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*MECHANICAL WAVE VELOCITY DISPERSION IN RODS AND
EXTENSION OF THE RESONANT BAR METHOD TO 800 KHz*

*A study of the velocity dispersion of longitudinal waves
in rods, and a feasibility study of extending the
Resonant Bar method of internal friction measurement
to 1 MHz*

A Thesis submitted for the Degree of

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by

RUDOLF EDWARD BOOKER

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Preface 1 deals with the theory of velocity dispersion in infinite cylindrical rods.

Preface 2 gives a summary of experimental work done by other investigators on the propagation and velocity dispersion of longitudinal waves in finite cylindrical rods.

The Thesis begins with the Abstract on page 31.