A Cooperative Game Approach to Patent Litigation, Settlement, and Allocation of Legal Costs

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A Cooperative Game Approach to Patent Litigation, Settlement, and Allocation of Legal Costs

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Abstract

We analyze litigation and settlement behavior in case of patent infringement using the Nash Bargaining Game framework. We show that litigation can be the Pareto efficient outcome. We also show that when there is settlement, the transfer payment from the defendant to the plaintiff is increasing in its own legal cost and decreasing in that of the plaintiff, reflecting the bargaining power on both sides. We also compare the American and English rules of cost allocation when legal costs are endogenously determined.

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1 Introduction

Between years 1980 and 1990, the number of patent litigation in the U.S. increased by 52% (Business Week (1991)). This is remarkable given that the ex-post probability of winning an infringement suit was 48% for years 1978 to 1985 (Hylton (1993a)). In addition, many high profile cases (Polaroid vs. Eastman Kodak, Honeywell vs. Minolta, Hyatt vs. North American Philips) have held out for verdicts instead of settling out of court. Traditional explanations for lack of settlement have resorted to some form of asymmetry. ¹

One common assumption has been that the plaintiff, the patent owner, has superior information. However the following facts prompt us to question this premise. Eastman Kodak’s R&D expenditure has been nearly ten fold of that of Polaroid. (Business Week (June 15, 1990, June 28, 1993). One measurement of technological strength (complied by the CHI Institute from number of patents controlling for importance by instances of citations. Business Week (August 3, 1992)) gives a score of 506 to Minolta while Honeywell’s is 146. ² How likely is it that an individual inventor such as Gilbert P. Hyatt has superior information than Philips which had a score of 781 in the same report. Why would such well informed defendants choose not to


²In the same report, Eastman Kodak’s score was 1186 and Polaroid’s was 41.
settle?

In this paper we show that not settling will be the Nash Bargaining Solution as long as the legal costs are not too high relative to profits. Thus the litigation outcome is Pareto efficient. This is because the sum of payoffs from a verdict is greater than the sum of payoffs achievable through settlement due to the 'efficiency effect'. \(^3\) While antitrust constraints pose restrictions on the settlement allocation, monopoly is possible under a verdict.

We also show that when there is settlement in equilibrium, the payment made by the defendant to the plaintiff is increasing in defendant's legal cost and decreasing in that of the plaintiff. This is consistent with the fact that avoidance of legal cost is the most commonly cited reason for settling out of court by both plaintiffs and defendants. Yet previous models in which settlement transfer is endogenous have concluded that the size of transfer does not depend on the legal cost of the defendant (Reinganum and Wilde (1986), Muerer (1989) \(^4\)). The fact that the defendant is just as eager to avoid heavy legal expenses was not reflected in the terms of settlement.

A common characteristic of strategic models of bargaining is that whoever makes the take-it-or-leave offer will collect all the surplus from a settlement. When the defendant makes his offer, \(^5\) he does not care about how much the plaintiff pays for the litigation but only whether the plaintiff will sue. So the defendant offers just enough to make the plaintiff indifferent between settling

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\(^3\)That is, that sum of duopoly profits is less than the monopoly profit.

\(^4\)In Muerer’s formulation, both parties have identical legal cost. However it is easy to see that the relevant cost is the one born by the plaintiff so that if the costs were different, the transfer is independent of defendant’s legal cost.

\(^5\)If the plaintiff makes an offer, we can give the same argument concerning defendant payoff and its litigation cost.
and not settling. Thus the defendant’s settlement offer will depend only on the legal cost of the plaintiff as long as we resort to a non-cooperative game framework.

This conclusion seems incomplete since the plaintiff knows that the defendant also wants to avoid the litigation cost and the more he has to pay, the more he wants a settlement. *Justice can be costly!* The bargaining power of each party depends on how costly it is for him to claim his right. The Nash Bargaining Solution reflects this aspect of bargaining. Although common knowledge of payoffs is assumed in both non-cooperative and cooperative games, players do not take into account joint payoffs in a non-cooperative game.

A non-cooperative approach to remedy this problem is to formulate the litigation and settlement process as a Rubinstein alternating offers model of strategic bargaining. As with all strategic models, the solution will be very sensitive to informational assumptions and order of play. Incomplete information usually generates inefficient outcome. Thus this approach would be very attractive when one is interested in specific laws or practices that dictate how settlement negotiations should be conducted, which is not our goal. On the other hand, both the Nash Bargaining Game and the Rubinstein alternating offers model capture essentially the same aspect of bargaining power. That is, the Nash Bargaining Solution is the limit (either when probability of breakdown goes to zero or when length of time between offers go to zero) of the subgame perfect Nash equilibrium of the Rubinstein alternating offers model. Nash Bargaining Solution focuses on what can be achieved while the
non-cooperative game focuses on how the equilibrium will deviate from the Solution due to institutional restrictions.

By taking a non-cooperative game approach to the litigation and settlement process, one interprets the litigation process as an opportunity to engage in strategic behavior, such as strategic revelation of information (P’ng (1983), Reinganum and Wilde (1986), Muerer (1989)). By taking the cooperative game approach⁶, we view the process as a negotiation processes with a credible bad outcome (verdict) where there is ample opportunities to exchange information, to propose settlement terms and to make counter offers. We also overcome the problem of who should move first when there is first mover advantage. In non-cooperative formulations it is always assumed that the defendant makes a settlement offer, which determines the equilibrium allocation of resources. However if the plaintiff knew it could collect all the surplus from settling if it makes an offer first, there is no reason why it should not. The plaintiff then just brings the suit to establish a credible threat. In reality, both sides are aware of this fact and it is taken into account in the negotiation process and the outcome should reflect it. The Nash Bargaining Game and the Solution capture exactly that aspect.

We also compare the American and the English rules of patent litigation and settlement. We find that the English rule encourages settlement in a patent lawsuit. But the bargaining power of the patent owner depends not only the choice of the rule of allocating legal costs but also the parameters

⁶Cooter and Rubinfeld (1989) have also used the Nash Bargaining Solution to explain nuisance suits. Prusa (1992) has applied Nash Bargaining Solution to the complicated process of trade negotiations.
of winning distribution function. Generally speaking, the American rule protects the patent right better when both parties thinks the defendant is more likely to win and the litigation can be affected at a wider range by the effort of the litigants. We do the comparison when the litigation costs are endogenous by simulation.

Litigation costs are exogenous and are allocated according to the American rule in the basic model. In the next section we formulate this basic model as a Nash Bargaining Game. The Nash Bargaining Solution is characterized in section 3. We do the same when the costs are allocated according to the English rule in section 4. Endogenous litigation costs are considered for both rules in section 5. Section 6 discusses the effect of risk aversion in our framework and possible extensions.

2 The Nash Bargaining Game Model of Litigation and Settlement with American System

In this section we formulate the litigation with settlement as a Nash Bargaining Game (See Owen (1982)). We consider a patent litigation when only the validity of the patent is at issue and the patentee is seeking an induction barring further use of the infringing product (Besen and Raskind (1991)). In the axiomatic approach of bargaining, the relative timing of settlement and litigation is irrelevant. The framework covers both cases when

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7 It is possible to generalize this framework to a case when damages are also considered. See comments following proposition 1.
settlement is reached after the suit has been filed (but no substantial legal expenditures have been realized) and when litigation is only a threat and the settlement is reached before litigation actually begins. Thus it is possible to interpret the settlement as a licensing agreement obtained to avoid litigation. (For the case when there is discounting and the two cases are differentiated, see Aoki and Hu (1995).)

We will first analyze the case of American rule of cost allocation. A two-person Nash Bargaining Game \((F, v)\) consists of the set of feasible payoff allocations, \(F\), and the disagreement payoff allocation, \(v\). \(F\) is a closed convex subset of \(\mathbb{R}^2\) and \(v = (v_1, v_2)\) is a vector in \(\mathbb{R}^2\) and the set

\[
FI = F \cap \{ (\pi_1, \pi_2) | \pi_1 \geq v_1, \pi_2 \geq v_2 \}
\]

is non-empty and bounded. A vector \((\pi_1, \pi_2)\) is a payoff allocation where player \(i\) gets \(\pi_i\). The non-emptiness of the set \(FI\) states that there is at least one feasible payoff allocation that guarantees both players a payoff equal to or more than what the player will get if there is disagreement (i.e., feasible and incentive compatible).

We assume that firm 1 is the patent owner (plaintiff) and firm 2 is the infringer (defendant). We define \(S\) to be the set of all possible profit pairs that the two firms may achieve together. A settlement will allow them to achieve one of these points. Given the antitrust restrictions, the most they can realize is duopoly profits. Thus

\[
S = \{ (\pi_1, \pi_2) | \pi_1 + \pi_2 \leq 2\pi_d \}
\]

It is possible for the payoffs to be negative. One or both firms making a loss
in the litigation and negotiation process is not ruled out \textit{a priori}. If there were no antitrust considerations, the maximum the two firms can achieve together would be the monopoly profit, $\pi_m$, instead of $2\pi_d$.

Payoff that each firm gets in the absence of negotiation is the payoff from going through with the litigation, i.e., the expected payoff of a verdict. We assume that firm $i$ believes the patentee (the plaintiff) will win with probability $\theta_i$. So firm 1 thinks itself will win with probability $\theta_1$ while firm 2 thinks itself will win with probability $1 - \theta_2$. For the time being, we assume the perceived probabilities are exogenous. If firm 1, the patent owner, wins then firm 1 will be the monopolist and firm 2 will get nothing. If firm 2 wins, then each firm will get duopoly profit. The disagreement point is defined as the expected payoffs from litigation,

\begin{align*}
    v_1 &= \theta_1 \pi_m + (1 - \theta_1) \pi_d - \ell_1, \\
    v_2 &= (1 - \theta_2) \pi_d - \ell_2.
\end{align*}

(3) (4)

where $\ell_i$ is the legal cost of firm $i$.

Note that the disagreement point will be in the set $S$ either if $\ell_i$’s are large or if $\theta_i$'s are sufficiently small (specifically, if $\ell_1 + \ell_2 > \theta_1 \pi_m - (\theta_1 + \theta_2) \pi_d$, see Figure 1). On the other hand, if the legal costs are small or if the firms believe that the probability of a successful infringement suit is high (specifically, if $\ell_1 + \ell_2 < \theta_1 \pi_m - (\theta_1 + \theta_2) \pi_d$, see Figure 2), then the disagreement point will be outside the set $S$. This is because the largest possible payoff, the monopoly profit, is possible only as result of a litigation.

Now we define the feasible set $F$. The most natural approach is to assume $F = A$. The set $S$ is closed and convex. If the disagreement point is an
element of the set $S$, then

$$FI = \{(\pi_1, \pi_2) | \pi_1 + \pi_2 \leq 2\pi_d, \pi_1 \geq v_1, \pi_2 \geq v_2\}.$$  

This is a set of payoff allocations which are achievable and each firm gets at least its payoff at the disagreement point.

However, it is not always the case that the set $FI$ is non-empty. A problem arises when the disagreement point is not in the set $S$. Then the set $FI$ is empty if we define $F$ by $F = S$. Instead, we define $F$ to be the convex hull of the union of the sets $S$ and $\{(v_1, v_2)\}$. That is,

$$F = \text{convex hull}(S \cup \{(v_1, v_2)\}).$$  \hspace{1cm} (5)$$

This corresponds to the shaded area in Figure 2. The set $FI = \{(v_1, v_2)\}$ is non-empty and bounded. We can actually use the definition (5) for both cases since the definition reduces to $F = S$ when the threat point is an element of $S$ (Figure 1).

3 The Nash Bargaining Solution under the American Rule

The Nash Bargaining Solution ($NSB$) is the only payoff allocation that satisfies the following five conditions: (1) strong efficiency, (2) independence of irrelevant alternatives, (3) symmetry, (4) individual rationality, and (5) scale invariance. Condition (1) guarantees that $NBS$ is Pareto efficient. Condition (2) requires that eliminating feasible points (except the disagreement point) that are not part of the solution will not change the solution. If the
players are symmetric \((v_1 = v_2)\), then the symmetry of \(NBS\) guarantees the same payoff to each player. Individual rationality requires that the Nash Bargaining Solution should make each player at least as well off as with the disagreement point. Scale covariance implies that the solution is independent of any risk-neutral utility specification. (See Meyerson (1990) for details.)

The scale covariance condition is required because the bargaining set is characterized in the Euclidean space. We denote the \(NBS\) payoff allocations by \((\pi_1^{NBS}, \pi_2^{NBS})\). The \(NBS\) is characterized by the following theorem.

**Theorem 1** The Nash Bargaining Solution is characterized as a solution to the following maximization problem:

\[
(\pi_1^{NBS}, \pi_2^{NBS}) \in \arg\max_{(\pi_1, \pi_2) \in F_I} \max(\pi_1 - v_1)(\pi_2 - v_2).
\]

**Proof**

See Meyerson (1990). \(\square\)

We characterize the \(NBS\) of our Bargaining Game using this theorem.

**Proposition 1** The Nash Bargaining Solution of the Nash Bargaining Game with American rule of cost allocation is the following:

Case 1: When \(\ell_1 + \ell_2 \geq \theta_1 \pi_m - (\theta_1 + \theta_2)\pi_d\), a settlement is reached and the payoffs are,

\[
\begin{align*}
\pi_1^{NBS} &= v_1 + \{\ell_1 + \ell_2 + (\theta_1 + \theta_2)\pi_d - \theta_1 \pi_m\}/2, \\
\pi_2^{NBS} &= v_2 + \{\ell_1 + \ell_2 + (\theta_1 + \theta_2)\pi_d - \theta_1 \pi_m\}/2.
\end{align*}
\]

Case 2: When \(\ell_1 + \ell_2 < \theta_1 \pi_m - (\theta_1 + \theta_2)\pi_d\), litigation results and the payoffs are,

\[
\pi_1^{NBS} = v_1 = \theta_1 \pi_m + (1 - \theta_1)\pi_d - \ell_1,
\]

10
\[ \pi_2^{NBS} = v_2 = (1 - \theta_2)\pi_d - \ell_2. \]

**Proof**

When the disagreement point is not in the set \( S \) (Case 2), then the solution is simple since \( FI = F \cap \{ (\pi_1, \pi_2) | \pi_1 \geq v_1, \pi_2 \geq v_2 \} = \{ (v_1, v_2) \} \). The \( NBS \) is the disagreement point, i.e., \( \pi_1^{NBS} = v_1 \) and \( \pi_2^{NBS} = v_2 \).

For Case 2, we use Theorem 1 to find \( NBS \) as the solution to the following constrained maximization problem.

\[
\max_{(\pi_1, \pi_2)} (\pi_1 - v_1)(\pi_2 - v_2) \tag{8}
\]

subject to

\[ \pi_1 \geq v_1, \]
\[ \pi_2 \geq v_2, \]
\[ \pi_1 + \pi_2 \leq 2\pi_d. \]

Since the disagreement point is an interior point of the set \( S \) constraints (9) and (9) are not binding. Let the Lagrange multiplier for (9) be \( \lambda \). Then the sufficient\(^8\) first order condition is,

\[
\begin{align*}
\pi_1 &= v_1 + \lambda, \\
\pi_2 &= v_2 + \lambda, \\
\pi_1 + \pi_2 &= 2\pi_d.
\end{align*}
\]

Equations (9) and (10) show that each player must receive something (\( \lambda \)) in addition to what it would get from a litigation. From all three equations,

\(^8\)It is easy to see that the second order condition is satisfied.
\[ \lambda = \left\{2\pi_d - (v_1 + v_2)\right\}/2. \] The two players split equally the surplus they achieve by a settlement. □

If the litigation costs are low enough or probabilities of a victory are large enough, firms get a litigation than settling. This is because monopoly profit is only achievable with a litigation. Of course if the plaintiff gets the monopoly profit, the infringer will be shut out of the market. However in order to settle when the plaintiff has such a large expected payoff from a litigation, the defendant must give up so much to satisfy the plaintiff that the expected payoff of a litigation is larger than settling. Each firm's payoff from settlement decreases with its own litigation and increases with rival litigation.

By definition, \(NBS\) is Pareto optimal. Lack of settlement in our model is not an inefficient outcome as result of strategic behavior or imperfect information as in non-cooperative game formulations. The Nash Bargaining Game describes a negotiation process where all relevant information are revealed. Particularly important is that both parties know each others' beliefs about winning. It is assumed that the negotiation process is such that all feasible payoff allocations are considered.

We can see from the proof that a patent owner can extract a greater settlement by suing for damages at the same time. Independent of how the damage \(D\) is determined (it may be endogenous or exogenous), \(D\) will be a transfer from defendant to plaintiff. In our formulation, \(v_1\) increases by \(\theta_1D\) and \(v_2\) decreases by \(\theta_2D\). The set \(S\) remains unchanged since \(D\) is
just a transfer. Thus the surplus, \(2\pi_d - (v_1 + v_2)\) increases by \((\theta_2 - \theta_1)D\). The equilibrium payoff with settlement for the plaintiff will increase by \(\theta_1D + (\theta_2 - \theta_1)D/2\) by suing for damages at the same time. The argument is similar to Landes's (1993) result that sequential reduces the likelihood that parties settle out of court by restricting range of possible settlements.

The settlement payoffs can be written as

\[
\pi_1^{NBS} = \pi_d + T, \quad \pi_2^{NBS} = \pi_d - T,
\]

where \(T = \{\theta_1\pi_m + (\theta_2 - \theta_1)\pi_d - \ell_1 + \ell_2\}/2\). \(T\) is the transfer payment from the infringer to the patentee. \(T\) is positive if the plaintiff's expected payoff of a litigation is no less that duopoly profit \((v_1 \geq \pi_d)\). Thus we have the following.

**Proposition 2** The settlement transfer payment from defendant to plaintiff \((T)\) is strictly increasing in \(\theta_1, \theta_2,\) and \(\ell_2\). \(T\) is strictly decreasing in \(\ell_1\).

The defendant is willing to pay more when its legal cost is high while it takes advantage of higher plaintiff legal costs. Either party believing a victory for plaintiff more likely increases the transfer payment. If the defendant believes that it is very likely that it will lose (\(\theta_2\) small), it is willing to pay more to avoid a litigation. If the plaintiff believes (and the defendant is convinced that the plaintiff really believes it) the likelihood of a win is very high (\(\theta_2\) large), it enhances its bargaining power.

It is important here that we understand the significance of a belief in a bargaining game where there is no informational asymmetry. A player
having a particular belief means that the player is able to credibly convince the other player that it has such a belief. In order for the plaintiff to extract a higher transfer payment, it is not sufficient that it just announces that it has believes that it can win with high probability. The plaintiff must convince the defendant that it truly believes that the probability is high.

Such a belief may be a source of why litigation outcomes differ depending on if the defendant is a domestic or a foreign firm. For instance, unfamiliar with the difference in the breadth of a patent and the jury system, Minolta might not only had a low \( \theta_2 \) (Honeywell was convinced it was low), but thought Honeywell’s \( \theta_1 \) was not very high. This would have made settlement impossible (Case 2) and thus a litigation. Had the defendant been a domestic firm which would perceive (and can be convinced that) \( \theta_i \)'s are low, such a patent infringement case might have settled out of court.

4 Comparison with the English Rule

The Nash Bargaining Game of litigation and settlement with the English rule litigation cost allocation is identical to the game presented in section 2 except for the disagreement point. The disagreement point is now

\[
\begin{align*}
\nu'_1 &= \theta_1 \pi_m + (1 - \theta_1)(\pi_d - \ell_1 - \ell_2), \\
\nu'_2 &= \theta_2(-\ell_1 - \ell_2 + (1 - \theta_2)\pi_d).
\end{align*}
\]  

(10)

The feasible allocation set \( (F') \) is defined as before using the disagreement point and the settlement set \( S \). The new disagreement point may or may not
be an element of the set \( S \).

\[
F' = \text{convex hull}(S \cup \{(v'_1, v'_2)\}). \quad (11)
\]

**Proposition 3** The Nash Bargaining Solution of the Nash Bargaining Game with English rule of cost allocation is the following:

**Case 1':** When \( (\ell_1 + \ell_2) (1 - (\theta_1 - \theta_2)) \geq \theta_1 \pi_m - (\theta_1 + \theta_2) \pi_d \), a settlement is reached and the payoffs are,

\[
\pi_{1 \text{NBS}'} = v'_1 + \frac{((\ell_1 + \ell_2)(1 + \theta_2 - \theta_1) + (\theta_1 + \theta_2)\pi_d - \theta_1 \pi_m)}{2}, \quad (12)
\]

\[
\pi_{2 \text{NBS}'} = v'_2 + \frac{((\ell_1 + \ell_2)(1 + \theta_2 - \theta_1) + (\theta_1 + \theta_2)\pi_d - \theta_1 \pi_m)}{2}.
\]

**Case 2':** When \( (\ell_1 + \ell_2) (1 - (\theta_1 - \theta_2)) < \theta_1 \pi_m - (\theta_1 + \theta_2) \pi_d \), a litigation results and the payoffs are,

\[
\pi_{1 \text{NBS}'} = v'_1, \quad (13)
\]

\[
\pi_{2 \text{NBS}'} = v'_2.
\]

**Proof**

When the disagreement point is not in the set \( S \) (Case 2'), then the solution is simple since \( F' \cap \{(\pi_1, \pi_2) | \pi_1 \geq v'_1, \pi_2 \geq v'_2\} = \{(v'_1, v'_2)\} \). The NBS is the disagreement point.

For Case 2', from Theorem 1 we solve the following constrained maximization problem.

\[
\max_{(\pi_1, \pi_2)} (\pi_1 - v_1)(\pi_2 - v_2) \quad (14)
\]

subject to
\[ \pi_1 \geq v_1, \]
\[ \pi_2 \geq v_2, \]
\[ \pi_1 + \pi_2 \leq 2\pi_d. \]

Let the Lagrangean multiplier for (9) be \( \lambda' \). Then from the necessary first order condition, we have \( \pi_1^{NBS'} = v_1' + \lambda' \) and \( \pi_2^{NBS'} = v_2' + \lambda' \) where \( \lambda' = \{2\pi_d - (v_1' + v_2')\}/2 = \{(\ell_1 + \ell_2)(1 + \theta_2 - \theta_1) + (\theta_1 + \theta_2)\pi_d - \theta_1\pi_m\}/2. \)

We first look at the conditions under which a settlement is reached with respect to the two cost allocation rules. From propositions 1 and 3 we have,

**Corollary 1** The minimum joint litigation cost \((\ell_1 + \ell_2)\) that induces settlement is larger [smaller] under American rule of cost allocation than under English rule if \( \theta_2 > \theta_1 [\theta_1 > \theta_2] \). The minimum litigation cost to induce settlement are equal under the two rules when \( \theta_1 = \theta_2 \).

There will be more settlement under English rule only if the defendant’s belief of a successful infringement suit is larger than that of the plaintiff.

Similarly, the payoffs with settlement under the English rule can be written as
\[ \pi_1^{NBS'} = \pi_d + T', \quad \pi_2^{NBS'} = \pi_d - T', \]
where \( T' = \{\theta_1\pi_m + (\theta_2 - \theta_1)\pi_d + (\theta_1 + \theta_2 - 1)(\ell_1 + \ell_2)\}/2 \). We make the following observation about the transfer payment.

**Corollary 2** Under English rule, the settlement transfer payment \((T')\) is strictly increasing in \( \theta_1 \) and \( \theta_2 \). \( T' \) is strictly increasing [decreasing] in \( \ell_1 + \ell_2 \).
if $\Theta_1 + \Theta_2 > 1$ [$\Theta_1 + \Theta_2 < 1$]. The transfer payment is independent of legal costs when $\Theta_1 + \Theta_2 = 1$.

The transfer payment depends on the total litigation cost since this what is paid by a single firm under English rule. Who ever is believed to be likely to lose is willing to give more to avoid a litigation higher the total litigation cost. That is, if firms believe that it is very likely that the plaintiff wins ($\Theta_1 + \Theta_2 > 1$), then the transfer payment increases and the plaintiff is better off higher the total litigation cost. Similarly, if the firms believe that it is very likely that the defendant wins ($\Theta_1 + \Theta_2 < 1$), then the defendant is better off higher the total litigation cost. As with the American rule, either party believing a victory for plaintiff more likely increases the transfer payment.

5 When Probability of Winning Depends on Litigation Costs

Several authors (Posner (1973), Hause (1989), Koo (1991)) have compared the effect of the American and English rules on settlement behavior when the probability of winning is determined by the amount of legal cost expended by both parties. Posner and Hause concluded that the English rule results in higher total Nash equilibrium litigation cost and induces more settlement when the probability distribution function is additively non-separable in litigation costs. Koo showed that when the distribution function is additively separable that although the English rule results in higher Nash equilibrium litigation costs but may not necessarily induce more settlement. However it seems unlikely that the marginal probability of winning is independent of the
opponent’s effort. Therefore we consider a distribution function which is not separable.

While factors such as the interpretation of the law by the courts will determine the perceived probability of winning ($\theta_1$) it is conceivable that the amount of legal cost expended ($\ell_i$) will also influence the probability. We follow the formulation by Hause (1989) of $\theta_i$ as a function of $\ell_1$ and $\ell_2$ which takes into account both of these factors. The probability $\theta_i$ has two parts: One is exogenous and determined by the administration and the court. The other is endogenous and determined by litigation costs of both parties. But to guarantee that the second-order conditions hold globally $^9$, we augment his distribution slightly:

$$
\theta_1(\ell_1, \ell_2) = A + b \frac{\ell_1}{\ell_1 + \ell_2},
$$

$$
\theta_2(\ell_1, \ell_2) = B + b \frac{\ell_1}{\ell_1 + \ell_2},
$$

(15)

where $0 \leq A + b \leq 1$ and $0 \leq b \leq 1$. $A$ and $B$ are the subjective beliefs on the lower bound of the plaintiff’s prevailing probability. $A + \frac{b}{2}$ and $B + \frac{b}{2}$ are the expected values of the probabilities. The parameter $b$ is the range the two litigants can effect around the means of subjective winning probabilities. When the perceived probability of winning is endogenous, the expected payoff of a litigation becomes endogenous. The disagreement point of the Nash Bargaining Game should be the expected payoff when litigation costs are chosen optimally. The optimal litigation cost will be defined to be the Nash equilibrium costs of a non-cooperative game in which firms choose the

\footnote{Hause used the distribution functions $\theta_1 = A + be^{-\frac{\ell_1}{B}}$ and $\theta_2 = B + be^{-\frac{\ell_2}{B}}$ with which the second-order conditions under the English rule may not be guaranteed globally.}
litigation costs simultaneously and the payoffs are the expected payoff with verdict.

5.1 Optimal Litigation Costs under American Rule

The expected payoffs for the plaintiff and the defendant under the American rule are,

\[ \theta_1(\ell_1, \ell_2) \pi_m + (1 - \theta_1(\ell_1, \ell_2)) \pi_d - \ell_1, \]
\[ (1 - \theta_2(\ell_1, \ell_2)) \pi_d - \ell_2. \]

The determination of optimal legal costs can be thought of as a non-cooperative game with simultaneous moves. We characterize the Nash equilibrium legal costs, \((\ell^*_1, \ell^*_2)\), and the beliefs about probability of plaintiff victory in equilibrium, \((\theta^*_1, \theta^*_2)\).

**Proposition 4** The Nash equilibrium litigation costs under the American system of cost allocation are

\[ (\ell^*_1, \ell^*_2)_{\text{American}} = \left[ \frac{b(\pi_m - \pi_d)^2 \pi_d}{\pi_m^2}, \frac{b(\pi_m - \pi_d)^2 \pi_d^2}{\pi_m^2} \right]. \]

Therefore the beliefs for the plaintiff to win at the Nash equilibrium are

\[ (\theta^*_1, \theta^*_2)_{\text{American}} = \left[ A + b \frac{\pi_m - \pi_d}{\pi_m}, B + b \frac{\pi_m - \pi_d}{\pi_m} \right]. \]

**Proof**

The first-order conditions of expected payoffs maximization given a level of rival legal cost are,

\[ \frac{b \ell_2}{(\ell_1 + \ell_2)^2} (\pi_m - \pi_d + \ell_1 + \ell_2) - (1 - A - b \frac{\ell_1}{\ell_1 + \ell_2}) = 0, \]
\[ \frac{b \ell_1}{(\ell_1 + \ell_2)^2} (\pi_d + \ell_1 + \ell_2) - (B + b \frac{\ell_1}{\ell_1 + \ell_2}) = 0. \]
Since the second-order conditions,

\[- \frac{2\ell_2}{(\ell_1 + \ell_2)^3} (\pi_m - \pi_d) < 0, \]

\[- \frac{2\ell_1}{(\ell_1 + \ell_2)^3} \pi_d < 0, \]

are satisfied, equations (17) and (18) characterize the best-response functions from which the unique pure strategy Nash equilibrium can be found.

By substituting the equilibrium legal costs into the probability functions (15), we attain \( \theta_1^* \) and \( \theta_2^* \). □

Note that the Nash equilibrium litigation costs \( (\ell_1^*, \ell_2^*)|_{\text{American}} \) are independent of the means \( A \) and \( B \) which can be public policy variables. The equilibrium litigation costs only depend on \( \pi_m \), \( \pi_d \), and \( b \). Therefore a legal reform to reduce the litigation fees under the American rule should reduce the marginal effect of legal cost on the probability of winning \( (b) \). An effort to shift the mean of winning probability \( (A \) or \( B ) \) has no effect on the Nash equilibrium litigation costs under the American rule.

**Corollary 3** Under the American rule, the plaintiff’s optimal litigation cost is no less than that of the defendant, as long as there is the efficiency effect. Specifically,

\[ \ell_1^*|_{\text{American}} > \ell_2^*|_{\text{American}} \iff \pi_m = 2\pi_d. \]

**Proof** It follows from \( (\ell_1^* - \ell_2^*)|_{\text{American}} = b \frac{(\pi_m - 2\pi_d)(\pi_m - \pi_d)(\pi_d)}{\pi_m^2} \). □
The Corollary implies that the efficiency effect induces the plaintiff to spend more than the defendant in a patent infringement litigation under the American rule. And if we observe the phenomenon that the plaintiff spends strictly more than the defendant, it implies that the efficiency effect exists. Efficiency effect plays an important role for both litigants to determine how much to spend in a patent suit under the American rule.

5.2 Optimal Litigation Cost under English Rule

The expected payoffs for the plaintiff and under the English rule, respectively, are

\[
\theta_1(\ell_1, \ell_2)(\pi_m + D) + (1 - \theta_1(\ell_1, \ell_2))(\pi_d - \ell_1 - \ell_2),
\]

\[
(1 - \theta_2(\ell_1, \ell_2))\pi_d - \theta_2(\ell_1, \ell_2)(D + \ell_1 + \ell_2).
\]

**Proposition 5** *The Nash equilibrium litigation costs under the English rule are*

\[
\ell_1|_{\text{English}} = \frac{bB}{\pi_d}
\left[\frac{\pi_m - \pi_d}{1 - A - b + B\frac{\pi_m - \pi_d}{\pi_d}}\right]^2,
\]

\[
\ell_2|_{\text{English}} = \frac{b}{1 - A - b + B\frac{\pi_m - \pi_d}{\pi_d}}.
\]

*Therefore the beliefs for the plaintiff to win at the Nash equilibrium are*

\[
\theta_1|_{\text{English}} = A + b\frac{B(\pi_m - \pi_d)}{\pi_d(1 - A - b + B\frac{\pi_m - \pi_d}{\pi_d})},
\]

\[
\theta_2|_{\text{English}} = B + b\frac{B(\pi_m - \pi_d)}{\pi_d(1 - A - b + B\frac{\pi_m - \pi_d}{\pi_d})}.
\]
Proof

The first-order conditions of expected payoffs maximization under the English rule are

\[
\frac{b\ell_2}{(\ell_1 + \ell_2)^2} (\pi_m - \pi_d + \ell_1 + \ell_2) - (1 - A - b\frac{\ell_1}{\ell_1 + \ell_2}) = 0, \tag{20}
\]

\[
\frac{b\ell_1}{(\ell_1 + \ell_2)^2} (\pi_d + \ell_1 + \ell_2) - (B + b\frac{\ell_1}{\ell_1 + \ell_2}) = 0. \tag{21}
\]

The second-order conditions under the English rule are

\[
-\frac{2\ell_2}{(\ell_1 + \ell_2)^2} (\pi_m - \pi_d) < 0,
\]

\[
-\frac{2\ell_1}{(\ell_1 + \ell_2)^2} \pi_d < 0.
\]

The analytical solution for the Nash equilibrium litigation expenses under the English rule can be solved from the two first order conditions. By substituting the equilibrium legal costs into the probability functions (15), we attain \(\theta_1^*\) and \(\theta_2^*\). \(\square\)

Under the English rule, the equilibrium litigation costs \((\ell_1^*, \ell_2^*)|_{\text{English}}\) are functions of \(A, B, \pi_m, \pi_d,\) and \(b\). Recall that \((\ell_1^*, \ell_2^*)|_{\text{American}}\) are independent of \(A\) and \(B\). Under the American rule, each party pays its own litigation cost, independent of the result. Under the English rule however, as long as there is a positive probability for each party to pay for the other’s legal cost, a change in \(A\) and \(B\) will affect the expected legal costs of each litigant.

A comparison of the American and the English rules is as shown in Table 1. From the optimal legal costs, we calculate the equilibrium perceived probabilities of patentee victory \((\theta_1^*, \theta_2^*)\). We then apply propositions 1 and 3 to
determine if there will be a settlement or not and also calculate the threat points \((v_1, v_2)\).

### 5.3 Which Rule Protects the Patent Right Better: American or English?

First we note in Table 1 that the English rule induces higher total litigation costs. However the perceived probabilities of patentee winning or losing may be lower or higher with the English rule. Since what is relevant under the English rule is the total litigation costs, the English rule results in higher litigation costs which encourage settlement to litigation (Posner (1973), Shavell (1982), and Hause (1989)). That is, whenever there is settlement under the American rule, there will be settlement under the English rule but not vice versa.

We also note that a symmetric belief for the plaintiff to prevail with less than a 50 percent probability would be low enough to make disagreement less probable under the English rule (Shavell (1982)). An increase in \(b\), the extent in probability of winning that litigants can actually effect, provokes both parties to spend more in litigation under both rules.

Under both rules, patentee's bargaining power increases \((v_1 - v_2)\) as \(A\), \(B\) and \(b\) increase. However, the rule under which the patentee has greater bargaining power depends on the size of parameters. If \(A\), \(B\), \(b\) are small, American rule gives greater bargaining power to the patentee. It is the English rule that favors the patentee otherwise. When \(A\), \(B\) are large, it is very likely that the defendant will pay the legal costs under the English
rule. Therefore, the defendant is willing to pay more to settle than under the American rule. When $b$ is large, it is less costly for the defendant to increase the probability of winning in his favor and to avoid the legal cost under the English rule. Thus the defendant is willing to pay less to settle under the English rule.

This result suggests that it is possible to increase the bargaining position of a patentee by changing the rule of cost allocation.  \(^{10}\)

6 Concluding Remarks

By defining the payoff allocations in terms of profits, we have assumed that both the defendant and plaintiff are risk-neutral. However by defining the payoff allocations in terms of utilities, our analysis can be extended to take different levels of risk aversion (including risk-loving) into consideration.

The relative positions of the disagreement point and the boundary of the set $S$ will change according to the utility function. The disagreement point for firm 1 denoting it's utility function by $u_1(\cdot)$ will be

$$\theta_1 u_1(\pi_m - \ell_1) + (1 - \theta_1) u_1(\pi_d - \ell_1).$$

A more risk-averse (concave) utility function will increases $u_1(\pi_d)$ more relative to $u_1(\pi_m)$. It has the effect of pulling the disagreement point towards the axis, moving the disagreement point from outside the set $S$ to inside it.

\(^{10}\)In India, where patent protection is generally weak ($A$ and $B$ are small), patentee’s position will be improved from the English rule to the American rule. On the other hand, if the patentee is already favored (perhaps U.S. very recently, see Waswofisky (1994)), patentee’s position can be decreased by switching to English rule.
This means a settlement is possible (for the same litigation costs) when it is not possible if the firm were less risk-averse.

Similarly, a risk-loving (convex) utility has the effect of moving the disagreement point away from the axis, putting the disagreement outside the set $S$ when it would be inside with a less risk-loving utility function. Thus litigation will result when a firm is risk-loving under circumstances in which there will be settlement with a less risk-loving firm (Landes (1971)).

We may apply our approach to cases where there are more than one infringers. Both the difference between winning and losing ($\pi_m - \pi_d$ when there is only one infringer) and the status quo profit ($\pi_d$ when there is only one infringer) will change. The disagreement point may move outward (if the change in the difference between winning and losing is large relative to that of status quo payoff) or inward (otherwise) relative to the origin. On the other hand, the boundary of the settlement set $S$ will shift towards the origin because what the plaintiff and the defendant can achieve together will be smaller when there are other firms in the market. It is not immediate whether there will be more or less litigation.

One may also change the number of infringers participating in the trial in order to compare the effect of unitary and sequential trials. When the trials are sequential, the beliefs ($\theta_i$) of the players in the second trial will be function of the outcome of the first trial.

---

11 The set $S$ also changes shape with $u_1$ but it only depends only on $u_1(\pi_d)$, not on $u_1(\pi_m)$. 

25
Table 1: Numerical Examples under the American and the English Rules

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Figure 1 (Case 1)