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The Mortensen Rule and Efficient Coordination Unemployment

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Abstract

We study the implementation of constrained-efficient allocations in labour markets where a basic coordination problem leads to an equilibrium matching function. We argue that these allocations can be achieved in equilibrium if wages are determined by *ex post* bidding. This holds true even in finite sized markets where the equilibrium matching function has decreasing returns to scale – where the "Hosios rule" does not apply – both with and without heterogeneity. This wage determination mechanism is similar to the one proposed by Mortensen (1982) in a different setting.

Key words: Coordination, price-posting, auctions, efficiency, directed search.

JEL Codes: D43, D44, D45, D83

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1. INTRODUCTION

Recent developments in search theory have uncovered a very simple coordination problem from which a matching function emerges as an equilibrium phenomenon. In particular, when sellers have a fixed amount to sell, and buyers choose which seller to approach, many asymmetric pure strategy equilibria exist, but only one symmetric equilibrium exists. In this symmetric equilibrium, buyers play mixed strategies and randomise over sellers. The randomisation implies a matching process, which has many natural properties that are similar to those imposed in the standard labour matching literature (Pissarides (2000)). For example, in large economies, in the limit, the equilibrium matching function from the coordination problem exhibits constant returns to scale – consistent with the assumption that is usually made. However, in any economy of finite size, the coordination equilibrium matching function has decreasing returns to scale: the coordination problem worsens as the scale of the market increases. This idea has been explored in papers by, among others, Julien, Kennes and King (2000), and Burdett, Shi and Wright (2001).[1]

This has implications for the efficiency of the equilibria considered.[2] In the context of the standard matching environment, Hosios (1990) argued that efficiency is possible only if the matching function has constant returns to scale. Under these conditions, efficiency is obtained if and only if the "Hosios Rule" holds: the share of the surplus going to agents (i.e., either workers or vacancies) equals their marginal contribution to matches. In that environment, with homogeneous agents, there is no reason why this condition should hold. However, as is now well known, this condition

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[1] Earlier work by Peters (1984) and Montgomery (1991) considered equilibria in capacity-constrained environments such as this without drawing out the implications for the matching function. Other recent papers that have explored this link are Shi (2001), and Shimer (2001). Recently, this type of model has been classified as "directed search". The class of directed search models is larger than the class considered, in this paper, where a coordination problem lies at the heart of the matching process. For example, Moen (1997) presents a directed search environment with many locations and a matching function at each location. There, search is "directed" in the sense that workers can choose locations, but "undirected" in the sense that workers face random assignment once reaching any particular location.

[2] In this literature, it has become conventional to examine whether or not equilibrium allocations are "constrained efficient" – that is, efficient given the friction associated with the matching function. In this paper, for the most part, we follow that tradition. Thus, whenever we use the word "efficient" we intend for the reader to understand that we mean "constrained efficient" in that sense.

is always satisfied in the *coordination* equilibrium environment in the limiting case when the market is large.[3]

When agents are heterogeneous, in the standard matching environment, Acemoglu (2001), Davis (2001), and Sargent and Ljungquist (2000) point out that, even in the presence of constant returns to scale in the matching function, an efficient allocation is simply not possible – that is, the Hosios Rule cannot be satisfied for all agents using a single matching technology. However, the equilibrium is efficient in large market models with heterogeneity, and coordination equilibrium matching (Shi (2001), Kennes, King, and Julien (2001)).

When the market is large, then, the coordination equilibrium matching function has constant returns to scale and the equilibrium allocations are efficient – either with or without the presence of heterogeneity. However, when the market has a finite size, the equilibrium matching function has decreasing returns to scale and the allocations are not efficient in the coordination equilibria considered so far in the literature.

In this paper, we argue that the inefficiency of coordination equilibria in markets of finite size hinges crucially on the particular wage determination mechanisms that have been used in the literature so far. Following Montgomery (1991), most models have assumed that wages are posted, and committed to, *ex ante* (that is, before the candidates and vacancies have been matched). In Julien, Kennes, and King (2000), we considered a similar mechanism but where *reserve* wages were committed to *ex ante*, while actual wages were determined *ex post* by a bidding game. We show here that if wages are determined entirely by *ex post* bidding, without any *ex ante* commitment to either wages or reserve wages, (that is, there is no reserve beyond the outside option) then allocations in coordination equilibria are always efficient – with or without heterogeneity, and even in finite-sized markets.

[3] See Julien, Kennes, and King (2000) and Shi (2001). Equilibrium allocations tend to be efficient more generally in large directed search economies. See, for example, Moen (1997) and Acemoglu and Shimer (2000).

We also note the resemblance between this wage determination mechanism and the rule proposed by Mortensen (1982) in the context of random search. He suggested a sharing rule that could be conditioned on the actions of the participants in the search process: at the local level (i.e., at the level of individual matches) the agent who "initiates" the match should be awarded the entire surplus. In the economies that we consider here, the buyers choose which seller to purchase from, and workers play the role of sellers of labour. Thus, buyers "initiate" the match. Mortensen's rule was devised with bilateral matches as the only possibility. When matches are bilateral in *our* framework, the buyers are indeed awarded all of the surplus from the match. However, when matches are not bilateral, (i.e., when more than one buyer approaches a seller) then buyers compete in a bidding game. In this case, the role of initiator, *ex post*, switches to the seller.[4]

Since, in these finite-sized environments, the matching process has decreasing returns to scale, the Hosios rule does not apply, while the Mortensen rule does apply for efficiency. However, as market size increases, in the limit, the matching process approaches constant returns to scale, and both the Mortensen and Hosios rules apply. In this limit, also, the equilibrium expected payoffs under the three different wage determination mechanisms (wage posting, reserve wage posting, *ex post* bidding) converge. Thus, for any finite-sized market, the *ex post* bidding allocation is always efficient and, the larger the market, the closer the equilibrium allocations of the other two mechanisms come to this allocation.

Following the literature, we consider efficiency along two different margins, which are conceptually quite distinct. The first margin is the one that is most commonly considered: whether or not the *entry* decision by agents is efficient. The second margin was considered by Montgomery (1991), when studying seller heterogeneity: given a fixed number of buyers and sellers, do the equilibrium prices induce visit probabilities that maximize expected aggregate surplus? These issues are covered under alternative heterogeneity conditions: homogeneity on both sides of the market, one-sided heterogeneity on either side of the market, and two-sided heterogeneity.

[4] Hosios (1990) also notes that Mortensen's rule can be interpreted as an auction.

In the case of two-sided heterogeneity we also find that, under fairly general conditions, with the *ex post* bidding mechanism, the mixed strategy equilibrium at the heart of the coordination matching approach does exist. This contrasts with earlier work by Coles and Eeckhout (1999) who show that this equilibrium does not exist for other mechanisms. In particular, the standard assumption of supermodularity in production is sufficient to rule out the mixing equilibrium for the mechanisms they consider, but not for this mechanism. The mixing equilibrium exists as long as the surplus from the best possible match is no larger than the sum of the surpluses from the next two best matches.

The paper is organized as follows. In the next section we present the basic model with finite numbers of homogeneous agents. Section three then introduces heterogeneity. We consider three different cases: one-sided heterogeneity (for both sides of the market) with arbitrary numbers of agents, and two-sided heterogeneity with two agents on each side of the market. In section four we present our conclusions and some discussion.

2. THE BASIC MODEL WITH HOMOGENOUS AGENTS

The assignment game is standard with $M \geq 2$ identical vacancies and $N \geq 2$ identical candidates. Here, workers sell, and firms buy, labour. To keep the analysis simple, we assume that each firm has one vacancy to fill, and refer to a firm as a vacancy. The productivity of a worker is $y_0 = 0$ if unemployed, and $y_1 = y > 0$ if employed. In all cases, we assume that all workers use identical selling mechanisms.

2.1 *Wage Determination (Ex Post Bidding)*

We assume here that pricing (the wage) is determined by ex post bidding. That is to say we assume that workers cannot commit to wages, or reserve wages, above their outside options. In this case the bidding game determines that the firm extracts all the surplus if she is alone in her offer to the worker and the worker extracts all the surplus if there are multiple offers.[5] Let w_j denote the payoff received by a worker whose *second best* alternative has productivity y_j. If the worker is employed, this payoff is his wage. Otherwise, the payoff is simply his productivity when unemployed (which we have normalized to zero). For $j \in \{0,1\}$, we have:

$$w_j = y_j \tag{2.1}$$

Thus, if a worker receives exactly one offer, his wage is $w_0 = y_0 = 0$, and if a worker receives two or more offers his wage is $w_1 = y_1 = y$. Notice, also, that *ex post*, a vacancy will receive the payoff *y* if she is alone when approaching a candidate, and zero otherwise.

2.2 *Equilibrium Vacancy Assignment*

Each vacancy chooses which candidate to visit, in order to maximize her expected payoff. Let π_n^m denote the probability that vacancy *m* approaches candidate *n,* and let V_n^m denote the expected payoff for vacancy *m* if she visits candidate *n*. In the symmetric mixed strategy equilibrium $\pi_n^m = \pi_n$ and $V_n^m = V_n$ for all $m = 1,2,...,M$. Since each vacancy receives *y* if alone when approaching a worker and zero otherwise, the expected payoff from visiting candidate i is:

$$V_i = (1-\pi_i)^{M-1} y$$

and the expected payoff from visiting candidate j is:

$$V_j = (1-\pi_j)^{M-1} y$$

[5] This scheme is not as extreme as it may sound. In a dynamic game, the outside option is the value of playing the game in the next period, which is not zero. In Kennes, King, and Julien (2001), we consider a calibrated example, where the length of a period is one week. In the stationary equilibrium, the weekly wage of a worker who receives only one offer is \$127, while the wage of a worker who receives multiple offers from the same type of job is \$150. (Two types of jobs exist in that model, and the figures reported here are from low productivity jobs.)

In the mixed strategy equilibrium, each vacancy selects π_i and π_j so that

$$V_i = V_j \quad \forall i, j \in \{1,2,...,N\}$$

which implies $\pi_i = \pi_j$. Since $\sum_{n=1}^{N} \pi_n = 1$ one obtains:

$$\pi = 1/N \tag{2.2}$$

and the expected number of matches (or the "matching function") is given by:

$$x(N,M) = N(1-(1-1/N)^M) \tag{2.3}$$

This matching function has decreasing returns to scale but, in the limit when *M* and *N* are large, has constant returns to scale. Figure 1 presents a graphical representation of this function, for *N* = 2, 3, …, 30 and *M* = 2, 3, …, 30.

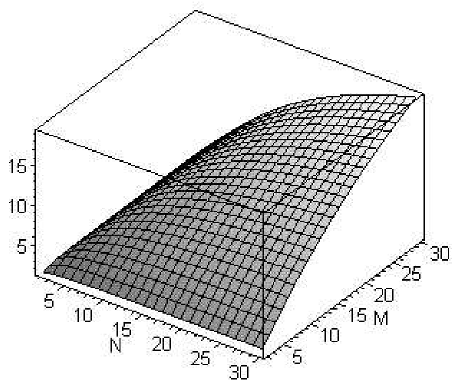

Figure 1: The Equilibrium Matching Function

We can now completely characterize the expected payoff distribution for workers. Let p_0 denote the probability that a worker receives no offers, (and so, is unemployed) p_1 the probability that he receives exactly one offer, and p_2 the probability that he receives at least two offers. The workers' payoff distribution is given by:

$$
w_i, p_i = \begin{cases} w_0 = 0 & p_0 = \left(\dfrac{N-1}{N}\right)^M \\[2ex] w_0 = 0 & p_1 = \dfrac{M}{N}\left(\dfrac{N-1}{N}\right)^{M-1} \\[2ex] w_1 = y & p_2 = 1 - p_0 - p_1 \end{cases} \tag{2.4}
$$

For vacancies, let r_0 and q_0 denote the *ex post* payoff and probability, respectively, of being alone when approaching a worker. Let r_1 and q_1 denote these figures when at least one other firm approaches the worker. The payoff distribution for a firm in this game is given by:

$$
r_i, q_i = \begin{cases} r_0 = y & q_0 = \left(\dfrac{N-1}{N}\right)^{M-1} \\[2ex] r_1 = 0 & q_1 = 1 - q_0 \end{cases} \tag{2.5}
$$

In the absence of endogenous entry, equations (2.4) and (2.5) fully characterize the equilibrium expected payoffs of the model. In Section 2.4, below, we consider the firms' entry decision. Before that, however, we can ask whether or not this equilibrium wage distribution is efficient in the following sense: given the numbers of vacancies and candidates, and that vacancies randomise in equilibrium, do they choose the probabilities that maximize total expected output?

2.3 *Efficient Assignment with Fixed Numbers of Agents*

Given M and N, for each vacancy m and worker n, consider a planner that chooses the visit probabilities $\pi_n^m \in (0,1)$ to maximize total expected output:

$$Y = Ny - \sum_{n=1}^{N} \prod_{m=1}^{M} \left(1 - \pi_n^m\right) y \qquad \text{s.t.} \qquad \sum_{n=1}^{N} \pi_n^m = 1 \qquad \forall m = 1, 2, \ldots, M$$

Clearly, to achieve this objective, the planner will choose the $\pi_n^m \in (0,1)$ that minimizes expected unemployment:

$$Min \sum_{n=1}^{N} \prod_{m=1}^{M} \left(1 - \pi_n^m\right) \qquad \text{s.t.} \qquad \sum_{n=1}^{N} \pi_n^m = 1 \qquad \forall m = 1, 2, \ldots, M \qquad (2.6)$$

This is precisely the "classical occupancy problem". The following theorem is simply a restatement of a well-known result in probability theory.

Proposition 1: Given any finite number M of homogenous vacancies, and N homogeneous candidates, the symmetric mixed strategy equilibrium with *ex post* bidding maximizes total expected output.

Proof: Solving the minimization problem in (2.6), one obtains (2.2).

2.4 *Vacancy Entry*

Suppose now that, given the number (N) of workers, and given a fixed entry cost $k > 0$ for each vacancy, firms can choose the number of vacancies to create. The equilibrium number of vacancies is determined by the zero profit free entry condition. Proposition 2 summarizes the main result in this section of the paper.

Proposition 2: Given any finite number N of homogeneous candidates, equilibrium entry of homogenous vacancies is efficient.

Proof: It is sufficient to show that, in equilibrium, the marginal private benefit of a new vacancy equals the marginal social benefit of a new vacancy. Using (2.5), if there are N candidates and M vacancies, the expected payoff for a new vacancy in the auction equilibrium is given by:

$$MPB(M,N) = y\left(\frac{N-1}{N}\right)^{M-1} - k \tag{2.7}$$

At the aggregate level, expected output in a market with M vacancies is given by:

$$Y(M,N) = Ny\left[1-\left(\frac{N-1}{N}\right)^{M}\right] - kM \tag{2.8}$$

Expected aggregate output with *M-1* buyers is given by:

$$Y(M-1,N) = Ny\left[1-\left(\frac{N-1}{N}\right)^{M-1}\right] - k(M-1) \tag{2.8'}$$

The marginal social benefit of an additional vacancy is given by

$$MSB(M,N) = Y(M,N) - Y(M-1,N) \; = y\left(\frac{N-1}{N}\right)^{M-1} - k \tag{2.9}$$

$$= MPB(M,N) \quad \blacksquare$$

2.5 *A Digression: Reserve Wages and Wage Posting*

At this point, it is worthwhile to contrast these results with those using alternative wage determination mechanisms. For example, in Julien, Kennes, and King (2000), candidates auction their labor in the same way as above, but can commit to a reserve wage. In Montgomery (1991) and Burdett, Shi and Wright (2001), wages are posted and committed to. In each of these environments, the unique symmetric equilibrium has all buyers randomising and visiting each seller with equal probability. Thus, equation (2.1) holds in all cases and, by the analysis of Section 2.3, given fixed numbers of candidates and vacancies, buyers are efficiently assigned.

When considering vacancy *entry* with finite numbers of players, however, Julien, Kennes, and King (2000) show that too few vacancies are created in equilibrium. Although neither Montgomery (1991) nor Burdett, Shi, and Wright (2001) consider entry under these conditions, it is straightforward to do.[6] When candidates commit to a posted wage then, from Burdett, Shi and Wright (2001), the equilibrium wage is:

$$w_P(M,N) = \frac{N - N\left(1+\dfrac{M}{N-1}\right)(1-1/N)^M}{N-\left(N+\dfrac{M}{N-1}\right)(1-1/N)^M}\,y \qquad (2.10)$$

In this case, the expected payoff to a vacancy is given by:

$$B_P(M,N) = \frac{N}{M}\left(1-(1-1/N)^M\right)\left(y - w_P(N,M)\right) - k \qquad (2.11)$$

The difference between this payoff and the social marginal benefit is illustrated in Figure 2:

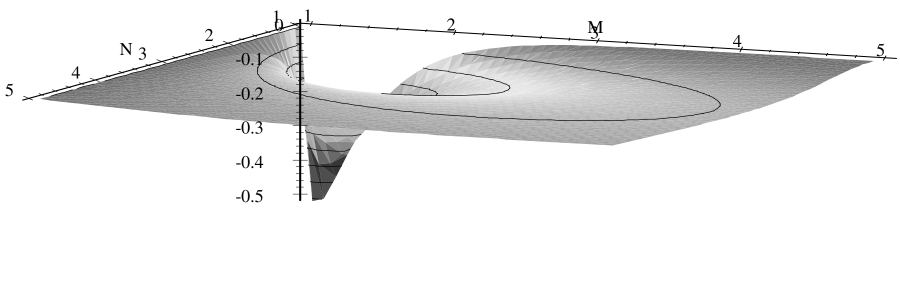

Figure 2: Inefficiency of Entry with Wage Posting

[6] This set-up is slightly different from the one studied by Montgomery (1991) and Burdett, Shi, and Wright (2001): the roles of workers and vacancies, as buyers and sellers, have been reversed. That is, in the Montgomery and Burdett *et al* papers, firms sell jobs. Here, workers sell labour. For the qualitative point that we make in this subsection, this distinction is irrelevant.

Figure 2 shows that, in any finite market, the private benefit from creating a new vacancy is always less than the social benefit – thus, as with auctioning with a reserve wage, too few vacancies are created in equilibrium. However, it is also clear from Figure 2 that this difference will be small for any significant size of the market.

3. HETEROGENEITY

We now extend the basic model by allowing for heterogeneity among agents. Specifically, we consider three different models, with different degrees of heterogeneity. First, we consider heterogeneous candidates facing homogenous vacancies, then homogenous candidates facing heterogeneous vacancies, then equilibria with heterogeneity on both sides of the market. In each case, as in the model with homogeneous agents, each candidate has one unit of labour to sell. Also, wages are determined by *ex post* bidding.

3.1 *Heterogeneous Candidates with Homogeneous Vacancies*

We start, as in Section 2 above, with M identical existing vacancies. Also, there are N candidates. However, the productivity of each candidate is unique, and these productivities can be ranked:

$$y_1 < y_2 < ... < y_N \tag{3.1.1}$$

Since vacancies are homogeneous, for each one, the payoff from visiting any of the candidates is either the whole surplus y_i if they are alone or 0 if not (due to the bidding structure). As before, let π_n^m denote the probability that vacancy m approaches candidate n and let V_n^m denote the expected payoff for vacancy m if she visits candidate n. In the symmetric mixed strategy equilibrium $\pi_n^m = \pi_n$ and $V_n^m = V_n$ for all $m = 1,2,...,M$. Thus, the expected payoff from visiting candidate i is:

$$V_i = (1-\pi_i)^{M-1} y_i$$

and the expected payoff from visiting candidate j is:

$$V_j = (1-\pi_j)^{M-1} y_j$$

In the mixed strategy equilibrium, each vacancy selects π_i and π_j so that

$$V_i = V_j$$

which yields $\pi_j = 1-(1-\pi_i)\left(\frac{y_i}{y_j}\right)^{\frac{1}{M-1}}$. Since $\sum_{\substack{j=1 \\ j\neq i}}^{N-1} \pi_j + \pi_i = 1$, one obtains

$$\pi_i = \frac{\sum_{\substack{j=1 \\ j\neq i}}^{N-1} \left(\frac{y_i}{y_j}\right)^{\frac{1}{M-1}} - (N-2)}{\sum_{\substack{j=1 \\ j\neq i}}^{N-1} \left(\frac{y_i}{y_j}\right)^{\frac{1}{M-1}} + 1} \qquad \forall i = 1,2,...,N \tag{3.1.2}$$

Equation (3.1.2) presents the equilibrium visit probabilities, for each candidate. We now consider the planner's problem with a fixed *M*. The planner chooses visit probabilities $\pi_n^m \in (0,1)$ to maximize total expected output:

$$Y = \sum_{n=1}^{N} \left(1 - \prod_{m=1}^{M} \left(1-\pi_n^m\right)\right) y_n \qquad \text{s.t.} \quad \sum_{n=1}^{N} \pi_n^m = 1 \qquad \forall m = 1, 2, \ldots, M. \tag{3.1.3}$$

Proposition 3: Given any finite number *M* of homogenous vacancies, and *N* heterogeneous candidates ordered as in (3.1.1), the symmetric mixed strategy equilibrium with *ex post* bidding maximizes total expected output.

Proof: Performing the maximization problem in (3.1.3), one obtains (3.1.2).■

This result contrasts with those in models with wage posting. For example, in the two-by-two case with one-sided heterogeneity, Montgomery (1991) shows that the equilibrium is inefficient. However, with auction and reserve prices, again in the two-by-two case, Julien, Kennes and King (2002) demonstrate efficiency. It is also worth pointing out that, while the visit probabilities in (3.1.2) maximize expected output for the market, they do not minimize unemployment. As in the homogeneous agent case, unemployment is minimized when the visit probabilities and simply $1/N$. Thus, as Montgomery noted, from a policy point of view, in the face of this type of heterogeneity, a potential trade-off exists if policymakers wish to both maximize expected output and minimize unemployment.

Vacancy Entry

We now consider whether or not the entry decision by vacancies is efficient. As in Section 2, we consider a fixed entry cost $k > 0$, and the equilibrium number of vacancies is determined by the zero profit free entry condition. The following proposition summarizes the main result.

Proposition 4: Given N candidates with productivities ordered in (3.1.1), equilibrium entry of homogenous vacancies is efficient.

Proof: It is sufficient to show that, in equilibrium, the marginal private benefit of a new vacancy equals the marginal social benefit of a new vacancy.

In equilibrium, due to the bidding structure, vacancies receive positive payoffs *ex post* if and only if they are alone when they approach a candidate. With M vacancies, the equilibrium expected marginal private benefit is given by:

$$MPB(M) = \sum_{n=1}^{N} y_n \pi_n (1-\pi_n)^{M-1} - k \tag{3.1.4}$$

where π_n is given in (3.1.2) for $n = 1,2,...,N$.

At the aggregate level, expected net output in a market with M vacancies is given by:

$$Y(M) = \sum_{n=1}^{N} \left(1 - \prod_{m=1}^{M} \left(1 - \pi_n\right) \right) y_n - Mk$$

where π_n is given in (3.1.2) for $n = 1,2,...,N$. This can be re-written as:

$$Y(M) = \sum_{n=1}^{N} \left(1 - \left(1 - \pi_n\right)^M \right) y_n - Mk \tag{3.1.5}$$

Similarly, with *M*-1 vacancies:

$$Y(M-1) = \sum_{n=1}^{N} \left(1 - \left(1 - \pi_n\right)^{M-1} \right) y_n - (M-1)k \tag{3.1.6}$$

Using (3.1.5) and (3.1.6), the expected marginal social benefit of the *M*th vacancy is therefore given by:

$$MSB(M) = Y(M) - Y(M-1) = \sum_{n=1}^{N} y_n \pi_n (1 - \pi_n)^{M-1} - k = MPB(M) \qquad \blacksquare$$

3.2 *Homogeneous Candidates with Heterogeneous Vacancies*

Suppose now there are *N* identical candidates, and *M* vacancies, where each vacancy has a different productivity level, and these levels are ranked as follows:

$$y_1 < y_2 < ... < y_M \tag{3.2.1}$$

Using the convention adopted in Section 2, let y_0 denote the productivity of a candidate when unemployed. We normalize so that $y_0 = 0 < y_1$. As before, let w_j denote the payoff received by a worker whose *second best* alternative has productivity y_j. Due to the bidding game, *ex post*, For $j \in \{0,1,2,...,M\}$, we have:

$$w_j = y_j \tag{3.2.2}$$

Again, let π_n^m denote the probability that vacancy *m* approaches candidate *n* and let V_n^m denote the expected payoff for vacancy *m* if she visits candidate *n*. Given the bidding outcome in (3.2.2), we can write these expected payoffs in the following way:

$$\begin{aligned} V_n^M &= [(1-\pi_n^1)(1-\pi_n^2)...(1-\pi_n^{M-1})]y_M + [\pi_n^1(1-\pi_n^2)...(1-\pi_n^{M-1})](y_M - y_1) \\ &\quad + [\pi_n^2(1-\pi_n^3)...(1-\pi_n^{M-1})](y_M - y_2) + ... + \pi_n^{M-1}(y_M - y_{M-1}) \end{aligned}$$

$$\begin{aligned} V_n^{M-1} &= [(1-\pi_n^1)(1-\pi_n^2)...(1-\pi_n^M)]y_{M-1} + [\pi_n^1(1-\pi_n^2)...(1-\pi_n^M)](y_{M-1} - y_1) \\ &\quad + [\pi_n^2(1-\pi_n^3)...(1-\pi_n^M)](y_{M-1} - y_2) + ... + \pi_n^{M-2}(y_{M-1} - y_{M-2}) \end{aligned}$$

$$\vdots$$

$$V_n^1 = (1-\pi_n^2)(1-\pi_n^3)...(1-\pi_n^M)y_1 \qquad \forall n = 1,2,...,N \tag{3.2.3}$$

In the mixed strategy equilibrium, each vacancy *m* chooses π_n^m so that:

$$V_i^m = V_j^m \qquad \forall i, j \in \{1,2,...,N\}$$

This then implies:

$$\pi_n^m = \pi^m \qquad \forall n = 1,2,...,N$$

That is, each vacancy assigns equal weight to each candidate. Hence:

$$\pi_n^m = \pi = 1/N \qquad \forall m = 1,2,...,M \quad \text{and} \quad \forall n = 1,2,...,N \tag{3.2.4}$$

Equation (3.2.4) presents the equilibrium visit probabilities. We now consider the planner's problem, choosing these probabilities, given a fixed number N of candidates, and given the vacancies in (3.2.1).

The planner chooses $\pi_n^m \in (0,1)$ to maximize total output:

$$Y = y_M + \sum_{m=1}^{M-1}\left(\sum_{n=1}^{N}\pi_n^m y_m \prod_{j=m+1}^{M}\left(1-\pi_n^j\right)\right) \quad \text{s.t.} \quad \sum_{n=1}^{N}\pi_n^m = 1 \quad \forall m = 1, 2, \ldots, M \tag{3.2.5}$$

Proposition 5: Given any finite number N of homogeneous candidates and M vacancies ordered as in (3.2.1), the mixed strategy equilibrium with *ex post* bidding maximizes total expected output.

Proof: Solving the maximization problem in (3.2.5), one obtains (3.2.4). ■

Notice that, in this case, since each vacancy approaches each candidate with equal probability, the unemployment rate is minimized in equilibrium. That is, in this setting, there is no potential policy trade-off between maximizing expected output and minimizing unemployment – as there was in Section 3.1 with heterogeneous candidates. This also implies the following corollary.

Corollary: In a game with fixed numbers of buyers and sellers, and heterogeneity on only one side of the market, expected output is higher if the agents on the homogeneous side of the market act as sellers rather than buyers.

Vacancy Entry

Suppose now that the creation of each vacancy has an associated cost. That is, let k_m denote the fixed cost of creating vacancy *m.* Once again, the equilibrium number of vacancies is determined by the zero profit free entry condition. We have the following proposition.

Proposition 6: Given *N* homogeneous candidates, and potential vacancies with productivities ordered as in (3.2.1), where each vacancy *m* has fixed cost k_m, equilibrium entry of these vacancies is efficient.

Proof: It is sufficient to show that, in equilibrium, the marginal private benefit of a new vacancy equals its marginal social benefit

Consider vacancy *M* as the marginal vacancy. In equilibrium, using (3.2.4) in (3.2.3), we have:

$$MPB(M) = (1-\pi)^{M-1} y_M + \pi \sum_{m=1}^{M-1} (1-\pi)^{M-m-1} (y_M - y_m) - k_M$$

Re-arranging this, we get:

$$MPB(M) = y_M - \pi \sum_{m=1}^{M-1} (1-\pi)^{M-m-1} y_m - k_M \tag{3.2.6}$$

At the aggregate level, using (3.2.4) in (3.2.5), expected net output with *M* vacancies is given by:

$$Y(M) = y_M + \sum_{m=1}^{M-1} (1-\pi)^{M-m-1} y_m - \sum_{m=1}^{M} k_m \qquad (3.2.7)$$

Similarly, with *M*-1 vacancies:

$$Y(M-1) = y_{M-1} + \sum_{m=1}^{M-2} (1-\pi)^{M-m-2} y_m - \sum_{m=1}^{M-1} k_m \qquad (3.2.8)$$

Using (3.2.7) and (3.2.8), the expected marginal social benefit of the *M*th vacancy is therefore given by:

$$Y(M) - Y(M-1) = y_M - y_{M-1} + \sum_{m=1}^{M-1} (1-\pi)^{M-m-1} y_m - \sum_{m=1}^{M-2} (1-\pi)^{M-m-2} y_m - k_M$$

Re-arranging, and using (3.2.6), we get:

$$MSB(M) = Y(M) - Y(M-1) = y_M - \pi \sum_{m=1}^{M-1} (1-\pi)^{M-m-1} y_m - k_M = MPB(M)$$

Using entirely analogous arguments, it is straightforward to show that this result holds for vacancies 1, 2, … *M*-1 when considered as marginal vacancies. ■

This result is very different from those derived in the standard matching function literature. For example, Sargent and Ljungquist (2000), Acemoglu (2001), and Davis (2001) argue that efficient entry is impossible in the presence of heterogeneous vacancies, using the standard sharing rule – even in large markets. In their environment, the matching technology has more than two arguments. This implies that the elasticity condition of the Hosios rule is no longer applicable. Essentially, there then exist three dimensions on which entry is determined – entry of good jobs, entry of bad jobs, and an extra externality concerning the composition of good and bad jobs in the market. Thus a sharing rule cannot generate a socially efficient outcome. What we have shown here is that the socially efficient outcome will be obtained, even in small markets, if wages are determined by *ex post* bidding.

3.3 *Two-Sided Heterogeneity*

We now consider allocations in the presence of heterogeneity on both sides of the market. Here, we restrict attention to the setting with two candidates and two vacancies. Let y_n^m denote the surplus of the match between vacancy *m* and candidate *n*, for $m,n \in \{1,2\}$. As before, y_0 denotes the payoff, to the candidate, when unemployed. Without loss of generality, we assume:

$$0 = y_0 \le y_1^1 \le y_1^2, y_2^1 \le y_2^2 \qquad (3.3.1)$$

The ex post bidding game implies the following payoff matrix for the vacancies, according to their choices of candidate to approach:

Vacancy 2

		Candidate 1	Candidate 2
Vacancy 1	Candidate 1	$0\ ,\ y_1^2 - y_1^1$	$y_1^1\ ,\ y_2^2$
	Candidate 2	$y_2^1\ ,\ y_1^2$	$0\ ,\ y_2^2 - y_2^1$

Table 1

Proposition 7:

a) If $y_2^2 > y_2^1 + y_1^2$ then there exists a unique Nash equilibrium. In this equilibrium both vacancies play pure strategies and the assignment has positive assortative matching.

b) If $y_2^2 \le y_2^1 + y_1^2$ then three Nash equilibria exist:

i) A pure strategy equilibrium with positive assortative matching.
ii) A pure strategy equilibrium with negative assortative matching.
iii) A mixed strategy equilibrium in which:

$$\pi_1^1 = \frac{y_2^1 + y_1^2 - y_2^2}{y_2^1 + y_1^1} \quad (3.3.2)$$

$$\pi_1^2 = \frac{y_1^1}{y_2^1 + y_1^1} \quad (3.3.3)$$

Proof: Parts (a) and (b(i),(ii)) are clear from inspection of the normal form game presented in Table 1. To prove part (b(iii)), let V_n^m denote the expected payoff for vacancy *m* when visiting candidate *n*. By inspection of Table 1, we have:

$$V_1^1 = (1-\pi_1^2)y_1^1 \qquad V_2^1 = \pi_1^2 y_2^1$$

$$V_1^2 = \pi_1^1(y_1^2 - y_1^1) + (1-\pi_1^1)y_1^2 \qquad V_2^2 = \pi_1^1 y_2^2 + (1-\pi_1^1)(y_2^2 - y_2^1)$$

Using the equilibrium conditions $V_1^1 = V_2^1$ and $V_1^2 = V_2^2$ we then obtain the result. ■

Part (a) of the above proposition states that, if the surplus associated with the best match is greater than the aggregate surplus of the next two best matches, then there will be no coordination problem in this market because there is only one Nash equilibrium. Moreover, in this equilibrium, vacancies play pure strategies, there is positive assortative matching and, from a surplus-maximizing perspective, the first-best allocation is achieved. This result is very similar to the one found in Coles and Eeckhout (1999), which they interpret as "heterogeneity as a coordination device".

However, there are some key differences between the results found here and those found by Coles and Eeckhout. First, they find that restricting the matching process to be strictly supermodular is sufficient to ensure the uniqueness of the equilibrium identified in part (a). In this setting, however, strict supermodularity is not sufficient to obtain this result. To see this, note that, here, the matching process is strictly supermodular if and only if:

$$y_2^2 > y_2^1 + y_1^2 - y_1^1 \quad (3.3.4)$$

By Proposition 7, then, all three equilibria exist when

$$y_2^1 + y_1^2 - y_1^1 < y_2^2 < y_2^1 + y_1^2$$

where matching is supermodular, by (3.3.4). Also, strict supermodularity is not necessary for the existence of the pure strategy equilibrium with positive assortative matching – as is clear from part (b) of Proposition 7. However, as should be clear, strict supermodularity is necessary for the uniqueness of that equilibrium.

Overall, then, the condition for heterogeneity to act as a coordination device (that is, to narrow down the number of equilibria to one in this type of setting) is stronger than the supermodular condition given by Coles and Eeckhout. While the supermodular condition is a common and fairly natural one to impose on a matching process, the stronger condition in part (a) of Proposition 7 is relatively extreme. Thus, the mixed strategy equilibrium, characterized in part (b) of the Proposition, exists fairly generally in these settings with two-sided heterogeneity. Also, this mixed strategy equilibrium is the analogue of the mixed strategy equilibrium studied in the cases when one side of the market is homogeneous (above). As with all of the mixed strategy equilibria, the allocation is not efficient (first-best). However, we can once again ask whether or not this equilibrium is constrained efficient in the usual way.

Proposition 8: Wherever it exists, the mixed strategy equilibrium characterized in Proposition 7 is constrained efficient.

Proof: The constrained planner chooses $\pi_n^m \in (0,1)$ to maximize:

$$Y = (1-\pi_1^2)y_2^2 + \pi_1^2 y_1^2 + \pi_1^1(1-\pi_1^2)y_1^1 + (1-\pi_1^1)\pi_1^2 y_2^1 \qquad (3.3.5)$$

Solving this maximization problem, one obtains (3.3.2) and (3.3.3). ■

As in all previous settings, then, given that vacancies randomise over workers, the mixed strategy equilibrium selects the visit probabilities that maximize expected surplus. One could argue, though, that the justification for focussing on the mixed strategy equilibrium is less compelling in this setting than in the others. First, this equilibrium exists only when the restriction $y_2^2 \leq y_2^1 + y_1^2$ holds. Second, even when this condition holds, (and so there is a coordination problem) there exists a unique Pareto dominant pure strategy equilibrium (with positive assortative matching). Using the traditional criteria for equilibrium selection in this case, one would not single out the mixed strategy equilibrium.

Fine and Coarse Heterogeneity

In this section, so far, the heterogeneity that we have considered has been quite *fine*: the surplus of each match can be entirely specific to the individual agents being matched. From sections 2, 3.1 and 3.2, we know that homogeneity on either side of the market ensures the existence of the mixed strategy equilibrium. This can also be seen by imposing restrictions on two-sided heterogeneity game in this section and by using Proposition 7. If all candidates are identical, this imposes the restrictions $y_1^1 = y_2^1$ and $y_1^2 = y_2^2$. Similarly, if all vacancies are identical, then $y_1^1 = y_1^2$ and $y_2^1 = y_2^2$. In either case, it is easy to see that these conditions rule out $y_2^2 > y_2^1 + y_1^2$.

While one-sided homogeneity is sufficient for the existence of the mixed strategy equilibrium, this degree of coarseness is not necessary. For example, if Candidate 2 looks the same to both vacancies, but not necessarily Candidate 1, then $y_2^1 = y_2^2$, which is sufficient to rule out $y_2^2 > y_2^1 + y_1^2$. Similarly, if Candidates 1 and 2 look the same to Vacancy 2, but not necessarily to Vacancy 1, then $y_1^2 = y_2^2$, which is also sufficient to rule out $y_2^2 > y_2^1 + y_1^2$. In both of these cases, then, the mixed strategy equilibrium exists.[7]

[7] However, neither having Candidate 1 look the same to both vacancies but not Candidate 2, (so $y_1^1 = y_1^2$ and $y_2^1 \neq y_2^2$) nor having the candidates look the same to Vacancy 1 but not to Vacancy 2 (so $y_1^1 = y_2^1$ and $y_1^2 \neq y_2^2$) are sufficient to rule out $y_2^2 > y_2^1 + y_1^2$.

4. CONCLUSIONS AND DISCUSSION

We argue here that, when the matching process is generated by the simple coordination problem, the equilibrium will be efficient if wages are determined by *ex post* bidding (or, the "Mortensen Rule"), regardless of the size of the market. This result holds both with homogeneous agents and with different types of heterogeneity. In limit large markets, with *homogeneous* agents on both sides of the market, the expected payoffs determined by this rule are identical to those determined by auctions with reserve wages (as in Julien, Kennes, and King (2000) and by wage posting (as in Burdett, Shi, and Wright (2001). However, in markets of finite size, these payoffs do not coincide and only the payoffs determined by *ex post* bidding are efficient.

The concept of efficiency that we used here is that of constrained-efficiency: when the planner can influence incentives, but cannot eliminate the coordination problem. We considered efficiency along two separate margins: when the number of agents is given and the visit probabilities are to be determined, and when the both the entry of vacancies and the visit probabilities are to be determined. We also considered different degrees of heterogeneity. We found that, in all cases considered, when the numbers of agents are given, the *ex post* bidding mixed strategy equilibrium (if it exists) implements the constrained-efficient allocation.

Whenever there is homogeneity on either side of the market, the *ex post* bidding mixed strategy equilibrium does exist and, when vacancy entry is endogenous, this equilibrium also implements the constrained-efficient allocation. We restricted attention to the cases where, on the heterogeneous side of the market, each agent is unique. It is not difficult to show, however, that the results derived here are preserved whenever two or more agents on the heterogeneous side of the market are identical.

With two-sided heterogeneity, to keep the analysis tractable, we only considered the case with two candidates and two vacancies. We showed that, while homogeneity on at least one side of the market is sufficient to ensure the existence of the mixed strategy equilibrium, it is not necessary. In particular, the mixed strategy equilibrium exists if and only if the value of the best match is no greater than the sum of the values of the next two best matches. Thus, the mixed strategy equilibrium is eliminated only when the best match is relatively extreme.

Overall, we believe that the introduction of *ex post* bidding or, loosely speaking, the Mortensen rule, into the coordination model of unemployment brings many advantages. First, as shown here, the equilibrium allocations, when using this wage determination mechanism, are typically efficient. Secondly, this is an extremely simple rule to use. In particular, relative to the mechanisms used in Montgomery (1991), Julien, Kennes, and King (2000), and Burdett, Shi and Wright (2001), this mechanism requires much less computation and since the modeller is not required to compute the Nash wage (or reserve wage) announcements. For this reason, this framework offers the promise of simple analytical solutions in quite rich environments.

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