



<http://researchspace.auckland.ac.nz>

### *ResearchSpace@Auckland*

#### **Copyright Statement**

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand).

This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author's right to be identified as the author of this thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author's permission before publishing any material from their thesis.

To request permissions please use the Feedback form on our webpage.

<http://researchspace.auckland.ac.nz/feedback>

#### **General copyright and disclaimer**

In addition to the above conditions, authors give their consent for the digital copy of their work to be used subject to the conditions specified on the Library Thesis Consent Form.

**SPREAD SPECTRUM SWITCHING:  
A LOW NOISE MODULATION TECHNIQUE  
FOR PWM INVERTER DRIVES**

by

P. G. Handley

A Thesis submitted in partial fulfilment  
of the requirements for the degree of  
Doctor of Philosophy in Electrical Engineering  
University of Auckland

1990

University of Auckland Library  
ENGINEERING LIBRARY

Thesis

1990-H19

Copy 1

916/954859/0/01

16/7/91

*'I wisdom dwell with prudence  
and find out knowledge of witty inventions'.  
Proverbs 8:12 (KJV).*

## ABSTRACT

Three phase AC drives controlling cage induction motors have become widely accepted in industry, but one extant problem with this technology is that of increased acoustic noise emitted from the driven motor.

This Thesis addresses the problem of the acoustic noise emitted from motors driven from voltage sourced PWM inverters and proposes a technique - Spread Spectrum Switching - for minimizing its effects. In the course of the work many other issues associated with real-time microprocessor-based PWM have also been advanced:

- efficient microprocessor based PWM waveform generation,
- harmonic analysis of generalized PWM waveforms,
- compensation for the effects of power switch timing delays, and
- compensation for the finite resolution of timers.

The Thesis uses a variety of computational and analytical methods, backed by experimental observations, to quantify the improvement gained in each of these areas.

Spread spectrum switching is a technique for eliminating the characteristically tonal structure of the acoustic noise emitted from a PWM inverter driven motor. Similar to the concept of spread spectrum communications, spread spectrum switching involves pseudo-randomly varying the instantaneous PWM switching frequency so that the energy of any PWM switching harmonics is dispersed over a wide bandwidth. This energy dispersion effectively eliminates any tonal components from the resultant motor acoustic noise while leaving the overall sound level largely unchanged; spread spectrum switching provides a significant qualitative yet minimal quantitative noise reduction.

The PWM generation paradigm used in this Thesis is the recently reported Space Vector Modulation. A novel algorithm for microprocessor based space vector PWM generation is proposed, providing a basis for fast, efficient generation, even when overmodulating - a situation where many algorithms operate significantly more slowly. Furthermore, it is shown that the space vector method inherently generates a near optimum - in terms of motor harmonic loss - PWM waveform. However, when physically realized on a practical inverter such ideal PWM waveforms are corrupted by timing errors associated with both the inverter's power switches, predominantly the lockout time, and the finite resolution of hardware

timers. Resolution corrected modulation is proposed for overcoming the problem of finite timer resolution and involves the use of integral feedback to account for any errors between ideal and physically realizable PWM switching times. This technique effectively provides 4 to 5 bits of added resolution to a given timer, allowing accurate waveform generation at low sinewave amplitudes and high switching frequencies using readily available, often microprocessor based, timers. Lockout times cause inverter output voltage errors, with consequent current zero crossing distortion, and a strategy for alleviating this problem is proposed and implemented in both a triangulation and space vector modulator.

Two harmonic analysis techniques are proposed for analyzing PWM waveforms. The first technique is suitable for the analysis of regularly sampled PWM waveforms and has been used here to obtain closed form expressions for the harmonics of both space vector and asymmetrical triangulation PWM. These expressions show that PWM harmonics occur as a series of "combs" centered on multiples of the switching frequency. A second technique - the Directional Rotational Transform - is proposed for numerical analysis of general PWM waveforms. This technique uses an equivalent space vector representation of the PWM waveform, yielding the magnitude, phase and sequence (positive or negative) of the harmonics, and is useful in situations where each of the three phase waveforms is different, as in these cases Fourier Transform analysis of a single phase or line voltage only approximates the harmonics actually seen by the motor. The spectra generated using both these techniques compare favourably with those measured experimentally and, for synchronous PWM, those evaluated from Fourier Transforms.

The culmination of modulation techniques presented in this Thesis yields a microprocessor based AC inverter drive featuring low acoustic noise emission at but a few kiloHertz switching rates and accurate PWM waveform generation using a single chip, low cost, micro-controller.

## ACKNOWLEDGMENTS

Looking back over the course of my PhD studies a number of people stand out as deserving special thanks and recognition, although at some time or another I have approached nearly every staff member or fellow postgraduate and without fail received welcome assistance.

In particular I wish to express my sincere gratitude and appreciation to Professor John Boys, for his supervision of this work, as without his on-going contributions, enthusiasm and answers to my (seemingly) endless questions this Thesis and the results it presents probably wouldn't have come to fruition.

Others who deserve special recognition are:

- Dr Andrew Green who both proof-read this Thesis and also assisted in obtaining the Voltage Sourced Reversible Rectifier measurements presented in Chapter 5.
- Mark Johnson, with whom many evenings were spent in evaluating the Spread Spectrum Switching technique of Chapter 7 using a chopper driven DC motor. The 56001 hardware mentioned in that Chapter was constructed (for other purposes) by Mark.
- Dr George Dodd (Acoustics Research Centre) and Mark Poletti (DSIR) who provided both sound measuring equipment and technical advice on acoustic matters relating to the Spread Spectrum Switching investigation.
- Fisher and Paykel (NZ) Ltd for their financial assistance and PDL Electronics (NZ) Ltd for both financial assistance and provision of the power electronic hardware used in the inverter detailed in Appendix B.

Finally I wish to thank my parents for the love and support they have given over the nineteen years of my education, and for always making "home" a welcome place to seek sanctuary.

Paul Handley  
December 1990.



# TABLE OF CONTENTS

Abstract	i
Acknowledgements	iii
Table of Contents	v
Nomenclature	viii

## Chapter 1 INTRODUCTION

1.1	AC Inverter Drives: A Perspective	1
1.2	Trends in 3 $\phi$ AC Drives	5
1.3	Acoustic Noise Sources in Variable Speed Drives	7
1.4	Psycho-acoustic Variables in the Inverter Drive Noise Problem	8
1.5	Thesis Outline	10
1.6	References	13

## Chapter 2 PULSE WIDTH MODULATION TECHNIQUES: A REVIEW

2.1	Introduction	15
2.2	PWM Terminology	16
2.3	Four Popular PWM Generation Techniques	19
2.4	Discussion	23
2.5	Summary	28
2.6	References	29

## Chapter 3 SPACE VECTOR MODULATION

3.1	Introduction	33
3.2	Space Vector Modulation	34
3.3	An Efficient Microprocessor Based PWM Generation Technique	36
3.4	Optimized Space Vector PWM Waveforms	41
3.5	Current Zero Crossing Distortion	45
3.6	Summary	51
3.7	References	51

**Chapter 4 AN ANALYTICAL HARMONIC ANALYSIS OF SPACE VECTOR PWM WAVEFORMS**

4.1	Introduction	53
4.2	Space Vector Equivalent Phase Waveform	54
4.3	Harmonic Analysis Technique	56
4.4	Space Vector PWM Harmonic Analysis	61
4.5	Experimental Verification of PWM Spectra	62
4.6	Significance of the PWM "A B C D" Constituent Waveforms	66
4.7	Summary	69
4.8	References	71
4.9	Appendices to Chapter 4	72

**Chapter 5 THE DIRECTIONAL ROTATIONAL TRANSFORM: A TECHNIQUE FOR NUMERICAL HARMONIC ANALYSIS OF PWM SPACE VECTORS**

5.1	Introduction	75
5.2	The Directional Rotational Transform	76
5.3	Verification of DRT PWM Spectra	82
5.4	Summary	85
5.5	References	85

**Chapter 6 MINIMIZATION OF VARIABLE SPEED DRIVE ACOUSTIC NOISE**

6.1	Introduction	87
6.2	Techniques for Minimizing Variable Speed Drive Acoustic Noise	93
6.3	Summary	98
6.4	References	98

**Chapter 7 SPREAD SPECTRUM SWITCHING**

7.1	Introduction	101
7.2	SSS applied to PWM Variable Speed Drives	102
7.3	SSS in DC Drives	107
7.4	SSS in AC Drives	112

7.5	Discussion and Summary	123
7.6	References	125
<b>Chapter 8</b>	<b>RESOLUTION CORRECTED MODULATION: THE PRACTICAL REALIZATION OF IDEAL PWM WAVEFORMS</b>	
8.1	Introduction	127
8.2	Concepts of Resolution Corrected Modulation	131
8.3	Simulated Results of Resolution Corrected Modulation	134
8.4	Experimental Verification of Resolution Corrected Modulation	136
8.5	Summary	141
8.6	References	142
<b>Chapter 9</b>	<b>CONCLUSIONS</b>	143
<b>APPENDICES</b>		
Appendix A:	Motor Parameter Data	149
Appendix B:	AC Inverter Circuit Diagrams	151

# NOMENCLATURE

## *Acronyms*

AC	-	Alternating "Current" (denotes a sinusoidal quantity)
BJT	-	Bipolar Junction Transistor
DC	-	Direct "Current" (denotes a constant quantity)
DFT	-	Discrete Fourier Transform
DRT	-	Directional Rotational Transform
EMF	-	Electro-Motive Force (V, pu)
EPROM	-	Erasable Programable Read Only Memory
FFT	-	Fast Fourier Transform
GTO	-	Gate Turn-Off Thyristor
IGBT	-	Insulated Gate Bipolar Transistor
MCT	-	MOS Controlled Thyristor
MMF	-	Magneto-Motive Force (A-t)
MOSFET	-	Metal-Oxide-Semiconductor Field Effect Transistor
NPC	-	Neutral Point Clamped
PRBS	-	Pseudo Random Binary Sequence
PU	-	Per Unit (pu)
PWM	-	Pulse Width Modulation
QSW	-	Quasi Square Wave
RCM	-	Resolution Corrected Modulation
RMS	-	Root Mean Square
RPM	-	Revolutions Per Minute (rpm)
SCR	-	Silicon Controlled Rectifier
SPL	-	Sound Pressure Level
SSS	-	Spread Spectrum Switching
SVM	-	Space Vector Modulation
$3\phi$	-	Three Phase

## *Symbols*

$a_n$	-	$n^{\text{th}}$ harmonic magnitude (pu, RMS or peak)
$A(t), B(t),$ $C(t), D(t)$	-	PWM pulse waveforms
$B$	-	instantaneous airgap flux density (T)
$B_r, B_t$	-	radial and tangential components respectively of $B$
$d(f)$	-	band-limited random dither variable (Hz, pu)

$E$	-	DC motor back emf or three level inverter DC bus voltage (V, pu)
$e_a, e_b$	-	error between ideal and realizable PWM switching times (sec)
$f_c, f_m$	-	PWM carrier and modulating waveform frequencies respectively (Hz)
$f_1$	-	fundamental frequency of modulating waveform (Hz)
$f_{av}$	-	average switching frequency of a SSS PWM modulator (Hz)
$J_n(x)$	-	Bessel function of first kind of order $n$
$L$	-	series inductance in DC motor equivalent circuit (H)
$L_m$	-	CIM equivalent circuit mutual inductance (H)
$l_s, l_r$	-	CIM equivalent circuit stator and rotor leakage inductances (H)
$n, p, q, r$	-	integers, i.e. 0,1,2,..
$P_{Cu}$	-	motor harmonic copper loss (W, pu)
$R, Y, B, N$	-	three phase waveform designators, and neutral
$R_s, R_r$	-	CIM equivalent circuit stator and rotor resistances ( $\Omega$ )
$t, t_n$	-	time (sec)
$\delta t_n$	-	width of $t_n^{th}$ pulse (sec)
$t_0, t_a, t_b, t_7$	-	duration at voltage vectors $\mathbf{u}_0, \mathbf{u}_a, \mathbf{u}_b$ and $\mathbf{u}_7$ respectively during a single space vector modulation cycle (sec)
$t_d$	-	lockout time (sec)
$T$	-	single space vector modulation cycle period {i.e. half a complete PWM cycle period, $(\frac{1}{2f_c})$ } in AC PWM or total PWM period in DC PWM (sec)
$T_1, T_2$	-	PWM pulse times for AMD9513 timing format
$T_m$	-	active modulation time (sec) $\{T_m = t_a + t_b\}$
$\mathbf{u}_s$	-	reference voltage space vector (pu)
$\mathbf{u}_a, \mathbf{u}_b$	-	generalized voltage space vectors; defined as the two active voltage space vectors immediately adjacent $\mathbf{u}_s$ (pu)
$\mathbf{u}_i$	-	voltage space vectors realizable with a two-level inverter, (pu), $i = 0..7$
$V$	-	voltage (V, pu)
$V_{DC}$	-	inverter DC bus voltage (nominally 560V for 400V 3 $\phi$ mains) (V)
$\pm V_{DC}$	-	voltage of a DC bus rail with respect to the bus midpoint, $\frac{V_{DC}}{2}$
$\alpha$	-	intrasextant angle of $\mathbf{u}_s$ (rad)
$\delta$	-	difference between circular and hexagonal space vector loci (pu)
$\Delta(\theta)$	-	SVM "curved" triangle distortion waveform (pu)
$\partial$	-	duty cycle, $0 \leq \partial \leq 1$ , (pu)

$\theta$	-	phase angle, $\omega_m t + \phi$ , (rad)
$\theta_i$	-	$i^{\text{th}}$ PWM switching angle, (rad)
$i_n$	-	$n^{\text{th}}$ harmonic current (A, pu)
$\Lambda$	-	motor airgap permeance (Wb/A-t)
$\mu_0$	-	permeability of free space, ( $\mu_0 = 4\pi * 10^{-7}$ T-m/A-t)
$\sigma$	-	harmonically weighted loss factor (pu)
$\sigma_B$	-	stress between two magnetic conductors (N/m <sup>2</sup> )
$\tau$	-	electrical torque (Nm, pu) or modulating waveform period (sec)
$v_n$	-	$n^{\text{th}}$ harmonic voltage (V, pu)
$\phi$	-	motor flux (Wb) or phase angle offset (rad)
$\omega$	-	angular frequency (rad/sec)
$\omega_c, \omega_m$	-	PWM carrier and modulating waveform angular frequencies respectively (rad/sec)
$\int e_a, \int e_b$	-	integral of timing error between ideal and realizable PWM switching times (sec)