NUMERICAL MODELLING OF FIBRE-REINFORCED THERMOPLASTIC SHEET FORMING

by

G. Richard Christie

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Department of Mechanical Engineering,
University of Auckland, New Zealand

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Abstract

Continuous Fibre Reinforced Thermoplastics (CFRTPs) combine high strength, stiffness, impact and chemical resistance with possibilities for efficient part production by various *thermoforming* processes. Sheet forming of molten CFRTP laminates has generated much interest, but problems of buckling, wrinkling and predicting fibre distribution have meant a deeper understanding of these processes is needed.

The first part of this study looks at the problem of gross buckling in homogeneous “trellis” flows of bidirectional laminates. Modelling the molten composite as a Newtonian fluid reinforced by inextensible fibres, linear stability analysis is used to determine the growth rate of small out-of-plane imperfections. Buckling is predicted when fibre tensions are negative, indicating that laminates must be kept in tension during forming to reduce such defects.

A new approach to *Grid Strain Analysis* is presented, which uses surface fitting to determine the deformations occurring in sheet forming. The new method improves analysis of smooth, inhomogeneous deformations, and allows greater flexibility in viewing the results. The technique has been used to visualise deformations in a blister fairing made from cross-ply PLYTRON laminates. Arrow diagrams produced from the part demonstrate the tendency of bidirectional composites to deform by trellis flow, while transverse flow results from the action of the diaphragms used to form the part.

The significance of inter-ply slip in CFRTP sheet forming provided the impetus to develop a finite element model for molten laminates, which treats each ply as a separate continuum. Contact between plies is modelled, with slip given a viscous response. Ply deformations are governed by a highly anisotropic elastic law, to handle the stiff fibres and as a first step towards a viscoelastic model of major intra-ply deformation modes.

The finite element model parameters were adjusted to fit the part shape and load response of unidirectional PLYTRON laminates in bending. However, a perfect fit is unobtainable due to local transverse flow occurring at the bend in the real laminate. Nevertheless, the bending of the remainder of the ply is well described by the elastic model, using a fibre direction stiffness 25000 times that in the transverse direction. With the present model, a somewhat less anisotropic set of parameters gives the best overall fit and has been applied in several thermoforming simulations.

As observed in experiments, matched-die bending simulations display ply buckling at high forming speeds. Hemispherical dome forming simulations exhibit out-of-plane buckling and near-inextensible fibre behaviour, with trellis-like deformation predominant in cross-ply laminates. In simulated double-diaphragm forming of bends and hemispherical domes, tension superimposed on the laminate from the stretching diaphragms is shown to eliminate buckling. However, high forming pressures and excessive transverse flow are a problem with current, stiff diaphragms.

Final discussions look at improving contact modelling, reducing model sizes by adopting thin shell assumptions, and improving the ply model.

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Contents

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List of Symbols

The following list defines all symbols used throughout this text, in the order they first appear. Since Chapter 2, Chapter 3 and Chapters 4 to 7 cover different methods of analysis, a slightly different set of symbols is employed in each to match those of the major references in their respective areas. To avoid confusion arising from the same symbol having a different meaning in different chapters, a separate symbol list is given for each of the three analyses.

By convention, all vector and matrix/tensor quantities are written in bold type, except where subscripted indices are used to refer to their individual components. Furthermore, repeated indices denote summation over the range of components involved, which is usually 3. For example, $F_{rS}X_S = F_{r1}X_1 + F_{r2}X_2 + F_{r3}X_3$.

Chapter	*Symbol*	
2	x_1, x_2, x_3	Cartesian coordinate axes
	$\boldsymbol{a}$	unit vector in direction of first family of fibres
	$\boldsymbol{b}$	unit vector in direction of second family of fibres
	ϕ	angle of fibre orientation in x_1-x_2 plane, measured from x_1
	$\boldsymbol{u}$	velocity vector
	$\boldsymbol{x}$	Cartesian coordinate vector
	$\boldsymbol{D}$	rate of deformation tensor
	$\boldsymbol{T}$	total stress tensor
	$\boldsymbol{R}$	reaction stress due to kinematic constraints
	$\boldsymbol{S}$	deviatoric or extra stress, dependent on the deformation
	T_a	tension reaction stress to inextensibility constraint in $\boldsymbol{a}$-direction
	T_b	tension reaction stress to inextensibility constraint in $\boldsymbol{b}$-direction
	p	hydrostatic pressure, reaction stress to incompressibility constraint
	δ_{ij}	Kronecker delta; 0 when $i \neq j$, 1 when $i = j$
	μ	isotropic shear viscosity
	ρ	density

Chapter	*Symbol*	

2 (cont.)	$D_1^0,\ D_2^0,\ D_3^0$	principal background rates of deformation
	t	time
	h	height of a fibre (in direction of axis x_3)
	h^0	unperturbed fibre height
	h'	form of the perturbation to the height of the fibre
	ε	small, dimensionless perturbation parameter
	$\boldsymbol{u}^0$, $\boldsymbol{a}^0$, etc.	unperturbed velocity, ***a***-fibre direction, etc.
	$\boldsymbol{u}'$, $\boldsymbol{a}'$, etc.	perturbation added to velocity, ***a***-fibre direction, etc.
	G	convected growth rate
	h^*, γ^*, etc.	initial amplitude of perturbation in fibre height, ***a***-fibre direction, etc.
	i	(when not used as indicial subscript) imaginary unit
	K	wave number of the buckle
	λ	buckle wavelength
	θ, x_θ	buckle angle and direction
	α, γ	in-plane and out-of-plane perturbations to ***a***-fibre direction
	β, δ	in-plane and out-of-plane perturbations to ***b***-fibre direction
	$\boldsymbol{T}^s$	stress tensor on the free surface, in the surface coordinate system
	$\boldsymbol{Q}$	transformation matrix
	T_θ	fibre tension reaction stresses resolved in the buckle direction
	κ	non-dimensional wave number squared
	τ	non-dimensional fibre tension factor
3	x, y, z	Cartesian coordinate axes
	$\boldsymbol{X}$	undeformed (reference, material) coordinates
	$\boldsymbol{x}$	deformed (spatial) coordinates
	$\boldsymbol{F}$	deformation gradient tensor
	$\boldsymbol{C}$	right Cauchy-Green deformation tensor
	λ_1, λ_2	principal stretches in the plane of the sheet
	ϕ_n	basis functions for interpolating over the surface element

Chapter	*Symbol*	
3 (cont.)	$\xi = (\xi_1, \xi_2)$	surface coordinate defined over the element
	$\boldsymbol{u}_n$	nodal vectors of position and slope, interpolated over the element
	S_i	arc length in direction of ξ_i
	$\xi^{(d)}$	surface coordinate of grid point *d*
	$\boldsymbol{K}$	global stiffness matrix
	$\boldsymbol{f}$	global force vector
	E	weighted, squared sum of fitting errors
	D	number of grid points
	$w^{(d)}$	weighting factor on grid point *d*
	$\hat{u}\left(\xi^{(d)}\right)$	unknown interpolated deformed position of grid point *d*, used interchangeably for *x*, *y* and *z*
	$u^{(d)}$	actual defomed position of grid point *d*, used interchangeably for *x*, *y* and *z* at *d*
	$\overline{E}$	modified least squares sum
	α	factor for controlling in-plane stretching of the fitted surface
	β	factor for controlling curvature of the fitted surface
	$\boldsymbol{V}$	matrix containing the eigenvectors of $\boldsymbol{C}$
	$\boldsymbol{\Lambda}$	matrix containing the eigenvalues of $\boldsymbol{C}$
	ε_i	engineering strains
	e_i	natural strains
	E_i	Lagrangian or Green's finite strains
	λ_3	through-thickness stretch
	$[0,90]_s$	denotes a symmetrical 4-ply laminate with 0° plies on the outer surfaces and two 90° oriented plies in the centre
4,5,6,7	$\boldsymbol{x}$	spatial (current) Cartesian coordinates
	$\boldsymbol{T}$	Cauchy (true) stress tensor
	$\boldsymbol{f}^{(body)}$	body force vector per unit volume
	$\boldsymbol{n}^{(S)}$	unit surface normal on boundary
	$\boldsymbol{t}^{(S)}$	boundary traction (force) vector

Chapter	*Symbol*	
4,5,6,7	$\delta\boldsymbol{u}$	virtual displacement
(cont.)	δW_{ext}	virtual work
	S	current boundary of body
	V	current volume occupied by body
	ε	infinitesimal strain tensor
	$\tilde{\boldsymbol{T}}$	symmetric second Piola-Kirchoff stress tensor, later expressed as a six-component vector
	$\boldsymbol{E}$	symmetric Lagrangian finite strain tensor, later expressed as a six-component vector
	V_0	reference (undeformed) volume originally occupied by body
	$\boldsymbol{X}$	material (reference) Cartesian coordinates
	$\xi = \left(\xi_1, \xi_2, \xi_3\right)$	parametric coordinates within an element
	$\boldsymbol{a}_j$	displacement or displacement slope vector at node j
	M	number of nodes in the element, or in the whole model as necessary
	$\psi^j(\xi)$	basis functions for interpolating nodal quantities over the volume of the element
	$\boldsymbol{F}^{utot}$	total displacement gradient tensor
	$\delta\boldsymbol{a}$	virtual nodal displacement vector
	$\delta\boldsymbol{E}$	virtual increment in Lagrangian strain tensor
	$\overline{\boldsymbol{B}}^i$	matrix for calculating incremental strains from nodal displacements
	$\boldsymbol{B}_0^i$	linear component of $\overline{\boldsymbol{B}}^i$
	$\boldsymbol{B}_L^i$	large strain, non-linear component of $\overline{\boldsymbol{B}}^i$
	$\Psi(\boldsymbol{a})$	residual vector
	$\boldsymbol{f}$	vector of generalised nodal forces
	$\boldsymbol{K}_T$	tangential stiffness matrix
	$\Delta\boldsymbol{a}$	increment of nodal displacements calculated in an iteration
	$\overline{\boldsymbol{K}}$	large strain global stiffness matrix
	$\boldsymbol{K}_\sigma$	initial stress matrix
	$\boldsymbol{K}_f$	load-correction matrix

Chapter	*Symbol*	
4,5,6,7	$\boldsymbol{D}$	incremental elasticity matrix
(cont.)	α	a small number
	x_1, x_2, x_3	Cartesian coordinate axes
	X_1, X_2, X_3	Material (convected) coordinate axes
	$\boldsymbol{a}^0$	unit vector in direction of undeformed fibres
	$d\boldsymbol{X}$	differential line element in the undeformed state
	$d\boldsymbol{x}$	$d\boldsymbol{X}$ transformed onto the deformed body
	$\boldsymbol{F}$	deformation gradient tensor
	$\boldsymbol{C}$	right Cauchy-Green deformation tensor
	W	strain energy function (per unit volume) for hyperelastic material
	I_1, I_2, I_3, I_4, I_5	invariants of tensors $\boldsymbol{C}$ and $\boldsymbol{a}^0 \otimes \boldsymbol{a}^0$
	δ_{ij}	(when not used as a prefix for virtual quantities) Kronecker delta; 0 when $i \neq j$, 1 when $i = j$
	$K_1, K_2, ... K_5$	material parameters in strain energy function
	E_f	Young’s modulus in fibre direction
	E_T	Young’s modulus in directions transverse to fibres
	G_L	longitudinal shear modulus
	G_T	transverse shear modulus
	$\overline{W}$	modified strain energy function
	p	hydrostatic pressure reaction stress to incompressibility constraint
	T	tension reaction stress to inextensible fibre constraint
	b	hydrostatic pressure at the node
	c	tension at the node
	$\hat{p}, \hat{T}$	interpolated pressure and tension, respectively
	$\overline{N}_i$	basis functions for interpolating pressure and tension
	$\boldsymbol{k}$	point on surface “A” touching some other surface “B”
	$\boldsymbol{p}$	point on surface “B” that $\boldsymbol{k}$ is in contact with
	$\boldsymbol{n}, \boldsymbol{t}^1, \boldsymbol{t}^2$	outward normal and tangent vectors to surface “B” at $\boldsymbol{p}$

Chapter	*Symbol*	
	d	distance between points $\boldsymbol{k}$ and $\boldsymbol{p}$ along normal $\boldsymbol{n}$
	S_n	normal contact stiffness (spring stiffness per unit area)
4,5,6,7	$\xi^{(\boldsymbol{k})}, \xi^{(\boldsymbol{p})}$	surface coordinates of points $\boldsymbol{k}$ and $\boldsymbol{p}$
(cont.)	w	integration/Gauss point weight
	A	area associated with a contact point
	$\boldsymbol{F}_n^{(\boldsymbol{k})}$	normal force vector transferred onto point $\boldsymbol{k}$ by a contact spring
	$\phi_i^{(\mathrm{A})}, \phi_i^{(\mathrm{B})}$	basis functions defined over surfaces A and B, respectively
	$\boldsymbol{g}_i^{(\mathrm{A})}$	generalised nodal contact force vector on element surface A
	$\boldsymbol{\kappa}$	3 × 3 contact stiffness matrix
	$\Delta\boldsymbol{k}$, $\Delta\boldsymbol{p}$	displacement of points $\boldsymbol{k}$ and $\boldsymbol{p}$ during an iteration
	$\boldsymbol{K}^{(\mathrm{AB})}$, etc.	contact stiffness terms
	$C(\boldsymbol{a})$	contact constraint equation
	π	penalty number for applying contact constraints
	S_{t1}, S_{t2}	tangential contact stiffnesses
	Δt	length of a time increment
	η	viscosity (in Pa.s) of the fluid in the inter-ply region
	θ	estimated inter-layer thickness for contact friction calculations
	τ	shear stress applied as a result of inter-ply slip
	v	relative velocity between two contacting surfaces
	c_f	coefficient of sliding friction, such that $\tau = c_f\, v$
	v_f	fibre volume fraction (ratio from 0.0 to 1.0)
	ρ_0	density in reference (undeformed) state
	$\ddot{\boldsymbol{u}}$	acceleration vector
	$\dot{\boldsymbol{u}}$	velocity vector