"NON-UNIFORMLY SPACED ARRAYS OF
DIRECTIONAL ELEMENTS"

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J.C. LIM
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PART I

NON-UNIFORMLY SPACED ARRAYS OF DIRECTIONAL ELEMENTS
INTRODUCTION

In February 1965, a research programme in radio astronomy was started by the Electrical Engineering Department of the University of Auckland. One of the main purposes of the programme was to provide a unified theme for post-graduate research in the department. The initial aim of the programme was to develop through graduate research, sufficient facilities for radio astronomy observations at frequencies below 100 Mhz.

Being among the first group of graduate students in the programme, the author was given the problem of studying the antenna requirements for the programme. At the frequencies concerned, the antenna systems are often large and expensive. As the programme is supported at present, only by funds for ordinary graduate research, there is a great need for an antenna array with good performance at minimal cost. This has led the author into his main field of study, viz. the synthesis of arrays with non-uniformly spaced directional elements. The use of directional elements together with non-uniform spacing technique permits larger inter-element spacings in the array without resulting in large sidelobes in the response pattern.

Available synthesis methods are inadequate despite the large number of papers published on the subject since its
introduction 7 years ago. The synthesis problem involves the
determination of a set of element positions to give a desirable
response pattern. Because the element position variables lie
in the arguments of the cosine terms in the pattern function,
the problem becomes highly non-linear. For simplicity, most
of the published works have assumed isotropic elements. The
methods proposed have been mainly centred on some form of
linear approximations to the problem. Consequently, these
methods are only effective over a limited region of space.*
As a result of this limitation, good pattern characteristics
can only be achieved with impractically small spacings.

For arrays with less than about 50 elements, the
element spacings can best be determined by an optimisation
procedure. This method involves the repeated application of
small perturbations to the element positions of a starting
array until maximum improvement to the sidelobe levels of
the array is achieved. An efficient perturbation method
has been proposed by Baklanov et al.\textsuperscript{29} using a matrix approach.

*The word 'space' used throughout the Introduction does not mean
the physical space, which is defined, in a 2-D case, by the zenith
angle $\theta$. Here, the space is defined by the parameter $\chi = 2\pi d_{av} \sin \theta/\lambda$
Thus with an average inter-element spacing $d_{av} = \lambda$, the visible
space is defined by $\chi = 0$ to $\pi$. 
Because of the inherent limitation of this method, Baklanov's arrays are mostly impractical due to the occurrence of small spacings. Such limitations are removed by the author through the use of a modified synthesis procedure. With this new procedure, the author was able to control the pattern over a considerably larger area in space. Thus arrays with average inter-element spacings up to two wavelengths can be synthesised with positive control over all sidelobes in the arrays. The sidelobe levels of the author's arrays are, as a whole, very close to the levels of the corresponding theoretically optimal patterns. The element directivities are taken into account in the synthesis process.

A total of 30 non-uniformly spaced arrays of varying sizes and sidelobe levels were synthesised using the method developed. Since all these arrays have near to optimal sidelobe characteristics, they provide a basis for a detailed study of the properties of non-uniformly spaced arrays as a whole. A number of interesting points are revealed when pattern parameters like gain, beamwidth, sidelobe level, etc., are studied in relation with the spacing characteristics of the arrays. A better understanding of the properties of non-uniformly spaced arrays is also gained by comparing the pattern characteristics of the synthesised arrays with that
of current tapered arrays.

The design and testing of a 16-element non-uniformly spaced array of Yagi antennas is described in Chapter 4. This array demonstrates one practical application of the synthesis work reported in this thesis.
CHAPTER I

Linear Arrays with Non-uniformly Spaced Elements

Antenna arrays are used in radio astronomy and many fields of communication engineering whenever increased gain or resolution are required. The basic form of antenna array is the linear array which consists essentially of a number of identical antennas distributed along a straight line. These elements are connected to the receiver, or transmitter, via a network of low loss transmission lines. The amplitude of excitation to any element can be controlled by varying the coupling between the antenna and the transmission line while the phase of the excitation can be controlled either by varying the length of signal path to each element, or by the introduction of a phase shifting network in the transmission path.

The aim of the array design is to ensure that the element locations and the amplitude and phase of the excitation to each element are such as to give a desirable spatial radiation pattern. In the case of a receiving array, the design problem involves the judicious choice of the locations at which radiations from the entire "visible" space are sampled. The signals from each of these sampling
points can be weighted appropriately both in amplitude and phase so that the resultant response corresponds closely to some prescribed spatial response characteristics.

1.1 Array Theory

The response of any practical antenna is a function of the direction of arrival of the signal. The voltage response pattern or alternately, the far field radiation pattern of an antenna array of isotropic elements can be computed easily by simple vector addition. Consider a linear array of point sources whose distances from an arbitrary reference point are as shown in fig. 1.

![Symmetrical Array of 2N Elements](image-url)
The voltage response pattern is, by vector addition,

\[ F_a(\theta) = \sum_{n=1}^{2N} I_n e^{-j \phi_n} e^{-j k x_n \sin \theta} \]  \hspace{1cm} (1.1)

where

\[ I_n = \text{the amplitude of the excitation to the nth element} \]

\[ \phi_n = \text{the additional phase delay introduced to the signal to the nth element.} \]

\[ x_n = \text{the distance of the nth element to the reference point.} \]

\[ k = \frac{2\pi}{\lambda}, \text{ the free space wave number.} \]

In practice, it is desirable to have a field pattern which is a real function of \( \theta \). In this case, the elements must be assumed symmetrically distributed about a reference phase centre. The pattern for such an array is given by

\[ F_a(\theta) = \sum_{n=1}^{N} I_n \cos(k x_n \sin \theta + \phi_n) \]  \hspace{1cm} (1.2)

Equation (1.2) results from the assumption that \( x_n = -x_n \), and \( \phi_n = -\phi_n \).

In practice, it is not possible to have an isotropic radiator. Assuming that each element has a pattern \( F_0(\theta) \) in the \( \theta \)-plane, the resultant field pattern is, by the principle of pattern multiplication,
Equation (1.3) is the most general form of expression for the field pattern of a symmetrical array of 2N elements. To prevent confusion of terminology, $F_a(\theta)$ will hereafter be called the array factor. Thus the field pattern of the array is the product of the array factor and the element pattern.

Equations (1.2) and (1.3) can also be derived using a well known formulation in antenna theory which expresses the far field radiation pattern of an antenna as simply the Fourier transform of the current distribution. The distribution function for a symmetrical array of 2N point sources is, as shown in Fig. 2b,

$$S_a(x) = \sum_{n=1}^{2N} I_n e^{j\phi_n} \delta(x - x_n)$$

where $\delta$ is the Dirac delta function.

The array factor $F_a(u)$, $u = \sin\theta$, is given by the Fourier transform of $S_a(x)$,

$$F_a(u) = \int_{-\infty}^{\infty} S_a(x) e^{jku} dx$$
Assuming that each element pattern is due to a continuous distribution $S_0(x)$, (fig. 2a), it can be seen that the resultant distribution is given by the convolution of $S_0(x)$ with $S_a(x)$, as shown in fig. 2c.

$$S(x) = S_0(x) \ast S_a(x)$$

---

Fig. 2a. Element distribution $- S_0(x)$

2b. Array distribution (isotropic sources) $- S_a(x)$

2c. Actual distribution $- S_0(x) \ast S_a(x)$
From Fourier Transforms theory, the transform of a convolution of two functions equals the product of their separate transforms. This is the principle of pattern multiplication, the application of which gives the array pattern as expressed in equation (1.3).

1.2 Array Design

From equation (1.3), there are three parameters within the control of an array designer. These parameters are:

(i) the element positions \( x_n \), 
(ii) the amplitude distribution \( I_n \), 
(iii) the phase distribution \( \phi_n \).

Given all the variables in the right hand side of equation (1.3) it is then a simple matter to compute the array pattern. However, the reverse problem, which incidently constitutes the design problem, is often not immediately apparent.

Two types of design problems are often encountered by the array engineers. The first is to design an array with a pattern which fits closely a prescribed radiation pattern. The second problem which is met more often in radio
astronomy, involves the synthesis of an array with response at angles outside the main beam below a certain prescribed level. Here, the exact form of the pattern is not critical. It is the second type of problem that will be considered in this thesis.

1.3 Uniformly Spaced Array

The classical approach to the problem of array synthesis has assumed uniform spacings between elements and zero or progressive phase shifts in order to provide facilities to steer the beam electrically. Thus \( x_n = nx \), and \( \phi_n = -k nx \sin \theta_0 \), where \( x \) is the constant inter-element spacing and \( \theta_0 \) is the angle to which the main beam is to be steered. The space factor of the array becomes

\[
F_a(\theta) = \sum_{n=1}^{N} I_n \cos(nkx(\sin \theta - \sin \theta_0))
\]

By introducing a new space variable \( u = \sin \theta - \sin \theta_0 \), equation (1.7) reduces to

\[
F_a(u) = \sum_{n=1}^{N} I_n \cos(nkxu)
\]

It is easy to recognise that equation (1.8) is simply a Fourier series expression. In the case where the
array factor has to be approximated to a certain prescribed pattern, the standard method of extracting Fourier components\(^5\) can be applied to equation (1.8) to evaluate the excitation \(I_n\) to each element.

In the case where the exact shape of the pattern is of little consequence, and the designer's aim is to keep all secondary lobe response below a prescribed limit, it is better to express equation (1.8) in the form of a polynomial of \(\cos(kxu)\). In the most general form, the space factor can be expressed as a polynomial of a complex variable \(z\), where \(z = e^{jkux}\).

\[
F_a(z) = \sum_{n=1}^{2N} I_n z^n
\]

The complex variable \(z\) lies on the unit circle in the complex Argand plane.

The polynomial approach, first introduced by S.A. Schelkunoff (1943)\(^7\), has formed the foundation of array design until the beginning of this decade. One of the most noteworthy applications of the polynomial technique was due to C.L. Dolph (1946)\(^8\) who, by comparing the array polynomial with the Chebyscheff polynomial of the corresponding order, was able to design arrays with equal side lobes. The
significant aspect of Dolph-Chebyscheff array is that the relationship between the beam width and sidelobe level of such an array is optimal. However, for arrays with large number of elements, computation becomes extremely laborious. Approximate methods for computing the excitation coefficients of the Chebyscheff array were developed by van der Maas (1954)\textsuperscript{9} and R.J. Stegen (1953)\textsuperscript{10}.

For large arrays, the excitation can be approximated to that of a continuous line source. The problem of constructing a line source with an optimal compromise between beam width and side lobe level was ably solved by T.T. Taylor (1955)\textsuperscript{11}. Also known results in Fourier transforms can be used to design long arrays with low sidelobe level. For example, excitation approximating to a truncated Gaussian distribution was used in the celebrated Mills' cross at Sydney\textsuperscript{12}. Extremely low sidelobe levels are required in the arrays forming the cross.

1.3.1 Limitations of the Uniformly Spaced Array

Although the uniformly spaced array can, in practice, be designed to meet very stringent pattern requirement, it is not the most efficient array from
the engineering point of view. The inefficiency arises from the fact that many elements are required to be mismatched in order to provide a prescribed current distribution. Consequently the gain of the array is, in general, less than that of a uniformly excited array with the same number of elements. Sidelobe suppression is often achieved at the expense of increasing the width of the main beam.

As a result of the uniform element spacings, the space factor of the array will be a periodic function of the space variable \( u \). In order that no secondary grating lobe appears in the "visible" space, the inter-element spacing must be less than \( \lambda \). Practical arrays have inter-element spacings ranging from \( .5\lambda \) to \( .8\lambda \). The larger spacing can be used only when limited electrical steerability is required\(^{23}\).

The inefficiency of feed together with the restrictions on the element spacings have made the uniformly spaced array uneconomical in many applications in radio astronomy. These limitations are of little consequence in the fields of communication engineering where arrays with relatively broad beamwidths and few
elements are required. But long arrays are often used in radio astronomy in order to achieve adequate resolving power. Here the increase in feed efficiency and ability to accommodate larger inter-element spacings will considerably reduce the total cost of the array. Besides reducing the number of structural supports required, the decrease in the number of elements used in an array also reduces the number of matching and phase shifting networks associated with the feeds to the elements. Beam steering is simplified. Also, by increasing the inter-element spacing, the effect of mutual coupling between adjacent elements is reduced and a wider array bandwidth can be realised.

In an attempt to overcome the inherent limitations of the uniformly spaced array, recent investigators have attempted array synthesis using other parameters such as the element positions and the phase of excitation to each element. Both these parameters have until recent years been kept under some restraints in order to simplify analysis.

The advantage of using the position parameter $x_n$ and the phase parameter $\phi_n$ as variables in the
synthesis is that the excitation can be kept constant so that the array is working with maximum gain. When the array is synthesised by relaxing the progressive phase shift restraint, the resultant array is called the non-uniform progressive phase shift (N.U.P.P.S.) array. The array synthesised through judicious design of the positions of the elements is known as the non-uniformly spaced (N.U.S.) array.

1.4 N.U.P.P.S. Array

Very few published results are available, to date, on the synthesis and properties of the non-uniform progressive phase shift array. In his pioneering investigation, M.T. Ma\textsuperscript{13} has shown that such an array can provide higher directive gain than the Hansen-Woodyard discrete end-fire array\textsuperscript{14}. Also, it is possible to synthesise equal ripple sidelobe response without any amplitude tapering.

One main disadvantage of the N.U.P.P.S. array is the periodicity of the array factor due to the periodic element distribution. This imposes the same restriction to the permissible inter-element spacing as in the case of the uniformly spaced array. For this reason, the N.U.P.P.S.
arrays are not as popularly investigated as the N.U.S. array, which is discussed in the following section.

1.5 N.U.S. Array

By relaxing the uniform element spacing restraint, the periodicity in the space factor is removed so that inter-element spacings larger than $1\lambda$ are permitted without incurring large grating lobes in the 'visible' space.

Each element can be accurately matched to the transmission system to give a uniform excitation in the array. The level of sidelobes can be controlled to a limited extent by careful design of the element spacings. In general, it can be said that for a given pattern specification and providing that the synthesis technique used can generate such a pattern, less elements will be used in a N.U.S. array than in a uniformly spaced array.

Although the N.U.S. array has many desirable properties, the synthesis of such an array was not attempted until recent years. The reasons for this lack of interest are quite clear. Before the advancement of radio astronomy and space research, antenna arrays were used only for the purpose of communications where the economic advantages of
the N.U.S. arrays were small compared with the simplicity in design of the uniformly spaced arrays. The space factor of the uniformly spaced array, as was seen in section 1.3, is amenable to linear analysis so that certain types of patterns can be synthesised exactly. On the other hand, the analysis of the space factor of the N.U.S. array (equation 1.10) is highly non-linear.

\[ F(u) = \frac{1}{N} \sum_{n=1}^{N} \cos (kx_n) \]  \hspace{1cm} 1.10

The design problem involves the determination of a set of arbitrary antenna positions, \( x_n \), which will give a satisfactory array factor. In practice, there is a certain restraint to the degree of freedom of the element locations. This is imposed by the requirement that no two elements in the array should be spaced less than half a wave-length apart. (Appendix A)

The main design difficulty lies in the fact that \( x_n \) occurs in the argument of each cosine term in the array series. No purely analytic solution is at present possible and N.U.S. array are often synthesised through the use of modern high speed digital computers.
A number of papers have been published over the past seven years on the synthesis of N.U.S. arrays. A brief historical survey of the work done, to date, in this field is given in the following section.

1.6 Historical Survey of N.U.S. Array Synthesis

Unz (1960) was among the first to investigate the synthesis of N.U.S. array. By Jacobi expansion, each term in the array factor can be expressed as

\[ e^{jkx_n \sin \theta} = \sum_{m=-\infty}^{\infty} e^{jm\theta} J_m(kx_n) \]  \hspace{1cm} (1.11)

where \( J_m \) is the Bessel function of the first kind and order \( m \).

Using this expansion, Unz was able to express the array factor of the N.U.S. array as an infinite Fourier series in which each Fourier component can, in turn, be expressed in terms of a series of Bessel functions. The Fourier components of the prescribed pattern can be equated to the corresponding components in the series expansion and the problem reduces to a matrix form. Limited design application is possible, however, from this expansion because the position variables \( x_n \) occur, in the final form,
within the arguments of the Bessel functions.

Later, King et al. (1960)\textsuperscript{17} studied the space factors of a few arbitrarily selected sets of element positions. Four types of spacing schemes were used, viz.

(i) spacings derived from a set of prime numbers
(ii) from an arithmetic progression
(iii) from the elimination of multiples and
(iv) from a 'control cosine' technique.

King's arrays were designed with minimum spacing greater than 1\(\lambda\), in order to demonstrate the wide band properties of such arrays. The schemes adopted seemed to be aimed at avoiding any periodicity in the array spacings. Because no attempt was made to optimise the spacings, the patterns obtained from these schemes generally had fairly high sidelobe levels.

In the same year, S.S. Sandler (1960)\textsuperscript{18} presented an analysis in which he expanded each term in the space factor of the N.U.S. array into an infinite Fourier series of a certain fundamental spatial frequency \(\omega_1\). By such an expansion the N.U.S. array pattern can be approximated by an equivalent uniformly spaced (E.U.S.) array. Apart from providing an insight into the property of N.U.S. array,
Sandler's expansion had little design possibilities since it would be extremely difficult to derive the spacings for the N.U.S. array when the E.U.S. array pattern is given.

A method of designing array spacings to give low level side lobes near to the main beam was presented by R.F. Harrington (1961)\(^{19}\). Briefly, the aim was to compute the amount each element in a uniformly spaced array had to be perturbed so that big sidelobes near the main beam would be reduced. To do this, Harrington suggested the application of a series of 'smoothed out' impulses of the appropriate amplitudes and signs to the pattern at the locations at which improvements to the sidelobe level were required. Using this technique, it was then a simple matter to compute the required perturbations to the element positions. The assumptions made by Harrington were such that only close-in sidelobes can be suppressed by this method. Harrington's arrays can be used whenever arrays of small inter-element spacings are required. If an average inter-element spacing larger than \(1\lambda\) are required, then the directivity of each element in the array must be sufficient to suppress the far out sidelobes. This method was used by the author to generate starting arrays in the synthesis of N.U.S. array with directional elements. The details of Harrington's
perturbation technique are given in Appendix B.

A.L. Maffett (1962)\textsuperscript{20} and Y.T. Lo (1962)\textsuperscript{21} simultaneously proposed an interesting approach to the design of N.U.S. array. The object in their method was to obtain a set of element positions which would give a pattern approximating to that of a continuous source distribution.

Assuming a continuous symmetrical current distribution \( h(x) \) over a length \( L \), the space pattern \( g(u) \) is

\[
g(u) = A \int_{0}^{\frac{L}{2}} h(x) \cos(kux) \, dx
\]

where \( A \) is a constant to normalise \( g(u) \).

Defining the cumulative current distribution \( y(x) \) as

\[
y(x) = \int_{-\frac{L}{2}}^{x} h(x) \, dx
\]

and assuming that \( h(x) \) is normalised such that \( y(\frac{L}{2}) = 1 \), equation 1.12 becomes

\[
g(u) = A \int_{\frac{1}{2}}^{1} \cos(kux(y)) \, dy
\]

The application of the trapezoidal rule for numerical integration will result in an approximation to \( g(u) \) as

\[
G(u) = \frac{A}{N} \sum_{n=0}^{N} \varepsilon_n \cos(kux_n)
\]
where \( \epsilon_n \) equals 1 when \( n \) is 0 or \( N \), and equals 2 otherwise. It can be seen that equation 1.15 is in the same form as the space factor of a uniformly excited, N.U.S. array of 2N elements.

Since numerical approximation is used, it is to be expected that the approximation will deteriorate as the angle from the main beam increases. The pattern of the array computed from the approximation to a continuous line source with a quadratic excitation taper was presented by Maffett in his paper. This pattern has sidelobes increasing gradually from a low level near to the main beam to a higher level away from the main beam. This type of pattern is ideally suited for an array where directional elements are used. For this reason, the "quadrically non-uniform arrays" were used by the author as starting arrays for the synthesis work given in Chapter 3.

It is interesting to note, at this juncture, that the application of Simpson's rule and other forms of numerical integrations will result in arrays with non-uniform excitations as well as non-uniform element spacings. Of particular interest is Lo's application of the Legendre-Gaussian quadrature technique\textsuperscript{22} to give patterns with very close fit
at angles near to the main beam. To reduce equation 1.14 to a form suitable to the application of the Gaussian quadrature method of numerical integration, it is necessary to change the limits of integration to ±1 by changing the variable $y$. Equation 1.14 then reduces to

$$g(u) = A \int_{-1}^{1} \cos(kux(z)) \, dz$$

$$= A \sum_{i=1}^{N} H_i \cos(kux_i)$$

where $x_i$ is computed from the known $i$th root of the Legendre polynomial of degree $N$. $H_i$ can be computed in the same manner as the computation for the Fourier components in a Fourier series expansion, by using the quadrature properties of Legendre polynomials.

Due to the lack of a simple analytic solution to the synthesis of N.U.S. arrays, modern high speed digital computers are obviously important tools in the array synthesis. Improvement to a pattern by systematic adjustments of the element positions was used by M.G. Andreason (1962)$^{23}$. There are two limitations to this method. The first arises from the tacit assumption made in the synthesis process that when one element is being adjusted, the other elements are
located in the 'optimal' positions. Consequently, even though an array may not be improved further by the process, the array is, in general, far from being optimal. It is very likely that further improvement to the pattern can be achieved by some other optimisation process where such limitations do not exist. The effectiveness of such an optimisation process where the elements are adjusted one by one is, therefore, very dependent on the type of starting array used. Secondly, limited sizes of arrays can be synthesised by this method owing to the tremendous amount of computations needed for very large arrays. The computation time required to synthesise large arrays usually extends to many hours even on a high-speed digital computer. For the limited and uncertain improvement to the array pattern, this represents an uneconomic use of the computational facilities. Nevertheless, such a program was also developed by the author in the initial phase of his research programme in the synthesis of N.U.S. array of directional elements. This method was later superseded by a more efficient method to be discussed later in this section.

Another systematic method of computer synthesis of N.U.S. arrays was suggested by M.I. Skolnik et al (1964)\(^{24}\), where the technique of dynamic programming\(^{25}\) was used to
seek a set of 'optimal' positions for the elements in an array. The advantage of this process lies in the fact that no initial array is required since the array is gradually built up one at a time. In an array of N elements each of which can be located say in m possible positions, the total number of combinations the computer will have to consider in a 'brute force' type of analysis is $m^N$ whereas in dynamic programming this figure is reduced to approximately $(N-1)m^2$. Lower sidelobe characteristics are shown by the arrays synthesised by Skolnik, et al than that by M.G. Andreason. But further improvements can, in practice, be obtained using improved optimisation techniques.

An interesting transformation of the space factor of the N.U.S. array was proposed by A. Ishimaru (1962)\textsuperscript{26}. The basis of Ishimaru's approach lies in the definition of two variables;

(i) the source position variable $s$ and
(ii) the source number variable $v$.

The function of the source position against the source number $s(v)$, is known as the source position function, and the function of source number against source position, $v(s)$, is known as the source number function. The design problem
lies in the determination of either the source position function or the source number function from which the actual positions of the elements can be generated. Through the use of Poisson's sum formula\textsuperscript{27} Ishimaru was able to express the N.U.S. array factor in terms of an infinite series expansion as follows.

\begin{equation}
F(\theta) = \sum_{m=-\infty}^{\infty} F_m(\theta) \quad 1.17
\end{equation}

where

\begin{equation}
F_m(\theta) = \int_{0}^{\infty} I(s) \frac{dv(s)}{ds} e^{j2\pi mv(\theta)} e^{jk s \sin \theta} ds \quad 1.18
\end{equation}

and \( I(s) \) is the excitation function.

It may seem at first that the transformation into an infinite series will tend to complicate the array function. The main strength of this expansion however, lies in the fact that, at any one region in space, the space factor is primarily controlled by one term in the series. For example, the space factor of a uniformly spaced array with constant excitation can be expressed by Ishimaru's expansion as an infinite series of shifted \( \sin \pi x / x \) type functions. Thus in the study of the pattern in the region near to the main beam, only the term corresponding to \( m=0 \) need be considered. At the region of the first grating lobe only \( F_1(\theta) \) is important. In this way, the N.U.S. array factor can be considered to be composed of a series of grating plateaux where \( m \) represents the
order of the grating plateaux. A synthesis procedure aiming at suppressing the level of these grating plateaux was developed by Y.L. Chow (1965) who discovered that an array with exponentially increasing inter-element spacing has flat plateaux.

There is, of course, considerable interaction between adjacent terms in Ishimaru's series. To represent the pattern more exactly, a number of terms need to be considered and analysis becomes very complicated. This is the main limitation of Ishimaru's method of array synthesis.

A matrix method to perturb the element positions was suggested by Baklanov, et al (1962). The aim was to synthesise N.U.S. arrays with Dolph Chebyscheff type pattern in which sidelobes are of equal level throughout the visible space. In this method, the desired perturbations to the positions of the elements are related to the required change in the pattern by a series of quasi-linear first order differential equations, the solutions to which can be obtained by the standard method of matrix inversion using the digital computer. In order that a solution be possible, there can be only as many equations as there are antenna positions. As a result, Baklanov's arrays invariably contained as many sidelobes in the 'visible space' as there were antenna
positions to juxtapose. This requirement imposed a serious limitation on the permissible average inter-element spacing of the array. Many of the spacings in Baklanov's arrays are less than $\frac{1}{2}\lambda$, so that such arrays cannot be used in practice. A small improvement to this limitation was made by M.T. Ma (1963)\textsuperscript{30} who showed that an additional sidelobe could be introduced into the 'visible space' by re-defining the space variable $u$. Even then, the element spacings are still small and the array is not much more economical than a corresponding current tapered uniformly spaced array.

A modified procedure to Baklanov’s method of synthesis was adopted by the author\textsuperscript{31} whose aim was to synthesise arrays which are practically realisable yet having near to optimum sidelobe level. The arrays within the author’s field of interest have average inter-element spacings ranging from $1.5\lambda$ to $2.5\lambda$. From the experience of previous investigators, it is clear that the main difficulty encountered in the synthesis of N.U.S. arrays lies in the suppression of the sidelobes away from the main beam. This difficulty is reduced to some extent if directional elements are used. Of course, the synthesis will be further complicated by the additional weighting factor in the array pattern. Using the method adopted in this thesis, the sidelobe levels of the synthesised
arrays are, as a whole, only slightly higher than that in the optimal patterns where the sidelobes are assumed of equal amplitudes. The details of the synthesis procedure will be discussed in chapter 2.

Another approach towards the analysis of N.U.S. array is through the use of the methods of Fourier transform. Such an approach was pursued by Butler and Unz (1965)\textsuperscript{32}. Later, Meyer (1966)\textsuperscript{33} was able to derive the same infinite series expansion of the space factor of the N.U.S. array as Ishimaru by the use of the convolution theorem and the generalised sampling theorem in Fourier transform.

Finally, it must be mentioned that any form of optimisation process will become exceedingly difficult if the array contains a large number of elements. Under such circumstances, the statistical approach becomes an important tool to array synthesis. Among the many investigators along this line were Rabinowitz and Kolar\textsuperscript{34}; Skolnik, Sherman and Ogg\textsuperscript{35}; Y.T. Lo\textsuperscript{36} and Maher and Cheng\textsuperscript{37}. Although the statistical approach is a potentially useful approach in the synthesis of arrays for radio astronomy where a great number of elements are usually required, it is not an efficient method when arrays with relatively few elements are concerned.
1.7 Use of Directional Elements in N.U.S. Arrays

High directivity antennas are sometimes used in arrays whenever a higher gain is required for a given number of antennas in the array. For example, arrays of Yagi antennas\textsuperscript{38} and, later, of wide band log periodic antennas\textsuperscript{39} were used in Stanford University for the purpose of studying the ionised trails of meteors using radar technique.

In low frequency radio astronomy, however, the forward gain of the array is secondary in importance to resolution so that little is gained from using directional elements unless large inter-element spacings can be used in the array without incurring large grating lobes in the 'visible' space. In this respect, the use of directional elements together with the application of non-uniform spacing technique in the array synthesis will result in an array with high resolution and minimum number of antennas. Such an array is clearly much more economical than the conventional current tapered array of closely spaced low directivity dipoles. The basis for the above statements will be developed in this section.

A detail analysis on the limitations of filled aperture antennas was developed by J.D. Kraus (1958)\textsuperscript{40}, in
connection with the requirements of radio astronomy. In his paper, Kraus showed that a filled aperture is always resolution limited at frequencies below 100 Mhz. It is to overcome this basic limitation that the cross type radio telescope was developed by B.Y. Mills in order to secure a balance between the resolving capacity and the detection capacity of the antenna system. For reasons which will be discussed in part II of this thesis, an interferometric system is preferred to the cross in the initial phase of the radio astronomy programme of this university. To improve the inherent confusion limitation of this system, an attempt is made in the array design to secure as small a beam solid angle as possible for a given collecting area of the array. The extent to which this can be carried out is, of course, limited by the requirement that the pattern must maintain an acceptably low sidelobe level.

Assuming each element has an axially symmetric pattern of \( \frac{1}{\phi} \) power beam width \( \phi \), the gain of the element is given by

\[
G = \frac{4\pi}{\phi^2}
\]

The effective collecting area in the direction of the main beam is

\[
A_e = \eta M G A_0
\]
where \( n \) = the aperture efficiency.

\( M \) = the number of elements in the array.

\[ A_0 = \frac{\lambda^2}{4\pi}, \] the average collecting area of an antenna.

The \( \frac{1}{2} \) power beam width of an array with uniform excitation is approximately \( \lambda/L \), where \( L \) is the total length of the array. Thus the beam solid angle is given by

\[ \Omega = \frac{\lambda \phi}{L} \]

1.21

Substituting equation 1.19 in equation 1.21,

\[ \Omega = \frac{2\lambda}{L} \sqrt{\frac{\pi}{G}} \]

1.22

From equation 1.22 it can be seen that the beam solid angle can be reduced by increasing either \( L \) or \( G \). Since the collecting area is more than adequate at these frequencies, the result of using high gain elements and increased feed efficiency is that the number of antennas needed can be reduced significantly. In fact, the extent to which the number of elements can be removed is limited only by requirement that the pattern sidelobe level must be acceptably low. The degree of sidelobe suppression required is governed by the ratio of flux densities of the radio sources against which the system is expected to discriminate.
CHAPTER II

Synthesis of N.U.S. Arrays of Directional Elements

Although a number of papers have been written on the synthesis of N.U.S. arrays with isotropic elements, very little work has been done on the synthesis and properties of such arrays when directional elements are used. One of the main difficulties encountered by previous investigators is that although it is relatively easy to optimise the space factor over a small region in space, it is extremely difficult to control all the sidelobes over a large area in space. In general, it is easier to suppress sidelobes near to the main beam than sidelobes at large angles from the main beam. Thus one can reason that perhaps with the help of the element directivity to suppress the far off sidelobes it may then be easier to synthesise N.U.S. arrays with uniformly small sidelobes over the entire 'visible' space. This tendency is found to be true from the author's experience indicating that the non-uniform spacing technique seems ideally suited for applications in arrays of directional elements.

The author's interest is confined to the synthesis of arrays having not more than 50 elements. It is unlikely that arrays any longer than this will be required in the
initial phase of the Auckland University radio astronomy programme. With this limited number of elements, it is of course unwise to consider a statistical approach to the array synthesis. The theoretical approach of Ishimaru will be greatly complicated by the inclusion of the element pattern in the array series. The same objection rules out the use of the numerical integration approach of Lo and Maffett. In fact it will not be wise, because of the relatively small number of elements involved, to adopt any method in which the element positions are generated from a continuous function derived from some form of theoretical approximation. In general, the pattern of such an array can be improved upon by an optimisation process.

An optimisation approach is, therefore, adopted in this thesis. The method used in the earlier part of the research programme was to systematically adjust the element positions one at a time until maximum improvement to the array pattern was obtained. This method was similar to the method used by Andreason\textsuperscript{23} except that the element pattern was taken into account. Only limited improvement to the pattern could be achieved by this method and a great deal of patience and careful selection of the type of initial array were needed before an array with reasonably good pattern could be
synthesised. Later, a more efficient optimisation process was developed. In this method a modification of the procedure used by Baklanov was used to synthesise arrays with near optimal patterns. The main strength of the later method was that a number of sidelobes could be simultaneously reduced by simultaneous perturbations to the element positions. This process did not have the limitation of the earlier method where an assumption was made when one element was adjusted that the other elements were located in the optimal positions.

The function of this chapter is to discuss the computer programs developed to optimise the arrays. In order to relate the spacing variables to the pattern characteristics in a physically more meaningful way, it is best to express the spacing variables in terms of the wave-length at which the array is operating. Thus assuming uniform excitation and no progressive phase shift being introduced, the array pattern is given by

\[ F(u) = F_0(u) \cdot \frac{1}{N} \sum_{n=1}^{N} \cos(2\pi x_n u) \]  

2.1 Systematic Adjustment of Element Positions

A computer program was developed in the early part of the author's research program to optimise N,U,S., arrays by
systematically adjusting the element positions one at a time until maximum improvement to the array was achieved. Although this method was later superseded by a more efficient method in which the elements were simultaneously perturbed, a discussion of the synthesis procedure is given in this section since the 16 element experimental array of Yagi antennas, as described in chapter 4, was synthesised using this method.

To start the computation an initial array with the desired average inter-element spacing is used. Clearly, a suitable array will be one whose space factor has sidelobes low near to the main beam and gradually increasing as the angle from the main beam increases. Such patterns are exhibited by the arrays synthesised through Harrington's perturbation technique, or the numerical integration technique of Maffett and Lo. The initial array used to synthesise the 16 element experimental array was derived using Harrington's method, which is discussed in Appendix B.

Having established a suitable initial array, it is then necessary to specify a simple function which is a good approximation to the space pattern of the element used. As will be discussed in the next chapter, an important property of N.U.S. arrays is their capacity to operate over
a wide frequency range. Thus an element pattern approximating to that of a log periodic antenna was assumed in this exercise. An end fire type radiation pattern can best be approximated by a cosine^n type radiation pattern. Thus

\[ F_0(\theta) = \cos^n(\theta) \]  

It is found that a good approximation to the pattern of a typical log-periodic antenna is when \( n=2 \) in equation 2.2. To make the approximation more realistic at large values of \( \theta \), \( F_0(\theta) \) is arbitrarily assumed to be at a fixed level of \( 0.2 \) for \( \theta > 67^\circ \).

The directivity of this assumed pattern can be calculated from

\[ D = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi F_0(\theta, \phi) \sin \theta \, d\theta \, d\phi} \]  

Assuming axial symmetry, this reduces to

\[ D = \frac{4\pi}{\int_0^\pi F_0(\theta) \sin \theta \, d\theta} \]  

Carrying out this integration, the directivity of the assumed pattern is 8.7 db above isotropic. This is a realistic figure for a log periodic antenna, and a conservative estimate for a 5-element Yagi.
2.1.1 Synthesis Procedure

In the following discussion of the synthesis procedure, the names of all variables used are as found in the computer program named 'OPTIM' in appendix C. Thus the position variable \( x_n \) is expressed in the computer program as \( D(n) \).

Starting with a set of element positions for the initial array the computer first computes the maximum sidelobe level of this array and stores it as a value \( PK \). The computer then shifts the \( I \)th element by a distance \( \text{DELT}\), by adding to the value \( D(I) \) the incremental value \( \text{DELT} \). After computing the peak sidelobe level, \( \text{FMAX} \), of this slightly modified setting, the computer then compares the value \( \text{FMAX} \), with the value \( PK \). If \( \text{FMAX} \) is less than \( PK \), an improvement is achieved, so that computer proceeds to shift the \( I \)th element by another increment, \( \text{DELT} \). At the same time, the value \( PK \) is reduced and made equal to \( \text{FMAX} \) of the last setting. Again \( \text{FMAX} \) is computed for the new setting and its value compared with \( PK \). This process continues as long as the space pattern keeps improving.
But, should the value of FMAX be greater than \( PK \), indicating that the forward shift has deteriorated the pattern, the computer then puts the element in question back to its previous position by subtracting \( \Delta \) from \( D(I) \). Having done so, the computer continues to shift the element forward by a smaller increment \( \Delta/10 \). It will keep on doing so until the pattern begins to deteriorate on any further shift. When this happens, the element will be put back to its last best position and the computer will proceed to adjust the next element, the \((I + 1)\)th element, by the same process.

If, however, none of the forward shift, even on a small increment, improves the pattern, the computer will proceed to shift the element first in coarse and then fine steps in the reversed direction until no further improvement can be obtained before going on to the next element.

The complete process can best be expressed in the form of a flow chart as given in fig. 3. The arrows indicate the directions of flow in the operation. The contents in the rectangular blocks describe the operations conducted. The contents in the rounded
rectangles describe the input-output operations while the diamond brackets contain the information over which some decisions are made. The program will flow into one of three possible paths depending on whether the quantity inside the diamond bracket is positive, zero, or negative.

This program was earlier compiled in P.D.Q. for the IBM 1620 digital computer. Later, it was altered to work in the IBM 1130 computer. The determination of the maximum sidelobe level FMAX was done by a subroutine named PAT, the function of which is to scan through the pattern in the sidelobe region, and to store the level of the biggest sidelobe as FMAX. The interval in the u-space at which the pattern function has to be computed so as not to miss out any significant sidelobe or to incur significant error in the estimate of the sidelobe levels, depends primarily on the total length of the array. Since the total length of the array also determines the maximum spatial frequency content in the pattern, an analogy to the Shannon sampling theorem in information theory will show that the pattern must be sampled at intervals not larger than \( \Delta u = \lambda/2L \), where \( L \) is the total length of the array. From experience, it was found that computation at intervals \( \Delta u = \lambda/4L \) was adequate.
Fig. 3 Flow Chart for "OPTIM"
2.1.2 Computing Experience

The aim of the optimisation process 'OPTIM' is to reduce the peak sidelobe level so that the resultant array will have sidelobes which are uniformly low in the entire visible space. In the limit when all the sidelobes are of the same level, the pattern will then be considered optimal.

Of course, the sidelobe level of the array synthesised with 'OPTIM' is in general not optimal. But the significant point is that the resultant array is always an improved version of the initial array, or, if the initial array is already 'optimal' to this process, then the array will be unchanged by the process.

Because the elements are adjusted one at a time, only limited improvement is expected from this process. Thus the quality of the pattern of the resultant array is very dependant on the type of initial array used. A good starting array should not only have reasonably low sidelobe level to start with, but the element distribution should also be such that further improvement can be made by adjusting the element positions one at a time. A few arrays generated by theories given by
various authors were tried. Among the most successful
ones were arrays synthesised from Harrington's perturbation\(^{19}\)
method and Maffett's "quadratically non-uniform array"\(^{20}\).
(Appendix B).

For any one starting array, a fair amount of
patience was found to be necessary to get a significant
improvement in the pattern characteristics. Usually
the process converged after two cycles so that little
improvement was achieved by further iterations. When
this occurred, it was often found helpful to disturb
the element positions slightly before the start of the
next cycle. These small disturbances were either
randomly assigned or were brought about by multiplying
the element positions through by a factor very near
unity.

The difficulties encountered with 'OPTIM'
had led the author to seek a more effective optimisation
scheme which would not have the limitations of the
process where the elements are adjusted one at a time.
The desired process must be capable of perturbing
the element positions simultaneously and to suppress
any sidelobe in the visible space. Such a process was
developed and this is discussed in the next section.
2.2 Perturbation using Matrix Technique

Basically, a perturbational approach to N.U.S. array synthesis involves the determination of a set of small perturbations to the existing element positions in order to improve the array pattern at certain points in space. Such a technique was developed by Harrington\textsuperscript{19} but his method is only applicable when the angle from the main beam is small. A more versatile approach was developed by Baklanov\textsuperscript{29} who showed that the desired element perturbations can be obtained by solving a set of quasi-linear differential equations. With this method, Baklanov was able to synthesise Chebyscheff type patterns where the sidelobes were of equal level over a limited region in space. However, Baklanov's arrays have little practical applications because they invariably have small inter-element spacings, some less than $\lambda/2$. This arises because of the inherent limitations of the method that only as many sidelobes as there are antenna positions can be optimised.

The arrays subjected to investigations in this thesis contain spacings averaging at $1.5\lambda$ to $2\lambda$. Such arrays will have more than twice the number of sidelobes as there are element positions to optimise. To overcome the analytic difficulties, the synthesis technique adopted is aimed not
at producing a Chebycheff type pattern, but to reduce a
given number of the bigger sidelobes in the pattern to a
prescribed level. The number of sidelobes that can be
simultaneously reduced at any one time is equal to the number
of element positions available for perturbation. The directions
and amplitudes of all sidelobes in the pattern are computed in
each cycle so that a fresh set of biggest sidelobes are
selected to be reduced at each cycle. Since a large amount
of computations are required to compute accurately the
locations of all the sidelobes in a long array, this limits
the maximum sizes of arrays that can be synthesised by this
method.

2.2.1 Theory of the Optimisation Process

Assuming either zero or progressive phase
shifts are permitted in the transmission system, the space
pattern of an arbitrary array can be expressed as a function
of three sets of variables, viz. \( u, I_n, \) and \( x_n \).
Thus

\[
F(u, I_n, x_n) = F_0(u), \quad \sum_{n=1}^{N} I_n \cos(2\pi x_n u)
\]

The change in the pattern function at a direction
\( u = u_1 \), due to small perturbations \( dx_n \) to the position

\[
\]
of each element and \( dI_n \) to the excitation to each element is given by

\[
dF_{u=u_i} = \frac{\partial F}{\partial u_i} \bigg|_{u=u_i} du_i + \sum_{n=1}^{N} \frac{\partial F}{\partial x_n} \bigg|_{u=u_i} dI_n + \sum_{n=1}^{N} \frac{\partial F}{\partial x_n} \bigg|_{u=u_i} dx_n \tag{2.6}
\]

If \( u_i \) corresponds to the point at which a secondary maxima occurs, \( \frac{\partial F}{\partial u_i} = 0 \). Also, only arrays with uniform excitation will be considered in this thesis as this represents the most efficient use of non-uniform spacing technique. Thus \( dI_n = 0 \) for all values of \( n \).

From the above assumption, equation 2.6 reduces to

\[
dF(u_i) = \sum_{n=1}^{N} \frac{\partial F}{\partial x_n} \bigg|_{u=u_i} dx_n \tag{2.7}
\]

Before computation is possible, it is necessary to express the element pattern in terms of a simple function the derivative of which must also be simple. A modified cosine-squared pattern is assumed in all arrays synthesised by this method. The function used is

\[
F_0(\theta) = \frac{0.2 + \cos^2 \theta}{1.2} \tag{2.8}
\]
The constants are arbitrarily assigned to make the pattern more realistic at values of $\phi$ nearing $90^\circ$. Expressed in terms of the space variable $u$,

$$F_0(u) = \frac{1.2 - u^2}{1.2} \quad 2.9$$

With this assumption, the normalised radiation pattern of the array is

$$F(u) = \frac{1.2 - u^2}{1.2N} \sum_{n=1}^{N} \cos(2\pi x_n u) \quad 2.10$$

Thus

$$\left. \frac{\partial F}{\partial x_n} \right|_{u=u_i} = \frac{2\pi u_i (1.2-u^2)}{1.2N} \sum_{n=1}^{N} \cos(2\pi x_n u_i) \quad 2.11$$

If $N$ values of $u_i$, $i = 1, N$ corresponding to the locations of $N$ biggest sidelobes in the pattern are known, equation 2.7 becomes a set of $N$ simultaneous equations which are best expressed in matrix form as

$$\begin{bmatrix}
    dF(u_1) \\
    \vdots \\
    dF(u_N)
\end{bmatrix} =
\begin{bmatrix}
    \frac{\partial F(u_1)}{\partial x_1} & \cdots & \frac{\partial F(u_1)}{\partial x_N} \\
    \vdots & \ddots & \vdots \\
    \frac{\partial F(u_N)}{\partial x_1} & \cdots & \frac{\partial F(u_N)}{\partial x_N}
\end{bmatrix}
\begin{bmatrix}
    dx_1 \\
    \vdots \\
    dx_N
\end{bmatrix} \quad 2.12$$
Fig. 4 The Idealised Element Pattern
The elements in the column matrix on the L.H.S. of equation 2.12 represent the desired corrections to the pattern at N points in space at which the N biggest sidelobes are located. Each element in the square matrix can be computed from equation 2.11. By the application of standard matrix inversion technique using the digital computer, the required perturbation to each element in the array can then be computed.

2.2.2 Computing Procedure

To start off the computation, an initial array is required. The array used by the author for this purpose is Maffett's "quadratically non-uniform array". Apart from a possible increase in the total computation time required, the wrong choice of an initial array will not affect the extent to which the sidelobes can be suppressed in the synthesised array. This is one feature which makes the present method superior to the method discussed in the last section in which the element positions are optimised one at a time.

The aim of the computation is to bring the N biggest sidelobes to as low a level as possible while at
the same time, keeping the total length of the array above a certain limit so that the required beam width can be obtained. One method of fixing the array length is to keep the position of the Nth element constant. This leaves N - 1 element positions for optimisation, and as a result only N - 1 lobes can be simultaneously reduced at any one cycle. In practice, it was not found necessary to fix the position of the Nth element. From experience, the array length tends to remain fairly constant until the sidelobes have been reduced to near optimal level for the given length after which any further reduction in the sidelobe level will be accompanied by a gradual shortening of the total length of the array. Thus no attempt is made in the computations to fix the position of the end element. If an array with a given sidelobe level is required, an initial array longer than the length of the theoretically 'optimal' array (section 3.4) is used so that by the time the sidelobes are reduced to the desired level, the length of the array will be near to optimal. Conversely, an array of a given length and near to optimal sidelobe level can be synthesised by using an initial array which is longer than the desired length. By the time the array length has been reduced to the required length,
the sidelobe level of the array will be near optimal.

Once the positions of the elements of the initial array are defined the next step is to compute the locations and the levels of all sidelobes of the array and to choose \( N \) of the biggest sidelobes in the pattern. This is done by a sub-program called DETPK which will be discussed in the next section.

Having computed the positions \( u_i \) and the amplitudes \( F(u_i) \) of the \( N \) biggest lobes, it is possible to compute \( dF(u_i) \) for \( i = 1, \ldots, N \) by computing the differences between the levels of these peaks and the level, \( \text{PMEAN} \), to which these peaks are desired to be reduced. The next step is to compute each term in the \( N \times N \) matrix from equation 2.11 by the appropriate specifications of the suffixes \( i \) and \( n \). By a subroutine named INVRT, this matrix is then inverted and the reciprocal matrix multiplied by the column vector \( \{dF(u_i)\} \) to give, from equation 2.12, the column vector \( \{dx\}_n \) the elements of which represent the perturbations required to be applied to the elements in order to achieve the desired corrections to the pattern.
It is necessary to limit the values of $dx_n$ in order to satisfy the assumption made in this process, that $dx_n$ is small. This is easily done by multiplying the column vector $[dx_n]$ through by a scaling factor, if necessary, such that the maximum value of $dx_n$ is $\leq 2$, a value which is arbitrarily prescribed and is found to be acceptable. From equation 2.12, it can be seen that the effect of multiplying the column vector $[dx_n]$ by a scaling factor is to scale down the vector $[dF(u_i)]$ by the same amount. This means that because small perturbations are only permitted in the element positions, only limited amount of corrections to the pattern can be applied at each cycle. Substantial improvement to the pattern of the starting can only be obtained by iteration of the above procedure. To do so, the scaled perturbations are added to the corresponding element positions and the directions and amplitudes of the new set of $N$ biggest sidelobes are computed by the use of the subroutine DETPK. Thus the whole cycle can be repeated until all the $N$ biggest sidelobes are reduced to the prescribed level.

The advantages of the above optimisation procedure are as follows:-
(i) The computations do not diverge. This is because the locations and levels of all sidelobes are computed at each cycle so that the values of \( u_i \) and \( F(u_i) \) are accurately defined at any stage of the computation.

(ii) Apart from affecting the total computation time required, the pattern characteristics of the synthesised arrays are independent of the type of initial array used. This is so because the element positions are simultaneously perturbed.

(iii) Sidelobes at any angle to the main beam can be reduced with equal efficiency. Hence arrays with average inter-element spacings considerably larger than \( 1 \lambda \) can be synthesised. Besides being practically realisable such arrays are usually very economical and are capable of operating over wider frequency bands.

(iv) The patterns of the synthesised arrays are, in general, close to 'optimal'. (The word 'optimal' is used here in the same sense as used in the Dolph-Chebyscheff arrays.) Thus for a given
length of the array the sidelobe level is
near optimal, and for a given sidelobe level, the
length hence the beam width of the array is near
optimal.

2.2.3 Peak Determination Sub-program

A sub-program named DETPK has been developed
to compute the locations and amplitudes of all sidelobes
in the visible space and to select the biggest sidelobes
for the main program TXOPT. From experience, it is
found necessary to locate the values of \( u_i \) to at least
the third decimal place otherwise, the assumption of
\[
\frac{\partial F}{\partial u} = 0 \quad u=u_i
\]

The method used involved the determination of
the zeros of the \( u \)-derivative of the pattern function.
A half-interval search technique is used to compute
the zero cross over points of the derived function,
which is

\[
\frac{\partial F}{\partial u} = -\frac{2}{1.2N} \sum_{n=1}^{N} \cos(2\pi x_n u) - \frac{1.2-u^2}{1.2N} \cdot 2\pi \sum_{n=1}^{N} x_n \sin(2\pi x_n u) \quad 2.13
\]

Thus the roots of equation 2.13 can be found by solving
for
\[ 2u \sum_{n=1}^{N} \cos(2\pi x_n u) + 2\pi(1,2-u^2) \sum_{n=1}^{N} x_n \sin(2\pi x_n u) = 0 \] 2.14

The function on the LHS of equation 2.14 is given the symbol SF in the program given in Appendix C.

The sub-program DETPK performs three functions. The first involves the determination of a set of adjacent values of \( u \), viz. UA(M) and UB(M), between which the Mth root of SF occurs. This is done by continually incrementing the value of \( u \) by a small increment DU and computing at each step the value SF. The signs of the function SF at adjacent values of \( u \) are compared as \( u \) is being incremented. If there is a change of sign, then a root of SF occurs between the two adjacent \( u \) values. These values of \( u \) are stored respectively as UA(M) and UB(M), where M signifies the number of the sidelobe which lies between them. Thus the maximum value for M equals the total number of sidelobes that occur in the pattern. Of course, the increment DU must be sufficiently small so that there can be only one sidelobe between UA(M) and UB(M). Again from Shannon's sampling theorem \(^{42}\), the increment should not be greater than half the period of the highest
spatial frequency component of the array pattern. Accordingly $DU$ must be less than $\lambda/2L$. In practice since $DU$ only defines a region over which a root occurs, it is sufficient to assume $DU$ to be equal to $\lambda/2L$.

Having computed a series of intervals, each of width $DU$, within which the zeros of the function $SF$ occur, the next step is then to find the exact locations of the roots within these intervals. A half-interval search method\textsuperscript{44} is used. Briefly, the search is as follows.

It is known that the $M$th root lies between $UA(M)$ and $UB(M)$. Let the values of $SF$ at these two points be respectively $SF1$ and $SF2$. At mid-way within this interval, $u=(UA(M) + UB(M))/2$, the value of $SF$ is computed. If the product $SF \times SF1$ is negative, the root lies between $UA$ and $u$. If the product is zero, the root occurs at $u$, while if the product is positive the root lies between $u$ and $UB$. By this means the $u$-interval within which the root occurs can be rapidly narrowed. The root will be considered as sufficiently accurately located if either $SF \leq .001$ or $UB - UA \leq .000005$. These limits are arbitrarily prescribed and were found satisfactory
from computing experience. The second limit is necessary because of the limited accuracy of the IBM 1130 digital computer. By incrementing the value of $M$, the computer can be made to compute by the above process, the locations of all sidelobes in the pattern.

Having found the locations of all the maxima in the pattern, it is then a simple matter to determine the values of the pattern function at these points. $N$ biggest peaks are selected and their locations and amplitudes are stored to be used by the main optimisation program TXOPT.

A flow chart for DETPK is given in fig. 5.

2.2.4 Matrix Inversion

The standard method of matrix inversion using the Gauss-Jordan elimination method with normalisation and the interchange of rows, is adopted in the optimisation programme. The sub-programme INVRT used is a slight modification of the programme called 'CHAP8' in the book "Numerical Methods and Computers" by S.S. Kuo. The programme is given in Appendix C. Detail discussion on the formulation of this programme can be found in
the reference book. Besides inverting matrices, the programme INVRT also provides the complete solution to the given set of simultaneous equations.
Fig. 5 Flow Chart for "DETIPK"
CHAPTER III

The Pattern Characteristics of Synthesised N.U.S. Arrays of Directional Elements.

A number of arrays of varying sizes were synthesised in the course of the author's research programme using the matrix perturbation technique as described in the previous chapter. This chapter presents the results of the computations and the analysis of the pattern characteristics of the synthesised arrays.

In the earlier sections of this chapter, a number of parameters viz. the beam-width the sidelobe level, the gain and the bandwidth of an array are discussed. These parameters serve to describe the pattern characteristics of the array, and can therefore be called the pattern parameters. The aim of the study is to attempt to find some correlations between the pattern parameters and the spacing parameters such as the average inter-element spacing, the minimum spacing and the 'degree' of non-uniformity in the element spacings. Since the pattern function bears a complicated relationship, mathematically speaking, to the position variables, it is to be expected that there will not be a simple and unique relationship between the spacing parameters and the pattern
parameters. The problem can only be studied in an empirical manner.

In practice some interactions are found to exist among the various array parameters. A stringent requirement in one can be met only at the expense of less favourable values for the other parameters. For example, it is found that a fair degree of non-uniformity in the element spacings is necessary in order to increase the effectiveness of the sidelobe suppression, but this will, in turn, decrease the bandwidth of the array. Very low sidelobe level can be achieved only at the expense of deteriorating both the bandwidth and beam-width of the array. Clearly, the most economical array is one which strikes the best compromise among the various pattern parameters. A few of the arrays presented in this chapter do exhibit this compromising characteristics. These arrays are ideally suited to a wide range of applications where good performance at minimum cost are required.

3.1 Beam-width

One of the main functions of an antenna array is to produce a narrow beam response. In radio astronomy, the beam-width defines the resolving power of the array, so
long as the sidelobe level is sufficiently low for the array not to be confusion limited. The beam-width of an array is defined throughout this thesis as the angle between the half power points in the main beam.

In a uniformly spaced array, assuming no mutual coupling between elements, the beam-width of the array depends only on the total length of the array and the current distribution in the array. Thus a convenient expression for the beam-width is given by

\[ \Delta_0 \text{h.p.} = c \cdot \frac{\lambda}{L} \text{ degrees} \] 3.1

where \( c \) is a constant for the particular array. The value for this constant depends on the type of distribution function used in the feed system. Equation 3.1 is also used in the study of the synthesised arrays described in this chapter. Since all arrays considered in this chapter have assumed uniform excitation, the value of the factor \( c \) will depend on the non-uniformity in the element spacing. By comparing the values of the factor \( c \) of a current tapered array with that of a N.U.S. array, a study can be made to see if space tapering in a N.U.S. array has a similar effect on the beam-width of the array as current tapering in a conventional uniformly spaced array. An interesting conclusion is arrived at by this comparison
and this will be discussed in section 3.7

In the case of a continuous line source with uniform distribution, \( c = 51 \). Thus

\[
\Delta \theta_{u.d} = 51 \cdot \frac{\lambda}{L} \text{ degrees}
\]

where \( \Delta \theta_{u.d} \) is the half power beam-width of the line source with uniform excitation.

It is convenient for the sake of comparison to define a parameter called the beam factor as

\[
B.F. = \frac{\Delta \theta_{h.i.d.}}{\Delta \theta_{u.d.}}
\]

Briefly, the beam factor is defined as the ratio between the half power beam-width of the array in question to the half power beam-width of an equally long line source with uniform distribution.

In general the beam factor is greater than unity in a current tapered array of discrete elements and the heavier the current tapering along the array, the larger will be the value for the beam factor. In the case of a uniformly spaced array with uniform excitations, the beam factor is equal to 1.

It will be interesting for the sake of comparison, to study
how the beam factor varies for a series of Dolph-Chebyscheff arrays designed for different values of sidelobe level \( r \).

The following table is compiled from Stegen's\(^{10}\) approximation to the factor \( c \) (eqn. 3.1) for Dolph-Chebycheff arrays longer than 5 wavelengths.

<table>
<thead>
<tr>
<th>( r ), db</th>
<th>( c )</th>
<th>B.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>51.1</td>
<td>1.00</td>
</tr>
<tr>
<td>-25</td>
<td>56.0</td>
<td>1.10</td>
</tr>
<tr>
<td>-30</td>
<td>60.6</td>
<td>1.21</td>
</tr>
<tr>
<td>-35</td>
<td>65.0</td>
<td>1.27</td>
</tr>
<tr>
<td>-40</td>
<td>68.7</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Table 1. Beam factor for long Dolph-Chebycheff arrays

It is interesting to note from Table 1 that the beam factor of the Dolph-Chebycheff array is very near unity for sidelobe level above -20 db. This point is often not fully realised by a few investigators who contend that sidelobe suppression using weighted excitation is achieved only at the expense of widening the width of the main beam. The reason for the near to unity beam factor in the above array lies in the fact that current distribution in a Dolph-Chebyscheff
array is not necessarily a uniform taper towards the end elements. In fact, at sidelobe levels higher than −20db, the currents in the end elements are often higher than that of the central elements.\textsuperscript{8,11}

Another point often brought up by some authors is that the beam-width of a N.U.S. array with uniform excitation is approximately that of a uniformly excited array, so that sidelobe suppression can be achieved without increasing the array beam-width. This is found to be not strictly true from the study made in this chapter. In fact, the B.F. is found, in general, to be greater than unity for N.U.S. arrays having average inter-element spacings larger than 1 wavelength. This point will be developed further when the results of the computations are analysed later in this chapter.

3.2 Sidelobe Level

To prevent confusion in the record, the response of the array to radiations arriving at angles outside the main beam must be well below the main beam response. The extent to which the response in the sidelobe region has to be suppressed will of course depend on the configuration in which the array is used and the ratio of the intensities of the radio sources against which the system is expected to discriminate.
In a uniformly spaced array, there is no limit to which the sidelobe level of an array can theoretically be suppressed. For example, an array with a binomial distribution has no secondary lobe at all. But the extremely large difference between the excitations to the central elements and that to the end elements makes such an array impossible to realise in practice. However, by using appropriate current tapering; for example, the use of a truncated Gaussian distribution, it is possible to design practical arrays with very low sidelobe level. In such arrays, the sidelobe suppression is achieved at the expense of widening the width of the main beam. These arrays invariably have high values of beam factor. A compromise between beam-width and sidelobe level is shown by arrays with Chebyscheff type response pattern. For these patterns, it is possible to estimate the minimum attainable sidelobe level from the given pattern gain and the required beam-width of the array. (Section 3.4) Such arrays will be considered as optimal in this thesis.

3.3 Gain and Gain Limitation

The directive gain of an array is defined as the ratio of the maximum intensity of radiation to the average intensity of radiation of the array. By the reciprocity principle, the gain of an array is the same both in transmission
and reception\(^1\). Thus

\[
G = \frac{4\pi}{\int_{0}^{2\pi} \int_{0}^{\pi} F^2(\theta, \phi) \sin \theta \, d\theta \, d\phi}
\]

In a linear array, it is convenient when computing the array gain, to assume the array pattern to be isotropic in the plane normal to the line of the array. The gain computed from this assumption will be called the pattern gain which from equation 3.4 is

\[
G = \frac{2}{\int_{-\pi/2}^{\pi/2} F^2(0) \cos \theta \, d\theta}
\]

where \(\theta\) is the angle from the main beam as shown in fig. 1.

Because the pattern is assumed symmetrical about the line of the array, equation 3.5 reduces to

\[
G = \frac{1}{\int_{-\pi/2}^{\pi/2} F^2(0) \cos \theta \, d\theta}
\]

Substituting in equation 3.6 the space variable \(u = \sin \theta\), the pattern gain becomes
\[ G = \int_{0}^{1} \frac{1}{F^2(u)} \, du \quad (3.7) \]

Thus the pattern gain can simply be computed by numerical integration of the power pattern in \( u \)-space over the range \( u = 0 \) to \( u = 1 \). In practice, there is a limit to the gain obtainable from an array of a given length. This limitation arises because elements cannot be placed too near to one another otherwise the prescribed excitations cannot be easily achieved because of the mutual coupling between the elements.

In theory, however, there is no limit to the gain of an array if small spacings and constant phase reversals between adjacent elements are permissible. Such arrays are known as super-gain arrays \(^{45,46}\) and are characterised by their extremely high array \( Q \)-factors. As a result, extremely large currents have to be fed to the elements. Most of these currents serve to cancel one another and only a small fraction of these currents contribute to the radiation of the array. Thus such arrays invariably have extremely small operative bandwidth and extremely low feed impedance which in turn results in extremely high ohmic loss. Obviously, these arrays are not possible to realise in practice.
A good estimate of the maximum gain of an array can be made by analogy with the limiting gain of a broad-side array with half-wave spacing and uniform excitations, when the array is assumed to have a large number of elements. Assuming the array to have $M$ elements, the array pattern is

$$F(u) = \frac{\sin \left( \frac{\pi}{2} Mu \right)}{M \sin \left( \frac{\pi}{2} u \right)} \quad 3.8$$

In a long array with large number of elements, most of the radiation occurs within an angle in which $u$ is small. With this assumption,

$$F(u) = \frac{\sin \left( \frac{\pi}{2} Mu \right)}{\frac{\pi}{2} Mu} \quad 3.9$$

Substituting equation 3.9 into equation 3.7, the array gain is given by

$$G = \frac{M\pi/2}{\int_{0}^{M\pi/2} (\sin \chi / \chi)^2 \, d\chi} \quad 3.10$$

where $\chi = \frac{\pi}{2} Mu$. In the limit when $M$ can be considered infinitely large, the integral in the denominator of equation 3.10 equals $\pi/2$ and
\[ G = M = \frac{2L}{\lambda} \]  

3.11

The same limiting relationship \( G = \frac{2L}{\lambda} \) is also obtained when a line source of uniform distribution is considered.

It follows from equation 3.11 that if each element is assumed to have a gain \( G_0 \) then the array gain is \( M \cdot G_0 \).

Although this is a reasonably good estimate of the gain of an arbitrary array of uniformly fed discrete elements, it does not represent the upper limit for the gain of the array. The reason is obvious, since the estimate is based on an analogy with an infinitely long broadside array of half-wave spaced elements. To illustrate the effect of using a finite number of elements and inter-element spacings other than half-wave a graph showing the variation of pattern gain with spacings 47 for a 16 element uniformly spaced array with uniform excitation is shown in fig. 6.
Fig. 6  Gain Variation in a 16-element Uniformly
    Dipole
    Spaced Broadside Array

From fig. 6 it can be seen that the gain of the 16 element
array at half-wave spacing is \( \frac{1}{6} \) as compared with 26 from the
estimate. This error will of course decrease as the number of
elements increases.

The gain of every N, U, S. array synthesised in this
chapter is computed by numerical integration of the power
pattern of the array and compared with the estimated gain $G = MG_0$, and in general, a close agreement is found. This point will be discussed in detail in section 3.8 when the results of the computations are examined.

3.4 Gain-Sidelobe level Relationship

It is important, for the sake of comparison, to be able to estimate the minimum sidelobe level that can be achieved in an array of a given length and a fixed number of elements. To do this, one must first specify the type of pattern which is assumed to be optimal. For applications in radio astronomy, it is desirable that the array has uniformly low response in the sidelobe region. Thus the pattern that is assumed to be optimal in this thesis is one which oscillates rapidly with equal amplitude in the sidelobe region. Although the exact shape of the pattern is of little consequence, it is reasonable to assume that the sidelobe level in the power pattern is equal to twice the average level in the sidelobe region. (fig. 7). With this assumption, a relationship between the sidelobe level and gain of the idealised pattern can be obtained.
In a long array of widely spaced discrete elements, it can be shown that the power radiated in the main beam is small compared with the total power radiated in the sidelobe region. Thus if the array gain is $G$, it is clear from equation 3.7 that the average sidelobe level of the array is $1/G$, if the main beam radiation is neglected. Following
the assumption of an equal ripple pattern in the sidelobe region, the sidelobe level of the optimal array will be $2/G$. As a result, the sidelobe suppression is, in db,

$$E = -10 \log (G/2)$$  \hspace{1cm} 3.12

In this thesis, however, the arrays considered are all not sufficiently long for the main beam power to be entirely neglected. A very simple and effective method of taking the main beam power into consideration was proposed by M.G. Andreason\textsuperscript{23}. Since Andreason's formulation has assumed the use of isotropic elements, a slight modification is necessary in order to extend his method to arrays with non-isotropic elements.

Assuming each element in the array is equally fed with a power $P$, the power per steradian radiated in the principal direction is

$$S = \frac{M \cdot G \cdot P}{4\pi}$$  \hspace{1cm} 3.13

where $G$ is the pattern gain and $M$ the total number of elements in the array. It must be noted that the pattern gain is computed with the assumption that the array pattern is isotropic in the plane normal to the line of the array. This is necessary because the directivity of the element in the
plane normal to the line of the array does not contribute to
the sidelobe suppression in the array pattern.

With the above assumption, the power radiated in
the main beam is

\[ P_m = 2\pi \Delta \theta_{h.p.} S \]

\[ = \frac{(\text{MP} \Delta \theta_{h.p.})}{2} \]  \[3.14\]

The final expression in equation 3.14 is derived by substituting
for \( S \) from equation 3.13.

The power radiated in the sidelobe region can then
be computed from the difference between the total power radiated
and the main beam power. Thus

\[ P_{s.l.} = \text{MP} - P_m \]

\[ = \text{MP}(1 - G \Delta \theta_{h.p.}/2) \]  \[3.15\]

If the sidelobe power is assumed uniformly distributed in space,
the power density in the sidelobe region is approximately
\( P_{s.l.}/4\pi \). The ratio, \( R \), of the power per steradian in the
direction of the main beam to the power per steradian in the
sidelobe region is
\[ R = \frac{4\pi S/P_s l}{1 - G\Delta \theta_{h,p}^2} \]

From the assumption of equal ripple response in the sidelobe region, the peak sidelobe intensity will be \( \frac{R}{2} \) times below the main beam intensity. Expressing in decibels, the maximum sidelobe suppression is

\[ E = -10 \log \left( \frac{G}{2} \right) - 10 \log \left[ \frac{1}{1 - (G/2)\Delta \theta_{h,p}} \right] \]

It is interesting to note that equation 3.17 differs from 3.12 by only an extra term which takes into account the main beam radiation. A good estimate for the half power beamwidth of the array is \( \Delta \theta_{h,p} = \lambda/L \), i.e., assuming \( c = 1 \) in equation 3.11. With this assumption, equation 3.17 reduces to

\[ E = -10 \log \left( \frac{G}{2} \right) - 10 \log \left[ \frac{1}{1 - \lambda G/(2L)} \right] \]

Equation 3.18 is used to estimate the maximum sidelobe suppression that can be achieved in an array of a given length \( L \) and pattern gain \( G \). The effectiveness of the synthesis process adopted can be studied by comparing the sidelobe level actually achieved with the level computed.
from equation 3.18.

A good estimate for the pattern gain $G$ is $G = MG_O$, where $G_O$ is the gain of each element whose pattern is assumed isotropic in the plane normal to the line of the array. (see section 3.3). Thus each element is assumed to have a doughnut type pattern. To simplify computation for $G_O$, the spherical coordinate system is so chosen that the pattern is constant w.r.t. $\phi$ (fig. 8).

Fig. 8 The Element Pattern Used to Compute $G_O$

$\left(F_O(\phi) = \text{constant}\right)$
The assumed element pattern in the orientation shown in fig. 8 is from equation 2.8,

\[ F_0(0) = \frac{1.2 - \cos^2 \theta}{1.2} \]

The gain \( G_0 \) computed from equations 3.19 and 3.4 is 1.72. Thus the estimated pattern gain for an array of \( M \) elements is 1.72. This is found to compare well with the pattern gain computed by pattern integration on the basis of equation 3.7.

3.5 Bandwidth and Steerability

One of the main limitations of a uniformly spaced array is its relatively small operative bandwidth. This arises from the fact that the inter-element spacings in such an array cannot exceed \( 1\lambda \) without incurring large grating lobes in the visible space. Thus the total number of elements needed in an array of a given length is largely dependent on the maximum permissible inter-element spacing. Since the elements have to be closely spaced in a uniformly spaced array, low directivity elements like simple dipoles are often used in the array. The bandwidth of such an array will be limited further if electrical steerability is required of
the array. This is illustrated simply by fig. 9a and 9b. It is easy to deduce from these figures, that to give near to 90° steerability, an array with half-wave spacings is needed which means that the array is operative only at a single frequency.

![Diagram](image)

**Fig. 9a** Broadside Pattern of a Uniformly Spaced Array. (Inter-element spacing close to $\lambda$).

**Fig. 9b** Big Grating Lobe Appears in the "Visible" Space if the Beam is Steered.
Limited steerability can be achieved without significant decrease in the operative bandwidth if directional elements with good front to back ratio and low sidelobes are used. This assumes of course, that the elements are mounted fully tiltable so that the forward direction of each element can be made to be aligned with the direction of maximum radiation of the array. (fig. 10a and 10b).

Fig. 10 (a) and (b) Beam Steering in N.U.S. Array. (--- Element Pattern)
The bandwidth of the array can be increased significantly if the elements are non-uniformly spaced. This is because large average element spacings can be used without being troubled by large grating lobes. The bandwidth of an array is defined throughout this thesis as the ratio of the frequency at which the array pattern has the designed sidelobe level to the frequency at which the minimum inter-element spacing in the array is half a wavelength. This definition is not strictly correct because of the increase in mutual coupling between elements with increased element directivity. (Appendix A). For lack of more exact information to date, in this field, the definition is assumed so as to provide a common frame of reference in the study of the pattern characteristics of the synthesised arrays.

Thus

$$\text{B.W.} = 2 \Delta x_{\text{min}}$$

3.20

where $\Delta x_{\text{min}}$ is the minimum inter-element spacing of the array expressed in terms of the wavelength at the design frequency.

To increase the bandwidth of the array, it is necessary to increase $\Delta x_{\text{min}}$ which means that the average inter-element spacing has to be increased to provide a fair degree of non-uniformity in the array which is needed for effective suppression of sidelobes. The result is that the array synthesised will
have a higher sidelobe level. This tendency is clearly shown from the patterns of the N.U.S. arrays synthesised by the author.

3.6 Synthesised Arrays

A number of arrays of varying sizes have been synthesised in the course of this research programme using the matrix perturbation technique as described in the previous chapter. Uniform excitations are assumed in all the arrays and the element pattern as shown in fig. 4 is adopted in all computations. Since this pattern is generally a conservative estimate of most end-fire type directive elements the results can be advantageously used in many applications in antenna engineering whenever an economical array with high electrical efficiency and good pattern characteristics is required. In all the arrays presented, the elements are assumed to be symmetrically located about the array centre. The advantage of a symmetrical array had already been discussed in section 1.1. The arrays are numbered alphabetically. Thus the arrays 32a and 32b are both 32-element arrays but these arrays have different design sidelobe level. The results are tabulated as in Table II.
<table>
<thead>
<tr>
<th>Array No.</th>
<th>Positions of elements from array centre expressed in terms of the wavelength at which the array operates</th>
</tr>
</thead>
<tbody>
<tr>
<td>16a</td>
<td>0.396 1.383 2.388 3.212 4.269 5.553 6.902 9.030</td>
</tr>
<tr>
<td>16b</td>
<td>0.440 1.423 2.388 3.233 4.259 5.502 6.769 8.834</td>
</tr>
<tr>
<td>16c</td>
<td>0.534 1.591 2.537 3.359 4.295 5.461 6.630 8.596</td>
</tr>
<tr>
<td>16d</td>
<td>0.533 1.438 2.182 2.847 3.762 4.908 5.872 7.476</td>
</tr>
<tr>
<td>16e</td>
<td>0.491 1.301 1.977 2.571 3.441 4.537 5.437 6.879</td>
</tr>
<tr>
<td>16f</td>
<td>0.437 1.184 1.840 2.436 3.275 4.320 5.204 6.560</td>
</tr>
<tr>
<td>24a</td>
<td>0.690 1.755 2.677 3.578 4.669 5.788 7.060 8.078</td>
</tr>
<tr>
<td>24b</td>
<td>0.703 1.801 2.753 3.586 4.523 5.511 6.743 7.718</td>
</tr>
<tr>
<td></td>
<td>9.311 10.899 13.637 18.765</td>
</tr>
<tr>
<td>24c</td>
<td>0.599 1.715 2.681 3.348 4.140 4.988 6.001 6.825</td>
</tr>
<tr>
<td>24d</td>
<td>0.603 1.503 2.376 3.214 3.880 4.733 5.727 6.383</td>
</tr>
<tr>
<td></td>
<td>7.561 8.754 10.937 14.777</td>
</tr>
<tr>
<td>24e</td>
<td>0.490 0.992 1.697 2.788 3.314 4.061 4.740 5.306</td>
</tr>
<tr>
<td></td>
<td>5.988 6.970 8.423 11.649</td>
</tr>
<tr>
<td>24f</td>
<td>0.462 1.123 1.719 2.726 3.373 4.033 4.812 5.590</td>
</tr>
<tr>
<td></td>
<td>6.237 7.248 8.608 12.193</td>
</tr>
<tr>
<td>32a</td>
<td>1.013 2.104 3.137 4.301 5.345 6.625 7.684 8.759</td>
</tr>
<tr>
<td>32b</td>
<td>0.898 2.056 3.011 3.990 4.853 5.887 6.861 7.904</td>
</tr>
<tr>
<td>32c</td>
<td>0.746 1.817 2.787 3.670 4.442 5.429 6.307 7.274</td>
</tr>
</tbody>
</table>

Table IIa  Element Positions in the synthesised Arrays
<table>
<thead>
<tr>
<th>Array No.</th>
<th>Positions of elements from array centre expressed in terms of the wavelength at which the array operates</th>
</tr>
</thead>
</table>

Table IIb  Element Positions in the synthesised Arrays
<table>
<thead>
<tr>
<th>Array No.</th>
<th>Positions of elements from array centre expressed in terms of the wavelength at which the array operates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>24.567 27.279 29.398 31.727 34.837 38.728 42.871 51.518</td>
</tr>
<tr>
<td></td>
<td>20.840 23.243 24.797 27.154 29.492 32.971 36.331 43.750</td>
</tr>
</tbody>
</table>

Table IIc  Element Positions in the synthesised Arrays

The core storage available in the IBM 1130 computer in Auckland will not permit synthesis of arrays much larger than 48 elements. Even if this is possible, the long computing time needed would make the method rather uneconomical. To give an order of the time required, the time taken to compute the six
48-element arrays presented in Table IIc was approximately 30 hours.

3.6.1 Pattern Characteristics of the Synthesised Arrays

The pattern characteristics of the synthesised arrays are presented in Table III. The parameters used to describe the pattern of each array are as follows.

(i) \( r \), the sidelobe level of the synthesised array. The value of \( r \) is numerically equal to the maximum absolute value of the normalised voltage response pattern, \( F(0) \), of the array, in the region outside the main beam.

(ii) \( r_{opt} \), the sidelobe level of the theoretically optimal array in which the pattern is assumed to oscillate rapidly in the sidelobe region with equal amplitude. Of course, this array must also be assumed to have the same length as the array in question. \( r_{opt} \) is computed on the basis of equation 3.18.

(iii) \%, the percentage by which the actual sidelobe level, \( r \), of the synthesised array exceeds the sidelobe level of the optimal array \( r_{opt} \).
Thus \( \% = 100 \cdot \frac{(r-r_{\text{opt}})}{r_{\text{opt}}}. \)

(iv) \( \Delta \theta \), the half power beamwidth of the synthesised array. The beamwidth is found from the computed pattern, \( F(\theta) \).

(v) \( B.F. \), the beam factor of the array, this is adequately discussed in section 3.1.

(vi) \( B.W. \), the bandwidth of the array as computed from equation 3.20.

(vii) \( G \), the pattern gain from pattern integration. Simpson's rule for numerical integration is used to compute \( G \). From experience, it was found that if the intervals in \( u \)-space were made smaller than \( \lambda / 2L \), the integration is reasonably accurate.

(viii) \( G_{\text{est}} \), the estimated gain computed from equation \( G_{\text{est}} = MG_0 \) as discussed in section 3.3. \( G_{\text{est}} \) has been used to compute \( r_{\text{opt}} \). The close agreement between the estimated gain with the gain computed from pattern integration justifies the use of the estimated gain in estimating to optimal sidelobe level of the array.
(ix) \( d_{av} \), the average inter-element spacing in the array, expressed in terms of the wavelength at which the array operates.

A pattern study program, 'PTEST' was developed which would compute the array pattern and some of the array parameters given in Table III once the element positions were defined.

<table>
<thead>
<tr>
<th>Array No.</th>
<th>( r )</th>
<th>( r_{opt} )</th>
<th>( % )</th>
<th>( \Delta\theta )</th>
<th>B.F.</th>
<th>B.W.</th>
<th>G</th>
<th>( G_{est} )</th>
<th>( d_{av} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16a</td>
<td>0.16</td>
<td>0.132</td>
<td>14.8</td>
<td>3(^\circ)</td>
<td>1.06</td>
<td>1.58</td>
<td>28.3</td>
<td>27.5</td>
<td>1.2</td>
</tr>
<tr>
<td>16b</td>
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<td>0.127</td>
<td>18.0</td>
<td>3(^\circ)</td>
<td>1.04</td>
<td>1.69</td>
<td>28.7</td>
<td>27.5</td>
<td>1.18</td>
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<td>0.120</td>
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<td>3(^\circ)</td>
<td>1.01</td>
<td>1.64</td>
<td>28.9</td>
<td>27.5</td>
<td>1.15</td>
</tr>
<tr>
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<td>71.0</td>
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<td>26.7</td>
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<td>1.00</td>
</tr>
<tr>
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<td></td>
<td>3.8 (^\circ)</td>
<td>1.03</td>
<td>1.19</td>
<td>25.5</td>
<td>27.5</td>
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<td></td>
</tr>
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<td>1.6(^\circ)</td>
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<td>1.80</td>
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<td>42.3</td>
<td>41.3</td>
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<td>1.9(^\circ)</td>
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<td>0.121</td>
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<td>2.0(^\circ)</td>
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Table IIIa  Pattern Characteristics of the Synthesised Arrays
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<tr>
<th>Array No.</th>
<th>$r$</th>
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<th>$%$</th>
<th>$\Delta \theta$</th>
<th>B.F.</th>
<th>B.W.</th>
<th>G</th>
<th>$G_{est}$</th>
<th>$d_{av}$</th>
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</thead>
<tbody>
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<td>.15</td>
<td>.142</td>
<td>6.1</td>
<td>1.1°</td>
<td>1.32</td>
<td>2.07</td>
<td>56.5</td>
<td>55.0</td>
<td>1.98</td>
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<tr>
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<td>.135</td>
<td>4.0</td>
<td>1.2°</td>
<td>1.30</td>
<td>1.73</td>
<td>58.0</td>
<td>55.0</td>
<td>1.78</td>
</tr>
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<td>.128</td>
<td>1.4</td>
<td>1.3°</td>
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<td>1.54</td>
<td>56.5</td>
<td>55.0</td>
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<td>.121</td>
<td>-1</td>
<td>1.4°</td>
<td>1.27</td>
<td>1.52</td>
<td>55.1</td>
<td>55.0</td>
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</tr>
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<td>1.6°</td>
<td>1.29</td>
<td>1.30</td>
<td>52.0</td>
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</tr>
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<td>1.8°</td>
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<td>1.60</td>
<td>69.7</td>
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</tr>
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<td>.119</td>
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<td>1.0°</td>
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<td>.108</td>
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<td>.097</td>
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<td>70°</td>
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<td>75°</td>
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<td>83.5</td>
<td>82.6</td>
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<td>80°</td>
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<td>85°</td>
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</tr>
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<td>.098</td>
<td>2.6</td>
<td>96°</td>
<td>1.28</td>
<td>1.00</td>
<td>82.3</td>
<td>82.6</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Table IIIb  Pattern Characteristics of the Synthesised Arrays
Fig. 11  Field Pattern of Array 16a
Fig. 14  Field Pattern of Array 32f
3.7 Analysis of Results

Many interesting properties of N.U.S. arrays can be revealed by careful study of the results tabulated in Table III. The main advantage of the method of synthesis used is that the array can be designed to give exactly the desired sidelobe level. The extent to which the sidelobes can be suppressed is limited only by the fact that the array spacings should not be impractically small. A discussion on this is given in Appendix A.

3.7.1 Pattern Characteristics

To illustrate the type of patterns typical of the synthesised arrays, the patterns of arrays 16a, 16f, 32a, 32f, 48a, 48f are plotted as shown in figures 11-16. Each pattern shows a clear plateau of peak sidelobe level which is the designed sidelobe level for the array. In general the pattern oscillates rapidly in the sidelobe region and the longer the array, the greater will be the number of sidelobes in the visible space. In the case where the computed theoretical optimal sidelobe level is approximately equal to the
designed sidelobe level, e.g. array 48f, it is clear from figure 16 that the pattern has approximately the same form as the Dolph-Chebyscheff array. There is also a tendency for the sidelobes near to the main beam to be considerably lower than the designed sidelobe level. This probably arises from the fact that the starting array used in the computation, (the quadratically non-uniform array); is characterised by low sidelobe level close to the main beam. Since the shape of the pattern function in the sidelobe region is free to assume any form, there is a possibility for the average sidelobe power level to be more than half the peak sidelobe power level, in which case, the sidelobe level of the array will be better than the theoretically optimal sidelobe level. There is only one case encountered in which the sidelobe level of the array effectively equals the level of the idealised array. This feature is shown in array 32d.

An interesting point arises from the computation of $r_{opt}$ for arrays 16e and 16f using equation 3.18. It is found that for these two cases, the denominator $(1 - \lambda G/2L)$ is negative resulting in a complex solution.
for $r_{\text{opt}}$. The physical meaning for this is that, if it were possible for the array to have a pattern gain $G$ and at the same time a beamwidth $\lambda/L$ radians, then the area under the power pattern in the sidelobe region must be negative for this condition to be satisfied. In other words, the basis by which the pattern gain $G$ and the half power beamwidth $\Delta \theta$ were estimated fails for these two cases. This is, however, only to be expected for in the derivation of equation 3.18 an assumption of a long array with widely spaced elements were made. In practice, it was found that the estimate was reasonably realistic as long as $\lambda G/2L$ is not too close to unity. This implies that a limited pattern gain for a given array length is only permitted, a situation closely analogous to super-gain effect in a broadside array. If the actual gain from pattern integration and the actual computed beamwidth of array 16e are substituted, $(1 - \lambda G/2L) = .84$ and $r_{\text{opt}} = .11$ as compared with the actual sidelobe level of .12 in the computed pattern.
3.7.2 Spacing Characteristics

In all the arrays synthesised, there is a distinct tendency for the inter-element spacings to be large at distances away from the array centre. For this reason, N.U.S. arrays are often called spaced tapered arrays. The nature of the space tapering is best shown by plotting the element position variable against the element number variable as was used by A. Ishimaru\textsuperscript{26}. The element position variable is defined as the ratio of the distance of the nth element from the array centre to the distance of the end element from the centre of the array. The value of the element number variable for the nth element is \( \frac{n}{N} \), where \( N \) is half the total number of elements in the array. A plot of the element position function is shown in figure 17. The curves presented are the average cases for the 16 element, the 32 element and the 48 element arrays respectively. A straight line joining the origin to the point \((1,1)\) represents the case for a uniformly spaced array. A curve below the straight line represents a space tapered array and a curve lying above the straight line indicates that the array has
spacings large near the array centre and gradually decreasing away from the array centre. It can be shown by analogy with a current tapered array that a N.U.S. array must be space tapered if good sidelobe characteristics are desired\textsuperscript{18}. For the sake of comparison a few points are also plotted in figure 17 for the 'quadratically non-uniform array', which was used as a starting array for the computation.

An attempt was made to generate the element positions by a smooth function derived from the computed positions. Although the pattern of the arrays so derived were often better than the case of a uniformly spaced, uniformly excited arrays, the results were far from being as good as those of the synthesised arrays. This arises because the element spacings in the synthesised arrays do not, in general, vary in a systematic manner. To illustrate this point, the inter-element spacings of the six selected arrays are plotted as shown in figures 18 and 19. These graphs are quite oscillatory and no clear pattern can be detected. However, if the ripples are smoothed, serious deterioration in the array pattern immediately
Fig. 27 The Approximate Source Position Functions for the 16 Element, 32 Element and 64 Element Arrays. (+ - Points for Quadratically-non-Uniform Array)
occurs. Consequently, no further attempt was made to seek for a simple position generating function. Most probably such a function does not exist.

As expected, the average inter-element spacings of the array decreases as the design sidelobe level decreases. The variation of average inter-element spacing with design sidelobe level for the five sets of arrays are as shown in fig. 20. Except for arrays 16e and 16f, all the other arrays have average inter-element spacings of at least $1\lambda$. For arrays larger than 16 elements it is difficult to have an average inter-element spacing less than $1\lambda$ without resulting in some spacings being less than half a wavelength.

3.7.3 Beamwidth

Contrary to popular belief, the beamwidth of the arrays are, in general, larger than the corresponding uniformly excited line source having the same aperture lengths. From the tabulated values for the beam factor of the synthesised arrays, up to 30% increase in beamwidth due to non-uniform spacing can be expected. Thus from the point of
Fig. 20 Graphs of Average Inter-element Spacings vs Sidelobe Level
view of the beamwidth alone, there is little advantage which the synthesised arrays have over the corresponding uniformly spaced arrays with weighted excitation. But the saving over the number of elements required is significant. In a uniformly spaced array, an inter-element spacing of about \( 0.7\lambda \) is commonly used, in which case a N.U.S. array with an average spacing of \( 2\lambda \) will have a 3 fold saving in the number of elements required. The saving in cost is probably higher, since with more elements, the feed system is necessarily more complicated.

3.7.4 Bandwidth

As a result of the gradual decrease in the average spacing with the decrease in design sidelobe level, the bandwidth of the array also decreases as the sidelobe level decreases. The bandwidth of the synthesised arrays are plotted in fig. 21. It is obvious that low sidelobe levels are achieved at the expense of a decreased bandwidth. In general, the sidelobe level increases rapidly outside the designed frequency band. This is illustrated in fig. 22.
From the point of view of cost, N.U.S. arrays are ideally suited to applications where the sidelobe level is not required to be very low. Here large spacings are permitted and the number of elements required in the array can be reduced by a factor of 3 or 4. On top of this, the array is operative over a bandwidth of about 2:1. This greatly enhances the economic value of the array. When low sidelobe levels are required, the N.U.S. array quickly loses its economic feature. For example, array 32f has a beamwidth approximately 20% wider but has about 50% less elements than a corresponding Dolph-Chebyscheff array with \(0.7\lambda\) spacing. But whereas the 20db sidelobe level is about the lowest sidelobe level that a practical N.U.S. array of that size can have, the sidelobe level can be further reduced with only small increase in beamwidth in the Dolph-Chebyscheff array. Thus whenever extremely low sidelobe level is required, the current tapered array is still the only solution. An interesting application of both uniform and non-uniform spacing in an array is found in a radio telescope at the University of Illinois\(^{49}\). The central portion of the array is a conventional current
Fig. 21 Variations of Array Bandwidths with Design S.L. Level

Fig. 22 Variation of Sidelobe Level with Operating Frequency
For Array 32a. ($f_{\text{min}}$ = the frequency at which the minimum spacing in the array is $\frac{1}{2} \lambda$)
tapered array which serves to provide effective control of the sidelobe level of the array, while the far out elements are non-uniformly spaced to achieve a high reduction in beamwidth without significant increase in the number of elements.

3.7.5 Pattern Gain

The method of estimating the pattern gain of the array from the principle of gain limitation proves to be sufficiently accurate. In general the gain of the array obtained from pattern integration tends to be higher than the estimated gain at higher design sidelobe levels (larger spacings) and lower than the estimated gain at lower design sidelobe levels. This is shown in fig. 23. Thus the estimated gain lies between the two extreme cases in the synthesised arrays. Comparing fig. 23 with the gain variation in a broadside uniformly spaced array, (fig. 6), it can be seen that this method of estimating the gain of an array is closer in the case of an N.U.S. array than in the case of a U.S. array. The reason for this is that there are no grating lobes in an N.U.S. array so that the array gain settles down to the limiting value rapidly
Fig. 23  Gain Variations for Different Design Sidelobe Levels
(-- ---, the Estimated Gain for the Given Number of Elements.)
when the average spacing is larger than about $1\lambda$.

However, because of the grating lobes in an uniformly spaced array, there is a large variation in gain whenever the spacing equals an integral number of wavelengths. Thus the gain is an oscillatory function of spacing as seen in fig. 6.
The 16-Element Experimental N.U.S. Array

An experimental array consisting of 16 non-uniformly spaced Yagi antennas was erected on the tarmac in the School of Engineering at Ardmore. A full scale model was decided upon since, besides providing an experimental verification to the properties of the array, the same array can be used to provide valuable experience in the operation of an array for radio astronomy application.

It may at first seem rather strange that narrow bandwidth antennas like the Yagis should be used after the advocation of the wide band feature of N.U.S. arrays in the previous chapters. The need to reduce the cost of the experimental array to a minimum was the main factor which called for the use of narrow band elements. A fair amount of experimental studies into the properties of log-periodic antennas and ferrite cored matching transformers have been carried in the School of Engineering and the results obtained have indicated that there is a good possibility that such elements can be introduced into the array if so desired to achieve a wide band operation. The discussion
of these elements is however, beyond the scope of this thesis.

The function of this chapter is to discuss the design of the array and the feed system and to present the results of the experiments conducted on the array.

4.1 Array Synthesis

The array was synthesised by systematic adjustments of the element positions of an initial array derived from the application of Harrington's perturbation technique, as was discussed in chapter 3. A considerably better array can be synthesised using the later method involving element perturbation by a matrix method. Unfortunately, this method was not developed at the time of erection of the array. Despite the higher sidelobe level, the pattern of the array is considerably better than the corresponding uniformly spaced array with uniform excitation. Note that in the latter array, the maximum sidelobe level is 0.21 whereas in the synthesised array the level is 0.19. Although the average inter-element spacing is 1.3λ, there is no grating lobe effect in the array pattern. The element positions and the computed pattern for the array are presented respectively in Table IV and fig. 24. If the improved synthesis method were used, the sidelobe level of 0.16
Fig. 24  Theoretical Pattern of the 16-Element Experimental Array
can be achieved for the same length and beamwidth of the array, as is seen in Table IIIa.

<table>
<thead>
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<th>(x_3)</th>
<th>(x_4)</th>
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<td>7.674</td>
<td>9.723</td>
</tr>
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</table>

Table IV  Computed Element Positions of the Experimental Array

It should be clear that the values given in Table IV are the distances of the various elements from the array centre expressed in terms of the wavelength at which the array is designed to operate. The design frequency adopted was 41 MHz so, to get the actual distances in meters, the given values must be multiplied through by 7.32.

4.2 Details of the Yagis

A modified version of the commercial channel 1 television Yagi antenna was used in the array. The Yagis were made by Hills Hoist Limited, Auckland. Each Yagi has 5 elements and the driven element is a folded dipole
All the elements are made from 1/4 inch rolled aluminium and the various dimensions of the antenna are, according to figure 25, given as follows:

\[
\begin{align*}
L_r &= 3.65\text{m} \ (0.5\lambda) \\
L &= 3.38\text{m} \ (0.463\lambda) \\
L_1, L_2, L_3 &= 3.08\text{m} \ (0.421\lambda) \\
d_r &= 1.12\text{m} \ (0.153\lambda) \\
d_1, d_2 &= 0.79\text{m} \ (0.108\lambda) \\
d_3 &= 1.12\text{m} \ (0.153\lambda)
\end{align*}
\]

(Note: \( \lambda \) = the free-space wavelength at 41 Mhz.)

Fig. 25 The 5-Element Yagi
The gain of the 5-element Yagi is estimated to be about 11 db above isotropic and approximately 3 db higher than the gain adopted in the array synthesis. This is desirable from the point of view of practical tolerance. It is clear that if the elements used have exactly the same pattern as that assumed in the synthesis, the array pattern will depend heavily on the accurate alignment of the element patterns with respect to the array factor. The permissible misalignment in the element orientation obtained through the use of higher gain elements can be estimated as follows. The estimate is of course only theoretical since mutual couplings between antennas are not taken into account.

If $\theta_1$ is the half power beamwidth of the assumed pattern and $\theta_2$ the half power beamwidth of the element actually used, the permissible misalignment of the second pattern with respect to the first before having a significant effect on the sidelobe level of the array is approximately $(\theta_1 - \theta_2)/2$. Let the gain of the element assumed in the synthesis be $G_1$ and that of the element used be $G_2$. Assuming each element to have a conical pattern, and using the approximate formula suggested by Kraus\textsuperscript{1} for end-fire type elements $G_1 = 41,250/\theta_1^2$ and $G_2 = 41,250/\theta_2^2$. 
Since $G_2$ is 3 db higher than $G_1$, $G_2/G_1 = 2$, resulting in $0_2 = 0.707 \cdot 0_1$ and $(0_1 - 0_2)/2 = 0.146 \cdot 0_1$. For the assumed pattern, $\theta_1 = 72^\circ$, the permissible tolerance in the alignment of the element pattern is approximately $10^\circ$.

4.3 Structural Support

The simplicity in the supporting structures was one of the economical features in the array. Each element was supported by a single 3" x 4" x 15' wooden post concreted to the tarmac, an ex-airport parking area in the campus of the School of Engineering. The element is mounted fully tiltable in the vertical angle on a steel pipe which is in turn clamped to the wooden post by two u-bolts. Thus the pipe can be rotated if necessary to adjust the horizontal orientation of the element. The entire array and the details of the supporting structures on each post are shown in Plate I and Plate II respectively.

The array was built on a east-west base-line. Care was taken to mount each Yagi in the same horizontal plane, since the tarmac was uneven. If the elements are not in the same horizontal plane, additional phase delays not accounted for in the pattern formula can be introduced.
PLATE I

THE 16-ELEMENT EXPERIMENTAL ARRAY
PLATE II
SUPPORTING STRUCTURES FOR EACH YAGI
4.4 Feed System

The main requirements for the feed system of the array are:

(i) It must be sufficiently stable to weather conditions.

(ii) The loss must be sufficiently low for astronomy applications.

(iii) The system should be capable of providing electrical beam steering facilities.

Two types of transmission lines can be used in the system, the open wire line and the coaxial cable. The open wire line has the advantages of lower loss and easily adjustable characteristic impedance. The main disadvantage of such a system is that it is relatively expensive to install, if the system is to be stable under varying weather conditions. A coaxial cable, on the other hand, can be run along the ground or near to a conducting surface without affecting the transmission line characteristics. It is also very stable to weather conditions. Although it has higher losses and lower power handling capacity than the open wire lines, this is not at all critical for radio astronomy applications at the frequency concerned.
For this reason, the coaxial cable was used.

The next problem is to decide on the type of feed system. There are two configurations which can be adopted. The first is to feed the antennas from a central transmission line. Such a system is popularly used in large arrays for radio astronomy because it is very economical and minimal lengths of transmission lines are required to feed the elements. One main disadvantage of such a system, which has discouraged its use in the experimental array, arises from the fact that the physical path length from each antenna to the array centre differs widely although the electrical phase delay can be made equal. As a result, more attenuation is encountered by signals from the far out antennas than from the antennas close to the array centre. Although this system may be suited for a current tapered array, it is not as suited for the present application where a uniform excitation in the array is required.

Another method of feeding the array is popularly known as the parallel feed system. Such a system was used in the experimental array and a schematic diagram of the system used is as shown in fig. 26. It can be seen
Fig. 26  The Array Feed System

$L_1 = 80$ ft.
$L_2 = 60$ ft.
$L_3 = 88$ ft.

To the other half of the array

Preamp

To radiometer
from the figure that the signal from each antenna travels through the same length of cable and thus suffers the same attenuation and phase delay before the preamplifier. Consequently the system is ideally suited for use in an array where uniform excitations to the elements are desired. Also, because the transmission paths from antenna to the preamplifier are equal, the feed system is basically wideband if the matching devices used are also capable of operating over a wide frequency bandwidth.

In order to reduce cost, the experimental array was designed to operate over a narrow frequency band. All impedance matchings were done through the use of shorting stubs and quarter wave transformers.

The balanced impedance across the folded dipole of the Yagi was first transformed to an unbalanced impedance through the use of a half wave coaxial balun. The transformed unbalanced impedance was measured with an RX meter, type 250A, manufactured by Boonton Radio Co. The readings taken on four arbitrary selected antennas were 34Ω/-17.5pf, 35Ω/-17.6pf, 34Ω/-20.2pf and 36Ω/-12.4pf. The readings are given in terms of a resistance in parallel with a capacitor. A negative value for the latter is taken
to mean that the parallel reactance is inductive. The average impedance from the four measured values was taken to be 35Ω//20pf. A single stub L-type matching section was designed to transform this impedance to 75Ω, the characteristic impedance of the cable used. The details of the half wave balun and matching section are shown in fig. 27a. The velocity factor of the cable was assumed to be 2/3.

Fig. 27a The Half-wave Balun and Matching Section for the Yagi
Fig. 27b Matching Section to Transform 37.5Ω Impedance to 75Ω.
The measured transformed impedance at point 'A' for the four antennas were 75Ω/-1pf, 75Ω/-3pf, 77Ω/-5pf, 76Ω/-4pf. Thus a variation of less than 2% was found in these measured impedances.

A fixed length of cable was connected between the point 'A' and the point 'B' which was located inside a weather box fixed to each post. (see fig. 26) Between B and C a coaxial cable of length up to one wavelength can be inserted if so desired for the purpose of electrically steering the main beam of the array. Since the impedance looking into both B and C are designed to be 75Ω, there can only be a very small standing wave ratio between B and C so that the insertion of a short length of coaxial cable will not affect the amplitude of excitation to the antenna.

To save the use of many L-type matching sections, the antennas are cabled together in groups of four via cables C-D, to four weather boxes located at appropriate posts in the array. Two quarter wave cables are used in each box to transform the impedances such that the impedance looking into E is 75Ω. It is well known that a quarter wave section will transform an impedance $Z_a$ to $Z_b$ such that $Z_0 = Z_aZ_b$, where $Z_0$ is the characteristic
impedance of the quarter wave section. Since the impedance at the point 'D' is approximately 37.5Ω, the transformed impedance after the quarter wave section is 150Ω, a parallel of which will give the required impedance of 75Ω at the point 'E'. The points 'E' are then joined in groups of two at two points 'F'. The impedance of 37.5Ω at F is transformed to 75Ω at G by another L-section matching cables as shown in figure 27b. The signals from the two halves of the array are finally combined via cables G-H, the point 'H' being the input of the preamplifier the function of which is to compensate for the loss in the long cable (approximately 250 yards) leading to the radiometer.

4.5 Electrical Beam Steering

The principle electrical beam steering has already been discussed in section 1.3. Briefly, if a phase delay \( \phi_n = 2\pi x_n \sin \theta_0 \) is inserted into the signal path to the nth element, the array pattern is given by

\[
F(\theta) = F_0(\theta - \theta_0), \quad \sum_{n=1}^{N} I_n \cos(2\pi x_n (\sin \theta - \sin \theta_0))
\]
It can be seen from equation 4.1 that the main beam of the array is pointed in the direction $\theta = 0_0$. It is assumed in the equation that the direction of maximum radiation of each element in the array is also $\theta = 0_0$. By this assumption, it is then necessary to consider only the case when $\theta_0 = 0$ in the array synthesis.

The beamwidth of the array is, however, dependent on the direction the main beam is steered; the smallest beamwidth occurs when $\theta_0 = 0$. This can be seen from the following analysis. Assuming the computed beamwidth in $u$-space be $\Delta u$, where $u = \sin \theta - \sin \theta_0$. If the corresponding beamwidth in the angular space is $\Delta \theta$, then

$$\frac{\Delta u}{2} = |\sin(\theta_0 + \Delta \theta/2) - \sin \theta_0|$$  \hspace{1cm} (4.2)

Assuming $\Delta \theta$ to be small so that the approximations

$$\sin \Delta \theta = \Delta \theta, \quad \cos \Delta \theta = 1 - \frac{1}{2} \Delta \theta^2$$

can be used, equation 4.2 reduces to

$$\Delta u = |\Delta \theta \cos \theta_0 - \Delta \theta^2 \sin \theta_0 / 2|$$  \hspace{1cm} (4.3)

For steering angles not nearing $90^\circ$, the term involving $\Delta \theta^2$ can be neglected in equation 4.3 and

$$\Delta \theta \approx \frac{\Delta u}{\cos \theta_0}$$  \hspace{1cm} (4.4)
Equation 4.4 shows that the beamwidth of the array increases as the beam is steered away from the broadside direction.

The proposed method of introducing the phase shift $\phi_n$ into the transmission path of the nth element is to insert the appropriate length of cable between points 'B' and 'C' as shown in fig. 26. Assuming $L_n$ to be the length of cable in wavelength, that is required to be inserted,

$$L_n = x_n \sin \theta_0 - \lambda$$  \hspace{1cm} (4.5)

where $X$ is the whole number immediately less than the value of $x_n \sin \theta_0$. Thus the length of the coaxial cable inserted is always less than one wavelength.

It will of course be inconvenient to have to work out the lengths of cables required every time the beam is steered. The best method is to pre-cut 100 lengths of cables with lengths ranging from $1/200 \lambda$ to $1/2 \lambda$. Thus cable number 50 means a cable of length $1/4 \lambda$. At least 16 half-wave cables (no. 100) must be prepared so that they can be seriesed with the other cables if a phase delay of greater than $180^\circ$ are required. A table is compiled using the computer to give the number of the cable required to be
inserted in the feed to each antenna for various steering angles $\theta_0$. There are a couple of points which will need to be explained in Table V before the significance of the figures can be understood. Firstly, cable number 125 is taken to mean that a series of cable 100 with cable 25 is needed to give the correct phase delay. Secondly, the antennas are numbered from the array centre. $A_5$ means the 5th antenna from the array centre in the direction to which the beam is steered. $A\overline{5}$ means the 5th antenna from the array centre in the other direction. Thus if a phase delay of $\phi_5$ is required for $A_5$ a phase advance of the same amount will be required for $A\overline{5}$. 
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Table V  Code Numbers of Beam-steering Cables
4.6 Losses in the Feed System

The coaxial cable used in the entire feed system was Telcon K19M Telcothene cable. The measured loss of the cable at 41Mhz was 7.4 db/100yds.

To test the efficiency in the matching, the feed system was terminated at points 'C' by 16 68ohm nominal resistances, which were approximately 75 ohm at 41 Mhz. A 41 Mhz signal was fed in at the feed centre 'H' and the attenuation in the signal between 'H' and 'C' was measured to be 17.75 db. Of course not all of this attenuation was due to losses; 12 db of the total attenuation was due to the fact that the resistances were fed in parallel. Thus the total losses between 'H' and 'C' was 5.75 db.

Now the total length of the signal path between 'H' and 'C' is approximately 228ft, which accounts for 5.6 db due to the coaxial cables. The loss due to the matching system is therefore about 0.15 db. It must be realised, however, that the figures given above can only be approximate owing to the relatively unsophisticated measurement technique used. The r-f voltages were measured using a Philips DC Micro-voltmeter GM6020 with the V.H.F. Diode Probe GM6050.
In a radio astronomy system, it can be shown (see Part II) that the minimum detectable flux density of the system is

\[ \Delta S_{\text{min}} = \frac{2kK_s(T_a + \alpha N T_0)}{A_e \sqrt{\Delta f \tau}} \]

where
- \( k \) = Boltzmann's constant (= 1.38 x 10^{-23} \text{ joule}^0\text{K}^{-1})
- \( K_s \) = Sensitivity constant which depends on the type of receiver used.
- \( T_a \) = Antenna temperature
- \( \alpha \) = Transmission loss, \( \alpha > 1 \)
- \( N \) = Receiver's noise factor
- \( T_0 \) = Ambient temperature
- \( A_e \) = Effective antenna collecting area
- \( \Delta f \) = Predetector bandwidth
- \( \tau \) = Postdetector integration time

From equation 4.6, it can be seen that if \( \alpha N T_0 < T_a \), the cable loss has little effect on the sensitivity of the system. At 41 Mhz it is reasonable to assume an antenna temperature of 10,000^0K. The noise figure of the radiometer used is 6 db, thus \( N=4 \). The total feed loss from the antenna to the feed centre 'H' is approximately 3 db,
i.e. \( \alpha = 2 \). Hence \( \alpha N T_0 = 2,400 \) which is only \( \frac{1}{4} T_a \).

Consequently, no preamplifier is needed in the parallel feed system. However, a preamplifier is needed to compensate for the 17.4 db loss over the approximately 220 yard cable used to bring the signal from the array centre to the laboratory. If uncompensated, \( \alpha N T_0 = 125,000^\circ K \) which is obviously unacceptably large. A preamplifier with a gain of approximately 20 db was inserted at the array centre. Two silicon npn high frequency transistors operating in a cascode configuration were used in the preamplifier, which was designed by G.A. Moyle.

4.7 Feed Tolerance

From measurements, the impedance of the Yagis showed a dispersion of about 5%. To test the tolerance of the parallel feed system, the system was again terminated by 16 resistances each with an r-f resistance of 75 ohm. A 41 Mhz signal was fed at the array centre and the r-f voltages on the resistances were found to vary only by about 3%. Thus assuming no poor contacts in the system, the rms excitation error to the antennas will not be more than 10%. 
A phase error can result from the cable lengths being not exactly the designed length. Assuming it was possible to make an error of 10 cm in measuring out the total lengths of the cables to each antenna, this represents a phase error of .13 radians.

It is reasonable to assume that the individual errors are independent of one another and distributed in a random manner. Assuming the rms amplitude error of the excitation to the nth element be $\Delta_n I_n$ and the rms phase error to the nth element is $\delta_n$, the resulting error term in the radiation pattern is

$$R(\theta) = \sum_{n=-N}^{N} \Delta_n I_n \exp(j2\pi x_n \sin\theta + j\delta_n)$$  \hspace{1cm} (4.7)

By a statistical analysis, Ruze\textsuperscript{52} showed that the difference between the power pattern $\bar{P}(\theta)$ of an "average system" with random error and the power pattern $P_0(\theta)$ of the system with no error is\textsuperscript{3}

$$\bar{P}(\theta) - P_0(\theta) = P_0^2(\theta) \cdot (\bar{\Delta}^2 + \bar{\delta}^2) \frac{\sum I_n^2}{(\sum I_n)^2}$$  \hspace{1cm} (4.8)

$\bar{\Delta}$ = mean-square amplitude error

$\bar{\delta}$ = mean-square phase error
\[ F_0(\phi) = \text{element field pattern}. \]

Thus the effect of random error in an array of discreet elements is to increase the power response of the array by a small amount weighted only by the slowly varying element power pattern. It is interesting to note from equation 4.8 that the factor \( \frac{\Sigma I_n^2}{(\Sigma I_n)^2} \) is a minimum for an array with uniform excitation so that the pattern deterioration due to a given mean square error in the feed system is smaller in a uniformly excited array than in a current tapered array.

To find the condition that \( \frac{\Sigma I_n^2}{(\Sigma I_n)^2} \) is minimum it is best to start from a well known formulation in statistics relating the standard deviation \( \sigma \) with the mean \( \bar{I} \). Assuming there are altogether \( M \) elements in the array, the statistical relation becomes

\[
\sigma = \frac{(\Sigma I_n^2 - M\bar{I}^2)}{M} \quad 4.9
\]

Dividing equation 4.9 through by \( M\bar{I}^2 \) and noting that
\[
M^2\bar{I}^2 = (\Sigma I_n^2)^2,
\]

\[
\frac{\sigma}{(M\bar{I}^2)} = \frac{\Sigma I_n^2}{(\Sigma I_n)^2} - \frac{1}{M} \quad 4.10
\]
Thus the required ratio will be a minimum if the standard deviation is zero, i.e. $\sigma = 0$, which will then result in

$$\frac{\sum I_n^2}{(\sum I_n)^2} = \frac{1}{M} \quad 4.11$$

From previous discussion in this section, it is reasonably conservative to assume a rms error of .1 in the current distribution and a rms error of .13 radians in phase distribution. The total mean square error is .027. From equation 4.8, it can be seen that the region in space in which the feed error will affect the pattern most severely is the region where $F_0(\theta)$ is near to unity. Substituting $F_0(\theta) = 1$ and $(\bar{\delta}^2 + \bar{\phi}^2) = .027$ in equation 4.8, the estimated increase in the sidelobe level in the power pattern is .0017. Thus if the sidelobe level in the field pattern of the synthesised array is .19, this pattern deterioration due to random error represents only about 4.7% of the power response in the biggest sidelobe. It must be noted that the increase in the power response due to random error is independent of the sidelobe level of the synthesised array so that if the array is designed for a lower sidelobe level it may then be necessary to impose a closer tolerance to the feed system. Very accurate feed tolerance can be obtained if necessary by
actually sampling the current from each antenna using a well
designed loop\textsuperscript{53}. This is a very tedious process and is only
needed in the case where a very low sidelobe level is required.
A picture of the sampling loop and the associated detecting
equipment designed by the author is shown in Plate V of Part II.
By such an adjustment, the prescribed excitation can be 
obtained despite mutual coupling between elements.

4.8 Pattern Testing

The main difficulty encountered in plotting the
pattern of an array is the necessity for the signal source to
be at a sufficient distance away from the array centre so
that the wave front from the source can be assumed plane
throughout the total length of the array\textsuperscript{1}. Using the
arbitrary criterion proposed by Cutler\textsuperscript{54} which states that
the difference between the distance from the source to the
array centre and the distance between the source and the edge
of the array should not exceed $1/16$th of the wavelength,
the minimum distance the source has to be away from the
array centre is

$$D_{\text{min}} = \frac{2L^2}{\lambda}$$

4.12
where \( L \) is the total length of the array. According to equation 4.12 the signal source must at least be 3700 meters away from the centre of the experimental array.

It will be clear that any terrestrial means of plotting the array pattern will be complicated and expensive. The only radio source that can emit sufficient power so as to bring out the sidelobe characteristics of the array is the sun when active. But the flux density from the active sun is very sporadic and is, therefore, not suited to be used in the pattern study.

Probably the most economical method of plotting the pattern of the array will be to use the signal transmitted by one of the satellites at present in orbit. The 41 Mhz signal from the American ionospheric research satellite Beacon C was used for this purpose. A few factors however, made it difficult for an accurate pattern measurement. Firstly, although at times the transit was quite nearly due east-west, it was not generally so, and an accurate pattern plot could only be obtained by tedious processing of the data obtained in order to take into account the exact orbital path of the satellite. Secondly, even if the satellite's orbit were to be taken into account, the periodic signal nulls due to Faraday effect at the ionosphere would make the data
processing very complicated indeed. To illustrate the Faraday effect, the satellite was 'observed' on a simple phase switched interferometer tuned to 41 Mhz. (See Part II for details of the interferometer system.) Each arm of the interferometer consists of two Yagis spaced .8λ apart. The baseline of the interferometer is approximately 20λ. A couple of records of the satellite transit are shown in fig. 28a and fig. 28b. The first figure shows distinct nulls caused by Faraday effect. The second, which occurred only occasionally, shows no signs of Faraday nulls at all. Since the exact pattern is of little consequence for the applications intended, it was decided not to attempt to find the exact radiation pattern of the array but to get a reasonably accurate estimate of the peak sidelobe level in the pattern. To do so, a number of transits of the satellite were observed. A typical record of such a transit is shown in fig. 29. A logarithmic response was used in the radiometer so that both the levels of the sidelobes and of the main beam could be estimated with reasonable accuracy. A calibration was made after each transit and the peak sidelobe level was estimated from the calibration. The calibration was done with the use of Hewlett Packard V.H.F. Signal Generator, Model 608c. The results from the calibrated records are as shown in Table VI.
<table>
<thead>
<tr>
<th>Date</th>
<th>Transit Time</th>
<th>Elevation Angle at Centre of Transit</th>
<th>Peak Sidelobe Level w.r.t. the Main Beam Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.8.67</td>
<td>1220</td>
<td>66.5°</td>
<td>-16.4 db</td>
</tr>
<tr>
<td>24.8.67</td>
<td>1610</td>
<td>39.5°</td>
<td>-19.0 db</td>
</tr>
<tr>
<td>26.8.67</td>
<td>1140</td>
<td>68.0°</td>
<td>-17.4 db</td>
</tr>
<tr>
<td>26.8.67</td>
<td>1447</td>
<td>56.4°</td>
<td>-16.3 db</td>
</tr>
<tr>
<td>27.8.67</td>
<td>1212</td>
<td>72.0°</td>
<td>-17.3 db</td>
</tr>
<tr>
<td>27.8.67</td>
<td>1406</td>
<td>65.7°</td>
<td>-15.4 db</td>
</tr>
<tr>
<td>29.8.67</td>
<td>1245</td>
<td>84.2°</td>
<td>-18.0 db</td>
</tr>
<tr>
<td>30.8.67</td>
<td>1204</td>
<td>87.6°</td>
<td>-18.3 db</td>
</tr>
<tr>
<td>30.8.67</td>
<td>1358</td>
<td>42.0°</td>
<td>-18.3 db</td>
</tr>
</tbody>
</table>

Table VI  Records of Satellite Transits

It is interesting to note that the peak sidelobe level recorded in the worst case is 15.4 db below the main beam level. Since there are a number of sidelobes in the array with levels near to the peak sidelobe level, it is unlikely that all these sidelobes are attenuated more than the main beam by Faraday effect. This weighting against the main beam level due to Faraday rotation tends to compensate for the reduction in signal strength from the satellite when it
Fig. 28 Interferometer Records of Satellite Transits.
(Antenna system as for fig. 52)

Fig. 29 Response of Experimental Array to Satellite Transit.
is in the sidelobe region. Thus it is reasonable to estimate that the actual sidelobe level of the array is approximately 15.4 db below the main beam level. This is better than the computed sidelobe level for the array. This is probably due to the fact that the elements used are about 3 db more directive than those assumed in the synthesis.

It must be admitted that the experiment carried out was quite unsophisticated and the results can only be taken as a rough estimate. To obtain an accurate pattern test a high frequency model is necessary, and there are no facilities at present in the University for such a test.

To test the array for radio astronomy application, the array was connected to a total power radiometer which tuned slightly off 41 Mhz so that the signal from the satellite was well suppressed. The results obtained were consistent with an array with a single main beam and well suppressed sidelobe level. A record of a transit of the milky way across the beam on 24th August 1967 is shown in fig. 30. The strong radio source, Centaurus A, is clearly seen in the record. It is not expected that many other radio sources can be 'observed' in a total power record because the beamwidth is still rather wide and the flux from radio sources
tend to be confused by the total noise power received in the main beam solid angle. This limitation will be discussed in greater detail in Chapter VI (Part II) of this thesis. The ionospheric sounder from the neighbouring Radio Research Centre also caused periodic pulses in the record tending to obscure any rapid modulations on the record. However, it is clear that the detail shown on the profile across the milky way is consistent with the $3^\circ$ beamwidth of the array. It was not possible with the facilities then available to measure accurately the antenna temperature at the peak in the record. To do so, a diode noise source followed by a wideband amplifier would be necessary in order to generate sufficient noise for calibration. Such facilities are currently being developed by graduate students in the radio astronomy programme. The maximum antenna temperature was estimated to be more than $30,000^\circ$K. A 6 db attenuation pad was inserted before the intermediate frequency amplifier in the radiometer in order to keep the output level at the first detector sufficiently low for good dynamic response at the transit of the galactic equator. This pad was removed for the same reason at the 'colder' region of the sky near the galactic pole where the antenna temperature was around $8,000^\circ$K. A schematic diagram of
the receiving system used is shown in fig. 31. The radiometer used was developed by J.R. Irving.

4.9 Assessment of the Results

Although only relatively unsophisticated tests were conducted on the array, the results from both the satellite and radio astronomy observations did show with a fair degree of certainty that there were no unexpectedly big sidelobes in the pattern. The beamwidth measured by comparing with the interferometric record of the satellite transit agreed closely with the computed beamwidth of the array. The array was found to be very stable to rain and other types of weather conditions. This stability was achieved through the use of coaxial cables in the feed system and through careful weather-proofing of all exposed terminals. Only the strongest radio source in the southern hemisphere, Centaurus A, could be 'observed' with the total power system. This was expected because such a system is highly resolution limited, although it had adequate sensitivity to detect many more sources. To obtain a significant astronomy contribution from the array, another such array would be needed to operate in an interferometric system.
16-Element N.U.S. Array

Preamp. (20db)

r-f amp.
and mixer
(43 db)

Attenu.
垫

10.7 Mhz log.
response

I.F. Amp.
(83 db Max.)

Detector
and R-C

Filter

d.c.

Amp.

RCDR

Fig. 31 The Receiving System for Array Testing
CHAPTER V

Some Conclusional Remarks

The main contribution of Part I of this thesis has been the synthesis of a series of N.U.S. arrays each designed to a specific sidelobe level. The advantage of the synthesis technique used is that the sidelobes can be designed exactly to the prescribed level. The arrays presented are practical and the method used for synthesis also permits an array to be designed with a prescribed average inter-element spacing. Since the locations and amplitudes of all sidelobes are computed at every cycle in the iteration, the process is, in general, convergent even though an occasional ill condition matrix may be encountered. At times, however, the computation can go into an endless loop but this problem can simply be rectified by randomly disturbing the spacings manually and then re-feed into the computer for optimisation. Another advantage of the synthesis technique used is that the end result is largely independent of the type of array used to start off the computation. A poor starting array tends only to increase the computation time required.
The analysis of the results in Chapter III has brought out several interesting features about the properties of the synthesised arrays. Firstly, the arrays have, in general, near to optimal sidelobe level; the optimal level being estimated from the beamwidth and pattern gain of the array, and the assumption of an equal ripple response in the sidelobe region. Secondly, the beamwidths of the arrays are, as a whole, widened by the adoption of non-uniform element spacings in the arrays. This shows that space tapering in N.U.S. arrays does have a similar effect on the beamwidth of the array as current tapering in a conventional uniformly spaced array. Unlike the uniformly spaced arrays, the computed pattern gain varies only slightly for different average inter-element spacing in the synthesised arrays. The pattern gain is approximately equal to \( MG_0 \) where \( M \) is the total number of elements in the array and \( G_0 \) the gain of the element with the pattern in the plane normal to the line of the array assumed isotropic.

The synthesised arrays are economically most attractive at higher sidelobe levels where up to 3 or 4 times saving in the number of elements in the array
can be achieved. The saving in cost will expectedly be greater than the saving in the number of elements. At low sidelobe levels, the advantages of the N.U.S. arrays over the corresponding current tapered arrays are small. Also the effects of mutual couplings between elements are expected to be high at low design sidelobe level. It is difficult to synthesise practical N.U.S. arrays to have sidelobe levels more than 20 dB below the main beam level.

The experimental array has shown the practical application of a N.U.S. array of directional elements. By decreasing the number of elements used in the array, the feed system is largely simplified. Beam steering is slightly complicated by the adoption of non-uniform spacings in the array, but this is largely overcome through the use of a table of computed phase delays. Within the limit of facilities and time available, the results obtained from the experimental array agree reasonably well with theoretical expectation.

5.1 Possible Approaches for Future Work

For arrays with less than 50 elements, the synthesis procedure adopted in this thesis has been very effective and
it is not expected that any other method will give pattern characteristics significantly better than those achieved in this thesis. For larger arrays, the 'load on call' facilities will have to be used in the IBM 1130 computer in order to prevent core storage overflow. If synthesis of very large arrays is attempted, the extremely large number of computations required to compute the locations and levels of all sidelobes in the array makes the method extravagant of computer time. Thus if arrays much larger than 50 elements are required, an attempt will have to be made towards finding a suitable position generating function which will generate the element positions such that the pattern characteristic is acceptable. Alternately, a statistical approach can be attempted.

Another approach which may also be suggested is to investigate the possibilities of relaxing the restraint of constant excitation in the array synthesis. The author, however, has doubts whether this will bring about significant advantage over the assumption of constant excitation as used in this thesis. Firstly, for a fixed size array, the matrix size will be increased by a factor of four to include the additional current perturbations. In other
words, for a given amount of core storage in the computer, the introduction of current perturbation will tend to decrease the maximum size of array the program can accommodate. Secondly, the introduction of current variation in the array will tend to decrease the pattern gain which in turn increases the theoretical optimal sidelobe level of the array. Thus no significant improvement to the array pattern is expected from the relaxation of the constraint on the excitation. Lastly a strong objection arises from practical consideration. The feed system will be greatly complicated if a non-uniform excitation function is prescribed in the array. The need to incorporate a mismatching or attenuating network to each antenna increases not only the capital cost of the feed system but also the maintenance cost of the array as a whole. Besides, any mismatching will decrease the electrical efficiency of the array and, in the case of an array for radar astronomy, the power handling capacity of the system.
APPENDIX A

On Mutual Coupling in N.U.S. Arrays

One of the problems with practical N.U.S. arrays arises from the mutual coupling between elements. In a long uniformly spaced array, each element can be considered to have the same coupling environment. Thus mutual coupling may lower the array efficiency but has little effect on the pattern of the array. On the other hand, in an N.U.S. array, every element has a different coupling environment. Consequently, feed point impedances will vary from element to element. Unless means are incorporated in the feed system to compensate for such impedance variations, mutual coupling can affect the pattern of the synthesised array. In arrays used for radio astronomy at meter and decameter wavelengths, the amplitude and phase of the excitation in each element are usually adjusted to the prescribed values in the presence of all the other elements. Hence the importance of mutual coupling in a radio astronomy is considerably less than in the case of a phased scanning array for radar applications. However, when directional elements are used in an N.U.S. array, care must be taken to ensure that the average inter-element spacing in the array is sufficiently large to avoid "supergain effects". The higher the element directivity, the larger must be the average inter-element spacing. Because of the
absence of grating lobes in the array factor, non-uniformly spaced arrays are ideally suited for use with directional elements if the synthesis procedure can accommodate large inter-element spacings. This is the characteristic feature of the synthesis procedure developed in this thesis.

In the field of array synthesis, mutual coupling has been universally neglected for the obvious reason that the synthesis problem becomes immensely complicated if mutual coupling were to be taken into account. The pattern of an array with mutual coupling is, in the most general form,

$$ F(\theta) = \sum_{n=-N}^{N} F_n(s_{nm}, \theta) \cdot e^{jk d_n \sin \theta} $$

where $s_{nm}$ is the spacing between the nth element and the mth element, i.e. m has values between -N and N but $m \neq n$. It is possible to compute the exact pattern of an array with non-uniformly spaced, infinitely thin dipole elements using the well known Carter formula to compute the mutual impedances between the elements. It is possible to compute the exact pattern of an array with non-uniformly spaced, infinitely thin dipole elements using the well known Carter formula to compute the mutual impedances between the elements. The process involves the solution of a set of $2N \times 2N$ simultaneous equations to determine the values of $F_n(s_{mn}, \theta)$. However, the synthesis problem even for this simplest case is immensely complex. Allen (1967) proposed an empirical formula for $F_n(s_{mn}, \theta)$ for the case where the element consists of a half-wave dipole placed $\frac{\lambda}{4}$. 
above a conducting ground plane, Allen's empirical formula is

\[ F_n(s_{mn}, \theta) = \frac{D_n/\lambda}{1 + D_n/\lambda \left(1 + |\sin \theta|\right)} \]

where \( D_n \) is the spacing between the \( n \)th element and the \( n-1 \) element. Essentially, Allen has assumed that \( s_{mn} \) is significant only when \( m = n - 1 \). Unsuccessful attempts were made by Allen to synthesise N.U.S. array using this grossly simplified assumption.

Mutual coupling affects the pattern of a N.U.S. array of directional elements in two ways. The first effect arises from the fact that there are stronger couplings among elements near the centre of the array than among elements away from the array centre. From Allen's experiment with a 16-element dipole array\(^{56}\), the effect of the non-uniform coupling is to increase the sidelobe level close to the main beam. As was mentioned earlier, such a problem less likely to affect a radio astronomy array since the prescribed excitation is usually achieved in the actual array by matching networks.

The second effect arises when the elements are positioned very close together so that it is no longer valid to assume the element directivity in the pattern synthesis.
There is at present no well defined criterion to estimate the minimum average inter-element spacing permitted in a N.U.S. array for a given element directivity. The author has used a criterion based on the estimated pattern gain of the array (Section 3.3). The estimated pattern gain neglecting mutual coupling is \( G = MG_0 \), where \( M \) is the total number of elements in the array and \( G_0 \), the 2-D pattern gain of the element. From a well known theorem due to Taylor\(^{11}\), the maximum gain a line source of length \( L \) can have without assuming a super-gain aperture is \( 2L/\lambda \). Thus, in a practical array,

\[
G < \frac{2L}{\lambda}
\]

With the element pattern as assumed in this thesis, the pattern gain \( G = 1.72M \). In a long array, \( L \approx M\overline{d}_{av} \), where \( \overline{d}_{av} \) is the average inter-element spacing in the array. It follows from substitutions into \( A2 \) that the average inter-element spacing in the synthesised array must at least be \( 0.86\lambda \) if the array performance were to be practically realisable.

A quick scan through Table III will show that excepting 16(e) and 16(f) the average spacing of the synthesised arrays are at least \( 1\lambda \).

The above criterion has been found consistent with the experimental evidence of King and Peter\(^ {58}\) and that of Allen\(^ {56}\).
King and Peters experimented with 5-element uniformly spaced arrays of 6λ polyrod elements. The free space directive gain of each element is 15.7 db. By the criterion used by the author, the minimum permissible element spacing in such an array is 2.1λ. The results of King and Peters show considerable broadening of the element beamwidth when the spacing is 2λ, but only small deterioration in the element pattern when the spacing is 1.5λ. This is consistent with theoretical expectation. Allen experimented with a 16-element N.U.S. array of dipole elements backed by a conducting ground plane. The directivity of an isolated element is thus approximately 4.5 db above isotropic. The minimum average inter-element for the array from the assumed criterion is .58λ. Allen's array has an average spacing of .64λ and a minimum spacing of .5λ. Allen reported serious increase in the levels of the sidelobes immediately adjacent to the main beam and small deterioration in the pattern away from the main beam. This tends to indicate that the pattern deterioration is due mainly to the non-uniform coupling in the array and not to the broadening of the element pattern. The non-uniform coupling can be compensated in the feed system of a radio astronomy array.
APPENDIX B

1 Harrington Perturbation Technique

The initial array used in the synthesis of the 16 element experimental array was obtained by a perturbation technique proposed by R.F. Harrington\textsuperscript{19}. Basically this method is useful for suppression of sidelobes close to the main beam. Its effectiveness is superseded by the matrix perturbation technique used for the majority of the synthesis work discussed in this thesis.

The starting array used by Harrington was the uniformly spaced, uniformly excited array. The space pattern of such an array is, assuming isotropic elements,

\[ F(u) = \frac{2}{M} \sum_{n=1,3}^{M-1} \cos(\nu/2) \]  \hspace{1cm} B1

where \( M \) equals the total number of elements in the array. \( M \) is assumed even in this case.

Supposing that the biggest sidelobe next to the main beam is to be suppressed. Let the desired new pattern, \( F_0(u) \), be everywhere the same as \( F(u) \) except when \( u \) is near \( u_k \), the position of the lobe to be
suppressed. (fig. 32a) Hence the difference between $F(u)$ and $F_c(u)$ will be a small pulse (fig. 32b) centred at $u = u_k$. Assuming that this new pattern can be obtained by shifting each element position from its position in the uniform array by an amount $\varepsilon_n$, where $\varepsilon_n$ can assume positive or negative values,

$$F_c(u) = \frac{2}{M} \sum_{n=1,3}^{M-1} \cos((n/2 + \varepsilon_n)u)$$  \hspace{1cm} (B2)

and

$$F(u) - F_c(u) = \frac{2}{M} \sum_{n=1,3}^{M-1} [\cos(nu/2) - \cos((n/2 + \varepsilon_n)u)]$$

$$= \frac{2}{M} \sum_{n=1,3}^{M-1} 2\sin(\varepsilon_n u/2) \sin((n + \varepsilon_n)u/2)$$ \hspace{1cm} (B3)

If $\varepsilon_n u$ is small, which makes the method effective only for close in sidelobes,

$$F(u) - F_c(u) = \frac{2}{M} \sum_{n=1,3}^{M-1} \varepsilon_n u \sin(nu/2)$$ \hspace{1cm} (B4)

i.e.

$$\frac{M}{2} \frac{F(u) - F_c(u)}{u} = \sum_{n=1,3}^{M-1} \varepsilon_n \sin(nu/2)$$ \hspace{1cm} (B5)
Fig. 32a  The Uniformly Spaced Array Pattern, $F(u)$; and the Desired Pattern, $F_c(u)$

Fig. 32b  The Function $F(u) - F_c(u)$

It can be seen that the R.H.S. of equation B5 is in the form of a Fourier series expansion. Using the standard method of extracting Fourier components,

$$
\varepsilon_n = \frac{M}{\pi} \int_{0}^{\pi} \left[ \frac{(F(u) - F_c(u))}{u} \right] \sin(nu/2) \, du \quad B6
$$
To enable computation of $\varepsilon_n$ from equation B6, Harrington assumes the function in fig. 32b to be an impulse function of amplitude $a_k$ and located at $u = u_k$, i.e.,

$$F(u) - F_c(u) = a_k \delta(u - u_k)$$

where $\delta$ is the Dirac delta function. Substituting the above into equation B6 and evaluating the integral, yields

$$\varepsilon_n = \frac{M}{\pi} \cdot \left(\frac{a_k}{u_k}\right) \cdot \sin\left(nu_k/2\right)$$  \hspace{1cm} B7

Through the use of equation B7, an N.U.S. array with arbitrarily low sidelobes close to the main beam can be generated. It is a simple matter to compute the perturbations $\varepsilon_n$ from equation B7 using a digital computer. The program is not included in this thesis.

2. Maffett's Quadratically Non-Uniform Array'

An array which gives reasonable pattern with directional elements and whose element positions can be computed very simply is the 'quadratically non-uniform array', which was first developed by A.L. Maffett$^{20}$. Maffett's approximate integral method has been briefly discussed in section 1.6 under the historical survey of N.U.S. array synthesis. The 'quadratically non-uniform
array' is derived by approximation to the pattern of a linearly tapered line source. It is well known from the Fourier Transform theory that the space pattern of such a distribution is the square of the space pattern of a line source with uniform distribution. The linearly tapered distribution function is assumed to be

\[ h(x) = \frac{2}{L} \cdot (1 - 2x/L) \quad 0 \leq x \leq L/2 \quad \text{B8} \]

where \( L \) is the total length of the array and symmetry about the array centre is assumed. The cumulative current distribution computed from equation 1.13 is

\[ y(x) = \frac{1}{2} + 2x/L - 2x^2/L^2 \quad 0 \leq x \leq L/2 \quad \text{B9} \]

The application of trapezoidal rule to the integral in equation 1.14 results in an N.U.S. array pattern (equation 1.15) which is an approximation to the pattern of the linearly tapered line source, at least for a limited range of \( \alpha \). The element positions for this N.U.S. array are obtained by inversion of equation B9. In this case, the inversion is simple for it involves a simple solution to a quadratic equation.
Thus

\[ x_n = \frac{L}{2} \left[ 1 - \sqrt{\left(1 - y_n^2\right)} \right], \quad \frac{1}{2} \leq y_n \leq 1 \]

\[ = \frac{L}{2} \left[ 1 - \sqrt{1 - \frac{n}{N}} \right], \quad n = 1, 2, \ldots, N \quad \text{B10} \]

Equation B10 gives the element positions of the 'quadratically non-uniform array' and these positions are obviously extremely easy to generate. From the pattern plotted by Maffett, it can be seen that such arrays are ideally suited for applications where directional elements are used. Consequently, these arrays are used as starting arrays for the synthesis of all the arrays presented in Chapter III.
APPENDIX C

Computer Programs

1. Systematic Optimisation Program

This program was used to synthesise the 16 element experimental array. Its effectiveness had since been superseded by the matrix perturbation technique, which was discussed in Section 2.2. A discussion on the Systematic Optimisation Program was given in 2.1. The program was originally used on the IBM 1620 digital computer, but the form presented here is designed to be used with the IBM 1130 computer.

Name: OPTIM

Meanings of variables:

D(I) - The distance in terms of wavelengths, of the Ith element from the array centre.

FI - The initial value of u to be used in the computation of the sidelobe level.

DU - The increment in u used to scan the pattern in the sidelobe region.
FMAX - The new maximum sidelobe level.

PK - The maximum sidelobe level before perturbation.

NUM - Half the total number of elements in the array.

J - The number of the element in the array on which the perturbation is to be effected.

M - A number, the value of which directs the computer whether or not to perturb the element in the reverse direction. M = 0, yes; otherwise no.

Program

C OPTIMISATION BY SYSTEMATIC ADJUSTMENTS OF ELEMENTS

DIMENSION D(32)

READ(2,20) NUM, J, DELTA, FI, DU

20 FORMAT (2I3, 3F8.4)

READ IN AND TYPE OUT POSITIONS OF INITIAL ARRAY

READ (2,1) (D(I), I=1,NUM)

1 FORMAT (8F8.4)

WRITE (1,2)

2 FORMAT (18H INITIAL SPACINGS.)

WRITE (1,1) (D(I), I=1,NUM)
C COMPUTE AND TYPE OUT MAXIMUM SIDELOBE LEVEL.
CALL PAT(D,FI,NUM,DU,FMAX)
PK=FMAX
WRITE (1,5) PK
5 FORMAT (4H PK=, F8.4)
C COARSE PERTURBATION IN THE FORWARD DIRECTION.
DO 64 I=J,NUM
   M=0
52 D(I)=D(I)+DELTA
   CALL PAT(D,FI,NUM,DU,FMAX)
   IF (FMAX-PK) 53,53,54
53 PK=FMAX
   GO TO 52
54 D(I)=D(I)-DELTA
C FINE PERTURBATION IN THE FORWARD DIRECTION
55 D(I)=D(I)+DELTA/10.
   CALL PAT(D,FI,NUM,DU,FMAX)
   IF (FMAX-PK) 56,56,57
56 PK=FMAX
   M=M+1
   GO TO 55
57 D(I)=D(I)-DELTA/10.
   IF (M) 58,58,64
C COARSE PERTURBATION IN THE REVERSE DIRECTION IF M=0

58 D(I)=D(I)-DELTA
   CALL PAT(D,FI,NUM,DU,FMAX)
   IF (FMAX-PK) 59,59,60

59 PK=FMAX
   GO TO 58

60 D(I)=D(I)+DELTA

C FINE PERTURBATIONS IN THE REVERSE DIRECTION

61 D(I)=D(I)-DELTA/10.
   CALL PAT(D,FI,NUM,DU,FMAX)
   IF (FMAX-PK) 62,62,63

62 PK=FMAX
   GO TO 61

63 D(I)=D(I)+DELTA/10.

64 WRITE(1,65) I, D(I), PK
65 FORMAT (I2, 2F8.4)
   CALL EXIT
   END

C SUBROUTINE PAT(D,FI,NUM,DU,FMAX)

DIMENSION D(32)

U=FI
FMAX=0.
TOTCR=NUM
2 PATT=0.
   B=6.28318*U
   DO 3 N=1,NUM
3 PATT=PATT+COS(B*D(N))
   IF (U-.92) 9,10,10
9 PATT=(1.-U*U)*PATT/TOTCR
   GO TO 11
10 PATT=2.*PATT/TOTCR
11 PATT=ABS(PATT)
   IF (PATT-FMAX) 14,14,13
13 FMAX=PATT
14 IF (U-1.) 15,16,16
15 U=U+DU
   GO TO 2
16 RETURN
END

2 Simultaneous Perturbations to Element Positions using the Matrix Approach.

This program is considerably more effective and
versatile than the previous program 'OPTIM'. A discussion of the derivation of the program is given in Section 2.2 of this thesis.

Name: TXOPT

Meanings of Variables

D(I) - The distance, in wavelengths, of the Ith element from the array centre.

C(I) - The excitation to the Ith element.

P(I) - The level of the Ith sidelobe.

PM(I) - The level of the Ith highest sidelobe.

UM(I) - The location of the peak of the Ith sidelobe.

DMU(I,1) - The computed perturbation to the Ith element. The number 1 signifies that DMU is a column matrix.

A(I,J) - An element of the square matrix.

UMAX(I) - The location of the Ith highest sidelobe.

PMEAN - The design sidelobe level.

DU - The increment in the space variable u used in the peak determination sub-program, DETPK.
F - A variable used to compute for the pattern level at a given u.

PMAX - A variable used to select a set of biggest sidelobes in the pattern.

DET - The value of the determinant of the square matrix. The determinant is computed to check the possible occurrence of an ill-condition matrix.

DXMAX - The maximum value of computed perturbation to the element positions.

N - The number of independent position variables in the array.

MAX - The maximum number of sidelobes in the 'visible space'.

Built-in facilities in the program.

Since the time taken in some of the synthesis work can be very long, some facilities must be designed into the program to make the running automatic, to suppress intermediate outputs if necessary and to make the computer type out the results in the core before the program is stopped. Once the design sidelobe level is prescribed, the computer will
keep perturbing the element positions until the peak
sidelobe level in the array is sufficiently close to the
design sidelobe. When this situation occurs, the computer
will decrease the design sidelobe level, PMEAN, if this
is not smaller than .1, by a value .01 and computations
will then continue to design an array to the new value
of PMEAN. At the same time, the computed positions for
the previous value of PMEAN are typed out. In this
way, a family of arrays designed to various sidelobe levels
can be generated without any manual attention to the
computer.

For overnight running, it is desirable to suppress
intermediate output, but before the program is stopped, it
is necessary to know the element positions arrived at in
the core storage of the computer, otherwise hours of
valuable computation time can be wasted. Hence the computer
is made to sense data switch 1 at every cycle. If switch
1 is on, the intermediate outputs are typed out.

The program can go into an endless loop sometimes,
but this is easily cured by randomly disturbing the element
positions slightly and then re-feed the modified array
spacings into the computer for optimisation.
Program

C SIMULTANEOUS PERTURBATIONS BY MATRIX TECHNIQUE
DIMENSION D(32), C(32), P(80), DMU(32,1), A(32,32), UM(80), PM(32)
DIMENSION UMAX(32)
READ(2,1) N, DU, PMEAN
1 FORMAT (I4, 2F8.4)
READ (2,2) (D(I),I=1,N)
READ (2,2) (C(I),I=1,N)
WRITE(1,16) PMEAN
16 FORMAT (29H PEAKS TO BE REDUCED TO LEVEL, F8.4)
WRITE(1,101)
101 FORMAT(16H INITIAL SPACING)
WRITE(1,2) (D(I),I=1,N)
2 FORMAT(8F8.4)

C COMPUTE LOCATIONS OF PEAKS IN THE SIDELobe REGION.
CN=N
8 CALL DETPK(D,DU,MAX,N,C,P,UM)

C COMPUTE LEVELS OF ALL PEAKS IN THE PATTERN.
DO 4 I=1,MAX
  F=0.
  DO 3 J=1,N
    3 F=F+Cos(6,28318*UM(I)*D(J))
P(I)=F*(1.2-UM(I)*UM(I))/1.2/CN
40 P(I)=0.
4 CONTINUE
C THE LOCATIONS AND POSITIONS OF N BIGGEST LOBES ARE SELECTED.
DO 11 I=1,N
   PMAX=0.
   DO 10 J=1,MAX
      IF(ABS(P(J))-PMAX) 10,10,9
10 JJ=J
      PMAX=ABS(P(J))
   CONTINUE
   PM(I)=P(JJ)
   UMAX(I)=UM(JJ)
11 P(JJ)=0.
C THE MAXIMUM SIDELOBE LEVEL IS COMPARED WITH THE DESIGN LEVEL.
   IF(ABS(PM(1)) -PMEAN-.0002) 26,181,181
C COMPUTE THE ELEMENTS OF COEFFICIENT MATRIX.
181 DO 19 I=1,N
   DO 19 J=1,N
19 A(I,J)=SIN(6.28318*D(J)*UMAX(I))
C THE DESIRED PATTERN PERTURBATIONS COMPUTED.
   DO 20 I=1,N
DPK = ABS(PM(I)) - PMEAN
DMU(I, 1) = PM(I)/ABS(PM(I)) * DPK

20 DMU(I, 1) = DMU(I, 1) * 1.2 * CN / (6.28318 * UMAX(I) * (1.2 - UMAX(I) * UMAX(I)))

C MATRIX INVERSION TO SOLVE SIMULTANEOUS EQUATIONS.
CALL INVRT(A, N, DMU, 1, DET)

C COMPUTED PERTURBATIONS LIMITED TO A MAXIMUM OF .2 IF NECESSARY.
DXMAX = 0.

DO 202 I = 1, N
IF (ABS(DMU(I, 1)) - DXMAX) 202, 202.201

201 DXMAX = ABS(DMU(I, 1))
202 CONTINUE
IF (DXMAX - .2) 205, 205, 203

203 DO 204 I = 1, N
204 DMU(I, 1) = DMU(I, 1) / DXMAX * .2

C COMPUTED PERTURBATIONS ADDED TO ELEMENT POSITIONS

205 DO 22 I = 1, N
22 D(I) = D(I) + DMU(I, 1)

C SENSE DATA SWITCH 1; PRINT OUT IF ON, CONTINUE ITERATION IF OFF.
CALL DATSW(1, MSW)
IF (MSW - 1) 8, 220, 8

220 WRITE(1, 12) N

12 FORMAT (I4, 23H BIGGEST SIDELOSES ARE)
WRITE(1, 13) (PM(I), I = 1, N)
13 FORMAT (12F8.4)
   WRITE(1,221) DET
221 FORMAT (5H DET=, F10.4)
   WRITE(1,23) (D(I),I=1,N)
23 FORMAT (14H NEW SPACINGS,/12F8.4)
   GO TO 8
26 IF(PMEAN-.095) 27,260,260
260 PMEAN=PMEAN-.01
   WRITE(1,16) PMEAN
   WRITE(1,2) (D(I),I=1,N)
   GO TO 181
27 STOP
END

2.1 The Peak Determination Subroutine

The proper functioning of the matrix perturbation program TXOPT depends critically on the accurate locations of the maxima in the pattern function. The peak determination sub-program developed for this purpose is capable of locating the peak of each sidelobe such that either the slope of the pattern function at that point is equal to or less than .001 or that the range over which the peak occurs is narrowed
down to a value of 0.00001. A discussion on the development of this program is given in Section 2.2.3.

Name of the Subroutine: DETPK
Meanings of variables: (Only variables not common with TXOPT are listed below)

\[ B(I) = 2\pi \cdot D(I) \]

\[ UA(I) \quad - \quad \text{One end of the u-range, within which the Ith peak of the pattern occurs. The width of the u-range is DU,} \]

\[ UB(I) \quad - \quad \text{The other end of the u-range within which the Ith peak of the pattern occurs. UB(I) > UA(I)} \]

\[ FUA(I) \quad - \quad \text{The value of the slope of the field pattern at } u = UA(I). \]

\[ FUB(I) \quad - \quad \text{The value of the slope of the field pattern at } u = UB(I). \]

\[ TOTCR \quad - \quad \text{The sum of the excitations in the array. The sub-program DETPK has been designed to be operative for arrays with both non-uniform spacings as well as non-uniform excitations. A uniform excitation, however, was assumed in the program TXOPT.} \]
A - A variable used to save excessive repetition in computation.

SFA - The slope of the field pattern at u.

SFB - The slope of the field pattern at u + DU.

SF - The slope of the field pattern at u = (UA(I)+UB(I))/2.

PSF - The product of SFA and SFB.

M - A fixed point variable used to count the number of sidelobes in the pattern.

Program:

```
SUBROUTINE DETPK(D,DU,MAX,N,C,P,UM)

DIMENSION D(32), B(32), P(80), UA(80), UB(80), FUA(80), FUB(80), UM(80)

DIMENSION C(32)

TOTCR=0.

DO 1 I=1,N
   TOTCR=TOTCR+C(I)
1   B(I)=6.28318*D(I)
   SF1=0.
   SF2=0.
   M=0

C NEXT STATEMENT DEFINES THE INITIAL VALUE OF U.
```
U = 0.5/D(N)

C COMPUTATION OF SFA

DO 10 I = 1, N
    A = B(I) * U
    SF1 = SF1 + C(I) * COS(A)
10    SF2 = SF2 + C(I) * B(I) * SIN(A)
    SFA = 2.0 * U * SF1 + (1.0 - U * U) * SF2

C COMPUTATION OF SFB

U = U + DU

11 SF1 = 0.

SF2 = 0.

DO 20 I = 1, N
    A = B(I) * U
    SF1 = SF1 + C(I) * COS(A)
20    SF2 = SF2 + C(I) * B(I) * SIN(A)
    SFB = 2.0 * U * SF1 + (1.0 - U * U) * SF2

C THE SIGNS OF THE TWO SLOPES ARE COMPARED.

PSF = SFA * SFB

IF (PSF) 30, 30, 40

30 M = M + 1

MAX = M

C THE TOTAL NUMBER OF SIDELOBS SCANNED IS LIMITED TO 80.

IF (MAX < 80) 302, 302, 301
301 GO TO 50

C DEFINES THE RANGE WITHIN WHICH A ROOT OCCURS.

302 UA(M)=U-DU
UB(M)=U
FUA(M)=SFA
FUB(M)=SFB

40 SFA=SFB
U=U+DU
IF(U-1.) 11,50,50

C THE REST OF THE STATEMENTS LOCATE ACCURATELY THE ROOTS WITHIN THE DEFINED RANGES.

50 DO 60 J=1,MAX

C COMPUTATION OF THE SLOPE AT THE MID-POINT OF A DEFINED U-RANGE.

51 U=(UA(J)+UB(J))/2.
SF1=0.
SF2=0.
DO 52 I=1,N
A=B(I)*U
SF1=SF1+C(I)*COS(A)
52 SF2=SF2+C(I)*B(I)*SIN(A)
SF=2.*U*SF1+(1.2-U*U)*SF2

C SLOPE OF .001 CONSIDERED CLOSE ENOUGH TO DETERMINE ROOT.
IF (ABS(SF)-.001) 59,59,53
C IF THE U-RANGE IS EQUAL TO OR LESS THAN .000005, ROOT IS LOCATED.

53 IF (UB(J)-UA(J)-.000005) 59,59,531

531 IF (FUA(J)*SF) 54,55,56

C U-RANGE HALVED ACCORDING TO THE SIGN OF SF.

54 UB(J)=U
   FUB(J)=SF
   GO TO 51

55 UM(J)=UA(J)
   GO TO 60

56 UA(J)=U
   FUA(J)=SF
   GO TO 51

59 UM(J)=U

60 CONTINUE
   RETURN
   END

2.2 Matrix Inversion Subroutine

Name: INVRT

Note: This program is taken from the book 'Numerical Methods and Computers' by S.S.Kuo\textsuperscript{45}. A listing of the program is only given here. For details and meanings of variables, the readers are

advised to refer to page 168 of the reference book.

Program:

C MATRIX INVERSION AND SIMULTANEOUS EQUATIONS SUBROUTINE
DIMENSION A(32,32), B(32,1), IPVOT(32), PIVOT(32), INDEX(32,2)
EQUIVALENCE (IROW, JROW), (ICOL, JCOL)

57 DET=1.
   DO 17 J=1,N
17 IPVOT(J)=0
   DO 135 I=1,N
      T=0.
      DO 9 J=1,N
         IF (IPVOT(J)=1) 13,9,13
13 DO 23 K=1,N
      IF (IPVOT(K)=1) 43,23,81
43 IF(ABS(T)-ABS(A(J,K))) 83,23,23
83 IROW=J
   ICOL=K
   T=A(J,K)
23 CONTINUE
9 CONTINUE
   IPVOT(ICOL)=IPVOT(ICOL)+1
IF (IROW-ICOL) > 3, 109, 73
73 DET = --DET
   DO 12 L = 1, N
   T = A(IROW, L)
   A(IROW,L) = A(ICOL, L)
12   A(ICOL, L) = T
   IF(M) 109, 109, 33
33   DO 2 L = 1, M
   T = B(IROW, L)
   B(IROW, L) = B(ICOL, L)
2    B(ICOL, L) = T
109 INDEX(I, 1) = IROW
    INDEX(I, 2) = ICOL
    PIVOT(I) = A(ICOL, ICOL)
    DET = DET * PIVOT(I)
    A(ICOL, ICOL) = 1.
   DO 205 L = 1, N
205   A(ICOL, L) = A(ICOL, L) / PIVOT(I)
   IF (M) 347, 347, 66
66   DO 52 L = 1, M
52    B(ICOL, L) = B(ICOL, L) / PIVOT(I)
347   DO 135 LI = 1, N
135    IF (LI-ICOL) > 21, 135, 21
21 T=A(LI, ICOL)
    A(LI, ICOL)=0.
    DO 89 L=1, N
89 A(LI, L)=A(LI, L)-A(ICOL, L)*T
    IF(M) 135, 135, 18
18 DO 68 L=1, M
68 B(LI, L)=B(LI, L)-B(ICOL, L)*T
135 CONTINUE
    DO 3 I=1, N
    L=N-I+1
    IF (INDEX(L, 1)-INDEX(L, 2)) 19, 3, 19
19 JROW=INDEX(L, 1)
    JCOL=INDEX(L, 2)
    DO 549 K=1, N
    T=A(K, JROW)
    A(K, JROW)=A(K, JCOL)
    A(K, JCOL)=T
549 CONTINUE
    3 CONTINUE
81 RETURN
END
Pattern Study Program

This program computes the voltage pattern of the given N.U.S. array at the prescribed intervals in u-space. It also performs pattern integration to compute for pattern gain. The sidelobe level of the corresponding equal ripple type pattern is also computed and compared with the actual sidelobe level of the array.

Since the important variables are all referenced by H-Format specifications, no explanation of symbols is given for this program.

Name: PTEST

Program Listing:

```
DIMENSION D(32), DN(32), DEL(32), PATTN(600)
10 READ(2,1) N,NODU
   IF(N) 16,100,16
16 READ(2,2) (D(I),I=1,N)
   DO 11 I=1,N
11 DN(I)=D(I)/D(N)
   DEL(1)=D(1)*2.
   DO 12 I=2,N
   IA=I-1
```
12 \text{DEL}(I) = \text{D}(I) - \text{D}(IA) \\
\text{AN}=2*\text{N} \\
\text{R} = \text{AN} * 1.72 / (1. - \text{AN} * .43 / \text{D}(\text{N})) \\
\text{RV} = \text{SQRT}(2. / \text{R}) \\
\text{RLOG} = \text{ALOG}(\text{R} / 2.) / .2303 \\
\text{WRITE}(3, 13) \\
13 \text{FORMAT (17H ARRAY SPACINGS = )} \\
\text{WRITE}(3, 2) (\text{D}(I), I=1, \text{N}) \\
\text{WRITE}(3, 3) \\
3 \text{FORMAT (31H NORMALISED ELEMENT POSITIONS = )} \\
\text{WRITE}(3, 2) (\text{DN}(I), I=1, \text{N}) \\
\text{WRITE}(3, 4) \\
4 \text{FORMAT (24H INTERELEMENT SPACINGS = )} \\
\text{WRITE}(3, 2) (\text{DEL}(I), I=1, \text{N}) \\
\text{WRITE}(3, 5) \text{RV} \\
5 \text{FORMAT (33H THEORETICAL VOLTAGE S.L. LEVEL =, F8.4) } \\
\text{WRITE}(3, 6) \text{RLOG} \\
6 \text{FORMAT (23H SIDELobe SUPPRESSION =, F8.4, 4H DB.) } \\
\text{DUNO} = \text{NODU} \\
\text{U} = 0. \\
\text{DU} = 1. / \text{DUNO} \\
\text{BN} = \text{N} \\
\text{U1} = 1. / \text{D}(\text{N})
SUM=0.
PK=0.
DO 8 I=1,NODU
   U=U+DU
   PATTN(I)=0.
DO 7 J=1,N
7   PATTN(I)=PATTN(I)+COS(6.2832*U*D(J))
   PATTN(I)=PATTN(I)*(1.2-U*U)/1.2/BN
IF (U-U1) 21,19,19
19  IF (ABS(PATTN(I))=PK) 21,20,20
20  PK=ABS(PATTN(I))
21  A=3.0-(-1.0)**I
8   SUM=SUM+A*PATTN(I)*PATTN(I)
     SUM=(SUM+1.0-PATTN(NODU))*DU/3.
     GAIN=1.0/SUM
     G=AN*1.72
     WRITE(3,80) PK
80  FORMAT (27H PEAK SIDELOBE IN PATTERN =, F8.4)
     RATIO=PK/RV
     WRITE(3,81) RATIO
81  FORMAT (17H SIDELOBE RATIO =, F8.4)
     WRITE(3,9) GAIN
9   FORMAT(30H GAIN FROM PATTERN INTEGRATION, F8.3)
WRITE(3,90) G
90 FORMAT (17H ESTIMATED GAIN = , F8.3)
WRITE(3,14) NODU
14 FORMAT (17H PATTERN PLOTTING, I4, 18H NO. OF POSITIONS.)
WRITE(3,15) (PATTN(I),I=1,NODU)
1 FORMAT(2I4)
15 FORMAT (12F8.4)
2 FORMAT (8F8.4)
GO TO 10
100 CALL EXIT
END

4 Program to select phasing cables.

100 graded lengths of cables are assumed to be available. The cables are coded such that cable number 100 is half a wavelength long and cable number 26 is \( \frac{1}{13} \) \( \lambda \) long. The print-outs are as follows:

MDEG - the angle by which the main beam is to be steered from the broadside direction.

NO(I) - the number of the cable to be inserted into the feed to the Ith antenna. (see Section 4.5)
Program Listing

    DIMENSION D(32), NO(64)
    READ(2,1) N
    READ(2,2)(D(I),I=1,N)
    1 FORMAT (I4)
    2 FORMAT(8F8.4)
    WRITE(3,4)
    4 FORMAT (45H BEAM STEERING BY INSERTION OF COAXIAL CABLES)
    WRITE(3,5)
    5 FORMAT (22H ELEMENT POSITIONS ARE)
    WRITE(3,2) (D(I),I=1,N)
    DO 100 J=1,20
       AJ=J
       TETA=3.*AJ*3.*1416/180.
       MDEG=3.*AJ
    DO 99 I=1,N
       CABL=D(I)*SIN(TETA)
       M=CABL
       CABL=CABL-M
       NN=N+1
       NO(NN)=200*CABL
    II=N+1-I
IF (200.*CABL-NO(NN)*.5) 99, 99, 101
101 NO(NN)=NO(NN)+1
99 NO(I)=200-NO(NN)
N2=2*N
100 WRITE(3,3) MDEG, (NO(I),I=1,N2)
3 FORMAT (30I4)
CALL EXIT
END
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PART II

THE DESIGN OF A 42MHZ PHASE SWITCHED INTERFEROMETER
CHAPTER VI

The Techniques of Radio Astronomy

Radio astronomy concerns the study of the radio emissions from celestial sources\(^1\)\(^-6\). Its origin dates back to December 1932, when Karl G. Jansky\(^7\) first reported the detection of radio noise from the direction of the milky way. The complete significance of Jansky's discovery was not realised for about 10 years during which time a lone survey of the sky was made by Grote Reber, using a home made steerable parabolic reflector, 30 ft in diameter. Reber published the first radio map of the sky at 160 Mhz in 1940\(^8\). Since then, radio telescopes have been rapidly set up all over the world, and aided by the advancement of antenna and receiver technology, radio astronomy grew to be a major contributor to the knowledge of astronomy in the present decade.

Radio telescopes capable of ambitious astronomy contributions are invariably very expensive. Indeed, costs up to a few million dollars are not uncommon in many of the better known radio telescopes in the world. Nevertheless, in certain fields, for example the measurements of flux densities of strong radio sources at meter and decameter
wavelengths\textsuperscript{9}, important contributions can still be made with relatively small research expenditure. The radio astronomy programme of the University of Auckland was started with this aim in mind.

A brief discussion on some radio astronomy fundamentals will be given in this chapter. A system study on the Auckland University radio astronomy programme is also presented.

6.1 The Nature of Cosmic Noise

Irrespective of the type of origin, cosmic radio waves have the characteristics of random noise. Apart from a few line emissions, cosmic radiations occur over the entire radio spectrum. A strong clue towards the mechanisms of these radio emissions lies in the study of the spectral variations of the cosmic radiations. Ground based radio astronomy is limited to wavelengths less than 30 meters by ionospheric reflections and greater than 3 cm by atmospheric absorptions.

6.2 The Functions of a Radio Telescope

A radio telescope consists of two parts; the antenna system and the radiometer.
The function of the antenna system is to transform the radiations from the radio sources into noise voltages on transmission lines. By sampling the radiations over a large area, the antenna system provides information concerning the spatial brightness distribution of the radio sky. The resolving power of the antenna system determines the details of the sky brightness that can be studied by the radio telescope.

The function of the radiometer is to measure the noise power received by the antenna system. This noise power is amplified and the desired information displayed on a recorder. The sensitivity of the radio telescope is determined by the radiometer, and the collecting area of the antenna system.

6.3 Spectral Flux Density

The intensity of radiation from a radio source is measured in terms of the power incident on a unit area normal to the direction of incidence. Thus the flux density at any instant of time is

\[ S(\nu, t) = \varnothing(\nu, t) \cdot \beta(\nu, t)/Z_0 \]  

6.1
where $E_0(x,t)$ is the electric field due to a wave incident from the direction of a unit vector $\hat{x}$, and $Z_0$ is the intrinsic impedance of free space.

Because of the stochastic nature of the field, only the time average value of $S(x,t)$ is meaningful. In practice, this average flux density is measured over a narrow frequency band centred about a particular frequency of observation. Thus the intensity of radiation of a radio source is specified by its spectral flux density, which is defined as \(^\text{10}\)

$$S_f(x) = \lim_{\tau \to \infty} \left( \frac{A_f^T(f) \cdot A_f^*(f)}{Z_0} \right)$$ \hspace{1cm} \text{6.2}$$

where $A_f^T(f)$ is the Fourier transform of $E_f(x,t)$, the electric field defined over a time interval $\tau$. In other words, the spectral flux density is defined as the time average spectral power density radiated by a source when the source is assumed to be observed over an infinite period of time.

6.4 System Sensitivity

Although equation 6.2 can be shown, from the theory of stochastic processes, to possess a finite limit, it is not
possible in practice to determine uniquely the spectral flux density of a source because observations can neither be carried out for an infinite length of time nor at a single frequency. As a result, there is always a random fluctuation in the record which determines the minimum flux density detectable by the system. An estimate of the r.m.s. fluctuation can be made from a well known theorem in statistics. Assuming the predetector bandwidth is $\Delta f$, there will be $\Delta f \cdot \tau$ independent noise pulses over an integration time $\tau$. From statistics, the standard deviation $\Delta R$ of the mean output level $R$, is given by

$$\frac{\Delta R}{R} = \frac{1}{\sqrt{\Delta f \cdot \tau}}$$  \hspace{1cm} (6.3)

The noise power from an antenna is best measured in terms of its antenna temperature, $T_a$, which is defined as the temperature to which a matched resistor replacing the antenna must be heated in order to produce the same amount of noise received by the antenna. The minimum detectable change in antenna temperature of the system is, by virtue of equation 6.3,

$$\Delta T_{\text{min}} = K_s T_{\text{sys}}/\sqrt{\Delta f \cdot \tau}$$  \hspace{1cm} (6.4)

where $T_{\text{sys}}$ is the system temperature, and $K_s$, a
sensitivity constant the value of which depends on the type of
receiver used and its mode of operation. \( T_{\text{sys}} = T_a + (\alpha N - 1)T_0 \),
where \( \alpha \) is the loss factor of the transmission line, \( \alpha \geq 1 \);
\( N \), the noise factor of the radiometer and \( T_0 \), the ambient
temperature of the system. Tables of values for \( K_g \) and
\( \tau \) for various types of radiometers and low pass filters are
given in Chapter VII of reference 2.

6.5 Brightness and Brightness Temperature

As with optical astronomy, the intensity of
radiation from a radio source is often expressed by its
brightness, which is defined as the spectral flux density per
unit solid angle of the source. Thus

\[
B(\nu) = \frac{dS(\nu)}{d\Omega}
\]

6.5

The brightness is also expressable in terms of the
brightness temperature of the source which is defined as the
temperature of a thermal black body which will give the same
brightness as the source at the frequency concerned. From
the Rayleigh-Jeans relation \(^2\),

\[
T_b(\nu) = \frac{\lambda^2}{2k} B(\nu)
\]

6.6
6.6 The Role of the Antenna System

It is easy to deduce that the relationship between the total power output of the antenna system and the brightness distribution of the sky is,

\[ P(\hat{r}_0) = A_e \Delta \nu \int_0^\infty g(\hat{r} - \hat{r}_0) \cdot B(\hat{r}) \, d\Omega \]

where \( \hat{r}_0 \) = the direction of a point in the sky where the brightness is to be determined,

\( A_e \) = the maximum effective collecting area of the antenna system,

\( g(\hat{r} - \hat{r}_0) \) = the normalised power pattern of the antenna system.

Thus the power pattern of the antenna system determines how accurately the total power record represents the brightness distribution of the sky. For least confusion in the record, the antenna system must have an extremely sharp main beam response with all secondary responses well below the main beam level. Although good resolution is easily obtained at optical frequencies, this is not the case at radio frequencies since the area over which the radiation need to be sampled to give a prescribed resolution increases proportionately as the wavelength of the observation. Indeed it would have been impossible to achieve good pencil beam resolution at decameter
wavelengths had it not been for the invention of the cross-
type radio telescope by B.Y. Mills\textsuperscript{11} of Australia.

Very often, details of source distributions finer
than that which can be provided by the cross or the dish are
required. One method of obtaining very high resolution is
to seek a set of practical antenna patterns such that the recorder
output bears a mathematically significant relationship with
the actual brightness distribution of the sky. It can be
seen from equation 6.7 that if the antenna pattern is a
rapidly oscillating cosine function, the power record, \( F(r_0) \),
is then a Fourier transform of the brightness distribution of
the sky. Such a pattern can be obtained by using two antennas
spaced many wavelengths apart operating as a phase switched
interferometer\textsuperscript{12}. The interferometer is also an economical
means of discriminating between 'point' source radiations
and gradual background brightness of the sky.

6.7 Mills Cross

At meter and decameter wavelengths, the resolution
requirement often necessitates the antennas to be spread over
a large area (possibly many acres), whereas only a fraction of
this area need be filled for adequate sensitivity\textsuperscript{13}. An
ingenious method of achieving high pencil beam resolution with
the minimal number of antennas was proposed by B.Y. Mills. By cross correlating the noise signals from two long arrays placed in the form of a cross, an effective pencil beam power response pattern, \( g(\theta, \phi) \), is obtained by the principle of pattern multiplication.

\[
g(\theta, \phi) = F_1(c, \phi) \cdot F_2(\theta, \phi)
\]

Since the power pattern in the cross is obtained by the multiplication of a highly directional field pattern with an essentially non-directional pattern, the sidelobe level of the field pattern of each array must be extremely well suppressed. For example, if the response of cross in a direction outside the main beam is to be 100 times less than the response in the direction of the main beam, the sidelobe level of each array must be 40 db below the main beam level. This is an extremely stringent practical requirement which can be achieved only by heavily tapering the excitations to the end elements and by careful adjustment of the excitation to each element.

Thus although the Mills cross is, at present, the most effective and economical method of cataloguing radio sources at meter and decameter wavelengths, it still requires
a substantial capital investment which is not likely to be met by a small research institute.

6.8 The Interferometer

In its most basic form, an interferometer consists of two antennas or arrays of antennas spaced many wavelengths apart and connected to the receiver by a pair of transmission lines. (fig. 33). Such an interferometer is called a simple additive interferometer or a total power interferometer.

![Diagram of a simple additive interferometer]

**Fig. 33** A Simple Additive Interferometer

Assuming the receiver is connected to the antennas by equal lengths of transmission lines, the power pattern of the interferometer is

\[ G(0) = \frac{1}{2} g_0(0) \cdot (1 + \cos(2\pi s \lambda \sin 0)) \]
where \( g_0(\theta) \) is the power pattern of each antenna, and \( s_\lambda \) is the spacings between the antennas expressed in terms of the wavelength of the 'observation'.

In general \( g_0(\theta) \) is a slowly varying function of \( \theta \). Hence the power pattern of the interferometer can be split up into two components:

(i) the slowly varying component, \( g_0(\theta) \) and

(ii) the rapidly varying component \( \frac{1}{2} g_0(\theta) \cos(2\pi s_\lambda \sin \theta) \).

The spatial frequency of oscillation of the latter component is defined by the spacing between the antennas.

Thus the output record of the total power interferometer will consist of two components by virtue of equation 6.7:

(i) the \textbf{average} sky brightness component and

(ii) the \textbf{fringe} component.

The first component is the weighted average of the sky brightness over the primary pattern of each arm of the interferometer. The fringe component is only significant for sources with angular widths less than \( 1/s_\lambda \) radian, since this component is derived from averaging the sky brightness with a highly oscillatory weighting function. (equation 6.7)
The response of a total power interferometer to a source transit is illustrated in figure 34.

![Diagram](image)

**Fig. 34** The Response of a Simple Interferometer to (a) a "Point" Source and (b) a relatively "Broad" Source

Since the flux from discrete radio sources is, at meter and decameter wavelengths, only a small fraction of the total flux within the primary pattern of the total power
interferometer, limited post-detector gain can be used without saturating the output record. Even if the total power component on the recorder can be cancelled by repeated applications of d-c off-sets, the gain stability of the radiometer will impose an upper limit to the total gain of the system. To overcome these limitations, a phase-switched interferometer was proposed by Ryle (1952)\textsuperscript{12}.

6.9 The Phase-switched Interferometer

If the phase of one antenna is periodically reversed, a modulation at switching frequency appears in the r-f stages when a discrete source transits the beams of the interferometer. This modulation can be amplified after the first detector and synchronously detected against the switching signal using a phase sensitive rectifier. The action of the phase switched interferometer can be simply explained as follows.

Consider the case when the phase of the signal from one antenna is reversed by the insertion of an extra half wavelength cable in the transmission path. The power pattern of the interferometer in this state is

\[
g(\theta) = \frac{1}{2} g_0(\theta) \cdot (1 - \cos(2\pi \lambda \sin \theta))
\]
A modulation at the switching frequency occurs at the output of the receiver if there is a difference in antenna temperatures corresponding to the two states of switching. Thus the effective power pattern of the phase-switched interferometer is given by the difference between equation 6.9 and equation 6.10.

\[ g(\theta) = g_0(\theta) \cdot \cos(2\pi s_\lambda \sin \theta) \]  \hspace{1cm} 6.11

As a result of the phase-switching, the effective antenna pattern of the phase-switched interferometer has no slowly varying component. Hence the interferometer is insensitive to the background brightness of the sky. Sources of angular dimensions less than \( 1/s_\lambda \) radians will give significant fringe response in the output record. The response of a phase-switched interferometer to the transit of radio sources is illustrated in fig. 35.

![Fig. 35 Response Pattern of a Phase Switched Interferometer](image-url)
The phase-switched interferometer can also be considered to be a band pass spatial frequency filter by the following considerations. It is well known that the spatial frequency response of an antenna system is given by the auto-correlation function of the aperture distribution. Assuming for simplicity a rectangular distribution for each antenna, the auto-correlation functions of the aperture distributions corresponding to the two states of switching are shown in fig. 36a and 36b respectively. The spatial frequency response of the phase-switched interferometer, according to the action of phase switching, is then the difference between the auto-correlation functions of the two distributions. (Fig. 36c).
Fig. 36  Phase Switched Interferometer as a Spatial Frequency Band-pass Filter

From fig. 36a, it can be seen that the phase-switched interferometer has a spatial frequency band pass characteristics. In contrast, the spatial frequency response of the total power interferometer has both low pass and band pass characteristics as shown in fig. 36a.
The same band pass characteristics as shown in fig. 36c can also be obtained by cross-correlating the aperture distribution of each arm of the interferometer using a correlation radiometer\textsuperscript{15}.

The main advantage of the phase-switched interferometer lies in the fact that there is no response to background brightness so that adequate post-detector gain can be applied to enhance the detection of discrete radio sources. Also, because the discrete source information is carried by the modulations on the r-f noise, the system is relatively insensitive to the slow fluctuations in the total gain of the receiver\textsuperscript{12}.

6.10 Low Frequency Flux Measurements of Discrete Radio Sources

As was discussed in Section 6.7, a detailed catalogue of discrete radio sources at meter and decameter wavelengths is best done through the use of the cross type radio telescope. However, the cost of a cross is usually very high because of the stringent array requirements.

A phase-switched interferometer is more economical when the aim is to measure the flux densities of a few strong discrete radio sources. The number of sources that can be
studied by the interferometer is usually limited by its primary resolution which is determined by the beam solid angle of each array. To have no confusion in the record, only one strong radio source should occur at any time within the envelope pattern of the interferometer. A good estimate of the maximum number of sources that can be 'observed' by the interferometer is the ratio of the solid angle of the 'visible' space to the main beam solid angle of each array.

Interferometers using very simple antennas are often advantageous in the study of strong variable sources like the active sun and the decameter bursts of Jupiter. Because of the relatively broad envelope pattern, these variable sources can be studied over a period of many hours without re-orientation of the element patterns. Also, in the absolute measurements of the flux densities from strong radio sources, simple antenna are desirable because the antenna gain can then be accurately determined.

Phase switching is essential at the frequencies concerned because of the intense background brightness. To illustrate this point, consider Centaurus A, the strongest radio source in the Southern Hemisphere. At 50 MHz, the flux density of Centaurus A is approximately $10^{-22}$ watts/m²/Hz,
immersed in a background of brightness temperature approximately 10,000°K. Assuming a predetector bandwidth of 100 Khz and 10 second post-detector integration time, the minimum detectable change in antenna temperature in a total power system is 10°K. For positive detection, it is desirable that the deflection due to the transit of the radio source be at least 10 times ΔT_{min}, i.e. 100°K. The effective collecting area needed in the antenna system is \( A_e = k\Delta T_a / S \), which is 13.8 m². This represents a gain of only 6.7 db above isotropic, hence an extremely simple antenna system is sufficient. Of course a single antenna with this gain will not distinguish the source because this deflection will then be very gradual and be completely confused by the variation of the sky brightness within the main beam of the antenna. Two such antennas spaced about 20 wavelengths apart can be used as a total power interferometer. Now there will be a fringe component superimposed on the gradual background variation on the record as the source transits the interferometer. As this fringe variation only represents a fluctuation of 200°K in a total system temperature of 10,000°K, a gain fluctuation of 2% will completely obscure the transit of the source. Also any terrestrial interference will tend to obscure this fringe modulation which is only about 2% of the output level.
By introducing phase-switching in one arm of the interferometer, the minimum detectable change in antenna temperature is increased by a factor of 4 so that the gain of each antenna must be increased by 6 db in order to maintain the same sensitivity as the total power interferometer. The significant improvement is that the system can now be made to operate at its maximum sensitivity without the necessity of extreme gain stability; unless, of course, when the gain varies synchronously with the switching frequency. By the principles of cross-correlation, only correlated signals from each antenna result in responses on the recorder. Thus the phase-switched interferometer has a greater capacity to reject terrestrial interference than the total power interferometer. At a relatively small spacing like 20 wavelengths, however, interference of a common origin does get into both antennas resulting in an unwanted response on the recorder.

A phase-switched interferometer was designed by the author using 4 Yagi antennas, two on each end, placed on an east-west baseline of 20 wavelengths. This interferometer was built primarily to provide practical experience within the programme. Results close to expectations were achieved. These are reported in Chapter VII of this thesis.
CHAPTER VII

A 42 Mhz Phase-switched Interferometer

This Chapter discusses the design of a 42 Mhz phase-switched interferometer. This instrument was primarily intended to provide initial 'observational' experience within the University.

A schematic diagram showing the various functional components in the interferometer is given in fig. 37. The author was responsible for the overall system design and the design of the r-f switching and the post-detector circuitry in the system. The pre-amplifiers were designed by G.A. Moyle\textsuperscript{19a} and the receiver by J.R. Irving\textsuperscript{19b}. The frequency adopted was the operating frequency of the receiver, which was designed as a total power radiometer.

Plate III is included to show some of the laboratory facilities available at the time of writing of this thesis. Plate IV shows the complete phase-switching radiometer.

7.1 The Antennas and Feeds

The interferometer was built on an east-west baseline of about 20\(\lambda\). Each arm of the interferometer
Fig. 37 Functions of a Phase-Switched Interferometer
PLATE III

SOME OF THE LABORATORY FACILITIES
PLATE IV

THE PHASE SWITCHED RADIOMETER
consists of two 5-element Yagis giving a gain approximately 15 db above isotropic. As was shown in Chapter VI, such a system would have more than adequate sensitivity for detection of strong radio sources with flux densities between $10^{-23}$ to $10^{-22}$ watts/m$^2$/Hz.

When the 16 element experimental array of non-uniformly spaced elements was erected, it was decided that a convenient interferometer could be formed by using two antennas from each end of the array. Since the spacing between the two antennas forming each arm of the interferometer is approximately $2\lambda$, the envelope pattern of the interferometer consists of three lobes. Another antenna is at present being erected between each pair of antennas in the existing system. The aim is to remove the grating lobes in the envelope pattern so that more sources can be detected without confusion.

The antenna site is approximately 200 yards from the radiometer. The total transmission loss between the antennas and the radiometer is estimated to be 20 db. The noise figure of the radiometer is 6 db, i.e. $N = 4$, so that $\alpha NT_0 = 120,000^oK$. Assuming $T_a = 10,000^oK$, the effect of the transmission loss is to decrease the sensitivity of the system by a factor of approximately 12. This is obviously not acceptable. A pre-amplifier with a gain of 20 db was therefore inserted in
each arm of the interferometer. The pre-amplifiers were
designed and built by G. A. Moyle\textsuperscript{19a}. 

7.2 Phase-switching

The action of phase-switching can be explained as follows. Let the noise voltages from each arm of the interferometer be $V_1$ and $V_2$ respectively. The voltage at the output of the square law detector is, assuming in-phase addition,

$$V_{\text{in}} = K(V_1 + V_2)^2$$ \textit{7.1}

where $K$ is a constant depending on the total gain of the radiometer.

The voltage output at the square law detector due to anti-phase addition is

$$V_0 = K(V_1 - V_2)^2$$ \textit{7.2}

Therefore the switch frequency modulation at the output of the square law detector is

$$V_{\text{mod}} = 4KV_1V_2$$ \textit{7.3}
Thus the amplitude of the modulation is proportional to the product of the r-f voltages from the antennas. When this product is averaged over a period much longer than the period of switching, the result is an analogue record of the cross-correlation function of the r-f noise voltages from each antenna.

7.2.1 Factors Affecting the Results of Phase-Switching

Two practical problems in phase-switching are analysed in this section. The first is the effect of unbalance between the two states of switching and the second, the effect of unbalance in the addition of the signals from the antennas.

The first problem can be caused by the fact that there is more transmission loss in one state of switching than in the other. Alternately, the switching may introduce large impedance variations and, consequently, a variation to the transmitted voltages in the two states of switching. To see the effect of such unbalance, let the in-phase voltage from switched arm of the interferometer be $V_2$ and the anti-phase voltage be $-(1 - \epsilon)V_2$. By a procedure similar to that in the previous section, the switch frequency modulation at the output of the square law detector is
\[ V_{\text{mod}} = 4K V_1 V_2 + 2K(V_1 - V_2)V_2 \epsilon - KV_2^2 \epsilon^2 \]  \hfill 7.4

From equation 7.4, it can be seen that the effect of unbalance in the two states of switching is to introduce an extraneous response proportional to the total power from the switched arm of the interferometer. Thus it is very important to ensure a proper balance between the two states of switching in order to realise the full advantage of phase-switching.

An unbalance in signal addition can be caused by an unbalance in the impedance level of the two arms of the interferometer. For the purpose of analysis, each arm of the interferometer can be replaced by a Thevenin voltage source. The equivalent circuit for the interferometer is, therefore as shown in fig. 38, where \( Z_1 \) and \( Z_2 \) are the output impedances of the two voltage sources and \( Z_3 \) their common load impedance.
Fig. 38 Equivalent Circuit Showing
Unbalance in Signal Addition

By application of Thevenin's theorem, the voltage, \( V_3 \), across the common load impedance \( Z_3 \), can be expressed as

\[
V_{31} = \gamma_3 (\gamma_1 V_1 + \gamma_2 V_2)
\]

where \( \gamma_1 = Z_2/(Z_1 + Z_2) \), \( \gamma_2 = Z_1/(Z_1 + Z_2) \) and \( \gamma_3 = Z_3/(Z_1 + Z_2) \), in which \( Z_{11} \) is the parallel impedance of \( Z_1 \) and \( Z_2 \).

Assuming no unbalance in the switching, the voltage across \( Z_3 \) when the phase of antenna 2 is reversed is

\[
V_{30} = \gamma_3 (\gamma_1 V_1 - \gamma_2 V_2)
\]
As a result, the modulation at the output of the square law detector is

\[ V_{\text{mod}} = 4K \gamma_1 \gamma_2 \gamma_3^2 V_1 V_2 \]  

7.7

Thus, unlike the unbalance in switching, an unbalance in signal addition does not introduce extraneous components in the system response. However, a large difference between \( Z_1 \) and \( Z_2 \) or a large mismatch between \( Z_{\parallel} \) and \( Z_3 \) can result in a considerable loss in the system sensitivity.

7.2.2 The R-F Switch

The best method of effecting phase reversal without resulting in an impedance transformation is to insert an extra half wavelength cable in the transmission path of one antenna. The method adopted by the author is one similar in principle to the method used by H.W. Wells (1956)\(^{18}\). The details of the r-f switch are as shown in fig. 39, and in Plate V. For reasons which will be discussed in the next section, silicon diodes are used in preference to the mechanically driven capacity switch used by Wells.

The action of the switching network shown in fig. 39 is as follows. A positive voltage on points C and D
Fig. 39  The r-f Phase Reversing Switch

Ant. 1

To Radiometer

Switching Signal

Ant. 2

0.02 μF

0.02 μF

1/2λ

72 m Stub shorted by 0.02 μF capacitor
PLATE V

RIGHT  THE  PHASE  REVERSING  SWITCH

LEFT  AN  ANTENNA CURRENT  SAMPLING  LOOP
will forward bias diode 1 and reverse bias diode 2. Assuming the conducting diode to be a r-f short circuit and the reverse biased diode a r-f open circuit, cables AC and CB will simply be quarter wave stubs. Transmission is made via A-D-B. Similarly, a negative voltage on C and D will gate the transmission via A-C-B. Since the two transmission paths differ by half an electrical wavelength, the effect is a reversal of the phase of the transmitted signal without affecting the impedance matching of the system. Assuming the diodes to be ideal switches, only a negligibly small unbalance occurs in the switching network arising from the small difference in the transmission losses of the two signal paths.

In practice, the diodes cannot be considered as perfect r-f switches. In the forward biased state, the diode has a small impedance while the reversed biased diode can be considered as a small capacitor. The effect of the 'imperfect switching' can be analysed as follows. Consider the state when diode 1 is conducting so that transmission is made through path A-D-B. The effect of the reverse biased diode at D is to load the transmission line at point D with a capacitor which should have a reactance higher than the characteristic impedance of the transmission line. Thus the transmitted
voltage via A-D-B can be considered to be affected by a factor \( \xi_1 \), where \( \xi_1 \) is, in general, complex with amplitude slightly less than unity due to the reflection at D. Also the effect of the non-zero impedance of the conducting diode is to allow a small fraction, \( \xi_2 \), of the signal to be transmitted via path A-C-B. If the impedance of the conducting diode is small compared with the characteristic impedance of the cable, \( \xi_2 \) will have amplitude much smaller than unity. Thus the voltage transmitted when diode 1 is conducting is

\[
V_{t1} = \xi_1 V_2 - \xi_2 V_2
\]  

7.8

Assuming the diodes to be identical, and neglecting the difference in the losses of the two paths, the transmitted voltage, \( V_{t2} \), when diode 2 is conducting is,

\[
V_{t2} = -V_{t1}
\]  

7.9

Hence, the modulation at the output of the square-law detector is

\[
V_{\text{mod}} = 4K(\xi_1 - \xi_2)V_1V_2
\]  

7.10
Equation 7.10 shows that as long as the diodes used are properly matched the finite on-off impedance ratio of the diodes does not introduce extraneous response to the system. The effect or the 'imperfection' of the diode as a r-f switch is to decrease the sensitivity of the system and to shift the fringe pattern of the interferometer due to the additional phase shift introduced.

7.2.3 Practical Considerations

For maximum economy of coaxial cables, it is desirable to have the switch at the centre of the interferometer baseline so that the modulated r-f signal can be transmitted to the radiometer site by means of a single cable. Because of the absence of power points at the antenna site, it would be inconvenient to use a mechanically driven capacity switch. Diode switching is ideally suited under such circumstances since the switching signal can be transmitted to the antenna site via the same cable that carries the modulated r-f noise to the radiometer.

The switching frequency adopted in the system is 430 Hz. In theory the switching period has to be much smaller than the time constant of the low pass filter used.
There must also be no significant gain variation over the period of switching. Since the integration time used in the output recorder is of the order of a few seconds, the switching frequency used is, in fact, higher than necessary. One reason for adopting a higher switching frequency is that stable amplification can be applied to the modulation before rectification. Care was taken to choose a switching frequency away from harmonics of the mains frequency. Although square wave switching was assumed in the analysis, satisfactory results were obtained with sine wave switching. The average switching current used in the system was approximately 10 mA.

The choice of the diode is mainly governed by the requirement that the forward biased impedance should be considerably less than the characteristic impedance of the coaxial cable. This requirement can be met through the use of a silicon diode. The diode used in the switching system is BA145, which is popularly used in clamp circuits of colour difference amplifiers in television receivers. The characteristic of a BA145, plotted using a transistor curve tracer, is shown in fig. 40. For comparison, the diode characteristic of a germanium diode, OC161, is also included in the same figure. At 10mA d.c. bias, the dynamic
Fig. 40  Diode Characteristics
resistance of the silicon diode is 5 ohm as compared with 70 ohm for the germanium diode. The impedance of BA145 at 42 Mhz was measured to be less than 10 ohm when biased with 10 mA d.c. When unbiased or reverse biased with a small d.c. voltage of less than 1.5V, the diode acts as a capacitor of 20 pf. Thus at 42 Mhz, the reverse biased diode has a reactance of approximately 200 ohm, which is less than 3 times the characteristic impedance of the coaxial cable. The effect of the capacitive loading of the diode is, as shown in the previous section, to introduce an unbalance in the addition of the signals from each arm of the interferometer and to shift fringe pattern of the interferometer. Since adequate sensitivity was found from the records obtained, no attempt was made to balance the signal addition. The shifting of the fringe pattern of the interferometer is of little consequence because the exact pattern of the interferometer can always be obtained from the transit record of a strong 'point' source like Cygnus A.

The function of the L-type matching section between point 'B' and point 'E' in fig. 39 is to match the measured impedance at B to the 75 ohm characteristic impedance of the coaxial cable to the radiometer.
The switching signal is injected at the radiometer end of the cable. Short circuiting of the switching signal is prevented wherever necessary with capacitors of the order of .01 μF. Often, these capacitors are slightly inductive at 42 Mhz; the capacitance being swamped by the lead inductance. However, the reactance is very small compared with the characteristic impedance of the cable, so that the r-f signal is transmitted with little attenuation.

Fig. 41 Switching Signal Injection Network
The circuit used to inject the switching signal into the coaxial cable is as shown in Fig. 41. The phase delay network used to supply the phase sensitive rectifier with a delayed and attenuated switching signal is also included in Fig. 41. To determine the amount of phase delay required, the two inputs of the phase switch (Fig. 39) are connected to a common noise source. By measuring the phase difference between the output of the selective amplifier (Section 7.3) and the switching signal, an appropriate phase shifting network can be designed to compensate for the phase shift in the modulation. From measurements, the required phase delay was found to be approximately 90°. This is easily obtained using a two-pole R-C network as shown in Fig. 41. The transfer function of the network is, assuming the two R-C stages to be identical,

\[ T.F. = \frac{1}{1 - \omega^2 \tau^2 + j3\omega \tau} \]

where \( \tau \) is the time constant of each R-C stage. By designing for \( \omega \tau = 1 \), the network provides a phase delay of 90° with an attenuation of 3.
7.3 The Selective Amplifier

The desired modulation in a phase switched receiver can be considered to be a square wave even when a sinusoidal switching signal is used. To reproduce all important harmonics in the modulation a wideband amplifier with bandwidth about 10 times the switching frequency must be used. Since the r.m.s. value of the fundamental component of a square wave is only .9 times the amplitude of the square wave, the use of a narrow band selective amplifier at the switching frequency instead of the wideband amplifier will result in only 10% loss in sensitivity of the system. The main advantage of using a selective amplifier is that it is less liable to be saturated by noise pulses away from the switching frequency.

The selective amplifier was designed to operate at 430 Hz. Because of component tolerance, the centre frequency of the amplifier was found to be 423 Hz. Since the exact frequency was not critical, no attempt was made to adjust the frequency to the designed value. The 3 db bandwidth of the amplifier is 12 Hz. Thus the nearest A.C. mains harmonic is suppressed by more than 3 db. The first harmonic of the switching frequency is suppressed by more than 35 db.
The frequency selectivity in the amplifier is achieved through the use of a parallel-T R-C network \(^{20}\) in the feedback loop. For simplicity in design, a symmetrical parallel-T network is used. (fig. 42). The conditions for zero transmission can be derived by simply equating the reverse transfer admittance \(y_{12}\) of the network to zero.

\[ C_1 \quad C_2 = C_1 \]
\[ R_1 \quad R_2 = R_1 \]
\[ R_3 \quad C_3 \]

Fig. 42 The Parallel-T Network

For a symmetrical network as shown in figure 42, the conditions for zero transmission are
\[ X_1^2 = 2R_1R_3 \]
\[ R_1^2 = 2X_1X_3. \]
The frequency at which zero transmission occurs is
\[ f_0 = \frac{1}{R_1 \sqrt{2C_1C_3}} \]
The input impedance of the network at resonant frequency is \( Z_i = R_i / \sqrt{2} \). The design method adopted is to compute the value of \( R_i \) to give an input resistance of 10K at resonance frequency. Assuming \( C_3 = 2C_1 \), the values of \( C_1 \) and \( C_3 \) can be computed from equation 7.12. These capacitances are rounded to the nearest preferred values, and the value for \( R_i \) recomputed from equation 7.12. The value of \( R_3 \) is then computed from the first condition for zero transmission viz. \( X_1^2 = 2R_1R_3 \). The values of the components used in the final circuit are; \( R_1 = 16.4K \), \( R_3 = 8.6K \), \( C_1 = 0.022\mu F \) and \( C_2 = 0.047\mu F \).

The circuit of the complete selective amplifier is shown in fig. 43. Transistors \( Q_1 \) and \( Q_2 \) are used in a two stage direct coupled amplifier in which negative feedback is effected from the collector of \( Q_2 \) to the emitter of \( Q_1 \) via the parallel-T network. Transistor \( Q_3 \) is used as an emitter follower to prevent excessive loading of the parallel-T network, since the selectivity of the network decreases rapidly with loading. Stanton\(^2\) suggests the load resistance to be at least 3 times \( R_1 + R_2 \) for satisfactory operation of the parallel-T network. Although the loading in the circuit used is approximately \( \frac{1}{3} \) the recommended value, the selectivity of the amplifier was found to be adequate. Transistor \( Q_4 \)
is used in the output stage to give an extra gain of approximately 3. Because of component tolerance, the centre frequency of the amplifier was 423 Hz instead of the design frequency, 430 Hz. The over-all gain of the amplifier at the centre frequency is 37.5 db. Variable attenuation can be introduced at the front end of the selective amplifier to provide an average output voltage level of approximately 1/10th the switching voltage at the phase-sensitive rectifier. The frequency response of the selective amplifier is shown in fig. 44.

![Frequency Response Diagram](image)

**Fig. 44** The Frequency Response of the Selective Amplifier
7.4 The Phase Sensitive Rectifier

As was shown in section 7.2, a modulation coherent with the switching signal will occur in the receiver if a radio source with angular dimension small compared with the fringe width of the interferometer lies either in a peak or a null of the antenna pattern of the interferometer. As the source transits the fringe pattern of the interferometer, the modulation varies periodically in amplitude and phase. The function of the phase sensitive rectifier is to measure the amplitude and phase of this modulation and to discriminate against the high level of noise modulations in the receiver. The d.c. output of the rectifier is proportional to the amplitude of the modulation and the polarity of the d.c. voltage gives the phase relationship between the modulation and the switching signal.

Phase sensitive detectors have been extensively studied for over 30 years\textsuperscript{21-23}. As a result, there are at present a large variety of practical forms of phase sensitive detectors. The word 'rectifier' is used in this section because of the long time constant used in the filtering network.
The requirements for a good phase sensitive rectifier are:

(i) A stable zero which should be relatively insensitive to the amplitude variation of the switching signal.

(ii) A stable and linear relationship between the d.c. output and the amplitude of the correlated signal.

(iii) Linear response over a wide range of input signal.

(iv) Strong rejection to signals uncorrelated or bearing a quadrature phase relationship with the switching signal.

The basic form of a practical phase sensitive rectifier is as shown in fig. 45. Diodes 1 and 2 are alternately switched on and off by the switching signal via the centre tapped transformer. Thus in the absence of a signal, each diode acts as a half wave rectifier to the alternate half cycle of the switching signal. If the switching waveform is symmetrical, the net d.c. voltage across points A and B is zero. The voltage pulses at A and B in the presence of a small in-phase signal are as shown in fig. 45. The net d.c. voltage across AB is therefore proportional to the amplitude of the in-phase signal.
The main disadvantage of the simple phase sensitive rectifier shown in fig. 45 is the occurrence of switching voltage at A and B. Consequently, if the capacitors at A and B are removed, the voltage waveform across AB is that of the switching signal. As a result, a ripple voltage due to the switching signal is introduced at the output of the rectifier. This problem can be solved simply by the use of a modified network as shown in fig. 46.
Fig. 46  A Ring Demodulator

It is easy to see from fig. 46 that no switching signal will appear on either A or B. At one half of the switching cycle, diode 1 and 2 conduct and at the other half of the switching cycle, diode 3 and 4 conduct. The series resistance R to each diode must be included to prevent short circuit to the switching signal.

The phase sensitive rectifier used by the author is adopted from a circuit used in a standard receiver designed
by the National Radio Astronomy Observatory, Green Bank, West Virginia\textsuperscript{24}. The principle used in the phase sensitive detector is similar to that in the circuit shown in fig. 46. The difference lies in the use complimentary signal injection so that one end of the output of the phase sensitive rectifier is at earth potential. The circuit diagram of the detector networks is given in fig. 47.

![Circuit Diagram](image)

\textbf{Fig. 47} Ring Demodulator with Complementary Signal Inputs
At one half of the switching cycle, diodes 1 and 2 are conducting and the signal voltage is fed into the load resistor via phase 'a'. At the other half of the switching cycle, diodes 3 and 4 conduct and the signal voltage is fed into the load resistor via phase 'b'. Thus if the signal is assumed to be in-phase with the switching signal, the waveform across the load resistor is the full-wave rectification of the signal waveform.

In the design, \( r = R \) has been assumed. Thus the circuit is completely symmetrical and the switch input and the signal input can be interchanged in the detector. To minimise attenuation, \( R < R_L \). Since the effective load across the complimentary outputs is approximately 2\( R \), the value for \( R \) must be chosen such that a good diode current is obtained from the switching signal while, at the same time, not to present excessive loading on the driving sources.

One advantage for using the phase sensitive rectifier shown in fig. 47 is that no transformer is needed. The complimentary driving sources can be obtained using standard phase splitting circuits. The circuit diagram of the complete phase sensitive rectifier designed by the author is shown in
fig. 48. Transistor $Q_1$ operates as a phase splitter providing anti-phase outputs at the collector and emitter. Transistors $Q_2$ and $Q_3$ are used as direct-coupled emitter followers to prevent loading of the outputs of $Q_1$. All the resistors used in the detecting network, fig. 47, are of 1% tolerance. A small amount of trimming of the values of the resistors in the collector and emitter of $Q_1$ was found necessary to secure a proper balance at the output of the rectifier.

The response of the phase sensitive rectifier to signal in phase and in anti-phase with the switching signal is plotted in fig. 49a and 49b. The time constant of the low pass filter used for the test was about 10 sec. Each reading was taken 1 minute after the change in signal level was made. The experimental results show that the phase sensitive rectifier has a linear response for coherent signal amplitude up to half the voltage of the switching signal. The signal voltage ratio over which the response is linear is approximately 50.

To test the rectifier for rejection to non-coherent noise, the signal was taken from the output of the selective
\[ R = 10\text{K} \text{ 1\% tolerance} \]
\[ R_L = 56\text{K} \]

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Fig. 48 The Phase Sensitive Rectifier
Fig. 49 Characteristics of the Phase Sensitive Rectifier ($V_{SW} = 2V$)
amplifier the input of which was connected to a white noise generator. For 1V r.m.s. noise signal input, the d.c. level at the output of the phase sensitive rectifier is found to fluctuate with amplitude less than 10 mV. Considering the fact that it requires only 30 mV in-phase signal to give a deflection of approximately 10 mV d.c. the rectifier can be said to operate satisfactorily in the presence of noise voltages 30 times higher than the signal level. Of course the level of the input signal must be adjusted such that the amplifiers are not saturated.

7.5 Low Pass Filter

The time constant of the low pass filter used for the radio astronomy observations conducted with the interferometer lies between 5 to 20 seconds. A simple R-C low pass filter was tried and found to be satisfactory. The main problem encountered in the use of a simple R-C filter lies the large value of capacitance required. For example, with a 27K resistor, a 400 μF capacitor is required to give a time constant of 10.8 seconds. To be physically small, an electrolytic capacitor must be used. Since the d.c. voltage across the capacitor is seldom greater in amplitude than 300 mV, satisfactory operation was
achieved using a capacitor with fairly high voltage rating. A simple experiment was carried out on a 400 μF, 40V electrolytic capacitor charged from a .3V source through a 27K resistor. The time taken to charge the capacitor to .63 times the battery voltage was found to be 16 seconds irrespective of the polarity of the battery voltage. Of course, a smaller capacitance value would be needed to give the same time constant if the series resistance is increased. However, the value of this resistance should not be greater than the leakage resistance of the capacitor or the input impedance of the recorder d.c. amplifier which is connected across the capacitor. The input impedance of the amplifier in the recorder used is 100K.

7.6 Some 'Observational' Results

At the time of writing, four months of continuous recording has been made with the equipment. Apart from the active sun, only three strong radio sources can be unambiguously detected. These sources are Cygnus A, Centaurus A, and Sagittarius A.

Cygnus A, the second strongest radio source in the sky, is a remote galaxy at a distance approximately 600 light years
away. At 42 Mhz, the flux density of the source is approximately $2 \times 10^{-22} \text{watts/m}^2/\text{Hz}$ and the source can be regarded as a point source to an interferometer of low resolution. Since the declination of Cygnus A is $40^\circ36'$, the antennas must be tilted northwards so that the source lies within the primary pattern of the interferometer.

Centaurus A, on the other is a relatively broad source consisting essentially of two emitting regions spaced $3^\circ$ apart. However, since these two sources lie approximately along the same right ascension, there should be no significant fringe attenuation on an interferometer with an east-west baseline 20 $\lambda$ long. The flux density of Centaurus A at 42 Mhz is estimated to be $10^{-22} \text{watts/m}^2/\text{Hz}$. The source is at a declination of approximately $-43^\circ$. Hence the transit is almost directly overhead in Auckland.

Sagittarius A is a radio source located at the nucleus of our galaxy. Its flux density according to Mills' catalogue is $4.5 \times 10^{-23} \text{watt/m}^2/\text{Hz}$ (at 85.5 Mhz).

A typical 24 hour record of the interferometer is shown in figure 50. It can be seen that small fringes occur almost continuously showing that the system is confusion limited due
Fig. 5C A Typical
(The recorder's No. 557)

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Contaurus A

Cygnus A
to the lack of primary resolution in the antenna pattern. Confusion also arises because the primary pattern of the antennas consists of three lobes owing to the $2\lambda$ spacing between the two antennas at each end of the non-uniformly spaced experimental array which was adapted to be used as an interferometer.

A number of strong bursts of the sun were recorded over the period of four months of continuous recording. A record of the active sun is given in fig. 51.

To indicate the over-all sensitivity of the system, a record of the transit of Centaurus A with increased recorder sensitivity and correct alignment of the antennas to the direction of the source is given in fig. 52.

No attempt was made to calibrate the flux densities of the radio sources recorded. This is not within the scope of the thesis. Accurate calibration at the frequency concerned is by no means a simple process. For absolute calibration, the coherent calibrating noise must be injected simultaneously with the sky noise. Also the antenna gain must be accurately known. However, if the flux density of one strong source is assumed known, the flux densities of the
Fig. 51 Record of the Active Sun on 21.3.68
(Antenna system as for fig.50)

Fig. 52 Record of Centaurus A on 23.4.67
other detected sources can be estimated by comparing the fringe amplitudes of the sources in question with that of the reference source. Care must be taken to take into account the possible variation of receiver gain with different level of background noise at the antennas.

7.7 Conclusions

A phase-switched interferometer at 42 Mhz has been designed and constructed. The system was found to have an expected sensitivity from the records of transits of known radio sources. Good rejection to the background brightness of the sky is indicated by the stable baseline in the output trace on the recorder. However, frequent terrestrial interference is encountered. Such an interference usually arises from an occasional man-made transmission at 42 Mhz.

The system has been in continuous operation for over four months at the time of presentation of this thesis. A record has been kept of the solar activities which occurred over this period.

The interferometer has achieved its principal aim which is to provide an initial observational experience within the radio astronomy research programme of the University of Auckland.
REFERENCES


