THE SOLUTION OF THE ORDER CONDITIONS FOR GENERAL LINEAR METHODS

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Abstract

The introductory chapter in this thesis examines briefly the nature of initial value problems and surveys the main types of methods used for their numerical solution. The general linear formulation of methods, first proposed by J.C. Butcher, is introduced, together with the definition of order for this class of methods. Finally in this chapter, the problem of stiffness and its effect on numerical procedures is considered.

Following a review of Butcher's algebraic approach to the theory of Runge-Kutta and general linear methods in Chapter 2, the theory is applied in Chapter 3 to the search for general linear methods of various orders. As in the case of Runge-Kutta methods, the use of so-called simplifying assumptions plays a significant role in the practical determination of general linear methods. From amongst the range of possible numbers of simplifying assumptions, two cases are chosen and investigated in detail. The important question of stability is considered in the final section of Chapter 3.

When a general linear method is used to approximate the solution of an initial value problem, special procedures are required to start and finish the integration. Whilst a major part of Chapter 4 is devoted to the determination of these procedures, the problems of the estimation of local truncation error and the implementation of general linear methods are also discussed.

Finally, the Appendices contain Algol 60 procedures for the most important of the algorithms developed in the main body of the thesis.

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