



<http://researchspace.auckland.ac.nz>

ResearchSpace@Auckland

Copyright Statement

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand).

This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author's right to be identified as the author of this thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author's permission before publishing any material from their thesis.

To request permissions please use the Feedback form on our webpage.

<http://researchspace.auckland.ac.nz/feedback>

General copyright and disclaimer

In addition to the above conditions, authors give their consent for the digital copy of their work to be used subject to the conditions specified on the Library Thesis Consent Form.

NUMERICAL MODELLING OF FIBRE-REINFORCED THERMOPLASTIC SHEET FORMING

by

G. Richard Christie

*A thesis submitted in partial fulfilment of the requirements
for the degree of Doctor of Philosophy in Engineering*

Department of Mechanical Engineering,
University of Auckland, New Zealand

September 1997

Abstract

Continuous Fibre Reinforced Thermoplastics (CFRTPs) combine high strength, stiffness, impact and chemical resistance with possibilities for efficient part production by various *thermoforming* processes. Sheet forming of molten CFRTP laminates has generated much interest, but problems of buckling, wrinkling and predicting fibre distribution have meant a deeper understanding of these processes is needed.

The first part of this study looks at the problem of gross buckling in homogeneous “trellis” flows of bidirectional laminates. Modelling the molten composite as a Newtonian fluid reinforced by inextensible fibres, linear stability analysis is used to determine the growth rate of small out-of-plane imperfections. Buckling is predicted when fibre tensions are negative, indicating that laminates must be kept in tension during forming to reduce such defects.

A new approach to *Grid Strain Analysis* is presented, which uses surface fitting to determine the deformations occurring in sheet forming. The new method improves analysis of smooth, inhomogeneous deformations, and allows greater flexibility in viewing the results. The technique has been used to visualise deformations in a blister fairing made from cross-ply PLYTRON laminates. Arrow diagrams produced from the part demonstrate the tendency of bidirectional composites to deform by trellis flow, while transverse flow results from the action of the diaphragms used to form the part.

The significance of inter-ply slip in CFRTP sheet forming provided the impetus to develop a finite element model for molten laminates, which treats each ply as a separate continuum. Contact between plies is modelled, with slip given a viscous response. Ply deformations are governed by a highly anisotropic elastic law, to handle the stiff fibres and as a first step towards a viscoelastic model of major intra-ply deformation modes.

The finite element model parameters were adjusted to fit the part shape and load response of unidirectional PLYTRON laminates in bending. However, a perfect fit is unobtainable due to local transverse flow occurring at the bend in the real laminate. Nevertheless, the bending of the remainder of the ply is well described by the elastic model, using a fibre direction stiffness 25000 times that in the transverse direction. With the present model, a somewhat less anisotropic set of parameters gives the best overall fit and has been applied in several thermoforming simulations.

As observed in experiments, matched-die bending simulations display ply buckling at high forming speeds. Hemispherical dome forming simulations exhibit out-of-plane buckling and

near-inextensible fibre behaviour, with trellis-like deformation predominant in cross-ply laminates. In simulated double-diaphragm forming of bends and hemispherical domes, tension superimposed on the laminate from the stretching diaphragms is shown to eliminate buckling. However, high forming pressures and excessive transverse flow are a problem with current, stiff diaphragms.

Final discussions look at improving contact modelling, reducing model sizes by adopting thin shell assumptions, and improving the ply model.

Acknowledgments

Completion of this work would not have been possible without the help of many people, and I take this chance to acknowledge their input.

Firstly, I would like to thank my joint supervisors, Associate Professor Debes Bhattacharyya and Professor Ian Collins for the knowledge and experience they have shared with me during this endeavour, and for the many times when their advice and encouragement helped me overcome obstacles to its completion. I wish to acknowledge equally the assistance of Associate Professor Peter Hunter both in the area of finite elements, and in proposing and helping with the application of surface fitting to Grid Strain Analysis.

I also acknowledge the technical assistance of numerous staff members and fellow students in the Department of Mechanical Engineering. I especially thank Simon Mander, Todd Martin and Russell Dykes for many helpful discussions and sharing of ideas in the area of thermoplastic composites. Thanks also to Ulf Hampel for digitising the grid strain components.

I owe many thanks to the German Academic Exchange Service (DAAD) for funding my year with the Institut für Verbundwerkstoffe in Kaiserslautern, Germany, where I had the freedom to begin the finite element component of this work. The friendly support of the IVW staff, in particular Professor Klaus Friedrich, has been greatly appreciated.

For their generous contributions towards the funding of this project I wish to thank the New Zealand Vice-Chancellors' Committee, and Fisher and Paykel Ltd.

Finally, thank you to my parents for their support throughout this lengthy study.

Contents

Abstract.....	ii
Acknowledgments.....	iv
Contents	v
List of Figures.....	ix
List of Symbols.....	xii
Chapter 1: Introduction	1
1.1 Continuous Fibre Reinforced Thermoplastics	1
1.1.1 GF/PP PLYTRON®	2
1.2 Thermoforming of CFRTP Laminates.....	4
1.2.1 Ideal Flow Processes	4
1.2.2 CFRTP Sheet Forming Processes	6
1.3 Thesis Outline	7
Chapter 2: Buckling Stability in Homogeneous Flows of Bidirectional Composites	10
2.1 Introduction.....	10
2.2 Governing Equations	12
2.2.1 Kinematic Constraints	12
2.2.2 Constitutive Equation	13
2.3 Linear Stability Analysis.....	14
2.3.1 Background Solution	14
2.3.2 Perturbed Flow	15
2.3.3 Linearised Governing Equations	17
2.3.4 Determination of Stability Criteria	20
2.4 Discussion.....	22
2.4.1 Influence of Wavelength	22
2.4.2 Practical Consequences	25

Chapter 3: Strain Measurement in Sheet Forming.....27

3.1 Grid Strain Analysis.....	27
3.1.1 Introduction	27
3.1.2 Current Techniques	28
3.1.3 Applications	30
3.2 Strain Analysis by Surface Fitting with the FEM.....	31
3.2.1 Elements for Representing Smooth Surfaces	32
3.2.2 Input and Preprocessing	35
3.2.3 Solution	37
3.2.4 Postprocessing	39
3.3 GSA Example: Composite Blister Fairing.....	41
3.3.1 [0,90] _s Laminate	42
3.3.2 [±45] _s Laminate	48
3.4 Further Uses.....	49

Chapter 4: A Finite Element Model of a Molten Laminate50

4.1 Numerical Modelling of Thermoforming	50
4.1.1 The Ply as a Continuous Material	50
4.1.2 Literature Survey	53
4.1.3 <i>SimForm</i> Model and Program	54
4.2 The Finite Element Method - Non-Linear Overview	57
4.2.1 The Principle of Virtual Work	57
4.2.2 Discretization into Finite Elements	59
4.2.3 Solution of Non-Linear Finite Element Equations	62
4.3 Constitutive Equation for Elastic, Anisotropic Plies	65
4.3.1 Material Coordinates and Strains	65
4.3.2 Transverse Isotropy	66
4.3.3 Incompressible and Inextensible Formulation	68
4.4 Modelling Contact	71
4.4.1 Normal Contact Constraints	71
4.4.2 Friction Models	75
4.4.3 Managing Contacts	77
4.5 Finite Element Algorithm	79

Chapter 5: Determining Model Parameters	81
5.1 Introduction.....	81
5.2 Inter-Ply Slip/Friction Parameters	82
5.2.1 Ply-Pull-Out Experiments	82
5.2.2 Simulated Ply-Pull-Out	84
5.3 Intra-Ply Properties	85
5.3.1 Free Bending Experiments	86
5.3.2 2-D, Extensible Assumption	87
5.3.3 3-D, Inextensible Assumption	91
5.4 Simulation Parameters	94
Chapter 6: Thermoforming Simulations	96
6.1 Introduction.....	96
6.1.1 Simulation Capability of <i>SimForm</i>	96
6.1.2 Post-Processing with <i>SimView</i>	97
6.2 2-D, 90° Bend Forming Examples.....	99
6.2.1 Matched-Die Forming	99
6.2.2 Double-Diaphragm Forming	103
6.3 3-D, Dome Forming Examples	107
6.3.1 Buckling and Inertia	107
6.3.2 Matched-Die Unidirectional Dome Forming	110
6.3.3 Matched-Die Cross-Ply Dome Forming	112
6.3.4 Double-Diaphragm Unidirectional Dome Forming	115
Chapter 7: Discussion On Numerical Modelling	119
7.1 Contact Stability and Limitations	119
7.2 Optimising Elements and Procedures	123
7.3 Material Model Issues.....	125
Chapter 8: Concluding Remarks	128
8.1 Main Findings	128
8.2 Improving CFRTP Thermoforming	131

References.....	134
Appendix A: GSA User Guide	139
A.1 Overview.....	139
A.2 Data Input.....	140
A.3 Mesh Creation	142
A.4 Solution	145
A.5 Post-Processing	146
A.5.1 Graphical Structures	146
A.5.2 Display Settings	148
A.5.3 Contour Plotting	149
A.5.4 Viewing Transformations	149
A.5.5 Plotting to File	150
A.5.6 Outputting Strains	151
Appendix B: <i>SimForm</i> Input File Format.....	152
Appendix C: <i>SimForm</i> Program Verification	159

List of Figures

<i>Page</i>	<i>Figure</i>
3	1.1 Micrograph cross sections of two PLYTRON GF/PP laminates.
5	1.2 Deformation mechanisms of molten continuous fibre-reinforced thermoplastic prepregs and their laminates.
6	1.3 Non-isothermal diaphragm forming.
7	1.4 Non-isothermal matched-die thermoforming.
10	2.1 Trellis flow of a bidirectional sheet showing coordinate system.
11	2.2 Instabilities occurring in forming flows of bidirectional composites.
20	2.3 Local coordinate system on perturbed sheet surface.
23	2.4 Stability plot for thickening trellis flows.
24	2.5 Stability plot for thinning trellis flows.
28	3.1 Deformation of a triangle described by three adjacent grid points.
28	3.2 Superimposed triangular grid elements from before and after deformation.
30	3.3 Principal strains as depicted in Arrow Diagrams.
32	3.4 Bicubic Hermite element showing the four nodal vector quantities.
33	3.5 One dimensional cubic Hermite basis functions.
35	3.6 Degenerate 3-noded bicubic Hermite element.
37	3.7 Finding the surface coordinate, ξ_d , of a data point in an element on the undeformed mesh.
42	3.8 Deformed over undeformed grid points for $[0,90]_s$ blister fairing.
43	3.9 Undeformed grid points and mesh for the $[0,90]_s$ blister fairing.
44	3.10 Elements on the fitted deformed mesh for the $[0,90]_s$ blister fairing.
45	3.11 Arrow diagram of principal Lagrangian strains for the $[0,90]_s$ blister fairing, plotted on the undeformed mesh.
46	3.12 Arrow diagram of principal Lagrangian strains on deformed surface for the $[0,90]_s$ fairing.
47	3.13 Contours of percentage thickness change for the $[0,90]_s$ fairing.
48	3.14 Principal strain plot for $[0,90]_s$ fairing example.

49	3.15	Arrow diagram of principal Lagrangian strains on the deformed surface of the $[\pm 45]_s$ blister fairing.
51	4.1	Micrograph of end of 8 x 0.5 mm PLYTRON bend specimen.
52	4.2	Continuous approximation to in-plane shear.
59	4.3	8-noded brick-type element.
65	4.4	Deformation of a fibre-reinforced sheet showing material coordinates.
72	4.5	Contact surface coordinate system.
73	4.6	6×6 contact points on bicubic element surface.
82	5.1	Schematic of Scherer's ply-pull-out experiments.
83	5.2	Influence of velocity on inter-ply slip resistance, with linear best fit.
84	5.3	Load variation during simulated ply-pull-out at 0.25 mm/s.
86	5.4	Schematic of free-bending experiments of Martin <i>et al.</i>
87	5.5	Free bending loads at 169.5 °C, 500 mm/min, from Martin <i>et al.</i>
89	5.6	2-D simulated free bending for friction relation $\tau = 2 \times 10^7 \cdot v$.
90	5.7	Load curves for 2-D simulated free-bending under three friction conditions.
92	5.8	3-D simulated free bending of a $[0_4]$ PLYTRON laminate.
93	5.9	Unstable behaviour in inextensible, incompressible free-bending model.
98	6.1	The <i>SimView</i> post-processor.
100	6.2	Set-up of matched-die, 90° bend forming simulations.
101	6.3	Matched-die, 90° bend forming example. Slow forming at 7 mm/s.
101	6.4	Matched-die, 90° bend forming example. Fast forming at 70 mm/s.
102	6.5	Fibre stresses early in 90° bend matched-die simulations.
103	6.6	Set-up of double diaphragm, 90° bend forming simulations.
104	6.7	2-D, double-diaphragm 90° bend forming of a unidirectional laminate.
106	6.8	Double-diaphragm forming without friction showing buckling at 0.039 s.
108	6.9	Matched-die forming a dome from a square, isotropic sheet.
111	6.10	Simulated matched-die dome forming of a $[0_4]$ laminate.
112	6.11	Final fibre orientation in top ply of $[0_4]$ matched-die dome.
<i>Page</i>	<i>Figure</i>	
113	6.12	Simulated matched-die dome forming of a $[0,90]_s$ laminate.
115	6.13	Final fibre orientations in $[0,90]_s$ matched-die dome.

- 116 **6.14** Double-diaphragm forming a dome from a single, thick ply.
- 118 **6.15** Fibre orientation in double-diaphragm formed dome at 1.506 s.
- 119 **7.1** Bending of two isotropic sheets showing normal contact stresses.
- 132 **8.1** Equipment for forming a cross-ply dome, using differential heating to encourage tension and limit transverse flow in surface plies.
- 139 **A.1** Screen shot of *GSA* showing undeformed mesh and grid points for the blister fairing example.
- 140 **A.2** Dialogue Box for creating .GSA files for data input.
- 146 **A.3** Options|Display dialogue box for selecting Graphical Structures for display.
- 148 **A.4** Options|Display Settings dialogue box.

List of Symbols

The following list defines all symbols used throughout this text, in the order they first appear. Since Chapter 2, Chapter 3 and Chapters 4 to 7 cover different methods of analysis, a slightly different set of symbols is employed in each to match those of the major references in their respective areas. To avoid confusion arising from the same symbol having a different meaning in different chapters, a separate symbol list is given for each of the three analyses.

By convention, all vector and matrix/tensor quantities are written in bold type, except where subscripted indices are used to refer to their individual components. Furthermore, repeated indices denote summation over the range of components involved, which is usually 3. For example, $F_{rS}X_S = F_{r1}X_1 + F_{r2}X_2 + F_{r3}X_3$.

Chapter *Symbol*

2	x_1, x_2, x_3	Cartesian coordinate axes
	\mathbf{a}	unit vector in direction of first family of fibres
	\mathbf{b}	unit vector in direction of second family of fibres
	ϕ	angle of fibre orientation in x_1 - x_2 plane, measured from x_1
	\mathbf{u}	velocity vector
	\mathbf{x}	Cartesian coordinate vector
	\mathbf{D}	rate of deformation tensor
	\mathbf{T}	total stress tensor
	\mathbf{R}	reaction stress due to kinematic constraints
	\mathbf{S}	deviatoric or extra stress, dependent on the deformation
	T_a	tension reaction stress to inextensibility constraint in \mathbf{a} -direction
	T_b	tension reaction stress to inextensibility constraint in \mathbf{b} -direction
	p	hydrostatic pressure, reaction stress to incompressibility constraint
	δ_{ij}	Kronecker delta; 0 when $i \neq j$, 1 when $i = j$
	μ	isotropic shear viscosity
	ρ	density

Chapter *Symbol*

2 (cont.)	D_1^0, D_2^0, D_3^0	principal background rates of deformation
	t	time
	h	height of a fibre (in direction of axis x_3)
	h^0	unperturbed fibre height
	h'	form of the perturbation to the height of the fibre
	ε	small, dimensionless perturbation parameter
	$\mathbf{u}^0, \mathbf{a}^0$, etc.	unperturbed velocity, \mathbf{a} -fibre direction, etc.
	\mathbf{u}', \mathbf{a}' , etc.	perturbation added to velocity, \mathbf{a} -fibre direction, etc.
	G	convected growth rate
	h^*, γ^* , etc.	initial amplitude of perturbation in fibre height, \mathbf{a} -fibre direction, etc.
	i	(when not used as indicial subscript) imaginary unit
	K	wave number of the buckle
	λ	buckle wavelength
	θ, x_θ	buckle angle and direction
	α, γ	in-plane and out-of-plane perturbations to \mathbf{a} -fibre direction
	β, δ	in-plane and out-of-plane perturbations to \mathbf{b} -fibre direction
	\mathbf{T}^s	stress tensor on the free surface, in the surface coordinate system
	\mathbf{Q}	transformation matrix
	T_θ	fibre tension reaction stresses resolved in the buckle direction
	κ	non-dimensional wave number squared
	τ	non-dimensional fibre tension factor
<hr/>		
3	x, y, z	Cartesian coordinate axes
	\mathbf{X}	undeformed (reference, material) coordinates
	\mathbf{x}	deformed (spatial) coordinates
	\mathbf{F}	deformation gradient tensor
	\mathbf{C}	right Cauchy-Green deformation tensor
	λ_1, λ_2	principal stretches in the plane of the sheet
	ϕ_n	basis functions for interpolating over the surface element

Chapter Symbol

3 (cont.)	$\xi = (\xi_1, \xi_2)$	surface coordinate defined over the element
	\mathbf{u}_n	nodal vectors of position and slope, interpolated over the element
	S_i	arc length in direction of ξ_i
	$\xi^{(d)}$	surface coordinate of grid point d
	\mathbf{K}	global stiffness matrix
	\mathbf{f}	global force vector
	E	weighted, squared sum of fitting errors
	D	number of grid points
	$w^{(d)}$	weighting factor on grid point d
	$\hat{u}(\xi^{(d)})$	unknown interpolated deformed position of grid point d , used interchangeably for x , y and z
	$u^{(d)}$	actual deformed position of grid point d , used interchangeably for x , y and z at d
	\bar{E}	modified least squares sum
	α	factor for controlling in-plane stretching of the fitted surface
	β	factor for controlling curvature of the fitted surface
	\mathbf{V}	matrix containing the eigenvectors of \mathbf{C}
	\mathbf{A}	matrix containing the eigenvalues of \mathbf{C}
	ε_i	engineering strains
	e_i	natural strains
	E_i	Lagrangian or Green's finite strains
	λ_3	through-thickness stretch
	$[0,90]_s$	denotes a symmetrical 4-ply laminate with 0° plies on the outer surfaces and two 90° oriented plies in the centre
4,5,6,7	\mathbf{x}	spatial (current) Cartesian coordinates
	\mathbf{T}	Cauchy (true) stress tensor
	$\mathbf{f}^{(body)}$	body force vector per unit volume
	$\mathbf{n}^{(S)}$	unit surface normal on boundary
	$\mathbf{t}^{(S)}$	boundary traction (force) vector

Chapter Symbol

4,5,6,7	$\delta \mathbf{u}$	virtual displacement
(cont.)	δW_{ext}	virtual work
	S	current boundary of body
	V	current volume occupied by body
	$\boldsymbol{\varepsilon}$	infinitesimal strain tensor
	$\tilde{\mathbf{T}}$	symmetric second Piola-Kirchoff stress tensor, later expressed as a six-component vector
	\mathbf{E}	symmetric Lagrangian finite strain tensor, later expressed as a six-component vector
	V_0	reference (undeformed) volume originally occupied by body
	\mathbf{X}	material (reference) Cartesian coordinates
	$\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$	parametric coordinates within an element
	\mathbf{a}_j	displacement or displacement slope vector at node j
	M	number of nodes in the element, or in the whole model as necessary
	$\psi^j(\boldsymbol{\xi})$	basis functions for interpolating nodal quantities over the volume of the element
	\mathbf{F}^{tot}	total displacement gradient tensor
	$\delta \mathbf{a}$	virtual nodal displacement vector
	$\delta \mathbf{E}$	virtual increment in Lagrangian strain tensor
	$\bar{\mathbf{B}}^i$	matrix for calculating incremental strains from nodal displacements
	\mathbf{B}_0^i	linear component of $\bar{\mathbf{B}}^i$
	\mathbf{B}_L^i	large strain, non-linear component of $\bar{\mathbf{B}}^i$
	$\boldsymbol{\Psi}(\mathbf{a})$	residual vector
	\mathbf{f}	vector of generalised nodal forces
	\mathbf{K}_T	tangential stiffness matrix
	$\Delta \mathbf{a}$	increment of nodal displacements calculated in an iteration
	$\bar{\mathbf{K}}$	large strain global stiffness matrix
	\mathbf{K}_σ	initial stress matrix
	\mathbf{K}_f	load-correction matrix

Chapter Symbol

4,5,6,7	D	incremental elasticity matrix
(cont.)	α	a small number
	x_1, x_2, x_3	Cartesian coordinate axes
	X_1, X_2, X_3	Material (convected) coordinate axes
	\mathbf{a}^0	unit vector in direction of undeformed fibres
	$d\mathbf{X}$	differential line element in the undeformed state
	$d\mathbf{x}$	$d\mathbf{X}$ transformed onto the deformed body
	F	deformation gradient tensor
	C	right Cauchy-Green deformation tensor
	W	strain energy function (per unit volume) for hyperelastic material
	I_1, I_2, I_3, I_4, I_5	invariants of tensors C and $\mathbf{a}^0 \otimes \mathbf{a}^0$
	δ_{ij}	(when not used as a prefix for virtual quantities) Kronecker delta; 0 when $i \neq j$, 1 when $i = j$
	K_1, K_2, \dots, K_5	material parameters in strain energy function
	E_f	Young's modulus in fibre direction
	E_T	Young's modulus in directions transverse to fibres
	G_L	longitudinal shear modulus
	G_T	transverse shear modulus
	\bar{W}	modified strain energy function
	p	hydrostatic pressure reaction stress to incompressibility constraint
	T	tension reaction stress to inextensible fibre constraint
	b	hydrostatic pressure at the node
	c	tension at the node
	\hat{p}, \hat{T}	interpolated pressure and tension, respectively
	\bar{N}_i	basis functions for interpolating pressure and tension
	\mathbf{k}	point on surface "A" touching some other surface "B"
	\mathbf{p}	point on surface "B" that \mathbf{k} is in contact with
	$\mathbf{n}, \mathbf{t}^1, \mathbf{t}^2$	outward normal and tangent vectors to surface "B" at \mathbf{p}

Chapter Symbol

	d	distance between points \mathbf{k} and \mathbf{p} along normal \mathbf{n}
	S_n	normal contact stiffness (spring stiffness per unit area)
4,5,6,7	$\xi^{(k)}, \xi^{(p)}$	surface coordinates of points \mathbf{k} and \mathbf{p}
(cont.)	w	integration/Gauss point weight
	A	area associated with a contact point
	$\mathbf{F}_n^{(k)}$	normal force vector transferred onto point \mathbf{k} by a contact spring
	$\phi_i^{(A)}, \phi_i^{(B)}$	basis functions defined over surfaces A and B, respectively
	$\mathbf{g}_i^{(A)}$	generalised nodal contact force vector on element surface A
	$\boldsymbol{\kappa}$	3×3 contact stiffness matrix
	$\Delta \mathbf{k}, \Delta \mathbf{p}$	displacement of points \mathbf{k} and \mathbf{p} during an iteration
	$\mathbf{K}^{(AB)}$, etc.	contact stiffness terms
	$C(\mathbf{a})$	contact constraint equation
	π	penalty number for applying contact constraints
	S_{t1}, S_{t2}	tangential contact stiffnesses
	Δt	length of a time increment
	η	viscosity (in Pa.s) of the fluid in the inter-ply region
	θ	estimated inter-layer thickness for contact friction calculations
	τ	shear stress applied as a result of inter-ply slip
	v	relative velocity between two contacting surfaces
	c_f	coefficient of sliding friction, such that $\tau = c_f v$
	v_f	fibre volume fraction (ratio from 0.0 to 1.0)
	ρ_0	density in reference (undeformed) state
	$\ddot{\mathbf{u}}$	acceleration vector
	$\dot{\mathbf{u}}$	velocity vector