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Assertibility and Coordination:

Analysing ‘and’ and/or ‘or’, some ‘some’, but not so much ‘but’, ‘not’, ‘so’, or ‘nor’, nor ‘much’.

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Abstract

This thesis explores how far classical logic, combined with formal restrictions that model some Grice-inspired maxims, can go towards describing the multiple distinct uses of the English natural language connectives ‘and’ and ‘or’. H.P. Grice famously claimed that ‘and’ and ‘if’ could be completely reduced to the truth-functional connectives ‘∧’ and ‘⊃’ plus his maxims, so this thesis partakes somewhat of the spirit of his project. However, it is neither Gricean nor neo-Gricean, as conversational implicature is ignored in favour of an appeal to formal and general predictions that can be drawn by an ideal rational cooperative Hearer. This allows the development of formal propositional systems based on norms inspired by Grice’s cooperative principle and subsequent maxims. These formal systems seek to capture assertibility, and require some new approaches to logical semantics that are interesting in their own right. Many types of pragmatic phenomena can be described in terms of which minor deviations are required to the utterance form to meet the assertibility criteria. Many of these deviations can be described precisely by formalising additional maxims or borrowing some concepts from cognitive linguistics and formal syntax. Some of the functional-typological distinctions within each of the coordinations ‘or’, ‘and’, and ‘but’ will be compared to a list of predictions produced by assertibility considerations. The principles behind propositional assertibility are then applied to predicate logic, negation, and speech acts such as interrogatives and arguments with some interesting results. An approach to conditionals (‘if’ clauses) and related subordinating conjunctions such as ‘when’ and ‘since’ is sketched based on linguistic classifications and assertibility considerations. The assertibility conditions for disjunction also suggest an analysis of the so-called paradox of Free Choice that suggests it is both far more pervasive and simple than previously believed. Finally, the results of this exploration are tallied, along with some puzzles, surprising parallels, and thoughts for further research.
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This thesis wouldn’t have been nearly as good without the wonderful support and assistance of those listed above, many of whom went above and beyond the call of duty; however, I will take full credit for two elements. All the crazy ideas, incoherent explanations, and far-fetched claims are the end-product of my own convoluted thought processes; and all errors are purely my own work, and have been included despite excellent advice to the contrary.
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1. Introductions

In this thesis I will investigate the degree to which the variation in what is conveyed by our usage of each of the English coordinations ‘and’ and ‘or’ can be predicted by formal systems modelling cooperative norms of conversation. These norms will chiefly apply to assertions, and so will be referred to as “assertion norms”. The resulting formal system will create a standard of assertibility that will be used to assess potential interpretations of an utterance for how accurately they capture what the Speaker could reasonably be taken to have conveyed, in particular by their use of coordinations. Coordinations are commonly represented by the truth-functions ‘∧’ and ‘∨’ in standard logic and hence often in traditional semantic theory (although the current logico-linguistic literature tends towards more complex descriptions). Grice (1978) claimed that facts about the usage of the English ‘or’ could be modelled by treating it as semantically equivalent to ‘∪’, and then deriving all additional information about the coordination from his Cooperative Principle and maxims, and similar pragmatic considerations. This is roughly the approach that I will be taking, including using minor variations on the Gricean maxims to motivate the formalisms. However, I will not appeal to an informal notion like Grice’s conversational implicature. Instead I will develop formal criteria for the assertibility of propositional formulas, rank the possible alternative interpretations, and from this ranking, produce a series of predictions about the use of coordinations in context. The assertibility system that I will develop in this thesis generates calculable predictions about the usage of these expressions without making any claims about which mechanisms are semantic and which are pragmatic.

1.1 Overview

This thesis investigates one approach to modelling the cooperative use of coordination in assertions. I will expound and evaluate a principled approach to categorising and predicting much of the polysemy of English coordinators. Functional approaches to this topic, such as Haspelmath (2000), Mauri (2008), and Dixon and Aikhenvald (2009), only propose ways of categorising connectives that are informed by cross-linguistic generalisations and exceptions. At some point this process of ‘butterfly collecting’ should be superseded by the development of systematic and principled theories that make testable predictions. Propositional connectives provided an early step towards this systematisation, but they appear simply insufficient, even when these connectives are augmented with simple pragmatic principles. Grice’s attempt to model standard usages of the English ‘or’ using ‘∪’ and conversational implicature is perhaps the best known approach using general principles. More recently, other researchers have addressed specific issues with connectives with equally specific analyses and formalisms. One example of this is the growing industry of producing formal models of disjunction specifically to explain the paradox of Free Choice, regardless of their applicability to other uses of disjunction. This piecemeal patching seems to ignore the intuitive conjecture that each coordinator represents a single simple concept that can be interpreted in different circumstances to produce almost all of the diverse range of usages that have been independently described in many different natural languages. The alternative is to propose some motivated principles that at least demonstrate the feasibility of a principled approach. The assertion norms will serve this role. If assertibility can be used to accurately predict a wider range of usages (and related pragmatic phenomena) than was built into the initial norms, then it may serve as an approximate partial explanation of coordination. I will provide new evidence that imposing a series of norms on classical disjunction and conjunction can explain a significant degree of the polysemy of natural language coordinators in a principled way.
Polysemy

A single word with multiple, related meanings, senses, or usages is polysemous; this is often contrasted with homonymy, where multiple words with identical pronunciation or spelling have unrelated meanings. Fillmore and Atkins (2000) suggest that polysemous senses have a common origin, and that the senses are related to each other (and usually the common origin) via a network that is recoverable by a competent native speaker. For example, the English noun ‘mole’ can refer to a burrowing animal, an embedded covert operative, a skin blemish, or a number of molecules. It is polysemous, as the first two senses are related, with the ‘spy’ meaning a metaphor derived from the ‘burrowing animal’. It is also homonymous, as the last three meanings are unrelated to each other or to the first pair, with separate root words and etymologies. The original sense of most polysemous words is relatively simple, and identifying the series of changes from the original that each sense has undergone will contribute to understanding the range of usage associated with the word. A word is strongly or weakly polysemous in proportion to how obvious the distinct senses are in typical usage, or even in a particular context, according to the intuition of native speakers. Polysemy originally referred to a word in isolation having multiple meanings, but I am more interested in the different messages that can be conveyed by a word in a context, sometimes called pragmatic polysemy.

The number of different senses, and their conceptual distance from the original sense, can vary between words, or between contexts. There can be just a pair of closely related senses, or a large number of only vaguely associated senses. One way of classifying polysemy is by whether these relationships between the senses, and the transformations each sense has undergone, are principled. Some polysemy is predictable and follows the same principles as other polysemous words, while other sense shifts are peculiar to that word or context, and appear to occur through an unpredictable series of accidents. One way to identify principled polysemy is by proposing a set of formal or rigorous principles that might underlie the polysemy, and test the accuracy of the ensuing predictions. This approach assumes that we can easily identify the right principles, which is putting the cart before the horse. Another approach is to check if word (or sense) distributions respect the same distinctions across languages, or conform to some proposed cross-linguistic classification system.

My primary approach for testing polysemy in English coordinators will be to propose a set of motivated principles of assertion and then investigate whether their predictions correspond to patterns of use in English coordinations. If so, we may have the basis of a principled polysemy for English coordinators. I will also test whether my assertion norms help to explain some of the distinctions proposed by researchers of cross-linguistic coordination. Here I will rely on Dik (1968), Haspelmath (2000), and Mauri (2008).

The Project

I will provide a formal account of some proposed conditions on cooperatively asserting for boolean connectives, and then introduce semi-formal methods for classifying the corresponding natural language connectives based on very generic principles. These principles include the linear ordering between and within utterances, folk-psychological notions of connection and causation, and the domains of discourse proposed by the cognitive linguist Sweetser (1990). The interaction of formal assertibility with the resulting symmetric/asymmetric, probabilistic/deterministic, direct/indirect and content/performance distinctions will provide a framework for predicting some of the pragmatic phenomena that are used to convey information about an utterance. I will primarily focus on the English coordinators ‘and’ and ‘or’, and later expand this treatment to include ‘but’, ‘so’, ‘if’, ‘since’, ‘when’, and several related connectives. Each connective is assumed to be polysemous rather than homonymous; that is, there is a single underlying general concept or procedure associated with each connective, which is modified by the context and by the (causal, temporal, logical, etc.) relationships between the coordinands. The resulting formalisms will be tested against a wide range of usages and classifications to evaluate the robustness of the proposed principled polysemy. A surprising range of pragmatic phenomena can be predicted, and most (though not all) of the polysemy of the connective terms can be explained by rigorous reasoning using on a small number of principles. The formal norms at the core of the analysis impose certain fundamental restrictions which roughly mir-
ror restrictions that are commonly reported in natural language. These include changes in available interpretations in downwards-entailment contexts, the chirality of asymmetric connectives, and the differing behaviour of so-called indicative and counterfactual conditionals. The vague nature of the cognitive and contextual distinctions regarding correlation, causation, etc., mean that many predictions are not completely formal, and so allow for more flexibility and so less defeasibility room than I would like. My account should be judged by how well my defeasible predictions match empirical data, along with the simplicity of the principles that generate these predictions.

In addition to describing my goals, it is equally important to draw clear boundaries around what I am not trying to achieve, to avoid potential confusion about the scope of this project.

- I am not engaged in empirical linguistic research, despite my heavy reliance on linguistic theories and classifications. I am presenting a philosophically and linguistically motivated pattern, and demonstrating its adequacy as a predictor of some of the variation in natural language coordination.
- I do not claim this pattern models human reasoning in language comprehension. I merely observe the correlation between the results of a formal model and certain pragmatic phenomena. While one explanation of this correlation is that human reasoning mirrors formal assertibility, neither I nor anyone else has engaged in the relevant empirical research in cognitive or neuro-psychology.
- I also wish to avoid any strong, radical or controversial claims on the feasibility of translation between natural and formal languages. The formalisation that I use is simply to fix selected connectives (‘and’, ‘or’, ‘if’, etc.,) in an uttered expression as logical constants and replace the rest of the utterance with variables. Apart from my focus on the utterance rather than the expression, this is a common approach for anyone modelling natural language in a formal system. I will assume that the meaning of natural language expressions is somewhat compositional, but will rely on this to a much lesser degree than most formal semanticists. The boldest claim I will make is that some aspects of what is conveyed in an utterance can be recovered from observing what has been omitted, as well as what has been uttered.
- There is not space to engage in a sustained or systematic effort to examine multiple languages, and so I will refrain from making strong claims about the cross-linguistic applicability of this theory. At times I appeal to the behaviour of other languages, and underlying my analysis is an assumption that much of what I present using examples from New Zealand English may also be applicable to other languages, and equally tendentiously, other cultures. There is strong evidence that a conjunctive ‘and’-type term is almost ubiquitous, and that an inclusive disjunctive ‘or’-type term is very common, across the 6,000 or so languages studied by linguists; some of the patterns presented here will apply to many of these other languages.
- Finally, I am not proposing a new candidate for relevant entailment or implication. Some of the inference relations that I describe share a number of formal properties with various relevant and connexive logic systems, but I will be working primarily with syntactic formulas, and communication rather than deduction will be my primary concern.

One final disclaimer: I will not take any particular position on where to locate the division between (linguistic) semantics and pragmatics, or how they interact. Some of the conditions that I will associate with the norms for a particular coordinator will be truth-conditional, while others will be modal, counter-factual, or procedural. Some readers may feel that I am confusing semantics (the meaning of linguistic expressions), with pragmatics (how they are used to communicate), or am not honouring this distinction. I simply have not found the semantics/pragmatics distinction particularly useful in this particular project, and as a result, I tend to avoid distinguishing between semantics and pragmatics, except when co-opting existing formal theories such as the linguistic type theory in §3.5. For those who prefer to maintain this division, my frequent appeals to context, likelihood, and common ground place most of my predictions fairly uncontroversially in pragmatics.

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1Semantics is roughly the theory of the meanings of expressions in a language and pragmatics has something to do with what Speakers do with expressions. Beyond this, however, there is much disagreement. For example, many expressions are underspecified, and are completed by appealing to the context. Is the appropriate expression for semantics to operate on the initial fragmentary expression, or the contextually-informed expression that was never uttered? As there are very deep disagreements on the theoretic terrain, and no unified view on where semantics stops and pragmatics starts, I will try to rely as little as possible on this distinction.
Thesis Outline

The thesis has five main chapters, a conclusion, and three technical appendices. Each chapter builds on earlier results, but takes the project in a new direction. Each chapter is built upon a different primary discipline, and so has its own distinct register. The rest of Chapter One is conceptual analysis in the tradition of Philosophy of Language. Chapter Two (and its appendices) is almost entirely concerned with formal systems of logic. Chapter Three flirts with a range of notions from psychology, cognitive linguistics and formal syntax. Chapter Four draws heavily on functional-typological semantics. Chapter Five extends earlier chapters’ findings for coordination to several other linguistic phenomena and logical constants, and so switches between several of these registers.

This first chapter frames the problem I am investigating, introduces a number of conventions and background notions, and positions the investigation with respect to Gricean pragmatics. Its final sections contain a piece of conceptual analysis about restrictions upon a cooperative Speaker making an assertion. This results in a series of syntactic assertion norms that are imposed on assertoric utterances. These norms primarily restrict the acceptable uses of disjunction and conjunction.

The second chapter provides four different formal semantics for the assertion norms: a bottom-up truth-table algorithm; a top-down set-theoretic decomposition using possible worlds; a modal semantics using ideals on lattices; and a preference relation between formulas. These semantics are all introduced incrementally, norm by norm, producing five systems which impose increasing restrictions on classical validity. That is, we end up with five incremental assertibility systems, each with four equivalent semantics. The semantic families are each inspired by a different quasi-Gricean maxim, each describes the constraints on assertibility in a different way, and each will be used to model different variations introduced in subsequent chapters. I demonstrate the equivalence of these seemingly-diverse families, and then explore some of their interesting formal properties including their soundness and completeness with respect to the system of syntactic norms from the first chapter. The assertibility family is also shown to be closely related to an existing family of connexive systems developed by Gerhard Schurz.

I start the third chapter by considering how assertibility informs the interpretation of an utterance by a hearer. I soon conclude that a little more flexibility is required in both the norms and notation to account for a hearer’s comprehension of assertions, as the interpretation of natural language coordination is more subtle than I had assumed in the initial analysis of assertion. I introduce a metalanguage for describing conjunctions and disjunctions, and then engage in increasingly informal discussions of how coordination is affected by: order and asymmetry; probability and relevance; phrasal syntax and semantics; and the psychological connexions between coordinands. These four discussions provide a motivated series of variations on the assertion norms, and consideration of which alternatives are satisfied generates further information about the sort of coordination being used in an utterance.

The fourth chapter discusses some of the classifications of natural language coordination, and then shows how the variant assertion norms enable the accurate prediction of approximately forty distinct pragmatic phenomena that aid the interpretation of disjunctive and conjunctive coordination. Some other English coordinators are also briefly discussed, along with some of the classes of disjunction that are not predicted by assertibility. I then review and summarise the explanatory and predictive power of the assertion norms for interpreting coordinations, completing the core project of providing a systematic and principled partial guide to the usage of coordinations.

Several obvious extensions of the project are explored in the fifth chapter. They are: the extension of assertibility from propositional to predicate logic, and how this allows a systematic approach for higher degrees of coordination; an investigation of assertion norms for negation; the behaviour of Boolean coordinators nested within modal operators, including the Free Choice ‘paradox’; the extension of this project to questions and other non-assertoric utterances, and in particular rules for assertible arguments; and finally subordination, including the paradoxes of material implication.

The final chapter draws some modest conclusions, suggests some wild speculations, and then outlines some potential lines of research.
1.2 A Gricean Spirit

The project I am undertaking is very much in the spirit of H.P. Grice, who conceived of communication as a cooperative, rational, goal-directed endeavour. Grice (1975) begins with his Cooperative Principle – that as a speaker you should “[m]ake your contribution such as it is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged”. He then introduces a number of slightly more specific maxims, such as “Do not say what you believe to be false” and “Be relevant”. Grice suggests that these maxims, along with the cooperative principle itself, are guidelines for successful communication. While he is not explicit, it seems that for Grice the maxims are specific ways of being cooperative in normal contexts. By either following the guidelines, or deviating from the guidelines in a marked manner, Speakers can convey information that is in addition to, or even a replacement for, the content of the expression they utter. Grice coined the term ‘conversational implicature’ for the information conveyed in this manner. Conversational implicatures share several key features: cancellability and reinforceability (they can disappear or be made stronger due to additional information); non-detachability (they are independent of particular word choice); and calculability (they can be calculated from the context, the cooperative principle, and associated maxims). Later writers added additional criteria: universality (cross-linguistic occurrence), and non-conventionality (the additional information is not merely present by convention). A hearer can then attempt to determine the Speaker’s likely intended message by evaluating the possible interpretations of an utterance in context, and using ampliative reasoning to recover the Speaker’s original intentions.

Grice’s ideas on the pragmatics of conversation have been developed in several different directions. For instance Levinson (1983) and Horn (1989) have each developed well-known semi-formalised heuristics based on subsets of Grice’s maxims. The neo-Gricean Bach (1999) defends Grice’s view of a minimal interpretation of semantic content as being compositionally determined from conventional meanings of words, with little dependence on context or unarticulated content. Relevance Theorists such as Carston (1988) are primarily engaged in a primarily psychological project, with their model of communication relying more on the psychology of inferences than semantic rules of language to generate their explanation of pragmatic information. Each of these research projects has developed Grice’s ideas in different ways.

Grice’s insights about communication also play a key role in my project. One of Grice’s major contributions is to place the recognition of Speakers’ intentions at the heart of communication. Speakers and Hearers also share common rules about natural language, such as the universal and language-specific constraints of syntax, and the range of lexemes available to them, and these constraints will heavily shape the choices that Speakers make. Even if a hearer’s reasoning can be described in terms of the conversational principle or maxims, intention-recognition is not simply a matter of following conventional rules, but requires ampliative reasoning about the contents of other minds. The implicatures or similar ampliative inferences must be objectively calculable so that the underlying theories can be tested for defeasibility and accuracy. This commitment to calculability is a key element in Grice’s vision of implicatures, but few of the current theorists put substantial effort into ensuring their predictions are calculable. This is the feature of pragmatic inferences that I will concentrate on in this thesis. I will produce formal systems that, when interpreted as applying to natural language, produce testable predictions about pragmatic phenomena. Here are the quasi-Gricean maxims that I use to motivate my various formal systems:

- **Expressivity**: Say what you mean.
- **Informativity**: Provide novel information.
- **Consistency**: Be consistent with what has been agreed.
- **Inclusion**: Only mention that which relates to what you mean.
- **Extensibility**: Don’t start what you can’t finish.
- **Relevance**: Mention something only when it may make a substantive difference.
- **Brevity**: Be brief.
- **Order**: Be orderly.
1.3 Conventions & Terminology

I will consistently make use of several terminological conventions to simplify my exposition. For instance, gendered pronouns will usually relate to utterance contexts, following the convention that the Speaker is female and the Hearer male.

*Entailment* and *validity* will always be classical, except in Appendix A.4's discussion of connexive logics. *Implication* (and cognates like ‘implies’) will always connote the defeasible inference between assertions first presented in Defn 2.105. A classical entailment claim will be represented by the symbol $\vdash_{cl}$, while $\vdash$ will be reserved for an unspecified inference relation. Thus `$p \vdash_{cl} p$' is an assertion that $p$ classically entails $p$, while `$p \vdash p$' refers to an inference whose properties are under discussion. When specific syntactic `$\vdash$' or semantic `$|= $' inference relations are defined, satisfaction of their conditions will be asserted using an appropriate subscript on the inference relation, e.g., $p \vdash_2 p$ or $p \models_{R} p$. *Atoms* include both propositional variables (e.g., $p, q, r, s, \ldots$) that stand for an arbitrary proposition, and the constant $\bot$. *Literals* includes atoms and negated atoms, and so $p$, $\neg q$, $\bot$ and $\top$ are all literals. The symbol `$\oplus$' will represent the exclusive disjunction truth function.

Linguists and logicians use the same terminology to describe different aspects of our subject matter. To avoid complete confusion I will restrict some polysemous terms to a single meaning, while relying on context to disambiguate the logical and linguistic usages of other terms. When the context allows either reading, I will prefix a polysemous term with ‘logical’ or ‘linguistic’ to avoid ambiguity. Following some linguists, I will use *Epistemic* in the broad sense referring to cognitive content in general, including beliefs and opinions. This is because neither truth nor justification plays a large role in conveying information via assertion, so knowledge holds no special place in my analysis. *Connective* refers either to the (linguistic) class containing all coordinating, subordinating, and correlative conjunctions, including the English terms ‘and’, ‘or’, ‘but’, ‘if’, and ‘since’, or to any (logical) symbol that designates a non-monadic propositional function. *Coordination* refers to the class of coordinating conjunctions, including the English terms ‘and’, ‘or’, and ‘but’, and to the associated conjoined phrase; when these need to be distinguished, the term will be called a *coordinator*. *Boolean Coordination* refers to those coordinators such as the English terms ‘and’ and ‘or’ that are often modelled by Boolean truth functions. *Disjunction* refers to the (linguistic) class of coordinations that presents alternatives, including those formed by concatenating phrases with the English ‘or’. It also refers to instances of the (logical) truth function ‘$\lor$’, and its assertible variants. *Conjunction* refers to the (linguistic) class of coordinations that presents two non-contrasting phrases that are both appropriate, including those formed by concatenating phrases with the English ‘and’. It also refers to instances of the (logical) truth function ‘$\land$’, and its assertible variants. It does not usually include ‘but’ or similar adversative conjunctions. I will not use its other common linguistic meaning, which I have reserved for ‘connective’. *Subordination* refers to connectives that join a main and a dependent clause such as the English terms ‘although’, ‘because’, ‘if’, ‘until’, and ‘when’, and their associated conjoined phrase. * Conditional* refers to the (linguistic) class of subordinations that use the English ‘if’ to join the main and subordinate phrase, and other subordinations that behave similarly, including those that use the English ‘since’, ‘when’, and ‘unless’. It may also cover instances of the (logical) material conditional truth function ‘$\supset$’.

I will refer to the types of phrases in natural language using the fairly standard nomenclature from Heim and Kratzer (1998). A sentence or independent clause is an Inflection Phrase (IP). An IP typically consists of a Determiner Phrase (DP) for the subject and a Verb Phrase (VP) containing everything else. A DP is either a Name, a Pronoun, or a Determiner (Det) such as ‘a’, ‘most’, or ‘three’ followed by a Noun Phrase (NP). A simple Noun Phrase may contain Adjective Phrases (AdjP) followed by a Noun. Other common phrase types include Adverbial Phrases (AdvP), Prepositional Phrases (PP), Complementiser Phrases (CP), and the Conjunctive Phrases (ConjP) which will be our primary concern. These phrase types, along with the generic phrase (XP), come from X-bar theory in generative grammar. Some phrase structure grammars transpose the role of NPs and DPs.
1.4 Assertion Norms

Natural language communication is a complex and layered phenomenon that requires cooperation between a speaker and a hearer, each with an awareness of their common ground. I will follow Grice and many others in focusing my analysis on that part of a conversation where a speaker is cooperatively conveying novel information to a hearer through a sequence of assertions. Assertion is often taken to be the central case of communication. One reason for this may be that questions, commands, and other types of speech act are more complex, and so allow for less idealisation of the roles of the Speaker and Hearer, and their interaction with the context. I will initially idealise away all the factors involved in successful assertion except: the illocutionary act performed by the Speaker; the propositional content the Speaker intends to communicate; and the context, being the set of propositions that are common ground between the Speaker and Hearer.

The general question that I seek to answer is: ‘What is the relationship between context, communicative intention, and assertion that characterises all typical cooperative assertoric information exchanges?’ This question has several elements that will be unpacked and analysed in stages in the remainder of this chapter. These stages are as follows: First I will describe how utterances are used to convey a speaker’s communicative content, and consider two measures for successfully conveying some of the content. Next, I will explore when an utterance is informative in a shared context. Consideration of the internal structure of utterances will lead to a decision to use the language of propositional logic, which in turn will allow a formal definition of three norms for assertibility: Expressivity, Informativity, and Consistency. These norms will be defined recursively through a decomposition of disjunctions and conjunctions. Finally, two more norms, Inclusion and Extensibility, will be briefly considered. All these norms are absolutes, in that failing one aspect of a norm to the tiniest degree constitutes a failure to meet the norm. In Chapter Three I will introduce complications by treating the norms as aspirational, where the Speaker will try to minimise their overall deviation from the norms by engaging in a series of trade-offs and compromises.

I am primarily interested in the propositions that are intended to be conveyed by the assertion (the intentional or communicative propositional content) rather than the particular locutions used to convey those propositions. To this end, I have selected the communicative or illocutionary act, as opposed to the mere linguistic expression uttered. By analysing the communicative act rather than the linguistic expression, I can bypass discussion of literal meanings and the semantics of expressions, and directly analyse the communicative intention associated with the action. For example, one action that in many contexts can communicate the desire that a window be closed is the utterance of the expression ‘it’s cold in here’. The essentially linguistic locutionary act will thus be ignored (until §3.7) in favour of the inherently communicative illocutionary act.

As assertions are actions, they are intentional, and the content of the associated intention(s) can be determined by reporting on the action. McDowell (2010) says “the content of an intention in action is given by what one would say in expressing it, and the proper form for expressing such an intention is a statement about what one is doing: e.g., ‘I am doing such-and-such’”. The choice of an action is often driven by a combination of multiple intentions operating at different levels of abstraction; for example, a football player may kick a ball as a way of attempting to satisfy their intentions to: pass a ball, put their right-wing in a gap, score a goal, win, and obey the rules of the game. The intention to communicate a piece of information is always present when asserting cooperatively, and so this ‘communicative intention’ is the second of our three factors. In this case, McDowell’s form simplifies to the explicit performative clause ‘I am asserting that such-and-such’, where ‘such-and-such’ is the linguistic expression that reports the intentional content of the relevant illocutionary act. This reporting linguistic expression might bear close similarities to the initial uttered expression, but it need not. For instance in the example above, the communicative content of the assertion is that the window should be closed, while the propositional content of the expression is that it’s cold in here. This is where an appeal to our third factor, the common ground, is required. The common ground between Speaker and Hearer includes specific propositions such as if a room is cold, act so as to warm it, and closing a window will make this room warmer, and it is this context that enables the expression to convey the intent of the Speaker.
1. INTRODUCTIONS

1.4.1 Utterances

I will stipulate that a complete utterance is the performance of a unit of speech that can stand alone grammatically and conceptually, such as any independent clause, sentence, or paragraph. Sentences are useful indicators of minimal complete and self-contained utterances, and thus of cooperative conversation, even if they are somewhat artificial constructs of linguistic analysis, and spoken natural language often consists of a series of sentential fragments. Perhaps the primary responsibility in cooperative communication is that of ‘truth-telling’, which is more accurately described as faithful reporting of one’s beliefs or chosen epistemic stance on an issue. Actual truth-telling about the world or one’s beliefs is not required for successful communication, which is fortunate given that it is unclear how we access truth about the world. I will assume the Speaker is not engaged in self-deception, so the communicative content will only consist of propositions that the Speaker wishes to convey as representative of their beliefs. The first norm I propose is *Expressivity*, which an assertion satisfies iff it successfully conveys the Speaker’s communicative content in a particular context. For the intended content to be successfully conveyed, ‘the content, the whole content, and nothing but the content’ must be communicated, except that any content that is already in the common ground need not be conveyed. This norm is in the spirit of Grice’s maxims of Quantity, which require that the correct amount of information for the situation is successfully conveyed.

**Definition 1.1 Conveying Content**

An utterance conveys a Speaker’s communicative content in a context iff given the context (including the common ground with the intended Hearer), the communicative content and the utterance’s propositional content entail each other.

A speaker’s intent to communicate content and their associated assertion are an example of setting a goal and acting to achieve it. Some goals are sufficiently complex that the intention to meet the goal can usefully be broken down into a sequence of smaller, self-contained intentions, each with an associated action, where the sequence of actions achieves the goal. Examples of this include making jam, or telling a long story. The primary complexity added by complex goals is defining progress towards a goal. I suggest there are two basic approaches to making progress: identifying and achieving sub-goals; and reducing the remaining activity required to achieve the complex goal. Given a complex goal, it is (usually) possible to identify and achieve some sub-goals without having a complete understanding of how to achieve the final goal. In contrast, reducing the activity required to achieve the final goal requires at least some high-level planning of the entire sequence of actions to avoid the need to undo some ill-timed earlier action (e.g., putting the lid on an empty jam jar before filling it with jam). This difference is fundamentally informational rather than psychological, and thus can be analysed a priori.

It seems both commonplace and perfectly reasonable for a Speaker intending to convey a complex idea to only plan part of what she wishes to say before constructing and performing an utterance, and then devise the next utterance in the sequence as she finishes performing the previous utterance. Some Speakers will plan their utterances further in advance than others. It is even possible that some Speakers may never need to communicate a complex proposition for which they cannot completely construct a series of utterances in advance that fully conveys the entire intended proposition. However many people will share my frustration in finding their ambition to communicate a complex idea exceeds their planning capacity. The norm for incomplete Expressivity will thus be that of achieving a sub-goal by partially conveying the overall communicative content, rather than the seemingly higher standard of partially achieving the overall goal by conveying a part of its communicative content.

**Definition 1.2 Partially Conveying, or Conveying Part of, the Content**

An assertion partially conveys a Speaker’s communicative content in a context iff the common ground and communicative content entail the content of the assertion.

An assertion conveys part of a Speaker’s communicative content in a context iff it is the first of a finite sequence of assertions, and performing this sequence of assertions conveys the Speaker’s communicative content in the context.
My second norm is Informativity, which requires that each new assertion be informative in context. Conveying information which is part of the contextually active common ground (as opposed to the vast subconscious store of shared history or cultural assumptions) is not cooperative, so the content of an assertion should not already be contained within the common ground. Also, an attentive and cooperative Hearer will typically comprehend each assertion fairly quickly, and in the order they are uttered. It is thus reasonable to assume that the propositional content of each assertion becomes part of the common ground for the next assertion. In practice, a Speaker often repeats new information until the Hearer understands it, but once the Speaker knows the Hearer has comprehended something, it is not cooperative for them to assert that information again. Cooperative communication thus requires that a Speaker does not convey information which is a consequence of the common ground, including the content of previous utterances.

Definition 1.3 Informatively Conveying Content

An assertion informatively conveys a Speaker’s communicative content in a context iff it conveys the communicative content in the context, and the common ground alone does not entail the propositional content of that assertion.

An assertion informatively partially conveys a Speaker’s communicative content in a context iff the common ground and communicative content entail the propositional content of the assertion, but the common ground alone does not.

An assertion informatively conveys part of a Speaker’s communicative content in a context iff it is the first of a finite sequence of utterances, the sequence conveys the entire communicative content in the context, and neither this assertion alone, nor the remainder of the sequence, conveys the entire communicative content.

The Informativity norm helps us to distinguish between the different types of partial Expressivity. For example, a Speaker might cooperatively communicate some complex and structured information by first providing an overview, and then further details that render the overview redundant. The overview may informatively convey the content, but it will not informatively convey part of the content. In general, while it is desirable for a later assertion to respect previous assertions by not merely repeating information, earlier assertions need not respect later assertions in this way. In contrast, providing the details first and then the overview will not satisfy either norm, although the overview might perform a useful role in confirming comprehension. The sequential performance and comprehension of utterances is a temporally asymmetric process, and this should be reflected in our choice of norms. The Expressivity norm I will formalise is that of partially conveying content. The additional complexity of distinguishing the information relationships between assertions from those within assertions is more than repaid by including temporal considerations in the analysis. This representation of order will prove useful in modelling natural language utterances, starting in §3.4.

The third norm I propose is the seemingly innocuous Consistency, which requires that the Speaker is not seeking to revise any previously-agreed information. In other words, the communicative content must be consistent with the common ground. The common ground can be thought of as what all parties are assuming the other parties are assuming. The Speaker need not actually believe the common ground any more than she needs to believe her communicative content, but the epistemic stance she chooses to communicate must be that of someone who does. That is, she must behave as if she is sincerely expanding the current common ground. If her new information deliberately contradicts what is assumed, then she would be not just providing novel information, but challenging the existing (putative) shared beliefs of herself and the Hearer. Ordinary single-agent belief revision is contentious, and the various formal methods arcane and obtuse; the revision of common beliefs is simply too complex to consider in this limited project. The Consistency norm will primarily be used to test that a series of utterances by a single Speaker is consistent, which should be expected from any cooperative Speaker, even those engaged in belief revision.

These three norms are sufficient for describing the relationship between assertions. However, I am interested primarily in what makes a single assertion acceptable in a context, and for that, I will need to delve within an assertoric utterance.
1.4.2 Structured Utterances

Utterances can have a complex internal structure. This structure is partially described by the syntax and semantics of the uttered expression. Expressions can be created by composing several existing related expressions into a new expression, either syndetically or asyndetically (that is, with or without the use of explicit lexical conjoining elements). I will focus on the syndetic case, as the explicit use of conjoining lexemes will simplify the analysis. The simplest examples of these lexemes in English are the coordinating conjunctions ‘and’, ‘or’, ‘but’, etc., and the subordinating conjunctions ‘if’, ‘when’, ‘since’, ‘unless’, etc. I will restrict my initial analysis to clausal expressions conjoined by ‘and’ and ‘or’, as these coordinators have the greatest similarity to truth-functional connectives, and so are the simplest connectives to analyse with a system whose restrictions are based on classical entailment. These coordinators are linguistic elements and hence part of the expression. Despite this, I will analyse the communicative content associated with each of the conjoined subutterances, rather than the linguistic expressions joined by the coordinators. This is because I am concerned not just with the linguistic properties of an assertion, but also the pragmatic properties due to its utterance in a particular context. This allows me to consider assertions that have a compound structure, where some subutterances are idiomatic, metaphorical, ironic, or otherwise convey information that is unrelated to the semantic content of the sub-expressions uttered. The relevant structure of the utterance is merely the relevant structure of the expression (or more correctly, of the logical form of the syntactic structure, or an equivalent abstraction depending on the preferred syntactic theory). This composite approach resolves a potential conflict, as connectives are linguistic objects, and hence part of the expression, but the content of what they connect is inherently intentional and propositional.

For example, if the expression ‘Either Melissa or Thomas will pick John up from the airport’ is uttered, then it has the form of a disjunction, and the propositional content of the utterance can be represented by a formula that has the corresponding structure \( \varphi \lor \psi \). This utterance form allows the utterance content to be decomposed alongside the expression. The symbols \( \varphi \) and \( \psi \) represent the propositional contents of the atomic subutterances expressed by ‘Melissa will pick John up from the airport’ and ‘Thomas will pick John up from the airport’ which in this case are atomic. One complication in this example is the use of phrasal rather than clausal coordination. In §3.5 I will present a standard method of representing phrasal coordination using boolean functions and lambda calculus, but until then I will ignore this wrinkle, describing norms and conditions for clausal coordination while continuing to use the more natural-sounding phrasal coordination in examples.

I will require a formal language that can describe atomic propositions, the negation of atomic propositions, and conjunction and disjunction between arbitrarily complex propositions, so that I can analyse the internal structure of utterances. This will allow me to represent the propositional structure of utterance forms. Given my choice of coordinations, the obvious candidate for this task is the language of propositional logic. Conjunction and disjunction will be structural connectives, but I will treat negation as a non-structural operator, and so any complex formula will be regarded as identical to its Negation Normal Form\(^2\). By not treating negation as imposing additional structure on utterances, I gain several advantages. These include: reducing the number of connectives to investigate; simplifying formal proofs; and avoiding the complexities of modelling downwards-entailing environments, negative scopes, and other difficulties imposed by negating complex propositions\(^3\).

The propositional structure of a formula is a tree structure corresponding to the Negation Normal Form (NNF) of the formula, with each leaf node being associated with a literal occurrence, and each interior node a truth-functional connective. In addition, each node has an associated proposition that can be described by the formula generated by an in-order traversal of the subtree with that node as the root. The root node is thus associated with the proposition described by the initial formula, so a formula is both sufficient to generate its propositional structure, and can be recovered from it. Formulas and their propositional structure are thus interchangeable. In addition, if we allow commutativity, the tree need not be ordered, and if we allow associativity, it need not be confined to binary branching.

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\(^2\)In Negation Normal Form, only atoms may be negated.

\(^3\)The decision to treat negation as a mere operator will be reviewed in §5.2.
I will use this propositional structure when claiming, for instance, that all utterances of the
form ‘A and A’ have a similarity that they do not share with atomic utterances of the form ‘A’ by
contrasting the formulas $p \wedge p$ and $p$. These formulas have different structures even though their
truth-conditions are identical, and the Speaker’s choice of structure will allow the Hearer to discern
additional information from the corresponding utterance.

The context and communicative content are sets of propositions without the propositional struc-
ture imposed on the content of utterances by linguistic considerations. As I want to relate these sets
of propositions to the utterance’s propositional structure, I will express them in the same (proposi-
tional) language. I will assume that a set of communicative intentions is always finite, due to the
finite size of the brain, finite length of actual utterances, and because I will occasionally wish to dis-
join the members of this set; infinite sets add technical complexity without adding any value in this
case. The communicative content, being finite, can be represented as a single proposition. However
my notational preference is to represent both this and the common ground as sets of propositions.
This notational contrast with the formula representing a propositional structure reminds us that
the communicative content and the common ground are semantic, rather than syntactic, objects.
The notation also discourages us from attempting contraposition, antilogism, or similar syntactic
manoeuvres. These manoeuvres would be peculiar in the extreme (except when engaged in purely
uninterpreted formalism), as sets of propositions are radically different types of objects than the
propositional structures associated with utterances.

1.4.3 Formalised Assertion Norms

With the selection of a propositional language, we can now introduce formal notation. An assertoric
utterance is represented by $U$, with or without a subscript ($U_1$, $U_2$). An utterance sequence is
simply a concatenation of utterances, $U_1...U_n$. The propositional structure of an utterance $U_i$ is
represented by the formula $\phi_i$. The propositional content of the communicative intention associated
with an utterance is represented by the set $\Gamma$. The propositional content of the common ground is
represented by the set $\Pi$. Our current definitions are restated below using this notation:

**Definition 1.4** Assertions Convey Intentions

1. An assertion $U$ **conveys** $\Gamma$ in $\Pi$ iff $\Pi |_{CL} \land \Gamma \leftrightarrow \varphi$.
2. An assertion $U$ **partially conveys** $\Gamma$ in $\Pi$ iff $\Pi, \Gamma |_{CL} \varphi$.
3. An assertion $U_1$ **conveys part of** $\Gamma$ in $\Pi$ iff there exists $U_2$ such that $U_1U_2$ conveys $\Gamma$ in $\Pi$.
4. An assertion $U$ **informatively conveys** $\Gamma$ in $\Pi$ iff $U$ conveys $\Gamma$ in $\Pi$ and $\Pi, \Gamma |_{CL} \varphi$.
5. An assertion $U$ **informatively partially conveys** $\Gamma$ in $\Pi$ iff $\Pi, \Gamma, \Gamma |_{CL} \varphi$ and $\Pi |_{CL} \varphi$.
6. An assertion $U_1$ **informatively conveys part of** $\Gamma$ in $\Pi$ iff there exists $U_2$ such that $U_1U_2$ informatively conveys $\Gamma$ in $\Pi$, but neither $U_1$ nor $U_2$ conveys $\Gamma$ in $\Pi$.

Some of the results mentioned earlier become trivial to confirm with this notation. For instance,
if $U$ (informatively) conveys $\Gamma$ in $\Pi$, or if $U$ (informatively) conveys part of $\Gamma$ in $\Pi$, then $U$ (informa-
tively) partially conveys $\Gamma$ in $\Pi$. The assertion norms can also be restated more formally using this
notation, demonstrating their relationship with Defn 1.4.5 for informatively partially conveying $\Gamma$:

**Definition 1.5** Assertion Norms for context $\Pi$, content $\Gamma$ and utterance form $\varphi$.

1. **Expressivity**: $\Pi, \Gamma |_{CL} \varphi$.
2. **Informativity**: $\Pi |_{CL} \varphi$.
3. **Consistency**: $\Pi, \Gamma |_{CL} \perp$.

**Lemma 1.6** An assertion respects the assertion norms of Expressivity, Informativity, and Consis-
tency iff it informatively partially conveys $\Gamma$ in $\Pi$, and $\Gamma$ is consistent in context $\Pi$.

**Proof**: By Defns 1.4.5 and 1.5. □
Every complex utterance has an associated structure consisting of a number of atomic subutterances linked by conjunctions and disjunctions. More finely-grained restrictions can be imposed on these structures than on unstructured utterances, as each subutterance can be recursively restricted based on the role it plays in conveying the overall contents of the utterance. The three assertion norms of Defn 1.5 should apply not just to the complex utterance, but also to each subutterance embedded within it, for exactly the same reasons as they apply to the complex utterance. The norms for an utterance refer to the communicative content, so these need to be defined for each subutterance. The communicative content partially conveyed by the complex utterance is the same for all subutterances, but the context varies between subutterances, being the common ground of the complex utterance plus some information about the rest of the utterance. I have previously stipulated that an utterance is an action that is completely planned before being performed. The Hearer can then expect the Speaker to have considered all subutterances of an utterance before uttering any, and so the order of the subutterances is not important for determining the subutterance context\(^4\).

The context of a subutterance is determined recursively using the structural connectives. If a complex utterance \(U_1 \text{ ‘and’ } U_2\) has the conjunctive utterance form \(\varphi_1 \land \varphi_2\), then uttering \(U_1\) occurs in a context where uttering \(U_2\) has also been planned, and both \(\varphi_1\) and \(\varphi_2\) are believed to be true. In this case, \(U_1\) needs to be informative even though \(U_2\) will also be uttered, and so whether \(U_2\) is uttered before or after \(U_1\), the putative truth of \(\varphi_2\) is part of the context for \(\varphi_1\). For example, the utterance conveying ‘I like all Schnauzers and I like all dogs’ is not informative. The disjunctive utterance conveying ‘I like all Schnauzers’, while informative by itself and even in the available context at that point, is not informative given the rest of the overall utterance (assuming that it is common knowledge that Schnauzers are dogs). Similarly, if a complex utterance \(U_1 \text{ ‘or’ } U_2\) has the disjunctive form \(\varphi_1 \lor \varphi_2\), then uttering \(U_1\) occurs in a context where uttering \(U_2\) has also been planned, and at least one of \(\varphi_1\) or \(\varphi_2\) is believed to be true. Again, \(U_1\) needs to be informative even though \(U_2\) will also be uttered, which requires there to be an informative case where \(\varphi_1\) is true and \(\varphi_2\) is not, and so whether \(U_2\) is uttered before or after \(U_1\), the falsity of \(\varphi_2\) is part of the content for \(\varphi_1\). The disjunctive utterance conveying ‘I like a Schnauzer or I like a dog’, is also not informative. In this case, the subutterance conveying ‘I like a dog’, while informative by itself and even in the available context at that point, is not informative given the rest of the overall utterance (and common knowledge about Schnauzers).

The following definition imposes the assertion norms recursively on a propositional structure. I will refer to conditions 2 and 3 as Disjunctive Compositionality and Conjunctive Compositionality respectively.

**Definition 1.7** Assertible Propositional Structure Relations \(\langle \Pi; \Gamma \rangle \vdash \varphi\).

1. \(\langle \Pi; \Gamma \rangle \vdash \varphi \iff \Pi, \Gamma \models_l \varphi, \Pi \models_{\ell_l} l, \text{ and } \Pi, \Gamma \models_{\ell_{\perp}} \perp, \text{ for } l \text{ a literal.}\)
2. \(\langle \Pi; \Gamma \rangle \vdash \varphi \land \psi \iff \langle \Pi, \neg \psi; \Gamma \rangle \vdash \varphi \text{ and } \langle \Pi, \neg \varphi; \Gamma \rangle \vdash \psi.\)
3. \(\langle \Pi; \Gamma \rangle \vdash \varphi \land \psi \iff \langle \Pi, \varphi; \Gamma \rangle \vdash \psi, \langle \Pi, \psi; \Gamma \rangle \vdash \varphi, \text{ and } \Pi, \Gamma \models_{\ell_{\land}} \varphi \land \psi.\)

**Lemma 1.8** If \(\langle \Pi; \Gamma \rangle \vdash \varphi\) then the assertion norms hold for context \(\Pi, \text{ content } \Gamma\) and form \(\varphi\).

**Proof:** By Induction on the complexity of \(\varphi\). The base case of literals is trivial, as Defn 1.7.1 is identical to Defn 1.5. For the inductive case, assume this Lemma holds for all formulas less complex than \(\varphi\). If \(\varphi\) is a disjunction \(\varphi_1 \lor \varphi_2\), then by Defn 1.7.2: (i) Expressivity: if \(\langle \Pi, \neg \varphi_2; \Gamma \rangle \models \varphi_1\) then \(\Pi, \Pi, \neg \varphi_2, \Gamma \models_{\ell_{\land}} \varphi_1, \text{ so } \Pi, \Pi \models_{\ell_{\land}} \varphi_1 \lor \varphi_2\). (ii) Informativity: if \(\langle \Pi, \neg \varphi_2; \Gamma \rangle \models \varphi_1\) then \(\Pi, \Pi, \neg \varphi_2 \models_{\ell_{\land}} \varphi_1, \text{ so } \Pi \Pi \models_{\ell_{\land}} \varphi_1 \lor \varphi_2\). (iii) Consistency: if \(\langle \Pi, \neg \varphi_2; \Gamma \rangle \models \varphi_1\) then \(\Pi, \Pi, \neg \varphi_2, \Gamma \models_{\ell_{\perp}} \perp, \text{ so } \Pi, \Pi \models_{\ell_{\perp}} \perp.\)

\(\varphi\) is a conjunction \(\varphi_1 \land \varphi_2\), then by Defn 1.7.3: (i) Expressivity: \(\Pi, \Pi \models_{\ell_{\land}} \varphi_1 \land \varphi_2\). (ii) Informativity: if \(\Pi, \varphi_2; \Gamma \models \varphi_1\) then \(\Pi, \varphi_2 \models_{\ell_{\land}} \varphi_1, \text{ so } \Pi \Pi \models_{\ell_{\land}} \varphi_1 \land \varphi_2\). (iii) Consistency: if \(\Pi, \varphi_2; \Gamma \models \varphi_1\) then \(\Pi, \varphi_2, \Gamma \models_{\ell_{\perp}} \perp, \text{ so } \Pi, \Pi \models_{\ell_{\perp}} \perp.\) By Defn 1.7.3.

Lemma 1.8 demonstrates that applying the Informativity and Consistency norms to each atomic subutterance in their recursively-defined contexts is sufficient to ensure Informativity and Consistency for the overall utterance. However atomic Conjunctive Expressivity is insufficient without the additional requirement that \(\Pi, \Gamma \models_{\ell_{\land}} \varphi \land \psi\). That is, imposing the norms in Defn 1.5 on each atomic

\(^4\)This idealisation is one of the less plausible assumptions in this analysis, and will be revisited in §3.4.
subutterance in a complex utterance is sufficient for imposing them on the complex utterance, if the complex utterance also satisfies Expressivity. If an utterance satisfies the norms when \( \Pi = \emptyset \), then it will satisfy them for any \( \Pi \) except if \( \Pi \) contains some particular proposition that contradicts or duplicates some parts of the utterance. That is, the utterance is assertible in any context unless there is a particular relevant piece of information in the context that may prevent it. An empty \( \Pi \) thus models the default context. Applying the constraints in Defn 1.5 to the utterance form \( \langle \emptyset; p \land q \rangle \vdash (p \land r) \lor (q \land \neg r) \), which satisfies Defn 1.7. An apple’s state of decay does not seem to be directly relevant to expressing the desired information; this is reflected by the propositional variable \( r \) representing ‘the apple is rotten’ not appearing in the communicative content or the common ground. It seems natural to prohibit these irrelevant variables from the utterance form, by adding an Inclusion norm that results in all propositional variables in an utterance form also occurring in its communicative content or context in the same scope (positive or negative). One approach would be to simply require that the literals in \( \varphi \) are a subset of those in \( \Pi \cup \Gamma \), but this still permits some unintuitive cases. For example one can partially convey that each apple is either brown and rotten or red and round, by asserting ‘each apple is brown and rotten, red and rotten, or round and not rotten’. In addition, this norm is quite different from those introduced so far, and it doesn’t take advantage of how close the existing norms come to imposing variable and even literal inclusion by themselves. I have chosen instead to strengthen some of the existing norms just enough to impose literal inclusion.

One way to augment the existing assertion norms to provide literal inclusion is to require that they are satisfied cetereis paribus; that is with all other things being held constant. Then for each literal occurrence in the utterance form there would be some pair of circumstances (possible worlds, for those who like such talk) which are identical except for the truth value of that literal, the utterance is true iff the literal is true, and in that case is true due to that literal occurrence, so that if we reduced the context of the subutterance represented by the literal occurrence to just those two possibilities, it would still be assertible. This cetereis paribus requirement, when imposed recursively, ensures that each literal in \( \varphi \) distinguishes a case where \( \varphi \) and \( \Gamma \) are true from an otherwise identical case where they are false.

**Definition 1.9** Propositional Structure Relations with Inclusion for \( \langle \Pi; \Gamma \rangle \vdash \varphi \)

1. \( \langle \Pi; \Gamma \rangle \vdash l \) iff there is a maximal consistent set \( \Sigma \subseteq LIT(\Pi, \Gamma) \setminus \{l, \neg l\} \) such that
   \( \Pi, \Sigma, \Gamma \vDash l; \Pi, \Sigma, \Gamma \vDash \neg l; \Pi \vDash l; \) and \( \Sigma \vDash \varphi \) (the complete utterance).
2. \( \langle \Pi; \Gamma \rangle \vdash \varphi \lor \psi \) iff \( \langle \Pi, \neg \psi; \Gamma \rangle \vdash \varphi \) and \( \langle \Pi, \neg \varphi; \Gamma \rangle \vdash \psi \).
3. \( \langle \Pi; \Gamma \rangle \vdash \varphi \land \psi \) iff \( \langle \Pi, \varphi; \Gamma \rangle \vdash \psi \), and \( \Pi; \Gamma \vDash \varphi \land \psi \).

The literal clause of Defn 1.9 differs from that of Defn 1.7 only by the addition of the set \( \Sigma \) (that characterises the two cetereis paribus cases) to the Expressivity and Consistency norms. Recursively applying the conditions reveals that \( \langle \emptyset; p \land q \rangle \vdash (p \land r) \lor (q \land \neg r) \) fails the strengthened norms. Proof that the cetereis paribus assertion norms are sufficient to impose literal inclusion on all assertions will have to wait until Theorems 2.94 and 2.102. It might seem natural to also impose a cetereis paribus requirement on the Informativity norm \( \Pi \vDash l \), making it \( \Pi, \Sigma \vDash l \). The system with all three cetereis paribus norms is called \( 5^+ \)-assertibility, and it is discussed in Appendix A.2, along with my reasons for rejecting it.
The last norm that we will consider in this chapter is Extensibility, which requires that any assertion that only partially conveys a Speaker’s communicative content should be extensible to an assertion that conveys their entire communicative content. This requirement has already appeared in the guise of conveying part of the communicative content, where it was rejected as being too demanding on a disorganised Speaker. One more formal reason to consider it is that it has an interesting and complex relationship with Inclusion. We will explore this relationship in §2.6.1.

**Definition 1.10** Assertion Norms with Extensibility. Extend Defn 1.5 with:

4. If $\Pi \not\vDash_{cl} (\wedge \Gamma \leftrightarrow \varphi)$ then $\exists \psi : \Pi \vDash_{cl} (\wedge \Gamma \leftrightarrow \varphi \land \psi)$ and $\langle \Pi; \Gamma \rangle \models \varphi \land \psi$.

**Lemma 1.11** Extensibility and Inclusion are independent of each other.

*Proof:* $\langle \emptyset; p \land q \rangle \not\vdash ((p \land r) \lor (q \land \neg r)) \land ((p \land \neg r) \lor (q \land r))$ satisfies Extensibility but not Inclusion. $\langle \emptyset; \{p \lor r, \neg r \lor q\} \rangle \models p \lor q$ satisfies Inclusion but not Extensibility. Both utterance forms satisfy all the other assertion norms. ■

**1.4.5 Cumulative Restrictions**

All of our assertion norms (except perhaps Extensibility) should be expected to hold in typical cases of cooperative assertion, but it is also instructive to consider the effects of individual norms and particular combinations. One reason for this is to produce simpler and more intelligible systems and proofs. Another reason is that the intermediate systems are useful in their own right, as they relate to existing logical relations. It is also often useful to be able to effectively suppress some of the less relevant norms. In the next chapter I will analyse five incremental sets of the norms, and their associated relations $\models_1$ through $\models_5$. In later chapters I will primarily use the system containing all the norms. The first set of norms $\langle \Pi; \Gamma \rangle \models_1 \varphi$ will be Expressivity, Disjunctive Compositionalcy, and Consistency of disjuncts. Next, $\langle \Pi; \Gamma \rangle \models_2 \varphi$ also imposes universal Consistency. Third, $\langle \Pi; \Gamma \rangle \models_3 \varphi$ adds Informativity. Then $\langle \Pi; \Gamma \rangle \models_4 \varphi$ introduces Conjunctive Compositionalcy from Defn 1.7.3 to complete the standard assertion norms. Finally, $\langle \Pi; \Gamma \rangle \models_5 \varphi$ adds the Inclusion norm. The classical conjunctive compositionalcy principle $(\langle \Pi; \Gamma \rangle \models \varphi \land \psi \iff \langle \Pi; \Gamma \rangle \models \varphi \land \langle \Pi; \Gamma \rangle \models \psi)$ applies to those systems without Conjunctive Compositionalcy.

**Definition 1.12** A Family of Cumulative Norms on Propositional Structure Relations

1. $\langle \Pi; \Gamma \rangle \models_1 \varphi \iff$ Expressivity, Disjunctive Compositionalcy and Disjunct Consistency all hold.
2. $\langle \Pi; \Gamma \rangle \models_2 \varphi \iff \langle \Pi; \Gamma \rangle \models_1 \varphi$ and Consistency holds.
3. $\langle \Pi; \Gamma \rangle \models_3 \varphi \iff \langle \Pi; \Gamma \rangle \models_2 \varphi$ and Informativity holds.
4. $\langle \Pi; \Gamma \rangle \models_4 \varphi \iff \langle \Pi; \Gamma \rangle \models_3 \varphi$ and Conjunctive Compositionalcy holds.
5. $\langle \Pi; \Gamma \rangle \models_5 \varphi \iff \langle \Pi; \Gamma \rangle \models_4 \varphi$ and Inclusion holds.

**Lemma 1.13** The Cumulative Family of Norms imposes the standard Assertion Norms

$\langle \Pi; \Gamma \rangle \models_4 \varphi \iff \langle \Pi; \Gamma \rangle \models \varphi$ satisfies Defn 1.7.

$\langle \Pi; \Gamma \rangle \models_5 \varphi \iff \langle \Pi; \Gamma \rangle \models \varphi$ satisfies Defn 1.9.

*Proof:* By Defns 1.5, 1.7, 1.9, and Lemma 1.8. ■

There are other combinations of norms that could be investigated instead of those above. For instance the first system only imposes Consistency on disjuncts which may seem rather odd, but it has very natural semantics and interpretations. The choice of Consistency or Informativity as the first norm is tied to the choice of Disjunctive or Conjunctive Compositionalcy as the first compositionalcy principle, as each norm gains most of its bite from recursive application. Informativity might seem more fundamental, but an inconsistent assertion can never be true in a consistent world, while Informativity only ensures that the assertion could have been false, and this asymmetry in truth and time makes Consistency a more basic requirement. Finally, although it is presented as an automatic choice above, Conjunctive Compositionalcy is not necessarily epistemically prior to Inclusion; both require Informativity, but neither depends on the other.
2. Formalisms

The cumulative assertion norms have a number of interesting semantics. In this chapter, I will present four of them, each of which is particularly suited to resolving some of the problems we will encounter. Licensed Contribution is an extension of classical truth table semantics that tracks which subformulas affect the truth of the conclusion. Relevance keeps track of the epistemic possibilities described by each part of the conclusion. The Modal semantics use Kripke semantics over a powerset of worlds with non-empty sets being accessible to all their supersets. Concision uses a partial preference order over formulas to ensure that every part of the conclusion is necessary for the inference’s validity. To simplify the semantics and proofs, I will initially assume that the common ground Π is empty, and not include it in the semantics until §2.6.

2.0.1 Standard Definitions

Unless stated otherwise, the formal systems throughout the thesis will use the language $L$ of classical propositional logic, with the propositional variables $\{p, q, r, s, t, \ldots\}$ and the constant $\bot$ for falsum. In addition, the following symbols are useful abbreviations: $\varphi \supset \psi := \neg \varphi \lor \psi$. $\top := \neg \bot$. $\varphi \oplus \psi := (\varphi \lor \psi) \land \neg (\varphi \land \psi)$. Section §2.3 will use the extended languages $L_M$ and $L_\Diamond$. The diamond symbols will be explained when the modal logics are defined.

$L : p | \bot | \neg \varphi | (\varphi \lor \psi) | (\varphi \land \psi)$.

$L_M : p | 0 | \neg \varphi | (\varphi \lor \psi) | \Diamond \varphi$.

$L_\Diamond : p | 0 | \neg \varphi | (\varphi \lor \psi) | \Diamond \varphi | \lozenge \varphi$.

In this chapter, the variable $n$ represents the cumulative assertibility norms applied to a particular inference relation, and will range over the values $\{1, 2, 3, 4, 5\}$. All formulas $\{\varphi, \psi, \ldots\}$ will belong to $L$. By stipulation any set $\Gamma$ will be a finite subset of $L$ (or at least will contain a finite number of variables). These conventions allow me to reduce the quantification required for presenting each lemma or definition.

Definition 2.1 Negation Normal Form

$\varphi$ is in Negation Normal Form (NNF) iff only its atoms fall in the scope of negations. By applying De Morgan Laws and Double Negation Elimination, any formula $\varphi \in L$ can be turned into NNF. The resulting formula is represented by $\text{NNF}(\varphi)$.

Definition 2.2 Sets of Propositional Variables and Literals

The set $\text{PROP}$ is the set of propositional variables in the language. The function $\text{PROP}(\varphi)$ returns the set of propositional variables that occur in $\varphi$. The function $\text{LIT}(\varphi)$ returns the set of literals that occur in $\text{NNF}(\varphi)$.

$\text{PROP}(\Gamma)$ is the union of the sets $\text{PROP}(\gamma)$ for all $\gamma \in \Gamma$, and similarly for $\text{LIT}(\Gamma)$.

$[\text{PROP}(\varphi)]$ is the intersection of $\text{PROP}(\psi)$ for all $\psi$ classically equivalent to $\varphi$.

$[\text{LIT}(\varphi)]$ is the intersection of $\text{LIT}(\psi)$ for all $\psi$ classically equivalent to $\varphi$.

The sets returned by $[\text{PROP}(\varphi)]$ and $[\text{LIT}(\varphi)]$ thus contain precisely those members of $\text{PROP}(\varphi)$ and $\text{LIT}(\varphi)$ that are necessary to represent the content of the formula $\varphi$.

Example: $\text{PROP}(p \land (q \lor \neg q)) = \{p, q\}$. $\text{LIT}(p \land (q \lor \neg q)) = \{p, q, \neg q\}$.

However, $[\text{PROP}(p \land (q \lor \neg q))] = [\text{LIT}(p \land (q \lor \neg q))] = \{p\}$.
2. Contribution & Licensing

Truth tables are an intuitive way to present the semantics for connectives for classical propositional logic, and to test for validity of inferences. Truth tables can also be adapted to other purposes, such as tracking whether the truth value of a particular occurrence of a subformula of the conclusion affects the overall truth value of the conclusion on a row. A contribution table is an extension of a truth table that records the contribution of a subformula on each row of the truth table alongside its truth value. Those subformulas that can take any truth value without affecting the inference’s validity do not contribute any content to the inference, and do not need to be evaluated. An inference is licensed if it is classically valid and each occurrence of an atom in its conclusion contributes some information about the conclusion’s truth value on some row.

2.1 Contribution Procedure

The procedure for completing a contribution table is an extension of that for completing a truth table, followed by the evaluation of an incremental series of criteria applied to the column for the main connective of the conclusion. Whether an inference complies with a particular licensing semantics depends on which of the criteria hold for its contribution table. Most of the criteria for licensing semantics are reasonably natural, apart perhaps from the very weakest (which is equivalent to an existing system, and thus interesting in its own right). Before I introduce all the steps for the contribution procedure, it will be helpful to look at examples of the end product. The inference on the left satisfies most licensing criteria, as can be seen by tracing the subscripted contribution of each atom occurrence to the truth value of each subformula. Each atom occurrence contributes at least once to the overall conclusion on a row where the premise and conclusion are true, and again where they are both false. The inference on the right is not licensed as the atom occurrence $q$ does not contribute to the truth of the conclusion on any row. These contribution tables will be explained as each step of their construction is introduced.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \oplus q$</th>
<th>$\vdash$</th>
<th>$p_a$</th>
<th>$\lor$</th>
<th>$q_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$1_a$</td>
<td>1</td>
<td>1</td>
<td>$1_b$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$1_a$</td>
<td>1</td>
<td>0</td>
<td>$0_b$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$0_a$</td>
<td>1</td>
<td>1</td>
<td>$1_b$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$0_a$</td>
<td>0</td>
<td>0</td>
<td>$0_b$</td>
</tr>
</tbody>
</table>

Provision

As we are concerned with the contribution of every subformula in the conclusion, each atom occurrence will be provided with a unique symbol, typically a sequentially assigned lowercase letter \{a, b, c, ...\}, representing its contribution. In variations where not all atoms in the conclusion are important, the remaining atoms will be provided with a non-specific contribution represented by ‘*’.

Definition 2.3 Standard Provision

An inference is provided with contributions by assigning a unique symbol (the contribution) to each atom occurrence in the conclusion of the inference.

The standard provision for $p \vdash p \lor (p \land q)$ is written $p \vdash p_a \lor (p_b \land q_c)$. There are several interesting variations on contribution provision. For instance, variable provision only provides specific contributions for occurrences of variables, rather than atoms. In this variant, the variable provision for $p \vdash p \lor \top$ is $p \vdash p_a \lor \top$. Schurzian relevance in Appendix A.5 ignores constants in this way.

Assignment

Once each contribution has been provided to an atom occurrence in the conclusion, it is then assigned to some or all of the rows of the inference’s contribution table.
Definition 2.4 *Simple Contribution Assignment*

The contribution of each atom occurrence is assigned to all rows of the contribution table.

Simple assignment is a little, well, simple. We can restrict the contribution assignment for an atom occurrence to only those rows where either the conclusion is false, or it would be false if the atom had a different truth value (*ceteris paribus*). Then the contribution of an atom occurrence is assigned only when its truth value affects the validity of the inference. Sharp-eyed readers may note this is a paraphrase of the Inclusion norm from Defn 1.5. To formalise this restriction, we need to define the notion of a pair of rows which are identical except for the value of a single variable.

Definition 2.5 *Atomic Twin Rows*

Two distinct rows of a truth table are twin rows for a particular atom if they share identical valuations for all other atoms. The function $twin(u,p)$ returns the $p$-twin row for row $u$.

Definition 2.6 *Restricted Contribution Assignment*

The contribution of each atom occurrence is assigned to a row of the contribution table iff the conclusion is false for that row or its twin row for that atom; other rows are assigned the non-specific contribution ‘*’.

Example: Simple and restricted contribution assignment for the inference $p \vdash (p \land \neg q) \lor q$. I have shaded the twin rows within an atom occurrence the same colour.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \vdash (p \land \neg q) \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$1_a$ 0 0 1 $1_c$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$1_a$ 1 $1_b$ 1 0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0 $a$ 0 $b$ 1 $1_c$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0 $a$ 0 $b$ 0 0 $c$</td>
</tr>
</tbody>
</table>

Compositionality

The contribution for a complex formula on a row is composed of the contributions from those of its direct subformulas which, if their truth values were changed (*ceteris paribus*), would change the truth value of the formula on that row\(^1\). The truth value of a conjunction will vary with a conjunct iff the other conjunct is true, that of a disjunction will vary with a disjunct iff the other disjunct is false, and that of a negation will always vary with its negand. Because of this compositional construction, the contribution of the conclusion on a row can be understood as the set of those atom occurrences that determine the truth value of the conclusion on that row.

As conjunction and disjunction are defined as dyadic rather than polyadic functions, the subformulas of a complex formula can be identified as either the ‘left’ or ‘right’ subformula (presentation conventions make the negand a ‘right’ subformula) by taking advantage of the linear ordering of writing. When describing the contribution-function for a connective, the contributions from the left and right subformulas are represented by the variables $l$ and $r$ respectively. When neither subformula contributes, the resulting non-specific contribution is represented by *. A non-specific contribution behaves like an empty set of contributions, so having a non-specific contribution is not the same as having no contribution.

Definition 2.7 *Compositional Contribution Tables*

The contribution of a complex formula on a row may consist of the contributions from any, all, or none of its subformulas:

<table>
<thead>
<tr>
<th>negation</th>
<th>conjunction</th>
<th>disjunction</th>
<th>conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\neg$</td>
<td>$\land$ $l_r$ $l_0$</td>
<td>$\lor$ $l_r$ $l_0$</td>
<td>$\supset$ $l_r$ $l_0$</td>
</tr>
<tr>
<td>$l_r$</td>
<td>$l_0$</td>
<td>$l_1$ $l_0$</td>
<td>$l_1$ $l_r$ $l_0$</td>
</tr>
<tr>
<td>$l_0$</td>
<td>$l_1$</td>
<td>$l_r$ $l_0$</td>
<td>$l_l$ $l_0$ $l_0$</td>
</tr>
</tbody>
</table>

\(^1\)This intuition is similar to that which motivated Körner (1947), see Appendix A.4.
Example: The contribution table for the inference \( p \vdash (p \land \neg q) \lor q \), using standard provision, simple assignment and standard compositional tables.

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \vdash (p_a \land \neg q_b) \lor q_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1_a 0_b 1_c 1_c</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1_a 1_ab 1_b 1_ab 0_c</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0_a 0_a 0_b 1_c 1_c</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0_a 0_a 1_b 0_ab 0_c</td>
</tr>
</tbody>
</table>

The standard compositional tables above are used for all the systems in this section, and preserve contribution over Commutativity, Associativity, the De Morgan laws and Double-Negation Introduction / Elimination.

Lemma 2.8 If a literal occurrence in positive scope contributes to \( \varphi \) on a row where \( \varphi \) is true (false), then every subformula of \( \varphi \) containing that literal occurrence is true (false) on that row.

Proof: This follows immediately from the contribution tables in Defn 2.7, regardless of the type of assignment used. This result does not apply if \( \varphi \) contains any asymmetric connectives from Defn 3.11, or other variations introduced in later chapters. \( \blacksquare \)

Licensing Evaluation

The final step in the algorithm is to evaluate whether all the atoms in the conclusion contribute appropriately to its truth value, and thus license its informativity as a conclusion for the inference. There are four criteria which help form a family of increasingly strong restrictions. The criteria test how variation in the truth value of an atom occurrence affects the truth value of the conclusion, either on rows where the premises and conclusion are both true, or those where they are both false. They are to be tested cumulatively against classically valid inferences.

Definition 2.9 Licensing Evaluation Criteria

The criteria for a classically valid inference to be contributing are:

[D1] Every contribution provided that is within the scope of a disjunction\(^2\) must be in the contribution for the conclusion on a row where the premise set and conclusion are true.

[D2] Every contribution provided must be in the contribution for the conclusion on a row where the premise set and conclusion are true.

[D3] Every subformula with a contribution must be true on some row, and false on another.

[D4] Every contribution provided must be in the contribution for the conclusion on a row where the premise set and conclusion are false.

Each criterion is independently motivated. [D1] rejects any inference with a disjunct whose truth value does not contribute on any row to the inference’s validity. [D2] rejects any inference with an atom occurrence that makes no difference to the truth value of the conclusion on any row where the premise set is true, and therefore varying its truth value on that row ceteris paribus cannot affect the inference’s validity. [D3] rejects any inference which has a contradiction or tautology as a subformula of the conclusion; no part of a contradiction or tautology can contribute meaningfully to the overall formula, as once a formula there is no need to consider the subformulas of a contradiction or tautology to determine its truth value. [D4] rejects any inference where at least one atom occurrence does not place restrictions on when the conclusion is true (Informativity) that are not already required by the rest of the conclusion. We can also refine [D2] and [D4] by requiring the atom occurrence to contribute to the conclusion on a row where changing the truth value of the atom will change the truth value of the conclusion. This ceteris paribus clause gives us literal inclusion.

\(^2\)More fully, a disjunction in positive scope, or conjunction in negative scope.
Example Inferences

Here are some examples of simple inferences along with which licensing criteria they satisfy. More information on these examples can be found in Appendix C.4.

Inferences that are not 1-licensed: $p \vdash p \lor q; p \vdash q \lor p; p \vdash \lnot p \lor q; p \vdash p \land q \lor (p \lor q); p \lor q \vdash (p \land q); p \land q \vdash (p \lor q) \land q;$ $p \land r \vdash p \lor (p \land q) \lor r; p \lor q \vdash p \lor r; p \vdash (p \land q)$ or $(p \land q) \lor r; p \vdash (p \lor q) \land q; p \vdash (q \lor p) \lor (q \land r)$; $p \vdash (p \land q) \lor (q \land r)$.

Inferences that are 1-licensed, but not 2-licensed: $p \vdash p \land q; p \vdash p \land r; p \lor q \vdash p \land q; p \lor q \vdash (p \land q) \land r$.

Example: The contribution tables for $p \land q \vdash p \lor q$ and $p \lor q \vdash p \land q$ with simple assignment are identical apart from the columns for premises, while they differ under restricted assignment. The first inference fails [D1] under simple assignment and so is not 1-licensed, while the second satisfies [D1]–[D4] under simple assignment (having each contribution on some row where the premise set is true, and again where the conclusion is false) and so is 4-licensed. The inference $p \lor q \vdash p \land q$ is an example of restricted assignment not affecting any of the contributions to the overall conclusion, so the inference is 5-licensed as well.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
<th>$p_a \lor q_b$</th>
<th>$p_a \lor q_b$</th>
<th>$p \lor q$</th>
<th>$p \lor q$</th>
<th>$p_a \land q_b$</th>
<th>$p_a \land q_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$1_a$</td>
<td>$1_a$</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>1</td>
<td>$0_a$</td>
<td>$0_a$</td>
</tr>
</tbody>
</table>

Example: The contribution tables for $p \land (q \lor r) \vdash (p \land r) \lor (q \land r)$ with simple and restricted assignment are similar. The inference is 4-licensed but not 5-licensed. The first contribution table uses simple assignment, and all atom occurrences contribute to the conclusion on a row where the premise is true, and another where the conclusion is false. The second table uses restricted assignment, and there are no rows where the conclusion is false and either occurrence of $r$ contributes to the overall conclusion. I have shaded some of the twin rows for each atom occurrence.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r$</th>
<th>$p \land (q \lor r)$</th>
<th>$(p_a \land r_b) \lor (q_c \land r_d)$</th>
<th>$(p_a \land r_b) \lor (q_c \land r_d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$1_a$</td>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>$1_a$</td>
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<td>$0_a$</td>
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</tr>
</tbody>
</table>
2. FORMALISMS

2.1.2 Licensing Properties

The various $n$-licensing criteria are closely related. $D1$ is entailed by $D2$, and I will prove below $D3$ is weaker than the combination of $D2$ $&$ $D4$. This means that 4-licensing and 5-licensing just require $D2$ $&$ $D4$. Furthermore, Lemma 2.12 shows we need only care about conclusions in negation normal form, and Lemma 2.13 shows that $D4$ and $D2$ are closely related. Finally, Lemma 2.14 demonstrates that $D4$ with restricted assignment imposes literal inclusion.

Lemma 2.11 Properties of $[D3]$

1. $[D3]$ holds iff no subformula of the conclusion is a tautology or contradiction.

Proof: 1. A subformula has a row where it is false iff it is non-tautologous, and a row where it is true iff it is non-contradictory. $[D3]$ applies to every subformula of the conclusion, as all of them will have a contribution on every row under standard contribution provision.

2. If $[D3]$ fails then a subformula $\psi$ of the conclusion $\varphi$ must be a tautology or contradiction. Suppose $\psi$ is a tautology. By the conjunction table in Defn 2.7, the contribution of a true conjunct can only be passed to its conjunction when the conjunction is true; and a true disjunct always results in a true disjunction, so any contribution from $\psi$ that is provided to $\varphi$ must occur on a row where $\psi$ is true. Then no contribution from atoms in $\psi$ will be provided to $\varphi$ on a row where $\varphi$ is false, and $[D4]$ fails. When $\psi$ is a contradiction, similar reasoning results in $[D2]$ failing.

Lemma 2.12 $\Gamma \vdash \varphi$ is an $n$-licensed inference iff $\Gamma \vdash NNF(\varphi)$ is an $n$-licensed inference.

Proof: By Defn 2.3, only atoms are provided with contributions. The contribution assignments in Defns 2.4 and 2.6 do not depend on $\varphi$, only its truth value and atom occurrences, and these are identical to those of $NNF(\varphi)$. By Defn 2.7 the contributions for $\varphi$ and $\lnot\varphi$ are the same, as are those for $\lnot(\varphi \land \psi)$ and $\lnot\varphi \lor \lnot\psi$, and those for $\lnot(\varphi \lor \psi)$ and $\lnot\varphi \land \lnot\psi$. Finally, Defn 2.9 imposes no connective-dependent conditions, except for $[D1]$, which respects the De Morgan laws. So $\varphi$ and $NNF(\varphi)$ are treated identically in every step of the algorithm.

Lemma 2.13 $\Gamma \vdash \varphi$ is 4-licensed iff $\Gamma \vdash \varphi$ and $\lnot\varphi \vdash \lnot\varphi$ are 2-licensed.

Proof: $\Gamma \vdash \varphi$ is 4-licensed iff $[D1]$ $- $ $[D4]$ are satisfied for $\Gamma \vdash \varphi$. $[D2]$ and $[D4]$ entail $[D3]$ (by Lemma 2.11.2), and $[D2]$ entails $[D1]$, so $\Gamma \vdash \varphi$ is 4-licensed iff $[D2]$ and $[D4]$ are satisfied. But $[D4]$ holds for $\Gamma \vdash \varphi$ iff $\lnot\varphi \vdash \lnot\varphi$ holds for $[D2]$ by Defn 2.9.$([D2]/[D4])$. So $[D2]$ and $[D4]$ hold for $\Gamma \vdash \varphi$ iff $\Gamma \vdash \varphi$ is 2-licensed and $\lnot\varphi \vdash \lnot\varphi$ is 2-licensed.

Lemma 2.14 Variable and Literal Inclusion

1. If $\Gamma \vdash \varphi$ is 5-licensed then every propositional variable occurring in $\varphi$ also occurs in $\Gamma$.
2. If $\Gamma \vdash \varphi$ is 5-licensed then every propositional variable occurring in positive (negative) scope in $\varphi$ also occurs in positive (negative) scope in $\Gamma$.

Proof: 1. This follows directly from Defn 2.6. Consider any occurrence of a propositional variable $p$ which is in $\varphi$ but not $\Gamma$. By restricted assignment, occurrences of $p$ will only be assigned a contribution on those rows where changing the value of $p$ will affect the truth value of $\varphi$. If $\varphi$ is true on a row, and changing the value of $p$ makes it false, then $\Gamma$ must be false on both rows. So there will be no rows where the premise set is true and any occurrence of $p$ provides a contribution to the conclusion, meaning $[D2]$ cannot hold under restricted assignment.

2. Suppose that a propositional variable $p$ occurs in negative scope in $\varphi$, and only occurs in positive scope in $\Gamma$. Under restricted assignment the only rows where $\varphi$ is true and occurrences of $p$ are assigned a contribution are those where changing the value of $p$ will make $\varphi$ false. If $\varphi$ is true on a row $u$ and false on its $p$-twin row $v$ due to an occurrence of $p$ in negative scope, then $p$ must be false on row $u$ by Lemma 2.8. For any occurrence of $\lnot p$ to contribute to $\varphi$ on $u$, $\Gamma$ must be true on $u$, and thus on $v$, violating classical entailment. So $[D2]$ cannot hold under restricted assignment for any such occurrence of $p$ in negative scope. A similar argument holds when the positive/negative scope roles are reversed.
2.1.3 Signatures

The central evaluation criteria for contributions ([D2] and [D4]) only reference two columns in the contribution table – the column of truth values for the premise set, and the column of truth values and contributions for the conclusion. I follow the conventions that truth table rows are ordered lexicographically and that contributions are assigned lexicographically throughout this thesis. By formalising these conventions, the information required for evaluation of an inference \( \Gamma \vdash \varphi \) for the conclusion. I follow the conventions that truth table rows are ordered reverse-

Definition 2.15 Signatures

1. The **signature** of \( \Gamma \vdash \varphi \) is an ordered \( 2^k \)-tuple \( \langle s_1, \ldots, s_{2^k} \rangle \), where \( k = |PROP(\Gamma \cup \{ \varphi \})| \). The element \( s_i \) in the signature is the entry for the \( i^{th} \) row of the conclusion in the contribution table for \( \Gamma \vdash \varphi \), and consists of a pair \( T \in C \), where \( T \) is the truth value of \( \varphi \) for that row and \( C \) is the corresponding set of contributions.

2. If the conclusion columns of the contribution tables for \( \Gamma \vdash \varphi \) under simple and restricted assignment differ, then \( \Gamma \vdash \varphi \) has distinct **simple** and **restricted signatures**.

3. The **negative signature** of \( \Gamma \vdash \varphi \) is the signature with no contributions listed when \( \varphi \) is true.

4. The **positive signature** of \( \Gamma \vdash \varphi \) is the signature with no contributions listed when \( \varphi \) is false.

Lemma 2.16 The simple and restricted negative signatures of \( \varphi \) are identical.

Proof: By Defns 2.4, 2.6, and 2.15.

Examples: The signature of \( p \lor q \) is \( \langle 1_s, 1_a, 1_b, 0_{ab} \rangle \).

The simple signature of \( (p \land r) \lor (q \land \lnot r) \) is \( \langle 1_{ab}, 1_{cd}, 1_{ab}, 0_{bc}, 0_{ad}, 1_{cd}, 0_a, 0_c \rangle \).

The simple signature of \( (p \lor \lnot r) \land (q \lor r) \) is \( \langle 1_a, 1_c, 1_{cd}, 0_{ab}, 1_{bc}, 0_{ab}, 0_{cd} \rangle \).

These two formulas share the restricted positive signature \( \langle 1_a, 1_c, 1_{ab}, 0, 0, 1_{cd}, 0, 0 \rangle \).

The signature of \( (p \land q) \lor (p \land r) \lor (q \land r) \) is \( \langle 1_s, 1_{ab}, 1_{cd}, 0_{bd}, 1_{ef}, 0_{af}, 0_{ce}, 0_s \rangle \).

The signature of \( (p \lor q) \land (p \lor r) \land (q \lor r) \) is \( \langle 1_s, 1_{ce}, 1_{af}, 0_{ef}, 1_{bd}, 0_{cd}, 0_{ab}, 0_s \rangle \).

Definition 2.17 Isomorphic Signatures

The signatures of \( \varphi \) and \( \psi \) are isomorphic iff \( \varphi \equiv \psi \), \( PROP(\varphi) = PROP(\psi) \), and there is an isomorphism from the contributions provided for \( \varphi \) to those for \( \psi \) such that each atomic contribution and its propositional image are assigned to occurrences of the same propositional variable, and the signature of \( \psi \) is identical to the signature of \( \varphi \) with each atomic contribution replaced by its image.

Examples (cont.): The signature of \( q \lor p \) is \( \langle 1_s, 1_b, 1_a, 0_{ab} \rangle \), so is isomorphic to that of \( p \lor q \).

The signatures of \( (p \land q) \lor (p \land r) \lor (q \land r) \) and \( (p \lor q) \land (p \lor r) \land (q \lor r) \) are also isomorphic.

The simple signatures of \( (p \land r) \lor (q \land \lnot r) \) and \( (p \lor \lnot r) \land (q \lor r) \) are not isomorphic, but their restricted positive signatures are. This result will later appear in the [B5] brevity operations.

Lemma 2.18 If \( \varphi \) and \( \psi \) have isomorphic simple negative, simple positive, or restricted positive signatures, then \( \Gamma \vdash \varphi \) satisfies \([D4], [D2]) under simple, and \([D2]) under restricted assignment respectively iff \( \Gamma \vdash \psi \) does, for any \( \Gamma \).

Proof: The only information from the conclusion that is used to determine the satisfaction of \([D2]\) and \([D4]\) is contained in the respective signature. If the signatures of \( \varphi \) and \( \psi \) are isomorphic, then \( \varphi \) and \( \psi \) are true on the same rows, and as \([D2]\) checks for the presence of contributions within those rows where \( \Gamma \) is true, and \([D4]\) within those rows where \( \varphi \) is false, only the positive signature is relevant for satisfying \([D2]\), and the negative signature for \([D4]\). As the isomorphism only permutes the contributions, it will not affect the satisfaction of \([D2]\) and \([D4]\), which each require the presence of all contributions.
2.2 Relevance & Possibility

The assertion norms can also be formalised by starting with considerations of relevance. I will develop semantics for them by starting with some existing relevance semantics, and extending them to cover all the norms we are interested in. There are several non-modal propositional deductive systems that are directly inspired by Grice’s maxim of Relevance as it relates to linguistic intuitions, rather than relevant inference or entailment, so I will extend some of these existing systems rather than start from scratch. The best systems for my purposes are RAD (Relevantly Assertable Disjunction) and C from Verhoeven (2007). The system RAD imposing restrictions on the relationships between disjuncts and the premise set to avoid vacuous disjuncts. Verhoeven uses RAD, and an extension C that also requires consistent premises, to formally derive several inferences that are closely related to conversational implicatures that rely on disjunction.

I will introduce the relevance systems as restrictions on propositional logic. To do that, I will first present classical propositional logic using Verhoeven’s set-theoretic semantics, before adding incremental restrictions on the non-emptiness of various sets. The first two relevance systems that I define, R1 and R2, are equivalent to Verhoeven’s systems. I will then extend the relevant assertibility restrictions to also account for tautologies, conjunction, and literal inclusion in three new systems that I call R3, R4 and R5. Each of the incremental restrictions impose another one of the assertion norms, as discussed in the last part of §1.4.3. The key to these systems is in understanding the role played by the non-emptiness of sets of possibilities associated with each part of the conclusion. Without this non-empty clause, each of the following systems collapses to classical propositional logic.

I have not provided independent motivation for each incremental restriction that I will impose. For those wanting additional motivation, this may be found by considering what makes a description relevant to the epistemic stance of the Speaker. Each semantic inference relation will be defined with respect to a set $S$ of possible worlds; $S$ can be understood as the epistemic stance of the Speaker, with each element of $S$ being an epistemic possibility that has not yet been ruled out. Each part of a description provides more detail, constraining or permitting possibilities. The possibilities described (and ruled out) by each part of the description should be real (non-vacuous) epistemic possibilities in the Speaker’s stance, so that each part of the description is relevant to the final epistemic constraints being imposed. The relevance semantics recursively defines this list of possibilities for each subformula, and tests whether each was a real epistemic possibility before the utterance. In the first two systems, these must remain possibilities after the utterance as well, but later systems also specify some possibilities that an utterance must rule out to be assertible.

2.2.1 CL Semantics

Modelling classical logic is a gentle way of acclimating ourselves to a new semantic approach.

**Definition 2.19** The $\varphi$-characterised Set $S_\varphi$

$W$ is the universe of epistemically available possible worlds.

For any context-free inference $\Gamma \vdash \varphi$, $W$ can be modelled by the powerset of $PROP(\Gamma \cup \{\varphi\})$.

1. $S_\varphi = \{w \in W \mid w \models \varphi\}$.
2. $S_\Gamma = \{w \in W \mid w \models \gamma, \forall \gamma \in \Gamma\}$.

**Definition 2.20** $S \models_{CL} \theta$

1. $S \models_{CL} l$ iff $S \subseteq S_l$, where $l$ is a literal.
2. $S \models_{CL} \varphi \land \psi$ iff $S \models_{CL} \varphi$ and $S \models_{CL} \psi$.
3. $S \models_{CL} \varphi \lor \psi$ iff $S \setminus S_\psi \models_{CL} \varphi$ and $S \setminus S_\varphi \models_{CL} \psi$.
4. $S \models_{CL} \neg \varphi$ iff $S \models_{CL} \varphi$.
5. $S \models_{CL} \neg(\varphi \land \psi)$ iff $S \models_{CL} \neg \varphi \lor \neg \psi$.
6. $S \models_{CL} \neg(\varphi \lor \psi)$ iff $S \models_{CL} \neg \varphi \land \neg \psi$. 
Example: $S_{p\land q} \models_{CL} p \lor q$. By Defn 2.19.1, $S_{p\land q} \models_{CL} p$, by Defn 2.20.1. Then $S_{p\land q} \setminus S_p \models_{CL} p$, and as $S_{p\land q} \setminus S_p = \emptyset$, we have $S_{p\land q} \setminus S_p \models_{CL} q$, so $S_{p\land q} \models_{CL} p \lor q$, by Defn 2.20.3.

Lemma 2.21 The valid inferences of CL are those of classical logic.
1. $S_\Gamma \models_{CL} \varphi$ iff $\forall w \in S_\Gamma, w \models \varphi$ iff $\Gamma \models_{CL} \varphi$.
2. $S \models_{CL} \varphi$ iff $S \subseteq S_\varphi$.
3. $S_\varphi^c = S \cap S_\neg \varphi$.

2.2.2 R1–R3 Semantics
The semantics for CL and R1 differ only in the recursive condition for disjunctions. A disjunction $\varphi \lor \psi$ is R1-deducible in $S_\Gamma$ iff it preserves truth (i.e., $\Gamma \models_{CL} \varphi \lor \psi$) and all parts of the disjunction are required to preserve truth. Each world in $S$ must satisfy $\varphi$ or $\psi$, with some worlds in $S$ not satisfying $\varphi$, and others not satisfying $\psi$. Also $\varphi$ and $\psi$ cannot contain disjuncts that entail any disjunct in the other formula, as $S \setminus S_\varphi$ and $S \setminus S_\psi$ are disjoint due to Defn 2.22.3.

Definition 2.22 $S \models_{R1} \theta$
1. $S \models_{R1} l$ iff $S \subseteq S_l$, where $l$ is a literal.
2. $S \models_{R1} \varphi \land \psi$ iff $S \models_{R1} \varphi$ and $S \models_{R1} \psi$.
3. $S \models_{R1} \varphi \lor \psi$ iff $S \setminus S_\varphi \models_{R1} \varphi$ and $S \setminus S_\psi \models_{R1} \psi$, where $S \setminus S_\varphi \neq \emptyset$, and $S \setminus S_\psi \neq \emptyset$.
4. $S \models_{R1} \neg \varphi \iff S \models_{R1} \varphi$.
5. $S \models_{R1} \neg(\varphi \land \psi)$ iff $S \models_{R1} \neg \varphi \lor \neg \psi$.
6. $S \models_{R1} \neg(\varphi \lor \psi)$ iff $S \models_{R1} \neg \varphi \land \neg \psi$.

Example: If $p$ is true in all worlds in $S$, then $S \models_{R1} p$, by Defn 2.22.1. However, as $S \setminus S_p = \emptyset$, $S \not\models_{R1} p \lor q$, by Defn 2.22.3. $S \not\models_{R1} p \lor p$, for the same reason. This universal rejection of Addition is the key to most of our later predictions of natural language disjunction usage.

Lemma 2.23 Some properties of R1.
1. If $S \models_{R1} \varphi$ then $S \models_{CL} \varphi$.
2. If $S \models_{CL} \varphi$ and $\varphi$ contains no disjunctions (in positive scope) then $S \models_{R1} \varphi$.
3. If $S \models_{R1} \varphi$, $S \subseteq S'$, and $S' \models_{CL} \varphi$ then $S' \models_{R1} \varphi$.
Proof: 1&2. Defns 2.20 and 2.22 differ only by an additional restriction on disjunction in Defn 2.22.3. 3. $S \models_{R1} \varphi$ requires $S \models_{CL} \varphi$ and that certain subsets of $S$ be non-empty. Increasing $S$ monotonically increases these subsets, so for any larger set $S'$ which preserves classical entailment, $S' \models_{R1} \varphi$.

One oddity of R1 is that it rejects an inference from inconsistent premises iff the conclusion contains a disjunction. The Consistency norm’s applicability should not depend on the choice of connective, and so we will introduce the next system R2, which requires that the premise set of an R1 inference is always consistent, by moving the check for non-emptiness from disjunctions to literals.

Definition 2.24 $S \models_{R2} \theta$
1. $S \models_{R2} l$ iff $S \subseteq S_l$ and $S \neq \emptyset$, where $l$ is a literal.
2. $S \models_{R2} \varphi \land \psi$ iff $S \models_{R2} \varphi$ and $S \models_{R2} \psi$.
3. $S \models_{R2} \varphi \lor \psi$ iff $S \setminus S_\varphi \models_{R2} \varphi$ and $S \setminus S_\psi \models_{R2} \psi$.
4. $S \models_{R2} \neg \varphi \iff S \models_{R2} \varphi$.
5. $S \models_{R2} \neg(\varphi \land \psi)$ iff $S \models_{R2} \neg \varphi \lor \neg \psi$.
6. $S \models_{R2} \neg(\varphi \lor \psi)$ iff $S \models_{R2} \neg \varphi \land \neg \psi$.

Example: If $S \neq \emptyset$ and $p$ is true in all worlds in $S$, then $S \models_{R2} p$, by Defn 2.24.1. However $S \models_{R2} p \lor q$ iff $S \setminus S_q \models_{R2} p$ and $S \setminus S_p \models_{R2} q$. Now $S \setminus S_p = \emptyset$, so the second conjunct is false by Defn 2.24.1, so $S \not\models_{R2} p \lor q$. In general, $S = \emptyset$ whenever the premises are contradictory, and if $S = \emptyset$, then $S \not\models_{R2} \varphi$, by Defn 2.24.1 and induction on the complexity of $\varphi$. 
Lemma 2.25 Some properties of $R_2$.
1. $S \models_{R_2} \varphi$ if $S \models_{R_1} \varphi$ and $S \neq \emptyset$.
2. $S \models_{R_1} \varphi$ if $S \models_{R_2} \varphi$, or $S \models_{C_L} \varphi$ and $\varphi$ is disjunction-free.
3. If $S \models_{R_2} \varphi$, $S \subseteq S'$, and $S' \models_{C_L} \varphi$ then $S' \models_{R_2} \varphi$.

Proof: 1. CL, R1, and R2 all recursively associate a subset of $S$ with each subformula of $\varphi$. The only difference between these three systems is that R1 requires each disjunct in $\varphi$ to have a non-empty subset of $S$, while R2 requires each literal occurrence in $\varphi$ to have a non-empty subset of $S$. But each literal has the same subset as the smallest disjunct in $\varphi$ containing that literal, or $\varphi$ if $\varphi$ contains no disjunctions. Conversely, if $S \models_{R_1} \varphi$ then every disjunct of $\varphi$ has a non-empty subset of $S$, and any formula not in the scope of a disjunction has the set $S$ by Defn 2.22.2, but $S \neq \emptyset$.

2. If $S \models_{R_1} \varphi$ and $S \not\models_{R_2} \varphi$ then some literal not in the scope of a disjunction has an empty subset of $S$, but that would be $S$ by Defn 2.22.2, so $S = \emptyset$. Then every subformula of $\varphi$ has an empty subset of $S$, and as all disjuncts must have a non-empty subset, $\varphi$ is disjunction-free. Conversely, if $S \models_{C_L} \varphi$ and $\varphi$ is disjunction-free then $S \models_{R_1} \varphi$ as the only difference between Defns 2.22 and 2.20 is in the respective disjunctive clauses.

3. As $S' \models_{C_L} \varphi$, when $\varphi$ is decomposed to its constituent literals as per Defn 2.20, each literal occurrence in $\varphi$ has a subset of $S'$ which is also a subset of $S_l$, thus satisfying the first condition of Defn 2.24.1 for $S' \models_{R_2} \varphi$. Also as $S \models_{R_2} \varphi$, each literal occurrence will have a subset of $S'$ and of $S$, and that subset of $S'$ will contain that of $S$. But each of these subsets of $S$ is non-empty as $S \models_{R_2} \varphi$, satisfying the second condition of Defn 2.24.1 for $S' \models_{R_2} \varphi$.

Lemma 2.26 $R_1$ is equivalent to Verhoeven’s RAD, and $R_2$ to her $C$.

Proof Sketch: In §2.2.2 and §2.11.5 of Verhoeven (2005), she defines RAD and $C$ identically to our Defns 2.22 and 2.24, except that she uses a more lengthy definition of the disjunctive clause involving partitions. However, in her Defn 2.15.4 she re-presents $C$ using the same disjunctive clause as our Defn 2.24, and in her §5.2.2, during the proof of her Theorem 5.1, she says that this disjunctive clause can be used for RAD as well.

Tautologies fail the Informativity norm, and are thus irrelevant even as part of a more complex formula. R2 does not check for these tautologies, so we introduce R3, which only accepts those R2 inferences which do not contain a tautology in the conclusion.

Definition 2.27 $S \models_{R_3} \theta$
1. $S \models_{R_3} l$ if either (a) $S \subseteq S_l$, (b) $S \cdot l = \emptyset$, and (c) $S \neq \emptyset$, where $l$ is a literal.
2. $S \models_{R_3} \varphi \land \psi$ iff $S \models_{R_3} \varphi$ and $S \models_{R_3} \psi$.
3. $S \models_{R_3} \varphi \lor \psi$ iff $S \setminus S_{\psi} \models_{R_3} \varphi$, $S \setminus S_{\varphi} \models_{R_3} \psi$, and $\not\models_{L} \varphi \lor \psi$.
4. $S \models_{R_3} \neg \varphi$ iff $S \models_{R_3} \varphi$.
5. $S \models_{R_3} \neg(\varphi \land \psi)$ iff $S \models_{R_3} \neg \varphi \lor \neg \psi$.
6. $S \models_{R_3} \neg(\varphi \lor \psi)$ iff $S \models_{R_3} \neg \varphi \land \neg \psi$.

Example: $S_p \not\models_{R_3} q \lor \neg q$, by Defn 2.27.3 as $\not\models_{C_L} q \lor \neg q$. Similarly, $S_p \not\models_{R_3} p \land (q \lor \neg q)$, but $S_p \models_{R_3} (p \land q) \lor (p \land \neg q)$, so distribution can make some formulas valid in R3, which is surprising.

Lemma 2.28 Some properties of $R_3$.
1. $S \models_{R_3} \varphi$ if $S \models_{R_2} \varphi$ and for all subformulas $\psi$ of $\varphi$, $W \not\models_{C_L} \psi$.
2. If $S \models_{R_3} \varphi$, $S \subseteq S'$, and $S' \models_{C_L} \varphi$ then $S' \models_{R_3} \varphi$.

Proof: 1. There are two differences between R3 and R2 (in Defn 2.24). The extra clause $W \not\models_{L} \varphi \lor \psi$ in Defn 2.27.3 prevents formulas containing disjunctions (in positive scope) that are tautologies from being valid in R3. The extra clause $S \cdot l \neq \emptyset$ in Defn 2.27.1(b) prevents formulas containing $\top$ from being valid in R3. But every tautology contains either a tautological disjunction in positive scope or $\top$, and $W$ is a powerset of variables, so all and only tautologies are true at every world in $W$.

2. If $S \models_{R_3} \varphi$ then $S \models_{R_2} \varphi$ by Lemma 2.28.1, and by Lemma 2.25.3, $S' \models_{R_2} \varphi$, so $S \models_{R_3} \varphi$ by Lemma 2.28.1 again.
2.2.3 R4–R5 Semantics

The relevance semantics for R4 and R5 are more complex than those for the earlier systems. To model an inference $\Gamma \vdash \varphi$, instead of only associating a set $S$ of worlds with each subformula $\psi$ of $\varphi$, we will use an ordered triple $\langle S, T, U \rangle$ of sets. The set $T$ is a set of worlds in which $\Gamma$ and $\varphi$ are false, and the falsity of $\varphi$ at any world in $T$ is due in part to the presence of $\psi$ in $\varphi$. The set $U$ is the context of $\psi$ within the overall formula $\varphi$. So $S \cap U$ can be thought of as a set within $\Pi \cap \Gamma$ and partially characterised by $\psi$, while $T \cap U$ is a set partially characterised by $\Pi \cap \{\neg \psi\}$. These restrictions ensure that $T \cap U \subseteq S_{\neg \psi} \subseteq S$. The symmetry gained by imposing the restriction $U$ both on $S$ and $T$ comes at a price. Classical entailment is not symmetric; otherwise it would be a type of equivalence relation. So either entailment needs to be imposed separately from the symmetric relevance conditions, as in Defn 2.29, or we require different relevance conditions for subformulas in positive and negative scope, as in Defn 2.31.

**Definition 2.29** $\langle S, T, U \rangle \models_{R4} \theta$

1. $\langle S, T, U \rangle \models_{R4} l$ if $S \cap U \neq \emptyset$ and $T \cap U \neq \emptyset$, for a literal $l$ in positive scope in $\theta$.
2. $\langle S, T, U \rangle \models_{R4} \varphi \land \psi$ if $\langle S, T \cap S_{\psi} \rangle \models_{R4} \varphi$ and $\langle S, T \cap S_{\psi} \rangle \models_{R4} \psi$.
3. $\langle S, T, U \rangle \models_{R4} \varphi \lor \psi$ if $\langle S, T \cap S_{\psi} \rangle \models_{R4} \varphi$ and $\langle S, T \cap S_{\psi} \rangle \models_{R4} \psi$.
4. $\langle S, T, U \rangle \models_{R4} \neg \varphi$ if $\langle T, S, U \rangle \models_{R4} \varphi$, for complex $\varphi$.

**Definition 2.30** $S_T \models_{R4} \varphi$ if $S_T \models_{CL} \varphi$ and $\langle S_T, S_{\neg \varphi}, W \rangle \models_{R4} \varphi$.

**Example:** $S_p \models_{R4} p$ as the literal (and its triple of sets of worlds) conforms to Defn 2.29.1.

**Lemma 2.32** $\langle S, T, 0 \rangle \models_{R4} \theta$ if $S \models_{CL} \theta$ and $\langle S, T, W \rangle \models_{R4} \theta$.

**Proof Sketch:** By induction on the complexity of $\theta$, and Defns 2.29 and 2.31. This proof is not spelled out, as each step is trivial except for conjunction where Defn 2.31.2(a) restricts $S$ to ensure classical entailment, and disjunction where Defn 2.31.3(b) does likewise with $T$. ■

**Lemma 2.33** If $\langle S, T, 0 \rangle \models_{R4} \text{NNF}(\varphi)$ then $S \models_{R3} \text{NNF}(\varphi)$.

**Proof:** The restrictions imposed on R3 but not R4+ are: (i) Defn 2.27.1(b) that $S_{\neg \cdot} \neq \emptyset$; and (ii) Defn 2.27.3 that no disjunctive subformula of $\varphi$ in positive scope is a tautology. But Defn 2.27.1(b) is subsumed in Defn 2.31.1(b) and (d) for literals in positive scope ($m=0$) as if $T \subseteq S_{\neg \cdot}$ and $T \neq \emptyset$, then $S_{\neg \cdot} \neq \emptyset$. Similarly, if $W \models_{CL} \varphi_1 \lor \varphi_2$ for some disjunctive subformula of $\varphi$ in positive scope, then as $S_{\neg (\varphi_1 \lor \varphi_2)} = \emptyset$, and the set $T$ for any subformula is a subset of the compliment of that subformula, $T = \emptyset$ for this subformula, and all its constituent literals. Note that the subformulas of $\varphi$ in positive scope have $m = 0$, while those in negative scope have $m = 1$. ■

**Lemma 2.34** Suppose $\langle S_1, T_1, U_1 \rangle \models_{R4} \varphi$.

1. If $S_1 \cap U_1 \subseteq S_2 \cap U_2 \subseteq S_{\varphi}$ and $T_1 \cap U_1 \subseteq T_2 \cap U_2 \subseteq S_{\neg \varphi}$ then $\langle S_2, T_2, U_2 \rangle \models_{R4} \varphi$.
2. If $\langle S_2, T_2, U_2 \rangle \models_{R4} \varphi$ then $\langle S_1, T_2, U_1 \rangle \models_{R4} \varphi$.

**Proof:** 1. The additional worlds in $S_2 \cap U_2$ and $T_2 \cap U_2$ cannot violate Defn 2.29.1 as it only checks for emptiness of non-decreasing sets. 2. As the sets $S$ and $T$ do not interact in Defn 2.29, the sufficiency of $\langle S_1, T_1, U_1 \rangle$ and $\langle S_2, T_2, U_2 \rangle$ demonstrates the joint sufficiency of $\langle S_1, T_2, U_1 \rangle$. ■
Lemma 2.35 \((S,T,0)\models_{R4}^+ NNF(\varphi)\) iff \(S\models_{R2} NNF(\varphi)\) and \(T\models_{R2} \neg NNF(\varphi)\).

Proof: By induction on the complexity of \(\varphi\). In the base case, \(\varphi\) is a literal. If \(\varphi = l\), then \(S\models_{R2} l\) and \(T\models_{R2} \neg l\) if \(S \subseteq S_l, T \not\subseteq S_l\), and \(T \not\subseteq 0\) by Defn 2.24.1 iff \((S,T,0)\models_{R4}^+ \neg l\) by Defn 2.31.1.

Our inductive hypothesis is that this lemma holds for all \(\psi\) of less complexity than \(\varphi\).

If \(\varphi = \varphi_1 \land \varphi_2\), then \(S\models_{R2} \varphi_1 \land \varphi_2\) and \(T\models_{R2} \neg((\varphi_1 \land \varphi_2))\) iff \(S\models_{R2} \varphi_1\) and \(S\models_{R2} \varphi_2\) and \(T\models_{R2} \neg\varphi_1\) and \(T\models_{R2} \neg\varphi_2\) by Defn 2.24.2/3/5 iff \((S,T \cap S_{\varphi_2},0)\models_{R4}^+ \varphi_1\) and \((S,T \cap S_{\varphi_1},0)\models_{R4}^+ \varphi_2\) by our inductive hypothesis iff \((S,T,0)\models_{R4}^+ \varphi_1 \land \varphi_2\) by Defn 2.31.2(a).

If \(\varphi = \varphi_1 \lor \varphi_2\), then \(S\models_{R2} \varphi_1 \lor \varphi_2\) and \(T\models_{R2} \neg((\varphi_1 \lor \varphi_2))\) iff \(S\models_{R2} \varphi_1\) and \(S\models_{R2} \varphi_2\) and \(T\models_{R2} \neg\varphi_1\) and \(T\models_{R2} \neg\varphi_2\) by Defn 2.24.2/3/6 iff \((S,S_{\varphi_2},T,0)\models_{R4}^+ \varphi_1\) and \((S,S_{\varphi_1},T,0)\models_{R4}^+ \varphi_2\) by our inductive hypothesis iff \((S,T,0)\models_{R4}^+ \varphi_1 \lor \varphi_2\) by Defn 2.31.3(a).

To define the semantics for R5, we need a new concept, the \(l\)-neutral extension of a set of worlds.

The additional condition imposed by \(l\)-neutral extensions in Defn 2.37.1 parallels the additional condition imposed by restricted assignment for 5-licensed inferences in \(\S\)2.1.2.

Definition 2.36 Set operations on Worlds

1. The world \(w\models_{\pm l}\) except the valuation of the literal \(l\) differs.
2. The \(l\)-neutral extension \(S_{\pm l}\) of \(S\) is the smallest superset of \(S\) where \(w\models_{\pm l}\) if \(w\models_{\pm l}\) in \(S_{\pm l}\).
3. The \(l\)-neutral contraction \(S_{\mp l}\) of \(S\) is the largest subset of \(S\) where \(w\models_{\mp l}\) iff \(w\models_{\pm l}\) in \(S_{\mp l}\).

Definition 2.37 \((S,T,U)\models_{R5} \theta\)

1. \((S,T,U)\models_{R5} l\) iff \(S \cap U \cap (S_{\neg l})_{\pm l} \neq \emptyset\) and \(T \cap U \cap (S_{\neg l})_{\pm l} \neq \emptyset\), for \(l\) a literal.
2. \((S,T,U)\models_{R5} \varphi \land \psi\) iff \((S,T,U \cap S_{\varphi})\models_{R5} \varphi\) and \((S,T,U \cap S_{\psi})\models_{R5} \psi\).
3. \((S,T,U)\models_{R5} \varphi \lor \psi\) iff \((S,T,U \setminus S_{\varphi})\models_{R5} \varphi\) and \((S,T,U \setminus S_{\psi})\models_{R5} \psi\).
4. \((S,T,U)\models_{R5} \neg \varphi\) iff \((S,T,U)\models_{R5} \varphi\), for \(\neg \varphi\).

Definition 2.38 \(S_{\Gamma} \models_{R5} \varphi\) iff \(S_{\Gamma} \models_{CL} \varphi\) and \((S_{\Gamma},S_{\neg \varphi},W)\models_{R5} \varphi\).

Example: \(S_{\rho \lor q} \models_{R5} p\). \(S_{\rho \lor q} \models_{R5} (p \land r) \lor (q \land \neg r)\) as both occurrences of \(r\) violate Defn 2.37.1(a).

Lemma 2.39 If \((S,T,U)\models_{R5} \varphi\) then \((S,T,U)\models_{R4} \varphi\).

Proof: As \(S \cap U \cap (S_{\neg \varphi})_{\pm l} \subseteq S \cap U\) and \(T \cap U \cap (S_{\neg \varphi})_{\pm l} \subseteq T \cap U\), if Defn 2.37.1 is satisfied, so is Defn 2.29.1. The recursive clauses of Defns 2.37 and 2.29 are identical.

Lemma 2.40 If \((S_1,T_1,U)\models_{R5} \varphi\), \(S_1 \subseteq S_2 \subseteq S_{\varphi}\), and \(T_1 \subseteq T_2 \subseteq S_{\neg \varphi}\), then \((S_2,T_2,U)\models_{R5} \varphi\).

Proof: As \(S_1 \subset S_2\) and \(T_1 \subset \subset T_2\), \(S_1 \cap U \cap (S_{\neg \varphi})_{\pm l} \subseteq S_2 \cap U \cap (S_{\neg \varphi})_{\pm l}\) and \(T_1 \cap U \cap (S_{\neg \varphi})_{\pm l} \subseteq T_2 \cap U \cap (S_{\neg \varphi})_{\pm l}\) as \(S_2 \subseteq S_{\varphi}, S_2 \models_{CL} \varphi\).

Lemma 2.41 If \(S_{\Gamma} \models_{Rn+1} \varphi\) then \(\Gamma\models_{Rn} \varphi\).

Proof: By Lemmas 2.25.1, 2.28.1, 2.33, and 2.39.

Lemma 2.42 If \(S_{\Gamma} \models_{Rn} \varphi\) then \(\Gamma\models_{CL} \varphi\).

Proof: By Lemmas 2.23.1 and 2.41, and Defns 2.29 and 2.37.

Lemma 2.43 \(S\models_{Rn} \varphi\) iff \(S\models_{Rn} NNF(\varphi)\).

Proof: For \(n \leq 3, S_{\models_{Rn} \neg \varphi}\), \(S_{\models_{Rn} \neg(\varphi \land \psi)}\), \(S_{\models_{Rn} \neg(\neg \varphi \land \psi)}\), and \(S_{\models_{Rn} \neg((\varphi \land \psi) \lor (\neg \varphi \land \psi))}\) iff \(S\models_{Rn} \neg \varphi\lor \neg \psi\), and \(S\models_{Rn} \neg(\varphi \lor \psi)\) iff \(S\models_{Rn} \neg \varphi\land \neg \psi\), by Defn 2.24.4/5/6, Defn 2.24.4/5/6, and Defn 2.27.4/5/6 for \(n = 1, 2, 3\) respectively. For \(n \geq 4, S\models_{CL} \varphi\) iff \((S_{\models_{CL} NNF(\varphi)}\) and \((S,S_{\neg \varphi},U)\models_{Rn} \varphi\) iff \((S,S_{\neg \varphi},U)\models_{Rn} \neg(\varphi \lor \psi)\) iff \((S,S_{\neg \varphi},U)\models_{Rn} \neg(\varphi \land \psi)\) by Defns 2.29.2/3/4 and 2.37.2/3/4, for \(n = 4, 5\) respectively. This lemma generalises the results of Lemmas 2.33 and 2.35.

We have provided a set-theoretic and decompositional characterisation of Relevance semantics. This is an ideal level of abstraction for introducing modality in 2.3, proofs of soundness and completeness for modelling assertion norms in 2.6.1, and probabilistic assertibility in 3.4.
2.3 Modal Semantics

This section provides a direct translation of the Relevance family of semantics into modal logic. The modal notation presented here will also turn out to be useful later, particularly when investigating asymmetric or non-commutative connectives, how speech acts are conjoined, and the interaction of modal verbs such as ‘can’, ‘must’, and ‘may’ with disjunction.

Definition 2.44 Model Properties
1. \( M = \langle W, R, V \rangle \) is a Kripke model iff \( R \subseteq W^2 \) and \( V : \text{PROP} \to \mathcal{P}(W) \).
2. \( M = \langle W, R, O, V \rangle \) is a Kripke O-model iff \( \langle W, R, V \rangle \) is a Kripke model; \( O \subseteq W \); \( \forall p \in \text{PROP} : \)
   \( O \subseteq V(p) \); and \( \forall w \in W, \not\exists v \in O : Rwv \).
3. \( \langle W, R, V \rangle \) is Persistent iff \( \forall u, v \in W : \) if \( u \in V(p) \) and \( Rwu \) then \( v \in V(p) \).
4. \( \langle W, R, V \rangle \) is Intuitionistic iff it is Persistent and \( R \) is a partial order.
5. \( \langle W, R, O, V \rangle \) is Recurrent iff \( \forall w \in W : \) if \( w \notin V(p) \) and \( w \notin O \) then \( \exists u \in W : Rwu, u \notin V(p) \), and \( \forall v \in W \) such that \( Rwv, v \notin V(p) \).
6. \( \langle W, R, O, V \rangle \) is Classic iff it is Intuitionistic and Recurrent.

Definition 2.45 Modal Semantics
A Kripke O-model \( M = \langle W, R, O, V \rangle \) for \( L_M \) is as follows:
1. \( R \subseteq (W \setminus O)^2 \).
2. \( M, w \models_M p \) iff \( w \in V(p) \).
3. \( M, w \models_M \neg \varphi \) iff \( M, w \not\models_M \varphi \).
4. \( M, w \models_M \varphi \lor \psi \) iff \( M, w \models_M \varphi \) or \( M, w \models_M \psi \).
5. \( M, w \models_M \square \varphi \) iff \( \exists u \in W, Rwu \) and \( M, u \models_M \varphi \).
6. \( M, w \models_M 0 \) iff \( w \in O \).

In addition, I will use the following helpful abbreviations:
7. \( \varphi \land \psi := \neg (\neg \varphi \lor \neg \psi) \).
8. \( \varphi \lor \psi := \neg \varphi \lor \psi \).
9. \( \square \varphi := \neg \neg \varphi \).

The set of points in \( O \) can be thought of as representing epistemically impossible scenarios. Under this interpretation, the constant 0 can be read as ‘inconceivable’, in the same way that \( \bot \) is often read as ‘logically impossible’ or ‘contradictory’.

2.3.1 Ideal Models

An ideal is the dual of a filter. An ideal on a set has some particularly elegant properties, which I will take advantage of to define O-Kripke models that have many of the properties from Defn 2.44. Given a set \( U \), a partial ordering of subset inclusion (\( \supseteq \)) can be defined on the powerset \( W = \mathcal{P}(U) \), making \( \langle W, \supseteq \rangle \) a lattice (and even a complete Boolean algebra).

Definition 2.46 Ideals
An ideal \( C \) on a set \( U \) is a subset of \( W = \mathcal{P}(U) \) with the following properties:
1. \( \emptyset \in C \). (\( C \) is non-empty)
2. \( U \notin C \). (\( C \) is proper)
3. If \( A \) and \( B \) are in \( C \), then so is their union. (\( C \) is closed under finite joins)
4. If \( A \in C \) and \( B \subseteq A \), then \( B \in C \). (\( C \) is a lower set)

A Kripke O-model \( \langle W, R, O, V \rangle \) can be constructed from a set \( U \), and a subset lattice \( \langle W, \supseteq \rangle \) as generated above by selecting: any \( O \subseteq W \); \( R \) as the maximal subrelation of \( \supseteq \) that does not relate any member of \( W \) to a member of \( O \); and for each \( p \in \text{PROP} \) \( V(p) \) as either \( W \) or any ideal \( C \) on \( U \) containing \( O \). The class of Kripke O-models constructed by applying ideals as described above is the class of Ideal models. All Ideal models are Classic as per Defn 2.44.6.
Lemma 2.47 **Properties of Ideal models**

All Ideal models $\langle W, R, O, V \rangle$ satisfy the following formulas:

1. $\Box (\varphi \lor \psi) \supset (\Box \varphi \lor \Box \psi)$
2. $\Box \varphi \supset \varphi$
3. $\Box \varphi \supset \Box \Box \varphi$
4. $\Box \Diamond \varphi \supset \Diamond \Box \varphi$
5. $p \supset \Box p$
6. $\Box \Diamond p \supset p$
7. $0 \supset p$

Proof: These properties follow from Defn 2.46. Note that formulas 5–7 are satisfied not just by atoms, but by all formulas in the positive fragment of $L_M$.

**Definition 2.48** $I_\Gamma = \langle W, R, O, V \rangle$ is the $\Gamma$-restricted Ideal model where:

1. $U = \{ v : PROP \rightarrow \{0, 1\} \}$.
2. $W = \wp(U)$.
3. $\forall w, u \in W: Rwu \text{ iff } u \subseteq w \text{ and } u \neq \emptyset$.
4. $O = \{ \emptyset \}$.
5. $\forall w \in W: w \in V(p) \text{ iff } \forall v \in w: v(p) = 1$.

**Definition 2.49** $M_\Gamma = \langle I_\Gamma, w_\Gamma \rangle$ is the $\Gamma$-rooted Ideal model and $w_\Gamma$ is its root where:

1. $I_\Gamma$ is the $\Gamma$-restricted Ideal model.
2. $w_\Gamma = \{ v \in U \mid \forall \gamma \in \Gamma: v(\gamma) = 1 \}$.

Lemma 2.50 Every $\Gamma$-restricted Ideal model $I_\Gamma = \langle W, R, O, V \rangle$ is an Ideal model.

Proof: $\langle W, R \rangle$ is a powerset lattice by Defn 2.46.2 and 2.46.3. For each $p \in \text{PROP}$: (1) $\emptyset \in V(p)$ iff $\forall v \in \emptyset, v(p) = 1$; (2) $W \in V(p)$ or $W \notin V(p)$; (3) $w, u \in V(p)$ iff $\forall v \in w, v(p) = 1$ and $\forall v \in u, v(p) = 1$ iff $w \cup u \in V(p)$; (4) If $w \in V(p)$ and $u \subseteq w$, then $\forall v \in w, v(p) = 1$, so $\forall v \in u, v(p) = 1$, and $u \in V(p)$.

Also, $O \subseteq V(p)$ as $O = \{ \emptyset \}$ and $\emptyset \in V(p)$.

$I_\Gamma$ will be a finite model, as PROP is a finite set of variables.

### 2.3.2 Embedding CL in Ideal models

The next step is to define satisfaction-preserving translations between the relevance-style semantics for CL over the set $S_\Gamma$ of possible worlds satisfying $\Gamma$, and the modal semantics for the $\Gamma$-rooted Ideal model $M_\Gamma$.

**Definition 2.51** Translation Function $c: L \rightarrow L_M$.

1. $c(p) = p$, where $p$ is an atom.
2. $c(\neg \varphi) = \neg c(\varphi)$.
3. $c(\varphi \lor \psi) = c(\varphi) \lor c(\psi)$.
4. $c(\varphi \land \psi) = \Box (\Diamond c(\varphi) \lor \Diamond c(\psi))$.

**Definition 2.52** $M_\Gamma \models_{MCL} \theta$ iff $M_\Gamma \models_M c(\theta)$.

**Lemma 2.53** $S_\Gamma \models_{CL} \theta$ iff $M_\Gamma \models_{MCL} \theta$.

Proof: This is proved in Lemma B.1 in Appendix B.1.
2.3.3 Embedding R1–R3 in Ideal models

R1 is a system that constrains disjunctions (in positive scope), and lacks the subformula property for negation. These two features result in the only two changes between the translation functions for CL and R1. The translation function $c$ from Defn 2.51 is also used in the definition of all subsequent translation functions.

**Definition 2.54** Translation Function $d : L \rightarrow L_M$.

1. $d(p) = p$, where $p$ is an atom.
2. $d(\neg p) = \neg \diamond d(p)$.
3. $d(p \land \psi) = d(p) \land d(\psi)$.
4. $d(\psi \land \psi) = \square(\diamond c(\psi) \lor \diamond c(\psi)) \land \diamond(d(\psi) \land \neg \diamond c(\psi)))$.
5. $d(\neg(p \land \psi)) = d(\neg \psi \lor \neg \psi)$.
6. $d(\neg(\psi \land \psi)) = d(\neg \psi \land \neg \psi)$.
7. $d(\neg \neg \psi) = d(\psi)$.

**Definition 2.55** $M1$

$M_\Gamma \models M1 \theta$ iff $M_\Gamma \models d(\theta)$.

**Lemma 2.56** $S_\Gamma \models R1 \theta$ iff $M_\Gamma \models M1 \theta$.

*Proof:* This is proved in Lemma B.2 in Appendix B.1.

R2 is identical to R1, except that the restriction of non-emptiness is extended to apply to all atoms, and not just disjuncts. Again, the only difference between the translation functions for R1 and R2 reflects this extension.

**Definition 2.57** Translation Function $e : L \rightarrow L_M$.

1. $e(p) = p \land \diamond p$, where $p$ is an atom.
2. $e(\neg p) = \neg \diamond p \land \neg \neg p$.
3. $e(p \land \psi) = e(p) \land e(\psi)$.
4. $e(\psi \land \psi) = \square(\diamond c(\psi) \lor \diamond c(\psi)) \land \diamond(e(\psi) \land \neg \diamond c(\psi)))$.
5. $e(\neg(p \land \psi)) = e(\neg \psi \lor \neg \psi)$.
6. $e(\neg(\psi \land \psi)) = e(\neg \psi \land \neg \psi)$.
7. $e(\neg \neg \psi) = e(\psi)$.

**Definition 2.58** $M2$

$M_\Gamma \models M2 \theta$ iff $M_\Gamma \models e(\theta)$.

**Lemma 2.59** $S_\Gamma \models R2 \theta$ iff $M_\Gamma \models M2 \theta$.

*Proof:* This is proved in Lemma B.3 in Appendix B.1.

**Example:** Suppose $\Gamma = \{p, q\}$, so $I_\Gamma = \langle W, R, O, V \rangle$, where $U = \{pq, p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$ and $W = \mathcal{P}(U)$, and $w_r = \{pq\}$. Then $M_{p,q} \models M2 \ p \lor q$ iff $M_{p,q} \models M2 \ (\diamond(p \lor q) \land ((p \land \diamond p) \land \diamond((p \land \diamond q) \land \neg \diamond q))$. By Defn 2.57.1/4, but $M_{p,q} \not\models M3 \ (\neg \diamond q)$. Thus $\{p, q\} \models p \lor q$ is not satisfied by M2.

In addition to the restrictions imposed by R2, R3 also requires that every subformula of the conclusion is contingent. Although contingency is an inherently modal property, it is not particularly well-suited to representation in $\Gamma$-restricted Ideal models, as the $\Gamma$-restriction potentially eliminates some or all of the worlds where a subformula is false. This results in M3 not being an interesting modal system in its own right, although it is included for the sake of completeness.

**Definition 2.60** $M3$

$M_\Gamma \models M3 \ \varphi$ iff $M_\Gamma \models M2 \ \varphi$ and no subformula of $\varphi$ is a contradiction or tautology.

**Lemma 2.61** $M_\Gamma \models M3 \ \theta$ iff $S_\Gamma \models R3 \ \theta$.

*Proof:* By Lemma 2.28 and Defns 2.58 and 2.60.
2.3.4 Embedding R4–R5 in Ideal models

To model R4, a point in the model corresponding to the set S needs to be able to interact with another point corresponding to the set T, where neither point is accessible to the other, even though accessibility is transitive. There are at least two natural ways to do this in a modal system. The approach I will take here is to extend the language to include a new modal operator ♦ that is the converse of ◊, and is analogous to the backwards-looking modalities in temporal logic.

**Definition 2.62** A Kripke O-model \( M = \langle W, R, O, V \rangle \) for \( L_\Diamond \) is as follows:

1. As per Defn 2.45.
2. \( M, w \models_M \Diamond \varphi \iff \exists u \in W, Ruw \ and \ M, u \models_M \varphi. \)

**Definition 2.63** Translation Function \( f : L \to L_\Diamond \):

1. \( f(p) = p \land \Diamond p \land \Diamond \neg p \), where \( p \) is an atom.
2. \( f(\neg p) = \neg \Diamond p \land \Diamond \neg p \land \Diamond p. \)
3. \( f(\varphi \land \psi) = f(\varphi) \land f(\psi) \land \Diamond (\neg \Diamond f(\psi) \land c(\varphi)) \land \Diamond (\neg \Diamond f(\varphi) \land c(\psi)). \)
4. \( f(\varphi \lor \psi) = \Box (\Diamond c(\varphi) \lor \Diamond c(\psi)) \land \Diamond (\neg f(\psi) \land \neg \Diamond c(\varphi)) \land \Diamond (\neg f(\varphi) \land \neg \Diamond f(\psi)). \)
5. \( f(\neg (\varphi \land \psi)) = f(\neg \varphi \land \neg \psi). \)
6. \( f(\neg (\varphi \lor \psi)) = f(\neg \varphi \land \neg \psi). \)
7. \( f(\neg \neg \varphi) = f(\varphi). \)

**Lemma 2.64** \( M_\Gamma \models_M f(\theta) \iff M_\Gamma \models_M 2 \theta \) and \( M_{\neg \theta} \models_M \neg 2 \theta. \)

**Proof:** This is proved in Lemma B.4 in Appendix B.1. ■

One disadvantage of the above approach is that using the ♦ operator hides the structure of the negative conditions, when they can be defined much more explicitly and directly. Another is that negation has to be defined via the De Morgan Laws. This makes sense for modelling R1–R3, which also use the De Morgan Laws explicitly, but not for R4 and R5 which define negation directly. A first attempt at a translation function \( g \) that defines negation directly over all formulas might be: \( g(\neg \varphi) = \neg \Diamond g(\varphi) \), but this gives \( M_\Gamma \models_M g(\neg (p \lor p)) \) for all \( \Gamma \) as \( p \lor p \) never holds. A more subtle attempt is to define \( g(\neg \varphi) = \Diamond \Diamond g(\varphi) \land \Diamond \neg \Diamond g(\varphi) \), which requires a formula to hold somewhere that does not include anywhere \( \Diamond \)-accessible. The flaw with this approach is equally subtle: if \( \varphi \) is a disjunct of a formula \( \theta = \varphi \lor \psi \), the points in the model where \( \varphi \) and \( \neg \Diamond \varphi \) contribute to satisfaction should both be restricted to those where \( \psi \) does not hold, as in Defn 2.29.3. In the proposed definition of \( g(\neg \varphi) \), the more we restrict the set of points where \( \varphi \) can be satisfied, the less we restrict the satisfaction of \( g(\neg \varphi) \). To have these restrictions increase for both \( g(\varphi) \) and \( g(\neg \varphi) \), we need \( g(\neg \varphi) = \Diamond \Diamond (c(\chi) \land g(\varphi)) \land \neg \Diamond \Diamond g(\varphi) \), where \( \chi \) is the conjunction of the various restrictions (in this case \( \neg \psi \) as \( \theta = \varphi \lor \psi \)) that apply to \( \varphi \). However, single-pointed Ideal models cannot ‘store’ this cumulative restriction. An alternative approach using multi-point Ideal models is described in Appendix B.1, and this allows us to produce modal systems M4 and M5 that are equivalent to R4 and R5, and which define negation directly rather than via De Morgan identities.

The equivalency results in Lemmas 2.56, 2.59, 2.61 and 2.64 are not interesting or surprising, as the modal systems were designed solely to be equivalent to their respective relevance semantic systems. Nor is taking a modal perspective for modelling disjunction particularly novel. The interesting results are the expansion of the conditions on disjunction in Defn 2.54, and the subsequent expansion of both conjunction and disjunction in Defn 2.63, and these expansions will reappear in various guises throughout the rest of this thesis. These expanded conditions provide a much finer-grained analysis of assertibility conditions on the boolean connectives than some of the other semantics in this chapter. This will prove particularly useful when defining a metalanguage in §3.2.1, defining weaker assertion norms for asymmetric connectives in §3.4, and investigating disjunctive Free Choice in §5.4.
2.4 Brevity & Concision

There are several inference systems that make some claim to removing redundancy and repetition, such as those in Schurz (1991) and Gemes (1994) in the field of modelling verisimilitude. I am not aware of any that have been designed specifically to implement linguistic intuitions of Brevity within a classical propositional framework. I will define some preference relations which model relative Brevity between formulas, while also requiring relative Informativity (i.e., the briefer formula must entail the less brief formula). These will be used to ensure that inferences have a conclusion which is a maximally brief way of describing (part of) the premise set. The family of brevity preference relations will thus lead to a family of concise inference relations whose conclusions lack certain kinds of redundancy or repetition, and which also, somewhat surprisingly, capture the assertion norms. I will then look at some formal properties of this family of concision deductive systems, including how different brevity operations combine to prevent standard properties of entailment such as monotonicity and transitivity, and allow ‘relevant’ properties such as variable inclusion to emerge.

The Gricean maxim ‘be brief’ is usually applied to the wording of a clause when evaluating whether it is succinct. Circumlocutions, redundant adjectives, flowery prose, indirect references, and so forth will usually only be used in an utterance as part of an attempt to communicate some implication, or when shorter alternatives are unavailable. For example, the phrase ‘be brief’ is to be preferred over ‘avoid unnecessary prolixity’, and ‘be clear’ over ‘eschew obfuscation, espouse elucidation’. However, there are no operators to represent the floridity of prose in the language of propositional logic, so all propositionally simple clauses are reduced to literals regardless of their locutionary brevity or lack thereof. All that remains for the maxim to operate on are entirely redundant or superfluous clauses. The following analysis of brevity will focus on defining the conditions for a formula to be relatively or absolutely brief, and thus whether an atom or subformula is unnecessary within a formula, or when shorter alternatives are unavailable. For example, the phrase ‘be brief’ is to be preferred over ‘avoid unnecessary prolixity’, and ‘be clear’ over ‘eschew obfuscation, espouse elucidation’. However, there are no operators to represent the floridity of prose in the language of propositional logic, so all propositionally simple clauses are reduced to literals regardless of their locutionary brevity or lack thereof. All that remains for the maxim to operate on are entirely redundant or superfluous clauses. The following analysis of brevity will focus on defining the conditions for a formula to be relatively or absolutely brief, and thus whether an atom or subformula is unnecessary within a formula.

To understand some of the decisions that will be made in constructing the brevity relations it will also be helpful to understand how they will affect the associated concision relations. Each brevity relation $\psi < \varphi$ will result in a different class of acceptable concise inferences. A conclusion $\varphi$ concisely describes (part of) the premise set $\Gamma$ iff $\Gamma \vdash \varphi$ and there is no formula $\psi$ such that $\psi < \varphi$ and $\Gamma \vdash \psi$. That is, $\varphi$ is a (locally) maximally brief formula for describing the part of $\Gamma$ that it does. These concision relations will be defined in Defn 2.79.

Notation

Some new notation is needed to compactly describe the kinds of permissible syntactic operations that either preserve or increase the brevity of a formula. Each operation will be performed on a particular occurrence of a subformula in positive scope within a formula $\varphi$. To identify the subformula occurrence being operated on, this occurrence will be replaced with a ‘dummy’ formula $x$. Universal substitution notation can then be used to represent the result of substituting just this subformula occurrence. For example, the (brevity-preserving, and thus reversible) substitution of an instance of $p \land q$ by $q \land p$ in $\varphi$ is represented as $\varphi[x/p,q] \equiv \varphi[q,p]$. Similarly, the (brevity-increasing and thus irreversible) substitution of an instance of $p \land q$ by $p$ in $\varphi$ as $\varphi[x/p,q,p] > \varphi[x,q]$. The placeholder variable can be dropped when it will not cause confusion, which simplifies the above substitution examples to $\varphi[p \land q] < \varphi[q \land p]$ and $\varphi[p \land q] > \varphi[p]$.

This new notation is used in the following definition of a permutation relation. Permutations are classically equivalent formulas that can be transformed into each other by operations that we have already shown are essentially irrelevant to the structure of an uttered expression, and which preserve the number of occurrences of each variable. The conceptual analysis of the previous chapter suggested that the assertion norms are indifferent to Commutativity and Associativity, and as negation is treated as a mere operator, the De Morgan Laws and Double Negation Introduction/Elimination are also transparent to assertibility. The new Brevity relation should also respect these laws, as they each preserve the number of occurrences of each variable or of dyadic connectives (as a whole), and also preserve classical equivalence. That is, there is no reason to prefer a formula over another if they can be transformed into each other via applications of these laws. As before, the number of negation occurrences is unimportant, as this is a mere operator.
Definition 2.65 Permutation: \( \varphi \equiv \psi \). Let \( \varphi \equiv \psi \) be the smallest reflexive, transitive relation that is closed under:

- [Commutativity] \( \varphi[A \times B] \triangleleft \triangleright \varphi[B \times A] \).
- [Associativity] \( \varphi[A \times (B \times C)] \triangleleft \triangleright \varphi[(A \times B) \times C] \).
- [De Morgan] \( \varphi[\neg(A \times B)] \triangleleft \triangleright \varphi[\neg A \circ \neg B] \).
- [Double Negation] \( \varphi[A] \triangleleft \triangleright \varphi[\neg \neg A] \).

where \(*, \circ \in \{\lor, \land\} \) and \(* \neq \circ \).

Example: \( p \land q \equiv q \land p \). \( p \not\equiv p \lor p \). \( \neg(p \land q) \lor r \equiv q \lor (r \lor p) \). \( p \lor (q \land r) \not\equiv (p \lor q) \land (p \lor r) \).

Lemma 2.66 Permutation as Equivalence

1. \( \equiv \) is an equivalence relation.
2. If \( \varphi \equiv \psi \) then \( \varphi \equiv \psi \).

Proof: 1. \( \equiv \) is transitive and reflexive, and all its operations are symmetric. 2. The operations of \( \equiv \) are all special cases of \( \equiv \). ■

2.4.1 Operational Brevity

The next step is to introduce some operations that increase brevity by shortening a formula while preserving the remaining structure. The transitive closure of combinations of these operations, along with the [Permutation] equivalence relation (\( \equiv \)), will produce different brevity relations and concision systems. Each of the operations [B1]–[B4] defined below either removes unnecessary disjuncts or conjuncts, or replaces complex formulas with simple constants. The shorter formulas produced by these operations will always entail the larger formulas they were derived from, which will preserve relative Informativity. These restrictions ensure each operation shortens the formula while preserving both structure and information. A formula is brief relative to another if it is a result of performing a series of operations on the second formula, each of which either preserve or increase brevity. Being strictly brief than another formula is a more complex relationship, requiring two distinct notions.

Definition 2.67 Brevity Operations (subformulas must be in positive scope):

[B1] \( \varphi[A \lor B] \triangleright \varphi[A] \).
[B2] \( \varphi[A] \triangleright \varphi[\bot] \).
[B3](a) \( \varphi[A] \triangleright \varphi[\top] \), when A is a tautology.
[B3](b) \( \varphi[A \land \bot] \triangleright \varphi[A] \).
[B4] \( \varphi[A \land B] \triangleright \varphi[A] \), when \( \varphi[A] \models_{\text{L}} \varphi[A \land B] \).

Definition 2.68 n-Brevity

\( \psi \leq_{n} \varphi \) iff there is a sequence \( \varphi_{1}, \ldots, \varphi_{k} \) such that \( \varphi_{1} = \varphi, \varphi_{k} = \psi \), and for each \( 1 \leq i < k \):

either (i) \( \varphi_{i} \equiv \varphi_{i+1} \), or (ii) \( \varphi_{i} \triangleright \varphi_{i+1} \) by some [Bm], \( m \leq n \).

Lemma 2.69 n-Brevity Relations.

\( \leq_{n} \) is the smallest relation which has [Permutation] and [B1]–[Bn] as subrelations, and is closed under transitivity. An operation \( \triangleright \) is a subrelation of \( \leq \) if whenever \( \varphi \triangleright \psi \), \( \psi \leq \varphi \).

Proof: By Defns 2.65, 2.67, and particularly 2.68, which ensures that [Permutation] and [B1]–[Bn] are subrelations of \( \leq_{n} \), and provides transitivity. ■

Definition 2.70 Strict n-Brevity Derivations

1. The sequence \( \varphi_{1}, \ldots, \varphi_{k} \) from Defn 2.68 is a derivation of \( \psi \leq_{n} \varphi \).
2. \( \psi \triangleright_{n} \varphi \) iff for some derivation \( \varphi_{1}, \ldots, \varphi_{k} \) of \( \psi \leq_{n} \varphi \), \( \exists i < k: \varphi_{i} \triangleright \varphi_{i+1} \) by some [Bm], \( m \leq n \).
3. \( \psi <_{n} \varphi \) iff for all derivations \( \varphi_{1}, \ldots, \varphi_{k} \) of \( \psi \leq_{n} \varphi \), \( \exists i < k: \varphi_{i} \triangleright \varphi_{i+1} \) by some [Bm], \( m \leq n \).

Example: \( \neg p <_{3} (\neg (p \land r) \land (q \lor \neg q) \lor (\neg q \lor \neg q) ) \) is \( (\neg p \land r) \land (q \lor \neg q) \), \( (\neg p \land r) \land \top, (\neg p \land r) \), \( (\neg p \lor \neg q, (\neg p \land r) \land \bot, (\neg p \land r) \), using the operations [B3](a), [B3](b), [De Morgan], and [B1], in that order. There is no derivation the other way, as \( \neg p \) contains fewer variables than \( (\neg p \land r) \land (q \lor \neg q) \), and no operation in [B1]–[B4] or [Permutation] increases the number of variables.
Lemma 2.71 For constant-free \( \phi, \psi \), \( \psi \leq_n \phi \) iff \( \psi \triangleleft_n \phi \).

**Proof:** If \( \phi \leq_n \psi \) then \( \phi \triangleleft_n \psi \). If \( \phi \triangleleft_n \psi \) and \( \phi \not\leq_n \psi \), then some derivations of \( \psi \) must use brevity operations, while others do not. All brevity operations [B1]–[Bn] reduce the complexity of the formula, except [B2] and [B3](a) when the substituted subformula is a [Permutation] of \( \perp \) or \( \top \) respectively. All [Permutation]s respect the complexity of the formula, so the only way for some (but not all) derivations of \( \psi \) from \( \phi \) to contain strict brevity operations is for the strict brevity operations to maintain complexity. Thus \( \psi \leq_n \phi \) and \( \phi \triangleleft_n \phi \) only differ when \( \phi \) is a [Permutation] of a logical constant.

Lemma 2.72 \( n \)-Brevity Properties.

1. \( \leq_n \) is a preorder.
2. \( \psi \leq_n \phi \) iff \( \psi \leq \phi \) and \( \phi \leq \psi \).
3. \( \leq_n \) is a strict partial order.
4. If \( \psi \leq \phi \) then \( \psi \leq_n \phi \).
5. \( \leq_n \) and \( \leq \) are subrelations of \( \leq_{n+1} \) and \( \leq_{n+1} \) respectively (when defined).

**Proof:** 1. Each \( \leq_n \) relation is transitive by Defn 2.68, and by [Permutation] is reflexive. 2. By Defn 2.70, the preservation of formula complexity by [Permutation], and the reduction in formula complexity that occurs in all instances of the operations in Defn 2.67 except \( \perp \perp \perp \) and \( \top \top \top \). 3. The \( \leq_n \) relations are irreflexive and transitive by Lemma 2.72.2. 4. By Lemma 2.66.2 [Permutation] is a subrelation of \( \equiv \), by Defn 2.67 each operation is a subrelation of \( \not\equiv \) (‘entailed by’), and by Lemma 2.69 \( \leq_n \) is the smallest transitive relation containing [Permutation] and [B1]–[Bn]. 5. By Defn 2.68 each \( \leq_n \) relation includes all operations in previous relations, and is generated from the (monotonically increasing) set of possible derivations using these operations. By Lemma 2.72.2 and as no operation has an instance \( \phi \supset \psi \) such that \( \phi \leq_n \psi \) for some \( n \), \( \leq_n \) is also monotonically increasing over \( n \). [B3](b) is a special case of [B4].

A natural simplification of the dyadic brevity relation is to reduce it to a monadic predicate that applies to formulas that lack a briefer equivalent formula. The restriction to equivalent formulas is necessary to avoid triviality, as \( \perp \perp \perp \) is 2-briefer than any other formula. A brief formula is one which cannot be any briefer without altering its information content. This brevity predicate applies not just to the shortest possible formula in an equivalence class, but to all those which cannot be made shorter without changing their structure, rather than just removing some elements.

Definition 2.73 \( n \)-brief formulas.

A formula \( \phi \) is \( n \)-brief iff \( \exists \psi : \phi \equiv \psi \) and \( \psi \leq_n \phi \).

Example \( n \)-brief formulas:

- \( p \vee p, p \vee (p \wedge q) \), and \( p \wedge (q \vee p) \) are equivalent and not 1-brief.
- \( p \wedge \neg p \) and \( q \wedge \neg q \) are equivalent and 1-brief but not 2-brief.
- \( p \vee \neg p \) and \( \neg (p \supset p) \supset p \) are equivalent and 2-brief but not 3-brief.
- \( p \wedge (p \wedge q) \vee (p \wedge \neg q) \) and \( (p \wedge \neg r) \vee (q \wedge r) \) are equivalent and 4-brief.

Lemma 2.74 Negation in 4-brevity.

1. \( \psi \leq \phi \) iff \( \neg \psi \leq_4 \neg \phi \), when \( \psi \equiv \phi \).
2. \( \phi \) is 4-brief iff \( \neg \phi \) is 4-brief.

**Proof:** [Permutation] and [B3](b) always preserve equivalence and brevity. [B1] can be applied to \( \phi \) while preserving equivalence iff [B4] can be applied to \( \neg \phi \) while preserving equivalence. [B2] can be applied to \( \phi \) while preserving equivalence iff [B3](a) can be applied to \( \neg \phi \) while preserving equivalence. These results do not hold for \( n < 4 \) as \( p \wedge p \) is 3-brief, but \( \neg p \leq_1 \neg p \vee \neg p \).

Definition 2.75 Formula Reductions and Alternatives

\( \psi \) is an \( n \)-brief reduction of \( \phi \) iff \( \psi \leq_n \phi \), \( \psi \equiv \phi \), and \( \psi \) is \( n \)-brief.

\( \psi \) is an \( n \)-brief alternative to \( \phi \) iff \( \psi \equiv \phi \), and \( \psi \) is \( n \)-brief.
2.4.2 5-Brevity

Two equivalent 4-brief formulas ‘almost always’ contain the same set of propositional variables. Perhaps the shortest exception to this appears above: \(((p \land r) \lor (q \land \neg r)) \land ((p \land \neg r) \lor (q \land r))\) is equivalent to \(p \land q\). A brevity relation that guarantees all equivalent and brief formulas have the same set of variables will result in a concision system with variable inclusion. As the strongest brevity relation introduced so far \((\leq_4)\) falls short of Inclusion, I will introduce an additional brevity relation. The 5-brevity relation will build on 4-brevity by adding some less intuitive operations containing specific instances of distributivity (4-brevity does not include any distributivity-like operations, such as \((p \land r) \lor (q \land r) \supset (p \lor q) \land r\)). We can produce a brevity relation where all equivalent brief formulas have the same set of variables, and the concision system has variable (and even literal) inclusion, by adding the brevity operations \([B5](a)\) and \([B5](b)\) that I will introduce below. They can be thought of as generalisations of the observation that \((p \land r) \lor (q \land \neg r)\) and \((p \lor \neg r) \land (q \lor r)\) are equivalent, equally brief, and contain identical numbers of occurrences of each literal. The claims that the brevity operations \([B1]–[B5](b)\) impose literal inclusion will eventually be proved in Theorems 2.92 and 2.94.

**Definition 2.76** 5-Brevity Operations (subformulas must be in positive scope):

- \([B5](a)\) \(\varphi[(A \land B) \lor (C \land D)] \supset \varphi[(A \lor C) \land (B \lor D)]\), when they are equivalent.
- \([B5](b)\) \(\varphi[(A \lor C) \land (B \lor D)] \supset \varphi[(A \land B) \lor (C \land D)]\), when they are equivalent.

The new brevity operations \([B5](a)\) and \([B5](b)\) are treated like the operations in Defn 2.67. This allows the relations \(\psi \leq_5 \varphi\), \(\psi \leq_5 \varphi\), \(\psi \equiv_5 \varphi\), and the predicate 5-brief to be defined as per Defns 2.68–2.73. Similarly, Lemmas 2.69–2.72 also hold for 5-brevity.

**Lemma 2.77** \(\varphi\) is 5-brief iff \(\neg \varphi\) is 5-brief.

**Proof:** By Lemma 2.74, \([\text{Permutation}]+[B1]–[B4]\) collectively treat \(\varphi\) and \(\neg \varphi\) identically. But \([B5](a)\) relates \(\varphi\) to a formula \(\psi\) iff \([B5](b)\) relates \(\psi\) to \(\varphi\). Furthermore, \([B5](a)\) relates \(\varphi\) to a formula \(\psi\) iff it also relates \(\neg \psi\) to \(\neg \varphi\), and similarly for \([B5](b)\).

2.4.3 Metric Brevity

Perhaps the most obvious candidate for a brevity relation is one that always prefers formulas with fewer symbol tokens to those with more (disregarding ‘\(\neg\)’). It is strong, simple, and intuitive.

**Definition 2.78** \(\psi \leq || \varphi\):

\(\psi \leq || \varphi\) iff the number of symbols in \(\psi\) is no more than the number of symbols in \(\varphi\), and \(\psi \preceq_{\text{CL}} \varphi\).

Metric brevity initially looks promising as simple way of testing redundancy in utterance forms. It has literal inclusion, and 4-brevity is a subrelation of Metric brevity, so all \(||\)-brief formulas are also 4-brief. I have three arguments against using Metric brevity to test assertions, which I find collectively, if not individually, compelling. First, \((p \land r) \lor (q \land r)\) is a formula that is 5-brief, but not \(||\)-brief. My linguistic intuitions are that it intuitively corresponds to an acceptable utterance form. Metric brevity rejects the distributed formula because \((p \lor q) \land r\) is \(||\)-briefer, which ignores the dissimilarity between the structures. Individual intuitions are a notoriously poor way to justify systematic accounts, but they can be useful guides in their development. Second, Metric brevity does not result in many interesting formal results compared with 4- or 5-brevity. Evidence for the relative complexity of the fine structure in 4- and 5-brevity includes: the delicacy of the partial orders; the near-distributivity as described in §2.4.5; and the equivalence with the other semantics introduced so far which will be proved in Theorem 2.92. Interesting systems should be preferred ceteris paribus. Finally, the assertion norms are related to 4- and 5-brevity, and not Metric brevity, due to the recursive Disjunctive and Conjunctive Compositionality conditions; a significantly different story would need to be told to justify a metric version of these norms.
2.4.4 Concision

A concise inference is one with a conclusion that cannot be any briefer without either losing information or causing the inference to be invalid. Concision depends on brevity and as there is a family of \(n\)-brevity preference relations, we can define a concomitant family of \(n\)-concision inference relations, along with the \(n\)-concision family that results from using the variant strict brevity from Defn 2.70.2.

Definition 2.79 \(n\)-Concision

\(\Gamma \vdash \varphi\) is \(n\)-concise iff \(\Gamma \vdash \varphi\) and \(\forall \psi \leq_n \varphi: \Gamma \not\vdash \psi\).

Γ ⊬ ϕ is \(n\)-concise iff Γ ⊬ ϕ and ∃ψ <n ϕ: Γ \not\models ψ.

Lemma 2.80 Concision Properties

1. \(\varphi\) is \(n\)-brief iff \(\varphi \vdash \varphi\) is \(n\)-concise.
2. If \(\Gamma \vdash \varphi\) is \(n\)-concise for some \(\Gamma\), then \(\varphi\) is \(n\)-brief.
3. If \(\Gamma \vdash \varphi\) is \((n+1)\)-concise then \(\Gamma \vdash \varphi\) is \(n\)-concise.
4. \(\Gamma \vdash \varphi\) is \(n\)-concise iff it is \(n\)-concise, and \(\varphi \not\models \perp\) (for \(n \geq 2\)), and \(\varphi \not\models \top\) (for \(n \geq 3\)).

Proof: By Defn 2.79, along with Defn 2.73 for 1/2, Lemma 2.72.5 for 3, and Defn 2.70 for 4. ■

Lemma 2.81 Concision Equivalences

1. \(\Gamma \vdash \varphi\) is 1-concise iff \(\Gamma \vdash \varphi\) is 2-concise, or \(\Gamma \not\vdash \varphi\) and \(\varphi\) is disjunction-free.
2. \(\Gamma \vdash \varphi\) is \(2^\bullet\)concise iff \(\Gamma \vdash \varphi\) is 1-concise and \(\Gamma\) is consistent.
3. \(\Gamma \vdash \varphi\) is 4-concise iff \(\Gamma \vdash \varphi\) is 2-concise and \(-\varphi \vdash \neg \varphi\) is 2-concise.

Proof: 1. (⇒) Suppose \(\Gamma \vdash \varphi\) is 1-concise but not 2-concise, and that \(\varphi\) contains a disjunction. Then \(\exists \psi: \psi < \varphi\), \(\Gamma \vdash \psi\) by Defn 2.79.

Now \(\varphi\) is 1-brief as \(\Gamma \vdash \varphi\) is 1-concise. This means there are no disjuncts in \(\varphi\) that can be removed (via \([B1]\)) without violating equivalence, and as neither \([B1]\), \([B2]\), nor \([\text{Permutation}]\) can add disjuncts, removing any disjuncts from \(\varphi\) will violate equivalence (and even entailment by \(\Gamma\)). Select any derivation \(⟨\varphi_1, \ldots, \varphi_k⟩\) of \(\psi \leq_2 \varphi\). No step in the derivation can use \([B1]\), and at least one step has to use \([B1]\) or \([B2]\). Suppose a step using the \([B2]\) operations transforms \(\varphi_1\) into \(\varphi_2\) by replacing some subformula \(\chi\) with \(\perp\). If \(\chi\) is in \(\varphi_1\), then removing that whole subformula would give a formula \(\varphi_3\) equivalent to \(\varphi_2\), so \(\varphi_3 < \varphi\) and \(\Gamma \vdash \varphi_3\), a contradiction. So no derivation step can ever apply \([B2]\) to (part of) a disjunct of \(\varphi\). This means that the \(\perp\) in \(\varphi_2\) is a conjunct of \(\varphi_2\), and so \(\varphi_2 \equiv \perp\). Then if \(\varphi_2\) had a disjunction, any disjunct could be removed via \([B1]\) and the resulting 1-briefer formula would be entailed by \(\Gamma\), making \(\Gamma \vdash \varphi\) 1-inconcise. Thus \(\varphi\) is disjunction-free.

(⇐) If \(\Gamma \vdash \varphi\) is 2-concise, it is 1-concise by Lemma 2.80.2. If \(\varphi\) is disjunction-free, then \([B1]\) can never be applied to \(\varphi\) or any permutation of \(\varphi\), so \(\not\exists \psi < \varphi\), and \(\Gamma \vdash \varphi\) is 1-concise.

2. (⇒) If \(\Gamma \vdash \varphi\) is \(2^\bullet\)concise, it is 1-concise by Lemma 2.80.2. If \(\Gamma\) is inconsistent, then as \(\perp <_2 \varphi\) and \(\Gamma \vdash \perp\) is not \(2^\bullet\)concise.

(⇐) If \(\Gamma \vdash \varphi\) is 1-concise then by Lemma 2.81.1, \(\Gamma \vdash \varphi\) is 2-concise or \(\Gamma \not\vdash \varphi\) and \(\varphi\) is disjunction-free. Suppose that \(\Gamma \not\vdash \varphi\) and \(\varphi\) is disjunction-free. As \(\Gamma\) is also consistent, then \(\varphi\) is consistent and \(\varphi \not\models \perp\), but any application of \([B2]\) will substitute \(\perp\) for \(\varphi\) or one of its conjuncts, and the resulting formula will be inconsistent, thus not entailed by \(\Gamma\), so \(\varphi\) is \(2^\bullet\)concise.

3. (⇒) Suppose \(\Gamma \vdash \varphi\) is not 4-concise. Choose a formula \(\psi < \varphi\) with \(\Gamma\) \not\vdash \psi\) and a derivation \(⟨\varphi_1, \ldots, \varphi_k⟩\) for \(\psi \leq_4 \varphi\) with the minimal number of applications of \([B1]–[B4]\) (i.e., one). If the operation in the derivation is \([B1]\) or \([B2]\), \(⟨\varphi_1, \ldots, \varphi_k⟩\) is also a derivation for \(\psi \leq_2 \varphi\), so \(\Gamma \vdash \varphi\) is not 2-concise. If the operation is \([B3]\) or \([B4]\), then \(⟨\neg \varphi_1, \ldots, \neg \varphi_k⟩\) is a derivation for \(\neg \psi \leq_2 \neg \varphi\), as any operation \([B3]\)(a) (\([B3]\)(b) or \([B4]\)) on \(\varphi_i\) is an operation \([B2]\) (\([B1]\)) on \(\neg \varphi_i\), respectively. But \(\varphi \equiv \psi\), so \(\neg \psi \not\vdash \neg \varphi\), and \(\neg \varphi \not\vdash \neg \varphi\) is not 2-concise.

(⇐) If \(\Gamma \vdash \varphi\) is not 2-concise then by Lemma 2.80.3 it is not 4-concise. If \(\neg \varphi \vdash \neg \varphi\) is not 2-concise then \(\exists \neg \psi < \neg \varphi, \neg \varphi \not\vdash \psi\), and there is some derivation \(⟨\neg \varphi_1, \ldots, \neg \varphi_k⟩\) for \(\neg \psi \leq_2 \neg \varphi\). Then \(⟨\varphi_1, \ldots, \varphi_k⟩\) is a derivation for \(\psi \leq_4 \varphi\), as any operation \([B1]\) (\([B2]\)) on \(\neg \varphi_i\) is an operation \([B4]\) (\([B3]\)(a)) on \(\varphi_i\) respectively if \(\varphi_i \equiv \varphi_{i+1}\), and as \(\psi \equiv \varphi\), \(\Gamma \vdash \psi\) is not 4-concise. ■
2.4.5 Some Concise Notes

One way of understanding a concision relation is as requiring the number of instances of each member of a set of logical constants and connectives to be *locally* minimised in the conclusion (as opposed to the global minimisation of Metric brevity). 1-concision removes all redundant disjuncts, so minimise the number of $\lor$ symbols. 2-concision removes all redundant disjuncts and contradictions, so minimise the number of $\lor$ and $\bot$ symbols. 3-concision also removes all tautologies, so also minimise the number of $\top$ symbols. The 5-concision relation also *globally* minimises the number of distinct literals, and as a result the literals in the conclusion must also appear in the premise set; support for this claim will have to wait until Lemma 2.14 gains wider application through Lemma 2.91.

The concision systems above do not exhaust the possibilities of interesting brevity-based inference relations. For example, a concision system based on Metric brevity minimises the number of literal instances, which is even stronger than literal inclusion. In Appendix A.5.3 I will present a variant of 5-concision that lacks [B4], and as a result has only variable, and not literal, inclusion.

**Distributivity**

No general distributivity operation has been included in Operational brevity, either as a reversible Permutation like $\varphi[(A \lor B) \land (A \lor C)] \iff \varphi[A \lor (B \land C)]$, or irreversible brevity operation like $\varphi[(A \land B) \lor (A \land C)] \supseteq \varphi[A \land (B \lor C)]$. Adding a general distributivity operator to any of the Operational brevity relations will lead to some counter-intuitive results. For instance, we will either have to reject the seemingly brief $(p \land r) \lor (q \land r)$, or accept the undesirable $((p \land q) \lor p) \land (p \lor r) \land (q \lor r)$. However, in 5-brevity we allow just enough distributive goodness to allow the repositioning of certain contrary subformulas, which can then be removed using other operations. The operations [B5](a) and [B5](b) each combine a distributive law with a combination of disjunct and conjunct removal to produce equivalent formulas with the same number of instances of each literal. The lack of general distributivity operations also means that all Operational brevity relations treat distributivity rather subtly. The two forms of distributivity have different effects, even in 4-brevity, which otherwise appears to treat conjunction and disjunction symmetrically.

**Consequences**

Each system, even the relatively simple 1-concision, contains many subtle relationships. that are quite hard to appreciate initially. Consider 1-concision simply as a consequence relation. Its properties include: being a subrelation of classical consequence, non-monotonicity, non-transitivity, non-reflexivity ($\varphi \vdash \varphi$ need not be concise), and the lack of a deduction theorem (when $\Gamma, \varphi \vdash \psi$ is concise, $\Gamma \vdash \varphi \supset \psi$ need not be concise). The first property follows directly from Defn 2.79, while the rest of the claims are discussed in §2.6.2. The stronger $n$-concision inference relations also have these properties. The lack of reflexivity, monotonicity, and transitivity is usually regarded as undesirable in a consequence relation that is intended to capture entailment. The same lack will prove to be useful for modelling certain pragmatic properties of the relationship between the communicative propositional content and the form of uttered expressions. I am not concerned about their absence as I am not trying to model entailment or implication, which is the primary motivation for desiring these properties.

The style of deductions used when making a formula brief should come naturally to any logician; it is precisely the sort of activity that they engage in when they simplify a formula by rearranging elements and removing redundancies. The more powerful brevity systems permit most common simplification techniques including: grouping like subformulas; removing duplicate conjuncts or disjuncts, contradictory disjuncts and tautological conjuncts; eliminating unnecessary terms; and even (restricted) applications of distributivity. Some more formal comments on simplification and brevity-producing algorithms, including the use of prime implicants, the Quine-McCluskey algorithm and Petrick's method, can be found in §5.5.4. Brevity is a standard part of the logician’s arsenal, and while it is usually regarded as a mere presentation issue rather than a feature that can be regimented by inference rules, it deserves to be more widely investigated.
2.5 Equivalence Proofs

Each of the semantic families we have just introduced is effectively equivalent to the family of cumulative Utterance Norms from the end of Chapter One. The obvious first step in proving this series of Soundness and Completeness results is to show that all the families of semantic inference relations are equivalent. This claim is very nearly true. There is one minor inconsistency between \(n\)-concision and the other families in how they treat inferences with a constant as the conclusion. To avoid this issue, the equivalences will be proved using \(n^*\)-concision, which uses a modified notion of strict brevity from Defn 2.70 that gives identical results to \(n\)-concision apart from disallowing \(\perp\) as a conclusion for \(2^*\)-concision, and \(\perp\) as conclusions for \(3^*\)-concision through \(5^*\)-concision.

**Lemma 2.82** \(S_T \models_{R2} \varphi\) iff \(\Gamma \models \varphi\) is a 2-licensed inference.

*Proof:* The equivalence of \(L2\) and \(R2\) follows by induction on complexity of \(\varphi\).

Suppose \(\varphi\) is an atom. Then \(\Gamma \models \varphi\) is 2-licensed iff \(\varphi \in \mathcal{L}_L\) and [D2] holds by Defn 2.10.1 if \(S_T \models_{CL} \varphi\) by Lemma 2.21.1 and there is a world where the premise set and conclusion are true by Defn 2.9. [D2] iff Defn 2.20.1 holds and \(S_T \cap S_p \neq \emptyset\) by Defn 2.19 iff Defn 2.24.1(a) and Defn 2.24.1(b) holds by Lemma 2.21.2 iff \(S_T \models_{R2} \varphi\).

Our Inductive Hypothesis is that \(\Gamma \models \psi\) is 2-licensed iff \(S_T \models_{R2} \psi\) for all \(\psi\) less complex than \(\varphi\).

Suppose \(\varphi = \varphi_1 \land \varphi_2\). Then \(\Gamma \models \varphi\) is 2-licensed iff \(\Gamma \cup \{\varphi_1, \varphi_2\} \models \varphi_1\) and \(\Gamma \cup \{\varphi_1, \varphi_2\} \models \varphi_2\) are 2-licensed (these are the conditions listed in Defn 2.7 by the contribution table for conjunction which are required for the conjunction to have contributions from both the left and right conjuncts when the conjunction is true, to satisfy [D2]) iff \(\Gamma \models \varphi_1\) and \(\Gamma \models \varphi_2\) are 2-licensed as \(\Gamma \models_{L2} \varphi_1 \land \varphi_2\) iff \(S_T \models_{R2} \varphi_1\) and \(S_T \models_{R2} \varphi_2\) by the inductive hypothesis iff \(S_T \models_{R2} \varphi\) by Defn 2.24.2.

Suppose \(\varphi = \varphi_1 \lor \varphi_2\). Then \(\Gamma \models \varphi\) is 2-licensed iff \(\Gamma \cup \{\varphi_1, \neg \varphi_2\} \models \varphi_1\) and \(\Gamma \cup \{\varphi_2, \neg \varphi_1\} \models \varphi_2\) are 2-licensed (these are the conditions listed in Defn 2.7 by the contribution table for disjunction which are required for the disjunction to have contributions from both the left and right disjuncts when the disjunction is true, to satisfy [D2]) iff \(S_T \cap (S_{\varphi_1} \setminus S_{\varphi_2}) \models_{R2} \varphi_1\) and \(S_T \cap (S_{\varphi_2} \setminus S_{\varphi_1}) \models_{R2} \varphi_2\) by the inductive hypothesis iff \(S_T \models_{R2} \varphi\) by Defn 2.24.3.

Suppose \(\varphi = \neg \varphi_1\). Then \(\Gamma \models \varphi\) is 2-licensed iff \(\Gamma \models \text{NNF}(\varphi)\) is 2-licensed by Lemma 2.12.2 if \(S_T \models_{R2} \text{NNF}(\varphi)\) by the inductive hypothesis iff \(S_T \models_{R2} \varphi\) by Lemma 2.43. □

**Lemma 2.83** \(S_T \models_{R1} \varphi\) iff \(\Gamma \models \varphi\) is a 1-licensed inference.

*Proof:* \(S_T \models_{R1} \varphi\) iff \(S_T \models_{R2} \varphi\) or \(\varphi\) is disjunction-free by Lemma 2.25.2 if \(\Gamma \models \varphi\) is 2-licensed or \(\varphi\) is disjunction-free by Lemma 2.82 iff \(S_T \models_{R1} \varphi\) is 1-licensed by Defn 2.10.1. □

**Lemma 2.84** \(S_T \models_{R3} \varphi\) iff \(\Gamma \models \varphi\) is a 3-licensed inference.

*Proof:* \(\Gamma \models \varphi\) is 3-licensed iff \(\Gamma \models_{L3} \varphi\) and [D2] and [D3] holds by Defn 2.10.2 if \(S_T \models \varphi\) is 2-licensed and no subformula of the conclusion in negation normal form is a tautology or contradiction by Lemma 2.11.1 and Lemma 2.12.2 if \(S_T \models_{R2} \varphi\) and there is no subformula \(\psi\) of \(\text{NNF}(\varphi)\) such that \(\in_{L3} \psi\) by Lemma 2.82 if \(S_T \models_{R3} \varphi\) by Lemma 2.28. □

**Lemma 2.85** \(S_T \models_{R4} \varphi\) iff \(\Gamma \models \varphi\) is a 4-licensed inference.

*Proof:* By Lemmas 2.13, 2.35, and 2.82. □

**Lemma 2.86** \(S_T \models_{R5} \varphi\) iff \(\Gamma \models \varphi\) is a 5-licensed inference.

*Proof:* By Lemmas 2.12.2 and 2.43 we can assume \(\varphi\) is in \(\text{NNF}\).  

\[\Rightarrow \] \(S_T \models_{R5} \varphi\) entails \(S_T \models_{R4} \varphi\), and so \(\Gamma \models \varphi\) is 4-licensed by Lemmas 2.39 and 2.85. It remains to show that any occurrence of an atom that satisfies both Defn 2.37.1 and contribution criterions [D2] and [D4] under simple assignment also satisfies [D2] under restricted assignment. By [D2] and [D4] under simple assignment, every atom occurrence must contribute to the conclusion on a row where \(\varphi\) and \(\Gamma\) are true, and on another where they are false. As Defn 2.37.1 is satisfied for each literal occurrence \(l\), we can disregard all but those worlds that are in \((S_{\neg \varphi})_l\), and thus those rows that satisfy \((S_{\neg \varphi})_{l\pm}\). But these are precisely the rows where \(\varphi\) is false, or changing the truth value of \(l\) makes \(\varphi\) false: the requirement for restricted assignment.
(\Leftrightarrow) \Gamma \vdash \varphi \text{ being 5-licensed entails that } \Gamma \vdash \varphi \text{ is 4-licensed, and so } (S_{\Gamma}, S_{\neg \varphi}, 0) \models^{+}_{R_{4}} \varphi \text{ by Lemmas } 2.12.4 \text{ and } 2.85. \text{ The only additional requirement for } \Gamma \models_{R_{5}} \varphi \text{ over } \Gamma \models_{R_{4}} \varphi \text{ is that the sets } S \cap U \text{ and } T \cap U \text{ intersect } (S_{\neg \varphi})_{\pm l} \text{ for each literal occurrence } l. \text{ But the sets } S \cap U \text{ and } T \cap U \text{ for each literal occurrence correspond to the rows of contributions satisfying } [D_{2}] \text{ and } [D_{4}] \text{ under simple assignment for that literal occurrence. And as discussed in the first half of this proof, } (S_{\neg \varphi})_{\pm l} \text{ corresponds to those rows where } \varphi \text{ is false, or changing the truth value of } l \text{ makes } \varphi \text{ false, the requirement for restricted assignment.}

**Lemma 2.87** \( \Gamma \vdash \varphi \) is \( 2^{\ast} \) concise iff it is a 2-licensed inference.

*Proof:* We assume \( \Gamma \vdash \varphi \) is \( 2^{\ast} \) inconcise and \( \Gamma \vdash \varphi \) is not 2-licensed.

The rest we show by induction on the complexity of \( \varphi \). Our base cases are where \( \varphi \) is a literal, \( \varphi = T \), and \( \varphi = \bot \). If \( \varphi \) is a literal or \( \varphi = T \), then \( \Gamma \vdash \varphi \) is \( 2^{\ast} \) concise iff \( \Gamma \) is consistent, as only \( \bot \in \varphi \); also \( \Gamma \vdash \varphi \) is 2-licensed iff there is a row where \( \varphi \) and \( \Gamma \) are both true, which is to say that \( \Gamma \) is consistent. If \( \varphi = \bot \), then \( \Gamma \vdash \varphi \) is \( 2^{\ast} \) inconcise and \( \Gamma \vdash \varphi \) is not 2-licensed, regardless of \( \Gamma \).

Our Inductive Hypothesis is that for all \( \chi \) less complex than \( \varphi \), \( \Gamma \vdash \chi \) is \( 2^{\ast} \) concise iff \( \Gamma \vdash \chi \) is 2-licensed. As both \( 2^{\ast} \) concision and 2-licensing respect De Morgan and double negation by [Permutation] and Lemma 2.12.2, we need only consider those complex formulas that are conjunctions and disjunctions. Also if \( \Gamma \) is inconsistent, then \( \Gamma \vdash \varphi \) is not \( 2^{\ast} \) concise and \( \Gamma \vdash \varphi \) is not 2-licensed for any \( \varphi \). Suppose \( \Gamma \) is consistent, \( \varphi \) is complex, and \( \Gamma \vdash \varphi \) is not \( 2^{\ast} \) concise. Then \( \exists \psi: \Gamma \vdash_{EL} \psi \) and \( \psi \in \varphi \). Consider the first application of either [B1] or [B2] in any derivation of \( \psi \in \varphi \); \( \exists \psi': \psi \triangleright \varphi' \) and \( \Gamma \vdash \varphi' \). [B1] removes a disjunct \( \chi \), while [B2] replaces any subformula \( \chi \) with \( \bot \). The resulting formula \( \varphi' \) is entailed by \( \Gamma \), so the subformula \( \chi \) cannot have contributed to the truth of \( \varphi \) in any case when \( \Gamma \) was true. As all literals in \( \chi \) are provided with contributions, [D2] will fail for \( \Gamma \vdash \varphi \). Conversely, if \( \Gamma \vdash \varphi \) is not 2-licensed then [D2] fails, there is a subformula \( \chi \) of \( \varphi \) which does not contribute to the truth of \( \varphi \) when \( \Gamma \) is true, and so \( \chi \) can be replaced by \( \bot \) without affecting entailment, so \( \Gamma \vdash \varphi \) is not \( 2^{\ast} \) concise.

**Lemma 2.88** \( \Gamma \vdash \varphi \) is \( 1^{\ast} \) concise iff it is a 1-licensed inference.

*Proof:* \( \Gamma \vdash \varphi \) is \( 1^{\ast} \) concise iff \( \Gamma \vdash \varphi \) is \( 2^{\ast} \) concise or \( \varphi \) is disjunction-free by Lemma 2.81.1 iff \( \Gamma \vdash \varphi \) is 2-licensed or \( \varphi \) is disjunction-free by Lemma 2.87 iff \( \Gamma \vdash \varphi \) is 1-licensed by Defn 2.10.1.

**Lemma 2.89** \( \Gamma \vdash \varphi \) is \( 3^{\ast} \) concise iff it is a 3-licensed inference.

*Proof:* \( \Gamma \vdash \varphi \) is \( 3^{\ast} \) concise iff \( \Gamma \vdash \varphi \) is \( 2^{\ast} \) concise (so no subformula is contradictory) and no subformula of \( NNF(\varphi) \) is tautological by Defns 2.65 and 2.67, [B3](a)/(b) iff \( \Gamma \vdash \varphi \) is 2-licensed and [D3] holds by Lemmas 2.87 and 2.11.1 iff \( \Gamma \vdash \varphi \) is 3-licensed by Defn 2.10.3.

**Lemma 2.90** \( \Gamma \vdash \varphi \) is \( 4^{\ast} \) concise iff it is a 4-licensed inference.

*Proof:* By Lemmas 2.81.3, 2.13, and 2.87.

**Lemma 2.91** \( \Gamma \vdash \varphi \) is \( 5^{\ast} \) concise iff it is a 5-licensed inference.

*Proof:* Justification of this claim is deferred until Lemmas B.15 and B.16 in Appendix B.2, as the former proof is long, complex, and unenlightening, and relies on Conjecture B.17.

**Theorem 2.92** The Contribution, Relevance, and Concision families are equivalent.

1. \( \Gamma \vdash \varphi \) is \( 1^{\ast} \) concise iff it is 1-licensed iff \( \Gamma \models_{R_{1}} \varphi \) iff \( M_{1} \models_{M_{1}} \varphi \).
2. \( \Gamma \vdash \varphi \) is \( 2^{\ast} \) concise iff it is 2-licensed iff \( \Gamma \models_{R_{2}} \varphi \) iff \( M_{2} \models_{M_{2}} \varphi \).
3. \( \Gamma \vdash \varphi \) is \( 3^{\ast} \) concise iff it is 3-licensed iff \( \Gamma \models_{R_{3}} \varphi \) iff \( M_{3} \models_{M_{3}} \varphi \).
4. \( \Gamma \vdash \varphi \) is \( 4^{\ast} \) concise iff it is 4-licensed iff \( \Gamma \models_{R_{4}} \varphi \) iff \( M_{4} \models_{M_{4}} \varphi \).
5. \( \Gamma \vdash \varphi \) is \( 5^{\ast} \) concise iff it is 5-licensed iff \( \Gamma \models_{R_{5}} \varphi \) iff \( M_{5} \models_{M_{5}} \varphi \).

*Proof:* By Lemmas 2.82–2.91, and B.11.

The equivalence of the four semantic deduction relation families presented in this chapter means that we can use whichever semantics are most convenient when investigating particular aspects of assertibility, which will prove extremely useful over the next few sections.
2.6 Assertibility Properties

I will demonstrate several interesting properties shared by the semantic families introduced so far. One such property is that they each form a sound and complete semantics for the family of cumulative assertion norms in Defn 1.12 when there is no common ground. As all the semantic families introduced in this chapter are equivalent, it does not matter which semantics we apply, adapt, or use to demonstrate their properties. This flexibility allows us to discuss the generic concept of assertibility while switching freely between these semantics, which is useful as each is more suited to proving the applicability of different properties.

Definition 2.93 \(n\)-Assertibility

\[ \Gamma \vdash \varphi \text{ is } n\text{-assertible iff } \psi \vdash \varphi \text{ is } n\text{-assertible.} \]

\[ \varphi \text{ is } n\text{-assertible iff } \varphi \vdash \varphi \text{ is } n\text{-assertible iff } \varphi \text{ is } n\text{-brief.} \]

By Theorem 2.92, \( \Gamma \vdash \varphi \text{ is } n\text{-assertible iff } S_{\Gamma} \models_{Rn} \varphi \text{ iff } \Gamma \models_{Mn} \varphi \text{ iff } \Gamma \vdash \varphi \text{ is } n\leq\text{concise.} \) A list of example inferences and which norms they satisfy can be found in Appendix C.4.

Theorem 2.94 Literal Inclusion

1. If \( \Gamma \vdash \varphi \text{ is 5-assertible then } \text{PROP}(\varphi) \subseteq [\text{PROP}(\Gamma)]. \)
2. If \( \Gamma \vdash \varphi \text{ is 5-assertible then } \text{LIT}(\varphi) \subseteq [\text{LIT}(\Gamma)]. \)

Proof: By Lemma 2.14 and Defn 2.93. \( \Box \)

Theorem 2.94 shows that 5-assertibility has the desirable properties of variable and literal inclusion – but they come at a cost of complexity and oddness in each semantics. 5-concision requires the quirky but well-behaved \([B5](a)\) and \([B5](b)\), both \(R5\) and \(M5\) require \(l\)-neutral extensions, while 5-licensing uses restricted assignment. In each case, a new kind of principle has to be added to the reasonably homogeneous semantics of the first four systems. The Inclusion norm also has another effect, as we can see by considering Lemma 2.14.2. If \( \Gamma \vdash \varphi \text{ is 5-assertible, and } \varphi \text{ is not } l\text{-neutral, then } l \text{ must be used in the premises, and } \Gamma \text{ must not be } l\text{-neutral.} \) The premise set \( \Gamma \) is \(l\)-neutral iff \( l \not\in [\text{LIT}(\Gamma)], \) and if \( \Gamma \) is \(l\)-neutral then \( l \not\in \text{LIT}(\varphi) \) either, which is the main argument in the proof of Lemma 2.14.2. Thus the Inclusion norm imposed by 5-assertibility also allows us to distinguish between a formula \( \varphi \) which simply mentions \( l \) and one that actually uses \( l \), and similarly with the corresponding utterance forms.

2.6.1 Assertion Norms

The assertion norms of Defns 1.7 and 1.9 describe the conditions not just on the cooperatively assertible utterance form \( \varphi \), but also on the common ground or context \( \Pi \), given a communicative content \( \Gamma \). The cumulative hierarchy of assertion norms from Defn 1.12 has a close relationship with the \(n\)-assertibility semantics presented in this chapter.

Context

We have ignored the common ground for most of this chapter. However, to relate our formal semantics to the assertion norms of Chapter One, we will need to consider how non-empty contexts will be modelled. Every assertion with content \( \varphi \) conveying some of the communication content \( \Gamma \) takes place on some common ground \( \Pi \), so any formal semantics for \(n\)-assertibility should really model the three-place relation \( (\Pi; \Gamma) \vDash \varphi \), and not just \( \Gamma \vdash \varphi \). Fortunately, altering the concision, contribution and relevance semantics to account for the common ground is quite simple, as all that is required is for references to tautologies to be altered to ‘being entailed by \( \Pi \)’. Each family of semantics will be extended below, and it will be obvious from the definitions that every result in this chapter will hold when \( \Pi = \emptyset \). It is not as clear that all the results will hold with non-empty common ground; in particular, I have not proved that the equivalences that lead to Theorem 2.92 will be maintained. I will assume that the formal equivalences will continue to hold under the respective extensions, or can be made to hold by minor technical adjustments.
**Definition 2.95** Implicit Contextual Relevance Semantics.

The set \( W \) of epistemically available possible worlds was defined in Defn 2.19. This definition can be refined to take account of the epistemically available worlds to those where \( \Pi \) is true. That is, for a given inference \( \Gamma \vdash \varphi \) in context \( \Pi \), the universe of possible worlds \( U \) can be modelled by the powerset of \( \text{PROP}(\Pi \cup \Gamma \cup \{\varphi\}) \), and the set \( W \) of epistemically available possible worlds consists of those worlds in \( U \) satisfying \( \Pi \).

An alternative approach is to represent the context explicitly. The semantics for R4 and R5 in Defns 2.29 and 2.37 already have facility to allow this by letting \( U \) be restricted by the context \( \Pi \). We simply need to extend Defns 2.30 and 2.38:

**Definition 2.96** Explicit Contextual Relevance Semantics.

1. \( \langle \Pi; \Gamma \rangle \models R4 \varphi \iff S_{\Pi,\Pi} \models CL \varphi \) and \( (S_{\Gamma}, S_{\sim \varphi}, S_{\Pi}) \models R4 \varphi \).
2. \( \langle \Pi; \Gamma \rangle \models R5 \varphi \iff S_{\Pi,\Pi} \models CL \varphi \) and \( (S_{\Gamma}, S_{\sim \varphi}, S_{\Pi}) \models R5 \varphi \).

**Definition 2.97** Contextual Licensed Semantics: \( \langle \Pi; \Gamma \rangle \vdash \varphi \) is \( n \)-licensed.

A contribution table is simply a truth table with extra subscripting. A contribution table in context \( \Pi \) is restricted to those rows of the contribution table where \( \Pi \) is true. This also affects restricted assignment, as both twin rows must belong to the restricted contribution table.

**Example:** \( \langle p \supset q; p \rangle \vdash p \) and \( \langle p \supset q; p \rangle \vdash q \) are 4-assertible, but \( \langle p \supset q; p \rangle \vdash p \land q \) is not, even though \( \langle p \supset q; p \rangle \vdash p \land q \) is true. That is, for a given inference \( \Gamma \vdash \varphi \) in context \( \Pi \), the universe of possible worlds \( U \) can be modelled by the powerset of \( \text{PROP}(\Pi \cup \Gamma \cup \{\varphi\}) \), and the set \( W \) of epistemically available possible worlds consists of those worlds in \( U \) satisfying \( \Pi \).

The brevity and concision systems refer to entailment and tautologies extensively. The definitions of concision and brevity must be amended to allow for context, a new brevity operation symbol \( \varphi \) introduced to replace \( \varphi \), and the definitions of brevity operations [B3](a)–[B5](b) amended to explicitly add \( \Pi \) to all entailments. These changes add clutter, but are otherwise straightforward.

**Definition 2.98** \( \langle \Pi; \Gamma \rangle \vdash \varphi \) is \( n \)-concise iff \( \Pi, \Gamma \models CL \varphi \), and \( \forall \psi <_{\Pi n} \varphi: \Pi, \Gamma \not\models CL \psi \).

**Definition 2.99** \( n \)-Brevity in Context

\( \varphi \) \( \leq_{n} \varphi \) iff there is a sequence \( \varphi_{1}, \ldots, \varphi_{k} \) such that \( \varphi_{1} = \varphi, \varphi_{k} = \psi \), and for each \( 1 \leq i < k \):

either (i) \( \varphi_{i} \equiv \varphi_{i+1} \), or (ii) \( \varphi_{i} \varphi_{i+1} \) by some \([Bm], m \leq n \).

**Definition 2.100** The revised operations in Defn 2.67 are:

[B3](a) \( \varphi[A] \triangleright \varphi[\top] \), if \( \Pi \models CL A \).
[B3](b) \( \varphi[A \land B] \triangleright \varphi[A] \), if \( \Pi \models CL B \).
[B4] \( \varphi[A \lor B] \triangleright \varphi[A] \), if \( \Pi \models CL B \).
[B5](a) \( \varphi[(A \land B) \lor (C \land D)] \triangleright \varphi[(A \lor C) \land (B \lor D)] \), when they are equivalent given \( \Pi \).
[B5](b) \( \varphi[(A \lor C) \land (B \lor D)] \triangleright \varphi[(A \land B) \lor (C \land D)] \), when they are equivalent given \( \Pi \).

**Lemma 2.101** \( \langle \emptyset; \Gamma \rangle \vdash \varphi \) is \( n \)-concise iff \( \Gamma \vdash \varphi \) is \( n \)-concise.

**Proof:** By Defns 2.79 and 2.98.

The above Defns 2.97–2.100 for context-sensitive licensing and concision will not be referred to again in this thesis, and are included simply to show that adding context to the various semantics is neither technically difficult nor interesting. In later chapters I will suppress the context and common ground in formal representations except where absolutely necessary.
2.6. ASSERTIBILITY PROPERTIES

Soundness and Completeness

When the context is empty (or only contains tautologies), the five cumulative syntactic inference relations defined by assertion norms from Defn 1.12 are equivalent to the semantic 1-assertibility through 5-assertibility inference relations. The striking and intentional parallels between the relevance semantics and assertion norms make the proofs for the simpler systems straightforward, so I will only prove this identity for 4- and 5-assertibility.

Theorem 2.102 \( \langle \Pi, \Gamma \rangle \models \varphi \) iff \( \langle \Pi, \Gamma \rangle \models \varphi \), for \( n \in \{4, 5\} \).

Proof: Both families of relations require \( \Pi, \Gamma \models \varphi \).

\( (n = 4) \) Defn 1.12.4 is equivalent to Defn 1.7. We will proceed by induction on complexity of \( \varphi \).

The base case is that \( \varphi \) is a literal. Defn 1.7.1 has three conditions: \( \Pi, \Gamma \models l \), \( \Pi \not\models l \), and \( \Pi, \Gamma \models \perp \).

These correspond to classical entailment, \( T \cap U \neq \emptyset \), and \( S \cap U \neq \emptyset \) respectively from Defn 2.29.1, as \( U = S_{\Pi} \), \( T = S_{\perp} \), and \( S = S_{\Gamma} \) by Defn 2.96. In the inductive case, \( \varphi \) is either a disjunction or a conjunction.

But Defn 2.29.3 is a paraphrase of Defn 1.7.2, so disjunction preserves equivalence, and Defn 2.29.2 and \( \Pi, \Gamma \models \varphi \) paraphrase Defn 1.7.3, so conjunction also preserves equivalence.

\( (n = 5) \) Defns 2.37 for \( R5 \) and 1.9 for the norms with Inclusion are just Defns 2.29 and 1.7 with strengthened conditions for literals. The inductive cases are thus the same as above.

Suppose a literal occurrence \( l \) satisfies Defn 2.37.1; then it satisfies Defn 2.29.1, and hence Defn 1.7 as above, so \( \Pi \models l \) and \( \Pi, \Gamma \models l \). Pick any \( w \) in the non-empty set \( S \cap U \cap (S_{\neg \varphi}) \). Let \( \Sigma \) be the set containing those elements of \( LIT(\Gamma \cup \Pi) \backslash \{l, \neg l\} \) that are satisfied by \( w \), so that \( w \models \Sigma \).

As \( w \models U \), \( w = \bigwedge \Pi \); as also \( w \models S, w \models \emptyset \), and thus \( w \models \Gamma \). So \( \Pi, \Sigma, \Gamma \models l \). As \( \Pi, \Gamma \models l \), we have \( \Pi, \Sigma, \Gamma \models l \). But also \( w_{\perp} = \bigwedge \Sigma \neg \varphi \), so \( \Sigma \not\models \varphi \). Thus \( \Sigma \) is a maximal consistent subset of \( LIT(\Gamma \cup \Pi) \backslash \{l, \neg l\} \) satisfying Defn 1.9.1. Conversely suppose \( \Sigma \) is a maximal consistent subset of \( LIT(\Gamma \cup \Pi) \backslash \{l, \neg l\} \) such that \( \Pi, \Sigma, \Gamma \models l \); \( \Pi, \Sigma, \Gamma \models \perp \); \( \Pi \not\models l \); and \( \Sigma \not\models \varphi \), as per Defn 1.9.1. By the above proof for \( (n = 4) \), we know that \( S \cap U \neq \emptyset \) and \( T \cap U \neq \emptyset \). But \( T \subseteq S_{\neg \varphi} \subseteq (S_{\neg \varphi}) \), so \( T \cap U \cap (S_{\neg \varphi}) \neq \emptyset \). As \( \Pi \cup \Sigma \cup \Gamma \) is consistent and entails \( l \), there is some \( w \) where \( \Pi, \Sigma, \Gamma, l \) are all true. But \( U = S_{\Pi} \), and \( S = S_{\Gamma} \) by Defn 2.96, so \( w \models S \cap U \). Also, \( w_{\perp} \models S_{\neg \varphi} \), so \( w \models (S_{\neg \varphi}) \). Then \( S \cap U \cap (S_{\neg \varphi}) \neq \emptyset \). This satisfies Defn 2.37.1. 

Extensibility

Lemma 2.103 shows that both 4- and 5-assertibility lack Extensibility; there is no \( \psi \) that can be conjoined to the conclusion of the 5-assertible inference \( \{p \lor r, \neg r \lor q\} \models p \lor q \) so that \( (p \lor q) \land \psi \) is equivalent to \( (p \lor r) \land (\neg r \lor q) \), and 5-assertible. However, 1-, 2- and 3-assertibility are Extensible. The imposition of Conjunctive Compositionality causes an assertibility system to lack Extensibility, but imposing Inclusion without Conjunctive Compositionality also appears to removes Extensibility (see Appendix A.5). I conjecture that any assertibility system that imposes double-variable sharing (see §2.6.2 below) cannot be Extensible. Another unexplained correlation is that all the non-Extensible inferences I have found involve some variation of \( p \lor q \lor r \lor q \lor r \), where a formula containing the ‘missing’ middle element cannot be conjoined to the conclusion without redundancy. There may be some deeper significance to the failure of this generalised ‘cut-rule’. Finally, I have observed that when \( \chi \models \varphi \) and \( \neg \varphi \models \neg \chi \) are both 4-assertible, \( \chi \models \varphi \) is Extensible iff \( \neg \varphi \models \neg \chi \) is 5-assertible (i.e., the contrapositive inference satisfies Inclusion). This relationship between the Inclusion and Extensibility norms requires further investigation, which I will not undertake.

Lemma 2.103 Assertibility and Extensibility

1. For \( n \leq 3 \), if \( \chi \models \varphi \) is \( n \)-assertible and \( \varphi \not\models \chi \) then \( \exists \psi: \chi \equiv \varphi \land \psi \) and \( \chi \models \varphi \land \psi \) is \( n \)-assertible.

2. The above result does not always hold for \( n \geq 4 \).

Proof: 1. By Defn 2.73 there is an \( n \)-brief formula \( \psi \) such that \( \psi \equiv \chi \lor \neg \varphi \). As \( \varphi \) and \( \psi \) are \( n \)-brief, hence \( n \)-assertible, so is their conjunction \( \varphi \land \psi \). To see this, consider only those instances of operations that are possible on \( \varphi \land \psi \) but not either \( \varphi \) or \( \psi \): [B1] and [B3] require \( \varphi \) or \( \psi \) to be (the 3-unassertible) \( T \), and [B2] would replace \( \varphi \land \psi \) with (the 2-unassertible) \( \perp \). [B4] applies to conjunctions, but is not available for \( n \leq 3 \).

2. An example is \( \chi = (p \lor r) \lor (q \lor \neg r) \) and \( \varphi = p \lor q \). 

2.6.2 Properties of Assertion

The $n$-assertibility inference relations have some properties (and lack others) that would be of particular interest if they were being used to capture intuitions about entailment. First, they lack several properties of a standard Tarski-style consequence relation. None of them are reflexive; e.g., $p \lor p \vdash p \lor p$ is unassertible. Nor are they monotonic; $p \lor q \vdash p \lor q$ is assertible but $p \lor q, p \vdash p \lor q$ is not. Transitivity also fails; $(p \lor q) \land r \vdash p \lor (q \land r)$ and $p \lor (q \land r) \vdash p \lor r$ are assertible, but $(p \lor q) \land r \vdash p \lor r$ is not. Finally, the deduction theorem does not hold for any of the systems.

Relevant Properties

Several properties of Classical Logic are retained by all $n$-assertibility systems, including forms of Simplification, Modus Ponens and Disjunctive Syllogism (when the premises are consistent). The connectives also satisfy Commutativity, Associativity, and the De Morgan Laws. Some assertibility systems also have other properties that have been sought by various non-classical logicians. These properties are engendered by particular combinations of norms; these are shown by indicating which of the associated brevity operations are required, as this is a conveniently modular way of presenting sets of restrictions. As operations are added, more and more properties accumulate. Many of these are associated with relevance or connexive logics. Although they have many properties that have been proposed as necessary or sufficient conditions for entailment, I do not suggest that any of the assertibility systems model (a type of relevant) entailment; this is simply not the project I am engaged in. However, the recovery of so many ‘relevant’ properties suggests a possible interpretation of some relevance projects: relevance logic may be thought of as (rightly or wrongly) adding cooperative or communicative restrictions to the purely truth-preserving endeavour of classical logic. Under this interpretation, if classical logic describes the rules for reasoning with truth, relevance logics would describes the rules for communicating about reasoning with truth. Some of the more interesting properties are listed below, along with the brevity operations $[B^k]$ that they require:

1 2 3 4 5
✓ – – – – The von Wright/Geach/Smiley criterion holds for the material conditional (see below).
– ✓ – – – Failure of Ex Falso Quodlibet. Aristotle’s Thesis in the form: if $A \vdash B$ then $A \not\vdash \neg B$.
– – ✓ – – No tautologies can occur within the conclusion.
– – ✓ – – No logical constants can occur within the conclusion. Strawson entailment.
✓ ✓ – – Variable sharing - at least one variable is shared by the premise set and conclusion.
✓ – – – – No redundant dyadic connectives can occur within the conclusion.
✓ ✓ ✓ – Double Variable sharing - if there are multiple propositional variables in the conclusion, then at least two of them are in the premise set.
✓ ✓ – – Variable Inclusion$^4$.
✓ ✓ ✓ ✓ ✓ All of the above. Also Literal Inclusion.

von Wright/Geach/Smiley Criterion

One desirable property of $n$-assertibility is the von Wright/Geach/Smiley criterion (WGS), proposed by von Wright (1957) and refined by Geach (1958) and Smiley (1959), which they claimed to be sufficient to eliminate the ‘paradoxes’ of the material and strict conditionals. Schurz (1991) describes the WGS criterion on p.416 as “$A \rightarrow B$ is a relevant implication (or $A \vdash B$ a relevant deduction, respectively), iff it is possible to prove $A \rightarrow B$ without proving $\neg A$ or $B$”. The principle $[B^1]$ directly implements WGS by ensuring that if it is possible to prove $\neg A$, or it is possible to prove $B$, then any proof of $A \supset B$ must be rejected due to the conclusion being inconcise. The most common criticism of WGS is the ambiguity of the modal condition ‘it is possible to prove’, which is resolved by identifying the possibility to prove a formula with its membership of the set of consequences. Thus all $n$-assertibility relations satisfy WGS for implication ($\supset$).

$^3$These counter-examples for R1 (RAD) come from pp. 348–350 of Verhoeven (2007).
$^4$The proof for this claim appears in Appendix A.5.
Witnesses

There are several ways to tell if an inference is n-assertible. One of the easiest ways is to compare it with another inference. If the conclusions stand in the right relationship, the second inference can serve as a witness to the assertibility or unassertibility of the former. For example, if $\psi \leq_n \varphi$ and $\psi \equiv \varphi$ then $\psi \vdash \varphi$ is n-unassertible. However, there are several other ways that formulas can serve as witnesses. The following Lemma is indicative of the possibilities, but by no means complete.

**Lemma 2.104 Witnesses to Assertibility**

1. If $\Gamma \vdash_1 \varphi$ and $\varphi <_n \psi$ then $\Gamma \vdash \psi$ is n-unassertible.
2. If $\Gamma \vdash \varphi$ is n-unassertible and $\varphi \leq_n \psi$ then $\Gamma \vdash \psi$ is n-unassertible.
3. If $\Gamma \vdash_1 \varphi$ then $\Gamma \vdash \varphi \lor \psi$ is n-unassertible.
4. If $\Gamma \vdash \varphi$ is n-assertible and $\psi$ is a subformula of $\varphi$ then $\psi$ is n-assertible.
5. If $\Gamma \vdash \varphi \land \psi$ is n-assertible then $\Gamma \vdash \varphi$ is n-assertible.
6. If $\Gamma \vdash (\varphi \land \psi) \lor (\varphi \lor \chi)$ is n-assertible then $\Gamma \vdash \varphi \land (\psi \lor \chi)$ is n-assertible, $n \neq 3$.
7. If $\Gamma \vdash (\varphi \lor \psi) \land (\varphi \lor \chi)$ is n-assertible then $\Gamma \vdash \varphi \lor (\psi \land \chi)$ is n-assertible.

**Proof:** 1/2. By $n$-concision. 3. By $n$-concision as $\varphi \land \psi \leq_1 \varphi$, and $\varphi <_n \varphi \lor \chi$. 4. By [Composition] and Lemma 2.80.2. 5. By [B4] and Lemma 2.80.2. 6. The simple signatures for $(\varphi \land \psi) \lor (\varphi \lor \chi)$ and $\varphi \land (\psi \lor \chi)$ are $\langle 1_s, 1_{ab}, 1_{cd}, 0_{bd}, 0_{ac}, 0_a, 0_c, 0_e \rangle$ and $\langle 1_a, 1_{ab}, 1_{ac}, 0_{bc}, 0_a, 0_a, 0_a, 0_a \rangle$ respectively, so [D2] and [D4] are satisfied for the latter formula if they are satisfied for the former, and this result holds for 2-, 4-assertibility. As the formulas are equivalent, restricted assigment affects each equally, so it holds for 5-assertibility. Finally, [D1] restricts the contributions of interest to those from $\psi \lor \chi$ in the second formula, and does not affect the first formula, so it holds for 1-assertibility. Note that $p \vdash (p \land q) \lor (p \land \neg q)$ is 3-assertible, but $p \vdash p \land (q \lor \neg q)$ is not. 7. The simple signatures for $(\varphi \lor \psi) \land (\varphi \lor \chi)$ and $\varphi \lor (\psi \land \chi)$ are $\langle 1_s, 1_c, 1_{ac}, 1_{bd}, 0_{cd}, 0_{ab}, 0_s \rangle$ and $\langle 1_s, 1_a, 1_{ac}, 1_{bc}, 0_{ac}, 0_{ab}, 0_a \rangle$ respectively. The same reasoning as in Lemma 2.104.6 above holds. In addition, if the latter formula contains a tautology, so does the former, so the result holds for 3-assertibility.

**Definition 2.105** $\varphi$ n-implies $\psi$ if $\varphi$ is n-assertible, and when $\Gamma \vdash \varphi$ is n-assertible, so is $\Gamma \vdash \psi$.

The relation of n-implication respects the usual rules for consequence relations, such as reflexivity, monotonicity, and even transitivity. It also supports Commutativity, Associativity, the De Morgan laws, and *ex falso quodlibet* but not Distributivity (as discussed in §2.4.5), nor *verum ex quodlibet*. As the implication relation does not rely on any particular $\Gamma$, it will be useful for describing relationships between utterance forms independently of any Speaker’s intentions.

**Finitude**

**Theorem 2.106** There is only a finite number of NNF formulas $\varphi$ such that $\Gamma \vdash \varphi$ is 5-assertible.

**Proof:** As $\Gamma \vdash \varphi$ is 5-assertible, $\varphi$ contains no logical constants. Only the literals in $[\text{LIT}(\Gamma)]$ can occur in $\varphi$ by Theorem 2.94.2. Suppose $|[\text{LIT}(\Gamma)]|=n$; then the maximum number of times a literal can appear in $\varphi$ is $2^{n-1}$ as otherwise each occurrence cannot fulfil both [D2] and [D4]. The number of permutations for combining a finite set of literals each occurring a finite number of times solely via the connectives $\land$ and $\lor$ is also finite.

The system of 4-assertibility is fairly elegant, satisfies the basic assertion norms, has fairly tidy and coherent concision, licensing, and relevance semantics, and demonstrates some natural symmetry in Lemmas such as 2.35 and 2.74. The 5-assertibility system lacks results like Lemma 2.35, but has Theorems 2.94 and 2.106 in its favour, and perhaps the quirky interest of the [B5] operations and $l$-neutrality. Fortunately we do not need to choose between them as they share the same rules for disjunction and conjunction in every compositional semantics, and it is only these compositional rules that will concern us for the next three chapters. Some alternatives to 5-assertibility are discussed in Appendix A.2.
3. Deviations

In this chapter I will show that the assertion norms can be used to generate some predictions about the information conveyed by natural language utterances. The limited information content of these predictions, along with acceptable simple coordinations that violate the assertion norms, will force us to add another layer to our analysis. The norms are appropriate for agents with no limitations on their rationality, information, and time to prepare and perform their utterances. Humans, being finite, fallible, hurried creatures, will often fall short of these norms, but still successfully communicate with each other. There are a number of common patterns to the ways that they fail to comply with the norms, and by taking advantage of these patterns, actually communicate more information in shorter utterances, at the cost of contextually-resolvable ambiguity. That is, by breaking the norms in recognisable ways, they actually obey the communicative goals of the norms more effectively than if they blindly followed the norms. This process is analogous to how specific forms of flouting of the Gricean maxims generate certain conversational implicatures. Like Grice, I will motivate a range of acceptable minor deviations from the norms via additional maxims. I will treat assertibility as a property that comes in degrees rather than as a simple success condition, and seek interpretations that maximise assertibility in the context. I will also call upon some techniques from both cognitive linguistics and formal theories of syntax to describe specific types of coordination with non-standard behaviour that becomes predictable when these techniques are combined with assertibility.

In §3.1 I will use the assertion norms to make some rather generic predictions about information recoverable from assertible and unassertible coordinated utterances. These predictions fall far short of what we will require to capture the range of variations in the common usage of ‘and’ and ‘or’, as we can only accuracy determine whether the norms are satisfied, and not why or where they fail. Some systematic types of norm violation in seemingly acceptable coordinations will become apparent during a review of some unassertible coordinations. The violations in these examples will eventually be codified as acceptable deviations from the standard assertibility conditions that convey additional information related to the specific norm failure. In the short sections §§3.2 through §3.4 I introduce new forms of notation for representing the norms that allow them to be generalised and weakened more easily, and present systematic deviations from the compositional coordination norms. These deviations are motivated by consideration of non-propositional methods for modelling additional requirements relating to the maxims of Informativity, Expressivity, Relevance and Order. The resulting variant coordinations will be used in the next chapter to provide further predictions of additional information conveyed by coordinations.

In §3.5 I present a type theory from Heim and Kratzer (1998), and show that the standard truth-functional conjunction and disjunction (along with set intersection and union) can be used to describe the semantics of all common phrasal coordinations. I will demonstrate how these descriptions of phrasal coordinations can be extended to the assertibility conditions for phrasal coordinations. Next in §3.6.1 I will introduce the psychological notion of a ‘connexion’ between coordinands, leading to five distinct modes of coordination (e.g., deductive, evocative, renunciative) based on the relationship between the coordinands’ content. These modes roughly parallel the discourse domains used by Sweetser (1990) and some other cognitive linguists for analysing coordination and modality. They will be extensively used in analysing both coordinations and conditionals in subsequent chapters. Two of the modes relate the propositional content of one coordinand with the felicity, relevance, illocutionary effects, or linguistic performance criteria of the other, and I will briefly explore some slightly unexpected logical properties of those coordinations in §3.7.
3. DEVIATIONS

3.1 Predictions & Shortfalls

Speakers can determine their communicative intentions easily, for they have direct access to them. A harder question is which of their communicative intentions Hearers can reliably predict from an utterance. I will present some basic predictions that Hearers can make about the communicative intentions of Speakers based on a coordinating utterance form, minimal common ground, and the assertion norms. I will also demonstrate that assertibility is too restrictive for its intended purpose, and will require some modification.

3.1.1 How Assertibility Influences Interpretation

We will switch from Chapter One’s analysis of utterance generation by a speaker to the interpretability of an utterance by a hearer. The assertion norms will still govern which utterances a cooperative speaker should assert, but our use of the formal assertibility systems will change slightly as a hearer only has access to the utterance (and the common ground Π), and not the speaker’s communicative intentions Γ or even the utterance form ϕ. An utterance can potentially have any of several different utterance forms, not only because of structural or semantic ambiguity, but also because of the underlying polysemy of its components, and a hearer must determine which of these forms was most likely intended by a cooperative Speaker. I will minimise the ambiguity in my examples, so that we can focus on determining which form of the coordination is (most) assertible. Each coordination form may deviate from the assertion norms in a different manner, and to a different degree, and so assertibility can no longer be a simple success/fail condition. We will instead seek the maximal degree of assertibility for an utterance by identifying the minimal deviation from the assertion norms that is required for a coordination form to be assertible.

This Hearer-centric measure of assertibility cannot use the Speaker’s intentions as a parameter, but Lemma 2.80.1/2 tells us that if there are any set of intentions for which an utterance is assertible, then it is assertible when the intention is just to communicate its propositional content. So we will check if ⟨Π; ϕ⟩ ⊩ ϕ holds for each candidate utterance form ϕ with its own variant coordination assertibility conditions. Formal descriptions of deviations from assertibility will still continue to use the triadic relation between Π, Γ, and ϕ. The Extensibility norm discussed in §1.4.4 and Lemma 2.103 becomes more important when the Speaker’s intentions are approximated using the utterance form, as the assertibility of the utterance form is affected by any (otherwise unknowable) remaining intentions. The impact of the Extensibility norm, or ‘what has not yet been said’, on interpretation is subtle and complex, and remains an open issue.

Some of the most common and interesting ways of adhering to, and violating, the formal assertion norms will be presented as numbered predictions. The predictions are simple comprehension heuristics derived from assertibility considerations that derive additional information a hearer could infer from an utterance in a given context, on the assumption that the Speaker is both competent and cooperative. They are similar to Grice’s Generalised Conversational Implicata, except that they need not be cancellable or reinforceable, and they are Hearer-centric and so rely only on what a hearer can reasonably infer about a speaker’s intentions, and not their actual intentions. When describing the predictions I will sometimes appeal to an initial ‘default’ or most obvious utterance interpretation and its corresponding default form, which serve a similar role to literal semantic content or sentence meaning, or a notional first pass at comprehension by the Hearer. All the predictions are motivated by consideration of the assertion norms, and have a common structure. Each prediction is given a unique number and name, and describes the conditions under which it may occur, as well as the extra information it conveys. These conditions may describe the utterance form, the context, or how the utterance is performed. When the conditions are met, the Hearer is entitled to infer the additional information listed as the effect of the prediction. I also provide an example utterance which can evoke the prediction in typical contexts, and usually discuss how the assertion norms motivate the prediction, the circumstances under which it may be cancelled, and often some general comments on the prediction. Most of these predictions are calculable. They are also usually more specific, more limited, and less prone to over-generation than Grice’s implicatures and many similar mechanisms.
3.1.2 Formal Interpretation

Suppose a speaker performs an assertoric utterance \( U \) in a shared context \( \Pi \), and a hearer wishes to recover the most reasonable communicative intention of the Speaker. Let the set \( \Phi(U) \) contain the candidate utterance forms for \( U \). These will include the default form(s), being the result of representing every ‘and’ with ‘\( \land \)’, and ‘or’ with ‘\( \lor \)’; standard issues with syntactic or semantic ambiguity may result in multiple default forms for some utterances. The remaining members of \( \Phi(U) \) will be deviations from the default form(s) that I will describe in the rest of this chapter to allow for linguistic, contextual, and cognitive factors that were ignored in Chapter One. Each of these deviations from simple coordination will add candidate utterance forms to \( \Phi(U) \). In addition, we will review some coordination-specific ambiguities that generate additional candidates for \( \Phi(U) \). Syntactic ambiguity can lead to a choice of which phrases are being coordinated. Similarly, semantic ambiguity can occur when it is unclear whether a predicate applies to the coordinands as a group, or each individually, resulting in ambiguous degrees of coordination (see §4.1.1).

Once we have \( \Phi(U) \), we impose a partial order over the candidate utterance forms by preferring \( \varphi \) over \( \psi \) iff \( \varphi \) implies \( \psi \) in context \( \Pi \), by extending Defn 2.105 to include the context \( \Pi \). Note that this preference relation has the traditional orientation (unlike brevity) in that the formula near the open end of the \( \leq \) symbol is preferred, as it has the richer information content. This information content is measured by the assertibility conditions imposed on each utterance form, not simply classical entailment; for example neither \( p \lor q \) nor \( p \oplus q \) imply each other in an empty context. The set of preferred interpretation(s) of an utterance for the context \( \Pi \) is \( I_{\Pi}(U) \). An interpretation of an utterance is preferred in a context iff it is a candidate utterance form in that context, and it is not strictly implied by any other candidate utterance form in the context.

Definition 3.1  
\[ \varphi \leq_{\Pi} \psi \; (\varphi \text{ implies } \psi \text{ in } \Pi) \iff \langle \Pi; \varphi \rangle \vdash \varphi, \text{ and } \forall \Gamma: \langle \Pi; \Gamma \rangle \vdash \varphi \text{ entails } \langle \Pi; \Gamma \rangle \vdash \psi. \]

Definition 3.2  
\[ \varphi \in I_{\Pi}(U) \iff \varphi \in \Phi(U) \text{ and } \not\exists \psi \in \Phi(U): \varphi \leq_{\Pi} \psi. \]

So far, this process is fairly deterministic, as each of the deviations from assertible coordination described later in this chapter will produce a finite number of deviant candidate forms, and the partial ordering of preferences and selection of the dominant preferences is purely algorithmic. Particular contexts will throw up the occasional unusual additional interpretation, and in many of these cases our prediction of the preferred interpretation will be incorrect. But the real underdetermination comes with the addition of three more factors. First, the common ground is not a single set \( \Pi \) of propositions, but a shifting ground of more-or-less obvious or relevant propositions, where what is obvious to the Speaker may not be to the Hearer, and vice versa. Generally, meaning postulates and perceptually salient information will be central examples of the common ground, with implicit aspects of shared culture or education being more peripheral. I will impose a partial order over proposition in \( \Pi \), with propositions that are more likely to be recalled with less effort and which are easier to relate to the utterance content preferred over those more difficult to recall or which are dependent on other propositions to relate to the utterance content. I will also weaken the current requirement for each subutterance to have 100% accuracy and >0% relevance by extending assertibility to use sliding scales of minimal error and maximal relevancy. Both the preference relation and the interpretation set will need to be relativised to these parameters in addition to context. The accuracy, quantity, and relevance of the information conveyed by the preferred interpretation(s) in various contexts might then be traded off against the reliability and effort required to successfully recover that context.

(3.1) Amanda and Steve or Linda will take the red car to the party.

The utterance in (3.1) is ambiguous. Two of its possible utterance forms are \( p \land (q \lor r) \) and \( (p \land q) \lor r \). Of these, the first form implies the second in an empty context. Because this interpretation is more Informative, if the Hearer updates their beliefs in accordance with it, their beliefs will also be consistent with the other interpretation, making this form the preferred interpretation. However, if it is common knowledge that Amanda and Linda actively avoid each other, then \( \neg(p \land r) \) is in \( \Pi \), and \( p \land (q \lor r) \) is no longer assertible, making \( (p \land q) \lor r \) the preferred interpretation. This interpretation process will remain unchanged as non-truth-conditional variant coordinations are added.
3.1.3 Some Example Reinterpretations

I will consider each of the following example utterances, and examine how the unassertibility of the default interpretation prompts the Hearer to test contextual assumptions. With all these utterances with unassertible default forms, it is easy to formally identify the need for an alternative interpretation, but more difficult to provide the correct interpretation. I will shortly provide predictions about the Speaker’s intentions based on the default utterance forms for each expression. They will identify the need for a new interpretation, but underdetermine the resolution. Many of the more specific predictions of later chapters will help to resolve some of this indeterminacy.

(3.2) a. I like fish and chips, or fish with hash browns or chips.
    b. Mary passed Logic 101, or both Mary and Tasha did.
    c. Mary and Tasha and Mary will be at the lake.
    d. I like your shoes, and your hat is a hat.
    e. Boys will be boys.
    f. I’m in New York, but I’m not in New York.

The default form for (3.2a) is $(p \land q) \lor (p \land (r \lor q))$, which is unassertible. One natural context for this utterance is where a menu lists the two disjuncts as distinct dishes, and a possible interpretation in this context is that the Speaker is happy to order either dish. In more generic contexts, as the two major disjuncts are very similar it is probably their differences rather than similarities that are most informative – in this case, the reference to hash browns.

The default form for the slightly odd (3.2b) is the unassertible $p \lor (p \land q)$. More information is conveyed to a hearer than is represented by the form. One possible interpretation is that Mary will pass Logic 101 no matter what Tasha does, while Tasha will only pass with Mary’s help, and this dependency on Mary is part of what is conveyed. Another is that there is an implicit ‘both’ following ‘Tasha’, so the two students are treated as a single unit in the second disjunct, not as two individuals; this might relate to some common knowledge about the outcome if both women pass the course. A third option is that the two instances of ‘Mary’ in the utterance refer to different people; then the resulting form is $p \lor (r \land q)$. Sometimes, as with this example, there is no single satisfactory interpretation without additional context. Whatever your preferred interpretation of (3.2b), the default interpretation appears unacceptable because of the unassertibility of its form.

A similar range of interpretations of the unassertible utterance occurs in (3.2c); for instance the repetition of ‘Mary’ might be a way of emphasising her importance to the Hearer (or Speaker), or the two instances of ‘Mary’ might again refer to different people.

Example (3.2d) cannot be adequately formalised in a propositional language, but can be approximated by the unassertible $p \land \top$, if we assume that trivial identity is a form of tautology. In this example, it is likely that the Speaker has nothing informative and appropriate to say about the Hearer’s hat, and as the preceding comment was complimentary, she has nothing nice to say about the hat. With the right intonation and context however, (3.2d) might be understood as conveying that the hat is proper, appropriate, or at least adequate, perhaps unlike others the Speaker could mention. As with any apparently unassertible utterance, a hearer will search for additional information in the context to allow them to interpret all elements of the utterance as informative.

The utterance in (3.2e) is a tautological utterance, and appears to have little or no informative value, apart from the identification of the referents’ gender (which is presumably already common ground, and thus uninformative). Perhaps the most likely interpretation is that the behaviour being commented on can best be understood as typical of boys. Alternatively, perhaps it provides some insight into boys, or any explanation of the behaviour would require some reference to intrinsically boyish properties. Of course, this expressions is an idiom and may have a conventionalised meaning, but even conventionalised meanings were initially derivable in context.

Finally, (3.2f) appears to have the contradictory form $p \land \neg p$, but can be made assertible by giving the two instances of ‘New York’ different referents, as hinted at by their differing intonation. Perhaps the Speaker is in upstate New York, and not the city, or in Brooklyn and not Central Manhattan, or in a generic hotel room rather than enjoying a specifically New York experience.
3.1.4 Predictions from Unassertible Forms

Here are some of the most general predictions about unassertible coordinations.

**Prediction 3.1 Non-Brevity.**
*Condition:* The utterance form is not brief.
*Effect:* Some additional pragmatic information is being conveyed. Word or phrase emphasis and the relationship between coordinands may provide additional information.
*Example:* See (3.2b)–(3.2f).

This very general prediction only serves as a ‘caution’ sign. The Hearer should be wary and pay close attention. More specific predictions must be identified to provide better information. This prediction appears almost vacuous, but it roughly corresponds to the detection of the potential flouting of a Gricean maxim, the signal for Relevance Theorists that a hearer should engage in additional processing of an utterance, and so forth.

**Prediction 3.2 Redundancy.**
*Condition:* The utterance form is not brief due to a coordinand entailing or contradicting another coordinand. The redundant coordinated phrase(s) usually have marked intonation.
*Effect:* Redundant phrases have increased importance.
*Example:* See (3.2b).

The redundant phrases correspond exactly to those propositions that can be removed while maintaining logical equivalence with the original form. There may be several possible combinations of redundant phrases, in which case the context may assist in disambiguating the most salient combination.

**Prediction 3.3 Repetition.**
*Condition:* There are deliberately repeated phrases, which are often indicated by marked intonation.
*Effect:* Repeated phrases are of greater importance or interest than other phrases.
*Example:* See (3.2c).

This is a specific subcase of Pred 3.2. The nature of the increased importance of repeated items is highly dependent on context. Deliberate repetition almost always uses simple sentence structure, whereas more complex forms of redundancy may be accidental. If a hearer believes the repetition is inadvertent, the entire utterance should be interpreted conservatively, with additional information inferred only when it is necessary to make sense of the utterance. This is because accidental repetition is perhaps the easiest violation of assertibility to detect and avoid, so the Speaker may not be meeting some of our idealised competency assumptions.

**Prediction 3.4 Duplication with Alteration.**
*Condition:* A phrase is repeated with minor alterations. The alterations usually have marked intonation.
*Effect:* The differences between near-synonymous phrases have increased importance.
*Example:* See (3.2a).

The effect of this prediction is almost the opposite of Pred 3.3, in that the novel, non-repeated elements are likely to be more important. The major difference between these cases is that here most of the utterance is repeated, not just a small component. The Speaker’s choice of emphasis can help to disambiguate these cases. The Speaker’s choice of intonation often resolves which prediction is intended in borderline cases.

The utterance form alone is insufficient to uniquely resolve which predictions apply to an utterance. This underdetermination prompts the Hearer to seek out other factors such as intonation and general context to assist in identifying the relevant predictions. The utterance (3.2a) has the form \((p \land q) \lor (p \land (q \lor r))\), and as \(p \land (q \lor r)\) and \((p \land q) \lor (p \land r)\) are both brief reductions (see Defn 2.75) of this form, Pred 3.2 predicts that either the clause represented by \(p \land q\), or just \(p\), has increased salience. On the other hand, the subformulas \(p \land q\) and \(p \land (q \lor r)\) could be seen as repetitions.
with minor alterations, so by Pred 3.4 the clause represented by \( r \) has increased salience. This leads to unresolved tension between contrary predictions. Underdetermination is not the same as pure arbitrariness; these predictions are still a reliable guide to the range of potential interpretations. One useful rule of thumb (which is not so applicable to the conflict between Preds 3.2 and 3.4) is that a prediction with detailed and precise conditions is likely to override a conflicting prediction with more general and vague conditions.

**Prediction 3.5** Inclusion 1.
*Condition:* An irrelevant proposition or term makes the utterance form fail to be brief.
*Effect:* The proposition has some relevance which has not been explicitly mentioned.
*Example:* See (3.2d).

This is another very general prediction that encourages the Hearer to examine the context more carefully to find the required relevance. For example the relevancy of the hat in (3.2d), is context-dependent, as previously discussed. A wide variety of more specific predictions can arise from consideration of how the seemingly irrelevant proposition relates to the context.

**Prediction 3.6** Tautology.
*Condition:* The presence of a logical truth makes the utterance form fail to be brief.
*Effect:* The propositions or terms in the tautology have some relevance.
*Example:* See (3.2d) and (3.2e).

Special attention should be paid to any part of an utterance that clearly violates assertibility. Including a tautological proposition allows other parts of the comprehension process to operate on the components of this tautology to extract more information from the common ground. The trivial identity in (3.2e) with form \((a = a)\) was selected as the ‘right kind’ of tautology is not easily captured in propositional logic; forms like \(p \lor \neg p\) tend to generate the soon-to-be-presented Pred 3.8 instead.

**Prediction 3.7** Conjunction 1.
*Condition:* The form contains an apparently contradictory conjunction (in positive scope).
*Effect:* The conjunction is reinterpreted to avoid the contradiction.
*Example:* See (3.2f).

A contradiction is never assertible, so an alternative interpretation must be sought for any apparent contradiction. Often a polysemous or homonymous conjunct will be reinterpreted to give a different meaning and thus avoid contradiction.

### 3.1.5 Predictions from Assertible Forms

Assertible utterance forms can also convey additional information through the presumption of their assertibility, either in default or unusual contexts. This is similar to those Gricean GCIs that arise through an utterance conforming to the maxims. As with the unassertible forms, I will describe some examples before presenting my formal predictions.

(3.3) a. There is a dairy around the corner.
    b. Jeremy is in China or Germany.
    c. Ralph or Jan will be at the party.
    d. Vicki and Lionel like fish.
    e. You have a real knack for golf.

The utterance in (3.3a) is assertible in any context where the Hearer does not know whether there is a dairy around the corner. However, it is not an appropriate utterance to simply drop into most conversations; the utterance must also be relevant to the topic of the conversation. Formal assertibility has no direct analogue to the conversational topic, and so cannot easily detect when the utterance is irrelevant. On the other hand, if the Speaker is cooperative, the Hearer can assume that the utterance is relevant, and when it appears to be only indirectly relevant, can fill in the gaps. For
example, (3.3a) is usually a relevant reply to “Do you know where I can get some milk?” if it is commonly known that dairies are usually reliable places to get milk. This proposition will sometimes not be relevant in a particular context, perhaps because there is a specific piece of information (such as the dairy being shut currently) that supersedes the default assumptions in this instance. However, the Hearer is normally entitled to infer that the Speaker does know where to get milk, and that one such location is just around the corner. This inference can be formalised as: when \((\Pi; \Gamma) \models \varphi\) is not assertible, but \(\exists \psi : (\Pi; \Gamma, \psi) \models \varphi\) is assertible, then \(\psi\) is conveyed in addition to \(\varphi\) (the inference requires \(\psi\) to be easily reconstructed from the context by the Hearer). This pattern of inference will be implicitly used throughout the remainder of the thesis.

One prediction specific to assertible disjunctive utterances is that disjunctions usually convey additional information about the epistemic state of the Speaker (or at least her chosen epistemic stance). In (3.3b), the Speaker is sure that Jeremy is in one of two countries (China and Germany), but is not sure which of these he is in, or perhaps does not want to reveal this information. This observation is the motivation behind [B1] and 1-assertibility in general. A second, closely related, property of assertible disjunctions is that a disjunct can always be true even when none of the others are, as described in Defn 2.22.3. This stronger property results in a correspondingly stronger prediction. For example, in (3.3c) the Speaker is indicating that Ralph might be at the party without Jan, or Jan without Ralph. While the utterance does not rule out the possibility that Ralph and Jan might both be at the party, the Speaker also does not confirm it. This property will turn out to be the basis for my treatment of ‘exclusive’ disjunction in §4.2.1 and (indirectly) for the paradox of Free Choice in §5.4. The asymmetric disjunctions that I will introduce in §3.4 lack this property.

Assertible conjunctions, like (3.3d) with its utterance form of \(p \land q\), also convey additional information, although this is a little more difficult to spot. The utterance is only assertible in a context where it was possible that Vicki liked fish and Lionel did not, and also possible that Lionel but not Vicki liked fish. If Vicki’s opinion of fish was already common ground, or it was known that they had the same opinion, then the conjunction would not be formally assertible for similar reasons to the disjunctive case above. On the surface, this restriction seems too strong for many uses, and it will be refined later in this chapter. The utterance (3.3e) has a default utterance form \(p\) that is assertible in a generic context. However when this interpretation contradicts the common ground (e.g., the Hearer has just missed the ball several times with errant swings of his golf club), it becomes unassertible in context. In this case, some other interpretation is required. One of the most easily accessible pieces of information in this context is the negation of the false claim; that is, that the Hearer is not good at golf, which is an appropriate commentary on his recent activities. That is, by asserting \(p\) the Speaker can instead convey \(\neg p\), which contradicts rather than simply adding to the original interpretation, and produces something akin to irony or sarcasm.

**Prediction 3.8 Inclusion 2.**

*Condition:* The utterance is assertible, but appears irrelevant to the conversation.

*Effect:* The context contains some information that makes the utterance relevant.

*Example:* See (3.3a).

This and the four following predictions rely not on deviations from the assertion norms, but on presumed adherence to them. The most common result is to evoke some common ground which connects this proposition with the rest of the conversation. However, there are also several subtle ways that a speaker can use this effect when they do not expect the Hearer to identify the connecting common ground. One is to mark a sharp switch in conversational topic: perhaps the Speaker finds the current topic grossly uninteresting or in poor taste, the conversation is becoming heated, or the subject of the discussion has appeared within hearing range. A less polite usage is to indicate that nothing is more relevant than this completely irrelevant proposition, and so communicate that the Speaker has nothing relevant to add; in this instance, the expressed proposition will often be absurd or starkly unrelated. A third use, confined almost exclusively to extended (usually written) utterances is a type of foreshadowing similar to ‘Chekhov’s gun’, where seemingly irrelevant details will later become relevant. All these variations within this broad family of predictions are dependent on assertibility having variable inclusion.
3. DEVIATIONS

Prediction 3.9 Disjunction 1.
Condition: The form contains a disjunction (in positive scope).
Effect: The Speaker is epistemically uncertain, indeterminate, or indifferent between the disjuncts.
Example: See (3.3b) and (3.3c).
This is a central prediction about disjunctions, and is difficult for other predictions to defeat, as each disjunct in an assertible disjunction must have an associated epistemic possibility where it is true. One example where it is overridden is by Pred 4.10, which applies to disjunctions with a final disjunct that is obviously very unlikely or seemingly impossible, such as in the utterance ‘It will rain tomorrow, or I’m a monkey’s uncle’.

Prediction 3.10 Disjunction 2.
Condition: The form contains a disjunction (in positive scope).
Effect: Each disjunct can hold true when none of the others do.
Example: See (3.3c).
For the utterance form to be brief, each disjunct must describe a possibility that none of the others do; otherwise that disjunct can be removed without affecting the truth of the utterance. For example, if the utterance form is \( p \lor q \lor r \), the situation must include possibilities \( \{pq\bar{r}, \bar{p}qr, \bar{p}q\bar{r}\} \) (though these need not be exhaustive). This prediction is much stronger than Pred 3.9, and thus more prone to being defeated; for instance, if a disjunction is sensitive to the order of its disjuncts, this prediction may be defeated. We can use this sensitivity to detect some weaker variants of disjunction.

Prediction 3.11 Conjunction 2.
Condition: The form contains a conjunction (in positive scope).
Effect: Each conjunct uniquely restricts the situation.
Example: See (3.3d).
Each conjunct restricts some possibilities from being part of the epistemic situation; otherwise that conjunct can be removed without affecting the falsity of the negated utterance (see Lemmas 2.13, 2.35, and 2.81.3). This prediction is cancelled when conjunctions are order-sensitive. This means that, like Pred 3.10, its failure can be used to detect some weaker conjunction variants.

Prediction 3.12 Irony/Sarcasm.
Condition: The utterance clearly contradicts the common ground.
Effect: The falsity of the literal is conveyed without being explicitly asserted. The emphatic nature of irony then relies on a strengthening litotes, which will be discussed in §5.2.1.
Example: See (3.3e).
By contradicting an obvious truth, that truth is alluded to, and thus conveyed without being asserted. This prediction relies on literal, rather than merely variable, inclusion. More complex subformulas can take the place of the literal, but the additional structure usually does not contribute to the effect. This condition is not sufficient by itself for either sarcasm or irony, as sarcasm also requires something like intent to wound, while the relevant type of irony may require deliberate ambiguity between opposite meanings, depending on the beliefs of the audience.

The imposition of assertion norms on utterances allows the rejection of many instances of odd or problematic conjunctions and disjunctions, and this detection of unassertibility can be used to infer additional information about the Speaker’s intentions. The twelve predictions listed above describe most of the inferences that can be made by solely considering the standard assertion norms. These predictions are rather generic, and are insufficient reward for the effort expended so far. The reason for this paucity of predictive power is that the assertion norms interact with other, along with non-logical features of coordination, including my maxims of Order and Relevance, and the causal, temporal, or similar relationships between coordinands. Also coordinated utterances often refer to the satisfaction and felicity criteria of other coordinands, so an investigation of performative and meta-linguistic coordinations is required. Once I have amended assertibility to allow for these additional parameters, it will be a much more flexible and powerful predictive tool.
3.1. PREDICTIONS & SHORTFALLS

3.1.6 Assertibility is too Strong

The assertion norms are too restrictive, as they require us to reject or radically reinterpret some seemingly straightforwardly acceptable utterances. This relates to why our predictions from the previous subsection are too generic and underdetermined; in both cases, more parameters will allow us to fit the data better. However, any data set can be fitted by adding sufficient degrees of freedom, so I will limit the number of parameters I use to modify assertibility, and I will also ensure that they are all independently motivated. The following examples include unacceptable, bizarre, and intuitively acceptable utterances, all of which have unassertible default forms. These examples and their suggested utterance forms demonstrate that the formal assertion norms are clearly too strong for modelling the kinds of natural language coordination polysemy, as they filter out many acceptable utterances, along with the undesirables. A useful set of norms and predictions should allow (most of) the intuitively acceptable utterances, and reject the rest. They also include several (not-yet discussed) features of coordination that the assertion norms do not allow for.

In these and future natural language examples, I will use the following linguistic conventions for indicating examples that are less than fully acceptable:

- ‘*’ is unacceptable, ungrammatical or semantically abnormal;
- ‘%’ is acceptable to only some native speakers;
- ‘?’ is odd, unusual, or its acceptability is heavily context-dependent.

(3.4) a. * John or Mary or John ate the cake. \{ p \lor q \lor p \}
   b. ? The boys, or both the boys and the girls, played rugby each lunchtime. \{ p \lor (p \land q) \}
   c. You may have ice-cream or pizza, or both. \{ p \lor q \lor (p \land q) \}
   d. He’s lying, or I’m Mother Teresa. \{ p \lor q, \text{where } \Pi \models \neg q \}

(3.5) a. * John and Mary and John ate the cake. \{ p \land q \land p \}
   b. ? We started moving faster and faster. \{ p \land p \}
   c. Turn left and right, and right again, and either left and right or right and left, and there you are. \{ p \land q \land q \land ((p \land q) \lor (q \land p)) \land r \}

(3.6) a. * If it is raining, it is not raining. \{ p \supset \neg p \}\(^1\)
   b. ? If Jill is sad, then Jill is sad. \{ p \supset p \}
   c. If it rains, it won’t rain hard. \{ p \supset \neg(q \land p) \}
   d. If you are hungry, there are biscuits on the sideboard. \{ p \supset q, \text{where } q \text{ is assertible} \}

The above examples provide a representative, though not complete, selection of the additional features that I wish to allow for. Formalising (3.4c) requires treatments of modality and phrasal coordination, and a special rule for formalising the anaphoric ‘both’ so that it is similar to but not quite synonymous with the associated conjunction (‘ice-cream and pizza’ in this example). A procedure is required to convert the disjunction with a patently false disjunct in (3.4d) into an assertible utterance without simply discarding the disjunct. An asymmetric (non-commutative) variant of conjunction is needed for (3.5c), while (3.6c) and (3.6d) are concessive and performative variants of the conditional respectively. All these deviations from the standard connectives can be predicted by extending our language, including some weaker formal connectives, and enhancing standard assertibility in a variety of more-or-less precise ways. The analysis of conditionals will require some further features, including renunciative negation and denial, the distinction between conjoining propositional contents and illocutionary acts, and a formal method for testing the assertibility and validity of arguments rather than just isolated utterances. I will describe most of these features in the remainder of this chapter. The treatment of modal auxiliaries, arguments, and conditionals are not essential to the primary task of describing coordinations, and will have to wait until §5.3, §5.5.4, and §5.6 respectively.

\(^1\)I will temporarily assume that the material conditional is the appropriate utterance form for conditional utterances. I reject this position in general, and will revisit conditional representation in §5.6.
3.2 Metalanguage & Encapsulation

Defining a metalanguage for assertibility will allow us to abstract away from particular semantics, emphasise symmetries and similarities, juxtapose semantic and pragmatic components, discuss inference procedures, and motivate our choice of connective variants. I will also introduce notation that forbids the application of the Compositional norms on logically complex forms of simple utterances.

3.2.1 A Metalanguage for Assertions

The propositional language we have used so far is sufficient for modelling the assertibility of utterances, but not for discussing the conditions for assertibility; for this we need a metalanguage which can describe the relationship between assertibility and truth. It will also be useful for describing the results of increasing or decreasing the Speaker’s set of epistemic possibilities. Connectives and modal operators with these expressive powers were already introduced in the modal semantics of §2.3, but I will provide equivalent definitions using the semantics-neutral assertion norms. The atoms of this base language operators with these expressive powers were already introduced in the modal semantics of L. There is a negation operator ‘~ϕ’ for ‘ϕ is not assertible’; an epistemic possibility operator ‘◊ϕ’ for ‘ϕ might be assertible with more specific information’; and a subjunctive epistemic possibility operator ‘♦ϕ’ for ‘ϕ might be assertible with less specific information’. The sole dyadic connective is conjunction: ‘ϕ & ψ’ for ‘both ϕ and ψ are assertible’. The operators and connective only apply to metalanguage expressions, and not expressions in L. This new metalanguage will allow us to decompose most of the conditions imposed by the assertion norms on disjunction and conjunction.

Definition 3.3 A Metalanguage for Assertibility.
1. If (II; Γ) ⊨ A_nϕ then (II; Γ) ⊨ ◊_nϕ.
2. If (II; Γ) ⊨ Aϕ then (II; Γ) ⊨ ♦_ϕ.
3. If (II; Γ) ⊨ Tϕ then (II; Γ) ⊨ [PROP(ϕ) ⊆ [PROP(Γ)].
4. If (II; Γ) ⊨ ◊ϕ then (II; Γ) ⊨ ℓϕ.
5. If (II; Γ) ⊨ ϕ & ψ then (II; Γ) ⊨ ϕ and (II; Γ) ⊨ ψ.
6. If (II; Γ) ⊨ ◊ϕ then (II; Γ; Δ ≥ ϕ & PROP(ϕ) ⊆ [PROP(Γ)].

The restrictions in Defn 3.3.6/7 on the variables in ϕ mean that the communicative content of the Speaker may change, but not the subject of the content. This will become crucial in §5.4.2.

Lemma 3.4 Basic Entailments
1. If (II; Γ) ⊨ A_nϕ then (II; Γ) ⊨ ◊_nϕ.
2. If (II; Γ) ⊨ Aϕ then (II; Γ) ⊨ Δϕ.
3. If (II; Γ) ⊨ [PROP(ϕ) ⊆ [PROP(Γ)].
4. If (II; Γ) ⊨ ◊ϕ then (II; Γ; Δ ≥ ϕ & PROP(ϕ) ⊆ [PROP(Γ)].

Proof: By Defn 3.3.

Theorem 3.5 Basic Connective Decomposition
1. If (II; Γ) ⊨ A_nϕ then (II; Γ) ⊨ ◊(A_nϕ & T(ϕ & ψ)) & ◊(A_nϕ & T(ϕ & ψ)).
2. If (II; Γ) ⊨ Aϕ then (II; Γ) ⊨ ♦(A_nϕ & A_nψ), n > 3.
3. If (II; Γ) ⊨ Aϕ then (II; Γ) ⊨ A_nϕ & A_nψ, n ≤ 3.
4. If (II; Γ) ⊨ Aϕ then (II; Γ) ⊨ A_nϕ & A_nψ & ♦(A_nϕ & Tψ) & ♦(A_nϕ & Tϕ), n > 3.

Proof: By Defns 2.54.3/4, 2.57.3/4, 2.60, B.5.3/4, B.9.3/4 and 3.3.

The main purpose of the operators introduced in Defn 3.3 is to allow for the decomposition of disjunction and conjunction in Thm 3.5. The decomposition parallels Defns 2.54.3/4, 2.57.3/4 and 2.63.3/4, but as it occurs outside the language, we are free to pick and choose which conjuncts apply.
3.2.2 Encapsulation

A simple natural language phrase can sometimes encapsulate a concept whose formalisation requires a complex and thus decomposable subformula. Such a subformula should not be decomposed further for the purposes of determining the utterance’s assertibility. Any complex subformula corresponding to a single lexeme in an utterance should be encapsulated, as the lexeme cannot be decomposed or simplified further, and so neither should its formal representation. Encapsulation is represented by placing the irreducible subformula in [square brackets].

It is quite common for some of the conjoined clauses or phrases in a coordination to contain anaphoric terms referring to objects or propositions in earlier or later phrases (forward-reference is also called cataphora, but anaphora is the more general term). These anaphoric terms include ‘both’, ‘one of’, and ‘only’. We will represent this conceptual encapsulation by bracketing the complex subformula representing the anaphoric term, so it is treated as atomic for the purposes of analysing the assertibility of the formula it is part of. For example, the utterance schema ‘ϕ or ψ, or both’ can be represented by \((ϕ ∨ ψ) ∨ [ϕ ∧ ψ]\), while the near-synonymous utterance schema ‘ϕ and ψ, or one of them’ can be represented by the form \(([ϕ ∧ ψ] ∨ [ϕ ∨ ψ])\). The determiners ‘both’ and ‘one of’ correspond to some generalised quantifiers, and indeed, encapsulation can be used for modelling many types of quantification, as a quantified clause cannot be syntactically decomposed into components in the same way as a conjunction or disjunction.

A more complex instance of encapsulation is required to represent ‘only ϕ’, which declares that all propositions ‘appropriately’ related to ϕ are not true. If we suppose that the set Ψ represents the set of propositions appropriately related to ϕ, then ‘only ϕ’ can be represented by \([ϕ] ∨ [ϕ ∧ ¬Ψ]\) \(2\). This conjunction is not decomposable, as we do not assert each conjunct in turn but rather the entire expression as a single unit, in the same way that we might say ‘all the leaves are brown’ or ‘most people are other people’. The above examples of ‘both’ and ‘one of’ fail to fully demonstrate the utility of encapsulation; neither \((ϕ ∨ ψ) ∨ [ϕ ∧ ψ]\), nor \((ϕ ∨ ψ) ∨ [ϕ ∧ ψ]\) is more assertible than the unencapsulated \((ϕ ∨ ψ) ∨ (ϕ ∧ ψ)\) and \((ϕ ∨ ψ) ∨ (ϕ ∧ ψ)\), as in all these formulas at least one disjunct is redundant. This redundancy will be resolved in §3.4, but in the meantime, contrast the form of ‘both ϕ and ψ, or only ϕ’, which (with the presupposition of ϕ made explicit) is the assertible \([ϕ ∧ ψ] ∨ [ϕ ∧ ¬ψ]\), with that of ‘ϕ and ψ, or ϕ and not ψ’, which is the unassertible \((ϕ ∧ ψ) ∨ (ϕ ∧ ¬ψ)\).

The semantic families from the previous chapter can all be extended to include encapsulation. These extensions to the various semantic families preserve the equivalence of their respective underlying semantics from Theorem 2.92, at least up to 4-assertibility. In contribution semantics we simply assign a contribution to the encapsulated subformula rather than its component literals. With concision semantics, we restrict brevity operations from removing, replacing, or rearranging part of an encapsulated subformula, although they may operate on the entire subformula. For relevance semantics, the encapsulated clauses are non-recursive; instead they have a base clause with conditions identical to those of literals. Problems occur however with the semantics for 5-assertibility. We can guarantee literal inclusion by imposing 5-assertibility on the non-encapsulated parts of the formula, as the encapsulated subformulas are anaphoric and so use a subset of the literals from the rest of the formula. But the overall inference is not automatically 5-assertible. Further investigation is required to resolve this open issue. In contribution semantics, for example, the role of encapsulated subformulas in determining the twin rows for standard literals is unclear, as are the twin rows for encapsulated subformulas themselves. When formulas with encapsulation would normally be assessed for 5-assertibility, they will instead be assessed separately against 4-assertibility and literal inclusion.

Encapsulation is a useful technique whenever a formal language has sufficient power to represent the content of a utterance, but cannot represent it in a manner that mirrors the utterance structure. The limitations of propositional logic mean that encapsulation is required to represent any kind of quantification. This technique is also needed for anaphoric references, as well as for generalised quantifiers such as ‘exactly three’ and ‘more than half’, even using the language of predicate logic.

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2 For those wondering why I did not model ‘only ϕ’ as asserting p, it is because ‘not only ϕ’ and ‘only ϕ’ both assume that ϕ is true, which is a reliable sign that ϕ is presupposed rather than asserted. This also explains the acceptability of expressions such as ‘ϕ and only ϕ’.
3.3 Probability & Contrariety

Sometimes Speakers wish to convey more than simple propositional content. Part of the information that we convey is that some epistemic possibilities are more likely than others. In addition, the common ground is often a vague probability distribution over propositions rather than a neat set of propositions. The assertion norms can be adapted so as to model some aspects of probability without needing to extend our propositional language. I will look at three ways this can be done. First I will consider how to convey an approximation of the communicative content by including only the most relevant possibilities, at the cost of introducing some potential for error. Then I will look at the different sorts of inference that can be made by appealing to a probabilistic context. Finally I will discuss the more general approach of modelling the communicative process as a probabilistic update.

3.3.1 Epistemic Probability

A Speaker may consider the epistemic possibilities that they wish to convey as more or less likely relative to each other, and this can influence how information about them is conveyed. I will call the Speaker’s stance on the probability of an epistemic possibility, or a formula representing a set of such possibilities, its weight. The weight of any possibility characterised by a formula $\varphi$ will be represented by $P(\varphi)$, and by abuse of notation, the weight of a set of epistemic possibilities $S_\varphi$ is $P(S_\varphi)$. The weight of a proposition is similar to comparative subjective probability from Carnap (1950). The relative weight of two formulas can only be compared if the formulas involve the same variables, and they stand in some logical relation such as entailment or (restricted) contradiction (e.g., $p \land q$ and $p \land \neg q$ are comparable). Carnap shows that such a probability system can still obey the basic Kolmogorov probability axioms such as Addition, so for example $P(p \land q) \leq P(p) \leq P(p \lor r)$.

Our three basic norms from Defn 1.5 are Expressivity ($\Pi, \Gamma \not\vdash \text{cl} \varphi$), Consistency ($\Pi, \Gamma \not\vdash \bot$), and Informativity ($\Pi \not\vdash \varphi$). As we are initially only imposing weight on the Speaker’s communicative content, it is only $\Gamma$ that is probabilistic, so the Informativity norm will not be affected. Consistency will be strengthened to a Relevance norm, which is a guarantee that the communicative content of each subutterance is relevant to the overall utterance in the context, not just consistent with the context. In turn, Expressivity will be weakened to a probabilistic condition to allow this strengthened Relevance norm. This weakening of Expressivity reflects our practice, as we often choose not to speak of outcomes that we believe to be far-fetched or unlikely, effectively trading off an accurate description of communicative content for a guarantee that the possibilities mentioned are non-trivial. Assertions are usually acceptable if they only cover the subjectively probable cases, and not acceptable if they dwell on the seemingly improbable cases.

(3.7)  

a. I will go to work tomorrow.

b. I will either go to the beach or the cinema tomorrow.

In the atomic utterance (3.7a) the Speaker knows that there are a number of unlikely possibilities that could prevent them from going to work tomorrow, such as illness, accident, death, earthquake, or war. However the probability of any one of these occurring is sufficiently low that it is not relevant (in context), and the combined probability of these exceptions is also sufficiently low that they do not collectively cast enough doubt on the proposition to prevent its assertion. In the disjunctive (3.7b) the probability of any other alternatives occurring is also too low to introduce significant error. In addition, the possibility of going to the beach (and not the cinema) must be relevant, as must the possibility of going to the cinema (and not the beach), for otherwise the respective disjunct would not have been uttered. The threshold for minimal relevance and maximal error are determined by context (e.g., the error threshold might be lower in a nuclear reactor than at a picnic).

I will modify the truth-preserving semantics of Defn 2.20 and the relevance semantics of Defn 2.37 by replacing the requirement that certain sets of epistemic possibilities be empty or non-empty with the requirement that the probabilistic measure over these sets exceeds or fails to exceed particular thresholds. When these relevance and error thresholds are set to zero, the probabilistic semantics will be equivalent to the standard relevance semantics.
The relevance threshold $\delta$ is the value below which a clause's weight or contribution to the utterance is exceeded by its cost (in time to utter, processing effort to comprehend, etc.), so that it is not worth uttering. A simple or expected clause may have a lower 'cost' than complex and surprising clauses, and the cost of a clause will vary with the context, so $\delta$ is a highly variable threshold value. The error threshold $\epsilon$ for an utterance is similarly context-dependent, as the consequences of inaccuracy can vary wildly. An utterance whose probability of error or inaccuracy exceeds $\epsilon$ varies too much from the Speaker's communicative content to reliably convey the content.

I will adapt the relevance semantics from §2.2 by strengthening the requirement for Relevance from possibility (non-emptiness) to contextually non-trivial possibility, and by weakening the requirement for Expressivity from classical entailment by allowing highly improbable counter-examples. Interpretations that roughly approximate the Speaker's communicative intent with some minor errors or omissions may then be preferred to ones where every minute detail is specified. The preferred interpretations that roughly approximate the Speaker's communicative intent with some minor errors or omissions may then be preferred to ones where every minute detail is specified. The preferred interpretations that roughly approximate the Speaker's communicative intent with some minor errors or omissions may then be preferred to ones where every minute detail is specified.

Defn 3.6 provides probabilistic Expressivity conditions by evaluating the weight of the possible error interpretations of an utterance (with respect to probability) minimise its overall error while maximising the relevance of its least relevant subutterances. These requirements are, of course, in direct conflict. Defn 3.6 provides probabilistic Expressivity conditions by evaluating the weight of the possible error in the content of an assertion against the weight of the propositions in the Speaker's communicative intention. Defn 3.7 extends $\delta$-assertibility to provide probabilistic relevance conditions, by evaluating the weight of each proposition in a complex assertion against the Speaker's communicative intention for that proposition in the context of the overall assertion.

**Definition 3.6** $S(\delta)\models_{CL} \theta$ is as per Defn 2.20 for $S\models_{CL} \theta$ except:
1. $S(\delta)\models_{CL} l$ if $P(S \setminus S_l)/P(S) \leq \epsilon$, where $l$ is a literal.

**Definition 3.7** $(S, T, U)(\delta)\models_{R5} \theta$ is as per Defn 2.37 for $(S, T, U)\models_{R5} \theta$ except:
1. $(S, T, U)(\delta)\models_{R5} l$ if $P(S \cap U \cap (S-\theta)_{\pm1} \cap S_l)/P(S) > \delta$, and $P(T \cap U \cap (S-\theta)_{\pm1} \cap S_l)/P(S) > \delta$.

The definitions show that $\epsilon > 0$ weakens the underlying classical logic in Defn 3.6, while $\delta > 0$ strengthens the R5 relevance conditions in Defn 3.7. Defns 3.6 and 3.7 are identical to Defns 2.20 and 2.37 for CL and R5 respectively when $\epsilon = \delta = 0$. To take a non-probabilistic context into account, simply use Defn 2.96 instead of Defn 2.38 when determining the initial value of $U$. The above definition of probabilistic relevance for the base clause of an utterance form may look unduly complex, but (apart from the appeal to $(S-\theta)_{\pm1}$) it is straightforward to use.

### 3.3.2 Contrariety

The communicated content of an assertion includes all the reliable consequences that a cooperative Hearer can reasonably be expected to draw by combining information in the context with the utterance content. The common ground includes knowledge of relative frequencies, correlations, and implications, many of which can be modelled using probability. The weighting of propositions in the common ground is similar to that of propositions in the Speaker's communicative content. However, the common ground is intersubjective and is equally available to both the Speaker and Hearer. We will model this probabilistic context as a probability function $\pi()$ that returns the relative weighting of some or all combinations of the relevant propositional variables, rather than a set $\Pi$ of formulas describing the common ground. A non-probabilistic $\Pi$ can be modelled by defining $\pi()$ as the partial function where for each formula $\varphi \in L$, $\pi(\varphi) = 1$ if $\Pi \models_{CL} \varphi$, $\pi(\varphi) = 0$ if $\Pi \not\models_{CL} \varphi$, and $\pi(\varphi)$ is undefined otherwise. Many individual epistemic possibilities will not have defined weights, but they will all belong to a set of epistemic possibilities with a collective weight. As with the subjective $P()$, the values of $\pi()$ are comparative, and we will mainly be comparing them with either $\delta$ to test for relevance, or $\epsilon$ to test for a significant chance of error. The Kolmogorov axioms will still hold, so for example $\pi(p \land q) \leq \pi(p) \leq \pi(p \land r)$, and in general $\pi(\varphi \lor \psi) + \pi(\varphi \land \psi) = \pi(\varphi) + \pi(\psi)$.

Suppose that an utterance form is $\varphi$, and part of the common ground is an expectation that whenever $\varphi$ holds, so does $\psi$; that is, $\pi(\varphi \rightarrow \psi)$ is high. Then it may seem reasonable to draw the conclusion that a cooperative Speaker's communicative intentions should include $\psi$. A cooperative
Hearer will not always be so bold, as they want to avoid drawing erroneous conclusions. If \(\pi(\varphi \rightarrow \psi) = 1\) then one can reliably draw the conclusion \(\psi\). If \(\pi(\varphi \rightarrow \psi) < 1\), then other inferences and predictions could result in additional information \(\chi\) where \(\pi((\varphi \land \chi) \rightarrow \psi)\) is much lower, making \(\psi\) an unreliable conclusion to draw. This cancellation of the inference to \(\psi\) can also occur based on the contents of subsequent assertions. Furthermore, modelling the common ground as intersubjective is an idealisation, and the Speaker and Hearer’s personal biases and different appreciations of the current context can lead to significantly different estimates of \(\pi(\varphi \rightarrow \psi)\) when some of the information is uncertain. For all these reasons, we will divide context-based inferences to \(\psi\) into those which are unquestionably reliable so \(\pi(\varphi \land \neg \psi) = 0\), and those where \(\pi(\varphi \land \neg \psi)\) is low but non-zero. The information that is reliable beyond question usually includes: logical laws; lexical meaning postulates and taxonomies; and the (assumed) physical and moral laws of the world. Other information in the common ground may be less absolute, either through being probabilistic or applicable in limited domains, such as culturally variable rules of politeness.

(3.8) a. Sharon drank one or two cups of tea.

b. Sharon drank from a cup which contained tea or coffee.

Both the examples in (3.8) exhibit some form of exclusive disjunction. In each case, suppose \(\varphi\) represents the proposition that both disjuncts are true, and \(\pi(\varphi)\) is the intersubjective weight of both disjuncts being true in the context, given that the disjunction is true. When \(\pi(\varphi) = 0\) I will call \(\varphi\) \(\ell\)-contrary. In (3.8a) the disjuncts are lexically contrary, as it is impossible to drink both exactly one and exactly two cups of tea; they cannot both be true. This effect occurs whenever the disjuncts are members of the same lexical hierarchy (e.g., ‘triangle or square’). A similar case occurs when a violation of natural laws is necessary for both disjuncts to be true (e.g., being in two places at once). Less frequently, a logical contradiction might be required for both disjuncts to hold (e.g., she is, and is not, undecided). In all these cases, \(\pi(\varphi) = 0\) and so it is in some sense irrational to contemplate the conjunction of the two contraries.

A proposition \(\varphi\) is \(c\)-contrary, or contrary to the common ground, if it is consistent but not sufficiently weighty to be worthy of consideration due to custom, convention, or context. In this case, expectation rather than pure reason is the basis for rejecting \(\varphi\), and \(0 < \pi(\varphi) < \epsilon\). In (3.8b) the disjuncts are contextually contrary, as (except in Hong Kong) it is not customary to mix tea and coffee in the same cup. The threshold \(\epsilon\) used with \(c\)-contrariety represents the Hearer’s judgement of the Speaker’s willingness to be non-conventional, counter-intuitive, or outré. This is a different threshold from the \(\epsilon\) used by the Speaker when determining whether their utterance accuracy reflects their intentions, but fulfils a parallel role. Unlike \(\ell\)-contrariety, a subsequent utterance can reinforce or cancel the Hearer’s \(c\)-contrariety-induced expectation (e.g., ‘but obviously not both’; ‘in fact, it contained both’; ‘she didn’t wash up between drinks’) without leading to Inconsistency. These characteristic properties of reinforcement and cancellation make \(c\)-contrariety a fertile ground for Grice’s conversational implicatures. I will not distinguish implicatures from other predictions, although they can be identified by considering the type of contrariety that is intended.

### 3.3.3 Probabilistic Update

A more radical shift in our model of communication is to treat an assertion as an update function from the old common ground \(\Pi\) to a new common ground \(\Pi_1\). This results in little change to the systems of Chapters One and Two when the common ground is non-probabilistic. The only change to the norms of §1.4.3 is that \(\Gamma\) is simply \(\Pi_1\), and we tighten the Consistency norm to \(\Pi_1 \vdash_{CL} \Pi\) to ensure that information is not lost.

When \(\Pi_0\) and \(\Pi_1\) are probabilistic, the propositional utterance form \(\varphi\) can only approximate the information required to transform one probability function into another, and a new form of error evaluation must be introduced. The error from §3.3.1 only measures the chance of an utterance being wrong, and does not allow for the utterance providing the wrong information about the relative probability of viable epistemic possibilities. Modelling this erroneous communication of relative weighting requires models of probabilistic belief revision, which is beyond the scope of this thesis.
3.4 Order & Asymmetry

The ordering of coordinands can be a rich source of information about an utterance. Some weaker variants of the boolean connectives arise naturally by imposing an order of evaluation on the subformulas. I will remove the requirement for symmetric assertibility conditions for coordinands, which will result in several connective variations and their concomitant predictions. This leads to a loss of commutativity, and weakens the Disjunctive and Conjunctive Compositionality norms of Defn 1.7.

\[(3.9)\]

a. Jessica took off her shoes and [then] jumped in the pool.
b. Jessica jumped in the pool and [then] took off her shoes.
c. Valerie will win the Olympic shotput competition, or [at least] get a medal.
d. Valerie will get a medal, or [even] win the Olympic shotput competition.

Many natural language coordinations are asymmetric or order-sensitive, as can be seen by considering the differences between (3.9a) and (3.9b). In these examples the conjunct order seems to convey additional temporal information, as it appears we evaluate the first conjunct without reference to the second conjunct, and then the second conjunct in the context of the first having just occurred. Conjunct order often relates to a specifically temporal or causal relationship; these relationships will be explored in the next section. The effects of ordering disjuncts are more subtle, as can be seen by comparing the very similar (3.9c) and (3.9d); the effects will be discussed in considerable detail in §4.2.2. These cases of natural language coordination are generally acceptable, despite one disjunct entailing the other in each of these examples and so causing the coordinations to fail the assertion norms. I will introduce weaker variants of conjunction and disjunction that allow one or even both coordinands to be evaluated without the other being part of the context, and so can model the behaviour of the examples in (3.9). A detailed justification of why I have selected these particular variants can be found in Appendix A.3.

**Definition 3.8 Asymmetric Compound Utterance Forms**

1. \(\langle \Pi; \Gamma \rangle \models \varphi \nleftrightarrow \psi \text{ iff } \langle \Pi; \Gamma \rangle \models \varphi \text{ and } \langle \Pi; \neg \varphi; \Gamma \rangle \models \psi.\)
2. \(\langle \Pi; \Gamma \rangle \models \varphi \nleftrightarrow \psi \text{ iff } \langle \Pi; \neg \psi; \Gamma \rangle \models \varphi \text{ and } \langle \Pi; \Gamma \rangle \models \psi.\)
3. \(\langle \Pi; \Gamma \rangle \models \varphi \rightarrow \psi \text{ iff } \varphi \text{ and } \langle \Pi; \Gamma \rangle \models \psi.\)
4. \(\langle \Pi; \Gamma \rangle \models \varphi \leftrightarrow \psi \text{ iff } \langle \Pi; \Gamma \rangle \models \varphi, \langle \Pi; \varphi; \Gamma \rangle \models \psi, \text{ and } \Pi, \Gamma \Vdash \varphi \land \psi.\)
5. \(\langle \Pi; \Gamma \rangle \models \varphi \leftrightarrow \psi \text{ iff } \langle \Pi; \psi; \Gamma \rangle \models \varphi, \langle \Pi; \Gamma \rangle \models \psi, \text{ and } \Pi, \Gamma \Vdash \varphi \land \psi.\)
6. \(\langle \Pi; \Gamma \rangle \models \varphi \leftrightarrow \psi \text{ iff } \varphi \text{ and } \Pi, \Gamma \Vdash \varphi \land \psi.\)

These deviant asymmetric connectives impose weaker conditions on the epistemic possibilities than the standard connectives. The assertibility claim \(A(\varphi \nleftrightarrow \psi)\), for example, only requires that \(A\varphi\) is epistemically possible, rather than the more restrictive \(A\varphi \land T(\neg \psi)\). Theorem 3.5 described some necessary modal conditions for the assertibility of disjunction and conjunction, which are restated in Lemma 3.9 below on the unnumbered lines. The modal conditions for each of the asymmetric connectives are similar but weaker, as indicated by whitespace in the Lemma. The predictions that these variations generate for natural language disjunction are detailed in §4.2.2.

**Lemma 3.9 Asymmetric Connectives**

If \(\langle \Pi; \Gamma \rangle \models A(\varphi \lor \psi)\) then \(\langle \Pi; \Gamma \rangle \models T(\varphi \lor \psi) \land \Diamond (A\varphi \land T(\neg \psi)) \land \Diamond (A\psi \land T(\neg \varphi)).\)

1. If \(\langle \Pi; \Gamma \rangle \models A(\varphi \rightarrow \psi)\) then \(\langle \Pi; \Gamma \rangle \models T(\varphi \lor \psi) \land \Diamond A\varphi \land \Diamond (A\psi \land T(\neg \varphi)).\)
2. If \(\langle \Pi; \Gamma \rangle \models A(\varphi \leftrightarrow \psi)\) then \(\langle \Pi; \Gamma \rangle \models T(\varphi \lor \psi) \land \Diamond (A\varphi \land T(\neg \psi)) \land \Diamond A\psi.\)
3. If \(\langle \Pi; \Gamma \rangle \models A(\varphi \nleftrightarrow \psi)\) then \(\langle \Pi; \Gamma \rangle \models T(\varphi \lor \psi) \land \Diamond A\varphi \land \Diamond A\psi.\)
4. If \(\langle \Pi; \Gamma \rangle \models A(\varphi \nleftrightarrow \psi)\) then \(\langle \Pi; \Gamma \rangle \models A\varphi \land A\psi \land \Diamond (A(\neg \varphi) \land T\psi) \land \Diamond (A(\neg \psi) \land T\varphi).\)
5. If \(\langle \Pi; \Gamma \rangle \models A(\varphi \nleftrightarrow \psi)\) then \(\langle \Pi; \Gamma \rangle \models A\varphi \land A\psi \land \Diamond (A(\neg \varphi) \land T\psi) \land \Diamond (A(\neg \psi) \land T\varphi).\)
6. If \(\langle \Pi; \Gamma \rangle \models A(\varphi \nleftrightarrow \psi)\) then \(\langle \Pi; \Gamma \rangle \models A\varphi \land A\psi \land \Diamond (A(\neg \varphi) \land T\psi) \land \Diamond (A(\neg \psi) \land T\varphi).\)

**Proof:** By Theorem 3.5.1/4, Defns 3.3 and 3.8, and Theorem 2.102. ■
Lemma 3.10 Asymmetric Connective Relationships

1. \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) and \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) if \( \langle \Pi; \Gamma \rangle \models A(\varphi \lor \psi) \).

2. \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) and \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) if \( \langle \Pi; \Gamma \rangle \models A(\varphi \land \psi) \).

3. If \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) or \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) then \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \).

4. If \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) or \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) then \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \).

5. \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) if \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \).

6. \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) if \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \).

7. \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) if \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \).

8. If \( \langle \Pi; \Gamma \rangle \models A(\varphi \rightleftharpoons \psi) \) then \( \langle \Pi; \Gamma \rangle \models A \varphi \land A \psi \).

Proof: By Defs 3.3 and 3.8.

The deviant asymmetric connectives are all weaker than the standard connectives; their entailment relationships are described in Lemma 3.10. An assertible utterance form that uses a weaker connective will be considered to have a lower degree of assertibility than one that uses a stronger connective, as the rules have been ‘bent’ to allow it. The upshot is that if you do not need to interpret a connective as asymmetric, you should not. If a connective has to be asymmetric for an utterance form to be assertible, then additional information can be predicted about the contents of the utterance. For instance, a redundant disjunct is usually particularly salient in some way, as its information content does not justify its inclusion.

The concision semantics for asymmetric connectives are complicated by the absence of commutativity and associativity, although Lemma 3.10.5–7 partially describe how the De Morgan laws must be extended. The \([B1],[B3](b),[B4] \), and \([B5] \) operations vary for each type of asymmetric connective, and so have been omitted. The relevance semantics for asymmetric variants of the connectives are straightforward, and given their close relationship with the formal assertion norms, they effectively duplicate Defn 3.8. The licensing semantics give the most informative presentation, with each asymmetric connective having a subtly different compositional contribution table:

Lemma 3.11 Asymmetric Contribution

The contribution tables for the asymmetric variants of disjunction and conjunction are:

<table>
<thead>
<tr>
<th>( \uparrow )</th>
<th>1</th>
<th>0</th>
<th>( \uparrow )</th>
<th>1</th>
<th>0</th>
<th>( \uparrow )</th>
<th>1</th>
<th>0</th>
<th>( \uparrow )</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (_l)</td>
<td>1 (_l)</td>
<td>1 (_l)</td>
<td>1 (_l)</td>
<td>1 (_l)</td>
<td>1 (_l)</td>
<td>1 (_l)</td>
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<td>1 (_l)</td>
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<td>1 (_l)</td>
<td>1 (_l)</td>
</tr>
<tr>
<td>0 (_l)</td>
<td>1 (_l)</td>
<td>0 (_l)</td>
<td>1 (_l)</td>
<td>0 (_l)</td>
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</tr>
<tr>
<td>0 (_l)</td>
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<td>1 (_l)</td>
<td>0 (_l)</td>
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<td>1 (_l)</td>
<td>0 (_l)</td>
<td>0 (_l)</td>
<td>0 (_l)</td>
<td>1 (_l)</td>
<td>1 (_l)</td>
</tr>
</tbody>
</table>

Utterances with forms that are ‘nearly assertible’, such as (3.9c) which has the form \( p \lor [\neg p \land q] \), can be made assertible by substituting in a weaker connective; in this case ‘\( \uparrow \)’ for ‘\( \lor \)’. This minor deviation to produce assertibility generates a prediction that (3.9c) should be interpreted as the asymmetric coordination ‘Valerie will win the Olympic shotput or if she does not, she will at least get a medal’. Similarly, (3.9d) is assertible only with the asymmetric form \([p \land \neg q] \rightleftharpoons q\), leading to a similar prediction about its interpretation. One simple method for testing asymmetry is to change the order of the coordinands and check for synonymy, but this does not help identify the direction of asymmetry. An indication that asymmetry is not simply a formal property of the system is that it is often lexicalised by coordinating phrases such as the phrases ‘or at least’ and ‘or even’ used above, and these phrases can be used to test for asymmetry. Some other useful asymmetry-testing phrases are ‘or else’ for ‘\( \uparrow \)’; ‘and then’, ‘and so’, or ‘before’ (depending on whether the order is causal, temporal, inferential, etc.) for ‘\( \uparrow \)’; and ‘after’ for ‘\( \uparrow \)’. The lack of an English adverbial modifier to ‘and’ that paraphrases ‘\( \uparrow \)’ conjunction provides more indirect support for its relative rarity.

The asymmetric deviant connectives also help with a problem raised in §3.2.2: how to model anaphoric determiners such as ‘both’ and ‘one of them’ so that the underlying utterances are assertible. Anaphora requires one clause to reference another, and so is intrinsically asymmetric. For example, the utterance schema ‘\( \varphi \) or \( \psi \)’ or both’ can be represented by the asymmetric assertible form \( (\varphi \lor \psi) \rightleftharpoons [\varphi \land \psi] \), while the similar schema ‘\( \varphi \) and \( \psi \), or one of them’ can be represented by the asymmetric assertible form \( (\varphi \land \psi) \rightleftharpoons [\varphi \lor \psi] \) (if \( \varphi \lor \psi \) and \( \varphi \land \psi \) are assertible, respectively).
3.5 Phrasal Coordination

The next extension to the assertibility norms is to add a systematic analysis of phrasal coordinations, as they are only defined for clausal coordinations. This will demonstrate that the underlying assertibility conditions for phrasal coordination are identical to those of clausal coordination, so our modelling of clausal coordination can be extended seamlessly to phrasal coordination. The demonstration will require some interaction with the expression’s syntax, particularly when determining which level of phrase structure should be coordinated. This will require a small detour into (linguistic) formal semantics. Independent clauses (inflection phrases, or IPs) of a sentence can usually be assigned a truth value, while other phrases cannot, so phrasal (non-clausal) coordination cannot be a simple function of the truth values of the phrases. Some phrasal coordinations are synonymous with the clausal coordinations from which they can be derived via moving the coordinator and deleting duplicate phrases, but most are persistently phrasal, meaning they cannot be derived from a synonymous clausal coordination. These phrasal coordinations are often considered counter-examples to the claim that all ‘and’ and ‘or’ coordination is truth-functional. To model phrasal coordination I will appeal to a formal system of compositional semantics that evaluates the truth-functional content of phrases, and show that there is a polymorphic function that combines the content of the coordinands appropriately for their phrase type. Finally, the initial restriction to truth-functional contents will be lifted, and a polymorphic function that includes the full assertibility constraints will be sketched.

There are several alternative compositional semantics theories accepted by different schools of functional-typographical or formal semanticists. I will use Heim and Kratzer’s particularly tractable take on type theory\(^3\). I will sketch the truth-functional semantics for a range of phrase types, while abstracting away from the details of the phrase structure trees and movement rules Heim and Kratzer use to generate and cross-check their proposal. There are two principles in §3.5.1 that are important for our project. First, each phrase type is assigned a lexical type that determines its treatment of coordination. Second, the nesting of types along with their interaction with the syntactic movement and deletion rules can explain the apparent non-truth-functionality of phrasal coordination. After this, we can drop the semantic-pragmatic distinction again, and extend the definition to all assertibility conditions for coordinating some of the main lexical types, by replacing the truth-functions with the more complex recursive expressions from Chapter Two.

3.5.1 Semantic Phrasal Coordination

Heim and Kratzer assign lexical types to each phrase type as part of their semantic evaluation. Sentences (and independent clauses) are assigned $t$ (for truth), meaning that semantic evaluation will always assign a truth value. Notionally, a linguistic expression designating a unique entity or object would be assigned $e$ (for entity), but I’ll be using the more general type $\langle \langle e, t \rangle, t \rangle$ for all determiner phrases (DPs) instead of following Heim and Kratzer’s position that any Name designates an object, and so is of type $e$. This change is primarily for technical simplicity\(^4\). Phrase types other than IPs cannot be evaluated in isolation, as they are mutually dependent on the evaluation of other phrases. For instance a noun phrase is assigned $\langle e, t \rangle$, a function that returns $true$ or $false$ ($t$) when it is applied to an entity ($e$). For example the function assigned to ‘cat’ returns $true$ when it is applied to a cat, and $false$ otherwise. The type $\langle e, t \rangle$ can be understood as a function from entities to truth values, or as a set of entities with that characteristic function. Heim and Kratzer say in footnote

\(^3\)Heim and Kratzer (1998) present a theory that represents conventional wisdom on type theories for formal generative semanticists and logicians. Heim and Kratzer belong to the transformational grammar tradition, and the high-level overview that I give is compatible with both the Principles & Parameters and Minimalist programmes. Other approaches such as HPSG and LFG still use phrase-structure trees, with minor differences such as NPs commanding DPs rather than the other way round. The semantic phrase types of lambda-calculus also have direct correlates in most alternative semantic theories. By deliberately keeping to a high-level and abstract explanation, I hope that the underlying approach taken in this section will be compatible with most current formal theories.

\(^4\)If we treat names like any other determiner phrase, they all have the same semantic type, so the conjunction in ‘Sue and her two friends arrived’ behaves like all other conjunctions. We also reduce ad hoc polymorphism in the system, leave type $e$ to apply to all and only DP traces, and ensure that every phrase type can be interpreted as a function returning a truth value.
2 of p.173, “we will indulge in a lot of set talk that you should understand as a sloppy substitute for the function talk that we would need to use to be fully accurate”. I will continue to indulge in this behaviour, although talk of composition only applies to functions, and not sets. Most of the complexity of their system relates to how phrases of different lexical types combine to provide the information for a third type of phrase to be evaluated, and what phrase-tree movement is required to ensure that the lexical types match. I will ignore most of this complexity, as we will only be concerned with how coordination combines phrases of the same type compositionally to produce another phrase of the same type. I will also disregard prepositions, adverbs, and pronouns, as these phrase types undergo coordination in the same way as the phrase types listed below, but other aspects of their semantics are more complex and this may distract from the task at hand. I will also avoid the open problems surrounding anaphora resolution by ignoring pronouns and similar anaphoric terms.

**Definition 3.12** Incomplete Table of Phrases and Phrase Types

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Abbrev</th>
<th>Example</th>
<th>Lexical Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sentence</td>
<td>IP</td>
<td>The cat sat</td>
<td>((e,t))</td>
</tr>
<tr>
<td>Determiner Phrase</td>
<td>DP</td>
<td>Every black cat</td>
<td>((e,t), (e,t), t)</td>
</tr>
<tr>
<td>Determiner</td>
<td>Det</td>
<td>Every</td>
<td>((e,t), (e,t), t)</td>
</tr>
<tr>
<td>Name</td>
<td>Name</td>
<td>John</td>
<td>((e,t), t)</td>
</tr>
<tr>
<td>Noun Phrase</td>
<td>NP</td>
<td>black cat</td>
<td>((e,t))</td>
</tr>
<tr>
<td>Noun</td>
<td>N</td>
<td>cat</td>
<td>((e,t))</td>
</tr>
<tr>
<td>Adjective</td>
<td>Adj</td>
<td>black</td>
<td>((e,t), (e,t))</td>
</tr>
<tr>
<td>Adjective Phrase</td>
<td>AdjP</td>
<td>big black</td>
<td>((e,t), (e,t))</td>
</tr>
<tr>
<td>Verb Phrase</td>
<td>VP</td>
<td>caught the ball</td>
<td>((e,t))</td>
</tr>
<tr>
<td>Verb</td>
<td>V</td>
<td>caught</td>
<td>((e,t))</td>
</tr>
</tbody>
</table>

I will decompose the following two simple sentences and reassemble their truth values from the lexical types of their phrases to demonstrate how the phrase types interact to produce a truth value for a sentence, and why this complicates coordination so much. In (3.10a), ‘Skye’ and ‘Sam’ are Names (DPs), with type \((e,t), t\), and ‘kiss’ is a Verb of type \((e,t)\). In addition, Heim and Kratzer postulate that there is a trace\(^6\) of type \(e\) preceding the verb where the direct object ‘Sam’ was originally located. The tense inflection ‘-ed’ makes no difference to the lexical type, so can be ignored. The bracketed expression in (3.10b) is a linear representation of the relevant layers of the phrase-tree for (3.10a).

(3.10)  

a. Skye kissed Sam.

b. \([|Skye|DP[-ed]infl[[]]trace[[kiss][V\omega [Sam]DP[V']VP]infl V']IP\].

c. Skye and Melisande kissed Sam.

The DP ‘Sam’ of type \((e,t), t\) takes the Verb ‘kiss’ of type \((e,t)\) as a parameter, returning a function of type \(t\) as a notional \(V\'). This combines with the trace of type \(e\) to produce the VP ‘kiss Sam’ of type \((e,t)\), and then the inflected VP ‘kissed Sam’ which is also of type \((e,t)\). Finally the DP ‘Skye’ of type \((e,t), t\) takes the inflected VP ‘kissed Sam’ of type \((e,t)\) as a parameter and returns a function of type \(t\) for the overall IP ‘Skye kissed Sam’. The same systematic composition of lexical types by walking the phrasal structure can be applied to most English sentences (or at least their deep structure analogues) with remarkably few special cases or rules for exceptions. In (3.10c), the coordination ‘and’ must take two parameters of type \((e,t), t\) from ‘Skye’ and ‘Melisande’ and return another of the same type for the DP ‘Skye and Melisande’. In other cases, it will take two parameters of type \(t\), or \((e,t)\), etc., but always return a value of the same lexical type as its parameters. I will need to define distinct coordination functions for each lexical type to account for the different parameters, while looking for commonalities to explain the underlying polysemy.

\(^5\) Heim and Kratzer assign the lexical type \((e,t)\) to intransitive, transitive and ditransitive verbs alike, and movement rules resolve the issues of multiple arguments. This is an advantage of Heim and Kratzer’s approach, as its nearest competitors have to assign some verbs a lexical type of \((e, (e,t))\) or \((e, (e, (e,t)))\).

\(^6\) A trace is null element in an expression that corresponds to a position in the phrase-tree which may have held a non-null phrase before or during phrase-tree movement.
The semantic content of a phrase $A$ is represented by $[[A]]$. As an example of semantic content, $[[\text{cat}]]$ can be thought of as a function that returns true whenever a cat is supplied as its parameter, or equivalently as the set of all cats. I will define the operation of the coordinators ‘and’ and ‘or’ by introducing $[[\text{and}]]$ and $[[\text{or}]]$. These functions will be polymorphic (take parameters of different types), but the type of polymorphism is crucial. It is important to have (bounded) parametric polymorphism that ignores the parameter function at higher levels of abstraction, and so avoid ad-hoc polymorphic functions which can vary arbitrarily depending on the parameter types. I will define coordination using lambda calculus, and represent the set of functions whose domain is the lexical type $\sigma$ by $D_\sigma$, and a generic phrase type by $XP$.

Definition 3.13 Phrasal Coordination

$$[[\text{and}]]_{XP} = \lambda f \in D_{(\sigma,t)}, \lambda g \in D_{(\sigma,t)}, \lambda x \in D_\sigma, f(x) \land g(x).$$

$$[[\text{or}]]_{XP} = \lambda f \in D_{(\sigma,t)}, \lambda g \in D_{(\sigma,t)}, \lambda x \in D_\sigma, f(x) \lor g(x).$$

where the coordinated phrases are all $XP$ phrases, which are of type $\langle \sigma, t \rangle$.

Independent Clauses and Sentences are assigned type $t$, so they are associated with a parameter-free truth value. Clause coordination is a special case of our general formulation of $[[\text{and}]]$ and $[[\text{or}]]$. In this case the parameter $\sigma$ is null, so the only parameter is a truth value, and the coordination functions are effectively reduced to the traditional truth-functional ‘and’ and ‘or’.

$$[[A \text{ and } B]]_{IP} = A \land B.$$

Nouns, Verbs, Noun Phrases, and Verb Phrases are assigned type $\langle e, t \rangle$ and associated with a function that returns a truth value when given an entity as a parameter; this function can also be understood as a set of entities. The first coordinand must also satisfy any grammatical agreement requirements, and (according to Johannessen (1998)) the remaining coordinand, if they differ from the first, “will have the default case of the language, and... will have the most general tense-aspect”. For instance, the subject-verb agreement in English means there must be number agreement between the first NP and the first VP (as per English’s head-order parameter), but NPs lack case and tense.

$$[[A \text{ and } B]]_{XP} = \lambda x \in D_e, A(x) \land B(x) = [[A]]_{XP} \cap [[B]]_{XP}.$$

Adjectives and Adjective Phrases are assigned type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$, and so modify a predicate to produce another predicate, or equivalently return a truth value (via Schönfinkelization/Currying) when supplied with a predicate and an entity. The context often supplies the predicate parameter $\lambda x$ in the formula below) when an adjective phrase is combined with a copula to form a VP. For example, to say ‘Sam is short’ is to say that ‘Sam is short compared to standard $\Psi$’, where $\Psi$ is determined by context, such as whether giraffes are currently salient.

$$[[A \text{ and } B]]_{AdjP} = \lambda x \in D_{(e,t)}, \lambda y \in D_e, A(x, y) \land B(x, y).$$

DPs (including Names and Pronouns) are assigned type $\langle \langle e, t \rangle, t \rangle$. That is, a DP returns a truth value when given a predicate as a parameter. For example, consider ‘Some cats and most dogs howl’. The conjunction returns true iff the set $[[\text{howl}]]$ is a superset of one of the sets in $[[\text{some cats}]]$ and one of those in $[[\text{most dogs}]]$, so contains a member of $[[\text{some cats}]] \cup [[\text{most dogs}]]$. There are further constraints on conjoining determiner phrases, such as the determiners being of the same monotonicity class.

$$[[A \text{ and } B]]_{DP} = \lambda x \in D_{(e, t)}, A(x) \land B(x) = [[A]]_{DP} \cap [[B]]_{DP}.$$

Determiners are assigned type $\langle \langle e, t \rangle, \langle e, t, t \rangle \rangle$ and so require two predicates as parameters to return a truth value, or one predicate to return a DP. Determiner coordination is only partially productive, which is hardly surprising given the constraints on determiner coordination described by Barwise and Cooper (1981) and Peters and Westerståhl (2006). Examples of determiner coordination include ‘some or all’, ‘every, or almost every’, and ‘some but not all’. A few determiner coordinations, such as ‘all and sundry’, have become frozen idioms and now behave like atomic DPs.

$$[[A \text{ and } B]]_{Det} = \lambda x \in D_{(e,t)}, \lambda y \in D_{(e,t)}, A(x, y) \land B(x, y) = \lambda x \in D_{(e,t)}, [[A(x) \land B(x)]]_{DP}.$$
3.5.2 Assertible Phrasal Coordination

In §3.5.1 I outlined a fairly conventional model of the boolean coordinators, where the semantics of coordination for each phrase type only require logical or set-theoretical connectives \( \{ \lor, \land, \cup, \cap \} \), and some syntactic agreement constraints. The following process can be adapted for these approaches as most also formalise phrasal coordination using the same set of connectives. I will define assertibility conditions for these coordinators by replacing the boolean connectives in Defn 3.13 with those from Defn 1.9. This new definition of assertible ‘and’ and ‘or’ is equivalent to that of standard assertibility for IP's (clausal coordination), and assertibility can also be extended to other phrase types via a single abstract definition. I claim that these coordinations are only weakly polysemous, at least with respect to the variations in meaning predicted by assertibility. Actual phrasal coordination instances roughly align with the assertibility restrictions and generated predictions for phrasal coordination, providing some support for this claim. I have appealed to some uncontroversial linguistic features, such as subject-verb case agreement and determiner monotonicity agreement, in constructing this reduction of phrasal coordination to clausal coordination. Case agreement is an independently motivated general principle applying to (almost) all phrase types. In the (rare) case that only one coordinand has case agreement, the specifier/complement binary structure from phrase structure trees provides the necessary asymmetry, as discussed in Johannessen (1998). Determiner monotonicity agreement also appears to be a fundamental restriction on determiner phrases in many ways. For example, Barwise and Cooper (1981) made a number of universal claims about the monotonicity of natural language determiners based on logical analysis, and most of these claims have been subsequently supported by cross-linguistic evidence. I feel entitled to appeal to both these linguistic features to support my arguments, as they are firmly established independently of assertibility theory.

I will restrict myself to modelling phrase types with the relatively simple \( \langle e, t \rangle \) and \( \langle \langle e, t \rangle, t \rangle \) lexical types, such as Noun Phrases, Verb Phrases, and Determiner Phrases, rather than formally define assertible coordination for every phrase type. These phrase types are sufficient to demonstrate the general approach, whilst the coordination of more complex phrase types requires a great deal more \( \lambda \)-calculus without providing any compensatory insight. I will represent the universal predicate by \([\lbrack U \rbrack]\). The simplified and context-free assertibility definitions for \( \langle e, t \rangle \) and \( \langle \langle e, t \rangle, t \rangle \) coordination with atomic coordinands are:

**Definition 3.14 Assertible \( \langle e, t \rangle \) and \( \langle \langle e, t \rangle, t \rangle \) Coordination**

\[
\lbrack\lbrack A \lor B \rbrack\rbrack = \lbrack\lbrack A \rbrack\rbrack \cup \lbrack\lbrack B \rbrack\rbrack, \text{ where } \lbrack\lbrack A \rbrack\rbrack \cup \lbrack\lbrack B \rbrack\rbrack \neq \emptyset.
\]

\[
\lbrack\lbrack A \land B \rbrack\rbrack = \lbrack\lbrack A \rbrack\rbrack \cap \lbrack\lbrack B \rbrack\rbrack, \text{ where } \lbrack\lbrack A \rbrack\rbrack \cap \lbrack\lbrack B \rbrack\rbrack \neq \emptyset.
\]

This definition is a combination of the semantic \( \langle e, t \rangle \) and \( \langle \langle e, t \rangle, t \rangle \) coordination conditions, so \( \lbrack\lbrack A \land B \rbrack\rbrack = \lbrack\lbrack A \rbrack\rbrack \cap \lbrack\lbrack B \rbrack\rbrack \) and \( \lbrack\lbrack A \lor B \rbrack\rbrack = \lbrack\lbrack A \rbrack\rbrack \cup \lbrack\lbrack B \rbrack\rbrack \), and a simplification of the conditions from Defn 2.29 for clausal/sentential \( t \) coordination of atomic subformulas. A truly general formulation of phrasal assertibility would need the recursive clauses to be amended to cope with the coordinands potentially containing further coordination, and to represent both the common ground and the immediate context imposed by the coordination being embedded in an utterance. Asymmetric and probabilistic variants of the phrasal coordinations are also derivable. Such amendments have already been made for clausal coordination in this and earlier chapters, and the formal application of these minor deviations to phrasal coordination is both linguistically and logically unedifying. Assertible coordination of the lexical types \( \langle\langle e, t \rangle, \langle e, t \rangle, t \rangle \) for Determiners (e.g., ‘each and every’, ‘all and sundry’, ‘some or all’) and \( \langle\langle e, t \rangle, \langle e, t \rangle \rangle \) for Adjective Phrases (e.g., ‘tall, dark, and handsome’) can be defined using the same approach. However I showed in §3.5.1 that there is no simple set-theoretic representation of \( \lbrack\lbrack A \land B \rbrack\rbrack_{AdjP} \) or \( \lbrack\lbrack A \land B \rbrack\rbrack_{Det} \). This means that the classical conjunction and disjunction truth-functions in their respective \( \lambda \)-calculus definitions would have to be replaced by the conjunctive and disjunctive recursive definitions from Defn 2.29, making for some rather unpleasant but calculable expressions. The important point here is not the details of the definitions, but the successful demonstration that any truth-functional conditions for phrasal coordination can, in principle, be amended to assertibility conditions through the systematic application of a common principle across all phrase types.
3.5.3 Phrase Depth Predictions

The phrase depth in an expression is a measure of how deeply nested a phrase is in the syntactic structure or phrase tree of the expression. Roughly synonymous expressions, such as those in (3.11a)–(3.11c), can be produced by placing the coordinating phrase at different depths, allowing the Speaker to choose the depth at which to coordinate. This choice enables them to convey additional information, independent of any information conveyed by the assertion norms. This information includes what I call the degree of coordination. Higher degrees of coordination ‘bind’ their coordinands more closely together, and usually correspond to coordinations at a greater depth in the phrasal structure. For example, (3.11a-i) and (3.11a-ii) are similar coordinations, but while (3.11a-i) merely informs us that Mary and John are not single, (3.11a-ii) with its deeper coordination and higher degree of coordination, conveys they are married to each other. The different degrees of coordination apply predominantly to conjunctions. This topic will be discussed in more detail in §4.1.1.

By the informal application of the Brevity and Informativity maxims, and the desire to avoid ambiguity, we would expect a Speaker to use the most deeply nested coordination phrase they can, as this maximises the information content, the potential degree of coordination, and the removal of duplicated phrases. They can then elide as much of the remaining utterance as possible while avoiding ambiguity. This is roughly what we observe.

(3.11) a. i. Mary is married and John is married.
    ii. Mary and John are married.
 b. i. John is in the kitchen, or he is in the bathroom.
    ii. John is in the kitchen or bathroom.
 c. i. Most officers and gentlemen are honourable.
    ii. Most officers and most gentlemen are honourable.
    iii. Most officers who are gentlemen are honourable.

Prediction 3.13 Phrasal Depth.
Condition: The coordination could be nested more deeply in the phrasal structure without violating the language’s syntactic and semantic constraints.
Effect: Some coordination predictions are cancelled, or the degree of coordination is not maximal.
Example: See (3.11a-i) and (3.11b-i).
Failing to maximise coordination depth results in one of two effects. If phrases are coordinated at a lesser depth than semantics permits, some of the coordination predictions are cancelled. The further the coordination lies from its maximal depth, the greater the extent of prediction cancellation. Unnecessarily clausal disjunction (3.11b-i) often lacks even the basic predictions of Pred 3.9. This effect also applies to conjunctions that are not at the minimal depth for any coordination degree, the first effect applied. In addition, conjunctions with different degrees of coordination have different minimal coordination depths, and so the phrasal depth can convey the maximal degree of coordination intended.

Prediction 3.14 Ambiguous Ellipsis.
Condition: The coordination is ambiguous between elided and non-elided phrasal coordinations.
Effect: The reading derived via elision is preferred, unless it is physically, semantically or logically contradictory, tautological, implausible, or otherwise in violation of Defn 3.14.
Example: See (3.11c-i)–(3.11c-iii).
Most potentially ambiguous expressions have one primary interpretation that is preferred to such an extent that the expression is not considered ambiguous unless an apparent contradiction or vacuity forces a reconsideration of the utterance. The elided interpretation is usually preferred if it is feasible, as the Speaker could have chosen not to elide if they thought that ambiguity would be a concern. The alternate interpretation is thus suppressed, and rephrasing is often required if the non-elided interpretation is intended, as in (3.11c-iii).

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7This reduction of coordination degree could also be seen as a prediction of cancellation or weakening.
3. DEVIATIONS

We can see the assertibility restrictions on coordinating Noun Phrases captured by Defn 3.14 in the following examples. The [square brackets] in each example indicate the scope of the NP coordination. Only (3.12a) and (3.13a) conform to Defn 3.14, although all are both grammatically and semantically correct. The other examples seem odd, or unacceptable, at least partially due to their violation of the assertion norms. These restrictions can also be seen in (3.12b), where many readers will self-correct, interpreting ‘mammals’ as ‘other mammals’ without consciously noticing the clash of disjuncts; this effect is less common for (3.12c) due to the disjunct order. We do not notice possible tautologies and contradictions if another interpretation is available, as shown in (3.13c), which is intended to exhibit a conjunction of contradictory NPs dominated by a single DP, although the default interpretation is as the elided conjunction of DPs ‘Most mammals and most amphibians...’.

(3.12)  a. Every [cat or dog] hunts birds.
        b. ? Most [cats or mammals] are viviparous.
        c. ? A [mammal or dog] killed this bird.

(3.13)  a. Every [officer and gentleman] shall see their soldiers well-horsed.
        b. ? Many [trees and oaks] shed their leaves.
        c. ! Most [mammals and amphibians] are herbivores.

**Prediction 3.15 Assertible Phrasal Coordination.**

*Condition:* The coordination is between non-clausal phrases of the same phrase type.

*Effect:* The standard clausal Preds 3.1–3.12 still apply to the phrasal coordination.

*Example:* See (3.12a)–(3.13a).

The reduction of phrasal coordination to the boolean coordination of truth functions or union of sets demonstrates that the principles of assertibility can be applied in full to coordination of phrases with the same phrase type. Johannessen (1998) extends this to heterophrasal (unbalanced) coordination of different phrase types that share the same lexical type.

**Syllepsis**

An utterance is usually ungrammatical when a coordination is nested too deeply in the phrase structure, due to the Speaker having broken some of the rules requiring duplicate phrase deletion. However sometimes, when this is done deliberately and skilfully, Speakers can produce effects that can be broadly classed as semantic zeugma or *syllepsis*. This can have comedic effect as the Hearer restructures their syntactic model of the utterance to restore grammaticality. Flanders’ and Swann’s song *Have Some Madeira, My Dear* has some wonderful examples of syllepsis. For example, in (3.14b) the VP ‘made up’ is split as if ‘up her mind’ was a prepositional phrase, but most native speakers can retrieve the correct grammatical structure, partially due to expectations raised by the less egregious grammatical sins in earlier lyrics such as (3.14a).

(3.14)  a. She lowered her standards by raising her glass,
        Her courage, her eyes and his hopes.
        b. When he asked ‘What in heaven?’ she made no reply,
           Up her mind, and a dash for the door.

**Prediction 3.16 Syllepsis.**

*Condition:* The coordination has been nested deeper in the phrasal structure than our syntactic and semantic constraints allow.

*Effect:* Parallelism between the coordinated clauses is highlighted, often to humorous effect.

*Example:* See (3.14a)–(3.14b).

The conditions for overly-deep coordinations to be reconstructed correctly are dependent on the specific context, and the effects are much more variable than those of Pred 3.13’s coordination at less than maximal depth. Rather than conveying additional information, syllepsis is primarily a demonstration of the Speaker’s linguistic virtuosity.
3.6 Connexion & Coordination Modes

Some of the information conveyed by uttering a coordination or subordination concerns the relationship between coordinated phrases. There is no standard way to describe these relationships, with each linguist coming up with their own groupings and key factors. I will refer mainly to a high-level classification into physical, mental, etc., domains of discourse by Sweetser (1990), and a more detailed analysis by Dixon and Aikhenvald (2009) that lists factors such as temporality, causality, contrast, and entailment. For example, the subordinator ‘because’ usually indicates a physical or epistemic causal link between clauses; in (3.15a) the link is causal while in (3.15b) it is epistemic (entailment). The coordinator ‘and’ allows for a wide range of relationships. In (3.15c) the conjunctive link appears to be one of temporal sequence, but ‘and’ often requires supporting lexemes such as ‘and then’ or ‘and so’ to resolve ambiguity. The coordinator ‘or’ supports still other relationships; e.g., in (3.15d), the similarities or commonalities between Paris and Milan are part of the information conveyed.

(3.15)  

a. I scratched my nose because it felt itchy.  
b. It is a rectangle because all squares are rectangles.  
c. Keith had a drink and sat down.  
d. Jane often travels to Paris or Milan.

The assertibility conditions of an utterance depend heavily on the type of relationship between the clauses in the corresponding expression. However, most of the possible relationships between clauses are not describable in terms of truth, so even an asymmetric probabilistic variant of assertibility cannot be modified to do justice to these concepts. In the rest of this section I will sketch a broad tripartite division of coordination relationships, and then use this to motivate five modes of coordination which are based on Sweetser’s “domains of discourse”. These coordination modes, along with information about their underlying relationships, will be used in later chapters to motivate the application of assertion norms to different aspects of meaning conveyed by utterances. This will go some way to predicting which of the myriad linguistic categories a particular instance of a coordination falls under, and hence its properties and usage.

Specific connectives (e.g., ‘thus’, ‘so’, ‘subsequently’), and connective modifiers (e.g., ‘and then’, ‘and so’) may provide some information about the type of link. The exact relationship (temporal, inferential, associative, causal, coincidental, etc.) between coordinands is usually strongly underdetermined by the choice of connective, and only partially determined by the lexical and syntactic content of the coordinands. The coordinands that are of specific interest to us (‘and’, ‘or’, ‘if’) are particularly generic, and can join clauses with a wide range of possible relationships.

3.6.1 Connexion

There are many classifications in the literature that describe how conjoined phrases are related, making both broad and fine distinctions, and some such as Dixon and Aikhenvald (2009) appear robustly cross-linguistic. However, these classifications are all relatively contentious, and so I will choose an account by Sweetser (1990), and modify it slightly to suit the logical applications of my project, rather than her cognitive and etymological project. I will introduce two privileged coordination relationships that underlie the logical distinctions I need: rule-based inferences and direct causal connections. These rough relationship classes will take us a surprising way towards mapping assertion norms onto natural language usage. I follow Strawson (1952), pp. 36–39 in using connexion to refer to types of relationships between coordinands; the spelling will help remind us that this is a term of art. Two propositions have an inferential connexion iff there is an entailment relationship between them or their negations, e.g., (3.15b). Two propositions about events have a direct connexion iff there is a perceived proximate correlative or causal relationship between the events, such as in (3.15a) and (3.15c). Two propositions have an indirect connexion if they are joined by a short chain of direct or inferential connexions between propositions that are part of the common ground or accessible in the shared context, but lack an inferential or direct connexion. Phrases joined by a connective such as ‘and’, ‘or’, ‘but’, or ‘if...’ almost always have some type of connexion.
(3.16) a. You should move your bishop to e6, or it’s checkmate. (Inferential)
b. Every prime number is odd or less than six. (Inferential)
c. Everyone should carry their own matches or lighter. (Direct)
d. I will pay you back on Tuesday or Wednesday (Direct)
e. She ate breakfast and then left the house. (Direct)
f. Someday I’d like to go to Hollywood or Nairobi. (Indirect)
g. Sit down, or you’ll have to see the headmaster. (Indirect)
h. Show me some identification, or should I say identificación. (Indirect)

The connexion class describes the sort of inference the Speaker expects the Hearer to make from one coordinand to the other, on the basis of logic, shared beliefs, context, defaults, etc. Each of the above coordination examples typically conveys one of the three connexion classes (although a different connexion could potentially be conveyed in a carefully chosen context).

**Inferential connexion** is the logically simplest of the classes, and applies when the meaning of the coordinands in a given context guarantees the epistemic possibilities of one coordinand are either completely contained within, or excluded from, those of the other. This includes all instances of entailment or contrariety, and can also appeal to both lexical content and the context of utterance. For example, in (3.16a) only one of the Hearer’s bishops may legally move to e6, so moving the other to e6 is not considered a possibility if the rules of Chess are being followed. Similarly, the disjunction in (3.16b) assumes some basic mathematical principles. Many inferential connexions are a result of definitional constraints such as mutually exclusive lexemes. If connectives such as ‘thus’ or ‘so’ are appropriate, the connexion is probably inferential.

**Direct connexion** is perhaps the most vague yet intuitive class, as it occurs between coordinands that belong to the same correlative web. A correlative web is an informal generalisation of a causal web, and includes cause-and-effect, associated co-causes of a relevant effect or co-effects of a relevant cause, cotemporaneous events, events that usually follow immediately after each other, any form of correlation that often generates a causal story (however spurious or improbable), repeated association or co-incidence, and even subjective imposition of patterns onto random events. Thus the connexion is often a subjective, correlative relation rather than strictly causal. The disjuncts in (3.16c) list the most common tools for causing small fires, and so their shared capacity for flame generation is the underlying connexion. In (3.16d) Tuesday and Thursday are exclusive, but consecutive, periods. Even a simple narrative association, such as in (3.16e), is an example of direct connexion. A direct connexion need not be objectively causal or even involve physical contact, as long as there is an element of prediction or association involved. Predictive direct connexions tend to be asymmetric, while associative direct connexions are usually symmetric.

**Indirect connexion** is the catch-all class, and is appropriate whenever the coordinands do not have a direct or inferential connexion. There is usually a fairly obvious and short chain of direct connexions that can be constructed given the context. When there is no apparent connexion between two coordinands, as in (3.16f), the Hearer will usually work hard to establish at least a spurious reasoning chain, and then use this connexion to help interpret the utterance. Indirect connexions are more tenuous or convoluted than direct connexions, as each link in the chain may involve a different default context, frame, or situation, and so the reasoning will be non-transitive and often highly defeasible. To construct an indirect connexion, the Hearer is expected to select a series of contexts and default assumptions which are sufficiently similar to the Speaker’s so that they arrive at the same implicit connexion chain. Any deviation from this can result in a wildly different interpretation of an utterance. One type of indirect connexion between propositional coordinands is via a direct connexion between the felicity conditions, presuppositions, or other performative success conditions of their associated illocutionary acts. In (3.16g) the second coordinand spells out the potentially disastrous consequences of failing to satisfy the first coordinand. Another indirect connexion occurs when a proposition is expressed repeatedly via distinct locutionary acts with different pragmatic effects, such as in (3.16h) where the underlying propositional identity relation links the locutions indirectly. These two types of indirect connexion will receive their own analysis in §3.7.
3.6.2 Modes of Coordination

Cognitive domains of discourse were introduced in Sweetser (1990) and further expanded upon in Dancygier (1998), and Dancygier and Sweetser (2005). Utterances containing modal verbs, coordinations and conditionals are classified based on the entities being related. These entities include objects in the physical world, concepts in the mental worlds of the Speaker and Hearer, the illocutionary acts being performed, the phrases used in the locutionary act, and the imagery and metaphor expressed. The coordination modes I will use are derived from Sweetser’s content, epistemic and speech act domains, plus Dancygier’s metalinguistic domain.

(3.17) Some Symmetric Disjunctions from each Domain/Mode: (Sweetser (1990))

a. Every Sunday, John eats pancakes or fried eggs. (*Content/Expositive*)
b. John is home or somebody is picking up his newspapers. (*Epistemic/Evocative*)
c. Seatbelts prevent many severe injuries or deaths in accidents. (*Epistemic/Deductive*)
d. Give me a hotdog or a salami sandwich. (*Speech Act/Performative*)
e. Everyone owns a pair of jandals or flip-flops. (*Metalinguistic/Renunciative*)

Sweetser’s content domain is concerned with possible states of affairs in the world: external, physical, concrete objects, events, and objective ‘facts of the matter’ such as (3.17a). It also includes connexions via temporality, causation, correlation, interaction, direct observation, and so forth. Her epistemic domain contains descriptions of reasoning rather than physical facts, including abductive and inductive inferences like (3.17b). Reasoning from cause-to-effect and from effect-to-cause are equally appropriate in the epistemic domain, as in each case the reasoning provides an interpretation or explanation. All just-so stories, speculative hypotheses, and generalisations appear to belong to the epistemic domain, as do the relations between concepts or propositional attitudes. The speech act domain is for describing the relationship between illocutionary acts that rely on actions by both Speaker and Hearer to fulfil their success conditions, such as (3.17d), where the Hearer can choose which request to satisfy. The metalinguistic domain is closely related to the speech act domain, as it concerns utterances about utterances, and the success criteria of the locutionary acts involved in performing them. The Hearer in (3.17e) may select which term is appropriate, but the basic meaning conveyed does not change. I have excluded Dancygier’s final domain, the meta-metaphorical which concerns reasoning via analogy, metaphor, and simile. This domain is problematic to characterise in a propositional setting, and does not seem to be required for discussing disjunction or conjunction (although it appears useful for some conditionals) I will also distinguish a particular mode of deductive reasoning that Sweetser places within her epistemic domain, as ‘analytic’ or deductive reasoning concerns artificial rule-based systems, laws, and taxonomies. The relationships in (3.17c) are far crisper and more logical than in the other examples, indicating that the coordinands have distinct logical properties and relationships. These domains affect how we interpret utterances, as Dancygier and Sweetser (2005) explain on p.29:

[T]he current domains of content, epistemic structure, speech-act structure, and metalinguistic structure are privileged domains, in that they are automatically and implicitly available for access in processing utterances, including conditional utterances. This fact is what accounts for the kind of diversity we see in conditional interpretation. Of course, if it is true that domains related to the current speech interaction are generally privileged with respect to mental space construction, we would expect that fact to have wide-ranging effects on interpretation, not limited to conditional constructions. And indeed, this is the case. As noted in Sweetser (1990), broad classes of linguistic forms show the same kind of possibility for multiple interpretations depending on the level or domain accessed. Parallel to conditionals are causal and adversative conjunctions, as well as many coordinate conjunction usages.
I have replaced Sweetser’s physical/mental distinction with that of direct/indirect connexion, as I have found it impossible to describe the distinction between her domains in terms of their logical features. The relata in an indirect connexion between coordinands can also be performance-based rather than propositional; this indicates the relationship is associated with their speech act and metalinguistic domains. This shift from domains to connexions, along with a desire to avoid using ‘epistemic’ in conflicting ways, has prompted me to replace their original terminology to avoid confusion. Instead of the content, epistemic, speech act, and metalinguistic domains outlined above, I will refer to the near-parallel *expositive, evocative, performative, and renunciative* coordination modes respectively, along with the new *deductive* mode. The different basis of classification and resulting shift in emphasis between Dancygier & Sweetser’s domains and my modes reflects the differing focus of our projects rather than any fundamental disagreement on where taxa boundaries should be drawn.

**Definition 3.15** *Modes of Coordination*

- **Deductive** coordination has an inferential connexion between the contents of the conjoined phrases.
- **Expositive** coordination has a direct connexion between the contents of the conjoined phrases.
- **Evocative** coordination has an indirect connexion between the contents of the conjoined phrases.
- **Performative** coordination has a direct connexion between the satisfaction conditions of an illocutionary act and the contents or satisfaction conditions of another, and thus an indirect connexion between their contents.
- **Renunciative** coordination has a direct connexion between the satisfaction conditions of a locutionary act and the contents or satisfaction contents of an illocutionary act.

**Deductive Mode**

The deductive mode of coordination is characterised by the association of concepts within a set of absolute rules. These rules are taken to be atemporal, all-encompassing, and infallible. Conjoining reasoning within logic and mathematics are prime examples of deductive coordination, as are those that appeal to the properties of taxonomies and lexical hierarchies such as (3.17c), although some steps of almost any chain of reasoning will be deductive. A simple example of deductive subordination is ‘if today is Tuesday then today is a weekday’. Features of this coordination mode include non-probabilistic reasoning, unlimited transitivity, entailment, *l*-contrariety, the use of conditionals as definitions, and the reversibility of each step via contraposition. This logical richness means that ‘indicative’ conditionals are often deductive. Deductive coordinations can also be regarded as a non-probabilistic form of expositive coordination, since the error threshold $\epsilon$ is 0.

**Expositive Mode**

Expositive coordinations typically involve the presentation of a temporal, causal chain of facts about the physical world, connecting information in a narrative. Sweetser (1990) calls the corresponding content domain “the fundamental discourse domain”. She also claims that most connectives and modal verbs in English were first applied in this domain, and then later adapted to other domains. Most simple examples of coordination such as ‘Jack and Jill’; ‘blue or red’; and ‘pancakes or eggs’ from (3.17a) are expositive. Asymmetric expositive conjunction and disjunction are often temporally ordered, as in ‘turn left and go up the hill’. Expositive conditionals are predictive and probabilistic, with different tenses explicitly marking that the antecedent precedes the consequent. There are no expositive adversatives coordinators such as ‘but’ as the contrast or denial of expectations is an indirect comment on contextual expectations or hypotheses.

**Evocative Mode**

The evocative coordination mode is useful when engaged in hypothesis building, default reasoning, inferences to the best explanation, or using metaphor, framing devices and intuitive leaps in reasoning to allow Hearers to fill in unspoken details. All speculation, explanations, and unsupported hypotheses such as (3.17b) are evocative, along with reasons for contrasts, surprises, and indirect relationships between facts. Inferences about the propositional attitudes of others are also evocative, unlike the}

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*Satisfaction conditions will be discussed further in §3.7.*
mere reporting of one’s own beliefs, thoughts or opinions. The use of ‘must’ or similar modal auxiliaries is a strong indicator of the evocative mode. Connectives have the fewest constraints when in the evocative mode, as evocative conjunction and disjunction can be symmetric or asymmetric, probabilistic or certain, temporally ordered or reversed, or atemporal, and may invoke c-contrariety or l-contrariety. The error threshold $\epsilon$ is also usually higher for the evocative than expositive mode. The fundamental mode of adversative coordinators appears to be evocative. Evocative conditionals are probabilistic, non-causal, and usually lack predictive, concessive or contrapositive elements. The evocative mode is thus the most logically impoverished of the three propositional coordination modes.

**Performative Mode**
The performative mode is distinguished by having one coordinand whose felicity is dependent on information available to the Hearer but not the Speaker. This information often relates to information about the external world, as well as the preferences, opinions, and mental states of the Hearer. Requests like (3.17d), orders, questions, recommendations, and other non-assertoric speech acts commonly rely on this sort of information for felicity. Assertions that involve performative connectives can often be rephrased as non-assertoric speech acts, such as a series of questions. Formalising assertions with performative connectives by using expositive connectives will often result in tautology, vacuity, triviality, or unlikely causal interpretations. Some paradoxes of material implication are dissolved in §5.6.1 by resolving this exact misunderstanding. The temporal ordering of the utterances means that performative conjunctions are rarely symmetric, although disjunctions and adversatives can be symmetric or asymmetric. All three types of coordination require the propositional component of both assertions to be true. Performative conditionals such as Austin (1962)’s infamous biscuit conditional rarely have predictive or contrapositive interpretations.

**Renunciative Mode**
Renunciative coordination always involves at least one of the coordinands being withdrawn after it is asserted, only to be replaced by a synonymous or closely-related element. This is due to the presence of a linguistic element in the withdrawn coordinand that the Hearer is invited to reject. There are two reasons why an utterance may be withdrawn: propositional renunciation is due to failure of informativity, extensibility, aptness, or assertibility; while locutionary renunciation such as (3.17e) is more a case of incorrect register, manner, accent, pronunciation, or emphasis. All renunciation relies on an unusual form of negation that acts as a withdrawal of an assertion. Renunciative disjunctions can be symmetric (offering two choices), or asymmetric (providing an alternative if the first disjunct is unsatisfactory). Renunciative adversatives are usually in the form ‘not $A$ but $B$’, and are asymmetric as they withdraw the first element in favour of the second. Renunciative conditionals simply allow the Hearer to reject the consequent as unsatisfactory for locutionary reasons (akin to failures of the maxims of Manner) proffered in the antecedent while still accepting the propositional content of the consequent. There do not appear to be any renunciative conjunctions.

Performative and renunciative coordinations usually contain some clauses or phrases that are uttered but not asserted, or asserted and then renounced. These unasserted phrases still have some pragmatic effects. Any presuppositions still apply, so a false presupposition for an unasserted phrase can still cause a violation of satisfaction conditions, unless the rest of the utterance addresses the presupposition failure. The Hearer may be offended, threatened, or confused by an unasserted phrase, as implicatures or other pragmatic effects that relate to relevance, style, register, politeness, pronunciation or other modes of delivery still hold. Predictions that are generated from the assertibility of coordinations in expositive, evocative, or deductive modes are less likely to arise from unasserted phrases, as these are generated by the information content and form of the utterance, not the mere performance of a locution. I am primarily concerned with the predictions generated by the presence of connectives in assertions, so I will not thoroughly investigate whether, when, or which pragmatic effects are still generated by an unasserted subutterance. What is worthy of further investigation is the structural restrictions in both performative and renunciative coordination imposed by the assertibility norms.
3.7 Satisfaction & Renunciation

Coordination in the performative and renunciation modes depends on the adequate performance of the illocutionary and locutionary acts associated with an utterance. Performative coordination is dependent on the success or failure of satisfaction conditions, being the presuppositions of the utterance and the felicity conditions for performing the illocutionary act. Austin (1962), Searle (1969), Gazdar (1979) and many subsequent theorists have provided several clear descriptions of presuppositions and felicity conditions. I will use these terms quite generically, deliberately remaining theory-neutral and appealing only to the broadest of intuitions regarding their success or failure. I am particularly interested in those conditions that appeal to information available only to the Hearer, such as their propositional attitudes, preferences, emotions, and the success or failure of prior illocutionary acts. This is because I (continue to) assume the general performative competence of both the Speaker and Hearer. Renunciative coordination depends on particular types of failure in either the illocutionary or the locutionary act that require an utterance to be retracted, and usually restated. Some conditions for the successful performance of the locutionary act are determined by the Hearer’s preferences on such factors as accent, register, and volume. Both these modes differ from expositive, evocative and deductive coordination, where the connexion occurs between the propositional contents of the coordinands.

3.7.1 Performative Coordination

The Hearer’s privileged access to some satisfaction conditions enables them to determine when an illocutionary act is unsatisfactory, even when the Speaker is sincere and the propositional content of the utterance is plausible. The Hearer may believe, for example, that something presupposed by the Speaker is false. Strategies for a Speaker wishing to avoid the failure of their illocutionary act include: explicitly indicating their dependence on specific prior utterances, presuppositions or felicity conditions; providing alternative utterances for the Hearer to select from; and indicating that a pair of contrasting utterances are not contradictory or collectively incoherent. These strategies are naturally represented by conditionals (3.18a) and conjunctions (3.18b), disjunctions (3.18c), and adversatives (3.18d) respectively. These connectives indicate the conjoining of illocutionary acts, and not merely their propositional contents.

(3.18) a. If you are hungry, there are biscuits on the sideboard. (Austin (1962))
    b. Glad to meet you, sir; and how may I help you. (Sweetser (1990))
    c. Give me a hotdog or a salami sandwich.
    d. George likes mu shu pork, but so do all linguists.

I will use \( \langle \phi \rangle \) to represent a particular illocutionary act whose utterance has a form \( \phi \). The proposition \( sat(\langle \phi \rangle) \) representing its satisfaction conditions plays a role in defining assertibility conditions for conjoined illocutionary acts similar to that of \( T \phi \) in the standard assertibility conditions. To highlight this parallel, I will represent performative connectives with the standard logical symbols, and present their definitions so as to emphasise their similarity with the respective propositional connectives. I will only present definitions for symmetric (commutative) disjunction and conjunction, although the asymmetric coordinator variants defined in §3.4 can also be used to model performative connectives. Performative adversatives are discussed in §4.5.1, and performative conditionals in §5.6.2.

Definition 3.16 Assertions as Illocutionary Acts \( \langle \phi \rangle \).

1. An assertoric utterance with utterance form \( \phi \) is represented by \( \langle \phi \rangle \).
2. The illocutionary satisfaction conditions for \( \langle \phi \rangle \) are represented by \( sat(\langle \phi \rangle) \).
3. The locutionary conditions for \( \langle \phi \rangle \) are represented by \( loc(\langle \phi \rangle) \).

Definition 3.17 Assertoric Performativ\ e Coordination

1. \( \langle \Pi; \Gamma \rangle \models A(\langle \phi \rangle) \iff \langle \Pi; \Gamma \rangle \models A\phi \& T(sat(\langle \phi \rangle)) \).
2. \( \langle \Pi; \Gamma \rangle \models A(\langle \phi \rangle \lor \langle \psi \rangle) \iff \langle \Pi; \Gamma \models \neg sat(\langle \psi \rangle); \Gamma \models A(\langle \phi \rangle) \) and \( \langle \Pi; \Gamma \models \neg sat(\langle \phi \rangle); \Gamma \models A(\langle \psi \rangle) \).
3. \( \langle \Pi; \Gamma \rangle \models A(\langle \phi \rangle \land \langle \psi \rangle) \iff \langle \Pi; \Gamma \models sat(\langle \psi \rangle); \Gamma \models A(\langle \phi \rangle) \) and \( \langle \Pi; \Gamma \models sat(\langle \phi \rangle); \Gamma \models A(\langle \psi \rangle) \).

3. DEVIATIONS
3.7. SATISFACTION & RENUNCIATION

Defn 3.17 only provides the recursive clauses for the simplest forms of assertoric performative coordination. Krifka (2001) claim to extend the formal semantic theory of Heim and Kratzer (1998) by incorporating assertoric and interrogative speech acts, including what appears to be a form of illocutionary conjunction.

**Lemma 3.18** Performative Coordination Entailments

1. If \( \langle \Pi; \Gamma \rangle \models A(\langle \varphi \rangle \lor \langle \psi \rangle) \) then \( \langle \Pi; \Gamma \rangle \models A \varphi \& A \psi \& T(sat(\varphi) \lor sat(\psi)) \) &
   \( \lo (A \varphi \& T(\neg sat(\psi))) \) &
   \( \lo (A \psi \& T(\neg sat(\varphi))) \).
2. If \( \langle \Pi; \Gamma \rangle \models A(\langle \varphi \rangle \land \langle \psi \rangle) \) then \( \langle \Pi; \Gamma \rangle \models A \varphi \& A \psi \& T(sat(\varphi) \land sat(\psi)) \) &
   \( \lo (A \neg \varphi \land T(sat(\psi))) \) &
   \( \lo (A \neg \psi \land T(sat(\varphi))) \).
3. If \( \langle \Pi; \Gamma \rangle \models A(\langle \varphi \rangle \lor \langle \psi \rangle) \) then \( \langle \Pi; \Gamma \rangle \models A(\varphi \land \psi) \).

Proof: By Defns 3.16 and 3.17, and Theorem 3.5.

Every kind of deviation of the assertion norms that I have introduced in this chapter is, in principle, applicable to performative coordination. For example, Declerck and Reed (2001) describe approximately 20 different performative conditionals, based on the type of connexion between the satisfaction conditions in the antecedent and the assertion in the consequent. I will examine one example class of deviations in depth, being the effects of asymmetry on disjunctions and conjunctions.

Both disjuncts are independently assertible in symmetric performative disjunctions such as (3.18c): the Speaker is potentially asserting either disjunct (or even both), based on which disjunct the Hearer deems satisfactory, useful in the current context, etc. The satisfaction conditions for the disjuncts are usually contrary in the utterance context, so at most one disjunct can be satisfied. For example, a Speaker recommending either of two restaurants for dinner tonight actually recommends both as a single disjunctive act, but expects to eat in at most one of them tonight, selected partially on the Hearer’s preferences. The logical structure of Lemma 3.18.1 parallels that of Theorem 3.5.1, apart from the conjunctive assertibility of the propositional contents. Performative coordinations are often asymmetric as the order of utterance applies to satisfaction conditions even more naturally than truth or assertibility conditions. Presuppositions, assumptions, and topicality are three examples of satisfaction conditions that are often established by the first coordinand and used in the second. This performative analogue to \( \varphi \not\leftrightarrow \psi \) is represented by \( \langle \varphi \rangle \not\leftrightarrow \langle \psi \rangle \). Its conditions are similar to \( \langle \varphi \rangle \lor \langle \psi \rangle \), except that the clause \( \lo (A \langle \varphi \rangle \land T(\neg sat(\psi))) \) is simplified to \( \lo A \langle \varphi \rangle \), which parallels Lemma 3.9.1. Note that the symmetric (3.18c) actually disjoins two imperatives, yet the assertibility conditions are effectively the same as for assertions. The surprising properties of asymmetric performative disjunction with only one imperative are investigated in §5.5.2.

The symmetric performative act conjunction \( \langle \varphi \rangle \land \langle \psi \rangle \) also requires both \( \varphi \) and \( \psi \) to be assertible. Their assertoric success conditions are also independent as neither conjunct’s satisfaction is dependent on the other’s. Defn 3.17.3 and Lemma 3.18.3 show that the conditions for a performative conjunction are almost identical to the standard (expositive) conjunction, although they have the additional requirement of independent satisfaction conditions in the counterfactual \( \lo \) clauses. Some analyses, including Sweetser (1990), deny the independent existence of the symmetric performative conjunction, and if it is virtually indistinguishable from the expositive, no unequivocal evidence can be offered in its support. There is however plenty of evidence for asymmetric, contrastive, and concessive variants of performative conjunction, so I posit the existence of this central case for theoretical economy. Unambiguously performative conjunctions such as (3.18b) is thus always asymmetric, usually as the performative analogue to \( \varphi \rightarrow \psi \) represented by \( \langle \varphi \rangle \rightarrow \langle \psi \rangle \). This asymmetric conjunction only differs from \( \langle \varphi \rangle \land \langle \psi \rangle \) by not requiring the second conjunct to be satisfiable independently of the first, so the clause \( \lo (A \langle \neg \varphi \rangle \land T(sat(\psi))) \) is simplified to \( \lo A \langle \neg \varphi \rangle \), as per Lemma 3.9.4. Later conjuncts are generally only assertible if the earlier conjuncts have been successfully asserted, since the presuppositions or satisfaction conditions of later conjuncts usually include the contents of earlier conjuncts, or build on earlier illocutionary effects. Conjunctive illocutionary acts where the assertibility of an earlier utterance is dependent on a later one are much rarer and primarily used for rhetorical effect, perhaps because the required order of comprehension differs from the order of utterance, making them more difficult to comprehend.
3.7.2 Renunciation

Renunciation describes a particular class of utterances that rely on metalinguistic negation, including coordinations which implicitly rely on this negation. A renunciative utterance such as (3.19a) denies that asserting its propositional content would be conversationally adequate, even though it is assertible. This differs from the non-renunciative (3.19b), which asserts its negation, and (3.19c) which denies its assertibility.

(3.19) a. It is not that Paul was a good man. (Renunciative)
   b. Paul was not a good man. (other)
   c. You can’t say that Paul was a good man. (other)
   d. It is not that Paul was a good man; he was a great man. (Propositionally Renunciative)
   e. Paul was not a good man; he was an evil son of a bitch. (other)
   f. It is not that Paul was a good man; he was a good man. (Locutionally Renunciative)

The inadequate nature of the first subutterance in (3.19d) can be represented propositionally, in that it accurately partially conveys the Speaker’s intention, but does not convey part of the Speaker’s intention. This utterance cannot be extended to convey the entirety of her intention in an assertible manner (see both §1.4.3 and the discussion of Extensibility preceding Lemma 2.103). Utterances which are corrected because the initial proposition is simply inaccurate, such as (3.19e), are simply assertions of negated propositions and these corrective utterances are not classified as renunciative. The first subutterance in (3.19f) fails due to an incorrect performance of the locutionary act, and its replacement subutterance uses a different emphasis to convey different information, even though the two expressions contain the same propositional content. The failure of a verbal locutionary act can be due to incorrect tone, intonation, emphasis, volume, pitch, speed, pauses, accent, pronunciation, enunciation, register, informality, archaism, or obscurantism. A written locutionary act can fail due to inappropriate hand-writing, capitalisation, font, size, italicisation, punctuation, justification, kerning, colour, etc. Failures of locution will be modelled similarly to performative satisfaction failures.

Definition 3.19 Renunciative Negation

1. An utterance is Propositonally Renunciably iff it is assertible but not extensible.
   \[ \langle \Pi; \Gamma \rangle \vdash R\varphi \iff \langle \Pi; \Gamma \rangle \vdash A\varphi; \exists \psi : \langle \Pi; \Gamma \rangle \vdash A\psi; \Pi, \varphi \vdash_{\text{L}} \psi; \text{ and } \exists \chi \vdash_{\Pi} \varphi \wedge \chi \equiv \psi \text{ and } \langle \Pi; \Gamma \rangle \vdash A(\varphi \not\equiv \chi). \]

2. An utterance is Locutionally Renunciably iff it is assertible, and could be uttered differently.
   \[ \langle \Pi; \Gamma \rangle \vdash R_{L}\varphi \iff \langle \Pi; \Gamma \rangle \vdash A_{L}\varphi; \text{ and } \exists \psi : \Pi \vdash_{\text{L}} \varphi \equiv \psi, \Pi \vdash_{\text{L}} \text{loc}(\varphi) \equiv \text{loc}(\psi). \]

(3.19d)–(3.19f) above show that it is fairly common to follow a renunciation with a replacement assertion in natural language. The propositionally renunciative (3.20a) is an example of how to proceed after a non-extensible assertion. Its utterance form is roughly \( R(p \supset r) \& A((p \supset q) \wedge (q \supset r)). \)

(3.20b)–(3.20e) are all locutionary renunciations with the simple utterance form of \( R(p) \& A(q) \).

Locutionary renunciation is primarily about ensuring smooth communication by withdrawing inappropriate elements of style or form (and usually replacing them) to reduce the inappropriate implications, connotations, and associations that are drawn. The renounced expression and its replacement are often conjoined using adveratives like ‘but’ or comparatives like ‘rather’.

(3.20) a. It’s not that drinking milk made his bones fragile; drinking milk helped him live until 93, and being 93 made his bones fragile.
   b. I didn’t eat a to-MAE-to; I ate a to-MAH-to.
   c. Grice didn’t avoid unnecessary prolixity; he was brief.
   d. Searle would never write ‘modus ponens’, only ‘modus ponens’.
   e. Barack Obama is not an ‘uppity nigger’, but a successful African-American politician.
Renunciative Disjunction

Disjunctions and adversatives are perhaps the most common connectives in renunciative mode. One advantage of disjunctions is that the renunciation is not explicitly voiced. The Hearer is instead offered a series of alternatives, at least one of which is (hopefully) acceptable. This allows all parties to ignore the unacceptable alternatives as if they had not been uttered, thus satisfying some politeness norm. Renunciative disjunctions can be propositional or locutionary, symmetric or asymmetric. (3.21a)–(3.21c) illustrate the three types of renunciative disjunctions. The Speaker offers two alternative locutions in the symmetric (3.21a), allowing the Hearer to choose whichever better matches his own preferences. The neutral and socially acceptable term ‘landlord’ is used in (3.21b), followed by the alternative and more evocative ‘slumlord’; the Hearer is thus informed about the Speaker’s attitude to the houseowner and can accept or ignore the offer to switch to a less neutral conversational register. The propositional content of the praise in (3.21c) may sound somewhat weak when voiced, so the Speaker has followed it with a stronger and preferred alternative. Finally, (3.21d) appears similar to (3.21c), but is corrective rather than renunciative as the second disjunct is weaker than the first. Renunciation could have been formalised more broadly so as to include corrective disjunctions like (3.21d), but I chose instead to have a class of renunciative negations, disjunctions and adversatives which share interesting logical properties.

(3.21)  a. Everyone owns a pair of jandals or flip-flops. (Symmetric locutionary)
        b. The landlord, or slumlord, wants his pound of flesh. (Asymmetric locutionary)
        c. He is a good dancer, or rather, very good. (Asymmetric propositional)
        d. He is a good dancer, or at least good enough. (Asymmetric non-renunciative)

An attentive reader might notice that there is no symmetric propositional renunciative disjunction. This is because the underlying propositional renunciative negation requires the negated proposition to be entailed by, but not actually entail, the replacement proposition; a decidedly asymmetric condition. The locutionary renunciative disjunction has no such restriction, and comes in both symmetric and asymmetric variants. Asymmetric locutionary disjunctions often occur when a Speaker doubts that an initial utterance was successful, and wishes to make a second attempt.

Definition 3.20 Renunciative Disjunction.

1. **Asymmetric Propositional Renunciative Disjunction**
   \((\Pi; \Gamma) \Vdash A(\varphi \not \Rightarrow_R \psi)\) iff \((\Pi; \Gamma) \Vdash A\varphi \land A\psi; \psi \vDash \varphi; \varphi \not \vDash; \psi;\) and there exists \(\chi\) such that \(\Pi \vDash \varphi \land \chi \equiv \psi\) and \((\Pi; \Gamma) \Vdash A(\varphi \not \Rightarrow \chi)\).

2. **Symmetric Locutionary Renunciative Disjunction**
   \((\Pi; \Gamma) \Vdash A(\langle \varphi \rangle \lor_R \langle \psi \rangle)\) iff \((\Pi; \Gamma) \Vdash A(\varphi) \land A(\psi); \Pi \vDash \varphi \equiv \psi;\)
   \((\Pi; \Gamma) \vDash \langle \varphi \rangle; \Pi \vDash \langle \psi \rangle; \) and \(\Pi \vDash \langle \varphi \rangle \lor \langle \psi \rangle\).

3. **Asymmetric Locutionary Renunciative Disjunction**
   \((\Pi; \Gamma) \Vdash A(\langle \varphi \rangle \not \Rightarrow_R \langle \psi \rangle)\) iff \((\Pi; \Gamma) \Vdash A(\varphi) \land A(\psi); \Pi \vDash \varphi \equiv \psi;\)
   \((\Pi; \Gamma) \vDash \langle \varphi \rangle; \) and \(\Pi \vDash \langle \varphi \rangle \lor \langle \psi \rangle\).

Lemma 3.21 Renunciative Disjunction Entailments.

1. If \((\Pi; \Gamma) \Vdash A(\varphi \not \Rightarrow \psi)\) then \((\Pi; \Gamma) \Vdash R\varphi\).
2. If \((\Pi; \Gamma) \Vdash A(\langle \varphi \rangle \lor_R \langle \psi \rangle)\) then \((\Pi; \Gamma) \Vdash R(\varphi) \land R(\psi)\).
3. If \((\Pi; \Gamma) \Vdash A(\langle \varphi \rangle \not \Rightarrow_R \langle \psi \rangle)\) then \((\Pi; \Gamma) \Vdash R(\varphi)\).
4. If \((\Pi; \Gamma) \Vdash A(\langle \varphi \rangle \lor_R \langle \psi \rangle)\) then \((\Pi; \Gamma) \Vdash A(\varphi \land A\psi \land T(\langle \varphi \rangle \lor \langle \psi \rangle) \land T(\langle \varphi \rangle \land \neg \langle \psi \rangle) \land T(\langle \psi \rangle \land \neg \langle \varphi \rangle)).\)
5. If \((\Pi; \Gamma) \Vdash A(\langle \psi \rangle \not \Rightarrow_R \langle \varphi \rangle)\) then \((\Pi; \Gamma) \Vdash A(\varphi \land A\psi \land T(\langle \varphi \rangle \lor \langle \psi \rangle) \land T(\langle \psi \rangle \land \neg \langle \varphi \rangle)).\)

Proof: By Defns 3.16 and 3.20, and Theorem 3.5. □

Renunciative adversatives and conditionals will be described in §4.5.1 and §5.6 respectively. There are no renunciative conjunctions, as conjunction does not directly rely on negation.
3.8 Interim Conclusions

We now have a highly flexible framework for determining the degree of assertibility for a range of rival interpretations of an utterance, with the expectation that the more assertible utterance form should correspond to the original communicative intentions of the Speaker.

Both conjunction and disjunction have been weakened in degrees, to allow for one or both of the coordinands to be interpreted without consideration of the other coordinand. Probabilistic considerations have been introduced, allowing for those cases where a little error is acceptable if it allows the elimination of unlikely and uninteresting epistemic possibilities. We’ve introduced a new meta-language that helps to describe both these extensions, along with encapsulation which allows for the representation of logically complex but linguistically irreducible concepts. The assertion norms have been extended to apply to phrasal coordination, which is essential as the vast majority of natural language coordination is non-clausal. In doing so, we have also garnered some tricks for learning more about coordinations based on their phrasal depth.

The deductive, expositive and evocative coordination modes vary primarily in their treatment of probability, and in the additional information that may be conveyed by their connexions. This affects assertibility, in that the acceptable $\langle \delta \rangle$ threshold ranges vary between the modes. In addition, some natural language phenomena appear to be confined to only one of these modes due to the coordinand relationships that it allows. This will affect some of the predictions we make in the next chapter. The performative and renunciative modes coordinate the success conditions of the coordinated illocutionary or locutionary acts as well as the propositional contents of the coordinands. This leads to two new and distinctive modes of coordination, which produce specific and unexpected predictions that result from the underlying assertion norms being applied to different sets of propositions.

The addition of phrasal coordination and the performative and renunciative modes significantly widens the range of natural language coordination that can be modelled, and the asymmetric and probabilistic connectives allow for degrees of assertibility. We can now compare rival interpretations for relative assertibility, rather than treating it as a threshold condition for a single interpretation. Our more flexible set of tools will allow us to produce more powerful and specific predictions, whose generation is based firmly on the formal semantics of Chapter Two. We are finally ready to analyse natural language coordination.
4. Coordinations

We are finally ready to produce a rich array of predictions about coordinations that are generated by our deviations from, and variations on, assertibility, and compare them with actual examples of English coordination. Coordination in natural language is complex, and appears messy and unsystematic, so it will prove useful to sketch some linguistic theories that may shed light on the underlying logical structure of coordination. These will include the degree of coordination (distributive, collective, relational, etc.) found in different conjunctions, the adicity of coordinations, and conditions under which ‘or’ and ‘and’ are intersubstitutable. After that, we will review each English coordinator, starting with ‘or’. We start with how assertibility distinguishes between inclusive and exclusive disjunction. Next we examine symmetric and asymmetric disjunction through the lens of assertibility, and repeat this process for each of the five coordination modes. There are also many other classes of disjunction, some of which can be partially described or predicted by appealing to assertibility. Each type of disjunction will have its own set of associated predictions. The conjunctive coordinator ‘and’ has similar assertibility criteria to ‘or’, and this allows us to recycle and adapt most of the disjunctive predictions for conjunction. The different coordinand relationships expressed by conjunction lead to a few additional predictions. The non-boolean coordinations of ‘but’, ‘so’, ‘nor’, ‘yet’, and ‘for’ will be studied last, and prove increasingly less amenable to analysis via assertibility. I will begin by considering how coordination fit into the wider linguistic family.

The linguistic category of conjunctions can be divided into coordinating, subordinating, and correlative conjunctions, although individual lexemes can sometimes slide between these categories. Coordinators, or coordinating conjunctions, are terms or constructions which join two or more clauses or phrases (of the same type) so that they are treated as a single item of that type. The most common examples of coordinators in English are ‘and’, ‘or’, ‘but’, ‘nor’, ‘so’, ‘for’ and ‘yet’. All languages appear to possess coordination constructions, but they differ in the number of lexical or morpho-syntactic items that are used and how these terms divide the coordination landscape. I will concentrate on the most common distinctions and uses for the English lexemes ‘and’ and ‘or’, but this does not mean that my analysis should be taken as language-specific; this is merely a convenient way to focus on an arbitrary cluster of related linguistic phenomena. As natural languages divide coordination in different ways, some of the distinctions that are implicit in English will be explicit in other languages – that is, context may be required to disambiguate concepts that other languages use distinct lexemes to distinguish.

I will mention the other types of conjunction only in passing. Common English subordinating conjunctions include ‘if’, ‘unless’, ‘although’, ‘because’, and ‘since’. A subordinating conjunction has a main independent (usually propositional) clause and a subordinated dependent (non-propositional) clause that modifies or restricts the main clause. In English, the subordinating conjunction usually marks the beginning of the dependent clause, which can be before, after, or embedded within the main clause. The antecedent of a conditional is the dependent clause, and the consequent is the main clause. Few subordinating conjunctions appear suited to being modelled by truth functions that take both clauses as propositional parameters; of these, if-conditionals are the best-known. Using a truth function to model if-conditionals is at best controversial, and this will be discussed further in §5.6. Common English correlative conjunctions include ‘both . . . and . . . ’; ‘either . . . or . . . ’; ‘neither . . . nor . . . ’; ‘not . . . but . . . ’; ‘not . . . nor . . . ’; and ‘not only . . . but (also) . . . ’. There is some evidence that these are, or have evolved from, conventionalised patterns of coordinators and pragmatic markers.
4.1 Boolean Coordination

In this section I will look at some properties that apply to the two English coordinators ‘or’ and ‘and’, which I will refer to collectively as ‘boolean’ coordinators. I will introduce a scale for the degree of coordination that predicts some of the complexities with predicate logic representations of boolean coordinations. I next present my position on the adicity of the boolean coordinators. Finally, I review some coordinations where the choice between using ‘or’ and ‘and’ appears arbitrary. The English coordinators ‘and’ and ‘or’ share a number of properties that other coordinators do not. These include a simplicity of structure, and near-universality in natural languages. Perhaps most relevant is their primarily truth-conditional nature in clausal coordination. Another key property is their tendency towards associativity, and to a lesser extent, commutativity. Grice (1975) and many others have even proposed that the meaning of ‘and’ and ‘or’ is adequately represented by the truth-functions ‘∧’ and ‘∨’.

4.1.1 Degrees of Coordination

When a coordinated phrase is the subject of a predicate, it can behave in any of several different ways. The predicate might apply to the coordination as a single unit, or to each individual member of the coordination in turn, or the predicate can even become a relation between the coordinands. In each case, a different logical structure seems to be applicable, and different truth conditions and syntactic constraints are evidenced. This distinction, which I call the degree of coordination, is usually treated as an unformalisable, vague, and contextual feature of coordination. I propose that it actually affects the truth and assertibility conditions, along with the applicability of some syntactic rules. The following distinctions are speculative, and not directly supported by independent linguistic research. The degree of coordination is determined by the logical properties of the coordinators, the relationship between the coordinands and related parts of the utterance such as the verb phrase (VP) for subject determiner phrase (DP) coordination, and contextual knowledge. The four major degrees of coordination are, from weakest to strongest:

1. Distributive Coordination. (individual elements contributing distributively)
   1a. Asynchronous Coordination. (individual elements contributing asynchronously)
2. Collective Coordination. (individual elements contributing to a collective predicate)
3. Relational Coordination. (elements of a relation collectively predicated as being in that relation)
4. Composite Coordination. (components of a hybridised individual)

I will explain each type of coordination in turn, but an overview of the degrees of coordination can be gained by considering typical interpretations of the following sentences:

(4.1) a. i. Hillary, Isobel, and Rikki are cats. (Distributive)
    ii. She went to Germany, or France, or somewhere in Europe. (Asynchronous)

b. Belinda, Muriel, and Steven carried my piano up the stairs. (Collective)

c. Ann and Bill are married. (Relational)

d. The film is black and white. (Composite)

Distributive Coordination is the archetypal coordination, coordinating two or more items of the same semantic class with or without regard for order. Typically, multiple items will form a list, with all but the last coordinator elided; while there may be a pragmatic reason for the order of list items, such as relative salience, changing the order will not change the meaning of the underlying sentence. This degree of coordination is ideal for listing independent objects or properties. Disjunctions are always distributive, as they present alternatives individuating rather than as a set.

(4.2) a. Hillary, Isobel, and Rikki are cats.
    b. Hillary or Rikki brought a dead bird inside.
    c. Belinda, Muriel, and Steven played my piano.
4.1. BOOLEAN COORDINATION

**Asynchronous Coordination** is a subclass of distributive coordination where the utterance is semantically and syntactically complete after each coordinand, so each additional coordinand can be understood as an additional dependent utterance referencing the earlier (accumulative) utterance. Coordination can occur with almost any phrase in the main utterance, as (4.3a) shows. Asynchronous coordination will often have comma intonation when spoken, or commas in written utterances. It is also common for the last coordinand to be at a higher level of taxonomic generalisation, and so entail the earlier coordinands, as in (4.3b). Asynchronous coordination is a result of a hearer’s ability to append new phrases to an utterance and have their syntactic nodes ‘attach’ to the correct part of the existing syntactic phrase structure, making it a syntactic rather than semantic phenomenon.

(4.3)  
a. Jane went to the shop, and [back/the park/bought food/?Kate].

b. She went to Germany or France, or somewhere in Europe.

**Collective Coordination** is the coordination of elements (typically a set of related DPs) into a whole, where predication is collective, not individual. In most coordinations it is pragmatic considerations, rather than semantics, that determines whether the elements are referred to distributively or collectively (e.g., pianos are more easily moved by a group of people, but more often played individually). Disjunction does not appear to have a collective interpretation, nor any stronger degree of coordination. One test for collective coordination is to paraphrase the coordination using ‘all’ to group the coordinands as in (4.4b), replace ‘all’ by ‘each’ (4.4c), and evaluate for synonymy. The successful paraphrasing to include ‘all’ indicates the degree is likely no stronger than collective, while its non-synonymy with the ‘each’ variant indicates a degree stronger than distributive. The determiner ‘all’ relates most closely to the collective sense, while ‘each’ is restricted to the distributive sense, as discussed in §5.1.3. Another test is to promote the coordinator to join clauses (IPs); synonymy indicates distributive coordination, as we see with (4.4d) and (4.4e).

(4.4)  
a. Belinda, Muriel, and Steven carried my piano up the stairs.

b. Belinda, Muriel, and Steven *all* carried my piano up the stairs.

c. ! Belinda, Muriel, and Steven *each* carried my piano up the stairs.

d. Hillary and Norgay climbed Mt Everest [together].

e. ? Hillary climbed Mt Everest, and Norgay climbed Mt Everest [separately].

**Relational Coordination** is the coordination of elements in order to ascribe a relation via predication. The relation and coordination are typically both dyadic, but polyadic relations and polyadic coordination of dyadic relations are both possible. In this last case, the relation usually holds between every possible pair in the coordination list. This is discussed further in §5.1.2. Distinguishing between distributive and relational coordination usually relies on considering the predicate being applied to the coordination. The predicate is often semantically polymorphic; examples include ‘are married’ and ‘are triplets’, as they can accept either a pair (triple, etc.) between which the relation holds (4.5a), or a list of entities to whom the term applies individually (4.5b). Quantifiers such as ‘both’, ‘each’ or ‘all’ usually indicate non-relational coordinations like (4.5c) when there are viable alternatives. There is no ambiguity for (4.5d), as can be seen by (4.5e) having to be interpreted as relating to the Speaker.

(4.5)  
a. Ann and Bill are married.

b. Ann, Bill, and Chris are married.

c. Ann and Bill are both married.

d. Ann, Bill, and Cathy are [all] friends.

e. ? Ann is a friend and Bill is a friend and Cathy is a friend [of mine].

**Composite Coordination** is the strongest degree of coordination, ascribing a conjunction of contrary properties to a single entity as a way of indicating that it is composed of parts that have the properties disjunctively. For example, as each feather of a penguin is either black or white, its plumage is black and white. This type of coordination is not a particularly productive grammatical
process, being primarily confined to combinations of colours, such as in herringbone and tartan patterns, and animal colouration. In addition, predicates (VPs and predicative AdjP) appear to undergo composite coordination when their subject is *implicitly* decomposed. These elements may be coloured areas in each frame for (4.6a), taste sensations in (4.6b), or even alternating time-slices in (4.6d) and (4.6e). The simplest test for composite coordination is to decompose the predicated object into its salient elements (e.g., penguin plumage → feathers), and test if the collection of these elements (e.g., each feather) satisfies the disjunction of the contrary properties (e.g., is black or white). Another less reliable test is whether the coordination can be hyphenated, such as ‘black-and-white’, ‘sweet-and-sour’, etc. Finally, native speakers of languages such as Afrikaans and Czech indicate that composite but non-decomposable coordination of AdvPs is often asyndetic, as can be seen in (4.6c), the Czech equivalent of (4.6b). Interestingly, while penguins are *black-white* in Czech, their plumage is *black and white*, and similar phenomena occur with other Composite Coordinations; see §5.1.2 for more details.

(4.6)  
\[ a. \quad \text{The film is black and white.} \]  
\[ b. \quad \text{Gherkins are sweet and sour.} \]  
\[ c. \quad \text{Okurky jsou sladkokyselé. [not ‘sladké a kyselé’]} \]  
\[ d. \quad \text{When Tim gets angry, he screams and shouts.} \]  
\[ e. \quad \text{Roller coasters go up and down.} \]

There appears to be a restriction on coordination nesting in English, where a coordination cannot occur within a coordination of stronger degree. This is probably due to a general compositional principle, where components with weaker ties (degrees of coordination) between them must be considered after those with stronger ties. Opposing asymmetric coordinations also seem to be prevented from occurring consecutively, so that the Hearer neither needs to comprehend both ends of a polyadic coordination before its middle, nor the middle before both ends. These impermissible cases can be formalised as \( \varphi \lor \psi \lor \chi \) and \( \phi \lor \psi \lor \chi \) respectively. Assertibility does not appear to predict these restrictions although they do seem intuitive, and perhaps a little more consideration of the principles behind asymmetry and connexion would provide a principled justification.

**Prediction 4.1 Maximal Coordination.**  
**Condition:** A conjunction is ambiguous between different degrees of coordination.  
**Effect:** The conjunction is of the maximal degree consistent with the type of phrase being conjoined, the semantics of the conjuncts, context, and common ground.  
**Example:** See (4.2c), (4.4a), (4.5a), and (4.6a).

A conjunction is assumed by default to be coordinated to the strongest degree compatible with the phrasal structure and relevant predicate, since weaker degrees of coordination can occur across higher levels of a syntactic phrasal structure. The variation in disjunctive meaning that occurs when phrases are disjoined at different syntactic levels is described in Pred 3.13.

**Prediction 4.2 Nested Coordinations.**  
**Condition:** A coordination is nested directly within another coordination of the same phrase type.  
**Effect:** The nested coordination will have a stronger degree of coordination.  
**Example:** See (4.8c) below.

Given two coordinations of the same phrase type, the one of stronger degree is always nested within the other (which is possible as all non-asynchronous(!) coordinations are associative), so that its meaning is resolved first. This issue does not arise when the coordinations are of different phrase types, as the semantic content of the nested phrase type is used as an argument for determining the semantic content of the higher phrase type.

Asynchronous coordination is just an asymmetric variant of distributive coordination, as covered in §3.4. The stronger degrees of coordination are all intrinsically phrasal, and some also require predicates to be modelled differently from the usual first-order logic approach, so the truth-functional element of their formalisation will be postponed until §5.1.2. My approach to composite coordination is foreshadowed in §4.1.3.
4.1. BOOLEAN COORDINATION

4.1.2 Dyadic or Polyadic Coordination?

The English coordinators ‘and’ and ‘or’ have a close relationship with the boolean truth-functions (\(\land\) and \(\lor\)), which are typically defined as dyadic functions. These functions can be extended to polyadic truth functions as the dyadic forms are associative. The assertibility connectives (\(\land\), \(\land\), \(\land\), \(\land\), \(\lor\), \(\land\), \(\land\), and \(\land\)) are also associative, and can trivially be extended to polyadic forms. A far more important and interesting question is whether the seemingly associative natural language coordinators are dyadic or polyadic. The only philosophical argument I am aware of that distinguishes dyadic and polyadic natural language coordination is due to Reichenbach (1947) in his discussion of exclusive disjunction. He points out that a sequence of dyadic exclusive disjunctions is true if and only if an odd number of the disjuncts are true, which differs from the common ‘exactly one’ understanding of exclusive disjunction in natural language. (4.7) has quite different interpretations under exclusive and ‘exactly one’ disjunction. Natural language ‘exclusive’ disjunctions must therefore be (variably) polyadic, unless we insist that dyadic exclusive disjunctions never occur nested within each other. When I discuss exclusivity in \(\S\) 4.2.1, I will reject all types of truth-functionally exclusive disjunction, and instead rely on implicature-like phenomena predicted by considerations of contrariety and probability applied to inclusive disjunction.

(4.7) Antony or Brutus or Cassius killed Julius.

(4.8) a. There is a man, several women, and two cats waiting outside. (Johannessen (1998))
   b. i. Either John or Mike’s parents live in that house.
   ii. Mike’s parents or John lives in that house.
   c. I have invited John, Mary or her twin, Jack and Jill, Greg, and Sonya.
   d. I will win this race, or at least come second, or third.

Our next argument comes from considerations of case and phrase structure grammar. Usually all coordinands satisfy case, number, and tense-aspect agreement, although when there is only partial agreement it is usually only the first (or last, in head-final languages) coordinand that is in agreement. The examples in (4.8), which are from corpora, exhibit different forms of partial agreement; for example, (4.8a) has plural agreement between the verb and the first coordinand only. Johannessen (1998) argues that coordinators are phrase-structure heads\(^1\), the first coordinand is its specifier, and the other coordinand(s) form its complement. She focuses her argument on showing that coordinators are heads and the first coordinand is a specifier, and her reasoning appears sufficiently convincing. However, she provides little evidence for the complement being another coordination node containing a specifier for the second coordinand and a complement containing the rest, etc., simply appealing instead to the (assumed) dyadic structure of phrase-tree structures. One argument against this is that some agreement occurs only with the last coordinand (in head-first languages), as we see in the minimal pair from (4.8b), and this coordinand occurs in the complement of the most deeply nested coordination phrase. I prefer an alternative analysis that privileges both the first and last coordinands by making the second through \((n-1)\)th coordinands adjuncts of the complement. It explains the syntactic behaviour of lists of coordinands such as (4.8c) where the coordinator occurs just before the final coordinand. This appears to be the only analysis which does not require any complex rules of coordinator deletion to remove all the ‘middle’ coordinators except those for “Jack and Jill” and “Mary or her twin”. These two coordinations occur at a different level in the phrase structure, and are either of higher degree or use a different coordinator. Unfortunately, both of these analyses fare poorly in explaining some instances of asymmetric coordination, such as (4.8d). Computational linguists point out that this is a property of the syntax-first approach of most transformational grammars, which require a complete syntactic tree before semantic interpretation is possible. I suggest that actually two dyadic disjunctions occur in (4.8d), the first asymmetric, and the second asynchronous. Overall, case agreement does not provide conclusive evidence as to whether we should model conjunction and disjunction as polyadic coordinators.

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\(^1\)The head of a phrase determines the syntactic type of that phrase, e.g., a verb is always the head of a verb phrase.
McCawley (1981) presents a slightly more convincing argument for polyadic coordination based on an analysis of deletions of sub-phrases in coordinations. Syntactic movement rules that allow the deletion of elements that are shared by all the coordinands in a coordination structure are demonstrated in (4.9)–(4.11). These rules from transformational grammar include Gapping, Ellipsis, Pseudo-gapping, Coordination Reduction and Right-Node Raising (or variants, combinations and reformulations of these). van Oirsouw (1983) collectively paraphrases these rules as ‘Delete under identity in coordinated structures’, where all but the first instance (or last instance, in head-final languages) of systematically repeated elements are removed.

These deletion rules can be divided into two broad categories: Coordinated Node Promotion raises one copy of the shared phrase node so that it is no longer dominated by the coordination, and removes any duplicates; while Coordinated Node Ellipsis replaces the shared phrase node with a phonetically null phrase of the same type in all coordinands except the first. In Coordinated Node Promotion the raised node (in (4.11), the DP ‘boiled fish’) still belongs to the first coordinand, as indicated by the subordinating commas in (4.11a). McCawley claims these rules preserve meaning when used universally, but not when applied to sub-coordinations. For example if a phrase is (only) repeated in the second and third coordinands, as in (4.12a), these coordinands may not be treated as a complete coordination phrase and the repeated phrase promoted, without the resulting sentence (4.12b) either becoming ungrammatical or shifting meaning. McCawley characterises (4.12a) as simply a list of predications, while he thinks (4.12b) is about people, and (4.12c) is about cosmetic disadvantages. Successful application of deletion rules within a coordination structure leads to topic shifts, a change in meaning, and hence persistently phrasal coordination

My final argument is based on the observation that a coordination is not permitted as a coordinand of another coordination of weaker degree or with different asymmetry. Determining whether a coordination can be nested within another coordination of the same degree and asymmetry would answer whether coordinations are dyadic or polyadic. Unfortunately, deciding whether a sentence such as (4.12a) contains a nested coordination requires us to take a position on this very issue. We can check which assumption leads to simpler restrictions, and this will at least give us a more elegant model which ceteris paribus is more likely to reflect the underlying linguistic rules. The above degree and asymmetry restrictions can be modelled simply by polyadic coordination phrases which only allow stronger degrees of coordination to occur as their children (specifier, complement, or adjunct). To model these restrictions with dyadic coordination would require several complex rules. One proposed middle ground is to only permit dyadic coordination syntactically but impose semantic constraints; this simply shifts the problem of dyadic vs. polyadic coordination to semantics, where the same issues will have to be addressed.

The arguments for each position appear inconclusive. I will treat disjunction and conjunction as polyadic as it leads to simpler models, and seems to raise no additional problems. The only exception is asynchronous coordination, which appears to always be dyadic due to its role of appending ‘one last thought’ to an utterance.

2By ‘persistently phrasal’, I mean that there is no synonymous clausal coordination that can be reduced through node deletion or promotion rules to the phrasal coordination, as discussed in §3.5.
4.1. BOOLEAN COORDINATION

4.1.3 Arbitrary Coordination

Some coordinated sentences appear to preserve their meaning when one boolean coordinator is replaced by the other, so that the choice of coordinator is arbitrary. In addition, there are many more where only minor syntactic changes are required to preserve synonymy when swapping between the boolean coordinators. These examples of arbitrary coordination appear to provide strong evidence against my position that the core usages of ‘and’ and ‘or’ are heavily dependent on their respective truth functions. Arbitrary coordination tends to fall into a small number of patterns, based on the phrase type being coordinated. I will analyse examples from several of these patterns and show that they actually provide additional support for both a truth-functional element to the coordinations, and the additional restrictions of assertibility.

Determiner Phrases

Examples (4.13a) and (4.13b) are identical apart from the choice of coordinator, and are usually treated as synonymous. However (4.13a) is the result of an Ellipsis from (4.13c), which is the result of a Node Promotion from (4.13d), while the interpretation of (4.13b) that is synonymous with (4.13a) is a genuine NP coordination rather than an Ellipsis of (4.13e). Node Promotion can convert a clausal coordination to any of a wide range of phrasal coordinations, but it cannot operate through Determiner Phrase heads. This barrier is not confined to DPs in subject position, as example (4.14) shows the same phenomenon occurring for DPs as objects.

(4.13) a. Every woman and child was saved.
   b. Every woman or child was saved.
   c. Every woman and every child was saved.
   d. Every woman was saved, and every child was saved.
   e. Every woman or every child was saved.

(4.14) a. I gave every boy and girl a gold star.
   b. I gave every boy or girl a gold star.
   c. I gave every boy a gold star and every girl a gold star.

There is a simple explanation for this restriction, which ties back into the differences between sets and predicates. Many phrases have a lexical type of $\langle e, t \rangle$ which represents a function on a set of entities, but the Determiner Phrase lexical type is $\langle \langle e, t \rangle, t \rangle$ which represents a function on a predicate. Each DP inside the coordination phrase acts as a restriction on the set of entities being predicated by the VP, so their disjunction is a union of restrictions which is closely related to the conjunction of those predicates representing the restrictions. The exact nature of this relationship is set by the determiner. When the determiner is ‘every’/‘all’/‘each’ they are synonymous, resulting in arbitrary coordination. When the determiner is ‘most’, ‘at least 3’, or any other upwards-monotonic determiner, the conjunction of predicates entails the disjunction of restrictions. When the determiner is ‘a few’, ‘at most 3’, or any other downwards-monotonic determiner, the converse entailment holds. There are some non-monotonic determiners, such as ‘exactly $n$’, where neither entailment holds. When the determiner is ‘some’ the relationship is entailment in one direction but only assertibility-based implication in the other, because for the disjunction ‘some $A$ or $B$’ to be assertible, both $A$ and $B$ must be possible when the other is not. This can also result in seemingly arbitrary coordination.

Adjective Phrases

Adjective Phrases also have a non-standard type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$, and (4.15) shows that the same set/predicate effect appears to be in play in this case too, with (4.15a) being synonymous to (4.15c) and not (4.15b). Of course, if (4.15b) was interpreted as having an elided ‘cats’ after ‘large’ rather than being a coordination of adjective phrases, it would be synonymous with the other two sentences. The above argument for arbitrary coordination in determiner phrases can be adapted to arbitrary adjective phrase coordination, given the lexical type theory from §3.5.
(4.15)  
  a. All large cats and all black cats scare mice.  
  b. All large and black cats scare mice.  
  c. All large or black cats scare mice.  

Inflectional Phrases

A slightly more subtle type of arbitrary coordination is raised in Jennings (1994), where he claims that the synonymy of (4.16a) and (4.16b) shows that disjunction is often conjunctively distributive. He presents (4.16a/4.16b) and (4.16c/4.16d) as a minimal pair to show that distribution over disjunction is unpredictable. However, consideration of (4.16e) and (4.16f) show that the second DP in a comparative like ‘older than’ falls under the scope of a second VP. The second verb (but not its subject DP) is elided in (4.16e), so ‘me’ has default case, but in (4.16f) it provides the case agreement for ‘I’. This VP must fall under an IP, which is of lexical type t, so there is a head of the wrong type forming a barrier above the phrase ‘Chloe or Phoebe’ in (4.16a) that prevents Coordinated Node Promotion, just as in the previous examples.

(4.16)  
  a. Gloria is older than Chloe or Phoebe. (Jennings (1994))  
  b. Gloria is older than Chloe and Gloria is older than Phoebe.  
  c. Gloria is the sister of Chloe or Phoebe.  
  d. Gloria is the sister of Chloe or Gloria is the sister of Phoebe.  
  e. Gloria is older than me.  
  f. Gloria is older than I am.

Modal Phrases

Jennings (1994) also conflates the failure of Coordinated Node Promotion out of non-⟨e, t⟩ phrase types with the almost unrelated Free Choice example (4.17). The relationship between (4.17a) and (4.17b) is not synonymy; rather it is a pragmatic implication involving the interaction of deontic and epistemic modals with disjunction. This complex phenomenon is discussed extensively in §5.4.

(4.17)  
  a. You may have tea or coffee. (Jennings (1994))  
  b. You may have tea and you may have coffee.

We can explain the otherwise mysterious intersubstitutability of ‘and’ and ‘or’ coordinators in a range of coordination patterns by distinguishing between elided and promoted nodes, and observing that the lexical type of phrases affects the promotion of nodes in a way that precisely correlates with the truth-conditional compositional generative semantics outlined in §3.5. I am not aware of any approach that accurately predicts the relationships between these sentence pairs without reference to the truth-functional and assertibility properties of ‘or’ and ‘and’. Now (4.13)–(4.17) become examples of usage that can only be explained via an appeal to their truth-functional elements, rather than being counter-examples to the truth-functional contribution to ‘and’ and ‘or’. This addresses a wide swathe of putative counter-examples, and concerns about phrasal coordinators being truth-functionally indistinguishable. Of course, I have not shown that every example of arbitrary coordination can, or should, be explained by an analysis of syntactic phrases and lexical types, but these common cases support, rather than undermine, the standard analysis.

Prediction 4.3 Marked Arbitrary Coordination.

Condition: In an arbitrary coordination, the coordinator used is ‘and/or’.

Effect: The coordination is marked as arbitrary in the sense discussed above.

Example: See (4.13)–(4.17).

Arbitrary coordination is often indicated by using ‘and/or’; in informal registers an abbreviated ‘and’ primarily consisting of a grunted ‘n’ can be used instead. This arbitrary coordinator can be replaced with either ‘and’ or ‘or’ synonymously. The ‘and/or’ construction here is serving as a metalinguistic disjunction, meaning you can use either ‘and’ or ‘or’.
4.2 Exclusive & Asymmetric Disjunction

For many linguists, disjunction is the language-neutral term for a coordinating conjunction that presents alternatives (such as most English phrases coordinated by ‘or’) as well as for any associated coordinator. Some logicians prefer to reserve the term ‘disjunction’ for the truth function \( \lor \) (and possibly \( \oplus \)), with Jennings and Hartline (2008) claiming that 95% of English sentences containing ‘or’ cannot be represented by disjunction. I would rephrase this as the claim that 95% of English disjunctions are not both clausal and simply truth-conditional (using either ‘\( \lor \)’ or ‘\( \oplus \)’), and then agree whole-heartedly. This is merely a terminological dispute, but one where any confusion may quickly prove fatal. In English, ‘or’ is the archetypal disjunctive coordinator. It can join phrases of almost any phrase type, creating a new phrase of the same type as its constituents, such as ‘tea or coffee’, ‘jumping or running’, ‘you or me’, ‘sad or lonely’, ‘in or out’, ‘some or all’, ‘blue, red, or yellow’, and ‘Ann went to Auckland or Bill left Birkenhead’. The reduction of the truth-conditional and assertibility conditions of phrasal disjunction to those of clausal disjunction has already been described in §3.5.1 and §3.5.2. This allows my formal analysis to operate at the clausal level, despite the relative rarity of clausal truth-conditional disjunction in natural utterances.

Linguists and philosophers draw several distinctions within the class of disjunctions, based on a range of syntactic, semantic, and pragmatic features. Perhaps the division best-known by philosophers is that between inclusive and exclusive disjunction; this distinction seems somewhat surprising as linguists are not currently aware of any natural language with a distinct exclusive disjunction coordinator. I will analyse a variety of disjunctions, and demonstrate that much of the polysemy of ‘or’ is highly principled and predictable, by combining contrariety from §3.3.2 with asymmetry from §3.4 and the coordination modes from §3.6.2. One major division is between symmetric disjunctions where disjunct order is unimportant and asymmetric disjunctions which rely on disjunct order to convey some key information. The potential asymmetry of the connective, along with its coordination mode, provides important clues as to the type of relationship (e.g., temporal, causal, alternative, comparative) between the disjuncts. In addition to these broad divisions, there are also plenty of special cases where disjunction appears to behave in an unexpected way. I will analyse some of these cases, to see whether these unexpected usages can be at least partially explained by an appeal to assertibility.

4.2.1 Exclusivity

Many philosophers, including Tarski (1936) and Quine (1995), have held that some instances of natural language disjunction are truth-functionally exclusive, although following Barrett and Steiner (1971) this position has become less common. It appears that nowadays most linguists consider disjunction to be semantically inclusive, although there are exceptions such as Seuren (2010). One reason for this is that there are no attested examples of lexemes for exclusive disjunction that contrast minimally with those for inclusive disjunction. Cross-linguistic studies, from the foundational Dik (1968) through to the recent writings of Haspelmath (2000) and Mauri (2008), have failed to find a single language showing distinct strategies for inclusive and exclusive disjunction. Ever since I first gave the form of a disjunctive utterance in §1.4.2, I have assumed that the truth-conditional component of disjunction’s assertibility conditions is inclusive. This assumption underlies all the semantics in Chapter Two, the asymmetric disjunctions in §3.4, the performative and renunciative disjunctions in §3.7, and so forth. However, the assertion norms are not a form of linguistic semantics, and are compatible with the notion that the semantics of some natural language disjunctions such as (4.18a) require an appeal to the ‘\( \oplus \)’ rather than ‘\( \lor \)’ truth-function. They are instead intended to capture the variation in disjunction usage, including those cases often referred to as ‘exclusive’. One caveat: sometimes a disjunction falls within the scope of modal auxiliaries and then behaves more like a conjunction of exclusive possibilities, as in (4.18b). This is an instance of Free Choice rather than exclusivity, and will receive its own extensive analysis in §5.4.

(4.18) a. The cup contains tea or coffee.
   b. You may have either tea or coffee.
Explanations of exclusivity as a pragmatic phenomenon which is parasitic on (truth-conditional) inclusive disjunction go back as least as far as Grice (1989) [originally 1967], Horn (1972), and Gazdar (1979). The standard position, from Grice (1975), Levinson (2000), and many others, is that exclusivity is a pragmatic phenomenon generated from a scalar implicature arising from the use of ‘or’ when using ‘and’ would be more informative, thus ruling out the possibility of both disjuncts being true. This position is not unanimously accepted in pragmatics; Geurts (2005), Sevi (2005), and Schulz (2006) are examples of linguists who have recently proposed other pragmatic mechanisms. Assertibility also conflicts with the standard position, as no disjunction is assertible when the conjunction of its disjuncts is assertible. Instead, I believe the presence of exclusivity depends primarily on the contrariety relationship between the disjuncts.

Two very desirable cooperative goals for a Speaker are to ensure their disjunction is maximally relevant and minimally ambiguous. The primary conditions for a \( \langle \delta, \epsilon \rangle \)-assertible disjunction \( \varphi \lor \psi \) are that \( \pi(\varphi \lor \psi) \geq 1 - \epsilon \), \( \pi(\varphi \land \neg \psi)/\pi(\varphi \lor \psi) > \delta \) and \( \pi(\neg \varphi \land \psi)/\pi(\varphi \lor \psi) > \delta \). The optimal strategy for maximising the relevancy \( \delta \) of both disjuncts is to select \( \varphi \) and \( \psi \) so as to maximise \( \pi(\varphi \land \neg \psi) \) and \( \pi(\neg \varphi \land \psi) \), which leads to the minimisation of \( \pi(\varphi \land \psi) \). Thus, whenever possible, a cooperative Speaker should select disjuncts whose intersection is of minimal weight to maximise the relevance of each disjunct. This minimisation of overlap is greatly eased by the partitioning effect of hierarchical lexical taxonomies, which means that comparable disjuncts are often \( l \)-contrary. Following this strategy also reduces the ambiguity between exclusive and inclusive readings. The disjunctive phrase becomes completely unambiguous if the Speaker can select contrary disjuncts, as although it is now completely indeterminate whether the disjunction is truth-conditionally inclusive or exclusive, there is no relevant scenario where this can matter. This relevance maximisation strategy is only possible if the context allows the Speaker to choose their disjuncts, rather than them being predetermined by convention or context. When it is unclear whether the disjuncts are sufficiently \( c \)-contrary, the Speaker may append ‘or both’ or ‘and not both’ to the disjunction so as to avoid confusion.

**Prediction 4.4 Exclusive Disjunction.**

**Condition:** The disjuncts of a symmetric disjunction are \( c \)-contrary or \( l \)-contrary.

**Effect:** The disjunction is read as exclusive; the conjunction of the disjuncts is treated as irrelevant.

**Example:** See (4.18a).

This prediction is only generated when the disjunction is symmetric and in positive scope. It can also apply to conjunctions in negative scope. When the disjunction is asymmetric, Pred 4.7 is applicable instead. Importantly, it is the semantic or pragmatic relationship between the disjuncts, rather than the disjunctive coordinator itself that generates the prediction.

### 4.2.2 Asymmetric Disjunctions

Natural language utterances are inherently linear sequences of simple elements (usually phonemes or graphemes). Both formal syntactic structures and propositional formulas are, however, essentially variations of binary trees. The most important factors when converting from a tree to a sequence are the order in which the tree nodes are walked and when each node is appended to the sequence, so it is not surprising that a significant amount of information can be conveyed by the choice of word or phrase order. In English and many other languages coordinand order often conveys an underlying epistemic ordering by temporality, causality, explication, generality, importance, likelihood, relevance, or complexity. The order underdetermines which of these is being communicated, and sometimes the potential orderings conflict. Much of the evidence for identifying which ordering(s) are conveyed comes from a combination of the utterance form, the semantics of the coordinands, their connexion, and the physical constraints of the situation being described.

(4.19) a. Vanessa is holidaying in Monaco, or somewhere in Europe.
    b. ? Vanessa is holidaying somewhere in Europe, or in Monaco.
    c. Jan, or both Jan and Jean, will take Jenny to the airport.
    d. Paul is in France or Germany.
4.2. EXCLUSIVE & ASYMMETRIC DISJUNCTION

Assertibility identifies (and rejects) redundant disjuncts, and when combined with connexion can predict the presence of an implicit ordering. For example in (4.19a) the first disjunct is strictly less general than the second disjunct, and the utterance form would be true in the same cases if it lacked the first disjunct, so it is truth-conditionally redundant. This redundancy suggests that Monaco is the first, preferred, or most likely country Vanessa is visiting, as there is no other reason to mention it apart from this underlying order. The reversed disjunction in (4.19b) doesn’t permit any of the standard connexion and ordering interpretations suggested above, which makes it very difficult to interpret. Notation for asymmetric disjunctions was introduced in §3.4, and (4.19a) has the form \( \varphi \rightarrow \psi \), with the arrow in the disjunction symbol indicating the ordering. Similarly, (4.19c) has the form \( \varphi \leftrightarrow \psi \), as the right disjunct ‘Jan and Jean’ can be interpreted without consideration of the first disjunct while the converse is not the case. The symmetric (4.19d) is of the form \( \varphi \vee \psi \), as each disjunct is potentially affected by the other, although in this case the disjuncts are \( l \)-contrary due to physical laws, and cannot both be true.

(4.20)  

a. Rachael always goes dancing at a fashionable club, or [at least] one with trendy music.  
b. Jeremy enjoys listening to Shostakovich, or [even] some Myaskovsky.  
c. Marco supports either the French or German football team.

The disjunctions in (4.19) all had \( l \)-contrary relationships between their disjuncts. The same patterns of reasoning apply when this relationship is the defeasible \( c \)-contrariety. The disjunctions (4.20a) and (4.20b) should be interpreted as having the forms \( \varphi \leftrightarrow \psi \) and \( \varphi \leftrightarrow \psi \) respectively, with the disjunctive modifiers ‘at least’ and ‘even’ indicating the direction of asymmetry. The symmetric form of either of these disjunctions is unassertible due to one of the disjuncts being \( c \)-contrary to the negation of the other. The symmetric (4.20c) also has a \( c \)-contrary possibility, as it is unlikely that anyone would support both the German and French teams, but this possibility does not affect the assertibility conditions. One reliable indicator of both (a)symmetry and intended contrariety is the use of particular disjunctive modifiers. English modifiers for \( \varphi \leftrightarrow \psi \) include ‘or else’, ‘or at least’, ‘or just’, and ‘or only’, while \( \varphi \leftrightarrow \psi \) can be marked by ‘or even’, ‘or both’, ‘or all [of them]’, and similar compound coordinators. In addition, ‘only’, ‘just’, ‘both’, and ‘all’ tend to be used when there is asymmetric \( l \)-contrariety, while the weaker ‘at least’ and ‘even’ are more commonly found with \( c \)-contrariety. The modifier ‘either’ often indicates a symmetric disjunction with contrary disjuncts.

(4.21)  

a. Jean has been to France or Germany.  
b. Each weekend, Jenny dances on Friday, or goes to church on Sunday.  
c. Either Jan will return the book promptly, or she really needs it.

Finally, there are disjunctions that are asymmetric due to factors other than contrariety. While all the examples in (4.21) are logically symmetric, the disjuncts in (4.21b) have a temporal connexion, and those of (4.21c) have an epistemically causal connexion. In both cases the temporally/causally prior disjunct can be understood without reference to the subsequent disjunct, and so the comprehension process is structurally identical to that of logically asymmetric disjunctions. A finer-grained classification of connexion, such as that of Dixon and Aikhenvald (2009), would assist us in specifying the connexive relationships and modes of coordination that can lead to non-contrary asymmetric disjunctions such as those just discussed\(^3\).

The interaction of asymmetry and contrariety also affects whether a disjunction is interpreted as exclusive. The strength of the exclusivity prediction for symmetric disjunctions increases with the degree of contrariety, with the \( l \)-contrary symmetric (4.19d) usually being understood as exclusive.

\(^3\)An alternative way of modelling these connexively asymmetric disjunctions is as subordinations rather than coordinations. Then (4.21b) would be read as ‘if Jenny doesn’t go dancing on Friday, she goes to church on Sunday’, and (4.21c) as ‘Jan will return the book promptly [unless/except if] she really needs it’. In some languages, the division between coordination and subordination is quite clear; in others it is not. Also, the divide can occur in different places in languages where the division is clear, see e.g., Johannessen (1998) and Cristofaro (2003). Cross-linguistically therefore, coordination/subordination is more of a punctuated continuum than a simple binary distinction.
Asymmetric disjunctions have the opposite tendency: the stronger the contrariety-induced implication from one disjunct to another, the more likely that both are true. For example, both (4.21b) and (4.21c) can be read as exclusive, while the $l$-contrary (4.19a) and (4.19c) and $c$-contrary (4.20a) and (4.20b) do not admit this interpretation, although they are all asymmetric.

The logical and connexive relationships between disjuncts are not the only indicators of asymmetry; further clues on asymmetry can be found in the syntactic structure. The cross-linguistic data in Mauri (2008) suggests that the degree of both syntactic and semantic parallelism (due to Ellipsis or Node Promotion) that occurs in the coordination structure correlates with the temporal independence of the coordinands. Disjunctions are more often symmetric, less often temporally ordered, and exhibit more parallelism than conjunctions, and their adherence to these three criteria is highly correlated. That is, disjunctions with non-parallel syntax are usually asymmetric.

Yet another common indicator is comma intonation\textsuperscript{4}. All the asymmetric disjunction examples above have a comma preceding the coordinator, and most spoken asymmetric disjunctions are pronounced with comma intonation. Comma intonation is neither a necessary nor sufficient condition for asymmetry, however, as it can indicate that any of a variety of factors are present. For instance, it might indicate that the coordination is behaving more like a subordination, which usually results in asymmetric coordination. Comma intonation is also common in polyadic disjunctions, and when disjuncts contain nested coordinations of higher degree such as in ‘tee-shirts are available in orange, or black and white’. In addition, almost all performative and renunciative coordinations will have comma intonation or a similar prosodic indicator. Sweetser (1990) says in her §4.1.2 that causal and adversative subordinating conjunctions (e.g., ‘because’, ‘although’, ‘since’) have comma intonation iff they belong to her epistemic or speech act domains (my evocative, performative, and renunciative modes). There also appears to be a similar, though weaker, correlation for coordinations.

**Prediction 4.5 Asymmetric Disjunction 1.**
*Condition:* The evaluation of one of the disjuncts is not affected by the evaluation of the other.
*Effect:* The disjunction is read as asymmetric.
*Example:* See (4.19a), (4.19c), (4.20a), (4.20b), (4.21b), and (4.21c).

Sometimes a disjunct can only be evaluated in the context of the other disjunct having been considered and rejected, or in the context of the other having not yet been considered and accepted. These asymmetric disjunctions have common logical properties under assertibility, and can result in different predictions from symmetric disjunctions. Asymmetry usually occurs when there is some type of contrariety between a disjunct and the negation of another.

**Prediction 4.6 Asymmetric Disjunction 2.**
*Condition:* There is an asymmetric modifier associated with the disjunction coordinator.
*Effect:* The disjunction is read as asymmetric. The modifier indicates the direction of ordering.
*Example:* See (4.19c) or (4.20b).

English modifiers for $\rightarrow\lor$-disjunction include ‘or else’, ‘or at least’, ‘or just’, and ‘or only’. Those for $\leftarrow\lor$-disjunction include ‘or even’, ‘or both’, ‘or all [of them]’. This prediction is not generated by assertibility, but as it refers to asymmetric disjunction, it is dependent on assertibility.

**Prediction 4.7 Exclusive Asymmetric Disjunction 1.**
*Condition:* An expositive or evocative disjunction is asymmetric due to the temporal or causal connexion between disjuncts, as opposed to a (probabilistic) entailment relationship.
*Effect:* The disjunction is read as exclusive; the conjunction of the disjuncts is treated as irrelevant.
*Example:* See (4.21b) and (4.21c).

Across most asymmetric disjunctions, left-to-right $\lor$-coordination is much more common than right-to-left, possibly as they are processed in the order that they are uttered, with only (and all) prior subutterances being relevant to its evaluation. This order also corresponds to the order of real-world events, and so reflects any causal, temporal, or epistemic dependencies of the disjuncts.

\textsuperscript{4}Comma intonation is an indication of prosody, and while it is highly correlated with the use of a comma in writing, punctuation conventions (such as Oxford commas) are independent of comma intonation.
4.3 Non-Expositive Disjunction

The disjunctions that first come to mind may be expositive, but many other coordination modes also have a number of properties attributable to the restrictions imposed by assertibility.

4.3.1 Evocative Disjunctions

The connexion between evocative disjuncts is due to an indirect and potentially nebulous chain of reasoning, making their relationship more tenuous than that of expositive disjuncts. Symmetric and asymmetric disjunction also occur in the evocative mode, but the indirect and non-deductive nature of the connexion allows for greater interplay within the \( \delta \) probabilistic thresholds. The examples in (4.22) contain symmetric evocative disjunctions, while those in (4.23) are asymmetric. Note that (4.22b) is also an open disjunction, and that (4.23c) has a seemingly impossible disjunct, so would appear to be unassertible at first glance. The non-transitive nature of the connexion chain and the decreased emphasis on accuracy, make the evocative mode ideal for describing the most weighty alternatives in a piece of extended reasoning. The common use of modal auxiliaries and adverbs such as ‘certainly’, ‘must’, and ‘has to be’ assist by indicating that the Speaker is anything but certain. An epistemic disjunction in a piece of otherwise close-knit or structured reasoning may indicate a missing step or the presence of a false dilemma. It can also indicate speculative or ampliative reasoning, as epistemic disjunctions always have indirect non-deductive connexions between their disjuncts, and so there must be a series of intermediate propositions that serve to connect the disjuncts via direct connexions. When these implicit intermediate propositions are already part of the topic or common ground, they will be made more prominent by the disjunction. When they are not, more imagination is required by the Hearer. The following examples, except for (4.23a), come from Sweetser (1990).

(4.22) Symmetric Evocative disjunction:
   a. John is home, or someone is picking up his newspapers.
   b. A: You were supposed to get the decision about that job you applied for.
      B: Yeah. Well the mail delayed it, or they got held up making their decision, or there
      was some problem . . .

(4.23) Asymmetric Evocative disjunction:
   a. John will be in the kitchen, or perhaps the wine cellar.
   b. John will be home for Christmas or I’m much mistaken in his character.
   c. John eats pancakes for breakfast, or I’m the Shah of Iran.

Prediction 4.8 Inference to Available Explanation(s).
Condition: Each disjunct in an evocative disjunction serves as a possible physical or epistemic cause, or explanation, for an event or proposition under discussion.
Effect: The disjuncts are collectively taken as the best, and possibly only relevant, explanations for the event or proposition being referred to.
Example: See (4.22) or (4.23).

If phrasal intonation or other markers indicate that the disjuncts may not capture all the relevant explanations, as in (4.22b), it can lead to the additional Pred 4.11.

Prediction 4.9 Inference to Best Explanation.
Condition: An asymmetric evocative disjunction satisfies Pred 4.8.
Effect: The disjuncts are ordered by decreasing subjective probability or plausibility, so the non-redundant (usually left) disjunct is the preferred and most weighty assertion.
Example: See (4.23a)–(4.23c), and possibly (4.22b).
The modal ‘perhaps’ in (4.23a) explicitly marks the disjunction as asymmetric. Similarly, the self-effacing nature of the second disjunct of (4.23b) reinforced by the use of ‘much’ to indicate a degree in excess of the Speaker’s expectation, and the degree of improbability of the second disjunct of (4.23c), indicate their respective asymmetry.
Prediction 4.10 Strengthening Disjunction.

Condition: An asymmetric evocative disjunction has an improbable or infelicitous final disjunct.

Effect: The remaining disjunct(s) are asserted more confidently than if the improbable disjunct had not been uttered.

Example: See (4.23c), and possibly (4.23b).

If a disjunction is asserted then it must be at least as relevant as any alternative disjunction, so there cannot be a weightier alternative to the final disjunct when the prior disjunct(s) are false. As the final disjunct is highly improbable, the $\delta$ threshold (and hence the $\epsilon$ threshold) for the assertion must be surprisingly low, making the probability $(1 - \epsilon - \delta)$ of the prior disjunct(s) higher than we would normally expect for a simple assertion in the current context. An alternative explanation of the prediction is that denying the prior disjuncts would lead to an implicit assertion of the final disjunct. As this disjunct is unassertible, either through improbability as in (4.23c), or infelicitous through rudeness as in (4.23b), the prior disjunct(s) are not only assertible, but unchallengeable.

Prediction 4.11 Open Disjunction.

Condition: A disjunction has a trailing ‘...’, ‘or...’, or ends in a high final intonation.

Effect: The disjunction is open, and hence evocative, but the remaining alternatives are too improbable or infelicitous to assert.

Example: See (4.22b).

Open Disjunctions can be symmetric or asymmetric, but are always evocative. If the disjunction is open because there is an obvious and relevant final disjunct that is too impolite or horrific to mention, the disjunction acts as a closed asymmetric disjunction that includes the unmentioned disjunct, but without the undesirable effects of mentioning it. More commonly, the remaining possibilities break down into a number of individually insignificant or unidentifiable but collectively weighty disjuncts that cannot be encapsulated by a relevant subordination class.

The difference between open and closed disjunction is that open disjunction does not exhaust the current epistemic possibilities. It is usually understood that the disjunction conveyed by the English ‘or’ is closed, and an open interpretation requires additional indicators such as a rising final intonation or a trailing ‘or...’.

languages such as Chinese avoid this potential ambiguity by having different particles for open and closed disjunction. Dixon and Aikhenvald (2009) claims that relatively few languages have a dedicated closed disjunction coordinator like English, instead using a modal particle or conditional indicator, roles fulfilled by words like ‘perhaps’ and ‘if’ in English. Those that use modal particles allow open disjunctions to be interpreted as closed either in specific contexts or by using non-standard intonation. Zimmermann (2000) and Geurts (2005) claim that there is no reason to privilege the semantics of closed disjunctions over open disjunctions in English. This is because the English ‘or’ can convey both open and closed disjunction, where the only phonetic difference is that closed disjunctions typically end in a low intonation while open disjunctions have a high final intonation. I agree, and am happy to consider the variation a form of polysemy.

Dixon claims that most languages only have open disjunctions of the form $\Diamond \varphi \lor \Diamond \psi$ (for the epistemic possibility operator $\Diamond$). If so, how does this form compare with the closed disjunction $\varphi \lor \psi$, for which we have developed assertibility? The answer is surprisingly elegant: the assertibility conditions for $\varphi \lor \psi$ are exactly those for $\Diamond \varphi \lor \Diamond \psi$, with the addition of $T(\Diamond \varphi \lor \psi)$ (to show this, merely substitute $\Diamond \varphi$ and $\Diamond \psi$ for $\varphi$ and $\psi$, and simplify). The assertibility conditions for open disjunctions are therefore identical to those for closed disjunctions, apart from their lack of truth conditions. For those keen on observing a strict divide between semantics and pragmatics, this suggests that some of the non-truth-conditional aspects of open (and hence all) disjunction are semantic. Taking this semantic appeal to non-truth-conditional properties to extremes, Zimmermann (2000) and Geurts (2005) regard open disjunction as the basic form in their work on Free Choice (see §5.4.1), and so have dispensed with truth conditions for all disjunctions. In contrast, I have just shown that assertibility can model uses of both open and closed disjunction through the same approach that many languages have taken (by the use of modal particles prefixing each disjunct), based on the classical truth-functional semantics for closed disjunction.
4.3. NON-EXPOSITIVE DISJUNCTION

4.3.2 Deductive Disjunctions

Deductive disjunctions are surprisingly and counter-intuitively restricted to instances like (4.24a), in which one disjunct is true only when the other is (in context). That is, one disjunct must be \( l \)-contrary to the negation of the other. It appears that there are no symmetric deductive disjunctions (every disjunction I have found where each disjunct entails the other is either performative or renunciative). Deductive disjunction (and conjunction) is usually not particularly interesting, but there are some predictions that only arise in this mode. There is a pronounced preference for deductive \( \leftrightarrow \)-disjunction over \( \rightarrow \)-disjunction, as there is with all asymmetric disjunctions.

\[\text{(4.24)}\]
\[\begin{align*}
\text{a. } & \text{A parallelogram with four right angles is a square or a rectangle.} \\
\text{b. } & \text{Jane’s pet ‘Dogtucker’ is either a cow or a bull.}
\end{align*}\]

\[\text{(4.25)}\]
\[\begin{align*}
\text{a. } & \text{* John was born in Paris or in France. (Singh (2008))} \\
\text{b. } & \text{John was born in Paris, or at least in France.} \\
\text{c. } & \text{John was born in Paris, or somewhere in France.}
\end{align*}\]

Some disjunctions such as (4.25a) seem to be unacceptable, even when minor variations of their wording produces near-synonymous disjunctions that are apparently fine. This unacceptability occurs when the disjuncts are in a hyponymous or meronymous relationship; that is, when one forms a subclass of the other in some lexical hierarchy. Singh (2008) over-generalises from the unacceptability of (4.25a) to the patently false claim that all asymmetric disjunctions are unacceptable. As both Kai von Fintel and Irene Heim point out in their paper, (4.25b) is an acceptable variation of (4.25a). Singh’s reply that these are not “actual disjunctions, but retractions, whereby the speaker weakens her initial assertion \ldots [(4.25b)] seem[s] to be felicitous only if pronounced with comma intonation” is essentially fallacious. He rejects this and all other instances of ‘or’ that do not match his theory as ‘not actual disjunctions’ by showing they do not fit his theory, which in turn is motivated primarily by its success in predicting which instances of ‘or’ are disjunctions. I have discussed in §3.7.2 why these corrective \( \leftrightarrow \)-disjunctions do not require a distinct formalisation as renunciative disjunctions (although corrective \( \rightarrow \)-disjunctions do), and I have also addressed comma intonation above. This still leaves us with (4.25a) as an interesting instance of an asymmetrically assertible disjunction that seems unacceptable due to its form. The relationships between the disjuncts in the unacceptable (4.25a) and in the acceptable (4.24a) both appear to be simple taxonomical inclusion, but the former is a matter of fact while the latter may be a choice of description and thus metalinguistic. The difference between (4.25a) and the relatively acceptable (4.25c) seems to be that a node in a taxonomic hierarchy cannot be directly disjoined with a higher node, although it may be disjoined with a grouping of its siblings, as in (4.25c). The contribution of ‘at least’ to (4.25b) that makes it acceptable is even more opaque. Whatever the correct explanation, some criteria other than assertibility are responsible for rejecting (4.25a) but not the other deductive disjunctions.

An interesting deductive variant occurs when a disjunct is polysemous, and one interpretation of this disjunct is entailed by the other interpretation and by the other disjunct. An example is (4.24b), where ‘cow’ is polysemous as ‘cow\(_2\)’ refers to an adult female ‘cow\(_1\)’, and [[bull]] and [[cow\(_2\)]] are disjoint subsets of [[cow\(_1\)]]. This contrastive autohyponomy or automeronymy occurs for many pairs, including ‘rectangle’/‘square’, ‘duck’/‘drake’ and ‘man’/‘woman’. The non-deductive interpretation is always preferred where available, resulting in a symmetric \( l \)-contrary disjunction, which generates an exclusive reading.

**Prediction 4.12 Exclusive Asymmetric Disjunction 2.**

*Condition:* In a potentially asymmetric deductive disjunction the disjuncts are \( l \)-contrary under an alternative interpretation of one of the disjuncts.

*Effect:* The disjuncts are interpreted as being \( l \)-contrary, the disjunction is read as symmetric and \( l \)-contrary, and so exclusive by Pred 4.4.

*Example:* See (4.24a) and (4.24b).

Wherever possible, disjunctions will be interpreted as having \( l \)-contrary disjuncts, as this minimises disjunctive ambiguity and maximises the relevance of each disjunct, as discussed in §4.2.1.
4.3.3 Performative Disjunctions

Performative disjunctions present two alternative illocutionary acts for the Hearer to select between. Symmetric and asymmetric performative disjunctions are formally similar, but generate quite distinct predictions. Symmetric performative disjunctions whose disjuncts are both assertions tend to have parallel distinct forms that are then elided, so are usually phrasal rather than clausal. Asymmetric disjunctions almost always involve non-assertions, such as an imperative or question, so are usually clausal disjunctions which modify the strength of the first disjunct. The symmetric performative disjunction was formalised in Defn 3.17, and the left-redundant asymmetric variant described shortly afterwards. I have not formalised the variations for disjoining speech acts other than assertion; they will be discussed in §5.5.

(4.26)  
a. Give me a hotdog or a sandwich. (Sweetser (1990))  
b. Have an apple turnover, or would you like a strawberry tart?  
c. King Tsin has great mu shu pork, or China First has good dim sum, or there’s always the Szechuan place just around the corner.  
d. King Tsin has great mu shu pork, and China First has good dim sum, and there’s always the Szechuan place just around the corner.

The symmetric performative disjunctions in (4.26) each provide a (putatively) complete list of assertible alternatives. This may be in the form of an imperative which can be satisfied in diverse ways (4.26a); multiple offers, only one of which you are expected to accept (4.26b); or a list of recommendations, only one of which can be taken up in the current circumstances (4.26c). In each of these examples every disjunct is assertible, and some information that is available to the Hearer but not the Speaker determines which is most satisfactory.

Prediction 4.13 Hearer’s Choice Disjunction.  
Condition: Each disjunct of a symmetric performative disjunction is assertible, and some satisfaction conditions for each disjunct are evaluable by the Hearer but not Speaker.  
Effect: The disjunction is interpreted as an assertion (or other appropriate speech act) of whichever disjunct the Hearer finds most satisfactory.  
Example: See (4.26a)–(4.26c).  
The disjuncts are usually interpreted as being $c$-contrary, generating exclusivity as per Pred 4.4.

The performative disjunction (4.26c) and the expositive conjunction (4.26d) seem almost synonymous, but they are not instances of arbitrary coordination as described in §4.1.3. Instead, the similarity follows from Lemma 3.18.1, which establishes that $A(\langle \phi \rangle \lor \langle \psi \rangle)$ entails $\Diamond A(\langle \phi \rangle)$ and $\Diamond A(\langle \psi \rangle)$. The effect is subtle because the speech act in (4.26c) is a recommendation of locales for a specific event and thus the satisfaction conditions for the disjuncts are $l$-contrary (bilocation is impossible). The corresponding speech act in (4.26d) is a more general recommendation of possible locales, so both conjuncts may be simultaneously satisfactory. That is, (4.26c) suggests that any single option would be acceptable, while (4.26d) provides a list of suggested acceptable options.

Prediction 4.14 Performative Arbitrary Coordination.  
Condition: A performative disjunction of requests or suggestions appears near-synonymous with the conjunction of the corresponding disjuncts presented as possibilities.  
Effect: The disjunction is interpreted as an exhaustive conjunction of possibilities.  
Example: See (4.26c) and (4.26d).  
A performative disjunction like (4.26c) that offers a range of options by the assertion of facts relevant to each alternative’s desirability implies, and can appear to be virtually synonymous with, the performative or expositive conjunction (4.26d). Note that the exhaustive nature of the conjunction is only due to the disjunction being closed; an open disjunction is even more closely related to the non-exhaustive conjunction of respective possibilities. This performative variant of Free Choice is discussed in more detail in §5.4.2.
4.3. NON-EXPOSITORY DISJUNCTION

Consider the following disjunctions of conditionals due to Johnson-Laird and Byrne (1991).

(4.27) a. If I have a Jack, I have a King, or if I don’t have a Jack, I have an Ace.
   b. If John is in Paris, he is in Turkey, or if he is in Istanbul, he is in France.

If we treat the above conditionals as material and the disjunctions as content-based (expositive, evocative or deductive) then (4.27a) is a tautology, and given our common knowledge that France and Turkey do not overlap, (4.27b) must always be true in the context. That is, they are uninformative, and so unassertible\(^5\). In (4.27a) the Speaker is asserting two bidding rules, both of which are true, and only one of which is applicable in any given scenario. The Hearer can then decide which scenario is likely. The disjunction in (4.27a) is thus a standard assertible symmetric performative disjunction. The performative interpretation of (4.27b) combined with some common knowledge, presents a choice between two conditionals, neither of which can ever be informative, and so it is still unassertible.

(4.28) a. Happy birthday! Or did I get the date wrong?. (Sweetser (1990))
   b. Give me liberty or give me death!
   c. Give me that book, or I’ll throw your cat into the lake.

Sweetser (1990) observes on p. 98 that in all the asymmetric cases of performative disjunction that she has found, the disjunction is a $\Rightarrow\lor$-disjunction, with the later disjunct being subordinate in one of two roles: “it either gives the Speaker a ‘loophole’ against potential infelicity . . . , or bolsters the primary conjunct by presenting an unacceptable alternative”. These roles are exemplified in (4.28a), and both (4.28b) and (4.28c) respectively. van Dijk (1979) points out that a common way of guarding against infelicity is to use later disjuncts to explicitly query the key presupposition for the first disjunct, as shown in (4.28a). The presentation of an unacceptable alternative, such as an imperative to kill the Speaker (4.28b) or an assertion that the Speaker will undertake a heinous act (4.28c), strengthens the other disjunct. This serves as a performative variant of the strengthening Pred 4.10 by providing as the ‘best’ alternative a disjunct which the Hearer wishes to make improbable, rather than one which is intersubjectively considered unlikely or insignificant.

**Prediction 4.15 Performative Strengthening Disjunction.**

**Condition:** An asymmetric performative disjunction has an unacceptable final disjunct.

**Effect:** The Hearer will act to ensure (one of) the remaining disjunct(s) is felicitous with more alacrity or vigour than if the unacceptable disjunct was not mentioned.

**Example:** See (4.28b) and (4.28c).

Performative disjunctions with an unacceptable final disjunct usually have an imperative (or other Hearer-satisfiable speech act) as an earlier disjunct. If the final disjunct is also an imperative (4.28b), the only asymmetry is in the Hearer’s preferences. If the other disjunct is an unacceptable assertion (4.28c), or a question with only unacceptable answers (e.g., ‘which family member would you like me to kill first?’), the overall disjunction behaves very much like a conditional. This pattern, along with the related conjunctive conditional, is discussed in §5.5.2.

4.3.4 Renunciative Disjunctions

Renunciative disjunctions offer an assertion followed by an alternative that provides (at least) the same information via a different locution. One of the disjuncts (usually the first) is then implicitly withdrawn, as the Hearer accepts the other, more informative or appropriate, option. Any predictions or inferences drawn from manner and politeness properties of either disjunct are part of what is conveyed by an utterance. If the first disjunct is offensive, disjoining it with an acceptable synonymous second disjunct may slightly reduce offence by an implicit acknowledgement of its inappropriateness. However, if the first disjunct is acceptable but the second offensive, any offence is compounded by

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\(^5\) Some types of procedural, non-truth-bearing clauses cannot be disjoined at a content level (i.e., in the expositive, evocative or deductive modes), only by presenting a choice of assertions. In §5.6 I will argue why I believe all conditionals fall under this category, even though very few of them should be modelled as material conditionals.
the knowledge that it was deliberately uttered although it added nothing to the conversation except to offend. Unsurprisingly, renunciation of a more acceptable disjunct in favour of a less acceptable disjunct is generally fairly offensive, if not done in jest.

The introductions in (4.29) provide minimal contrasts between the different types of renunciative disjunction, while (4.30) presents some near-misses. Further examples can be found in (3.21).

(4.29)
a. This is Sue, or Susie.
b. This is Sue, or [rather] Sioux.
c. This is Sue, or [rather] Susan.
d. This is Sue, or rather Sue and Mark.

(4.30)
a. This is Susan rather than Sue.
b. * This is Sue, or (rather/should I say) Stephanie.

Symmetric disjunctions such as (4.29a) provide a choice of two equally acceptable locutions. Asymmetric locutionary disjunctions are about preferences; when the preference is that of the Speaker, the coordination is often followed by pragmatic markers indicating preference, such as ‘rather’. This preference can be one of spelling or pronunciation (4.29b), a register of formality (4.29c), or other subtleties of locution; a fuller list can be found at the start of §3.7.2. The preference can be made explicit by reversing the order of the coordinands and using the comparative conjunction ‘rather than’ instead of a disjunction (4.30a). Propositional disjunctions are more restricted as they can only be asymmetric. They strengthen a weak claim so that a full description of the situation would not involve further redundancy, thus avoiding incorrect exhaustiveness predictions via the Extensibility norm. In (4.29d) Sue is presented as part of a couple, perhaps to forestall single men from assuming that she is available, or roving aunts from finding said men to get her safely married off. The line between renunciation and correction is often contextual; if Mark appears at Sue’s elbow partway into the introduction, then the second disjunct is more likely to serve as a correction to the original introduction. One clear-cut case of corrective, non-renunciative disjunction is (4.30b), where the Speaker has one of those unfortunate brain-freeze moments that we can all relate to, and utters the wrong name. This is not a standard renunciative disjunction as the first disjunct is simply incorrect, and the disjunction is unassertible as the first disjunct does not accurately represent any part of the Speaker’s epistemic stance. Corrective disjunction and its relationship to renunciative disjunction will be discussed further in §4.3.5.

**Prediction 4.16 Hearer’s Choice of Wording Disjunction.**

*Condition:* The disjuncts are roughly synonymous, but have different locutions.

*Effect:* The Hearer may select any information, implications, and insinuations from the non-accepted locution, including both positive and negative aspects, except for those explicitly excluded by the accepted disjunct.

*Example:* See (4.29a) and (4.29b).

If there is a tension between disjuncts (i.e., c-contrariety), the second disjunct overrides the first.

**Prediction 4.17 Addendum Disjunction.**

*Condition:* Both disjuncts are true (and assertible), and the propositional content of one is a strict subset of the other. Indicators such as intonation show that the later disjunct supersedes the earlier.

*Effect:* The Hearer will regard the assertion as consisting only of the second disjunct.

*Example:* See (4.29c) and (4.29d).

The conditions for this prediction are similar to those for the standard asymmetric Pred 4.5, and the appropriate prediction for an utterance is usually only identifiable by reference to the mode and resulting intonation. If the additional information in the second disjunct cannot be conjoined to the first disjunct, as described in Lemma 2.103, then this is probably the applicable prediction. If the conditions for Pred 4.12 are met, then the disjuncts are interpreted as l-contrary, and the conditions for this prediction are not met.
4.3.5 Exceptional Disjunctions

There are many other ways of classifying disjunction, and some of them do not neatly fit within one of the coordination modes outlined above. I will examine some classes suggested by Haspelmath (2000), Eckardt (2007), and Mauri (2008). I have found assertibility to be useful for describing some aspects of each of these classes. Its inability to fully describe, and thus predict, all these classes indicates that additional aspects may need to be added to the list of deviations in Chapter Three, perhaps by introducing finer discrimination into the connexion classes.

Strictly Inclusive Disjunction

The existence of the English compound coordinator ‘and/or’ is sometimes presented as evidence that natural language disjunction is either exclusive by default, or hopelessly ambiguous. Its use reduces ambiguity by explicitly permitting the possibility of both disjuncts being true rather than leaving them to be determined by pragmatic or contextual considerations. Many grammarians, teachers, internet bloggers, and other self-appointed guardians of English rail against ‘and/or’, but its 800 million google hits indicates it may be here to stay. This disjunction indicates when potentially c-contrary disjuncts can both be true, which otherwise would require a triadic disjunction ‘A or B or [both]’. It seems that the need to be more precise in parts of our highly regulated society has led to increased pressure on the inclusive/exclusive ambiguity in ‘or’, and eventually this resulted in a new lexeme, in the way of many semantic shifts. The existence of a new coordinator ‘and/or’ does not in any way undercut my analysis that the disjunctive coordinator ‘or’ has a truth-conditionally inclusive condition. Moreover, this new coordinator can be modelled as a conjunction of the assertible ‘and’ and either the inclusive or exclusive ‘or’ coordinators (with appropriate encapsulation), as presented in §3.4 and discussed further in Appendix A.3.

\[(4.31)\] Every owner of a cat and/or a dog should provide regular food and affection for their pet.

**Prediction 4.18 Strict Inclusive Disjunction.**

*Condition:* The disjunctive coordinator used is ‘and/or’

*Effect:* Any potential exclusive disjunction prediction is cancelled before it can occur.

*Example:* (4.31).

The use of ‘and/or’ only really makes sense when an exclusive interpretation of the disjunction would exclude some relevant possibility, and contextual considerations are insufficient to rule out this interpretation. This generally happens in a regulative or evaluative environment, where context is not sufficiently allowed for, producing ambiguity unless the desired interpretation is stipulated.

Quantificational Disjunction

Some disjunctions range over individuals rather than epistemic possibilities. These quantificational disjunctions appear to challenge our conception of disjunctions as describing alternative epistemic possibilities. Eckardt (2007) discusses this issue, and provides the following examples:

\[(4.32)\]

a. Everyone ordered a beer or a pizza.

b. Someone ordered a beer and someone ordered a pizza.

Eckardt claims that (4.32a) can be asserted by a waitress who has perfect knowledge about the order. However this assertion cannot be intended to convey all her information about the order, nor even sufficient information for other staff to take effective action in preparing the order. It may instead be intended to convey that the owner’s new upscale menu of bruschetta and saltimbocca is not popular, or that none of the unruly customers may be ejected as they have all placed orders. I agree with Eckardt that the waitress is not expressing a global epistemic uncertainty about who ordered a beer vs. a pizza. However the other staff will be in a state of local epistemic uncertainty with respect to each customer after the assertion. The epistemic uncertainty thus relates to the communicative content, not the waitress’s knowledge. As an aside, (4.32b) follows from (4.32a), and in §5.4.2 I will argue that this Free Choice effect is grounded firmly in epistemic possibility.
**Disjunctions in Proofs**

Mathematical and logical reasoning is deductive, but a proof imposes more constraints on the use of a disjunction than it simply being licensed to follow from previous steps. A proof by cases is a form of disjunction, and the Speaker must assume that each case is possible for the purposes of the proof, even if she knows that it will lead to a contradiction. That is, when a disjunction $\varphi \lor \psi$ is asserted as part of a proof, both $\Diamond A\varphi$ and $\Diamond A\psi$ are part of the meaning of the disjunction, even if one of them is actually inconsistent in the context. This neutral epistemic stance to each disjunct during the utterance of the disjunction is essential to the operation of Disjunctive Syllogism. Proofs by cases may occasionally have either $\varphi \land \neg \psi$ or $\psi \land \neg \varphi$ as a case (that is, $\Diamond (A\varphi \land \neg T\psi)$ or $\Diamond (A\psi \land \neg T\varphi)$), but never both unless $\varphi \land \psi$ is a third case as *every* epistemic possibility must be accounted for in a proof. Interestingly, exclusive disjunction is not a standard part of mathematical reasoning; satisfying both disjuncts is always sufficient to satisfy a disjunction. This may be due to the overriding importance of truth and correspondingly low priority to maximise relevancy of disjuncts in the deductive mode (particularly in maths), which means the exclusivity-generating conditions of Pred 4.4 do not occur.

**Corrective Disjunction**

Asynchronous disjunction can be used to correct an erroneous first disjunct, such as in (4.33), rather than to provide additional options. Corrective disjunction appears to violate assertibility, as there is no chance that the first disjunct is true. However, the Speaker thought that it was an epistemic possibility *at the time* that the first disjunct was spoken; it is only after having spoken (and possibly heard her own utterance) that the Speaker realises her error and wishes to correct herself. If the error is caught in time, disjunction is a possible, face-saving, correction strategy. Most other corrective approaches involve an explicit admission of misspeaking, but disjunction avoids this through its ambiguity. Corrective disjunction could thus be thought of as a form of asymmetric propositional renunciative disjunction where the second disjunct is preferable to the first by virtue of being true. Strictly speaking, renunciative disjunction as defined in Defn 3.20 requires both disjuncts to be considered true at the end of the utterance, and not just at the time of uttering each disjunct. The current definition excludes this form of corrective disjunction, which then falls under evocative asymmetric disjunction. A dynamic epistemic logic could be developed to model corrective disjunction more accurately, and this might also help address several other idealisations that the current assertibility approach makes for the sake of simplicity.

(4.33)  
\begin{itemize}
  \item a. This is Sue, or rather, Stephanie.
  \item b. My favourite colour is blue, or actually red.
\end{itemize}

**Generalised Disjunction**

Disjunctions like (4.34) that are also generalisations observe the basic assertibility conditions for disjunction. One of these conditions is that each disjunct must be satisfied by at least one instance of the generalisation. For example, in the arbitrary coordination (4.34a) at least one jury must have the potential to include at least one man, even if no jury to date ever has, in addition to the condition that every jury must consist of twelve individuals who are either men or women. One easy way to misinterpret the sentence is to apply this generalisation to a specific jury after its constituents have been determined. The disjunction in (4.34a) would still be assertible in reference to a particular jury panel containing twelve women, even though there is no epistemic possibility that there is a man empanelled. However the more specific (4.34b) would not be assertible in that same context, as the quantification is restricted to the members of that panel. A charitable Hearer would likely treat (4.34b) as an elided disjunction of the DPs ‘twelve men’ and ‘twelve women’ rather than a disjunction of the NPs ‘men’ and ‘women’. This charitable interpretation is potentially assertible only in a context where the jury is known to be single-sex, as a result of Pred 3.14.

(4.34)  
\begin{itemize}
  \item a. A jury consists of twelve men or women.
  \item b. This jury consists of twelve men or women.
\end{itemize}
4.3. NON-EXPOSITIVE DISJUNCTION

**Alternative Disjunction**

Alternative disjunctions occur when the disjuncts generate contextually contrasting consequences which need to be considered separately rather than conjointly. Alternative disjunction provides different options, while the more common conjoining disjunction provides a single ‘disjunctive’ option. I prefer the use of ‘alternative’ by Dik (1968) over that of ‘interrogative’ by Haspelmath (2000) and ‘choice-alternative’ disjunction from Mauri (2008) for this distinction. Mauri demonstrates that many languages have different coordinators for alternative and collective disjunction. She claims that natural languages with alternative coordinators always have a conjoining coordinator, but that the converse is not true, as alternative disjunctions are often asyndetic. I interpret this as showing that collective disjunction is a more basic or central kind of disjunction when used as a coordinator.

Any example of alternative disjunction requires some scene-setting to establish the context:

\[(4.35)\] Scene: It is a dark and stormy night in a remote forest. A distraught family, having just lost one of their siblings while fleeing in terror from an armed maniac, huddle in a cabin. There is a horrific scream just outside, the sound of a body collapsing on the porch, and then a pounding on the door. It’s my sister or the axe-murderer.

The propositions that are relevant in the circumstance where my sister is knocking on the door are distinctly different from those where an axe-murderer is knocking. Once we know who is outside, the scope of our epistemic concerns will rapidly collapse due to this reduction of relevant propositional variables. Modelling the assertibility of alternative disjunction requires a different set of variables for each disjunct being tested, to ensure appropriate restrictions on variables. The formal systems from Chapter Two and the variants in Chapter Three use the same set of propositional variables for every subformula. The additional propositional variables will not affect propositional assertibility requirements in most circumstances, but their irrelevance affects the assertible possibilities, such as those modal assertions generated by Free Choice in §5.4. As the set of propositional variables is not reduced in the recursive disjunctive clause, assertibility is too simple to model alternative disjunction.

**Temporal Alternation**

Temporal Alternation describes cyclical variation over time, and can be represented by disjunction, conjunction, or correlative coordinators, as these roughly synonymous examples show:

\[(4.36)\]

\[a.\] The temperature was now hotter, now colder.
\[b.\] It kept getting hotter or colder throughout the day.
\[c.\] It got hotter and colder during the day.

The possibilities in the disjunction (4.36b) are temporal (the-world-at-a-time) rather than epistemic. This is an open disjunction, as the description is compatible with there being times during which the temperature remained constant. The assertibility conditions for an open disjunction are those of a conjunction of exclusive possibilities, and the resulting near-synonymy of (4.36b) and (4.36c) forms yet another variation of modal Ambiguous Coordination, although (4.36a) shows that neither coordination is actually needed. The conjunction in (4.36c) is a composite coordination summarising across all times, while the disjunction is distributed over each of the times. The temporal correlative coordinator ‘now’ in (4.36a) conveys the additional meaning that the temperature has fluctuated up and down several times, which (4.36b) also conveys using ‘kept’, unlike the less informative (4.36c). Representing points-in-time as epistemic possibilities may not be a safe cross-linguistic generalisation, as Haspelmath claims (4.36a) is the only one of these three that is acceptable in Russian. Temporal alternation can occur when any linear measure can be imposed on the relevant possibilities, such as describing a road that undulates across the terrain, and need not be restricted to temporal ordering. Neither the linear ordering of possibilities nor the disjuncts’ truth values alternating with respect to this ordering are represented in the semantics of assertibility, although a dynamic epistemic model may aid in capturing its cyclical nature.
4.4 Conjunction

The Consistency and Informativity restrictions imposed by assertibility are identical for disjunction and conjunction. However, the multiple ways that a disjunction can be true correspond to multiple ways that a conjunction can be false, which makes for a far less informative utterance. The result is that almost every prediction described above for disjunction is also present for conjunction, although it is usually less informative and less interesting, and thus less discussed in the literature. Conjunction has some additional predictions, as the lack of alternatives permitted by conjunction encourages the connexion between conjuncts to be linear, and this allows temporality and causality to play a bigger part in the interpretation of asymmetric conjunction.

4.4.1 Asymmetry and Coordination Modes

Like disjunction, we can identify and predict a range of interpretations for a conjunction by considering the conjunct order in combination with connexion, contrariety, and any discourse markers that are present. As (4.37a) and (4.37b) have the same events presented in a different order, we can interpret them as describing the events occurring in a different temporal order. Adding the discourse marker ‘then’ reinforces this interpretation. Conjunctions that describe events that tend to occur sequentially usually have a temporal connexion, and this encourages the hearer to interpret the conjuncts as forming a temporal sequence. The events in (4.37c) are temporally ordered, but the addition of the marker ‘so’ imposes a different primary ordering, making the conjunction epistemically causal (and evocative). A hearer must be cautious when identifying these connexion markers, as not all instances of ‘then’, ‘so’, ‘even’, ‘yet’, and other potential conjunctive modifiers perform this role, even when they occur directly after ‘and’. For instance the adverb ‘so’ is strongly polysemous, behaving as a VP-specific anaphor placeholder in (4.37d) while also indicating both evocative causality and expositive temporal ordering (or perhaps correlation), both of which are aspects of ‘so’ that are independent of conjunction. To make matters more complex, ‘so’ and ‘yet’ are also coordinating conjunctions in their own right.

(4.37) a. I took off my shoes and [then] got into bed.
b. I got into bed and [then] took off my shoes.
c. It started snowing and [so] we went inside.
d. Marks and Spencer returns to Paris – and so does the orderly queue. (The Guardian)

Prediction 4.19 Asymmetric Conjunction 1.
Condition: The evaluation of one conjunct is not affected by the evaluation of the other.
Effect: The conjunction is read as asymmetric, usually temporally or causally ordered.
Example: See (4.37a) – (4.37d).
Asymmetric conjunctions are usually $\wedge$-conjunctions due to the underlying temporal ordering. This prediction parallels the disjunctive Pred 4.5.

Prediction 4.20 Asymmetric Conjunction 2.
Condition: There is an asymmetric modifier associated with the conjunction coordinator.
Effect: The conjunction is read as asymmetric; the modifier indicates the type of connexion.
Example: See (4.37a) – (4.37c).
English modifiers for $\wedge$-conjunction include ‘and then’, ‘and so’, ‘and thus’, ‘and also’ and ‘and even’. The much rarer $\wedge$-conjunction does not appear to have conventionalised modifiers, relying instead on temporal adverbs such as ‘previously’ being applied to the later conjuncts. This prediction is dependent on assertibility in the same way as Pred 4.6, as it relies on asymmetric conjunction and Pred 4.19, but it is not directly generated by the assertibility conditions.

Like disjunction, conjunction has a range of behaviours depending on the underlying connexion between the conjuncts, and the corresponding coordination mode. The following examples from Sweetser (1990) cover most of the combinations of conjunctive (a)symmetry and mode:
Symmetric and Asymmetric Expositive Conjunctions

(4.38) a. John eats apples and pears.
   b. King Tsin has great mu shu pork, and China First has good dim sum, and there’s always the Szechuan place just around the corner.
   c. John took off his shoes and jumped in the pool.
   d. Mary must have left for London last night, as nobody has phoned from England to ask why she didn’t come, and her suitcases are gone.
   e. Don’t learn basket weaving; Mary got an MA in basket weaving, and she joined a religious cult.
   f. Go to bed now! And no more backtalk!
   g. Glad to meet you sir; and how may I help you?

Dixon and Aikhenvald (2009) claim cross-linguistic evidence strongly suggests that most languages use the same coordinator for representing both the (symmetric) addition and the (asymmetric) temporal succession relationships. Symmetric expositive conjunctions like (4.38a) contain a direct connexion that is not of a linear temporal or causal nature, and are often used for reporting multiple properties or states of affairs. Dixon and Aikhenvald (2009) say that this unordered or same-event addition or elaboration is the central and fundamental case of conjunction, and thus coordination. The polyadic conjunction (4.38b) is also symmetric and expositive, but has an interesting relationship with (4.26c), as discussed in §4.3.3. Asymmetric expositive conjunctions like (4.38c) rely on the direction of the connexion to capture temporal or consequential links. They can only be differentiated by the underlying (a)symmetric logical relationship when using my broad connexion classes. In the symmetric evocative (4.38d) the conjuncts are premises in a non-deductive argument for a previous assertion. The asymmetric evocative (4.38e) consists of an initial situation and a putatively correlated outcome, which collectively form a cautionary tale. There are no unequivocal examples of symmetric performative conjunctions, for reasons given in §3.7.1. There are a wide range of asymmetric performative conjunctions, as assertions, imperatives, questions, and other speech acts can be conjoined in the performative mode whenever the felicity of an illocutionary act is partially dependent on the successful performance of earlier illocutionary acts(s). These conjoined speech acts need not be assertions, and can be homogenous (4.38f), or mixed (4.38g). One interesting combination is an imperative conjoined with an assertion; the resulting threat or promise behaves like a conditional rather than a conjunction and is discussed in §5.5.2. Finally there are no renunciative conjunctions, as explained in §3.7.2.

4.4.2 Further Parallels

Exclusive conjunction occurs via the same mechanism as exclusive disjunction (see §4.2.1), though to more subtle effect. When the context or common ground has already ruled out the possibility that both conjuncts are false, a regular conjunction behaves ‘exclusively’. This prediction only describes the transitional states of reasoning that the conversational participants have already gone through. It is a useful inference for Hearers who have recently joined an ongoing conversation, or as a reminder of the reasoning chain. The use of ‘both’ to prefix the conjunction, as in (4.39), strengthens this prediction in the same way that ‘either’ can assist in interpreting a disjunction as exclusive.

(4.39) It turns out that Aaron and Bruce are both coming to the wedding.

Prediction 4.21 Exclusive Conjunction.

Condition: The negated conjuncts of a symmetric conjunction are $c$-contrary or $l$-contrary.

Effect: The conjunction of the negated conjuncts is treated as having been irrelevant; the informative element of this conjunction is that not one but both conjuncts hold.

Example: See (4.39).

It is the semantic or pragmatic relationship between the conjuncts, rather than the conjunction itself that generates the prediction; the conjunction merely enables the prediction to satisfy assertibility.
Conjunctive weakening occurs when (usually) the final conjunct is patently unlikely or untrue, and this weakens the plausibility of the other conjuncts. A Speaker can indicate she is sceptical of an assertion by conjoining it with a patently false clause. This is most often done by appending a conjoining phrase to someone else’s assertion they disagree with (4.40a), in self-deprecation, or after they have had time to reflect on their previous statement. This prediction parallels the disjunctive Pred 4.10, while the performative version of the prediction parallels Pred 4.15.

(4.40)  

a. Yeah, Sarah is an honest politician. And pigs will fly.

b. Move and I’ll shoot.

Prediction 4.22 Weakening Conjunction.  
Condition: An asymmetric evocative conjunction has an improbable or infelicitous final disjunct.  
Effect: The plausibility of previous conjunct(s) is greatly reduced.  
Example: See (4.40a).

A weakening conjunction undermines the entire assertion, so the last conjunct is usually uttered after a change of mind by the Speaker, or by a second Speaker seeking to discount a previous utterance via an asynchronous conjunction. This rarely occurs in English except in informal spoken discourse. The error threshold is relatively fixed within a content-mode conjunction (compared to the variation between non-conjoined utterances), so if one conjunct has a very high probability of falsehood then the assertion provides no reason to believe that the other conjunct(s) are likely to be true.

Prediction 4.23 Performative Weakening Conjunction.  
Condition: An asymmetric performative conjunction has an unacceptable final conjunct.  
Effect: The Hearer will act to ensure an earlier conjunct is unsatisfactory, by disobeying an imperative, disbelieving an assertion, etc.  
Example: See (4.40b).

Performative conjunctions with an unacceptable final conjunct usually have an earlier imperative conjunct. When the final conjunct is an unacceptable assertion (4.40b), or a question where any answer is unacceptable, such as ‘which family member would you like me to kill first?’, the conjunction behaves very much like a conditional. Further discussion can be found in §5.5.2.

Conjunctions rarely provide every detail of a situation, so the conjunctive equivalent of Pred 4.11 for Open Disjunction is slightly more subtle. Most conjunctions include all sufficiently weighty conjuncts of the same type, leading to the ‘exhaustivity’ prediction in Pred 4.24, but some conjunctions are deliberately left incomplete. In (4.41a) the Speaker has probably invited only the six people listed, while in (4.41b) they have invited an indefinite number more.

(4.41)  

a. I’ve invited Sam and Tam and Jim and Tim and Van and Jan.

b. I’ve invited Sam and Tam and Jim and Tim and Van and Jan and …

Prediction 4.24 Exhaustive Conjunction.  
Condition: Any expositive or evocative conjunction that is not an open conjunction.  
Effect: Every conjunct in the conjunction is more weighty than any missing conjunct.  
Example: See (4.41a).

This follows directly from \( \binom{n}{\delta} \) weightings for conjunction. The standard exhaustivity implicature in the pragmatics literature refers to the special case where \( \delta = 0 \), so all conjuncts have been listed.

Condition: A conjunction has a trailing ‘…’, ‘and …’, or ends in a high final intonation.  
Effect: The conjunction is open, and hence evocative. That is, the explicitly uttered conjuncts do not eliminate all weighty but incorrect epistemic possibilities.  
Example: See (4.41b).

This parallels Pred 4.11 and cancels Pred 4.24.
Some predictions generated by disjunctions do not have a corresponding prediction generated by conjunctions. For instance, conjunctions do not seem to generate inferences to the best explanation such as Preds 4.8 and 4.9, although explanations can obviously include conjunctions. I initially thought that the corresponding conjunctive inference was one to the best evidence, as seen in the evocative conjunction (4.38d); however this inference is not related to the conjunction. The lack of a readily identifiable symmetric performative conjunction means there is no prediction corresponding to Pred 4.13. Similarly there are no conjunctive predictions corresponding to Pred 4.16 or Pred 4.17, due to the absence of conjunctive renunciation.

Arbitrary coordinations that are disjunctions are often interpreted as conjunctions, due to predictions like Preds 4.3 and 4.14, but the converse is much more unusual in English. An assertible disjunction always implies the conjunction of its possibilities. In contrast, a conjunction must have modal conjuncts to imply an open disjunction, and also be exhaustive to imply a standard disjunction, and these conditions on conjunctions are individually uncommon and collectively rare. The example (4.42a) is my best reconstruction of such a conjunction, but as it expresses possible alternatives, it is at best an open form of (4.42b). The same problems occur in the performative mode, for as we saw with (4.26c) and (4.26d), a performative disjunction can become an expositive conjunction, allowing the connexion to be expressed directly. The converse transformation would both decrease the directness of the connexion and increase ambiguity due to the alternative-bearing nature of disjunction, neither of which are generally desirable.

(4.42) Q: Who will pick me up from school?
   a. Maybe your mother, and maybe your father.
   b. Your mother or father.

4.4.3 Exceptional Conjunctions

Each linguist seems to classify conjunctions in their own way, and many use categories that don’t correspond to any of the above predictions. For example, Haspelmath (2000) proposes natural, representative, augmentative, inclusory, and summary conjunction. As a test of its descriptive power, I will explore whether assertibility predicts these classes of conjunction.

(4.43) a. I like fish and chips.
   b. He makes chairs and tables and stuff.
   c. The lists of example sentences go on and on.
   d. maaua ko te rata. [Maori: us and the doctor; we two]
   e. The stamp duty, legal fees and commission all need to be paid.
   f. Agus m´ e ag teacht abhaile, chonaic m´ e madra mor.
      [Irish: When I was coming home, I saw a big dog.]

Natural Conjunction

A natural conjunction (4.43a) joins parts together into a conceptual or conventional whole. It is usually dyadic, and sometimes hyphenated or asyndetic. Haspelmath’s examples include: mother and father, husband and wife, boys and girls, bow and arrows, needle and thread, and house and garden. These are to be contrasted with related but not-complementary pairs such as ‘mother and uncle’, or ‘bow and spear’. The part-whole and relationship elements of natural conjunction mean that most composite coordination and some relational coordination is natural. Neither assertibility nor connexion helps predict which conjunctions are natural.

Representative Conjunction

These are open conjunctions where the conjuncts are archetypal examples of a longer list of conjuncts, and the other conjuncts are evoked rather than tediously listed. It can be used even with a single representative conjunct. English has the dedicated marker ‘etc.’ for representative conjunction, as well as the ever-charming ‘and stuff’ which has become conventionalised in this role in less-formal registers, as in (4.43b). This type of conjunction is described by a variant of Pred 4.25.
Augmentative Conjunction
A conjunction is augmented when a conjunct is repeated for emphasis or to indicate intensity or repetition (4.43c). There does not appear to be a special marker of augmentative coordinator in any natural language, perhaps because it is predicted by the very basic violation of assertibility via redundancy that was described in Pred 3.3.

Inclusory Conjunction
An inclusory conjunction has one conjunct which is a plural Pronoun that includes another DP conjunct, such as (4.43d). This is a particular type of deductive conjunction where the inclusion can only be determined by resolving the anaphora in the Pronoun, which requires the otherwise redundant conjunct. Haspelmath (2000) identifies some languages, such as Russian, Tzotzil, and Maori, which make frequent use of inclusory conjunction, but only with plural pronouns. The Informativity norm would need to be extended to allow for anaphora and indexical resolution before we could model inclusory conjunction.

Summary Conjunction
Summary or list-and-count conjunction occurs when a determiner such as a numeral or ‘all’ is appended to a conjunction like (4.43e) to indicate that the conjunction is closed. One advantage of using a numeral is that it helps a hearer recall all the items. Assertibility does not provide much information on this class of conjunctions, although it is an explicit instance of Pred 4.24.

Degrees of Coordination
In §4.1.1 I introduced four degrees of coordination. Standard assertibility only models Distributive coordination, which is applicable to both disjunction and conjunction, but not the conjunction-specific Collective, Relational, or Composite coordination. Most Composite and some Relational conjunctions appear to also be natural conjunctions. The best formal model of these higher degrees that I have found is by Krifka (1990). He analyses these ‘non-boolean’ conjunctions as conjoining lexical items of type $e$ (entities) by object fusion. This analysis applies to DPs only (and only those where its $\langle\langle e,t\rangle,t\rangle$ lexical type could be collapsed into a simple $e$ type, such as Names, Pronouns, and other singular DPs). For example, the individuals ‘Peter’ and ‘Mary’ can be conjoined to form a new entity, the couple ‘Peter-and-Mary’. A number of the assumptions of assertibility are implicit in his treatment of entities, but I believe this and similar approaches are flawed, as they do not distinguish between different degrees of coordination, and are restricted to an overly small range of phrase types. I will outline my own preferred analysis using an extension of predicate logic in §5.1.2.

Phrasal Conjunction
The choice of conjunction can vary based on the phrase type being conjoined. For instance, Haspelmath (2000) reports on p.20 that Welmers, an expert in African languages, “is not aware of any African language that expresses [DP] conjunction and sentence conjunction in the same way”. On the other hand, a single conjunction lexeme is used in English, and most other European languages, to conjoin entities, events, and facts (DPs, VPs, and IPs). The distribution of shared DP and IP conjunction on other continents falls somewhere between these two extremes. The formal phrasal coordination described in §3.5 treats both boolean coordinations identically. However the underlying relationship between disjuncts is essentially just that of alternatives, regardless of the disjoined phrase types, while conjunctive relationships can include temporal succession, physical or epistemic cause, addition or union, same-event, elaboration, and contrast. Many of these relationships are confined to particular phrase types, and are associated with specific conjunctions in some languages. The English ‘and’ is a very general conjunction, but it is not maximally general. For instance, ‘while’ is generally preferred when describing an event that occurs completely within the duration of another. The Irish conjunction ‘agus’ serves as both the neutral ‘and’ and this type of ‘while’ in (4.43f). A complete and thorough description of connexions should both distinguish and link these relationships, and indicate the fault lines where many languages use distinct conjunctions to distinguish the underlying relationships.
4.5 Non-Boolean Coordination

The assertion norms do not appear to play as large a role in constraining the use of the remaining common English coordinators, "but", "so", "nor", "yet", and "for". A close examination does reveal traces of their influence in the usage patterns of some coordinators.

4.5.1 But

The English coordinating conjunction 'but' is often treated as being truth-functionally equivalent to a dyadic 'and', with the additional meaning that the second coordinand invokes a 'denial of expectation' or 'contrast' with the first coordinand. See for instance R.Lakoff (1971) and Bach (1999) for variations on this approach. This is not the only view; Relevance Theorists such as Blakemore (2000) and Iten (2005) claim 'but' is more procedural than conceptual, in that it communicates information about how to interpret the rest of the sentence, rather than a concept that can be evaluated directly. I am not trying to fully define the meaning of 'but', only to describe some of the patterns of constraints in its usage. This allows me to remain somewhat agnostic as to which theory provides the best description. Adversative coordination is dyadic, and has a truth-functional component similar to that of conjunction. This means that the underlying restrictions imposed by the combination of assertibility and connexion are still generally applicable, and many of the predictions for conjunction will still apply. My highly abstract description for 'but' is that the current context suggests a small number of salient scales of evaluation, the first coordinand resolves any underdetermination in the choice of scale and strongly suggests one value (or range of values) on that scale, and the second coordinand suggests a different value on that scale at least as strongly (even if it would not normally be evaluated on that scale). This description is broadly compatible with most 'conjunction plus denial/contrast' explanations, and even with Iten’s procedural view. She summarises her procedural account for ‘P but Q’ on p.236: “[p]rocess what follows (that is, Q) as a denial of a manifest assumption” generated by P or the context). The simplest and perhaps most common evaluation scale is truth or falsity, although more graduated scales include the degree of appropriateness or plausibility, the temperature scale [‘boiling’, ‘hot’, ‘warm’, ‘cool’, ‘cold’, ‘freezing’], and numerical ranges. The following examples are from Sweetser (1990).

(4.44) a. Do you think John will make Jenny a good husband?
   i. John is rich but dumb.
   ii. John is dumb but rich.
   b. John is rich but Bill is poor.
   c. John keeps six boxes of pancake mix on hand, but he never eats pancakes.
   d. Do you know if Mary will be in by nine this evening?
      Well, she’s nearly always in by then, but she has a lot of work to do at the library.

R.Lakoff (1971) claims that (4.44a-i) is symmetric, where the question sets the relevant scale, and the conjuncts in (4.44a-i) provide conflicting, non-commensurate, and hence seemingly irresolvable evidence for his being a good or bad husband. In New Zealand English (4.44a-i) and (4.44a-ii) are not synonymous, and in each case the second coordinand overrides the first when determining John’s rating as a husband, making them both asymmetric. A less controversially symmetric example is (4.44b), where our expectation is that similar predicates would be applicable to connected subjects, but contrasting predicates on the same scale are ascribed instead. Haspelmath (2000) notes that in languages such as Polish and Russian this symmetric contrast-without-conflict has its own opposite coordinator that can be translated into English as either ‘and’ or ‘but’. Asymmetric adversative coordination also takes two forms, the concessive (4.44c) whose second conjunct always trumps its first, and the simple adversative (4.44d) where the final evaluation is indeterminate. The evidence or information in the second conjunct of a simple adversative is usually sufficient to cancel any inferences drawn from the first conjunct alone. As these examples show, there are several pressures on ‘but’ to be asymmetric. General narrative ordering principles suggest that the second coordinand will tend to
supersede the first in cases of unresolved conflict, for the same reason as that $\wedge$-conjunction is more common than $\wedge$-conjunction. Both orderings must also use the same scale for the coordination to be symmetric, so either the context must explicitly determine the relevant scale or each coordinand must make the same scale salient. The two coordinands must therefore also provide identical (or possibly non-commensurate) degrees of evidence for their respective values on their common scale.

The examples above are all evocative, but adversative coordination can occur in other modes. The relevant connexion between the coordinands is their relationship to the predicate or scale of evaluation, and so they cannot have a direct connexion. This indirect connexion prevents any expositive adversatives. This result parallels the finding in Sweetser (1990) that ‘but’ cannot occur in the content domain. Similarly, adversative coordinands cannot entail each other, and this prevents deductive coordination. There are multiple types of both performative and renunciative adversatives.

(4.45) a. King Tsin has great mu shu pork, but China First has excellent dim sum.

b. Please look up that phone number – but don’t bother if it will take you very long.

c. Go to bed, but clean your teeth first!

d. It’s not good, but great!

e. It’s not ‘good’, but ‘bon’.

f. It’s not good, but bad.

Performative adversative coordination has both symmetric and asymmetric forms. The symmetric performative contains two illocutionary acts with conflicting satisfaction conditions, either of which would be satisfactory without the other. The inability in (4.45a) for a single Hearer to accept both recommendations for eating out tonight means that accepting one requires the rejection of the other. Asymmetric adversatives like (4.45b) have an illocutionary act as the first coordinand, and a satisfaction condition exception to that act in the second coordinand, typically in the form of a ‘only if . . . ’, ‘not if . . . ’, ‘not until . . . ’, or other explicit negation and conditional or subordination. A second type of asymmetric performative adversative has illocutionary acts with conflicting satisfaction conditions for its conjuncts, as in (4.45c), so the tension is in resolving the order that the Hearer responds to the coordinands. Symmetric coordination cannot occur between an illocutionary act and one of its satisfaction conditions.

Adversative coordination can also be renunciative, as ‘but’ can be used for correction, unlike ‘and’ conjunction. Renunciative adversatives are always asymmetric, as they occur only in variations of the schema ‘not A, but B’. In the propositional renunciation (4.45d), the predicate ‘good’ is satisfied but inapt, as the referent is great, and calling it good would violate the Extensibility norm. Asymmetric adversative locutionary renunciations like (4.45e) are also common. Not all corrective adversatives are renunciative: (4.45f) is also corrective, but the second conjunct asserts that the first is false, rather than true-but-inappropriate, so the adversative is evocative. Syndetic adversative corrective coordination is rarely between IPs, preferring small simple NP/DP/VP/PP/CPs without subordinate clauses, while adversative coordination between IPs is usually asyndetic. The adversative corrective is a separate coordinator in Spanish (‘sino’ rather than ‘pero’), German (‘sonder’ rather than ‘aber’), Hebrew, and many other languages, and can coordinate in both the evocative and renunciative modes. This reminds us that the distinction between renunciative and corrective coordinations in §3.7.2 is based on logical rather than linguistic properties, and is not a natural division for polysemy. A putative corrective mode would capture an apparently cross-linguistic category of adversatives, at the cost of destroying the logical unity of renunciations. A project focused on adversatives should probably include renunciations under a broader corrective mode, particularly if it were more cross-linguistic and less focused on logical properties and patterns than my analysis.

R. Lakoff (1971) and Sweetser (1990) both observe that the symmetric and asymmetric uses of ‘but’ build on the corresponding symmetric and asymmetric ‘and’. However, the restrictions imposed by assertibility are mostly superseded by the stronger requirements of adversative coordination, and the restrictions described by specific modes of coordination are primarily the result of connexion considerations rather than assertibility. It seems that the English ‘but’ is compatible with assertibility, but hardly a poster child for its centrality in shaping coordination.
4.5. NON-BOOLEAN COORDINATION

4.5.2 So

The English causative coordinator ‘so’ typically coordinates IPs where the second clause expresses a consequence of the event described in the first clause. The coordination ‘so’ is usually asymmetric, which is clearly appropriate when describing physical causation and many epistemic connexions. It can even be asymmetric when reasoning between equivalences, as the reasons why each entails the other may be different. There does not appear to be a distinct symmetric causative lexeme in English, perhaps for the same reason that ‘iff’ is only a term of art. Causative coordination can be expositive (4.46a), evocative (4.46b), deductive (4.46c), or even performative (4.46d), but it does not coordinate in the renunciative mode as there is no negation element in ‘so’ (paralleling ‘and’).

The clausal nature of the coordinands also provides another barrier to the use of ‘so’ in locutionary renunciation. In utterances like (4.46e) the subordinating compound connective ‘so that’ allows the subordinated coordinand to take other phrase types, but this should not be confused with the simple coordinator ‘so’ allowing a complementiser phrase (CP) as a second coordinand.

(4.46) a. He was tired and comfortable, so he fell asleep.
   b. Radon is not regulated in Minnesota, so it is up to you to decide when to seek help.
   c. It is an equilateral triangle, so it is a regular polygon.
   d. You seem alright now, so what were you doing last night?
   e. Come here, so that I may see you better.

The coordinator ‘so’ is often glossed as a coordinating variant of ‘if’. However, the assertibility restrictions for ‘so’ are more closely related to those for ‘and’. One difference between ‘so’ and ‘if’ is that there is no hypothetical component to ‘so’; any expression of the form ‘A so B’ asserts A, asserts B, and also asserts their causal relationship (asserting connexions is discussed further in §5.5.1). Conjunctions that use the compound coordinator ‘and so’ instead of an unadorned ‘and’ have a similarly restricted connexion. The coordinator ‘so’ is always dyadic because the pair-wise relationship between cause and effect is part of what is asserted, although a causal chain can be described by sequential use of the coordinator. The coordination ‘A so B’ can be used when A and even B is known, at which point the informative content of the utterance consists solely of the causal relationship from A to B. The coordinands for ‘so’ must have an explicable direct causal, temporal, or inferential connexion, not just an association or correlation like ‘and’, and this results in a more restrictive range of connexions than that described in §3.6.1. This physically or epistemically causal connexion is also responsible for the restriction on phrase types, as neither objects nor properties cause another. It is rather the change in an object’s properties (such as position, momentum, etc.,) that causes the change in the properties of a (potentially) different object, so both the cause and effect must have properties predicated of an object. This requires both coordinands to be IPs or other phrases that can express a proposition, not simply two phrases of the same type. Because of the significant constraints imposed by its connexive relationship, and the Informativity provided by asserting this connexion, assertibility does not generate any significant predictions for ‘so’. This does not mean that the norms are inapplicable.

4.5.3 Nor, Yet, For

Assertibility does not provide many illuminating insights for the remaining standard English coordinating conjunctions, ‘nor’, ‘yet’, and ‘for’. The modern English ‘nor’ acts as an ‘and not’ which requires that the first conjunct is negative and the second positive (on the salient scale of evaluation); that is, it requires an implicit ‘not’ in front of a hypothetical positive first conjunct. The schemas ‘neither A nor B’ and ‘[not-A] nor B’ then behave just like ‘not A and not B’. The coordinator ‘nor’ appears to inherit the conjunctive assertibility restrictions from ‘and’ (being those of a negated ‘or’).

The coordinators ‘yet’ and ‘for’ are strongly polysemous, as they have many uses other than as a connective. There are disagreements between linguists about whether their core meaning (if any) includes a truth-conditional component, without which the assertion norms would not be applicable. Further investigation awaits developments in the linguistic literature.
4.6 Preliminary Conclusions

I have shown that the assertion norms, when adapted to accommodate logical asymmetry, intersubjective weighting, contrariety, connexions, and modes of coordination, generate a substantial number of predictions about the information that is conveyed by natural language coordination. This demonstrates that it is possible to describe much of the contribution that these coordinations make to a conversation via formalised cooperative principles and their interaction with cognitive linguistics.

I have described a much wider range of coordinative phenomena than Grice (1975), but there are still several classes of boolean coordination that the current set of principles does not predict. There are at least four possible reasons for this: It is possible that coordinations are not even partially predictable by consideration of their utterance forms, and the apparent success of the above predictions is fortuitous or even forced. More optimistically, the principles could be correct and complete, although I have not yet extracted all the possible predictions, and once this is done boolean coordination will be fully described. I do not believe either of these extreme positions are correct. It is more likely that some of the central core cases are formalisable, but that the entire range of pragmatic uses of any term is unformalisable in principle. This seems analogous to filling a circle with squares; the first 90% is easy, but the task can never be completed as there are always exceptions hiding in the corners. If this is accurate, it would be because pragmatic phenomena are highly-sensitive to details of context. Even if just about all the details of context that we normally encounter will fit a relatively simple pattern, there will be exceptions, and these outliers could make a complete description impossible. Finally, it is also probable that some incorrect principles have been used, or at most a subset of the right principles, and that further development of this general approach will lead to descriptions of many of the missing cases.

The principles I have proposed are only intended to demonstrate that general cooperative principles can be formalised to a greater degree than they have been historically, and that these principles may serve as part of a common core for interpreting the natural language coordinators, along with some slightly nebulous cognitive principles. I feel that I have succeeded at this task. One important technique that I have employed is the use of aspirational norms with fall-back positions. Several independent axes each contain an ordered series of conditions, and the best interpretation(s) of an utterance can be found by trading off between these axes. Each axis then works like a Horn Scale, providing additional information based on an interpretation’s position on that axis. These axes include symmetry, relevance and error thresholds, phrasal depth, and degrees of coordination. While I may not have identified the correct axes, the general approach provides the richness of analysis required to capture most disjunction and conjunction classes.

Assertibility provides no information about some coordination classes, and in other cases it makes conflicting predictions without clear resolution procedures. Both of these problems should be expected for a simple model that is trying to predict a complex phenomenon. Many of the unresolved conflicts fall within some fairly specific restrictions, and in many of these cases fluent English speakers are equally unsure of the correct interpretation of the utterance if they lack a specific context. The failure of some predictions in some circumstances is also a good sign, as it shows that the theory is not unfalsifiable, which can be a danger with theories with multiple parameters, each with room for interpretation. Many informal appeals to Grice are of this kind; whenever a prediction appears wrong, a second maxim can be appealed to, along with an ad hoc explanation of why (in this particular context) the prediction must be changed. The formal nature of assertibility means that it is very easy to specify what would count as a false prediction. The predictions I have listed can come into conflict, or simply be wrong, and these failures can hopefully lead to refinements of the axes of evaluation, which will then produce more accurate defeasible predictions.

I have had to put aside some topics, with a promise to investigate them later. These include the nesting of disjunction within modal operators, modelling higher degrees of coordination for conjunction, and the behaviour of both disjunctions and conjunctions in negative scope. Each of these issues will be covered in the next chapter, along with a sketch of how assertibility might be extended to encompass negation, predicate logic, and the historically troublesome class of conditionals.
5. Extensions

This chapter extends my investigation of assertibility into several new areas. The development of the assertibility semantics in Chapter Two and the deviations and variations on assertibility in Chapter Three will allow some sophisticated conclusions to be drawn relatively quickly.

I investigate predicates and quantifiers in §5.1. The formal language is altered to that all quantification is constrained by predicates, to model the behaviour of restricted quantification in natural language. The higher degrees of coordination from §4.1.1 are reproduced by varying how the restricting predicates are defined, and this leads to a description of the differences between the English determiners ‘all’, ‘each’, and ‘every’. Some brevity operations are proposed for quantifiers and the identity relation. Lastly, polar and scalar predicates are described, and some associated pragmatic inferences sketched.

In §5.2 I review how negation is used in English and other natural languages, with the aim of finding some assertion norms for negation. In natural language very few instances of multiple negations are used to cancel each other out; additional negations typically either reinforce the negated assertion, or strengthen or weaken the unnegated assertion. As a result, I reject my earlier assumption that \( \neg\neg \varphi \cong \varphi \). Cross-linguistic evidence about ‘nand’ also calls into question whether the De Morgan law that identifies negated conjunctions with disjoined negations applies to natural language. These changes result in new [Double Negation] and [De Morgan] brevity operations.

I sketch a simple treatment of assertibility for modal auxiliaries in §5.3. Assertibility does not provide any particularly keen insights into modality, but it aids an investigation into the interaction of coordination and modality. One well-known issue with modal disjunction is that it gives rise to the paradox of Free Choice, where a modal disjunction \( \Diamond(\varphi \vee \psi) \) seems to entail its modal disjuncts \( \Diamond \varphi \) and \( \Diamond \psi \). Recent literature shows that this is a wide-spread but fairly subtle implicature-like phenomenon, which can be cancelled under a range of circumstances. In §5.4 I describe and critique some of the more successful investigations in this literature, and then propose my own theory of propositional Free Choice, which depends on the nature of assertible disjunction and epistemic possibility.

Assertions are not the only central type of speech act, and so imperatives, interrogatives, explanations, and arguments are briefly considered in §5.5. Imperatives appear to obey similar norms to assertions, but my primary interest in them lies in how they coordinate with other speech acts. Interrogatives have a weaker form of Informativity, which applies only to potential answers. Explanations also describe the connexion between propositions, so their Informativity norm is not simply propositional. Finally, arguments require more than validity; their conclusions must also be assertible whenever all their premises are assertible.

In §5.6 I provide an assertibility-based solution to one construal of the paradoxes of material implication. There are several classes of argument which contain conditionals in the premises and conclusion, whose premises are obviously true and acceptable, and conclusions bizarre, seemingly false, and unacceptable. I demonstrate that in these cases the premises and conclusion are never assertible in the same situation, so that the arguments from premises to conclusion are not assertible. This is followed by a quick sketch of how probabilistic asymmetric mode-specific assertibility can be applied to conditionals in a way that closely maps several corpus-driven linguistic theories of conditionals, and yet describes classes primarily by their associated pragmatic inferences.

These formal and informal extensions of assertibility demonstrate that the underlying norms are potentially useful for more than just describing features of coordination, and that they may assist in explaining a wide variety of linguistic phenomena.
5.1 Predicate Logic

The language of propositional logic is an effective tool for analysing natural language coordination, and a great way to introduce the assertion norms. However, there have been a few topics where the extra power of a predicative language would have been useful, and more where I have carefully steered around issues that required discussion of quantification, predication, taxonomic hierarchies, and so forth. I will not attempt to provide a full predicate assertibility system, only sketch some possible initial moves, and provide concrete examples of some benefits of considering a predicative analysis. I will also fulfil my promise to model higher degrees of coordination.

5.1.1 Extending the Formal Language

The standard first-order syntax for quantification and predication is not ideal for my project as it requires dyadic connectives that do not correspond to any connectives in the linguistic expression. In addition, first order predicate logic is inadequate for representing the full range of English determiners. These determiners include not just ‘all’, ‘some’, and ‘no’, but also terms such as ‘most’, ‘less than half’, ‘a few’, and ‘both’. Quantifiers should not represent determiners (Dets) like ‘all’ and ‘some’, but instead determiner phrases (DPs) consisting of a determiner applied to a noun phrase like ‘all cats’. Implementing this fully would require Generalised Quantifiers as defined in Barwise and Cooper (1981) and the comprehensive Peters and Westerståhl (2006). I have chosen not to engage with this level of complexity, and will confine myself to discussing those English determiners usually represented by the universal and existential quantifiers. I have modified the first-order language syntax to ensure that these quantifiers operate over a restricted domain of quantification. This alternative syntax does not require spurious connectives, has the same expressive power as the standard syntax, and is in the spirit of generalised quantification. I will simply present a schema and some examples of simple English sentences, as a full definition of this syntax is not required.

<table>
<thead>
<tr>
<th>Quantifier</th>
<th>English</th>
<th>First Order</th>
<th>Generalised Quant.</th>
<th>Restricted Quant.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal</td>
<td>Each A is a B</td>
<td>$(\forall x)(Ax \supset Bx)$</td>
<td>$\forall x(Ax,Bx)$</td>
<td>$\forall(x</td>
</tr>
<tr>
<td>Existential</td>
<td>Some A is a B</td>
<td>$(\exists x)(Ax \land Bx)$</td>
<td>$\exists x(Ax,Bx)$</td>
<td>$\exists(x</td>
</tr>
</tbody>
</table>

(5.1) a. All ravens are black. \(\forall(x|Raven(x)): Black(x)\).

b. Any person who laughs, weeps. \(\forall(x|Person(x) \land Laugh(x)): Weep(x)\).

c. Everything is spatially extended. \(\forall(x|\exists x): Extended(x)\).

With this new syntax, any connectives in the formula should now correspond to natural language connectives, and so can be constrained by assertibility. For instance, the only connective in the form of (5.1b) is the conjunction ‘\(\land\)’ representing the subordinating conjunction ‘who’. In those rare cases like (5.1c) where quantification is unrestricted, a trivial predicate can be used to restrict the quantified variable. The main disadvantage of this syntax is that the rules of inference will be more complex, but I am interested in faithful representation of coordinations, not ease of deduction.

I will also extend the language to allow new ways of constructing objects, in line with common natural language practises. This will allow us to represent the various degrees of coordination discussed in §4.1.1. Some predicates apply to sets rather than individuals, so I will introduce a set-construction function \(\{x_1, \ldots, x_k\}\) that takes a collection of \(k\) objects and returns the set as an object\(^1\). This set can then be the subject of a predicate like any constant or variable. For example, in (5.2a) the three people carry the piano collectively, while in (5.2b) they each play the piano individually. Some sets are defined by a characteristic predicate, and we can directly indicate this by enclosing the predicate in set braces, so \(\{P\}\) is short for \(\{x|P(x)\}\), as in (5.2c). Finally, to avoid using universal quantifiers in the utterance form that do not correspond to universal determiners in the utterance, I will use the form \(B(\in\{A\})\) for ‘the As are \(B\)’ when the form of ‘each \(A\) is a \(B\)’ is \(\forall(x|A(x)): B(x)\).

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\(^1\) The use of sets can be avoided by approaches such as plural quantification from Boolos (1984), which extends the formal language to allow for collective quantification. I will use sets as I am not worried about metaphysical implications so much as adequate representations of natural language forms.
5.1.2 Degrees of Coordination

(5.2) a. Belinda, Muriel, and Steven carry the piano. $\text{Carry}(\{b,m,s\}, p).
    b. Belinda, Muriel, and Steven play the piano. $\text{Play}(b, p) \land \text{Play}(m, p) \land \text{Play}(s, p).
    c. The students carried the piano. $\text{Carry}(\{\text{Student}\}, p).
    d. The students played the piano. $\text{Play}(\in\{\text{Student}\}, p)$.

The expression in (5.2a) uses Collective coordination, while that of (5.2b) uses Distributive coordination. The utterance forms of these similar expressions are quite different. Using characteristic predicates in the utterance form, as in (5.2c) and (5.2d), preserves the similarities of the expressions.

(5.3) a. John is married.
    b. John and Chris are married.

A natural language predicate or relation can be overloaded by representing multiple senses with different adicities. The utterance form usually represents the sense with the highest adicity. For example, ‘married’ can be transitive, so (5.3a) has the utterance form $\exists(x)\text{Body}(x)$. $\text{Married}(j, x)$ rather than $\text{Married}(j)$. Combining overloading with conjoined parameters often produces ambiguity between Distributive and Relational coordination. The utterance form for (5.3b) could be either $\exists(x)\text{Body}(x)$. $\text{Married}(j, x) \land \exists(x)\text{Body}(x)$. $\text{Married}(c, x)$ or the Relational $\text{Married}(j, c)$.

(5.4) a. The customer who ordered a ham sandwich has paid.
    b. The ham sandwich has paid.

Objects are often referred to indirectly in natural language, by deixis, or reference to their properties, relations, etc. This indirect reference is usually represented by a context-dependent function that returns the desired object, so the utterance form for (5.4a) might be $\text{Paid}($ordered($\text{ham}$)). More interestingly, in (5.4b) the reference function is elided and must be reconstructed from the context. I will represent an elided function by $\_\_\_$, making the utterance form for (5.4b) $\text{Paid}(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.)$.

(5.5) a. Every cat or dog is a mammal.
    b. All cats and all dogs are mammals.
    c. Cats and dogs are mammals.

Predicates can be conjoined in natural language before being applied to objects. An interesting case of this is Arbitrary coordination from §4.1.3, where the universal disjunction of NPs is often equivalent to the conjunction of universal DPs. I will define the conjoined predicate $\lambda(x)(A(x) \lor B(x))$ as $\forall(x)(A(x) \lor \text{Dog}(x))$. Then $\forall(x)(A(x) \lor B(x))$: $C(x)$ is equivalent to $(\forall(x)A(x))$: $C(x)$ and $(\forall(x)B(x))$: $C(x)$. For example, (5.5a)–(5.5c) are roughly synonymous, and their respective utterance forms are $\forall(x)(\text{Cat} \lor \text{Dog}(x))$: $\text{Mammal}(x)$, $(\forall(x)\text{Cat}(x))$: $\text{Mammal}(x) \lor (\forall(x)\text{Dog}(x))$: $\text{Mammal}(x)$ and $\text{Mammal}(\in\{\text{Cat} \lor \text{Dog}\})$, which are all equivalent to each other.

(5.6) a. Each of Gus’s feathers are black or white.
    b. Gus’s plumage is black and white.
    c. Gus is black and white.

We now have all the tools to represent Composite Coordination. If the utterance form for (5.6a) is $\forall(x)\text{Feather}(x, gus)$: $(\text{Black}(x) \lor \text{White}(x))$, then it can also be written as $(\text{Black} + \text{White})$ $(\in\{\text{Feather}\{\text{gus}\}\})$, and the utterance form for (5.6b) is $(\text{Black} + \text{White})(\in\{\text{plumage}\{\text{gus}\}\})$, assuming that Gus’s plumage is the set of his feathers. Finally, the utterance form for (5.6c) is $(\text{Black} + \text{White})(\in\{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.)$, with the elided function returning some meronym, metonym, or metaphor of Gus which is a set whose members are coloured. In languages such as Afrikaans and Czech, Gus’s plumage would be described as black and white, but Gus himself would simply be black-white, using asyndetic coordination. I had expected this natural boundary between syndetic and asyndetic coordination to correspond to the use of arbitrary coordination, as specifying the coordinator becomes irrelevant at this point. Native speakers tell me that the elided function is required; this also occurs in Czech with sweet-sour gherkins, which taste sweet-and-sour. I do not understand why this would be.
5.1.3 Degrees of Quantification

The techniques used to represent the various degrees of coordination may also shed some light on an oft-overlooked distinction in universal quantifiers. The English determiners ‘all’, ‘each’, and ‘every’ are usually all formalised by the quantifier \( \forall \), but they have different syntactic and semantic restrictions, and these restrictions occur cross-linguistically according to Seuren (2010). The determiners presumably convey significant conceptual distinctions relating to different formal abstractions.

<table>
<thead>
<tr>
<th>Example</th>
</tr>
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<tbody>
<tr>
<td>(5.7a) A. Each soldier had a 10% chance of dying.</td>
</tr>
<tr>
<td>B. A tenth of all the soldiers died.</td>
</tr>
<tr>
<td>C. Every tenth soldier was killed.</td>
</tr>
</tbody>
</table>

The quantification in (5.7a) is over individual soldiers, with each soldier independently having a chance of failure. This is similar to distributive conjunction. The soldiers are treated collectively in (5.7b), as only the effect on the group is described. This set-predication is like collective conjunction. In (5.7c) it is the sequence of soldiers which is primary, and their position in the sequence determines their fate. The determinant ‘every’ is grammatically distinct from ‘all’ and ‘each’, as it cannot be suffixed with ‘of them’. Nor can it be freely moved around in the sentence like the other (e.g., ‘I gave them [each/all/*every] an apple’), although ‘every[one/thing/body]’ can often be substituted. ‘Every’ behaves as if it were only a part of a determiner, with the complementary sequential information removable only in certain syntactic locations. Seuren (2010) has a nice discussion of this phenomenon.

There is no simple formal representation in the language of first-order predicate logic for the sequence quantification of ‘every’. Conversely, there do not appear to be universal quantification determiners in English that correspond to either relational or composite coordination. This is also not surprising, as these higher degrees of coordination are not concerned with the coordinands, as they model higher-order abstractions, being the relationship and the composite object respectively.

Definition 5.1 Universal Determiners

1. ‘All Ps are Q’ is represented by \( Q(\in \{P\}) \) (Collective Universal).
2. ‘Each P is a Q’ is represented by \( \forall(x|P(x)) : Q(x) \) (Distributive Universal).
3. ‘Every P is a Q’ lacks a first-order representation (Sequential Universal).

Example: ‘All ravens are black’ from (5.1a) is formalised as \( Black(\{x|Raven(x)\}) \), while ‘Each raven is black’ is \( \forall(x|Raven(x)) : Black(x) \). The sentence ‘Every raven is black’ would not be formalisable without implicit or explicit reference to a sequence containing each-and-every raven, such as ‘Every raven I have ever seen is black’ or ‘Every raven, from the far east to the uttermost west, is black’.

5.1.4 Predicative Brevity

Propositional brevity was described in §2.4.1, and the operations \([B1]–[B5]\) and [Permutation] still apply in the language of first-order predicate logic. This brevity is one of the major reasons for the introduction of the restricted quantification notation and the resulting removal of the additional conjunctions and conditionals associated with unrestricted quantification. These operations are sufficient for ensuring that many predicate formulas are brief; for example by \([B1]\) \( Fa \) is briefer than \( Fa \lor Gb \), and \( \forall(x|F(x)) : G(x) \) is briefer than both \( \forall(x|F(x)) : (G(x) \lor H(x)) \) and \( \forall(x|F(x)) : G(x) \lor (\forall(x|H(x)) : J(x)) \). I propose the following brevity operations for quantifiers and identity:

Definition 5.2 Proposed First-Order Brevity Operations

[Identity] If \( \Pi \models_{EL} a = b \) then \( \varphi \lessdot \varphi[b/a] \).
[B6] \( \varphi[\exists(x|F(x)) : G(x)] \lhd \varphi[F(a) \land G(a)] \).
[B7] \( \varphi[\forall(x|F(x)) : G(x)] \land (\forall(x|F(x)) : H(x))] \lhd \varphi[\forall(x|F(x)) : (G(x) \land H(x))] \).

These new brevity operations respect most of the basic intuitions from section §2.4.1. [Identity] captures the synonymy of known identicals, [B6] the desirability of specific concrete examples when they are available, and [B7] the desirability of avoiding multiple generalisations. However [B7] globally eliminates a form of distributivity, and thus may be too strong. It is likely that several more operations are required to fully characterise first-order brevity.
5.1.5 Predicative Scales

Natural language predication conveys more than simply the assignment of an object to a class, or a property to an object. The predicated class or property may also stand in some relation to other classes or properties, and this affects the information being conveyed by the predication. Most natural language predicates belong to one of the following classes: scalar, hierarchical, polar, and monopolar predicates. Cross-linguistically, predicates with similar meanings tend to belong to the same class, and this is particularly likely for scalar and hierarchical predicates.

Scalar predicates represent a position on a multi-valued scale. The scale can be fuzzy, as in ['scorching', 'hot', 'warm', 'cool', 'cold', 'freezing'], or precise [. . . , 34°C, 35°C, . . . ]. One of the specialist uses of scalar predicates is for the concessive asymmetric disjunctions of §4.2.2 (e.g., 'hot, or at least warm') and the concessive conditionals of §5.6.2 (e.g., ‘even if the tea is only Luke-warm, it is better than nothing’). Scalar predicates are also used in the generation of scalar implicatures, which are perhaps the most widely discussed set of generalised conversational implicatures in the literature. Numbers are usually considered to fall under this class, along with any set of terms that can form a Horn Scale as described in Horn (1972).

Hierarchical predicates describe membership in a class that stands in a hierarchical relationship with many other classes. Most of the categories of ancient and medieval logic are hierarchical, as are taxonomies such as that for household goods, which contains ‘fork’, ‘knife’, ‘cutlery’, ‘table’, ‘chair’, and ‘furniture’ amongst many other predicates. Any object satisfying a hierarchical predicate will always satisfy every predicate above it in the hierarchy. (e.g., each square is also a rectangle and a polygon). These entailment relations mean that the most specific applicable predicate is usually the most Informative. However, there are two additional factors that can complicate matters. First, the context may contain information relating to more general categories. For example when considering whether a whale has hair, it may be better to describe it as a mammal, as this makes facts about mammals more salient, such as the fact that all mammals (even whales) have hair. Second, some predicates describe ‘basic-level’ categories such as ‘shoe’, ‘banana’, and ‘dog’, which are relatively common and simple terms whose members are easily distinguishable from those in sibling categories and have a strong intra-category resemblance. A large amount of information is usually associated with these basic-level categories. Cognitive psychologists claim that these properties collectively make it faster and easier to process information concerning basic-level categories, thus decreasing the required relevance threshold for associated predicates in the sense of §3.3.1. More information on basic-level categories can be found in §3.3.1.5 and §7.2.3.6 of Cruse (2004).

Polar predicates are non-scalar predicates that have natural opposites or antonyms, so the two predicates represent exclusive, and sometimes exhaustive, states. These predicate pairs include: ‘up’ and ‘down’, ‘dead’ and ‘alive’, ‘encouraging’ and ‘discouraging’, and ‘light’ and ‘dark’. Many such pairs are generated by a lexical form of predicate negation, such as ‘happy’ and ‘unhappy’, and the aforementioned ‘encouraging’ and ‘discouraging’, while others have distinct lexical roots for each predicate. In the first case, the unnegated predicate is called the positive polar predicate, and its negation is the negative polar predicate. These negative polar predicates are the standard source of the double negation litotes we will discuss in §5.2.1. They are also the only negations applied directly to the predicative phrase – all other negations of predicates will be either propositional or renunciative, or occur at a higher phrasal level. If a pair of polar predicates has distinct lexical roots, they are both treated as positive predicates.

Monopolar predicates are non-scalar and non-hierarchical predicates without antonyms; that is, they do not stand in any simple relation to other predicates. These include predicates such as ‘horrific’, ‘wistful’, and [chilli] ‘hot’ that describe a state for which there is no current English antonym. For example, there is no English predicate for ‘unhorrific’, ‘wistless’, or [chilli] ‘cold’ (‘mild’ is too generic to serve as the antonym to [chilli] ‘hot’). Monopolar predicates are not directly related to any other predicates, and so cannot be used in preference to them. This means that they generate relatively few pragmatic inferences.

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2I have already expressed my doubts about some Horn Scales containing coordinations or determiners, but the Horn Scales containing predicates appear to work as advertised.
5.2 Negation

There are many forms of negation in natural language, including sentential, predicate, and meta-linguistic negations. Horn (1989) gives an extensive and excellent analysis of the various sorts of negation, and their treatment by logicians, grammarians, and linguists throughout history. Grammatical polarity, which categorises most simple clauses into the affirmative and negative, is perhaps the broadest category related to negation in language. Negative clauses like those in (5.8) are usually denials, complements, or antonyms of the affirmative clauses, constructed by inflecting a verb, or adding a negative particle such as the English ‘not’.

(5.8)  
a. I have no cellos.  
b. I haven’t a cello anymore.  
c. That’s not a cello.  
d. My cello don’t play jazz.  
e. No cello stays in tune for long.  
f. There ain’t no cellos nowhere.

One reliable indication of being in the scope of a negative polarity (often shortened to ‘negative scope’) is that the context is downwards-entailing. In upwards-entailing contexts a clause such as (5.9a-i) is always entailed by almost any restriction (5.9a-ii) of the same clause. In downwards-entailing contexts the unrestricted clause (5.9b-i) entails almost any restriction (5.9b-ii) of the clause. There are also clauses such as (5.9c) which are neither upwards nor downwards-entailing and so neither sentence entails the other. There is a reasonably close, although not 100%, match between the grammatical property of negative scope and the logical property of being a downwards-entailing context, and so I will use the latter to stand in for the former.

(5.9)  
a. i. Helen slept.  
   ii. Helen slept soundly.  
b. i. Nobody cried.  
   ii. Nobody cried loudly.  
c. i. Exactly three people died.  
   ii. Exactly three people died bravely.

The presence of polarity-sensitive items, being words and phrases that are restricted to negative, non-positive, positive, or non-negative contexts, is another useful indicator of grammatical polarity. One of the most common negative polarity items (NPIs) in English is ‘any’, and its variants such as ‘anything’, and ‘anyone’. By testing for the acceptability of this NPI, we can see that negative polarity contexts include negative particles (5.10a), negative connectives (5.10b), the antecedents of most conditionals (5.10c), polar questions (5.10d), negative quantifiers (5.10e), some subjunctive verbs (5.10f), comparatives, minimisers and superlatives (5.10g), and ‘only’ (5.10h).

(5.10)  
a. After the party Jasmine was not in any condition to drive.  
b. You can exercise at home without any equipment.  
c. If anyone cares, I’ll be in my trailer.  
d. Would you like any cake?  
e. No one gave a fig about Mary anymore.  
f. I doubt that anybody can fly by flapping their arms.  
g. Our burgers are better than anywhere else.  
h. Only alcohol gave Janice any solace after her loss.

Each negative polarity context in the above examples is also a downwards-entailing context, as adding adjectives or adverbs will demonstrate. If one such context is placed within another, it becomes positive; a third item makes it negative again, and so on. The only practical limits to nesting negative polarities involve issues of general comprehension.
5.2. Double Negation

‘Double negation’, when applied to natural language expressions, is an imprecise and even vague term. It is often used to describe the occurrence of multiple negative particles within the same phrase or sentence when there has historically been discussion about whether each is made redundant by the other, such as when they both negate the same phrase, or when one immediately negates the other. Many sentences with more than one negative particle do not have any form of double negation. When the negations have a DP, IP, CP, or PP phrase head between them (5.11a), or apply to separate parts of the utterance (5.11b), they are never redundant.

(5.11)  

| a. She was not about to hang him for not being likable.  |
| b. I do not want chicken and I dislike fish. |

(5.12)  

| a. There’s not a day that goes by that I don’t wonder what might have been. |
| b. Every day I wonder what might have been. |
| c. There’s a day that goes by that I wonder what might have been. |

One explanation for this irredundancy is that the lexical types associated with CPs, DPs, and PPs in the formal semantics from §3.5.1 are sufficiently complex that the negations on either side of these phrase heads cannot be rearranged to apply to the same phrase. For many sentences with multiple NPIs like (5.12a), there will be a synonymous, simpler, and no less acceptable sentence with fewer negations (5.12b), but this is never simply the result of removing negative particles (5.12c).

Using our modified language of predicate logic, if the form of (5.12a) is \( \neg \exists (x | Dx) \): \( \neg W(i,x) \), then the form of (5.12b) is \( \forall (x | Dx) \): \( W(i,x) \), and the form of (5.12c) is the non-equivalent \( \exists (x | Dx) \): \( W(i,x) \).

(5.13)  

| a. No quiero nada. (Spanish) |
| b. I didn’t do anything. |
| c. I didn’t do nothing. |

(5.14)  

| a. She would never, [n]ever say that in front of her mother. |
| b. Neither a lender nor a borrower be. |

(5.15)  

| a. It never occurred to me for a moment to doubt that your work . . . would not advance our common object in the highest degree. (Darwin to Haeckel (1867)) |
| b. I always had confidence that your work would not advance our common object greatly. |
| c. I repeat that I can feel no doubt that your work will greatly advance our subject. |

Two distinct phenomena are often described as double negation: negative concord and litotes. Negative concord is a type of grammatical agreement, where inflections or negative particles are added to multiple elements within a sentence to produce a single negation. According to Cheshire (1998), most languages, including all dialects of English, have some form of negative concord. The Spanish negative concord sentence (5.13a) translates as (5.13b) in Standard English, but some dialects allow or even require the literal transliteration (5.13c). When concordance is grammatically optional, the repetition of negative particles intensifies the negation, as can be seen in (5.14a) where the second occurrence of ‘ever’ can be either positive or negative. One contentious example of negative concord in Standard English is the correlative conjunction ‘neither . . . nor . . . ’ (5.14b), which may have originated as ‘not either . . . or . . . ’, with the ‘or’ changed to ‘nor’ for concordance. Jespersen (1917) discusses (5.15a) from a letter of Charles Darwin as an example of negative concord used for emphasis. At first glance this sentence may appear to be synonymous with the rather rude (5.15b), but later in the same paragraph Darwin writes (5.15c). Nor is this a mistake by Darwin; Noland (1991) explains that two of the three negative particles in (5.15a) are in concord, and that this concordance occurs in all dialects of English under the following conditions. “[T]he higher clause has to have a verb of negation or mentation or an existential subject; the lower clause is either a conditional or denies the reality of a conceivable; the lower clause is almost always a complement”. Negative concordance occurs when there are multiple negative particles or markers due to grammatical agreement, but from a logical viewpoint there is only a single negation present.
Some eighteenth century prescriptive grammarians were decidedly against the use of negative concord, despite it being commonly used by many writers such as Shakespeare. Bishop Lowth (1762) described negative concord as “a relic of the ancient style”, because “[t]wo negatives in English destroy one another, or are equivalent to an affirmative”. As ver der Wouden (1995) explains, two negatives are often equal to some affirmative, but they do not behave as simply as is suggested by the commonly-offered analogies of multiplying two negative numbers or applying two propositional negation operators to a formula. Instead, the two negative particles combine to create a proposition that is either weaker (5.16a) or stronger (5.16b) than the negative-free proposition. These are both types of litotes, although the term originally applied only to the weakening form. Double negation is not required for either weakening (5.17a) or strengthening (5.17b) litotes, but it is both a very common form of litotes, and the only way to produce an affirmative from two negatives.

\[(5.16)\]
\[\begin{align*}
\text{a.} & \quad \text{I don’t dislike you.} \\
\text{b.} & \quad \text{He sure doesn’t dislike you!}
\end{align*}\]

\[(5.17)\]
\[\begin{align*}
\text{a.} & \quad \text{He is not as young as he once was.} \\
\text{b.} & \quad \text{No love is lost between them.}
\end{align*}\]

Double negation litotes like (5.18a) usually have a negative particle like ‘not’ followed by a term with a negating prefix such as ‘un-’, ‘in-’, and ‘dis-’, or suffix such as ‘-less’. The term is often the antonym of a positive polar predicate (see §5.1.5) that is contingently monopolar\(^3\) and so is the most natural candidate for a negating prefix or suffix. If the negation is a simple denial, then either the original predicate or some unnamed middle ground holds, and this results in the assertion of a weak form of the predicate. On the other hand, if the negation is an assertion of opposition to an extreme value on a scale, then it indicates that the opposite extreme (a strong version of the original predicate) must hold. The prefixes ‘non-’, ‘a-’, and ‘anti-’ can also be used in double negation litotes, but because they do not usually denote a state on the opposite end of the scale from the original term, they are generally only used in weakening litotes. Double negation litotes can occur with scalar predicates as well, such as in (5.18b), but not with more categorical predicates such as the colours in the famous (5.18c) from Orwell (1946). To be ‘not ungreen’ is not much more meaningful than to be ‘not an unfish’. Finally, double negation litotes can also contain negative connectives (5.18d) or adverbs (5.18e) rather than negating prefixes.

\[(5.18)\]
\[\begin{align*}
\text{a.} & \quad \text{Nobody can be uncheered with a balloon. (Milne (1926))} \\
\text{b.} & \quad \text{That chilli is not un-hot.} \\
\text{c.} & \quad \text{* A not unblack dog was chasing a not unsmall rabbit across a not ungreen field.} \\
\text{d.} & \quad \text{You all did love him once, not without cause.} \\
\text{e.} & \quad \text{If words come from the heart, they are not only words but compassion.}
\end{align*}\]

Litotes can appear ambiguous, as they often require either contextual or intonation clues to determine whether they are weakening or strengthening litotes. There appears to be a cultural bias as well; for instance British English tends towards weakening litotes like (5.16a) as part of a culture of understatement, while strengthening litotes like (5.16b) are often used by the more boisterous speakers of American English. Despite litotes being both versatile and widely used since at least Homer’s *Iliad*, it does not seem to have sunk into ‘folk’ understandings of grammar. Instead, the idea that the two negations simply cancel each other out is still widespread, as can be seen by the insistence on this ‘gospel truth’ by many of the authors of the Wikipedia Double Negation page (as of 2006-2012). Cheshire (1998) addresses this myth directly on p.110: “[T]he system of negation in English has never, in any case, been one in which two negatives cancel each other out to make an affirmative” (my emphasis). Our formal system of assertibility should respect this observation, and reject any formulas that contains $\neg\neg\phi$, as we very rarely interpret the content of a utterance with two negations as simply being equivalent to that of the unnegated utterance.

\(^3\)That is, it has a non-compositional antonym in other languages, so its monopolarity is a contingent fact of English rather than due to the nature of the predicate.
5.2. **Concise Negation**

Most natural languages, including English, do not allow two negatives to cancel each other out, except perhaps in some unusual contexts. This principle is broadly cross-linguistic, although it may not be completely universal. No restrictions have been applied to negation in any of our assertibility systems. The simplest and most intuitive way to modify assertibility so as to prohibit (logical) double negation is to alter the brevity definitions in our concision semantics. These changes will apply to all the different variations on assertibility, as they occur in the definitions of the [De Morgan] and [Double Negation] principles in Defn 2.65, which apply universally. These changes will shift negation from being a mere logical operator to a structural connective in the same sense as conjunction and disjunction, as explained in §1.4.2. One concern is that the proofs of the various Lemmas and Theorems in Chapter Two rely heavily on the standard [De Morgan] and [Double Negation] principles, and it is possible that some of the results will not hold under the proposed restrictions.

**Single-Nested Negation**

Perhaps the simplest change to the brevity restrictions on negation operators is to prohibit any atom from falling within the scope of two or more negations. This restriction may be too severe if implemented across predicates and quantifiers as well as our original propositional language, as it may then forbid any double negation litotes. However it is both simple to implement in propositional logic and fits with folk intuitions. The replacement operations for Defn 2.65 are:

**Definition 5.3 Single-Nested Negation**

The definition of $\varphi \equiv \psi$ is modified by replacing [Double Negation] with the asymmetric:

[$\text{Double Negation Elim}$] $\varphi[-\neg A] \triangleright \varphi[A]$.

**Locally-Brief Negation**

Defn 5.3 does not minimise the number of negation symbols in a formula. In contrast, 4- and 5-brevity locally minimise the number of dyadic (disjunction and conjunction) symbols. By these same minimisation intuitions, we should prefer e.g., $\neg\varphi \lor \psi$ over $\neg(\varphi \land \neg\psi)$ as this reduces the number of negations in the formula. The replacement operations for Defn 2.65 are:

**Definition 5.4 Locally-Brief Negation**

The definition of $\varphi \equiv \psi$ is modified by replacing [De Morgan] and [Double Negation] with:

[$\text{De Morgan A}$] $\varphi[-\neg A \circ \neg B] \triangleright \varphi[-(A \ast B)]$.

[$\text{De Morgan B}$] $\varphi[-(A \circ \neg B)] \triangleright \varphi[-A \ast B]$.

[$\text{Double Negation Elim}$] $\varphi[-\neg A] \triangleright \varphi[A]$.

where $\ast, \circ \in \{\lor, \land\}$ and $\ast \neq \circ$.

**Metrically-Brief Negation**

Perhaps the strongest form of brevity over negation is to globally minimise the total number of negation symbols. This is identical to Defn 2.78’s Metric Brevity, except that I will no longer disregard negation symbols in the symbol count. Defn 2.78 is repeated as:

**Definition 5.5 Metrically-Brief Negation**

$\psi \leq_{||} \varphi$, $\psi \leq_{||} \varphi$ iff the number of symbols in $\psi$ is no more than the number of symbols in $\varphi$, and $\psi \vdash_{\text{cl}} \varphi$.

Each of these definitions treat the conjunctive and disjunctive [De Morgan] laws symmetrically. However, there is evidence that natural language does not treat negated disjunctions and conjunctions so evenly. Once I have investigated this phenomenon, I will define a final form of negation brevity.

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4. The lack of distributivity prevents a global minimisation; only the overly-strong $||$-brevity imposes this condition.

5. This approach will not globally minimise the total number of negation symbols across a complex formula. For example $\neg(\neg \neg p \land q) \lor (r \land \neg s)$ and $(\neg \neg p \lor q) \land (\neg r \lor s)$ are equivalent, have different numbers of negation symbols, and yet are both locally-brief.
5.2.3 Nand

Many linguists have observed that conjunction and disjunction are cross-linguistic near-universalists, and negated disjunction (‘nor’) also appears in the vast majority of natural languages. However, negated conjunction (‘nand’) has been attested to in at most a handful of languages. Horn (1989), Jaspers (2005), Moeschler (2007), and Katzir and Singh (2007) are amongst those that have presented hypotheses for the unacceptability of ‘nand’ which involve a combination of logic and pragmatics. Perhaps the only widely accepted theory is that of Horn (1989), which relies on (nor, nand) being a Horn scale like (and, or). The result is that either ‘or’ or ‘nand’ is redundant in a language, and the unnegated ‘or’ is then preferred over ‘nand’ in every natural language. My analysis of ‘and’ and ‘or’ is incompatible with the latter Horn scale; Moeschler (2007) also provides many counter-examples to Horn’s analysis. For further information, I recommend the book-length treatment in Jaspers (2005).

The following observations about how ‘nand’ differs from the other three connectives do not rely on the details of assertibility, only on some of the foundational principles and assumptions. A coordination provides information about each coordinand both individually and in light of the other coordinand(s). To affirm ‘A and B’ is to affirm A, and moreover to affirm A even when B is true (and similarly for B mutatis mutandis). To affirm ‘A or B’ is to affirm the possibility of A, and moreover to affirm this possibility holds when B is false (and m.m. for B). To affirm ‘A nor B’ is to deny A and moreover, to deny A even when B is false (and m.m. for B). But to affirm ‘A nand B’ only denies A given that B is true (and m.m. for B). A ‘nand’ connective does not provide any information about A independently from that about B, and so fuses the coordinands rather than coordinating them. This fusion appears similar to encapsulation from §3.2.2, and indeed some encapsulation marker such as ‘both’ appears mandatory for ‘nand’-type coordinations in English such as (5.19a), unlike the optional prefixing of ‘nor’-coordination by ‘neither’. Without the use of ‘both’, the utterance (5.19b) becomes an arbitrary coordination in the sense of §4.1.3, and easily interpreted as the ‘nor’ coordination (5.19c).

(5.19) a. I can’t fit both Hillary and Isobel in the cat box.
   b. I can’t fit Hillary and Isobel in the cat box.
   c. I can’t fit [either] Hillary or Isobel in the cat box.

No-Nand Negation

For whatever reason, ‘nand’ is not a naturally-occurring syndetic coordinator in English, and to reflect this, all formulas matching the schema ¬(ϕ ∧ ψ) should be non-brief and hence unassertible. This additional restriction still permits the negation of a conjunction via encapsulation; that is, ¬[ϕ ∧ ψ] is still a potentially assertible form, as reflected by the assertibility of (5.19a), with its encapsulating ‘both’. The resulting brevity operations derivations share the three De Morgan rules with Intuitionistic Logic; I am not sure what to make of this co-incidence.

Definition 5.6 No-Nand Negation

The definition of ϕ ≡ ψ is modified by replacing [De Morgan] and [Double Negation] with:

[De Morgan 1] ϕ[¬(A ∨ B)] ≪ ϕ[¬A ∧ ¬B].
[De Morgan 2] ϕ[¬(A ∧ B)] ≫ ϕ[¬A ∨ ¬B].

As assertibility is merely a heuristic model for aspects of the usage of the logical connectives in natural language, its features cannot serve as an explanation of why there is no ‘nand’ lexeme or redundant double negation in natural language; at best it can reflect actual language patterns.

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7 Here I treat disjunction as open, although the English ‘or’ is usually regarded as closed. Open disjunction is more common cross-linguistically, which is the level at which the absence of ‘nand’ must be analysed. In addition, assertibility treats closed disjunction as a special case of open disjunction, so this analysis holds either way. See Zimmermann (2000) for further arguments in favour of modelling English ‘or’ as open.
8 At least with the modern English ‘nor’; see §4.5.3 for more discussion.
5.3 Modal Auxiliaries

Coordinations sometimes interact with modal auxiliary verbs such as ‘might’, ‘can’, ‘may’, ‘would’, ‘must’, ‘ought’ and ‘should’ in interesting and even surprising ways. Before we can discuss the effects of modal auxiliaries on coordinations, it is important to have a clear grasp of natural language modal auxiliaries, and not simply conflate them with the logical modal operators. A modal auxiliary can be used to indicate the Speaker’s stance on the possibility/certainty, weight, significance, probability, permissibility/necessity, capability/ability, advisability, legality, intention, or futurity of a proposition. Most of the ten or so modal auxiliaries in modern English have multiple interpretations that present the Speaker’s stance along different axes, and other languages have lexemes with different combinations of these interpretations, so I will not try to define the individual English modal auxiliaries. Instead, I will investigate interpretations that correspond to distinctive positions along some of the modal axes. These axes include the alethic (logical/metaphysical), deontic (moral), legal, physical (nomological), and epistemic (conceptual); some of these will be subsets of others.

5.3.1 Epistemic Modality

The first axis I will consider is the epistemic, which tracks the epistemic stance of a Speaker with respect to a proposition, being their putative degree of belief. The English modal auxiliaries each express vague and sometimes overlapping degrees of epistemic commitment. I will avoid discussion of exactly when each should be used, and simply designate the three auxiliaries in (5.20a)–(5.20c) as corresponding to no, low, and very high epistemic commitment respectively; some roughly commensurate adverbs can be found in (5.20d). This loose categorisation of degrees of commitment to an epistemic stance will be sufficient for our purpose.

\[(5.20) \ a. \ \text{John won’t be at the beach.} \]
\[b. \ \text{John might be at the beach.} \]
\[c. \ \text{John must be at the beach.} \]
\[d. \ \text{John is [possibly/certainly] at the beach.} \]

A Speaker’s choice of epistemic modal often indicates the strength of their argument; that is, whether the premises are intended to weakly or strongly suggest the conclusion. Simple deductive arguments tend not to require an indication of strength, so the conclusion of a reasoning chain only requires an epistemic modal if there is some uncertainty through probabilistic or default reasoning, (scientific) induction, analogy, or complexity leading to a chance of error. Sweetser (1990), Brezina (2009), and many others concur that the presence of even the strongest epistemic modal usually results in a reduction in the assertoric force of a proposition. That is, saying ‘A must be the case’ is by default a weaker claim than simply ‘A’. Even when the modal auxiliary is used as an emphatic marker, the utterance is not significantly stronger than a simple modal-free assertion, as it merely indicates a high degree of confidence in the uttered proposition. Utterances containing emphatic modal auxiliaries will usually be the evocative mode. As an aside, readers may be concerned that we are talking about modes of coordination for modalities. Modalities rely on connexions just as much as coordinations; it is just the connexion from the currently described situation to that being described within the modality that is at stake. Similarly, Sweetser’s original domains of discourse were introduced to model the different usage patterns of both coordinations and modal auxiliaries.

You might think that we already have a modal operator $\Diamond()$ in our language that corresponds to epistemic possibility in formal assertibility; and that, when some conditions on $\Diamond()$ are added, it can represent degrees of epistemic probability. However $\Diamond A\varphi$ is metalinguistic, meaning ‘$\varphi$’ is possibly assertible, while we need to model the assertibility of the linguistic modal ‘possibly $\varphi$’. The first linguistic, rather than metalinguistic, modal operator I will introduce is $\Diamond \varphi$ which represents that $\varphi$ is a possibility with some weight, but not enough weight to rule out all other alternatives, or eliminate any chance of being mistaken. This roughly corresponds to the English modal ‘possibly’, as in natural language stronger modal claims may render weaker modal claims unassertible, so if ‘certainly $\varphi$', or even just ‘$\varphi$’ is assertible, then ‘possibly $\varphi$’ should not be.
Definition 5.7 Epistemic Modal Assertibility

1. \( \langle \Pi, \Gamma \rangle \models \phi \iff A\phi \text{ iff } \langle \Pi, \Gamma \rangle \models \Diamond \phi \text{ and } 1-\epsilon > P(\phi) > \delta. \)
2. \( \langle \Pi, \Gamma \rangle \models \Box \phi \iff \langle \Pi, \Gamma \rangle \models A\phi \text{ and } 1-\epsilon \leq P(\phi). \)
3. \( \langle \Pi, \Gamma \rangle \models T\phi \iff \langle \Pi, \Gamma \rangle \models A\phi \text{ and } 1-\epsilon \leq P(\phi). \)
4. \( \langle \Pi, \Gamma \rangle \models T\Box \phi \iff T\phi \text{ and } \sim T\neg \phi. \)

Epistemic modal auxiliaries can occur in the evocative, expositive, and deductive modes, indicating that a proposition is possible or necessary. Performative modals can also affect the type of speech act being performed, such as making a change in a command into a request (e.g., ‘you bring that here’, vs. ‘perhaps you could bring that here’). Modals can also serve as propositional renunciative indicators, as something offered as a mere possibility can be ignored if it seems inappropriate, in a way that a non-modal assertion cannot. However, we have already seen in §3.7.1 and §3.7.2 that there are complications with modelling coordinations in these modes, and the challenges with modals are of a similar kind. As this analysis will primarily be used in §5.4 to model propositional Free Choice, I will only analyse modal auxiliaries as modifiers to propositional content, and not to illocutionary or locutionary acts. In the deductive mode \( \epsilon = \delta = 0 \), so a deductive \( A\Box \phi \) requires that \( P(\phi) = 1 \), making it equivalent to \( A\phi \). However in the evocative and expositive modes \( \epsilon > 0 \), so \( A\Box \phi \) not being in a deductive mode implies \( P(\phi) < 1 \); that is, \( \phi \) is not necessarily true.

5.3.2 Other Modalities

There are several modal axes besides the epistemic, each associated with its own sets of possibilities. Some of these modalities are independent of each other, while others are restrictions of more general modalities (e.g., all physical possibilities are also metaphysical). For the purposes of modelling modalities in natural language, I will take the epistemic to be the most general modality, and define the others by imposing restrictions upon it. This is because communication about possibilities is generally limited to what is conceivable, and any conceptually accessible possibility can be expressed with an epistemic modal auxiliary (recall that I use ‘epistemic’ to refer to beliefs and concepts, rather than knowledge). Some philosophical conversations might fall outside this simplified model which takes talk about an inconceivable possibility to be contentless, or to only have content to the degree that we can conceive of approximations to the possibility. To simplify matters, I will assume that all non-epistemic modalities can be defined by applying appropriate restrictions to epistemic possibility, and that these restrictions will all have the same sort of structure. This assumption falls between contentious and foolish philosophically, but is a common assumption in linguistics, presumably due to the focus on communication and expression as opposed to conceptual analysis of conceptualisation. Making this assumption allows me to define two parameterised generic modal operators, to which I can apply appropriate restrictions to represent each modal auxiliary. The use of generic modal operators is based on the general agreement by linguists that there is a common semantic core to all modal verbs; see e.g., Sweetser (1990); Kratzer (1991); Zimmermann (2000). The generic modal operators will be \( \Delta_p \phi \) representing ‘it is possible-in-sense-\( p \) that \( \phi \)’ and \( \nabla_p \phi \) representing ‘it is necessary-in-sense-\( p \) that \( \phi \)’. I will drop the subscript whenever the type of modality is not relevant.

Definition 5.8 Metalinguistic Generic and Deontic Modality

1. Modal predicates are predicates of the form ‘\( \phi \) is an acceptable state of the world’, where acceptability is a generic placeholder for such conditions as possibility, permisibility, moral acceptability, legitimacy, or potential futurity.
2. \( \langle \Pi, \Gamma \rangle \models \Delta_p A \phi \iff \langle \Pi, \Gamma \rangle \models \Diamond (A\phi \& T(P\phi)), \) where \( P \) is a modal predicate.
3. \( \langle \Pi, \Gamma \rangle \models \Delta_p T\phi \iff \langle \Pi, \Gamma \rangle \models \Diamond (T\phi \& T(P\phi)), \) where \( P \) is a modal predicate.

Definition 5.9 Generic Modal Assertibility

1. \( \langle \Pi, \Gamma \rangle \models A\Delta_p \phi \iff \langle \Pi, \Gamma \rangle \models \Delta_p A\phi, \) and \( 1-\epsilon > P(\phi) > \delta. \)
2. \( \langle \Pi, \Gamma \rangle \models A\nabla_p \phi \iff \langle \Pi, \Gamma \rangle \models \Delta_p A\phi, \) and \( 1-\epsilon \leq P(\phi). \)
3. \( \langle \Pi, \Gamma \rangle \models T\Delta_p \phi \iff \langle \Pi, \Gamma \rangle \models \Delta_p T\phi. \)
4. \( \langle \Pi, \Gamma \rangle \models T\nabla_p \phi \iff \langle \Pi, \Gamma \rangle \models \sim \Delta_p T\neg \phi. \)
In the above definition of the metalinguistic $\triangle p\varphi$ operator and its two linguistic counterparts $\triangle p\varphi$ and $\triangledown p\varphi$, I have made two major assumptions that may be controversial or even unreasonable. I have already discussed my assumption that any set of modal possibilities is always a subset of the epistemic possibilities. Lewis (1970) and Kamp (1973) argue convincingly that command and permission sentences often primarily change and extend rather than restrict our obligations or permissions: that is, permission statements may change the preference order between worlds, and hence the permissibility set. Changes to the permissibility set need not require changes to an actor’s set of epistemic possibilities, but changes to the assertible set of permissible states do. Dynamic extensions to the underlying set of epistemic possibilities, rather than simple contractions of an existing set, requires complex modelling, such as dynamic epistemic models of belief revision. My solution is to treat permissibility revision in the same way that I treat belief revision and similar problems; these issues need to be addressed at some point, but my current formalisms for the semantics and pragmatics of individual utterances make no specific allowance for them.

I have also assumed that the possibility of a set of outcomes occurring entails the possibility of any subset of these outcomes occurring. As an example, if one is permitted to take a slice of cake, one is permitted to take a slice with a candle. Clearly this is not always the case, as social norms discourage taking the largest slice of cake, a slice trapped between several others, the most distant slice, and so on. As there may be outcomes with insignificant weight included in a set, and the permissibility is usually only tested against all weighty outcomes, if an insignificant or unlikely outcome occurs, this monotonicity requirement may fail. However, I have already discussed similar issues when introducing $\langle \delta \rangle$ thresholds in §3.3.1, and I expect that the same approach will resolve any issues that may arise here.

One standard assumption that I have avoided is that the Speaker has competence in the modal subject matter. There is no more need for a Speaker to be reliable on, say, deontic matters than epistemic or factual matters. All assertions are taken to be reliable expressions of what the Speaker wishes to communicate about the state of the world, not true propositions. This is why I used Expressivity as an assertion norm, rather than Grice’s maxim of Quality; both wilful deception and woeful ignorance can affect the relationship between what a Speaker wishes to say and reality, without impacting assertibility. The assertibility of the modalised proposition $\triangle p\varphi$ is only dependent on the Speaker’s intention with respect to $\varphi$, and not its truth. Also this predicate need not be expressed in the language of propositional logic, so the entire resources of classical logic, and its various predicate and modal extensions, can be used to describe the modal criteria.

\begin{align*}
(5.21) \quad & \text{a. It is moral for me to help Tim across the road.} \\
& \text{b. It is moral for me to help Tim across the road or rob him.} \\
& \text{c. It is moral for me to help Tim across the road and then rob him.}
\end{align*}

**Example:** Suppose $D(\cdot)$ is an appropriate predicate for deontic modality, $q$ represents ‘I will help Tim across the road’, and $r$ represents ‘I will rob Tim’. Then (5.21a) does not imply (5.21b) as $A\triangle dq$ does not imply $A\triangle d(q \lor r)$; nor does it imply (5.21c) as $A\triangle dq$ does not imply $A\triangle d(q \land r)$. Both of these implications fail independently of the specific modality, as $A(q)$ does not imply either $A(q \lor r)$ or $A(q \land r)$. However, $T\triangle dq$ is equivalent to $\Delta T(q \land D(q))$, which implies $\Delta T((q \lor r) \land D(q \lor r))$, which is equivalent to $T\triangle d(q \lor r)$; so if it is true that it is moral to help someone across the road, then it is true that it is moral to help them across the road or rob them. This example demonstrates that irrelevant and unassertible truths can be as misleading in morality as they are in any other subject.

Many of the ‘paradoxes’ of deontic reasoning can be treated by analysing them with assertibility rather than simple truth preservation, whereupon the usual puzzles and problems fail to arise. Schurz (1991) presents an approach to dissolving several deontic paradoxes by using a system similar to 5-assertibility (see Appendix A.5 for details of his system). There is no need to introduce special deontic operators to resolve these issues, as the same ‘paradoxes’ occur with epistemic modalities. However the inter-related ‘paradoxes’ of Free Choice, Free Choice Permission, and Free Choice Quantification that we consider next will require this fine-grained level of analysis.
5.4 Free Choice

The ‘paradox’ of Free Choice Permission was raised by von Wright (1968) and Lewis (1970), and first investigated thoroughly by Kamp (1973). I will confine my investigation to the propositional aspect of this puzzle, which can be exemplified by (5.22a) from Kamp (1973):

\[(5.22) \quad \text{a. You may go to the beach or the cinema. } \therefore \text{ You may go to the beach.} \]
\[(5.22) \quad \text{b. You must go to the beach or the cinema. } \therefore \text{ You might go to the beach.} \]
\[(5.22) \quad \text{c. I might go to the beach or the cinema. } \therefore \text{ I might go to the beach.} \]
\[(5.22) \quad \text{d. All of us went to the beach or the cinema. } \therefore \text{ I might have gone to the beach.} \]

Native English speakers indicate that the disjunction in (5.22a) implies each of its disjuncts. But as $\varphi \models \varphi \lor \psi$, classical logic would appear to demand that permission to go to the cinema entails permission to go to the cinema or the beach. Their combination results in permission to go to the beach following from permission to go to the cinema. No wonder the kids are running wild these days! More seriously, although the conclusion in (5.22a) appears to follow from its premise, it cannot be due to the properties of a classical disjunction, or even alternatives such as intuitionistic or relevant disjunction. This phenomenon is not confined to permission, as the obligation in (5.22b), the epistemic uncertainty in (5.22c), and even variation across individuals in the non-modal (5.22d) is sufficient to generate some kind of Free Choice inference. Instead it seems that we require a new approach to analysing the commitments conveyed by asserting a disjunction that does not rely on truth-conditional disjunction. And indeed, almost every theorist to write on the subject has proposed a novel formal analysis specifically to disarm the Free Choice paradoxes, starting with Kamp.

5.4.1 Free Choice Theories

Kamp (1973) proposed that Free Choice occurs due to the nature of permission, and more generally modality. He claims that asserting the conclusion of (5.22a) or (5.22b) lifts a prohibition on a set of epistemically possible worlds in which you go to the beach. For Kamp, asserting the associated premise similarly lifts a prohibition on the union of the possible worlds above and another set in which you go to the cinema. Kamp treats disjunctions as a set-theoretic union, which is roughly akin to classical disjunction. He also extends his analysis to include permission statements involving the quantifier ‘any’ which he characterises as set-theoretic union of a set of sets, rather than a pair of sets. Kamp has successfully identified Free Choice Permission as a problem, and in his later Kamp (1979) he extends his discussion of Free Choice to include non-permission based examples of Free Choice such as (5.22c), but (5.22d) shows Free Choice can occur without deontic, or any, modality present. This means his otherwise reasonable-sounding solution does not generalise across the natural problem domain. However he has linked propositional and quantificational Free Choice (indicated by the use of ‘or’ and ‘any’ respectively), and any general solution should provide a single explanation for both. Many later theorists concentrate purely on disjunction, and are not concerned whether their solutions extend to the quantificational case, ignoring one of Kamp’s important findings.

A number of approaches to (disjunctive) Free Choice Permission have been taken since Kamp, from the semantic to the pragmatic to the logical. The logical approaches tend to involve refining deontic logics using modern dynamic or non-monotonic models of strong permission, such as Dignum, Meyer and Wieringa (1996), and Asher and Bonevac (2005). Similar applications of dynamic modal semantics by linguistically motivated researchers result in quite different systems, including those of Aloni (2005), Schulz (2005), van Rooij (2006), and Eckardt (2007). Linguists such as Kratzer and Shimoyama (2002), Kratzer (2005), and Chierchia (2006) have made cross-linguistic investigations of Free Choice. Zimmermann (2000) and Geurts (2005) take a more radical approach by removing the usual truth-conditional element of disjunction, instead treating it as a conjunction of modal propositions. An excellent description of some of the key treatments up to 2005 can be found in Schulz (2005), with additional detail and commentary in the book containing Schulz (2006). I will sketch the theories in Zimmermann (2000), Geurts (2005), Asher and Bonevac (2005), Schulz (2005), and Eckardt (2007), as these represent some of the more successful, or at least oft-cited, approaches.
Zimmermann (2000) and Geurts (2005) regard the core meaning of ‘or’ to be an epistemic modality of the form $\Diamond \varphi_1 \land \ldots \land \Diamond \varphi_n$ rather than a truth-function, where $\Diamond$ is both an epistemic possibility operator and a representation of an epistemic linguistic modal auxiliary. For them, the basic form of ‘or’-type coordination is an open disjunction (as discussed in §4.3.1). Closed disjunctions occur through intonation and contextual effects indicating that the epistemic options mentioned are exhaustive. This fits with claims by linguists like Dixon and Aikhenvald (2009) that most languages have an open disjunction or use modal operators that are joined either asyndetically, or syndetically using a conjunction rather than disjunction. Zimmermann (2000) proposes some principles that produce Free Choice in cases like (5.22a) and (5.22c) with deontic or epistemic modalities, but Geurts (2005) subsequently shows these principles are too strong. The first principle entails both $\Diamond \varphi \Rightarrow \Diamond \varphi$, which is desirable in all their examples, and $\Box \Diamond \varphi \Rightarrow \Box \varphi$, which is not. The second principle is that when a Speaker is authoritative on a subject, the Hearer may assume that $\varphi \Rightarrow \Box \varphi$. This causes problems in the example ‘It must be here or it must be there’, which by Zimmermann’s analysis entails ‘It must be here’. His reversal of the usual semantic and pragmatic elements when defining disjunction is also surprising for the English ‘or’, although it would have been more reasonable as a cross-linguistic analysis of disjunctions.

Geurts rejects Zimmermann’s principles, but adopts roughly the same ‘conjunction of modals’ definition of disjunctions. He varies this definition by allowing the modal operators to range over epistemic, deontic, and other forms depending on the context, and by making each modal operator dependent on a particular context, in the style of Kratzer (1991). A disjunction $\varphi_1 \lor \varphi_2$ is thus $S_1 \Diamond \varphi_1 \land S_2 \Diamond \varphi_2$, where $S_i$ is a contextual relevant situation for $\varphi_i$, and $\Diamond$ is a modal operator operating within the context of $S_i$ ($\Diamond$ represents epistemic possibility by default, but can be any kind of possibility or necessity operator, depending on the context). He also resolves formulas in the scope of multiple modal operators by simply saying that sometimes the outer one fuses to the inner modality (that is, conveniently disappears), and sometimes it does not. He adds three more near-universal constraints: exhaustivity (giving a closed disjunction), disjointness (exclusivity), and non-triviality (epistemic non-emptiness of each disjunct, which should sound familiar by now). Each of these constraints is pragmatically motivated on independent grounds – that is, independent of the meaning of disjunction. With his convenient ‘optional fuse’ principle and constraints that ‘usually hold’, Geurts can post facto predict a wide range of Free Choice phenomena, including those from necessity modal operators such as (5.22b), and provide reasonable-sounding explanations for their occurrence. His only systematic failures relate to negated disjunctions and disjunctions in the antecedents of conditionals. My resolution of these failures in §§5.4.3 uses the De Morgan laws, but these require a truth-conditional analysis of disjunction. Until he can predict Free Choice without appealing to rules if they are useful, his system remains an overly-flexible just-so story. His later developments of this theory may address some of these issues.

Asher and Bonevac (2005) claim that representing disjunctive assertions using a defeasible, ‘faint-hearted’ modal conditional in a non-monotonic logic is the best way to model Free Choice effects. Treating $\varphi \lor \psi$ as $\neg \varphi \rightarrow \psi$ has its obvious advantages, and most of the problems with the classical version of this equivalence come from the converse claim that all conditionals should be treated as disjunctions. Their conditional was introduced in Asher and Morreau (1991) and Morreau (1997), and uses default reasoning. It is non-monotonic and respects a defeasible form of modus ponens (which applies iff the conclusion does not contradict the premise set). It resolves conflicts based on two principles: if default rules conflict, the most specific rule applies; and if conflicting default rules are of the same specificity, neither can be used. Most of the complexities of their system deal with temporality of permission, and the interaction of multiple permissions and prohibitions. This reliance on permission means that like Kamp (1973), they fail to address the simpler Free Choice that arises from disjunctions with no modal operators like (5.22d), or even those with epistemic operators like (5.22c), let alone quantificational Free Choice. Their treatment of disjunctions as conditionals fails to hold up when modelling negated disjunctions, which have quite different properties from negated (linguistic) conditionals. Conditionals are also inherently asymmetric, while disjunctions with Free Choice have a higher tendency to symmetry than normal disjunctions. Still, they do show the power of non-monotonic default reasoning in addressing many of the issues in Free Choice.
Schulz (2005) introduces a pragmatic ‘entailment’ operator $\models_S \frac{\varphi}{\psi}$ for deducing Free Choice as a formal conversational implicature. She first defines a partial order over worlds in Kripke models, where $⟨M, w⟩ \preceq ⟨M', w'⟩$ iff $\forall p \in PROP : M,w \models \varphi \Rightarrow M',w' \models \varphi$. Then for a class $S$ of models, $\varphi \models_S \psi$ iff $\forall ⟨M, w⟩ : M,w \models \varphi \Rightarrow \exists ⟨M', w'⟩ : (M',w') \models \varphi \Rightarrow ⟨M, w⟩ \preceq ⟨M', w'⟩$, then $M,w \models \psi$ (that is, all globally $\varphi$-minimal worlds satisfy $\psi$). This is sufficient to produce all epistemic Free Choice inferences such as (5.22c) and (5.22d). A little more work is required for Free Choice Permission inferences like (5.22a) and (5.22b), as the relationship between the epistemic and deontic accessibility relations in each model needs additional restrictions. This approach is both elegant and reasonably successful, but it has several minor problems. Like Zimmermann, Schulz requires the undesirable principle $\Diamond \Box \varphi \Rightarrow \Box \varphi$, and runs into the same problems as a result. She also initially requires that the Speaker conveys the entirety of their communicative intention, which causes problems not only with differentiating exclusive vs. inclusive disjunction, but also causes her system to reject any statement with a deontic modality; she later removes this requirement, but only for deontic modalities. She requires deontic competence in the Speaker, which is odd as epistemic and factual competence are not required, so only the deontic deductions need to be true; other deductions only reveal the opinion of the Speaker. Certainly, deontic incompetence can cancel Free Choice implicatures, but so can epistemic or factual incompetence. Perhaps most fatally, her system predicts that $p \lor q$ implies both $\Diamond r$ and $\Delta r$: everything irrelevant is possible in every modal axis. She provides an example of this, which I include below as (5.26d). Schulz admits that this last problem requires more development, but does not seem concerned by the other issues I have listed. Despite these problems, I believe her system is the most reliable predictor of disjunctive Free Choice in the literature. To avoid the problem of irrelevant possibilities being assertible, she needs something like the variable inclusion of Theorem 2.94.1, and this must be maintained under epistemic uncertainty, such as in Defn 3.3.6. My use of preference relations over formulas rather than preference relations over worlds leads to principles that are distinct from, but recognisably similar to hers, so I feel the approach I will present below almost parallels Schulz’s.

Eckardt (2007) claims that the fundamental cause of many free choice effects is implicit (or even explicit) existential claims rather than disjunction. For her, the typical assertion of a disjunction involves making epistemic existential claims (‘there is a possibility that . . . ’). These same existential claims also arise from the use of modalities or explicit quantification, such as in (5.22d) where the disjunction does not convey any epistemic uncertainty, and yet Free Choice still occurs. I have already discussed the effect of existential quantification on modelling epistemic possibilities in §4.3.5. I suggested that in this case the epistemic uncertainty resides in the common ground, and this approach can also apply to modelling Free Choice phenomena. Independently of her claim that existential quantification is at the root of Free Choice, she also investigates the interesting case of anaphoric conditionals, and notes that Free Choice can occur in their antecedents, unlike most conditionals. She also reemphasises Kamp’s point that Free Choice can occur through quantifiers such as ‘any’ as well as disjunction. Her examples are more diverse and troublesome than most, and raise several problems for most of the analyses of Free Choice since Kamp. Any complete explanation should address all the issues she raises. However her own analysis is informal, and appeals to Gricean considerations on an ad hoc basis. It may be sufficient for many cases, and illuminates some data that other writers have not taken full account of, but it is not satisfying from a formal modelling perspective. Her work raises the bar for others, but does not provide a general and formal solution.

Many of these solutions appeal to Grice: Kamp (1979) appeals to his submaxim of Brevity, Zimmermann and Schulz appeal to the maxims of Quantity, Eckardt appeals to the maxims of Quality, Quantity, and Manner, while Schulz also claims to be modelling Grice’s generalised conversational implicatures. A close look at the formal semantics each author outlines reveals strong similarities with one or another of my semantics for assertibility. It does not seem far-fetched to suppose that assertibility can generate some fairly reliable predictions about Free Choice inferences. In addition, I will test assertibility against those examples that highlight the shortcomings of some of these solutions.

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9The variable restrictions in Defn 3.3 were added purely to avoid replicating Schulz’s inference $p \lor q : \Diamond r$. 
Disjunctive Free Choice

I will investigate whether predictions of Free Choice can arise from the assertion norms, and in particular, the recursive conditions on disjunction. The predictions for the original Free Choice Permission examples like (5.22a) are more subtle than for epistemic Free Choice, and require the consideration of additional factors. I will thus start by considering epistemic Free Choice cases like (5.22c) and (5.23b), and those like (5.22d) and (5.23a) which occur despite the disjunction lacking any explicit modality.

(5.23)  
\begin{enumerate}  
\item a. Peter or Mary took the beer from the fridge. (Schulz (2005))  
\item b. We [might] go to France or stay put next summer.  
\item c. Mr X might be in Victoria or in Brixton. (Zimmermann (2000))  
\item d. Mr X might be in Victoria or he might be in Brixton.  
\item e. Mr X might be in Victoria or in Brixton – but I don’t know which.  
\item f. Mr X might be in Victoria or he might be in Brixton – but I don’t know which.  
\end{enumerate}

The sentence (5.23a) implies that it is possible that Peter took the beer from the fridge, but that this is not certain, as Mary may have taken it instead. The explicit epistemic modals in (5.23b) and (5.23c) provide similar levels of information, being that each disjunct has non-trivial weight, and as the disjuncts are contrary, neither disjunct is overwhelmingly likely. In the wide disjunction (5.23d) each disjunct is a mere possibility, making it an open disjunction. The epistemic doubt applied to the choice of disjuncts in both (5.23e) and (5.23f) is almost transparent to the epistemic modal ‘might’, although it does close both of these open disjunctions, making them essentially synonymous at the level of abstraction we can model.

Lemma 5.10 Simple Free Choice

1. $A(\varphi \lor \psi)$ implies $\Diamond A\varphi$.
2. $A(\varphi \leftrightarrow \psi)$ implies $\Diamond A\varphi$, as do $A(\varphi \dashv \psi)$ and $A(\varphi \nabla \psi)$.
3. $A(\varphi \lor \psi)$ implies $\Diamond A\varphi$.

Proof: 1. By Theorem 3.5.1 $A(\varphi \lor \psi)$ implies $\Diamond (A\varphi & T(\neg \psi)) & \Diamond (A\psi & T(\neg \varphi))$, which implies $\Diamond A\varphi$. 2. By Lemma 3.9.3 $A(\varphi \leftrightarrow \psi)$ also implies $\Diamond A\varphi$. 3. By Lemmas 3.4.3 and 5.10.1.

The conclusion drawn in each part of Lemma 5.10 is only $\Diamond A\varphi$, which is not quite sufficient for $A(\varphi \lor \psi)$; that is, to assert that ‘it might be $\varphi$’. But $\Diamond A\varphi$ implies $\Diamond T\varphi$ and hence $T\Diamond \varphi$ by Defn 5.7.3, so $A(\varphi \lor \psi)$ implies that $T\Diamond \varphi$; that is, it is true that it might be $\varphi$. For example, in (5.23a), a hearer is entitled to infer it is possible that Peter took the beer from the fridge, but cannot assert that Peter might have taken the beer until they also infer that Mary might have taken it, that this reduces the chance of Peter taking, and that is eliminates the option of using a stronger modal auxiliary like ‘must’. The analysis gets a little more interesting when explicit epistemic modality is introduced.

Lemma 5.11 Epistemic Free Choice

1. $A(\Diamond (\varphi \lor \psi))$ implies $A(\Diamond \varphi)$.
2. $A(\Diamond \varphi \lor \Diamond \psi)$ implies $A(\Diamond \varphi)$.

Proof: 1. $A(\Diamond (\varphi \lor \psi))$ implies $\Diamond A(\varphi \lor \psi)$, which implies $\Diamond A\varphi$ by Lemma 5.10.3; and as $\varphi \geq P(\varphi \lor \psi) \geq P(\varphi)$, $\varphi \geq P(\psi)$, so $A(\Diamond (\varphi \lor \psi))$ also implies $A(\Diamond \varphi)$. 2. $A(\Diamond \varphi \lor \Diamond \psi)$ is the open disjunction that is equivalent to the closed $A(\Diamond (\varphi \lor \psi))$ without the conjunct $T(\varphi \lor \psi)$, as discussed in §4.3.1, so it also implies $A(\Diamond \varphi)$ by the same reasoning as Lemma 5.11.1.

Assertibility correctly predicts that every one of the examples (5.22b)–(5.23f) generate a full Free Choice effect, as each of them can be represented by either $A(\Diamond (\varphi \lor \psi))$ or $A(\Diamond \varphi \lor \Diamond \psi)$, and so by Lemma 5.11, $A(\Diamond \varphi)$ holds in each case. So we might go to France, while Mr X might be in Victoria.

\footnote{Schulz uses ‘may’, with an explicitly epistemic reading.}
The fairly transparent interaction between disjunction and epistemic modal operators, or between multiple epistemic modal operators, is described in Lemma 3.4 and Theorem 3.5. This does not extend to other modalities. The operator scope is thus significant for determining what can be inferred from an assertion now that we consider the full generality of Free Choice Permission.

\[(5.24)\] a. You may go to the beach or go to the cinema. (Kamp (1973))

b. You may send it by post or by email. (Schulz (2005))

c. Detectives may go by bus or boat. (Zimmermann (2000))

d. Detectives may go by bus or they may go by boat.

In (5.24a)–(5.24d) Kamp, Schulz, and Zimmermann all present central examples of Free Choice Permission, where a deontic modality has scope over a disjunction that does not contain any other modal information. In these circumstances, each disjunct is permissible (Free Choice holds) unless additional information is provided before or after the disjunction that changes the interpretation of the disjunction, and so cancels the inference. However, Zimmermann notes that (5.24d) can only be interpreted as having Free Choice in specific contexts, such as when someone is paraphrasing the disjunction, and so cancels the inference. However, Zimmermann notes that (5.24d) can only be interpreted as having Free Choice in specific contexts, such as when someone is paraphrasing from a rulebook. In this case, the disjunction may be an arbitrary coordination as discussed in §4.1.3. Performative disjunction with wide scope over modalities can also exhibit Free Choice. In this case, each disjunct is possible (or permissible, etc.), and the Hearer may select which choice is most felicitous.

**Lemma 5.12 Free Choice Permission**

1. \(A \Delta (\varphi \lor \psi) \implies A \Delta \varphi\).
2. \(A (\Delta \varphi \lor \Delta \psi) \implies \Diamond A \Delta \varphi \) but not \(A \Delta \varphi\).
3. \(A \Delta (\varphi \lor \psi) \implies \Delta A \varphi, \Delta T \varphi, \) and \(\Delta T \Delta \varphi\); also \(A \Delta \varphi\) when \(\delta \geq \epsilon\).

**Proof**: 1. \(A \Delta (\varphi \lor \psi) \implies \Diamond A (\varphi \lor \psi)\), which implies \(\Diamond A (\varphi \lor \psi)\) and \(T (P (\varphi \lor \psi))\). But \(\Diamond A (\varphi \lor \psi)\) implies \(\Diamond A \varphi, \) and \(T (P (\varphi \lor \psi))\) implies \(T (P (\varphi))\) by assumption, and these give \(\Diamond A \varphi\). Now \(\Diamond A \varphi\) ensures \(P (\varphi) > \delta\), and \(1 - \epsilon > P (\varphi \lor \psi) > P (\varphi),\) so \(A \Delta \varphi\). 2. First, by Lemma 5.10.1. Second, \(A (\Delta \varphi \lor \Delta \psi)\) can hold in scenarios such as (5.25a) where the Speaker knows they do not know whether permission is confined to cases where \(\neg \varphi \land \psi\) is true. 3. \(A \Delta (\varphi \lor \psi)\) implies \(\Delta A (\varphi \lor \psi)\) by Defn 5.9.2, and thus \(\Delta A \varphi\). Also as \(P (\psi \land \neg \varphi) > \delta, P (\varphi) < 1 - \delta,\) so when \(\delta \geq \epsilon\) this implies \(A \Delta \varphi\). □

Each of (5.24a)–(5.24c) have the form \(A \Delta (\varphi \lor \psi)\), and so by Lemma 5.12.1 have Free Choice Permission. In contrast, (5.24d) has the form \(A (\Delta \varphi \lor \Delta \psi)\), so Lemma 5.12.2 correctly predicts only that ‘it is possible that detectives may go by bus’, and not that ‘detectives may go by bus’. Here are more examples of Free Choice Permission failing:

\[(5.25)\] a. You may take an apple or a pear, but I don’t know which. (Schulz (2005))

b. Detectives may go by bus or boat, I forget which. (Zimmermann (2000))

c. Detectives may go by bus. So, detectives may go by bus or boat.

Examples (5.25a)–(5.25b) all explicitly mention the epistemic doubt that was implicit in (5.24d), and so have the same form \(A (\Delta \varphi \lor \Delta \psi)\), and their Free Choice failure is also predicted by Lemma 5.12.2. Zimmermann’s (5.25c) presents information about how the disjunction was derived, showing that the Speaker is only attempting to convey the disjunction’s truth, rather than asserting the disjunction in compliance with the full set of assertion conditions. The best utterance form for the disjunction in (5.25c) is thus \(T \Delta (\varphi \lor \psi)\) which, even when combined with the Speaker’s earlier assertion \(A \Delta \varphi\), fails to imply \(\Diamond A \psi\), let alone \(A \Delta \psi\).

\[(5.26)\] a. I can drop you at the next corner or drive you to the bus stop. (Schulz (2005))

b. Mr X must take a taxi or a boat.

c. Peter is in love or I’m a monkey’s uncle.

d. You may take an apple or a pear. ∴ You may take a banana.
In (5.26) I list several other examples from Schulz (2005) that warrant consideration. First, (5.26a) uses a capability modality ‘can’ rather than the deontic ‘may’ of, say, (5.23b), but Lemma 5.12.1 only relies on the generic assertibility properties of non-epistemic modalities, so Free Choice still holds. The stronger deontic modality ‘must’ is used in (5.26b), and by Lemma 5.12.3 one can assert ‘it is true that Mr X might take a taxi’, and depending on context and the values of \( \delta \), may also be able to assert ‘Mr X might take a taxi’. The distinction between asserting a statement and asserting its truth is often blurred in reasoning where truth is the main goal. The example (5.26c) parallels (4.23c) from §4.3.1, and so satisfies Pred 4.10. This prediction results in the strengthening of the disjunct ‘Peter is in love’ because the possibility that the second disjunct is true is extraordinarily low. For ‘I’m possibly a monkey’s uncle’ to be assertible there would need to be a separate motivation to use such a low \( \delta \), and this motivation is not supplied or even implied by the disjunctive assertion. Finally, the oddity of (5.26d) is an undesirable artefact of the formal system in Schulz (2005). The permission to take an apple or a pear does not extend into permission to do anything else not explicitly forbidden, and neither does assertibility predict that it should. This is not simply due to 5-assertibility’s literal inclusion, as even 3-assertibility is sufficiently strong to prevent the inference from \( A\varphi \) to \( A(\varphi \land (\psi \lor \neg \psi)) \) and thus to \( \Box A\psi, \Delta A\psi, \text{ or } A\Delta \psi \). However it does rely on, and motivates, the propositional restrictions on epistemic possibility imposed in Defn 3.3.6/7.

**Performative and Renunciative Disjunction**

Symmetric Performative and Renunciative disjunctions support predictions resulting from similar reasoning to that behind Free Choice, but the outcome is completely different. In both these types of disjunction, each disjunct must be assertible, so a strict Free Choice result would have no effect. Also, these disjunctions do not represent the Speaker’s epistemic possibilities, but possible acceptability to the Hearer, so the Free Choice belongs to the Hearer, not the Speaker. For example, in the performative (4.26c) the Hearer can freely choose any of the recommendations. Similarly, with a renunciative disjunction such as (4.29a), the Hearer can freely choose to refer to their new acquaintance by any of the names proffered. An analogue of Lemma 5.10.2 would show that the Hearer also has Free Choice with \( \vee\leftarrow, \vee\rightarrow, \) or even \( \leftrightarrow\)-disjunction, although politeness or maximal aptness norms may be violated. This Hearer’s Free Choice is something that I have not seen in the literature, and explains some of the utility of both Performative and Renunciative disjunctions.

**Conclusions**

Free Choice is a complex and nuanced phenomenon. Even within the narrower classes of disjunctive Free Choice and Free Choice Permission we have seen several different utterances forms for which it occurs, and some very similar forms for which it does not. Careful consideration of the restrictions assertibility imposes on disjunction, and of the nesting of modalities, has helped to show that a disjunctive utterance form implies its modalised disjuncts exactly when the corresponding utterance has Free Choice. This analysis follows directly from the more general principles of assertibility, and correctly predicts every disjunctive Free Choice utterance I have provided. I have also tested my predictions against all the examples in every paper that I have cited above, except for those the authors admitted were ambiguous. From these results, assertibility appears to be a good candidate for modelling Free Choice.

There are at least two further ways to test this Free Choice model. The formal nature of assertibility allows some more general claims about Free Choice to be derived. One consequence of assertibility is that any disjunction of epistemic possibilities (‘might’) and almost any disjunction with no modal verbs generates Free Choice, while only a specific subclass of disjunctions with permission (‘may’) generate Free Choice Permission. Assertibility predicts that a disjunctive utterance will generate a Free Choice effect if there are no non-epistemic modalities embedded within either an epistemic modal or a disjunct, the disjunction is in positive scope, and the disjunction was not derived via a weakening inference. Any counter-example to this claim would cause considerable difficulty for my theory. Assertibility also generates a number of predictions relating to non-disjunctive propositional Free Choice, which we will investigate next.
5.4.3 Free Choice and other Connectives

Free Choice can occur in negated conjunctions such as (5.27a) and (5.27b), as well as in disjunctions. Eckardt (2007) also claims that Free Choice does not arise for negated disjunctions like (5.28), including those in the antecedent of most conditionals, such as (5.29a) and (5.29b). However she points out that some conditionals, such as her (5.29c) and Zimmermann’s (5.29e), do permit disjunctive antecedents with Free Choice. If these exceptional conditionals do not generate a downwards-entailing context in their antecedents, then most of these disparate observations can be modelled by treating disjunctions in negative scope as conjunctions, and vice versa.

(5.27)  
a. Ann and Bill did not go to court.  
b. Ann and Bill did not both go to court.  
c. Both Ann and Bill did not go to court.

(5.28) Nobody was bored or annoyed. (Eckardt (2007))

Conjunctions in negative scope often appear to behave like disjunctions in positive scope, so we might expect negated conjunctions to exhibit Free Choice. With ambiguous utterances like (5.27a), it can be tricky to spot whether a negation is applying to a conjunction or its conjuncts. Disambiguating, we can see that the subject DP is negated in (5.27b), while it is the predicate VP that is negated in (5.27c), explaining why (5.27b) has Free Choice and (5.27c) does not.

(5.29)  
a. If you get an A or a B in the exam, I will take you out for dinner. (Eckardt (2007))  
b. If some pupils take drugs or steal jewellery, then the teacher will be fired.  
c. If Gordon sometimes drinks beer or wine, then we can offer him a good bottle of Bordeaux as a present.  
d. If Gordon sometimes drinks beer, then we can offer him a good bottle of Bordeaux as a present; and if Gordon sometimes drinks wine, then we can offer him a good bottle of Bordeaux as a present.  
e. If Mr. X might be in Chelsea or Hyde Park, then we can as well give up. (Zimmermann (2000))  
f. If Mr. X might be in Chelsea, then we can as well give up; and if Mr. X might be in Hyde Park, then we can as well give up.

Eckardt explains the difference between those conditionals with Free Choice in the antecedent and those without: “The conditionals above [(5.29c) and (5.29e)] are not the law-like uses of conditionals like in ‘if it rains, then the street gets wet’. The conditionals in question are used in a discourse structuring function. The antecedent takes up a statement that was part of the preceding discourse, and the consequent names the conclusion the speaker wants to draw. . . . What is important here is that the content of the if-clause is taken up anaphorically from the discourse”. One reliable diagnostic for Free Choice in disjunctive antecedents is whether the original utterance such as (5.29c) or (5.29e) is synonymous with its associated clausal conjunction (5.29d) or (5.29f) respectively. Synonymy generally indicates a ‘standard’ conditional with a downwards-entailing antecedent whose disjunctions do not exhibit Free Choice. Neither (5.29c) nor (5.29e) maintains synonymy upon expansion, and both also exhibit antecedent Free Choice. In contrast, negated disjunctions within an antecedent only exhibit Free Choice within ‘standard’ conditionals. Eckardt’s anaphoric conditionals will be discussed further in §5.6.2, where we will see that they form a significant subclass of those conditionals where the antecedent does not produce a downwards-entailing environment.

Combining these observations, assertibility accurately predicts that a conjunction in positive scope within an antecedent of a ‘standard’ conditional generates Free Choice, while it must be in negative scope to generate Free Choice in an anaphoric conditional. All these observations (apart from the behaviour of anaphoric conditionals) follow immediately from the propositional De Morgan laws, the successful approximate formalisation of many conditionals of the form ‘if ϕ then ψ’ by ‘ϕ ⊃ ψ’, the truth-functional equivalence of ϕ ∨ ψ and ¬ϕ ⊃ ψ, and the simplest kind of Free Choice from Lemma 5.10.1.
5.5 Other Speech Acts

Some of the assertion norms are applicable to illocutionary acts other than assertion. Explanations are similar to assertions, but often appear to violate the Informativity norm. The assertion norms interact with imperatives and interrogatives in interesting ways, and again most apply in most cases. Arguments or chains of reasoning usually contain a conclusion that is intuitively informative, but which violates the Informativity norm. I will investigate assertible arguments more formally.

5.5.1 Explanations

An explanation can be understood as a type of assertion where the primary information being communicated includes the connexion between facts, as well as the truth of those facts. This means that the propositional content is often second-order, containing claims such as ‘that \( p \) is true is caused by the truth of \( q \)’, which is a more informative proposition than the first-order \( q \supset p \). Tautologies are never Informative, but that a tautology is part of the reason for the truth of another assertion can be Informative, and the tautology can be assertible in this explanatory context. For example, one way to explain that ‘\( p \) is true whenever \( (p \supset q) \supset p \) is’ is to mention Pierce’s Law that \(( (p \supset q) \supset p ) \supset p \). Similarly, if a proposition that is already part of the common ground is the reason behind another proposition’s truth, then it can be mentioned as part of an explanation. For example, (5.30a) and (5.30b) are both common knowledge, and yet (5.30c) provides an explanation of their relationship, and so is perfectly acceptable as an assertion. Note too the use of a rhetorical question, a common device for mentioning background assumptions while avoiding asserting them. Translating explanations into appropriate propositions that include the relationships between other propositions is difficult. The underlying assertibility principles do not change. However, the technical Informativity requirements must be altered to allow propositions to be mentioned rather than simply asserted. Because of the shift in focus to propositions about propositions, we will have for first-order propositions being mentioned and described in second-order propositions.

(5.30) a. Many birds fly (from New Zealand) to the Northern Hemisphere during the winter.
   b. The Northern Hemisphere is too far away (from New Zealand) for birds to walk.
   c. Why do birds fly north during the winter? Because it’s too far to walk.

Another way of understanding how explanations differ from standard assertions is to consider them as assertions with tighter requirements on coordinand connexion (see §3.6.1). Indirect connexions are not generally acceptable; instead, every link in the chain of direct connexions that comprise the indirect connexion must be asserted. This reduces the reliance on a shared context that enables the Speaker and Hearer to make the same chain of connexions. For the same reason, the contextual dependence of direct connexions is usually reduced or made explicit. The reduction of reasoning chains even applies to inferential connexions, which may be broken down into a series of more trivial reasoning steps, some of which are described just so that the Speaker and Hearer can be assured they are making the same connexions. This decomposition allows us to weaken the unrealistic assumption that the Hearer has both logical closure and encyclopaedic knowledge of the relevant lexical definitions and law-like regularities. This approach, while different from that above, also allows the basic precepts of assertibility to apply to explanations and proofs.

Under either model of explanations, conjunction and disjunction have the same kind of restrictions imposed as for generic assertibility, and it is not surprising that almost every kind of coordination described in §4.2–§4.4 occurs in explanations. Perhaps the only exception is performative disjunction, which is used only to offer alternative incompatible explanations when either of them might be acceptable to the Hearer, such as the various interpretations of quantum mechanics. As long as one of the parallel explanations is acceptable to the Hearer, the Speaker has been successful. Of course, the pragmatic effects of the Speaker presenting a highly objectionable explanation as an option for the Hearer may have consequences even if the other explanation is perfectly acceptable, just like any other performative or renunciative disjunction.
5.5.2 Imperatives

An imperative is typically an utterance which describes a state of affairs and indicates that the Speaker expects the Hearer to make it come about. Part of what makes an imperative felicitous is the appropriate social or institutional relationship between Speaker and Hearer which motivates the expectation that the Hearer will attempt to satisfy the command. Most commands are short, with little coordination. The coordination within an imperative is usually limited to conjunction of NPs or DPs as objects or subjects of the imperative. Multiple imperatives are typically either conjoined by performative conjunction, or occur as separate utterances (sentences); disjunctive imperatives are uncommon. An imperatives with a complex objective can require substantial descriptions of the objective, circumstances to avoid, and methods or preparation. However these descriptions appear to behave like assertions embedded within the imperative act, and their coordination restrictions thus follow those of assertions. Apart from the tendency for extremely brief imperatives (excluding any descriptive component), coordination within imperatives is not particularly distinctive.

What is interesting, however, is the coordination of an imperative with an assertion or interrogative. In almost every case, the imperative is uttered first, followed by the consequence of obeying (conjunction) or disobeying (disjunction) the imperative. This form of coordination almost always forms a paratactic conditional (a conditional using coordination rather than subordination). Russell (2007) provides an excellent overview of this issue.

(5.31) During a holdup, there are four bank robbers, each pointing a firearm at a bank teller.
   a. Robber 1: Don’t move, or I’ll shoot.
   b. Robber 2: Don’t move, and I’ll not harm you.
   c. Robber 3: Move, and I’ll shoot.
   d. Robber 4: * Move, or I’ll not harm you.

(5.32) a. If you move, I’ll shoot.
   b. If you don’t move, I’ll not harm you.
   c. It is not the case that if you don’t move, I’ll shoot.

In (5.31a), Robber 1 commands the teller not to move; the second disjunct being unacceptable to the teller, he should try to ensure the satisfaction of the first disjunct, as then the second need not come about. In (5.31b), Robber 2 reassures the teller that obeying the imperative will lead to a relatively favourable outcome; the conjunctive assertion norm informs the teller that harm must be a real possibility if the first conjunct did not hold, giving him incentive to ensure its success. In (5.31c), Robber 3 asserts that obeying the apparent imperative will have negative consequences for the teller. Again, conjunctive assertibility informs the teller that there must be some chance that the second conjunct will be false when the first conjunct is false, and so he should ensure that the first conjunct does not hold. Finally the panicked Robber 4 in (5.31d) utters an imperative then provides a second, acceptable alternative, leading to a bewildered teller.

Examples (5.31a)–(5.31d) are all asymmetric coordinations, with a direct causal connexion between coordinands. The first three utterances behave like conditionals, which is relatively unusual for non-deductive coordinations. Both (5.31a) and (5.31c) appear synonymous with (5.32a), while (5.31b) appears synonymous with (5.32b), and (5.31d) with the equally confusing (5.32c). This is fine for (5.31a), and possibly also for (5.31d). However, the conjunctive utterance forms for (5.31b) and (5.31c) are not classically equivalent to the utterance forms for the corresponding conditionals. It is only the control that the Hearer has over the satisfaction of the first conjunct that reduces the four possibilities of a conjunction to the two required by non-deductive conditionals. In (5.31b) if the teller does not move, either he will not get shot by the robber, or the robber will break her word and the Expressivity norm. In (5.31c) if the teller also elects not to move, the robber may choose whether to shoot him without breaking her word either way. The teller is relying on the conditional being expositive rather than merely evocative, and thus the falsity of the consequent following from the falsity of the antecedent.

11Symmetric deductive disjunctions are often synonymous with deductive conditionals as \((A \lor B) \equiv (\neg A \supset B)\).


5.5. OTHER SPEECH ACTS

5.5.3 Interrogatives

For our purposes, questions can be divided into two main types: closed questions which are used to elicit one of a small number of answers, and open questions which are used to request less predictable information. I will ignore rhetorical questions, as they are more like assertions than questions. Open questions such as (5.33a) and most other ‘Wh-’ questions – those using ‘who’, ‘what’, ‘when’, ‘where’, and especially ‘how’ and ‘why’ – can be modelled as assertions that have some critical informational component missing, and which invite the Hearer to complete the assertion. Apart from a lack of Informativity, these questions appear to comply with the assertion norms. The class of closed questions is far more interesting for my project. There are two main subclasses of closed questions; polar questions such as (5.33b) which expect an affirmation or denial, and alternative questions such as (5.33c) which expect one of the provided options as a response.

(5.33) a. What did you eat this morning for breakfast?
   b. Did you watch last week’s Test match at Lords?
   c. Who is a better actor: Brad Pitt or Charlie Chaplin?

Alternative questions commonly present a disjunctive list of alternatives (5.34a), or an invitation to choose from a conjunctive list of contrary possibilities (5.34b). The invitation to select from the conjunctive list produces an effect very similar to open disjunction, as I have already discussed in §4.3.1 and §5.4.1. A polar question (5.34c) is usually answered with ‘Yes’ or ‘No’ (or similar affirmations and denials, depending on its wording). Some closed questions such as (5.34d) are ambiguous between alternative and polar interpretations, and a cooperative Hearer may choose to answer either or both potential questions (e.g., ‘yes; tea please’; ‘coffee for me, thanks’; ‘no thanks’). With a little more effort, both (5.34a) and (5.34b) can also be interpreted ambiguously. In comparison, (5.34e) is always unambiguously polar, although general cooperativeness may encourage Hearers to volunteer additional information about the type of drink they desire.

(5.34) a. Would you like tea, or coffee instead?
   b. Would you like a drink? – we have tea and coffee.
   c. Would you like a drink?
   d. Would you like tea or coffee?
   e. Would you like any tea or coffee?
   f. Wouldn’t you like [any] tea or coffee?

Let Q_{alt} and Q_{polar} represent metalinguistic question operators analogous to the existing A for assertibility and T for truth. Then I suggest Q_{alt}(\varphi \lor \psi) holds for a given context when T(\varphi \lor \psi) & \Diamond (A(\varphi) & T(\lnot \varphi)) & \Diamond (A(\psi) & T(\lnot \psi)) holds, which is A(\varphi \lor \psi) without the Informativity condition. Similarly Q_{polar}(\varphi) is just Q_{alt}(\varphi \lor \lnot \varphi), which holds when \Diamond A(\varphi) & \Diamond A(\lnot \varphi) holds for that context. This means the Expressivity, Consistency and Disjunctive Compositionality norms are appropriate for questions. In addition, Informativity needs to hold for any possible answer to the question, a requirement that is not trivial to formalise. The question (5.34d) can be disambiguated as either [Q_{alt}(you want tea \lor you want coffee)]; or [Q_{polar}(you want tea) \lor Q_{polar}(you want coffee)]; in the second case the Hearer can answer either polar question, as long as they indicate which.

One clue for ambiguity resolution is the NPI ‘any’ (see §5.2 for a discussion of NPIs), which is permissible in polar questions like (5.34e) but not alternative questions, presumably indicating that there is some negative element to polar questions. This negative element is unaffected by negating the verb, as is seen in (5.34f). However, what does change is the epistemic stance taken by the Speaker; in (5.34f) the Speaker assumes the Hearer would like a drink, and the query is in response to contrary evidence. With the more common positive phrasing in (5.34e), the Speaker does not assume the Hearer wants a drink, usually taking a neutral stance on the question. The contrary relationship between the stance of the Speaker and the positive response to the question appears to be what licenses the presence of NPIs in polar questions. In contrast, alternative questions always presume a positive stance, while seeking clarification within that assumption, and do not allow NPIs.
5.5.4 Assertible Arguments

The norms of argumentation appear to be significantly more complex than those of assertoric utterances. A lot of work has been done by argumentation theorists on the semantics and pragmatics of argumentative discourse. Some arguments are fragmentary, complicated, probabilistic, or involve multiple Speakers, but I will avoid these complexities. I will confine my analysis to simple premise-conclusion arguments (i.e., ones with no intermediate subconclusions), with all premises and the conclusion being asserted by the same Speaker. I will only analyse classically valid arguments, although the general approach can be extended to probabilistic arguments. One role that arguments can play is as explanations by explicitly listing a series of direct or inferential connexions between premises and conclusions. I will ignore this aspect as it has already been covered in §5.5.1. These restrictions mean that the formal assertibility theory developed in Chapter Two can be applied directly to arguments, allowing me to produce some formal results about algorithms for generating and testing assertible arguments, and to resolve some paradoxes of material implication in §5.6.1.

The conclusion of a valid argument may appear novel and acceptable, but either the conclusion, or some of the premises if they follow it, will fail to satisfy the assertion norms\textsuperscript{12}. For example, the conclusion of the argument \( p \supset q; p; \therefore q \) is not assertible because it is entailed by the premises, and thus not informative. We are usually indifferent as to whether a conclusion is presented first or last in an argument, so the Informativity requirement for argument conclusions will need to be changed via a simple principled exception. A more interesting issue occurs where a conclusion is not assertible in every, or even any, context where the premises are all assertible (but not yet asserted). This inability for the premises and conclusion to be assertible in the same context appears to be the common factor that makes many of the paradoxes of material implication odd or ‘paradoxical’ even though they are classically valid arguments. These paradoxes are discussed further in §5.6.1.

A simple definition of an argument in logic is: a set of formulas, one of which is distinguished as the conclusion, while the rest are premises. I will model an argument with the ordered pair of a set of premise utterance forms followed by a conclusion utterance form, and represent it as \( \langle \{p_1, \ldots, p_n\}; q \rangle \) where \( p_1, \ldots, p_n \) are the forms of the premises, and \( q \) is the form for the conclusion. For example, the general argument form for Simplification can be represented by the pair \( \langle \{p \wedge q\}; p \rangle \). The assertibility of the conclusion of an argument will be evaluated in isolation from the premises, but it cannot contain more information than the premises. To evaluate this we will check not only for validity, but also that every assertibility constraint imposed by the conclusion is also imposed by the set of premises. That is, any information that we can infer about logical and epistemic possibilities from the conclusion must also be derivable from the context and the premise set. We can now put these evaluation conditions for the conclusion together with the standard conditions for asserting the premises. An argument is assertible in a particular context iff the argument is valid in the context, each premise is assertible in that context, and asserting the conclusion partially conveys the premises in the context. And yes, treating the premises as communicative intentions is somewhat odd.

Given the above definition of assertible arguments, it is relatively straightforward to determine whether any given argument is assertible, as we already have simple algorithms for determining propositional validity and assertibility. The decision procedure is trivial: simply check if the premises are assertible in the context, the conclusion is assertible, and the truth and assertibility conditions of the conclusion are contained in those of the premise set in the context. Each step can be done deterministically in polynomial time using contribution or relevance semantics. It is not as trivial to produce an efficient algorithm for generating all and only the assertible arguments given a set of premises in a context. In fact, brute-force searches running in exponential time (with respect to the number of literals) will probably be required to find all possible conclusions. Even producing a deterministic general algorithm that produces all three of the following assertible arguments \( (p, p \supset q; q \therefore q); (p \vee (q \wedge r); (p \vee q) \wedge (p \vee r)); \) and \( (p \supset q, q \supset r; p \supset r) \) is non-trivial. I will sketch some algorithms that produce broad classes of conclusions for a given premise set, but will not try to imitate the conclusion-selection preference orderings of natural language arguers.

\textsuperscript{12}The conclusion may be assertible if it precedes (some of) the premises, but then at least one of the premises that follow it will not be assertible, because of the same underlying Informativity issue.
5.5. OTHER SPEECH ACTS

Algorithms for Assertible Arguments

In the following algorithms for producing assertible arguments, I will assume a null or empty context, as I did in most of Chapter Two. I will also assume that the conclusions are in NNF to ensure there are a finite number of assertible conclusions for any argument, by Theorem 2.106, although assuming the conclusion contains no double-negations would suffice. I will start with the simplest argument forms, as arguments with fewer premises are easier to analyse. No valid zero-premise argument is 3-assertible, as it would require the conclusion to be tautological, and thus not assertible. The algorithm for one-premise arguments is a generalisation of Simplification \( \{p \land q\} : p \) and Permutation from Defn 2.65. This algorithm does not produce all 5-assertible one-premise arguments (e.g., it cannot produce \( \{p \lor (q \land r)\} : (p \lor q) \land (p \lor r)\)), but it only produces arguments with 5-assertible conclusions.

**Lemma 5.13 Assertible Single-premise Arguments**

The argument \( \langle \psi :, \varphi \rangle \) is 5-assertible if \( \psi \) is 5-brief, and \( \varphi \) is the result of removing zero or more conjuncts (in positive scope) from a permutation of \( \psi \).

**Proof:** Any permutation of a 5-brief formula is 5-brief. Removing a conjunct from a 5-brief formula produces a 5-brief formula entailed by the original formula, by Lemma 2.104.  

I will only present two algorithms that produce assertible multi-premise arguments from assertible finite premise sets. The first algorithm produces all assertible arguments whose conclusion is the result of removing disjuncts and conjuncts from the premises, and so works in polynomial time. By replacing brief reductions with brief alternatives, all assertible arguments can be derived from a premise set, but deterministic algorithms for generating brief alternatives are exponential in \( |\text{Lit}(\Gamma)| \).

**Lemma 5.14 Assertible Arguments via Brief Reductions**

The argument \( \langle \Gamma :, \varphi \rangle \) is 5-assertible if \( \Gamma \) is consistent, each \( \gamma \in \Gamma \) is 5-brief, and \( \langle \psi :, \varphi \rangle \) is a 5-assertible argument, for some \( \psi \) a 5-brief reduction of \( \bigwedge \Gamma \).

**Proof:** If \( \psi \) is a 5-brief reduction of \( \bigwedge \Gamma \) then \( \psi \) is 5-brief and \( \bigwedge \Gamma \equiv \psi \not\equiv \bot \). Then if \( \langle \psi :, \varphi \rangle \) is a 5-assertible argument, and each \( \gamma \in \Gamma \) is 5-brief, \( \langle \Gamma :, \varphi \rangle \) is also a 5-assertible argument.

**Example:** \( \langle \{p, p \lor q\} :, q \rangle \) is 5-assertible, as \( \{p, p \lor q\} \) is consistent: \( p \) and \( p \lor q \) are 5-brief; \( p \land q \) is a 5-brief reduction of \( p \land (p \lor q) \); and \( \langle p \land q :, q \rangle \) is a 5-assertible argument.

The next algorithm finds all 4-assertible, literal-inclusive conclusions in Disjunctive Normal Form for a given premise set, by using minimal forms and prime implicants. A minimal form is any disjunction of prime implicants which has no redundant disjuncts. Quine defines a prime implicant \( \psi \) of a formula \( \varphi \) as a conjunction of literals that entails \( \varphi \), where removing any conjunct from \( \psi \) produces a formula that does not entail \( \varphi \) (i.e., \( \neg \varphi \vdash \neg \psi \) is 1-assertible). Any minimal form of prime implicants is 4-brief (as removing any disjunct or conjunct will change its truth value for some valuation) and contains only the literals of the original formula. Minimal forms of prime implicants are usually found by either Karnaugh maps or the Quine-McCluskey algorithm along with Petrick’s method. Karnaugh maps are non-algorithmic, while the Quine-McCluskey algorithm requires exponential time.

**Lemma 5.15 Assertible Arguments via Prime Implicants**

The argument \( \langle \Gamma :, \varphi \rangle \) is 4-assertible with literal inclusion if \( \Gamma \) is consistent, each \( \gamma \in \Gamma \) is 4-brief, and \( \langle \psi :, \varphi \rangle \) is a 4-assertible argument, for some \( \psi \) a minimal form of \( \bigwedge \Gamma \).

**Proof:** As \( \psi \) is a minimal form of \( \bigwedge \Gamma \), \( \bigwedge \Gamma \not\equiv \psi \not\equiv \bot \) and \( \psi \) is 4-brief, with \( \text{LIT}(\psi) \subseteq \text{LIT}(\Gamma) \). The result then follows from Lemma 5.13. 5-assertibility is much harder to prove.

**Example:** \( \langle \{p \lor q, q \lor r\} :, p \lor r \rangle \) is assertible, as \( p \lor q \) and \( q \lor r \) are 4-brief, \( (\neg p \land \neg q) \lor (q \land r) \) is a minimal form of \( (p \lor q) \land (q \lor r) \), and \( \langle (\neg p \land \neg q) \lor (q \land r) :, \neg p \lor r \rangle \) is an assertible argument.

As the number of potential 5-assertible conclusions increases exponentially with the number of literals in the premises, there is no polynomial time algorithm that produces all assertible arguments for an arbitrary premise set.
5.6 Conditionals

In natural language, a conditional is a subordinating conjunction consisting of a subordinate clause and a main clause whose acceptability is dependent on the subordinated hypothesis. The subordinated clause is the protasis or antecedent, and the main clause is the apodosis or consequent. The archetypical conditional subordinator in English is ‘if’, but conditionals can sometimes use any one of ‘when’, ‘since’, ‘unless’, or ‘in case’ instead, while less central cases might use the compounds ‘even if’, ‘only if’, or ‘whether . . . or not’. Modelling natural language conditionals is one of the largest and most controversial industries in the Philosophy of Language (it is also investigated widely in the Philosophy of Logic, metaphysics, and natural language semantics), and engaging fully with the literature to argue for a particular position would consume a thesis in itself. I will instead provide an analysis of a well-known and well-behaved subclass of conditionals that is highly suited to the assertion norms described in earlier chapters. I will then sketch an example theory of conditionals using the assertibility principles and coordination modes. This rough sketch fits much of the data derived by linguists from corpus analysis.

5.6.1 Material Implication

There is wide agreement amongst linguists and philosophers that some conditionals appear to be near-synonymous with their corresponding disjunctions, when transformed under the schema ‘if $A$, $B$’ $\leftrightarrow$ ‘either not $A$, or $B$’. The controversy comes when characterising this class and describing its extent; for some, this class might simply be the indicative conditionals, while for others it is a narrow class found primarily in mathematics and logic textbooks. Putting this controversy aside, conditionals where probability and doubt play no role, and contraposition produces synonymy (i.e., the corresponding disjunction is commutative), form an important subset of this class. These are the conditionals that can most adequately be represented by material implication. When one of these conditionals fits the schema ‘if $A$, $B$’, I will represent its form by $A \supset B$, which is equivalent to $\neg A \lor B$, $B \lor \neg A$, $\neg B \supset \neg A$, and $\neg (A \land \neg B)$.

Paradoxes of Material Implication

Using material implication to model even this hand-picked subset of natural language conditionals is known to lead to several counter-intuitive results. For example:

(5.35)  
\begin{enumerate}
  \item If $A$ is false, then $A \supset B$ is true for any $B$.
  \hspace{1cm} \text{Today is not Tuesday. } \therefore \text{If today is Tuesday, today is Wednesday.}
  \item If $B$ is true, then $A \supset B$ is true for any $A$.
  \hspace{1cm} \text{The sun is now shining. } \therefore \text{If it is night, the sun is now shining.}
  \item If $A \supset B$ is true, then $(A \land C) \supset B$ is true for any $C$.
  \hspace{1cm} \text{If it rains, I will lend you my umbrella. } \therefore \text{If it rains and I die, I will lend you my umbrella.}
  \item Either $A \supset B$ or $B \supset A$ is true, for any $A$ and $B$.
  \hspace{1cm} \text{Either if it is Easter, Gregory is the pope, or if Gregory is the pope, it is Easter.}
\end{enumerate}

Examples (5.35a)–(5.35c) are all valid arguments, but their conclusions are unassertible (given the truth of the premises) as each has redundant disjuncts. This means the arguments are not assertible, regardless of whether the premises are assertible. Example (5.35d) is supposed to be tautologous, but under this interpretation it is not assertible as it has the form $(\neg A \lor B) \lor (\neg B \lor A)$. As I will explain shortly, disjunctions of conditionals are always performative, and so the Hearer is actually being given a choice of two putatively true assertions to apply; as at least one of the rules must be incorrect, the expression is unassertible. The principles behind assertible arguments and performative coordinations can also be applied to some far more challenging paradoxes, which have been used to argue against material implication being appropriate for representing this central class of conditionals. Hunter (1983) has assembled the following examples from several sources in which intuitively invalid natural language arguments become valid when ‘if . . . then’ is formalised by material implication.
(i) (Adams 1965) If switch S is closed and switch T is closed the motor will start. So either if switch S is closed the motor will start, or if switch T is closed the motor will start.

(ii) (Stevenson 1970) If this figure is rectangular and equal-sided then it is a square. So either if it is rectangular but not equal-sided it is a square, or if it is equal-sided but not rectangular it is a square.

(iii) (Cooper 1968) If John is in Paris, then he is in France. If he is in Istanbul, then he is in Turkey. So if he is in Paris he is in Turkey, or if he is in Istanbul he is in France.

(iv) (Cooper 1968) If Goldbach’s conjecture is correct, then it is false that if the mayor’s telephone number is an even number then it cannot be represented as the sum of two primes. So if the mayor’s telephone number is not an even number then Goldbach’s conjecture is not correct.

All four involve only indicative conditionals, and the first three make no use of negated conditionals. It may be that Grice’s theory can explain how the addition of ‘conversational implicatures’ to the (valid) $\supset$ ‘translations’ of those arguments can make them invalid: but I don’t see how, and so far as I know no-one has even tried to show how.


Hunter uses these examples to justify his claim that material implication does not capture natural entailment, as part of his argument against the claim from Grice (1975) that the truth-functional semantics of ‘$\supset$’ plus pragmatic conversational principles are sufficient to characterise most uses of ‘if’. I claim instead that these are not assertible arguments as introduced in §5.5.4, by demonstrating there are situations in which the premises are assertible but the conclusion is not (in several of these examples there is no situation where the premises and conclusion are all assertible).

(i) The form of the premise is $(p \land q) \supset r$. It is unclear whether one disjunctive conclusion is proposed, or if a performative disjunction provides a choice of two, so I’ll examine both approaches. If the form of the conclusion is $(p \supset r) \lor (q \supset r)$, then the conclusion is not 1-brief, and so never assertible. If the Speaker is proffering a choice of conclusions which have the forms $p \supset r$ and $q \supset r$, then the assertibility of the premise $(p \land q) \supset r$ requires that both $pqr$ and $pq\bar{r}$ be epistemically possible, but these are counterexamples to the truth of $p \supset r$ and $q \supset r$ respectively.

(ii) The form of the premise is $(p \land q) \supset r$. Again, the conclusion is ambiguous. If the form of the conclusion is $((p \land \neg q) \supset r) \lor ((\neg p \land q) \supset r)$, then it is not 1-brief, and so never assertible. If the disjunction is performative, the potential conclusions are $(p \land \neg q) \supset r$ and $(\neg p \land q) \supset r$, but for the premise $(p \land q) \supset r$ to be assertible, it requires the epistemic possibilities $pq\bar{r}$ and $p\bar{q}\bar{r}$, which are counterexamples to the truth of $(p \land \neg q) \supset r$ and $(\neg p \land q) \supset r$ respectively.

(iii) The forms of the premises are $p \supset q$ and $r \supset s$. If the conclusion’s form is $(p \supset s) \lor (r \supset q)$, then it will be true when either premise is true, but not assertible when either premise is assertible. If the Speaker is offering a choice of the conclusions $p \supset s$ and $r \supset q$, then the argument to either conclusion is invalid, with reality serving as a counter-example. Our common knowledge of geography constrains the epistemic possibilities to members of $\{pq\bar{r}s, p\bar{q}\bar{r}s, pq\bar{r}s, p\bar{q}\bar{r}s\}$, and while both premises are assertible in this situation, neither conclusion is true.

(iv) In this example, the premise is ambiguous between $p \supset \neg(q \supset r)$ and $p \supset \neg(q \land r)$. The form $p \supset \neg(q \land r)$ is equivalent to $p \supset (q \land r)$, which formalises the claim “If Goldbach’s conjecture is correct, then the mayor’s telephone number is an even number and it can be represented as the sum of two primes”, making it too strong to represent the premise. The weaker form $p \supset \neg(q \supset r)$ is only assertible when $pq\bar{r}$ is an epistemic possibility along with one of $p\bar{q}\bar{r}$, $pq\bar{r}$, and $p\bar{q}\bar{r}$, but any other epistemic possibility may also be present except $pq\bar{r}$, including $p\bar{q}\bar{r}$ and $pq\bar{r}$. As the form of the conclusion is unambiguously $\neg q \supset \neg p$, and the premise allows the possibility of $pq\bar{r}$ and $p\bar{q}\bar{r}$, this argument is not valid.

These representative examples demonstrate that assertibility, assertible arguments, and performative disjunction collectively resolve many of the paradoxes of material implication. Whether all such paradoxes are resolvable, or if this is the correct solution, are much deeper questions.

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13By Goldbach’s conjecture, $r$ is true when $p \land q$ is true, so $(p \land q) \supset r$ is part of the shared context.
5.6.2 A Theory of Conditionals

There are several popular ways to formally model natural language conditionals, although some linguists such as Carston (1988) and Dancygier and Sweetser (2005) claim that they cannot be formalised. My preference is to use conditional probability in the tradition of Ramsey (1926) and Adams (1970). A related position, espoused by several philosophers including Edgington (1995) and Bennett (2003), is that conditionals are not the sort of linguistic object that is always associated with a truth value. For example, conditionals such as (5.36c) may not have a truth value when their antecedent is false. This position chimes well with some of the deviations I have introduced, and is particularly useful for my performative and renunciative conditionals, and for modelling counterfactuals.

I will represent a conditional ‘if $A$, $B$’ whenever $A$, etc., as $A \rightarrow B$ where $A$ represents the antecedent and $B$ the consequent. The meta-variables $A$ and $B$ need not stand for truth-bearing propositions. The simplest restrictions on assertion for a conditional $A \rightarrow B$ are that $P(A) > \delta$, and $P(B) < 1-\epsilon$. The content of the assertion includes the claim that $P(B|A) > 1-\epsilon$. That $P(A) > \delta$ means that, for the purposes of this utterance only, the Speaker’s stance towards $A$ is that its epistemic possibility is worthy of consideration; this can be done even with a logical contradiction (e.g., ‘if neither $p$ nor $\neg p$, then . . . ’). That $P(B) < 1-\epsilon$ similarly means that $B$ is considered informative. These restrictions are a fallible two-sided variant of the robustness introduced in Jackson (1987).

There are several arguments against probabilistic conditionals being modelled by conditional probabilities. Perhaps the most famous is Lewis (1976), which shows that $P(B|A) = P(B)$, given a couple of assumptions such as $(A \land B) \rightarrow C$ being equivalent to $A \rightarrow (B \rightarrow C)$. However, the condition that $P(B) < 1-\epsilon$ causes Lewis’s argument to fail (except for the special case of ‘whether or not’ conditionals which require $P(B) \geq 1-\epsilon$, allowing Lewis’s argument to succeed, but in that case we want $P(B|A) \approx P(B)$ anyway). There are more subtle arguments along the same lines, such as that of Hájek (1994), but these arguments assume that the conditional always has a truth value based on valuations of the antecedent and consequent. But recall that assertible conditionals describe the Speaker’s epistemic stance, and this must be indeterminate, as both $P(A \land B) > 0$ and $P(\neg A \land \neg B) > 0$ given the conditions above. A conditional is not evaluated over a single valuation (any more than a disjunction is), so there are no ‘facts of the matter’ about whether the antecedent or consequent are true. None of these arguments allow for simultaneous consideration of multiple epistemic possibilities.

The properties of a particular conditional utterance will also be strongly influenced by both the Speaker’s epistemic stance to the antecedent, and the connexion between the antecedent and consequent (and hence the conditional’s mode). This groups conditionals in ways that are roughly similar to categories proposed separately by both Dancygier and Sweetser (2005) and Declerck and Reed (2001)\(^{14}\). I will start by considering conditionals in each of the five modes from §3.6.2.

(5.36)  
a. If a closed planar polygon is a square, it is a rectangle.
b. If you throw a boomerang properly, it will come back.
c. If Vaughn is late, it’s because he missed the bus.
d. If you are hungry, there are biscuits on the sideboard. (Austin (1962))
e. Good morning, if we can still call it morning.

When $A \rightarrow B$ is assertible and $P(B|A) = 1$, such as in (5.36a), the conditional is deductive, and as $P(B|A) = 1$ iff $P(\neg A | \neg B) = 1$, its contrapositive $\neg B \rightarrow \neg A$ is also assertible. The deductive conditional also has transitivity $(A \rightarrow B, B \rightarrow C \therefore A \rightarrow C)$, monotonicity $(A \rightarrow C, (A \land B) \rightarrow C)$, and other standard properties of the material conditional. The conditions $P(A) > \delta$ and $P(B) < 1-\epsilon$ prevent ex falso quodlibet and verum ex quodlibet conditionals from being assertible. Deductive conditionals form a large subset of both Dancygier and Sweetser’s generic conditionals, and Declerck and Reed’s direct inferentials.

\(^{14}\) These two pairs of linguists bitterly disagree with each other on why the lines between conditional classes are drawn as they are, exactly where to draw them, and what level of detail is key for describing conditionals. Neither party would agree with my attempt to motivate some of the categories via logical principles.
When $A \rightarrow B$ is assertible, $P(B|A) < 1$, there is a direct causal, correlative, or temporal connexion between $A$ and $B$, and the tense of $A$ is prior to that of $B$, like in (5.36b), the conditional is in the expositive mode. It is not synonymous with its contrapositive $\neg B \rightarrow \neg A$, as that describes an indirect effect-to-cause relationship and so is evocative. Its inverse $\neg A \rightarrow \neg B$ is expositive however, and is implied by $A \rightarrow B$ in many contexts, due to the same relevance min-max pressures as described in Pred 4.4 for exclusive disjunction. Dancygier and Sweetser (2005) characterises these conditionals as predictive, while Declerck and Reed (2001) prefers to call them actualization conditionals.

When $A \rightarrow B$ is assertible and there is an indirect connexion between the contents of $A$ and $B$, as in (5.36c), the conditional is in the evocative mode. Its contrapositive may be an expositive or evocative conditional. Either way, whether it is assertible or not will depend on independent reasoning; that is, the assertibility of $A \rightarrow B$ cannot imply that of $\neg B \rightarrow \neg A$ nor its inverse $\neg A \rightarrow \neg B$, as an evocative conditional provides no information about the probability of $B$ when $A$ is false. Dancygier and Sweetser (2005) calls these conditionals epistemic, and Declerck and Reed (2001) describes them as having case-specifying antecedents. When $A$ is presented with a positive stance, and it has been suggested by another Speaker (or has occurred in the immediate environmental context), the conditional is backgrounding or anaphoric as discussed in §5.4.1 and Eckardt (2007).

When $A$ is a felicity, relevance, or satisfaction condition for $B$, like (5.36d), $A \rightarrow B$ is a performative conditional. Similarly, when $A$ is a locutionary condition for $B$ as in (5.36e), $A \rightarrow B$ is a renunciative conditional. For either conditional, $B$ is true regardless of the truth of $A$, but the Speaker’s cooperativeness in asserting it is not guaranteed should $A$ be false. Also the indirect connexion prevents the assertibility of $A \rightarrow B$ from implying anything about either its contrapositive or inverse, just like evocative conditionals. While Dancygier and Sweetser (2005) parallels our distinction with their speech act and metalinguistic conditionals, Declerck and Reed (2001) bundle both these conditionals into their broad class of rhetorical conditionals, with its twenty-odd subtypes.

These five coordination modes cover most conditionals, but there are plenty of interesting variations within them. Milne (1926) provides a range of non-standard conditionals in typical contexts:

(5.37) a. You can’t help respecting anybody who can spell TUESDAY, even if he doesn’t spell it right.

b. We could say ‘Aha!’ even if we hadn’t stolen Baby Roo. [emphasis mine]

c. When having a smackerel of something with a friend, don’t eat so much that you get stuck in the doorway trying to get out.

d. If I thought you would [forget me] I’d never leave.

A conditional $A \rightarrow B$ is concessive when $P(B|A) < P(B)$. This is usually expressed in English by ‘$B$ even if $A’$, although the ‘even’ is often elided. The antecedent of a concessive conditional usually expresses a predicate which falls upon a scale, whether of the predicate as in (5.37a), or an associated object, as in (5.37b). The appropriate scale is evoked by the context and the antecedent. These scalar predicates have been discussed in §§4.5.1 and 5.1.5. Any proposition $C$ that falls higher on this scale than $A$ is assumed to satisfy $P(B|C) \geq P(B|A)$ (although other linguistic or contextual factors may render $C \rightarrow B$ unassertible). The antecedent rarely falls at either extreme of the evoked scale, and usually just low enough so that $P(B|D)<1-\epsilon$ for any $D$ lower than $C$, so $D \rightarrow B$ is not assertible. When the antecedent falls or at near the low end of the scale, both $P(B|A)$ and $P(B|\neg A)$ are highly likely, and if $A \rightarrow B$ and $\neg A \rightarrow B$ share the same mode, they can be expressed using a conditional of the form ‘whether $A$ or not, $B’$. Jackson’s robustness conditions are based upon concessive conditionals.

The antecedents of most conditionals are presented with a neutral stance. These conditionals, along with those with a negative stance – so-called counterfactuals such as (5.37d) – create a downwards-entailing environment in their antecedents. In contrast, antecedents presented with a positive stance such as (5.37c) do not. I speculate that the doubt represented by antecedents with a neutral or negative stance evokes the same effects as a polar question (see §5.5.3), thus making the antecedent a downwards-entailing environment unless a positive stance is taken.
Conditionals like (5.37c) that present the antecedent with a positive stance can be expressed using ‘since’, ‘as’, ‘when’, or ‘whenever’ instead of ‘if’. Like evocative conditionals, they do not provide any information about the consequent were the antecedent to be false. Antecedents presented with a negative stance (typically by using the subjunctive mood) such as (5.37d) are usually counterfactual, although the subjunctive can also convey tentativeness due to politeness or diffidence. Counterfactual conditionals are notoriously difficult to formalise, and the best description I have for them is as the hypothetical epistemic past conditional ♦(A → B) ∧ ¬A, similar to that described (and rejected) in the fourth chapter of Adams (1975). Modelling the class of positive stance conditionals presents similar issues to those for counterfactuals, as the Speaker is explicitly presuming that A is true. In both cases, we can think of a broader (and in some sense epistemically prior) situation where A → B is assertible and A unknown, with the later addition of either reliable information or a presupposition about the appropriate stance to take towards A. As I do not have a full dynamic epistemic semantics for assertibility, I will model a positive stance as the presupposition that the antecedent is true.

One final troublesome feature of English conditionals is that they cannot be embedded in other coordinations and subordinations arbitrarily, in the same way as conjunctions and disjunctions. This property has been appealed to by Adams, Edgington, and several others to argue for their lack of truth values, or at least the inadequacy of the material conditional. By appealing to the five conditional modes, we can make some finer distinctions about embedding conditionals.

(5.38) a. If Amy goes to the party then if Beth goes Chris won’t.
   b. If Amy and Beth go to the party then Chris won’t.
   c. If Beth goes to the party if Chris does, then Amy won’t go.
   d. If Beth and Chris go to the party, then Amy won’t go.
   e. If Beth goes to the party or Chris doesn’t, then Amy won’t go.
   f. If Amy goes to the party Beth goes, or if Amy doesn’t go, Chris won’t.
   g. If Amy goes to the party Beth won’t, and if Beth goes, Chris won’t.

Any conditional can be nested in the consequent of a non-concessive conditional. If two conjoined conditionals have the same mode and the same stance, as with (5.38a), the utterance is equivalent to a conditional whose antecedent is the conjunction of the two antecedents, and whose consequent is the nested consequent, which for this example gives us (5.38b). When an evocative, performative, or renunciative conditional is within the antecedent of another conditional with the same stance like (5.38c), the outer conditional is synonymous with a simpler conditional. This new conditional has the same consequent as the outer conditional, and its antecedent is the conjunction of the antecedent and consequent of the inner conditional (5.38d); not their material implication (5.38e). That is, (A → B) → C is congruent to (A ∧ B) → C and not (¬A ∨ B) → C, as these conditionals carry no information about ¬A. Performative and deductive conditionals are difficult to interpret when used as the antecedents of other conditionals, as the information implied by the possibility their antecedent is false is not available for use in the antecedent of the outer conditional, and so the conditional becomes non-compositional, ambiguous and even incoherent. An assertible disjunction of conditionals (5.38f) is always performative (or renunciative), as both conditionals will be true, so the assertion norms rule out every other sort of disjunction. This was observed in §5.6.1 during the analysis of Hunter’s first three ‘paradoxical’ arguments. A conjunction of conditionals is usually assertible iff its conjuncts are. The formal indistinguishability of (symmetric) expositive and performative conjunctions and my intuitions when looking at examples like (5.38g) lead me to conclude it is possible that, just like disjunctions, only performative conjunctions allow conditional conjuncts. Finally, negated conditionals are never synonymous with the conjunction of the antecedent and the negated consequent, although a negated evocative conditional ¬(A → B) is roughly synonymous with A → ¬B, and by litotes, often with A → ¬B. All these results are what we would expect if conditionals were assertions of conditional probability rather than conditional assertions.

\(^{15}\)Conditionals using ‘since’, ‘when’, or ‘whenever’ present a positive stance towards their antecedents, as do those if-conditionals that are synonymous with one of these conditionals, and any that presuppose their antecedents.
5.7 Penultimate Conclusions

The assertion norms and the resulting assertibility systems were first defined with the goal of modelling the assertion of natural language coordinations. In this chapter, I have shown that the general mechanisms developed for this purpose can easily be adapted to model a number of related linguistic phenomena. These include:

- Some restrictions on determiners, and on how the degrees of coordination can be modelled by different types of predicates and relations.

- Litotes, and the missing natural language coordinator ‘nand’.

- Modal auxiliaries.

- Disjunctive, conjunctive, and conditional Free Choice phenomena across all types of modality.

- The semantics of ‘any’ that is compatible with the disjunctive Free Choice mechanisms, and should allow the prediction of quantificational Free Choice.

- Common types of speech acts apart from assertion. Many of the assertion norms are applicable across these speech acts, with minor alterations.

- The assertibility conditions of conditionals that is compatible with recent research in cognitive linguistics, while bearing similarities to some mainstream philosophical models.

From a formal perspective, I have also provided extensions to brevity and concision so as to include negation, identity, and quantifiers, and a theory of assertible arguments that defuses the ‘paradoxes’ of material implication. None of these proposed extensions to assertibility are as fully developed as the theory of coordination, but I hopefully have described them in sufficient detail to demonstrate that assertibility is no one-trick pony.
6. Conclusions

My project was to demonstrate that English coordinations have principled polysemy, by using the degree of deviation from formal cooperative assertion norms to predict the information that is conveyed over a wide range of coordinations. In Chapter One I motivated and formalised some cooperative norms for assertoric utterances. In Chapter Two I provided truth-table, set-theoretic, modal, and preference systems that are all sound and complete with respect to these norms. In Chapter Three I switched perspective to the interpretability of an utterance, then modified the assertion norms to allow Hearers to take advantage of shared information about probability, order, relationships between coordinands, and syntax. In Chapter Four I predicted many natural and cross-linguistic categories in disjunctions, conjunctions and other coordinations using these richer assertion norms. Finally in Chapter Five I sketched several ways that the assertion norms could be extended, including predication, negation, free choice, arguments, and conditionals.

I have shown that the Expressivity, Informativity, Consistency, and Compositional norms can be used to model some cross-linguistic constraints on natural language coordinations, and that when combined with psychological factors they can predict some quite fine-grained communicative phenomena. None of these norms are universal: we have already seen that explanations often bypass Informativity, and belief revision must momentarily side-line Consistency, while sometimes politeness or other norms may override Expressivity. These norms have an advantage over the Gricean maxims and several similar proposals, as they have been formalised (at least for coordinations), and this formalisation results in a relatively narrow class of fairly firm predictions, which are easier to confirm or falsify. However, many of the predictions make necessary reference to vagaries of context, and these may need to be made more precise before being truly testable. Another promising sign for these norms is that they appear applicable not just to coordinations, but also to a wider class of related phenomena. The principled extensions to modality, predicate logic, negation, and conditionals are based solely on the same underlying considerations that motivated the original norms.

I have presented a range of evidence to suggest that the assertion norms correspond to some linguistic, psychological, or pragmatic principles that underlie linguistic practise. To investigate this more thoroughly would require further analysis of corpora in multiple languages, and reviewing a wider range of proposed coordination patterns, then evaluating whether the norms adequately describe these patterns. However, even if the assertion norms are merely formal devices which do not correspond to any psychological restrictions, they appear extensionally adequate, in that they successfully predict many of the different forms of disjunction and conjunction in some detail. This consists of a single core formal description for each of disjunctive and conjunctive coordination, along with the cognitive factors modelled by coordination modes, reliability, asymmetry, etc., that cause them to be used in many different ways. I have thus met the stated goal of the thesis. Some of the types of disjunction in §4.3.5 and conjunction in §4.4.3 are characterised by properties that aren’t modelled by assertibility and coordination modes; other parameters would be required to create a complete, rather than merely accurate, characterisation of coordination. The assertibility norms have been applied primarily in default, empty, or near-empty contexts, but there appears no reason why ‘particularised’ as well as ‘generalised’ predictions could not be drawn from the assertion norms by appealing to the interaction of the norms with the particular context.

The use of preference relations between formulas, rather than possible worlds, appears to be a novel formal technique. The inference relations of 4-assertibility and 5-assertibility have a number of elegant formal properties, and are formally interesting in their own right, particularly as they demonstrate several behaviours more commonly associated with relevant entailment.
6.0.1 Unwarranted Speculations

My formal machinery is used to predict what people think should be said. This is what I have tested for, and it does not require corroboration by a theory of linguistic processing. Suppose, however, that the assertion norms have some psychological reality, meaning that they fairly accurately describe aspects of some heuristic used in human linguistic processing. This would provide an explanation for their predictive power. I have no direct evidence to support this speculative claim, but assertibility has a number of striking similarities with Relevance Theory, a psychologically-motivated programme of pragmatics research proposed by Sperber and Wilson (1986).

Neither psychological nor biological processes follow formal algorithms, as best we can understand, but idealised versions of their results can often be produced by formal methods. These range from slime moulds ‘solving’ minimal solutions to mazes, to edge-detection in vision processing, to the natural language grammars that can be approximated with recursive rule-based systems. A simple iterative algorithm such as ‘find an utterance that conveys the content, then remove phrases until removing any more prevents it from conveying the content’ is relatively modest compared with many processes that appear to occur within our brains. Its biggest assumption is that we have some semantic module that can determine the literal content of a particular expression, but this assumption underlies most work in semantics and pragmatics. This algorithmic simplicity has been an ongoing design consideration of mine. Its influences include: my presentation of licensing semantics as the truth-table-based algorithm of §2.1.1 rather than the recursive predicates of Appendix A.1.3; my presentation of brevity as a series of operations in a derivation in §2.4.1, rather than a syntactic axiomatic formalisation; my rejection of metric brevity as non-procedural in §2.4.3; my desire for finitude in §2.6; my focus on transitivity in §3.6.1; and my discussion of algorithmic efficiency when evaluating arguments in §5.5.4. As a result, assertibility can be thought of as procedural as much as predicative.

Relevance theory is a mainstream theory of communication that emphasises both the procedural nature of meaning and the psychological processing aspect of linguistic comprehension. It describes communication in terms of a hearer extracting the maximum information from the minimum processing. The central relevance principles, as described on pp.383–4 of Cruse (2004), are:

The cognitive principle of relevance is that human cognition is geared towards maximal relevance – the achievement of as many cognitive effects as possible for as little processing effort as possible. The communicative principle of relevance is that every fact is initially assumed to be of optimal relevance. The procedure for using a fact is to check deductive hypotheses in order of their accessibility; that is, follow the path of least effort, until either a deduction which satisfies the expectation of relevance is found or the candidate is deemed to be suboptimal and rejected; then stop.

I follow the same min/max approach when weighing up the various possible interpretations, relevant contexts and \( \rho \) thresholds, as explained in §3.1.2. The shared judgements in relevance, weighting, and ease of extracting information from the context that are key to Relevance theory have also been deliberately incorporated into \( \rho \) weighting in §3.3.1.

There is one final similarity between Relevance Theory and assertibility. There is an oft-overlooked chapter in Sperber and Wilson (1986) dedicated to inference patterns. There they classify different types of natural deduction rules, and declare several times, most clearly on p.96, that ‘the only rules which in any interesting sense form part of the basic deductive equipment of humans – are elimination rules’. They claim that basic human reasoning only ever reduces the complexity of concepts. If we charitably also allow the rearrangement of concepts into forms of equivalent complexity and content, then this restriction of only allowing rules that produce a smaller formula also describes the basis of brevity and concision. I presented the concision semantics primarily because of their algorithmic properties discussed above, but also because of this parallel. I am not aware of any other single-premise natural deduction system that operates only by reducing complexity, and so these similarities make me wonder if assertibility may be modelling some of the same cognitive processes as Relevance theory.
6.0.2 Future Developments

Some of the suggested principles and formalisms have shortcomings that indicate there may be better solutions to the problems they are designed to address. One aspect that requires further teasing out is the relationship between the Inclusion and Extensibility norms, as discussed in §1.4.4 and Lemma 2.103. However, my primary concern is with the aesthetics of 5-assertibility. The inference relation of 4-assertibility captures our three initial assertion norms along with the recursive disjunctive and conjunctive restrictions, has a natural representation in many different semantics, and can be defined in terms of 2-assertibility (see Lemmas 2.13, 2.35, and 2.81.3). It also has a brevity relation where briefer formulas are always shorter, which applies to a formula if it applies to its negation, and only require the removal of disjuncts or conjuncts, and so on. Adding the Inclusion norm invalidates many of these desirable properties, in return for providing literal inclusion and a finite number of assertible conclusions for each premise set. Aesthetic considerations suggest that there be a better solution to literal inclusion and finitude than 5-assertibility. And yet it does not seem appropriate to model literal inclusion as a separate requirement like connexion or weighting, rather than as an assertion norm.

A number of lines of research have only been lightly sketched, or even skipped over altogether. Each of the principles for deviating from assertibility could be replaced with concepts that better reflect current psychological or linguistic research, and then formalised to whatever extent is appropriate to each concept. I have taken a step towards this by modifying Sweetser’s domains of discourse into modes that depend on connexion classes, which in turn are described in terms of transitivity and subjective causal relations. However weight, illocutionary satisfaction, locutionary success, and connexion itself all need further development based on psychological research. Similarly, phrasal coordination is only described via a theory of syntactic types; when this theory falls out of favour, phrasal coordination will need to be reconstructed for the next set of candidates. In short, the entire third chapter needs to be grounded in linguistic research that has not yet been performed.

The coordination category predictions are not defeasible. While they are a step up from the vague justifications that are often associated with Gricean maxims, they still frequently call upon unspecified context to determine their applicability or information content. One reason for this is that common ground and utterance contexts are rich and nuanced, while I have modelled them as a single set of propositions. Another cause of vagueness is that the Hearer has no access to the Speaker’s intentions, and so cannot determine whether the Speaker has accidentally broken the norms, broken them to aid their communication, or violated them with flagrant disregard. These cases lead to quite different interpretations of utterances. Better theories of context and relevance may reduce the vagueness or ambiguity, and thus remove many of the just-so stories masquerading as predictions.

Extending the assertion norms beyond conjunctive and disjunctive assertion to include negation, predicates and quantifiers, modality, and conditionals, as well as different types of speech acts, is a series of projects, each potentially as large as this assertible coordination project. It is possible that some of these extensions will produce incompatible models, but my initial investigation indicates that the research appears both feasible and fruitful. Declerck and Reed (2001), Dancygier and Sweetser (2005), and other corpus-based analyses that classify conditionals into several dozen categories based on their syntax and semantics provides a basis for perhaps the most challenging and philosophically relevant of these projects.

Finally, representing Schurzian relevance and other connexive logics with assertibility-style semantics has been a useful task as it sheds additional light on both projects. 5-assertibility provides a range of semantics that can be applied to verisimilitude, finitude, and restrictions on entailment, while Schurz’s relevant deduction has helped me to develop 5-assertibility. Exploration of the philosophical underpinnings of these relationships may link assertibility into several additional lines of enquiry, possibly providing further insight into our intuitions about implication.

At the end of this research I have far more questions than when I started, and an awareness of the limitations of the few answers I have provided, but many of my new questions are more precise, searching, and informed than the questions I initially proposed.
Appendix A. Additional Semantics

It is a universal truth that one cannot have too many semantics. Here are some more semantics and connectives for assertibility, and licensing semantics for some well-known connexive systems which show that some of them are quite closely related to assertibility.

A.1 Alternative Semantics

The semantics provided in Chapter Two are not the only assertibility semantics, merely the most convenient for the variations and applications in Chapters Three & Five. The following are variations on relevance, concision, and licensing semantics respectively. Some of the semantics do not define all the members of the assertibility family.

A.1.1 Sets of Non-empty Sets

The relevance semantics in §2.2 take sets of possibilities representing the premises, recursively operates on them by decomposing the conclusion, and then requires the resulting sets to be non-empty. An alternative approach is to recursively define sets of formulas based on the structure of the conclusion, and then check that the formulas stand in appropriate relations to the premises.

Definition A.1  \( \Gamma \vdash \theta \) is 2-assertible iff \( \Gamma \vdash \text{cl}(\theta) \) and \( \forall \langle \sigma, \tau, \lambda \rangle \in \Delta(\theta) : S_{\Gamma} \cap S_{\sigma} \neq \emptyset \).

\[ \Gamma \vdash \theta \] is 4-assertible iff \( \Gamma \vdash \text{cl}(\theta) \) and \( \forall \langle \sigma, \tau, \lambda \rangle \in \Delta(\theta) : S_{\Gamma} \cap S_{\sigma} \neq \emptyset \) and \( S_{\neg \theta} \cap S_{\tau} \neq \emptyset \).

\[ \Gamma \vdash \theta \] is 5-assertible iff \( \Gamma \vdash \text{cl}(\theta) \) and \( \forall \langle \sigma, \tau, \lambda \rangle \in \Delta(\theta) : S_{\Gamma} \cap S_{\sigma} \cap (S_{\neg \theta})_{\pm \lambda} \neq \emptyset \) and \( S_{\neg \theta} \cap S_{\tau} \neq \emptyset \).

Definition A.2 \( \Delta(\theta) \)

1. \( \Delta(l) = \{<l, \neg l, l>\} \).
2. \( \Delta(\varphi \land \psi) = \{<\psi \land \chi_1, \psi \land \chi_2, l> : \langle \chi_1, \chi_2, l \rangle \in \Delta(\varphi) \} \cup \{<\varphi \land \chi_1, \varphi \land \chi_2, l> : \langle \chi_1, \chi_2, l \rangle \in \Delta(\psi) \} \).
3. \( \Delta(\varphi \lor \psi) = \{<\neg \psi \land \chi_1, \neg \psi \land \chi_2, l> : \langle \chi_1, \chi_2, l \rangle \in \Delta(\varphi) \} \cup \{<\neg \varphi \land \chi_1, \neg \varphi \land \chi_2, l> : \langle \chi_1, \chi_2, l \rangle \in \Delta(\psi) \} \).
4. \( \Delta(\neg \varphi) = \Delta(\text{NNF}(\neg \varphi)) \) for complex \( \varphi \).

A.1.2 Adaptive Logics

Adding a relative \( n \)-brevity requirement to any classical natural deduction system will produce an Adaptive logic for \( n \)-concision, and hence \( n \)-assertibility. Any Adaptive logic can be defined in four steps. (1) Select a base logic with natural deduction. For all \( n \)-assertibility systems, the base logic is classical propositional logic. (2) Define conditions for rejecting derived natural deduction lines. Using \( n \)-brevity, this is simple: a deduction line containing a formula \( \varphi \) is rejected iff it is possible to derive a formula \( \psi \) from the same premises and assumptions such that \( \psi <_n \varphi \). Rejected lines may still be used to derive further lines in the deduction. (3) Define conditions for reaccepting rejected deduction lines. No assertibility logic ever reaccepts any rejected deduction lines. (4) Any formula on an assumption-free deduction line and which cannot be rejected, is the conclusion of a valid deduction from the premises, or in our case an \( n \)-assertible inference. Verhoeven (2007) also uses Adaptive logic, but takes a slightly different approach as she defines rejection conditions for R1 (her RAD) and R2 (her C) using nested multi-sets. Adaptive logic provides a methodology for using natural deduction to derive the non-monotonic, non-transitive assertibility inference relations.
A.1.3 Contribution Predicates

Licensed inferences can be defined by using predicates representing contributions of formulas and sets of variables, rather than procedurally using contribution tables. Each of the following definitions has a direct correspondence with a step in the algorithm described in §2.1.1.

Definition A.3 Given \( \theta \in L \) containing \( k \) atom occurrences, and \( \Gamma \subseteq L \):
1. \( \Sigma = \text{PROP}(\Gamma \cup \theta) \). This is the set of variables for the truth table.
2. \( \Lambda(\theta) = \{p_1, \ldots, p_k\} \). These are the unique sequential names for the atom occurrences in \( \theta \).
3. \( f(\theta) \) is the result of replacing the atom occurrences in \( \theta \) from left to right by \( p_1, \ldots, p_k \).
4. \( g(\theta, \varphi) \) is a function from the subformula \( \varphi \) of \( f(\theta) \) to the corresponding subformula of \( \theta \).

Example: \( f((p \land q) \lor (p \land \bot)) = (p_1 \land p_2) \lor (p_3 \land p_4) \), and \( g((p \land q) \lor (p \land \bot), p_3 \land p_4) = p \land \bot \).

Definition A.4 Truth Tables
\( T_\Sigma = \{u \mid u : \Sigma \mapsto \{\text{true, false}\}\} \). \( T_\Sigma \) corresponds to the truth table over \( \Sigma \).

\( p : T_\Sigma \mapsto T_\Sigma \) is the function where \( \forall p, q \in \Sigma : u_p(q) = u(q) \iff q \neq p \). \( u_p \) is the \( p \)-twin of \( u \) in \( T_\Sigma \).

\( u_p \) is the valuation function in \( T_\Sigma \); that is identical to \( u \) except for the value of \( p \). Note that \( u = (u_p)_p \).

The function \( u_p \) in Defn A.4 corresponds to the \( p \)-twin row in Defn 2.5.

Definition A.5 Restricted Assignment
\( \forall u \in T_\Sigma, p \in \Sigma : A(u,p,\Gamma) \iff \neg u(\Gamma) \lor \neg u_p(\Gamma) \).

\( A(u,p,\theta) \) is true when an occurrence \( p \) of an atom in \( \theta \) is assigned to at most one of valuation function \( u \) and its \( p \)-twin \( u_p \), and corresponds to restricted assignment from Defn 2.6. Simple assignment from Defn 2.4 applies to all rows, so a corresponding predicate is not needed.

Definition A.6 Compositional Contribution
\( \forall u \in T_\Sigma, \forall p_i, p_j \in \Lambda(\theta), \forall \varphi_1, \varphi_2 \in L : \)
1. \( C(u,p_i,p_j) \iff p_i = p_j \).
2. \( C(u,p_i,\neg \varphi_1) \iff C(u,p_i,\varphi_1) \).
3. \( C(u,p_i,\varphi_1 \land \varphi_2) \iff (C(u,p_i,\varphi_1) \land u(g(\theta,\varphi_2))) \lor (C(u,p_i,\varphi_2) \land u(g(\theta,\varphi_1))) \).
4. \( C(u,p_i,\varphi_1 \lor \varphi_2) \iff (C(u,p_i,\varphi_1) \land u(g(\theta,\neg \varphi_2))) \lor (C(u,p_i,\varphi_2) \land u(g(\theta,\neg \varphi_1))) \).

\( C(u,p_i,\varphi_j) \) is true when the atom occurrence \( p_i \) contributes to the subformula occurrence \( \varphi_j \) on row \( u \). Defn A.6.1 corresponds to contribution provision in Defn 2.3, while Defn A.6.2–4 corresponds to the contribution tables in Defn 2.7.

Definition A.7 Evaluation Criteria
1. \( D_1(\Gamma; \theta) \iff D_2(\Gamma; \theta) \) or \( \theta \) is disjunction-free.
2. \( D_2(\Gamma; \theta) \iff \forall p_i \in \Lambda(\theta), \exists u \in T_\Sigma : u(\Gamma), u(\theta) \) and \( C(u,p_i,\theta) \).
3. \( D_3(\Gamma; \theta) \iff \forall \varphi \ a \ subformula \ of \ \theta, \exists u_1, u_2 \in T_\Sigma : u_1(\varphi) \neq u_2(\varphi) \).
4. \( D_4(\Gamma; \theta) \iff \forall p_i \in \Lambda(\theta), \exists u \in T_\Sigma : \neg u(\Gamma), \neg u(\theta) \) and \( C(u,p_i,\theta) \).
5. \( D_5(\Gamma; \theta) \iff \forall p_i \in \Lambda(\theta), \exists u \in T_\Sigma : u(\Gamma), u(\theta), C(u,p_i,\theta), \) and \( A(u,\theta(p_i),\theta) \).

The predicates \( D_1(\Gamma; \theta) \ldots D_4(\Gamma; \theta) \) in Defn A.7 correspond to the criteria \([D1] \ldots [D4]\) in Defn 2.9 under simple assignment, while \( D_5(\Gamma; \theta) \) corresponds to \([D2]\) under restricted assignment.

Conjecture A.8 \( \Gamma \vdash \theta \) is \( n \)-licensed \( \iff \Gamma \vdash_t \theta \) and \( D_1(\Gamma; \theta) \ldots D_n(\Gamma; \theta) \).

Proof Sketch: Defn 2.3’s contribution provision and Defn 2.7’s compositionality are captured by the \( C(u,p_i,\varphi_j) \) predicate in Defn A.6. Defn 2.4 assigns contributions for all rows, while Defn 2.6 uses \( p \)-twin rows from Defn 2.5 to restrict contributions by assigning them to exactly those rows specified in \( D_5(\Gamma; \theta) \) by the predicate \( A(u,p,\theta) \) from Defn A.5 using the twin row function \( u_p \) from Defn A.4.

This leaves the evaluation criteria from Defns 2.9 and A.7. \([D2] \ldots [D4]\) and \( D_2(\Gamma; \theta) \ldots D_4(\Gamma; \theta) \) are obviously direct translations of their respective notations, while \([D1]\) and \( D_1(\Gamma; \theta) \) are equivalent by Lemma 2.81.1 and Theorem 2.92.1. ■
A.1.4 Unique Atomic Assertibility

Assertibility can be defined by some very simple licensing-type semantics without contributions, by moving most of the complexity into $\Pi$. This semantics can also be easily extended to include features such as encapsulation. We will initially restrict the conclusion $\varphi$ to $L'$, being that constant-free, positive fragment of $L$ where the $i$th atom occurrence in $\varphi$ is $p_i$. This restriction ensures that no atom occurs more than once, no constant occurs at all, and that we can readily identify each atom by its position.

**Definition A.9** Two rows of a truth table are $p$-sibling rows iff they have different valuations for each atom $q$ where $\Pi, p \not\models_{\mathfrak{F}} q$ and $\Pi, p \not\models_{\mathfrak{F}} \neg q$. Two rows of a truth table are $p$-changeling (maximally different sibling) rows iff they are $p$-sibling rows and both satisfy $\Pi$.

**Definition A.10** Licensing criteria where $\Gamma \models_{\mathfrak{F}} \varphi$.

[D2'] For each atom $p$ in $\varphi$, there is a truth table row where $\Pi$ and $\Gamma$ are true, and $\varphi$ is false on its $p$-twin row.

[D2''] For each atom $p$ in $\varphi$, there is a truth table row where $\Pi$ and $\Gamma$ are true, and $\varphi$ is false on its $p$-changing row.

[D2''''] For each atom $p$ in $\varphi$, there is a truth table row where $\Pi$ and $\Gamma$ are true, and $\Gamma$ is false on all its $p$-sibling rows.

[D4'] For each atom $p$ in $\varphi$, there is a truth table row where $\Pi$ is true and $\varphi$ is false, and $\varphi$ is true on its $p$-twin row.

To extend this approach to allow $\varphi$ to be any formula in the full language of $L$, we simply add a series of biconditionals to $\Pi$. We first define $f(\varphi)$ as the unique formula in $L'$ that is identical to $\text{NNF}(\varphi)$ apart from the choice of variables or constants. Now suppose that $\varphi$ is a formula in $L$ whose $m$ literal occurrences are $(l_1, \ldots, l_m)$ (recall that a literal can be a variable or constant, and may be negated). Then for each literal occurrence $l_i$, we add $(l_i \leftrightarrow p_i)$ to $\Pi$.

**Conjecture A.11** $n$-licensing for $(\Pi; \Gamma) \vdash \varphi$.

1. $(\Pi; \Gamma) \vdash \varphi$ is $2$-licensed iff [D2'] holds for $(\Pi, \bigcup_{i \leq m} (l_i \leftrightarrow p_i); f(\Gamma)) \vdash f(\varphi)$.
2. $(\Pi; \Gamma) \vdash \varphi$ is $4$-licensed iff [D2'] and [D4'] hold for $(\Pi, \bigcup_{i \leq m} (l_i \leftrightarrow p_i); f(\Gamma)) \vdash f(\varphi)$.
3. $(\Pi; \Gamma) \vdash \varphi$ is $5$-licensed iff [D2'''] and [D4'] hold for $(\Pi, \bigcup_{i \leq m} (l_i \leftrightarrow p_i); f(\Gamma)) \vdash f(\varphi)$.
4. $(\Pi; \Gamma) \vdash \varphi$ is a relevant deduction of Schurz’s third kind iff

[D2'''] holds for $(\Pi, \bigcup_{i \leq m} (l_i \leftrightarrow p_i); f(\Gamma)) \vdash f(\varphi)$, for constant-free $\varphi$.

**Proof Sketch**: The $p$-twin rows in [D2'] and [D4'] in Defn A.10 check if varying the valuation of the atomic occurrence is sufficient to affect the truth of $\varphi$. The $p$-changeling row in [D2'''] from Defn A.9 works like a $p$-twin row in standard licensing, in that it varies the valuation of all occurrences of the atom. These observations are sufficient to construct inductive proofs for 2-, 4-, and 5-licensing on the complexity of $\varphi$. The proof for Schurz’s third kind follows directly from Defns A.29 and [D2'''], as the set of $p$-sibling rows from Defn A.9 effectively mimics substitutions of any combination of one or more occurrences of $p$. □

**Example**: $p \land q \vdash (p \land r) \lor (q \land \neg r)$ is 5-licensed iff $(\varphi \equiv \{\{(p \leftrightarrow p_1), (r \leftrightarrow p_2), (q \leftrightarrow p_3), (\neg r \leftrightarrow p_4)\}; p_1 \land p_2\} \vdash (p_1 \land p_2) \lor (p_3 \land p_4)$ is 5-licensed, by Conjecture A.11. But $(\varphi \equiv \{\{(p \leftrightarrow p_1), (r \leftrightarrow p_2), (q \leftrightarrow p_3), (\neg r \leftrightarrow p_4)\}$ is false on the $p_2$-changeling row of each of the four truth table rows where $\Pi$ is true and $(p_1 \land p_2) \lor (p_3 \land p_4)$ is false, so neither inference is 5-licensed.

Finally, we can extend this definition further to include encapsulation from §3.2.2. If a subformula $\psi$ of $\varphi$ is encapsulated, simply replace $\varphi$ with $\varphi q_\psi$ for a new variable $q$, and then add $(q \leftrightarrow \psi)$ to $\Pi$. This directly captures the essence of encapsulation, by treating $\psi$ as if it were atomic.
A.2 Rival Semantics

The 5-assertibility inference relation combines the restrictions of 4-assertibility and literal inclusion, by extending the basic norms of §1.4.3 and the respective 4-assertibility licensing, relevance, and concision semantics. It appears to be the best of several viable candidates. I will describe briefly some other options below, and justify my choice of Inclusion norm. All candidate norms require restrictions on contribution assignment in a manner similar to that of Defn 2.6.

Definition A.12 Restricted Assignment with Variants

Restricted Assignment: The contribution of each atom occurrence is assigned to a row of the contribution table iff the conclusion is false for that row or its twin row for that atom; other rows are assigned the non-specific contribution ‘$\ast$’.

Premise Restricted Assignment: The contribution of each atom occurrence is assigned to a row of the contribution table iff the premise set is true for that row or its twin row for that atom; other rows are assigned the non-specific contribution ‘$\ast$’.

Definition A.13 Variant 5-Licensing Evaluation Criterion

\[ [D2]^+ \text{ Every contribution provided must be in the contribution for the conclusion on a row where the premise set and conclusion are true, and on its twin-row.} \]

\[ [D4]^+ \text{ Every contribution provided must be in the contribution for the conclusion on a row where the premise set and conclusion are false, and on its twin-row.} \]

Definition A.14 Variants in Defn 2.10.5 for $\Gamma \vdash \varphi$

\[ \Gamma \vdash \varphi \text{ is a 5-licensed inference iff } [D2] \text{ and } [D4] \text{ hold under restricted assignment.} \]

\[ \Gamma \vdash \varphi \text{ is a } 5_T\text{-licensed inference iff } [D2] \text{ and } [D4] \text{ hold under premise restricted assignment.} \]

\[ \Gamma \vdash \varphi \text{ is a } 5^+\text{-licensed inference iff } [D2]^+ \text{ and } [D4]^+ \text{ hold under restricted assignment.} \]

\[ \Gamma \vdash \varphi \text{ is a } 5^+_T\text{-licensed inference iff } [D2]^+_T \text{ and } [D4]^+_T \text{ hold under premise restricted assignment.} \]

Lemma A.15 $\Gamma \vdash \varphi$ is a $5^+_T$-licensed inference iff it is a $5^+_T$-licensed inference

Proof: Both semantics require precisely that that each atom occurrence in $\varphi$ contribute on a row where $\Gamma$ and $\varphi$ are true, and on its twin-row where $\Gamma$ and $\varphi$ are false. □

The candidate semantics all have reasonably compact representations in relevance semantics:

Definition A.16 Variant Atomic Clauses in Defn 2.37 for $\Gamma \models R_5 \theta$

5. \[ \langle S, T, U \rangle \models_{R_5} l \text{ if } S \cap U \cap (S_\theta)_{\pm l} \neq \emptyset \text{ and } T \cap U \cap (S_\theta)_{\pm l} \neq \emptyset. \]

5_T. \[ \langle S, T, U \rangle \models_{R_{5T}} l \text{ if } S \cap U \cap (S_T)_{\pm l} \neq \emptyset \text{ and } T \cap U \cap (S_T)_{\pm l} \neq \emptyset. \]

5^+. \[ \langle S, T, U \rangle \models_{R_{5^+T}} l \text{ if } S \cap U_{\mp l} \cap (S_\theta)_{\pm l} \neq \emptyset \text{ and } T \cap U_{\mp l} \cap (S_\theta)_{\pm l} \neq \emptyset. \]

5^+_T. \[ \langle S, T, U \rangle \models_{R_{5^+_T}} l \text{ if } S \cap U_{\mp l} \cap (S_T)_{\pm l} \neq \emptyset \text{ and } T \cap U_{\mp l} \cap (S_T)_{\pm l} \neq \emptyset. \]

These systems are not identical. For example, \[ (p \lor \neg q) \land (q \lor r) \vdash (p \lor r) \land (q \lor r) \] is 5-licensed but is not $5_T$ or $5^+_T$-licensed. As a result, any concision semantics for one of the other candidates should require that \[ (A \lor \neg B) \land (B \lor C) < (A \lor C) \land (B \lor C). \] If $C$ was shorter than $\neg B$, the partial order would be inconsistent. Another distinguishing example is \[ (p \land q) \lor (p \land r \land s) \lor (p \land \neg r \land \neg s) \lor (q \land r \land s) \lor (p \land \neg s) \lor (q \land s) \lor (r \land s), \] which is $5_T$-licensed, but not 5 or $5^+_T$-licensed. $5_T$-licensing thus does not respect the [B5](b) brevity principle.

These two example inferences demonstrate that 5 and $5_T$-licensing are not subrelations of each other and, along with Defn A.13, that $5^+_T$-licensing is a strict subrelation of both 5-licensing and $5_T$-licensing. As 5 and $5_T$-licensing both impose literal inclusion, $5^+_T$-licensing imposes unnecessarily strong restrictions, and I have chosen to reject it despite its elegant ceteris paribus Informativity Norm $\Pi, \Sigma \text{ } \parallel l$. In addition to its representability in concision semantics, I prefer 5-licensing over $5_T$-licensing (at least for my intended linguistic application) as its contributions do not depend on the set $\Gamma$ representing the Speaker’s communicative intentions, and of course these intentions are not available for the Hearer’s consideration.
A.3 Rival Connectives

I defined three variants for each of disjunction and conjunction in §3.4, and claimed they were sufficient to capture the asymmetry of natural language coordination. There are many alternative candidate variants for each connective, so in this appendix I will review and reject some of their closest rivals. I will only review potential disjunction candidates and not conjunctions, as it is easier to present and follow reasoning about a disjunction’s redundant truth conditions than the parallel reasoning about a conjunction’s redundant falsity conditions. The conclusions drawn will also apply to conjunctions. I will only consider candidates that arise from consideration of the compositional contribution tables, which may leave me vulnerable to implicit restrictions imposed by my choice of semantics. However, I have also applied a similar process to several other semantics, and while the pool of candidates differed, the best four disjunctions remained constant.

There are $4^4 = 256$ possible contribution tables for inclusive disjunction, as each of the four cells in the compositional table for any binary function selects a contribution from $\{l, r, \{l, r\}, *\}$. However, [D2] and [D4] from Defn 2.9 require there to be at least one contribution from each of the left and right disjuncts when the disjunction is true, and again when it is false, which restricts us to 49 candidates. The default disjunction of Defn 2.7 inherits contributions from the left disjunct exactly when the right disjunct is false, and vice versa. I am seeking connectives that are weaker than this disjunction for reasons explained below, so I will require every disjunction candidate to provide these contributions as a minimum. This leaves sixteen contribution tables where each of the disjuncts contributes whenever they are false, and possibly also when they are true. Eliminating those candidates where a false disjunct contributes to a true disjunction leaves the four contribution tables from Lemma 3.11.

I should also address the apparent success of some rejected candidates in capturing natural language disjunction schemas such as ‘one or both of $p$ and $q$’. This particular disjunction can be modelled in two ways: as a disjunction of the underlying propositions $p$ and $q$; and as a disjunction of the quantifiers ‘one of $p$ and $q$’ and ‘both of $p$ and $q$’. The first approach requires a novel disjunction $\lor_1$, while the second approach uses the standard disjunction. Below is the compositional table for $\lor_1$, along with the completed contribution tables for the schema ‘one or both of $p$ and $q$’ using the two approaches described. I will use xor with encapsulation to represent the anaphoric ‘one of’ operator. This use of xor does not mean that I believe logically exclusive disjunction occurs in natural language; it is merely standing in for the appropriate quantifier.

$$
\begin{array}{c|cc|c|c|c|c}
\lor_1 & 1_r & 0_r & 1_l & 1_l & 1_l & 0_l, r \\
\hline
1_l & 1_l & 1_l & 1_l & 1_l & 1_l & 0_l, r \\
0_l & 0_l & 0_l & 0_l & 0_l & 0_l & 0_l, r \\
\end{array}
$$

The second table shows that even seeming counter-examples to the adequacy of the four standard disjunctive connectives can easily be modelled by those disjunctions and encapsulation. Other examples of reducing complex disjunctions to the standard connectives can be found in §3.2.2 and §3.4. Linguistic surveys such as Haspelmath (2000) have not found any lexicalised disjunctions that correspond to this or other more complex forms, and it is tempting to say that this is because they can always be constructed from the basic natural language disjunctions. The compound conjunction ‘and/or’ discussed in Preds 4.3 and 4.18 may be an exception to this generalisation.

Default Disjunction Candidates

One option I seriously considered was to make Free Choice the intuitive core of disjunction instead of alternation, which would make $\leftrightarrow$-disjunction the default, with any strengthening of it being an acceptable variant. However assuming that disjuncts were equivalent unless context indicates otherwise makes the default forms very rare, while deviations like $\leftrightarrow$-disjunction and $\Rightarrow$-disjunction are much more common and intuitive. This seems like a sufficiently good reason for me to provisionally reject Free Choice as the basis for assertible disjunction.
A.4 Connexive Logics

All connexive logics share a desire for some connection between the premise set and the conclusion of inferences, although the class of connexive logics lacks an agreed definition. Most connexive logics reject either Simplification: \((\varphi \land \psi) \rightarrow \varphi\) or Addition: \(\varphi \rightarrow (\varphi \lor \psi)\), while accepting Disjunctive Syllogism: \(\varphi, (\neg \varphi \lor \psi) \rightarrow \psi\). Their cousins the relevant logics typically accept the former principles and reject the latter. Another common connexive principle is that no formula entails its own negation: \(\neg (\varphi \rightarrow \neg \varphi)\). Equivalences such as \((\varphi \rightarrow \neg \psi) \leftrightarrow (\neg \varphi \rightarrow \psi)\) are also characteristic of connexive logics. Many of the better-known connexive logics only have axiomatic presentations, and no known semantics. In addition to these axiomatic systems, there are several connexive logics that restrict classical logic by imposing principles about the nature of entailment, where these principles are similar to the assertion norms. I will model some of these logics by adapting the licensing semantics so that it can apply restrictions to inference premises via contraposition and antilogism.

Parry

Parry (1932) introduced Analytic Implication, which is one of the earliest and best-known twentieth-century connexive logics. The first-degree implication fragment of Analytic Implication consists of all and only the variable-inclusive classical entailments between formulas in the constant-free fragment of \(L\). The restrictions on higher-degree implications are more complex, as one would expect from any axiomatically presented system. Both 5-assertibility and Schurz’s third kind of relevance are subrelations of first-degree Analytic Implication, as are any other variable-inclusive restrictions of classical logic that only have negation, conjunction, and disjunction as connectives. I am not aware of any literal-inclusive logic that has 5-assertibility as a subrelation; Analytic Containment from Angell (1974) is the closest I have found, being a literal-inclusive subrelation of Analytic Implication, but it lacks Disjunctive Syllogism.

Strawson

Strawson (1948) claims that entailment statements cannot be necessary, although each entailment statement is associated with a necessary statement. For Strawson, an entailment is contingent, leads from a contingent statement to another contingent statement, and is either true or false, but never necessary. For instance, "'p' entails 'q'" is a contingent entailment statement that is about the contingent statements 'p' and 'q', and is true iff "'p ⊃ q' is necessary" is true. As he says on p.188 of Strawson (1948), 'where “p” and “q” are necessary, “ ‘p ⊃ q’ ” has no sense'. Contradictions may still appear in the premise(s), and tautologies in the conclusion, as long as the overall premise(s) and conclusion are contingent. Strawson entailment has no theorems (entailments of the form \(\vdash \varphi\)), and lacks uniform substitution for variables; it also has the non-classical thesis: if \(\psi \vdash \varphi\) then \(\psi \not\vdash \neg \varphi\). Despite these oddities, Transitivity and Contraposition still hold. It also has variable sharing, so at least one propositional variable must be shared by a premise and the conclusion.

**Definition A.17 Strawson entailment**

If \(\Gamma \vdash_L \varphi\), then \(\Gamma \vdash \varphi\) is a Strawson entailment iff \(\Gamma\) is consistent and \(\varphi\) is not a tautology.

Lehrer

Lehrer (1974) proposes two restrictions on classical entailment: that every premise of an inference must be consistent, and that its inclusion as a premise must also be necessary for the validity of the inference. These restrictions result in premise sets always being consistent, and conclusions never tautologous. Lehrer entailment is thus a restriction on Strawson entailment. Lehrer’s restrictions are strictly stronger than Strawson’s. For example, \(\{p \land q\}\) Lehrer-entails \(p\), but \(\{p, q\}\) does not, as \(\{p\}\) alone can entail \(p\). Lehrer entailment is also not transitive, as \(\{p, \neg p \lor q\}\) Lehrer-entails \(q\), and \(\{q\}\) Lehrer-entails \(\neg p \lor q\), but \(\{p, \neg p \lor q\}\) does not Lehrer-entail \(\neg p \lor q\).

**Definition A.18 Lehrer entailment**

If \(\Gamma \vdash_L \varphi\) and \(\Gamma\) is finite, then \(\Gamma \vdash \varphi\) is a Lehrer entailment iff \(\Gamma \cup \{\neg \varphi\}\) is an inconsistent set, but no proper subset of it is inconsistent.
Lemma A.19 $\psi \vdash \varphi$ is a Lehrer entailment iff $\psi \vdash \varphi$ is a Strawson entailment.


Körner, Cleave, and Smiley

Körner (1947) suggests that the antecedent of any entailment $\psi \vdash \varphi$ should be internally consistent, and that this property should be preserved under contraposition and antilogism. He calls any classical entailment with this property a Pure entailment. This is equivalent to the condition that every strict subset of the set of conjuncts in the premise and negated disjuncts in the conclusion being consistent, which I will call Körner entailment. Cleave (1974) tidies up a technical oversight in Pure entailment, and generalises the restriction for multiple premises, calling it Rigid entailment. Finally, Timothy Smiley has investigated numerous types of entailment, and one of his ‘alternative concepts of deducibility’ from footnote 18 of section 5 of Smiley (1959) is also equivalent to Körner entailment; I will call this approach Irredundant entailment.

Definition A.20 Körner entailment

1. If $\psi \Vdash_{CL} \varphi$, then $\psi \vdash \varphi$ is a Pure entailment iff $\psi$ is consistent, and the antecedent’s consistency is preserved by contraposition ($\psi \vdash \varphi$ iff $\neg \varphi \vdash \neg \psi$) and antilogism ($\psi \Vdash_{CL} \varphi$ (where $\psi$ is $\psi_1 \and \ldots \and \psi_n$ and $\varphi$ is $\varphi_1 \or \ldots \or \varphi_m$), then $\psi \vdash \varphi$ is a Körner entailment iff there is no inconsistent strict subset of $\{\psi_1, \ldots, \psi_n, \neg \varphi_1, \ldots, \neg \varphi_m\}$.

2. If $\psi \Vdash_{CL} \varphi$, then $\psi \vdash \varphi$ is a Rigid entailment iff there is no single instance of an atom in the inference that can be replaced with its negation while maintaining classical validity.

3. If $\psi \Vdash_{CL} \varphi$, then $\psi \vdash \varphi$ is an Irredundant entailment iff omitting any disjunct of any subformula of the tautology $\text{NNF}(\neg \psi) \or \text{NNF}(\psi)$ results in a non-tautologous formula.

Lemma A.21 $\psi \vdash \varphi$ is a Pure entailment iff $\psi \vdash \varphi$ is a Körner entailment iff $\psi \vdash \varphi$ is a Rigid entailment iff $\psi \vdash \varphi$ is an Irredundant entailment.

Proof: These equivalences are proved in Körner (1947), Cleave (1974), and Kielkopf (1977). □

Contraposition and Antilogism

A form of $n$-assertibility can be applied to a one-premise inference by contraposing the inference, and then applying the standard $n$-assertibility restrictions. This technique has already been used implicitly in Lemma 2.13, amongst others. Similarly, antilogism ($\Gamma, \psi \vdash \varphi$ iff $\Gamma, \neg \varphi \vdash \neg \psi$) can be applied to multi-premise inferences. By effectively applying assertibility restrictions to premises, we can produce licensing semantics for all the connexive logics that we have defined above.

Definition A.22 $\overline{n}$-assertibility

$\psi \vdash \varphi$ is $\overline{n}$-assertible iff $\neg \varphi \vdash \neg \psi$ is $n$-assertible.

$\Gamma \vdash \varphi$ is $\overline{n}$-assertible iff $\Gamma \and \varphi$ is $n$-assertible.

$\Gamma \vdash \varphi$ is $\overline{n}$-assertible iff $\Gamma \without \{\gamma\}, \neg \varphi \vdash \neg \gamma$ is $n$-assertible for all $\gamma \in \Gamma$.

Examples: $p \and q \vdash p$ is $\overline{1}$-unassertible, as $\neg p \vdash \neg p \or \neg q$ is 1-unassertible. $p \vdash q \or \neg q$ is $\overline{2}$-unassertible, as $q \and \neg q \vdash \neg p$ is 2-unassertible, while $q \and \neg q \vdash p$ is $\overline{3}$-unassertible, as $\neg p \vdash q \or \neg q$ is 3-unassertible.

Finally, $\{p, q\} \vdash p$ is $\overline{2}$-unassertible as $p \vdash \neg q \or (p \and \neg q)$ is 2-unassertible.

It appears $\overline{1}$-assertibility is too restrictive to model a useful deviant entailment, as it prohibits modus ponens, modus tollens, argument by cases, and Disjunctive Syllogism. In contrast the only common deductive rule prohibited by $\overline{1}$-assertibility is Simplification.

We will need Encapsulation to model some of the entailment relations. When a subformula of the conclusion is encapsulated by enclosing it in [square brackets], it is tested for assertibility as a single unit. See §§3.2.2 for further details on the associated changes to the licensing semantics.
A.4.1 Connexive Assertibility variants

**Lemma A.23** \( \Gamma \vdash \varphi \) is a Strawson entailment iff \( [\wedge \Gamma] \vdash [\varphi] \) is 2, \( \Xi \)-assertible iff it is 3-assertible.

*Proof:* (1 \( \Rightarrow \)) If \( \Gamma \vdash \varphi \) is a Strawson entailment then \( \Gamma \vDash_\Lambda \varphi \), and \( \Gamma \) and \( \varphi \) are contingent. As \( \varphi \) is encapsulated, its internal structure cannot be tested, so 2-assertibility will only test that \( \Gamma \) is consistent; similarly, 2-assertibility can only test for the consistency of \( \neg \varphi \). (2 \( \Rightarrow \)) If \( [\wedge \Gamma] \vdash [\varphi] \) is 2-assertible then \( \neg \varphi \) is consistent, so \( \varphi \) is contingent, which with 2-assertibility makes \( [\wedge \Gamma] \vdash [\varphi] \) 3-assertible. (3 \( \Rightarrow \)) If \( [\wedge \Gamma] \vdash [\varphi] \) is 3-assertible then \( \Gamma \) is consistent and \( \varphi \) is contingent. ■

**Lemma A.24** \( \Gamma \vdash \varphi \) is a Lehrer entailment iff \( \{\varphi\}_{\gamma} \in \Gamma \vdash [\varphi] \) is \( \Xi \)-assertible.

*Proof:* (\( \Rightarrow \)) Suppose that \( \Gamma \vdash \varphi \) is a Lehrer entailment. Then \( \Gamma \cup \{\neg \varphi\} \) is inconsistent, so \( \Gamma \vDash_\Lambda \varphi \). If \( \gamma_0 \in \Gamma \), then \( (\Gamma \setminus \{\gamma_0\}) \cup \{\neg \varphi\} \) is consistent by Lehrer entailment, and \( (\Gamma \setminus \{\gamma_0\}) \cup \{\neg \varphi\} \vDash_\Lambda \neg \gamma_0 \) by antilogism. Then \( \{\varphi\}_{\gamma} \in \Gamma \setminus \{\gamma_0\}, \neg \varphi \vdash \neg \gamma_0 \) is 2-assertible, so \( \{\varphi\}_{\gamma} \in \Gamma \vdash [\varphi] \) is \( \Xi \)-assertible.

(\( \Leftarrow \)) Suppose that \( \{\varphi\}_{\gamma} \in \Gamma \vdash [\varphi] \) is \( \Xi \)-assertible. Then \( \Gamma \vDash_\Lambda \varphi \) by antilogism, so \( \Gamma \cup \{\neg \varphi\} \) is an inconsistent set. But for any \( \gamma_0 \in \Gamma \), \( \{\varphi\}_{\gamma} \in \Gamma \setminus \{\gamma_0\}, \neg \varphi \vdash \neg \gamma_0 \) is 2-assertible, so \( \{\varphi\}_{\gamma} \in \Gamma \setminus \{\gamma_0\} \cup \{\neg \varphi\} \) is consistent, and so is \( (\Gamma \setminus \{\gamma_0\}) \cup \{\neg \varphi\} \). ■

**Lemma A.25** \( \psi \vdash \varphi \) is a Körner entailment iff \( \psi \vdash \varphi \) is 2, \( \Xi \)-assertible.

*Proof:* (\( \Rightarrow \)) Suppose that \( \psi \vdash \varphi \) is a Körner entailment. Then by Lemma A.21 it is a Rigid entailment, and \( \psi \vDash_\Lambda \varphi \). Substitution of any single atom occurrence \( p \) in \( \varphi \) with \( \neg p \) will cause the inference to become classically invalid. Then any such substitution causes \( \varphi \) to become false on a row where \( \psi \) is true, so every atom occurrence \( p \) in \( \varphi \) contributes to the validity of the inference on at least one row where \( \psi \) is true; that is, \( [D2] \) is satisfied for \( \psi \vdash \varphi \), so \( \psi \vdash \varphi \) is 2-licensed. Similarly, substituting \( \neg p \) for any atom occurrence \( p \) in \( \psi \) causes \( \psi \) to become true on a row where \( \varphi \) is false. Thus every atom \( p \) in \( \varphi \) contributes to the validity of the inference \( \neg \varphi \vdash \neg \psi \) on at least one row where \( \varphi \) is false; that is, \( [D2] \) is satisfied for \( \neg \varphi \vdash \neg \psi \), so \( \neg \varphi \vdash \neg \psi \) is 2-licensed, and \( \psi \vdash \varphi \) is \( \Xi \)-assertible.

(\( \Leftarrow \)) Suppose \( \psi \vdash \varphi \) is 2, \( \Xi \)-assertible. As \( \psi \vdash \varphi \) is 2-assertible, it is 1-concise, so if \( \varphi' \) is the result of omitting any disjunct from \( NNF(\varphi) \), then \( \psi \vDash_\Lambda \varphi' \). Similarly, as \( \neg \varphi \vdash \neg \psi \) is 1-concise, if \( \psi' \) is the result of omitting any conjunct from \( NNF(\psi) \), then \( \neg \varphi \vDash_\Lambda \neg \psi' \). If the entire premise \( \psi \) or conclusion \( \varphi \) were omitted, classical entailment would also fail, as \( \psi \) and \( \varphi \) are both contingent. Thus, \( \psi \vdash \varphi \) is an Irredundant entailment, and by Lemma A.21, a Körner entailment. ■

**Theorem A.26** Assertibility and Connexive Logics

1. \( \psi \vdash \varphi \) is 2, \( \Xi \)-assertible iff \( \psi \vdash \varphi \) is 1, \( \Xi \)-assertible and a Strawson entailment.

2. If \( \psi \vdash \varphi \) is (3- or \( \Xi \)-assertible) and (2- or 3-assertible) then it is a Strawson entailment.

3. Strawson, Lehrer, and Körner entailment all have variable sharing.

*Proof:* 1. Where \( \psi \) is consistent, \( \psi \vdash \varphi \) is 1-assertible iff it is 2-assertible. 2. If \( \psi \vdash \varphi \) is 3-assertible or \( \Xi \)-assertible then \( \varphi \) is non-tautologous. 3. Strawson entailment has variable sharing, and by Lemmas A.19 and A.25, and Thm A.26.1, Lehrer and Körner are subrelations of Strawson. ■
A.5 Schurzian Relevance

There are other inference relations that are similar to the assertibility family. In Schurz (1991) and Schurz (1995), Gerhard Schurz uses truth-preserving substitution to introduce three kinds of conclusion-relevance as restrictions on propositional and predicate logic. The propositional form of Schurz's kinds of relevant deduction can be modelled using concision semantics, revealing that they are closely related to 2-assertibility and 5-assertibility. Schurz needs to restrict inferences from any finitely axiomatisable theory so they only have a finite set of consequences, so that he can compare the verisimilitude of scientific theories. His third kind of relevance imposes variable inclusion but allows redundant conjuncts, so to achieve finitude he must impose the unrelated restriction that conclusions must be in conjunctive normal form. 5-assertibility has the desired finitude properties by Theorem 2.106, and so is potentially applicable to his verisimilitude project. Conversely, Schurzian relevance provides a new way to look at assertibility via validity-preserving substitution. To allow closer comparisons, I will convert members of each into semantics developed for the other.

A.5.1 Kinds of Relevance

Schurz's first kind of relevance restricts valid inferences to those which maintain classical validity under substitution of a single occurrence of a propositional variable in the conclusion.

Definition A.27 Relevance of the first kind

\[ \Gamma \vdash \varphi \text{ is a relevant deduction of the first kind iff } \Gamma \vDash_{cL} \varphi \text{ and no single occurrence of a propositional variable in } \varphi \text{ is replaceable by an arbitrary propositional variable, salva validitate of } \Gamma \vdash \varphi. \]

Schurz's second kind of relevance restricts valid inferences to those which do not allow substitution of strictly multiple instances of a propositional variable while preserving logical equivalence with the original conclusion, unless some of those instances can be substituted individually salva validitate. His second kind of relevance does not directly relate to any of the \( n \)-assertibility family, mainly because it excludes most instances of relevance of the first kind. Those inferences that are relevant of both the first and second kinds form a more natural comparison class: for constant-free conclusions, they will turn out to be the same as 3-assertibility closed under Distributivity.

Definition A.28 Relevance of the first-and-second kind

\[ \Gamma \vdash \varphi \text{ is a relevant deduction of the first-and-second kind iff: } \Gamma \vDash_{cL} \varphi, \text{ and simultaneously replacing two (or more) occurrences of a propositional variable in } \varphi \text{ by an arbitrary propositional variable results in a formula non-equivalent to } \varphi. \]

Schurz's third kind of relevance is a generalisation of his earlier kinds as it allows any number of occurrences of a single propositional variable in the conclusion to be simultaneously substituted for by a new variable. It is less restrictive than 5-assertibility, as it allows several forms of redundant conjuncts and, like all Schurzian substitutive systems, it does not place restrictions on constants. It also has variable but not literal inclusion. For example the inferences \( p \vdash p \land p, p \lor q \vdash p \lor (\neg p \land q), p \vdash p \land \top, p \vdash \top, \text{ and } p \land (q \lor r) \vdash (p \land r) \lor (q \land \neg r) \) are relevant of the third kind but not 5-assertible.

Definition A.29 Relevance of the third kind

\[ \Gamma \vdash \varphi \text{ is a relevant deduction of the third kind iff } \Gamma \vDash_{cL} \varphi, \text{ and no propositional variable can be replaced on some of its occurrences in } \varphi \text{ by an arbitrary propositional variable, salva validitate } \Gamma \vdash \varphi. \]

Lemma A.30 \( \Gamma \vdash \varphi \text{ is a relevant deduction of the first-and-second kind iff it is a relevant deduction of the first kind and } \varphi \vdash \varphi \text{ is a relevant deduction of the third kind.} \)


Lemma A.31 Relevant deduction of the third kind has variable inclusion but not literal inclusion.

Proof: Any variable in \( \varphi \) but not \( \Gamma \) can be universally substituted for salva validitate. A counter-example to literal inclusion is \( p \land (q \lor r) \vdash (p \land r) \lor (q \land \neg r). \)
A.5.2 Kinds of Assertibility

The first four assertibility relations all translate easily into substitution salva validitate. As these substitutions are closely related to the brevity operations [B2] and [B3](a), the equivalence results are essentially trivial. 5-assertibility does not translate in a straight-forward manner.

Lemma A.32 n-Assertibility salva validitate

1. \( \Gamma \vdash \varphi \) is 1-assertible iff \( \Gamma \not\vdash \varphi \), and for all \( \psi \) a disjunct in \( \varphi \): \( \Gamma \not\vdash \varphi \).
2. \( \Gamma \vdash \varphi \) is 2-assertible iff \( \Gamma \not\vdash \varphi \), and for all \( \psi \) a subformula in \( \varphi \): \( \Gamma \varphi \).
3. \( \Gamma \vdash \varphi \) is 3-assertible iff \( \Gamma \not\vdash \varphi \), and for all \( \psi \) a subformula in \( \varphi \): \( \Gamma \varphi \), and \( \varphi \neq \varphi \).
4. \( \Gamma \vdash \varphi \) is 4-assertible iff \( \Gamma \not\vdash \varphi \), and for all \( \psi \) a subformula in \( \varphi \): \( \Gamma \varphi \), and \( \varphi \neq \varphi \).

Proof: By Defs 2.67, 2.68, 2.79, and 2.93.

A.5.3 Kinds of Concision

All Schurzian deduction ignore constants, so we will strengthen the Permutation relation to render equivalent those formulas which only differ by redundant constants. Our initial brevity intuitions can be preserved by regarding constants as of zero length. The simplest way to disregard non-brief constants is to define any formula \( \varphi \) as being as brief as \( \varphi \land \top \) and \( \varphi \lor \bot \), which we do in [Top] and [Bottom]. These additional equivalences allow most of the operations from Defs 2.67 and 2.65 to be recycled. The lack of restrictions on conjunction means that [B4] and most of the [B5] operations are not required, leaving only a fragment of [B5](a) for the stronger kinds of relevant deduction.

Definition A.33 \( \top \)-Permutation: \( \varphi \equiv \top \psi \).

Let \( \varphi \equiv \top \psi \) be the smallest reflexive, transitive relation that is closed under:

- Permutation \( \varphi[A] \equiv \top \varphi[B] \) for \( A \equiv B \).
- [Top]: \( \varphi[A] \equiv \top \varphi[A \land T] \) for \( T \) any variable-free tautology.
- [Bottom]: \( \varphi[A] \equiv \top \varphi[A \lor F] \) for \( F \) any variable-free contradiction.

Definition A.34 Additional Brevity Operations for when \( \varphi[(A \lor C) \land (B \lor D)] \equiv \varphi[(A \land B) \lor (C \land D)] \), and there exists \( \alpha, \delta \) such that \( \alpha < i, A, \delta < i, D, |PROP(\alpha, \delta)| < |PROP(A, D)| \).

\[ [B5](\alpha') \varphi[(A \land B) \lor (C \land D)] \equiv \varphi[(\alpha \lor C) \land (B \lor \delta)], \text{ when they are equivalent.} \]

\[ [B5](\alpha'') \varphi[(A \land B) \lor (C \land D)] \equiv \varphi[(\alpha \lor C) \land (B \lor \delta)]. \]

Definition A.35 More n-Brevity Relations.

1. \( \leq_{\text{i}} \) is the smallest relation which has [\( \top \)-Permutation], [B1], and [B2] as subrelations, and is closed under transitivity.
2. \( \leq_{\text{ii}} \) is the smallest relation which has [\( \top \)-Permutation], [B1], [B2], [B3](a), and [B5](a') as subrelations, and is closed under transitivity.
3. \( \leq_{\text{iii}} \) is the smallest relation which has [\( \top \)-Permutation], [B1], [B2], [B3](a), and [B5](a'') as subrelations, and is closed under transitivity.
4. \( \psi <_{n} \varphi \) iff \( \psi \leq_{n} \varphi \) and \( \varphi \not<_{n} \psi \).

Example: \( p <_{i} p \lor q. q \lor p \leq_{\top} p \land \top. p \land p \equiv_{\top} (p \lor \bot) \land (p \lor \bot) <_{ii} (p \lor q) \lor (p \lor \neg q). \neg p \lor q \equiv_{\top} (p \lor \bot) \lor (q \lor \bot) <_{ii} (p \lor q) \lor (\neg p \lor q). p \lor (p \lor q) <_{\text{ii}} (p \lor \bot) \lor (q \lor \bot) \equiv_{\text{ii}} (p \lor \bot) \lor (q \lor \bot). \) And although \( p \land q <_{5} (p \land \bot) \lor q. \neg p \lor q <_{\top} (p \lor \bot) \lor (q \lor \bot) <_{ii} (p \lor q) \lor (\neg p \lor q). \) See Appendix C.4 for more examples.

Lemma A.36 Properties of i-Brevity and i-Concise

1. If \( \psi <_{i} \varphi \) then \( \psi <_{2} \varphi \).
2. If \( \varphi \) is constant-free then \( \psi <_{i} \varphi \) iff \( \psi <_{2} \varphi \).
3. If \( \Gamma \vdash \varphi \) is 2-concise then it is i-concise.
4. If \( \varphi \) is constant-free then \( \Gamma \vdash \varphi \) is i-concise iff it is 2-concise.

Proof: 1/2: By Defs 2.65, 2.67, 2.68, and A.35.1/4. 3/4: By Lemma A.36.1/2 and Defn 2.79. Counter-examples to the converse include: \( p \vdash p \lor \bot, \top \vdash \top \lor \bot, \) and \( \bot \vdash \bot \land \top. \)
Lemma A.37 Π ⊨ φ is a relevant deduction of the first kind iff it is i-concise.

Proof: Π ⊨ φ is i-inconcise iff ∃ψ: Π ⊨ψ ψ and ψ < ϕ by Defn 2.79 iff ∃ψ: Π, ψ and ψ <i ϕ and ψ is the result of (a) removing at least one disjunct from ϕ containing propositional variables, or (b) replacing a subformula of ϕ (in positive scope and containing propositional variables) with ⊥ iff there is at least one occurrence of a propositional variable in ϕ whose truth value does not affect the validity of Π ⊨ ϕ under any valuation iff there is at least one occurrence of a propositional variable in ϕ which is replaceable by an arbitrary propositional variable, salva validitate of Π ⊨ ϕ iff Π ⊨ ϕ is an irrelevant deduction of the first kind.

Lemma A.38 Properties of ii-brevity and ii-Concision

1. If ϕ is constant-free and ψ <3 ϕ then ψ <ii ϕ.
2. T <ii ϕ iff ϕ is a tautology and ϕ is not variable-free.
3. If ϕ is constant-free and Π ⊨ ϕ is ii-concise then it is 3-concise.
4. That Π ⊨ ϕ is 4-concise does not entail that it is ii-concise.
5. If Π ⊨ ϕ is 5-concise then it is ii-concise.
6. If Π ⊨ ϕ is i-concise, then it is ii-concise.

Proof: 1. By Defns 2.67, 2.68, and A.35, as all operations in ≤3 are in ≤ii, and [\top\text{-}\text{Permutation}] and [\text{Permutation}] are identical for the constant-free fragment. 2. By Defns 2.67, A.33, and A.35.2/4.
3. By Lemma A.38.1 and Defn 2.79. However, p ⊨ p ∧ T is ii-concise but not 3-concise. 4. p ∧ q ⊨ ((p ∧ r) ∨ (q ∧ ¬r)) ∧ ((p ∧ ¬r) ∨ (q ∧ r)) is 4-concise but not ii-concise. 5. By Defns 2.67, 2.68 and A.35.2, all operations of ii-brevity are operations of 5-brevity, except [B5](\alpha') which is a special case of [B5](\alpha)+[B1]+[B4]. Although [\text{Permutation}] is a strict subrelation of [\top\text{-}\text{Permutation}], they are identical when restricted to the set of 5-brief formulas (which is a subset of the constant-free fragment plus \{\top, \bot\}). Thus ii-brevity is a subrelation of 5-brevity over the set of 5-brief formulas: if ϕ is 5-brief then ϕ is ii-brief. Concision inclusion follows by Defn 2.79. 6. If ψ <i ϕ then ψ <ii ϕ.

By Defns A.35, as ii-brevity includes i-brevity. Concision inclusion follows by Defn 2.79.

Lemma A.39 Properties of iii-Brevity and iii-Concision

1. If ψ <i ϕ or ψ <ii ϕ then ψ <iii ϕ.
2. If Π ⊨ ϕ is iii-concise, then it is i-concise and ii-concise.
3. If Π ⊨ ϕ is 5-concise then it is iii-concise.
4. If Π ⊨ ϕ is 4-concise, it need not be iii-concise.
5. If Π ⊨ ϕ is iii-concise, it need not be 4-concise.

Proof: 1. By Defns A.35, as the iii-brevity principles include those of i-brevity and ii-brevity. 2. By Lemma A.39.1, and Defn 2.79. 3. By Defns 2.67, 2.68 and A.35.3, all operations of iii-brevity are operations of 5-brevity, except [B5](\alpha'') which is a special case of [B5](\alpha)+[B1]+[B4]. Although [\text{Permutation}] is a strict subrelation of [\top\text{-}\text{Permutation}], they are identical when restricted to the set of 5-brief formulas (which is a subset of the constant-free fragment plus \{\top, \bot\}). Thus iii-brevity is a subrelation of 5-brevity over the set of 5-brief formulas. Concision inclusion follows by Defn 2.79. 4. p ∧ q ⊨ (p ∧ r) ∨ (q ∧ ¬r) is 4-concise but not iii-concise. 5. p ⊨ p ∧ p and p ∨ q ⊨ p ∨ (¬p ∧ q) are iii-concise but not 4-concise.

Lemma A.40 Π ⊨ ϕ is a relevant deduction of the third kind iff it is iii-concise.

Proof: (⇒) If Π ⊨ ϕ is relevant of the third kind then it is relevant of the first kind by Defns A.27 and A.29 and i-concise by Lemma A.37. Suppose it is not iii-concise; then there is some ψ such that Π ⊨ ψ and ψ <iii ϕ. But any derivation of ψ <iii ϕ must use [B3](\alpha) or [B5](\alpha'') as ψ <i ϕ. If a derivation of ψ <iii ϕ uses [B3](\alpha), but not [B5](\alpha''), then ϕ is a permutation of a formula containing a non-variable-free tautology, as otherwise by [B1] and [\top ϕ = \top ϕ]. But then all instances of any variable within the tautology can be substituted for salva validitate, and Π ⊨ ϕ is not relevant of the third kind. So every derivation of ψ <iii ϕ must use [B5](\alpha''), and so have a step of the form ϕ[(A ∧ B) ∨ (C ∧ D)] ⊨ ϕ[(α ∨ C) ∧ (B ∨ δ)] where α <i A, δ <i D, and |PROP(α, δ)| < |PROP(A, D)|. Let p be in PROP(A, D) \ PROP(α, δ). Every formula in the sequence forming a brevity derivation is entailed by the next formula in the sequence by Defn 2.68 and Lemma 2.72.4 (as extended to Defns
2.76 and A.34). Then $\Gamma \vdash \varphi$ $\vdash \varphi[(\alpha \lor C) \land (B \lor \delta)] \vdash \varphi[(A \land B) \lor (C \land D)]$ $\vdash \varphi$. So by transitivity, $\Gamma \vdash \varphi[\varphi(\alpha \lor C) \land (B \lor \delta)]$. But $\Gamma \vdash \varphi[(A[p] \land B) \lor (C \land D[q])]$ for any $q$ by [B5](a′), so $\Gamma \vdash \varphi[(A \land B) \lor (C \land D)]$ is not a relevant deduction of the third kind, and neither is $\Gamma \vdash \varphi$.

(⇐) If $\Gamma \vdash \varphi$ is $\text{iii}$-concise then it is $\text{i}$-concise by Lemma A.39.2 and relevant of the first kind by Lemma A.37. Suppose it is $\text{iii}$-concise but not relevant of the third kind, then some formula $\psi$ is the result of replacing multiple occurrences of a variable $p$ in $\varphi$ by an arbitrary variable and $\Gamma \vdash \varphi$. Also, occurrences of $p$ in both positive and negative scope must have been replaced, as otherwise each occurrence could be replaced individually salva validitate, making it irrelevant of the first kind.

As any variable can be substituted for $p$ salva validitate, the occurrences of $p$ in positive scope can be replaced by $\bot$ and those in negative scope by $\top$ (of those multiple occurrences collectively substitutable salva validitate). That is, there are subformulas $A$ and $D$ of $\varphi$ and non-universal substitutions $E = A[E[p]]$ and $F = D[D[p]]$ such that $\Gamma \vdash \varphi(E \land D)$. Suppose $\chi$ is the smallest subformula of $\varphi$ containing $A$ and $D$. If $\chi$ is a conjunction, then as $\Gamma \vdash \varphi(A \land D)$, $A$ and $D$ must have disjuncts that can be replaced by $\bot$ salva validitate, so $\Gamma \vdash \varphi$ could not be even $\text{i}$-concise. So $\varphi$ must be a disjunction. Now, whenever $A$ is false and $\chi$ needs to be true to ensure $\varphi$ is true, $D$ must be true, so $\varphi[p] \vdash \varphi[\chi_A \land D]$. But neither $A$ nor $D$ can be a tautology, as otherwise variables in the other disjunct could be substituted individually salva validitate, again making it irrelevant of the first kind. The simplest case where these conditions are met is: $\chi \equiv \top \land D$. So $\chi$ is a tautology and [B3](a) can be applied, making $\Gamma \vdash \varphi$ $\text{iii}$-concise. More generally, we can assume both disjuncts of $\chi$ are conjunctions (if not, apply the $\top$ operation so they are). Then $\chi \equiv \top \land D$. So $\chi$ is a $\text{iii}$-brief reduction of $A$ and $D$. Then $\alpha < A$, $\delta < D$, $p \in \text{PROP}(A, D)$ and $p \notin \text{PROP}(\alpha, \delta)$ by construction, so $\text{PROP}(\alpha, \delta) = \text{PROP}(A, D)$. As the conditions for [B5](a′) are met, $\varphi[(\alpha \lor C) \land (C \lor D)] < \text{iii} \varphi[(A \land B) \lor (C \land D)],$ but as $\Gamma \vdash \varphi[\varphi(\alpha \lor C) \land (C \lor D)], \Gamma \vdash \varphi$ is $\text{iii}$-concise.

Lemma A.41 $\Gamma \vdash \varphi$ is a relevant deduction of the first-and-second kind iff it is $\text{ii}$-concise.

Proof: $\Gamma \vdash \varphi$ is a relevant deduction of the first-and-second kind iff it is relevant of the first kind and $\varphi \vdash \varphi$ is relevant of the third kind by Lemma A.30 iff $\Gamma \vdash \varphi$ is $\text{i}$-concise by Lemma A.37, and $\varphi$ is $\text{iii}$-concise by Lemma A.40. Suppose $\varphi$ is $\text{iii}$-concise. Then there does not exist $\psi < \text{iii} \varphi$ where $\psi \equiv \varphi$, so if $\Gamma \vdash \varphi$ is $\text{iii}$-concise, then any derivation of any $\psi$ such that $\Gamma \vdash \varphi$ and $\psi < \text{iii} \varphi$ must use an $\text{iii}$-brief operation that does not preserve equivalence and is not a $\text{i}$-brief operation. As [B3](a) always preserves equivalence, this must be [B5](a′). But [B5](a′) is just [B5](a″) with an equivalence requirement, so $\psi < \alpha \varphi$, and $\Gamma \vdash \varphi$ is $\text{iii}$-concise. Conversely, suppose $\Gamma \vdash \varphi$ is $\text{ii}$-concise but $\varphi$ is $\text{iii}$-concise. Then for any $\psi$ such that $\psi < \alpha \varphi$ and $\psi \neq \alpha \varphi$, $\psi < \alpha \varphi$ by the same reasoning, so $\Gamma \vdash \varphi$ is $\text{i}$-concise.

Schurz’s third kind of relevant deduction is thus similar to 5-concision, apart from his lack of restrictions on constants and conjunction, which leads to $\text{iii}$-brief lying the operation [B4], and having weakened forms of the [B5] principles. Both the odd and complex criteria for [B5](a′) and [B5](a″) and the absence of [B5](b) follow directly from the absence of [B4]. This is because they are precisely the elements of [B5](a) & (b) that can be derived by using distributivity while preserving brevity, and without using [B4]. [B5](a) is derived by distributing $(A \lor C) \land (B \lor D)$ to $(A \land B) \lor (A \land D) \lor (B \land C) \lor (C \land D)$, then removing the second and third disjuncts with [B1]. [B5](b) is derived by distributing $(A \land B) \lor (C \land D)$ to $(A \lor C) \land (A \lor D) \land (B \lor C) \land (B \lor D)$, then removing the second and third conjuncts with [B4]. All Schurzian systems lack [B4], which prevents the derivation of [B5](b). Without [B5](b), the full-strength [B5](a) is inappropriate as it creates inequalities between equivalent equi-length formulas. In $\text{iii}$-concision, a weaker [B5](a) is derivable by using [B1] to remove the first disjunct of the expanded form instead of the third, leaving $(A \land D) \lor (B \land C)$ and then reducing the number of variables in $A$ and $D$ via additional uses of [B1]. Thus [B5](a″) is precisely the operation that can be recovered from the derivation of [B5](a) given the absence of [B4]. As all $\text{i}$-brief operations except those in $\text{i}$-brief must preserve equivalence, the $\text{ii}$-brief operation [B5](a′) derived from [B5](a″) requires that the related formulas are equivalent.
A.5.4 Kind of Licensed?

Schurz’s first kind of relevance is so similar to 2-concision that it is not surprising it also has a licensing semantics. There are no standard licensing semantics for Schurz’s second or third kind of relevance, although a variation on 5-licensing comes close. Examining this near-miss helps us appreciate some subtle features of both Schurzian relevance and licensing.

Definition A.42 Licensing Semantics for Schurz’s first

Γ ⊢ ϕ is a i-licensed inference iff [D2] holds under simple assignment whenever each propositional variable occurrence in the conclusion is provided with a specific contribution and each constant occurrence in the conclusion is provided with a non-specific contribution.

Lemma A.43 Γ ⊢ ϕ is a relevant deduction of the first kind iff it is a i-licensed inference.

Proof: Γ ⊢ ϕ is a relevant deduction of the first kind iff no occurrence of a variable in the premises or conclusion is replaceable by a literal whose truth value may differ while maintaining classical validity by Defn A.27 iff every occurrence of a variable in the conclusion contributes to the validity of the inference on a row where the premise set is true iff [D2] holds under simple assignment for every variable in the conclusion under the contribution provision specified in Defn A.42 iff Γ ⊢ ϕ is a i-licensed inference.

To approximate Schurz’s third kind of relevance with licensing semantics, we will adjust Defn 2.3 so that each constant occurrence in the conclusion is provided with a non-specific contribution, as we did for i-licensing above. We will also tweak Defn 2.7 so that the contribution of a conjunction is always the union of the contribution of its conjuncts.

Definition A.44 near-Schurzian Licensing

Γ ⊢ ϕ is a iii~ licensed inference iff [D2] and [D4] hold under restricted assignment whenever each variable occurrence in the conclusion is provided with a specific contribution, each constant occurrence with a non-specific contribution, and conjunctions inherit all the contributions of their conjuncts.

Γ ⊢ ϕ is a i-licensed inference iff it is a i-licensed inference and ϕ ⊢ ϕ is a iii~-licensed inference.

Lemma A.45 Not all inferences which are relevant of the third kind are iii~-licensed.

Proof: ϕ ⊢ ϕ is a relevant deduction of the first kind, but is not ii~ or iii~-licensed, as the first occurrence of ϕ fails [D2] under restricted assignment.

Conjecture A.46 If Γ ⊢ ϕ is iii~-licensed then it is relevant of the third kind.

Proof Sketch: This is simple speculation. It appears that iii~-licensing imposes the restrictions of relevance of the third kind. In addition, Lemma 2.14.2 still holds for iii~-licensing, so it imposes literal inclusion. This makes iii~-licensing slightly too strong, as it rejects inferences such as ϕ ∨ q ⊢ (p ∧ ¬q) ∨ q and p∧(q∨r) ⊢ (p∧r)∨(q∧¬r) due to a non-contributing conjunct which can be removed, but not substituted for due to the presence of a contributing disjunct which is its negation.

One way to weaken iii~-licensing is to replace the restricted assignment with premise-restricted assignment, from Defn A.12 in Appendix A.2. This has literal inclusion using the standard conjunction contribution, but only variable inclusion with the modified conjunction contribution function defined in Defn A.44. This system is equivalent to a relevance of the third kind that only allows substitution of either one or every occurrence of a variable. Thus, it allows the irrelevant inference p ∨ q ⊢ (p ∧ q) ∨ (p ∧ ¬q) ∨ (¬p ∧ q), in which exactly two occurrences of either p or q can be substituted salea validatate. In Appendix A.1.4 I introduce a way of defining licensed inferences that doesn’t require the recursive calculation of contributions. This approach also allows the definition of a variant of premise restricted assignment using ‘sibling rows’, being twin rows across some or all occurrences of a variable, effectively allowing substitution of any number of occurrences of a variable. This condition is described in Defn A.10.[D2*m]. Conjecture A.11.4 claims that relevance of the third kind is equivalent to this variant licensing system.
A.5.5 Schurz and Assertibility

The Schurzian and assertibility families are quite closely related, in that each can be expressed in semantics designed for the other family. The Schurzian system could easily be adapted to restrict constants, or the assertibility family to ignore them. However, Schurzian relevance is only concerned with preserving validity and not information redundancy, and the absence of any restrictions on conjunctive redundancy results in the more restrictive systems of the two families being less closely related than we might wish.

The figure above summarises the relationships between assertibility and Schurzian relevance in terms of the increasing restrictions on conclusions, as shown by the strict brevity operations used in each. The relationship marked with ‘∼⊤’ is an equivalence when limited to constant-free conclusions. For further understanding of the relationships, Appendix C.4 contains a list of inferences along with the assertibility and Schurzian systems they satisfy.

Schurz uses his relevant deductions of the third kind in a number of projects, most notably to provide a solution to the Tichý-Miller paradox of verisimilitude in the Philosophy of Science, but also to address deontic paradoxes (a topic I will return to in §5.3.2). However, he does not explore any applications in the Philosophy of Language. Ken Gemes suggests on p.470 of Gemes (1994) that Schurz’s relevance systems might be better suited to modelling Gricean maxims than to their collective goal of modelling the verisimilitude of scientific theories. This may not be as dismissive a comment as it seems, as in a footnote he also suggests that applying a form of concision over his own system might also make it appropriate for modelling Gricean maxims. The concision system he sketches is a variant of ||-concision (see Defn 2.78) that also requires equivalence between brevity relata. Unfortunately, neither Schurz nor Gemes seems to have taken this linguistic possibility further.

We will now put these alternative systems aside and return to the development of assertibility for the purposes of modelling natural language coordination.
Appendix B. Additional Proofs

Some Lemmas referred to in the main text have proofs that are more tedious than informative. These include the proofs of the equivalence of R1–R3 and their corresponding Ideal model systems M1–M3 in Lemmas 2.53, 2.56, 2.59 and 2.64 from §2.3, as well as the definitions and equivalence Lemmas for the two-dimensional modal semantics for R4 and R5. In addition, the proof of the equivalence between 5-concision and 5-licensing in Lemmas B.15 and B.16 requires a number of preparatory lemmas, and is fairly long and involved. In the following proofs we will assume that the conclusion of all inferences are in NNF.

B.1 Modal Equivalences

The equivalence proofs from §2.3 are fiddly but fundamentally uninteresting proofs by induction on the complexity of the conclusion. Each equivalence follows by decomposing all the Relevance semantics definitions, and reassembling them into the valuations that hold for members of sets of worlds or points in the ideal models so as to match the translation functions for the modal semantics.

Lemma B.1 $S_{\Gamma} \models_{\text{CL}} \theta$ iff $M_{\Gamma} \models_{\text{MCL}} \theta$.

Proof: By induction on complexity of $\theta$. The base case is $\theta = p$. The inductive cases are $\theta = \neg \varphi$, $\theta = \varphi \land \psi$, and $\theta = \varphi \lor \psi$.

1. $S_{\Gamma} \models_{\text{CL}} p$
   - if $\forall v \in S_{\Gamma}: v = p$
   - if $\forall v \in u_{\Gamma}: v(p) = 1$
   - if $I_{\Gamma}, u_{\Gamma} \models_{M} p$
   - if $M_{\Gamma} \models_{M} c(p)$.

2. $S_{\Gamma} \models_{\text{CL}} \neg \varphi$
   - if not $\exists v \in S_{\Gamma}: v \models \varphi$
   - if not $\exists S' \subseteq S_{\Gamma}: S' \neq \emptyset$ and $S' \models_{\text{CL}} \varphi$
   - if not $\exists v: Rw_{\Gamma}, v$ and $I_{\Gamma}, v \models_{M} c(\varphi)$ [by I.H. for some $\Gamma'$: $w_{\Gamma}' = v$]
   - if $M_{\Gamma} \not\models_{M} \Diamond c(\varphi)$
   - if $M_{\Gamma} \models_{M} \neg \Diamond c(\varphi)$.
   - if $M_{\Gamma} \models_{M} c(\neg \varphi)$.

3. $S_{\Gamma} \models_{\text{CL}} \varphi \land \psi$
   - if $S_{\Gamma} \models_{\text{CL}} \varphi$ and $S_{\Gamma} \models_{\text{CL}} \psi$
   - if $M_{\Gamma} \models_{M} c(\varphi)$ and $M_{\Gamma} \models_{M} c(\psi)$ [by I.H.]
   - if not $(M_{\Gamma} \not\models_{M} c(\varphi)$ or $M_{\Gamma} \not\models_{M} c(\psi))$
   - if $M_{\Gamma} \models_{M} c(\varphi) \land c(\psi)$
   - if $M_{\Gamma} \models_{M} c(\varphi \land \psi)$.

4. $S_{\Gamma} \models_{\text{CL}} \varphi \lor \psi$
   - if $S_{\Gamma} \backslash S_{\varphi} \models_{\text{CL}} \varphi$ and $S_{\Gamma} \backslash S_{\psi} \models_{\text{CL}} \psi$
   - if $\forall S' \subseteq S_{\Gamma}$ where $S' \neq \emptyset$ then $\exists S'' \subseteq S'$: $S'' \neq \emptyset$ and $(S'' \models_{\text{CL}} \varphi$ or $S'' \models_{\text{CL}} \psi)$
   - if $\forall u \in W$ where $Rw_{\Gamma}, u$ then $\exists v: Rw_{\Gamma}, v \models_{M} c(\varphi)$ or $I_{\Gamma}, v \models_{M} c(\psi)$ [by I.H. on $\varphi$]
   - if $\forall u \in W$ where $Rw_{\Gamma}, u$ then $I_{\Gamma}, u \models_{M} c(\varphi)$ or $I_{\Gamma}, u \models_{M} c(\psi)$
   - if $I_{\Gamma}, u_{\Gamma} \models_{M} \Box(c(\varphi \lor c(\psi))$
   - if $M_{\Gamma} \models_{M} c(\varphi \lor \psi)$. $\square$
Lemma B.2  \( S_\Gamma \models_{R_1} \theta \) iff \( M_\Gamma \models_{M_1} \theta \).

*Proof:* By induction on complexity of \( \theta \). We can restrict ourselves to formulas in NNF by Lemma 2.43, as Defn 2.54.5/6/7 ensures \( d(\varphi) = NNF(d(\varphi)) \). The base cases are \( \theta = p \) and \( \theta = \neg p \), which proceed as per Lemma B.1.1/2. The inductive cases are \( \theta = \varphi \land \psi \) and \( \theta = \varphi \lor \psi \), for \( \varphi, \psi \) in NNF. The conjunctive case is as per Lemma B.1.3. The disjunctive case is more interesting:

1. \( S_\Gamma \models_{R_2} p \)
   - if \( S_\Gamma \neq \emptyset \) and \( \forall v \in S_\Gamma : v \models p \)
   - if \( w_\Gamma \neq \emptyset \) and \( \forall v \in w_\Gamma : v(p) = 1 \)
   - if \( I_\Gamma, w_\Gamma \models_{M_2} p \)
   - if \( I_\Gamma \models_{M_2} e(p) \).

2. \( S_\Gamma \models_{R_2} \neg p \)
   - if \( S_\Gamma \neq \emptyset \) and \( \forall v \in S_\Gamma : v \not\models p \)
   - if \( w_\Gamma \neq \emptyset \) and \( \forall v \in w_\Gamma : v(p) = 0 \)
   - if \( \exists w \in w_\Gamma \) and \( \exists u : \text{if } Rw_\Gamma u \text{ then } I_\Gamma, u \not\models_{M_2} p \)
   - if \( I_\Gamma \models_{M_2} \neg e(p) \).

Lemma B.3  \( S_\Gamma \models_{R_2} \theta \) iff \( M_\Gamma \models_{M_2} \theta \).

*Proof:* By induction on complexity of \( \theta \). Again we restrict ourselves to formulas in NNF by Lemma 2.43, as Defn 2.57.5/6/7 ensures \( e(\varphi) = NNF(e(\varphi)) \). The base cases of \( \theta = p \) and \( \theta = \neg p \) are more interesting:

1. \( S_\Gamma \models_{R_2} p \)
   - if \( S_\Gamma \neq \emptyset \) and \( \forall v \in S_\Gamma : v \models p \)
   - if \( w_\Gamma \neq \emptyset \) and \( \forall v \in w_\Gamma : v(p) = 1 \)
   - if \( I_\Gamma, w_\Gamma \models_{M_2} p \)
   - if \( I_\Gamma \models_{M_2} e(p) \).

2. \( S_\Gamma \models_{R_2} \neg p \)
   - if \( S_\Gamma \neq \emptyset \) and \( \forall v \in S_\Gamma : v \not\models p \)
   - if \( w_\Gamma \neq \emptyset \) and \( \forall v \in w_\Gamma : v(p) = 0 \)
   - if \( \exists w \in w_\Gamma \) and \( \exists u : \text{if } Rw_\Gamma u \text{ then } I_\Gamma, u \not\models_{M_2} p \)
   - if \( I_\Gamma \models_{M_2} \neg e(p) \).

Lemma B.4  \( M_\Gamma \models_{M_2} f(\theta) \) iff \( M_\Gamma \models_{M_2} \theta \) and \( M_{-\theta} \models_{M_2} \neg \theta \).

*Proof:* By induction on complexity of \( \theta \). Again we restrict ourselves to formulas in NNF by Lemma 2.43, as Defn 2.63.5/6/7 ensures \( f(\varphi) = NNF(f(\varphi)) \). The base cases are \( \theta = p \) and \( \theta = \neg p \). The inductive cases are \( \theta = \varphi \land \psi \) and \( \theta = \varphi \lor \psi \).

1. \( M_\Gamma \models_{M_2} p \) and \( M_{-p} \models_{M_2} \neg p \)
   - if \( M_\Gamma \models_{M_2} p \) and \( M_{-p} \models_{M_2} \neg p \) and \( M_{-p} \models_{M_2} \neg p \)
   - if \( M_\Gamma \models_{M_2} p \) and \( M_{-p} \models_{M_2} \neg p \) and \( M_\Gamma \models_{M_2} \neg p \) [by Defn 2.62 and Lemmas 2.25.3 and B.3]
   - if \( M_\Gamma \models_{M_2} p \) and \( M_{-p} \models_{M_2} \neg p \)
   - if \( M_\Gamma \models_{M_2} f(p) \).

2. As per 1, substituting \( \neg p \) for \( p \).

3. \( M_\Gamma \models_{M_2} \varphi \land \psi \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} \neg(\varphi \lor \psi) \)
   - if \( M_\Gamma \models_{M_2} \varphi \land \psi \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} \neg(\varphi \land \psi) \)
   - if \( M_\Gamma \models_{M_2} \varphi \land \psi \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} \neg(\varphi \land \psi) \)
   - if \( M_\Gamma \models_{M_2} \varphi \land \psi \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} \neg(\varphi \land \psi) \)
   - if \( M_\Gamma \models_{M_2} \varphi \land \psi \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} \neg(\varphi \land \psi) \) [by Defn 2.25.3 and B.3]
   - if \( M_\Gamma \models_{M_2} e(\varphi) \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} e(\varphi) \) and \( M_{-\psi} \models_{M_2} e(\varphi) \) and \( M_{-\varphi \land \psi} \models_{M_2} e(\varphi) \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} e(\varphi) \) and \( M_{\neg(\varphi \lor \psi)} \models_{M_2} e(\varphi) \) [again by Lemmas 2.25.3 and B.3]
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iff \(\mathcal{M}_1 \models_M f(\varphi) \land f(\psi)\) and \(\mathcal{M}_{\varphi \land \psi} \models_M f(\neg \varphi) \land c(\psi)\) [by I.H.]
iff \(\mathcal{M}_1 \models_M f(\varphi) \land f(\psi) \land \Diamond (\neg \Diamond f(\varphi) \land c(\psi)) \land \Diamond (\neg \Diamond f(\varphi) \land c(\psi))\)
iff \(\mathcal{M}_1 \models_M f(\varphi \land \psi)\).

4. \(\mathcal{M}_1 \models M \varphi \lor \psi\) and \(\mathcal{M}_{\varphi \lor \psi} \models M \neg(\varphi \land \psi)\)
iff \(\mathcal{M}_1 \models \Box (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi))\) and \(\mathcal{M}_{\varphi \lor \psi} \models M \neg(\varphi \land \psi)\)
iff \(\mathcal{M}_1 \models \Box (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi))\)
iff \(\mathcal{M}_1 \models \Box (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi))\)
iff \(\mathcal{M}_1 \models \Box (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi)) \lor \Diamond (\Diamond c(\varphi) \lor \Diamond c(\psi))\).

Modal semantics for R4 and R5 can be defined directly, and not just via the equivalence proved in Lemma B.4. The following approach comes straight from Defn 2.31 for \(\models^+_R\), while the formulas used in Defn 2.63 were read off the licensed truth tables of Defn 2.7. The formulas will be evaluated at an ordered pair of points \((w, u)\), rather than evaluating a formula at a single point in the T-rooted Ideal model. This pair can be thought of as the points witnessing the truth and falsity of the conclusion respectively; they serve the same role as the sets \(S\) and \(T\) in relevance semantics.

**Definition B.5** \(\mathcal{M}_1 \models M_4 \theta\) if \(\langle I_\Gamma, \langle w, u \rangle \rangle, 0 \rangle \models^+_M \theta\).

1. \(\langle I_\Gamma, \langle w, u \rangle \rangle, 0 \rangle \models^+_M \theta\) if \(I_\Gamma, u \models_M l \land \Diamond l\) and \(I_\Gamma, v \models_M \neg \Diamond l \land \neg \Diamond l\).
2. \(\langle I_\Gamma, \langle w, u \rangle \rangle, m \rangle \models^+_M \neg \varphi\) if \(\langle I_\Gamma, \langle w, u \rangle \rangle, 1-m \rangle \models^+_M \varphi\).
3. \(\langle I_\Gamma, \langle w, u \rangle \rangle, m \rangle \models^+_M \varphi \land \psi\) if \(\exists \nu': \text{Ru}^\nu, \langle I_\Gamma, \langle w, u \rangle \rangle, m \rangle \models^+_M \varphi, I_\Gamma, u \models M c(\psi), I_\Gamma, v' \models^+_M c(\psi); \) and \(\exists \nu': \text{Ru}^\nu, \langle I_\Gamma, \langle w, u \rangle \rangle, m \rangle \models^+_M \psi, I_\Gamma, u \models^+_M c(\varphi), I_\Gamma, v' \models^+_M c(\varphi).\)
4. \(\langle I_\Gamma, \langle w, u \rangle \rangle, m \rangle \models^+_M \varphi \lor \psi\) if \(\exists \nu': \text{Ru}^\nu, \langle I_\Gamma, \langle w, u \rangle \rangle, m \rangle \models^+_M \varphi, I_\Gamma, u' \models M \neg \Diamond c(\psi), I_\Gamma, v \models M \neg \Diamond c(\psi); \) and \(\exists \nu': \text{Ru}^\nu, \langle I_\Gamma, \langle w, u \rangle \rangle, m \rangle \models^+_M \psi, I_\Gamma, u' \models^+_M \neg \Diamond c(\varphi), I_\Gamma, v \models^+_M \neg \Diamond c(\varphi).\)

**Lemma B.6** \(\mathcal{S}_\Gamma \models R_4 \theta\) if \(\mathcal{M}_1 \models^+_M \theta\).

*Proof:* By induction on complexity of \(\theta\) over \(\models^+_R\). We will only provide the proof for complex formulas in positive scope, as the definitions are symmetric. The base case is \(\theta = p\). The inductive cases are \(\theta = \neg \varphi, \varphi = \varphi \land \psi, \) and \(\theta = \varphi \lor \psi\).

1. \(\langle S_\Gamma, S_{\varphi}, 0 \rangle \models^+_R p\)
iff \(S_\Gamma \neq \emptyset, \forall v \in S_\Gamma:\ v \models p, S_{\neg \varphi} \neq \emptyset \) and \(\forall v \in S_{\neg \varphi}: v \not\models p\).
iff \(w_r \neq \emptyset, \forall v \in w_r: v(p) = 1, w_{\neg \varphi} \neq \emptyset \) and \(\forall v \in w_{\neg \varphi}: v(p) = 0\).
iff \(I_\Gamma, w_r \models_M p, \exists u: \text{Ru}^u, I_\Gamma, w_{\neg \varphi} \models M \neg p \) and \(\exists v: \text{Ru}^v, I_\Gamma, w_{\neg \varphi} \models M \neg \Diamond (\neg p)\).
iff \(\langle I_\Gamma, \langle w_r, u \rangle \rangle, 0 \rangle \models^+_M p\).
iff \(\mathcal{M}_1 \models M_4 p\).

2. \(\langle S_\Gamma, S_{\varphi}, 0 \rangle \models^+_R \neg \varphi\)
iff \(\langle S_{\neg \varphi}, S_\Gamma, 1-m \rangle \models^+_R \varphi\).
iff \(\langle I_\Gamma, \langle w_{\neg \varphi}, u \rangle \rangle, (1-m) \rangle \models^+_M \varphi\).
iff \(\langle I_\Gamma, \langle w_{\neg \varphi}, u \rangle \rangle, m \rangle \models^+_M \neg \varphi\).
iff \(\mathcal{M}_1 \models M_4 \neg \varphi\).

3. \(\langle S_\Gamma, S_{\neg \varphi}, 0 \rangle \models^+_R \varphi \land \psi\)
iff \(\langle S_{\neg \varphi}, S_\Gamma, 1-m \rangle \models^+_R \varphi\) and \(\langle S_{\neg \varphi}, S_{\varphi \land \psi} \cap S_\varphi \rangle, m \rangle \models^+_R \varphi\).
iff \(\langle S_{\neg \varphi}, S_{\varphi \land \psi} \cap S_\varphi \rangle, m \rangle \models^+_R \psi\).
iff \(\langle S_{\varphi \land \psi} \cap S_\varphi \rangle, m \rangle \models^+_R \varphi\).
iff \(\langle S_\varphi \cap S_\psi \rangle, m \rangle \models^+_R \psi\).
iff \(\langle S_\Gamma, \langle w_{\neg \varphi}, u \rangle \rangle, m \rangle \models^+_M \varphi; I_\Gamma, w_r \models M c(\psi); I_\Gamma, w_{\neg \varphi} \models M c(\psi)\) and \(\langle I_\Gamma, \langle w_{\neg \varphi \land \varphi} \rangle \rangle, m \rangle \models^+_M \psi; I_\Gamma, w_r \models M c(\varphi); I_\Gamma, w_{\neg \varphi \land \varphi} \models M c(\varphi)\).
Lemma B.7 $\mathcal{M}_\theta \models_{M4} \theta$ iff $\mathcal{M}_\theta \models_{M} f(\theta)$.


The inference relation $R5$ is just $R4$ with a single additional restriction on literals imposed by using $l$-neutral extensions, $M5$ can similarly be defined by adding an additional restriction to $R4$, and the similarity of the $M4$ and $M5$ definitions reflects this.

Definition B.8 $u_{\pm l}$ is the $l$-neutral extension of the set of valuations functions $u$.

1. The $l$-reversed valuation function $v_{\pm l}$ is identical to the valuation function $v$ except the valuation of the atom in $l$ is reversed.

2. The $l$-neutral extension $u_{\pm l}$ of a set $u$ of valuations functions is the smallest extension of $u$ containing $v_{\pm l}$ for each $v \in u$.

Definition B.9 $\mathcal{M}_\theta \models_{M5} \theta$ iff $\langle I_\theta, (u_\theta^{w_\theta \land \phi}), 0 \rangle \models_{M5} \theta$.

1. $\langle I_\theta, (u_\theta), 0 \rangle \models_{M5} l$ iff $I_\theta, u \models_{M} l \land \phi$ and $I_\theta, v \models_{M} \neg \phi \land \neg \phi$ and $u_{\pm l} \cap w_\theta \neq \emptyset$ and $v_{\pm l} \cap w_\theta \neq \emptyset$.

2. $\langle I_\theta, (u_\theta), 1-m \rangle \models_{M5} \neg \phi$ iff $\langle I_\theta, (u_\theta), 1-m \rangle \models_{M5} \phi$.

3. $\langle I_\theta, (u_\theta), m \rangle \models_{M5} \phi \land \psi$ iff $\exists v': Rv', \langle I_\theta, (u_\theta), m \rangle \models_{M5} \phi, I_\theta, u \models_{M} c(\psi), I_\theta, v' \models_{M} c(\psi)$; and $\exists v''': Rv'', \langle I_\theta, (u_\theta), m \rangle \models_{M5} \psi, I_\theta, u \models_{M} c(\phi), I_\theta, v'' \models_{M} c(\phi)$.

4. $\langle I_\theta, (u_\theta), m \rangle \models_{M5} \phi \lor \psi$ iff $\exists u': Ru', \langle I_\theta, (u_\theta), m \rangle \models_{M5} \phi, I_\theta, u' \models_{M} \neg \neg \phi, I_\theta, v \models_{M} \neg \neg \psi$; and $\exists u'': Ru'', \langle I_\theta, (u_\theta), m \rangle \models_{M5} \psi, I_\theta, u'' \models_{M} \neg \neg \phi, I_\theta, v \models_{M} \neg \neg \psi$.

Lemma B.10 $\mathcal{S}_\theta \models_{R5} \theta$ iff $\mathcal{M}_\theta \models_{M5} \theta$.

Proof: By induction on complexity of $\theta$ over $R5$. Only the proof for the base case of $R5$ is needed, as the induction clauses for $R4$ and $R5$, and $M4$ and $M5$, are identical.

1. Where $S = S_\theta$ and $T = S_{\pm l}, \langle S, T, U \rangle \models_{R5} l$
   if $S \cap U \cap (S_{\pm l})_{\pm 1} \neq \emptyset$ and $T \cap U \cap (S_{\pm l})_{\pm 1} \neq \emptyset$.
   if $\langle S, T, U \rangle \models_{R4} l$ and $(S_{\pm l})_{\pm 1}$ intersects both $S \cap U$ and $T \cap U$.
   if $\langle I_\theta, (\emptyset), 0 \rangle \models_{M5} l$ and $u \cap (w_{\theta})_{\pm 1} \neq \emptyset$ and $v \cap (w_{\theta})_{\pm 1} \neq \emptyset$.
   if $\mathcal{M}_\theta \models_{M5} l$.

Lemma B.11 $\mathcal{S}_\theta \models_{Rn} \theta$ iff $\mathcal{M}_\theta \models_{Mn} \theta$, for $n \in \{1, \ldots, 5\}$.

Proof: By Lemmas B.1-B.10.
B.2 5-Concision Equivalences

The justification of Lemma 2.91 from §2.5 is the most complex argument in this thesis. I will assume a high degree of familiarity with the licensing semantics of §2.1, including the effects of restricted assignment from Defn 2.6, and the various types of signature from Defn 2.15.

Lemma B.12 Suppose $\Gamma \models \varphi$, $\varphi$ is 4-licensed, and a literal occurrence $l$ contributes to $\varphi$ on a row $u$ where $\varphi$ is true, and $\varphi$ is also true on $u$’s l-twin row $v$.

1. $l$ does not contribute to $\Gamma \vdash \varphi$ on $u$ under restricted assignment.
2. $l$ does not contribute to $\varphi$ on $v$.
3. $\varphi$ has a subformula $\psi_1 \lor \psi_m$ to which $l$ contributes on $u$ but not $v$, yet $l$ contributes to $\psi_1$ on $v$.
4. $\psi$ is true on $u$ and $\psi_m$ is false on $u$.
5. $\psi_1$ is false on $v$ and $\psi_m$ is true on $v$.
6. $\psi_m$ contains a literal occurrence $m$ such that $l \equiv \neg m$.

Proof: 1. By Defn 2.6 $l$ is not assigned a contribution on $u$ under restricted assignment as $\varphi$ is also true on $v$. 2. As $l$ is false on $v$ but $\varphi$ is true on $v$, $l$ cannot contribute to $\varphi$ on $v$ by Lemma 2.8. 3. As $l$ is false on $v$, but $\varphi$ is true on $v$, there is a minimal subformula $\psi$ of $\varphi$ containing $l$ such that $\psi$ is true on $v$. If $\psi$ were a conjunction then its conjunct containing $l$ would also be true, so $\psi$ is a disjunction $\psi_1 \lor \psi_m$ where $\psi_1$ contains $l$. 4. $l$ contributes to $\varphi$ on $u$, so every subformula of $\varphi$ containing $l$, including $\psi_1$, is true on $u$ by Lemma 2.8. Also $\psi_1$ must contribute to $\psi_1 \lor \psi_m$ on $u$, and so $\psi_m$ is false on $u$ by Defn 2.7. 5. As $\psi$ is the minimal formula containing $l$ which is true on $v$, $\psi_m$ is false on $v$. But $\psi$ is true on $v$, and then so is $\psi_m$. 6. As $\psi_m$ is true on $v$ and false on $u$, it must contain a literal occurrence $m$ whose truth value differs between these two rows. The rows $u$ and $v$ are l-twin rows, making the only variable that differs between them the one in $l$, so $l \equiv m$ or $l \equiv \neg m$. But $\varphi$ is in NNF, so $m$ is true when $\psi_m$ is, making it true on $v$ when $l$ is false, and false on $u$ when $l$ is true, so $l \equiv \neg m$. ■

Lemma B.13 Suppose $\varphi[(A \land B) \lor (C \land D)] \equiv \varphi[(A \lor C) \land (B \lor D)]$. Then $\Gamma \vdash \varphi[(A \land B) \lor (C \land D)]$ is 5-licensed iff $\Gamma \vdash \varphi[(A \lor C) \land (B \lor D)]$ is 5-licensed.

Proof: The restricted signature of $(p \land q) \lor (r \land s)$ is $(1_s, 1_{ab}, 1_{ab}, 1_{cd}, 0_{bd}, 0_{bc}, 0_{b}, 1_{cd}, 0_{ad}, 0_{ac}, 0_{a}, 1_{cd}, 0_{d}, 0_{c}, 0_s)$ and that of $(p \lor r) \land (q \lor s)$ is $(1_s, 1_c, 1_a, 1_{ac}, 1_d, 0_{cd}, 1_{ad}, 0_{cd}, 1_b, 1_{bc}, 0_{ab}, 0_{bd}, 0_{bc}, 0_{a}, 0_s)$. These signatures have different truth values for rows seven and ten; by our assumption that $\varphi[(A \land B) \lor (C \land D)] \equiv \varphi[(A \lor C) \land (B \lor D)]$, we constrain the formulas so they are equivalent on these rows, and thus cannot contribute to the truth/falsity of $\varphi$. Not only can we ignore their truth value on these rows, but also their contributions, and hence that of their twin rows. Under this assumption their restricted signatures are: $(1_s, 1_b, 1_a, 1_{ab}, 1_{cd}, 0_{bd}, 0, 0_{bc}, 0_{a}, 1_{cd}, 0_{d}, 0_{e}, 0_s)$ and $(1_s, 1_c, 1_a, 1_{ac}, 1_d, 0_{cd}, 0_{a}, 0_{b}, 0, 0_{bc}, 0_{a}, 1_{cd}, 0_{e}, 0_s)$ respectively. These are almost isomorphic; the second signature has an extra ‘b’ contribution in row 12 and an extra ‘c’ contribution in row 14. Suppose these differences meant that $D4$ failed in $\varphi[(A \land B) \lor (C \land D)]$ but held in $\varphi[(A \lor C) \land (B \lor D)]$. If row 12 is the only row where ‘b’ contributes to the inference when $\varphi$ is false, then ‘b’ can’t contribute to $\varphi$ on either rows 11 or 15 due to $\varphi$ being true on those rows, meaning that ‘b’ wasn’t assigned to their C-twin rows 9 and 13. But these are the only rows where $\varphi$ is true and ‘b’ potentially contributes to $\varphi$, so $\Gamma \vdash \varphi[(A \lor C) \land (B \lor D)]$ would fail $D2$ under restricted assignment, and not be 5-licensed. A similar argument for the ‘c’ contribution in row 14 results in ‘c’ not contributing to $\varphi$ on either row 2 or 4, and hence $\Gamma \vdash \varphi[(A \lor C) \land (B \lor D)]$ failing $D2$. ■

Lemma B.14 If $\psi \equiv \varphi$ then $\Gamma \vdash \varphi$ is 5-licensed iff $\Gamma \vdash \psi$ is 5-licensed.

Proof: The relation $\equiv$ is the smallest equivalence relation closed under [Commutativity], [Associativity], [De Morgan], and [Double Negation], by Defn 2.65. As $\Gamma \vdash \varphi$ is 5-licensed iff $\Gamma \vdash NNF(\varphi)$ is 5-licensed by Lemma 2.12, 5-licensing is closed under [De Morgan] and [Double Negation]. The compositional tables in Defn 2.7 are symmetric, and the signatures for $A \ast (B \ast C)$ and $(A \ast B) \ast C$ (for $\ast \in \{\lor, \land\}$) are identical, demonstrating that 5-licensing is closed under [Commutativity] and [Associativity] respectively. ■
Lemma B.15 If $\Gamma \vdash \varphi$ is $5^\bullet$concise then it is a 5-licensed inference.

Proof: Let $\varphi$ be the smallest NNF formula such that for some $\Gamma$: $\Gamma \vdash \varphi$ violates this lemma. Suppose $\Gamma \vdash \varphi$ is $5^\bullet$concise but not 5-licensed. By Lemmas 2.80.3 and 2.90, $\Gamma \vdash \varphi$ is 4-licensed. Then [D2] and [D4] hold for $\Gamma \vdash \varphi$ under simple assignment, but [D2] fails under restricted assignment.

Let $\chi$ be a maximal subformula of $\varphi$ such that no literal in $\chi$ contributes to $\Gamma \vdash \varphi$ under restricted assignment on any row where $\Gamma$ and $\varphi$ are true. As [D2] holds for $\Gamma \vdash \varphi$ under simple assignment, every literal occurrence in $\varphi$ must contribute to $\Gamma \vdash \varphi$ on some row where $\Gamma$ and $\varphi$ are true. Let $l$ be a literal occurrence in $\chi$, $u$ be a row where $\Gamma$ and $\varphi$ are true and $l$ contributes to $\Gamma \vdash \varphi$ under simple assignment, and $v$ be the $l$-twin row of $u$. If $\varphi$ is false at $v$, then $l$ is assigned to rows $u$ and $v$ under restricted assignment, and so satisfies [D2]; $\varphi$ must thus be true at $v$.

Then by Lemma B.12 there is a literal occurrence $m$ in $\varphi$ such that $l \equiv \neg m$ and the minimal subformula of $\varphi$ containing $l$ and $m$ is a disjunction $\psi_l = \psi_l \lor \psi_m$ where the disjunct $\psi_l$ containing $l$ is true on $u$ and false on $v$, and the disjunct $\psi_m$ containing $m$ is true on $v$ and false on $u$. An example of an inference satisfying these conditions, with $l$, $u$, etc., labelled, is provided after this proof.

If $\psi_l$ is a disjunction, then the disjunct of $\psi_l$ containing $l$ is true on row $u$, and so the other disjuncts of $\psi_l$ can be moved outside the disjunction with $\psi_m$ by [Associativity], reducing $\psi_l$ to a conjunction or a literal, and similarly with $\psi_m$. [Associativity] is a subrelation of [Permutation], so preserves licensing by Lemma B.14. This leaves four cases:

Case 1: $\psi_l$ and $\psi_m$ are both literals. Then $\psi = l \lor m$ which is a tautology as $l \equiv \neg m$, so $\varphi$ is not 3-brief, a contradiction;

Case 2: $\psi_l$ is a literal and $\psi_m$ a conjunction. Then all instances of $m$ can be removed from $\psi_m$ without affecting the truth of the disjunction by [B4], so $\varphi$ is not 4-brief, also a contradiction;

Case 3: as per Case 2, with the roles of $\psi_l$ and $\psi_m$ reversed;

Case 4: both $\psi_l$ and $\psi_m$ are conjunctions. That is, $\psi_l$ is of the form $A \land B$ where $A$ contains $l$, and $\psi_m$ is of the form $C \land D$, where $D$ contains $m$. This is the only potentially consistent case. An example of an inference that satisfies these conditions follows this proof.

Let $\theta = \varphi[2]^\bullet$, so that $\varphi = \theta[(A \land B) \lor (C \land D)]$, and $\varphi \equiv \theta[(A \lor C) \land (B \lor D)]$. If both $A$ and $D$ contribute to $\varphi$ on a row, then $\psi$ must be false on that row by Defn 2.7, making both $A$ and $D$ false on that row. Similarly, if both $B$ and $C$ contribute to $\varphi$ on a row, then $\psi$ must be false on that row by Defn 2.7, making both $A$ and $D$ true on that row. But as $\varphi$ is 4-brief, there are no redundant conjunctions or disjunctions in $A$ or $D$. This rules out any case where they are both false, and $\psi$ contributes to $\varphi$. Therefore $\theta[B \land C] \equiv \theta[(A \lor C) \land (B \lor D)]$. By Conjecture B.17 $\theta[A \land B] \equiv \theta[\bot]$, so $A$ and $D$ are not both true on a row where $\psi$ contributes to $\varphi$.

So, there are no rows where either $A$ and $D$ both contribute to $\varphi$, or both $B$ and $C$ contribute to $\varphi$, even under simple assignment. When we consider the restricted signature of $\varphi$ we must then disregard the contributions on these rows, and also on their twin rows if the original rows are false, as these contributions would not have been assigned under restricted assignment. The restricted positive signature of $\theta[(A \land B) \lor (C \land D)]$ is $\{1_s, 1_{ab}, 1_{cd}, 0, 1_{ab}, 1_{ab}, 0, 0, 1_{cd}, 0, 1_{cd}, 0, 0, 0, 0\}$, so by the previous paragraph rows 4 and 13 must never contribute to $\varphi$. The restricted positive signature becomes: $\{1_s, 1_b, 1_c, ?_s, 1_a, 1_{ab}, 0, 0, 1_d, 0, 1_{cd}, 0, ?_s, 0, 0, 0\}$. But this is isomorphic to the restricted positive signature of $\theta[(A \lor C) \land (B \lor D)]$ under the same conditions, so $\Gamma \vdash \theta[(A \lor C) \land (B \lor D)]$ must also be not 5-licensed due to the failure of [D2] under restricted assignment.

Now $\theta[(A \lor C) \land (B \lor D)]$ lacks the subformula $(A \land B) \lor (C \land D)$, and by our assumption of minimality, there are no further instances of this form in $\varphi$. Also, if $A \lor C$ or $B \lor D$ serve as instances of $\psi$ or $(A \lor C) \land (B \lor D)$ serves as an instance of $\psi_l$ or $\psi_m$ in $\Gamma \vdash \theta[(A \lor C) \land (B \lor D)]$ as described above for $\Gamma \vdash \varphi$, the above procedure can be repeated until all such instances of the form required by Lemma B.12 have been removed. The resulting inference will then fail to satisfy [D2] under restricted licensing, contradicting Lemma B.12.

Lemma B.16 If $\Gamma \vdash \varphi$ is a 5-licensed inference then it is $5^\bullet$concise.

Proof: If $\Gamma \vdash \varphi$ is 5-licensed then it is $4^\bullet$concise, by Defn 2.10 and Lemma 2.90. By Lemma B.13 5-licensing respects [B5](a)&(b). Thus if $\Gamma \vdash \varphi$ is 5-licensed and $4^\bullet$concise, it is $5^\bullet$concise.
Example Case 4: The contribution table of $p \land (q \lor r) \vdash (r \land p) \lor (q \land \neg r)$ is 4- but not 5-licensed. Occurrence $l = \chi = r_a$, $m = \neg r_d$, $\psi_l = r \land p$, $\psi_m = q \land \neg r$, $u$ is the first row, and $v$ the second.

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\[
(r_a \land p_b) \lor (q_c \land \neg r_d)
\]

\[
(r_a \land p_b) \lor (q_c \land \neg r_d)
\]

Conjecture B.17 If $\varphi[(A \land B) \lor (C \land D)]$ is 5-brief and $\varphi[B \land C] \equiv, \varphi[(A \land B) \lor (C \land D)]$, then $\varphi[A \land D] \equiv \varphi[\bot]$.

**Proof Sketch:** There is no need for $A$ and $D$ to be true under any valuation that does not affect the truth of $\varphi$. As $\varphi[(A \land B) \lor (C \land D)]$ is 5-brief, brevity operations [B1]–[B4] must have removed all redundant disjunctions and conjunctions. The only remaining form for $A$ or $D$ such that $\varphi[A \land D]$ to be true when $\varphi[\bot]$ is false is described by Lemma B.12, which we have shown in the proof of Lemma B.15 is a disjunction of conjuncts that satisfies the conditions of [B5](a); then by applying the brevity operation [B5](a), the resulting formula will have redundant disjuncts (or possibly conjuncts) that can be removed by [B1]–[B4]. These operations all fall under 5-brevity, so $\varphi[A \land D] \equiv \varphi[\bot]$. $\blacksquare$

The restricted signatures of $\varphi[(A \land B) \lor (C \land D)]$ and $\varphi[(A \lor C) \land (B \lor D)]$ in the proof of Lemma B.13 are not isomorphic. To address this, we can change the restricted contribution assignment in Defn 2.6 so that a contribution for an atom occurrence is assigned to a row if the conclusion is true for exactly one of that row and its twin row. This restricts the contributions on rows where the conclusion is false in the same way that it already restricts them on rows where the conclusion is true. The restricted signature for the first formula is unchanged, while the restricted signature for the second formula becomes $(1_s, 1_c, 1_a, 1_ac, 1_d, 0_cd, 0_x, 0_c, 0_x, 0_c, 0_c, 0_c, 0_c, 1_d)$, making the signatures isomorphic.

Conjecture B.18 Further restricting the contribution assignment on rows where the conclusion is false in the same manner as restricted assignment restricts the contribution assignment on rows where the conclusion is true will have no effect on which inferences are licensed.

**Proof Sketch:** I do not have an outline of a proof for this claim. However, under this stronger assignment restriction, the formulas $\varphi[(A \land B) \lor (C \land D)]$ and $\varphi[(A \lor C) \land (B \lor D)]$ from the [B5] brevity operation have isomorphic signatures if and only if they are equivalent. This stronger contribution restriction thus appears to have some explanatory power for its equivalence with the [B5] concision operation than standard restricted assignment lacks. In addition, a substantial number of inferences have been evaluated under both forms of restricted licensing, and no counter-examples to their equivalence have been identified. $\blacksquare$

If Conjecture B.18 is true, then Lemma B.13 becomes an instance of Lemma 2.18, making the proof of Lemma B.16 more direct. It would also provide another reason to favour 5-assertibility over its rivals discussed in Appendix A.2. However like Conjectures A.8, A.11, and A.46, if B.18 turns out to be false, there is no impact on the rest of the claims in this thesis. Conjecture B.17 is thus the only substantial formal claim that is not proved in this thesis, it being necessary for proving 5-concision is a form of 5-assertibility.
Appendix C. Quick References

I have found it helpful when working with Assertibility to have a quick list of the most commonly referenced semantics, definitions and results, and some key example inferences.

C.1 Assertibility Definitions

Definition 2.37 \(\langle S,T,U \rangle \models_{R_{A}} \theta\)

1. \(\langle S,T,U \rangle \models_{R_{A}} l\) iff \(S \cap U \cap (S_{\neg l}) \neq \emptyset\) and \(T \cap U \cap (S_{\neg l}) \neq \emptyset\), for \(l\) a literal.
2. \(\langle S,T,U \rangle \models_{R_{A}} \varphi \land \psi\) iff \(\langle S,T,U \cap S_{\varphi} \rangle \models_{R_{A}} \varphi\) and \(\langle S,T,U \cap S_{\psi} \rangle \models_{R_{A}} \psi\).
3. \(\langle S,T,U \rangle \models_{R_{A}} \varphi \lor \psi\) iff \(\langle S,T,U \setminus S_{\psi} \rangle \models_{R_{A}} \varphi\) and \(\langle S,T,U \setminus S_{\varphi} \rangle \models_{R_{A}} \psi\).
4. \(\langle S,T,U \rangle \models_{R_{A}} \neg \varphi\) iff \(\langle T,S,U \rangle \models_{R_{A}} \varphi\).

Definition 2.65 Permutation: \(\varphi \cong \psi\)

\(\varphi \cong \psi\) is the smallest reflexive, transitive relation that is closed under:

- [Commutativity] \(\varphi[A \land B] \cong \varphi[B \land A]\).
- [Associativity] \(\varphi[A \land (B \land C)] \cong \varphi[(A \land B) \land C]\).
- [De Morgan] \(\varphi[\neg (A \land B)] \cong \varphi[\neg A \lor \neg B]\).
- [Double Negation] \(\varphi[A] \cong \varphi[\neg \neg A]\).

where \(\land, \lor \in \{\lor, \land\}\) and \(\forall \neq \circ\).

Definition 2.67 Brevity Operations (subformulas must be in positive scope)

- [B1] \(\varphi[A \lor B] \cong \varphi[A]\).
- [B2] \(\varphi[A] \cong \varphi[A]\).
- [B3(a)] \(\varphi[A] \cong \varphi[A]\), for \(A\) a tautology.
- [B4] \(\varphi[A \lor B] \cong \varphi[A]\), for \(\varphi[A] \models \varphi[B]\).
- [B5(a)] \(\varphi[(A \lor B) \land (C \land D)] \cong \varphi[(A \land D) \lor (B \land C)]\), when they are equivalent.
- [B5(b)] \(\varphi[(A \lor C) \land (B \land D)] \cong \varphi[(A \lor B) \land (C \land D)]\), when they are equivalent.

Definition 3.7 Probabilistic Relevance

Only the literal clauses differ from the standard defns for \(\langle S,T,U \rangle \models_{R_{A}} \theta\):

1. \(\langle S,T,U \rangle \models_{\theta_{S}} \land \text{iff } P(S \cap S_{\theta})/P(S) \leq \epsilon\), where \(l\) is a literal.
2. \(\langle S,T,U \rangle \models_{\theta_{S}} \land \text{iff } \langle S,T,U \cap S_{\theta} \rangle \models_{\theta_{S}} \land \text{and } P(S \cap S_{\theta})/P(S) \geq \epsilon\).
3. \(\langle S,T,U \rangle \models_{\theta_{S}} \land \text{iff } P(S \cap S_{\theta} \cap S_{\neg l})/P(S) > \delta\) and
   \[P(T \cap U \cap (S_{\neg \theta} \cap S_{\neg l})/P(S) > \delta\).

Definition 3.8 Asymmetric Compound Utterance Forms

1. \(\langle \Pi; \Gamma \rangle \models \varphi \lor \psi\) iff \(\langle \Pi; \Gamma \rangle \models \varphi\) and \(\langle \Pi; \neg \varphi; \Gamma \rangle \models \psi\).
2. \(\langle \Pi; \Gamma \rangle \models \varphi \land \psi\) iff \(\langle \Pi, \neg \psi; \Gamma \rangle \models \varphi\) and \(\langle \Pi, \Gamma \rangle \models \psi\).
3. \(\langle \Pi; \Gamma \rangle \models \varphi \lor \psi\) iff \(\langle \Pi, \Gamma \rangle \models \varphi\) and \(\langle \Pi; \Gamma \rangle \models \psi\).
4. \(\langle \Pi; \Gamma \rangle \models \varphi \land \psi\) iff \(\langle \Pi, \Gamma \rangle \models \varphi\), \(\langle \Pi, \varphi; \Gamma \rangle \models \psi\), and \(\Pi, \Gamma \models \varphi \land \psi\).
5. \(\langle \Pi; \Gamma \rangle \models \varphi \lor \psi\) iff \(\langle \Pi, \Gamma \rangle \models \varphi\), \(\langle \Pi, \varphi; \Gamma \rangle \models \psi\), and \(\Pi, \Gamma \models \varphi \land \psi\).
6. \(\langle \Pi; \Gamma \rangle \models \varphi \lor \psi\) iff \(\langle \Pi, \Gamma \rangle \models \varphi\), \(\langle \Pi, \Gamma \rangle \models \psi\), and \(\Pi, \Gamma \models \varphi \land \psi\).
C.2 Definitions & Results

Definition 1.4.5: An assertion $U$ with utterance form $\varphi$ partially conveys $\Gamma$ in $\Pi$ iff $\Pi, \Gamma \vdash_{CL} \varphi$.

Definition 1.5: Basic Assertion Norms
1. Expressivity: $\Pi, \Gamma \vdash_{CL} \varphi$.
2. Consistency: $\Pi, \Gamma \not\vdash_{CL} \bot$.
3. Informativity: $\Pi \not\vdash_{CL} \varphi$.

Definition 2.9: Licensing Evaluation Criteria
[D2] Every contribution provided must be in the contribution for the conclusion on a row where the premise set and conclusion are true.
[D4] Every contribution provided must be in the contribution for the conclusion on a row where the premise set and conclusion are false.

Definition 2.79 $n$-Concision
$\Gamma \vdash \varphi$ is $n$-concise iff $\Gamma \vdash_{CL} \varphi$, and $\forall \psi <_n \varphi$: $\Gamma \not\vdash_{CL} \psi$.

Theorem 2.92: Equivalence of Semantic Families
The Contribution, Relevance, Modal, and $\Diamond$Concision families are equivalent.

Theorem 2.94: Literal Inclusion
If $\Gamma \vdash \varphi$ is 5-assertible then $\text{LIT}(\varphi) \subseteq [\text{LIT}(\Gamma)]$.

Theorem 2.102: Soundness and Completeness
$\langle \Pi, \Gamma \rangle \models_n \varphi$ iff $\langle \Pi, \Gamma \rangle \models_{RN} \varphi$.

Theorem 2.106: Finitude
There is only a finite number of NNF formulas $\varphi$ such that $\Gamma \vdash \varphi$ is 5-assertible.

Definition 3.15 Modes of Coordination
Deductive coordination has an inferential connexion between the contents of the conjoined phrases. Expositive coordination has a direct connexion between the contents of the conjoined phrases. Evocative coordination has an indirect connexion between the contents of the conjoined phrases. Performative coordination has a direct connexion between the satisfaction conditions of an illocutionary act and the contents or satisfaction conditions of another. Renunciative coordination has a direct connexion between the satisfaction conditions of a locutionary act and the contents or satisfaction conditions of an illocutionary act.

Definition 3.17 Assertoric Performative Coordination
2. $\langle \Pi; \Gamma \rangle \models A(\langle \varphi \rangle \lor \langle \psi \rangle)$ iff $\langle \Pi, \neg \text{sat} (\langle \psi \rangle); \Gamma \rangle \models A(\langle \varphi \rangle)$ and $\langle \Pi, \neg \text{sat} (\langle \varphi \rangle); \Gamma \rangle \models A(\langle \psi \rangle)$.
3. $\langle \Pi; \Gamma \rangle \models A(\langle \varphi \rangle \land \langle \psi \rangle)$ iff $\langle \Pi, \text{sat} (\langle \psi \rangle); \Gamma \rangle \models A(\langle \varphi \rangle)$ and $\langle \Pi, \text{sat} (\langle \varphi \rangle); \Gamma \rangle \models A(\langle \psi \rangle)$.

Definition 3.20 Renunciative Disjunction
1. Asymmetric Propositional: renounces a proposition in favour of a stronger alternative.
2. Symmetric Locutionary: presents two alternative synonymous locutions.
3. Asymmetric Locutionary: renounces a locution in favour of a synonymous alternative.
C.3 List of Predictions

The following predictions about coordinations in context all require appeals to assertibility.

Prediction 3.1: Non-Brevity.
Prediction 3.2: Redundancy.
Prediction 3.3: Repetition.
Prediction 3.4: Duplication with Alteration.
Prediction 3.5: Inclusion 1.
Prediction 3.6: Tautology.
Prediction 3.7: Conjunction 1.
Prediction 3.8: Inclusion 2.
Prediction 3.9: Disjunction 1.
Prediction 3.10: Disjunction 2.
Prediction 3.11: Conjunction 2.
Prediction 3.14: Ambiguous Ellipsis.
Prediction 3.15: Assertible Phrasal Coordination.

Prediction 4.1: Maximal Coordination.
Prediction 4.2: Nested Coordinations.
Prediction 4.3: Marked Arbitrary Coordination.

Prediction 4.4: Exclusive Disjunction.
Prediction 4.5: Asymmetric Disjunction 1.
Prediction 4.6: Asymmetric Disjunction 2.
Prediction 4.7: Exclusive Asymmetric Disjunction 1.
Prediction 4.8: Inference to Available Explanation(s).
Prediction 4.9: Inference to Best Explanation.
Prediction 4.10: Strengthening Disjunction.
Prediction 4.11: Open Disjunction.
Prediction 4.12: Exclusive Asymmetric Disjunction 2.
Prediction 4.14: Performative Arbitrary Coordination.
Prediction 4.15: Performative Strengthening Disjunction.
Prediction 4.16: Hearer’s Choice of Wording Disjunction.
Prediction 4.17: Addendum Disjunction.
Prediction 4.18: Strict Inclusive Disjunction.

Prediction 4.19: Asymmetric Conjunction 1.
Prediction 4.20: Asymmetric Conjunction 2.
Prediction 4.21: Exclusive Conjunction.
Prediction 4.23: Performative Weakening Conjunction.
Prediction 4.24: Exhaustive Conjunction.
Prediction 4.25: Open Conjunction.
## C.4 Example Inferences

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<th>#</th>
<th>Inference</th>
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