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Scalar field as a model describing the birth of the Universe

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Preliminary results

Abstract. We investigate a scalar field described by a cubic nonlinear Klein-Gordon equation. Using approximate solutions we study the models for the scalar fields and the scalar potentials. The evolution of the field is determined by the quantum perturbations, bifurcations and resonances. We believe that the solutions give coherent model for the emergence and initial evolution of the Universe. A scenario is developed, when the Universe begins in a state that differs greatly from that of the theories of the Big Bang and the inflation.
1. The nonlinear Klein-Gordon equation (NKGE) and its approximate solutions

The classical NKGE has the following form

\[ \dot{\Phi}_i - c^2 I \sum_{i=1}^{I} \dot{\Phi}_i = -\partial \Phi \partial V \partial \Phi. \]  

(1)

Here \( c \) denotes a constant, \( I \) is an integer value \((I \geq 1)\) and \( V \) is the scalar field potential. Index \( i \) denotes the differentiation; \( \dot{\Phi}_i = \partial^2 \Phi / \partial x_i^2 \), here \( x_i \) are the coordinates. Different expressions for the function \( V \) can be found in many books.

We assumed that

\[ V(\Phi) = V_0 + \alpha (\Phi^2 - 1)^2 + \beta \Phi^4. \]  

(2)

Here \( V_0, \alpha \) and \( \beta \) are the constants.

Equation (1) is not integrable in the general case. Therefore we sought approximate solutions. The following new variables were used to describe different scenarios of the evolution of the field:

\[ \xi = B \sin^2 \omega t - K \sum_{i=1}^{I} (a_i + \sin \vartheta x_i)^2, \quad \eta = B \sin^2 \omega t + K \sum_{i=1}^{I} (a_i + \sin \vartheta x_i)^2. \]  

(3)

In (3) the values \( B, \ K, \ \vartheta, \ a_i \) and \( \omega \) are constants, \( n \) is an integer. It is assumed that \( n = 1 \) or \( n = 2 \).

In this case equation (1) may be rewritten approximately in the form

\[ \frac{1}{2} \omega^2 B^2 - 2c^2 K^2 \sum_{i=1}^{I} a_i^2 \dot{\Phi} \xi + \partial V(\Phi) \partial \Phi = 0. \]  

(4)

This equation allows us to study the change of the scalar field inside the multidimensional spacetime.

Remark 1. The linear Klein-Gordon equation was named after the physicists Oskar Klein and Walter Gordon who in 1926 proposed that it describes relativistic electrons. This equation is considered as relativistic version of the Schrödinger equation. The nonlinear Klein-Gordon equation (1) is used in nonlinear optics, plasma physics, fluid mechanics and cosmology.

Remark 2. It is important that the Klein-Gordon equation explicitly contains the d'Alembertian operator. This operator can equal zero or a very small value (the "small" divider or the "resonance" term). In particular, in (4) the value
$\frac{1}{2} \omega^2 B^2 - 2c_b \mathcal{E}^2 K^2 \left( \sum_{i=1}^{L} a_i^2 + \frac{1}{4} \right)$ may be the small divider. At the same time this value determines the width of the resonant band.

2. The potential oscillations and the multidimensional spacetime

We look for a solution of (3) as a sum
\[ \hat{\Phi} = \Phi + \phi. \]  

(5)

Here $\Phi$ is a stationary component and $\phi$ is a dynamic component of the scalar field. Let us assume that an interaction of $\Phi$ and $\phi$ is weak. The localised solution of (4) is written in the form
\[ \hat{\Phi} = A \sec h(\mathcal{E}) \sec h(\mathcal{E}) + A \sec h(\mathcal{E}). \]  

(6)

The coordinate $\xi$ (3), the equation (4) and the expression (6) allow us to study the scalar field inside the multidimensional spacetime. The stationary part of the scalar field potential describes a landscape which consists of the hills and the valleys (Fig. 1). The highest energy density is reached at the top of the hill. The lowest energy density is reached in the valleys. At the hill tops there are craters.

Fig. 1. The two-dimensional landscape of the stationary part of the potential. Number 1 corresponds to the hill tops, and the number 2 - the valleys of the potential.

The dynamic part of the potential corresponds to a multidimensional sphere which has a very thin wall. The dimension of the sphere is less than the Planck dimension. The scalar field changes only near the sphere wall. Within the sphere the field is practically constant.
Fig. 2. The two-dimensional map of the combined landscape.

The actual landscape is determined by the sum of the stationary and the dynamic components of the potential. The landscape corresponding to a moment of oscillations of the multidimensional sphere is shown in Fig. 2. The dynamic part of the potential oscillates inside of an energy barrier, which is formed by the crater wall (Fig. 3).

Fig. 3. The two-dimensional maps of the combined landscape calculated for different times.

We can tell that the energy clot (the dynamic part - the sphere) oscillates inside of the energy well. This clot cannot cross the barrier unless it is given a large enough energy influx.

**Remark.** There are a certain similarity between the oscillations of the dynamic part of the potential (Fig 3) and the data from the experiments studying the wave processes at the Bose-Einstein condensate and on a granular layer, shown in Figs. 4 and 5.
Fig. 4. Oscillations of a rotating Bose-Einstein condensate. (Stock et al 2004)

Fig. 5. The typical wave formation on granular layers arising at strong vertical vibrations. The periodical granular peak and crater on a surface of the vertically excited layer: b and c are a bird view, d and e, are a side view (the left), Localized excitations in a vertically vibrating granular layer. (Umbanhowar et al 1996, Galiev 2011).

3. The tunnelling of the energy clot through the potential wall

Now we consider equation (4) subject to a quantum fluctuation. In this case we have

\[
\left[ \frac{1}{2} \omega^2 B^2 - 2c^2 \partial^2 K^2 \left( \sum_{i=1}^{\frac{1}{2}} a_i^2 + \frac{1}{2} \right) \right] \Phi_{xx} + \partial V(\Phi)/\partial \Phi = f(\Phi) C \delta(\xi).
\]

(7)

Here \( \delta(\xi) \) is the Dirac delta function (the impulse function), \( C \) is the amplitude of the quantum fluctuation and \( f(\Phi) \) is an arbitrary function. Let

\[ \Phi = A \sec h \xi \quad \text{and} \quad f(\Phi) = \text{sech}^2 K \xi_j \sinh K \xi_j. \]

(8)
Our calculations show that if
\[
\frac{1}{2} \omega^2 B^2 \approx 2c_i^2 \vartheta^2 K^2 \left( \sum_{i=1}^I a_i^2 + \frac{1}{4} \right),
\] (9)
then even a small fluctuation - the amplitude of an order of $10^{-43} - 10^{-30}$ - can increase the amplitude of the scalar field from $10^{-43}$ to $10^{-7}$.

Again we note that NKGE explicitly contains the d'Alembertian operator. The amplitude of the dynamic part can change very strongly when the value
\[
\frac{1}{2} \omega^2 B^2 - 2c_i^2 \vartheta^2 K^2 \left( \sum_{i=1}^I a_i^2 + \frac{1}{4} \right)
\]
corresponding to this operator changes its sign. Thus, the field changes when the resonant condition (9) takes place. This resonant situation corresponds to a bifurcations (Fig. 9).

Fig. 6. Bifurcations of the amplitude of the dynamic part calculated for different values $\frac{1}{2} \omega^2 B^2 - 2c_i^2 \vartheta^2 K^2 \left( \sum_{i=1}^I a_i^2 + \frac{1}{4} \right)$.

The bifurcation diagram shown in Fig. 6 was calculated for $\frac{1}{2} \omega^2 B^2 - 2c_i^2 \vartheta^2 K^2 \left( \sum_{i=1}^I a_i^2 + \frac{1}{4} \right)$ changing from $-4 \times 10^{-40}$ to $+8 \times 10^{-40}$. This diagram shows the possibility of a very strong change in the amplitude of the dynamic part.

It becomes possible for the energy clot to escape the potential well. However, we think that this process of the tunnelling through the energy barrier is not instant. Let us assume that the quantum action is described by a function $f(\xi)$. In this case the equation (4) yields
\[
\left[ \frac{1}{2} \omega^2 B^2 - 2c_i^2 \vartheta^2 K^2 \left( \sum_{i=1}^I a_i^2 + \frac{1}{4} \right) \right] \Phi_{\xi\xi} + \partial V(\Phi) / \partial \Phi = f(\xi). \] (10)

Changing $f(\xi)$ we will model the tunnelling processes. For the calculations $\frac{1}{2} \omega^2 B^2 - 2c_i^2 \vartheta^2 K^2 \left( \sum_{i=1}^I a_i^2 + \frac{1}{4} \right) = -10^{-42}$ in (10).
The function \( f(\xi) \) can describe a group of waves (Fig. 7). During the quantum action the amplitude and the form of oscillations of the dynamic part change strongly inside the potential well (Fig. 8). There are three different fields if \( \xi < -4 \times 10^{-40} \). The fields begin to interact and yield other fields, when \( \xi \) changes from \(-4 \times 10^{-40}\) to \(-10^{-40}\). As a result of this interaction the initial spacetime may be disrupted.

In Fig. 9 the function \( f(\xi) \) describes a group of peaks.
The ways of tunnelling through the potential wall may be quite different. One of them is illustrated by Fig. 11. It shows a picture of the crossing of the energy barrier by the energy clot.

Thus, due to the quantum fluctuation the field oscillations may be amplified very strongly and the initial dynamic field can escape from the potential well. The scalar fields begin to interact. We think that, as a result, the multidimensional spacetime may be destroyed and a new spacetime may be generated. The multidimensional spacetime fragments and new kinds of scalar fields appear during the tunnelling.

4. The birth of the Universe and the fragmentation of the multidimensional spacetime
The above model can be viewed as describing the birth of the Universe. The dynamic part of the scalar field or the energy clot is the "seed" of the Universe and the moment it crosses the potential barrier - the moment of its birth.

**Remark.** According to (4) and (5) the interaction of the components $\Phi$ and $\Phi'$ may be important for the tunnelling and a form of the potential landscape. The landscape of the static part of the scalar field influence the tunnelling process (Fig. 11). The shape of the energy barrier could be marked in the cosmic microwave background (CMB) radiation emitted in the very early stages of the formation of the Universe. Perhaps, the influence of the interactions noted above is reflected in a spectacular new map which was presented recently by the European Space Agency (http://www.bbc.co.uk/news/science-environment-21866464) (Fig. 12).

![Fig. 12. The north/south differences and a "cold spot" in CMB.](image)

We assume that in the course of destruction of the multidimensional spacetime the size of the Universe increased. When certain dimension got destroyed it “unfolded” over the surviving dimensions of the spacetime. The process could be visualised by imagining a three-dimensional drop of oil spreading over a two-dimensional surface of water. As a result the particles of oil occupy in the two-dimensional space much bigger volume than they had in the initial moment.

It is possible to assume that the curled up dimensions in the course of the destruction were separated into elements which occupied much bigger volume. These elements also possess extremely high energy.

Generally speaking, the energy field can have more dimensions than we are aware of, which are curled up into tiny but complicated shapes. We think that they can undergo transformations in which their fabric is being fragmented, and then repairs itself within new number of dimensions. The string theory proclaims that the number of the dimensions may be different, for example, 5, or 11 or 26.

5. **The birth of the particles of energy and matter.**

   The beginning of the expansion the Universe during the tunnelling
A large number of small finite elements appear in the scalar field as a result of the fragmentation of the spacetime. They begin to vibrate.

We can study these vibration as a strongly nonlinear problem which is described by NKGE (1). This equation has the cubic nonlinear term and the d'Alembertian operator. Near the resonant frequencies the influence of this operator is small. Using this we constructed multivalue models describing the birth of the highly energetic particles. Figs. 13 and 14, 15 illustrate two possibilities.

Fig. 13. Case 1. Resonant oscillations of the scalar field elements according to the second resonant form. A periodic formation of the particles of energy and matter. The thin lines correspond to the linear oscillations.

Fig. 13 illustrates the emergence of particles of energy and matter. The particles swim above or below of the scalar field element. However there are moments (0.065 and 0.175) when the particles can separate from the energy level of the element.
Fig. 14. Case 2-1. Resonant oscillations of the scalar field elements according to the second resonant form. Periodic formation of the particles of energy and matter. The thin lines correspond to the linear oscillations.

Figs. 14 and 15 illustrate a localised mechanism of the emergence of particles of energy and matter. The particles can appear and separate from the energy level of the element at certain moments.

It is important that the appearance of the energetic particles (see Figs. 13 and 14) incorporates the important quantum concept - the momentary creation of pairs of particles. However, a periodic radiation of a sole particle is also possible if the element oscillates according to the first resonant form (Fig. 15).
The elements noted above vibrate according to their resonant frequencies and can radiate clots of energy. If these clots possess some critical energy, they form matter particles. The great number of particles can appear at any point of the pra-Universe during the tunnelling. As a result the size of the pra-Universe began to increase very rapidly.

According to the described scheme the tunnelling is accompanied by unimaginably fast emergence of matter, reduction of energy of the pra-Universe.

Perhaps, during the tunnelling both positive and negative particles of energy and matter appeared. However the number of the positive and negative particles might be different.

Thus, the Universe can appear as a result of the tunnelling through the energy barrier. During this process the number of spacetime dimensions decreased and the Universe began rapidly expanding. The destruction of the spacetime dimensions for our Universe stopped on four-dimensions. That moment can be defined as the beginning of our Universe. Certainly it is very difficult to define the moment of origin of time. Let us try to do this within the developed theoretical model.

6. The appearance of the four-dimensional spacetime

There is no concept of space and time during the tunnelling. Our language is too coarse to express this idea, for, in fact, there is no notion of before and after. The individual elements noted above are only ‘shards’ of space and time. The three-dimensional space and time emerged only after the appearance of enough matter which assembled the spacetime.
In order to describe the emergence of time and the four-dimensional spacetime we will again consider the equation (7) (or (10)). This is the ordinary differential equation. It is the same in multidimensional and four-dimensional spacetimes. However, we previously assumed that the four-dimensional spacetime was a result of the quantum fluctuation.

In particular, the emergence of matter could determine the emergence of “time arrow”. We consider the following variable so that to describe formally this beginning

\[ \xi = B \sin \omega t - K \sum_{i} (a_i + \sin \vartheta x_i)^2. \]  

(11)

In (11) we have approximately

\[ \sin \omega t \approx \omega t - \frac{1}{6} \omega^3 t^3 + \frac{1}{120} \omega^5 t^5 - \ldots. \]  

(12)

and

\[ \sin \vartheta x_i = \vartheta x_i - \frac{1}{6} \vartheta^3 x_i^3 + \frac{1}{120} \vartheta^5 x_i^5 - \ldots. \]  

(13)

Further we consider very small values of time and space, but these values are larger than the Planck-scale sizes. Let us assume that

\[ t >> \frac{1}{6} \omega^2 t^2 - \frac{1}{120} \omega^4 t^4 + \ldots, \]  

(14)

\[ x_i >> \frac{1}{6} \vartheta^2 x_i^2 - \frac{1}{120} \vartheta^4 x_i^4 + \ldots \]  

(15)

Thus, the multidimensional spacetime could tunnel into the four-dimensional spacetime if (14) and (15) take place during the quantum action. In this case, (11) may be presented as

\[ \xi = \overline{B}t - \sum_{i=1}^{3} \overline{a}_i x_i. \]  

(16)

Here \( \overline{B} \) and \( \overline{a}_i \) are constants. Before the tunnelling the time arrow periodically changes the direction. Therefore it is impossible to talk about time in the conventional sense. The one directional time arrow, which always increases in value, could appear due to the quantum fluctuation, the tunnelling and the emergence of matter.

7. The expansion of the Universe after the tunnelling

The appearance of time and the three-dimensional space did not mean the expansion of the Universe stopped. It continued to possess a huge amount of energy. High energy elements vibrating with resonant frequencies continued to generate particles of matter. Of course, the energy of those vibrations kept reducing. Therefore the
particles appeared with less and less energy (Fig. 16). On the other hand the high-energy particles that appeared earlier were breaking up into smaller-energy particles. As a result the Universe was being filled by the particles more and more familiar to us.

Fig. 16. A cascade of bifurcations which formed our Universe (the left). The birth of particles and the matter during the bifurcations (the right).

The subsequent global evolution of the Universe was probably determined by the initial parameters of the scalar fields and the matter, in particular, by their initial heterogeneity. The dynamic part of the scalar field which was originally approximately uniform, gained some weak heterogeneity during the tunnelling through the potential barrier. Similar heterogeneity could also arise due to quantum effects in the course of the formation of the Universe.

Fig. 17. Scheme of the bifurcations and the birth of strongly nonlinear waves.
The initial heterogeneities of the energy and the matter could begin to oscillate generating waves. Because of the nonlinearity of the substance these waves would evolve strongly into extraordinary forms. Perhaps, these waves were reminding extreme ocean waves. These waves propagated with a speed close to the speed of photons. Therefore there was the resonant condition for a very strong amplification of the waves, their breaking and the formation of vortices. The process could remind the transformation of waves into vortices which are observed in the Karman “vortex street”.

The bifurcations could be accompanied by the formation of waves and vortices (Figs. 17 and 18). In particular, the waves and the vortices may appear as a result of the interaction of different scalar fields. Figs. 19 and 20 illustrate these structures generated because of the resonant interaction of the scalar and the temperature fields.

Perhaps, similar transresonant processes could have formed the seeds of the galaxies and the clusters in the early Universe.
We emphasize that the wave evolution shown in Fig. 20 was observed during experiments (Fig. 21).

**Conclusion**

We considered a version of the nonlinear Klein-Gordon equation and the approximate solutions, which describe the evolution of the Universe as an evolution of the clots (the bubbles) of energy (Fig. 22). Almost all energy of these bubbles is concentrated on their surface. Inside the bubble the energy is almost uniform. However, there are traces of non-uniformity. Such traces could appear in the course of the tunnelling of the energy bubble through the energy barrier (Fig. 11) which resulted from a quantum fluctuation.

During the tunnelling the bubble is being filled by matter. Some spacetime elements begin to interact forming new four-dimensional space-time. As a result our Universe appear. Its size is much greater than the Planck size.

The tunnelling process leads to the transformation of the multi-dimensional spacetime into our familiar three-dimension space and time. The moment this process finishes is the moment of birth of our Universe. The Universe is then filled by different kinds of energies and particles. High-energy particles float in four-dimensional spacetime. However, some of these particles can have multidimensional structure hereditary from the multidimensional spacetime.

The production of the particles continues after the birth of the Universe. Inside of the expanding Universe, the heavy highly energetic particles decay into lighter particles and radiation. The Universe size continued to increase. According to the well known...
terminology the described fast expansion of the Universe may be called the inflation. Although according to our representations the speed of the expansion was much slower than it follows from the known inflation models. According to our model the Universe could form rather large, almost evenly filled by the particles of energy and matter. Therefore the special stage of the superfast expansion is not required for our model (Fig. 23).

The speed at which these particles are produced reduces all the time. It is possible that the particles of matter form in the depths of cosmos even now. Certainly, the volume of their production is not comparable to the volumes in the first moments of the existence of the Universe. This production may support and can even accelerate the continuing expansion of the Universe.

Thus, we explain qualitatively the emergence of the Universe and describe its initial evolution. It is assumed that the tunnelling was a process at which the multidimensional timespace collapsed and the newly formed spacetime elements began to vibrate with resonant frequencies creating energy and matter particles. Then the waves and the vortices were formed (see Figs.17 and 18).

Our Universe appeared with its four-dimensional spacetime. The Universe size was much greater than the Planck size and it was substantially evenly filled with highly energetic particles of matter.

**Acknowledgment**

We thank Professor Mace B. (the University of Auckland). We were discussing with him certain key questions of this research.
References

There are many articles and books which are devoted to subject of this research. Here we will note only a small number of publications, which had the most influence on this research.


Martin, A., Parker, N., Gardiner, S., Adams, C. Solitons and vortices in atomic BECs. Durham University.