Economics Department

Economics Working Papers

The University of Auckland *Year* 2002

Asymmetric Information and R&D Competition

Tina Kao
University of Auckland, tina.kao@anu.edu.au

Asymmetric Information and R&D Competition

Tina Kao*

Department of Economics

The University of Auckland

August 9, 2002

Abstract

This paper analyses R&D competition among firms with incomplete information. In a stochastic R&D game, firms possess private information regarding their R&D progress. They can only observe the rival's R&D investments, but not its actual R&D position. R&D investments thus carry both investment and signalling effects. In this two-period model, there are two possible regimes for the second period game: the complete information regime and the signalling regime. In the signalling regime, in order to credibly convey to the rival its first period research success, the first mover has to over-invest. Both firms have higher profits in the complete information regime. The game is in the signalling regime if the difference between monopoly and duopoly profit is sufficiently large and if the possibility of leapfrogging is high. For some parameter ranges, the choice of the information regime is endogenous.

JEL Classification: D23, D82, L15.

Keywords: Asymmetric information, R&D, patent race, signalling.

*Email: t.kao@auckland.ac.nz. I am grateful to Reiko Aoki and John Hillas for very helpful suggestions. Comments from Aaron Schiff, participants in a workshop at the University of Auckland at July 26 2001, and game theory conference at Stony Brook at July 25 2002 are also appreciated. Financial support by the Bright Future Scholarship, file number 432, is gratefully acknowledged.

1 Introduction

In the literature of research and development(R&D) competition, many different issues have been studied. However, the problem of incomplete information in a R&D race is not well addressed. In reality, firms' R&D progress is difficult to observe especially if there is some unpatentable intermediate invention and if firms cannot credibly announce its technology advance without spill-overs of its research to the rivals. Even if such means of announcement are available, the incentive for firms to disclose such information is also not straightforward. Depending on the nature of the competition, a firm may want to send out different signals to its rival. If a technology lead triggers more aggressive behaviour from the rival, the leading firm may want to send out the signal that its technology progress is poor. For example, in Grossman and Shapiro (1986), the R&D competition is most vigorous when firms are even in the technology race. Their model is a complete information one, but we might expect that if firms can hide their technology progress, they may have the incentive to do so. If, on the other hand, a technology lead softens competition, the leading firm may want to convince the rival that its past R&D has been successful. Some examples include Fudenberg *et al.* (1983), Harris and Vickers (1985), and Fudenberg and Tirole (1984)'s example of the top dog strategy. It is also often argued that Microsoft use preannouncements for its operating system upgrades strategically to discourage investments by its rivals.[1] Some empirical support for this strategic announcement effect in the Digital Versatile Disc (DVD) player industry is given in Dranove and Gandal (2001).

In this paper, a two-stage model with sequential move is presented where firms' R&D progress is not observable to its rival. In each stage, each firm makes one R&D investment decision which is observable to the rival. Therefore, R&D investments have both investment and signalling effects. Firms are symmetric in the beginning of the game. That is, they stand at the same starting point for the R&D race. After the first period R&D investment, their own technology progress is private information to each firm. Depending on the parameter values, the second period game can be in the complete information regime where firms

[1]Some references on this include Lopatka and Page (1995) and Shapiro and Varian (1999).

do not have incentive to deviate from the complete information equilibrium[2]. Or the game could be in the signalling regime where the first mover, if succeeded in the first period R&D, needs to over-invest to signal its type. We identify the conditions for the change of regimes and characterise the unique signalling equilibrium. Furthermore, we show that for some parameter range, the choice of information regimes in the second period is endogenous which depends on firms' first period investments.

The rest of the paper is organised as follows. We review some related literature in the second section. The set up of the model is presented in the third section. Firms' second period behaviour is analysed in the fourth section. We then study the first period game with comments on the endogenous choice of the second period game regime, followed by conclusions.

2 Literature Review

There are a few different strands of models studying the technology innovation problem. The most popular and recent one is that of a patent race.[3] Such a modelling captures some characteristics of R&D competition. For example, in both a race and R&D competition, the largest prize is awarded to the first one to cross a well defined finishing line and competitors adjust their efforts according to their relative positions to their rivals.[4]

Many different questions have been analysed in the framework of a patent race.[5] Loury (1979), Dasgupta and Stiglitz (1980), Lee and Wilde (1980), and Reiganum (1981, 1982a, b) are models with memoryless or Poisson patent races where firms' probability of making a discovery today conditional on no one else having done so depends only on their current level of R&D investment. In such models, firms' accumulated R&D or knowledge stock have no effect

[2]The equilibrium is the same as that in the case where there is no incomplete information about the the first mover's position. Although we term it as complete information, the second mover's position is still unknown.

[3]The other two main streams of models are deterministic auction models and contest models.

[4]There is also some difference between these two. In a race, there is usually a winner. But in a stochastic R&D game, it is possible that no firms would make the discovery.

[5]A good survey is in Reinganum (1989), but more papers have appeared since then.

on its current likelihood of discovery. Gilbert and Newbery (1982) consider a deterministic model where an arbitrarily small advantage allows the lead firm to act as a monopolist and still preempt the entry. Fudenberg *et al.* (1983) have a model with information lags. Firms invest simultaneously and only observe each other's investment with a time lag. By introducing this information lags and leapfrogging, it is possible for the technology laggard to win the R&D race. They conclude that the possibility of leapfrogging is important for the lagging firm to stay in the race. Grossman and Shapiro (1987) have a two stage model based on Lee and Wilde with firms adjusting their R&D investments depending on their relative positions in the race. The general conclusion is that competition in R&D is most fierce when firms are close in the race.

There are a few papers which incorporate elements of incomplete information. One that addresses a similar question as this paper, discussed in more detail later, is Aoki and Reitman (1992). They propose a model with two-sided private information where firms' initial marginal costs are not observable to the rival. Firms play a Cornot game in the final product market. The R&D technology is deterministic. They focus on a partial pooling equilibrium where firms under-invest in R&D.

Some papers deal with firms' information disclosure decisions. In such models, firms possess some private information, costly or costlessly acquired, which is not observable to the rival. Firms decide whether or not to disclose their private information. Some examples include Ulph(1998), Gośalbez and Díez(2000), and Rosenkranz (2001), where firms' information disclosure incentive in a research joint venture is studied. Anton and Yao (1999) study firms' decision on information disclosure with imperfect patent protection and the possibility of imitation. In the model, only the leading firm engages in R&D. Other papers on information disclosure include Admati and Pfleiderer (2000), Dewatripont and Tirole (1999), Shin (1998), and Jansen (2002a, b). All the papers deal with the case where some credible announcement is available. If firms are willing, they can simply convey to the rival the private information they have. In our model, we assume that such an announcement instrument is not available. Firms can only manipulate the rival's belief about their R&D progress through adjusting their R&D effort. Therefore R&D investment carries both investment and signalling effects in our model.

In this paper, the main issue we deal with is incomplete information of firms' R&D positions. We incorporate a few elements from some papers mentioned above. We specify an intermediate invention following Grossman and Shapiro to indicate technology leader and follower. The probability of leapfrogging is crucial to keep the technology laggard active in the game and is also included. This paper tries to complement the analysis of Aoki and Reitman. In their model, firms have initial private information about their costs. This can be viewed in this current paper as firms have completed the first period R&D investment. In this sense, the paper include the analysis of the first period investment behaviour which is not modelled in Aoki and Reitman. Finally, Aoki and Reitman employs a deterministic R&D technology. Reinganum (1983) demonstrates that the results from a patent race could be quite different depending on if the invention process is deterministic or stochastic. The R&D technology is assumed to be stochastic in the paper. The game is a sequential move one to emphasise the role of signalling. We don't model the product competition explicitly and focus on firms' R&D behaviour. But the profit setting is general enough to consider different market structures.

3 The Model Set up

The model is a two-stage R&D race game. The first stage is the primary research stage which is not necessary for the final invention but the success brings one closer to the final goal. The reason for specifying this primary research stage is to define the technology leading and lagging firms and analyse how their R&D behaviour differs. There are two firms, A and B. They make one R&D investment decision in each period. The R&D technology is the same for both firms. A firm's R&D progress is illustrated by its position in the R&D race, which is defined as $s_i \in \{0, 1, 2\}$, $i = A, B$. Position $s_i = 0$ indicates that firm i is at the starting point of the R&D process, $s_i = 1$ means that firm i has finished the primary research, and $s_i = 2$ indicates that firm i has reached the finishing line for the invention and a patent is granted to guarantee its profit. We assume that there is no uncertainty and delay in patent application.

The probability of advancing in the patent race depends on firms' R&D investments, $r_{it} \in (0,1)$, for $t = 1,2$. At the end of the first period, with probability r_{i1}, firm i's first period research is successful and it advances to position one. With probability $1 - r_{i1}$, the firm stays at position zero. Firms can only advance one step in the first period. In the second period, they can move forward for two steps, which introduces the possibility of leapfrogging. Denote the probability of firm i advancing x steps, $x \in \{0,1,2\}$, by f_i^x. Assume that when r_i increases, the probability of firm i staying at the same position, f_i^0, decreases and the probability of advancing two steps, f_i^2, is non-decreasing.[6] In this model, we construct a discrete probability distribution for f_i^x from a nested Binomial distribution. For an R&D investment r_{i2}, with probability r_{i2}, firm i moves forwards. If the firm moves forward, with probability $1-q$, it moves one step, and with probability q , it moves forwards for two steps. Therefore, with investment r_{i2}, $f_i^0 = (1 - r_{i2})$, $f_i^1 = r_{i2}(1-q)$, and $f_i^2 = r_{i2}q$.

A firm's gross profit depends on its own and the rival's R&D success. The monopoly profit, π_M, is what one firm can enjoy when it is the only one to reach position two at the end of the second period. The duopoly profit, π_d, is what both firms have when they both reach the finishing line at the end of the second period. Firms receive positive profits only when they complete the final goal.[7] To guarantee the incentive to be the monopolist, assume that $\pi_M > 2\pi_d$. The game finishes after two periods, whether or not firms have reached the finishing line.

To emphasise the signaling effects, a sequential move model is analysed. Firm A is the first mover in each period. After A invests in the first period, Nature moves and determines if the R&D investment is successful with the probability of success being r_{A1}. This R&D outcome is not observable to B.

[6] We do not post the restriction that f_1 should be non-decreasing. If $s_1 = 0$, only the probability of leapfrogging matters. If $s_1 = 1$, advancing one step is as good as advancing two steps. It could be specified that for some certain range of r_i, the possibility of moving two steps increases while the possibility of advancing one step decreases. In that case, firms can target the size of innovation. This is left for future extension. In this model, there is only one innovation to be made.

[7] This can be thought as some normalisations. If we model the market competition period explicitly, it may make a difference whether or not firms' costs are revealed in the end of the second period. The market competition period is suppressed to focus on the R&D competition.

Observing A's first period investment, B has a prior belief on A's position. B makes its first period investment based on this belief. In the second period, knowing A's own position and based on its belief of B's position derived from B's first period investment, A makes its second period investment. Finally, B moves for the second time with the updated belief derived from A's second period investment. The game finishes at the end of the second period and firms' profits are determined.[8]

Let $C : r \rightarrow R_+$ be the cost function for R&D investment for both firms. Assume that R&D costs in the two periods are additively separable, $C_i = c_{i1} + c_{i2}$, and take the quadratic functional form,

$$C_i = \frac{kr_{i1}^2}{2} + \frac{kr_{i2}^2}{2},$$

where k, $k > 0$, is a parameter.

Assumption 1

$$\left.\frac{\partial \pi_i}{\partial r_{it}}\right|_{r_{it}=0} > 0,$$

and

$$\pi_m < \frac{k}{2}.$$

This assumption guarantees that the solutions solved by first-order conditions are interior. That is, $0 < r^* < 1$. Given there is no fixed cost in production, $\left.\frac{\partial \pi_{it}}{\partial r_t}\right|_{r_{it}=0} = 0$, the only assumption we need to guarantee positive investment is $\left.\frac{\partial \pi_i}{\partial r_{it}}\right|_{r_{it}=0} > 0$. To guarantee that the investment level is less than one, a sufficient condition is to assume that even in case of maximum gain, firms do not invest $r_i = 1$. The maximum gain for firms is the monopoly profit. Given r_{i1} and focusing on the second period, the condition follows that

$$\pi_m < C_i|_{r_{i2}=1} = \frac{kr_{i1}^2}{2} + \frac{k}{2}.$$

A sufficient condition for this is $\pi_m < \frac{k}{2}$.

[8] In a previous paper, we analysed a one-sided private information model. In the separating equilibrium, the R&D investment levels are the same as those that would prevail in a complete information model due to the assumption that it is impossible to leapfrog.

4 Second Period Competition

Since firms have private information regarding their R&D positions, they form beliefs about the rival's R&D progress. The equilibrium concept used here is sequential equilibrium which consists of a pair (r_i, μ_i) where r_i is the equilibrium R&D investment and μ_i is a consistent assessment of the rival's position. As illustrated in Section 4.3.1, the freedom of specifying the off-equilibrium beliefs permits plenty of sequential equilibria in the signalling regime. Some refinement concepts with which we can focus on the more "sensible" equilibria are discussed later. We solve the game backwards. In the second period, after observing A's second period investment decision, B makes its R&D investment depending on its updated belief and B's own position .

In the second period, given firm i's own research outcome in the first period, its expected profit depends on the rival's first period R&D position. Let $\mu_i \in [0,1]$ be firm i's belief of firm j being at position one at the beginning of the second period. Firms form this belief based on the rival's past R&D investment. If i's first period research was not successful, when calculating its expected profit after the second period investment, i assigns probability μ_i to getting profit $\pi_i(s_i = 0, s_j = 1)$ and probability $(1 - \mu_B)$ to getting profit $\pi_i(s_i = 0, s_j = 0)$. Its expected profit is

$$E\pi_i(s_i = 0, s_j) = \mu_i \pi_i(s_i = 0, s_j = 1) + (1 - \mu_i)\pi_i(s_i = 0, s_j = 0) - C_i.$$

Since i is at position zero, it only gets positive profit if it advances two steps in second period. That is, with probability f_i^2. If i advances for two steps and gets to the finishing line at the end of the second period, its profit depends on j's performance. If j is at position one in the end of the first period, as long as j does not stay at position one after the second period investment, both firms get to the finishing line at the end of the second period and they share π_d. If j does not move and stay at position one, i gets π_m. If j is at position zero in the end of the first period, firms share π_d if j moves for two steps. If j does not move for two steps, no matter j stays at position zero or it moves to position one, i has the monopoly profit. To summarise, for type zero of i, the expected profit is

$$\begin{aligned} E\pi_i\left(s_i=0,s_j\right) &= \mu_i f_i^2\left(f_j^0\pi_m+\left(1-f_j^0\right)\pi_d\right) \\ &\quad +\left(1-\mu_i\right)f_i^2\left(\left(1-f_j^2\right)\pi_m+f_j^2\pi_d\right)-C. \quad (4.0.1) \end{aligned}$$

Similarly, if i's first period research is successful,

$$\begin{aligned} E\pi_i\left(s_i=1,s_j\right) &= \mu_i\pi_i\left(s_i=1,s_j=1\right)+\left(1-\mu_i\right)\pi_i\left(s_i=1,s_j=0\right)-C \\ &= \mu_i\left(1-f_i^0\right)\left(f_j^0\pi_m+\left(1-f_j^0\right)\pi_d\right) \\ &\quad +\left(1-\mu_i\right)\left(1-f_i^0\right)\left(\left(1-f_j^2\right)\pi_m+f_j^2\pi_d\right)-C. \quad (4.0.2) \end{aligned}$$

The profit function is the same for both firms. The game is sequential. Therefore we solve for B's best responses first and substitute them into A's profit function to get its optimal investment.

4.1 Firm B's best responses

Using Equation 4.0.1 and 4.0.2 to solve for B's best replies, we observe that as μ_B increases, π_B decreases for both types of B. Firm B's profit decreases in μ_B since as the probability of A being at position one increases, the likelihood of B getting the monopoly profit decreases.

Index B's best response by two firms' positions, $R_B^{s_A s_B}$. Solving for the FOC for Equations 4.0.1 and 4.0.2, B's best responses in the second period depending on B's own position are

$$\begin{aligned} R_{B2}^{s0} &= \frac{q}{k}\left(\mu_B\left(\left(1-r_{A2}\right)\pi_m+r_{A2}\pi_d\right)+\left(1-\mu_B\right)\left(\left(1-r_{A2}q\right)\pi_m+r_{A2}q\pi_d\right)\right) \\ R_{B2}^{s1} &= \frac{1}{k}\left(\mu_B\left(\left(1-r_{A2}\right)\pi_m+r_{A2}\pi_d\right)+\left(1-\mu_B\right)\left(\left(1-r_{A2}q\right)\pi_m+r_{A2}q\pi_d\right)\right). \end{aligned}$$

Note that B's first period position does not affect anything else except for B's second period investment. Therefore, we can move nature's move after B's first period investment to be just before B's turn to make its second period investment. In particular, put nature's move for B's first period investment outcome to be after A' second period investment. In the original game, A would have two nodes (indicating B's position which is not known to A) in each of its two information sets (depending on A's own position). That is, A's decision would be indexed by the four possible states in the beginning of the second period with weights assigned by A's belief. After we move nature's move, A would only have two nodes depending on its own position. Firm A still forms μ_A based on r_{B1}, but it is independent of s_{B1} since it is not observable. This does not change the outcome of the game, but it simplifies the structure. Furthermore, for both types of B, the best responses are monotonic in μ_B and there is a one-to-one relationship between R^{s0}_{B2} and R^{s1}_{B2}. We can write r^{s0}_{B2} in terms of r^{s1}_{B2} and solve everything in terms of r^{1}_{B2}. From R^{s0}_{B2} and R^{s1}_{B2},

$$R^{s0}_{B2} = qR^{s1}_{B2}. \tag{4.1.1}$$

Note that firms' beliefs in the second period are quite different. Since A is the first mover, when it is A's turn to move in the second period, the only action that B has taken is r_{B1}. Given that both firms start at the same position in the beginning of the game, there is no signalling component in r_{B1}. Given the probability of nature's move, μ_A should be equal to r_{B1}. However, when it is B's turn to move in the second period, to form μ_B, B has observed r_{A1} and r_{A2}. When A chooses r_{A2}, A is either at position zero or one. Given that the optimisation problem would be different for different types of A, A would have incentive to act differently at different positions.[9] Knowing this and knowing that B knows this, r_{A2} is the signal that A can send to B to convey or to hide its type. Therefore, after observing r_{A2}, B updates its belief about A's position. As mentioned in the previous paragraph, we solve the game for when B is in position one. If B is successful in the first period and if $\mu_B\left(r_{A2}|_{s_B=0}\right) = 0$,

$$R^{01}_{B} = \frac{1}{k}\left[\left(1 - r_{A2}q\right)\pi_m + r_{A2}q\pi_d\right]. \tag{4.1.2}$$

When $\mu_B\left(r_{A2}|_{s_B=1}\right) = 1$,

$$R^{11}_{B} = \frac{1}{k}\left[\left(1 - r_{A2}\right)\pi_m + r_{A2}\pi_d\right]. \tag{4.1.3}$$

These are the best responses B has against two types of A. The best responses are depicted in Figure 4.1. If B is not sure about A's type, that is, $\mu_B \in (0,1)$, the best response is some weighted average of these two. Observe that $R^{01}_{B} > R^{11}_{B}$. When its first period research is successful, B is more aggressive when A

[9] The type represents firms' R&D position. Type zero means that the firm failed in the first period research and type one indicates that the firm is at position one in the beginning of the second period.

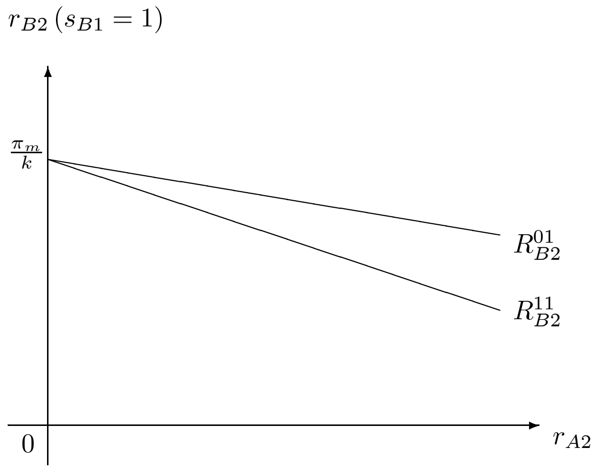

Figure 1: Firm B's best responses

failed in the first period since it is more likely for B to have monopoly profit. When B's first period R&D is not successful, $R_{B2}^{0} = qR_{B2}^{1}$ and $R_{00}^{B} > R_{10}^{B}$. Note that $\frac{\partial R_B^{01}}{\partial r_A} < 0$ and $\frac{\partial R_B^{11}}{\partial r_A} < 0$. The more A invests, the less aggressive B is, and firms' R&D investments in the second period are strategic substitutes. Given B's own position, B is more aggressive when A did not succeed in the first period research.[10]

4.2 Firm A's second period investment

Substituting B's best replies into A's profit function, Equation 4.0.1 and 4.0.2, and differentiate the equations for the two types of A with respect to r_{A2}, we get

$$\begin{aligned}
\frac{\partial \pi_A^{0s}}{\partial r_{A2}} &= \mu_A \left(\frac{\partial \pi_{01}^A}{\partial r_{A2}} + \frac{\partial \pi_{01}^A}{\partial R_{01}^B} \frac{\partial R_{01}^B}{\partial r_{A2}} \right) + (1-\mu_A) \left(\frac{\partial \pi_{00}^A}{\partial r_{A2}} + \frac{\partial \pi_{00}^A}{\partial R_{01}^B} \frac{\partial R_{01}^B}{\partial R_{00}^B} \frac{\partial R_{00}^B}{\partial r_{A2}} \right) \\
&= \mu_A \left(q\pi_m \left(1 - \frac{1}{k} (\pi_m - \pi_d) \right) + r_{A2} \frac{2}{k} q^2 (\pi_m - \pi_d)^2 \right) \\
&\quad + (1-\mu_A) \left(q\pi_m \left(1 - \frac{1}{k} q^2 (\pi_m - \pi_d) \right) + r_{A2} \frac{2}{k} q^4 (\pi_m - \pi_d)^2 \right) \\
&\quad - k r_{A2}, \qquad (4.2.1)
\end{aligned}$$

[10]From FOCs of Equations 4.0.1 and 4.0.2, for a given μ_B, $R_B^{1s} \gtreqless R_B^{0s}$ if $\frac{\partial f_2^B}{\partial r_B} \lesseqgtr -\frac{\partial f_0^B}{\partial r_B}$. In the second period, given the belief about A's position, B invests more when it is at position one if the probability of not advancing at all is greater than the probability of advancing two steps. Given our distribution, $\frac{\partial f_B^2}{\partial r_B} = q$ and $-\frac{\partial f_B^0}{\partial r_B} = 1$, for any given belief of A's position, B invests more if it is at position 1. The conclusion may be changed if we employ some different R&D technology.

and

$$
\begin{aligned}
\frac{\partial \pi_A^{1s}}{\partial r_{A2}} &= \mu_A \left(\frac{\partial \pi_A^{11}}{\partial r_{A2}} + \frac{\partial \pi_A^{11}}{\partial R_B^{11}} \frac{\partial R_{11}^B}{\partial r_{A2}} \right) + (1-\mu_A) \left(\frac{\partial \pi_A^{10}}{\partial r_{A2}} + \frac{\partial \pi_A^{11}}{\partial R_B^{11}} \frac{\partial R_B^{11}}{\partial R_B^{10}} \frac{\partial R_B^{10}}{\partial r_{A2}} \right) \\
&= \mu_A \left(\pi_m \left(1 - \frac{1}{k} (\pi_m - \pi_d) \right) + r_{A2} \frac{2}{k} (\pi_m - \pi_d)^2 \right) \\
&\quad + (1-\mu_A) \left(\pi_m \left(1 - \frac{1}{k} q^2 (\pi_m - \pi_d) \right) + r_{A2} \frac{2}{k} q^2 (\pi_m - \pi_d)^2 \right) \\
&\quad - k r_{A2}. \qquad (4.2.2)
\end{aligned}
$$

For any given μ_A, $\frac{\partial \pi_A^{1s}}{\partial r_{A2}} > \frac{\partial \pi_A^{0s}}{\partial r_{A2}}, \forall r_{A2}$. This means that firm A is more aggressive when its first period research is successful. Setting Equations 4.2.1 and 4.2.2 equal to zero gives the FOCs. Denote the optimal r_A that solve the FOCs to be r_A^0 and r_A^1 for two types of A respectively. Let $\delta \equiv \pi_m - \pi_d$. Firm A's optimal investments are

$$
\begin{aligned}
r_A^0 &= \frac{\mu_A q \pi_m \left(1 - \frac{1}{k}\delta\right) + (1-\mu_A) q \pi_m \left(1 - \frac{1}{k} q^2 \delta\right)}{k - \frac{2}{k} q^2 \delta^2 \left(\mu_A + (1-\mu_A) q^2\right)} \\
r_A^1 &= \frac{\mu_A \pi_m \left(1 - \frac{1}{k}\delta\right) + (1-\mu_A) \pi_m \left(1 - \frac{1}{k} q^2 \delta\right)}{k - \frac{2}{k} \delta^2 \left(\mu_A + (1-\mu_A) q^2\right)}.
\end{aligned}
$$

Investment levels r_A^0 and r_A^1 are solved by the profit maximising FOCs. They are what A would choose if A's type were observed by B. When there is incomplete information regarding A's type, there may be incentives for type zero A to mimic the behaviour of type one, and the separating equilibrium would deviate from the optimal level. To check if r_A^0 and r_A^1 are also equilibrium in the presence of private information, we examine if the two types' incentive constraints are violated at r_A^0 and r_A^1. That is, if they would rather to be recognised as the other type at the equilibrium $\left(r_A^0, r_A^1\right)$.

4.3 Characterising the signalling equilibrium

Being the first mover in the game, A's second period investment has both investment and signalling effects. When A makes its second period investment decision, it could be at position zero or position one. Therefore, apart from the investment effect, A may want to convey or disguise its type to B by the second period investment. After A's second period investment, B revises its belief about A's position. Based on this posterior belief and B's own position, B makes the second period investment. Note that since no one moves after

B's second period investment, there is no need for B to do any signalling. Two firms' second period investments carry quite different information. In the second period, the equilibrium should satisfy the following conditions.

1. The equilibrium should be sequential.

2. The equilibrium for B consists a pair of investment strategy and belief with B's strategy optimal given A's strategy, B's own position, and B's belief about A's position.

3. The equilibrium for A consists of a pair of investment strategy and belief with A's strategy optimal given A's prior belief, μ_A, B's best responses, and A's own position. This optimality should take into account the influence A's investment has upon B's investment and the consequent profit changes for A.

4. Beliefs are derived from the Bayes' rule wherever possible.

In a separating equilibrium, the incentive constraints are satisfied for both types of A so that neither type would have the incentive to mimic the other. Upon observing the equilibrium investment level, B assigns beliefs 0 or 1 and correctly identifies A's type. In a pooling equilibrium, both types of A choose the same level of second period investment. B learns nothing from observing the second period investment and still holds its prior belief based on r_{A1}.

Take r_{A1} and r_{B1} and therefore both firms' prior beliefs as given. Before characterising the signalling equilibrium, we first verify that the single crossing property and thus the stability condition for the equilibrium holds. Totally differentiating two types of A's profit functions,

$$\begin{aligned} d\pi_A^{0s} &= \left(\mu_A q\left(\pi_m - r_{B2}\delta\right) + \left(1-\mu_A\right) q\left(\pi_m - r_{B2}q\delta\right) - kr_{A2}\right) dr_{A2} \\ &\quad + \left(-\mu_A r_{A2} q\delta - \left(1-\mu_A\right) r_{A2} q^2 \delta\right) dr_{B2} \end{aligned}$$

and

$$\begin{aligned} d\pi_A^{1s} &= \left(\mu_A\left(\pi_m - r_{B2}\delta\right) + \left(1-\mu_A\right)\left(\pi_m - r_{B2}q\delta\right) - kr_{A2}\right) dr_{A2} \\ &\quad + \left(-\mu_A r_{A2}\delta - \left(1-\mu_A\right) q r_{A2}\delta\right) dr_{B2}. \end{aligned}$$

The slope of type zero of A's iso-profit curve in (r_{A2}, r_{B2}) space is

$$\left.\frac{\partial r_{B2}}{\partial r_{A2}}\right|_{\pi_A^{0s}=\bar{\pi}_A} = -\frac{\mu_A q\left(\pi_m - r_{B2}\delta\right) + \left(1-\mu_A\right) q\left(\pi_m - r_{B2}q\delta\right) - kr_{A2}}{-\mu_A r_{A2} q\delta - \left(1-\mu_A\right) r_{A2} q^2 \delta}. \quad (4.3.1)$$

For type one of A, the slope is

$$\left.\frac{\partial r_{B2}}{\partial r_{A2}}\right|_{\pi_A^{1s}=\bar{\pi}_A} = -\frac{\mu_A\left(\pi_m - r_{B2}\delta\right) + \left(1-\mu_A\right)\left(\pi_m - r_{B2}q\delta\right) - kr_{A2}}{-\mu_A r_{A2}\delta - q\left(1-\mu_A\right) r_{A2}\delta}.$$

Comparing the slopes of the two types,

$$\begin{aligned} & \left.\frac{\partial r_{B2}}{\partial r_{A2}}\right|_{\pi_A^{1}=\bar{\pi}_A} - \left.\frac{\partial r_{B2}}{\partial r_{A2}}\right|_{\pi_A^{0}=\bar{\pi}_A} \\ = & -\frac{\mu_A\left(\pi_m - r_{B2}\delta\right) + \left(1-\mu_A\right)\left(\pi_m - r_{B2}q\delta\right) - kr_{A2}}{-\mu_A r_{A2}\delta - q\left(1-\mu_A\right) r_{A2}\delta} \\ & - \left(-\frac{\mu_A q\left(\pi_m - r_{B2}\delta\right) + \left(1-\mu_A\right) q\left(\pi_m - r_{B2}q\delta\right) - kr_{A2}}{-\mu_A r_{A2} q\delta - \left(1-\mu_A\right) r_{A2} q^2\delta}\right) \\ > & \ 0 \end{aligned}$$

Type one's iso-profit curves are everywhere steeper than those of type zero in the second period and the single crossing property holds. That is, increasing the level of R&D investment is less costly for type one of A.

4.3.1 Sequential equilibrium with refinements on B's beliefs

As in most signalling games, there are a large number of sequential equilibria, including separating and pooling ones. The existence of multiple equilibria is due to the freedom of specifying the signal receiver's off-equilibrium beliefs. However, by strengthening restrictions on the off-equilibrium beliefs, we can pin down a subset of the equilibria which are more plausible. We use the refinement concepts proposed in Cho and Kreps (1987) and Kreps (1990). Consider the following refinements for the off-equilibrium beliefs.

Criterion 1 *Suppose that for a type h of A, investment levels r_A and r'_A are such that $\pi_h\left[r_A, R_B\left[s_A = 0\right]\right] > \pi_h\left[r'_A, R_B\left[s_A = 1\right]\right]$, then in any Nash equilibrium, it must be possible to sustain the equilibrium outcome with beliefs that put zero probability on investment level r'_A being selected by type h.*

Since given an investment level r, the best A can do is to convince B that it is indeed a type one. If what A gets from investing r_A when B thinks it is a type zero (the worse outcome from investing r_A) is greater than what it gets from investing r'_A when B thinks it is a type one (the best outcome from investing r'_A), type h of A would never choose to invest r'_A. Therefore upon seeing r'_A, B should never assess that it comes from type h of A. This is essentially Cho and Kreps' "Intuitive Criterion".

Criterion 2 *Fix a sequential equilibrium and let π_h^* be the profit level at this equilibrium for type h of A. Suppose that an investment level r'_A is such that $\pi_h [r'_A, r_B [s_A = 1]] < \pi_h^*$, then it must be possible to sustain the given equilibrium with beliefs that put zero probability on investment level r'_A being chosen by type h of A.*

This is Cho and Kreps' "equilibrium domination test".

The game has a separating equilibrium if we have an equilibrium $\left(\hat{r}_A^0, \hat{r}_A^1\right)$ with no type having the incentive to mimic the behaviour of the other, that is, type zero always chooses $\hat{r}_A^0$ and type one always chooses $\hat{r}_A^1$. Therefore, for any proposed separating equilibrium $\left(\hat{r}_A^0, \hat{r}_A^1\right)$, the following incentive constraints for two types of A must be satisfied:

$$\begin{aligned} & \mu_A \pi_A \left[\hat{r}_A^0, R_B^{01}\left[\hat{r}_A^0\right] | s_A = 0, s_B = 1\right] \\ & + (1-\mu_A)\,\pi_A \left[\hat{r}_A^0, R_B^{00}\left[\hat{r}_A^0\right] | s_A = 0, s_B = 0\right] \\ \geq \;& \mu_A \pi_A \left[\hat{r}_A^1, R_B^{11}\left[\hat{r}_A^1\right] | s_A = 0, s_B = 1\right] \\ & + (1-\mu_A)\,\pi_A \left[\hat{r}_A^1, R_B^{10}\left[\hat{r}_A^1\right] | s_A = 0, s_B = 0\right] \end{aligned}$$

and

$$\begin{aligned} & \mu_A \pi_A \left[\hat{r}_A^1, R_B^{11}\left[\hat{r}_A^1\right] | s_A = 1, s_B = 1\right] \\ & + (1-\mu_A)\,\pi_A \left[\hat{r}_A^1, R_B^{10}\left[R_B^{11}\left[\hat{r}_A^1\right]\right] | s_A = 1, s_B = 0\right] \\ \geq \;& \mu_A \pi_A \left[\hat{r}_A^0, R_B^{01}\left[\hat{r}_A^0\right] | s_A = 1, s_B = 1\right] \\ & + (1-\mu_A)\,\pi_A \left[\hat{r}_A^0, R_B^{00}\left[\hat{r}_A^0\right] | s_A = 1, s_B = 0\right]. \end{aligned}$$

The incentive constraints state that for either type, it must be better off playing its equilibrium strategy than playing the other type's strategy and being recognised as the other type. Since $R_B [s_A = 1]$ is everywhere below $R_B [s_A = 0]$, $r_A^{1s} > r_A^{0s}$, and $\frac{\partial \pi_A}{\partial r_B} < 0$, only type zero of A would have the incentive to mimic type one. Type one would never want to mimic the behaviour of type zero and the incentive constraint for type one is always satisfied.

Definition 1 *Define* $f[r_A]$ *to be the profit difference for type zero of* A *from mimicking type one and from being recognised as a type zero by investing its optimal investment level along* B*'s best response against type zero of* A*. That is,*

$$f[r_A|s_A=0] \equiv \pi_A\left[r_A, R_B^{1s}|s_A=0\right] - \pi_A\left[r_A^0, R_B^{0s}|s_A=0\right].$$

Furthermore, denote the investment level that solves $f[r_A]=0$ *for* $r_A > r_A^0$ *as* $\hat{r}_A$*. That is,* $f[\hat{r}_A]=0$*.*

The function $f[r_A]$ gives the gains from mimicking for type zero of A against its optimal investment when being recognised as a type zero. When $f[r_A] \leq 0$, type zero has no incentive to mimic the behaviour of type one. The profit level, $\pi_A\left[r_A^0, R_B^{0s}|s_A=0\right]$, can be thought of as the reservation profit for type zero. Type zero only has the incentive to mimic type one if what it gets from deviating is greater than this level. Since the game is sequential, substituting B's best responses into type zero of A's profit function, Equations 4.0.1, A's reservation profit is

$$\begin{aligned}
\pi_A^{0s}\left[r_A^0, R_B^{0s}\right] &= \mu_A r_A^0 q\left(\left(1-R_B^{01}\right)\pi_m + R_B^{01}\pi_d\right) \\
&\quad + (1-\mu_A) r_A^0 q\left(\left(1-q^2 R_B^{01}\right)\pi_m + q^2 R_B^{01}\pi_d\right) - C \\
&= -\mu_A r_A^0 q \frac{1}{k}\pi_m\delta + \mu_A\left(r_A^0\right)^2 \frac{1}{k} q^2\delta^2 + r_A^0 q\pi_m \\
&\quad - (1-\mu_A) r_A^0 \frac{1}{k}\pi_m q^3\delta + (1-\mu_A)\left(r_A^0\right)^2 \frac{1}{k} q^4\delta^2 - C.
\end{aligned}$$

On the other hand, if type zero of A mimics the behaviour of type one of A, substituting B's best responses against type one into type zero of A's profit function, Equation 4.0.1, A's expected payoff is

$$\begin{aligned}
\pi_A^{0s}\left[r_A, R_B^{1s}\right] &= \mu_A \pi_A\left[r_A, R_B^{11}|s_A=0, s_B=1\right] \\
&\quad + (1-\mu_A)\pi_A\left[r_A, R_B^{10}|s_A=0, s_B=0\right] - C \\
&= \mu_A r_A q\left(\left(1-R_B^{11}\right)\pi_m + R_B^{11}\pi_d\right) \\
&\quad + (1-\mu_A) r_A q\left(\left(1-q^2 R_B^{11}\right)\pi_m + q^2 R_B^{11}\pi_d\right) - C \\
&= -\mu_A r_A q\frac{1}{k}\pi_m\delta + \mu_A r_A^2 q\frac{1}{k}\delta^2 + r_A q\pi_m \\
&\quad - (1-\mu_A) r_A \frac{1}{k}\pi_m q^3\delta + (1-\mu_A) r_A^2 q^3 \frac{1}{k}\delta^2 - C.
\end{aligned}$$

From Definition 1, $f[r_A]$ is the difference of these two profit levels:

$$\begin{aligned} f[r_A] &= r_A^2\left(\mu_A q\frac{1}{k}\delta^2+(1-\mu_A)q^3\frac{1}{k}\delta^2-\frac{k}{2}\right) \\ &\quad -r_A\left(\mu_A q\frac{1}{k}\pi_m\delta-q\pi_m+(1-\mu_A)\frac{1}{k}\pi_m q^3\delta\right) \\ &\quad -\left(r_A^0\right)^2\left(\mu_A\frac{1}{k}q^2\delta^2+(1-\mu_A)\frac{1}{k}q^4\delta^2-\frac{k}{2}\right) \\ &\quad -r_A^0\left(q\pi_m-(1-\mu_A)\frac{1}{k}q^3\pi_m\delta-\mu_A q\frac{1}{k}\pi_m\delta\right). \qquad (4.3.2) \end{aligned}$$

This function is quadratic in r_A. There are two values of r_A which satisfy $f[r_A]=0$, one less than r_A^0 and one greater than r_A^0, which we term as $\hat{r}_A$. Given the single crossing property, for $r_A<r_A^0$, type one of A would prefer investing r_A^0 and being identified as type zero than investing at this lower level and pretending to be a type one. This could not be sustained as a separating equilibrium. Given the quadratic functional form, $f[r_A]>0$ for $r_A^0\leq r_A<\hat{r}_A$ and $f[r_A]\leq 0$ for $r_A\geq\hat{r}_A$.

Proposition 1 *With refinements in Criteria 1 and 2, if the incentive constraints are satisfied at $\left(r_A^0,r_A^1\right)$, the separating equilibrium is that type zero chooses r_A^0 and type one chooses r_A^1. This equilibrium is the same as the one when B knows A's position. If the incentive constraint is violated at $\left(r_A^0,r_A^1\right)$, the separating equilibrium consists of type zero choosing r_A^0 and type one choosing $\hat{r}_A$.*

Proof. In a separating equilibrium, type zero is correctly recognised as type zero. The best thing it can do is the point that maximises its profit along B's best response against type zero. Therefore it invests r_A^0. If the incentive constraint is satisfied at $\left(r_A^0,r_A^1\right)$, that is, if $f\left[r_A^1\right]\leq 0$, type zero would have no incentive to mimic type one when type one invests r_A^1. The optimal levels solved by the FOCs are the equilibrium investment levels for two types in this case. If the incentive constraint is violated at $\left(r_A^0,r_A^1\right)$, that is if $f\left[r_A^1\right]>0$, type zero has the incentive to mimic type one when the latter invests r_A^1. To separate itself from type zero, type one needs to invest more so that type zero does not have the incentive to imitate. That is, type one chooses the investment level which maximises its profit along B's best response against type one, subject to

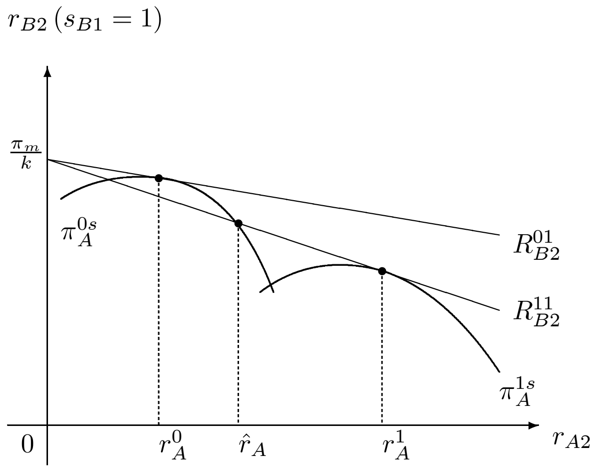

Figure 2: Complete information equilibrium

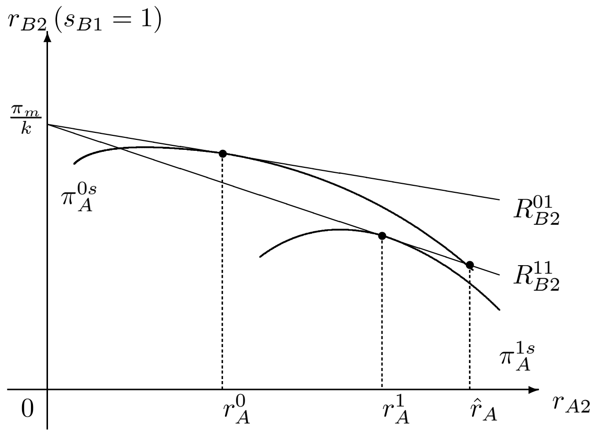

Figure 3: Signalling equilibrium

the condition that type zero does not strictly prefer this chosen investment level to r_A^0.

From Definition 1, for investment levels greater than $\hat{r}_A$, $f\,[r_A] < 0$. For $r_A \geq \hat{r}_A$, type zero does not have incentive to imitate type one. Therefore, B should have the belief $\mu_B\,(r_A | r_A > \hat{r}_A) = 1$. In equilibrium type one of A chooses $\hat{r}_A$. ■

The complete information and signalling equilibria are illustrated in Figures 2 and 3 respectively.

As Cho and Kreps demonstrate, there are no pooling equilibria that can survive the criteria.

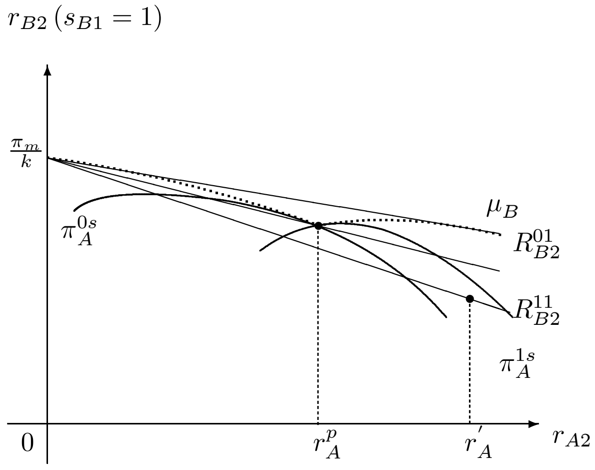

Figure 4: Pooling equilibrium

Proposition 2 *With the refinement in Criteria 1, there is no pooling equilibrium in this game.*

Proof. After the first period, B's prior belief is that with probability r_{A1}, A is a type one. In a pooling equilibrium, B learns nothing from A's second period investment and still holds its prior beliefs. For any pooling equilibria, the two types of A face an average best response weighted by r_{A1}. With single-crossing property, for any pooling equilibrium, r_A^P, there must exist some out of equilibrium investment level, $r_A^{'} > r_A^P$ such that $\pi_A^1\left[r_A^{'}\right] > \pi_A^1\left[r_A^P\right]$, and $\pi_A^0\left[r_A^P\right] > \pi_A^0\left[r_A^{'}\right]$. Hence by Criterion 1, type one of A can safely choose $r_A^{'}$ and B should form the belief $\mu_B\left[r_A^{'}\right] = 1$. This breaks any proposed pooling equilibrium. ■

This is illustrated in Figure 4.

As we demonstrate in the following section, the second period game can be either a complete information game or a signalling game depending on the parameter values and firms' first period investments.

4.3.2 The second period game regime

To determine if the second period game is a complete information game or a signalling game, we check if the incentive constraint for type zero is violated at $\left(r_A^0, r_A^1\right)$. From Proposition 1, for the incentive constraint to be violated, we need $f\left[r_A^1\right] > 0$ or $\hat{r}_A > r_A^1$. Let $E = \pi_m\left(1 - \frac{1}{k}\delta\left(\mu_A + (1-\mu_A)\,q^2\right)\right)$ and $D = 2\frac{1}{k}\delta^2\left(\mu_A + (1-\mu_A)\,q^2\right)$. Then, $r_A^0 = \frac{qE}{(k-q^2D)}$, $r_A^1 = \frac{E}{(k-D)}$, and

$$f\left[r_A\right] = r_A^2\left(\frac{qD}{2} - \frac{k}{2}\right) + qEr_A - \frac{1}{2}\frac{q^2E^2}{k-q^2D}.$$

We first solve for $\hat{r}_A$, the investment level for type one in a signalling game. Setting $f\left[\hat{r}_A\right] = 0$, we get

$$\begin{aligned}\hat{r}_A &= \frac{-qE - \sqrt{q^2E^2 + 4\left(\frac{qD}{2} - \frac{k}{2}\right)\left(\frac{1}{2}\frac{q^2E^2}{k-q^2D}\right)}}{2\left(\frac{qD}{2} - \frac{k}{2}\right)} \\ &= \frac{qE\left(1 + \sqrt{\frac{qD(1-q)}{k-q^2D}}\right)}{k-qD}.\end{aligned}$$

Proposition 3 *If* $q > \frac{\sqrt{3}}{2}$ *and* $\underline{X} < \delta < \frac{k}{2}$, *where* $\underline{X} = \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q(\mu_A+(1-\mu_A)q^2)}}$, *the second period game is in the signalling regime. Otherwise, the game is in the complete information regime.*

Proof. The game is in the signalling regime if the incentive constraint is violated at r_A^1 or if $\hat{r}_A > r_A^1$. That is, if

$$\frac{qE}{k-qD} + \frac{\sqrt{q^2E^2 - (k-qD)\left(\frac{q^2E^2}{k-q^2D}\right)}}{k-qD} > \frac{E}{k-D}.$$

After some re-arranging, the above holds if

$$q^2D^2 + kq\,(1-2q)\,D + (1-q)\,k^2 < 0.$$

This holds for

$$\begin{aligned}&\frac{kq\,(2q-1) - \sqrt{k^2q^2\,(1-2q)^2 - 4q^2\,(1-q)\,k^2}}{2q^2} \\ <\;& D < \frac{kq\,(2q-1) + \sqrt{k^2q^2\,(1-2q)^2 - 4q^2\,(1-q)\,k^2}}{2q^2}.\end{aligned}$$

Substituting in D, this holds for

$$\begin{aligned}&\frac{k\,(2q-1) - \sqrt{k^2\,(1-2q)^2 - 4\,(1-q)\,k^2}}{2q} \\ <\;& 2\frac{1}{k}\delta^2\left(\mu_A + (1-\mu_A)\,q^2\right) \\ <\;& \frac{k\,(2q-1) + \sqrt{k^2\,(1-2q)^2 - 4\,(1-q)\,k^2}}{2q},\end{aligned}$$

or

$$\frac{(2q-1)\,k^2 - k^2\sqrt{4q^2-3}}{4q\,(\mu_A + (1-\mu_A)\,q^2)} < \delta^2 < \frac{k^2\,(2q-1) + k^2\sqrt{4q^2-3}}{4q\,(\mu_A + (1-\mu_A)\,q^2)}.$$

For $q > \frac{1}{2}\sqrt{3}$, the game is in the signalling regime if

$$\frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q\,(\mu_A + (1-\mu_A)\,q^2)}} < \delta < \frac{k}{2}\sqrt{\frac{2q-1+\sqrt{4q^2-3}}{q\,(\mu_A + (1-\mu_A)\,q^2)}}.$$

Let $\underline{X} \equiv \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q(\mu_A+(1-\mu_A)q^2)}}$.

It can be verified that $\frac{k}{2}\sqrt{\frac{2q-1+\sqrt{4q^2-3}}{q(\mu_A+(1-\mu_A)q^2)}} > \frac{k}{2}$ and $\underline{X}$ may be greater or less than $\frac{k}{2}$ depending on μ_A and q. When $\underline{X} < \delta < \frac{k}{2}$,[11] the game is in the signalling regime and the equilibrium investment levels are $\left(r_A^0, \hat{r}_A\right)$. When $\delta \leq \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q(\mu_A+(1-\mu_A)q^2)}}$ or $\underline{X} > \frac{k}{2}$, the game is in the complete information regime. The equilibrium investment levels are $\left(r_A^0, r_A^1\right)$. ■

Observe that when $q = 0$, $r_A^0 = \hat{r}_A = 0$. Since it is not possible to reach the finishing line, there is no point for type zero to invest in the second period. When $q = 1$, $r_A^0 = r_A^1 = \hat{r}_A$. If the probability of advancing two steps is one, type zero and type one have the same incentives. Type zero only has the incentive to mimic type one and hence enforces type one to signal by over-investment when the probability of leapfrogging is high. From our profit setting, by convincing B that it is a type one, type zero doesn't get any profit unless it can reach the finishing line in the end of the second period. Therefore, the q required is high.

An alternative way of explaining this is that the parameter q measures how costly sending out the signal is for type zero relative to that for type one. When q is small, it is very costly for type zero to pretend to be type one, therefore the incentive to mimic is small and the complete information equilibrium prevails. For larger q, on the other hand, trying to mimic type one is not that costly. Therefore, type zero would have more incentive to mimic type one, and this makes type one having to over-invest and the game is in the signalling regime.

Given that the probability of leapfrogging is high, type zero does have the incentive to mimic the behaviour of type one. Since by being more aggressive, it reduces B's investment and increases the probability of A being the monopolist. The larger the profit difference, the more incentive for firms to invest. Therefore,

[11] Given $q > \frac{\sqrt{3}}{2}$, $\frac{k}{2} > \underline{X}$ for not too small μ_A. If $q > 0.87$, $\frac{k}{2} > \underline{X}$ for all μ_A.

we also observe that there is a lower bound for δ for the incentive constraint to be violated.

Furthermore, this lower bound depends on A's belief about B's position. Type zero of A's incentive to mimic type one depends on where B is, or, more precisely, where it believes B is in the race. As μ_A increases, $\underline{X}$ decreases and the range of δ for the signalling regime widens. This is because as μ_A decreases, it is more likely for B to be at position zero. This belief induces type one of A to increase its investment level even without signalling, that is, r_A^1 increases. This makes the condition $\hat{r}_A > r_A^1$ more difficult to be met.

Note that in both regimes, A invests more at position one regardless of B's positions. Firm B invests more if it is ahead, $r_B^{01} > r_B^{11}$, in both cases. This is a different result from Grossman and Shapiro where firms compete more vigorously when they are even. This difference comes from the set up of the model. R&D investments are strategic substitutes in this model and are strategic complements in their model.

4.3.3 R&D investments and A's prior beliefs

In the second period, given B's own position, B is always more aggressive when A does not succeed in the first period research. Unlike B who gets to see A's second period investment before its second move, A's belief about B's position, μ_A, depends only on B's first period investment. We characterise the effects of B's first period investment on A's second period behaviour in the following proposition.

Proposition 4 *As μ_A increases, both r_A^0 and r_A^1 decrease, and the effects on $\hat{r}_A$ is uncertain. Comparing two iso-profit curves for type zero of A with different μ_A, the single crossing property does not hold.*

Proof. Differentiate r_A^0 and r_A^1 with respect to μ_A,

$$\begin{aligned}
\frac{\partial r_A^0}{\partial \mu_A} &= \frac{q\pi_m \frac{1}{k}\delta\left(q^2-1\right)\left(k-\mu_A\frac{2}{k}q^2\delta^2-(1-\mu_A)\frac{2}{k}q^4\delta^2\right)}{\left(k-\mu_A\frac{2}{k}q^2\delta^2-(1-\mu_A)\frac{2}{k}q^4\delta^2\right)^2} \\
&\quad -\frac{q^3\pi_m\frac{2}{k}\delta^2\left(\mu_A\left(1-\frac{1}{k}\delta\right)+(1-\mu_A)\left(1-\frac{1}{k}q^2\delta\right)\right)\left(q^2-1\right)}{\left(k-\mu_A\frac{2}{k}q^2\delta^2-(1-\mu_A)\frac{2}{k}q^4\delta^2\right)^2} \\
&= -\frac{q\pi_m(1-q)(1+q)\delta\left(k-2q^2\delta\right)}{k\left(k-\frac{2}{k}q^2\delta^2\left(\mu_A+(1-\mu_A)q^2\right)\right)^2} \\
&< 0.
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial r_A^1}{\partial \mu_A} &= \frac{\pi_m\delta\frac{1}{k}\left(q^2-1\right)\left(k-\mu_A\frac{2}{k}\delta^2-(1-\mu_A)\frac{2}{k}q^2\delta^2\right)}{\left(k-\mu_A\frac{2}{k}\delta^2-(1-\mu_A)\frac{2}{k}q^2\delta^2\right)^2} \\
&\quad -\frac{\pi_m\delta^2\frac{2}{k}\left(\mu_A\left(1-\frac{1}{k}\delta\right)+(1-\mu_A)\left(1-\frac{1}{k}q^2\delta\right)\right)\left(q^2-1\right)}{\left(k-\mu_A\frac{2}{k}\delta^2-(1-\mu_A)\frac{2}{k}q^2\delta^2\right)^2} \\
&= -\frac{\pi_m(1-q)(1+q)\delta(k-2\delta)}{k\left(k-\frac{2}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)\right)^2} \\
&< 0.
\end{aligned}$$

For the effect of μ_A on $\hat{r}_A$, if the single crossing property holds for type zero's iso-profit curves with different μ_A, then if the slope increases as μ_A increases, $\hat{r}_A$ would increase. If, on the other hand, the slope decreases as μ_A increases, $\hat{r}_A$ would decrease. Totally differentiating Equation 4.3.1,

$$\begin{aligned}
\frac{\partial\left(\left.\frac{\partial r_{B2}}{\partial r_{A2}}\right|_{\pi_A=\overline{\pi}_A}\right)}{\partial \mu_A} &= \frac{q^2 r_{A2}\delta\left(r_{2B}(q-1)\delta\right)\left(\mu_A+(1-\mu_A)q\right)}{\left(\mu_A r_{A2}q\delta+(1-\mu_A)\left(r_{A2}q^2\delta\right)\right)^2} \\
&\quad -\frac{(1-q)\delta\left(\mu_A q\left(\pi_m-r_{B2}\delta\right)+(1-\mu_A)q\left(\pi_m-r_{B2}q\delta\right)-kr_{A2}\right)}{r_{A2}q\left(\mu_A\delta+(1-\mu_A)q\delta\right)^2}.
\end{aligned}$$

The slope for type zero of A's iso-profit curve decreases as μ_A increases and therefore the investment level decreases if $\frac{\partial\left(\left.\frac{\partial r_{B2}}{\partial r_{A2}}\right|_{\pi_A=\overline{\pi}_A}\right)}{\partial \mu_A} < 0$. That is, if

$$\begin{aligned}
& qr_{B2}\delta(q-1)\left(\mu_A+(1-\mu_A)q\right) \\
& -\left(\mu_A q\left(\pi_m-r_{B2}\delta\right)+(1-\mu_A)q\left(\pi_m-r_{B2}q\delta\right)-kr_{A2}\right)(1-q) \\
<\; & 0.
\end{aligned}$$

This is satisfied if

$$r_A < q\frac{\pi_m}{k}$$

or from the total differentiation, if

$$\frac{\frac{\partial\left(\frac{\partial \pi_A^0}{\partial r_{A2}}\right)}{\frac{\partial \pi_A^0}{\partial r_{A2}}}}{\frac{\partial \mu_A}{\mu_A}} > \frac{\frac{\partial\left(\frac{\partial \pi_A^0}{\partial r_{B2}}\right)}{\frac{\partial \pi_A^0}{\partial r_{B2}}}}{\frac{\partial \mu_A}{\mu_A}}.$$

This condition says that whether $\hat{r}_A$ increases or decreases as μ_A increases depends on the elasticity of the marginal effect of r_A to π_A and r_B to π_A with respect to μ_A. Given that $q\frac{\pi_m}{k} < 1$, the investment level increases as μ_A increases for large r_A. It is uncertain whether or not $\hat{r}_A < q\frac{\pi_m}{k}$ and hence the effect of μ_A on $\hat{r}_A$ is uncertain. ■

We have seen the reason why the effect of an increase of μ_A on $\hat{r}_A$ is uncertain. Let's try to work on the relationship between μ_A and $\hat{r}_A$ by Equation 4.3.2. Totally differentiating Equation 4.3.2 gives [12]

$$\frac{\partial f\left[r_A\right]}{\partial r_A} d\hat{r}_A + \left(\frac{\partial f\left[r_A\right]}{\partial \mu_A} + \frac{\partial f\left[r_A\right]}{\partial r_A^0}\frac{\partial r_A^0}{\partial \mu_A}\right) d\mu_A = 0.$$

The effect of μ_A on $\hat{r}_A$ is

$$\frac{d\hat{r}_A}{d\mu_A} = -\frac{\left(\frac{\partial f[r_A]}{\partial \mu_A} + \frac{\partial f[r_A]}{\partial r_A^0}\frac{\partial r_A^0}{\partial \mu_A}\right)}{\frac{\partial f[r_A]}{\partial r_A}}.$$

Since r_A^0 is the optimal response given $\mu_B = 0$, $\frac{\partial f\left[r_A, R_B^{0s}\right]}{\partial r_A^0} = 0$. It follows that

$$\frac{d\hat{r}_A}{d\mu_A} = -\frac{\frac{\partial f[\hat{r}_A]}{\partial \mu_A}}{\frac{\partial f[\hat{r}_A]}{\partial r_A}}. \tag{4.3.3}$$

Recall that $\pi_A\left[r_A^0, R_B^{0s}\right]$ is what type zero can get by choosing the optimal investment level and revealing its type. By definition of $f\left[r_A\right]$, $f\left[r_A\right] > 0$ for $r_A^0 < r_A < \hat{r}_A$. Therefore $\frac{\partial f[\hat{r}_A]}{\partial r_A} < 0$. For the numerator,

$$\begin{aligned}\frac{\partial f\left[r_A\right]}{\partial \mu_A} &= q\frac{1}{k}\delta^2\left(r_A^2 - \left(r_A^0\right)^2 q\right)\left(1-q^2\right) \\ &\quad - q\frac{1}{k}\pi_m\delta\left(r_A - r_A^0\right)\left(1-q^2\right).\end{aligned}$$

The derivative $\frac{\partial f(\hat{r}_A)}{\partial \mu_A} < 0$ if

$$\delta\left(\hat{r}_A^2 - \left(r_A^0\right)^2 q\right) < \pi_m\left(\hat{r}_A - r_A^0\right).$$

[12] The derivative of $\hat{r}_A$ with respect to μ_A is complicated. The expression is in the appendix and is used for numerical simulations.

or if

$$\frac{\delta}{\pi_m} < \frac{\left(\hat{r}_A - r_A^0\right)}{\left(\hat{r}_A^2 - \left(r_A^0\right)^2 q\right)}.$$

Alternatively, by solving the quadratic function, $\frac{\partial f(\hat{r}_A)}{\partial \mu_A} < 0$ if

$$\frac{\pi_m - \sqrt{\pi_m^2 - 4\left(\pi_m \delta r_A^0 - q\delta^2 \left(r_A^0\right)^2\right)}}{2\delta} < \hat{r}_A < \frac{\pi_m + \sqrt{\pi_m^2 - 4\left(\pi_m \delta r_A^0 - q\delta^2 \left(r_A^0\right)^2\right)}}{2\delta}.$$

Denote these two investment levels as $\underline{r}$ and $\bar{r}$.

Proposition 5 *For $q > 0.92$, $\underline{X} < \delta < \frac{k}{4q}$, $\hat{r}_A$ decreases as μ_A increases. The effect of μ_A on $\hat{r}_A$ is uncertain outside this parameter range.*

Proof. Note that $\frac{\pi_m}{2\delta} = \frac{\underline{r}+\bar{r}}{2}$ and $\underline{r} < \frac{\pi_m}{2\delta} < \bar{r}$. We prove this proposition in two steps. First we show that for $q > 0.92$ and $\underline{X} < \delta < \frac{k}{4q}$, $\frac{\pi_m}{2\delta} > \hat{r}_A$. The second step is to show that for $\delta < \frac{k}{4q}$, $\hat{r}_A > r_A^1 > \underline{r}$. Combining the two parts gives the sufficient condition for $\underline{r} < \hat{r}_A < \bar{r}$ or $\frac{\partial f(\hat{r}_A)}{\partial \mu_A} < 0$.

Part one:

$$\frac{\pi_m}{2\delta} > \hat{r}_A$$

if

$$\frac{\pi_m}{2\delta} > \frac{qE + qE\sqrt{\frac{qD(1-q)}{k-q^2D}}}{k - qD}.$$

Substituting in E and D and re-arrange, this holds if

$$\frac{k - 2q\delta}{2\delta\left(k - 2q\frac{1}{k}\delta^2\left(\mu_A + (1-\mu_A)q^2\right)\right)} > \frac{q\left(1 - \mu_A\frac{1}{k}\delta - (1-\mu_A)\frac{q^2}{k}\delta\right)\sqrt{\frac{2q(1-q)\frac{1}{k}\delta^2(\mu_A+(1-\mu_A)q^2)}{k-2q^2\frac{1}{k}\delta^2(\mu_A+(1-\mu_A)q^2)}}}{k - 2q\frac{1}{k}\delta^2\left(\mu_A + (1-\mu_A)q^2\right)}.$$

Since both sides are positive, after some rearrangement, this holds if

$$\left(\frac{k-2q\delta}{2q\delta\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)}\right)^2 > \frac{2q(1-q)\frac{1}{k}\delta^2\left(\mu_A + (1-\mu_A)q^2\right)}{k - 2q^2\frac{1}{k}\delta^2\left(\mu_A + (1-\mu_A)q^2\right)}.$$

Expanding the LHS, the inequality holds if

$$\begin{aligned}
&\frac{k^2}{4q^2\delta^2\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)^2}-\frac{k}{q\delta\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)^2}\\
&+\frac{1}{\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)^2}-\frac{2q(1-q)\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)}{k-q^2 2\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)}\\
> \; & 0.
\end{aligned}$$

Taking the first two terms,

$$\frac{k^2}{4q^2\delta^2\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)^2}>\frac{k}{q\delta\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)^2}$$

holds if

$$\frac{k}{4q}>\delta.$$

Given the constraints $0<q<1$ and $0<\mu_A<1$, the LHS falls into the relevant range,$\frac{k}{4q}>\underline{X}$, for $q>0.92$.

The remaining two terms,

$$\frac{1}{\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)^2}-\frac{2q(1-q)\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)}{k-q^2 2\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)}>0,$$

if

$$\frac{k-q^2 2\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)}{\left(1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta\right)^2}>2q(1-q)\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right).$$

Since $0<1-\mu_A\frac{1}{k}\delta-(1-\mu_A)\frac{q^2}{k}\delta<1$, the above holds if

$$k-q^2 2\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)>2q(1-q)\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right).$$

This holds for

$$\frac{k^2}{\left(2q(1-q)\left(\mu_A+(1-\mu_A)q^2\right)+2q^2\left(\mu_A+(1-\mu_A)q^2\right)\right)}>\delta^2.$$

This condition always holds since $2>q\left(\mu_A+(1-\mu_A)q^2\right)$. Therefore for $q>0.92$, $\underline{X}<\delta<\frac{k}{4q}$, $\frac{\pi_m}{2\delta}>\hat{r}_A$ and $\hat{r}_A<\bar{r}$.

Part two:

$$r_A^1>\underline{r}$$

if

$$\frac{E}{(k-D)} > \frac{\pi_m - \sqrt{\pi_m^2 - 4\left(\pi_m \delta r_A^0 - q\delta^2 \left(r_A^0\right)^2\right)}}{2\delta}.$$

Substituting in E and D and re-arrange, the inequality holds if

$$\frac{\sqrt{1-4q\left(\frac{\delta\left(1-\frac{1}{k}(\pi_m-\pi_d)(\mu_A+(1-\mu_A)q^2)\right)}{(k-q^2D)} - \left(\frac{q\delta\left(1-\frac{1}{k}(\pi_m-\pi_d)(\mu_A+(1-\mu_A)q^2)\right)}{(k-q^2D)}\right)^2\right)}}{2\delta}$$

$$> \frac{1}{2\delta} - \frac{\left(1-\frac{1}{k}\delta\left(\mu_A+(1-\mu_A)q^2\right)\right)}{\left(k-2\frac{1}{k}\delta^2\left(\mu_A+(1-\mu_A)q^2\right)\right)}.$$

After some re-arrangement, this holds if

$$\delta < \frac{-\left(q^2+q+1\right)+\sqrt{\left(q^2+q+1\right)^2+4\left(\mu_A+(1-\mu_A)q^2\right)\left(q^2+q+1\right)}}{2\frac{1}{k}\left(\mu_A+(1-\mu_A)q^2\right)\left(q^2+q+1\right)}$$

In can be verified that

$$\frac{-\left(q^2+q+1\right)+\sqrt{\left(q^2+q+1\right)^2+4\left(\mu_A+(1-\mu_A)q^2\right)\left(q^2+q+1\right)}}{2\frac{1}{k}\left(\mu_A+(1-\mu_A)q^2\right)\left(q^2+q+1\right)}$$

$$> \frac{k}{4q}.$$

From Proposition 3, $\hat{r}_A > r_A^1$ for the given parameter range and therefore $\hat{r}_A > r_A^1 > \underline{r}$. Combining the two steps, we have $\bar{r} > \hat{r}_A > r_A^1 > \underline{r}$ and $\frac{\partial f(\hat{r}_A)}{\partial \mu_A} < 0$. Therefore $\frac{d\hat{r}_A}{d\mu_A} < 0$. ■

Analytically, we have demonstrated that $\frac{d\hat{r}_A}{d\mu_A} < 0$ for $q > 0.92$ and not too large δ. We carry out some simulation exercises and the results suggest that for most of the parameter range, $\frac{d\hat{r}_A}{d\mu_A} < 0$ holds as long as δ is not very close to $\frac{k}{2}$. The results of the simulation are depicted in Figure 6 with $\frac{d\hat{r}_A}{d\mu_A}$ plotted against δ. Four different combinations of μ_A and q are examined.

Claim 1 *For large q or small μ_A, it is more possible for $\hat{r}_A$ to increase as μ_A increases. For large q and δ, $\hat{r}_A$ can increase as μ_A increases. $\frac{d\hat{r}_A}{d\mu_A}$ can be positive when δ is large.*

Intuitively, type zero has more incentive to mimic the investment behaviour of type one when the probability of leapfrogging is high and when the probability of B being a type zero is high. That is, when q is high or when μ_A is low, type one has to invest more to separate itself from type zero and $\hat{r}_A$ increases. Since μ_A depends on r_{B1}, as long as the game stays in the signalling regime, B may push up the investment level for type one to signal its position by under-investing in the first period.

Figure 5: Plot $\frac{d\hat{r}_A}{d\mu_a}$ against δ for various parameter combinations. In the horizontal axis, δ is represented as a fraction of $\frac{k}{2}$. For example, 1 stands for $\sigma = \frac{k}{2}$, which is the upper boun, and 0.54 stands for $\delta = 0.54\frac{k}{2}$. The simulation results given in the Figure is for $k = 1$.

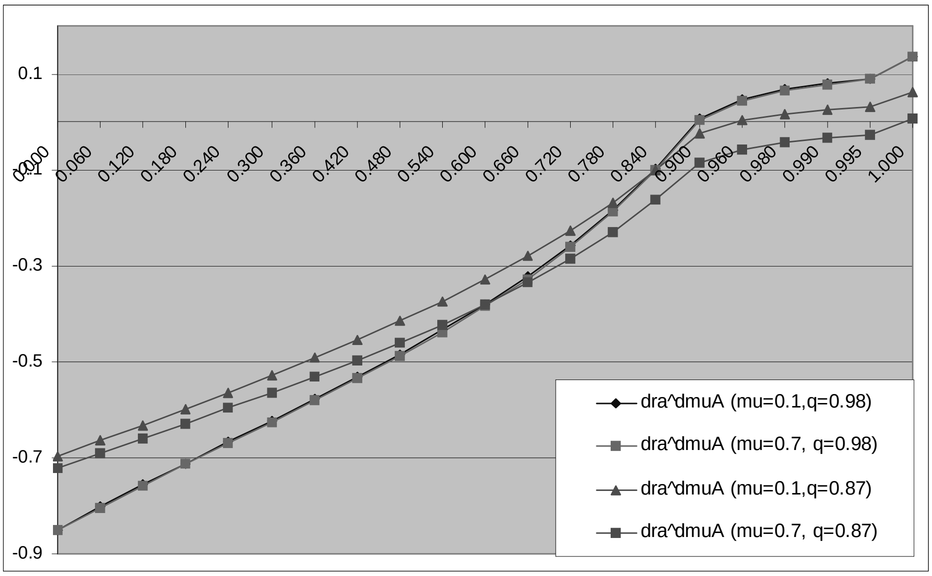

Figure 6: Plot $\frac{d\hat{r}_A}{d\mu_A}$ against δ for various parameter combinations. In the horizontal axis, δ is represented as a fraction of $\frac{k}{2}$. For example, 1 stands for $\delta = \frac{k}{2}$, which is the upper bound, and 0.54 stands for $\delta = 0.54\frac{k}{2}$. The simulation results given in the Figure is for $k = 1$.

5 First Period Investments

We have seen that a firm's second period behaviour depends on its and the rival's first period positions. In this section, we carry out the analysis on the first period R&D investment. Note that in the first period, both firms start at position zero, there is no signalling component in their investment. In the first period, after A's move, B forms a belief about A's position μ_{B1}. Since with investment r_{A1}, A moves to position one with probability r_{A1}, $\mu_{B1} = r_{A1}$.

Firm B's expected profit at the beginning of the first period is that with probability $r_{A1}r_{B1}$, both firms would advance to position one and B's profit is π_B^{11}. Similarly, B gets π_B^{01} with probability $(1 - r_{A1})\, r_{B1}$, π_B^{10} with probability $r_{A1}(1 - r_{B1})$, and π_B^{00} with probability $(1 - r_{A1})(1 - r_{B1})$. Since A's investment differs in the second period depending on if it is in the complete information regime or in the signalling regime, B's expected payoffs are different in the two game regimes. If the second period game is a signalling game and type one of A invests $\hat{r}_A$, firm B's expected payoff is

$$\begin{aligned}
\pi_{B1}[., \hat{r}_A] &= r_{A1}r_{B1}\left(r_{B2}^{11}[\hat{r}_A](\pi_m - \hat{r}_A\delta) - \frac{k\left(r_{B2}^{11}[\hat{r}_A]\right)^2}{2}\right) \\
&\quad + (1 - r_{A1})\, r_{B1}\left(r_{B2}^{01}\left[r_A^0\right]\left(\pi_m - qr_A^0\delta\right) - \frac{k\left(r_{B2}^{01}\left[r_A^0\right]\right)^2}{2}\right) \\
&\quad + r_{A1}(1 - r_{B1})\left(q^2 r_{B2}^{11}[\hat{r}_A](\pi_m - \hat{r}_A\delta) - \frac{k\left(qr_{B2}^{11}[\hat{r}_A]\right)^2}{2}\right) \\
&\quad + (1 - r_{A1})(1 - r_{B1})\left(q^2 r_{B2}^{01}\left[r_A^0\right]\left(\pi_m - qr_A^0\delta\right) - \frac{k\left(qr_{B2}^{01}\left[r_A^0\right]\right)^2}{2}\right) \\
&\quad - \frac{k(r_{B1})^2}{2} \\
&= r_{A1}\left(r_{B1} + q^2(1 - r_{B1})\right)\frac{1}{2k}(\pi_m - \hat{r}_A\delta)^2 \\
&\quad + (1 - r_{A1})\left(r_{B1} + q^2(1 - r_{B1})\right)\frac{1}{2k}\left(\pi_m - r_A^0 q\delta\right)^2 - \frac{k(r_{B1})^2}{2}.
\end{aligned}$$

Similarly, in the complete information regime, type zero and type one of A invest r_A^0 and r_A^1 respectively and B's expected profit at the beginning of the first period is

$$\begin{aligned}\pi_{B1}\left[.,r_A^1\right] &= r_{A1}\left(r_{B1}+q^2\left(1-r_{B1}\right)\right)\frac{1}{2k}\left(\pi_m-r_A^1\delta\right)^2\\&\quad+\left(1-r_{A1}\right)\left(r_{B1}+q^2\left(1-r_{B1}\right)\right)\frac{1}{2k}\left(\pi_m-qr_A^0\delta\right)^2-\frac{k\left(r_{B1}\right)^2}{2}.\end{aligned}$$

Proposition 6 *For any given r_{A1} and r_{B1}, B's profit level in the complete information regime is always greater than or equal to its profit in the signalling regime. It follows that, for any given r_{A1}, the optimal profit level in the complete information regime is no less than the optimal profit level in the signalling regime, that is, $\pi_{B1}^*\left[.,\hat{r}_A\right]\leq\pi_{B1}^*\left[.,r_A^1\right]$.*

Proof. From the profit functions, for any given $r_{A1}$and r_{B1}, since

$$\left(\pi_m-\hat{r}_A\delta\right)^2\leq\left(\pi_m-r_A^1\delta\right)^2,$$

we have $\pi_{B1}\left[r_{B1},\hat{r}_A\right]\leq\pi_{B1}\left[r_{B1},r_A^1\right]$.

It follows that

$$\pi_{B1}^*\left[r_{B1}^*\left[\hat{r}_A\right],\hat{r}_A\right]\leq\pi_{B1}\left[r_{B1}^*\left[\hat{r}_A\right],r_A^1\right].$$

By definition,

$$\pi_{B1}^*\left[r_{B1}^*\left[r_A^1\right],r_A^1\right]\geq\pi_{B1}\left[r_{B1}^*\left[\hat{r}_A\right],r_A^1\right].$$

Therefore

$$\pi_{B1}^*\left[r_{B1}^*\left[r_A^1\right],r_A^1\right]\geq\pi_{B1}^*\left[r_{B1}^*\left[\hat{r}_A\right],\hat{r}_A\right].$$

The equality holds when $r_A^1=\hat{r}_A$. ■

Firm B enjoys higher profit if the second period game is in the complete information regime. Since the lower bound for the profit difference for the signalling regime depends on r_{B1}, B may have an incentive to push the game into the complete information regime by under-investing in R&D.

Differentiating B's profit function with respect to r_{B1} in the two regimes,

$$\begin{aligned}\frac{\partial\pi_{B1}}{\partial r_{B1}}\left[.,\hat{r}_A\right] &= r_{A1}\left(1-q^2\right)\frac{1}{2k}\left(\pi_m-\hat{r}_A\delta\right)^2\\&\quad-r_{A1}\left(r_{B1}+q^2\left(1-r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m-\hat{r}_A\delta\right)\frac{\partial\hat{r}_A}{\partial r_{B1}}\\&\quad+\left(1-r_{A1}\right)\left(1-q^2\right)\frac{1}{2k}\left(\pi_m-qr_A^0\delta\right)^2\\&\quad-\left(1-r_{A1}\right)\left(r_{B1}+q^2\left(1-r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m-qr_A^0\delta\right)q\frac{\partial r_A^0}{\partial r_{B1}}\\&\quad-kr_{B1},\end{aligned}\tag{5.0.4}$$

for the signalling regime, and

$$\begin{aligned}
\frac{\partial \pi_{B1}}{\partial r_{B1}} \quad & \left[.,r_A^1\right] = r_{A1}\left(1-q^2\right)\frac{1}{2k}\left(\pi_m - r_A^1\delta\right)^2 \\
& -r_{A1}\left(r_{B1}+q^2\left(1-r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m - \hat{r}_A\delta\right)\frac{\partial r_A^1}{\partial r_{B1}} \\
& +\left(1-r_{A1}\right)\left(1-q^2\right)\frac{1}{2k}\left(\pi_m - q r_A^0\delta\right)^2 \\
& -\left(1-r_{A1}\right)\left(r_{B1}+q^2\left(1-r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m - q r_A^0\delta\right) q\frac{\partial r_A^0}{\partial r_{B1}} \\
& -k r_{B1}. \qquad (5.0.5)
\end{aligned}$$

for the complete information regime. For given r_{A1}, we can guarantee $r_{B1}^*\left[r_A^1\right] \geq r_{B1}^*\left[\hat{r}_A\right]$ if

$$\frac{\partial \pi_{B1}}{\partial r_{B1}}\left[.,\hat{r}_A\right] \leq \frac{\partial \pi_{B1}}{\partial r_{B1}}\left[.,r_A^1\right].$$

The inequality holds if

$$\begin{aligned}
& \left(r_{A1}-q^2 r_{A1}\right)\frac{1}{2k}\left(\pi_m - \hat{r}_A\delta\right)^2 \\
& -\left(r_{A1}r_{B1}+q^2 r_{A1}\left(1-r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m - \hat{r}_A\delta\right)\frac{\partial \hat{r}_A}{\partial r_{B1}} \\
\leq\ & \left(r_{A1}-q^2 r_{A1}\right)\frac{1}{2k}\left(\pi_m - r_A^1\delta\right)^2 \\
& -\left(r_{A1}r_{B1}+q^2 r_{A1}\left(1-r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m - r_A^1\delta\right)\frac{\partial r_A^1}{\partial r_{B1}}. \qquad (5.0.6)
\end{aligned}$$

Lemma 1 *If* $\frac{\partial \hat{r}_A}{\partial r_{B1}} > \frac{\partial r_A^1}{\partial r_{B1}}$*, then* $\frac{\partial \pi_{B1}}{\partial r_{B1}}\left[.,\hat{r}_A\right] < \frac{\partial \pi_{B1}}{\partial r_{B1}}\left[.,r_A^1\right]$ *and* $r_{B1}^*\left[\hat{r}_A\right] < r_{B1}^*\left[r_A^1\right]$*. If* $\frac{\partial \hat{r}_A}{\partial r_{B1}} < \frac{\partial r_A^1}{\partial r_{B1}}$*, the sign is uncertain.*

The choice of r_{B1} is influenced by three effects. The first one is the investment effect. When anticipating a higher r_{A2}, the probability of getting the monopoly profit decreases, and r_{B1} decreases. This effect says that B should invest more in the first period if the second period competition is in the complete information regime. The second effect is the strategic effect of r_{B1} on r_{A2}. If as r_{B1} increases, r_{A2} decreases, B should invest more to discourage A's second period investment. The third effect is the effect of r_B on the realisation of the second period game regime which we would discuss in the next section. Take the second game regime as given for now, if $\frac{\partial \hat{r}_A}{\partial r_{B1}} > \frac{\partial r_A^1}{\partial r_{B1}}$, the strategic effect says that as r_{B1} increases, $\hat{r}_A$ increase or decreases less then the decrease of r_A^1. The investment and strategic effects both suggest that $r_{B1}^*[\hat{r}_A] < r_{B1}^*[r_A^1]$. That is, B is more aggressive in the first period if the second period game is in the complete information regime.

If $\frac{\partial \hat{r}_A}{\partial r_{B1}} < \frac{\partial r_A^1}{\partial r_{B1}}$, due to the investment effect, B would invest less when A invests $\hat{r}_A$ in the second period. However, by increasing r_{B1}, $\hat{r}_A$ decreases more than r_A^1 does. The strategic effect induces B to invest more in the first period. The overall effect is uncertain. We can show that as long as $\frac{\partial \hat{r}_A}{\partial r_{B1}}$ is not too small, the investment effect dominates the strategic effect, and B is more aggressive in the first period when it anticipates that the second period game is in the complete information regime.

Lemma 2 *If*

$$\frac{\partial \hat{r}_A}{\partial r_{B1}} > -\frac{\left(1-q^2\right)\left(\left(2\pi_m - \delta\left(r_A^1 + \hat{r}_A\right)\right)\right)\left(\hat{r}_A - r_A^1\right)}{2\left(r_{B1} + q^2\left(1 - r_{B1}\right)\right)\left(\pi_m - \hat{r}_A\delta\right)} + \frac{\left(\pi_m - r_A^1\delta\right)}{\left(\pi_m - \hat{r}_A\delta\right)}\frac{\partial r_A^1}{\partial r_{B1}},$$

then $\frac{\partial \pi_{B1}}{\partial r_{B1}}[.,r_A^1] > \frac{\partial \pi_{B1}}{\partial r_{B1}}[.,\hat{r}_A]$ *and* $r_{B1}^*[r_A^1] > r_{B1}^*[\hat{r}_A]$. *Firm B is more aggressive in the first period if the second period game is in the complete information regime.*

Proof. Note that

$$-\frac{\left(1-q^2\right)\left(\left(2\pi_m - \delta\left(r_A^1 + \hat{r}_A\right)\right)\right)\left(\hat{r}_A - r_A^1\right)}{2\left(r_{B1} + q^2\left(1 - r_{B1}\right)\right)\left(\pi_m - \hat{r}_A\delta\right)} + \frac{\left(\pi_m - r_A^1\delta\right)}{\left(\pi_m - \hat{r}_A\delta\right)}\frac{\partial r_A^1}{\partial r_{B1}} < \frac{\partial r_A^1}{\partial r_{B1}}.$$

From Equation 5.0.6, $r_{B1}^*[r_A^1] > r_{B1}^*[\hat{r}_A]$ if

$$\begin{aligned}
& -r_{A1}\left(r_{B1} + q^2\left(1 - r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m - \hat{r}_A\delta\right)\frac{\partial \hat{r}_A}{\partial r_{B1}} \\
< \;& r_{A1}\left(1-q^2\right)\frac{1}{2k}\left(\left(\pi_m - r_A^1\delta\right)^2 - \left(\pi_m - \hat{r}_A\delta\right)^2\right) \\
& -r_{A1}\left(r_{B1} + q^2\left(1 - r_{B1}\right)\right)\frac{1}{k}\delta\left(\pi_m - r_A^1\delta\right)\frac{\partial r_A^1}{\partial r_{B1}}.
\end{aligned}$$

After some re-arrangement, this holds if

$$\begin{aligned}
\frac{\partial \hat{r}_A}{\partial r_{B1}} \;>\; & -\frac{\left(1-q^2\right)\left(\left(2\pi_m - \delta\left(r_A^1 + \hat{r}_A\right)\right)\right)\left(\hat{r}_A - r_A^1\right)}{2\left(r_{B1} + q^2\left(1 - r_{B1}\right)\right)\left(\pi_m - \hat{r}_A\delta\right)} \\
& + \frac{\left(\pi_m - r_A^1\delta\right)}{\left(\pi_m - \hat{r}_A\delta\right)}\frac{\partial r_A^1}{\partial r_{B1}}.
\end{aligned}$$

■

We cannot work out analytical conditions on primitives for this inequality to hold. Simulation results show that this holds for most cases when $k > q$ and when δ is not very close to $\frac{k}{2}$.

Claim 2 *Simulations show that B's first period best response is downward sloping in* r_{A1}*, and the investment levels are small for most of the parameter values.*

This may be due to two effects. The first one is B's incentive to push the game into the complete information regime for the second period by under-investment. The second one is that due to the high probability of leapfrogging, the first period success is not that critical.

5.1 Firm B's ability to affect the regime of the second period game

The only endogenous variable that B can choose to affect the second game regime is r_{B1}, through the effects on the lower bar for the incentive constraint to be violated at $\left(r_A^0, r_A^1\right)$, $\underline{X}$. Recall that the second period game will be in the signalling regime, that is $\hat{r}_A > r_A^1$, if $\delta > \underline{X}$. Since in the first period, by investing r_{B1}, with probability r_{B1}, B advances to position one. In the second period, A should assess that with probability r_{B1}, B is at position one. That is, A's second period belief, μ_A, is equal to r_{B1}. Substitute μ_A for r_{B1} into $\underline{X}$ gives the condition for the second period game to be in the signalling regime:

$$\delta > \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q\left(r_{B1}+\left(1-r_{B1}\right)q^2\right)}}.$$

With some rearrangements, for $q > \frac{\sqrt{3}}{2}$, this condition holds if

$$r_{B1} > \frac{\left(2q-1-\sqrt{4q^2-3}\right)k^2}{4q\left(1-q^2\right)\delta^2} - \frac{q^2}{\left(1-q^2\right)}.$$

Let r_{B1}^C denote this critical investment level. If B wants to push the game into the complete information regime, the only instrument available to it is to under-invest, that is, to invest less than r_{B1}^C. At $r_{B1} = r_{B1}^C$, $r_A^1 = \hat{r}_A$, and $\pi_{B1}\left[., r_A^1\right] = \pi_{B1}\left[., \hat{r}_A\right]$.

Firm B's profit is

$$\pi_{B1} = \begin{cases} \pi_{B1}\left[., r_A^1\right] & \text{if } r_{B1} \leq r_{B1}^C \\ \pi_{B1}\left[., \hat{r}_A\right] & \text{if } r_{B1} \geq r_{B1}^C \end{cases}.$$

It can be verified that r_{B1}^C is always less than one for the relevant parameter range and $r_{B1}^C < 0$ if

$$\delta^2 > \frac{\left(2q - 1 - \sqrt{4q^2 - 3}\right) k^2}{4q^3}.$$

For large δ, the game is in the signalling regime. Otherwise, r_{B1} can impact on the choices of the game regime. We divide the game regimes according to the profit differences as follows:

$$\begin{cases} \delta < \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q(r_{B1}+(1-r_{B1})q^2)}} & \text{Complete information regime.} \\ \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q(r_{B1}+(1-r_{B1})q^2)}} < \delta < \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q^3}} & \text{Signalling regime.} \\ \frac{k}{2}\sqrt{\frac{2q-1-\sqrt{4q^2-3}}{q^3}} < \delta & \text{Signalling regime with } r_{B1}^C < 0. \end{cases}$$

Proposition 7 *If $r_{B1}^* < r_{B1}^C$, then $r_{B1}^* = r_{B1}^*\left[r_A^1\right]$, and the second period game is in the complete information regime. If $r_{B1}^* > r_{B1}^C$, then $r_{B1}^* = r_{B1}^*\left[\hat{r}_A\right]$, and the second period game is in the signalling regime.*

Proof. By Proposition 3, when $\delta = \underline{X}$, $r_A^1 = \hat{r}_A$. Therefore $\pi_{B1}\left[r_A^1\right] \geq \pi_{B1}\left[\hat{r}_A\right]$ with the equality holding when $r_{B1} = r_{B1}^C$. Given that the profit functions are continuous for both cases, $\pi_{B1}\left[r_A^1\right]$ and $\pi_{B1}\left[\hat{r}_A\right]$ are tangent to each other at r_{B1}^C and $\left.\frac{\partial \pi_{B1}\left[r_A^1\right]}{\partial r_{B1}}\right|_{r_{B1}^C} = \left.\frac{\partial \pi_{B1}\left[\hat{r}_A\right]}{\partial r_{B1}}\right|_{r_{B1}^C}$. If $\left.\frac{\partial \pi_{B1}\left[r_A^1\right]}{\partial r_{B1}}\right|_{r_{B1}^C} = \left.\frac{\partial \pi_{B1}\left[\hat{r}_A\right]}{\partial r_{B1}}\right|_{r_{B1}^C} < 0$, r_{B1}^C is greater than the optimal investment levels for both regimes. Since for $r_{B1} < r_{B1}^C$, the game falls into the complete information regime and $r_{B1}^* = r_{B1}^*\left[r_A^1\right]$. If $\left.\frac{\partial \pi_{B1}\left[r_A^1\right]}{\partial r_{B1}}\right|_{r_{B1}^C} = \left.\frac{\partial \pi_{B1}\left[\hat{r}_A\right]}{\partial r_{B1}}\right|_{r_{B1}^C} > 0$, r_{B1}^C is less than the optimal investment levels for both regimes. Since $r_{B1} > r_{B1}^C$, the game falls into the signalling regime and $\pi_B\left[r_{B1}^*\left[\hat{r}_A\right]\right] \geq \pi_B\left[r_{B1}^C\right]$. The optimal investment is $r_{B1}^* = r_{B1}^*\left[\hat{r}_A\right]$. ■

The two cases are depicted in the following diagrams.[13]

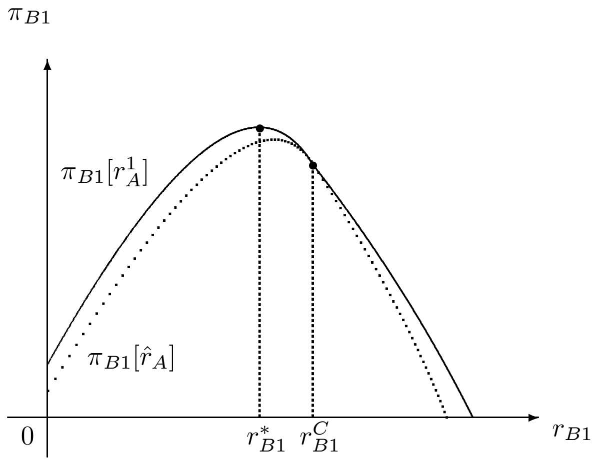

Figure 7: When $\left.\frac{\partial \pi_{B1}}{\partial r_{B1}}\right|_{r_{B1}=r^C_{B1}} < 0$, $r^*_{B1} = r^*_{B1}[r^1_A]$.

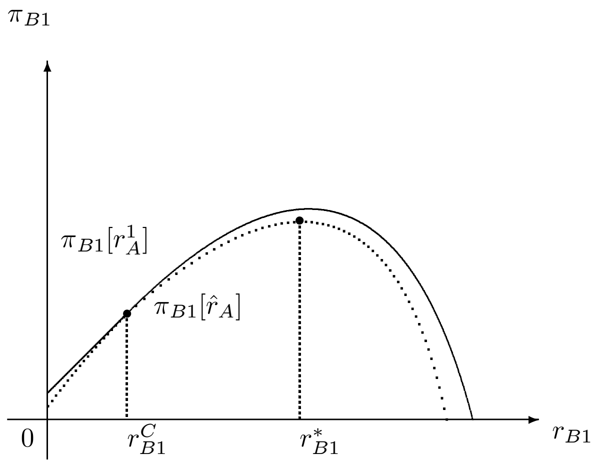

Figure 8: When $\left.\frac{\partial \pi_{B1}}{\partial r_{B1}}\right|_{r_{B1}=r^C_{B1}} > 0$, $r^*_{B1} = r^*_{B1}[\hat{r}_A]$.

Depending on the derivative of B's profit function with respect to r_{B1} at r^C_{B1}, we can know the game regime for the second period. In the following proposition, we show that the game falls into a complete information regime if r_{A1} is sufficiently small.

Proposition 8 *The derivative,* $\left.\frac{\partial \pi_{B1}\left[r^1_A\right]}{\partial r_{B1}}\right|_{r^C_{B1}} = \left.\frac{\partial \pi_{B1}\left[\hat{r}_A\right]}{\partial r_{B1}}\right|_{r^C_{B1}} < 0$, *and the game falls into the complete information regime with* $r^*_{B1} = r^*_{B1}\left[r^1_A\right]$ *if* $r_{A1} < \frac{M}{N}$, *defined below. The derivative,* $\left.\frac{\partial \pi_{B1}\left[r^1_A\right]}{\partial r_{B1}}\right|_{r^C_{B1}} = \left.\frac{\partial \pi_{B1}\left[\hat{r}_A\right]}{\partial r_{B1}}\right|_{r^C_{B1}} > 0$, *and the game falls into the signalling regime with* $r^*_{B1} = r^*_{B1}\left[\hat{r}_A\right]$ *if* $r_{A1} > \frac{M}{N}$. *When* $r_{A1} = \frac{M}{N}$, $\left.\frac{\partial \pi_{B1}\left[r^1_A\right]}{\partial r_{B1}}\right|_{r^C_{B1}} = \left.\frac{\partial \pi_{B1}\left[\hat{r}_A\right]}{\partial r_{B1}}\right|_{r^C_{B1}} = 0$ *and* $r^*_{B1} = r^C_{B1}$.

Proof. From Proposition 7, to show that the game falls into the complete information regime with $r^*_{B1} = r^*_{B1}\left[r^1_A\right]$, it suffices to show that $\left.\frac{\partial \pi_{B1}\left[r^1_A\right]}{\partial r_{B1}}\right|_{r^C_{B1}} < 0$. Substituting r^C_{B1} into Equation 5.0.5 gives

$$\begin{aligned}
\left.\frac{\partial \pi_{B1}}{\partial r_{B1}}\left[r^1_A\right]\right|_{r^C_{B1}} &= \left(1-q^2\right) r_{A1}\frac{1}{2k}\left(\pi_m - r^1_A\delta\right)^2 \\
&\quad - \left(\left(\frac{\left(2q-1-\sqrt{4q^2-3}\right)k^2}{4q(1-q^2)\delta^2} - \frac{q^2}{(1-q^2)}\right)\left(1-q^2\right)+q^2\right) r_{A1}\frac{1}{k}\delta\left(\pi_m - r^1_A\delta\right)\frac{\partial r^1_A}{\partial r_{B1}} \\
&\quad + \left(1-r_{A1}\right)\left(1-q^2\right)\frac{1}{2k}\left(\pi_m - q r^0_A\delta\right)^2 \\
&\quad - \left(1-r_{A1}\right)\left(\left(\frac{\left(2q-1-\sqrt{4q^2-3}\right)k^2}{4q(1-q^2)\delta^2} - \frac{q^2}{(1-q^2)}\right)\left(1-q^2\right)+q^2\right)\frac{q\delta\left(\pi_m - q r^0_A\delta\right)}{k}\frac{\partial r^0_A}{\partial r_{B1}} \\
&\quad - k\left(\frac{\left(2q-1-\sqrt{4q^2-3}\right)k^2}{4q\left(1-q^2\right)\delta^2} - \frac{q^2}{\left(1-q^2\right)}\right).
\end{aligned}$$

[13] The concave shape for the profit functions is verified in our simulation exercises.

With some re-arrangements,

$$\begin{aligned}
\left.\frac{\partial \pi_{B1}}{\partial r_{B1}}\left[r_A^1\right]\right|_{r_{B1}^C} &= \left(1-q^2\right) r_{A1} \frac{1}{2k}\left(\pi_m - r_A^1\delta\right)^2 \\
&\quad - \frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\delta} r_{A1}\left(\pi_m - r_A^1\delta\right)\frac{\partial r_A^1}{\partial r_{B1}} \\
&\quad + \left(1-r_{A1}\right)\left(1-q^2\right)\frac{1}{2k}\left(\pi_m - q r_A^0\delta\right)^2 \\
&\quad - \left(1-r_{A1}\right)\frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\delta}\left(\pi_m - q r_A^0\delta\right) q\frac{\partial r_A^0}{\partial r_{B1}} \\
&\quad - \frac{\left(2q-1-\sqrt{4q^2-3}\right)k^3}{4q\left(1-q^2\right)\delta^2} + \frac{q^2 k}{\left(1-q^2\right)}.
\end{aligned}$$

The condition $\left.\frac{\partial \pi_{B1}}{\partial r_{B1}}\left[.,r_A^1\right]\right|_{r_{B1}=r_{B1}^C} < 0$ holds if

$$\begin{aligned}
& \left(1-q^2\right) r_{A1} \frac{1}{2k}\left(\pi_m - r_A^1\delta\right)^2 \\
& - \frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\delta} r_{A1}\left(\pi_m - r_A^1\delta\right)\frac{\partial r_A^1}{\partial r_{B1}} \\
& - r_{A1}\left(1-q^2\right)\frac{1}{2k}\left(\pi_m - q r_A^0\delta\right)^2 \\
& + r_{A1}\frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\delta}\left(\pi_m - q r_A^0\delta\right) q\frac{\partial r_A^0}{\partial r_{B1}} \\
< \; & \frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\delta}\left(\pi_m - q r_A^0\delta\right) q\frac{\partial r_A^0}{\partial r_{B1}} \\
& - \left(1-q^2\right)\frac{1}{2k}\left(\pi_m - q r_A^0\delta\right)^2 \\
& + \frac{\left(2q-1-\sqrt{4q^2-3}\right)k^3}{4q\left(1-q^2\right)\delta^2} - \frac{q^2 k}{\left(1-q^2\right)}.
\end{aligned}$$

This holds if $r_{A1} < \frac{M}{N}$ where

$$\begin{aligned}
M &= \frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\left(\pi_m-\pi_d\right)}\left(\pi_m - q r_A^0\delta\right) q\frac{\partial r_A^0}{\partial r_{B1}} - \left(1-q^2\right)\frac{1}{2k}\left(\pi_m - q r_A^0\delta\right)^2 \\
&\quad + \frac{\left(2q-1-\sqrt{4q^2-3}\right)k^3}{4q\left(1-q^2\right)\delta^2} - \frac{q^2 k}{\left(1-q^2\right)},
\end{aligned}$$

and

$$
\begin{aligned}
N &= \left(1-q^2\right)\frac{1}{2k}\left(\pi_m - r_A^1\delta\right)^2 - \frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\left(\pi_m-\pi_d\right)}\left(\pi_m - r_A^1\delta\right)\frac{\partial r_A^1}{\partial r_{B1}} \\
&\quad -\left(1-q^2\right)\frac{1}{2k}\left(\pi_m - qr_A^0\delta\right)^2 + \frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\left(\pi_m-\pi_d\right)}\left(\pi_m - qr_A^0\delta\right)q\frac{\partial r_A^0}{\partial r_{B1}} \\
&= \left(1-q^2\right)\frac{1}{2k}\left(2\pi_m - \delta\left(r_A^1 + qr_A^0\right)\right)\delta\left(qr_A^0 - r_A^1\right) \\
&\quad + \frac{\left(2q-1-\sqrt{4q^2-3}\right)k}{4q\delta}\left(\pi_m\left(q\frac{\partial r_A^0}{\partial r_{B1}} - \frac{\partial r_A^1}{\partial r_{B1}}\right) - \delta\left(q^2 r_A^0\frac{\partial r_A^0}{\partial r_{B1}} - r_A^1\frac{\partial r_A^1}{\partial r_{B1}}\right)\right).
\end{aligned}
$$

■

If $r_{A1}$is sufficiently small, the second period game is in the complete information regime.

6 A's First Period Investment

We analyse A's first period investment behaviour in this section. Standing at the starting point of the first period, A needs to take the expectation of all the possible outcomes in the end of the second period. With probability $r_{A1}r_{B1}$, A has expected profit π_A^{11} in the end of the first period. Similarly, it gets π_A^{01} with probability $\left(1-r_{A1}\right)r_{B1}$, π_A^{10} with probability $r_{A1}\left(1-r_{B1}\right)$, and π_A^{00} with probability $\left(1-r_{A1}\right)\left(1-r_{B1}\right)$. If the second period game is a signalling game, A's expected profit is

$$
\begin{aligned}
\pi_{A1}\left[.,\hat{r}_A\right] &= r_{A1}r_{B1}\left(\hat{r}_A\left(\pi_m - r_{B2}^{11}\left[\hat{r}_A\right]\delta\right) - \frac{k\left(\hat{r}_A\right)^2}{2}\right) \\
&\quad + \left(1-r_{A1}\right)r_{B1}\left(qr_A^0\left(\pi_m - r_{B2}^{01}\left[r_A^0\right]\delta\right) - \frac{k\left(r_A^0\right)^2}{2}\right) \\
&\quad + r_{A1}\left(1-r_{B1}\right)\left(\hat{r}_A\left(\pi_m - q^2 r_{B2}^{11}\left[\hat{r}_A\right]\delta\right) - \frac{k\left(\hat{r}_A\right)^2}{2}\right) \\
&\quad + \left(1-r_{A1}\right)\left(1-r_{B1}\right)\left(qr_A^0\left(\pi_m - q^2 r_{B2}^{01}\left[r_A^0\right]\delta\right) - \frac{k\left(r_A^0\right)^2}{2}\right) \\
&\quad - \frac{k\left(r_{A1}\right)^2}{2}.
\end{aligned}
$$

Substitute in B's second period best responses,

$$\begin{aligned}
\pi_{A1}\left[.,\hat{r}_A\right] &= r_{A1}\hat{r}_A\pi_m + (1-r_{A1})\,qr_A^0\pi_m - r_{A1}r_{B1}\hat{r}_A\frac{1}{k}\left(\pi_m - \hat{r}_A\delta\right)\delta \\
&\quad -(1-r_{A1})\,r_{B1}qr_A^0\frac{1}{k}\left(\pi_m - r_A^0 q\delta\right)\delta \\
&\quad -r_{A1}\left(1-r_{B1}\right)\hat{r}_A q^2\frac{1}{k}\left(\pi_m - \hat{r}_A\delta\right)\delta - r_{A1}\frac{k\left(\hat{r}_A\right)^2}{2} \\
&\quad -(1-r_{A1})\left(1-r_{B1}\right)r_A^0 q^3\frac{1}{k}\left(\pi_m - r_A^0 q\delta\right)\delta \\
&\quad -(1-r_{A1})\frac{k\left(r_A^0\right)^2}{2} - \frac{k\left(r_{A1}\right)^2}{2}.
\end{aligned}$$

If the second period game is in the complete information regime, A's expected profit in the first period is

$$\begin{aligned}
\pi_{A1}\left[.,r_A^1\right] &= r_{A1}r_A^1\pi_m + (1-r_{A1})\,qr_A^0\pi_m - r_{A1}r_{B1}r_A^1\frac{1}{k}\left(\pi_m - r_A^1\delta\right)\delta \\
&\quad -(1-r_{A1})\,r_{B1}qr_A^0\frac{1}{k}\left(\pi_m - r_A^0 q\delta\right)\delta \\
&\quad -r_{A1}\left(1-r_{B1}\right)r_A^1 q^2\frac{1}{k}\left(\pi_m - r_A^1\delta\right)\delta - r_{A1}\frac{k\left(r_A^1\right)^2}{2} \\
&\quad -(1-r_{A1})\left(1-r_{B1}\right)r_A^0 q^3\frac{1}{k}\left(\pi_m - r_A^0 q\delta\right)\delta \\
&\quad -(1-r_{A1})\frac{k\left(r_A^0\right)^2}{2} - \frac{k\left(r_{A1}\right)^2}{2}.
\end{aligned}$$

It is uncertain if $\pi_A\left[r_A^1\right] > \pi_A\left[\hat{r}_A\right]$. In the complete information regime, A does not have to over-invest in the second period. However, from Lemma 2, B may be more aggressive in the first period in the complete information regime. If this investment effect is greater than the distortion effect from signalling, A's profit may be higher in the signalling regime.

Lemma 3 *If*

$$\frac{\partial\hat{r}_A}{\partial r_{B1}} < -\frac{\left(1-q^2\right)\left(\left(2\pi_m - \delta\left(r_A^1+\hat{r}_A\right)\right)\right)\left(\hat{r}_A - r_A^1\right)}{2\left(r_{B1}+q^2\left(1-r_{B1}\right)\right)\left(\pi_m - \hat{r}_A\delta\right)} + \frac{\left(\pi_m - r_A^1\delta\right)}{\left(\pi_m - \hat{r}_A\delta\right)}\frac{\partial r_A^1}{\partial r_{B1}}$$

then $\pi_A\left[r_A^1\right] > \pi_A\left[\hat{r}_A\right]$. *Otherwise, the sign is uncertain.*

From Lemma 2 and given the range of $\frac{\partial\hat{r}_A}{\partial r_{B1}}$, B is less aggressive in the complete information regime. Both the investment effect and distortion effect suggest $\pi_A\left[r_A^1\right] > \pi_A\left[\hat{r}_A\right]$. Outside this range of $\frac{\partial\hat{r}_A}{\partial r_{B1}}$, B is more aggressive in the complete information regime. The investment effect and the distortion effect run at different directions. Whether A's profit would be higher in the complete information depends on the relative magnitude of the two effects. Note that when $r_{A1} = \frac{M}{N}$, $r^*_{B1} = r^C_{B1}$, $r^1_A = \hat{r}_A$, and A's profits in the two regimes coincide.

We use some simulations to analyse how the parameter values would impact on the game regimes and the variables. The simulation results for some selected parameter ranges are reported in the appendix and some figures are reported below. In the simulation, we specify two additional parameters, d and d_2, with $\pi_m = \frac{k}{d}$ and $\pi_d = \frac{\pi_m}{d_2}$. The parameter d measures the relative size of π_m and k while d_2 measures the relative size of π_m and π_d.

The first observation is that B always has higher profit in the complete information regime. In most cases, A's profit is higher in the complete information regime. For some parameter ranges, A has higher profit in the signalling regime. In both regimes, A has higher profit than B. There is a first mover advantage despite the fact that A discloses its position by the second period investment and that A needs to signal when the second period game is in the signalling regime. The profit difference between the two regimes for A when varying q and holding other parameters constant is plotted in Figure 9. When q is small, A has higher profit in the signalling regime. In such a case, although the game is in the signalling regime, type zero does not have much incentive to mimic type one and the distortion effect is not that great. We have $r^*_{A1} > \frac{M}{N}$ for small q and the second period game is in the signalling regime. As q increases, the probability of moving for two steps increase, both A and B's profit increase. A's profit in the complete information regime gets higher initially and then the profit for the two regimes converge when q gets very large. When q gets large, there is not much difference between being a type zero or a type one and the profit levels are similar. Firm B's first period investment decreases as q increase in both regimes. As q increases, A's first period investment decreases in the signalling regime and the effect is uncertain in the complete information regime.

When we vary k, $\frac{M}{N}$ stays constant and the game is always in the complete information regime for the chosen parameter range. Both firms' profit increases in both regimes. This is due to the way we specify our parameters in the

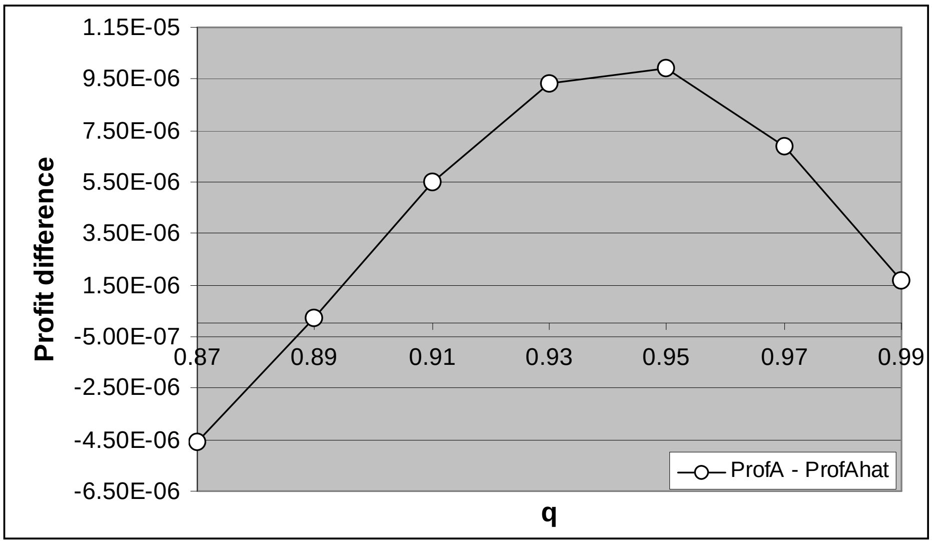

Figure 9: Profit difference for A between the complete information and signalling regimes, $\pi_A\left[r_A^1\right] - \pi_A\left[\hat{r}_A\right]$, when varying q.

simulations. Holding d and d_2 constant, as k increases, both monopoly profit and duopoly profit increases and firms' expected profit increases.

The profit difference for two firms in the two regimes when varying d is plotted in Figure 10. For $d < 2.5$, the game is in the complete information since $r_{A1}^* < \frac{M}{N}$. For $d \geq 2.5$, $\hat{r}_A < r_A^1$ and the game is in the complete information regime. In almost all the cases, both firms' first period investments decrease as d increases. Both firms' profit levels decrease as d increases since the monopoly profit decreases.

When we vary d_2, the profit difference for A and B are plotted in Figure 11. When d_2 is small ($d_2 < 9$), $\hat{r}_A < r_A^1$ and the second period game is in the complete information regime. When $d_2 \geq 9$, $r_{A1} < \frac{M}{N}$ and the game is in the complete information regime. Firm B's first period investment decreases as d_2 increases in both regimes. But the effect on r_{A1} is uncertain. Both firms' expected profit decreases as d_2 increases.

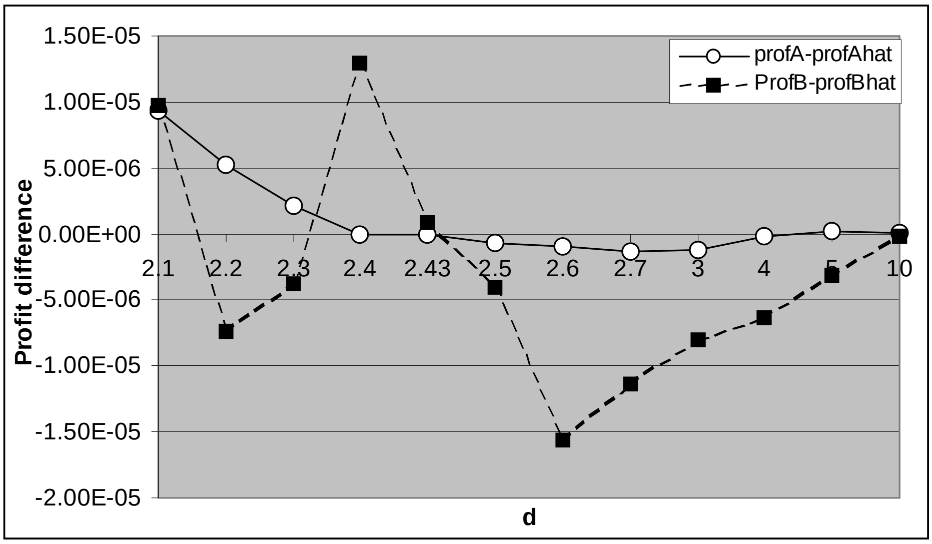

Figure 10: Profit difference in two regimes for A and B when varying d.

7 Conclusions

When R&D positions are unknown, the second game is in the signalling regime if both the probability of leapfrogging and the profit difference between the monopoly and duopoly are sufficiently high. The probability of leapfrogging matters in R&D competition, as in the conclusion in Fudenberg *et al* (1983). In this game, the high probability of leapfrogging keeps the follower active in the game and may also makes it necessary for type one of A to signal in the second period game. Unlike the results in Grossman and Shapiro (1987) where firms compete more aggressively when they are equal, firms always invest more when they succeed in the first period. Given A's over-investment in the signalling case, B invests less at $(1, 1)$ than at $(0, 1)$.

When taking into consideration the first period behaviour, it is shown that since firms may prefer the complete information regime to the signalling regime, with the ability to affect the regime by their first period investments, we observe that the second period game is often in the complete information regime in the simulation exercises.

There are some papers studying government's information disclosure regula-

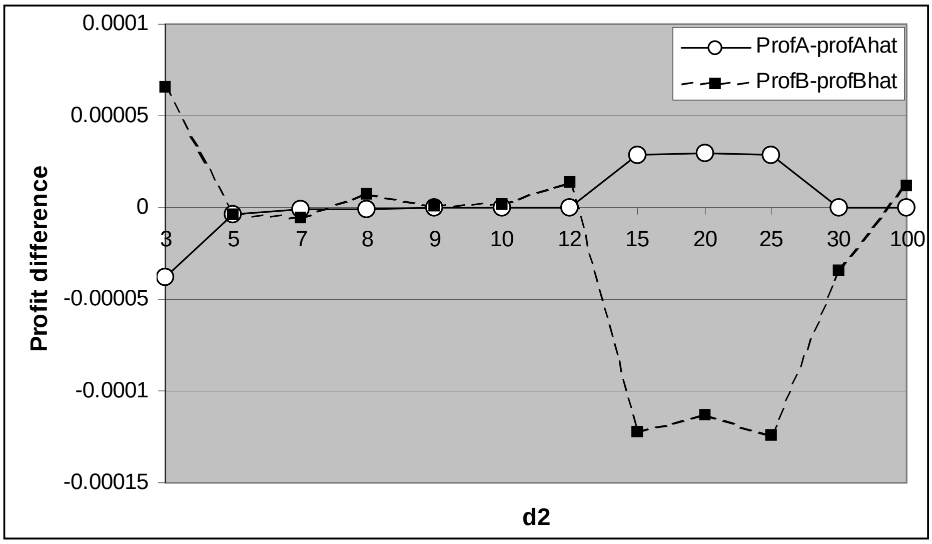

Figure 11: Profit difference for A and B in two regimes as d_2 varies.

tion. The results of this paper suggest that in many cases, such regulation would not be necessary since firms have the incentive to actively disclose their private information. In this model, firms may choose not to get into the signalling regime by choosing their first period investments. On the other hand, the regulation would matter most in industries characterised by large profit difference between monopoly and duopoly and high turnovers in technology leaders. There is still a first mover advantage despite the fact that A discloses its position by the second period investment.

There are some possible directions for further extensions. The first one is to make q related with the investment level. By doing so, by choosing the R&D investments, firms can target their innovation sizes. In this model, if successful in the first period research, A has the incentive to signal the R&D advantage, leading to the result of over-investment. We assume that there is no spill-over between firms in R&D. If there is some spill-over, we can expect that while the firm has the incentive to over-invest to signal its type, it also has the incentive to cut its investment since its investment benefits the rival. Whether or not firms over-invest would depend on the relative strength of the two effects. On the other hand, with the spill-over, even if the firm do need to signal its type, due to the externality the investment creates, the overall industry profit may be higher in the asymmetric information regime compared with the complete information regime. Finally, the problem of firms cooperating with each other in some market, R&D in this case, and competing in some other market, product market, has long been a center question in the literature of cooperative R&D. The problem of R&D coordination with asymmetric information is not yet well addressed. It may be interesting to extend the model in this direction.

References

[1] Admati, A. R. and P. Pfeiderer (2000). Forcing firms to talk: Financial disclosure regulation and externalities. *Review of Financial Studies*, **13**: 479-519.

[2] Anton, J. and D. A. Yao (1999). Little patents and big secrets: managing intellectual property. Memio. University of North Carolina and University of Pennsylvania, Philadephia.

[3] Aoki, R. and D. Reitman (1992). Simultaneous signalling through investment in an R&D game with private informaiton. *Games and Economic Behaviour*, **4**: 327-46.

[4] Cho, I. K. and D. M. Kreps (1987). Signalling games and stable equilibria. *Quarterly Journal of Economics*, **102(2)**: 179-222.

[5] Dasputa, P. and J. Stiglitz (1980). Uncertainty, industrial structure, and the speed of R&D. *Bell Journal of Economics*, **11**: 1-28.

[6] Dewatripont, M. and J. Tirole (1999). Advocates. *Journal of Political Economy*, **107(1)**: 1-39.

[7] Dranove, D. and N. Gandal (2001). The DVD v.s. DIVX standard war: Network effects and empirical evidence of preannouncement effects. *memeo*, Tel Aviv University.

[8] Fudenberg, D., R. J. Gilbert, J. Stiglitz, and J. Tirole (1983). Preemption, leapfrogging and competition in patent races. *European Economic Review*, **22**: 3-31.

[9] Fudenberg, D. and J. Tirole (1984). The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. *AEA Papers and Proceedings*, **74**: 361-66.

[10] Gilbert, R. J. and D. M. G. Newbery (1982). Preemptive patenting and the persistence of monopoly. *American Economic Review*, **72**: 514-26.

[11] Grossman, G. M. and C. shapiro (1987). Dynamic R&D competition. *Economic Journal*, **97**: 372-87.

[12] Jasen, J. The effects of disclosure regulation on innovative firms: Common values. Discussion paper FS IV 02-04, Wissenschaftszentrum Berlin.

[13] Jasen, J. The effects of disclosure regulation on innovative firms: Private values. Discussion paper FS IV 02-05, Wissenschaftszentrum Berlin.

[14] Kreps, D. M. (1990). *A Course in Microeconomics Theory.* Harvester Wheatsheaf.

[15] Lee, T. and L. L. Wilde (1980). Market structure and innovation: A reformulation. *Quarterly Journal of Economics*, **94**: 429-36.

[16] Lopatka, J. E. and W. H. Page (1995). Microsoft, monopolisation, and externalities: Some uses and abuses of economic theory in antitrust decision making. *The Antitrust Bulletin*, **Summer**: 317-370.

[17] Loury, G. C. (1979). Market structure and innovation. *Quarterly Journal of Economics*, **93**: 395-410.

[18] Reinganum, J. F. (1981). Dynamic games of innovation. *Journal of Economic Theory*, **25**: 21-41; (1985). Corrigendum, **35**: 196-97.

[19] Reinganum, J. F. (1982a). A dynamic game of R and D: Patent protection and competition behaviour. *Econometrica*, **50**: 671-88.

[20] Reinganum, J. F. (1982b). Strategic search theory. *International Economic Review*, **23**: 1-15.

[21] Reinganum, J. F. (1983). Uncertain innovation and the persistence of monopoly. *American Economic Review*, **73(4)**: 741-48.

[22] Reinganum, J. F. (1989). The timing of innovation: Research, development, and diffusion. Chapter 14 of *Handbook of Industrial Organisation*, **1**, Schmalensee, R. and R. D. Wilig (eds.). Elsevier Science Publisher B. V.

[23] Shapiro, C. and H. R. Varian (1999). *Information rules: A strategic guide to the network economy.* Harvard Business School Press.

[24] Shin, H. S. (1998). Adversarial and inquisitorial procedures in arbitration. *RAND Journal of Economics*, **29(2)**: 378-407.

8 Appendix

8.1 Derive $\frac{\partial \hat{r}_A}{\partial r_{B1}}$.

$\hat{r}_A = \frac{qE}{k-qD} + \frac{qE\sqrt{\frac{qD(1-q)}{k-q^2D}}}{k-qD}$.

$$\begin{aligned}\frac{\partial \hat{r}_A}{\partial r_{B1}} &= \frac{q\frac{\partial E}{\partial r_{B1}}(k-qD) + q^2 E\frac{\partial D}{\partial r_{B1}}}{(k-qD)^2} \\ &\quad + \frac{\frac{\partial\left(qE\sqrt{\frac{qD(1-q)}{k-q^2D}}\right)}{\partial r_{B1}}(k-qD) + q^2E\sqrt{\frac{qD(1-q)}{k-q^2D}}\frac{\partial D}{\partial r_{B1}}}{(k-qD)^2}.\end{aligned}$$

$$\begin{aligned}\frac{\partial\left(qE\sqrt{\frac{qD(1-q)}{k-q^2D}}\right)}{\partial r_{B1}} &= q\frac{\partial E}{\partial r_{B1}}\sqrt{\frac{qD(1-q)}{k-q^2D}} + qE\frac{\partial\left(\sqrt{\frac{qD(1-q)}{k-q^2D}}\right)}{\partial r_{B1}} \\ &= -q\left(1-q^2\right)\pi_m\frac{1}{k}\delta\left(\frac{q(1-q)D}{k-q^2D}\right)^{\frac{1}{2}} \\ &\quad + \frac{q^2\left(1-q^2\right)(1-q)E(\pi_m-\pi_d)^2}{(k-q^2D)^2\left(\frac{q(1-q)D}{k-q^2D}\right)^{\frac{1}{2}}}\end{aligned}$$

$$
\begin{aligned}
&\frac{\partial \hat{r}_A}{\partial r_{B1}} \\
= \quad & \frac{q\left(-\pi_m \frac{1}{k}\delta\left(1-q^2\right)\right)(k-qD)}{(k-qD)^2} + \frac{q^2 E 2\frac{1}{k}\delta^2\left(1-q^2\right)}{(k-qD)^2} \\
& + \frac{\frac{\partial\left(qE\sqrt{\frac{qD(1-q)}{k-q^2D}}\right)}{\partial r_{B1}}}{(k-qD)} + \frac{q^2 E\sqrt{\frac{qD(1-q)}{k-q^2D}}2\frac{1}{k}\delta^2\left(1-q^2\right)}{(k-qD)^2} \\
= \quad & -\frac{q\left(1-q^2\right)\pi_m\frac{1}{k}\delta}{(k-qD)} \\
& + \frac{2q^2\left(1-q^2\right)\pi_m\frac{1}{k}\left(1-\frac{1}{k}\left(\pi_m-\pi_d\right)\left(\mu_A+\left(1-\mu_A\right)q^2\right)\right)\delta^2}{(k-qD)^2} \\
& - \frac{q\left(1-q^2\right)\pi_m\frac{1}{k}\delta\left(\frac{q(1-q)D}{k-q^2D}\right)^{\frac{1}{2}}}{(k-qD)} \\
& + \frac{q^2\left(1-q^2\right)(1-q)\pi_m\left(1-\frac{1}{k}\delta\left(\mu_A+\left(1-\mu_A\right)q^2\right)\right)\delta^2}{(k-qD)\left(k-q^2D\right)^2\left(\frac{q(1-q)D}{k-q^2D}\right)^{\frac{1}{2}}} \\
& + \frac{2q^2\left(1-q^2\right)\frac{1}{k}\pi_m\left(1-\frac{1}{k}\delta\left(\mu_A+\left(1-\mu_A\right)q^2\right)\right)\sqrt{\frac{qD(1-q)}{k-q^2D}}\delta^2}{(k-qD)^2}.
\end{aligned}
$$

8.2 Simulation result tables.

8.2.1 Vary q.

Other parameter values given are: $k = 4$, $d = 2.43$, $d_2 = 9$. Note that $\pi_m = \frac{k}{d}$, $\pi_d = \frac{\pi_m}{d_2}$, and $\delta = \pi_m - \pi_d$. In the complete information regime:

q	**0.87**	**0.89**	**0.91**	**0.93**	**0.95**	**0.97**	**0.99**
r_{A1}	0.0159	0.013502	0.011131	0.008673	0.006227	0.003758	0.001286
r_{B1}	0.0141	0.0121	0.010097	0.008201	0.006379	0.00465	0.003035
r_A^0	0.3183	0.326431	0.33487	0.343719	0.353052	0.36296	0.37355
r_A^1	0.3886	0.38737	0.386047	0.384653	0.383182	0.381631	0.379993
π_A	0.158139	0.162371	0.166565	0.170727	0.17487	0.179007	0.183159
π_B	0.145367	0.147351	0.148905	0.149976	0.150495	0.150388	0.149571
r_B^C	1.956	-0.0776	-1.6992	-3.6946	-6.8566	-13.7643	-47.2953
$\frac{M}{N}$	-568.893	37.1402	1069.943	3810.644	13715.96	75557.93	2305112

In the signalling regime:

q	**0.87**	**0.89**	**0.91**	**0.93**	**0.95**	**0.97**	**0.99**
r_{A1}	0.015979	0.013514	0.010977	0.008569	0.00601	0.00327	0.000875
r_{B1}	0.014056	0.012049	0.010093	0.008198	0.006378	0.00465	0.003035
r_A^0	0.318336	0.326431	0.33487	0.343719	0.353052	0.36296	0.37355
$\hat{r}_A$	0.381209	0.387789	0.393635	0.398453	0.401731	0.402399	0.397023
π_A	0.158144	0.162371	0.166559	0.170718	0.17486	0.179	0.183157
π_B	0.1454	0.147349	0.148882	0.149938	0.150459	0.150369	0.149567

8.2.2 Vary k.

Other parameter values given are: $q = 0.889$, $d = 2.43$, $d_2 = 9$. In the complete information regime:

k	**0.5**	**1.5**	**2.5**	**3.5**	**5**	**6**	**7**
π_m	0.205761	0.617284	1.028807	1.440329	2.057613	2.469136	2.880658
π_d	0.022862	0.068587	0.114312	0.160037	0.228624	0.274348	0.320073
r_{A1}	0.013244	0.010002	0.013604	0.013638	0.013648	0.013595	0.013634
r_{B1}	0.012154	0.012162	0.012154	0.012154	0.012154	0.012154	0.012154
r_A^0	0.326018	0.326018	0.326018	0.326018	0.326018	0.326018	0.326018
r_A^1	0.387435	0.387435	0.387435	0.387435	0.387435	0.387435	0.387435
π_A	0.02027	0.0608	0.10135	0.14189	0.2027	0.243241	0.283781
π_B	0.018409	0.055267	0.092039	0.128853	0.184076	0.220894	0.257707
r_B^C	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035
$\frac{M}{N}$	3.149782	3.149782	3.149782	3.149782	3.149782	3.149782	3.149782

In the signalling regime:

k	**0.5**	**1.5**	**2.5**	**3.5**	**5**	**6**	**7**
r_{A1}	0.010634	0.010001	0.010028	0.013642	0.013636	0.010005	0.01362
r_{B1}	0.012157	0.012158	0.012158	0.012149	0.012149	0.012158	0.012148
r_A^0	0.326018	0.326018	0.326018	0.326018	0.326018	0.326018	0.326018
$\hat{r}_A$	0.387475	0.387475	0.387475	0.387475	0.387475	0.387475	0.387475
π_A	0.020268	0.0608	0.101334	0.14189	0.202701	0.243201	0.283781
π_B	0.01842	0.055267	0.092112	0.128853	0.184076	0.221069	0.257699

8.2.3 Vary d.

Other parameter values given are: $q = 0.889$, $k = 4$, $d_2 = 9$. In the complete information regime:

d	2.1	2.2	2.3	2.4	2.5	2.6	3	4	5	10
π_m	1.904762	1.818182	1.73913	1.666667	1.6	1.538462	1.333333	1	0.8	0.4
π_d	0.21164	0.20202	0.193237	0.185185	0.177778	0.17094	0.148148	0.111111	0.088889	0.044444
π_m-π_d	1.693122	1.616162	1.545894	1.481481	1.422222	1.367521	1.185185	0.888889	0.711111	0.355556
r_{A1}	0.018841	0.01698	0.015332	0.013688	0.012847	0.011791	0.008897	0.005144	0.003394	0.000927
r_{B1}	0.015408	0.01432	0.013321	0.012411	0.011582	0.01083	0.008437	0.005045	0.003363	0.000926
r_A^0	0.382617	0.363008	0.345724	0.330323	0.316476	0.30393	0.263381	0.199823	0.161963	0.084178
r_A^1	0.466248	0.438262	0.414208	0.393227	0.374703	0.358181	0.306302	0.228769	0.184196	0.094939
$\hat{r}_A$	0.47251	0.4423	0.416336	0.393713	0.373777	0.356038	0.300722	0.219747	0.174468	0.087256
π_A	0.203991	0.189652	0.176861	0.165378	0.155019	0.14563	0.115593	0.07153	0.048622	0.0138
pi_B	0.173712	0.165553	0.157447	0.149575	0.142011	0.134836	0.11008	0.070059	0.048084	0.013775
r_B^C	-0.9516	-0.6769	-0.3894	-0.0891	0.224	0.5498	1.9809	6.4533	12.2034	60.1212
$\frac{M}{N}$	0.136782	3.375026	11.51947	7.532438	-30.2851	-120.74	-1276.16	-14460.3	-57353.9	-2064153

In the signalling regime:

d	2.1	2.2	2.3	2.4	2.5	2.6	3	4	5	10
r_{A1}	0.018208	0.016291	0.014956	0.013995	0.012829	0.011677	0.009109	0.00517	0.003377	0.000904
r_{B1}	0.015399	0.014313	0.013315	0.012405	0.011578	0.010828	0.008436	0.005045	0.003363	0.000926
r_A^0	0.382617	0.363008	0.345724	0.330323	0.316476	0.30393	0.263381	0.199823	0.161963	0.084178
$\hat{r}_A$	0.472511	0.442301	0.416336	0.393713	0.373777	0.356038	0.300722	0.219747	0.174468	0.087256
π_A	0.203981	0.189647	0.176859	0.165378	0.15502	0.145631	0.115594	0.07153	0.048622	0.0138
π_B	0.173702	0.165561	0.157451	0.149562	0.142015	0.134852	0.110088	0.070066	0.048088	0.013775

8.2.4 Vary d_2.

Other parameter values given are: $q = 0.889$, $k = 4$, and $d = 2.43$. In the complete information regime:

d_2	3	5	7	9	10	12	15	20	25	30
π_d	0.548697	0.329218	0.235156	0.182899	0.164609	0.137174	0.109739	0.082305	0.065844	0.05487
$\pi_m - \pi_d$	1.097394	1.316872	1.410935	1.463192	1.481481	1.508916	1.536351	1.563786	1.580247	1.591221
r_{A1}	0.01	0.013222	0.013568	0.013975	0.01326	0.013405	0.013752	0.0135	0.013922	0.014743
r_{B1}	0.013945	0.013005	0.012482	0.012153	0.012032	0.011841	0.011641	0.011432	0.011301	0.011209
r_A^0	0.344843	0.334573	0.329251	0.326018	0.324837	0.323014	0.321128	0.319177	0.317974	0.317158
r_A^1	0.398929	0.39273	0.389451	0.387435	0.386693	0.385544	0.384349	0.383105	0.382334	0.38181
π_A	0.179765	0.168678	0.164421	0.16216	0.161384	0.160234	0.159099	0.157976	0.157308	0.156863
π_B	0.169433	0.155256	0.149973	0.14725	0.146364	0.145034	0.143744	0.142524	0.141797	0.141303
r_B^C	2.9378	0.8884	0.2881	0.0035	-0.0891	-0.2217	-0.3473	-0.4663	-0.5347	-0.5792
$\frac{M}{N}$	-1721.93	-411.626	-119.235	3.149782	40.55841	92.00739	138.3337	180.0143	202.9706	217.4719

In the signalling regime:

d_2	3	5	7	9	10	12	15	20	25	30
r_{A1}	0.014001	0.013561	0.013562	0.013995	0.013258	0.013681	0.010005	0.010005	0.010134	0.013389
r_{B1}	0.013938	0.013	0.012478	0.012148	0.012027	0.011835	0.011646	0.011437	0.011307	0.011207
r_A^0	0.344843	0.334573	0.329251	0.326018	0.324837	0.323014	0.321128	0.319177	0.317974	0.317158
$\hat{r}_A$	0.389243	0.389089	0.388214	0.387475	0.38717	0.386664	0.386098	0.385468	0.385058	0.384772
π_A	0.179803	0.168683	0.164422	0.16216	0.161384	0.160234	0.15907	0.157947	0.15728	0.156863
π_B	0.169367	0.155259	0.149979	0.147249	0.146362	0.14502	0.143866	0.142638	0.141921	0.141337