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An Experimental Investigation of Turbulence and Unsteady Loading on Tidal Turbines

Ian Angus Milne

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering, The University of Auckland, 2014
Abstract

This research addresses the need for an improved characterisation of the onset flow turbulence and the unsteady hydrodynamic blade loads on tidal turbines for the purposes of predicting fatigue life.

A new, extensive set of parameters which characterise the magnitudes of the turbulent fluctuations, the anisotropy and the scales of the turbulence at a tidal energy site have been presented. A novel application of rapid distortion theory estimated the velocity fluctuations to be amplified by 15% due to the presence of the turbine. The turbulence was also predicted to be well correlated over the outer span of a turbine blade at the frequencies of interest. Together, these results enabled a set of non-dimensional parameters describing the turbulence induced forcing on a turbine blade to be established.

A model-scale horizontal-axis turbine was used to investigate the unsteady blade load response in a still-water towing tank. A set of wind tunnel tests of the S814 foil were also conducted and used to demonstrate that the lift on the blades could have been degraded by 10% at the relatively low Reynolds numbers at which the turbine was tested, relative to full-scale. This was owing to dominant laminar separation bubbles.

Single frequency planar oscillations of the turbine were used to quantify the contribution of hydrodynamic unsteadiness to the blade-root bending moment. For attached flow, the unsteady bending moment was found to amplify the steady loads by up to 15%. The total hydrodynamic added mass was up to 2.7 times larger than from non-circulatory forcing and decreased with frequency. Dynamic inflow theory and a returning wake model were able to provide qualitative predictions of these results at low frequencies. At low tip-speed ratios, phenomena consistent with delayed separation and dynamic stall were characterised and the unsteady loading was up to 25% larger than the steady load. Linear superposition of the single frequency responses was also demonstrated to offer a reliable technique to model the response to a multi-frequency forcing and to a large eddy.
Acknowledgements

It is a pleasure to acknowledge the wealth of support that I have been fortunate to receive whilst undertaking my Ph.D. This has been from various sources whom without this research would not have been possible.

Firstly, I would like to acknowledge the support received from The University of Auckland. I would like to thank my supervisors, Dr Rajnish Sharma, Professor Richard Flay and Associate Professor Simon Bickerton. They provided me with the opportunity and support to pursue a line of research which I am passionate about, despite the fact that such a research topic had not been previously undertaken at the university.

I wish to also thank Dr John Cater for providing valuable feedback on the thesis, and Associate Professor Peter Richards for his assistance during the wind tunnel testing. The aerofoils were manufactured by Mr Joel Glass at The University of Auckland Engineering Workshop. The high quality of the models was a reflection of his care and expertise in such complex jobs. I also appreciate the support of Nick Velychko for his technical assistance during the wind tunnel experiments.

This research was funded through the Bright Futures Top Achiever Doctoral Scholarship. I wish to also acknowledge the additional sources of funding from the Energy Institute at The University of Auckland, the International Network on Offshore Renewable Energy, the Marine Technology Society and the IEEE Oceanic Engineering Society. Together, this provided the financial support to undertake significant parts of the research in the UK.

The turbulence data was kindly provided by Andritz Hydro Hamnafest and ScottishPower Renewables in the UK. I would like to acknowledge the effort of Craig Love in facilitating access to the data, and Peter Wilson and Dr Emmanuel Osalusi at Partrac Limited in Glasgow for their technical assistance on matters related to the data processing.

The turbine experiments were made possible owing to the generosity of Pro-
fessor Sandy Day at the University of Strathclyde. His support and guidance for this research has been invaluable. The time I spent conducting these experiments was amongst the most challenging, yet most rewarding and enjoyable during this Ph.D. The technical expertise of Charles Kay, Grant Dunning, Bill McGuffie and Bill Wright at the Kelvin Hydrodynamic Laboratory is also much appreciated.

It has also been a privilege to have had the support of Professor J. Michael Graham and the opportunity to spend a short period of time at Imperial College in London. His guidance on both the turbulence and turbine analyses and feedback on the thesis is very much appreciated. He also provided the inspiration for investigating the effect of the turbine on the free-stream turbulence.

The insightful comments on the turbulence analyses by Dr Mark Trevethan is gratefully acknowledged. The kindness of Professor David Peters at Washington University in taking the time to provide resources and answer queries related to the analysis of dynamic inflow and Loewy theory is also very much appreciated. Furthermore, I wish to acknowledge the support of Mat Thomson and Dr Robert Rawlinson-Smith at DNV-GL over the duration of this research. They generously provided a license for GH-Tidal-Bladed software which greatly facilitated the numerical analysis.

Throughout this research I have also been fortunate to make a lifelong friend in Dr Paul Harper, whose support and encouragement over the course of the Ph.D. has been invaluable. I also thank Dr Joanna Chetwood, whose interest and support in various aspects of this research has been instrumental. Additionally, it has also been great getting to know Drs Duncan McNae, Tim Divett, Craig Stevens and Ross Vennell, whom together have put New Zealand on the map in the tidal energy sector.

On a personal note, a big thank you is in order to my family for the love and support that they have provided over my years of study. This extends to the most supportive and generous second family and in-laws I could ever have asked for.

Last but certainly not least, a huge thank you to my wife Farah whom I dont believe I could have done this without. It is difficult to express how much her support has meant throughout this research and I appreciate the sacrifices she has had to make during this Ph.D.
# Contents

Abstract iii

Acknowledgements v

Contents vii

List of Figures xi

List of Tables xvii

List of Research Achievements xix

Nomenclature xxiii

1 Introduction 1

1.1 Tidal Stream Energy . . . . . . . . . . . . . . . . . . . . . . . . 1
1.2 Tidal Stream Turbines . . . . . . . . . . . . . . . . . . . . . . . 2
1.3 Challenges Faced by Industry . . . . . . . . . . . . . . . . . . . 6
1.4 Research Objectives, Scope & Novelty . . . . . . . . . . . . . . 8
1.5 Thesis Structure . . . . . . . . . . . . . . . . . . . . . . . . . . . 10

2 Literature Review 11

2.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 11
2.2 Fundamentals of Turbulence Theory . . . . . . . . . . . . . . . 12
2.3 Experimental Measurements of Turbulence . . . . . . . . . . . . 16
2.4 Unsteady Hydrodynamic Loading of Rotors . . . . . . . . . . 23
2.5 Experimental Investigations of Tidal Turbine Loading . . . . . . 29
2.6 Performance of Foils at Low Reynolds Numbers . . . . . . . . 33
2.7 Justification for the Experimental Methodologies . . . . . . . . 35
## Contents

3 Acquisition & Processing of the Turbulence Data 39
   3.1 Introduction ............................................. 39
   3.2 Acquisition of the Turbulence Data ..................... 40
   3.3 Data Processing Methodology ........................... 45
   3.4 Computation of Turbulence Statistics ................... 48
   3.5 Summary .................................................. 56

4 Characterisation of the Turbulence at the Sound of Islay 59
   4.1 Introduction ............................................. 59
   4.2 Observed Hydrodynamics ................................ 60
   4.3 Observed Turbulence Properties ........................ 65
   4.4 Discussion of the Turbulence Properties .............. 83
   4.5 Implications for Tidal Turbine Loading ................ 86
   4.6 Summary .................................................. 93

5 Set-Up of the Turbine Tank Testing 95
   5.1 Introduction ............................................. 95
   5.2 Facility & Towing Carriages .............................. 95
   5.3 Turbine & Instrumentation .............................. 96
   5.4 Control Strategies, Data Acquisition & Quality of Motion 103
   5.5 Experimental Uncertainties ............................ 107
   5.6 Characterisation of the Turbine in Steady Flow ....... 111
   5.7 Summary .................................................. 116

6 Performance of the Foil at Low Reynolds Number 117
   6.1 Introduction ............................................. 117
   6.2 Wind Tunnel Facility ................................... 118
   6.3 Methodology of the Force Measurements ................. 119
   6.4 Methodology of the Pressure Distribution Measurements 121
   6.5 Analysis & Discussion of the Experimental Measurements 124
   6.6 Insights for the Tidal Turbine .......................... 133
   6.7 Summary .................................................. 138

7 Blade Loads for Unsteady Forcing 141
   7.1 Introduction ............................................. 141
   7.2 Methodology ............................................. 142
   7.3 Single Frequency Oscillations for Attached Flow ....... 149
List of Figures
1.1

Schematic of the Andritz Hydro Hammerfest HS1000 turbine . . .

3

1.2

Co-ordinate system and the principal loads for the tidal turbine .

5

2.1

Schematic of the sub-division of an open channel flow . . . . . . .

12

2.2

Two-dimensional lift coefficient of the S814 foil for oscillations in
pitch . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

28

Schematic diagram of a laminar separation bubble on an aerofoil
and the corresponding pressure distribution . . . . . . . . . . . .

35

3.1

Map of the Sound of Islay . . . . . . . . . . . . . . . . . . . . . .

40

3.2

Map of the bathymetery at the location of the data acquisition . .

41

3.3

Sound of Islay near peak tidal flow . . . . . . . . . . . . . . . . .

42

3.4

Deployment of the turbulence rig at the Sound of Islay . . . . . .

43

3.5

Schematic diagram of the configuration of the ADV . . . . . . . .

44

3.6

Schematic diagram of the ADCP beam geometry . . . . . . . . .

45

4.1

Water level and mean velocity at 30 m over the entire data acquisition period . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

60

2.3

4.2

Tidal compass of the mean velocity at 5 m and 30 m above the bed 61

4.3

Time histories of the water level and mean velocity at 5 m during
the period where turbulence data were acquired . . . . . . . . . .

62

4.4

Profiles of the mean streamwise velocity . . . . . . . . . . . . . .

63

4.5

Profiles of the shear and of the logarithmic fit to the mean velocity 64

4.6

Time histories of the turbulence intensity and ratios of the standard deviations of the turbulent velocities at 5 m above the bed .

66

Time histories of the normalised Reynolds shear stresses at 5 m
above the bed . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

68

4.7

xi


List of Figures

4.8 The Reynolds shear stress as a function of the depth-averaged mean velocity squared ........................................ 70
4.9 The standard deviation of the streamwise velocity and Reynolds shear stress throughout the water column .... 71
4.10 Profiles of the turbulence intensity and ratio of the standard deviation of the streamwise velocity to the friction velocity ........ 73
4.11 Profiles of the Reynolds shear stress ........................................ 74
4.12 Time histories of the integral length-scales of velocity ............. 75
4.13 Time histories of the fluctuating velocities ................................. 77
4.14 Autocorrelation function of velocity at an elevation of 5 m .......... 78
4.15 Velocity auto-spectra at an elevation of 5 m above the seabed ... 79
4.16 Co-spectral and quad-spectral density at an elevation of 5 m above the seabed ..................................................... 81
4.17 Schematic of an eddy illustrating the quad-spectrum .................... 82
4.18 Co-spectra at elevations of 20 m and 30 m from the seabed ........ 82
4.19 Estimates of the amplification of the turbulence fluctuations for a sudden expansion ............................................. 88
4.20 Rotational sampled spectra ...................................................... 90
5.1 Towing tank main-carriage and sub-carriage ................................. 97
5.2 Schematic diagram of the set-up of the turbine experiments ........... 98
5.3 The turbine prior to installation on the carriage ......................... 99
5.4 The chord and twist distribution along the blade ......................... 100
5.5 The alignment of the blade pitch using dual dial gauges .............. 101
5.6 The turbine hub showing the strain-gauges and the water sealing 102
5.7 Kinematics for a single frequency forcing, demonstrating the quality of oscillatory motion able to be achieved .............. 104
5.8 Kinematics for a multi-frequency forcing, demonstrating the quality of oscillatory motion able to be achieved .......... 105
5.9 Examples of the spectra of the carriage velocity and out-of-plane blade-root bending moment .............................. 106
5.10 Performance coefficients of the turbine in steady flow ................ 112
5.11 Effective centre of loading on the blade for steady flow ............. 114
5.12 Bending moment coefficient normalised by the rotor speed .......... 115
6.1 Plan view of the de Bray wind tunnel ........................................ 118
6.2 Set-up of the aerofoil model to measure forces in the wind tunnel 120
List of Figures

6.3 Pressure taps on aerofoil section .............................................. 122
6.4 The aerofoil model which was used to acquire the pressure distributions installed in the test-section .............................................. 123
6.5 Two-dimensional lift and drag coefficients of the S814 foil measured in the wind tunnel .................................................. 126
6.6 Profile of the S814 aerofoil and pressure distributions for $\alpha = 0^\circ$ .................................................. 128
6.7 Pressure distributions for the S814 foil at angles of attack of $\alpha = 5^\circ$ and $\alpha = 8^\circ$ .................................................. 130
6.8 Pressure distributions for the S814 foil at angles of attack of $\alpha = 10^\circ$, $\alpha = 12^\circ$ and $\alpha = 15^\circ$ .................................................. 132
6.9 Comparisons of the bending moment coefficients against BEM theory .................................................. 135
6.10 Estimates of the angle of attack and axial induction factor using BEM theory .................................................. 136
6.11 The Prandtl tip-loss factor for the outer half of the blade span using BEM theory .................................................. 137

7.1 Convergence of the phase-averaged out-of-plane bending moment response .................................................. 144
7.2 Removal of the gravitational component from the in-plane blade root bending moment response .................................................. 144
7.3 Effect of the filter applied to the forcing and bending moment response .................................................. 145
7.4 Identification of a seiche wave using estimates of the amplitude and phase obtained from a cycle-by-cycle fit .................. 148
7.5 Effect of the oscillatory frequency on the bending moment responses for $\mu = 0.150$ .................................................. 150
7.6 Effect of the Current number on the bending moment responses for $f = 0.40$ Hz .................................................. 152
7.7 Effect of the Current number on the bending moment responses for $f = 0.67$ Hz and 1.00 Hz .................................................. 153
7.8 Effect of the tip-speed ratio on the bending moment responses for $\mu = 0.250$ .................................................. 154
7.9 Estimates of the amplitude and phase corresponding to the single frequency bending moment response .................................................. 155
7.10 Ratio of the out-of-plane bending moment component in-phase with acceleration to the non-circulatory bending moment .................................................. 156
List of Figures

7.11 Comparison of the bending moment response predicted using dynamic inflow theory ........................................ 158
7.12 Loewy lift deficiency function corrected for the singularity ...... 161
7.13 Effect of the oscillatory frequency on the bending moment response for low tip-speed ratios .................................. 163
7.14 Bending moment response exhibiting dynamic stall for $\mu = 0.100$ .............................................................. 165
7.15 Comparison of the out-of-plane bending moment response with a dynamic-stall model ........................................... 167
7.16 The bending moment response for a forcing comprising the frequencies of $f = 0.40\,\text{Hz}$ and $0.50\,\text{Hz}$ with a Current number of $\mu = 0.075$ ............................................................... 169
7.17 The bending moment response to a forcing comprising oscillatory frequencies of $f = 0.40\,\text{Hz}$ ($\mu = 0.075$) and $0.50\,\text{Hz}$ ($\mu = 0.125$) ................................................................. 170
7.18 Bending moment response to a forcing comprising the frequencies $f = 0.40\,\text{Hz}$, $0.50\,\text{Hz}$ and $0.67\,\text{Hz}$ ................................................................. 171
7.19 Bending moment response to a forcing comprising the oscillatory frequencies of $f = 0.67\,\text{Hz}$ and $0.80\,\text{Hz}$ ................................................................. 171
7.20 Estimates of the amplitude and phase from a linear fit to a multi-frequency forcing response ......................................... 172
7.21 Bending moment response to a discrete half-sinusoidal forcing for $f = 0.40\,\text{Hz}$ and $f = 0.50\,\text{Hz}$ ................................................................. 173
7.22 Bending moment response to a positive half-sinusoidal forcing for $f = 0.40\,\text{Hz}$ and $0.50\,\text{Hz}$ ................................................................. 174

A.1 Identification and removal of bias in the ADCP variances ...... 199
A.2 Time histories of the mean streamwise velocity and the standard deviation of the streamwise, transverse and vertical velocities at an elevation of 5 m from the seabed ........................................ 200
A.3 Time histories of the ratios of the along-channel and across-channel Reynolds shear stresses to $2\sigma$ at an elevation of 5 m ........................................ 201
A.4 Profiles of the mean streamwise velocity for the second and third tidal cycles ......................................................... 202
A.5 Profiles of the streamwise turbulence intensity over the second and third tidal cycles ......................................................... 202
A.6 Profiles of the Reynolds shear stress over the second and third tidal cycles ......................................................... 203
List of Figures

C.1 A schematic of the geometry from which the rotational sampled correlation function is based. . . . . . . . . . . . . . . . . . . . . . . . . . . . 208

D.1 Aerofoil forces of the S814 aerofoil for Re = 1.5 × 10^6 . . . . . . . 214

D.2 Pressure distributions for the S814 aerofoil measured for between
α = 0° and 5° for Re = 1.5 × 10^6 . . . . . . . . . . . . . . . . . . . . . . 215

D.3 Pressure distributions for the S814 aerofoil measured for between
α = 6° and 10° for Re = 1.5 × 10^6 . . . . . . . . . . . . . . . . . . . . . . 215

D.4 Pressure distributions for the S814 aerofoil measured for between
α = 11° and 15° for Re = 1.5 × 10^6 . . . . . . . . . . . . . . . . . . . . . . 216

D.5 Lift and drag measurements of the S823 aerofoil . . . . . . . . . . . . 217

G.1 The steady flow separation point used in the dynamic-stall model 227
# List of Tables

1.1 Operational specifications for three commercial-scale horizontal-axis tidal turbines ................................................. 4

3.1 Specifications of the ADV and ADCP ........................................ 44

4.1 Summary of the observed turbulence properties at the Sound of Islay 84
4.2 Estimates of the non-dimensional parameters used to describe the forcing due to turbulence. ........................................ 92

5.1 Summary of the uncertainties associated with the turbine experiments ................................................................. 107

D.1 Summary of the wind-tunnel corrections .......................... 212
Research Achievements

The following articles were derived from the research presented in this thesis

Journal Publications


Conference Publications


List of Research Achievements


Summary of Contributions

• A new, extensive set of statistics characterising the magnitudes, anisotropy and scales of the turbulence at a tidal energy site have been obtained. Not all of the turbulence properties presented in this thesis have been previously reported for tidal streams. However, a subset of these parameters could be compared to literature, and these were found to be consistent with other tidal energy sites, idealised open-channel flows and theoretical models.

• In a novel application of rapid distortion theory, it was found that streamwise velocity fluctuations could be amplified by 15% at the rotor due to the presence of the turbine.

• A new set of analyses revealed for the first time that turbulence as perceived the turbine blades, was well correlated over the outer sections of a turbine blade. This implies that the planar oscillations of the turbine can be used to study the response of the blade loads to turbulence.

• Blade-root bending moment responses have been obtained for frequencies higher than those previously reported in the literature. The unsteady loads were found to exceed the steady loads by 15% when the boundary layer remains attached and 25% when dynamic stall is observed.

• The total hydrodynamic contribution to the bending moment in-phase with acceleration was found to decrease with frequency. At relatively low frequencies, it was up to 2.7 times as large as the non-circulatory forcing.

• The blade load response to oscillations at low tip-speed ratios have been shown to exhibit phenomena consistent with delayed separation and dynamic stall of foils.
List of Research Achievements

- The unsteady loads for a multi-frequency forcing and discrete half-sine forcing were demonstrated to be able to be reconstructed using the single frequency responses.

- At the low Reynolds numbers at which the turbine experiments were conducted, the lift was found to be 10% smaller and the minimum drag 400% larger relative to full-scale, due to dominant laminar separation bubbles. Laminar separation of the boundary layer on pressure surface of the foils was also identified as limiting the range of tip-speed ratios at which useful data could be obtained.
## Nomenclature

The primary symbols and acronyms used in this thesis are listed below in alphabetical order. It can be noted that certain symbols have multiple definitions, depending on whether they relate to turbulence or tidal turbines.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Axial induction factor</td>
</tr>
<tr>
<td>$a'$</td>
<td>Tangential induction factor</td>
</tr>
<tr>
<td>$b_i$</td>
<td>Fluctuating velocity measured along the $i^{th}$ ADCP beam (m s$^{-1}$)</td>
</tr>
<tr>
<td>$b_X$</td>
<td>Systematic uncertainty of variable $X$</td>
</tr>
<tr>
<td>$C_{i,i}$</td>
<td>Co-spectral density function, where $i = u, v, w$ (m$^2$ s$^{-1}$)</td>
</tr>
<tr>
<td>$C_{Mx}$</td>
<td>Blade root in-plane bending moment coefficient</td>
</tr>
<tr>
<td>$C_{My}$</td>
<td>Blade root out-of-plane bending moment coefficient</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Bottom drag coefficient; Two-dimensional drag coefficient</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Two-dimensional lift coefficient</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Turbine axial force (thrust) coefficient</td>
</tr>
<tr>
<td>$C_P$</td>
<td>Turbine power coefficient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Pressure coefficient</td>
</tr>
<tr>
<td>$c$</td>
<td>Aerofoil chord length (m)</td>
</tr>
<tr>
<td>$d$</td>
<td>Depth of water in the towing tank (m)</td>
</tr>
<tr>
<td>$e$</td>
<td>Ratio of cross-sectional area of the rotor to the cross-sectional area of the stream-tube in the free-stream</td>
</tr>
<tr>
<td>$h$</td>
<td>Channel depth (m)</td>
</tr>
<tr>
<td>$F_x$</td>
<td>Rotor axial (thrust) force (N)</td>
</tr>
<tr>
<td>$f$</td>
<td>Oscillatory frequency (Hz)</td>
</tr>
<tr>
<td>$Fr$</td>
<td>Froude number</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration (m s$^{-2}$)</td>
</tr>
</tbody>
</table>
Nomenclature

KC Keulegan-Carpenter number, \( \hat{u}T/2R \), where \( T \) is the oscillatory period

\( k \) Reduced frequency, \( \pi fc/W \)

\( k_x \) Streamwise radial wave-number, \( 2\pi n/U \) (m\(^{-1}\))

\( M \) Number of data points in a velocity sample

\( M_x \) Blade root in-plane bending moment (about an axis normal to the shaft) (N m)

\( M_y \) Blade root out-of-plane bending moment (about an axis parallel to the shaft) (N m)

\( I_u \) Streamwise turbulence intensity, \( \sigma_u/U \)

\( L_i \) Integral length-scale of turbulence in the longitudinal direction, where \( i = u, v, w \) (m)

\( n \) Frequency of the turbulence fluctuations (Hz)

\( n_s \) Sampling frequency (Hz)

\( P \) Rotor power (W)

\( p_\infty \) Static pressure

\( Q \) Rotor torque (N m)

\( Q_{ij} \) Quad-spectral density function, where \( i \neq j = u, v, w \) (m\(^2\)s\(^{-1}\))

\( R \) Tip radius of the turbine with respect to the hub centre (m)

\( \text{Re} \) Reynolds number, \( UL/\nu \), where \( U \) and \( L \) are the characteristic velocity and length scales

\( r \) Local radius of the blade section with respect to the hub centre (m)

\( S_{ii} \) Spectral density of velocity component \( i = u, v, w \)

\( s_X \) Random uncertainty of variable \( X \)

\( U, V, W \) Streamwise, transverse and vertical mean velocity; Mean carriage velocity \( (U) \); Mass flow parameter \( V \), Relative incident velocity at blade section \( (W) \) (m s\(^{-1}\))

\( \overline{U} \) Streamwise velocity averaged over the channel depth; (m s\(^{-1}\))

\( u, v, w \) Fluctuating component of the streamwise, transverse and vertical velocity; Velocity of the auxiliary towing carriage \( (u) \) (m s\(^{-1}\))

\( \hat{u} \) Oscillatory velocity amplitude (m s\(^{-1}\))

xxiv
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_* )</td>
<td>Friction velocity (m s(^{-1}))</td>
</tr>
<tr>
<td>( u_{TX} )</td>
<td>Total uncertainty of variable ( X )</td>
</tr>
<tr>
<td>( \dot{u} )</td>
<td>Acceleration of the auxiliary towing carriage (m s(^{-2}))</td>
</tr>
<tr>
<td>( v_i )</td>
<td>Axial induced velocity (m s(^{-1}))</td>
</tr>
<tr>
<td>( \bar{v}_i )</td>
<td>Mean axial induced velocity (m s(^{-1}))</td>
</tr>
<tr>
<td>( z )</td>
<td>Elevation above the sea-bed (m)</td>
</tr>
<tr>
<td>( z_0 )</td>
<td>Roughness length (m)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Kolmogorov constant; Angle of attack ((^\circ))</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Twist angle</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Anisotropic ratio incorporating the ADCP beam angle</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Strain imposed on a turbulent stream</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Rate of eddy dissipation (m(^2) s(^{-3}))</td>
</tr>
<tr>
<td>( \theta )</td>
<td>ADCP beam angle relative to the vertical ((^\circ))</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>von Kármán constant</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Tip-speed ratio, ( \Omega R/U )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity (m(^2) s(^{-1}))</td>
</tr>
<tr>
<td>( \nu_i )</td>
<td>Normalised axial induced velocity</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density (kg m(^{-3}))</td>
</tr>
<tr>
<td>( \rho_{ii} (\tau) )</td>
<td>Autocorrelation function, ( C_{ii} (\tau) / \sigma_i^2 )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Shear stress (Pa); Non-dimensional time of the oscillatory cycle, ( \tau = t/T )</td>
</tr>
<tr>
<td>( \tau_i )</td>
<td>Integral time-scale (s)</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>Phase angle ((^\circ))</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Inflow angle (rad)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Current number ( \tilde{u}/U )</td>
</tr>
<tr>
<td>( \sigma_i^2 )</td>
<td>Variance of the fluctuating velocity, where ( i = u, v, w ) (m(^2) s(^{-2}))</td>
</tr>
<tr>
<td>( \sigma_{ij}^2 )</td>
<td>Reynolds shear stress normalised by density, where ( i \neq j = u, v, w ) (m(^2) s(^{-2}))</td>
</tr>
<tr>
<td>( \sigma_i )</td>
<td>Standard deviation of the fluctuating velocity, where ( i = u, v, w ) (m s(^{-1}))</td>
</tr>
<tr>
<td>( \sigma_M )</td>
<td>Standard deviation of the bending moment (N(^2) m(^2))</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Angular velocity of the turbine (rad s(^{-1}))</td>
</tr>
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**Nomenclature**

**Acronyms**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ADV</td>
<td>Acoustic Doppler velocimeter</td>
</tr>
<tr>
<td>ADCP</td>
<td>Acoustic Doppler current profiler</td>
</tr>
<tr>
<td>BEM</td>
<td>Blade element-momentum</td>
</tr>
<tr>
<td>EMEC</td>
<td>European Marine Energy Centre</td>
</tr>
<tr>
<td>HATT</td>
<td>Horizontal-axis tidal turbine</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite Impulse Response</td>
</tr>
<tr>
<td>MCT</td>
<td>Marine Current Turbines Ltd.</td>
</tr>
<tr>
<td>NACA</td>
<td>National Advisory Committee for Aeronautics</td>
</tr>
<tr>
<td>NREL</td>
<td>National Renewable Energy Laboratory</td>
</tr>
<tr>
<td>SEE</td>
<td>Standard error in an estimate</td>
</tr>
<tr>
<td>TKE</td>
<td>Total turbulent kinetic energy</td>
</tr>
<tr>
<td>2-D</td>
<td>Two-dimensional</td>
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</table>
Co-Authorship Form

This form is to accompany the submission of any PhD that contains research reported in published or unpublished co-authored work. **Please include one copy of this form for each co-authored work.** Completed forms should be included in all copies of your thesis submitted for examination and library deposit (including digital deposit), following your thesis Acknowledgements.

Please indicate the chapter/section/pages of this thesis that are extracted from a co-authored work and give the title and publication details or details of submission of the co-authored work.

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<th>Nature of contribution by PhD candidate</th>
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**CO-AUTHORS**

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<th>Nature of Contribution</th>
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<tr>
<td>Dr. Rajnish Sharma</td>
<td>Proof reading and comments</td>
</tr>
<tr>
<td>Prof. Richard Flay</td>
<td>Proof reading and comments</td>
</tr>
<tr>
<td>Ass. Prof. Simon Bickerton</td>
<td>Proof reading</td>
</tr>
</tbody>
</table>

**Certification by Co-Authors**

The undersigned hereby certify that:

- the above statement correctly reflects the nature and extent of the PhD candidate’s contribution to this work, and the nature of the contribution of each of the co-authors; and
- in cases where the PhD candidate was the lead author of the work that the candidate wrote the text.

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<tr>
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<td>___</td>
<td>21/08/2013</td>
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<tr>
<td>Dr. Rajnish Sharma</td>
<td>___</td>
<td>22/08/2013</td>
</tr>
<tr>
<td>Prof. Richard G.J. Flay</td>
<td>___</td>
<td>23/08/2013</td>
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<td>Extent of contribution by PhD candidate (%)</td>
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Chapter 1

Introduction

What we know is a drop, what we don’t know is an ocean.

— Isaac Newton

1.1 Tidal Stream Energy

Tidal energy is a renewable and highly predictable source of electricity generation. Harnessing the energy of the tides is not new. Tidal mills existed centuries ago and tidal barrage schemes, which extract potential energy, have been in operation since the 1960s (Devaux, 1964). However, in response to a global demand for renewable energy generation, there has recently been an increased interest in technologies that exploit the kinetic energy from fast flowing tidal streams. This form of generation is perceived to have significantly lower ecological impacts than tidal barrage schemes and require a lower capital investment (Rourke et al., 2010). Furthermore, a tidal stream turbine offers a comparatively higher energy density per unit size, four times greater than a typical wind turbine and thirty times greater than a solar photovoltaic array (Fraenkel, 2002).

Arguably the most activity within the tidal stream energy industry has been witnessed in the UK. The world’s first marine energy test facility, the European Marine Energy Centre (EMEC, 2013) was opened in 2003 in Orkney, Scotland. It has proved popular with device developers, with several devices having been tested at or near full-scale. The Carbon Trust (Black & Veatch Consulting Ltd., 2011) identified 30 key sites within the UK where electricity from tidal energy could be economically generated. One of the most significant resources is the
Pentland Firth, where up to 1.9 GW could potentially be generated (Adcock et al., 2013).

However, other locations worldwide also possess notably larger tidal stream resources. An assessment by Karsten et al. (2008) suggested that 2.5 GW (from a maximum of 7 GW) could theoretically be extracted by tidal turbines in Minas Passage at the Bay of Fundy in North America. On the West Coast of North America, Sutherland et al. (2007) evaluated that over 2 GW could potentially be generated at Johnstone Strait, Vancouver. In China, a total installed generation of 14 GW is considered to be possible from 130 locations (Wang et al., 2011). The tidal flows in New Zealand are also considered to be suitable for electricity generation and tidal turbine farms have been proposed at the Kaipara Harbour (Crest Energy, 2013) and off the Cook Strait (Neptune Power, 2013).

It is also important not to overlook the potential role of tidal turbines for smaller countries and remote communities. For example, countries in the South Pacific, rely substantially on fossil fuels for electricity generation, which is subject to increasing commodity prices (Jafar, 2000). Energy demand at these locations is generally only between 100 kW to 500 kW. This generation capacity could be met by smaller or fewer tidal turbines and these could potentially operate in slower flows, such as at the inlets of atolls.

1.2 Tidal Stream Turbines

1.2.1 Concepts & Turbines Currently Deployed

Various concepts have been proposed to extract tidal stream energy (see Hardisty, 2009; Rourke et al., 2010). These have ranged from horizontal-axis turbines, to vertical-axis and transverse-axis rotors, to oscillating hydrofoils. Whilst a consensus has not yet been reached in terms of the preferred technology to extract tidal stream energy, horizontal-axis tidal turbines (HATTs) have arguably been the most popular to date (Rourke et al., 2010).

At the most fundamental level, the principle of a HATT is to convert kinetic energy, or momentum from the free-stream to rotational motion. This is achieved by means of a cambered foil which induces lift from a relative velocity. The similarity between HATTs and modern wind turbines has attributed to the popularity of HATTs as knowledge has been able to be transferred.

As at 2013, the longest operating commercially accredited HATT is the SeaGen
1.2. Tidal Stream Turbines

(MCT, 2013). It was developed by Marine Current Turbines Limited and has exceeded 1,000 operational hours. The twin rotor turbine is deployed at Strangford Loch in Northern Ireland and has a total rated power output of 2.4 MW, which approximately equates to the electricity demand of 1,000 households.

There are also a number of HATT devices currently being tested at or near full-scale. Of note is the 500 kW single rotor, tri-bladed turbine developed by Tidal Generation Limited (TGL, 2013). It has operated at EMEC since September 2010 and a scaled-up 1 MW device is currently under development. A 1 MW turbine (the HS1000), developed by Andritz Hydro Hammerfest (Andritz, 2013) has also been deployed at EMEC since 2011 and is shown in Figure 1.1. It is based on a 300 kW device that was tested for 14,000 operational hours.

![Figure 1.1: Schematic of the Andritz Hydro Hammerfest HS1000 turbine as deployed on the seabed. Reproduced with permission of Andritz Hydro Hammerfest.](image-url)
1.2.2 General Specifications

The general specifications of the three aforementioned turbines are compared in Table 1.1. These are typically governed by the physical constraints imposed by the sites, such as the water depth and flow speed.

A tidal turbine typically begins to generate power at a mean flow speed (cut-in speed) of around 1 m s\(^{-1}\) to 2 m s\(^{-1}\). Rated power is achieved at flow speeds of between 2.5 m s\(^{-1}\) and 4.0 m s\(^{-1}\). However, the rated speed does not necessarily correspond to the maximum flow speed observed at the site.

The hub-heights of the rotors are generally selected to position the turbine at sufficiently high elevation to avoid the slower mean velocity near the seabed. A sufficient distance below the water line is also typically required for the passing of marine-craft. For example, at Sound of Islay in Scotland, the turbines are proposed to have a hub-height of \(z = 22\) m. This provides a clearance of approximately 20 m below the free-surface (ScottishPower Renewables, 2010). However, there are exceptions to this, such as Verdant Power who have deployed turbines at \(z/h \approx 0.2\) above the bed, where \(h\) is the channel depth (Thomson et al., 2012). Also interesting is, Scotrenewables (2013), who have developed a floating turbine with a hub in relatively close proximity to the surface. In the analyses presented in this thesis, a hub-height of between \(z/h = 0.4\) and 0.5 above the bed is assumed. This is consistent with the idea of keeping the turbine away from the bed and the surface.

The control strategy is generally indicative of the developer’s perception of the required level of complexity of the turbine. Variable speed and pitch regulation systems are implemented for all these turbines in an attempt to optimise efficiency. However, simpler turbines with less complexity have also been developed. These have been perceived to offer increased robustness against component

<table>
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<tr>
<th>Specifications</th>
<th>Seagen TGL 1MW</th>
<th>HS1000</th>
</tr>
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<tr>
<td>Rotor diameter (m)</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>Number of blades</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Rated rotor speed (rpm)</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Cut-in flow speed (m s(^{-1}))</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>Rated flow speed (m s(^{-1}))</td>
<td>2.5</td>
<td>2.7</td>
</tr>
<tr>
<td>Water depth (m)</td>
<td>24</td>
<td>35 to 80</td>
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</tbody>
</table>

Table 1.1: Operational specifications for three commercial-scale horizontal-axis tidal turbines (where available).
failures in the harsh operating environment (DTI, 2007). Composite materials are typically employed for the blades of tidal turbines. These are favoured for their superior performance against fatigue compared to metallics (Marsh, 2009).

1.2.3 Terminology & Significance of the Blade Loading

The terminology and conventions used to describe the rotor performance and loading in this thesis are generally consistent with those of horizontal wind-turbines (see GL, 2005). Referring to Figure 1.2, the coordinate system which defines the blade-root bending moment, rotates with the blade. The out-of-plane bending moment ($M_y$) can be considered to be the moment which arises predominantly from the rotor thrust ($F_x$) and tends to deflect the blades downstream. The in-plane bending moment ($M_x$) is imparted by the torque and tangential force distribution which produces the power, as well the gravitational and hydrostatic forces.

![Figure 1.2: Co-ordinate system and the principal loads for the tidal turbine used in this thesis, adapted from GL (2005). Legend: axial thrust force ($F_x$), in-plane blade root bending moment ($M_x$), out-of-plane blade root bending moment, ($M_y$) and rotational speed of rotor ($\Omega$)](image-url)
Chapter 1. Introduction

The out-of-plane blade-root bending moments are expected to be significantly larger than the in-plane bending moments. This is due to the large thrust loads and which can be up to 8 times greater than for a wind turbine of the equivalent rotor diameter (Fraenkel, 2002). The out-of-plane hydrodynamic loading is also not able to be as efficiently relieved by the in-plane loads than for a wind turbine. This is due to the comparatively smaller centrifugal forces, given the slower rotational speeds required to prevent cavitation in water turbines.

Therefore, characterising the out-of-plane bending moment is critical for the design of the blade and predicting the fatigue life (Marsh, 2009). The need to understand the unsteady loading is compounded by the fact that composite materials are inherently sensitive to large load cycles. This can be contrasted with metallics, where the fatigue is generally due to an accumulation of smaller loads (Sutherland, 1999).

1.3 Challenges Faced by Industry

There are now several examples of early-generation turbines that have experienced catastrophic blade failures at their root. These failures have been attributed to a poor understanding of the magnitudes and the spectral characteristics of the hydrodynamic loads (Liu and Veitch, 2012). In an attempt to mitigate premature failures and amidst a lack of knowledge of the unsteady blade loads, some blade designers have resorted to over-engineering their turbines by up to 30% (Marsh, 2009). This has consequently increased the cost of the turbines.

If the resource potentials cited in Section 1.1 are to be met, tidal turbines must prove to be economical to manufacture and also operate reliably over their design life of at least 20 years (GL, 2005; Wolfram, 2006). Reliability is an even greater concern for turbines which are deployed in more remote communities. This is because replacement components and expertise is not likely to be readily available and electricity supply would be jeopardized (Anyi et al., 2010).

It should also be noted that the use of horizontal-axis turbines is not exclusive to tidal streams. Such challenges are shared by designers of turbines which are deployed in rivers and canals (Khan et al., 2009). Therefore, providing an improved understanding of the unsteady blade loads is expected to draw wide interest.

As discussed in the following sections, solving these issues is underpinned by a need for an improved characterisation of the onset flow turbulence and of the
1.3. Challenges Faced by Industry

unsteady hydrodynamic loading which is imparted on the blades.

1.3.1 Inflow Characterisation

Whilst the characteristics of the mean flow in tidal streams are relatively well understood, much less is known about the onset turbulence (Gant and Stallard, 2008). This is indicative of the inherent technical difficulty in acquiring measurements of turbulence in fast flowing currents (Grant et al., 1962) and the relative infancy of the tidal stream industry.

The lack of validated industrial guidelines for tidal turbines has added to the difficulty in establishing the turbulence parameters at a tidal site. These currently rely substantially on theories that were developed for atmospheric turbulence (GL, 2005). Tidal streams are influenced by a free-surface, in addition to the topography and the terrain at the seabed. Therefore, it is questionable whether atmospheric models provide a true representation of the turbulence. This emphasises the need to obtain measurements of the turbulence at tidal energy sites for validation purposes.

The streamwise turbulence intensity ($I_u = \sigma_u/U$) has typically been the primary turbulence inflow parameter used by designers to predict the dynamic loading on tidal turbines (McCann et al., 2008). Therefore, acquiring measurements of this statistic is of most importance. However, Sutherland (1999) considers that the inclusion of other parameters is likely to be necessary for accurate predictions of fatigue. Extensive experimental studies of horizontal-axis wind turbines, including those of Kelley (1994), Mouzakis et al. (1999) and Sutherland and co-workers (Sutherland, 2002; Sutherland et al., 2001), have indicated that statistics which describe coherent structures (such as the Reynolds shear stresses) are likely to be influential. However, these parameters have not been extensively reported for fast tidal flows. Given the relatively high degree of similarity that exists between wind turbines and tidal turbines, these are likely to also be of interest.

It is also important to note that the turbulence observed by a rotating blade differs from that which would be observed at a fixed point in the free-stream. This can be attributed to two phenomena. The first is the action of the rotor to slow down the mean flow, resulting in the expansion of the stream-tube. This will effectively impart a strain on the free-stream turbulence and would amplify the streamwise velocity fluctuations. Secondly, a rotating blade passes through a turbulence field that is not fully coherent across the rotor plane, giving rise
to energy peaks corresponding to integer multiples of the rotational-speed of the rotor. Therefore, establishing the significance of these effects is necessary for predicting the character of the unsteady loads and therefore the fatigue life of tidal turbine blades.

1.3.2 Unsteady Hydrodynamic Blade Loading

For unsteady flows there are additional contributions that have an effect on the hydrodynamic loading. These include non-circulatory added mass forcing as well as circulatory forcing from the dynamic response of the wake. For turbines operating at or near peak power, additional circulatory contributions from delayed separation and dynamic stall of the foil can also be present (Leishman, 2002).

Currently, there are only limited experimental data available on the unsteady loading of tidal turbines to quantify the significance of these effects. This has led to questionable assumptions and safety factors being used in predicting the fatigue loads of tidal turbine blades (GL, 2005; Zhou et al., 2013). Furthermore, it has not yet been extensively verified whether commonly employed models for dynamic inflow and dynamic stall, or a linear superposition of the single frequency response are suitable for predicting the hydrodynamic response to turbulence.

The lack of experimental data is arguably attributed to the high cost of large, high quality facilities which are required to account for the effects of scaling and reducing the effects of blockage. This calls for techniques to be developed in order to efficiently attain suitable measurements of unsteady blade loads and at the required level of accuracy to allow for the underlying hydrodynamic phenomena to be studied.

1.4 Research Objectives, Scope & Novelty

The overall aim of this research was to quantify the unsteady hydrodynamic loading on tidal turbine blades that would be imparted by turbulence in the onset flow. In order to achieve this, the study was divided into the following objectives.

The first objective was to quantify the magnitudes of the turbulent fluctuations and the scales of the turbulent energy at a tidal energy site. The intention was to use this data to establish a description of the forcing imposed on a turbine blade.
The second objective was to establish the unsteady hydrodynamic contribution on the blade loads at frequencies and amplitudes that were representative of the turbulence, for a range of turbine operating conditions.

Furthermore, in achieving both these objectives the research also aimed to investigate whether low fidelity models could be applied to describe the turbulence and the unsteady loads.

It is important to note what has been deemed to be out-of-scope in this research. No attempts were made to use vortex wake-based theories, or to apply computational-fluid dynamics (CFD) to model either the turbulence or the hydrodynamic response of the turbine. This is because lower-computationally demanding models are widely employed by industry at present and their verification is considered to be most useful for turbine designers. Predictions from CFD-based models are also deemed to require extensive validation, particularly for cases involving significant flow separation. Due to the time constraints imposed on this research, developing a CFD model as well as performing experiments necessary for validation was not feasible.

The structural loading, hydro-elastic behaviour of the blades and electrical characteristics of the rotor, which can vary significantly between turbines, has also not been considered. Additionally, the effect of any axial misalignment (yaw) of the turbine with respect to the mean flow and any possible interaction between multiple turbines in a farm was also excluded.

There are several novel contributions to the literature from this research. Firstly, a new set of parameters which describe the forcing that is expected to be imparted from turbulence have been obtained. These have been obtained from experimental data and through a new application of rapid distortion theory.

Secondly, blade-root bending moment responses have been obtained for frequencies higher than those previously reported in the literature. The author believes that this is the first study that has specifically investigated the hydrodynamic blade loads for conditions where the boundary layer on the foil was both attached and experienced separation, and quantified the relative unsteady contribution. Furthermore, experimental data for a multi-frequency forcing have been used to investigate the applicability of a stochastic approach to model unsteady loading.

Additionally, this research provides new insights for the industry into the implications of employing relatively thick foils for model-scale turbine tests. Experimental data of both two-dimensional force and the pressure distributions of the
Chapter 1. Introduction

S814 aerofoil at Reynolds numbers of approximately $1 \times 10^5$ have been presented. These have demonstrated the role of laminar separation on the performance of the foil.

1.5 Thesis Structure

Following this introduction, a review of the literature on tidal stream turbulence, hydrodynamic loading of tidal turbines and low Reynolds number behaviour of aerofoils is presented in Chapter 2. The review is also used to justify the methodologies employed to meet the research objectives.

Chapter 3 provides an overview of the site at which the turbulence data were acquired and details the set-up of the sensors. It then discusses the methodology employed to process the data and obtain the relevant turbulence statistics. Chapter 4 presents an analysis of the mean flow hydrodynamics and the observed turbulence properties. These data, together with analytical models for the amplification and rotational sampling of turbulence are used to obtain estimates of the forcing parameters for a tidal turbine.

The methodology and set-up of the model-scale turbine experiments are presented in Chapter 5. The quality of the oscillatory motion is demonstrated, the uncertainties are quantified, and the turbine performance and loading are characterised for steady flow. In Chapter 6, the implications of the low Reynolds numbers at which the turbine experiments were conducted are investigated with the aid of wind-tunnel measurements of both the forces and pressure distributions of the S814 aerofoil.

Chapter 7 presents a parametric analysis of the blade loads in response to unsteady forcing. The relative contribution of the unsteady hydrodynamic component to the bending moment is quantified as a function of the oscillatory frequency, the velocity amplitude of the forcing and the tip-speed ratio of the turbine. These single frequency responses are used to explore the applicability of dynamic inflow models, classical unsteady foil theories and semi-empirical models for delayed separation and dynamic stall for tidal turbines. The chapter concludes by demonstrating the applicability of the single frequency response to reconstruct a more general, multi-frequency forcing as well as the forcing from an isolated large coherent eddy.

Finally, Chapter 8 presents the main conclusions of this research, summarises the achievements and cites aspects where further work may be directed.
Chapter 2

Literature Review

The beginning of knowledge is the discovery of something that we do not understand

— Frank Herbert

2.1 Introduction

This chapter begins by providing an overview of the fundamentals of turbulent boundary layer theory, introducing scaling laws, the wavenumbers of interest and semi-empirical models. Turbulence statistics which have been previously reported in the literature for tidal streams are then identified. Specific consideration is given to studies which have employed modern acoustic-Doppler based measurement techniques and have obtained data in well-mixed tidal flows with mean velocities exceeding 1 m s\(^{-1}\).

Following this, the phenomena associated with unsteady hydrodynamic loading of tidal turbines are discussed and relevant theories and models which have been developed for helicopter rotors and wind turbines are cited. Experimental studies that have investigated unsteady loading of tidal turbines are reviewed and issues pertaining to model-scale testing are identified. The effect of Reynolds number on the aerofoil performance is also discussed.

The chapter concludes by drawing on the review of the literature to justify the type of experimental data and the methodologies which are deemed to be most suitable for meeting the objectives of this thesis.
2.2 Fundamentals of Turbulence Theory

Comprehensive discussions of the fundamental theories of turbulence have frequented the literature, including in the texts by Batchelor (1953), Tennekes and Lumley (1972), Hinze (1975) and Townsend (1976); the latter specifically treating shear flows. Given this extensive coverage, the theories are only summarised here for brevity.

2.2.1 The Boundary Layer & the Law of the Wall

The turbulence in a fast flowing and well mixed tidal stream is typically driven by the dynamics of the sea-floor boundary layer. Experimental studies on tidal channel flows have shown that the boundary layer can exist through a significant portion of the water column (Soulsby, 1983 and Lueck and Lu, 1997). This includes the depths at which a tidal turbine is likely to be deployed.

To characterise an open channel flow dominated by a boundary layer, Nezu and Nakagawa (1993) sub-divided the vertical profile into a wall region, an intermediate region and a free-surface region, as depicted in Figure 2.1.

![Figure 2.1: Schematic of the sub-division of an open channel flow, as depicted by Nezu and Nakagawa (1993) and where z is the elevation above the bed.](image-url)
It was considered that the region near the wall extends upwards from the viscous sub-layer to approximately $z/h = 0.15$ to $0.20$, where $z$ is the elevation above the bed and $h$ is the channel depth. In this region, the dominant velocity scale is the friction velocity, $u_*$. At relatively high Reynolds numbers (i.e. $R_* = u_* h/\nu \gg 1$, where $\nu$ is the kinematic viscosity), which is generally the case at tidal energy sites, the near-bed shear stress ($\tau = -\rho \sigma_{uw}$) is approximately constant (Townsend, 1976). Given this, the friction velocity can be related to the shear stress through the expression

$$\sigma_{uw} \approx u_*^2$$

and to the shear of the mean flow velocity as

$$\frac{\partial U}{\partial z} = \frac{u_*}{\kappa z}.$$  \hspace{1cm} (2.2)

The von Kármán constant ($\kappa$) is generally assumed to have a value of $\kappa = 0.408 \pm 0.004$ (Long et al., 1993). The integration of equation 2.2 yields the logarithmic velocity distribution, or the law of the wall

$$U(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right),$$

where $z_0$ is a constant of integration, typically denoted as the roughness length. The distribution is often simplified by turbine designers to a power-law form (Fraenkel, 2007).

Tidal turbines are most likely to be sited at an intermediate elevation between the near-bed and the free-surface regions (i.e. $0.15 < z/h \leq 0.6$), where the turbulence structure is not considered to be strongly dependent on either the seabed or the free-surface. Consequently, the turbulence in this region is expected to be near equilibrium (i.e. $P \approx \varepsilon$, where $P$ is the rate of turbulent production and $\varepsilon$ is the rate of dissipation), for which the friction velocity may also be obtained from the expression

$$u_*^3 = \varepsilon \kappa z.$$  \hspace{1cm} (2.4)

At a sufficiently high elevation above the bed ($0.6 < z/h \leq 1$) the channel depth and maximum free-stream velocity ($U_{max}$) are expected to become the dominant parameters characterising the turbulence structure. At these elevations, a wake function may be incorporated to account for deviations from the logarithmic law profile (see Townsend, 1976 and Nezu and Nakagawa, 1993). However, as the
maximum free-stream velocity can be related to the friction velocity through the law of the wall, Nakagawa et al. (1975) maintain the use of friction velocity as the characteristic velocity scale in the region. This emphasises the need to obtain the friction velocity for characterising the turbulence onset to tidal turbines.

### 2.2.2 Energy Cascade

Turbulence encompasses a wide range of scales. The physical size of the largest eddies which generally dominate the spectral energy, or variance can be characterised by the integral length-scales \( L_i \), where \( i = u, v, w \), as defined in Section 3.4.3). Inertial forces transfer the momentum from the large eddies to smaller scales in a cascade-like process within a wavenumber \( (k) \) band denoted the inertial subrange (Pope, 2000). Within the inertial sub-range the turbulence is expected to become increasingly isotropic. As such, its average properties do not depend on the position or the direction of the axis of reference (Batchelor, 1953).

Kolmogorov (1941) has shown that within the inertial subrange, the spectral energy \( S(k) \) of the turbulence is expected to conform to the universal expression

\[
S(k) = \alpha \varepsilon^{2/3} k^{-5/3},
\]

where \( \alpha \approx 1.5 \) is the three-dimensional Kolmogorov constant (Sreenivasan, 1995). As the turbulence structure tends towards isotropy in the subrange, the Reynolds shear stresses are expected to decay rapidly at the rate of \( k^{-7/3} \) (Wyngaard and Cote, 1972).

At high Reynolds numbers the inertial subrange may extend to one or more wavenumber decades, but conversely the region may not be identifiable at low Reynolds numbers (Nezu and Nakagawa, 1993). From a theoretical basis and assuming that the velocity profile conforms to the law of the wall, Tennekes and Lumley (1972) (see also Voulgaris and Trowbridge, 1998) estimate isotropy to be developed at non-dimensional wavenumbers exceeding, \( k_x z = 4.5 \), where \( k_x = 2\pi n/U \) and \( n \) is the frequency in Hertz.

The turbulent energy at wavenumbers higher than those corresponding to the subrange is finally dissipated by viscosity as heat (Pope, 2000). These scales are important for inferring the turbulent energy budget. However, they are not regarded by the author to significantly affect the tidal turbine loads. Therefore, only the energy encompassed by waveumbers up to the inception of the inertial
2.2. Fundamentals of Turbulence Theory

subrange are of primary interest for this study.

2.2.3 Homogeneous Isotropic Turbulence

The assumption that the turbulence is homogeneous and isotropic greatly facilitates a theoretical model of turbulence. This implies that the Reynolds shear stresses (which are regarded in this thesis as the co-variance velocity terms $\sigma_{ij}$, where $i \neq j$) are zero. An explicit ratio between the longitudinal to lateral (or vertical) spectral energy of $S_{vv} = (4/3) S_{uu}$ is also expected (Batchelor, 1953). Similarly, the length-scales can be defined solely by a knowledge of the streamwise or lateral scales as $L_x^v = 2L_x^u$. Therefore, given the inherent simplifications to the description of turbulence with isotropy, it is of interest to identify the degree of anisotropy present in a tidal stream.

With the assumption that the turbulence is homogeneous and isotropic, von Kármán proposed a spectral model which interpolates between the regions of energy production and the inertial subrange (which is described in ESDU, 1974). The spectrum is non-dimensionalised by the variance of the streamwise velocity ($\sigma^2_u$) and the integral length-scale of the streamwise velocity ($L_x^u$). The streamwise spectrum is defined as a function of frequency ($n$) by the expression

$$\frac{nS_{uu}}{\sigma^2_u} = \frac{4\eta_u}{(1 + 70.8\eta_u^2)^{5/6}},$$  (2.6)

where $\eta_u (= nL_x^u/U)$ is a non-dimensional frequency. At low frequencies, the normalised model produces an $n^1$ spectrum. At high frequencies above the inception of the inertial subrange, the spectrum conforms to the Kolmogorov (1941) ($n^{-5/3}$) description.

The attractiveness offered by the theoretical description of the von Kármán spectrum has resulted in it being frequently employed to synthesise a turbulent flow field for simulating the dynamic response of wind turbines and more recently, tidal turbines (Bossanyi, 2009). Connell (1982) and Burton et al. (2001) have also employed this model and the corresponding spatial correlations to develop a theoretical rotational spectrum for a point on a wind turbine blade. This was used to demonstrate that the energy peaks which occur at integers of the rotor frequency are sensitive to the integral length-scales common in the atmosphere. Given this finding, it is of interest to apply a similar technique to investigate the response for tidal turbines.
Chapter 2. Literature Review

2.3 Experimental Measurements of Turbulence

2.3.1 Pioneering Studies of Open Channel Turbulence

For open channel flows with width-to-depth ratios greater than 5, the flow may be considered to be approximately two-dimensional, with secondary currents being negligible (Nezu, 2005). The assumption that the flow is two-dimensional and the turbulence is in equilibrium has facilitated a range of empirically-based correlations for relevant turbulence parameters.

The correlations which are provided in the following sections were developed by Nezu and Nakagawa, and presented in a series of papers (see Nakagawa et al., 1975; Nezu, 1977a,b; Nezu, 2005) as well as in a widely cited monograph (Nezu and Nakagawa, 1993). They were based on an extensive series of laboratory flume experiments at Reynolds numbers of the order of $10^4$. These Reynolds numbers are several orders of magnitude lower than expected in a fast flowing tidal channel. However, as stated by Nezu and Nakagawa (1993), the correlations for the turbulence intensity are considered to be independent of both the Reynolds number and the Froude number ($Fr = U/\sqrt{gh}$). The correlation for the integral length-scale is also believed to be only a weak function of the Reynolds number. Therefore, it is useful to compare these correlations with the turbulence statistics acquired from an actual tidal channel.

Turbulence Intensities

For elevations of between $0.1 < z/h \leq 0.9$, the proposed universal expressions for the standard deviations of the velocity fluctuations are in the form of exponential functions. These are scaled by the friction velocity and the elevation above the bed. The normalised standard deviation of the streamwise velocity is expressed as

$$\frac{\sigma_u}{u_*} = 2.30 \exp\left(-\frac{z}{h}\right)$$  \hspace{1cm} (2.7)

and the ratios of the standard deviations of the transverse and vertical velocities to the standard deviation of the streamwise velocity are expressed respectively as

$$\frac{\sigma_v}{\sigma_u} = 0.71 \text{ and } \frac{\sigma_w}{\sigma_u} = 0.55.$$  \hspace{1cm} (2.8)

These ratios imply that the anisotropy is maintained throughout the water column. As discussed in Nezu (2005), due to the presence of the free surface,
2.3. Experimental Measurements of Turbulence

these results are in contrast to the anisotropy that is expected for flows in conduits, where \((\sigma_v \approx \sigma_w)\). This has significant implications for tidal turbines. For instance, the structure of turbulence at the rotor plane would not be expected to be isotropic at all scales, irrespective of the hub height. However, a constant value for the anisotropy also implies that if it is obtained at one point, it may also be able to be applied throughout the water column. As discussed later in Section 2.3.3, this assumption facilitates an estimate of the turbulence intensities from a four-beam Acoustic Doppler Current Profiler (ADCP).

As discussed previously in Section 1.3.1, industrial guidelines for tidal turbines currently rely substantially on models for atmospheric turbulence. Therefore, it is interesting to note that for mechanical turbulence in the atmosphere (i.e. when convection is neglected), the ratio of the standard deviation of the streamwise velocity to the friction velocity is assumed to be independent of elevation (Panofsky and Dutton, 1984). For flat homogeneous terrain, Panofsky and Dutton (1984) reported that the ratio \(\sigma_u/u_*\) range is between 2.2 to 2.5 and is greater for rolling terrain, where it can range between 2.7 and 4.5. Whilst for flat terrain the ratios would be nearly equivalent to those of equation 2.7 at \(z/h = 0.1\), the difference would be more significant at greater elevations from the bed.

The ratios of the standard deviations of the transverse and vertical velocities to the standard deviation of the streamwise velocity for the atmosphere have also been reported for flat terrain by Panofsky and Dutton (1984). These are \(\sigma_v/\sigma_u = 0.80\) and \(\sigma_w/\sigma_u = 0.52\) respectively, being 13% larger and 10% smaller, respectively than those predicted by equation 2.8. Cook (1985) has also presented ratios for atmospheric turbulence, which are \(\sigma_v/\sigma_u = 0.67\) and \(\sigma_w/\sigma_u = 0.45\). These are both smaller than the corresponding open-channel flow correlations. In light of these differences, the anisotropy relevant to atmospheric turbulence may be dubious for tidal turbines. This emphasises the need for experimental data in tidal channels for validation.

Reynolds Stresses

The Reynolds shear stresses are expected to exhibit a linear profile throughout the water column (Tennekes and Lumley, 1972; Townsend, 1976). As such, the ratio of the Reynolds stress to the total turbulent kinetic energy, \(q = 0.5(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)\) can be expressed as

\[
\frac{-\sigma_{uw}}{2q} \approx \frac{(1 - z/h)}{9.56 \exp \left(-2z/h\right)},
\]  
(2.9)
Chapter 2. Literature Review

The model predicts that within the near-bed region the ratio is approximately $-\sigma_{uw}/2q = 0.10$ and that in the intermediate region, it becomes a near constant value of 0.15. These correlations provide a useful means by which the Reynolds stresses and intensities can be related, independent of the friction velocity.

**Integral Scales**

The correlations that were developed for the streamwise integral length-scale were based on a relatively slow increase in the length-scale with elevation. For $z/h < 0.6$ these are expressed as

\[
\frac{L_u^x}{h} = B_1 \left( \frac{z}{h} \right)^{1/2},
\]

where $B_1$ is approximately equal to 1. A lack of data has meant that correlations for the integral length-scales of the lateral and vertical velocities have not been developed.

It should also be noted that these correlations were developed for smooth channels. Nezu and Nakagawa (1993) have shown that the effect of surface roughness on the intensities is also most pronounced near the bed i.e. $z/h < 0.3$. It was suggested that this was due to larger roughness elements having a greater ability to break down large eddies. This is associated with a decrease in the integral length-scale of turbulence and consequently lower streamwise intensities. Tidal streams may have a comparatively more irregular seabed than those of the experimental studies used to develop the correlations. This further emphasises the need for experimental data for validation, specifically if these correlations are to be used for predicting the turbulence onset to a turbine.

### 2.3.2 Early Studies of Turbulence in Tidal Flows

Compared to the atmospheric boundary layer, there are substantially fewer measurements available in the literature of turbulence in fast flowing tidal streams. Pioneering experimental studies from circa 1960s provided initial insights into the turbulence characteristics in the bottom boundary layer of tidal flows. Grant et al. (1962) utilised hot film and electrocurrent techniques to find strong support for the $k_{-5/3}$ model predicted by Kolmogorov (1941) at high wavenumbers. However, the acquisition of reliable data for the low wavenumber eddies proved difficult. Instead, the eddies were aptly described as being ‘so large that a ship
2.3. Experimental Measurements of Turbulence

Measurements of turbulence at Red Wharf Bay in the UK were reported by Bowden and Fairbairn (1956) and Bowden and Howe (1963), in which one of the first quantifications of the low frequency spectral range of turbulence near the seabed in a tidal current was provided. The mean flow was of the order of 0.50 m s\(^{-1}\). Whilst this is slower than typically necessary for tidal energy conversion, estimates of many parameters relevant to this thesis were nevertheless provided. These included the turbulence intensity, integral length-scales, Reynolds stresses and spatial correlations. In particular, the ratio of the vertical to streamwise length-scale was found to increase with elevation above the bed, implying that the vertical scale of the eddies was constricted by the presence of the bed.

Heathershaw (1979) provided additional measurements at Red Wharf Bay. Their study, together with that of Soulsby (1977) and Gross and Nowell (1985) also gave insights into spectral scaling laws. It was found that when the near-bed velocity wavenumber spectra were scaled using the distance from the bed \(z\), the spectra collapsed to theoretical forms expected for boundary layer turbulence (see Townsend, 1976).

Additional important insights into the spectral scaling laws were also provided by Lien and Sanford (2000), who presented velocity spectra from the constant stress region of an unstratified channel in the Pickering Passage in Washington, USA. The mean velocity was approximately \(U = 0.8\) m s\(^{-1}\). At relatively high wavenumbers, a \(k^{-5/3}_x\) spectral slope for the streamwise and vertical velocities, as well as a convergence to \(S_{ww}/S_{uu} = 4/3\) and a \(k^{-7/3}_x\) slope in the co-spectra were observed. These findings are all consistent with the existence of an inertial subrange for isotropic turbulence (see Section 2.2.2). It was also found that most of the variance of the momentum flux spectra (a proxy of anisotropy) was present around a peak corresponding to a normalised wavenumber of \(k_x z = 0.6\). Interestingly, the streamwise spectrum tended towards a \(k^{-5/3}_x\) spectral slope at non-dimensional wavenumbers of \(k_x z = 3\) to 4, which was slightly lower than that for the vertical spectrum. Similar characteristics have been observed in the atmospheric boundary layer (Panofsky and Dutton, 1984).

2.3.3 Applications of Acoustic Doppler Current Profilers

Advances in instrumentation such as those which have incorporated Acoustic Doppler techniques, have facilitated a greater understanding of the structure of
Chapter 2. Literature Review

turbulence in tidal channels.

Acoustic Doppler Current Profilers (ADCPs) were introduced in the 1980s, and a brief historical account of their development is presented by Simpson et al. (2005). ADCPs utilise the Doppler shift principle to compute the velocity from a signal which is transmitted, reflected from plankton and then received along a beam that penetrates through the water column. The beams of an ADCP are generally inclined at an angle of 20° or 30° from the vertical. The measurements are acquired at vertical intervals (bins) from the transducer of between 0.1 m and 1 m and sampled at rates of up to 2 Hz (Rusello, 2009).

ADCPs were originally intended for acquiring the profile of the mean flow as an average over the beam spread. For a 60 m deep channel at mid-depth, the beam spread is approximately equal to a rotor diameter. For fast and highly turbulent flows, the turbulence is not expected to be homogeneous across the beam spread. As such, the three-dimensional instantaneous velocities cannot be reliably obtained from a four-beam ADCP (Osahusi et al., 2009).

However, by assuming that the higher-order statistical moments of velocity are homogeneous over the beam spread, researchers have been able to acquire estimates of the Reynolds stresses and the turbulent kinetic energy (Lohrmann et al., 1990). This is commonly referred to as the variance technique and can be applied if the variances are dominated by eddies that are at least as large as the size of the ADCP bin. It is also dependent on the Doppler noise, which is more pronounced for smaller bin sizes (Simpson et al., 2005).

Stacey et al. (1999) applied this variance method to a data set collected from an ADCP penetrating downwards from water surface at Three Mile Slough in the USA. This is a relatively narrow and straight unstratified tidal flow with very little wave activity and a flow speed of approximately 0.8 m s⁻¹. The technique was used to demonstrate that reliable estimates of the Reynolds stresses, friction velocity and the turbulent kinetic energy could be acquired. However, the turbulent kinetic energy were obtained by assuming the intensity ratios proposed by Nezu and Nakagawa (1993). Errors due to bias and spread in the Reynolds stresses and turbulent kinetic energy were also identified, and which were found to be aligned with those predicted by the sensor manufacturer. Therefore, the study provided confidence in the use of the variance method, though for relatively slow flows.

Lu and Lueck (1999a) also employed the variance method to data collected from an upwards profiling ADCP that was deployed on the seabed in the Cordova
2.3. Experimental Measurements of Turbulence

Channel, USA. The channel was approximately 30 m deep and the mean velocity was approximately 1 m s\(^{-1}\). Reynolds stresses were estimated, and in order to quantify the turbulent kinetic energy, a parameter \( S = \gamma q \) (where \( \gamma \) is a function of the flow anisotropy and beam angle) was introduced. The estimates of \( \gamma \) again incorporated the intensity ratios proposed by Nezu and Nakagawa (1993).

Spectral analyses of \( S \) plotted as a function of wavenumber, indicated a shift in the peak towards smaller wavenumbers with elevation from the bed. This is consistent with the hypothesis that the size of the energy-producing eddies near the bed is dependent on elevation. In the study, the uncertainties associated with the variance method were further explored, and in particular the importance of acquiring measurements from a rigidly mounted ADCP was reinforced.

Osalusi (2010) (see also Osalusi et al., 2009) provided one of the first ADCP-based studies that had the specific objective of obtaining turbulence properties to improve the understanding of the transient performance of tidal turbines. Measurements were acquired from a bottom-mounted ADCP at the Fall of Warness within EMEC. The water depth was 46 m and the mean flow was approximately 2 m s\(^{-1}\). Once again, the variance method was employed to estimate the Reynolds stresses, the variance \( S \) and the friction velocity. The profiles for the flood flows showed that the Reynolds stresses decreased relatively linearly with elevation and were consistent with 2-D channels. However, the profiles for ebb flows were more complex and indicated that there were regions of relatively high stress near the surface and at \( z/h = 0.1 \). Whilst the profile of the variance \( S \) decreased with elevation, it revealed that regions of high stress were associated with relatively high turbulence intensity. The integral-scales could have provided useful information on the size of the dominant scales related to the high intensities and Reynolds stresses, but these were not reported in their study.

Li et al. (2010) also provided estimates of the turbulence intensity measured from an ADCP deployed at the East River, New York. This corresponded to the site of the proposed tidal turbine deployment by Verdant Power. The mean velocity was approximately 2.5 m s\(^{-1}\) and the water depth was comparatively shallow at 9 m. Relatively high turbulence intensities of up to \( I_u = 0.35 \) were reported. The intensity is up to 3.5 times greater than the intensity at the Fall of Warness, which can be derived by applying the intensity ratios of Nezu and Nakagawa (1993) to the measurements of \( S \) presented by Osalusi (2010). These high intensities could be due to the estimates having neglected the non-homogeneity across the beam spread. Furthermore, the East River is subjected
to stratification (Blumberg and Pritchard, 1997), which could imply that the turbulence they observed was not driven solely by the boundary layer on the seabed.

2.3.4 Applications of Acoustic Doppler Velocimeters

Acoustic Doppler Velocimeters (ADVs) were introduced in the 1990s (see Lohrmann et al., 1994). Similar to ADCPs, they utilise the Doppler principle to estimate the velocities. However, the three-dimensional point measurements they can provide are arguably more suitable for analysing the turbulence at the scales relevant to tidal turbines than the measurements from an ADCP. The sampling rates are also typically higher than those of ADCPs and therefore the turbulence can be estimated for higher wavenumbers. However, this is dependent on the Doppler noise, which can limit the maximum wavenumber and is more pronounced for the horizontal components (Voulgaris and Trowbridge, 1998).

There are several studies which have utilised ADVs to acquire measurements of variances, Reynolds shear stresses and turbulence spectra in natural tidal streams of less than 1 m s$^{-1}$ (see, e.g. Kim et al., 2000, Sherwood et al., 2006, Trevethan, 2007 and Walter et al., 2011). Furthermore, the studies of Kim et al. (2000) and Sherwood et al. (2006) demonstrated how a direct measurement of the Reynolds stress as well as the ability to resolve the inertial sub-range can provide estimates of the friction velocity. These provide a further independent check of the friction velocity aside from using a log-law fit to the velocity profile or the variance method to compute the Reynolds stresses from an ADCP dataset.

One contributing factor to the scarcity of ADV based studies in fast flowing tidal streams is the difficulty in obtaining accurate measurements. This is due to the sensor movement and vibration which can compromise data quality. The engineering technicalities involved in installing these sensors in fast flowing tidal streams restricts their use for collecting data within the near-bed region and up to elevations much lower than typical turbine hub heights.

The study by Thomson et al. (2012) is of particular interest as data were collected in a relatively fast flow using an ADV, which sampled at 32 Hz and which was positioned at an elevation of 4.7 m. The measurements were made at the Puget Sound, Washington where the water depth was approximately 22 m and the mean velocity at the sensor was between 1 to 1.5 m s$^{-1}$. The streamwise turbulence intensity at $z/h = 0.21$ was found to be $I_u = 0.084$. Whilst the intensities
in the transverse and vertical directions were not reported, the frequency spectra of the streamwise and vertical velocities showed that the streamwise energy was significantly more dominant at frequencies $n < 0.1$ Hz compared to the vertical energy. This is consistent with the vertical energy having been suppressed by the seabed.

The streamwise turbulence intensity was found to comprise energy that was distributed across a broad range of scales. The peak scale corresponded to approximately three times the water-depth. These large scales were attributed to eddies which were shed from headlands near the deployment site. As these length-scales may have significant implications for the unsteady loads on tidal turbines, there is a need for further data from other sites to evaluate how site dependent they could be.

At high frequencies, the spectra presented by Thomson et al. (2012) also showed that an approximate $n^{-5/3}$ slope was present between $0.2 < n \leq 2$ Hz. This supports the development of an inertial subrange at these frequencies. As the spectra were averaged across all non-slack flow velocities in the frequency domain, the wavenumber that corresponded to the inception of the inertial subrange is somewhat difficult to ascertain. However, based on the reported mean velocities, a frequency of 0.2 Hz would imply that the wavenumber for which the $k_z^{-5/3}$ slope was first identified was between $k_{xz} = 3.9$ to 5.9. This concurs with the theoretical estimates of Tennekes and Lumley (1972) (see Section 2.2.2) as well as the experimental observations of Lien and Sanford (2000) and provides a useful metric for comparisons of the turbulence characteristics measured at the Sound of Islay for this thesis.

## 2.4 Unsteady Hydrodynamic Loading of Rotors

The present understanding of the unsteady hydrodynamic loading of tidal turbines has been mostly generated from investigations of helicopter rotors and more recently, wind turbines. Extensive coverage of the theories and models have been provided by Leishman (2002, 2006) and by Peters and co-workers (e.g. Gaonkar and Peters, 1988, Peters et al., 1989, Peters et al., 2007 and Peters, 2009). The fundamental concepts and their applicability to horizontal-axis tidal turbines are discussed in the following sections.
2.4.1 Attached Flow

The unsteady hydrodynamic loads of a rotor that is subjected to an axial flow perturbation include both non-circulatory and circulatory components. The non-circulatory loads arise due to the pressure forces which are required to accelerate the fluid in the vicinity of the blade (Leishman, 2002). This forcing is independent of the presence of the rotor wake and acts 180-degrees out-of-phase with acceleration. This implies that the non-circulatory force gives rise to a phase-lead in the loading over the velocity and is perceived as a positive added mass.

For a tidal turbine, the magnitude of the non-circulatory loading has previously been estimated by integrating the individual contributions from a series of cylindrical sections along the blade span and incorporating standard added mass coefficients (Whelan, 2010). Sarpkaya and Isaacson (1981) have provided the hydrodynamic added mass for a cylindrical element from potential flow theory as $m_a = \frac{0.25\rho\pi c^2}{2}$. For a turbine blade, $c$ is taken to be the local chord length.

The unsteady circulatory forces are attributed to the shedding of vorticity into the wake. For a rotor subjected to relatively low frequency perturbations with respect to the rotational frequency of the rotor, the circulatory loading is commonly associated with a dynamic inflow effect (provided that the boundary layer on the foil remains attached). This phenomenon arises from the induced flow in the trailed wake taking time to reach a new equilibrium state following a change in loading at the rotor plane. As discussed by Peters (2009), these effects were first observed in the 1950s for helicopter rotors that were subjected to changes in pitch angles. An experimental campaign for wind turbines concluded that the unsteady inflow effects were more significant for variations in pitch compared to variations in axial velocity (Snel, 1995). However, Whelan (2010) argued that the effects of an axial velocity perturbation may be more significant for tidal turbines. This is owing to the ratio of the fluid to structural density being much closer to 1 than for helicopter rotors or wind turbines. Therefore, there is a need to quantify its contribution.

As discussed by Leishman (2002), there are a variety of techniques in which the unsteady circulatory forcing can be modelled, each varying in complexity up to a full CFD-based analysis. An objective of this thesis is to establish whether unsteady loading can be qualitatively modelled using relatively simple approaches. These were identified as including a two-dimensional dynamic inflow theory and a classical unsteady aerofoil theory based on a lift-deficiency function. These two
approaches are discussed further in the following sections.

Dynamic Inflow Models

Dynamic inflow models are fundamentally based on the concept of accounting for the lag in induced inflow in the wake by incorporating an apparent mass. A historical account of the development of dynamic inflow models has been presented by Peters (2009). Their original inception is attributed to the work of Carpenter and Friedovich (1953), who found that the added mass of an impermeable disk correlated well with the delay in the development of the collective inflow mode (i.e. all blades perturbed in phase with each other). This mode is analogous to a planar axial perturbation of a rotor.

In the 1970s, Peters (as reported by Peters, 2009) incorporated these apparent mass terms into the momentum theory for an actuator disk and developed a set of dynamic-wake equations for a hovering helicopter rotor. For a perturbation in the thrust acting on the rotor plane, this model was expressed as

\[
\frac{8}{(3\pi)} \frac{d\nu_i}{d\tau} + 2V\nu_i = C_T. \tag{2.11}
\]

where \( \nu_i \) is the normalised induced velocity perturbation, \( \tau \) is a normalised time, \( V \) is a mass flow parameter (originally assumed to be equal to the normalised induced velocity, \( \nu_i/\Omega R \)) and \( C_T = F_x/0.5\rho\pi R^2(\Omega R)^2 \) is the rotor thrust normalised by the rotor speed.

The model may be extended to account for a rotor extracting momentum from the free-stream (analogous to a tidal turbine) by assuming that the mass flow parameter \( V = (U - 2\pi\tau)/(\Omega R) \). A time constant for the decay can also be obtained as \( \tau_{\text{decay}} = 0.424/V \) (Leishman, 2002). Based on the full-scale rotor specifications in Table 1.1 and assuming an optimal inflow of \( \nu_i = 1/3U \) (which gives \( V \approx 0.07 \), this would equate to a decay of \( \tau_{\text{decay}} \approx 6s \). This implies that the effect can persist for several rotor revolutions before decaying.

Dynamic inflow models have proved popular due to their relatively high computational efficiency and fundamentally, their ability to separate the induced flow (momentum) theory from the aerofoil theory. This means that they do not depend on the method by which the forces at the blade-elements are computed. Dynamic inflow models also form the basis of commercial simulation codes for tidal turbines such as GH-Tidal Bladed (Bossanyi, 2009). It is important to note
Chapter 2. Literature Review

that within such codes the non-linear version of dynamic inflow presented by Peters and HaQuang (1988) is applied in terms of total inflow and loading. Furthermore, an allowance is made to account for a non-zero rotor skew angle based on the work of Pitt and Peters (1981). However, as only axial perturbations are considered in this thesis, accounting for the skew angle was not necessary.

As Leishman (2002) has emphasised, the use of a non-circulatory mass term to model a circulatory effect is not physically representative of the underlying dynamics. It is also important to note other shortcomings of the model. For instance, as the model is based on a thrust coefficient, the pressure distribution across the blade is not incorporated and tip-loss is not accounted for. Some of these limitations have been addressed as the model has evolved over time to incorporate more flow states and modes (Peters and He, 1995). As these adaptations have not been rigorously verified, they are deemed to be out-of-scope of this research.

Classical Vortex Based Models

Whilst dynamic inflow theory is applied in the time domain, a model for the circulatory response of an oscillating two-dimensional thin aerofoil was presented in the frequency domain by Theodorsen (1935). The model was based on vortex theory and assumed that the trailed wake remained planar and that the forcing was of small amplitude. Theodorsen’s model is applied as a complex lift-deficiency function, which introduces a time delay to the quasi-steady loads.

In an attempt to account for the helical form of rotor wake, Loewy (1957) proposed an extended model which modified the lift deficiency function. The model was developed for a rotor in hover and represented the wake as a series of returning sheets of vorticity below the foil. Depending on the spacing of the wakes, the induced flow could amplify or attenuate the total induced flow. Peters et al. (1989) have demonstrated that the Loewy (1957) model is also fundamentally more appropriate for a rotor than Theodorsen (1935) theory at low frequencies, such as those relevant for tidal turbines.

However, it is important to note that Peters et al. (1989) have also shown that the original Loewy (1957) model exhibits a singularity in the zeroth harmonic (analogous to thrust) at these important frequencies. This gives a physically unrealistic infinite apparent mass at zero frequency (quasi-steady flow). This shortcoming was not acknowledged by Whelan (2010) who applied the theory in
its original form to estimate the magnitude of the returning wake contribution for tidal turbines and as such, this warrants further investigation. A more realistic estimate of the returning wake effect could potentially be obtained by incorporating the correction proposed by Peters et al. (1989). This gives more appropriate time constants and gains at low frequency which are in-line with dynamic inflow theory.

### 2.4.2 Separated Flow

For a turbine operating at or near maximum power, oscillations can induce additional unsteady circulatory phenomena. These arise from a delay in the onset of boundary-layer separation on the pressure surface of the foil, and at sufficiently high angles of attack, the formation and the shedding of a vortex from the leading edge. Much of the current understanding of delayed separation and dynamic stall has been derived from experimental tests of two-dimensional aerofoils of relatively thin sections which separate from the leading edge. Comprehensive reviews of such studies have been provided by McCroskey et al. (1976) and Carr (1988).

Experimental data for relatively thick foils, such as those used on tidal turbines and which are dominated by trailing edge separation, are comparatively more limited. However, Janiszewska et al. (1996) investigated the response of the National Renewable Energy Laboratory (NREL) S814 foil to oscillations in pitch angle. This foil has a 24% thickness with respect to the chord and was used in the turbine experiments in this thesis. Examples of the two-dimensional lift coefficient measured by Janiszewska et al. (1996) in response to a \( \alpha = 5 \pm 5.5^\circ \) sinusoidal oscillation at a relatively high Reynolds number of \( 0.7 \times 10^6 \) are shown in Figure 2.2. These serve to demonstrate the general characteristics of dynamic stall in that an increase in the reduced frequency delays the separation and reattachment to higher angles of attack. Furthermore, the maximum lift-coefficient is significantly higher (in this case up to 20%) compared with steady flow.

However, there are no data available for the S814 foil oscillating at lower Reynolds numbers corresponding to a typical model-scale rotor (see Section 2.6), which makes drawing quantitative comparisons difficult. The majority of the experimental studies reported in the literature have also only considered the response of foils to a change in pitch angle. In contrast, the forcing applied to a tidal turbine due to turbulence is more analogous to an oscillatory plunge (or heave). Importantly, an oscillation in pitch can have a lower leading edge pres-
Chapter 2. Literature Review

Figure 2.2: The two-dimensional lift coefficient of the S814 foil for a $\alpha = 5 \pm 5.5^\circ$ sinusoidal oscillation in the pitch angle at the reduced frequencies of $k = 0.04$ (○) and $k = 0.07$ (□), presented by Janiszewska et al. (1996). The two-dimensional lift coefficient for steady flow presented by Somers (1997) is also shown for comparison, (●). All data correspond to a Reynolds number of $0.7 \times 10^6$.

sure gradient compared to an oscillation in plunge and as such the separation is instigated at higher angles of attack (Leishman, 2006).

Delayed separation and stall of a rotor is inherently more complex than for the idealised two-dimensional flow situations described above. Each section experiences a different reduced frequency and mean angle of attack. There appears to be no literature which has focused specifically on the boundary layer separation and dynamic stall for tidal turbines. Useful insights into the effect of three-dimensionality have been provided from studies on wind turbines. For instance, Barnsley and Wellicome (1992) identified that the effect of rotation and three-dimensionality was to suppress the leading edge suction peaks on a blade of a 1 m diameter wind turbine. This effectively slowed the advancement of the separation point towards the leading edge. The implications of the greater delay is that there will be an increase in the lift and an increase in the magnitude of the loading. However, the investigation by Barnsley and Wellicome (1992) was restricted to steady flow. Studies that have experimentally characterised the delay in separation and dynamic stall of wind turbine blades in unsteady flow conditions have also been seldom reported in literature.
As the unsteady effects associated with delayed separation and dynamic stall occur at the blade section level, they generally have a characteristic time-scale which is an order of magnitude shorter than that of dynamic inflow. Consequentially, the two effects are typically modelled independently. The Beddoes-Leishman model (Leishman and Beddoes, 1989) has arguably proved to be the most popular method for predicting the delayed separation and the dynamic stall of foils. The model was originally devised for thin helicopter blades and is based on a series of modules for attached flow, the delay in the boundary-layer separation, and the initiation and shedding of the leading edge vortex. A set of time constants are used to account for the delays in both the boundary layer separation and vortex shedding processes.Whilst the model is semi-empirical due to these time constants, the popularity of the model has resulted in its application to predict the unsteady loads of rotors.

In an attempt to model the unsteady delayed separation and delayed stall of a wind turbine, Pierce (1996) modified the original Beddoes-Leishman model by removing the need to account for compressibility and to incorporate user defined aerofoil data. However, there are a number of other aspects of the model which require consideration for its application to a rotor. For instance for a plunging foil, the attached flow module is based on the planar wake model developed by Theodorsen (1935) for thin aerofoils. As discussed in Section 2.4.1, this theory is not considered to be applicable to rotors. The trailing edge separation module is also based on a flat-plate assumption, which may be dubious for thick foils. Furthermore, no explicit allowance is made for three-dimensional effects which as discussed above, can act to further delay the separation. Therefore, these issues also reinforce the need for experimental data on dynamic stall events of tidal turbine blades for validation purposes.

2.5 Experimental Investigations of Tidal Turbine Loading

2.5.1 Steady Flow

The steady loads of a 800 mm diameter, tri-bladed tidal turbine were reported by Bahaj et al. (2007b). They characterised the performance and loads from a series of tests in both a cavitation tunnel and a towing tank. At tip-speed ra-
tios of approximately $\lambda = \Omega R/U = 6$, which corresponded to a maximum power coefficient of $C_p = P/0.5\rho AU^3 = 0.46$, the thrust coefficient was approximately $C_T = F_x/0.5\rho AU^2 = 0.8$. However, more importantly, the thrust coefficient was highly sensitive to the pitch angle, with its peak value decreasing by approximately 30% for a 5° increase in the pitch angle at the equivalent flow speed.

The results of Bahaj et al. (2007b) provide a basis from which to compare the quasi-steady loads of a model-scale rotor against. However, the study also emphasises the importance of establishing the steady loads at the equivalent set-up and flow speeds as used in an unsteady test, such that the unsteady hydrodynamic contribution can be accurately isolated from the steady loading.

The study of Bahaj et al. (2007a) found that the general characteristics of the experimentally measured loads for steady flow presented by Bahaj et al. (2007b) could be predicted using blade element-momentum (BEM) theory, which is commonly employed within industrial software codes (e.g. GH-Tidal Bladed). The findings demonstrate that BEM could likely serve as a means of estimating important parameters which could not be measured experimentally. These parameters include the angles of attack and induced velocity, which can be used to characterise the flow over the rotor blades and the inflow through the rotor.

However, the investigations of Bahaj et al. (2007a) also found that the model tended to under-predict the observed thrust by at least 5%, with the disparity increasing with an increase in the tip-speed ratio. These differences may have arisen from the semi-empirical corrections that were employed within the BEM model to account for the tip-losses and the turbulent wake state, as well as the quality of the aerofoil data. The effect of the free surface which was not included within the model may have also contributed to the errors (see Whelan et al., 2009). Therefore, the study by Bahaj et al. (2007a) also serves to show that the uncertainties inherent in a BEM model are likely to be too large to allow for it to be applied in this thesis to isolate and quantify the exact quasi-steady flow contribution from the experimental measurements of the dynamic loads, particularly at model-scale. This reiterates the need to acquire these steady loads experimentally.

### 2.5.2 Unsteady Flow

A variety of techniques have been employed by investigators to study the unsteady loading of horizontal-axis tidal turbines. Maganga et al. (2010) acquired
measurements from the tower structure of a 700 mm diameter (approximately 1/30th scale) tidal-turbine in a water flume. At typical operating states, they observed that the mean thrust loads increased by approximately 15% when the turbulence intensity in the flow was increased from 8% to 25%. However, the spectral characteristics of the turbulence and the dominant length-scales were not reported. Therefore, it is difficult to relate these results to a full-scale turbine.

Galloway et al. (2010) reported on experiments of a 800 m diameter model tidal turbine towed at a constant velocity in a still-water tank, where the unsteady forcing was instead imparted from oncoming single-frequency surface waves. These tests were conducted for an intrinsic wave frequency of 0.75 Hz, wave height of 0.8 m (approximately 1.6 m height full-scale) and with a carriage speed of 0.9 m s$^{-1}$. They observed that this forcing induced a cyclic shaft thrust that had a range of 37% of the mean. Whilst their results provided a quantification of the unsteady thrust perturbation, inferring the loading on the individual blades is difficult. This is because the non-coherent loading imparted by the surface waves was effectively averaged out across all three blades. Furthermore, it could be expected that there were losses through the shaft.

Barltrop et al. (2006) also conducted experiments with a 350 mm diameter tidal turbine in a towing tank subjected to surface waves, but provided measurements of the blade-root bending moments. Time histories were presented for intrinsic wave frequencies between 0.5 Hz and 1.0 Hz, at a wave height 150 mm and a carriage speed of 1.0 m s$^{-1}$. At a rotational speed of 200 rpm, this forcing corresponds to reduced frequencies of between $k = 0.020$ and 0.048 at the spanwise location of 0.75$R$. For the highest frequency case, the dynamic load amplitudes were found to be 50% and 100% of the mean in the out-of-plane and in-plane directions, respectively. For the low frequency surface waves, the bending moment response was found to compare reasonably well with a prediction using a quasi-steady numerical BEM model with no acceleration effects included. However, for the higher frequency cases the amplitudes were of the order of twice that of the model predictions. This implies that the unsteady hydrodynamic contribution for the low frequency cases was relatively small but became more significant as the frequency was increased.

The study by Whelan (2010) involved perturbing a 300 mm diameter twin-bladed rotor using a towing carriage. These were conducted in a flume with a steady flow and the axial thrust was inferred from the force applied to the rotor support structure. It was the first investigation that attempted to establish the
Chapter 2. Literature Review

relative contribution of the forcing components in-phase and out-of-phase with velocity of a tidal turbine. As the quasi-steady loading would also appear in-phase with velocity and was not removed, inferring the unsteady hydrodynamic contribution from these results is difficult.

Whilst the study conducted by Whelan (2010) showed promise, as for the previous experimental studies cited, separated flow cases and multi-frequency forcing were not specifically investigated. Furthermore, the experimental set-up used by Whelan (2010) was subject to not only the complications of variable speed control and high blockage, but also the presence of a low frequency wave and which restricted the maximum oscillatory frequency that was able to be investigated to \( f = 0.1 \text{Hz} \). This corresponded to a reduced frequency limit of approximately \( k = \pi fc/\Omega r = 0.02 \) at the radial location of \( r = 0.75R \). As is discussed in Chapter 4, based on the specifications of the full-scale turbines listed in Table 1.1, these reduced frequencies are relatively low and are considered to be representative of large eddies. Therefore, it is possible that the unsteady effects were inherently small at these relatively low frequencies.

2.5.3 Scaling Laws

Inferring useful insights into the unsteady loading of tidal turbines from model-scale tests requires that both the operational state of the rotor and the unsteadiness are scaled appropriately. The tip-speed ratio is a fundamental non-dimensional parameter for a rotor and its replication implies that the angles of attack are equivalent between full and model-scale. However, whilst tip-speed ratio similarity can generally be achieved, it is much more technically challenging to replicate the Reynolds number of the foil sections. For the aforementioned experimental studies, the Reynolds number at the outer-blade sections was generally an order of magnitude smaller than is expected at full-scale. As is discussed further in Section 2.6, this can affect the lift and drag of the foil and influence the performance of the rotor.

There is no definitive approach for scaling the unsteadiness of a rotor between full-scale and model-scale and a compromise between several applicable laws is generally required. For single frequency forcing, the reduced frequency is one means of scaling the oscillatory frequency, particularly the unsteadiness at the blade-section level.

However, with respect to the model of Loewy (1957), it may also be appropri-
2.6. Performance of Foils at Low Reynolds Numbers

To replicate the wake-spacing. For a typical rotor in steady flow, the average wake spacing may be expected to be approximated by

\[
\frac{h}{Bc} = \frac{2\pi U (1 - a)}{\Omega Bc} = \frac{(1 - a)}{\lambda_r \sigma},
\]

(2.12)

where \( \sigma = \frac{Bc}{2\pi r} \) is the local solidity, \( \lambda_r \) is the ratio of the free-stream velocity to the rotational speed at the blade-section, \( a \) is the axial induction factor and \( B \) is the number of blades. Therefore, replicating the wake-spacing is possible provided that both \( \lambda_r \) and \( \sigma \) are equivalent between full-scale and model-scale.

Whelan (2010) also identified the Current number and the Keulegan-Carpenter number as a means of scaling the oscillatory forcing. The Current number is defined as the ratio of the maximum velocity perturbation amplitude to the steady free-stream velocity, \( \mu = \tilde{u}/U \). This can be generally interpreted as a metric associated with the turbulence intensity and it is straightforward to replicate this at model-scale using a towing carriage. The Keulegan-Carpenter number is defined as \( KC = \tilde{u}T/D \), where \( T \) is the oscillatory period. It is representative of the distance in terms of the rotor diameter that a fluid particle would be perturbed by in a steady flow. For the experiments of Whelan (2010), no relationship between the \( KC \) number and the velocity and inertia coefficients was definable, despite full-scale values of the \( KC \) number being replicated. However, it is important to consider that this may be a reflection of the relatively low frequencies that were tested. Therefore, it may be possible that trends would have been observed for the same \( KC \) number had the oscillatory frequency been greater and the velocity perturbation smaller. This suggests that the \( KC \) number may not be the most appropriate scaling parameter for characterising the blade loading.

2.6 Performance of Foils at Low Reynolds Numbers

As cited in the previous section, the relatively low Reynolds number at the blade section is an important consideration for model-scale testing of tidal turbines. At low Reynolds numbers, laminar separation bubbles can be present over a substantial portion of the chord on both the pressure and suction surfaces of the foil (Shyy et al., 2008). These bubbles can give rise to significantly degraded performance relative to that expected for Reynolds numbers of the order of \( 1 \times 10^6 \).
It is important to distinguish these bubbles from short separation bubbles which can represent the laminar-turbulence mechanism at high Reynolds numbers. These are typically located near the leading edge of the foil. Their lengths are generally less than 3% of the chord and they do not significantly influence the aerofoil performance (Lissaman, 1983).

As Lissaman (1983), Shyy et al. (2008) and Genç et al. (2012) have discussed, the larger laminar separation bubbles are a consequence of a laminar boundary layer being comparatively more susceptible to the pressure gradient than a turbulent boundary layer. This results in a separation of the boundary layer prior to natural transition. For Reynolds numbers exceeding approximately $7 \times 10^4$, the energetic flow can subsequently overcome an adverse pressure gradient leading to reattachment as a turbulent boundary layer and the formation of a bubble.

The general characteristics of a laminar separation bubble are depicted by the schematic diagram shown in Figure 2.3. This serves to demonstrate that during the transition the velocity of the flow is approximately constant across the extent of the bubble, which gives rise to a constant pressure region or ‘plateau’. Depending on the extent of the bubble it can effectively modify the aerofoil shape (Lissaman, 1983) and influence the downstream turbulent boundary layer development (Van Ingen and Boermans, 1986). It is this action which can reduce the hydrodynamic lift and increase the drag, as well as lead to hysteresis if the bubble bursts.

Given these characteristics, it is important to ascertain the performance of the individual foil used in the experiments. The 24% thick NREL S814 section which was employed in the turbine experiments in this thesis is from a family of foils which were designed for extensive laminar flow (Somers, 1997). Two-dimensional wind tunnel measurements of the pressure distribution for this foil reported by Somers (1997), showed that at the Reynolds numbers of $1.5 \times 10^6$ the laminar separation bubbles were relatively small. These were observed at angles of attack of up to approximately $\alpha = 7^\circ$. These neither degraded the aerodynamic performance significantly, nor interacted with the process of stall which was from trailing-edge separation. However, of most concern is that no data for the S814 foil at Reynolds numbers of less than $0.7 \times 10^6$ have been published in the literature.

Low Reynolds number data for laminar flow foils are generally very limited. This makes inferring the significance of laminar separation bubbles and any other low Reynolds number effects very difficult. However, Selig et al. (1995) have pro-
Figure 2.3: Schematic diagram of the characteristics of a laminar separation bubble on an aerofoil and the corresponding pressure distribution, as depicted by Roberts (1980).

provided lift and drag data for the 21% thick NREL S823 foil which arguably is geometrically similar to the S814 foil. At a Reynolds number of $1 \times 10^5$, the lift was observed to be degraded by approximately 10% relative to the measurements for $\text{Re} \geq 2 \times 10^5$. The drag coefficients at this low Reynolds number were also at least an order of magnitude greater than those expected at full-scale. Interestingly, the data also showed that at relatively low angles of attack the lift tended to remain relatively high and the drag increased substantially. These characteristics suggest that other phenomena were also present at the lowest Reynolds numbers. However, no pressure distributions were reported for the NREL S823 foil at these Reynolds numbers. Such data would allow for the role of the laminar separation bubbles and their potential effect on the dynamics of flow separation to be established more confidently.

2.7 Justification for the Experimental Methodologies Employed in this Research

This literature review has demonstrated that there is a need for more measurements of turbulence in tidal flows of the order of $2\text{m}\text{s}^{-1}$. This is in order to
establish the magnitudes, anisotropy and scales of turbulence at tidal energy sites with more confidence.

Data from an ADV was sought as it was considered to provide the most appropriate means of acquiring direct estimates of the relevant turbulence parameters and with sufficiently low uncertainties for the purposes of this research. The data were selected on the basis that the sampling frequency needed to be sufficiently high, as to enable the turbulence spectrum to be resolved at the eddy producing scales and up to those which correspond to the low-wavenumber end of the inertial sub-range. However, it was acknowledged that ADV data would only be available for elevations relatively close to the bed.

Measurements further from the bed were sought from an ADCP which profiled throughout the water column. A dataset was selected on the basis that the ADCP needed to have been mounted rigidly at the seabed and data acquired in a coordinate system aligned with the transducer beams. This would provide the most appropriate means of estimating the Reynolds stresses and turbulence intensities from the variance method. Furthermore, data from an ADCP with bin sizes of the order of 1 m were considered to be most desirable. This was a compromise between maximising the wavenumber at which the energy could be resolved and ensuring that the bin size was not too small so that the Doppler noise would not significantly affect the quality of the data.

In terms of characterising the unsteady hydrodynamic loads on a turbine blade, the review has demonstrated that there is a need to develop a methodology which enables the unsteady hydrodynamic contribution of the loading to be quantified. Additionally, the set-up should allow for both the attached and separated flow conditions to be studied. Separated flow in particular is expected to be associated with the largest load amplitudes. The ability to examine the response to a more general forcing, consisting of multiple frequencies is also considered to be highly beneficial. This is in order to evaluate the suitability of simple, stochastic models.

Testing turbines in surface waves involves a highly complex flow varying over the rotor plane, which may be compromised by free-surface effects and limited by the achievable wave height. Acquiring measurements of relatively small phase differences (which may be expected for attached flow) from measurements of the free surface is also technically difficult. In order to achieve an appropriate and controllable unsteady flow which is uniform over the rotor plane, it appears to be most beneficial to oscillate the turbine axially in the flow. This is because
the forcing kinematics are known exactly and the phases can be estimated with greater confidence. Whilst there is no Froude-Krylov force (which is induced by the pressure field from the surface wave), for a foil section with relatively low volume, the effect is expected to be small. However, if oscillatory motion is combined with a mean velocity generated by the current in a flume, boundary layer effects and background turbulence can lead to non-uniformities in the flow unless the flume is extremely large and well-conditioned.

The most promising approach for generating an unsteady flow with the appropriate level of complexity is deemed to be by superimposing an unsteady surging motion upon a steady forward speed in calm water in a towing tank. This approach is adopted in this thesis. Inducing a fully coherent loading over the rotor plane is an idealisation of the flow in nature. However, it is an attempt to reduce an already complex situation, whilst providing useful data to study the underlying hydrodynamics as well as to validate models. Furthermore, as is identified in Chapter 4, the effect of rotational sampling of turbulence may be expected to impart a loading that is relatively coherent across the outer-section of the blade. These blade sections dominate the out-of-plane bending moment response. Measuring the blade loads directly is also considered to be more favourable than the rotor thrust, as losses from components such as the shaft do not need to be accounted for.

As has been identified in Section 2.6, in order to better understand the implications of conducting turbine experiments at model scale, there is a need for data characterising the performance of thick foils at low Reynolds numbers. Measurements of the two-dimensional lift and drag at a range of angles of attacks are most critical. This is because the data is required for numerical models such as BEM, dynamic inflow and dynamic stall. However, acquiring pressure distributions across the chord of the foil is also highly desirable for identifying the role of laminar separation bubbles and flow separation. Therefore, measurements of both the force and pressure distributions were obtained in this research. As the characteristics may be specific to the individual foil, data was also obtained for the S814 foil which was used in the turbine experiments. Whilst there were limitations with the wind tunnel available which restricted these tests to steady flow, the data that could be acquired were deemed to provide useful new insights into the low Reynolds number characteristics of the foil.
Chapter 3

Acquisition & Processing of the Turbulence Data

Where under this beautiful chaos can there lie a simple numerical structure?

— Jacob Bronowski

3.1 Introduction

A set of velocity measurements from both an ADV and ADCP have been used in this thesis to characterise the turbulence at a tidal energy site in Scotland, at the Sound of Islay. The data were acquired by Partrac Limited (Partrac, 2013) and were made available in raw format for this research through a collaboration with Andritz Hydro Hammerfest (Andritz, 2013) and ScottishPower Renewables (ScottishPower Renewables, 2013). Together they have proposed the installation of ten 1 MW tidal turbines at the Sound of Islay.

This chapter begins with an overview of the deployment site and the set-up of the experiment. The strategies and techniques which were utilised to sample the data, verify its quality, and replace erroneous signals, are then discussed. This is followed by the methodology employed to compute the relevant turbulence statistics. The implications of the sampling frequency and sensor volume on the estimates are also discussed.
Chapter 3. Acquisition & Processing of the Turbulence Data

3.2 Acquisition of the Turbulence Data

3.2.1 Overview of the Deployment Site

The Sound of Islay is a relatively narrow strait located between the Isles of Islay and Jura in the Inner Hebrides of Scotland, as shown in Figure 3.1. It is approximately 30 km in length and the currents flow approximately in a North-South direction. The velocity data were acquired at grid-reference N 55 50.432’, W 006 05.900’, where the strait is approximately 800 m wide as indicated in Figure 3.1. This corresponded approximately to the proposed location of the tidal turbine deployment.

Figure 3.1: A map of the Sound of Islay, aligned North and depicting the location of the data acquisition. The geographic location of the Sound of Islay in the Inner Hebrides of Scotland is shown by the inset at the lower right. These maps were adapted from Admiralty Charts which were acquired by permission from the United Kingdom Hydrographic Office.

As illustrated in Figure 3.2, at the location where the data were collected, the seabed drops steeply from the shore to the basin. The characteristics of the bathymetry and seabed have been reported by Howson and Mercer (2012), using a video survey of the proposed cable routes from the turbines. They found
3.2. Acquisition of the Turbulence Data

low-lying ridges and the seabed comprised predominately of rock, boulders and cobble, with relatively little sediment.

![Map of the bathymetry at the location of the data acquisition, aligned North and with the approximate water depth denoted in metres. The map has been adapted from an Admiralty Charts acquired by permission from the United Kingdom Hydrographic Office.](image)

Figure 3.2: Map of the bathymetry at the location of the data acquisition, aligned North and with the approximate water depth denoted in metres. The map has been adapted from an Admiralty Charts acquired by permission from the United Kingdom Hydrographic Office.

The Sound of Islay is considered to be well suited for tidal energy generation. This is due to the water being sufficiently deep to accommodate turbines of approximately 20 m in diameter and the mean velocity exceeding 2.5 m s\(^{-1}\) (Andritz, 2013). The tidal variation in the water level is relatively low and is generally less than 1.5 m. The high velocity flows arise due to the different tidal wave speeds which travel through the Irish Sea and the shelf in the West of Ireland (Inall and Sherwin, 2006). This creates a driving head between the two larger bodies of water through the narrow constriction of the Sound of Islay.

Another attribute of the Sound of Islay considered to be favourable by turbine manufacturers is the relatively sedate wave climate (Andritz, 2013). Large surface waves with relatively long periods, can otherwise be a significant source of unsteady loading on a tidal turbine (Milne et al., 2010). The terrain of both the Isles of Islay on the West of the Sound, and Jura on the East, acts to protect the site from significant wind action (Howson and Mercer, 2012). A photograph
Chapter 3. Acquisition & Processing of the Turbulence Data

taken at the approximate deployment location during maximum flow, demonstrating the typically sedate wave action is provided in Figure 3.3. The software programme WavesMon (Teledyne RDI, 2010b) that is provided by Teledyne was used to estimate the wave state during the time period for which the turbulence data were analysed. The significant wave heights were less than 1 m and the dominant wave periods were generally between 6 s and 10 s. Therefore, these estimates provide support for the wave action being relatively insignificant.

Figure 3.3: Sound of Islay during maximum flood flow at a location corresponding approximately to the ADV and ADCP deployment. The photograph was taken at Port Askaig in an Eastward direction and the Isle of Jura can be observed in the background. Supplied by I. Milne and taken on March 20, 2012.

3.2.2 Configuration of the Sensors

The design of the turbulence rig and acquisition of the velocity data were undertaken by Partrac Limited (Partrac, 2013). Data were collected over a period of approximately 17 days between the 9th and 25th of September, 2009, encompassing both a spring and a neap tide. Point measurements of the near-bed velocities were acquired using a Nortek Vector ADV (Nortek, 2005) and the velocities through the water column were collected using a 300 kHz TRDI Workhorse ADCP (Teledyne RDI, 2009). Both these sensors were positioned closely together (within approximately 1.5 m) on a steel frame at the seabed. The upwards looking ADCP was located inside a hollow tube to provide additional rigidity. The
ADV was mounted vertically (i.e. the probe was above the canister) and on top of a mast at $z = 5\,\text{m}$ above the seabed. The mast was fitted with a helical profile to mitigate vortex shedding. The arrangement of the sensors on the frame is shown in Figure 3.4, prior to the deployment during slack tide.

![Deployment of the turbulence rig at the Sound of Islay. The ADV is mounted to the mast at top right of the photo and the ADCP is partially obscured at lower left. Reproduced with permission of Partrac Limited (Partrac, 2013).](image)

The specifications of the ADV and ADCP are summarised in Table 3.1. The sampling frequencies for the ADV and ADCP were 4 Hz and 2 Hz respectively, with the ADCP operating in Mode 1.\(^1\) These sampling rates and settings were set

\(^1\)Mode 1 is data collection method that is specific to TRDI ADCPs and is generally recommended for deep channels by the manufacturer Teledyne RDI (2009).
by Patrac, which ensured that the memory and batteries permitted continuous sampling over the entire deployment period. The ADV had three receivers spaced at 120° intervals around the transmitter and the sampling size was 14.8 mm in diameter and at a distance of 150 mm from the head of the probe. The transducers of the ADCP were arranged at intervals of 90° and each projected upwards through the water column at an angle of 20° relative to the vertical. The ADCP acquired data at 1 m vertical intervals through the water column from approximately $z = 4$ m. A schematic diagram of the transducer geometry of the ADV and the ADCP is provided in Figures 3.5 and 3.6 respectively.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Nortek ADV</th>
<th>TRDI Workhorse ADCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of transducers</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Operational frequency</td>
<td>6000 kHz</td>
<td>300 kHz</td>
</tr>
<tr>
<td>Configuration</td>
<td>Canister below</td>
<td>Upright</td>
</tr>
<tr>
<td>Sampling frequency</td>
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<td>2 Hz</td>
</tr>
<tr>
<td>Sample diameter/bin size</td>
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<td>1 m</td>
</tr>
<tr>
<td>Height above bed/first bin</td>
<td>5 m</td>
<td>4 m</td>
</tr>
<tr>
<td>Operational mode</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>Sampling mode</td>
<td>Continuous</td>
<td>Continuous</td>
</tr>
<tr>
<td>Sampling reference system</td>
<td>Beam</td>
<td>Beam</td>
</tr>
</tbody>
</table>

Figure 3.5: Schematic diagram of the configuration of the ADV.
3.3 Data Processing Methodology

3.3.1 Measurement of Velocity & Co-ordinate Transformations

For the ADV, the fluctuating velocities for the ADV were analysed in a Cartesian co-ordinate system aligned with the principal directions of the flow. This involved two intermediate transformations from the original beam co-ordinate system which was aligned with the transducer beams. A benefit of obtaining the velocities in the beam coordinate system is that poor data signals can be readily traced to an individual transducer. Furthermore, for the ADCP in particular, acquiring the beam velocities directly is more suitable for estimating the velocity variances. This is because the sensor tilt was sampled at a lower rate than the velocity. If the velocities had been obtained in the Cartesian reference system, an average tilt angle would have been incorporated. Consequently, when transforming the Cartesian velocities back to a reference system aligned to the beams (to apply the variance method), the higher frequency content would be less reliable.
system. Data were first transformed to a Cartesian system aligned to the sensor using a transformation matrix that was unique to the sensor. These velocities were subsequently transformed to an Earth-fixed reference frame, which incorporated the heading and tilt angles which accounted for any movement of the sensor. The final transformation to obtain the principal directions of the flow (streamwise, $U$, transverse, $V$ and vertical, $W$) was based on a rotation of the East and North velocities about the vertical axis. This used the mean flow direction for each individual sample. The transformation matrices which were applied are provided in Appendix A.

For a four-transducer ADCP aligned with the principal directions of the flow and with zero tilt, the beam velocities ($b_i$, where $i = 1$ to $4$), are related to the Cartesian velocity components by

$$b_1 = -u_1 \sin(\theta) - w_1 \cos(\theta),$$
$$b_2 = u_2 \sin(\theta) - w_2 \cos(\theta),$$
$$b_3 = -v_3 \sin(\theta) - w_3 \cos(\theta),$$
$$b_4 = v_4 \sin(\theta) - w_4 \cos(\theta),$$

where $u$, $v$ and $w$ are the fluctuating velocities and where $\theta$ is the beam angle. The mean flow velocities were estimated by assuming that the average velocities between opposite beams were equal over the sampling period i.e. $\bar{U}_1 = \bar{U}_2$, which enabled $U$, $V$, and $W$ to be explicitly resolved. The effect of the non-zero tilt was also corrected for. The transformation matrices that were applied to the ADCP data are also provided in Appendix A.

However, as subsequently discussed in Section 3.4.2, the implication of the non-homogeneity of the flow is that the fluctuating velocities at scales smaller than the beam spread, could not be acquired directly. Instead, only the second moment statistics of the ADCP velocities were estimated and were affected by the sensor tilt.

### 3.3.2 Sampling Strategy

The turbulence analysis was performed using a truncated data set which encompassed three tidal cycles coinciding with the spring tide. This significantly reduced the computational overhead necessary to process the turbulence data. It was deemed to be sufficient to allow for the variability of turbulence statistics
between cycles to be assessed.

Various sample durations over which to compute the turbulence statistics were evaluated. A 5-minute period for the ADV and 10-minute period for the ADCP (both 1200 data points) were selected. This was based on a test for stationarity using the ‘runs method’ (Bendat and Piersol, 1971), in which over 94% of the samples of both sets were considered to be stationary. The sampling duration of the ADV was equal to that used by Thomson et al. (2012) for analysing the turbulence at the Puget Sound. Osalusi (2010) also considered a 10-minute period as appropriate for analysing the ADCP data set at the Fall of Warness.

An overlapping sampling strategy was employed to compute the turbulence statistics. Samples were extracted from the ADV data set at 1-minute intervals and for the ADCP at 2-minute intervals. A similar strategy was also employed by Trevethan (2007) to analyse a data set of ADV measurements acquired in the field, where it was considered to be appropriate for capturing short term events. Following Larsen and Hansen (2004), in an attempt to remove any bias, each data set was de-trended by fitting a linear curve and subtracting this from the raw data samples.

3.3.3 Sensor Motion

The motion of the sensors during the sampling period was inferred from the compass and tilt data recorded by the ADV. The pitch and roll angles were relatively constant during the deployment and were approximately 4° and 2° respectively.

The standard deviation of the sensor tilt angles over the cycles of interest were relatively small and typically less than 0.2°. Samples for which the standard deviation of either the heading, pitch or roll angle exceeded $\sigma = 0.4^\circ$ were not analysed. In general, the excluded samples comprised less than 3% of the total number of samples analysed. The majority of these samples corresponded to the acceleration of the tide and where the mean velocities were relatively low.

3.3.4 Signal Validation & Spike Removal

The pre-processing of the samples included a further step to identify any instances of poor quality signals. For the ADV, communication-type errors were identified using the correlation (COR) and the energy strength in each beam inferred from the signal-to-noise ratio (SNR). Data points corresponding to either a $\text{COR} \leq 70\%$
or SNR ≤ 15 dB were replaced for all the ADV beams using a linear interpolation to prevent bias. The number of replaced data points due to these errors was relatively low, comprising less than 1% of all the data points.

For the ADCP, instantaneous communication-type errors were identified using the correlation and the percentage-good (PG) parameters. PG is a proxy encompassing a range of rejection criteria including low-correlation, large error-velocity and fish detection (Rusello, 2009). Data points where COR < 64 counts (out of 128) or PG < 100% were flagged and replaced using a linear interpolation between adjacent data points. At the elevations of 3 m and 40 m from the transducer, the average number of data points that were flagged as being of poor quality totalled approximately 1% and 3% of the total number of ensembles respectively. Samples where the percentage of poor data points exceeded 5% of the total were not considered. Only the first 43 bins of the ADCP were analysed (up to 50 m). The data at elevations above this were deemed to be of poor quality due to the effect of the free-surface.

The use of the phase-space despiking technique of Goring and Nikora (2002), which has been previously applied to both ADV and ADCP data sets (see e.g. Trevethan, 2007 and Osalusi, 2010) was considered. The total number of data points which were identified as being poor using the phase-space technique was relatively small for both the ADV and ADCP data sets in this thesis. For instance, for the ADV they comprised 5.4% of the total samples. The resulting effect on the standard deviation of the fluctuating velocity of all the samples was small, with the median difference being 0.6%, 1.5% and 2.5% in the streamwise, transverse and vertical components respectively. It was also found that the removal and replacement of this erroneous data affected the spectral energy at high wave numbers, such that a tendency towards an inertial subrange was slightly less apparent. It is possible that this was associated with the relatively small number of data points in each sample, compared to that for which the despiking method was originally developed (approximately 30,000) Wahl (2003). Given the small effect on the variances and the influence on the high wavenumber spectra, the phase-space technique was ultimately not applied to the data in this thesis.

### 3.4 Computation of Turbulence Statistics

The turbulence statistics of interest were computed from the pre-processed samples, according to the formal definitions presented by Bendat and Piersol (1971).
3.4. Computation of Turbulence Statistics

3.4.1 Estimates of Second-Order Moments from the ADV

The variance and its positive square root (the standard deviation), were the primary statistics of interest in this thesis. For a random stationary process the variance is defined as

\[ \sigma^2_x = \frac{1}{M} \sum_{m=0}^{M} (x(m) - \bar{x})^2, \]

where \( M \) is the number of realisations of a variable \( x \). The co-variance between two variables \( x \) and \( y \) is similarly defined as

\[ \sigma_{xy} = \frac{1}{M} \sum_{m=0}^{M} (x(m) - \bar{x})(y(m) - \bar{y}). \]

For length-scales larger than the sampling volume, these statistics can be computed directly from the Cartesian velocities measured by the ADV. However, these estimates are affected by noise which if assumed to be spectrally white, is relatively larger at the highest frequencies. The magnitude of the noise is a function of the alignment of the beam geometry and is specific to the ADV used. Following Voulgaris and Trowbridge (1998), the contribution of noise to the variance estimates for the sensor used in this analysis can be expressed in the form

\[ \sigma^2_{u*} = \sigma^2_u + 11.67\sigma^2_N, \]

where \( \sigma^2_{u*} \) is the measured streamwise variance, \( \sigma^2_u \) is the true streamwise variance and \( \sigma^2_N \) is the contribution from noise. For the transverse and vertical velocities the variance components due to noise were 11.67\( \sigma^2_N \) and 0.35\( \sigma^2_N \) respectively, and for the covariance term, was 0.03\( \sigma^2_N \). The comparatively smaller effect of noise on the vertical velocity variance and the covariance, relative to the noise for the variances of the horizontal velocities, is a consequence of the sensor geometry.

Voulgaris and Trowbridge (1998) have also shown that for rapid flows, the uncertainty is dominated by turbulence in the flow as opposed to that originating from the inherent noise in the sensor. This is consistent with the noise floor not being pronounced in the ADV spectral densities for mean velocities of the order of \( U = 2 \text{ m s}^{-1} \) observed in this thesis (see Section 4.3.5), but it is also acknowledged that this could have been a consequence of the relatively low sampling frequency of 4 Hz. By averaging multiple data points, the error is reduced by a factor inversely proportional to the square-root of the number of data points, i.e. for
Chapter 3. Acquisition & Processing of the Turbulence Data

this thesis, \( \sigma_{N\text{mean}} = \sigma_{N\text{data}} / \sqrt{1200} \).

3.4.2 Estimates of Second-Order Moments from the ADCP

Variance Method

As the flow is not expected to be fully coherent across the spread of the beams of the ADCP, the Cartesian variances cannot be computed directly (Osalusi et al., 2009). The fluctuating velocities of the ADCP were instead analysed in their beam reference frame, i.e.

\[
b_1 = -u \sin (\theta) - w \sin (\theta), \quad b_2 = u \sin (\theta) - w \sin (\theta).
\]

(3.5)

Following Lohrmann et al. (1990), for a four-beam ADCP of the same configuration as used in this thesis, the Reynolds stresses (co-variances) and the total kinetic energy were obtained from the variances of opposing beam velocities. This assumed that the second order statistics were approximately homogeneous between the beams. For instance, when beam 1 is aligned to the direction of the positive streamwise flow, the variances of the velocities measured along beams 1 and 2 are expressed as

\[
\sigma_{b_1}^2 = \sigma_u^2 \sin (\theta) - 2\sigma_{uw} \sin (\theta) \cos (\theta) + \sigma_w^2 \cos^2 (\theta),
\]

(3.6)

\[
\sigma_{b_2}^2 = \sigma_u^2 \sin (\theta) + 2\sigma_{uw} \sin (\theta) \cos (\theta) + \sigma_w^2 \cos^2 (\theta),
\]

(3.7)

where higher order terms are neglected. An expression for the covariance between the streamwise and vertical velocities can then be derived by subtracting the variance of the velocities from beam 1 from beam 2, i.e.

\[
\frac{\sigma_{b_1}^2 - \sigma_{b_2}^2}{4 \sin (\theta) \cos (\theta)} = -\sigma_{uw}.
\]

(3.8)

The covariance between the transverse and vertical velocities can also be derived as

\[
\frac{\sigma_{b_3}^2 - \sigma_{b_4}^2}{4 \sin (\theta) \cos (\theta)} = -\sigma_{vw}.
\]

(3.9)

Similarly, in the process of summation of the opposing beam variances the covariance is cancelled out. The variances from beams 1 and 2, together with those of beams 3 and 4 provide two independent expressions for the relationship between
3.4. Computation of Turbulence Statistics

The Cartesian variances. These were denoted by Stacey et al. (1999) as $d_u^2$ and $d_v^2$ and are expressed as

$$d_u^2 = \frac{\sigma_{b1}^2 + \sigma_{b2}^2}{2} = \sigma_u^2 \sin^2 (\theta) + \sigma_w^2 \cos^2 (\theta),$$

(3.10)

and

$$d_v^2 = \frac{\sigma_{b3}^2 + \sigma_{b4}^2}{2} = \sigma_v^2 \sin^2 (\theta) + \sigma_w^2 \cos^2 (\theta).$$

(3.11)

To solve these equations explicitly for the individual variance components, a third relationship is required, such as the anisotropy in the flow. In this thesis, the anisotropy was obtained from the ADV measurements at 5 m from the bed. As such, it was assumed that the anisotropy did not vary with elevation, which is consistent with Nezu and Nakagawa (1993). It is important to also consider that whilst the use of a five-beam ADCP (which have only very recently become commercially available) would have allowed the anisotropy to be explicitly determined throughout the water column, the instantaneous three-dimensional velocity vector could still not have been obtained due to non-homogeneity.

Corrections for Bias

The bias in the estimates of the variances and the Reynolds stresses from the ADCP were established following the approaches outlined previously by Lohrmann et al. (1990), Stacey et al. (1999) and Williams and Simpson (2004).

These investigators have demonstrated that the estimates of $d_u^2$ and $d_v^2$ are biased proportionately by the variance of the Doppler noise, e.g.

$$d_u^2* = d_u^2 + \sigma_N^2.$$

(3.12)

As is demonstrated in Appendix A, for the ADCP used in this analysis the bias was equal to a ping-by-ping error of $\sigma_N = 0.067\ m/s$. This was equivalent to the standard deviation predicted for the sensor using the software package PlanADCP (Teledyne RDI, 2006). This value was subtracted from both $d_u^2$ and $d_v^2$, prior to incorporating the anisotropic ratios and resolving the product to estimate the streamwise intensities.

By taking differences of the beam velocity variances, the estimates of the momentum fluxes are, to the first order, not biased by Doppler noise. However, any non-zero tilt introduces bias to both the momentum flux and to $d_u^2$ and $d_v^2$. 

51
Chapter 3. Acquisition & Processing of the Turbulence Data

Lohrmann et al. (1990) has shown that this bias can be estimated to the first order as
\[
(\sigma_{uw})_{Bias} = \phi_3 (\sigma_w^2 - \sigma_u^2) - \phi_2 (\sigma_{uv}).
\] (3.13)

In this thesis, the first and second terms on the RHS of equation 3.13, were estimated from the variances and covariance measured by the ADV. These values can be inferred from the observations that are subsequently presented in Section 4.3.1 as \(\sigma_u^2 \approx 0.07 \text{ m}^2 \text{ s}^{-2}\), \(\sigma_w^2 \approx 0.02 \text{ m}^2 \text{ s}^{-2}\), and \(\sigma_{uv}^2 \approx 0.01 \text{ m}^2 \text{ s}^{-2}\). For a pitch angle of \(\phi_2 \approx 4^\circ\) and a roll angle of \(\phi_3 \approx 2^\circ\) this bias was estimated to be approximately \((\sigma_{uw})_{bias} \approx 0.002 \text{ m}^2 \text{ s}^{-2}\). This is deemed to be sufficiently small for the purposes of analysis here to not compromise the results.

### 3.4.3 Correlation Functions & Integral Scales

The correlation function provides a measure of the degree of similarity between two signals which in general are separated in time and space. As single point measurements in space are acquired by the ADV, the correlations that were able to be measured were those for a time lag, \(\tau\). Assuming that the mean has been removed from the respective time histories, these correlations are defined as
\[
R_{ij} (\tau) = \frac{1}{T} \int_0^T i(t) j(t + \tau) \, dt \quad \text{(for } i,j = u,v \text{ or } w). \tag{3.14}
\]

The correlation \(R_{ii}(\tau)\) is the auto-correlation function and \(R_{ij}(\tau)\) \((i \neq j)\) is the cross-correlation function. These are equal to the variance and covariance at a time lag of zero respectively. The auto-correlation is an even function of \(\tau\) and satisfies the relationship
\[
R_{ii} (-\tau) = R_{ii} (\tau), \tag{3.15}
\]
whilst the cross-correlation satisfies the condition
\[
R_{ij} (-\tau) = R_{ji} (\tau). \tag{3.16}
\]

The cross-correlation may be considered to be a combination of a symmetric part and an anti-symmetric part. The symmetric component can be expressed as
\[
\{R_{ij}(\tau)\}_\text{sym} = \frac{1}{2} (R_{ij}(\tau) + R_{ij}(-\tau)), \tag{3.17}
\]
whilst the anti-symmetrical component is given as
\[
\{R_{ij}(\tau)\}_{\text{asym}} = \frac{1}{2}(R_{ij}(\tau) - R_{ij}(-\tau)).
\] (3.18)

When the correlation functions are normalised by the respective variances they give rise to the auto-correlation and cross-correlation coefficients
\[
\rho_{ii} = \frac{R_{ii}(\tau)}{\sigma_i^2},
\] (3.19)
and
\[
\rho_{ij} = \frac{R_{ij}(\tau)}{\sigma_i \sigma_j}.
\] (3.20)

The integral time-scale associated with the auto-correlation provides a measure of the time duration in which the largest eddies remain correlated. It is defined formally as
\[
\tau_i = \int_0^\infty \rho_{ii}(\tau) \, d\tau.
\] (3.21)
The integration was taken to be between \(\tau = 0\) and the lag that corresponded to the first instance in which \(\rho_{ij}(\tau) = 0\). By invoking Taylor’s frozen wake hypothesis (Taylor, 1938), the corresponding integral length-scales were obtained from the integral time-scales using the expression
\[
L_i^\pi = U\tau_i.
\] (3.22)

An integral length-scale which provides a measure of the size of the Reynolds stresses was also estimated. This scale was estimated from the symmetrical part of the cross-correlation. Chanson et al. (2008) have previously defined its associated time-scale from the integration of the cross-correlation function between the limits
\[
\tau_{ij} = \int_{\tau = \tau(\rho_{ij}=0)}^{\tau = \tau(\rho_{ij}=\rho_{ij})_{\text{max}}} (\rho_{ij} \tau)_{\text{sym}} \, d\tau
\] (3.23)
and which has been adopted in this thesis. This time-scale was multiplied by the mean velocity to obtain the length-scale.
3.4.4 Spectral Densities

The velocity spectral densities are defined formally as

\[ S_{ij}(n) = 2 \int_{-\infty}^{\infty} R_{ij}(r, r'; \tau) e^{-(2\pi ni\tau)} d\tau. \]  

(3.24)

As the autocorrelation function is even, the one-sided auto-spectrum can be expressed as

\[ S_{ii}(n) = 4 \int_{0}^{\infty} R_{ii}(r, r'; \tau) e^{-(2\pi ni\tau)} d\tau. \]  

(3.25)

As the cross-correlation is anti-symmetrical, the cross-spectral density is a complex quantity and can be expressed as

\[ S_{ij}(n) = C_{ij}(n) - iQ_{ij}(n), \]  

(3.26)

where \( C_{ij}(n) \) is the symmetrical part, denoted as the co-spectrum and \( Q_{ij}(n) \) is the anti-symmetrical part, or quad-spectral density function. These are expressed in terms of the cross-correlations, i.e.

\[ C_{ij}(n) = 2 \int_{0}^{\infty} [R_{ij}(r, r'; \tau) + R_{ij}(r, r'; -\tau)] \cos (2\pi n\tau) d\tau \]

\[ = 2 \int_{0}^{\infty} [R_{ij}(r, r'; \tau) + R_{ji}(r, r'; \tau)] \cos (2\pi n\tau) d\tau \]  

(3.27)

and

\[ Q_{ij}(n) = 2 \int_{0}^{\infty} [R_{ij}(r, r'; \tau) - R_{ij}(r, r'; \tau)] \sin (2\pi n\tau) d\tau. \]  

(3.28)

Estimates using the ADCP Beam Velocities

For the ADCP, the co-spectra were estimated from the auto-spectra of opposing beam velocities. This approach has been previously employed by Lien and Sanford (2000) to obtain the co-spectra at various elevations through a tidal channel. Kirincich et al. (2010) also used the co-spectra to calculate the Reynolds stresses from ADCP velocities in the presence of surface waves.

Following Bendat and Piersol (1971), the autospectral density of the beam velocity \( S_{ii}(n) \) can be expanded as a sum of two processes,

\[ S_{11}(n) = a^2 S_{uw}(n) + ab [S_{uw}(n) + S_{wu}(n)] + b^2 S_{ww}(n), \]  

(3.29)
3.4. Computation of Turbulence Statistics

\[ S_{22}(n) = a^2 S_{uu}(n) - ab [S_{uw}(n) + S_{wu}(n)] + b^2 S_{ww}(n), \quad (3.30) \]

where, \( a = \sin(\theta) \) and \( b = \cos(\theta) \). The second terms in the enclosed brackets of the above equations are equal to twice the co-spectral density. When the second equation is subtracted from the first, the co-spectrum is isolated and can be expressed as

\[ -C_{uw}(n) = \frac{S_{11}(n) - S_{22}(n)}{4 \sin(\theta) \cos(\theta)}. \quad (3.31) \]

However, the normalisation of the co-spectral density was not possible as the standard deviations of \( \sigma_u \) and \( \sigma_w \) were not known explicitly. This also implies that an equivalent integral length-scale for the shear stresses could not be obtained.

Computation of Spectral Density Estimates

A Fast-Fourier Transform approach with 2048 ensembles was utilised to compute the densities. A Hamming window function was applied to taper the time histories. Individual spectra were converted between the frequency and wavenumber \((k_x)\) domain by invoking Taylor’s frozen turbulence hypothesis (Taylor, 1938), such that

\[ S(k_x) = \frac{2\pi}{U} S(n), \quad (3.32) \]

where \( U \) was the mean velocity of the sample.

The spectral densities were interpolated such that they corresponded to a set of 2048 universal wavenumbers. Multiple samples were used to estimate an average density at these wavenumbers. The need for averaging multiple samples is particularly relevant for the spectral densities which were computed from the ADCP beam velocities. This is due to a comparatively greater contribution from Doppler noise in the ADCP than for the ADV.

3.4.5 Implications of the Sampling Frequency

The sampling frequency of the ADV of 4 Hz may be considered to be relatively low compared to that used in other studies of turbulence in the natural environment (e.g. Trevethan, 2007 and Thomson et al., 2012). Therefore, it is important to identify the implications of this sampling rate on the measurements of the turbulence statistics.

As was discussed in Section 2.2.2, the inertial sub-range is theoretically expected to be developed at nondimensional wavenumbers exceeding \( k_{xz} = 4.5 \),
after which point the spectral energy decays rapidly. At an elevation of 5 m, this corresponds to a wavenumber of \( k_{\text{sub}} = 0.9 \text{ m}^{-1} \). For a mean velocity of 2.5 m s\(^{-1}\) such as observed during ebb tide at the Sound of Islay site (see Section 4.2.1), the maximum wavenumber that is able to be resolved at a sampling frequency of 4 Hz is approximately \( k_{\text{max}} = 2\pi n_{\text{Nyquest}}/U = 3.28 \text{ m}^{-1} \), which is \( 3.6k_{\text{sub}} \). Therefore, a region of the sub-range may be able to be resolved.

For the ADCP, the theoretical maximum wavenumber which could be resolved at 2 Hz is half of that able to be resolved by the ADV at an equivalent flow speed. At a flow speed of \( U = 2.5 \text{ m s}^{-1} \), the maximum resolvable wavenumber corresponds to a wavelength of \( L = 2\pi/k = 7.33 \text{ m} \). Simpson et al. (2005) note that for a 1 m bin size which is used in this thesis, the cut-off length-scale is approximately 2 m. Therefore, for this study and during peak flows, the ADCP measurements near the bed were expected to be limited by the sampling frequency, rather than by the sampling volume.

As is subsequently shown in Section 4.3.5 a pronounced noise floor was not observed in the ADV spectra for flow-speeds of \( U \approx 2 \text{ m s}^{-1} \). As such, no account was made to correct for Doppler noise. However, it is interesting to consider the implications of the reduction in the variance owing to the relatively low sampling frequency. This additional variance was estimated by extrapolating the observed spectra for an additional decade, assuming that the spectral density conformed to the proportionality predicted by Kolmogorov (1941) (equation 2.5). This contribution was found to be approximately 2%, 5% and 7% of the measured streamwise, transverse and vertical variances respectively. Whilst this is relatively large for the vertical component in particular, it is the streamwise variance which is most relevant to the unsteady loading of turbines. The error in the streamwise variance is deemed to be sufficiently small for the requirements of this thesis.

The sensitivity of the integral-scales to the sampling frequency was also investigated by re-sampling the data set at a lower rate of 1 Hz. A negligible difference (i.e. less than 1%) was identified in the streamwise scales. Therefore, a 4 Hz sampling frequency was also deemed to be sufficient for estimating this parameter.

### 3.5 Summary

A data set of velocity measurements from an ADV and ADCP were obtained from the Sound of Islay in Scotland, where a tidal turbine deployment is proposed. The site is characterised by relatively strong currents and a relatively low tidal
variation in the water level. The seabed is comprised predominately of rock and gravel and the wave climate is relatively sedate. The ADV was mounted at 5 m above the bed and sampled at 4 Hz. The ADCP data were obtained at 1 m intervals upwards through the water column and were sampled a rate of 2 Hz.

Data were processed using samples of 5-minute and 10-minute duration for the ADV and ADCP respectively and poor quality signals were identified and replaced. The methodology employed to estimate the intensities, Reynolds stresses, integral scales and spectra near the bed from the ADV was presented. The variance technique was applied to the ADCP velocities to estimate the streamwise intensity and the Reynolds stresses throughout the water column. Despite the relatively low sampling rates of the sensors, the dominant energy producing scales are expected to be able to be resolved at a mean velocity of 2 m s\(^{-1}\). The errors associated with the streamwise variance and integral-scales are also sufficiently small for the purpose of this thesis.
Chapter 4

Characterisation of the Turbulence at the Sound of Islay

Big whorls have little whorls,
which feed on their velocity; and
little whorls have smaller whorls,
and so on to viscosity.
— Lewis Fry Richardson

4.1 Introduction

This chapter establishes the magnitudes of the fluctuations, the anisotropy and the scales of the turbulence at the Sound of Islay. These results are used to describe the forcing that a turbine blade would be subjected to by turbulence.

It begins with a characterisation of the mean flow at the site and the evolution of the boundary layer. The turbulence intensities, Reynolds shear stresses, integral length-scales and spectra observed at spring tide are then presented. Comparisons with other tidal energy sites and with theoretical models are used to evaluate the universality of the turbulence characteristics. The analyses are an extension of those which were published in the journal *Philosophical Transactions of the Royal Society, Part A* (Milne et al., 2013b).

Theoretical models for turbulence are then used to predict the amplification of the velocity fluctuations at the turbine due to the expansion of the stream-tube, as well as the turbulence coherence across a turbine blade. Together with the measured turbulence data, these results are used to establish the range of values
of key non-dimensional parameters characterising the unsteadiness on a turbine blade.

4.2 Observed Hydrodynamics

4.2.1 Mean Velocity Regime

The time history of the 10-minute mean of the streamwise velocity over the entire data acquisition period is presented in Figure 4.1. The data were obtained for an elevation of $z = 30\,\text{m}$ above the seabed and the corresponding water level variations are also shown.

![Figure 4.1: Ten-minute mean of the water level and the streamwise velocity at an elevation of 30 m over the entire data acquisition period. The turbulence data were analysed for the tidal cycles which are highlighted in bold.](image)

The spring tide was observed approximately 7 days after data acquisition commenced. At spring tide, the water level ranged by approximately 1.5 m between the low and high tides. These tidal cycles were associated with the greatest mean velocities over the entire deployment. The turbulence data were analysed for the
three cycles around the spring tide which are highlighted in bold. For these cycles, the maximum velocities were approximately 2.7 m s\(^{-1}\) during the flood and 2.9 m s\(^{-1}\) during the ebb.

The neap tides were observed around day 15, during which the maximum velocities were approximately 1.5 m s\(^{-1}\). The variation in the water level corresponding to the neap cycles was relatively small and approximately 0.5 m. It was also comparatively more erratic and less sinusoidal than was observed at spring tide.

The directionality of the 10-minute mean velocity over the entire data acquisition period is demonstrated using tidal compasses for elevations of \(z = 5\) m and 30 m in Figure 4.2. These show that the principal flow directions were generally well defined throughout the water column during the tidal cycle, varying by approximately 178° between the flood and ebb flow. These observations are consistent with the near North-South orientation of the channel. This also concurs with no dominant underwater formations being near the site, which could otherwise have affected the flow throughout the column.

The 5-minute means of the streamwise and vertical velocities at an elevation of \(z = 5\) m, corresponding to the tidal cycles for which the turbulence was analysed are presented in Figure 4.3. These have been estimated from both the ADV and
ADCP data, and the corresponding water level is also provided for comparison. This shows that the estimates between the two sensors agree well, providing confidence in their measurements. The maximum velocity was approximately 2 m s\(^{-1}\) during the flood and 2.3 m s\(^{-1}\) during the ebb, whilst the vertical velocities were approximately zero.

The time histories demonstrate that low water was observed approximately 0.15 of a tidal cycle after the commencement of the flood currents. High water was observed approximately 0.05 of a cycle after the first ebb currents. There were no significant differences observed in the mean velocity histories between consecutive periods of flood and ebb flow. Fluctuations in the mean velocity of approximately 0.2 m s\(^{-1}\), typically lasting less than 20 min were observed during each tidal cycle. These fluctuations did not appear to be correlated with the fluctuations observed in the water level, which were relatively small.
4.2. Observed Hydrodynamics

4.2.2 Boundary Layer Dynamics

Profiles of the mean streamwise velocity throughout the water column are presented in Figure 4.4. These have been obtained at 1-hour intervals over the first tidal cycle and demonstrate the evolution of the boundary layer. These profiles were generally representative of those for the subsequent two tidal cycles for which the turbulent data were analysed, as shown in Appendix A.

![Figure 4.4: Profiles of the mean streamwise velocity computed every hour during the first tidal cycle and averaged over 10 samples, each of 10-minute duration. The profiles are labelled sequentially, with the first profile corresponding to the first hour of the flood tide. Velocities during flood tide and ebb tide are denoted as positive and negative respectively. They are presented as a function of the ratio of the elevation above the bed to the water level, z/h.](image-url)

The 1st profile corresponds to the beginning of the flood flow. Near the bed, the velocity distribution was approximately uniform and it increased slightly towards the surface. The 2nd profile corresponds to the period in the tidal cycle where the cut-in velocity of the turbine would be exceeded and power would begin to be generated. During this period, the height of the boundary layer was approximately \( z/h = 0.4 \) (which was assumed to correspond to an elevation where \( U(z) = 0.99U_{max} \)). The height of the boundary layer increased with velocity and extended up to \( z/h = 0.85 \). The boundary layer was maintained to at least this height during the deceleration of the current.

For the ebb, the height of the boundary layer also increased during the accel-
eration of the flow. As the maximum ebb currents were approached, the height of the boundary layer was lower than for the flood and was approximately equal to $z/h = 0.6$. This is interesting, as the water level variation between the flood and the ebb was relatively small. Nevertheless, the boundary layer still extended to elevations corresponding to a typical tidal turbine.

The 11th profile demonstrates that the height of the boundary layer was greater during the initial deceleration of the ebb tide than at peak flow. This suggests that there was a slight lag in the development of the boundary layer. As the ebb current continued to decelerate, the boundary layer height began to decrease. It tended to be approximately equal to that observed during the acceleration of the flow.

![Figure 4.5: The profiles throughout the water column of the mean velocity shear, (a) and of the mean streamwise velocity, with a logarithmic ordinate (log $(z)$), (b). The data for the flood and ebb (grey) are denoted as positive and negative respectively. An estimate of the mean velocity based on the law of the wall up to an elevation of log$(2.5)$ is presented in (b) as solid lines. All data have been averaged over ten samples of 10-minute duration, which were acquired every 2-minutes.](image)

The corresponding profiles of the mean streamwise velocity shear ($\delta U/\delta z$) are presented in Figure 4.5(a). The shear during the flood flow and ebb flow are denoted as positive and negative respectively. For clarity, the profiles for slack tide, where $U \leq 1 \text{ m s}^{-1}$ have also been excluded. The maximum shear generally
occurred at the lowest elevations resolvable. At elevations up to $z/h < 0.2$, the shear decreased at a rate that was approximately inversely proportional to the elevation, which is consistent with theory (see Section 2.2.1, equation 2.2). There were also no indications of any subsurface shear reversals, as has been observed at other tidal channels (e.g. Lueck and Lu, 1997 and Li et al., 2010). Together these observations are consistent with the turbulence having been predominately generated within the bottom boundary layer through the action of shear forces.

The mean velocity profiles are presented with a logarithmic ordinate in Figure 4.5(b). They collapse to a near linear slope for $z/h < 0.2$, which provides additional support for the conformity with the law of the wall. The friction velocity and the roughness length were estimated using a least-squares technique to fit the velocities to equation 2.3 for elevations up to $z/h = 0.2$. Near peak flood flow (profiles 3 to 5 in Figure 4.4), the friction velocities ranged between $u_\text{*} = 0.10 \text{ m s}^{-1}$ and $0.14 \text{ m s}^{-1}$ and the roughness lengths ranged between $z_0 = 0.01 \text{ m}$ and $0.02 \text{ m}$. During the peak ebb flow (profiles 9 to 11), the friction velocities were approximately equivalent to those for the flood. The roughness lengths for the ebb were up to an order of magnitude smaller and between $z_0 = 0.002 \text{ m}$ and $0.008 \text{ m}$. As is subsequently shown in Section 4.3.2, the smaller roughness lengths at ebb are consistent with the bottom drag coefficient (estimated using the Reynolds stresses) also being smaller during the ebb tide than during the flood. In general, the roughness lengths are representative of a seabed that comprised gravel and rocks (Health & Safety Executive, 1990) and thus in agreement with the seabed characteristics described in Section 3.2.1.

### 4.3 Observed Turbulence Properties

#### 4.3.1 Near-Bed Intensities

The magnitudes of the higher frequency components of the flow at the Sound of Islay are firstly quantified in terms of the turbulence intensity. As discussed in Section 1.3.1, the turbulence intensity is generally regarded to be the turbulence parameter most relevant for tidal turbine loading. It provides a quantification of the turbulent kinetic energy present in the flow. Therefore, it is statistically related to the amplitudes of the unsteady forcing imparted on a turbine.

The time history of the streamwise intensity, $I_u = \sigma_u/U$ an elevation of $z = 5 \text{ m}$ is presented in Figure 4.6. The intensities have been computed from the ADV
data and the 5-minute means of the streamwise velocity are also shown for reference. The time has been non-dimensionalised with respect to consecutive tidal cycles (flood then ebb) in the mean velocity. For mean velocities exceeding approximately \( U = 1 \text{ m s}^{-1} \), the median turbulence intensity was nearly constant during consecutive flood and ebb flows. During the flood, it had a value of \( I_u = 0.13 \) and it was slightly lower for the ebb, approximately equal to \( I_u = 0.11 \). The turbulence intensities ranged between 0.10 and 0.18 during the flood and between 0.08 and 0.16 during the ebb. The maximum and minimum intensities were typically associated with events that had a duration of 20 minutes or less.

Figure 4.6: From top: Time histories of the mean streamwise velocity, streamwise turbulence intensity, ratio of the standard deviations of the transverse and streamwise velocity and the ratio of the standard deviations of the vertical and streamwise velocities, at an elevation of 5 m. Data were averaged over a period of 5-minutes, computed at 1-minute intervals from the ADV data, and have been truncated at slack tide for clarity.

The time histories of the absolute values of the standard deviations of the
transverse and vertical fluctuating velocities are presented in Appendix A. Like
the standard deviation of the streamwise velocity, these values were observed
to vary in-phase with the mean velocity and were approximately equal to $\sigma_u = 0.20 \text{ m s}^{-1}$ and $\sigma_w = 0.15 \text{ m s}^{-1}$ at peak flow. For this study, the ratios of the
standard deviations of the transverse and vertical fluctuating velocities to the
standard deviation of the streamwise fluctuating velocity are of particular interest
and are presented in Figure 4.6. For isotropic flow, the ratios are theoretically
equal to 0.5. Therefore, a deviation from this value provides a quantification of
the anisotropy of the large-scale eddies present in the flow.

For mean velocities exceeding $U = 1 \text{ m s}^{-1}$, the values of both ratios exhibited
a relatively small decrease with the mean velocity. Near peak flow, the median
of the ratio $\sigma_v/\sigma_u$ was approximately equal to 0.75 during the flood. Its value
was slightly lower during the peak ebb and approximately equal to 0.70. The
ratio $\sigma_w/\sigma_u$ was comparatively smaller, with a median value of approximately
$\sigma_w/\sigma_u = 0.55$ during the peak flood and 0.51 during the peak ebb.

Short term fluctuations were also observed in both these ratios over the tidal
cycle. The ratio $\sigma_v/\sigma_u$ ranged between 0.60 and 1.00, whilst $\sigma_w/\sigma_u$ ranged be-
tween 0.40 and 0.70. The largest fluctuations in both the ratios were typically
observed at equivalent times during the tidal cycle. This suggests that they were
associated with coherent eddies in the flow.

### 4.3.2 Near-Bed Reynolds Shear Stresses

**Ratios of the Shear Stresses to the Turbulent Kinetic Energy**

The Reynolds stresses provide an additional metric by which to characterise the
anisotropy in the flow due to the mixing. The stresses are theoretically equal to
zero in isotropic flow, and in a 2-D open channel flow they are expected to be
dominated by the $-\sigma_{uw}$ component. In this thesis, the ratio of the Reynolds shear
stresses to $2q$, where $q = 0.5 (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$ is the total turbulent kinetic energy
measured by the ADV, has been computed. Therefore, this ratio serves to also
provide a useful measure of the relative energy contained by the stresses. The
time histories of the ratios at an elevation of 5 m are presented in Figure 4.7. The
time histories of $\sigma_{uw}/2q$ and $\sigma_{vw}/2q$ for the first tidal cycle are also reproduced
in Appendix A for clarity and to facilitate comparisons between the ADV and
ADCP data sets.

The ratio $-\sigma_{uw}/2q$ represents the relative energy due to shearing in a vertical
plane which lies parallel to the streamwise flow. The independent estimates from both the ADCP and ADV align well. During the peak flood flow, the median ratio was approximately equal to 0.10 and the ratio generally ranged between 0.08 and 0.20. The median ratio at peak ebb flow was slightly smaller, and approximately 0.08. The range of ratios observed for the ebb were generally between 0.02 and 0.15.

The ratio $-\sigma_{vw}/2q$ represents the relative energy due to shearing in a vertical plane parallel to the transverse flow. During the flood tide, both the ADV and ADCP estimates of this ratio were approximately equal to zero. However, during the ebb tidal flow, the ratio was non-zero. The ADV estimate was approximately equal to 0.05 and the ADCP estimate was larger, equating to approximately 0.10. A non-zero Reynolds stress implies that there may have been either shedding of vortices off the frame or secondary currents present during the ebb. The difference
4.3. Observed Turbulence Properties

between the estimates from the two sensors suggests that the dominant length-scales of the stress were too small to be fully resolved by the ADCP.

The ratio $-\sigma_{uv}/2q$ was only able to be acquired from the ADV velocity. However, it enables the Reynolds stresses to be completely determined near the bed. The ratio was not significantly different between the flood and the ebb and its median value during peak flow was approximately 0.03.

**Estimates of the Friction Velocity & Drag Coefficient**

The Reynolds stresses were also used to provide an independent check of the friction velocity, as well to obtain an estimate of the drag coefficient. The Reynolds stress components $-\sigma_{uw}$ are presented as a function of the square of the depth-averaged mean velocity ($|\bar{U}|$) in Figure 4.8. The depth-averaged velocity was computed from the expression

$$U = \frac{1}{h} \int_{z=0}^{z=h} U(z) \, dz.$$  \hspace{1cm} (4.1)

Within the constant stress region, the square root of the Reynolds stress is approximately equal to the friction velocity (equation 2.1). Near peak flood and ebb flow, the friction velocities tended to values of between approximately $u^* = \sqrt{-\sigma_{uw}} \approx 0.11 \text{ m s}^{-1}$ and $0.14 \text{ m s}^{-1}$. Therefore, this is within the range of friction velocities that were obtained from the mean velocity distributions (see Section 4.2.2).

The slope of the linear curves that were fitted to the scatter plots can be interpreted as the bottom drag coefficient, $C_d$ (Rippeth et al., 2002). At the Sound of Islay, the drag coefficient was approximately $C_d = 0.0023$ during the flood and 0.0020 during the ebb. The lower drag coefficient at ebb is consistent with the estimates of the smaller roughness lengths during the ebb presented in Section 4.2.2.

The coefficients are between 8% and 20% lower than the drag coefficient of $C_d = 0.0025$ that has been reported for slower open channel flows (Stacey et al., 1999, Rippeth et al., 2002, Li and O’Donnell, 2005). Therefore, the bottom drag coefficient at Islay was not significantly different than other tidal channels. It implies that it is likely to have a near constant value in high Reynolds number flows.
Figure 4.8: The Reynolds shear stress measured by the ADCP at an elevation of 5 m as a function of the depth-averaged mean velocity squared. Data were averaged from 10-minute samples acquired every 2-minutes over the first tidal cycle. For clarity, the Reynolds shear stresses for the ebb are denoted negative. A linear fit to the data for flood and ebb tides is superimposed (- -), with the gradient corresponding to the estimate of the bottom drag coefficient.

4.3.3 Intensities & Stresses Throughout the Water Column

The time histories of the standard deviation of the streamwise velocity throughout the water column over the three tidal cycles are presented in Figure 4.9(a). The estimates incorporate the median values of the standard deviations of the fluctuating velocities that were measured by the ADV.

The magnitude of the standard deviation of the streamwise velocity was largest near the bed. The variation in its value throughout the water column was nearly equivalent between consecutive flood and ebb flow cycles. During the flood and ebb flows, relatively large velocity fluctuations were observed to penetrate up through the water column to elevations of approximately $z/h = 0.5$. These fluctuations occurred over relatively short periods of time (i.e. less than 10 minutes) and appeared to be more severe during the ebb. The magnitudes of the fluctuations at $z/h = 0.4$ were up to $\sigma_u = 0.25$ m s$^{-1}$. This was equivalent to
Figure 4.9: The standard deviation of the streamwise velocity, $\sigma_u$ (a), the Reynolds shear stress, $-\sigma_{uw}$ (b) and the Reynolds shear stress, $-\sigma_{vw}$ (c) throughout the water column over 3 tidal cycles. Data were averaged over 10-minute periods and were acquired at 2 minute intervals.
a streamwise turbulence intensity of $I_u \approx 0.10$.

The corresponding time histories of the Reynolds shear stresses in vertical planes parallel and perpendicular to the free-stream direction are presented in Figures 4.9(b) and (c) respectively. The magnitudes of the Reynolds stresses $-\sigma_{uw}$ were largest near the bed. These stresses were comparatively more variable over the tidal cycle than the standard deviations of the streamwise velocities. Stresses of relatively large magnitude were observed to penetrate through the water column at irregular intervals during the tidal cycle. These events appear to correspond with the relatively large values in the standard deviations of the streamwise velocities that were observed. Therefore, the high turbulence intensities were believed to be associated with the eddies which dominated the turbulent mixing.

The time histories of the Reynolds shear stress $-\sigma_{vw}$ reveal that the relatively large shear stresses that were observed during the ebb tide were localised near the bed (i.e. at elevations up to $z/h = 0.1$). This provides support for the stresses having arisen from topographical features on the bed or shedding from the frame. No significant transverse stresses were observed at elevations corresponding to those of a typical tidal turbine.

Profiles of the streamwise turbulence intensity are presented in Figure 4.10(a). These were averaged over a 20-minute period and were acquired at 1-hour intervals during the first tidal cycle. For clarity, the profiles for mean velocities of $U < 1 \text{ m s}^{-1}$ are excluded. The profiles serve to elucidate the distributions of the velocity fluctuations. During the flood and ebb flows, the averaged turbulence intensity is shown to have remained relatively consistent. The turbulence intensities decreased slightly with increased elevation but remained relatively large at elevations which correspond to a typical rotor. For example, at $z/h = 0.4$ they obtained values of approximately $I_u = 0.08$.

The corresponding profiles of the ratio of the standard deviation of the streamwise velocity to the friction velocity are shown in Figure 4.10(b). These demonstrate that the profiles were able to be approximated relatively well using an exponential function of the form $A \exp (-z/h)$, where $A$ is a constant (as depicted by the dotted lines). The two profiles which are poorly described by this function correspond to the 2nd and 8th mean velocity profile in Figure 4.4. As demonstrated, the boundary layer height was comparatively smaller during these sections of the tidal cycle. This is expected to have attributed to the poor conformance to the exponential model described above, except near the bed.
4.3. Observed Turbulence Properties

Figure 4.10: The streamwise turbulence intensity, (a) and the ratio of the standard deviation of the streamwise velocity to the friction velocity, (b) throughout the water column. The profiles were computed at hourly intervals from 10, 10-minute samples over the first tidal cycle, where data during the ebb is denoted as negative for clarity. An exponential model of the form $A \exp\left(-\frac{z}{h}\right)$ has been fitted to the profiles for peak flow in (b) and is depicted by the dotted lines.

The corresponding profiles of the ratio of the Reynolds shear stress $-\sigma_{uw}$ to the square of the friction velocity are presented in Figure 4.11. These exhibit a comparatively greater degree of variability than the standard deviation of velocity. They generally depict an inversely proportionate relationship with elevation, which is consistent with theory. As for the profiles of the velocity fluctuations, the profiles of the Reynolds stress are approximately symmetrical between the flood and the ebb.

There is evidence of a weak reduction in the magnitude of the shear stress ratios during the flood and a weak increase in the magnitude during the ebb at an elevation of approximately $z/h = 0.2$. These stresses would occur at an elevation too low to affect a tidal typical turbine. However, it does suggest that localised secondary currents could have been present in the flow (Nezu and Nakagawa, 1993).
Figure 4.11: The ratio of the Reynolds shear stress to the square of the friction velocity throughout the water column. The profiles were computed at hourly intervals from 10, 10-minute samples over the first tidal cycle, where data for the ebb is denoted negative for clarity. A model of the form \((1 - z/h)\) has been fitted to the profiles for peak flow and is depicted by the dotted lines.

### 4.3.4 Integral Scales & Correlation Functions

The integral scales provide a useful quantification of the average size of the largest eddies in the flow. They are generally associated with the eddies which contribute the greatest amount to the observed intensities and stresses.

The time history of the integral length-scale of the streamwise velocity \(L_u^x\) observed at the elevation of 5 m is presented in Figure 4.12(a). It shows that the length-scales varied in-phase with the velocity. At peak flood and ebb flows, they tended towards median values of approximately 11 m and 13 m respectively. The integral length-scales exhibited a relatively large degree of variability and they ranged between 5 m and 30 m. The smallest and largest values of the integral scales were typically observed for short periods of time and were seen to occur irregularly throughout the tidal cycle.

The ratios of the integral length-scales of the transverse and vertical velocities to the integral length-scale of the streamwise velocity are presented in Figure 4.12(c) and (d). For isotropic flow, these ratios are expected to be \(L_v^z/L_u^x = L_w^z/L_u^x = 0.5\). The observed ratios decreased with an increase in the
Figure 4.12: The time histories of the integral length-scale of the streamwise velocity, (a) the ratio of the integral length-scales of the transverse and streamwise velocity, (b) the ratio of the integral length-scales of the vertical and streamwise velocity, (c) and the ratio of the integral length-scales of the Reynolds shear stress and streamwise velocity (d). The ratios were truncated near slack flow for clarity.
mean velocity during the tidal cycle. They also exhibited a relatively large degree of variability. At peak flood and ebb flows, the median ratio of the transverse to streamwise scales was approximately \( \frac{L_x}{L_u} = 0.3 \). The median ratio of \( \frac{L_x}{Lu} \) was approximately 0.20 during peak flood and 0.15 during peak ebb.

The median ratio of the integral length-scales of the Reynolds stress to the integral length-scale of the streamwise velocity (\( \frac{L_{uw}}{L_u} \)) is shown in Figure 4.12(d). It provides a useful measure of the relative size of the coherent structures that are associated with the mixing. It was observed to have a value of approximately 0.4 during the flood and 0.2 during the ebb.

The relatively large values observed for the integral length-scales are believed to be attributed to low frequency oscillations in the mean velocity. To demonstrate this, the time histories of the low-pass filtered fluctuating velocities for two 5-minute samples are presented in Figure 4.13. The first sample corresponded to a streamwise integral length-scale (averaged from consecutive 5 samples) of approximately \( L_u = 11 \text{ m} \) and is shown in black. The second sample corresponded to a relatively large integral length-scale of \( L_u = 20 \text{ m} \) and is shown in grey. The velocity time histories reveal that relatively long oscillations (order of 200 s) were present in the streamwise velocity for the second sample.

The autocorrelations and the cross-correlations corresponding to these samples are shown in Figure 4.14. As low frequency oscillations were not removed in the linear de-trending process, they are shown to have given rise to relatively long decays in the correlations. However, the figure also serves to demonstrate that, in general, the streamwise velocity decorrelated much more slowly than for the transverse and vertical fluctuations. The symmetrical part of the cross-correlation function also dominated over the asymmetrical part.

### 4.3.5 Velocity Spectra

The velocity autospectra and co-spectra provide an alternative means of characterising the dominant scales of turbulence. In comparison to the correlation functions, they are also less sensitive to the effect of long eddies and the de-trending processes (Panofsky and Dutton, 1984).

The normalised auto-spectral density corresponding to peak flood and ebb flows at an elevation of 5 m are shown in Figure 4.15. These were averaged from approximately 100 samples during the first cycle and are presented as a function of the non-dimensional streamwise wavenumber (\( k_{xz} \)). These spectra serve to
4.3. Observed Turbulence Properties

Figure 4.13: Examples of the low-pass filtered velocity time histories for two samples analysed. Two cases are presented. The first correspond to a length-scale approximately equal to $L_u^x = 11$ m (black). The second, correspond to a relatively large length-scale of $L_u^x = 25$ m (grey) and show that there is a low frequency oscillation present in the streamwise velocity.

elucidate the distribution of energy across the wavenumbers of interest.

The spectral peak of the streamwise velocity corresponds to $k_{xz} = 0.6$ during the flood flow. The dominant scales for the ebb flow, appear to have been slightly larger and the peak corresponds to $k_{xz} = 0.5$. The spectral peaks of both spectra are approximately 6 times greater than the median integral length-scales observed at peak flow. The streamwise spectral densities for the flood and ebb flow are approximately equal for non-dimensional wavenumbers exceeding $k_{xz} \geq 3$ to 4. For these wavenumbers, the spectral energy appears to tend towards the proportionality predicted by Kolmogorov (1941) within the inertial-subrange.

For both the flood and ebb flow, the peaks of the transverse and vertical velocity spectra correspond to non-dimensional wavenumbers of approximately $k_{xz} = 2$ and $k_{xz} = 3$ respectively. Their ratios to the wavenumber that corresponded to the peak of the streamwise spectra are generally consistent with the
Figure 4.14: Examples of the autocorrelation function of the streamwise, (a) transverse, (b) and vertical, (c) velocity and of the cross-correlation function, (d). The data shown in black correspond to a length-scale approximately equal to $L_u = 11$ m. The data in grey correspond to a relatively large length-scale of $L_u = 25$ m (grey). All data were computed as a mean of five, 5-minute samples.
4.3. Observed Turbulence Properties

(a) Streamwise

(b) Transverse

(c) Vertical

Figure 4.15: Normalised velocity spectra of the streamwise, (a) transverse, (b) and vertical, (c) velocities at an elevation of 5 m above the seabed. Data were averaged from 100, 5-minute samples at maximum flood (black) and ebb flow (grey) and were smoothed over 5 data points. The Kolmogorov (1941) $k_x^{-2/3}$ proportionality is denoted by the dash-dot line.
ratios of the integral length-scales presented in Section 4.3.4. The peak of the vertical spectrum is also comparatively narrower than the streamwise dominant energy peak. This implies that the dominant scales associated with the vertical velocities were more constrained by the presence of the seabed than for the streamwise velocity. The spectra for the transverse and vertical velocities appear to begin tending towards the Kolmogorov (1941) model at higher wavenumbers of approximately $k_x z \geq 4$ to 5.

**4.3.6 Co-Spectra & Quad-Spectra**

**Near-Bed Observations**

The co-spectra and quad-spectra of velocity assist in charactering the stochastic distribution of anisotropy in the flow. The normalised co-spectral and quad-spectral densities at an elevation of $z = 5$ m at peak flood and ebb flows are presented in Figure 4.16. The data were obtained from the same samples as for the velocity spectra.

The peaks of the normalised co-spectral density $-k_x C_{ij}/\sigma_u \sigma_w$ correspond to a normalised wavenumber of approximately $k_x z = 0.8$ during the flood and $k_x z = 1$ during the ebb. Therefore, not all of the dominant streamwise fluctuations were associated with the turbulent mixing. For wavenumbers higher than approximately $k_x z \geq 4$, the co-spectral density decays rapidly at a rate approximately proportional to $k_x^{-7/3}$. This behaviour is consistent with that expected at the low wavenumber end of the inertial subrange.

The quad-spectra have seldom been reported for fast flowing tidal flows. During both the flood and the ebb flows, the normalised quad-spectral density $(k_x Q_{uw}/\sigma_u \sigma_w)$ were negative at low normalised wavenumbers of around $k_x z = 1$. This implies that the streamwise fluctuations ($u(t)$) that were associated with the dominant turbulent mixing lead the vertical fluctuations ($w(t)$).

To interpret this result, it is useful to consider the schematic diagram in Figure 4.17. This shows two eddies near the seabed which are convected with the mean velocity (left to right) without changing their structure (i.e. the frozen turbulence hypothesis of Taylor, 1938 is invoked). Following Panofsky and McCormick (1954), relative to a fixed point near the bottom of the eddy $A$, a positive horizontal fluctuation would be followed by a negative vertical velocity a quarter of a period later. For eddy $B$, a negative horizontal fluctuation would be followed by a positive vertical velocity a quarter of a period later. Based on the definition
4.3. Observed Turbulence Properties

Figure 4.16: Co-spectral density, (a) and quad-spectral density, (b) of the Reynolds stress at an elevation of 5 m above the seabed. Data has been averaged from 100, 5 minute samples at maximum flood (black) and ebb flow (grey) and smoothed using a triangle window 5 data points wide.

of the quad-spectrum in Section 3.4.4, both situations correspond to a negative density value. Therefore, the negative value that was observed implies that the measurements were obtained in their lower half and that their vertical size was likely to be larger than the measurement height.

Observations Throughout the Water Column

The co-spectra at the elevations of $z = 20$ m and $z = 30$ m are presented in Figure 4.18. These were estimated from the ADCP data as an average of approximately 50 samples at peak flood and ebb flow during the first tidal cycle. They are presented in terms of the co-spectral density, multiplied by the wavenumber ($-k_x C_{uu}$).
Figure 4.17: Schematic diagram illustrating the quad-spectrum, adapted from Panofsky and McCormick (1954). To an observer at point $A$ or $B$ near the bottom of the eddy which is convected with the mean velocity (left to right) the quad-spectral density is negative, irrespective of the eddy vorticity.

Figure 4.18: Co-spectral density at elevations of 20 m and 30 m from the seabed. Note that the density has not been normalised by the variance. The $k_z^{-7/3}$ proportionality is denoted by the dashed line.
These co-spectra provide insight into the variation in the dominant scales associated with the Reynolds stresses at the elevations corresponding to a turbine. At an elevation of \( z = 20 \text{ m} \), the peaks of the co-spectra correspond to a non-dimensional wavenumber of \( k_x z = 1.5 \) and for \( z = 30 \text{ m} \), to a non-dimensional wavenumber of \( k_x z = 2.5 \). These values correspond to length-scales of approximately 80 m, which are four-times larger than a typical tidal turbine diameter.

At both elevations, the co-spectral density begins to decay rapidly at wavenumbers exceeding \( k_x z \gtrsim 4 \). The rate of the decay appears to be consistent with a proportionality of \( k_x^{-7/3} \). However, it is difficult to ascertain this relationship confidently due to the relatively low wavenumbers that were resolvable. Furthermore, the effect of the auto-spectral densities on these estimates becomes significant at high frequencies. These terms arise from sensor tilt and are only cancelled out to first-order during the subtraction of beam velocity spectra.

4.4 Discussion of the Turbulence Properties

The key turbulence properties observed at the Sound of Islay are summarised in Table 4.1. These provide the industry with a comprehensive set of statistics that describe the turbulence at a fast flowing tidal energy site. In the following sections, comparisons are made with the limited data that is available for other tidal energy sites as well as theoretical models. This assists in ascertaining the variability in the turbulence between sites and identifying universal characteristics.

4.4.1 Magnitudes of the Turbulent Fluctuations

In general, the streamwise turbulence intensities observed at the Sound of Islay compare favourably with those reported for other fast flowing tidal energy sites. For example, at an elevation of \( z/h = 0.1 \), they were only 10% greater than the intensities at the Fall of Warness, which can be derived from the total turbulent kinetic energy presented by Osalusi et al. (2009) and which incorporate the anisotropic ratios of Nezu and Nakagawa (1993). The intensities at an elevation of \( z/h = 0.21 \) at the Sound of Islay were also only 15% greater than those for the Puget Sound reported by Thomson et al. (2012).

It is also encouraging to find that reliable estimates of the magnitudes of the velocity fluctuations could be obtained using semi-empirical models for turbu-
Chapter 4. Characterisation of the Turbulence at the Sound of Islay

Table 4.1: A summary of the observed turbulence properties at the Sound of Islay at an elevation of 5 m.

<table>
<thead>
<tr>
<th></th>
<th>Flood</th>
<th></th>
<th>Ebb</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>Range</td>
<td>Median</td>
<td>Range</td>
</tr>
<tr>
<td>$u_*$ (m s$^{-1}$)</td>
<td>0.11</td>
<td>0.10 to 0.14</td>
<td>0.11</td>
<td>0.10 to 0.14</td>
</tr>
<tr>
<td>$I_u$</td>
<td>0.13</td>
<td>0.12 to 0.15</td>
<td>0.11</td>
<td>0.09 to 0.14</td>
</tr>
<tr>
<td>$\sigma_u/\sigma_u$</td>
<td>0.75</td>
<td>0.60 to 0.85</td>
<td>0.70</td>
<td>0.6 to 0.85</td>
</tr>
<tr>
<td>$\sigma_w/\sigma_u$</td>
<td>0.55</td>
<td>0.50 to 0.62</td>
<td>0.50</td>
<td>0.45 to 0.62</td>
</tr>
<tr>
<td>$-\sigma_{uw}/2q$</td>
<td>0.11</td>
<td>0.07 to 0.17</td>
<td>0.09</td>
<td>0.05 to 0.15</td>
</tr>
<tr>
<td>$L_u^z$ (m)</td>
<td>11</td>
<td>8 to 20</td>
<td>13</td>
<td>9 to 20</td>
</tr>
<tr>
<td>$L_u^z/L_u$</td>
<td>0.3</td>
<td>0.2 to 0.8</td>
<td>0.3</td>
<td>0.2 to 0.8</td>
</tr>
<tr>
<td>$L_w^z/L_u^z$</td>
<td>0.2</td>
<td>0.1 to 0.5</td>
<td>0.15</td>
<td>0.1 to 0.2</td>
</tr>
<tr>
<td>$L_{uw}^z/L_u^z$</td>
<td>0.25</td>
<td>0.2 to 0.6</td>
<td>0.1</td>
<td>0.05 to 0.3</td>
</tr>
<tr>
<td>$k_{x,z,peak}$</td>
<td>0.5</td>
<td>–</td>
<td>0.6</td>
<td>–</td>
</tr>
<tr>
<td>$k_{x,z,Kolmogorov}$</td>
<td>–</td>
<td>3 to 4</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Turbulence in steady, idealised open channel flows. For instance, the distribution of the standard deviation of the streamwise velocity, normalised by the friction velocity ($\sigma_u/u_*$) conforms well with the exponential model proposed by Nezu and Nakagawa (1993). More specifically, at an elevation of $z/h = 0.08$, the predicted streamwise intensity is equal that observed during the flood flow. It is only 10% greater than that observed during the ebb flow.

The turbulence intensity is a key parameter for tidal turbine loading. The good agreement in its value between these different sites and the semi-empirical correlations could now lead to the magnitudes of the unsteady forcing on a turbine being predicted with more certainty. The Islay measurements also serve to demonstrate that the intensities of between $I_u = 0.20$ and 0.30, which were reported by Li et al. (2010), are uncharacteristically high. As mentioned in Section 2.3.3, these high values may be due to the effects of stratification in the East River, New York. Furthermore, the Sound of Islay measurements imply that the assumption that the ratio $\sigma_u/u_*$ is constant with elevation, based on atmospheric turbulence observations, is less applicable to tidal flows.

4.4.2 Anisotropy in the Flow

The ratios of the standard deviations of the velocities provide a useful quantification of the anisotropy of the flow. However, they have seldom been reported previously for fast flowing tidal streams. The observed ratios of the standard
deviations of the transverse and vertical velocities to the standard deviations of the streamwise velocities are within only 5% of the ratios of $\sigma_v/\sigma_u = 0.71$ and $\sigma_w/\sigma_u = 0.55$ that are predicted for open channel flows (Nezu and Nakagawa, 1993). This good alignment with these channel flow corrections implies that the ratios observed for atmospheric turbulence, of $\sigma_v/\sigma_u = 0.67$ and $\sigma_w/\sigma_u = 0.45$ are also less valid for strong tidal flows.

Furthermore, the observed ratio $-\sigma_{uw}/2q$ at $z/h = 0.1$ at the Sound of Islay was nearly equal to that found near the bed of open channel flows in the laboratory (see Nezu and Nakagawa, 1993) and in natural tidal streams (Walter et al., 2011). The Reynolds stress profiles at Islay were also reasonably consistent with boundary layer theory. Therefore, it is postulated that the comparatively more irregular profiles observed at the Fall of Warness were influenced by factors that were specific to that site. However, this serves to reinforce that the collection of turbulence data at the actual site at which the turbine is to be deployed is still favourable, given that it could reveal important site-specific characteristics.

4.4.3 Scales of Turbulence & Distribution of the Energy

Observations of integral-scales for fast flowing tidal sites in the literature are also scarce. However, it is interesting to find that streamwise integral-scales between 30% to 50% larger than observed at Islay could be estimated from the correlations proposed by equation 2.10. Therefore, in spite of its inherent assumptions associated with an idealised steady flow and a relatively smooth bed, the model could provide reasonable estimates of this important scale.

The relationships between the co-spectra at 5 m and 30 m provide useful information into the variation in the scales with elevation. These showed that the peak co-spectral density did not scale exactly with elevation above the bed, as observed for atmospheric turbulence (Lumley and Panofsky, 1964). The higher wavenumber is consistent with the free-surface having constrained the size of the eddies associated with the turbulent mixing.

It is important to consider that the energy-producing scales demonstrated a large degree of variability over the tidal cycle, which would not be accounted for using a simplistic model. The observations that the energy-producing eddies encompass a broad range of scales appears to be a characteristic of tidal channels in general (see e.g. Trevethan, 2007, Walter et al., 2011 and Thomson et al., 2012). As demonstrated in Section 4.3.4, the relatively large integral scales are
Chapter 4. Characterisation of the Turbulence at the Sound of Islay

associated with oscillations at length-scales several orders of magnitude greater than the channel depth. Given this finding, they are not expected to be due to the turbulence generated from the bed. It is postulated that the low frequency perturbations reflect the unsteady nature of the mean velocity over the tidal cycle, or that there are very slow but large meandering structures in the flow.

The form of the streamwise spectra at high wavenumbers is of interest for identifying the scales at which the turbulence tends towards being isotropic. Evidence for the inception of a near $k_x^{-5/3}$ behaviour at wavenumbers higher than $k_x z = 3$ to 4 is consistent with that observed at Puget Sound (which were derived as $k_x z = 3.9$ to 5.9; see Section 2.3.4) and at the Pickering Passage (Lien and Sanford, 2000), as well as with the theoretical predictions of $k_x z = 4.5$ (Tennekes and Lumley, 1972). Therefore, the Sound of Islay spectra provide further support for the non-dimensional wavenumber at which the subrange occurs, being a universal property at a fast flowing natural tidal stream. At elevations of $z/h = 0.4$ to 0.5, $k_x z = 5$ corresponds to scales of the order of the rotor diameter at the Sound of Islay. Therefore, this would imply that the Kolmogorov (1941) model could be employed to predict the turbulent fluctuations at scales relevant to a tidal turbine.

4.5 Implications for Tidal Turbine Loading

It is important to also consider that the turbulence observed at a fixed location in the free-stream differs from that which will be perceived by the turbine blade. In the following sections, theoretical models are used to estimate the amplification of the turbulent fluctuations, as well as the coherence of the turbulence across the rotor blade. These results assist in establishing the values of the non-dimensional forcing parameters that are relevant to a tidal turbine blade.

4.5.1 Amplification of the Turbulence Fluctuations

During normal operation, a tidal turbine would act to extract momentum from the free-stream. This leads to an expansion of the stream-tube bounding the rotor plane. The effect is considered to be analogous to flow having passed through a diffuser.

Batchelor (1953) used rapid distortion theory to predict the decrease in the turbulent kinetic energy for a turbulent stream subjected to a sudden contraction.
The solution can also be applied to a sudden expansion (i.e. a diffuser). In a novel approach, this solution was applied in this thesis to predict the amplification of the velocity fluctuations due to a turbine.

The application of the linearised theory requires that the inertia and viscous forces can be neglected during the distortion. This implies that the ratio of the standard deviation of the streamwise velocity \( \sigma_u \) to the mean velocity \( U \) in the free-stream must be much smaller than the ratio of the length-scale of the turbulence \( L \) to the diameter of the distortion tube \( D \) (analogous to the turbine diameter), i.e.

\[
\frac{\sigma_u}{U} \ll \frac{L}{D}.
\] (4.2)

At the Sound of Islay, the former ratio corresponded to \( \sigma_u/U \approx 0.1 \). The integral length-scales were also of the order of the turbine diameter or larger. Therefore, the condition expressed by equation 4.2 is considered to be sufficiently met.

The solution presented by Batchelor (1953) also assumed that the turbulence was isotropic. The largest scales of the turbulence observed at the Sound of Islay were not isotropic. Nevertheless, at an elevation of \( z/h \approx 0.5 \), the turbulence was tending towards isotropy at scales corresponding to the rotor diameter (see Figure 4.18). This was deemed to be sufficient for the application of this model in this thesis to provide a first-approximation of the amplification.

A turbine would act to impart a symmetrical distortion of the turbulence. Batchelor (1953) showed that in this case, the principal strains in the axial and lateral directions reduce to \( \epsilon_1 = 1/e \) and \( \epsilon_2 = \sqrt{e} \) respectively. The expansion ratio \( e = A_d/A \) is assumed here to be equal to ratio of the cross-sectional areas of the stream-tube in the free-stream \( A \) to the rotor disk \( A_d = 0.25\pi D^2 \).

From continuity, the expansion ratio can be expressed in terms of the free stream velocity upstream and at the rotor plane \( U_d \), i.e.

\[
e = \frac{A_d}{A} = \frac{U}{U_d}.
\] (4.3)

By applying one-dimensional linear momentum theory to an actuator disk, the velocities can be related through the axial induction factor \( a \) (see e.g. Burton et al., 2001). Therefore, the expansion ratio can be expressed in terms of the unaffected upstream velocity as

\[
e = \frac{U}{U_d} = \frac{U}{U (1-a)} = \frac{1}{(1-a)}.
\] (4.4)
Chapter 4. Characterisation of the Turbulence at the Sound of Islay

For a turbine operating in optimal conditions, the axial induction factor is equal to \( a = 1/3 \) (Burton et al., 2001). This corresponds to an expansion ratio of \( e = 1.5 \).

The ratios of the standard deviation in the streamwise and lateral turbulence velocity and the kinetic energy at the rotor, to their respective values to that upstream \( (\sigma'_u/\sigma_u, \sigma'_v/\sigma_v, \text{ and } q'/q) \) are presented as a function of the expansion ratio in Figure 4.19. The expressions which were used to compute these values are presented in Appendix B.

![Figure 4.19: Estimates of the amplification of the turbulence fluctuations for a sudden expansion as a function of the expansion ratio, \( e = A_d/A \). A typical turbine is expected to correspond to an expansion ratio of \( e = 1.5 \). Legend: \( \sigma'_u/\sigma_u (\text{-}), \sigma'_v/\sigma_v (\text{- -}), q'/q (\text{-.-}), \) where the superscript denotes conditions at the rotor plane.

For an expansion ratio corresponding to an ideal turbine, the amplification of the standard deviation of the streamwise velocity is approximately 15\%. This is considered to be a significant increase. It is also important to consider that with respect to the rotor plane, the streamwise turbulence intensity is amplified by \( I'_u/I_u = (\sigma'_u/U_d) / (\sigma_u/U) = (\sigma'_u/\sigma_u) e = 2.0 \). This is a consequence of the mean velocity having been reduced relative to the free stream.

There is a corresponding decrease in the lateral fluctuations by 20\%. However, for a turbine this is not significant, as the magnitudes of the transverse and vertical fluctuations are relatively small. The amplification in the streamwise direction is also sufficiently large that the total turbulent kinetic energy of the turbulence would also increase.
4.5. Implications for Tidal Turbine Loading

4.5.2 Rotational Sampling of Turbulence

During each revolution, the turbine blade rotates through a turbulence field that is not fully coherent. Lower frequency eddies can also be re-encountered on subsequent rotations. This action gives rise to significant energy contributions at multiples of the rotational frequency of the rotor. As previously discussed in Section 1.3.1, this effect is considered to be a primary contributor to the unsteady loading of a tidal turbine.

By linearising the loading, Burton et al. (2001) provided a stochastic expression for the out-of-plane blade root bending moment. This was defined as

\[ S_{my}(n) = \left( \frac{1}{2} \rho \Omega \frac{dC_l}{d\alpha} \right)^2 \int_0^R \int_0^R S_u(r_1, r_2, n) c(r_1) c(r_2) r_1^2 r_2^2 dr_1 dr_2, \]

where \( \rho \) is the water density, \( \Omega \) is the rotor speed, \( \frac{dC_l}{d\alpha} \) is the two-dimensional lift slope and \( c(r_1) \) is the chord at a blade section at a radius distance \( r_1 \) from the hub. The rotational cross-spectrum of the streamwise turbulence between two points on the blade at distances of \( r_1 \) and \( r_2 \) from the hub \( (S_u(r_1, r_2, n)) \) is of primary interest. A theoretical expression for the cross-spectrum has been presented by Burton et al. (2001) and is provided in Appendix C for reference. This was derived from the isotropic von Kármán spectrum and its corresponding correlation functions.

As stated in the previous section, the large-scale turbulence at the Sound of Islay was not isotropic. However, as shown in Figure 4.20(a), the von Kármán spectrum provides a relatively good description of streamwise frequency spectra that were observed at 5 m at the Sound of Islay during the flood flow. Therefore, the theoretical model was considered to be useful for providing new qualitative insights into the spectra that could be observed by a tidal turbine blade.

The theoretical cross-spectra corresponding to the tip of a 10 m turbine blade and two further in-board sections at 6 m and 4 m are also shown in Figure 4.20(a). These are based on a tip-speed ratio of \( \lambda = \Omega R/U = 5 \) and an integral length-scale of 11 m. This integral length-scale is equivalent to that which was observed at an elevation of \( z = 5 \) m at the Sound of Islay.

These spectra show that the rotational energy at the low frequencies is significantly reduced relative to the fixed-point spectrum and the energy is instead concentrated at the rotational frequency of the rotor. The cross-correlation function demonstrates that for frequencies near the rotational frequency, the correlation of
Figure 4.20: Rotational sampled streamwise auto-spectral density observed at a point at \( r = 10 \) m (solid black line) and the rotationally sampled cross-spectrum between points at \( r_1 = 10 \) m and \( r_2 = 6 \) m (- -) and \( r_2 = 4 \) m (-.-), assuming the integral-length scales of \( L_u = 11 \) m, (a) and \( L_u = 42 \) m, (b). The model is derived from the von Karman spectrum and assumes a tip-speed ratio of \( \lambda = 5 \). The corresponding auto-spectrum are shown in grey, with the observed spectrum at 5 m also shown in (a) in grey (bold).
the turbulence remains relatively high across the outer-blade sections. However, in contrast to the auto-spectra, the energy of cross-spectra decays more rapidly at higher frequencies than this. Therefore, the forcing at frequencies corresponding to higher multiples of the rotational-frequency of the rotor is not considered to be significant.

An integral length-scale of 11 m may be considered to be relatively small to that a typical rotor hub. For example, the integral length-scale is estimated to be 42 m at mid-depth at the Sound of Islay using the correlations proposed by Nezu and Nakagawa (1993). Furthermore, the time histories have demonstrated that short term fluctuations the integral length-scales occur at irregular intervals over the tidal cycle. Therefore, gaining insight into the effect of a range of integral-length scales on the rotational spectrum is warranted.

Given this, the auto-spectrum and the cross-spectrum for the equivalent rotor parameters, but for an integral length-scale of $L_u^* = 42$ m are presented in Figure 4.20(b). The rotational sampling effect (i.e. the distortion of the spectrum) at very low frequencies is shown to be significantly reduced following a decrease in the ratio $r/L_u^*$. However, the turbulent energy at the revolution frequency and the correlation of the turbulence across the outer blade sections remains relatively high.

The total variance remains constant between the Eulerian and rotational systems (Connell, 1982). This implies that the turbulence intensity measured in the free-stream is now concentrated at the rotational frequency. Therefore, the dominant frequencies of interest for a tidal turbine are expected to range up to $f = 0.4$ Hz.

An important consequence of the relatively high correlation across the blade length predicted for combinations of tip-speed ratios and of blade length to integral length-scale ratios typical for tidal turbines, is that the forcing on a blade could be approximately replicated by subjecting the rotor to an oscillatory planar forcing. This is because the out-of-plane bending moment is dominated by the loading over a relatively small region at the outer-span of the blade.

### 4.5.3 Non-Dimensional Forcing Parameters

An objective of the turbulence analysis was to establish key non-dimensional parameters that would describe the unsteady forcing that would be imparted on the blades due to turbulence. In Section 2.5.3, the reduced frequency and
frequency ratio were identified as being appropriate for describing the frequency of the unsteady fluctuations. Estimates of the reduced frequency \( k = \frac{\pi f c}{\Omega r} \) were obtained for a rotor diameter of 20 m and rotor speed of 15 rpm. They were based on a blade section at \( r = 0.75 R \), where the chord length was assumed to be 1 m, which is consistent with Whelan (2010).

The minimum reduced frequency applicable was considered to correspond to the integral length-scale of \( L_u^x = 42 \text{ m} \) and a streamwise mean velocity of \( U = 2.5 \text{ m s}^{-1} \). The maximum reduced frequency was deemed to correspond to the rotational frequency of the rotor, owing to the effect of rotational sampling of turbulence. Therefore, the relevant reduced frequencies were expected to range between approximately \( 0.02 < k \leq 0.10 \). It follows that the range of frequency ratios of interest are between \( m = 0.24 \) and 1.

The Current number \( (\mu = \bar{u}/U) \) was used to scale the velocity amplitude. Its lower value was assumed to be equivalent to the turbulence intensity observed at the Sound of Islay at an elevation of \( z/h = 0.4 \). Its maximum value was estimated by incorporating the amplification of the streamwise velocity fluctuations, as predicted in Section 4.5.1.

The Keulegan-Carpenter number \( (KC = \bar{u}T/D) \) was also cited in Section 2.5.3, as a parameter by which the frequency and velocity amplitude could be scaled together. Estimates of the relevant range of \( KC \) numbers were based on the frequencies and velocity amplitudes that were incorporated into the estimates of the reduced frequencies and the Current numbers. The ranges of all these parameters are summarised in Table 4.2.

Table 4.2: Estimates of the non-dimensional parameters used to describe the forcing due to turbulence.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced frequency, ( k_{0.75R} )</td>
<td>0.02 to 0.10</td>
</tr>
<tr>
<td>Frequency ratio, ( m )</td>
<td>0.25 to 1</td>
</tr>
<tr>
<td>Current number, ( \mu )</td>
<td>0.10 to 0.25</td>
</tr>
<tr>
<td>Keulegan-Carpenter number, ( KC )</td>
<td>0.02 to 0.13</td>
</tr>
</tbody>
</table>

The ranges of the aforementioned parameters were based solely on the higher frequency characteristics of the flow. However, it also important to consider that the variation in the mean velocity with depth would also induce a once-per-revolution forcing. An estimate of this corresponding velocity fluctuation was obtained from the velocity profiles presented in Figure 4.4. For a 20 m diameter
rotor with a hub at an elevation of \( z = 30 \) m, the amplitude of the velocity fluctuation observed at a blade radius of \( r = 0.75R \) would be approximately equal to \( u = 0.1 \) m s\(^{-1}\). Therefore, the Current numbers could be 50% larger than are listed in Table 4.2.

Furthermore, whilst the flow was near bi-directional at the Sound of Islay, this may not be the case at other sites. Flow misalignment to the rotor is expected to also impart an appreciable once-per-revolution load on turbines that do not have a yaw mechanism. Together, these two effects reinforce the need to investigate the rotor loading corresponding to relatively large Current numbers.

### 4.6 Summary

A comprehensive new set of parameters characterising the magnitudes and scales of the turbulence fluctuations at a tidal energy site have been obtained from the Sound of Islay during the spring tide. During this period, the mean velocity exceeded 2.5 m s\(^{-1}\) and the water level ranged between 59 m and 60 m. The turbulence was generated from shear on the seabed. The boundary layer height typically exceeded \( z/h \geq 0.8 \) and the law of the wall applied up to \( z/h = 0.2 \).

Near the bed at an elevation of 5 m (\( z/h \approx 0.1 \)), the streamwise turbulence intensity was between \( I_u = 0.11 \) and 0.13. Its value remained relatively large throughout the water column. At elevations representative of a tidal turbine of \( z/h = 0.4 \) to 0.5, the turbulence intensity was approximately equal to 0.08. The magnitudes of the turbulence intensities were generally consistent with those reported from other fast flowing tidal streams. The anisotropy in the flow near the bed was also consistent with that expected for idealised open channel flows and natural tidal streams.

The streamwise integral length-scales at an elevation of \( z = 5 \) m were estimated to be between \( L_x = 11 \) m and 13 m. They exhibited a large degree of variability and were up to 20 m. The largest integral-length scales were a consequence of low frequency oscillations in the flow. The dominant scales of the transverse, vertical velocities, and of the Reynolds stresses were comparatively much smaller and typically less than \( 0.3 L_x \).

The spectra of the streamwise velocity near the bed tended towards inertial subrange behaviour at non-dimensional wavenumbers between \( k_x z = 3 \) and 4. The co-spectral density at elevations corresponding to typical hub heights also tended to decrease rapidly from this non-dimensional wavenumber. At these
elevations, the turbulence was tending towards isotropic at the length-scales corresponding to a tidal turbine blade.

The standard deviation of the upstream velocity was predicted to be amplified by 15% at the rotor with respect to the free-stream, which is a significant increase. It was also established that the turbulent eddies are likely to be well correlated along the outer-span of the blade at once-per-revolution frequencies. This supports the use of planar oscillations to study the effects of turbulence on tidal turbine blade loads.

Most importantly, the analyses in this chapter revealed that the unsteady forcing on a blade due to turbulence is characterised by reduced frequencies of between $k = 0.03$ and 0.10, frequency ratios between $m = 0.25$ and 1, Current numbers of up to $\mu = 0.25$ and $KC$ numbers between 0.02 and 0.13.
Chapter 5

Set-Up of the Turbine Tank Testing

If you cannot measure it you cannot improve it
— Lord Kelvin

5.1 Introduction

A model-scale tidal turbine in a towing tank was used to study the unsteady hydrodynamic blade loads due to turbulence.

This chapter details the set-up and methodology of these experiments. It describes the specifications of the facility, the configuration of the towing carriages, the design of the turbine and the control strategy that was employed. An analysis of the uncertainties that were inherent in the experiment is also presented.

The performance and loading of the turbine in steady flow is then characterised. These measurements provide a basis to quantify the relative contribution of the unsteady hydrodynamics to the blade loads.

5.2 Facility & Towing Carriages

The turbine experiments were conducted at the Kelvin Hydrodynamics Laboratory towing tank at the University of Strathclyde in Glasgow over two campaigns, one in September 2010 and one in March 2012. The still water tank has a working
Chapter 5. Set-Up of the Turbine Tank Testing

length of 76 m, width of 4.6 m and a maximum depth of 2.5 m. The water depth was maintained at 2.23 m and 2.00 m for the 2010 and 2012 tests respectively. The effect of reflected waves was mitigated, in-part, by a 9 m long beach with a reflection coefficient of less than 5%, and by absorbers positioned on the side walls of the tank.

The experiments were performed by using a combination of two towing carriages. An auxiliary carriage which was attached to the main carriage, was used to generate a planar perturbation. It was driven by a computer controlled Parker ETB 100 linear actuator, with a maximum displacement range of 1.5 m. A prescribed displacement time history was input to the controller to generate the desired perturbation kinematics. Owing to the comparatively larger inertia of the main carriage, its speed was held constant during the tests, replicating a mean flow past the turbine. The facility and the towing carriages are shown in Figure 5.1 and a schematic diagram of the test set-up is provided in Figure 5.2.

Prior to the commencement of each individual test run, data were collected for a period of three rotor revolutions with zero axial velocity and a rotational speed of 3 rpm. At this low rotational speed, the blade loading was considered to be due to gravitational forcing only, which allowed the hydrodynamic response to be isolated (see Section 7.2.2). For each oscillatory test, the auxiliary carriage motion commenced whilst the main carriage was accelerating. This facilitated the decay of non-periodic transients in the kinematics and the blade loading histories and maximised the useful test period.

The first test run of the day served to agitate the water in the tank and any data from these tests were discarded. Absorbers were raised on the sides of the tank following each individual test to dampen any induced wave energy, particularly in the transverse direction. This enabled subsequent test runs to be performed after a relatively short settling period (approximately 5 minutes).

5.3 Turbine & Instrumentation

5.3.1 Turbine Design

A horizontal-axis, tri-bladed turbine with a diameter of 780 mm and a 120 mm diameter hub was used in the experiments and is shown in Figure 5.3. The blockage of the rotor, based on the projected frontal area, was relatively low at 4.7% compared to previous studies, such as those of Bahaj et al. (2007a) (between
Figure 5.1: From top, the towing tank facility, the configuration of the towing carriages, and the turbine as installed on the auxiliary carriage (submerged in the water).
Chapter 5. Set-Up of the Turbine Tank Testing

Figure 5.2: Schematic diagram of the set-up of the turbine experiments. The water depth and distance of the hub from the water surface correspond to the 2010 trials.

8% and 17%) and Whelan (2010) (35%). Bahaj et al. (2007a) estimated that the effects of blockage in the towing tank increased their measured thrust coefficient by 5%. Therefore, as the blockage ratio for the turbine used in this research was 40% smaller than for the study by Bahaj et al. (2007a), its effects were deemed to be sufficiently small as to not necessitate a correction. Additionally, following the approach of Whelan et al. (2009), the likely influence of the free-surface was considered to be negligible.

The mechanical components were housed in a submerged rectangular enclosure, at a distance approximately 2-diameters downstream of the rotor. This was mounted rigidly to the auxiliary carriage from above. The rotor axis was 0.70 m and 0.47 m below the mean free water surface for the 2010 and 2012 trials respectively. A nose cone was fitted to the hub, which was adapted from a previous model turbine and manufactured from high-density foam.

A Mclennan M600 series DC servo motor was connected in-line to the rotor shaft. This set-up was deemed to be more favourable for control purposes than, for instance, using a 90-degree bevel shaft as employed in other experimental studies (e.g. Myers and Bahaj, 2006, Bahaj et al., 2007b and Galloway et al., 2010). The rotor torque and thrust were measured using a Futek MBA500 biaxial sensor. This was installed between the hub and the 20 channel, GAT ESR 100 RotorFlex slip-ring.
Figure 5.3: The turbine prior to installation on the carriage.
5.3.2 Blade Profile & Hydrofoil Selection

The turbine blades tested were originally intended for use in a contra-rotating tidal turbine, which was also tested at the University of Strathclyde (see Clarke et al., 2007). These had a non-uniform profile, with the chord and twist varying along the span as depicted in Figure 5.4. The pitch angle between the blades was aligned using two dial gauges to a tolerance within 0.2°. This process is demonstrated in Figure 5.5.

The blade sections conformed to the 24% thick (defined with respect to the chord) NREL S814 profile. This was selected given its high strength and roughness insensitivity (Somers, 1997) and was deemed to be representative of a section which is likely to be employed in full-scale tidal turbines. These attributes were also cited for the use of the S814 foil in a previous model-scale tidal turbine study of Barltrop et al. (2006).

![Figure 5.4: The chord and twist distribution along the blade.](image-url)
5.3.3 Measurement of the Blade-Root Bending Moment

The three blades were instrumented with strain gauges in a Wheatstone bridge configuration at the blade root, at a radius of \( r_{sg} = 36 \text{ mm} \) from the rotor axis. These were sealed water-tight and were enclosed within the rotor hub, as is shown in Figure 5.6. One blade was used to measure the in-plane bending moment and a second blade was used to measure the out-of-plane moment. The measurements from the strain gauge on the third blade were not able to be acquired due to an insufficient number of channels available on the slip-ring. However, the installation of strain-gauges on all three blades ensured that the hub was balanced and provided a degree of redundancy.

As Papadopoulos et al. (2000) discuss, strain gauges are sensitive to a number of errors. The installation on a cylindrical blade root was an attempt to minimise the effect of stress concentrations. Utmost care was taken to ensure that they were correctly bonded and aligned to the in-plane and out-of-plane axes. During testing, measurements were acquired only when the temperature had stabilised and no drift in the signals was observed.
Amplification of the strain gauge signals (and of the thrust and torque) was performed after the slip-ring, using a six channel Ogawa Seiki DSA100 strain amplifier. Whilst amplification of the signal in the hub was considered, it was deemed to be too problematic. This would have required the construction of a miniature amplifier that would be prone to water ingress and less likely to be linear and stable. It is acknowledged that the slip-rings could have potentially added resistance and noise to the un-amplified signals. Upon recommendations by the slip-ring manufacturer, these effects were deemed to be minor and, assuming that the noise was Gaussian with zero mean, could be filtered out. A high signal quality was confirmed in the preliminary tests which were performed in situ, in and out of the tank. The calibration of the signals was also performed with all components connected, both prior and post-testing.
5.4 Control Strategies, Data Acquisition & Quality of Motion

5.4.1 Methodology

A range of strategies was considered for the speed control of the rotor. The use of a closed-loop constant torque system would require the measured shaft torque to be incorporated into the control system. This was deemed to be potentially problematic in the context of an already complex system. Using a generator to control speed effectively restricts the experiment to the simulation of behaviour of a particular system, which is in conflict with the aim of studying idealised hydrodynamics. Hence, the tests were performed at constant rotational speed, using a closed-loop digital controller. An EM200 DC servo system, with tacho-generator feedback, was used to provide bi-directional positional control.

A Cambridge Electronic Design Power1401 data acquisition system was used to acquire all signals except the velocity of the main carriage. The sampling rates for the 2010 and 2012 trials were $n_s = 400$ Hz and 376 Hz, respectively. A low-pass digital filter with a cut-off frequency of 300 Hz was applied to all signals in real time.

5.4.2 Achievable Reynolds Numbers

The use of a tachometer in the speed control loop and a limitation on the maximum voltage output of 10 V, effectively restricted the maximum rotor speed to 96 rpm for both the 2010 and 2012 experiments. The maximum carriage velocity was also limited to approximately $1.00 \text{ m s}^{-1}$. This was due to the need to prevent water spilling into the turbine enclosure and to not overload the torque transducer. Together, this restricted the minimum tip-speed ratio able to be considered for each rotor speed.

It was not practical to replicate the full-scale Reynolds numbers at model-scale. For the 2010 tests, a rotor speed of 96 rpm corresponded to a Reynolds number at $0.75 R$ of $1.05 \times 10^5$. Therefore, the model scale Reynolds numbers were an order of magnitude lower than would be expected at full-scale (see Section 2.5.3). The implications of these relatively low Reynolds numbers on the foil performance was, ultimately, the motivation for conducting the wind tunnel tests (presented in Chapter 6).
It is also important to note that the temperature of the water in the tank differed between the two test campaigns. For the 2010 and 2012 tests, the temperatures nominally were 16°C and 13°C respectively. As a consequence of the difference in the kinematic viscosity, the Reynolds numbers for the 2012 tests were approximately 8% lower than those conducted in 2010.

5.4.3 Quality of Motion & Achievable Forcing Parameters

The quality of the forcing that was able to be achieved in the experiment is demonstrated in Figures 5.7 and 5.8. The time histories show that the kinematics were approximately sinusoidal for both a single frequency and a multi-frequency forcing. However, irregularities were present for combinations of relatively high frequencies and low amplitudes. This was a consequence of the physical limitations with the actuator used to drive the auxiliary carriage and the controller.

![Figure 5.7: Time histories of the kinematics for a single frequency forcing, demonstrating the quality of oscillatory motion that was able to be achieved in the experiment.](image)

Legend: $f = 0.50$ Hz, $\mu = 0.1$ (..), $f = 0.89$ Hz, $\mu = 0.1$ (−−), $f = 0.50$ Hz, $\mu = 0.2$ (−−), and $f = 0.89$ Hz, $\mu = 0.2$ (−).

Example of the spectra of the kinematics and of the out-of-plane bending moment for a relatively high, single frequency oscillation are presented in Figure 5.9. For this case, the blade is believed to have experienced dynamic stall, which corresponds to the peak in the bending moment spectra at $2f$ and $3f$. The spectra show that whilst structural vibrations were minimal, the irregularities in the kinematics appear as energy peaks at higher multiples of the dominant frequency (i.e. $5f$, $6f$ and $7f$). In this example the spectral energy associated with
5.4. Control Strategies, Data Acquisition & Quality of Motion

Figure 5.8: Time histories of the kinematics for a multi-frequency forcing corresponding to an oscillation which comprised the frequencies of \( f = 0.40 \text{ Hz} \) and \( 0.50 \text{ Hz} \), with Current numbers of \( \mu = 0.750 \) and \( 0.125 \) respectively. To illustrate the quality of motion that was able to be achieved, a linear harmonic fit is superimposed in grey, with the corresponding instantaneous difference the actual and model-fit shown in the figures below.

Dynamic stall is more dominant than of the irregularities. However, the underlying hydrodynamics is increasingly difficult to separate (or filter out) from the forcing as the irregularities become more severe.

The need to minimise these irregularities restricted the maximum frequency at which the blade loading could be investigated. The limiting frequency was deemed to be \( f = 1.40 \text{ Hz} \), for which the velocity amplitude was required to be relatively large and at least \( \mu/U \geq 0.2 \). This corresponded to a reduced frequency at \( r = 0.75R \) of \( k = \pi f c/\Omega r \leq 0.08 \) and a frequency ratio of \( m = 1 \), which was based on a rotor speed of \( \Omega = 84 \text{ rpm} \).

However, for lower Current numbers, the majority of the tests were limited to a frequency of \( f = 0.80 \text{ Hz} \). For a rotor speed of \( \Omega = 96 \text{ rpm} \), this corresponded
to a reduced frequency of $k = 0.04$ and a frequency ratio of $m = 0.5$. Therefore, the maximum reduced frequencies and frequency ratios that were achieved in the experiment were typically 40% and 50% of their values that could be expected at full-scale turbine, respectively. Despite this, these reduced frequencies were nearly equivalent to the maximum reduced frequency of $k = 0.048$ in the surface-wave tests reported by Barltrop et al. (2006). They were also twice as high as the maximum value of $k = 0.02$ ($m = 0.10$) which was achieved in the planar oscillatory experiments by Whelan (2010) (see Section 2.5.2).

![Figure 5.9: Examples of the spectra of the oscillatory velocity, (a) and of the out-of-plane blade-root bending moment, (b). The frequency has been non-dimensionalised by the oscillatory frequency of $f = 0.64$ Hz. The oscillation was performed at a Current number of $\mu = 0.2$ and tip-speed ratio of $\lambda = 3.6$. The spectral energy in the bending moment around the peaks corresponding to $2f$ and $3f$ arises from the dynamic stall of the blade (see Section 7.4).]
5.5 Experimental Uncertainties

5.5.1 Methodology

An uncertainty analysis was conducted to estimate the experimental errors associated with the measurement of the out-of-plane bending moment and other relevant parameters. Experimental uncertainties for unsteady testing of tidal turbines have seldom been reported in the literature and a methodology specific to unsteady turbine testing was not able to be identified. Therefore, the methodology that was subsequently employed is based on that presented by Coleman and Steele (2009) for general model testing and the recommended guidelines published by the International Towing Tank Committee (ITTC, 2006, 2008a,c).

The uncertainties of a variable \( X \) were categorised as being either systematic \((b_X)\), in that they were inherent and equal in all tests, or random \((s_X)\) and varied between each test. The total uncertainty of variable \( X \) can then be expressed as

\[
u^2_X = s^2_X + \sum_{k=1}^{M} b^2_k, \tag{5.1}\]

where \( k \) is the number of systematic sources of uncertainty. For variables that were correlated, the uncertainty was estimated using the Taylor method of propagation (see Coleman and Steele, 2009). The systematic and random uncertainties are summarised in Table 5.1 and are discussed further in the following sections.

Table 5.1: Summary of the systematic and random uncertainties associated with the measurement of the blade root bending moment, its coefficient for steady flow, and the tip-speed ratio.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Systematic, ( b_X )</th>
<th>Random, ( s_X )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady</td>
<td>Oscil. 1</td>
</tr>
<tr>
<td>( M_y ) ((10^{-3} \text{ N m}))</td>
<td>17</td>
<td>0.65</td>
</tr>
<tr>
<td>( U ) ((10^{-3} \text{ m s}^{-1}))</td>
<td>3</td>
<td>0.39</td>
</tr>
<tr>
<td>( u ) ((10^{-3} \text{ m s}^{-1}))</td>
<td>0.07</td>
<td>–</td>
</tr>
<tr>
<td>( \Omega ) ((10^{-3} \text{ rad s}^{-1}))</td>
<td>0.38</td>
<td>0.52</td>
</tr>
<tr>
<td>( C_{M_y} ) ((10^{-3}))</td>
<td>0.6</td>
<td>0.025</td>
</tr>
<tr>
<td>( \lambda ) ((10^{-3}))</td>
<td>1.2</td>
<td>1.6</td>
</tr>
</tbody>
</table>
5.5.2 Systematic Uncertainties

Bending Moment

The systematic error in the measurement of the root out-of-plane bending moment was considered to be composed of four uncorrelated elemental errors. These were due to calibration of the strain gauge using M1 standard weights and a lever arm ($b_{My, calib} = 0.015 \text{ N m}$), scatter (twice the standard error 2SEE) in the least-square fit of the strain gauge calibration ($b_{My, curvefit} = 2 \text{SEE} = 0.008 \text{ N m}$), the 16-bit analogue-digital conversion of the signal ($b_{AD} = 6 \times 10^{-5} \text{ N m}$) and the misalignment of the strain-gauge. The total systematic error of the bending moment was found to be dominated by the calibration uncertainty and, more specifically the uncertainty from the weights. Any transient effects from the strain gauges were considered to be negligible as they would have a first order response (ITTC, 2008c).

Velocity

Two independent measurements of the velocity of the main carriage were acquired. These were using both an encoder on the driving motor and a counter on the trailing wheel of the carriage. Ascertaining the systematic uncertainties associated with the velocity is challenging due to the complexities inherent in the systems and the physical difficulties involved in measuring the key components. Direct comparisons between the two measurement sources were found to agree well. As a case in point, for a desired velocity of $U = 0.894 \text{ m s}^{-1}$, which is the maximum mean velocity used in the oscillatory tests, the measurements from the encoder and trailing wheel from one particular run were 0.13\% greater and 0.40\% lower than the desired velocity respectively. However, the encoder measurement was generally deemed to be more accurate and was subsequently used in all tests. Furthermore, previous experiments have shown that the speed measurements are within the thresholds recommended by ITTC (2008b) for resistance tests.

The uncertainty in the auxiliary carriage velocity arose from the mechanical system and the encoder measurement. The repeatability of the actuator displacement was stated by the manufacturer as $\pm 0.07 \text{ mm}$ and which is attributed to the backlash in the 40 mm pitch ballscrew. Another source of error was due to the variation in pitch of the ballscrew, which was approximately 0.1 mm over the entire 1500 mm length. For the maximum displacement considered in the exper-
iment, of approximately 80 mm, this was a relatively small source of error and ±0.008 mm. The number of turns of the screw were measured using a 4096 pulse-per-revolution digital encoder. The uncertainty was assumed to be ±1 pulse (i.e. the resolution of the measurement). Therefore, this contributes another relatively small displacement error of 40 mm/4096 = ± 0.0097 mm.

As the systematic uncertainty of the auxiliary carriage velocity is derived from the displacement, it varies with the oscillatory frequency. Its magnitude was based on the highest frequency case typically investigated, of $f = 1$ Hz. As the errors in the timing were negligible, a displacement error of approximately 0.07 mm corresponds to a systematic uncertainty in the velocity of approximately $7 \times 10^{-5}$ m s$^{-1}$. This was 0.04% of the oscillatory velocity amplitude for a Current number $\mu = 0.2$ and is relatively small.

**Blade Geometry & Water Temperature**

The rotor blades were manufactured from aluminium alloy using a CNC machine to within a tolerance of $b_R = 0.5$ mm. The temperature was measured with a probe to an accuracy of 0.1°C. Following ITTC (2006), at a typical tank temperature of 16.0°C, the uncertainty in the fresh water density was estimated as $b_{\rho} = 0.021$ kg m$^{-3}$.

**Non-Dimensional Coefficients**

The bending moment coefficient for the steady loading was defined as

$$C_{M_y} = \frac{M_y}{0.5\rho A U^2 R}, \quad (5.2)$$

where $A = \pi R^2$ is the swept area of the rotor, $R$ is the tip radius and $U$ is the carriage speed. The contribution to the uncertainty of the bending moment coefficient from the uncertainty in the blade length was approximately twice the magnitude of the bending moment and the velocity. There was negligible contribution from the uncertainty in the water density.

The rotational speed of the rotor was calibrated by comparing the output measured by the tachometer with a pulse count from a proximity sensor attached to the rotor shaft. The uncertainty in the rotational speed was predicted as $b_\Omega = 0.0004$ rad s$^{-1}$. For a constant speed of $U = 0.894$ m s$^{-1}$ the systematic uncertainty in the tip-speed ratio ($\lambda = \Omega R/U$) was estimated as $b_\lambda = 0.012$. The
uncertainty associated with the velocity contributed to nearly 100% of this error.

**Sources of Additional Bias**

For the 2010 tests, a relatively small (approximately 2%), but notable difference was observed in the bending moment (and the shaft thrust and torque) between repeat tests performed at the initial and final stages of the test programme. It is believed that this was a systematic-type error. It was attributed to an unknown small change in the blade pitch angle. This may have occurred during the removal and re-installation of the turbine rig on the carriage prior to performing the repeated runs. As this event was identified as having occurred late in the test programme, it did not affect any of the data which were presented for the unsteady experiments. No such error was observed during the 2012 trials.

5.5.3 Random & Total Uncertainties

The random uncertainties of the velocities of the carriages and of the out-of-plane bending moments were estimated for a 95 percent confidence level from multiple repeated runs of selected cases, including for both steady and oscillatory motion.

The uncertainties for the steady loads were computed for a mean velocity of $U = 0.890 \text{ m s}^{-1}$ and tip-speed ratio of $\lambda = 3.6$. These were evaluated as the standard deviation of the mean of the signals from five repeats. The random uncertainties of the bending moment and velocity were an order of magnitude smaller than the systematic uncertainties. This implies that the total uncertainty of these measurements was approximately equal to the systematic error.

The random uncertainty of the rotor speed was of the same order of magnitude as the systematic error and the total uncertainty was approximately $u_\Omega = 0.0005 \text{ rad s}^{-1}$. However, the uncertainty of the rotor speed has negligible influence on the error of the bending moment coefficient. Given this, the total uncertainty of the bending moment coefficient was approximately equal to its systematic error. For this particular repeated case, the error in the bending moment coefficients was approximately 0.5% of the measurement. Whilst the total uncertainty in the tip-speed ratio was $u_\lambda = 0.002$, owing to the random error in the rotor speed, the error was still negligible (i.e. 0.04% of the measurement).

The uncertainties for the unsteady experiments were inferred from four repeated runs of two oscillatory tests. These comprised $U = 0.780 \text{ m s}^{-1}$, $\lambda = 3.6$, $f = 0.89 \text{ Hz}$, $\mu = 0.2$ and $U = 0.890 \text{ m s}^{-1}$, $\lambda = 3.6$, $f = 0.45 \text{ Hz}$, $\mu = 0.2$. For both
these cases the blades experienced dynamic stall. Due to the bending moment not being perfectly sinusoidal, the random uncertainty in the bending moment was computed from the root-mean-square of the amplitude. The random errors were smaller than the systematic error, but were significant and implied that the total uncertainty was approximately equal to 0.02 Nm. For the two unsteady cases, this corresponded to an error of approximately 0.3% of the measurement, which is deemed to be sufficiently small.

The uncertainty of the auxiliary carriage velocity was also computed from the root-mean-square of the amplitude for consistency. Both random and systematic errors were dominant in the total uncertainty of the velocity, which was approximately equal to 0.0015 m s$^{-1}$. For the higher frequency case, the random uncertainty in the rotor speed was comparatively more significant than for the steady case. However, the total uncertainty in the rotor speed was still relatively small and equal to only 0.07% of the nominal rotor speed.

## 5.6 Characterisation of the Turbine in Steady Flow

### 5.6.1 Performance & Loading Coefficients

Measurements of the performance and loads in steady flow were conducted in both the 2010 and 2012 campaigns. These were performed at constant carriage speeds of between $U = 0.45$ m s$^{-1}$ and 1.01 m s$^{-1}$, and for a series of fixed rotor speeds of between $\Omega = 63$ rpm and 96 rpm. This approach was taken because the unsteady tests were conducted at constant rotor speed. Therefore, it allowed for a quasi-steady load to be reconstructed from the experimental data with minimal influence of the Reynolds number.

The coefficients of the power ($C_P = P/0.5\rho AU^3$), thrust ($C_T = F_x/0.5\rho AU^2$) and the blade-root bending moment response ($C_M = M/0.5\rho AU^2 R$) in steady flow are presented in Figure 5.10 for various rotor speeds as a function of the tip-speed ratio.
Chapter 5. Set-Up of the Turbine Tank Testing

Figure 5.10: The coefficients of the steady rotor power, (a), thrust (b), in-plane blade root bending moment, (c) and out-of-plane blade root bending, (d) as a function of the tip-speed ratio. Legend: constant rotor speeds of 62 rpm (⋆), 72 rpm (△), 84 rpm (○), 90 rpm (○) and 96 rpm (□ for the 2010 trials and + for the 2012 trials).

Maximum power coefficients of $C_P = 0.38$ and 0.35 were obtained for the 2010 and 2012 tests respectively, which corresponded to a tip-speed ratio of approximately $\lambda = 4$. Power was generated by the rotor at tip-speed ratios ranging between approximately $\lambda = 2.5$ and 6.5. The magnitude of the out-of-plane bending moment coefficient was approximately 5 times larger than the coefficient of the in-plane bending moment. This is consistent with the expectation that the thrust forces, which give rise to the out-of-plane bending moment, dominate over the hydrostatic and centrifugal inertial forces (Fraenkel, 2007).
The 2010 trials, specifically, are useful for demonstrating the sensitivity of the coefficients to the rotor speed. For a tip-speed ratio of $\lambda = 4.1$, the out-of-plane blade root bending moment coefficient was 20% lower for a rotor speed of $\Omega = 62 \text{ rpm}$ compared to 96 rpm. As the pitch angle was consistent for all trials, these observations are consistent with a Reynolds number dependency in the performance of the hydrofoil.

It is also interesting to observe that there is a significant increase in the bending moment coefficient (as well as thrust) between the tip-speed ratios of $\lambda = 5$ and 5.5. Whilst an increase in the loads at high tip-speed ratios would be expected as the rotor enters the brake state (Burton et al., 2001), the sudden increase suggests that additional lift may arise from other phenomena existing at low Reynolds numbers. One possibility is flow separation on the pressure surface of the foil (see Section 6.5.3).

The Reynolds number sensitivity reinforces the notion that constant speed is the most desirable form of rotor control for unsteady turbine tests. This is because the Reynolds number would be maintained approximately constant during the perturbation. This would help isolate the effect of the unsteadiness due to the rotor wake, added mass and flow separation.

A difference in the magnitudes of the coefficients between the 2010 and the 2012 tests for a rotor speed of 96 rpm is also noted. It is acknowledged that the Reynolds numbers of the 2012 tests were comparatively lower than for the 2010 tests for $\Omega = 96 \text{ rpm}$ (see Section 5.4.2). However, the 2010 results for $\Omega = 90 \text{ rpm}$ correspond approximately to the equivalent Reynolds number of the 2012 tests. This implies that the Reynolds number difference had a relatively small effect on the loads.

This difference is instead believed to be due to a relatively small variation ($\delta\phi \approx 1.5^\circ$) in the pitch angle between the trials. A new set of blades were manufactured for the 2012 trials, as the previous set were required for the testing of another model turbine. Although these new blades were nominally identical to those used in 2010, they were attributed to having introduced a minor pitch difference. However, these differences in the loading curves serve to highlight that the pitch angle is critical at model-scale. From the author’s experience, setting the pitch angle accurately at model-scale was also challenging due to the relatively small hub.

It is also useful to observe that the out-of-plane bending moment is also generally consistent with the measurement of the thrust. The effective centre of
loading on the blade can be derived by assuming that the bending moment of each blade is equal and by incorporating the offset of the strain gauge from the rotor axis. It is expressed as

\[
\frac{x}{R} = \frac{3CM_y}{C_T} - \frac{r_{sg}}{R}.
\] (5.3)

This ratio is presented in Figure 5.11, where for tip-speed ratios of between \(\lambda = 3\) and 5, it tends to a value of \(x/R \approx 0.63\). This is only 5\% lower than the ratio \(x/R = 0.67\) which is predicted for an ideal rotor (Hansen, 2008) and provides further confidence in the measurements.

![Figure 5.11: Effective centre of loading on the blade for steady flow, non-dimensionalised by the radius of the blade. Legend as for Figure 5.10.](image)

**5.6.2 Local Linearity**

A range of tip-speed ratios for which the steady out-of-plane bending moment was approximately linear with axial velocity were also identified. These ratios were of interest for investigating whether the bending moment response could be modelled through a linear superposition of the single frequency response. (see Chapter 7). This was facilitated by normalising the out-of-plane blade root bending moment coefficient by the rotor speed, instead of the axial velocity, i.e. \(K_{M_y} = M_y / (0.5\rho(\Omega R)^2AR)\). These coefficients are presented as a function of the inverse of the tip-speed ratio in Figure 5.12.

For the relatively high rotor speeds of 90 rpm and 96 rpm, the out-of-plane
bending moment is shown to exhibit an approximately linear relationship with velocity between $1/\lambda = 0.2$ and $0.28$ (equivalent to tip-speed ratios ranging between $\lambda = 3.5$ and $5$ respectively). Therefore, these rotor speeds and these range of operating states were selected as being most appropriate for acquiring data to study linear superposition.

![Figure 5.12](image)

Figure 5.12: Bending moment coefficient normalised by the rotor speed and presented as a function of the inverse of the tip-speed ratio. A linear fit to the coefficients for the 2012 tests is shown in grey. Legend as for Figure 5.10.

### 5.6.3 Comparison With Other Rotors

In comparison to a full-scale tidal turbine, the maximum power coefficient achieved by the model-scale turbine in the 2010 and 2012 tests was 25% and 27% lower respectively than the power coefficient of $C_P = 0.48$ by the twin-bladed SeaGen reported by MCT (2010). The maximum power coefficient was approximately 20% lower than was reported for the 800 mm diameter turbine tested by Bahaj et al. (2007b). The thrust coefficient achieved at maximum power was approximately 40% lower for the turbine studied in this thesis than that reported by Bahaj et al. (2007b).

The reduced performance compared to the SeaGen and the turbine used by Bahaj et al. (2007b) is believed to be attributable to the relatively low Reynolds number of the blade section in the present tests, which is explored further in Chapter 6. It is important to consider that the blades were originally intended
Chapter 5. Set-Up of the Turbine Tank Testing

for a contra-rotating turbine system, as cited in Section 5.3.2. Additionally, the pitch angle may have been slightly sub-optimal. Despite the reduced performance, as no significant irregularities were observed in the steady state performance characteristics, the turbine was deemed to be suitable for the purposes of this research.

5.7 Summary

This chapter has demonstrated that the experimental set-up used in this research has enabled the blade-root bending moment to typically be measured at reduced frequencies up to $k = 0.05$ and frequency ratios up to $m = 0.6$. These reduced frequencies and frequency ratios are significantly greater than have been reported previously for tidal turbines. Furthermore, the methodology has enabled the response to multi-frequency forcing to be investigated.

The uncertainty analysis demonstrated that the experimental errors were sufficiently low that accurate measurements of the blade-root bending moment were obtained.

The performance and loading of the rotor were characterised for rotor speeds up to 96 rpm. The maximum power coefficient of $C_p = 0.38$ was achieved at a tip-speed ratio of approximately $\lambda = 4$. The out-of-plane bending moment exhibited an approximately linear relationship with $1/\lambda$ (analogous to velocity) for tip-speed ratios between $\lambda = 3.5$ and 5. Given this approximate linearity, these range of operating states are considered to be the most suitable for assessing whether a multi-frequency response can be modelled using a linear superposition of the responses to several single frequency velocity perturbations.

The performance and the loading were found to be sensitive to the rotor speed. This implies that the performance of the foil was likely to have been affected by the relatively low Reynolds number at which the experiments were performed. This emphasises the need for constant rotor speed experiments for investigating unsteady hydrodynamic blade loads.
Chapter 6

Performance of the Foil at Low Reynolds Number

However beautiful the strategy, you should occasionally look at the results

— Winston Churchill

6.1 Introduction

The previous chapter demonstrated that the maximum power and thrust coefficients achieved by the model turbine used in the towing tank experiments were sensitive to the rotor speed. This is consistent with a degradation in the aero/hydrodynamic performance of the foils as the Reynolds number is reduced to this range of relatively low Reynolds numbers at which the turbine was tested.

However, no data are available in the literature for the NREL S814 foil for Reynolds numbers of approximately $1 \times 10^5$, which could have assisted in verifying this. The previous turbine studies which have employed the S814 foil (Barltrop et al., 2006 and Clarke et al., 2007) have also not specifically addressed the role of the Reynolds number at model-scale. Accurate aerofoil data are essential if BEM theory is to be applied to predict useful turbine parameters which are not able to be measured experimentally. Additionally, the data are needed to develop a dynamic inflow and dynamic-stall model for describing the unsteady blade loads.

This provided the motivation for performing a series of wind-tunnel experiments to characterise the performance of the S814 foil at Reynolds numbers of
Chapter 6. Performance of the Foil at Low Reynolds Number

between approximately $1 \times 10^5$ and $3 \times 10^5$. Due to the limited capabilities of the wind tunnel, the two-dimensional aerofoil forces and the pressure distributions across the chord were acquired in two separate experiments. Both these were restricted to steady flow.

This chapter begins with a description of the wind tunnel facility used and the experimental methodology. This is followed by an analysis of the aerofoil forces and pressure distributions. New insights into the role of the laminar separation bubbles and flow separation of the foil at relatively low Reynolds numbers are provided. The data are then incorporated within a BEM model and are used to predict the angles of attack, inflow and the tip-loss of the turbine under various conditions.

6.2 Wind Tunnel Facility

The aerofoil experiments were conducted in the de Bray wind tunnel at The University of Auckland. The wind tunnel has a horizontal closed-loop configuration and two working sections located on either side of the tunnel. A schematic diagram of the wind tunnel is provided in Figure 6.1.

![Figure 6.1: Plan view of the de Bray wind tunnel, adapted from Guha (2013).](image)

The higher-speed test section has a maximum velocity of approximately $50 \text{ m s}^{-1}$ and a working section 2000 mm long, 767 mm wide and 615 mm high. An external mechanical load balance is installed in this section and was used to acquire the
6.3. Methodology of the Force Measurements

The lower-speed section of the wind tunnel was designed for wind engineering studies. It has a maximum velocity of approximately $12 \text{ m s}^{-1}$ and a larger working section, 8000 mm long, 1800 mm wide and 1200 mm high. This section was used to measure the pressure distributions across the aerofoil chord.

6.3 Methodology of the Force Measurements

6.3.1 Aerofoil Model & Apparatus

A test rig and an aerofoil model conforming to the NREL S814 section were designed in this research for the purposes of obtaining the aerofoil forces. The aerofoil had a chord of 120 mm and spanned the test section horizontally, so that the effective aspect ratio was $b/c = 6.5$. It was manufactured from solid aluminium alloy using a CNC machine to within a tolerance of $0.5 \text{ mm}$. A high quality surface was achieved by using 400 grit emery paper. This was also expected to reduce the likelihood of premature transition of the boundary layer (Shyy et al., 2008).

The clearances between the ends of the model and the wind tunnel walls were less than 1 mm. This gap was within the tolerance recommended by Rae and Pope (1984) (given for a semi-span model). The models were rigidly connected to the load balance about a pivot point at the 1/4 chord point from the leading edge of the aerofoil. The connecting struts protruded through slots in the side walls of the wind tunnel with clearances of less than 1 mm.

The angle of attack was adjusted and fixed by inserting a pin into pre-drilled holes corresponding to $0.5^\circ$ increments in the angle of attack. As the profile of the S814 is non-symmetrical, the zero angle of attack in the test section was identified by performing a limited set of tests on a symmetrical NACA 0012 foil (see Abbott and Von Doenhoff, 1959). This foil had the equivalent chord and span as the S814 model.

The force balance was calibrated using M1-grade ($\pm 10 \text{ mg}$) standard masses. The systematic uncertainty was found to be dominated by the scatter in the least-square fit to the calibration curve. For the lift force, this error (assumed as equal to 2SEE) was equal to 0.12 N and corresponded to 0.12% of the maximum lift force measured at $Re = 0.85 \times 10^5$. The error in the drag force was equal to 0.05 N and equated to 0.50% of the drag force measured at $Re = 0.85 \times 10^5$. 

119
6.3.2 Flow Quality & Measurement of the Dynamic Pressure

The flow passed through a series of wire-mesh screens and a honeycomb immediately upstream of the contraction. This improved the flow straightness and the homogeneity of the turbulence. Tapered fillets were also installed in the corners of the test section. These reduced the effect of secondary flows and assisted in compensating for any boundary layer growth along the tunnel walls.

The turbulence intensity in the test section was measured in the empty test section using a TRI Cobra Probe which sampled at a rate of \( n_s = 1250 \text{ Hz} \). The streamwise turbulence intensity was 0.17\% at the location corresponding to the aerofoil centre. No dominant spectral energy at scales corresponding to the aerofoil chord \( (n = U/c \geq 125 \text{ Hz}) \) were identifiable. Insights into the effect of a model in the wind tunnel on the turbulence intensity can be drawn from the investigations of Selig et al. (2011). They found that an aerofoil and its apparatus acted to induce low frequency fluctuations into the flow. For this thesis, it was deemed that the possibility of similar effects occurring would not significantly compromise the steady state test data.

The pressure difference across the wind tunnel contraction was measured us-
6.4 Methodology of the Pressure Distribution Measurements

6.4.1 Aerofoil Model

A model was also designed and manufactured in this research for acquiring the pressure distributions across the chord of the foil. The model was installed vertically in the wind tunnel and was mounted to a turntable in the wind tunnel floor. It was assembled from three sections with spans of 300 mm and chords of 300 mm, as well as a 150 mm base. The effective aspect ratio of the complete model was 3.2. The lower aspect ratio compared to the model used in the force experiments was due to the need to maximise the chord length in order to achieve the desired Reynolds numbers. All the components were machined from aluminium alloy on a CNC machine to within a tolerance of 0.5 mm and the surfaces were smoothed using 400 grit emery paper.

The pressure taps were located on the central section of the aerofoil. These were limited to within the region $0 < x/c \leq 0.76$ from the leading edge. This was because it proved to be too technically challenging to install any taps near the relatively narrow trailing edge. The orifices were 0.8 mm in diameter and were drilled normal to the aerofoil surface. Thin metallic tubes were inserted into the orifices with a press fit and were filed flush with the aerofoil surface. The plastic...
tubing which lead to the acquisition system were connected onto these steel tubes within the interior of the model.

The pressure taps were staggered at an angle of inclination of 5° to the chord, as is shown in Figure 6.3. This was an attempt to minimise the wake from upstream taps interfering with downstream taps. A total of 64 taps were used for the investigations. The extra taps, which were drilled for redundancy, were sealed as to not affect the flow.

As the model did not completely span the total height of the test section, an end-plate was installed on the top of the model. The circular end-plate extended approximately two-chord lengths upstream and downstream of the foil and had chamfered edges to reduce flow separation. The set-up is shown in Figure 6.4, where the instrumentation is concealed below the aerofoil.

6.4.2 Flow Quality

The streamwise turbulence intensity in the test section that was used to acquire the aerofoil pressure distributions was approximately 3 times larger (0.6%) than that the force measurements (0.2%). As the model was positioned near the downstream end of the test section, the boundary layer at the wind tunnel floor
6.4. Methodology of the Pressure Distribution Measurements

was also considerably thicker. The thickness of this boundary layer at the location of the model (δ) was estimated using the approximation for a turbulent boundary layer on a flat plate (see Kothandaraman and Rudramoorthy, 1999). This is expressed as

\[ \delta \approx 0.382x / Re_x^{0.2}, \]

where \( Re_x = Ux/\nu \), \( x \) is the distance from the tunnel inlet (8 m) and \( \nu \) is the kinematic viscosity of air. For the wind speeds at which the experiments were conducted, the boundary layer thickness was estimated as approximately 130 mm.

The implication of the boundary layer was that near the base of the aerofoil, a span-wise pressure gradient was present. The extent of the span-wise effect on the lower part of the foil was also expected to be greater than near the upper part, as the end-plate would have induced the formation of a new boundary layer. The reason that the pressure taps were positioned at the centre of the span was to attempt to minimise the effect of any span-wise pressure gradient on the measurements. Tufts were used to visually confirm that the flow was approximately two-dimensional. These tufts were removed prior to acquiring the data.

The pressure distributions were acquired at four Reynolds numbers, nominally \( 1.1 \times 10^5 \), \( 1.5 \times 10^5 \), \( 1.8 \times 10^5 \) and \( 2.7 \times 10^5 \). Therefore, these encompassed a
Chapter 6. Performance of the Foil at Low Reynolds Number

wider range of Reynolds numbers than the experiment to measure the aerofoil forces.

6.4.3 Data Acquisition

A digital system was used for the acquisition of the aerofoil pressures. It comprised a series of Honeywell XSCL04DC differential pressure transducers and a National Instruments 16-Bit PCI-6255 A/D card. The pressures were sampled at a rate of $n_s = 600 \text{ Hz}$ and were recorded over a 30 s period. The calibration factors were re-established frequently and all tests were repeated at least twice.

The pressures were acquired relative to static pressure, which was measured at a distance of 5 m upstream of model in the low speed test section using a Pitot-Static tube. At this location, the measurement was found to be unaffected by the aerofoil. The total pressure from the Pitot-Static tube was acquired together with the aerofoil pressures. As this was also referenced to the static pressure, the reading corresponded to the dynamic pressure.

6.5 Analysis & Discussion of the Experimental Measurements

6.5.1 Data for Benchmarking

As the maximum achievable Reynolds number in the wind tunnel was $2.7 \times 10^5$, comparisons with the full-scale Reynolds number performance of the S814 foil were made using the data presented by Somers (1997) for $\text{Re} = 1.5 \times 10^6$ (see also Somers, 1994 and Somers and Tangler, 1996).

It should be noted that Somers (1997) acquired data in a wind tunnel with a turbulence intensity between 0.02\% and 0.04\%, which is lower than for the present study. An increase in turbulence intensity can promote earlier transition, resulting in thinner and shorter laminar separation bubbles (Shyy et al., 2008 and O’Meara and Mueller, 1987) as well as eliminate any hysteresis in the lift and drag (Mueller et al., 1983). Selig et al. (2011) considered a turbulence intensity of between 0.10\% and 0.15\% to be adequate to study the low Reynolds number aerofoil performance. Given that the turbulence intensity of the present tests is of the same order of magnitude as this, it was deemed to be sufficiently small as
6.5. Analysis & Discussion of the Experimental Measurements

to not compromise the measurements significantly. However, comparisons with
the results of Somers (1997) are made in light of these potential differences.

Numerical predictions of the flow over the S814 foil were also made using
the panel code Xfoil (Drela, 1989). The code incorporates a two-equation lagged
dissipation integral boundary-layer formulation. It allows for regions of limited
flow separation on the surfaces of the foil to be modelled. Numerical predictions of
the force and pressure were found to be in good alignment with the experimental
data for Re = 1.5 × 10^6 presented by Somers (1997). However, for the relatively
low Reynolds number cases of Re ≤ 1.5 × 10^5, the predictions of the code were
unstable and were found to be too unreliable to use. Therefore, the code was
deeded to provide useful comparisons with the experimental data in this thesis
for Re ≥ 1.8 × 10^5 only. In particular, it was used to estimate the pressure
distribution for the region of the chord where experimental data were not obtained
(i.e. x/c > 0.76).

Whilst experimental data for thick foils at low Reynolds numbers is relatively
scarce, as identified in Section 2.6, Selig et al. (1995) have published lift and drag
data for the 21% thick S823 foil at Reynolds numbers as low as Re = 1 × 10^5.
As the S823 foil has a somewhat similar shape to the S814 and is from the same
family of foils, the S823 data provide a useful basis for establishing any consistent
performance characteristics. The data for the S823 presented by Selig et al. (1995)
were acquired using a force balance and the turbulence intensity was less than
0.1%.

A summary of the experimental data for the S814 foil at high Reynolds number
reported by Somers (1997) and the S823 foil at low Reynolds numbers from Selig
et al. (1995) are provided in Appendix D for reference.

6.5.2 Lift & Drag Coefficients

The two-dimensional lift and drag coefficients for the S814 foil which were mea-
sured in the wind tunnel at the Reynolds numbers of 0.85 × 10^5, 1.1 × 10^5 and
1.5 × 10^5 are presented in Figure 6.5. The lift and drag coefficients are expressed
as

\[ C_l = \frac{L}{\frac{1}{2} \rho U^2 cb}, \quad C_d = \frac{D}{\frac{1}{2} \rho U^2 cb}, \]  \hspace{1cm} (6.2)

where c is the aerofoil chord and b is the span. The data have been corrected for
wall effects, as shown in Appendix D.
For low Reynolds numbers, the lift coefficients remained relatively large at low angles of attack and for $\alpha = 0^\circ$ were approximately $C_l = 0.5$. At low angles of attack, the drag coefficient also increased significantly and was up to $C_d = 0.09$. For a Reynolds number of $0.85 \times 10^5$, the lift coefficient exhibited an approximately linear relationship with angle of attack over $4^\circ < \alpha < 9^\circ$. The linear region extended to lower angles of attack as the Reynolds number was increased. The magnitudes of both the lift and drag coefficients were sensitive to the Reynolds number at moderate angles of attack. The maximum lift coefficient of approximately $C_l = 1.28$ was observed at an angle of attack of around $\alpha = 11^\circ$, which was nearly independent over the Reynolds number range of the tests.

These data exhibit discernible differences compared to the measurements of Somers (1997) at the full-scale Reynolds number of $1.5 \times 10^6$ which are superimposed in Figure 6.5 for comparison. For instance, the full-scale data showed that the lift coefficient was approximately linear over a larger range of angles of attack, $-3^\circ < \alpha < 7^\circ$. At an angle of attack of $\alpha = 8^\circ$, the lift coefficient at $Re = 1.5 \times 10^6$ was also nearly 10% higher and the drag 400% lower than for $Re = 0.85 \times 10^5$ measured. However, it can also be noted that the maximum lift
coefficient for the low Reynolds number tests was only 2% lower compared with full-scale.

It is interesting to find that the S814 forces have qualitatively similar characteristics to the low Reynolds number performance of the S823 foil presented by Selig et al. (1995). For instance, for Re ≤ 2 × 10^5, at angles of attack less than α ≤ 0°, the lift coefficient of the S823 foil remained almost constant at approximately C_l = 0 whilst the drag increased significantly up to C_d = 0.05. That suggests such that this high lift and high drag behaviour at low angles of attack may be characteristic of thick foils at low Reynolds numbers. At moderate angles of attack, the effect of the Reynolds number on the magnitudes of the lift and drag coefficients are also generally consistent between the two foils. For example, at an angle of attack of 8° the lift coefficient on the S823 foil was reported by Selig et al. (1995) to be 9% lower and the drag coefficient at least 400% higher at Re = 1 × 10^5 compared to Re = 1 × 10^6. Furthermore, at high angles of attack the lift coefficients of the S823 were approximately independent of Reynolds number, for Re ≥ 1.5 × 10^5. This is consistent with the observations of the S814 presented here.

6.5.3 Pressure Distributions for Low Angles of Attack

The pressure distributions provide valuable new insights into the phenomena responsible for the characterisation of the forces observed at low Reynolds numbers. The pressure distributions for an angle of attack of nominally α = 0° are presented in Figure 6.6, where distributions for Re = 1.1 × 10^5 have been annotated. The profile of the S814 foil and the locations of the pressure taps have also been provided for reference. The pressures are presented in terms of the pressure coefficient, defined as

\[ c_p = \frac{p - p_\infty}{q}, \]

where \( p_\infty \) is the reference static pressure and the dynamic pressure \( q = 0.5 \rho U^2 \) at the model location was corrected for wall effects, as shown in Appendix D.

A pronounced difference is observed in the pressure distributions between those for Re ≤ 1.8 × 10^5 and those for Re = 2.7 × 10^5. At the lower Reynolds numbers, the flow is revealed to have separated from the pressure surface (the lower surface) of the foil upstream of the maximum thickness location of the foil. This has the effect of significantly reducing the suction pressures on the
Chapter 6. Performance of the Foil at Low Reynolds Number

Figure 6.6: Profile of the S814 aerofoil and location of pressure taps, (a). The pressure distributions measured at an angle of attack of $\alpha = 0^\circ$ at the Reynolds numbers of $1.1 \times 10^5$ (..), $1.5 \times 10^5$ (-.-), $1.8 \times 10^5$ (- -) and $2.7 \times 10^5$ (–), (b). The pressure distributions on the suction surface are in bold for clarity. The labels are with respect to the Re $= 1.1 \times 10^5$ data.

lower surface of the foil and also increasing the suction over the upper surface of the foil, relative to Re $= 2.7 \times 10^5$. This is an important observation and implies that at these lower Reynolds numbers, the laminar boundary layer on the pressure surface had insufficient energy to overcome the adverse pressure gradient and to reattach as a turbulent boundary layer.

This flow separation elucidates the source of the relatively large lift, in that the downwards force generated by the pressure surface would have been comparatively weak. To demonstrate this further, the pressure coefficients on both the upper ($c_{pu}$) and lower ($c_{pl}$) surfaces of the foil were integrated using the trapezium rule over $0 < x/c \leq 0.76$. The difference in the integrated pressures between the two
surfaces yields the net lift contribution, i.e.

\[ C_l = \int_0^{0.76c} [c_{pl}(x/c) - c_{pu}(x/c)] d(x/c). \] (6.4)

For Reynolds numbers of $1.1 \times 10^5$, the lift in this region is 50% greater than for $Re = 2.7 \times 10^5$. A large separated flow region is also consistent with the relatively large drag coefficients that were measured.

The pressure distributions also suggest that relatively long laminar separation bubbles were present on the suction surface of the aerofoil at all Reynolds numbers investigated. The location of the laminar separation bubble on the suction surface was approximately $x/c \approx 0.5$ at all Reynolds numbers. The location was irrespective of whether the flow had separated from the lower surface.

As the Reynolds number was decreased, the location of the attachment of the turbulent boundary layer on the suction surface shifted towards the leading edge. For instance, for $Re = 1.1 \times 10^5$ reattachment was observed at $x/c \approx 0.7$ and for $Re = 2.7 \times 10^5$ reattachment was observed at $x/c \approx 0.6$. This is attributed to the lower Reynolds number cases having less energy in the separated region to promote transition and reattachment (Shyy et al., 2008).

The pressure distributions for a Reynolds number of $2.7 \times 10^5$ agree qualitatively with the distributions on the foil for $Re = 1.5 \times 10^6$ which were presented by Somers (1997). For example, the peak suction coefficients on the upper surface of $C_p = -1.00$ and on the lower surface of $C_p = -1.30$ were only 5% and 4% lower magnitude, respectively than were observed by Somers (1997). For both these Reynolds numbers, the laminar separation occurred at approximately the same locations on both the upper and lower surfaces of the foil, which was at $x/c = 0.40$ and $x/c = 0.30$, respectively. However, the bubbles on the upper and lower surfaces were comparatively longer for $Re = 2.7 \times 10^5$, where they extended to $x/c = 0.6$ and 0.52. In contrast, the distributions for $Re = 1.5 \times 10^6$ indicate that they extended to only $x/c = 0.5$ and 0.4. This implies that even at $2.7 \times 10^5$, the low Reynolds number effects on the laminar separation bubbles were not totally avoided.

Pressure distributions were not reported by Selig et al. (1995) for the S823 foil. However, it is postulated that full-boundary layer separation on the pressure surface was also present in those tests. It is interesting to note that for the S823 foil, the high lift and drag behaviour at low angles of attack was avoided at Reynolds numbers higher than $2 \times 10^5$. The pressure distributions on the S814
Chapter 6. Performance of the Foil at Low Reynolds Number

presented in this thesis also indicate that the separation is avoided near these Reynolds numbers. This provides support for having identified a universal range of critical Reynolds numbers for which separation of the pressure surface may be expected for thick laminar flow foils.

6.5.4 Pressure Distributions for Moderate Angles of Attack

The pressure distributions corresponding to angles of attack of nominally $\alpha = 5^\circ$ and $\alpha = 8^\circ$ are presented in Figure 6.7. The numerical predictions from Xfoil at Reynolds numbers of $1.8 \times 10^5$ and $2.7 \times 10^5$ are also presented for comparison.

![Pressure Distributions for S814 Foil](image)

Figure 6.7: Pressure distributions for the S814 foil at angles of attack of $\alpha = 5^\circ$, (a) and $\alpha = 8^\circ$, (b). Data are presented for Reynolds numbers of $1.1 \times 10^5$ (..), $1.5 \times 10^5$ (-.-) (not available for $\alpha = 8^\circ$), $1.8 \times 10^5$ (- -) and $2.7 \times 10^5$ (–). The XFoil predictions for Re = $1.8 \times 10^5$ and $2.7 \times 10^5$ are denoted as (□) and (○) respectively.

The distributions demonstrate that at these angles of attack the flow on the pressure surface of the foil was able to reattach as a turbulent boundary layer.
However, it can be noted that at $\alpha = 5^\circ$, the pressures on the lower surface of the foil remained relatively weak for $\text{Re} = 1.1 \times 10^5$. This is consistent with the force measurements which showed that as the Reynolds number was reduced, the region of the relatively high lift and drag persisted to higher angles of attack.

As the angle of attack was increased between $\alpha = 5^\circ$ and $8^\circ$, the laminar separation bubble on the pressure surface moved forward towards the leading edge and shortened in length. In contrast, the bubbles on the pressure surface moved towards the trailing edge and increased in length. An increase in the Reynolds number appears to have had an effect similar to an increase in angle of attack. For instance, it expedited the laminar separation and reattachment process, shortening the bubble on the suction surface. These effects are consistent with those expected for foils at low Reynolds numbers (Shyy et al., 2008). These characteristics are also replicated qualitatively by Xfoil. However, the numerical solution predicts that turbulent attachment occurs further downstream, giving rise to a longer bubble.

At an angle of attack of $\alpha = 8^\circ$, a suction peak had developed at the leading edge. The pressures at the peak and in the region of the adverse pressure aft of the peak decreased in magnitude as the Reynolds number was reduced. For example, at a Reynolds number of $1.1 \times 10^5$ the suction peak at $x/c = 0.10$ was approximately 13\% lower than for $\text{Re} = 2.7 \times 10^5$.

The full-scale Reynolds number data of Somers (1997) at $\alpha = 8.12^\circ$ show that there is a spike in the suction pressure coefficient, which obtains a value of $C_p = -2.7$ at $x/c < 0.07$. Pressure measurements were not available between $0 < x/c \leq 0.01$ for the S814 foil in the present study. Nevertheless, for $\text{Re} = 2.7 \times 10^5$, the suction pressure at $x/c = 0.1$ of $C_p = -2.7$ is approximately equal to that reported by Somers (1997).

It is also interesting to note that the full-scale data presented by Somers (1997) also show that laminar separation bubbles on the suction surface had disappeared at an angle of attack of $\alpha = 7.2^\circ$. The laminar separation bubbles appear to have a significantly larger role on the pressure distributions at low Reynolds numbers. To quantify their effect on the lift, the pressure distributions at an angle of attack of $\alpha = 8^\circ$ were integrated using equation 6.4. This integration was performed over the entire chord, where data for $x/c > 0.76$ were predicted using the Xfoil. For Reynolds numbers of $1.1 \times 10^5$ and $2.7 \times 10^5$ the lift coefficients were estimated as $C_l = 0.99$ and 1.07 respectively (i.e. an increase of 10\%). The former value is also approximately equal to that measured by the force balance. The latter value
is approximately equal to the data presented by Somers (1997) at \( \text{Re} = 1.5 \times 10^6 \). Therefore, these results also provide further confidence in the experimental data collected in this thesis.

### 6.5.5 Pressure Distributions for High Angles of Attack

The pressure distributions across the aerofoil chord at relatively high angles of attack of \( \alpha = 10^\circ, 12^\circ \) and \( 15^\circ \) are presented in Figure 6.8. The numerical predictions from Xfoil are also provided for comparison.

![Pressure Distributions for High Angles of Attack](image)

Figure 6.8: Pressure distributions for the S814 foil at angles of attack of \( \alpha = 10^\circ, 12^\circ \) and \( 15^\circ \). Data are presented for Reynolds numbers of \( 1.1 \times 10^5 \) (..), \( 1.5 \times 10^5 \) (-.-) (not available for \( \alpha = 12^\circ \)), \( 1.8 \times 10^5 \) (- -) and \( 2.7 \times 10^5 \) (–). The Xfoil predictions for \( \text{Re} = 1.8 \times 10^5 \) and \( 2.7 \times 10^5 \) are denoted as (□) and (○) respectively.
6.6 Insights for the Tidal Turbine

These distributions demonstrate that the suction peak and the adverse pressure gradient continued to increase in strength with an increase in angle of attack. This further expedites the movement of the laminar separation bubbles on the pressure surface. At a Reynolds number of \( \text{Re} = 1.1 \times 10^5 \), the laminar separation bubbles remained present at the relatively high angle of attack \( \alpha = 15^\circ \). It is also interesting to note that whilst a laminar separation bubble was not significant in the distributions for \( \text{Re} = 2.7 \times 10^5 \), there was a relatively small bubble evident at \( \alpha = 15^\circ \). This suggests that at these relatively large angles of attack, laminar separation still occurred prior to natural transition.

At these low Reynolds numbers, the process of stall of the S814 foil appears to still remain as a consequence of trailing edge separation. However, separation at the trailing edge was only observed in the pressure distributions for an angle of attack of \( \alpha = 15^\circ \), corresponding to approximately \( x/c = 0.65 \). The trailing edge separation appears to be delayed relative to that expected at full-scale Reynolds numbers. For instance, the data by Somers (1997) show separation occurring at \( x/c = 0.55 \) and \( x/c = 0.40 \) for \( \alpha = 12^\circ \) and \( \alpha = 15^\circ \), respectively.

In explaining this difference it should be noted that during acquisition of the data presented in this thesis, the trailing edge separation was observed to move rapidly with an increasing angle of attack. This sensitivity may have contributed to the differences observed between these low Reynolds number measurements and those of Somers (1997). Furthermore, at these relatively high angles of attack the effect of blockage becomes more pronounced. The corrections for blockage and wall effects that were employed may have also introduced uncertainties in the angle of attack.

6.6 Insights for the Tidal Turbine

6.6.1 Application of a BEM Model

A standard blade-element momentum (BEM) model was used in this thesis to estimate the angles of attack, induced velocity and the tip-loss of the model-turbine used in the towing-tank experiments. The model incorporated the lift and drag data that were acquired at low Reynolds numbers in the wind tunnel experiments. The implementation of the model is provided in Appendix E.

As the predictions from BEM theory are dependent on the accuracy of the lift and drag, the Reynolds number sensitivity presented challenges in the implemen-
Chapter 6. Performance of the Foil at Low Reynolds Number

tation of the model. For example, given that data were only acquired at three Reynolds numbers, an assumption was necessary to estimate the lift and drag coefficients at the Reynolds numbers applicable to the blade elements. In this instance, the coefficients were estimated by using a linear interpolation between the measured data. Furthermore, it is important to consider the uncertainties that arise from the wall corrections that were applied and any effect the turbulence intensity had on the data. Similar issues are also expected to be faced by other experimentalists who attempt to incorporate experimentally measured aerofoil data into BEM theory.

The data were also modified slightly so that the region of the relatively high lift and drag at low angles of attack corresponded to $\alpha \leq 2^\circ$. With this modification the alignment between the rotor coefficients for both the 2010 and 2012 tests was improved at the very high tip-speed ratios (i.e. low angles of attack). It is postulated that there may be increased inertia in the boundary layer or secondary 3-D flows for a rotating blade. This could have enabled the boundary layer to remain attached to the pressure surface of the foil at lower angles of attack compared to a two-dimensional foil in steady flow. However, the modification to the data did not have any significant influence on the estimates for tip-speed ratios between $3 < \lambda \leq 5$, which were the operational states of most interest.

As demonstrated in Figure 6.9, reasonable predictions of the observed out-of-plane and in-plane blade-root bending moments were able to be achieved by the model. Given this, the model was deemed appropriate to be applied to estimate the other relevant parameters that were not able to be measured in the experiment.

6.6.2 Predictions of the Operational State of the Turbine & Tip-Loss

The BEM estimates of the angles of attack at blade sections at $r = 0.50R$, $0.75R$ and $0.90R$ for both the 2010 and 2012 tests are presented as a function of the tip-speed ratio in Figure 6.10(a). For tip-speed ratios between $\lambda = 4$ and 6, the angles of attack at these outer-sections of the blade were predicted to range between approximately $\alpha = 4^\circ$ to $8^\circ$. Referring to Figure 6.5, at these angles of attack the two-dimensional lift coefficient was observed to vary approximately linearly with angle of attack.

The predicted angles of attack from the BEM model support the observation
6.6. Insights for the Tidal Turbine

Figure 6.9: Comparison of the observed and BEM predictions of the out-of-plane bending moment coefficients, (a) and the in-plane bending moment coefficients, (b) for a rotor speed of $\Omega = 96$ rpm. Observed loads are denoted by the symbols (□) and (×) and BEM predictions by the solid and dashed lines for the 2010 and 2012 trials respectively.

in Chapter 5, in that the out-of-plane bending moment response exhibited a nearly linear relationship with axial velocity between these tip-speed ratios. This can be demonstrated by linearising the bending moment coefficient, which yields the expression $C_{My} \approx C_l + C_d \phi$. It was shown that for a Reynolds number of $1.1 \times 10^5$, a lift coefficient of $C_l \approx 0.5$ corresponded to $C_d \approx 0.07$. At moderate angles of attack, the inflow angle at the blade sections is expected to be of the order $\phi = 0.1$ rad. This would imply that $C_d \phi \approx 0.007$, and that the contribution from the drag is small. The pressure distributions also showed that laminar separation bubbles may be present at these angles of attacks, which can affect the lift. This also supports the sensitivity in the bending moment coefficients to the Reynolds number that was observed.

However, it is also interesting to briefly consider the significance of the relatively large aerofoil drag coefficient at low Reynolds numbers on the in-plane bending moment. The linearised in-plane bending moment can be expressed as $C_{Mx} \approx C_l \phi - C_d$. For the same lift and drag coefficients and inflow angle as previously mentioned, the contributions from the lift and drag forces will be of the same order of magnitude as $C_l \phi \approx 0.05$. Therefore, the higher drag has a more dominant role in the in-plane bending moment, particularly at low angles of attack where it will act to significantly reduce the bending moment. Whilst the out-of-plane loading is of most relevance to the objectives of this thesis, it is
Chapter 6. Performance of the Foil at Low Reynolds Number

Figure 6.10: BEM theory estimates of the angle of attack, (a) and axial induction factor, (b) as a function of tip-speed ratio for $r/R = 0.5$ (−), 0.75 (−−) and 0.9 (−−−). All estimates correspond to a rotor speed of $\Omega = 96$ rpm and are denoted in grey and black for the 2010 and 2012 trials respectively.

an important consideration if model-scale testing is used to investigate rotational inertia effects.

At a tip-speed ratio of $\lambda = 3$, the angles of attack at these outer blade sections are predicted to be approaching the angle at which the aerofoil is expected to achieve maximum lift. The predictions from the BEM model are likely to be weakest in this region. This is because the effects of flow three-dimensionality at the blade section is expected to become more pronounced (Burton et al., 2001). However, the estimates serve to demonstrate that for axial perturbations of the rotor around this tip-speed ratio, unsteady contributions from delayed separation and possibly dynamic stall may be present.

Estimates of the axial induction factor ($a$) at the blade sections $r = 0.50R$, 0.75$R$ and 0.90$R$, for both the 2010 and 2012 tests are presented as a function of tip-speed ratio in Figure 6.10(b). For tip-speed ratios between $\lambda = 4$ and 6, the axial induction factor is predicted to range between $0.23 < a \leq 0.28$. These are below the value that is expected for an ideal rotor, which is $a = 1/3$ (Burton et al., 2001) and imply that the rotor was somewhat lightly loaded. It is also useful to note that at a tip-speed ratio of $\lambda = 4.5$ and at 0.75$R$, the axial induction factors for the 2010 and the 2012 tests both correspond to induced velocities of approximately $v_i \approx 0.2$ m s$^{-1}$. This value of induced velocity was incorporated with the dynamic-inflow and Loewy models that are subsequently presented in
6.6. Insights for the Tidal Turbine

The estimates of the Prandtl tip-loss factor ($F$) at the tip-speed ratios of $\lambda = 3.5$ and 4.5 are shown in Figure 6.11. These are presented as a function of the non-dimensional span ($x/R$) and correspond to both the 2010 and 2012 tests. The Prandtl tip-loss factor represents the ratio between the azimuthally averaged induced velocity at the rotor and the local induced velocity (see Appendix E). For both the 2010 and 2012 tests, the value of $F$ is less than 1 for approximately the last 40% of the blade, which is significant.

However, the extent of this loss in circulation is in general alignment with theoretical results. For instance, Goldstein (1929) provided the distribution of circulation for a 2-bladed and a 4-bladed propeller operating at a tip-speed ratio of $\lambda = 5$. Interpolating between the results for a 2 and 4 blade rotor gives loss of circulation over the outer span of the blade which qualitatively supports the prediction from the BEM model.

### 6.6.3 Additional Considerations

A significantly large tip-loss would imply than any laminar separation bubbles on the turbine foils would have been subjected to a relatively large spanwise gradient. This three-dimensional effect may be more significant than that due
to centrifugal effects at the in-board sections. Whilst the wind tunnel tests were restricted to a two-dimensional flow, characterising this 3-D effect at the outer-sections would be of interest. Additionally, it would also be useful to conduct unsteady tests of the aerofoils to investigate the effect of the laminar separation bubbles on the dynamic stall of the foil. This would assist in establishing the role of the Reynolds number in the model scale tests conducted at low tip-speed ratios.

It is also interesting to briefly consider whether full-scale aerofoil behaviour could be better replicated at model-scale. A large laminar separation bubble or premature flow separation on the pressure surface could possibly be avoided by promoting early transition of the boundary layer by using a mechanical turbulator. For instance, Faudot and Dahlhaug (2012) applied a trip wire to the surface of the blades of a model-scale tidal turbine. However, it is important to consider that at model-scale, a turbulator can also increase the thickness of the boundary layer and subsequently alter the profile of the aerofoil (Lissaman, 1983). Therefore, the desired full-scale lift and drag coefficients may be very difficult to achieve without the utmost care. This was the reason for not applying a turbulator to the model turbine used in the experiments in this thesis.

6.7 Summary

The two-dimensional lift and drag coefficients and the pressure distributions of the S814 foil have been obtained at steady flow in a wind tunnel. These Reynolds numbers at which data were obtained are applicable to the outer-blade sections of the model-scale turbine tested in the towing-tank.

The data provided new insights into the aerodynamic performance characteristics of the S814 foil at Reynolds numbers of the order of $1 \times 10^5$. At low angles of attack below $\alpha < 4^\circ$, full-separation occurs on the pressure surface of the foil which in turn, increases the lift and drag forces. This characteristic is believed to be a characteristic of thick laminar-flow foils.

The lift coefficient exhibited an approximately linear relationship with angle of attack between $4^\circ < \alpha < 9^\circ$. At an angle of attack of $8^\circ$, the lift coefficient was up to 10% lower and the minimum drag coefficient was at least 400% higher than is expected at full-scale (i.e. $Re = 1 \times 10^6$). The reduction in the lift is due to dominant laminar separation bubbles which are present across both surfaces of the foil. These laminar separation bubbles persist for much higher angles of
attack than is expected at full-scale, including the angles at which trailing-edge separation occurs.

At a tip-speed ratio of $\lambda = 4.5$, the angles of attack corresponding to the outer blade sections of the turbine are within the range where the aerofoil lift is approximately linear and the rotor is also predicted to be lightly loaded. An analysis revealed that the tip-loss is expected to affect up to 40\% of the span, which is significant. The identification of the implications of the low Reynolds number on the model-scale testing may also warrant a consideration of the effects of three-dimensionality on laminar separation bubbles.
Chapter 7

Blade Loads for Unsteady Forcing

Simplicity is the ultimate sophistication

— Leonardo da Vinci

7.1 Introduction

This chapter provides designers of tidal turbines with new insights into the unsteady hydrodynamic blade loading due to onset turbulence. These results extend those which were published in the journal *Ocean Engineering* (Milne et al., 2013a).

The responses to single frequency oscillations are used to demonstrate the sensitivity of the underlying hydrodynamic phenomena to the forcing. Operating conditions where the boundary layer on the blades is attached and undergoes separation are both specifically investigated. The contribution of the unsteady loading is quantified relative to the steady load. The data are used to explore the applicability of dynamic inflow theory, the returning wake model of Loewy (1957) and a semi-empirical dynamic stall model. Multi-frequency and discrete half-sinusoidal forcing experiments are used to demonstrate how the single frequency oscillatory response can be applied to model the response to a more general multi-frequency or gust-type forcing.
Chapter 7. Blade Loads for Unsteady Forcing

7.2 Methodology

7.2.1 Parameter Selection

Single Frequency Oscillations

The single frequency oscillatory experiments were performed in both the 2010 and 2012 test series. The 2010 tests were primarily focused on examining the blade load response when the turbine was operating near maximum power and boundary layer separation was expected on the blades. These were conducted at mean tip-speed ratios of $\lambda = 3.6$ and $\lambda = 4.1$ which correspond to rotor speeds of 84 rpm and 96 rpm. Four oscillatory frequencies of $f = 0.50, 0.64, 0.89$ and $1.40$ Hz and three velocity amplitudes of $\mu (= u/U) = 0.100, 0.200$ and $0.300$ were investigated.

The 2012 experiments entailed a more extensive parametric study of the sensitivity of the blade loads to the frequency and the velocity amplitude of the forcing. The majority of these tests were performed at a mean tip-speed ratio of $\lambda = 4.5$ and a rotor speed of $\Omega = 96$ rpm. At this operating state, the boundary layer was believed to remain attached on the outer sections of the blades. The frequencies that were investigated ranged between $0.40 < f \leq 1.00$ Hz. These correspond to reduced frequencies at a span of $0.75R$ in the range of $0.02 < k \leq 0.05$. The Current numbers that were considered ranged over $0.075 < \mu \leq 0.250$.

Multiple Frequency Oscillations

The multi-frequency forcing experiments were conducted in the 2012 programme. The out-of-plane bending moment responses were used to investigate the applicability of a linear superposition of the single frequency oscillations to describe the response. Given this, the forcing comprised a superposition of the equivalent frequencies that were tested in the single frequency experiments. The tests were also performed about the same mean tip-speed ratio of $\lambda = 4.5$ and at a rotor speed of 96 rpm.

The Current numbers for each oscillatory component were between $\mu = 0.075$ and 0.125. This was necessary to ensure that the kinematics of the forcing were sufficiently similar to the anticipated full-scale forcing and to avoid significant flow separation at the blade sections. These constraints effectively limited the number of frequencies in each multi-frequency case to a maximum of three.
7.2. Methodology

Discrete Half-Sinusoidal Forcing

A limited set of trials were conducted in the 2012 tests whereby the rotor was subjected to an individual, half-sinusoidal axial perturbation superimposed onto a steady velocity. This forcing was intended to represent isolated large eddies (analogues to gusts in the atmosphere). Two relatively low frequencies of $f = 0.40$ and $0.50$ Hz were considered, with a relatively large Current number of $\mu = 0.250$.

Both positive and negative sinusoidal motions were investigated. This forcing resulted in the turbine being perturbed towards stall and the turbulent wake state respectively. As for the multi-frequency tests, the experiments were performed about a mean tip-speed ratio of $\lambda = 4.5$ and a rotor speed of 96 rpm.

7.2.2 Data Processing

The oscillatory data analysed in the subsequent sections were obtained once the main carriage had achieved a constant velocity and after at least two oscillations had been completed. Due to the finite length of the towing tank, the total number of oscillations that comprised each test varied with the oscillatory frequency. For the lowest frequency considered, at least 10 oscillations were possible. A cycle-by-cycle analysis (as described in Section 7.2.4) showed that this was a sufficient number for inferring the hydrodynamic blade load response. The relatively quick convergence of the phase-averaged out-of-plane bending moment response for two typical oscillatory cases is also demonstrated in Figure 7.1.

As the hydrodynamic loading was of interest in this investigation, the once-per-revolution, gravitational contribution was removed. A sinusoidal function was fitted to the bending moment response that corresponded to zero carriage velocity and a low rotational speed (see Section 5.2). This was then subtracted from the bending moment that was measured during the unsteady tests. An example of the procedure is shown for a test case in Figure 7.2.

A 5th-order Butterworth infinite impulse response (IIR) digital filter was applied off-line to all signals co-currently. This was applied in both forward and backward directions to eliminate any phase shift. The effect of the cut-off frequency is demonstrated in Figure 7.3 and a cut-off frequency of $4f$ was selected for all signals (in the case of a multi-frequency test, this corresponded to the highest frequency component). This was found to reduce the majority of the erroneous noise, whilst also retain the essential underlying non-linear hydrodynamics associated with the boundary layer separation at the blade.
Figure 7.1: The convergence of the phase-averaged out-of-plane bending moment response for a forcing corresponding to $f = 0.25 \text{ Hz}, \lambda = 4.5$ (a) and $f = 0.89 \text{ Hz}, \lambda = 3.6$ (b). Legend: response average over 2 cycles (- -), 4 cycles (solid black line) and 6 cycles (solid grey line).

Figure 7.2: An example of the original low-pass filtered in-plane bending moment response (black) and the response with the gravitational component removed (grey), for a forcing corresponding to $f = 0.25 \text{ Hz}, \mu = 0.15$ and $\lambda = 4.5$. The full-time history is shown in (a) and a subset of the data is shown in (b) for clarity.
7.2. Methodology

Figure 7.3: The effect of the IIR digital filter that was applied to post-process the data. Examples of the time history of the velocity, (a) out-of-plane bending moment, (b) acceleration (c) and the in-plane bending moment, (d). Legend: cut-off frequencies of \( f_c = 16f \ldots \), \( 8f \ldots \) and \( 4f \ldots \). The oscillation corresponds to \( f = 0.64 \text{ Hz} \), \( \mu = 0.2 \), \( \lambda = 3.6 \) and \( \Omega = 84 \text{ rpm} \) and dynamic stall is observed in the bending moment response.

7.2.3 Presentation of the Dynamic Response

Hysteresis Loops

Several approaches were used to analyse the influence of unsteady hydrodynamics on the blade loads. Hysteresis loops have been frequently employed in the literature to elucidate the unsteady phenomena associated with aerofoils subjected to angle of attack forcing (see e.g. Leishman, 2006). However, they have not been previously applied to study the unsteady load on tidal turbine blades.

In this research, hysteresis loops are formed by presenting the (phase-averaged) instantaneous bending moment as a function of the instantaneous velocity. The width of the loops reveal the contribution of the bending moment component due to hydrodynamic inertia, which appears in-phase with acceleration. The unsteady component of the bending moment which appears in-phase with velocity acts to modify the gradients of the loops. For cases where the boundary layer
was believed to remain predominately attached to the foil, the ratio of the un-
steady bending moment to the bending moment which was measured in steady 
flow at the equivalent mean tip-speed ratio and rotor speed is presented on the 
ordinate. This provides a quantification of the magnitude of the bending moment 
perturbation in response to onset turbulence.

The corresponding steady loads at the equivalent axial velocity are also pre-
sented together with these hysteresis loops. Comparisons with these loads allow 
the unsteady hydrodynamic contribution to the bending moment to be quantified, 
which was a primary objective of this research. A key advantage of quantifying 
the contribution directly with the experimentally measured steady loads is that 
there are no errors that could potentially have arisen from the use of a numerical 
model or from using data from a different rotor.

**Non-Dimensional Coefficients**

For the out-of-plane bending moment, estimates of the amplitude and phase (anal-
ogous to added mass and damping coefficients) were obtained for cases where the 
response was approximately linear. To provide a quantification of the unsteady 
contribution, these amplitudes were normalised by the bending moment amplit-
tude for steady flow. This steady flow amplitude was obtained from the linear fit 
that was presented in Figure 5.10, where the gradient was equal to $\delta M_y = 11.9\delta U$.

It is important to consider that this approach is only useful if the steady bending 
moment response itself remains linear over the oscillation. This is a reasonable as-
sumption if the flow separation is not significant (i.e. in the range $3.5 < \lambda < 5.5$).

As shown in Section 6.5.2, the lift coefficient (and hence the out-of-plane 
bending moment) was reduced by a near constant factor at low Reynolds numbers 
relative to full-scale. Therefore, a key advantage of normalising the amplitudes 
as well as the ordinates of the hysteresis loops) by the load measured for steady 
flow is that they are likely to be approximately independent of the Reynolds 
number. This implies that results can be readily applied to a full-scale turbine.

An estimate of the non-circulatory bending moment was used to provide a 
quantification of the added hydrodynamic contribution to the inertia. The non-
circulatory forcing was predicted on a section basis using the expression

$$M_y|_{nc} = \frac{1}{4} \rho \pi \int_{r_{sg}}^{R} \alpha^2 r \, dr.$$  \hspace{1cm} (7.1)
The approach to compute the non-circulatory forcing is consistent with that used by Whelan (2010) and has been discussed in Section 2.4.1. The ratio of the component of the out-of-plane bending moment which appears in-phase with acceleration, to the bending moment due to non-circulatory forcing, is analysed in Section 7.3.3.

7.2.4 Approach Used to Estimate the Amplitude & Phase

It was necessary to fit a linear model to the loading time history to estimate the non-dimensional coefficients. Whelan (2010) applied a model of the form

$$F_x(t) = A_1 u(t) + A_2 \dot{u}(t),$$

(7.2)

to relate the oscillatory thrust to the oscillatory velocity and acceleration, where $A_1$ and $A_2$ were time-invariant coefficients. The advantage of this approach is that it may be applied to any motion history, including one which may not necessarily be perfectly sinusoidal.

However, this also implies that the contributions from individual frequency components are not able to be distinguished. Therefore, the technique cannot assist with the verification of linear superposition for multi-frequency oscillations. As the forcing was nominally harmonic in the present study, a model of the form

$$M(t) = \sum_{i=1}^{N} Z_i \cos(2\pi f_i t + \Phi_i),$$

(7.3)

that relates the oscillatory bending moment ($M(t)$) to the velocity forcing was considered to be more suitable and was subsequently employed. In this model, $Z_i$ is the amplitude, $\Phi_i$ is the phase of the $i^{th}$ oscillatory frequency component relative to the velocity and $N$ is the number of fundamental frequency components.

Various approaches are available to estimate the coefficients (Chakrabarti, 1987). For this thesis, a least-squares method was selected. It was deemed to be preferable over a Fourier Transform, due to relatively low number of oscillations in the data. A routine was written in Matlab and iterations were performed until the value of the residual sum of squares satisfied the condition $R^2 \geq 0.99$. The model was fitted to two oscillatory cycles at a time and the coefficients were averaged across all estimates. Estimates of the normalised amplitude and phase lead for a typical test case of $f = 0.40$ Hz are demonstrated in Figure 7.4. These
remain relatively stable over all the oscillatory samples.

Acquiring estimates for two cycles at a time also allowed for the effect of any underlying surface wave action to be identified and accounted for. As a case in point, for a single oscillatory frequency of $f = 0.50 \, \text{Hz}$, the auxiliary-carriage was found to induce a surface wave of equivalent frequency as the axial forcing. This was confirmed from measurements of the free surface displacement using a sonic probe. By applying linear wave theory (see Sarpkaya and Isaacson, 1981), the magnitude of the wave induced velocity was estimated to be an order of magnitude smaller than the axial velocity perturbation. However, the wave induced perturbation was sufficient to adversely affect the estimates of the phase had they been evaluated from the complete time history. As is also shown by Figure 7.4, a cycle-by-cycle analysis reveals the underlying wave motion and its effect on the amplitude and phase. In instances where the wave amplitude was small, the estimated coefficients were found to agree well with the expected trends based on the other frequency cases (as well as when estimated from multi-frequency forcing).

![Figure 7.4: Estimates of the normalised out-of-plane blade-root bending moment amplitude and phase for an oscillatory frequency of $f = 0.4 \, \text{Hz}$ and $(-\circ-) f = 0.5 \, \text{Hz}$ ($-\square-$.). The effect of a seiche wave is identifiable in the values for the $f = 0.5 \, \text{Hz}$ case.](image)

For cases where a seiche wave was identified as having been present during the test run, the estimates were based on those corresponding to the initial cycles or when the amplitude of the wave was relatively small. It is also important to note that the presence of a seiche wave was not observed for the multi-frequency
cases. Therefore, for a multi-frequency case the estimates of the amplitude and phase were found to be consistent when either applying the fit on a cycle-by-cycle basis or over the complete history.

7.3 Single Frequency Oscillations for Attached Flow

The effect of a variation in the oscillatory frequency and velocity amplitude is first demonstrated for a mean-tip speed ratio of $\lambda = 4.5$. Given the inherent complexity of the hydrodynamics, the sensitivity of the unsteady blade loading to the oscillatory forcing was investigated by varying each parameter individually.

7.3.1 Sensitivity to the Oscillatory Frequency

The hysteresis loops of the out-of-plane and in-plane blade-root bending moments for oscillatory frequencies of $0.40 < f \leq 0.80$ Hz at a fixed Current number of $\mu = 0.150$ are shown in Figure 7.5.

The unsteady response of the out-of-plane bending moment exhibits an approximately linear relationship with velocity. This is consistent with the expectation that the boundary layer remains attached across the outer sections of the blades in these conditions. These reveal a relatively small increase in the bending moment range with the frequency; increasing from $M_y/M_{yQS} = 0.54$ at $0.40$ Hz to $0.60$ at $0.80$ Hz. This implies that with an increase in frequency, the unsteady hydrodynamics increases the magnitudes of the loads on the blades.

The hysteresis loops of the out-of-plane bending moment are circumvented clockwise, implying a phase-lead and positive hydrodynamic added axial inertia. Both an overshoot of the steady load in the out-of-plane bending moment and a phase-lead over velocity are consistent with the effects attributed to dynamic inflow and the non-circulatory forcing (see Section 2.4.1). The loops become narrower and the phase-lead reduces as the frequency (and axial acceleration) is increased.

The identification of an underlying frequency dependency in the phase of the out-of-plane bending moment is an interesting new observation. Whelan (2010) was unable to experimentally identify any frequency dependency in the axial inertia. This may have been due to the relatively low frequencies in which that
The effect of the oscillatory frequency on the blade-root bending moment responses for a Current number of $\mu = 0.150$. The out-of-plane and in-plane responses are presented in (a) and (b) respectively. The oscillations were performed at a mean tip-speed ratio of $\lambda = 4.5$ and a rotor speed of 96 rpm. From bottom: $f = 0.40 \text{ Hz}$, $0.57 \text{ Hz}$, $0.67 \text{ Hz}$ and $0.80 \text{ Hz}$ (offset in increments of 0.1 for $M_y$ and 0.2 for $M_x$). The bending moments that were measured at the equivalent axial velocity in steady flow are depicted by the bullets.

Experimental data were obtained. A reduction in the phase-lead suggests that the induced velocity in the wake may have been less able to adapt to the changes in loading at the rotor plane at higher frequencies, tending towards a frozen-wake state. Whelan (2010) did not specifically consider the unsteady hydrodynamic influence on the loading in-phase with velocity relative to the steady loads. As the loading component in-phase with acceleration was relatively small, it is not believed to have contributed to the change in the slope of the loops that was observed. Instead, the unsteady circulatory forcing (dynamic inflow) in-phase with velocity is considered to have predominately given rise to the increased amplitudes of the unsteady loads.

The response of the in-plane bending moment is comparatively more non-linear than the out-of-plane bending moment. There appears to be a relatively
small increase in the amplitude of the in-plane bending moment with frequency. For the in-plane bending moment, a phase-lag over velocity is observed for all but the lowest frequency case. The magnitude of the component in-phase with acceleration increases with frequency (i.e. the loops widen). A phase-lag is consistent with the expectation that if there is no misalignment in the system, there should be negligible contribution to the in-plane bending moment from either the non-circulatory forcing or dynamic inflow that could otherwise impart a phase-lead.

### 7.3.2 Sensitivity to the Velocity Amplitude

The bending moment responses to Current numbers in the range $0.100 < \mu \leq 0.250$ at an oscillatory frequency of $f = 0.40\text{Hz}$ ($k = 0.02$) is presented in Figure 7.6.

At this relatively low frequency, the out-of-plane bending moment remained approximately linear for all the Current numbers considered. For the out-of-plane bending moment, the magnitude of the component which appears in-phase with acceleration increases with frequency (i.e. the loops become wider). This is consistent with the non-circulatory contribution having increased with an increase in the axial acceleration. For the in-plane bending moment, the component in-phase with velocity is relatively small for all the Current numbers considered. This also concurs with notion of the in-plane loads being insensitive to the non-circulatory forcing.

As demonstrated in Figure 7.7, similar trends in both the out-of-plane and in-plane bending moment responses to an increase in Current number are observed at the higher frequencies of $f = 0.67\text{Hz}$ (solid curves) and $f = 1.00\text{Hz}$ (dotted curves).

Figure 7.7 also shows that the non-linearities in the responses become pronounced at high Current numbers as the frequency is increased. These non-linearities give rise to a relatively large bending moment compared to the steady load. For example, for a Current number of $\mu = 0.250$, the bending moment range increased from $M_y/M_{yQS} = 0.83$ at $f = 0.40\text{Hz}$ to 0.98 at $f = 1.00\text{Hz}$, which is an 18% increase.

As the non-linearities are only present for a part of the oscillatory cycle, they are primarily attributed to unsteady circulatory effects at the blade-section level. To understand their origin, it is useful to consider that at high instantaneous velocity the rotor nears maximum power. As was shown by Figure 6.10, the angles
of attack are predicted to be relatively large and there may be flow separation at the blade sections. It is postulated that in unsteady flow there is a delay in the boundary layer response on the foil and the movement of the trailing-edge separation point, to the excitation. This could have increased the lift generated by the foil relative to that expected for steady flow. An increase in the lift is also consistent with the in-plane bending moment responses, which show a phase-lead at high instantaneous velocity. These effects are explored further in Section 7.4.

The bending moment responses for the frequencies of \( f = 0.40 \) Hz and 0.67 Hz and a Current number of \( \mu = 0.250 \) but at a higher mean tip-speed ratio of \( \lambda = 5.0 \) are presented as the dotted curves in Figure 7.8. These have been superimposed on the corresponding oscillatory responses obtained for \( \lambda = 4.5 \) for comparison and are presented in terms of the inverse of the instantaneous tip-

![Figure 7.6](image-url)

Figure 7.6: The effect of the Current number on the blade-root bending moment responses for an oscillatory frequency of \( f = 0.40 \) Hz. The out-of-plane and in-plane responses are presented in (a) and (b) respectively. The oscillations were performed at a mean tip-speed ratio of \( \lambda = 4.5 \) and a rotor speed of 96 rpm. From bottom: \( \mu = 0.100, 0.125, 0.175, 0.200 \) and 0.250 (offset in increments of 0.1 for \( M_y \) and 0.2 for \( M_x \)). The bending moments that were measured at the equivalent axial velocity in steady flow are depicted by the bullets.
Figure 7.7: The effect of the Current number on the blade-root bending moment responses for the oscillatory frequencies of $f = 0.67$ Hz (−) and 1.00 Hz (..). The out-of-plane and in-plane responses are presented in (a) and (b) respectively. The oscillations correspond to a mean tip-speed ratio of $\lambda = 4.5$ and rotor speed of 96 rpm. From bottom: $\mu = 0.125, 0.200, 0.250, 0.300, 0.350, 0.400$ (offset in increments of 0.2 for $M_y$ and 0.5 for $M_x$). The bending moments that were measured at the equivalent axial velocity in steady flow are depicted by the bullets.

speed ratio ($1/\lambda$). These serve to demonstrate that at high values of $1/\lambda$ (i.e. high instantaneous velocities) the component of the loading which appears in-phase with the acceleration now exhibits a greater degree of linearity. This provides additional evidence that these non-linearities that were observed in the previous cases are attributable to the operational state of the rotor and flow separation.

However, it can also be noted that non-linearities are conversely observed in the response for $\lambda = 5.0$ at low values of $1/\lambda$ (i.e. low instantaneous velocities). This is believed to be due to the flow separation on the pressure surface of the foils (see Chapter 6). Non-linearities may also arise from the rotor having neared the turbulent-wake state, which would be expected to change the induced flow significantly. This serves to reinforce the idea that for the model-scale rotor, there are a relatively small range of tip-speed ratios for which the out-of-plane bending
moment is approximately linear, and where linear superposition of the loading can be reliably investigated.

### 7.3.3 Quantification of the Unsteady Contribution

The estimates of the normalised amplitude and phase corresponding to the out-of-plane bending moment response for Current numbers of $0.100 < \mu \leq 0.175$ and frequency ratios of $0.25 < m \leq 0.50$ ($0.40 < f \leq 0.80$ Hz) are presented in Figure 7.9.

As discussed in Section 7.2.3, this normalisation was based on the equivalent linear bending moment amplitude for steady flow. The normalised amplitude increases with frequency from approximately 1.05 to 1.15, whilst the relatively small phase-lead decreases from approximately $\Phi = 4.5^\circ$ to $1.5^\circ$. 
The normalised amplitudes decrease with the Current number, an effect which appears to be more significant at low frequencies. To explain this, it can be noted that at relatively large Current numbers the steady loads were more non-linear at higher instantaneous velocities, with the local gradient decreasing. As the frequency was increased, the delayed separation was expected to have increased the lift coefficient. This would have effectively increased the amplitude of the response. As such, the response would have been in closer agreement to the linear, steady flow load that was used for normalisation.

There is very limited literature for tidal turbines to compare these estimates of the amplitude and phases against. However, it is interesting to contrast the component of the bending moment that appears in-phase with the acceleration, with the added inertia coefficients that were reported at relatively low frequencies by Whelan (2010). These were \( C_I = \frac{\{F_x\}_u}{\rho V \dot{u}} \leq 0.05 \), where \( \{F_x\}_u \) was the component of the axial thrust that appeared in-phase with acceleration and \( V \) was the volume enclosed by the rotor. An equivalent coefficient was estimated from the data herein. This assumed that the three blades contributed equally to the thrust and gave rise to an effective axial force acting at \( x/R = 0.63 \) (see Section 5.6.1). For the lowest frequencies investigated, a coefficient of \( C_I = 0.03 \) is predicted from the experimental results in this thesis. Therefore, the value of

![Figure 7.9: Estimates of the amplitude, (a) and phase, (b) corresponding to the single frequency out-of-plane bending moment response, as a function of the frequency ratio. Legend: \( \mu = 0.100 (\square), 0.125 (\circ), 0.150 (\Delta) \) and \( 0.175 (+) \). All data correspond to an oscillation at a mean-tip speed ratio of \( \lambda = 4.5 \) and rotor speed of 96 rpm.](image-url)
the coefficient is within the bounds that was estimated by Whelan (2010). This is despite the inconsistencies between the experimental set-ups and the control strategies in the different sets of experiments.

The component of the bending moment that appears in-phase with acceleration contains both circulatory and non-circulatory forcing contributions. Its sensitivity to the forcing provides interesting new insights into the role of the underlying hydrodynamic phenomena. As discussed in Section 7.2.3, the ratio of the bending moment in-phase with acceleration to the bending moment due to the non-circulatory forcing was estimated. This ratio is presented as a function of the frequency ratio \( m \) in Figure 7.10.

\[
M_y \dot{u} / M_{ync} = 2.7
\]

\( m = 2\pi f / \Omega \)

**Figure 7.10:** The ratio of the out-of-plane bending moment component in-phase with acceleration to the non-circulatory bending moment, as a function of the frequency ratio. Data for single frequency planar oscillations about a mean tip-speed ratio of \( \lambda = 4.5 \). The symbols denote Current numbers of \( \mu = 0.100 (\square), 0.125 (\circ), 0.150 (\triangle) \) and 0.175 (+).

At low frequency ratios, the ratio \( M_y \dot{u} / M_{ync} \) is approximately equal to 2.7 and its value decreases with frequency ratio to 0.5 for \( m = 0.5 \). This implies that the net circulatory forcing tends to act in opposition to the non-circulatory forcing at relatively high frequency ratios. It is also interesting to find that the coefficients are generally independent of the Current number. This suggests that the circulatory forcing is predominately a function of the frequency.
7.3. Single Frequency Oscillations for Attached Flow

7.3.4 Comparisons with a Dynamic Inflow Model

In Section 2.4.1, dynamic inflow theory was identified as a possible technique to predict the unsteady circulatory response in attached flow conditions. Its application to the model-turbine is demonstrated here using both the perturbation and the non-linear forms of the models which are described by Peters and HaQuang (1988).

However, one significant issue in its application to the model rotor used in these experiments is that both models do not account for the tip-loss. The tip-loss was predicted to influence approximately the outer 40% of the blades of the model turbine (see Section 6.6.2). There is a scarcity of literature with regards discussions on the implications of the tip-loss when applying the theory. Given this situation, consideration was made to incorporate the tip-loss into the model in this thesis.

Perturbation Model

For the perturbation version of the dynamic inflow model, one option investigated was to assume that the unsteady loading was a small perturbation to the mean loading and to incorporate the loss into the mean loading only. On this basis, the dynamic-inflow model described by equation 2.11 was applied according to

\[ \delta F = 4\pi r \rho \bar{v}_i \delta u = \frac{8}{3} \pi r^2 \rho \frac{dv_i}{dt} + 4\pi r \rho (U - 2\bar{v}_i) \delta v_i, \]  

(7.4)

where, \( \delta u \) and \( \delta v_i \) are small increments in the axial and induced velocities respectively. The mean induced velocity, \( \bar{v}_i \) was obtained using equilibrium BEM theory (which included the tip-loss). An estimate of the increment in the axial thrust, per unit length (\( \delta F \)), can be obtained by assuming that \( dC_l/d\alpha = 2\pi \), neglecting drag and swirl, and approximating the resultant velocity as \( W \approx \Omega r \).

For small angles, this increment in force on the blade element, per unit length is expressed as

\[ \delta F = \rho \Omega R \pi c (\delta u - \delta v_i). \]  

(7.5)

This provides the circulatory thrust from which the circulatory response to the blade root bending moment was obtained. The total circulatory bending moment response was added to the non-circulatory contribution estimated using equation 7.1. However, the non-circulatory forcing is not incorporated into this dynamic-inflow model.
The prediction of the model is compared with the observed out-of-plane bending moment response for frequencies $f = 0.40 \text{ Hz}$ and $0.57 \text{ Hz}$ and a mean Current number of $\mu = 0.15$ in Figure 7.11. The unsteady loading which appears in phase with acceleration is able to be reconstructed relatively well for $f = 0.40 \text{ Hz}$, but the amplitude is over-predicted by approximately 20%. The over-prediction in the amplitude is a consequence of not accounting for the tip-loss in the perturbation. For blade design, the over-prediction in the amplitude does introduce conservativeness. However, the model was found to be unable to reproduce the decrease in the component of the loads which appears in-phase with the acceleration at the higher frequencies that were investigated.

![Figure 7.11: Comparison of the observed normalised oscillatory part of the bending moment (–), with a prediction using the perturbation version of the dynamic inflow model with no tip-loss (..), and the non-linear dynamic inflow model with a tip-loss (−−). The oscillations comprise frequencies of $f = 0.40 \text{ Hz}$ and $f = 0.57 \text{ Hz}$, a Current number of $\mu = 0.15$ and a mean-tip speed ratio of $\lambda = 4.5$. The hysteresis loops are all circumvented clockwise and the latter bending moment is offset upwards by 0.2.]

**Non-Linear Model**

In the case of the non-linear version of the model, it is the total induced velocity and loading which is modelled. The model is expressed as

$$F = \frac{8}{3} \pi r^2 \rho \frac{dv_i}{dt} + 4\pi r \rho (U + u - v_i) v_i.$$ (7.6)
To incorporate the effect of the tip-loss in the model, one approach investigated was to apply the Prandtl loss factor as a lift deficiency function to the forces, as opposed to the momentum, at the blade elements. A similar approach was used by Peters et al. (2007), where the lift-deficiency was used to obtain an equivalent additional induced velocity contribution. The benefit of this was that the additional-induced-velocity contribution could be used to correct the drag.

However as the lift-deficiency is applied to the loading at each blade element, the mean loading is not equivalent to that predicted using the quasi-steady BEM model that was used in Section 6.6.1. For the aforementioned cases the mean loading is reduced by approximately 7% relative to the quasi-steady BEM model. Therefore, comparisons with the dynamic-inflow model were based on the oscillatory part of the response. As is also shown in Figure 7.11, the predictions of the oscillatory response using the non-linear model agree well with the component in-phase with the acceleration, but under-predict the bending moment amplitude. As for the perturbation model, the non-linear model was found to be unable to replicate the decrease in the loading in-phase with acceleration that was observed at higher frequencies.

Therefore, whilst qualitative predictions of the response at low frequencies could be obtained using a dynamic inflow model, a more rigorous modelling of the induced flow field would be desirable at higher frequencies. The generalised dynamic wake model of Peters and He (1995) could potentially allow the azimuth and radial variation in the inflow to be more accurately modelled. Vortex-wake based techniques (see Leishman, 2006), are also becoming increasingly popular and would also inherently account for the presently challenging three-dimensional effects near the tip.

7.3.5 Comparisons with Loewy Theory

The returning wake model presented by Loewy (1957) was also used in this thesis to predict the unsteady circulatory loading. It was applied to estimate the unsteady lift amplitude for a heaving motion, which is compared with the out-of-plane bending moment response.

As shown by Whelan (2010), for a constant speed rotor subjected to an oscillatory heaving motion, the circulatory oscillatory lift \( (L') \) is expressed as

\[
L' = \pi \rho \Omega r c C''(k) [\hat{u} \exp (i2\pi ft)].
\]  

(7.7)
The deficiency factor, $C'(k) = F(k) + iG(k)$ is complex and its value as given by Loewy (1957) is provided in Appendix F.

The approach to compute the deficiency factor for the model tidal turbine was based on the recommendations of Gaonkar and Peters (1988) and Peters et al. (1989). Specifically, the near-wake approximation was used as the frequency ratios were relatively low (i.e. $m < 0.5$). At these frequencies, the wake wraps around itself during the axial perturbation and the near-wake approximation is more applicable than the full-wake model. Given this, the lift deficiency function was approximated as

$$C'(k) = \frac{1}{1 + A(k)}, \quad (7.8)$$

where

$$A(k) = 2\pi k (1 - 2W). \quad (7.9)$$

The wake spacing is incorporated within the parameter $W$, which is also defined in Appendix F. Furthermore, the correction proposed by Peters et al. (1989) was employed to account for the singularity at low frequencies. This modifies the expression for the reduced frequency to be $k = b(m + 1.5)/\Omega$.

Another issue in the application of the model for tidal turbines is that Loewy (1957) devised the model for a helicopter rotor in hover. Loewy (1957) expressed the wake spacing as $h = 4v_i/\Omega \sigma$, where $v_i$ is the increase in the velocity due to the work done by the rotor. In this thesis, the velocity term was assumed to be equal to $U - 2v_i$, which accounted for the induced velocity due to the turbine, acting to slow the mean flow.

The real and imaginary terms of the lift deficiency function predicted from the near-wake model, incorporating the modification proposed by Peters et al. (1989) are shown in Figure 7.12. These terms correspond to a blade-section at $r = 0.75R$, where the non-dimensional mass flow is $V = 0.15$, the wake-spacing $h/b = 4.2$ and the solidity $\sigma = 0.15$. The full uncorrected Loewy (1957) model is also depicted for comparison.

At zero frequency (i.e. for steady flow) the modified model provides a better representation of the induced velocity in the wake compared to the original theory. For example, the magnitude of the real component is reduced, such that it corresponds more closely to momentum theory. The imaginary term is also zero and the local gradient is finite and corresponds approximately to the pseudo added mass associated with dynamic inflow (see Peters et al., 1989).

For the range of frequency ratios that are applicable to the experiments
7.3. Single Frequency Oscillations for Attached Flow

\[ \frac{L_I}{L_{nc}} = \frac{\text{Im}(L_c) + L_{nc}}{L_{nc}} = 1 + \frac{\rho \pi r c G(k)}{\frac{1}{4} \rho \pi c^2 \omega \tilde{u}} + \frac{2G(k)}{k}. \]  

This ratio is presented for the aforementioned parameters as a function of the reduced frequency in Figure 7.12(c). It is interesting to compare these ratios from the model with those from the experiment that are presented in Figure 7.10. This is based on the assumption that the loading on the element at 0.75\( R \) is representative of the total loading of the blade. For the lowest frequencies, the model agrees qualitatively with the experiments, but over-predicts the ratio by approximately 40\%. However, the model ratios do not decrease as rapidly with an increase in frequency compared to the experiment, i.e. for \( m = 0.25 \) the ratio is equal to 2, compared to 0.5. Therefore, whilst the modified Loewy model could be useful for qualitative predictions of the contribution from hydrodynamic inertia at low frequencies, it is too limited to provide useful predictions at higher frequencies.
Chapter 7.  Blade Loads for Unsteady Forcing

7.4 Single Frequency Oscillations for Separated Flow

7.4.1 Sensitivity to the Oscillatory Forcing Parameters

The observations presented in Section 7.3.2 indicated that the unsteady loads associated with separated flow are typically of greater magnitude than for attached flow. Characterising the bending moment response for cases where flow separation is dominant across the blade sections is expected to be particularly important for stall-regulated tidal turbines.

The bending moment responses to oscillations about lower mean tip-speed ratios of \( \lambda = 3.6 \) and 4.1 and a rotor speed of \( \Omega = 84 \text{ rpm} \) are presented in Figure 7.13. These correspond to oscillatory frequencies of \( f = 0.50, 0.64 \) and 0.89 Hz and a Current number of \( \mu = 0.200 \). The hysteresis loops are presented by plotting the total bending moment on the ordinate and the inverse of the instantaneous tip-speed ratio (i.e. \( (U + u)/\Omega R \)) on the abscissa. This allows the operational state of the rotor during the oscillation to be identified.

The oscillations at a mean tip-speed ratio of \( \lambda = 4.1 \) exhibit similar trends to the responses that were previously shown for oscillations about \( \lambda = 4.5 \). At low frequencies, the unsteady out-of-plane bending moment response is approximately linear. There is a phase-lead in the out-of-plane bending moment and a phase-lag for the in-plane bending moment response. As the frequency is increased, the response of the out-of-plane bending moment becomes increasingly non-linear. The out-of-plane bending moment response for \( f = 0.89 \text{ Hz} \) also shows that at high instantaneous velocities, the loops are circumvented counter-clockwise.

The responses at the lower tip-speed ratio of \( \lambda = 3.6 \) exhibit characteristics which are synonymous with the dynamic stall of oscillating foils (e.g. see Figure 2.2). Key stages in the out-of-plane bending moment response for \( f = 0.89 \text{ Hz} \) have been identified as (1) to (5). Insights into the underlying phenomena which are believed to be associated with these stages have been drawn from Leishman (2006).

At stage 1, the bending moment exceeds the maximum bending moment that was observed for steady flow conditions. This is consistent with a delay in the onset and the progression of the trailing-edge separation on the boundary layer of the foils. An increase in oscillatory frequency appears to further delay the movement of the separation point.
7.4. Single Frequency Oscillations for Separated Flow

Figure 7.13: Effect of the oscillatory frequency on the blade root out-of-plane, (a) and the in-plane, (b) bending moment response for low mean tip-speed ratios of $\lambda = 3.6$ and $\lambda = 4.1$ (grey) at a rotor speed of 84 rpm and a Current number of $\mu = 0.200$. From bottom: $f = 0.50 \text{ Hz}$, $0.64 \text{ Hz}$, $0.89 \text{ Hz}$ and $1.40 \text{ Hz}$, offset in intervals of $3 \text{ Nm}$ for clarity and presented as a function of the inverse of the instantaneous tip-speed ratio. The loads measured for steady flow are denoted by the solid markers and the out-of-plane response for $f = 0.89 \text{ Hz}$ is annotated to correspond with the description of the underlying flow phenomena provided in Section 7.4.

Between stages 2 and 3, a vortex is believed to have separated from the leading edge and travelled across the chord, during which time additional lift was induced. For the highest frequencies investigated, the maximum out-of-plane and in-plane bending moments are approximately 25% greater than for steady flow. This emphasises that the unsteady hydrodynamic contribution is significant and can be greater than for attached boundary layer conditions. Between points 3 and 4, the vortex is believed to have reached the trailing edge and has separated from the foil, resulting in lift stall. Full separation of the boundary layer is attributed to the sudden and severe decrease in the bending moment and the subsequent
Chapter 7. Blade Loads for Unsteady Forcing

pronounced non-linear hysteresis.

The boundary layer is able to re-establish on the surface of foil as the angle of attack is reduced (i.e. as $1/\lambda$ has increased), which corresponds to point 5. An increase in frequency can also be observed to have delayed the onset of flow reattachment to lower values of $1/\lambda$. For the out-of-plane bending moment, the highest frequency cases show that the reattachment occurs near the minimum oscillatory velocity. This shows that flow separation can be present for a significant portion of the load cycle.

For the lowest frequency case, the bending moment in-phase with acceleration is approximately equal to that observed for oscillations performed at both $\lambda = 3.6$ and 4.1 when the boundary layer is attached. This is an interesting observation and it suggests that at low frequencies the phenomena associated with delayed separation and dynamic stall have a relatively small influence on the unsteady response under attached flow conditions.

The responses for Current numbers of $\mu = 0.100$ and 0.300, corresponding to frequencies of $f = 0.50$ Hz and 0.89 Hz are presented in Figure 7.14. These oscillations were performed about a tip-speed ratio of $\lambda = 3.6$ and a rotor speed of 84rpm. The responses for Current numbers of both $\mu = 0.100$ and $\mu = 0.300$ show characteristics consistent with those observed for $\mu = 0.200$. For both the out-of-plane and in-plane bending moment, the separation and reattachment are delayed with an increase in frequency. For both Current number cases, there is also a phase-lead in the out-of-plane bending moment, and a phase-lag in the in-plane bending moment at low frequencies. Furthermore, the oscillations for $\mu = 0.300$ also show that for attached flow conditions, the phase of the out-of-plane bending moment alternates in direction with an increase in frequency.

7.4.2 Modelling the Delayed Separation & Dynamic Stall

As discussed in Section 2.4.2, the semi-empirical model by Leishman and Beddoes (1989) has proved popular for describing the delayed separation and dynamic stall of thin aerofoils. The model is also incorporated within commercial simulation codes for tidal turbines (Bossanyi, 2009). However, the model has yet to be validated for tidal turbines.
7.4. Single Frequency Oscillations for Separated Flow

Figure 7.14: Blade root out-of-plane and in-plane bending moment responses to an oscillation about a mean tip-speed ratio of $\lambda = 3.6$ and a current number of $\mu = 0.100$ (a and b), and $\mu = 0.300$ (c and d). The oscillations correspond to frequencies of $f = 0.50\,\text{Hz}$ and $0.89\,\text{Hz}$ (offset by 0.2 for clarity). The bending moment measured in steady flow at the equivalent rotor speed is denoted by the bullets. The unsteady bending moment is relative to the bending moment measured in steady flow at the mean flow speed.
Chapter 7. Blade Loads for Unsteady Forcing

Forcing Conditions & Implementation

As a first attempt at validation in this research, the model was applied to predict the lift coefficient response at a blade section at $r = 0.75R$. At this blade section the out-of-plane bending moment is considered to be approximately proportional to the component of the lift coefficient in the axial direction ($C_l \cos \phi$).

Several assumptions were made to simplify the analysis and to elucidate the performance of the model. For example, the wake was assumed to be frozen (i.e. the induced velocity remains constant) and dynamic inflow was not considered. This assumption is deemed to be more applicable for the higher frequency cases, where the effect of dynamic-inflow is believed to be comparatively small. The contribution from the tangential induced velocity (i.e. wake swirl) was also deemed to be sufficiently small to be able to be neglected. Therefore, the angle of attack history of the forcing at the blade element was estimated as

$$\alpha (t) = \tan^{-1} \left( \frac{U - \overline{v}_i + u(t)}{\Omega r} \right) - \beta.$$  (7.11)

The mean induced velocity was obtained from the BEM model described in Section 6.6.1. The blade section was considered to be sufficiently out-board to neglect three-dimensionality and delayed separation on the steady coefficients due to centrifugal forcing. Furthermore, the blade-section was considered to be sufficiently in-board such that the loss from the tip-loss would be relatively small.

The implementation of the model was based on that presented by Pierce (1996) and Minnema (1998) for a wind turbine and is detailed in Appendix G. It is important to note that the unsteady attached flow model was not applied, given that the wake was assumed to be frozen. The decision to exclude the attached flow module was also supported by the fact that the attached flow model (Leishman and Beddoes, 1989) replicates the Theodorsen (1935) response when applied in heave. As stated previously in Section 2.4.1, the Theodorsen (1935) model is not believed to be appropriate for a rotor.

Evaluation of the Model

The bending moment for oscillatory frequencies $f = 0.50, 0.64$ and $0.89$ Hz and a Current number of $\mu = 0.200$ measured in the experiment are compared with the model predictions in Figure 7.15. The model duly predicts an increase in the maximum lift and of the vortex induced component with an increase in frequency.
For the lowest frequency case, the hysteresis predicted by the model agrees qualitatively with that observed in the bending moment (including the attached flow regions of the response). However, the discrepancy between the measurements and the model becomes more apparent as the frequency is increased. The hysteresis is significantly under-predicted, with the reattachment occurring at higher angles of attack compared to the experiments.

Figure 7.15: Comparison of the out-of-plane bending moment response (a) and the steady and the dynamic lift coefficient (resolved in the axial direction) as predicted by the Beddoes-Leishman dynamic stall model, (b). The cases correspond to oscillations about a mean tip-speed ratio of $\lambda = 3.6$, rotor speed of $\Omega = 84$ rpm and a Current number of $\mu = 0.2$. From bottom: oscillatory frequencies of 0.50, 0.64 and 0.89 Hz, which are consecutively in intervals of 3 Nm and $C_l \cos(\phi)$ 0.2 respectively for clarity.

Having access to only the bending moment at the root makes it difficult to identify the source of the difference. It is expected that sections further inboard may experience a greater degree of hysteresis due to the relatively larger angles of attack and higher reduced frequencies. However, the contribution to the net root bending moment from these sections is also comparatively low.

It is postulated that the process in which the boundary layer is re-established
after the vortex has been shed is significantly different from that for a flat-plate, on which the trailing-edge model is based. Whilst not observed in the wind-tunnel experiments, it is also possible that the reattachment of the boundary layer of the S814 foil occurs at a lower angle of attack than that at which it separated. If so, then this would imply that there is hysteresis in the steady response at high angles of attack.

If hysteresis were present in the steady lift coefficients, this could be incorporated into the dynamic stall model by computing a separate steady separation point for both increasing and decreasing angles of attack. To qualitatively demonstrate this, the steady lift coefficient for decreasing angle of attacks in the region of \( \alpha \geq 9^\circ \) were reduced by up to 10% to approximate the hysteresis (see Appendix G). The predictions from the modified model are also shown in Figure 7.15. They show that an improved estimate of the hysteresis has been obtained at the lower frequencies. However, the modification is not sufficient to predict the delay in the reattachment observed at the highest frequencies. Therefore, a more rigorous modelling approach is deemed to be necessary.

7.5 Applications of the Single Frequency Response

7.5.1 Multi-Frequency Forcing

The bending moment response to multi-frequency forcing that comprised two relatively low frequencies of \( f = 0.40 \) Hz and \( 0.50 \) Hz, with a relatively low Current numbers of \( \mu = 0.075 \) is presented in Figure 7.16. To quantify the contribution from this unsteadiness, the time history is compared with a reconstruction based on the steady loads. This demonstrates that there is a relatively small overshoot of the steady load. This is in qualitative agreement with the relatively small overshoots that were observed at these frequencies for single frequency forcing, as shown in Figure 7.9.

As discussed in Section 1.3.2, large load cycles are relevant for predicting the fatigue of tidal turbine blades manufactured from composite materials. Given this, three relatively large load cycles were identified in the time history. These correspond to the response between the symbols (○) and (●) respectively. The hysteresis loops corresponding to these cycles have been presented. These are
7.5. Applications of the Single Frequency Response

Figure 7.16: The normalised bending moment response for a forcing comprising the frequencies of $f = 0.40\text{Hz}$ and $0.50\text{Hz}$ with a Current number of $\mu = 0.075$. The unsteady bending moment response (black) is compared with a reconstruction based on the steady load (grey) in (a). The unsteady bending moment (black) is compared with a reconstruction based on the amplitudes and phases for single frequency forcing (grey) in (b). The hysteresis loops correspond to the cycles identified in the time history between the points ◦ and ●. They are offset by intervals of 0.2 for clarity and are all circumvented clockwise.

compared with a reconstruction comprising a linear superposition of two single frequency oscillations with the equivalent amplitudes and phases that were estimated for the respective single frequency forcing cases, shown in Figure 7.9. These demonstrate that the reconstruction was able to replicate both the amplitude and phase-lead of the multi-frequency response.

The bending moment response to multi-frequency forcing comprising these same frequencies, but with a Current number of $\mu = 0.125$ for the 0.50 Hz component is shown in Figure 7.17. The time histories show that the unsteady bending moment still exhibits a relatively small overshoot from the steady load case. The hysteresis loops corresponding to three relatively large cycles are similarly presented. These demonstrate that by using the amplitudes and phases corresponding to the individual components for single frequency forcing, the multi-frequency response was able to be reconstructed again. This is in spite of the relatively large velocity amplitudes.

A case where the forcing comprised a combination of three frequencies of
0.40 Hz, 0.50 Hz and 0.67 Hz, each with a Current number of $\mu = 0.075$ is presented in Figure 7.18. As for the previous cases, the time histories demonstrate an overshoot relative to the steady loading. The hysteresis loops for four relatively large loading cycles also show that the response is able to be recovered using the amplitudes and phases of the single frequency forcing. Therefore, Figures 7.16 to 7.18 provide support for the use of the single frequency response method to model the lower frequency band of the turbulence spectrum relevant to tidal turbines.

A limitation of the use of the single frequency amplitudes and phases to reconstruct the multi-frequency response is demonstrated in Figure 7.19. For this case the forcing comprised two relatively high frequency components of $f = 0.67$ Hz and $f = 0.80$ Hz and Current numbers of $\mu = 0.100$. The hysteresis loops demonstrate that through the use of a linear superposition of single frequency responses, the overshoot of the steady loads that are observed in the time history have been able to be modelled. However, the phase-lead of the observed multi-frequency response is under-predicted.

As an independent check, the amplitude and phases of the individual forcing were also estimated from the multi-frequency forcing response using a regression technique (see Section 7.2.4). These are shown for a range of test cases that were performed in Figure 7.20. The figure demonstrates that for multi-frequency force-
7.5. Applications of the Single Frequency Response

Figure 7.18: Bending moment response to a forcing comprising the frequencies $f = 0.40\text{Hz}$, $0.50\text{Hz}$ and $0.67\text{Hz}$, each with a Current number of $\mu = 0.075$. The presentation of the time histories and hysteresis loops are consistent with Figure 7.16.

Figure 7.19: Bending moment response to a forcing comprising the oscillatory frequencies of $f = 0.67\text{Hz}$ and $0.80\text{Hz}$, each with a Current number of $\mu = 0.100$. The presentation of the time histories and hysteresis loops are consistent with Figure 7.16.

...ing, the amplitudes of the individual components are approximately equivalent to the single frequency forcing functions. However, at relatively high frequencies the phase-lead is larger, similar to the previous observations.
Chapter 7. Blade Loads for Unsteady Forcing

Figure 7.20: Estimates of the amplitude and phase from a linear fit to a multi-frequency forcing response. The symbols denote frequency pairs, with the Current numbers of the individual constituents provided in the legend (lowest frequency above).

The ability to predict the amplitudes of the multi-frequency forcing is significant. It implies that it could offer blade designers a relatively simple technique to obtain the fatigue loading from a turbulence spectrum. The ability to recover the amplitudes of a multi-frequency forcing is also significant. It suggests that reliable estimates of the single amplitudes (that are required to develop the amplitude part of the transfer function) could be obtained by performing a limited series of multi-frequency tests. As conducting experiments are expensive and time-consuming, test programs could be shorter in duration or more tests could be completed within a fixed time frame.

It is also important to consider that the phase is typically ignored in a stochastic modelling approach, as its effects are assumed to be able to be averaged out. These findings demonstrate that for a tidal turbine, the phase contribution is not only present but can be of greater magnitude than observed for single frequency cases. Whilst the magnitude of the inertia component is relatively small compared to the total load, it is important to consider that the fluid and structural
7.5. Applications of the Single Frequency Response

inertias are expected to be of the same order of magnitude (Whelan, 2010). The added inertia may also have significant implications for control purposes.

It is postulated that the circulatory forcing is the source of the larger-phase leads observed. This is because the non-circulatory forcing is expected to be linear and proportional to the acceleration. It is possible that at high frequencies the large load cycles induce circulatory transients which act to increase the phase. These would not be included in the amplitude and phases estimated for the single frequency oscillatory experiments, which were averages from multiple oscillations.

7.5.2 Discrete Half-Sinusoidal Forcing

The time histories of the out-of-plane bending moment response to a half-sinusoidal axial perturbation corresponding to frequencies of 0.40 Hz and 0.50 Hz, with a Current number of \( \mu = 0.250 \) are shown in Figure 7.21. These are compared with a reconstruction based on the steady loads. It can be observed that there is a rel-

![Figure 7.21: Time histories of the bending moment response to discrete half-sinusoidal forcing corresponding to a frequency of \( f = 0.40 \) Hz, (a–b) and 0.50 Hz, (c–d). A reconstruction based on the steady load is shown by the dotted line.](image-url)
Chapter 7. Blade Loads for Unsteady Forcing

At relatively small over-shoot of the magnitude of the steady loads for both a positive and a negative perturbation. A relatively small overshoot of the steady loads at the end of the perturbation is also identifiable. The magnitudes of the overshoots are also larger for the higher frequency case. In general, the unsteadiness decays relatively quickly after the perturbation and do not appear to be significant.

The corresponding hysteresis loops of the respective positive and negative half-sinusoidal perturbation are presented together in Figure 7.22. They elucidate the phase-lead that is present in the bending moment response for each case. The oscillatory responses for the equivalent frequencies and Current number are also shown for comparison. The amplitude and the phase-lead over velocity of the discrete half-sinusoidal forcing is generally consistent with the oscillatory case. The agreement in the phase-lead implies that the combined contributions from dynamic inflow and the non-circulatory forcing are similar, despite the differences in the forcing.

These comparisons demonstrate that a blade designer could use the single frequency oscillatory response to obtain a reliable estimate of the response to large, coherent eddies. They also emphasise that the single frequency response offers an inherently better prediction of the unsteady response than could be achieved by using only the steady loading.

\[
\begin{array}{c}
\text{Figure 7.22: Bending moment response to a positive half-sinusoidal forcing for } \nu = 0.40 \text{ Hz, (a) and 0.50 Hz, (b). The corresponding oscillatory response is depicted by the dotted line and the loads for steady flow are also shown.}
\end{array}
\]
7.6 Summary

This chapter has provided a quantification of the contribution of hydrodynamic unsteadiness to the blade-root bending moment response. For single frequency oscillations and attached boundary layer conditions, the amplitude of the out-of-plane blade root bending moment increased with frequency and was up to 15% greater than for steady flow. At low frequency ratios there was phase-lead in the unsteady out-of-plane bending moment response, which reduced with frequency. In contrast, the in-plane bending moment exhibited a phase-lag which increased with frequency.

At relatively low frequencies, the hydrodynamic contribution to the out-of-plane bending inertia was approximately a factor of 2.7 greater than the non-circulatory contribution. At higher frequencies the circulatory effects are believed to act in opposition to the non-circulatory forcing. Dynamic inflow theory and a modified version of Loewy theory were found to provide qualitatively correct predictions of the hydrodynamic added inertia at low frequencies. Quantitative predictions from dynamic inflow are compromised by the need to account for the tip-loss of the rotor. Both models also significantly over-estimated the ratio of the total added inertia to the non-circulatory added inertia at higher frequencies.

Phenomena consistent with delayed separation and dynamic stall were observed for oscillations at relatively low tip-speed ratios. For these cases the unsteady hydrodynamic contribution to the bending moment exceeded 25% of the steady load. Therefore, the unsteady hydrodynamic contribution from these effects is more significant than that from dynamic inflow or the non-circulatory forcing. Whilst the overshoot was predicted qualitatively using a modified version of Beddoes-Leishman model, the hysteresis post-stall was under predicted.

The amplitudes of the multi-frequency responses were able to be reconstructed using the summation of single frequency oscillatory responses. Therefore, a blade designer could potentially use this approach to synthesise a fatigue load spectrum due to turbulence. The single frequency oscillatory response could also provide a useful description of the response to a large low frequency eddy passing over the rotor. Importantly, the hydrodynamic added-inertia remains present for these general forcing cases and cannot be averaged out.
Chapter 8

Conclusions

Reasoning draws a conclusion, but does not make the conclusion certain, unless the mind discovers it by the path of experience

— Roger Bacon

8.1 Conclusions & Research Achievements

This research has presented new experimental data and analyses that can assist designers of tidal turbines in quantifying the unsteady hydrodynamic blade loads due to onset turbulence. It is envisaged that the outcomes of this research could lead to improved predictions of the fatigue life of turbine blades and ultimately more robust and economical turbines.

A new, extensive set of important statistics which characterise the magnitudes of the turbulent fluctuations, the anisotropy and the scales of the turbulence in a tidal stream has been provided in this research, based upon a comprehensive analysis of data from the Sound of Islay. It was established that:

- The turbulence, at a typical site such as the Sound of Islay, is mechanically driven by the shear at the sea-bed. The law of the wall is valid for the lower 20% of the water column. The boundary layer height typically exceeds $z/h = 0.85$, implying that tidal turbines will always operate within the boundary layer region.
Chapter 8. Conclusions

- The friction velocity corresponding to a flow velocity of 2 m s$^{-1}$ at $z/h = 0.1$ was between $u_* = 0.11$ m s$^{-1}$ and 0.15 m s$^{-1}$. The bottom drag coefficients of approximately $C_d = 0.0020$ to 0.0023, were not significantly different from other tidal streams, suggesting that the extended set of turbulence measures provided in this research will have wide application.

- Near the seabed, the median streamwise turbulence intensities were in the range $I_u = 0.11$ to 0.13. However, the turbulence intensity remained relatively large throughout the water column and at typical rotor hub-heights was approximately $I_u = 0.08$. These values are consistent with those reported for other tidal energy sites.

- The turbulence anisotropy near the bed was similar to an idealised open-channel flow, and dissimilar to that for the atmosphere, upon which many designers rely on. This brings into question the validity of the atmospheric turbulence models for fast flowing tidal streams.

- The median integral length-scales of the streamwise velocity were between 10 m and 13 m at $z/h = 0.1$, being comparative in size to a typical tidal turbine diameter. The range of integral scales was large, due to low frequency oscillations in the flow. At elevations corresponding to a typical tidal turbine hub, the dominant scales associated with the coherent mixing scales were approximately 4 times greater than a typical turbine diameter.

- At non-dimensional wavenumbers of approximately $k_x z = 3$ to 4, the velocity spectra and the co-spectra tended towards the Kolomogrov proportionality. The non-dimensional wavenumbers corresponding to the inception of the inertial subrange are consistent with other tidal streams as well as theory for boundary layer turbulence. The Kolmogorov model could be useful for predicting the turbulence at length-scales corresponding to a turbine blade length.

The tidal stream turbulence characteristics that were thus established, helped inform an analysis of the implications of turbulence on the unsteady turbine blade loading; define important non-dimensional input parameter ranges; and determine a suitable method of model-scale testing for unsteady hydrodynamics. This analysis, together with a novel application of the rapid distortion theory of turbulence, has shown that:
8.1. Conclusions & Research Achievements

- The standard deviation of the streamwise turbulence is amplified by approximately 15% at the rotor. This would have significant implications for unsteady loading amplitudes.

- The turbulence could be highly correlated over the outer-blade sections at the rotational frequency of the rotor. An important implication of this, which was utilised in this research, is that the unsteady blade load response of a tidal turbine can be studied using a planar oscillation of the turbine.

- The relevant range of reduced frequencies due to turbulence are between $k = 0.03$ and 0.10, at a typical tidal turbine blade radius of $0.75R$, the frequency ratios are between $m = 0.25$ to 1, and the Current numbers could be as high as $\mu = 0.25$.

In addition to establishing an extended set of inflow turbulence properties, and the novel analytical results, the research has provided important new insights into the unsteady hydrodynamic loading of tidal turbine blades from model-scale tests in a towing tank. However, as model-scale testing of tidal turbines is typically carried out at relatively low Reynolds numbers of approximately $1 \times 10^5$, the implications of this were first established through wind tunnel experiments of the S814 foil (that formed the basis of the blades of the model-scale tidal turbine). It was found that:

- The lift reduced by 10% and the drag increased by at least 400% relative to full-scale, owing to dominant laminar separation bubbles on the foil. However, the lift coefficient of the foil remained approximately linear at angles of attack between $4 < \alpha \leq 9^\circ$ and the foil stalls from the trailing edge, as at full scale.

- At low angles of attack, the boundary layer on the pressure surface can experience full-separation. This phenomenon appears to be a characteristic of thick foils at Reynolds numbers below approximately $2 \times 10^5$. This gives rise to relatively high lift and drag, thus limiting the range of tip-speed ratios at which useful data can be obtained from a turbine.

These findings imply that it is important to carefully consider the implications of the relatively low Reynolds numbers during model-scale testing of tidal turbines. Indeed, the data that was obtained assisted in the application of existing models against experimental data from model-scale testing, to gain insight into the unsteady hydrodynamic response of tidal turbine blades.
Chapter 8. Conclusions

The model-scale tidal turbine testing in this research enabled an extensive set of blade load responses to be obtained, at frequencies significantly beyond those found in the literature. Specifically, the relative contribution of the hydrodynamic unsteadiness to the blade loads was quantified and as a function of the operational state of the rotor, which has previously not been established for tidal turbines. The analyses based on unsteady fluid dynamic models and theories that were also conducted in parallel, enabled new insights to be gained into the unsteady hydrodynamic response of tidal turbine blades. The main conclusions and findings from this part of the research were that:

- For single frequency oscillations where the boundary layer remains attached, the unsteady blade loads increase with frequency and exceed the steady loads by up to 15%. The out-of-plane bending moment also exhibits a phase-lead over the velocity forcing. These effects are consistent with the effects of dynamic inflow and non-circulatory forcing.

- The total hydrodynamic bending moment in-phase with acceleration was approximately a factor of 2.7 greater than the non-circulatory contribution at relatively low frequency. However, it decreases with frequency, and circulatory effects act in opposition to the non-circulatory forcing at high frequencies.

- Models based on dynamic inflow theory and a modified Loewy theory can provide a qualitatively good prediction of the bending moment response at low frequencies, but not at higher frequencies. The accuracy of the numerical models is also significantly limited by the lack of an inherent allowance for the effect of the significant tip-loss from the blades of the turbine.

- Phenomena consistent with delayed separation and dynamic stall of oscillating foils were observed in the blade load responses at low tip-speed ratios. These effects result in a bending moment that exceeds the steady load by up to 25%. Therefore, the unsteady hydrodynamic contribution from these effects can be more significant than from dynamic inflow or the non-circulatory forcing.

- Whilst the overshoot due to delayed separation and dynamic inflow is able to be predicted qualitatively using a modified version of the Beddoes-Leishman
model, the hysteresis post-stall is significantly under-predicted. Nevertheless, this research provides a valuable data-set of unsteady blade loads by which more rigorous dynamic stall models can be validated against.

- The amplitudes of multi-frequency responses can be reconstructed using a superposition of the amplitudes and phases of the constituent single frequency oscillatory loading. The oscillatory response can also be applied to model the response to a large eddy passing over the rotor. Therefore, blade designers could potentially use the single frequency response to synthesise the unsteady load amplitudes from a turbulence spectrum. It also implies that a blade designer could obtain the amplitude part of the transfer function efficiently from a limited series of multi-frequency tests.

- Importantly, for multi-frequency responses that comprise relatively high frequencies, the phase is under-predicted using a superposition of the single frequency response. Therefore, not only can the phase not be averaged out in a stochastic model, it is larger than would be predicted using a single frequency response. This is significant as the fluid to structural inertias are of the same magnitude.

8.2 Recommendations for Future Work

The author believes that with the tidal energy industry continuing to develop, more measurements of turbulence in tidal flows with speeds of the order of $U = 2\, \text{m s}^{-1}$ will become available. By following a similar analysis as demonstrated in this thesis to acquire the relevant turbulence parameters, the data will assist in establishing the variability in the turbulence among sites with more certainty.

As the amplification of the turbulence due to the turbine is believed to be significant, it will be useful to acquire measurements of turbulence in the presence a full-scale turbine. This data could be obtained by projecting an acoustic beam in a direction upstream from the hub, and would provide a means to validate the theoretical-based estimate developed in this thesis.

With suitable modifications to the surging carriage, the experimental methodology used in this the towing tank experiments herein could be applied to investigate the unsteady loads due to a misalignment of the rotor axis to the mean flow (i.e. a non-zero yaw angle). Provided that the effects of the free-surface
are small, an effective yaw misalignment could be achieved by tilting the rotor vertically.

Furthermore, it would be interesting to repeat the experiments for a two-bladed rotor, or using a different blade profile. This would allow for the effect of solidity to be considered. It would also assist in quantifying the variability in the unsteady loading between different turbine designs.
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184


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189


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Appendix A

Turbulence Analysis Supplement

This appendix provides supporting material for the processing of the ADV and ADCP data and the analysis of the turbulence.

A.1 Transformation Matrices

A.1.1 ADV Transforms

The ADV data was first transformed from a non-orthogonal co-ordinate system aligned with the transducers to a Cartesian co-ordinate system that was aligned with the sensor. The transformation matrix was unique to the individual ADV that was used, and was provided by the sensor manufacturer (Nortek, 2005). A second transformation matrix was applied to transform the ADV velocities from a Cartesian co-ordinate system that was aligned with the sensor to a co-ordinate system that was aligned East, North, Up (ENU). This matrix is provided below and was supplied by Nortek (2005). It is applicable to all Nortek Vectors that have the transducers above the canister (as in the case in this research).

\[
\begin{bmatrix}
E \\
N \\
U
\end{bmatrix} =
\begin{bmatrix}
(CHCP + SHSPSR) & (CPSH - CHSPSR) & -CRSP \\
-CRSH & CHCR & -SR \\
(CHSP - CPSHSHR) & (SHSP + CHCPSR) & CPCR
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix},
\]

(A.1)

where $CH = \cos(H)$, $SH = \sin(H)$ etc. and $H$, $P$ and $R$ are the heading, pitch and roll angles, respectively.
Appendix A. Turbulence Analysis Supplement

A.1.2 ADCP Transforms

The data from the ADCP were acquired in directions aligned with the transducers beams. The mean velocities were obtained from the ADCP by applying two transformations, which were both provided by Teledyne RDI (2010a). The first was applied to transform the velocities from the beam system \( b_i \), where \( i = 1, 2, 3, 4 \) to a Cartesian co-ordinate system \( X, Y, Z \) and error, \( e \) that was aligned with the sensor. This matrix that was used is expressed as

\[
\begin{bmatrix}
X \\
Y \\
Z \\
e
\end{bmatrix} =
\begin{bmatrix}
a (b_1 - b_2) \\
a (b_4 - b_3) \\
b (b_1 + b_2 + b_3 + b_4) \\
c (b_1 + b_2 - b_3 - b_4)
\end{bmatrix},
\]

where \( a = 1.4619 \), \( b = 0.2660 \) and \( c = 1.0337 \).

A second transformation matrix was applied to obtain the velocities in a co-ordinate system that was aligned in an East, North, Up (ENU) frame. This transformation matrix is expressed as

\[
\begin{bmatrix}
E \\
N \\
U
\end{bmatrix} =
\begin{bmatrix}
(CHCR + SHSPSR) & SHCP & CHSR - SHSPCR \\
(-SHCR + CHSPSR) & CHCP & -SHSR - CHSPCR \\
-CPSR & SP & CPCR
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}.
\]

It is important to note that the internal pitch angle measured by the ADCP, \( P' \) was not equal to that for a gimble. The pitch angle was modified for use in equation A.3, using the relation

\[
P = \arctan \left[ \tan \left( P' \right) \tan \left( R \right) \right].
\]

A.2 Identification & Removal of Bias in ADCP Variances

As discussed in Section 3.4.2, the raw estimates of the variances terms, \( d_u^2 \) and \( d_v^2 \) were biased proportionally by Doppler noise. Estimates of \( d_u^2 \) and \( d_v^2 \) at an elevation of 5 m from the ADCP are presented in Figure A.1(a). These demonstrate that a bias of approximately 0.005 m\(^2\)s\(^{-2}\) was present in both the \( d_u^2 \) and \( d_v^2 \) estimates. The was equal to the error that was predicted for the sensor using
the software package PlanADCP (Teledyne RDI, 2006).

Figure A.1: The raw estimates of the variances $d_{u,v}^2$ (black) and $d_{u}^2$ (grey) as measured by the ADCP, (a) and the estimate of $d_{u}^2$ from the ADV (black) compared with the estimate of $d_{u}^2$ from the ADCP having subtracted a bias of $\sigma^2 = 0.005 \text{m}^2\text{s}^{-2}$ (grey), (b). For the ADCP, data was acquired from 10-minute samples, every 2-minutes, whilst data for the ADV were collected from 5-minute samples, every 1-minute.

The bias was removed from both the estimates of $d_{u}^2$ and $d_{u,v}^2$, prior to estimating the streamwise turbulence intensities.

As is demonstrated in Figure A.1(b), the corrected estimates of the variance $d_{u}^2$ generally agrees well with an estimate based on the ADV velocities, particularly for the flood flows. The differences between the two measurements that are observed are believed to arise due to the spectral resolution of the ADCP (see Section 3.4.5), which is more severe during the ebb because of the higher velocities. This error is expected to be less significant at elevations from the bed. Additionally, there is also expected to be a relatively small bias error due to the non-zero tilt of the ADCP sensor.
A.3 Time histories of Near-Bed Velocity Fluctuations

The 5-minute averaged standard deviations of the streamwise, transverse and vertical velocities, together with the corresponding streamwise mean velocity are presented in Figure A.2. These data were used to compute the turbulence intensities and the anisotropic ratios at an elevation of 5 m above the seabed. The absolute values of the standard deviations of velocity were observed to vary approximately in-phase with the mean velocity and obtain magnitudes of $\sigma_u = 0.20 \text{ m s}^{-1}$ and $\sigma_w = 0.15 \text{ m s}^{-1}$ at peak flow.

![Figure A.2](image)

Figure A.2: Time histories of the mean streamwise velocity and the standard deviation of the streamwise, transverse and vertical velocities at an elevation of 5 m from the seabed. Data were averaged over a period of 5-minutes, computed at 1-minute intervals from the ADV data, and have been truncated at slack tide for clarity.

Time histories of the ratios of the along-channel and across-channel Reynolds shear stresses to $2q$, where $q = 0.5(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$ at an elevation of 5 m were presented in Figure 4.6. The time history for the first tidal cycle is reproduced in
A.4. Profiles of Mean Velocity & Turbulence Properties

Figure A.3 for clarity and to facilitate comparisons between the ADV and ADCP data.

Figure A.3: Time histories of the ratios of the along-channel and across-channel Reynolds shear stresses to $2q$, where $q = 0.5 \left( \sigma_u^2 + \sigma_v^2 + \sigma_w^2 \right)$ from the ADV (black) and ADCP (grey) data at an elevation of 5 m during the first tidal cycle. Data were averaged over a period of 5-minutes and 10-minutes for the ADV and ADCP respectively and have been truncated at slack tide for clarity.

A.4 Profiles of Mean Velocity & Turbulence Properties

The hourly profiles of the mean velocity throughout the water column for the second and third tidal cycles for which the turbulence data were analysed are presented in Figures A.4.

The profiles of the streamwise turbulence intensity and of the ratio of the Reynolds shear stress to $u^2$ are presented in Figures A.5 and A.6, respectively. The profiles of these turbulence statistics correspond to the profiles of the mean velocity in Figure A.4, except for those of $U < 1 \text{ m s}^{-1}$, which have been excluded for clarity. In general, the profiles of the mean velocities and the turbulence properties can be observed to be relatively consistent between the tidal cycles.
Appendix A. Turbulence Analysis Supplement

Figure A.4: Profiles of the mean streamwise velocity for the second and third tidal cycles. The data have been averaged over a 20-minute period and acquired at intervals of 1-hour. The estimates for the ebb tide are shown as negative for clarity.

Figure A.5: Profiles of the streamwise turbulence intensity over the second and third tidal cycles. The data have been averaged over a 20-minute period and acquired at intervals of 1-hour. The estimates for the ebb tide are shown as negative for clarity.
Figure A.6: Profiles of the ratio of the Reynolds shear stress to the friction velocity squared, over the second and third tidal cycles. The data have been averaged over a 20-minute period and acquired at intervals of 1-hour. The estimates for the ebb tide are shown as negative for clarity.
Appendix B

Amplification of the Turbulence Fluctuations

Rapid distortion theory was applied to predict the amplification of standard deviation of the velocity across the rotor in Chapter 4. Batchelor (1953) has presented the solution for a sudden contraction. The solution remains the same for a diffuser, which is the application used in this thesis. The key equations are presented in this Appendix.

The solution is based on the assumption that the strain imposed on the flow by the distortion is constant and that the turbulent field is isotropic. Furthermore, as it is a linearised theory it requires that the inertia and viscous forces can be ignored during the distortion. This implies that the ratio of the standard deviation of streamwise velocity to the mean velocity, must be much smaller than the ratio of the length scale of the turbulence, $L$ to the diameter of the distortion tube, $D$ (analogous to the rotor diameter), i.e.

\[ \frac{\sigma_u}{U} \ll \frac{L}{D} \quad (B.1) \]

The problem was solved through vorticity considerations and by applying a pure, constant strain which is independent on the turbulent motion and the position of the fluid particle. The components of the principals strains are equal to $\epsilon_1 = c$, $\epsilon_2 = \epsilon_3 = 1/\sqrt{c}$, where $c = A/A' > 1$ is the contraction ratio and $A$ and $A'$ are the cross-sectional areas upstream and downstream of the contraction, respectively. In this thesis, the amplification is presented as a function of the expansion ratio ($e = A'/A > 1$). However, the following formulae are presented in terms of $c$, for consistency with the solution of Batchelor (1953).
Appendix B. Amplification of the Turbulence Fluctuations

The spectrum tensor in the longitudinal direction after the distortion can be expressed as
\[ \Phi_{11}'(\chi) = \frac{E(\kappa)}{4\pi \chi^4} \left( \kappa_1^2 + \kappa_3^2 \right), \quad (B.2) \]
where the wave number vector, \( \chi^2 = \kappa_1^2/c^2 + c (\kappa_2^2 + \kappa_3^2) \) and \( E(\kappa) \) is the wavenumber spectral energy. The sum of the spectrum tensors is required for establishing the transverse velocity variance and after the distortion. It is given as
\[ \Phi_{22}'(\chi) + \Phi_{33}'(\chi) = \frac{E(\kappa)}{4\pi \kappa^2 \chi^4} \left( c\chi^4 + \kappa_1^2 \kappa_2^2 \right). \quad (B.3) \]

The ratio of the variance of the streamwise velocity after the distortion to that upstream of the distortion is of most interest and can be expressed as a function of the spectrum tensors as
\[ \frac{\sigma_{u}'^2}{\sigma_u^2} = \frac{ \int \Phi_{11}'(\chi) \, d\chi }{ \int \Phi_{11}(\kappa) \, d\kappa } = \frac{ \int E(\kappa) \kappa_1^{\kappa_2^2 + \kappa_3^2} \, d\kappa }{ \int E(\kappa) \kappa_1^{\kappa_2^2 + \kappa_3^2} \, d\kappa }. \quad (B.4) \]

When a polar coordinate system is introduced, the ratio for the streamwise variances can be reduced to the expression
\[ \frac{\sigma_{u}'^2}{\sigma_u^2} = \frac{3}{4\epsilon^2} \left[ \frac{1 + \alpha^2}{2\alpha^3} \log \frac{1 + \alpha}{1 - \alpha} - \alpha^{-2} \right], \quad (B.5) \]
where \( \alpha^2 = 1 - c^{-3} \). The ratio of the variances in the transverse direction can be reduced to
\[ \frac{\sigma_{v}'^2 + \sigma_{w}'^2}{\sigma_v^2 + \sigma_w^2} = \frac{3\epsilon}{4} + \frac{3}{4} \epsilon^{-2} \left[ \frac{1}{2\alpha^2} - \frac{1 - \alpha^2}{4\alpha^3} \log \frac{1 + \alpha}{1 - \alpha} \right]. \quad (B.6) \]

The amplification of the standard deviation of streamwise and transverse velocities and are equal to the square-root of these two ratios, respectively. The ratio of the total kinetic energy after the distortion, \( q^* \) to the upstream total kinetic energy, \( q \) can also be expressed as
\[ \frac{q'}{q} = \frac{c}{2} + \frac{1}{2c^2} (1 - c^{-3})^{-\frac{1}{2}} \log c^{\frac{3}{2}} \left( 1 + (1 - c^{-3})^{\frac{1}{2}} \right). \quad (B.7) \]
Appendix C

Rotational Sampled Turbulence Spectra

The theoretical rotational sampled auto-spectra and co-spectra are employed in Chapter 4 as a basis for estimating the magnitude of the turbulence energy and the coherence observed by the rotor blade.

The mathematical derivations of the auto-spectra and co-spectra observed by a rotating blade are provided by Connell, 1982 (the auto-spectrum only) and Burton et al., 2001 and are summarised here. These are developed from the solutions for homogeneous, isotropic turbulence of von Kármán (1948) and Batchelor (1953) and also assume that the frozen turbulence hypothesis (Taylor, 1938) is valid.

The rotationally sampled normalised auto-spectrum observed at a point at a radius $r$ is defined through the Fourier Transform of the rotational auto-correlation function $\rho^0_u(r, \tau)$,

$$
\frac{S^0_u(n)}{\sigma^2_u} = 4 \int_{-\infty}^{\infty} \rho^0_u(r, \tau) \cos(2\pi n \tau) d\tau,
$$

(C.1)

When deriving the rotational auto-correlation function it is useful to consider the situation depicted schematically by Figure C.1. Let points $A$ and $B$ be the locations of the point on the rotating blade at a radius $r$ from the rotor axis at $\tau = 0$ and at $\tau = \tau$ respectively. Invoking Taylor’s frozen turbulence hypothesis the velocity at point $C$ is equal to the velocity at point $B$ at $\tau = 0$, which is at a distance of $\tau U$ upstream. The rotational auto-correlation function can, therefore,
Appendix C. Rotational Sampled Turbulence Spectra

Figure C.1: The geometry from which the rotational sampled correlation function is developed, adapted from Burton et al. (2001) and based on a plan view of the rotor. In a time period $\tau$ point $A$ rotates about the axis (the dash-dot line) to point $C$.

be related to the non-rotational zero-lag cross-correlation function as

$$\rho_u^0 (r, \tau) = \rho_u (s, 0),$$  \hspace{1cm} (C.2)

where $s$ is the separation distance between the two points $A$ and $B$ and noting that for isotropic turbulence the directionality of the vector $\vec{s}$ can be neglected.

For homogeneous, isotropic turbulence the non-rotational zero-lag cross-correlation function is given by Batchelor (1953) as

$$\rho_u (s, 0) = (\rho_L (s) - \rho_T (s)) \left( \frac{s_1}{s} \right)^2 + \rho_T (s),$$  \hspace{1cm} (C.3)

where $\rho_L (s)$ and $\rho_T (s)$ are streamwise and transverse non-rotational cross-correlations, $s$ is the separation distance between points $A$ and $B$, defined as

$$s^2 = U^2 \tau^2 + 4r^2 \sin^2 \left( \frac{\Omega \tau}{2} \right),$$  \hspace{1cm} (C.4)

and $s_1 = \tau U$ is the separation distance projected in the streamwise direction.

For homogeneous, isotropic turbulence von Kármán (1948) has shown that
the streamwise autocorrelation function is expressed by

$$\rho_L(\tau) = f = \frac{2}{\Gamma\left(\frac{1}{3}\right)} \left(\frac{\tau/2}{T'}\right)^{\frac{1}{3}} K_{\frac{1}{3}}\left(\frac{\tau}{T'}\right), \quad (C.5)$$

where \(\Gamma\) is the gamma function, \(K_{\left(\frac{1}{3}\right)}(x)\) is a modified Bessel function of the second kind of order \(v = \frac{1}{3}\), and \(T'\) is a function of the integral length-scale, where

$$T' = \frac{\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{5}{6}\right)} \sqrt{\frac{\pi}{U}}. \quad (C.6)$$

For isotropic turbulence, Batchelor (1953) has also shown that the lateral autocorrelation function is related to the longitudinal autocorrelation function by

$$\rho_T(s) = \rho_L(s) + \frac{s}{2} \frac{d\rho_L(s)}{ds}. \quad (C.7)$$

Employing this relationship, the transverse (or vertical) autocorrelation function can, therefore, be expressed as

$$\rho_T(\tau) = \frac{2^{\frac{2}{3}}}{\Gamma\left(\frac{1}{3}\right)} \left(\frac{\tau/2}{T'}\right)^{\frac{1}{3}} \left[ K_{\frac{1}{3}}\left(\frac{\tau}{T'}\right) + \frac{1}{2} \left(\frac{\tau/2}{T'}\right) K_{\frac{2}{3}}\left(\frac{\tau}{T'}\right) \right]. \quad (C.8)$$

The rotational sampled auto-correlation function is obtained by substituting equations C.5, C.6 and C.8 into equation C.3, yielding the expression

$$\rho_0^u(r, \tau) = \frac{2}{\Gamma\left(\frac{1}{3}\right)} \left(\frac{s/2}{1.34L_u^x}\right)^{\frac{1}{3}} \left[ K_{1/3}\left(\frac{s}{1.34L_u^x}\right) \right. \left. + \frac{s/2}{1.34L_u^x} K_{2/3}\left(\frac{s}{1.34L_u^x}\right) \left(\frac{2r \sin(\tau\Omega/2)}{s}\right)^2 \right]. \quad (C.9)$$

The rotationally-sampled normalised cross-spectrum rotational correlation function is developed using a similar approach to that of the rotationally sampled auto-spectrum. It is defined as

$$\frac{S_0^u(r_1, r_2, n)}{\sigma_2^u} = 4 \int_{-\infty}^{\infty} \rho_0^u(r_1, r_2\tau) \cos(2\pi n\tau) d\tau \quad (C.10)$$

where \(\rho_0^u(r_1, r_2\tau)\) is the corresponding cross-correlation function. This cross-
correlation function may be obtained by replacing the separation distance by

\[ s^2 = U^2 \tau^2 + r_1^2 + r_2^2 - 2r_1r_2 \cos(\Omega \tau), \]  

(C.11)
such that

\[
\rho_u^0 (r_1, r_2 \tau) = \frac{2}{\Gamma(\frac{1}{3})} \left( \frac{s}{1.34L_u^*} \right)^{\frac{1}{3}} \left[ K_{1/3} \left( \frac{s}{1.34L_u^*} \right) \right. \\
+ \left. \frac{s}{1.34L_u^*} K_{2/3} \left( \frac{s}{1.34L_u^*} \right) \left( \frac{r_1^2 + r_2^2 - 2r_1r_2 \cos(\tau \Omega)}{s} \right) \right]^2.
\]  

(C.12)
Appendix D

Aerofoil Analysis Supplement

This appendix provides further details of the methodology which was employed to correct the wind tunnel measurements for wall effects. It also provides the supplementary aerofoil data for the NREL S814 and S823 foils for reference.

D.1 Corrections for Wall Effects

The wind tunnel measurements were corrected for wall effects following the methodology provided by Rae and Pope (1984) for a closed test section. Correction factors for the dynamic pressure ($\epsilon_q$) and the lift ($\epsilon_{C_l}$) and drag ($\epsilon_{C_d}$) coefficients were estimated from the equations

$$\epsilon_q = \frac{q}{q_u} = 1 + 2(\epsilon_{sb} + \epsilon_{wb}),$$

$$\epsilon_{C_l} = \frac{C_l}{C_{lu}} = 1 - \sigma_{sc} - 2(\epsilon_{sb} + \epsilon_{wb}),$$

$$\epsilon_{C_d} = \frac{C_{do}}{C_{dou}} = 1 - \epsilon_{sb} - 2\epsilon_{wb},$$

where $\epsilon_{sb}$, $\epsilon_{wb}$ and $\sigma_{sc}$, are correction factors for solid blockage, wake blockage and streamline curvature respectively, and the subscript $u$ donates uncorrected values.

The solid blockage correction factor was computed using the expression

$$\epsilon_{sb} = \frac{\xi V}{c^{3/2}},$$

where $V$ is the aerofoil volume, $c$ is the chord and the constant, $\xi = 0.54$ for a
model installed horizontally.

The wake blockage correction factor was computed following the approach of Maskell (1963). This is expressed as

\[ \epsilon_{wb} = \frac{c}{2h} C_{du}, \]  

(D.3)

where \( h \) is the height of the wind tunnel test-section. The streamwise correction factor was obtained through the expression

\[ \sigma_{sc} = \frac{\pi^2}{48} \left( \frac{c}{h} \right)^2. \]  

(D.4)

The angle of attack \( \alpha \) was corrected through the expression

\[ \alpha = \alpha_u + \frac{57.3}{2\pi} \sigma_{sc} \left( C_{lu} + 4C_{m_{1}u} \right). \]  

(D.5)

As the bending moment coefficient \( (C_{m_{1}u}) \) was unable to be measured experimentally, its value was estimated from the XFOil predictions at a Reynolds number of \( 1.8 \times 10^5 \).

The magnitudes of these correction factors are summarised in Table D.1. In this instance, the correction for the angle of attack was based on an uncorrected drag coefficient of \( C_{du} = 0.04 \).

Table D.1: Summary of 2-D wind tunnel corrections corresponding to the measurements of the aerofoil forces and pressure distributions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Force</th>
<th>Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid blockage ( \epsilon_{sb} )</td>
<td>0.003</td>
<td>0.008</td>
</tr>
<tr>
<td>Wake blockage ( \epsilon_{wb} )</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>Streamline curvature ( \epsilon_{sc} )</td>
<td>0.008</td>
<td>0.006</td>
</tr>
<tr>
<td>Lift coefficient ( \epsilon_{C_l} )</td>
<td>0.977</td>
<td>–</td>
</tr>
<tr>
<td>Drag coefficient ( \epsilon_{C_d} )</td>
<td>0.985</td>
<td>–</td>
</tr>
<tr>
<td>Angle of attack ( \epsilon_{\alpha} )</td>
<td>0.071</td>
<td>0.050</td>
</tr>
<tr>
<td>Dynamic pressure ( \epsilon_{q} )</td>
<td>1.014</td>
<td>1.022</td>
</tr>
</tbody>
</table>
D.2 Performance of the NREL S814 Foil at High Reynolds Numbers

The aerofoil forces (including the pitching moment) and pressure distributions across the aerofoil chord of the S814 at \( \text{Re} = 1.5 \times 10^6 \) by Somers (1997) (see also Somers, 1994 and Somers and Tangler, 1996) are provided in Figure D.1 to Figure D.4.

It is important to note that turbulence in the wind-tunnel was reported as between 0.02% and 0.04%. This at least 7.5 and 15 times lower than measured in the wind tunnel test section that was used to acquire the force and pressure distributions in this thesis. Somers (1997) also obtained the lift and drag by integrating the pressure distribution and using a wake rake.

D.3 Performance of the NREL S823 Foil at Low Reynolds numbers

The lift and drag coefficients for the NREL S823 foil were presented by Selig et al. (1995) which are provided in Figure D.5 for reference. These were measured using a mechanical force balance and the turbulence intensity was approximately 0.1%.
Figure D.1: Two-dimensional lift and pitching moment coefficient (top) and lift and drag polar (below) for the S814 aerofoil $Re = 1.5 \times 10^6$ reported by Somers (1997).
D.3. Performance of the NREL S823 Foil at Low Reynolds numbers

Figure D.2: Pressure distributions for the S814 aerofoil measured for $\alpha = -0.05^\circ, 0.98^\circ, 2.02^\circ, 3.05^\circ$ and $4.09^\circ$ for Re = $1.5 \times 10^6$ reported by Somers (1997).

Figure D.3: Pressure distributions for the S814 aerofoil measured for $\alpha = 6.15^\circ, 7.19^\circ, 8.22^\circ, 9.24^\circ$ and $10.26^\circ$ for Re = $1.5 \times 10^6$ reported by Somers (1997). The labels have been enhanced for clarity.
Figure D.4: Pressure distributions for the S814 aerofoil measured for $\alpha = 11.27^\circ, 12.23^\circ, 13.22^\circ, 14.21^\circ$ and $15.21^\circ$ for $Re = 1.5 \times 10^6$ reported by Somers (1997). The labels have been enhanced for clarity.
D.3. Performance of the NREL S823 Foil at Low Reynolds numbers

Figure D.5: Lift and drag coefficients of the S823 foil, reproduced from Selig et al., 1995. For the lift plots the solid-triangle and open-circle symbols donate increasing and decreasing angles of attack respectively.
Appendix E

Blade Element-Momentum Model

The implementation of the blade element-momentum (BEM) model was based on the commercial software code GH-Tidal Bladed (Bossanyi, 2009). The key equations and the solution strategy are summarised below.

During normal operation, a turbine will impart a change in the momentum of the free stream velocity. For an annulus the rate of change of axial momentum can be expressed as an incremental axial (thrust) force, i.e.

\[ dF_x = 4a(1 - a)\rho U^2 \pi r dr. \]  

(E.1)

where \( a \) is the axial induction factor, \( U \) is the free-stream velocity and \( r \) is the radius of the element from the hub. The change in the angular momentum of the flow also imparts a torque, which can be expressed as

\[ dQ = \pi \Omega r^3 (1 - a) a' \rho U \pi r dr. \]  

(E.2)

where \( a' \) is the tangential induction factor and \( \Omega \) is the rotational speed of the rotor.

The force and torque is assumed to be balanced by equivalent axial and tangential forces acting on the blade element. These forces can be expressed, respectively as

\[ dF_x = \frac{1}{2} \rho W^2 c (C_L \cos \phi + C_D \sin \phi) dr, \]  

(E.3)
Appendix E. Blade Element-Momentum Model

and

\[ dF_y = \frac{1}{2} \rho W^2 c (C_L \sin \phi - C_D \cos \phi) \, dr, \]  

(E.4)

where \( W \) is the resultant velocity incident at the chord, defined as

\[ W = \sqrt{[U (1 - a)]^2 + [\Omega r (1 + a')]^2}. \]  

(E.5)

The two-dimensional lift and drag coefficients \( C_l \) and \( C_d \) were obtained from a look-up table. This look-up table comprised the low Reynolds number aerofoil data acquired from the wind tunnel force measurements of the S814 foil that were presented in Section 6.6.1. The inflow angle \( \phi (= \alpha - \beta) \) was obtained from the relationship

\[ \tan \phi = \frac{U (1 - a)}{\Omega r (1 + a')}. \]  

(E.6)

By equating equations E and E.3 an expression for the axial induction factor can be obtained as

\[ a = \frac{g_1}{1 + g_1}, \]  

(E.7)

where

\[ g_1 = \frac{B c (C_l \cos \phi + C_d \sin \phi)}{8 \pi r \sin^2 \phi}. \]  

(E.8)

Similarly, an expression for the tangential induction factor can be obtained using equations E.4 and E.4 as

\[ a' = \frac{g_2}{1 - g_1}, \]  

(E.9)

where

\[ g_1 = \frac{B c (C_l \cos \phi - C_d \sin \phi)}{8 \pi r \sin \phi \cos \phi}. \]  

(E.10)

The induction factors solved using an iterative process until they had convergence to within 0.05% of their previous values. Once the factors were obtained, the blade root out-of-plane and in-plane bending moments were computed by integrating the axial and tangential forces along the span, i.e.

\[ M = \int_{r_{xy}}^R r dF \, dr. \]  

(E.11)

Since its inception of BEM theory a set of empirical corrections have been developed in an attempt to overcome some of its shortcomings. For instance, as the model is applied in a two-dimensional sense, there is no allowance for
the increase in the induced flow due to vortices shed from the blade tips. This is associated with a loss in the lift (which drops to zero at the tip) and an increase in the drag. The model proposed by Prandtl (refer to Betz, 1919) was implemented in this thesis. The model is an approximation of the exact solution for the tip-loss of lightly loaded rotors in axial flow which was later presented by Goldstein (1929). It represents the helical vortex wake pattern by vortex sheets which are convected by the mean flow. The Prandtl model introduces a correction factor to the momentum equation for the thrust. This factor $F$ has two parts, a model for the loss at the tip

$$F_{\text{tip}} = \frac{2}{\pi} \cos^{-1} \exp \left( -\frac{B}{2} \frac{(R - r)}{r \sin \phi} \right).$$  \hspace{1cm} (E.12)

and a similar correction to account for losses due to the hub

$$F_{\text{tip}} = \frac{2}{\pi} \cos^{-1} \exp \left( -\frac{B}{2} \frac{(R - r)}{r \sin \phi} \right).$$  \hspace{1cm} (E.13)

These were multiplied together to obtain $F$, i.e.

$$F = F_{\text{tip}} F_{\text{hub}}.$$  \hspace{1cm} (E.14)

The strategy that was employed to apply this loss in BEM model followed that which was outlined by Leishman (2006) (p.143). It involved firstly obtaining a solution for the induced inflow for the case where there was no loss (i.e. $F$ is set equal to 1). This induced inflow angle was then used to compute the Prandtl loss factor, $F$. The uncorrected momentum thrust force was then multiplied by $F$ and a new solution for the induced flow was then computed using BEM, which now included the tip-loss.

The momentum theory also fails at high tip speed ratios, where the rotor enters the turbulent wake, or brake state. Equilibrium-wake theory predicts that some of the flow returning upstream, when in reality more flow is entrained from outside the wake and the thrust increases (Leishman, 2006). In light of this, Glauert (1935) developed a correction factor to the thrust coefficient for the entire rotor disc, which has now been applied on an annular basis. The thrust coefficient is expressed as

$$C_T = \frac{8}{9} + \left(4F - \frac{40}{9}\right) a + \left(\frac{50}{9} - 4F\right) a^2.$$  \hspace{1cm} (E.15)
This correction was implemented by replacing the equation for the rate of change of momentum with equation E.15 when the axial induction factor was found to exceed a value of $a = 0.352$, as recommended by Bossanyi (2009).
Appendix F

Loewy Theory for a Returning Wake

Loewy (1957) provided a linearised theory for a rotor in hover that was based on an oscillating thin foil that was influenced by a returning, planar wake. The theory is consistent with the model proposed by Theodorsen (1935) and replaces the lift deficiency function for the circulatory forcing. The key equations associated with the Loewy (1957) model are provided by Leishman (2006) and are summarised below.

The lift deficiency function, \( C'(k) \) function is expressed in terms of reduced frequency \( k \) as

\[
C'(k) = \frac{H^{(2)}_1(k) + 2J_1(k)W}{H^{(2)}_1(k) + iH^{(2)}_0(k) + 2(J_1(k) + iJ_0(k))W},
\]

(F.1)

where \( H \) is the Hankel function, defined as \( H^{(2)}_v = J_v - iY_v \), with \( J \) and \( Y \) being Bessel functions of the first and second kind respectively. The function \( W \) is complex valued and for the collective mode (i.e. all blades in perturbed in-phase) is given as

\[
W(k, s, m) = \left( \exp^{ks} \exp^{i2\pi(m/N)} - 1 \right)^{-1}.
\]

(F.2)

where \( m \) is the frequency ratio and \( s \) is the spacing between consecutive planar sheets of vorticity which are convected below the foil. The original model of Loewy (1957) was applied to model a rotor in hover. For a tidal turbine retarding the mean flow, it was assumed in this thesis that the wake is convected downstream at a velocity of \( U - 2\pi v_i \), where \( U \) is the free-stream velocity and \( \pi_i \) is the mean...
Appendix F. Loewy Theory for a Returning Wake

induced velocity through the rotor. This is a minor difference between the model used by Whelan (2010), which did not include the effect of the induced velocity in the wake velocity. It follows that the wake spacing is expressed as

\[ s = \frac{4\pi (U - 2\pi v_i)}{cB\Omega}. \]  

(F.3)

where \( c \) is the chord of the blade section, \( B \) is the number of blades and \( \Omega \) is the rotational speed of the rotor. As the wake spacing becomes large the returning wake has little effect and the solution of Loewy (1957) tends to that presented by Theodorsen (1935).

For heaving motion, which is analogous to a tidal turbine subjected to axial forcing, the Loewy (1957) deficiency factor can be applied to compute the circulatory lift forcing on an oscillating foil. Whelan (2010) gives this lift force as

\[ L' = \pi \rho \Omega rcC'(k) [\tilde{u} \exp (i\omega t)], \]  

(F.4)

where \( \rho \) is the density of water, \( \Omega \) is the rotor speed, \( c \) is the chord length and \( \tilde{u} \) is the axial velocity amplitude, and which was adopted in this thesis.
Appendix G

Dynamic Stall Model

An overview of the modified Beddoes-Leishman (Leishman and Beddoes, 1989) model which was applied in this thesis to describe the effects of delayed separation and dynamic stall on the lift-coefficient is provided in this appendix. Its implementation was based on that presented by Pierce (1996) and Minnema (1998) for a wind turbine rotor. The original model consists of a series of modules, which each describe the attached flow response, the delays in the separation and vortex development and shedding. However, as discussed in Section 7.4.2, the rotor wake was assumed to be frozen. As such, the unsteady attached flow model was not applied in this thesis.

Separated Flow

For unsteady flow there is a delay in the leading edge pressure response. A first order lag is applied to the steady circulatory normal force coefficient. This is expressed by

\[ C'_n = C_n - D_p. \]  \hspace{1cm} (G.1)

The function \( D_p \) is expressed in terms of an increment in time \( (\Delta t) \) which has been non-dimensionalised by the semi-chord \( (\Delta s = 2W\Delta t/c, \text{ where } W = \Omega r) \)

\[ D_{pn} = D_{pn-1} \exp\left(-\frac{\Delta s}{T_p}\right) + (C_{nn} - C_{n_{n-1}}) \exp\left(-\frac{\Delta s}{2T_p}\right), \]  \hspace{1cm} (G.2)

The indices \( n \) and \( n - 1 \) correspond to the current and previous time-steps, respectively.

The empirically determined leading edge pressure time-constant, \( (T_p) \) was assumed as \( T_p = 1.7 \), which is consistent with Pierce (1996). It is considered to
be relatively independent of the aerofoil shape (Leishman and Beddoes, 1986). Leading edge separation is considered to occur when $C'_n$ exceeds a predetermined critical value $C_{N1}$. As the foil is expected to stall slowly from the trailing edge, $C_{N1}$ was assumed to be equal to the value of $C_n = C_{N\alpha} (\alpha_{C_{n_{\text{max}}}} - \alpha_0)$, where $C_{N\alpha}$ is the slope of the normal coefficient and $\alpha_{C_{n_{\text{max}}}}$ is the angle of attack corresponding to maximum value of the experimentally measured normal coefficient ($C_n$). This is consistent with the approach of (Pierce, 1996).

The effects of trailing edge separation on the lift are modelled according to Kirchhoff/Helmholtz theory for a flat plate (Thwaites, 1987)

$$C_N = C_{N\alpha} \left( \frac{1 + \sqrt{f}}{2} \right)^2 \sin \alpha.$$ (G.3)

where $f$ is a dimensionless separation point. It is important to note that this separation point is not necessarily a physical point on the aerofoil. It only serves to relate the measured normal force to that of a flat plate. For steady flow the separation point is obtained from the experimental measurements of $C_N$ by re-arranging equation G.3 and it is shown in Figure G.1. For unsteady flow, the separation point is modified and is instead calculated at an angle of attack corresponding to the lagged normal force coefficient, i.e.

$$\alpha_f = \frac{C'_n}{C_{N\alpha}} + \alpha_0.$$ (G.4)

A time delay is also applied to the separation point to account for a lag in the boundary layer response, giving rise to the final modified separation point

$$f'' = f' - D_{f_n},$$ (G.5)

where

$$D_{f_n} = D_{f_{n-1}} \exp \left( -\frac{\Delta s}{T_f} \right) + (f'_n - f'_{n-1}) \exp \left( -\frac{\Delta s}{2T_f} \right).$$ (G.6)

The empirically determined time-constant associated with the boundary layer drag is assumed to be $T_f = 3.0$. Combining these aforementioned effects the coefficient of the circulatory normal force is expressed as

$$C_{N_n}^S = C_{N\alpha} \left( \frac{1 + \sqrt{T_n}}{2} \right)^2 \alpha_{E_n},$$ (G.7)
The angle of attack, $\alpha_{En}$, is an effective angle of attack and typically incorporates unsteady attached flow effects (see Leishman and Beddoes, 1989). However, as the unsteady attached flow effects were not considered, $\alpha_{En}$ was assumed to be equal to the angle of attack estimated by the BEM model.

**Dynamic Stall**

As discussed by Leishman and Beddoes (1989), dynamic stall of a foil is characterised by the formation of a vortex at the leading edge which then separates and passes across the chord. This vortex lift is modelled in the model as the difference between the linearised value of the circulatory lift $C_{n}^{C}$, and the non-linear lift from the Kirchhoff approximation, i.e.

$$C_{vn} = C_{n}^{C} (1 - K_{Nn}), \quad (G.8)$$

where

$$K_{Nn} = \left(1 + \sqrt{f''_{n}}\right)^{2}/4. \quad (G.9)$$

The vortex decays with time but can also be updated by a new increment. This process is modelled according to

$$C_{Nn}^{V} = C_{Nn-1}^{V} \exp \left(-\frac{\Delta s}{T_{v}}\right) + \left(C_{vn} - C_{vn-1}\right) \exp \left(-\frac{\Delta s}{2T_{v}}\right), \quad (G.10)$$
Appendix G. Dynamic Stall Model

where the vortex time constant is assumed to be $T_v = 4.0$. When the critical pressure coefficient is obtained (i.e. $C_n' \geq C_{n1}$) the vortex begins to convect across the rotor chord. This process is modelled using a non-dimensional vortex time parameter, defined in semi-chords. At $t = \tau_{v0}$ the vortex departs from the leading edge and it reaches the trailing edge at an assumed time of $t = \tau_{vl} = 6.0$, at which point the vortex lift does not increase further.

The time constants associated with the vortex, ($\tau_v$) and ($\tau_{vl}$) correspond to those presented by Sheng et al. (2010). They obtained semi-empirical constants for the S814 aerofoil from pitching experiments in a wind tunnel at a high Reynolds number of $Re = 1.5 \times 10^6$. It is acknowledged the Reynolds numbers are an order of magnitude greater than for the model turbine. They were an attempt to incorporate constants that were specific to used to the foil used in the experiments, in lieu of low Reynolds number data being available.

The total normal forces were obtained as a superposition of the contribution of the lift due to delayed separation, the vortex induced lift and the non-circulatory ($C_{Nn}^l$) i.e.

$$C_N = C_{Nn} \left( \frac{1 + \sqrt{f_n'}}{2} \right) \alpha_{En} + C_{Nn}^V + C_{Nn}^I.$$  \hspace{1cm} (G.11)

The lift coefficients are obtained by resolving the normal force coefficients according to

$$C_l \approx C_N \cos \alpha.$$  \hspace{1cm} (G.12)

As a first approximation, the contribution from the chord wise force (predominately due to the foil drag) was considered to be sufficiently small to be able to be neglected for the purposes of the analyses.