control (Figure 5-10) to be implemented in this TET system has been previously given in Chapter 5.

Additional circuitry on the primary side of a closed-loop system includes the Nordic NRF48LE transceiver which is used to receive feedback from the secondary pickup. In order for the received feedback to be useful, it is first converted back into an analogue signal via a digital to analogue converter IC. The analogue signal is then put through a PID controller where the output of the PID controller is the current limit signal $V_{ref}$ that is fed into the peak current control circuit shown in Figure 5-10. The schematic of the primary side feedback processing circuit is illustrated in Figure 6-6.
Since the system is standalone the different auxiliary voltages required for the control circuitry and the Nordic transceiver needs to be generated on-board. On this primary circuit there is a 12V and 5V buck converter IC and a negative -5V charge pump to supply the analogue control circuits. For the Nordic transceiver a linear 3.3V regulator was used for its supply. A system which generates auxiliary supply voltage rails on-board will result in a lower overall end to end efficiency, when compared to using separate external supplies for each voltage rail. However, this gives a better indication of the efficiency that can be expected of the final standalone product. A 4-layer PCB was designed for the primary circuit of this IPT system and the populated PCB measures 70mm by 65mm and has a height of 30mm. Photographs of the front and back of the populated primary circuit with the various component labels are given in Figure 6-7.

**Figure 6-6: Primary side feedback processing circuit of the standalone IPT system**

**Figure 6-7: Photograph of primary circuit of the standalone IPT system; Left) Top side, Right) Bottom side.**
6.3.2 Secondary circuit

In order to deliver power to the implantable heart pump, the power picked up by the secondary circuit must first be converted to a DC voltage. In this standalone TET system, this was achieved by using a full-bridge Schottky diode rectifier followed with a LC smoothing filter as shown in Figure 6-8. Schottky diodes were used to decrease conduction losses, due to their lower forward voltage; the Schottky diodes used had a maximum forward voltage of 0.2V when conducting up to 2A of continuous current. A smoothing inductor $L_{DC}$ is used in conjunction with a parallel tuned pickup to increase the voltage across the load $R_2$, since the inductor reduces the current ripple and tends to keeps the current draw constant from the pickup [107].

In order to send the required feedback for the primary power flow controller the output voltage $V_{out}$ was connected to the analogue to digital converter of a Nordic NRF24LE1-Q32 micro-controller which also has an in-built radio transceiver. Feedback for the primary controller is transmitted to the Nordic transceiver on the primary at 400µs intervals. Accounting for pack loss and sampling time in between the transmission intervals, the equivalent sampling rate of the output voltage at the primary is approximately 1 kHz. Such a sample rate is sufficient for any physiological change that may affect the loading of the heart pump [108]. A PCB was designed and made for the secondary circuit, the whole circuit excluding the pickup coil measures 40mm by 25mm. Photographs of the front and back of the secondary circuit are shown in Figure 6-9.

![Secondary pickup schematic of the standalone IPT system](image-url)
6.3.3 Encapsulation

In a practical TET system for powering implantable heart pumps, each component must be encapsulated with medical grade materials. Following the standard medical industry best practice, a titanium enclosure should be used for all long term active implantable devices [24]. Although the secondary circuit can be encapsulated in titanium, the power transfer coils may not as the titanium enclosure would shield the secondary pick up coil from the oscillating magnetic flux generated by the primary. Hence, the power transfer coils will be encapsulated in biocompatible silicon.

For the purpose of bench testing and future animal trials, the power transfer coils were encapsulated with Med-4011 biocompatible silicon using injection moulds designed especially for the coils. However, putting the circuits in a titanium enclosure was not a feasible option for this prototype implementation. Thus, ABS enclosures printed on a 3D printer was designed for the circuits. The primary enclosure was designed to fit tightly around the primary circuit with openings for the input DC voltage socket and the leads of the power transfer coil $L_1$. Vents above the power converter were incorporated in the enclosure to avoid the build up of heat. The secondary circuit was fitted into a 55mm by 38mm by 30mm enclosure, which has a volume of 62.7cc; however, the actual volume of the components is less than 30cc and the bulk of the mass comes from the resonant capacitance $C_2$. The fully assembled standalone IPT system is shown in Figure 6-10.
6.4 Performance

The performance of the implemented IPT system was determined and is presented in this section in terms of its regulation range, efficiency and heating.

6.4.1 Regulation range

The voltage regulation range of the standalone TET system was tested at a fixed input voltage of 22Vdc across a displacement range of 5mm to 38mm, which corresponds to a coupling range of $k=0.44$ to 0.058. The average output voltage across six load resistors in between the load range of $R_2=12$ to 33Ω was measured across this displacement range and the results are shown in Figure 6-11.

The operating specifications given in section 6.2.1 specified that the system must be able to supply at least 12Vdc to the output load $R_2$ which varies from 12Ω to 28Ω up to a maximum displacement of 20mm. This specified operating region is denoted by the green box shown in Figure 6-11. The standalone TET system implemented in this study was able to sustain an average voltage of 12.5V over this specified operating region and was even able to maintain this power flow control up to 25mm of concentric displacement. Note that the minimum concentric displacement is 5mm accounting for minimum encapsulation and skin thickness.
The concentric displacement range of 5mm to 20mm corresponds to the coupling range of $k=0.44$ to 0.14. This is a considerably large change in coupling as maximum coupling is over three times the minimum coupling. This demonstrates the wide control range of an energy injection converter.

![Average output voltage (Vdc)](image)

**Figure 6-11:** Average output voltage regulation capability of standalone IPT system, with specified displacement and load range indicated by the green box.

### 6.4.2 Power efficiency

At each of the coupling and load points shown in Figure 6-11, the end to end efficiency of the TET system was measured and is shown in Figure 6-12. The end to end efficiency refers to the average DC output power across load $R_2$ divided by the average input power of the primary circuit. Note that this efficiency includes the power consumption of all the auxiliary voltage regulators, control circuitry and rectification loss. Thus, the efficiency may not seem very high when compared to other systems reported in literature which may not be standalone systems.

The efficiency of the system across the specified coupling and load range is boxed in green in Figure 6-12, and shows that the IPT system is able to maintain an efficiency of at least 70% across this range. The average efficiency of the points within the specified coupling and load range is 75.3% and the maximum efficiency of 80% is achieved when delivering 10W of power ($R_2=15\Omega$) at coupling levels of $k>0.2$ (<11mm of displacement). Note that efficiency
decreases as output power decreases (increasing $R_2$) since the auxiliary losses of the system become more significant.

![Efficiency Diagram](image)

**Figure 6-12:** End to end efficiency of standalone IPT system, with specified displacement and load range indicated by the green box.

The difference between maximum and minimum efficiency of this system is approximately 10%. Since a 10% decrease in efficiency is expected due to changes in coupling, which affect the losses in the primary transfer coil due to the operating frequency and the primary resonant current changing. This implies that the energy injection controller maintains the same efficiency throughout its control range.

### 6.4.3 Loss breakdown

The loss breakdown of the standalone closed-loop IPT system presented here can be approximately broken down by measuring DC power losses with electrical methods and by measuring the power transfer component losses with the calorimeter presented in Chapter 3.

The power loss breakdown of this standalone TET system consists of the flowing categories:

**Primary side**
- Regulator losses
  - This represents the power consumed by the voltage regulators which generate the 12V, 5V and -5V auxiliary supply rails for the control circuitry.
• Control circuitry losses
  o This takes into account the power consumption of the Op-amps, flip-flops and resistive losses of the control circuit. Also including the power consumption of the Nordic transceiver used to obtain feedback from the secondary circuit.

• Main converter losses
  o The converter losses represent the losses of the MOSFETs and the input power supply capacitors on board the primary circuit. This includes the switching and conduction loss of the MOSFETs and the charge/discharge loss of the supply capacitors.

• Resonant circuit losses
  o The losses in this category will be broken down into the losses of the power transfer coil and the primary resonant capacitor.

**Secondary side**

• Resonant circuit losses
  o The losses in this category will be broken down into the losses of the pickup coil and the secondary resonant capacitor.

• Rectifier losses
  o This takes into account the power loss in the AC to DC rectification circuit, which includes the conduction and turn on/off losses of the Schottky diodes and the losses in the smoothing LC filter.

• Regulator losses
  o This represents the power consumed by the voltage regulator which generates the 3.3V auxiliary supply rail for the control circuitry.

• Control circuitry losses
  o This represents the power consumption of the Nordic micro-controller which functions as the analogue to digital converter and sends feedback to the primary circuit.

The power losses of the regulators and control circuit were measured with electrical instruments since their input voltages and currents are approximately DC. The losses of the power transfer coils and resonant capacitors were measured with the Peltier device balance calorimeter presented in Chapter 3.
The main converter losses were difficult to measure electrically due to switching transients and the components could not be placed individually in the calorimeter. Thus, its power consumption was estimated by subtracting the measurable losses off the standby input power on the primary converter. The rectifier losses were also difficult to measure due to the same difficulties encountered with the main converter components. Hence, the full-bridge rectifier losses were calculated with Equation (3-17) and the LC smoothing filter losses with Equation (3-18), using the output current flowing through $R_2$ and a diode forward voltage of $V_f=0.2V$.

The loss breakdown of the standalone series-parallel TET system across the displacement range of 5mm to 21mm when the system was delivering the maximum power of 13W (12.5V across 12Ω load) is shown in Figure 6-13. From this breakdown it can be seen that over 50% of the losses come from the primary side of the system. Since Figure 6-13 shows the loss breakdown when the system is delivering a constant power of 13W the secondary side losses remain roughly the same throughout the displacement range. It can also be seen that the secondary rectifier accounts for about 70% of the losses on the secondary side. The primary side losses increase as the coil displacement increases (coupling decreases), and is mostly due to the increased power transfer coil and resonant capacitor losses. This increased dissipation in the resonant circuit is due to the increase in primary resonant current required to supply the output power when coupling decreases.

![Figure 6-13: Power loss breakdown of standalone TET system when delivering 13W](image-url)
6.5 Summary

In this chapter the implementation of a standalone closed-loop TET system and puts the analysis in previous chapters of this thesis into a practical design context. A design specification which would meet the expectation of most people was put together using information obtained in published literature and from communication with heart pump companies. The implemented standalone closed-loop IPT system was able to regulate the output voltage to 12.5V from 5mm to 20mm of concentric displacement or 20mm of off-axis displacement while maintaining at least 70% end to end efficiency, across a controllable output power range of 5 to 12W.
Chapter 7

Conclusions and suggestions for future work

This thesis presents a comprehensive study in the characterisation and power flow control of Inductive Power systems for powering implantable heart pumps. While all aspects for the development of a practical IPT system was considered in this work, specific attention has be given to the following aspects:

- The selection of resonant topology and parameters of the IPT system
- The accurate power loss determination of power transfer components
- The development of primary side power flow control methods.

7.1 General conclusions

Chapter 1 of this thesis presents a general overview of the role of implantable heart pumps such as LVADs in the world of modern medical therapies for heart disease. A health risk of current implantable heart pump systems is the use of a percutaneous power delivery cable and their susceptibility to infection. Power delivery via Inductive Power Transfer (IPT) would eliminate this health risk and promote the use of implantable heart pumps. The power requirements of current state-of-the-art heart pumps are presented and the practical challenges associated with an IPT system for this application have been addressed. It is challenging to maintain a consistent coupling between the implanted coil and the external coil located on the body surface, so a system tolerant to variability in alignment is crucial for wide spread uptake. The system is further complicated by the need to manage heat generated carefully.
The design of a resonant system for moderately coupled IPT systems has always been a challenging task due to the complex relationships that exists between resonant parameters, coupling, topology and resonant frequency. The importance of proper selection of resonant topology and parameters has been presented in Chapter 2 and the characteristics of basic resonant topologies under moderately coupled conditions have been analysed. The design challenges and constraints of the resonant system in particular for the application of powering implantable heart pumps have been discussed. Frequency domain models were developed to analyse the steady-state characteristics of resonant circuit configurations. A design methodology called the Maximum efficiency tuning (MET) procedure was developed, which aids the design engineer to select the most suitable resonant topology, tuning parameters and operating frequency for a given set of power transfer coils and operating conditions. Finally, a case study was presented which demonstrated that the resonant configuration determined with the MET procedure increased the minimum efficiency point by 15% when compared to a traditionally tuned resonant configuration.

Chapter 3 was a comprehensive investigation of power loss measurement techniques of electrical components. The power dissipation of an IPT system for implantable heart pumps, especially the power loss of the power transfer coils, must be accurately known in order for it to be safe. The determination of power loss for components under high frequency voltage/current excitation can be highly inaccurate with electrical measurement techniques. This is due to the limitations of digital electronic instruments, since at high frequencies insufficient sampling speed or delays in capturing the high frequency waveforms will generate large errors in the calculation of power.

Calorimetric techniques of power measurement, which rely on measuring the heat released by a component, were proposed for the power loss measurement of power transfer coils. Since power loss must eventually be released as conducted or radiated heat, calorimetric measurement techniques are unaffected by the high frequency electrical excitation of the component to be measured. Several calorimeters have been proposed and analysed. A novel cost effective calorimeter based on using a Peltier device as a heat balance sensor was proposed and implemented. The proposed calorimeter had a power measurement accuracy of ±0.042W for components dissipating more than 0.2W of power and the calorimeter was shown to maintain this accuracy across starting coolant temperatures of 22°C to 27°C. It was shown that the loss of power transfer coils were overestimated by approximately 10% and the loss of the resonant capacitors were underestimated by approximately 50%. A comparison
between the power losses of resonant components measured by different electronic instruments showed that the measurement can be more than 100% different.

An overview of primary side power flow control based on varying the frequency of the resonant circuit of an IPT system was undertaken in Chapter 4. The effect of frequency power flow control was analysed and different methods to vary the frequency of the resonant tank was discussed. Practical converters based on frequency power flow control via the use of switched capacitors were applied on both parallel tuned and series tuned resonant tanks. The practical implementation has been improved such that a larger switched capacitance could be used which extends the power flow control range. The output voltage measured in practice driven by a switched capacitance converter matched the theoretical simulation output voltage of a variable capacitance resonant tank. A maximum output voltage control range of 30% from maximum voltage was shown to be possible with frequency power flow control.

In Chapter 5 the concept of energy injection control was applied as a primary side power flow controller for regulating the output voltage of an IPT system. Methods of how energy injection control can be applied to ZVS/ZCS power converters were presented. A new energy injection control method based on generating PDM control patterns that provides maximum control range, minimum Crest factor waveforms and linear control action was proposed. Energy injection power converters and control circuitry was designed for driving both series and parallel tuned primary resonant tanks. State-space representations of energy injection converter systems were created to simulate time domain resonant waveforms, such that different control methods could be compared and visualised. Different energy injection power flow control methods were practically implemented, and an output voltage control range of over 75% from maximum voltage was shown to be achievable with energy injection power flow control.

Chapter 6 of this thesis presents the implementation of a standalone closed-loop IPT system for powering implantable heart pumps. Practical considerations for each part of the TET system were taken into account. The resonant topology and tuning parameters of the practical TET system were determined with MET procedure presented in Chapter 2. Due to the superior control range of energy injection power flow control, it was applied to the selected topology. A wireless communications link was established between primary and secondary circuits, to obtain feedback for primary side output control. The performance of the
standalone TET system was able to provide 5 to 12W at an output voltage of 12V across a coupling range of $k=0.14$ to 0.44 at above 70% end to end efficiency.

7.2 Contributions

The main contributions of this thesis include:

- Developing an iterative numerical procedure (MET) for the selection of resonant topology, tuning parameters and operating frequency of an IPT system.
- Inventing a novel balance calorimeter using a Peltier device as the balance sensor. This calorimeter is cost effective and the implemented calorimeter was able to obtain a power loss estimate with an accuracy of ±0.042W.
- Accurate power loss determination of power transfer coils and resonant capacitors during operation.
- Improved implementation of switched capacitor frequency power flow control converters for both series and parallel tuned primary resonant tanks.
- Introduction of energy injection converters for IPT systems powering implantable heart pumps.
- Development of energy injection controllers for converters driving both series and parallel tuned primary resonant tanks.
- Development of multi-part state-space simulations for analysing the control of non-linear energy injection converters.
- Invented a new energy injection control method based on PDM patterns. The new PDM pattern control provides maximum control range, minimum Crest factor waveforms and linear control action.
- Implementation of a fully standalone closed-loop IPT system based on energy injection control. Demonstrating that 5 to 12W of power can be controlled over a coupling range of $k=0.14$ to 0.44 (5mm to 20mm separation) with an end to end efficiency of 70 to 80%.
7.3 Recognitions

Work undertaken over the duration of this PhD has produced the following journal publications:


Work undertaken has also produced the following conference papers:


Following awards were received for the work carried out in this thesis:

2. Top 20 finalist in EXPOSURE (Postgraduate Research Exposition) 2012 in the Poster Category.

### 7.4 Suggestions for future work

The work carried out in this thesis focused on the design and improvement of resonant IPT systems in terms of the selection of resonant parameters, primary power flow control and the power dissipation of resonant components. The following areas were identified where further research would be valuable:

**System level energy management**

Although a heart pump such as an LVAD is supplementary to a natural heart, the pump must not stop; any formation of blood clots could be fatal on re-starting. It is to be expected that an external primary coil - not fixed on to the skin of the patient – will at times be removed and power delivered over the TET system will be interrupted. In addition, when a change of the primary energy source is required (replace battery source, or change to wall power), the IPT link could be disrupted for a short period of time. Hence, the TET system must have a backup energy source which could power the heart pump for short periods of time. This backup source is necessary in terms of safety and operational reasons. The development of an energy management system is required for the practical implementation of a TET system for implantable heart pumps. Research in analysing different rechargeable implantable energy sources such as batteries and super-capacitors needs to be conducted. The design of charging circuits and charging schemes will also need to be developed. Since many rechargeable sources degrade after many cycles of charging and discharging a guideline must also be provided on how often the backup source can be used. This is important since operating the heart pump off an implanted source allows the patient to perform activities such as showering, which would otherwise be inconvenient with a tethered primary coil.
SAR and EMI analysis of implantable TET system

One area of concern regarding the use of an IPT system for delivering power to implantable devices is whether or not the magnetic field of an IPT system imposes a health risk on the patient. Since an IPT system works by generating high frequency magnetic fields to transfer power through the skin, analysis must be carried out to find if this has any effect on nearby tissue. This includes the analysis of Specific Absorption Rate (SAR) values due to the IPT system and whether this is sufficient to damage or heat up surrounding tissue. In addition, the field strength of the power transfer coils should also fall under the guidelines of medical device standards. Another area of concern is the amount of Electromagnetic interference (EMI) that the IPT system introduces. Tests must be carried out under conditions specified under electromagnetic compatibility (EMC) standards, to find the conducted and radiated emissions of the TET system. Results from such tests will indicate the necessary shielding and filtering required for the TET system.

Power efficiency improvements

The work presented in this thesis primarily focuses on improving power efficiency through proper selection of resonant parameters and the development of efficient power flow controllers for power converters. It has been observed during this work that a full-bridge diode rectifier used on the secondary circuit to generate a DC voltage for the load can decrease the efficiency of the system by up to 5% when compared to a system driving an AC load. Therefore, improvements to the AC to DC conversion of the secondary circuit will greatly improve the end to end efficiency and decrease the heat generation of the implanted secondary circuit. Synchronous rectification should be investigated to reduce the rectifier losses.

Tissue heating due to power transfer components

Although Chapter 3 of this thesis presents a calorimeter to accurately measure the power dissipation of power transfer components of the TET system. Further research is needed to investigate whether this level of power dissipation is safe for implantation. This work will involve creating a model to predict the effects of heating on skin tissue due to the power transfer components and the model will have to be verified in animal trials. Research should be carried out to determine a safe operating temperature range and relate this to the size and power dissipation of the IPT power transfer coils.
Appendices

Appendix A: MATLAB code for finding zero crossings

```matlab
% This function is used to find the zero crossing points of an expression
% 1) finds adjacent points with opposite polarity
% 2) use linear interpolation to find zero crossing point
function X0 = findzeros(X,Y)

Y_length = length(Y); indix = [];
Y1 = Y(2);
for i=2:Y_length
    if Y(i)*Y1 < 0
        Y1=Y(i);
        indix = [indix, i];
    end
end

Zeropoints=[];
for i=1:length(indix)
    K = (X(indix(i))-X(indix(i)-1))/(Y(indix(i))-Y(indix(i)-1));
    x= X(indix(i))- K * Y(indix(i));
    Zeropoints = [Zeropoints x];
end
X0 = Zeropoints;
end
```

%----------------------------------------------------------
%----------------------------------------------------------
Appendix B: MATLAB code for Maximum efficiency tuning (MET) procedure

```matlab
%--USER INPUT------------------------------------------------------------------
kmin = 14;  %minimum coupling k (x100)
kmax = 44;  %maximum coupling k (x100)
V2d = 12;   %desired RMS output voltage (Vrms)
L1=11.2e-6; %primary coil inductance (H)
L2=3.3e-6;  %secondary coil inductance (H)
C1=0;      %C1 Primary capacitor
C2=0;      %C2 Secondary capacitor
f=[0:1e2:2000e3];
w=2*pi.*f;

RL1 = 6e-13.*f.*f + 5e-8.*f + 0.026;  %Primary coil ESR profile (ohms)
RL2 = 9e-14.*f.*f + 4e-8.*f + 0.014;  %Secondary coil ESR profile (ohms)

C1min=15;   %Minimum C1 search value (nF)
C1max=50;   %Maximum C1 search value (nF)
C1step=5;   %Step size for C1 search value (nF)
C2min=60;   %Minimum C2 search value (nF)
C2max=150;  %Maximum C2 search value (nF)
C2step=5;   %Step size for C2 search value (nF)
R2min = 10; %Minimum load R2 (ohms)
R2max = 24; %Maximum load R2 (ohms)
R2step = 2; %Load increment step size (ohms)

%--Initialisation----------------------------------------------
Z1pp=zeros(20001,kmax);
Z1ps=zeros(20001,kmax);
Z1ss=zeros(20001,kmax);
Z1sp=zeros(20001,kmax);
ZApp=ones(1,kmax);
ZAps=ones(1,kmax);
ZAsp=ones(1,kmax);
ZAss=ones(1,kmax);

max_aveeffpp = 0;
max_aveeffps = 0;
max_aveeffsp = 0;
max_aveeffss = 0;

%--Impedance calculations------------------------------------
for y=C1min:C1step:C1max  %C1 search range
    C1=y*1e-9
    surfy = surfy + 1;
    surfx = 0;
    for z=C2min:C2step:C2max  %C2 search range
        C2=z*1e-9;
        surfx = surfx +1;
        for x=kmin:1:kmax
            k = x/100;
            M=k.*sqrt(L1*L2);
            for R2=R2min:R2step:R2max
                RC1 = 2e-7*C1^0.73;  %Primary coil ESR profile
                RC2 = 2e-7*C2^0.73;  %Secondary coil ESR profile
                %--Impedance calculations--------------------------
            end
        end
    end
end
```

%--USER INPUT------------------------------------------------------------------
\[
Z_{2p} = i \cdot w \cdot L_2 + RL_2 + R_2 \cdot (i \cdot w \cdot C_2 \cdot RC_2 + 1) \\
\quad ./(i \cdot w \cdot C_2 \cdot (R_2 + RC_2) + 1); \\
Z_{2s} = i \cdot w \cdot L_2 + RL_2 + 1/(i \cdot w \cdot C_2) + RC_2 + R_2; \\
Z_{rp} = w \cdot w \cdot M \cdot M/Z_{2p}; \\
Z_{rs} = w \cdot w \cdot M \cdot Z_{2s}; \\
Z_{lpp}(;x) = (1/(i \cdot w \cdot C_1 + RC_1) \cdot (i \cdot w \cdot L_1 + RL_1 + Z_{rp}) \\
\quad ./(i \cdot w \cdot L_1 + RL_1 + 1/(i \cdot w \cdot C_1) + RC_1 + Z_{rp}); \\
Z_{lps}(;x) = (1/(i \cdot w \cdot C_1) + RC_1) \cdot (i \cdot w \cdot L_1 + RL_1 + Z_{rs}) \\
\quad ./(i \cdot w \cdot L_1 + RL_1 + 1/(i \cdot w \cdot C_1) + RC_1 + Z_{rs}); \\
Z_{lass}(;x) = 1/(i \cdot w \cdot C_1) + RC_1 + i \cdot w \cdot L_1 + RL_1 + Z_{rs}; \\
Z_{lss}(;x) = 1/(i \cdot w \cdot C_1) + RC_1 + i \cdot w \cdot L_1 + RL_1 + Z_{rs}; \\
\%--------------------------------------------- \\
\%-- find highest zero phase angle frequency--------- \\
freqpp = findzeros(f, angle(Z_{lpp}(;x))); \\
freqps = findzeros(f, angle(Z_{lps}(;x))); \\
freqpp = findzeros(f, angle(Z_{lps}(;x))); \\
freqps = findzeros(f, angle(Z_{lss}(;x))); \\
\%-- if bifurcation occurs repeat with freqxx(length(freqxx)) = \\
\% Z_{App}=round(freqpp(1)/100); \\
\% Z_{App}=round(freqps(1)/100); \\
\% Z_{App}=round(freqpp(1)/100); \\
\% Z_{App}=round(freqps(1)/100); \\
\%-- find input required for desired output voltage V2------ \\
\%-- PP-------------------------------------------------
V_{lpp} = abs(V_{2d} \cdot (i \cdot w \cdot Z_{App}) \cdot L_1 + RL_1 \cdot Z_{App}) + Z_{rp}(Z_{App}) \\
\quad ./((i \cdot w \cdot Z_{App}) \cdot M \cdot (1 -(i \cdot w \cdot Z_{App}) \\
\quad \cdot L_2 + RL_2 (Z_{App}) / Z_{2p}(Z_{App}) )); \\
I_{lpp} = V_{lpp} / (i \cdot w \cdot Z_{App}) \cdot L_1 + RL_1 (Z_{App}) + Z_{rs}(Z_{App})); \\
I_{lpp} = e^{i \cdot w \cdot Z_{App}} \cdot M \cdot I_{lpp} / Z_{2p}(Z_{App}); \\
V_{lpp} = e^{i \cdot w \cdot Z_{App}} \cdot M \cdot I_{lpp} - e^{i \cdot w \cdot Z_{App}} \cdot L_2 \cdot I_{lpp} \\
\quad - RL_2 (Z_{App}); \\
R_{powerpp}(R_2/R_2step) = abs(I_{lpp}) \cdot abs(I_{lpp}) \cdot (RL_1(Z_{App}) + RC_1) \\
\quad + abs(I_{lpp}) \cdot abs(I_{lpp}) \cdot (RL_2(Z_{App}) + RC_2); \% not exact \\
R_{effpp}(R_2/R_2step) = (abs(V_{2pp}) \cdot abs(V_{2pp}) / R_2) / \\
\quad (R_{powerpp}(R_2/R_2step) + abs(V_{2pp}) \cdot abs(V_{2pp}) / R_2); \\
\%-- PS-------------------------------------------------
V_{lps} = abs(V_{2d} \cdot Z_{2s}(Z_{App} \cdot L_2 + RL_2 (Z_{App}) + Z_{rs}(Z_{App})) \\
\quad ./((i \cdot w \cdot Z_{App}) \cdot M \cdot Z_{rs}(Z_{App})); \\
I_{lps} = V_{lps} / (i \cdot w \cdot Z_{App}) \cdot L_1 + RL_1 (Z_{App}) + Z_{rs}(Z_{App}); \\
I_{lps} = e^{i \cdot w \cdot Z_{App}} \cdot M \cdot I_{lps} / Z_{2s}(Z_{App}); \\
R_{powerps}(R_2/R_2step) = abs(I_{lps}) \cdot abs(I_{lps}) \cdot (RL_1(Z_{App}) + RC_1) \\
\quad + abs(I_{lps}) \cdot abs(I_{lps}) \cdot (RL_2(Z_{App}) + RC_2); \% not exact \\
R_{effps}(R_2/R_2step) = (abs(I_{2ps}) \cdot abs(I_{2ps}) \cdot R_2) / \\
\quad (R_{powerps}(R_2/R_2step) + abs(I_{2ps}) \cdot abs(I_{2ps}) \cdot R_2); \\
\%-- SP-------------------------------------------------
V_{lsp} = abs(V_{2d} \cdot Z_{lsp}(Z_{App}, x)) / (i \cdot w \cdot Z_{App}) \cdot M \cdot (1 -(i \cdot w \cdot Z_{App}) \\
\quad \cdot L_2 + RL_2 (Z_{App}) / Z_{2p}(Z_{App})); \\
I_{lsp} = V_{lsp} / Z_{lsp}(Z_{App}, x); \\
I_{lsp} = e^{i \cdot w \cdot Z_{App}} \cdot M \cdot I_{lsp} / Z_{2p}(Z_{App}); \\
V_{lsp} = e^{i \cdot w \cdot Z_{App}} \cdot M \cdot I_{lsp} - e^{i \cdot w \cdot Z_{App}} \cdot L_2 \cdot I_{lsp} \\
\quad - RL_2 (Z_{App}); \\
R_{powersp}(R_2/R_2step) = abs(I_{lsp}) \cdot abs(I_{lsp}) \cdot (RL_1(Z_{App}) + RC_1) \\
\quad + abs(I_{lsp}) \cdot abs(I_{lsp}) \cdot (RL_2(Z_{App}) + RC_2); \% not exact \\
R_{effps}(R_2/R_2step) = (abs(V_{2sp}) \cdot abs(V_{2sp}) / R_2) / \\
\quad (R_{powersp}(R_2/R_2step) + abs(V_{2sp}) \cdot abs(V_{2sp}) / R_2); \\
\%-- SS-------------------------------------------------
V_{lss} = abs(V_{2d} \cdot Z_{lss}(Z_{App}, x)) \cdot Z_{2p}(Z_{App}) / (i \cdot w \cdot Z_{App}) \cdot M \cdot R_2); \\
I_{lss} = V_{lss} / Z_{lss}(Z_{App}, x); \\
I_{lss} = e^{i \cdot w \cdot Z_{App}} \cdot M \cdot I_{lss} / Z_{2s}(Z_{App});
\[ \text{Rpowerss}(R2/R2\text{step}) = \text{abs}(I1ss) \cdot \text{abs}(I2ss) \cdot \text{abs}(I2ss) \cdot (RL1(ZAss)+RC1) + \text{abs}(I2ss) \cdot \text{abs}(I2ss) \cdot \text{abs}(I2ss) \cdot R2/\left(\text{Rpowerss}(R2/R2\text{step}) + \text{abs}(I2ss) \cdot \text{abs}(I2ss) \cdot R2\right) \]

\%

\[ \text{Reffss}(R2/R2\text{step}) = (\text{abs}(I2ss) \cdot \text{abs}(I2ss) \cdot R2)/\left(\text{Rpowerss}(R2/R2\text{step}) + \text{abs}(I2ss) \cdot \text{abs}(I2ss) \cdot R2\right) \]

%----------------------------------------------------------
end

Kpowerpp(x) = mean(Kpowerpp([R2min/R2\text{step}:1:R2max/R2\text{step}]));
Kpowerps(x) = mean(Kpowerps([R2min/R2\text{step}:1:R2max/R2\text{step}]));
Kpowersp(x) = mean(Kpowersp([R2min/R2\text{step}:1:R2max/R2\text{step}]));
Kpowerss(x) = mean(Kpowerss([R2min/R2\text{step}:1:R2max/R2\text{step}]));
Keffpp(x) = mean(Keffpp([R2min/R2\text{step}:1:R2max/R2\text{step}]));
Keffps(x) = mean(Keffps([R2min/R2\text{step}:1:R2max/R2\text{step}]));
Keffsp(x) = mean(Keffsp([R2min/R2\text{step}:1:R2max/R2\text{step}]));
Keffss(x) = mean(Keffss([R2min/R2\text{step}:1:R2max/R2\text{step}]));

end

%calculate coil power loss across coupling and load range
%for current capacitor configuration
avepowerpp = mean(Kpowerpp([kmin:1:kmax]));
avepowerps = mean(Kpowerps([kmin:1:kmax]));
avepowersp = mean(Kpowersp([kmin:1:kmax]));
avepowerss = mean(Kpowerss([kmin:1:kmax]));
aveeffpp = mean(Keffpp([kmin:1:kmax]));
aveeffps = mean(Keffps([kmin:1:kmax]));
aveeffsp = mean(Keffsp([kmin:1:kmax]));
aveeffss = mean(Keffss([kmin:1:kmax]));

%building up surf diagram
surfpowerpp(surfx,surfy) = avepowerpp;
surfpowerps(surfx,surfy) = avepowerps;
surfpowersp(surfx,surfy) = avepowersp;
surfpowerss(surfx,surfy) = avepowerss;
surffeffpp(surfx,surfy) = aveeffpp;
surffeffps(surfx,surfy) = aveeffps;
surffeffsp(surfx,surfy) = aveeffsp;
surffeffss(surfx,surfy) = aveeffss;

if (aveeffpp > max_aveeffpp)
    max_aveeffpp = aveeffpp;
    Clpp=C1;
    C2pp=C2;
end
if (aveeffps > max_aveeffps)
    max_aveeffps = aveeffps;
    Clps=C1;
    C2ps=C2;
end
if (aveeffsp > max_aveeffsp)
    max_aveeffsp = aveeffsp;
    Clsp=C1;
    C2sp=C2;
end
if (aveeffss > max_aveeffss)
    max_aveeffss = aveeffss;
    Clss=C1;
    C2ss=C2;
end
end
end

%--Display results---------------------------------------------------------------
disp('k min =');
disp(kmin/100);
disp('k max =');
disp(kmax/100);
disp('highest ave eff for parallel parallel is');
disp(max_aveeffpp);
disp('when Cl =');
disp(C1pp);
disp(' and when C2 =');
disp(C2pp);
disp('highest avg eff for parallel series is');
disp(max_aveeffps);
disp('when C1 =');
disp(C1ps);
disp(' and when C2 =');
disp(C2ps);

disp('highest avg eff for series series is');
disp(max_aveeffss);
disp('when C1 =');
disp(C1ss);
disp(' and when C2 =');
disp(C2ss);

disp('highest avg eff for series parallel is');
disp(max_aveeffsp);
disp('when C1 =');
disp(C1sp);
disp(' and when C2 =');
disp(C2sp);

figure(1);
subplot(2,2,1);
surf([C1min:C1step:C1max],[C2min:C2step:C2max],surfeffpp)
ylabel('Secondary capacitor C2');
xlabel('Primary capacitor C1')
title('Parallel Parallel - efficiency vs C1 and C2');

subplot(2,2,2);
surf([C1min:C1step:C1max],[C2min:C2step:C2max],surfeffsp)
ylabel('Secondary capacitor C2');
xlabel('Primary capacitor C1')
title('Series Parallel - efficiency vs C1 and C2');

subplot(2,2,3);
surf([C1min:C1step:C1max],[C2min:C2step:C2max],surfeffps)
ylabel('Secondary capacitor C2');
xlabel('Primary capacitor C1')
title('Parallel Series - efficiency vs C1 and C2');

subplot(2,2,4);
surf([C1min:C1step:C1max],[C2min:C2step:C2max],surfeffss)
ylabel('Secondary capacitor C2');
xlabel('Primary capacitor C1')
title('Series Series - efficiency vs C1 and C2');

%---------------------------------------------------------------
Appendix C: MATLAB code for the time domain waveforms of a cycle-by-cycle output voltage controller on a series tuned half-wave energy injection converter.

```matlab
%--USER INPUT---------------------------------------------------------------
V2d = 12; % desired RMS output voltage (Vrms)
V1i = 20; % DC input voltage to half-wave converter (V)
L1 = 12e-6; % Primary coil inductance (H)
L2 = 11.45e-6; % Secondary coil inductance (H)
Rl1 = 0.05; % Primary coil ESR (ohms)
Rl2 = 0.03; % Secondary coil ESR (ohms)
Cl = 53e-9; % Primary capacitor (nF)
C2 = 16e-9; % Secondary capacitor (nF)
RC1 = 0.01; % Primary capacitor ESR (ohms)
RC2 = 0.01; % Secondary capacitor ESR (ohms)
R2 = 12; % Output load R2
k = 0.3; % Coupling coefficient

M = k*sqrt(L1*L2); % Mutual inductance M

%--State-space formulation in the form Ax' + Bx = C
%--x = [Il1 vC1 Il2 vC2]
%--For SP topology----------------------------------------------------------
Ap = [ 0   -C1  0   0   ];
Ap = [Ap; L1  0   M  0   ];
Ap = [Ap; M/R2 0   L2/R2  C2   ];
Ap = [Ap; M  0   L2   -RC2*C2   ];
InAp = inv(Ap);
Bp = [ 1  0   0   0   ];
Bp = [Bp; (RL1+RC1) 1   0   0   ];
Bp = [Bp; 0   0   (1+RL2/R2) 0   0   ];
Bp = [Bp; 0   0   RL2  -1   ];
Ct1p = [0 Vin 0 0]';
Ct2p = [0 0 0 0]';

%--For SS topology-----------------------------------------------------------
As = [ 0   -C1  0   0   ];
As = [As; L1  0   M  0   ];
As = [As; 0   0   0   -C2   ];
As = [As; M  0   L2  0   ];
InAs = inv(As);
Bs = [ 1  0   0   0   ];
Bs = [Bs; (RL1+RC1) 1   0   0   ];
Bs = [Bs; 0   0   1   0   ];
Bs = [Bs; 0   0   (RL2+RC2+R2) 1   ];
Ct1s = [0 Vin 0 0]';
Ct2s = [0 0 0 0]';

%--Put into the form
%--x'=Ax + B
%--for SP configuration (used either one)------------------------------------
A = -InAs*Bs;
B1 = InAs*Ct1s;
B2 = InAs*Ct2s;
%--for SS configuration (used either one)------------------------------------
A = -InAs*Bs;
B1 = InAs*Ct1s;
B2 = InAs*Ct2s;
%--Initialisation------------------------------------------------------------
cycles = 200; % Number of resonant cycles to simulate
options = odeset('RelTol',1e-8,'AbsTol',1e-8);
t=[0 0 0 0];
index = 1;
CT = 1e-5; % Search period for zero crossings
toggle = 1; % 1 indicates positive current phase, 0 indicates negative current phase
```

---

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Appendices

\[ xx = [0 0 0 0]; \quad %x=[IL1 VC1 IL2 VC2] \]
\[ x0 = [0 0 0 0]; \quad %initial conditions for x \]
\[ tt = [0]; \quad %time vector \]
\[ sw = [0]; \quad %switch vector \]
\[ Vout = [0]; \quad %output voltage vector \]
\[ on = ones(200,1); \]
\[ off = zeros(200,1); \]
\[ VoutRMS = 0; \quad %Most recent RMS output voltage \]

\[ \text{for cyc=1:1:cycles} \]
\[ \quad \text{if (cyc == 1)} \]
\[ \quad \quad \text{tspan} = [0 CT]; \]
\[ \quad \text{else} \]
\[ \quad \quad \text{tspan} = [\text{zero_crossing}(1) \text{ zero_crossing}(1)+CT]; \]
\[ \quad \text{end} \]
\[ \text{if (toggle == 1)} \quad %positive current phase \]
\[ \quad \quad \text{if (VoutRMS < V2d)} \quad %RMS Output voltage less than desired output \]
\[ \quad \quad \quad \quad [t,x] = \text{ode45}(\text{@}(t,x) \text{ makeode}(t,x,A,B1),\text{tspan},x0); \]
\[ \quad \quad \text{else} \quad %RMS Output voltage larger than desired output \]
\[ \quad \quad \quad \quad [t,x] = \text{ode45}(\text{@}(t,x) \text{ makeode}(t,x,A,B2),\text{tspan},x0); \]
\[ \quad \text{end} \]
\[ \quad \text{toggle} = 0; \]
\[ \text{else} \quad %negative current phase \]
\[ \quad \quad \text{if (VoutRMS < V2d)} \quad %RMS Output voltage less than desired output \]
\[ \quad \quad \quad \quad [t,x] = \text{ode45}(\text{@}(t,x) \text{ makeode}(t,x,A,B2),\text{tspan},x0); \]
\[ \quad \quad \quad \quad \text{toggle} = 1; \]
\[ \quad \text{end} \]
\[ \text{end} \]
\[ \%find primary current zero crossing, x(:,1) is primary current \]
\[ \text{[zero_crossing,index] = findzeros(t,x(:,1));} \]
\[ \%find next set of initial values at time of 1st zero crossing \]
\[ \text{x0} = \text{interp1}(t,x,\text{zero_crossing}(1)); \]
\[ \%--For SP configuration Vout=VC2, x(:,4) is VC2 (use either one)----- \]
\[ \text{Vout} = [\text{Vout}; x(1:1:(index(1)-1),4)]; \]
\[ \%--For SS configuration Vout=I2*R2, x(:,3) is I2 (use either one)----- \]
\[ \text{Vout} = [\text{Vout}; x(1:1:(index(1)-1),3)*R2]; \]
\[ \%--Build time vector for display \]
\[ \text{tt} = [tt; t(1:1:(index(1)-1))]; \]
\[ \%building output vector I1 V1 I2 V2 for display \]
\[ \text{xx} = [xx; x(1:1:(index(1)-1),:)]; \]
\[ \%build switch vector for display \]
\[ \text{if (toggle == 0) \&\& (VoutRMS < V2d)} \]
\[ \quad \text{sw} = [\text{sw}; \text{on}(1:1:(index(1)-1))]; \]
\[ \text{else} \]
\[ \quad \text{sw} = [\text{sw}; \text{off}(1:1:(index(1)-1))]; \]
\[ \text{end} \]
\[ \%Find current RMS output voltage \]
\[ \text{VoutRMS} = \text{findpeakabs}(x(1:1:(index(1)-1),:))/sqrt(2); \]
\[ \text{end} \]
\[ \%--Display results----------------------------- \]
\[ \text{figure(1);} \]
\[ \text{plot(tt,xx(:,1),tt,xx(:,3),tt,sw);} \]
\[ \text{title('Resonant currents');} \]
\[ \text{legend('show');} \]
\[ \text{legend('I L1','I L2','switch');} \]
\[ \text{figure(2);} \]
\[ \text{plot(tt,xx(:,2),tt,xx(:,4),tt,sw*20,tt,Vout);} \]
\[ \text{title('Resonant voltages');} \]
\[ \text{legend('show');} \]
\[ \text{legend('V C1','V C2','switch','Vout');} \]
\[ \%----------------------------------------------- \]
Appendix D: MATLAB code for the time domain waveforms of a cycle-by-cycle peak current controller on a series tuned half-wave energy injection converter.

```matlab
%Use same code for the part before Initialisation in Appendix C
%---Initialisation---------------------------------------------------------------
cycles = 200; %Number of resonant cycles to simulate
options = odeset('RelTol',1e-8,'AbsTol',1e-8);
t=[0 0 0 0];
index = 1;
CT = 1e-5; %Search period for zero crossings
toggle = 1; % indicates positive current phase, 0 indicates negative current phase
xx = [0 0 0 0]; %x=[IL1 VC1 IL2 VC2]
x0 = [0 0 0 0]; %initial conditions for x
tt = [0]; %time vector
sw = [0]; %switch vector
Vout = [0]; %output voltage vector
limit = [0]; %current limit vector
I1peak = 0; %primary current peak
on = ones(200,1);
off = zeros(200,1);
Climit = 3; %current limit (A)
%---------------------------------------------------------------
for cyc=1:1:cycles
  if (cyc == 1)
tspan = [0 CT];
  else
    tspans = [zero_crossing(1) zero_crossing(1)+CT];
  end
  if (toggle == 1) %positive current phase
    if (I1peak < Climit) %primary resonant current peak less than limit
      %solve for injection phase
      [t,x] = ode45(@(t,x) makeode(t,x,A,B1),tspan,x0);
    else %primary resonant current peak larger than limit
      %solve for free oscillation phase again
      [t,x] = ode45(@(t,x) makeode(t,x,A,B2),tspan,x0);
    end
    toggle = 0;
  else %negative current phase
    %solve for free oscillation phase
    [t,x] = ode45(@(t,x) makeode(t,x,A,B2),tspan,x0);
    toggle = 1;
  end
  %find primary current zero crossing, x(:,1) is primary current
  [zero_crossing,index] = findzeros(t,x(:,1));
  %find next set of initial values at time of 1st zero crossing
  x0 = interp1(t,x,zero_crossing(1))
  %---For SP configuration Vout=VC2, x(:,4) is VC2 (use either one)------
  Vout = [Vout; x(:,1:1:(index(1)-1),4)]
  %---For SS configuration Vout=I2*R2, x(:,3) is I2 (use either one)------
  Vout = [Vout; x(:,1:1:(index(1)-1),3)*R2]

%---build time vector for display
tt = [tt; t(1:1:(index(1)-1))];
%building output vector I1 V1 I2 V2 for display
xx = [xx; x(:,1:1:(index(1)-1),:)];
%build switch vector for display
if (toggle == 0) && (I1peak < Climit)
sw = [sw; on(1:1:(index(1)-1))];
else
    sw = [sw; off(1:1:(index(1)-1))];
end

%Find primary current peak
if (cyc > 1)
    I1peak = findpeakabs(x(1:1:(index(1)-1),1));
End

%build current limit vector for display
limit = [limit; ones(index(1)-1,1)*Climit];
end

%--Display results----------------------------------------------------------
figure(1);
plot(tt,xx(:,1),tt,xx(:,3),tt,sw,tt,-limit);
title('Resonant currents');
legend('show');
legend('I L1','I L2','switch');
figure(2);
plot(tt,xx(:,2),tt,xx(:,4),tt,sw*20,tt,Vout);
title('Resonant voltages');
legend('show');
legend('V C1','V C2','switch','Vout');
%----------------------------------------------------------
Appendix E: MATLAB code for the time domain waveforms of a 16 bit PDM pattern controller on a series tuned half-wave energy injection converter.

%Use same code for the part before Initialisation in Appendix C

%--Initialisation-----------------------------------------------
cycles = 200; %Number of resonant cycles to simulate
options = odeset('RelTol',1e-8,'AbsTol',1e-8);
t=[0 0 0 0];
index = 1;
CT = 1e-5; %Search period for zero crossings
toggle = 1; % indicates positive current phase, 0 indicates negative current phase
x0 = [0 0 0 0]; %initial conditions for x
tt = [0]; %time vector
sw = [0]; %switch vector
Vout = [0]; %output voltage vector
Vpres = [0]; %resonant voltage vector
limit = [0]; %current limit vector
I1peak = 0; %primary current peak
on = ones(200,1);
off = zeros(200,1);
Climit = 3; %current limit (A)
Inject_seq = [1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0]; %16 bit PDM pattern
bit_sel = 1; %pattern bit indicator
dispinj = 0; %flag for converter state, 1 for injection, 0 for free oscillation

%-------------------------------------------------------------
for cyc=1:1:cycles
    if (cyc == 1)
        tspan = [0 CT];
    else
        tspan = [zero_crossing(1) zero_crossing(1)+CT];
    end
    if (toggle == 1) %positive current phase
        if (Inject_seq(bit_sel) >= 1) %1 in PDM pattern
            %solve for injection phase
            [t,x] = ode45(@(t,x) makeode(t,x,A,B1),tspan,x0);
dispinj = 1; %flag 1 for injection instance
        else %0 in PDM pattern
            %solve for free oscillation phase again
            [t,x] = ode45(@(t,x) makeode(t,x,A,B2),tspan,x0);
dispinj = 0; %flag 0 for injection instance
        end
    else %negative current phase
        %solve for free oscillation phase
        [t,x] = ode45(@(t,x) makeode(t,x,A,B2),tspan,x0);
dispinj = 0; %flag 0 for injection instance
    end
    if (bit_sel < 16) %Next bit in pattern
        bit_sel = bit_sel + 1;
    else
        bit_sel = 1;
    end
    toggle = 0;
else %negative current phase
    %solve for free oscillation phase
    [t,x] = ode45(@(t,x) makeode(t,x,A,B2),tspan,x0);
toggle = 1;
end

%find primary current zero crossing, x(:,1) is primary current
[zero_crossing,index] = findzeros(t,x(:,1));

%find next set of initial values at time of 1st zero crossing
x0 = interp1(t,x,zero_crossing(1));

%--For SP configuration Vout=VC2, x(:,4) is VC2 (use either one)------
\( V_{out} = [V_{out}; x(1:1:(index(1)-1),4)]; \)

\%--For SS configuration \( V_{out} = I_2 \cdot R_2 \), \( x(:,3) \) is \( I_2 \) (use either one)------
\( V_{out} = [V_{out}; x(1:1:(index(1)-1),3) \cdot R_2]; \)

\% build time vector for display
\( tt = [tt; t(1:1:(index(1)-1))]; \)

\% building output vector \( I_1 V_1 I_2 V_2 \) for display
\( xx = [xx; x(1:1:(index(1)-1),:) ;] ; \)

\% build switch vector for display
\textbf{if} (toggle == 0) && (dispinj == 1)
\( sw = [sw; on(1:1:(index(1)-1))]; \)
\textbf{else}
\( sw = [sw; off(1:1:(index(1)-1))]; \)
\textbf{end}

\% build resonant voltage vector, \( x(:,2) \) is \( V_{C1} \) capacitor voltage
\textbf{if} ((toggle==0) && (dispinj ==1)) \% Injection state
\( V_{pres} = [V_{pres}; Vin-x(1:1:(index(1)-1),2)]; \)
\textbf{else} \% Free oscillation state
\( V_{pres} = [V_{pres}; -x(1:1:(index(1)-1),2)]; \% x(:,2) is \( V_{C1} \) capacitor voltage
\textbf{end}

\%-- Display results-------------------------------------------------------------------------------
\textbf{figure(1);}
\textbf{plot}(tt,xx(:,1),tt,xx(:,3),tt,sw);
\textbf{title}('Resonant currents');
\textbf{legend}('show');
\textbf{legend}('I L1','I L2','switch');

\textbf{figure(2);}
\textbf{plot}(tt,xx(:,2),tt,Vpres,tt,sw*20,tt,Vout);
\textbf{title}('Resonant voltages');
\textbf{legend}('show');
\textbf{legend}('V C1','V P-res','switch','Vout');
\textbf{%-----------------------------------------------------------------------------------------------}
Appendix F: MATLAB code for the time domain waveforms of a fixed frequency PWM controller on a parallel tuned push-pull energy injection converter.

```matlab
%--USER INPUT-----------------------------------------------------------------------------------------------
V2d = 12; %desired RMS output voltage (Vrms)  
Vin = 20; %DC input voltage to half-wave converter (V)  
L1 = 1e-3; %Split inductor inductance (H)  
L2 = 1e-3; %Split inductor inductance (H)  
Rl1 = 0.05; %Split inductor ESR (ohms)  
RL2 = 0.03; %Split inductor ESR (ohms)

Lp = 12e-6; %Primary coil inductance (H)  
Ls = 11.45e-6; %Secondary coil inductance (H)  
RLp = 0.05; %Primary coil ESR (ohms)  
RLs = 0.03; %Secondary coil ESR (ohms)  
Cp = 53e-9; %C1 Primary capacitor (nF)  
Cs = 16e-9; %C2 Secondary capacitor (nF)  
RCp = 0.01; %Primary capacitor ESR (ohms)  
Rcs = 0.01; %Primary capacitor ESR (ohms)  
R2 = 12; %output load R2  
k = 0.3; %coupling coefficient

M = k*sqrt(Lp*Ls); %Mutual inductance M

%--------------------------------------------------
%State-space formulation in the form %--Ax' + Bx = C
%--x = [I1 I2 LTP VCP ILS VCS]
%--For PP topology-------------------------------------------

A1pp = [ 0 L2 0 0 0 0 ];
A1pp = [A1pp; L1 0 0 Cp*RCp 0 0 ];
A1pp = [A1pp; 0 0 -Lp Cp*RCp -M 0 ];
A1pp = [A1pp; 0 0 0 -Cp 0 0 ];
A1pp = [A1pp; 0 0 0 0 M 0 Ls Cs*RCs ];
InA1pp = inv(A1pp);

A2pp = [ L1 0 0 Cp*RCp 0 0 ];
A2pp = [A2pp; 0 0 Lp -Cp*RCp M 0 ];
A2pp = [A2pp; 0 0 0 -Cp 0 0 ];
A2pp = [A2pp; 0 0 0 M 0 Ls Cs*RCs ];
A2pp = [A2pp; 0 1 0 0 0 0 ];
InA2pp = inv(A2pp);

A3pp = [ L1 0 0 0 0 ];
A3pp = [A3pp; 0 L2 0 -Cp*RCp 0 0 ];
A3pp = [A3pp; 0 0 -Lp -Cp*RCp -M 0 ];
A3pp = [A3pp; 0 0 0 Cp 0 0 ];
A3pp = [A3pp; 0 0 0 0 M 0 Ls Cs*RCs ];
InA3pp = inv(A3pp);

A4pp = [ 0 L2 0 -Cp*RCp 0 0 ];
A4pp = [A4pp; 0 0 Lp -Cp*RCp M 0 ];
A4pp = [A4pp; 0 0 0 Cp 0 0 ];
A4pp = [A4pp; 0 0 0 0 M 0 Ls Cs*RCs ];
A4pp = [A4pp; 1 0 0 0 0 ];
InA4pp = inv(A4pp);

B1pp = [ 0 RL2 0 0 0 0 ];
B1pp = [B1pp; RL1 0 0 1 0 0 ];
B1pp = [B1pp; 0 0 -RLp 1 0 0 ];
B1pp = [B1pp; 1 0 0 0 0 ];
```

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Appendices

\[ B_{1pp} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1/R_2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \]
\[ B_{2pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ R_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ B_{3pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ R_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ B_{4pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]

\[ C_{1pp} = \begin{bmatrix} V_i & V_i & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ C_{2pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ C_{3pp} = \begin{bmatrix} V_i & V_i & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ C_{4pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]

% For PS topology-----------------------------------------------------------
\[ A_{1ps} = \begin{bmatrix} 0 & L_1 & 0 & 0 & 0 & 0 \\ L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ A_{2ps} = \begin{bmatrix} 0 & L_1 & 0 & 0 & 0 & 0 \\ L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ A_{3ps} = \begin{bmatrix} 0 & L_1 & 0 & 0 & 0 & 0 \\ L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ A_{4ps} = \begin{bmatrix} 0 & L_1 & 0 & 0 & 0 & 0 \\ L_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]

% For BP topology-----------------------------------------------------------
\[ B_{1pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ B_{2pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ B_{3pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ B_{4pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]

% For BP topology-----------------------------------------------------------
\[ C_{1pp} = \begin{bmatrix} V_i & V_i & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ C_{2pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ C_{3pp} = \begin{bmatrix} V_i & V_i & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ C_{4pp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\begin{verbatim}
\input{appendices}
\end{verbatim}
if (Inj_toggle == 1) %check PWM energy injection state
  tpwm = tpwm + PWM_off;
  Inj_toggle = 0; %put into PWM free oscillation state
else
  tpwm = tpwm + PWM_on;
  Inj_toggle = 1; %put into PWM injection state
end

while (tspan(1) ~= tpwm) %while within the same PWM state
  if (Inj_toggle == 1) %PWM injection state
    if (ZVS_toggle == 1) %solve for PWM injection, S1 ON, S2 OFF
      [t,x] = ode45(@(t,x) makeode(t,x,A1,B1,6),tspan,x0);
      ZVS_toggle = 0; %flag 0 S1 ON, S2 OFF
    else %solve for PWM injection, S1 OFF, S2 ON
      [t,x] = ode45(@(t,x) makeode(t,x,A3,B3,6),tspan,x0);
      ZVS_toggle = 1; %flag 1 S1 OFF, S2 ON
    end
  else %PWM free oscillation state
    if (ZVS_toggle == 1) %solve for PWM free oscillation, S1 ON, S2 OFF
      [t,x] = ode45(@(t,x) makeode(t,x,A2,B2,6),tspan,x0);
      ZVS_toggle = 0; %flag 0 S1 ON, S2 OFF
    else %solve for PWM free oscillation, S1 ON, S2 OFF
      [t,x] = ode45(@(t,x) makeode(t,x,A4,B4,6),tspan,x0);
      ZVS_toggle = 1; %flag 1 S1 OFF, S2 ON
    end
  end

  %find primary voltage zero crossing, x(:,4) is primary current
  [zero_crossing,index] = findzeros(t,x(:,4));

  if (zero_crossing(1)<tpwm) %if still within same PWM state
    %find next set of initial values at time of 1st zero crossing
    x0 = interp1(t,x,zero_crossing(1));
    %--For PP configuration Vout=VC2, x(:,6) is VCs (use either one)
    Vout = [Vout; x(1:1:(index(1)-1),6)];
    %--For PS configuration Vout=I2*R2, x(:,5) is ILS (use either one)
    Vout = [Vout; x(1:1:(index(1)-1),5)*R2];
  end

  %build time vector for display
  tt = [tt; t(1:1:(index(1)-1))];
  %building output vector IL1 IL2 ILP VCP ILS VCS for display
  xx = [xx; x(1:1:(index(1)-1),:)];

  %build push-pull switch vector for display
  if (ZVS_toggle == 0)
    sw = [sw; on(1:1:(index(1)-1))];
  else
    sw = [sw; off(1:1:(index(1)-1))];
  end

  %build PWM switch vector for display
  if (Inj_toggle == 1)
    pwms = [pwms; on(1:1:(index(1)-1))];
  else
    pwms = [pwms; off(1:1:(index(1)-1))];
  end

  %next period to solve for
  tspan = [zero_crossing(1) zero_crossing(1)+1e-5];
else %if passed current PWM state
% find time when PWM state should change
for id=1:length(t)
    if t(id) > tpwm
        break;
    end
end

% find values at time of PWM change
x0 = interp1(t,x,tpwm);

% -- For PP configuration Vout=VC2, x(:,6) is VCs (use either one)
Vout = [Vout; x(1:1:(id-1),6)];
% -- For PS configuration Vout=I2*R2, x(:,5) is ILS (use either one)
Vout = [Vout; x(1:1:(id-1),5)*R2];

% build time vector for display
tt = [tt; t(1:1:(id-1))];
% building output vector IL1 IL2 ILP VCP ILS VCS for display
xx = [xx; x(1:1:(id-1),:)];

% build push-pull switch vector for display
if (ZVS_toggle == 0)
    sw = [sw; on(1:1:(id-1))];
else
    sw = [sw; off(1:1:(id-1))];
end

% build PWM switch vector for display
if (Inj_toggle == 1)
    pwms = [pwms; on(1:1:(id-1))];
else
    pwms = [pwms; off(1:1:(id-1))];
end

% next period to solve for
tspan = [tpwm tpwm+1e-5];
if (ZVS_toggle == 1)
    ZVS_toggle = 0;
else
    ZVS_toggle = 1;
end

end
end

% -- Display results -----------------------------------------------
figure(1);
plot(tt,xx(:,1),tt,xx(:,2),tt,sw,tt,pwms);
title('Split inductor current');
legend('show');
legend('I L1', 'I L2', 'switch', 'PWM');

figure(2);
plot(tt,xx(:,3),tt,xx(:,5),tt,sw,tt,pwms);
title('Resonant currents');
legend('show');
legend('I Lp', 'I Ls', 'switch', 'PWM');

figure(3);
plot(tt,xx(:,4),tt,Vout,tt,sw,tt,pwms);
title('Resonant voltages');
legend('show');
legend('V Cp', 'Vout', 'switch', 'PWM');

%------------------------------------------------------------------
Bibliographies


