NON-COMMENSURATE REALIZATION OF COMPACT BROADBAND R.F. CIRCUITS

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ABSTRACT

A new method for the synthesis of broadband impedance transformers having predictable passband frequency response is presented. The technique is based on the use of non-commensurate (i.e. unequal element length) transmission line networks. Through the use of approximations and computer optimization studies non-commensurate circuits are shown to be advantageous in distributed circuit design. The new method derives a non-commensurate circuit from a conventional commensurate prototype in such a way that the transmission matrix of pairs of elements in both circuits is made equal at one frequency. The transformation used ensures that the frequency response of the derived circuit closely matches that of the prototype circuit. Limitations on section characteristic impedances imposed either by constructional constraints or other practical realizability considerations are more easily met using the new design technique than when using conventional commensurate networks alone. Moreover, wide harmonically related passbands are largely avoided. The method finds greatest application where there are circuit length (or size) restrictions which must be met. Where such restrictions do not apply, conventional commensurate techniques are usually sufficiently flexible. The non-commensurate technique can however still provide benefits of even greater flexibility or better stop-band attenuation. The method presented is only an approximate equivalence and so an analysis of the technique is presented. The analysis establishes the degree of approximation. The use and application of the non-commensurate design technique are supported by an experimental investigation.

DEDICATION

In honour of the God who created all things so that we might one day discover them. May their discovery honour Him.

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LIST OF SYMBOLS AND ABBREVIATIONS

A, B, C, D	Modified transmission parameters, see equation (5.9)
B	susceptance (Ω^{-1})
BW	bandwidth = $2(f_2-f_1) / (f_2+f_1)$
C, C_s	shunt capacitance (F)
C_f	fringing capacitance
d, D	denominator (subscripts e and o refer to even and odd parts)
f	real frequency (Hz)
f_o	quarter-wave frequency, i.e. element lengths are $\lambda/_4$ at f_o
f_1	frequency of lower edge of passband
f_2	frequency of upper edge of passband
f_c	centre frequency = $(f_1+f_2)/2$
f_e	equivalencing frequency
G	gain parameter; see equation (2.4)
GBW	Gain-bandwidth theory
h	ripple height factor, see (2.18, 2.19) also microstrip dielectric thickness (m)
ℓ	physical length (m)
L, L_s	series inductance (H)
LHP	left-half-plane
m	number of open circuit-series and short circuit-parallel stubs
MIL	minimum insertion loss

n	number of unit elements
n, N	numerator (subscripts e and o refer to even and odd parts)
op	open circuited - parallel stub
os	open circuited - series stub
p	complex frequency variable (distributed) $= \Sigma + j\Omega$ ($\Omega = \tan\theta$)
P	$= \lvert\rho\rvert$
P_{in}	power into a network
P_{inc}	power incident at input of a network
P_L	power loss ratio; see equation (1.4)
P_{out}	power into a resistive network termination
q	constant factor in the factorization of $\lvert\rho\rvert^2$
r	number of short circuit - series and open circuit - parallel stubs
r_i, r_o	coaxial transmission line inner and outer conductor radii
R	resistance (Ω)
RHP	right-half-plane
s	non-commensurate length variable $= \sin\theta_1 \sin\theta_2$ also $s = j\omega$
sp	short circuited - parallel stub
ss	short circuited - series stub
S_{ij}	scattering parameters
t	impedance scaling factor also transmission coefficient

T	= $\|t\|$
[T]	transmission matrix
UE	unit element
v	speed of wave propagation (ms^{-1})
V	voltage
VSWR	voltage standing wave ratio= $(1+\|\rho\|)/(1-\|\rho\|)$
W	microstrip conductor width (m)
x	frequency variable; see equation (2.11)
X	reactance (Ω^{-1})
Y	admittance (Ω^{-1})
z	z-transform operator
Z	impedance or element characteristic impedance (Ω)
α	bandwidth parameter (eg $\alpha.\cos\Theta$)
	also attenuation constant (Np m^{-1})
β	phase constant = $2\pi/\lambda$
γ	propagation constant = $\alpha+j\beta$
Δ	small finite change
ε	ripple parameter; see equations (2.42 - 2.44)
Θ	electrical length (radians) = $\beta \ell$
Θ_o, Θ_1, Θ_2, Θ_c	electrical lengths corresponding to f_o, f_1, f_2, f_c
Θ_{diff}	difference length = $\Theta_1-\Theta_2$
Θ_{tot}	total length = $\Theta_1+\Theta_2$
λ	wavelength (m) $f\lambda = v$

ξ_1	frequency variable; see equation (2.9)
ξ_2	frequency variable; see equation (2.10)
ρ	input reflection coefficient
ϕ	frequency variable; see equation (2.8)
	also = $\underline{/\rho}$
ω	radian frequency = $2.\pi.f$
Ω	frequency variable = $\tan\theta$