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CAPACITY ANALYSIS OF SIGNALISED INTERSECTIONS: EFFECTS OF GREEN TIME ON SATURATION FLOW RATE

MOHSIN SHAHZAD CHAUDHRY

A thesis submitted in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

The University of Auckland,

2013.
ABSTRACT

Capacity is an important parameter on the basis of which other operational measures are calculated for signalised intersections. The capacity is estimated based on the green time ratio and saturation flow rate. The basic model of traffic operations at signalised intersections is based on an assumption that, when a signal changes to green, the flow across the stop line increases rapidly to a maximum queue discharge rate, also termed as saturation flow rate, which remains constant until either the queue is exhausted or the green period ends. In recent years, this assumption has been challenged by some field observations. These observations depict fluctuations in the queue discharge rate for different queue positions and show a marginal increase in the queue discharge rate along the queued vehicles’ position.

This research investigates the reported variations in queue discharge rate at signalised intersections. To verify this proposition, a pilot study was conducted to investigate the queue discharge behaviour at signalised intersections based on video data collected from one of the busiest at-grade signalised intersections in Auckland. A comparison was made between the queue discharge rate observed in the field and the saturation flow rate estimated based on the methodologies proposed in the HCM (2000) and the Australian Road Research Board Report 123. The results from the pilot study conducted in Auckland were similar to those observed in Taiwan and the USA.

A detailed field study was conducted at six signalised intersections in Auckland to collect data verifying the variable nature of queue discharge rate at signalised intersections. The observed trends are then modelled using empirical models and assessed by comparing against the models developed using the Gene Expression Programming (GEP) algorithm. The proposed model shows compatibility with the existing traffic signal design
methodologies for calculating different performance measures. More importantly, it shows prospects to overcome the shortcomings of the traditional method for practical applications.

The proposed model is investigated for the purpose of real world applications, which will require calculations relating to capacity and cycle time. The model is implemented to compute uniform delay using a modified formulation for a D/D/1 case. The modified uniform delay model is compared against the existing delay models based on traditional methodologies. The results revealed that the modified model can overcome the deficiencies of the existing ones.

Finally, a microscopic car-following model is proposed to replicate the increasing queue discharge behaviour observed at signalised intersections. The proposed model is verified using the GEP algorithm. It exhibits the potential to overcome the shortcomings of the existing car-following models in replicating the queue discharge behaviour observed in the field.
ACKNOWLEDGEMENTS

First of all, I would like to express my gratitude for my academic supervisor, Dr Prakash Ranjitkar, for his mentorship, continuous support and unfailing patience. This thesis would not have been possible without his expert advice and guidance throughout my PhD research. He remained a source of motivation and support for me.

I would also like to acknowledge the contributions of my co-supervisor Dr. Douglas Wilson for his useful advice and guidance. I am so glad to have the opportunity to work with them and collaborate on research and publishing papers.

My time at the Department of Civil and Environmental Engineering has been very productive, particularly the support of faculty and staff of the Transportation Research Centre made a difference to my research outcomes. Here, I would like to extend special thanks to Sujith Padiyara and Noel Perinpanayagam for helping me tirelessly and patiently.

I am eternally grateful to my family, particularly my wife, who remained a staunch support and raised our three boys on her own for the last two years of my doctoral journey. Special thanks to my children who brought immense joy and light into my life whenever darkness threatened to prevail.

Finally, I have to acknowledge my doctoral companions for their generous friendships and enduring encouragement during this most challenging and taxing time in my life. Special thanks to Stephan Hassold, Dr. Jawad Hussain, Dr. Najif Ismail, Mudassar Khan. Without them, my PhD study would be much harder.
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NOMENCLATURE

\( A \) \hspace{1cm} \text{constant acceleration of queued vehicles} \\
\( a_1, a_2, \ldots \) \hspace{1cm} \text{pcu coefficients} \\
\( A_n \) \hspace{1cm} \text{maximum acceleration} \\
\( c \) \hspace{1cm} \text{capacity of intersection approach (veh/h)} \\
\( C \) \hspace{1cm} \text{traffic signal cycle length} \\
\( C_0 \) \hspace{1cm} \text{optimal minimum delay cycle length} \\
\( C_p \) \hspace{1cm} \text{practical cycle time} \\
\( d \) \hspace{1cm} \text{average delay per vehicle} \\
\( D \) \hspace{1cm} \text{density of traffic per lane} \\
\( d_1 \) \hspace{1cm} \text{average delay per vehicle due to uniform arrival} \\
\( d_2 \) \hspace{1cm} \text{average delay per vehicle due to random arrival} \\
\( d_3 \) \hspace{1cm} \text{average delay per vehicle due to initial queue at start of analysis time period} \\
\( d_{\text{avg}} \) \hspace{1cm} \text{average delay per vehicle} \\
\( d_i \) \hspace{1cm} \text{distance between vehicles in a stopped queue} \\
\( d_{\text{imax}} \) \hspace{1cm} \text{the distance travelled to reach speed } v_q \\
\( D_i \) \hspace{1cm} \text{aggregate uniform delay} \\
\( g \) \hspace{1cm} \text{effective green time} \\
\( G \) \hspace{1cm} \text{Displayed green time} \\
\( g_i \) \hspace{1cm} \text{demand for green time by a lane group i} \\
\( g_r \) \hspace{1cm} \text{green time demand (unitless)} \\
\( h_m \) \hspace{1cm} \text{minimum average headway recorded after a number of cycles} \\
\( h_n \) \hspace{1cm} \text{headway of the } n\text{th vehicle} \\
\( h_s \) \hspace{1cm} \text{average saturation headway} \\
\( I \) \hspace{1cm} \text{inter green time} \\
\( K \) \hspace{1cm} \text{incremental delay factor} \\
\( k \) \hspace{1cm} \text{stop penalty parameter} \\
\( L \) \hspace{1cm} \text{lost time} \\
\( l \) \hspace{1cm} \text{lost time} \\
\( l_s \) \hspace{1cm} \text{initial lost time} \\
\( n \) \hspace{1cm} \text{the queue position} \\
\( PF \) \hspace{1cm} \text{progression factor} \\
\( PF2 \) \hspace{1cm} \text{Adjustment factor for effects of progression} \\
\( q \) \hspace{1cm} \text{arrival flow rate} \\
\( Q \) \hspace{1cm} \text{Back of queue size} \\
\( Q_i \) \hspace{1cm} \text{first term back-of-queue aize}
$Q_s$ second term back of queue size in veh/ln
$Q_{lb}$ lane group initial queue at the start of the analysis
$q_s$ queue discharge rate
$r$ red time
$s$ saturation flow rate (veh/h)
$S_1$ distance of the first vehicle from the stop line
$s_{an}$ average distance of acceleration of $n$th vehicle from start of motion to time $t_a^n$
$S_n$ distance from the stop line
$T$ analysis period
$T_s$ saturated green time
$t_a^n$ average acceleration time
$t_D$ time required to completely dissipate the queue
$td_a$ average discharge time at the stop line of the $n$th vehicle in queue
$T_f$ flow period
$t_n$ discharge time from the $n$th vehicle in queue
$T_r$ driver starting response time
$U$ intersection green time ratio
$v$ arrival flow rate
$v_L$ lane group flow rate per lane
$V_{\text{max}}$ the maximum speed
$v_q$ desired speed of queued traffic
$V_{sL(n)}$ stop line speed of the $n$th vehicle
$w_n$ start-up waiting time for the $n$th vehicle in a queue
$X$ critical volume to capacity ratio
$x$ volume to capacity ratio
degree of saturation below which the average over flow queue is approximately zero
$x_0$ ratio of flow rate to capacity ($v_L/c_L$ ratio)
y flow ratio
$Y$ sum of Critical flow ratios
$z$ $x-1$
$\alpha$ average initial rate of acceleration
$\beta$ average rate of decrease in acceleration with increasing speed
$\gamma$ a correction factor
$\Delta v_n$ difference between the speed of the leader and follower vehicles
$\theta$ time ratio ($t/ta$)
$\lambda$ sensitivity factor
$\tau$ excess start-up time of the lead vehicle in a queue
1 INTRODUCTION

The operational efficiency of traffic movements at signalised intersections is highly dependent on the queue discharge flow rate. The phenomenon of queue discharge flow rate is a complex process and is composed of several driver and vehicle performance characteristics including acceleration, speed and the driver’s perception and reaction time. However, this complex process is simplified using the assumption of a steady-state saturation flow rate concept.

1.1 Background

Capacity is the most widely used concept in traffic engineering practice. In the planning, design, and operation of signalised intersections, an estimate of lane capacities under prevailing roadway geometric, traffic and signal control conditions is required. The U.S. “Highway Capacity Manual (2010),” published by the Transportation Research Board, and a research report (ARR 123) on “Traffic Signals: Capacity and Timing Analysis,” published by the Australian Road Research Board, are two important references on this concept (2010, Akçelik, 1981). The capacity of a signalised intersection is a function of the saturation flow rate and the green time ratio. Saturation flow rate represents the single most important parameter in the capacity and signal timing analysis of signalised intersections (Akçelik, 1981). It is defined as the steady maximum rate at which queued vehicles can be discharged into an intersection while the traffic signal is green.

The basic model of traffic operations at signalised intersections is based on an assumption that when the traffic signal changes to green, the flow across the stop line increases rapidly to the saturation flow rate, which then remains constant until either the queue is exhausted or the green period ends (Akçelik, 1981). The model is shown in Figure 1.1. This convenient assumption makes it considerably easier to calculate the lane group capacity and other performance measures. Several factors are identified in the
literature as factors influencing saturation flow rate, however the basic concept remains the same. In recent years, this assumption has been challenged by a considerable number of researchers observing large variations in queue discharge rate (Chaudhry and Ranjitkar, 2013, Chaudhry et al., 2011, Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004, Tarko and Tracz, 2000, Teply, 1983).

Many of the past studies reported a decreasing headway or increasing saturation flow rate towards the back of the queue. Figure 1.2 shows the mean headway for queued vehicles at the stop line of signalised intersections as reported in the literature. Interestingly, a decreasing trend of mean discharge headway can be spotted towards the back of queue. Gerlough and Wagner (1967) and Carstens (1971), however, reported comparatively flatter trends in headway. The queue discharge rates in those studies were
either nearly constant or fluctuating slightly up and down. The rest of the studies in this figure depict a clear indication of a decreasing trend in the mean headway and hence an increasing queue discharge rate towards the back of queue.

![Figure 1.2: Departure headways with respect to queue position](image)

A clearer picture of this decreasing trend in headway is observed in some recent field studies conducted in Taiwan and the USA (Hawaii and New York) (Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004). The average headway between successive vehicles in these studies was measured to have a decreasing trend along the back of the platoon through to the last queued vehicle discharged from the intersection, causing a marginal increase in saturation flow rate. These large variations observed in saturation flow measurements challenged the presumption that the base saturation flow rate of 1,900 passenger cars per hour of green time per lane (as specified in HCM (2000)) is either stable or a constant value.
1. Introduction

1.2 Problem Statement

While significant research has been conducted on factors affecting the saturation flow rate assuming it as a constant maximum flow rate at signalised intersections, not much research has addressed variability in queue discharge rate. All of the existing methodologies calculate the operational efficiency of a traffic stream at a signalised intersection based on the minimum discharge headway. It is important to consider how this minimum discharge headway is reached after an onset of green time. The assumption of a constant saturation flow rate, used in traditional model of traffic signal design, is questionable in the light of several observations conducted in Taiwan, the USA and other places showing strong evidence of a decreasing trend for discharge headway and an increasing trend for queue discharge rate along the back of queue. This demands revisiting and re-assessing traffic signal design theory based on an improved understanding of driving behaviour and queue discharge behaviour at signalised intersections. As a first step, it is important to verify the published research outcomes on queue discharge behaviour at signalised intersections. Subsequently, the postulations developed in the literature need to be examined for underlying processes, including driver perception and reaction time, headway, and the development of the queue discharge rate in green time. These processes are primary indicators of how the traffic stream passes through the stop line at a signalised intersection. Based on these observations, a sound theoretical and empirical relationship can be established in order to develop a more realistic model of traffic signal design that is expected to be more efficient and more responsive to actual driving behaviour. Establishing a methodology that can be implemented in practice without making calculations excessively cumbersome is another challenging task yet to be accomplished.

1.3 Research Objectives

The specific objectives of this research are in five-folds:

1. Investigate queue discharge behaviour at signalised intersections and its contributing factors.
1. Introduction

2. Verify whether or not queue discharge behaviour observed at signalised intersections in Auckland is similar to what is reported in the literature that is an increasing trend.

3. Develop a mathematical model to approximate the observed queue discharge behaviour at signalised intersections.

4. Modify cycle time and uniform delay formulations to incorporate the observed queue discharge behaviour at signalised intersections.

5. Develop a microscopic car-following model to replicate the observed queue discharge behaviour at signalised intersections.

1.4 Research Contributions

This study primarily investigates queue discharge behaviour at signalised intersections, which forms the basis for the analysis of capacity and other performance measures at signalised intersections. It is anticipated that the work will help to improve the existing techniques, which are based on a constant headway and a constant saturation flow rate. It is expected that the knowledge provided in this dissertation will open up new methodological and practical propositions for the analysis of traffic operations at signalised intersections and will provide valuable information to transportation planners and policy makers. This research will enable people to understand the possible impacts of the implementation of a variable queue discharge rate methodology in the future, instead of the traditional practices.

1.5 Scope of the Research

The scope of this study is limited to traffic operations at isolated signalised intersections. Six intersections were selected for this study. The data was collected from only the most suitable approaches considering several factors including the intersection geometry, demand and traffic signal control conditions. This study focused on the variability of queue discharge rate as the green times pass. A simple scenario was established in which variability in queue discharge rate was encapsulated into a model based on the green time. The model has been tested rigorously against the data collected in Auckland, however,
further verification of the proposed method will be needed for other traffic flow conditions. A car-following model on signalised intersections is also proposed. However, the model requires a more detailed validation with actual field data.

1.6 Organization of the Dissertation

This dissertation is organised into 10 chapters. This chapter has introduced the background and objectives of this study. Chapter 2 summarises the literature on topics related to this study. The topics included are as follows: characterisation of traffic flow at signalised intersections, traffic signal operations, saturation flow rate, variations in saturation flow rate, capacity and degree of saturation, cycle time formulation, delay, and the gene expression methodology that has been used in the formulation of proposed model.

Chapter 3 presents the methodology adopted to perform the investigation and model the formulation in this study. Initially, a pilot study was designed to establish the further steps for the investigation of the variability in the saturation flow rate. The detailed data collection methodology is presented in a later section.

Chapter 4 delineates the data collection at six signalised intersections. This chapter discusses the problems faced during data collection and the modifications made in the data collection setup. The processing of the data collection is later discussed and some results are shown.

Chapter 5 presents an article published in the journal of the Eastern Asia Society for Transportation Studies (EASTS) with some modifications to fit the scope of this dissertation. The chapter discusses the results of the pilot study conducted at one intersection at Auckland, New Zealand.

Chapter 6 presents the results of the detailed field study conducted at six signalised intersections in Auckland, New Zealand. Linear and nonlinear models are presented based on the observed variations in the queue discharge rate. This chapter is extracted from an article accepted for publication in the journal of the Asian Transport Studies (ATS) with some modifications to fit in scope of this dissertation.
Chapter 7 presents an unpublished article quantifying the impacts of a nonlinear model, proposed based on the variable queue discharge rate, on the capacity and cycle time calculations at signalised intersections.

Chapter 8 presents an article conditionally accepted for publication in the journal of the Eastern Asia Society for Transportation Studies (EASTS) with some modifications to fit the scope of this dissertation. This chapter discusses the impact of variable saturation flow rate on the delay calculations.

Chapter 9 presents an unpublished article proposing a new car-following model able to replicate the queue discharge behaviour observed at signalised intersections.

Chapter 10 concludes this dissertation, highlighting the main contributions of this study along with some recommendations for future research directions.
2 LITERATURE REVIEW

This chapter discusses the concept of saturation flow rate, variability and potential impact on signalised intersection in general transportation concept. The signalised intersection is a facility to safely allocate right of ways for conflicting traffic movements. These intersections ensure safety of vehicular and pedestrian traffic, however they are also considered as bottlenecks in urban street network. The efficient system of urban streets is highly dependent on how signalised intersections are operating. Most traffic related delays occur on signalised intersections. A 2007 mobility study report indicated that the cost of congestion is exceeding $78 billion in US (Schrank and Lomax, 2007). The study was based on observations made on 437 cities of US. Due to the imperative nature of its operation, signalised intersections are considered one of the critical areas on roadway networks. Consequently, the capacity of the signalised intersection has been the most important factor in determining its overall performance and efficiency. Typically, the procedures involved in the capacity estimation, particularly the signalised intersection, are based on an assumption of an “ideal” maximum traffic flow rate over a saturated green time. This assumption of an ideal maximum traffic flow makes it considerably easier to calculate the lane group capacity which is equal to saturation flow rate multiplied by green to cycle time ratio (HCM, 2010). Several researchers have shown that saturation flow rate is not a constant value.

2.1 Characterisation of Traffic Flow at Signalised Intersections

Traffic flow along the highway is usually described as a function of space and time. However in signalised intersections, the functions of traffic flow are usually described in terms of arrivals and departures. These functions are further translated in terms of a sequence of arrivals, stop and departures. The control at signalised intersections regulates the arrivals, stop and departures to provide a safe and smooth environment for vehicular
and pedestrian traffic. The ultimate objective is to determine the signal settings that minimize the expected delay to the motorists.

Signalised intersection is a shared space where two or more conflicting traffic streams are separated in terms of time with alternatively allowed movements to use the facility. During this process the control at signalised intersections interrupts traffic flow in orderly and deterministic manner. Vehicles are stopped during the red interval and the interrupted stream starts building up as a queue. One of the first objectives is to clear that queue when the interrupted stream is given a green signal. The accumulated queue is then allowed to depart by providing green time. To establish a stable traffic flow, the green time must be long enough to clear the queue accumulated during the red interval. Once the queue is cleared, the departure flow in the remaining green time equals the arrival flow.

The efficiency of a signalised intersection is evaluated by how smoothly these queues accumulated during red interval are dissipated during green period. Any inconsistency in dissipation results in longer delays and more stops. The determination of the efficiency of a signalised intersection is different than the uninterrupted flow facilities. The delay perceived by the drivers waiting in a queue is the only parameter that governs quality of service at signalised intersections (Allsop, 1971). Two regimes are established due to traffic control at signalised intersections. On the upstream side, the flow is cut off by the traffic control and a new traffic state builds up at red signal. A queue starts building up until the green signal appears that allows this queue to move along.

2.1.1 Arrival Process

The arrival patron on signalised intersection depends on various factors. This pattern of arriving vehicles at a signalised intersection establishes how a queue forms. It depends on various conditions at upstream of the intersection, including the close vicinity to an upstream intersection, coordination among the intersection and site specific factors (such as shift based work areas etc.). This information is a key component in determining what strategy of traffic control is appropriate. Adams (1936) was the pioneer who used probability theories in traffic flow. The arrivals are usually considered on the basis of the distribution of inter-arrival times at the signalised intersection. Arrivals could be platooned, semi platooned, or random and the type and density of arrivals depicts the level
of congestion on the upstream side. During peak traffic hours, the platoons’ density becomes more consistent. On the basis of these conditions, the arrivals could be categorised into three classes, including free movers, joining or leaving a platoon and followers or member of a platoon. These classes form the following types of arrivals typically used in traffic engineering.

a) **Random Arrival**

Ideally, traffic arrives randomly and free movers at the upstream section of a road form this type of arrivals. This situation appears when there is no other intersection at the upstream side, or any platoon formed at upstream intersection diminishes due to a large distance. This arrival pattern is formed in absence of accumulation of vehicles due to signalization at upstream side and is usually found in isolated intersections.

b) **Grouped Arrivals**

A presence of another intersection at a close distance gathers traffic during the red phase and then releases it at the green phase, which makes vehicles follow each other and leads to the formation of a denser platoon. These platoons are formed during the stop phase of an upstream intersection, and they tend to disperse as they depart a signalised intersection.

c) **Mixed Arrivals**

These types of arrivals happen when the adjacent intersections are at such a distance that they cause some of the vehicles in the platoons to disperse and some of the vehicles from other sections of the road to join. Due to this dispersion and joining, this arrival type carries the characteristics of both random and grouped arrivals.

2.1.2 **Queue Process**

The signalised intersections regulate the conflicting streams of traffic. The controller at signalised intersections halts flow at an approach to permit movement to a conflicting approach. A desirable approach in traffic signal design is to provide green time that is just
enough to dissipate the traffic queue completely. This is because delays that are experienced by traffic vehicles due to signalised intersection control are affected by the number of vehicles in the queue waiting for movement. The formation of the queue can be analysed with models derived mainly from queuing and shockwave theories (Alexiadis et al., 2004). Further discussion on the queue process can be found in Section 0 at Page 35.

### 2.1.3 Departure Process

The main factor in the departure process at signalised intersections is the traffic control. A number of studies have indicated that drivers maintain their following distance based on the time headway that is independent of vehicle speed (Winsum and Heino, 1996, Van Winsum, 1999, Chang and Joseph, 2001). In the departure process, the capacity of the traffic stream is a basic parameter. The capacity parameter is determined based on the saturation discharge headway (Akçelik, 2008, Allsop, 1972b). The saturation discharge headway is defined as the headway in presence of the queue and can be shown as follows:

\[
h_s = \frac{3600}{s} \tag{2.1}
\]

where

- \( h_s \) is the queue discharge (saturation) headway in seconds
- \( s \) is the saturation flow rate in vehicles per hour

### 2.2 Headway

Discharge headway is defined as the time interval between two successive vehicles noted at some reference point. For signalised intersections, this reference point is generally taken to be the stop line. Headways are used to determine saturation flow rates and lost times which are important parameters in capacity analysis for signalised intersections. These parameters are sensitive to various internal and external factors including geometric, traffic and driver characteristics.

Greenshield et al. (1946) examined the effects of queue position and traffic composition on discharge headways at signalised intersections. He observed that the average discharge headway decreases for the first five vehicles then attains a constant level until either the
queue is exhausted or signal phase changes, as shown in Figure 2.1. This concept is widely accepted in traffic engineering and practiced for signal design.

![Figure 2.1: Variation in headway](image)

Several studies on headway at signalised intersections report a decrease in headway for the first four or five vehicles and then a levelling off until the queue is exhausted. The typical trend in these studies showed that headway becomes constant, however, some recent studies showed a gradual reduction in headways towards the back of the queue. These studies also reported a gradual reduction in start-up lost time, which is sometimes attributed to a better acceleration performance for new vehicles and sometimes attributed to the aggressive behaviour of drivers (Allsop, 1972a). While examining the sensitivity of different factors affecting the discharge headway, some studies indicated that the queue length significantly influenced the discharge headway. The studies on the effect of heavy vehicles on the discharge headway reported that vehicle size significantly influenced discharge headway (Buckholz, 2009, Dowling, 2007).

Shao and Liu (Shao and Liu, 2012) noted the traditional estimation of saturation headway does not accurately reflect the discharge headway. They observed the average value of queue discharge headways greater than the median value. Their study was based on the
data collected from 11 intersections.

2.3 Traffic Signal Operations

Traffic signals are an effective means for controlling and regulating conflicting flows of vehicles and pedestrians. The first traffic signal which used coloured lights was installed at the intersection of the Bridge Street and George Street outside the House of Parliament, London (Engineer, 1868). This signal was manually operated and its objective was to address safety concerns that were arising due to the rapid increase of traffic. Although an accident halted the further deployment, it proved a remarkable way to provide a safe “Right of Way” to conflicting streams of traffic. Since its inception by a Railway Engineer John Peake Knight, a gradual evolution and improvements made it an integral part to regularise the traffic at urban road network (Galviz, 2013). Over time, traffic signals which were introduced as a two light system gradually evolved into a three light signal system of red, amber and green.

Traffic signals periodically halt flow in each movement or set of movements. The primary purpose of this periodic halt for a certain movement or set of movements is to assign right of way to intersecting traffic streams. This phenomenon ensures the safe and efficient operation of traffic signals where all traffic streams cross the intersection without facing excessive delays. Performance of traffic operations at signalised intersections greatly influences the efficiency of the whole transportation network. A properly designed intersection reduces not only delays but also minimises fuel consumption without compromising safety (Smith et al., 2002). Due to their significance, signalised intersections are always a matter of great concern for traffic engineers and planners. Intersections are primary elements in any road network and traffic signals facilitate conflicting traffic movements in the same space by splitting the time share. Safe operation of road networks depends more on intersections than on any other single element (Yin, 2008).

The basic capacity model on which all the capacity methods are based assumes that when the signal changes to green, the flow across the stop line increases rapidly to a maximum level where it is sustained until either the queue is exhausted or the green period ends. The basic model replaces the actual departure flow curve by a rectangle of equal
area, the height of which is equal to the saturation flow rate and width of which is equal to the effective green time (Akçelik, 1979).

At the start of the green period, the queue of vehicles will start to accelerate across the intersection. The flow rate will increase rapidly to reach a maximum theoretical flow rate in a short span of time. This maximum discharge flow rate will continue until the queue is fully dissipated, or the phase aspect changes. This theory assumes that the saturation flow of vehicles remains constant after a certain short period has elapsed. The concept of lost time is introduced in order to simplify the constant saturation flow rate theory, making a rectangular shape for ease of calculations. The flow rate drops instantaneously at the end of the green phase. The lost time at the start of green phase is termed as start loss and the time lost by phase change to amber is termed as end loss. The effective green time is then determined by the equation:

\[ g = G + I + \ell \]  

where

- \( g \) is the effective green time (s),
- \( I \) is the inter green time (s),
- \( \ell \) is the lost time (s), and

In discussing capacity for controlled intersections, the first model was proposed by Clayton (1941). He introduced the concept of the most efficient cycle time as the shortest time which will handle the traffic that the headway at some distance downstream from the intersection can be assumed to be constant. He also introduced the term “saturation density” and defined it as the flow when vehicles are running as close together as possible (Clayton, 1941). His model for intersections is published in two papers. In his first paper, Clayton (1941) introduced the intersection factor which is:

\[ \text{Intersection Factor} = \frac{g - \ell}{C} \]

where

- \( \ell \) is an allowance for the delay in starting up (s),
- \( g \) is the effective green time (s), and
- \( C \) is the cycle length (s).
Later, Clayton (1955) further introduced the following equation:

$$S = \frac{Dc}{g - \ell}$$  \hspace{1cm} (2.4)

where $D$ is the density of traffic per lane.

Wardrop (1952) raised some questions concerning the basic model and argued the use of maximum density or saturation flow rate for the calculation of capacity. He maintained that the heavily trafficked intersections performed well below the theoretical absolute value of maximum density or saturation flow rate. He suggested a value of 80%-90% of maximum density for the efficient performance of an intersection.

Webster (1958) extended the concepts of previous research and adopted a fully saturated phase diagram to calculate the start loss and end loss as shown in Figure 2.2. He developed a model for delay based on a simulation, which is referred as the Webster formula for the delay imposed on a single traffic stream by a traffic signal operating on a fixed-time schedule.

$$c_0 = \frac{aL + b}{1 - Y}$$  \hspace{1cm} (2.5)

where

- $c_0$ is the optimum cycle time
- $Y$ is the sum of critical flow ratios $(\Sigma y)$
- $L$ is the total lost time per cycle
- $a$ & $b$ are constants

The work of Webster was extended later and the variance of the number of Passenger Car Units (PCUs) was introduced to allow the randomness of arrivals (Miller, 1963, Miller, 1968).

Webster’s formula was derived from the simulation of traffic flow. Later, it was followed by Webster and Cobbe (1966) and became a standard reference for fixed time signal control. This formula is used to justify a simple rule for calculating fixed-time settings which approximately minimizes the total of all the delays experienced by all vehicles passing through the junction in a unit time. Webster’s work involved following three phases;
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Theoretical modelling

Simulation to validate the delay formula

Optimisation to estimate reasonable signal settings

Most studies succeeding Webster’s pioneering work included all three elements in their research framework. As a result of the computer simulation of flow at traffic signals carried out by the Road Research Laboratory, it was possible to show the variations of average delay with cycle time that occur at any given intersection when the flows on the approaches remain constant. Figure 2.3 explains the relationship between delay and cycle time.

The methodology of equilibrium of degrees of saturation or v/c ratio is adopted by HCM. A critical v/c ratio can be calculated by combining all critical flow ratios for the available cycle time. This methodology makes the calculations simplified as performance of the intersection can be determined by a single measure of v/c ratio.
The central assumption in modelling the cycle time of signalised intersections in many studies is that the average flow rate is fixed along each lane approaching the intersection. This fixed rate of flow remains unaffected by the timing chosen. Unfortunately, however, this simple assumption became too popular and was extensively used in design of almost every signalised intersection on the planet. It has become the single greatest barrier to the effective design of traffic signal timings to achieve reasonable targets today (Button and Hensher, 2001, Abidin, 2007).

The procedures for the optimum cycle time calculation described above assume that saturation flow remains constant with increasing green time. However several researchers recorded some situations where they observed an increase in saturation flow with increasing green time. In those circumstances, longer cycle time seems to be preferable. These circumstances should be studied further.
2.4 Saturation Flow Rate

Saturation flow rate is considered one of the most important elements in the design of traffic signal systems. The concept of saturation flow rate is devised based on the earlier work in the late 1950s and early 1960s (Webster, 1958, Webster, 1964, Webster and Cobbe, 1966). Three methods of saturation flow rate are TRRL method which was published in 1963, Regression method proposed by Branston and Van Zuylen in 1978 and Headway method used in HCM.

2.4.1 TRRL 1963 Method

This procedure uses the methodology defined in the Transport and Road Research Laboratory Road Note 34 (1963). In this method, vehicles are counted in three saturated green intervals and the saturation flow rate is calculated as the number of vehicles in the middle interval divided by the length of the interval.

In this procedure, vehicles are counted in periods of 5 seconds each and average is taken. The first time period is ignored and all time periods that are not fully saturated are also omitted. The rest of the periods are considered as saturated and taken into account for calculations.

2.4.2 Regression Method

Branston and Van Zuylen (1978) proposed the regression method to simultaneously determine saturation flow rate, passenger car units (pcu), and start and end lags. Multiple linear regression technique estimates the saturation flow rate by vehicle car departure observations. In this approach, multiple linear regression techniques are employed for the estimation of saturation flows and passenger car equivalents. The number of passenger cars departing straight through is regressed against the span of the counting period and the number of other vehicles departing. Branston and Van Zuylen (1978) further evaluated the effects of two different counting procedures on the resulting saturation flow rates. They termed these counting procedures synchronous and asynchronous counting. For synchronous counting, vehicle departure is terminated exactly at the time of departure for the specified vehicle, while for asynchronous counting, vehicle departure can be
terminated at any arbitrary point. When both approaches were tested, utilizing data from a North London location, they observed that there was no remarkable difference between the estimates of saturation flow rates established by both approaches and documented an average saturation flow rate of 1750 passenger cars per hour of green time per lane (pcphgl) for the 10ft lane that was observed. The method of saturation green time is given as:

\[ T = a_1 n_1 + a_2 n_2 + \ldots + a_n n_n \]  

(2.6)

where

- \( T \) is the saturated green time (s);
- \( a_1, a_2, \ldots, a_n \) are pcu coefficients;
- \( n_1, n_2, \ldots, n_n \) are number of vehicles in each group crossing the approach during the time

\( T \) is a time at which traffic flow becomes saturated. This is determined in pce/hour/lane passenger car equivalent (PCE) units instead of vehicles, which is computed by converting all vehicles passing through the stop line at each five seconds time interval into, calculated using a factor of 0.25 for motorcycles, mopeds and scooters, 1.0 for passenger cars, vans and taxis and 2.0 for buses. (Minh and Sano, 2003).

### 2.4.3 Headway Method

The Headway Method is the method most commonly used in practice, using time headways after a certain time or number of vehicles to determine saturation flow rate. Greenshields et al (1946) were among pioneers who used this method to estimate the saturation flow rate at signalised intersections. The technique uses the average time headway between successive vehicles in the queue at the stop line. This headway is averaged after removing the headways of the first few vehicles in order to avoid inclusion of lost time during acceleration from stationary to moving position (Akcelik et al., 1999, Boumediene et al., 2009).

The HCM recognises the minimum discharge headway as a measure of the maximum capacity of a traffic lane. In the Headway Method, this minimum discharge rate is
measured in units of seconds per vehicle and represents the inverse of saturation flow rate. The discharge headway reported to be varying during the initial portion of green time. This methodology illustrates the headway eventually stabilises at a constant value after the first few vehicles.

2.5 Variations in Saturation Flow Rate

The Highway Capacity Manual (2000) reports a constant headway of 1.9 seconds after the fourth or fifth vehicle, which corresponds to about 1900 passenger cars per hour of green time per lane (pcphgpl). Clayton (1941) suggested 1500 pcphgpl for saturation flow rate based on his field observations. The same value was proposed in the first edition of the HCM published in 1950 but was later revised in the second edition of the HCM published in 1965 and increased to 1,800 pcphgpl (HCM, 1950, HCM, 1965). The value remained unchanged in the 3rd Edition of the HCM (HCM, 1985). In a special report 209 published by the Transportation Research Board in 1997, the value was further increased to 1900 pcphgpl (HCM, 1998). This value has been retained for the base saturation flow rate in the latest edition of HCM (2000). The saturation flow rate can be estimated from the base saturation flow rate after applying some adjustment factors to better represent the prevailing roadway geometric, traffic and signal control conditions (HCM, 2000). There is no better substitute for accurately collected field data. When such field data is available, saturation flow rate can be directly measured from the average headway measurements between successive vehicles entering the intersection.

Since the research of Clayton (1941), a great deal of research has been conducted on the saturation flow rate, proposing values ranging from 1500 to 2500 pcphgpl (Lin et al., 2004, Teply, 1983). The base saturation flow rate has been assumed to be constant for traffic signalised intersections during the green time. The methodology proposed in HCM (2000) measures saturation flow rate by averaging the saturation headways, which is defined as the average headway after the fourth vehicle in the queue, until the queue dissipates. The saturation flow rate is then determined by the Eq. (2.1).

Lee and Chen (1986) documented that the minimum discharge headway is significantly dependent on the queue length. They noted reduction in headway with longer queue lengths. Their study was based on the data collected from 16 intersections with 1899 single
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lane traffic queues. They also observed that the neither signal type nor time of the day has any significant influence on the discharge headway. However speed limit affects discharge headway.

In recent years, field observations conducted in Hawaii, Taiwan and USA have challenged the assumption that the saturation flow rate remains constant after the first three to five cars have passed through the stop line until either the queue is exhausted or the green period ends (Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004). Li and Prevedouros (2002) observed fluctuations in the queue discharge rate for different queue positions in Hawaii. Lin et al. (2004) indicated, based on their study in Taiwan, that the queue discharge flow rate does not rise quickly to maximum level; instead, it rises gradually. This rising trend of maximum discharge flow rate is also observed in another study of three intersections at Long Island, New York (Lin and Thomas, 2005). Teply (1983) also noted, based on his field observations conducted in Canada, that the saturation flow rate varied with the length of green time.

Long (2007) presented an analytical model to replicate queue discharge behaviour at signalised intersections and verified his model against data collected in the USA. The significance of this model is that, unlike others, it did not assume the saturation flow rate to be a constant rate. However, although the gradual increase in saturation flow rates observed in the field data was well replicated by the model, Long (2007) did not present the complete mathematical expression of his model. It was mentioned that the analytical solution for the model is difficult to obtain and it must be solved by some root-search techniques.

2.6 Existing Models for Saturation Headway

A great deal of study has been conducted in the past to quantify the maximum queue discharge rate at signalised intersections. Clayton (1941) stated that the saturation flow rate at signalised intersections typically ranges from 1200 to 1800 pcphgpl. Greenshield et al. (1946) observed a queue discharge rate of 1714 pcphgpl after 6 vehicles passed through the stop line. The HCM (1950) suggested a value of 1500 pcphgpl for saturation flow rate, which was later revised to 1800 pcphgpl in 1965. In 1997, this value was revised again to 1900 pcphgpl, which remained the same for all subsequent editions of the manual (HCM,
The assumption of a constant saturation flow rate is easy and uncomplicated to apply, making it convenient for use in capacity and cycle time equations. However, there were several studies questioning the validity of this assumption. Several models were also proposed to cater for an increasing trend in saturation flow rate. Briggs (1977) was first to propose a deterministic model to incorporate an increasing trend in queue discharge rate. His model assumes a constant acceleration for queued vehicles and takes the following form;

\[ h_n = T_r + \frac{\sqrt{2.7}}{A} \left( \frac{2.7 \cdot n}{A} - \frac{2.7 \cdot (n - 1)}{A} \right) \]  \hspace{1cm} (2.7)

For \( d \cdot n < d_{i,\text{max}} \), otherwise

\[ h_n = T_r + \frac{d_i}{V_q} \]  \hspace{1cm} (2.8)

where
- \( h_n \) is the headway of the \( n \)th vehicle
- \( n \) is the queue position
- \( V_q \) is the desired speed of queued traffic
- \( d_i \) is the distance between vehicles in a stopped queue
- \( d_{i,\text{max}} \) is the distance travelled to reach speed \( V_q \)
- \( T_r \) is driver starting response time
- \( A \) is constant acceleration of queued vehicles

This model has two sections. The first consists of vehicles trying to achieve a desired speed, \( V_q \), at the beginning of the green phase, while the second is comprised of a state where vehicles have already achieved the desired speed and travel at a constant speed. The first section gives a good approximation of queue discharge behaviour at the beginning of the green phase, however, the second section gives a constant headway. The values used in this model suggest that the minimum headway is achieved after 6 vehicles. The predictive ability of the Briggs’ (1977) headway model is demonstrated in Figure 2.4. Briggs model yields a relatively good comparison with the past studies and appears to explain the trend towards decreasing headways till 6th vehicle. Afterwards the trend is flat.
and shows a constant headway.

Bonneson (1992) reported that headway continues to decrease until the 8th or 9th vehicle in the queue crosses the stop line, after which a constant level is attained. He presented his model based on driver reaction time, vehicle speed and acceleration. Unlike Briggs’ (1977) model, Bonneson’s model was devised based on a non-constant acceleration rate. The headway of \( n \)th vehicle according to this model is:

\[
h_n = \tau N_1 + T_r + \frac{d_i}{V_{\text{max}}} + \frac{V_{\text{sl}(n)} - V_{\text{sl}(n-1)}}{A_{\text{max}}}
\]

(2.9)

![Figure 2.4: Comparison of Briggs'(1977) headway model with the results of other studies](image)

Figure 2.4: Comparison of Briggs’(1977) headway model with the results of other studies

where

- \( \tau \) is the additional response time of the first queued vehicle
- \( N_1 \) is 1 if \( n=1 \), or 0 if \( n>1 \)
- \( T_r \) is the driver response time
- \( d_i \) is the distance between vehicles in a stopped queue
- \( V_{\text{sl}(n)} \) is the stop line speed of the \( n \)th queued vehicles
- \( V_{\text{max}} \) is the maximum speed
- \( A_{\text{max}} \) is the maximum acceleration
For the calculation of $V_{sl}$, an equation is presented:

$$V_{sl(n)} = V_{max} \cdot (1 - e^{-n \cdot k}) \quad (2.10)$$

where $k$ is $\beta/V_{max}$, and $\beta$ is an empirical calibration constant.

Long (2007) presented a driver behaviour model of queue discharge rate in terms of the discharge times of each vehicle in queue. His model considers the distances at which vehicles are stopped in a queue from the stop line, acceleration, and average start up lost time, taking the following form:

$$td_n = w_n + ta_n = \tau + n \cdot T_r + t_a(sa_n, \alpha, \beta) \quad (2.11)$$

where

- $td_n$ is the average discharge time at the stop line of the $n$th vehicle in queue,
- $w_n$ is the start-up waiting time for the $n$th vehicle in a queue,
- $ta_n$ is the average acceleration time,
- $\tau$ is the excess start-up time of the lead vehicle in a queue,
- $T_r$ is the uniform or average start-up response time of each driver,
- $sa_n$ is the average distance of acceleration of the $n$th vehicle from the start of motion to time $ta_n$,
- $\alpha$ is the average initial rate of acceleration and
- $\beta$ is the average rate of decrease in acceleration with increasing speed.

It is difficult to implement this method for practical applications as equation (2.11) cannot be solved easily and needs root-search method to be applied.

Although there are a few models proposed in the literature to approximate variations in queue discharge rate which were calibrated and validated against field observations data with reasonable accuracy, they could not get much attention for practical applications mainly due to the complexity of the models and the number of assumptions made in those models. It would be wise to develop a model which is relatively simple with a minimum number of variables used to capture the observed queue discharge behaviour. Such a model should be easy to implement in practice as well.
2.7 Capacity and Degree of Saturation

The capacity of a lane group is defined as the number of vehicles that can be discharged through the intersection per hour during the allocated green time. Mathematically, it can be expressed as:

\[ c = s \frac{g}{C} \]  \hspace{1cm} (2.12)

where

- \( c \) is the capacity of a lane group,
- \( s \) is the saturation flow rate,
- \( g \) is the green time and
- \( C \) is the cycle time.

The Eq (2.12) clearly indicates that the capacity of a lane group can be increased by increasing the amount of green time. This fact is realised and investigated in several studies in past (Bonneson, 1992). However, the increase of green time at one approach may decrease the available green time for the other approach which will likely cause increase in delay for the traffic on other phases. Therefore an overall delay reduction criteria is set to allocate optimum cycle time to the signal phases.

The degree of saturation (DOS), also termed as v/c ratio, is an important variable in traffic signal design, which is considered an indicator of the adequacy of intersection capacity. It can be expressed as follows:

\[ X = \frac{v}{c} = \frac{v.C}{s.g} \]  \hspace{1cm} (2.13)

where \( v \) is the arrival flow rate. The DOS is calculated based on the assumption that the maximum discharge rate will prevail during the allocated green times. While this indicator has shown reasonable results in the highway sections where no complete stop and go scenarios exist, Gilbert (1984) criticized the application of the v/c ratio based on the full saturation flow rate for signalised intersections. In the later versions of the HCM (2000), an effective green time is used instead of total green time to address this problem.
2.8 Cycle Time Formulation

Cycles times are allocated to complete the sequence of signal directions. The allocation of cycle time is conducted based on the volume, intersection configuration, approach speed and coordination with nearby intersections if they are located too close to another intersection or part of a series of intersections on a long stretch of a road. Webster (1958) proposed the following cycle time formulation based on a mathematical simulation for a range of traffic conditions to minimize delays at signalised intersections.

Based on the highest volume to capacity ratio, Webster (1958) presented the minimum cycle time model to minimise overall delay at an intersection which is presented in Eq. (2.5) on page 15.

The results observed in the simulation confirmed that a close approximation can be achieved by replacing constants $a$ and $b$ with 1.5 and 5 to form the following equation:

$$c_0 = \frac{1.5L + 5}{1 - Y}$$  (2.14)

Depending on the traffic volume, this formula can give anything from less than 20 seconds to over 200 seconds. However, typical cycle times are selected between 30 seconds and 180 seconds for the stable operation of an intersection. Cycle time can influence capacity as well as the operational characteristics of signalised intersections. More cycle time can add to the capacity of intersections, as increased green time per hour for an intersection reduces lost time, which is necessary for the phase change. This increase in cycle time can be attributed to decreases in delays to some extent. However, providing increased cycle time to a degree can increase delays. The relationship of cycle time with delay is shown in Figure 2.3 on page 17.

Akçelik (1995) extended this formulation and showed that the optimum cycle time to the signal phases is dependent on the criteria being optimised. The different values of $a$ and $b$ constants result in different level of optimised signal timings for a particular site. He proposed a modified formulation as follows:
\[ c_0 = \frac{(1.4 + k)L + 6}{1 - Y} \]  

(2.15)

where \( k \) is a stop penalty parameter, and a typical values set as \( k = 0.4 \) for minimum fuel consumption, \( k = 0.2 \) for minimum cost and \( k = 0 \) for minimum delay. Another approximate method for cycle time calculation is based on balancing the degrees of saturation of all critical movements. This equilibrium method does not guarantee minimum delays however it is considered relatively efficient method to reduce delays at signalised intersections. Akçelik (1995) proposed the minimum cycle time formulation by ensuring the degrees of saturation of all movements are below the maximum acceptable degrees of saturation \((x < x_p)\):

\[ c_P = \frac{L}{1 - U} \]  

(2.16)

where

- \( c_P \) is the practical cycle time
- \( U \) is the intersection green time ratio.

In the HCM (2010), the cycle time is calculated based on the equalization of the volume to capacity ratio as follows:

\[ c = \frac{L.X}{X - \sum y_i} \]  

(2.17)

where \( X \) is the critical volume to capacity ratio.

### 2.9 Delay

In traffic engineering terms, delay is termed as the sum of the delay due to the operation of signal control. Delay at signalised intersections is defined as the additional time required by a vehicle due to signal control. In other words, it is the time difference when a vehicle is unaffected by the controlled intersection and when a vehicle is affected by the controlled intersection. Delay is an important measure widely used in traffic engineering to evaluate the operational efficiency of signalised intersections. It is the only factor on which the level of service rating is determined. However the measurement of the delay in the field is a challenging task. Teply (1993) noted in a study that delay cannot be measured accurately.
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In the field and therefore it is not always possible to correlate field delay measurements with analytical delay formulations.

In evaluating the performance of signalised intersection, control delay is an important measure which depicts the portion of total delay attributed to traffic signal control. Control delay includes the delay due to deceleration, delay due to stationary vehicles during red interval, and acceleration delay. A number of approaches has been proposed in literature to estimate the delay at signalised intersections.

2.9.1 Queuing Theory

On signalised intersections, two types of methods are usually employed to analyse the formation and dissipation of queues and to determine the amount of delay incurred by motorists while waiting in or moving through a queue. These methods are deterministic queuing analysis and stochastic queuing analysis. In deterministic queuing analysis, the arrival and departures are considered as determinist. Figure 2.5 shows a typical scenario on which deterministic queuing model is analysed. This model predicts average delay experience by a vehicle within a single cycle. The model assumes that arrivals at signalised intersection are uniform and at constant flow rate. In this model, the effects of acceleration and deceleration are neglected and it is assumed that vehicles decelerate and accelerate instantaneously. The queue formed during the red time at the rate, and dissipates fully if approach is operating in under-saturated conditions.

In over-saturated conditions, the number of vehicles arrivals at the intersections exceeds the number of departures. Therefore a residual queue appears at the end of each cycle. This phenomenon is shown in Figure 2.6. This growing residual queue propagates to the subsequent cycles. If the demand does not change, this process of increase in queue will continue infinitely. To make a check on this, an analysis time period \( T \) is used which is usually taken as 15 minutes.

Unlike deterministic queuing that assumes uniform arrival rate, stochastic queuing analysis deals with non-uniform arrival process. In fact, this phenomenon is much closer to actual field conditions where vehicles arrive at signalised intersections at more of random or other complex patterns of arrivals. This makes the accurate estimation of vehicle delay...
difficult and therefore several simplifications are normally made for stochastic analysis. One of the major simplifications made is to use Poisson process for vehicles arrivals with constant mean arrival flow rate throughout the period of analysis (Kyte et al., 2008).

These theories are implemented in delay estimation formulations. Beckmann et al. (1956) were among the pioneers who studied and presented delay formulas for signalised intersections based on queuing theory. Later, Webster (1958) extended his work on a signalised intersection capacity and delays estimation, which was based on the simplified assumption of a constant saturation flow rate. Several factors were previously identified as influencing the saturation flow rate; however the concept remained the same as the one proposed by Webster (1958).

**Figure 2.5:** Deterministic queuing at signalised intersections – under-saturated case
Figure 2.6: Deterministic queuing at signalised intersections – over-saturated case

Delay is the single most important factor that drivers can perceive while on road and because of this it has drawn a lot of attention from researchers in the past. Webster (1958) was among those who presented a delay formula expressed as follows;

\[
d = \frac{c(1 - \lambda)^2}{2(1 - \lambda x)} + \frac{x^2}{2q(1 - x)} - 0.65 \left(\frac{c}{q^2}\right)^{1/3} \times x^{(2 + 5\lambda)}
\]

(2.18)

where

- \(d\) is the average delay per vehicle in seconds
- \(C\) is the cycle time in seconds
- \(\lambda\) is the portion of the cycle which is effectively green for the phase under consideration i.e. \(g/C\).
- \(x\) is the degree of saturation.
- \(q\) is the arrival rate in vehicles per seconds

The first two terms were derived based on queuing theory, where the first term estimates delay using a uniform arrival rate and the second term accounts for the randomness in arrival rate that leads to overflow in queues. The third term was proposed as a correctional term based on empirical observations to compensate for the overestimation of delay when
using only the first two terms.

While a great number of investigations were conducted in the past to quantify the effects of varying arrival flow rate on delay at signalised intersections, the variability in departure flow rate did not get much attention. Despite indications of the presence of a variable discharge flow rate in some studies, the convenient assumption of a constant discharge flow rate was one of the reasons for the lack of attention. The strong evidence for variation in queue discharge flow rate reported in the literature suggests a need for further investigation to verify such trends and to incorporate them into the delay formulation, which might contribute to the improvement of the accuracy of delay estimation. The model is implemented to estimate delay at signalised intersections and then compared with the existing methods of delay estimation.

2.9.2 Shock Wave Theory

Shock wave theory describes the traffic in term of a fluid mass. The changes of the different states on traffic streams are attributed to shock waves. Lighthill and Whitham (1955) and Richards (1956) applied the theory of shock wave first time to transportation facilities. The Shock waves are defined as boundary conditions in the time-space domain that mark off discontinuity in flow density conditions on a traffic stream. They reflect the transition zones between two traffic states that move through a traffic environment like a propagating wave. The control at traffic signal breaks the traffic stream and different flow states generates which can be analysed using shock wave theory by applying flow-density relationship. Figure 2.7 shows the two conditions at signalised intersection. One condition is queue condition (Condition A) where vehicles are in stop condition and \( K_A \) and \( q_A \) are representing the density and flow conditions respectively. Condition B is on the upstream side of the queue, and representing free flowing condition with \( K_B \) and \( q_B \) are density and flow respectively. Shockwaves are very useful in estimating time of formation and dissipation of the flow conditions. They have some limitations as well. They assume that the demand and capacity over the length and the entire time duration of the study section either constant or changes instantaneously to specific constant values at pre-specified points in time.
2. Literature Review

2.10 Delay Models

At signalised intersections, delay is the difference between the actual travel time a vehicle experiences due to traffic control and the travel time in the absence of any traffic control signal. A typical process of delay occurrence is illustrated in the Figure 2.8. The calculation of the delay depends on several factors including the probabilistic distribution of arrival flow (demand), signal timings and departure flow rate (supply), and the time at which the vehicle arrives at the approach. Many of these factors are highly variable, thus making estimation of delay fairly complicated (Kang, 2000). The level of service criteria for signalised intersections is set on estimation of delay. This level of service (LOS) is an indicator of the operational efficiency of an intersection by which the quality of service is determined. The value of LOS is represented by letters A through F, with A being the best and F being the worst. Delay is used to determine the level of service of a signalised intersection, being the only element that is clearly perceived by the drivers.

Figure 2.7: Shock waves at signalised intersections
Different components of delay at signalised intersection are shown in Figure 2.8. Delay experienced at traffic signals can be divided into two components: uniform delay and incremental delay. Uniform delay can be estimated using a deterministic queuing approach considering a simple case of D/D/1. It is a well-known fact that during the congested period, the arrival flow rate approaches a uniform state. In the traditional approach, assuming a uniform rate for arrivals and departures makes it a simple case of area calculation to calculate delay. For this purpose, an assumption is made that all vehicles accumulated during phase pass during green time. Solving this case results in formulation of the first term of the Webster equation of delay - Eq (2.18). However, this assumption cannot be implemented for isolated signalised intersections, where the flow pattern is randomly distributed. To resolve this issue, a component of the random delay equation is introduced that assumes the Poisson distribution for arrivals (Kendall, 1951). The random delay (or overflow delay) component represents that the portion of delay caused by temporary overflow queues resulting from the random nature of the arrivals. The random delay is an additional term introduced to incorporate a component of delay additional to the uniform delay. This random delay component is adopted as a second term in the Webster delay equation.
Theoretically, incorporation of these two terms should represent actual delay, however it was observed that the first two terms produces a higher value of delay. Therefore a third term was introduced which was empirical in nature, derived from the simulation of traffic flow and generally referred to as the correction term (Webster, 1958). The Webster formula for delay was later refined to eliminate this correction term and a factor was introduced instead to reduce the sum of first and second term by 10% (Courage and Papapanou, 1977).

One of the major drawbacks of Webster’s (1958) model was its inability to compute delay at saturation level (x=1). Webster’s model performs reasonably well in under-saturated (x<1) conditions. However, when the approach frequently faces a condition during which the accumulated queue cannot fully dissipate in one cycle, a phenomenon of a growing queue is developed, which is termed overflow delay or incremental delay, as referred in the HCM (2010). Akcelik (1981) developed a formula for the overflow delay component;

\[ d_2 = \frac{cT}{4} \left[ (x - 1) + \sqrt{(x - 1)^2 + \left(\frac{12(x - x_0)}{cT}\right)} \right] \tag{2.19} \]

where \( T \) is the analysis period duration (h) and \( c \) is the capacity (veh/h) and other variables are as previously defined.

This model was incorporated in the HCM (2000) with some modifications. The HCM (2000) model for signalised intersections contains three terms;

\[ d = d_1 PF + d_2 + d_3 \tag{2.20} \]

where

- \( d \) is the average signal delay per vehicle in seconds
- \( d_1 \) is the average delay per vehicle due to uniform arrivals in seconds
- \( PF \) is the progression adjustment factor
- \( d_2 \) is the average delay per vehicle due to random arrivals in seconds
- \( d_3 \) is the average delay per vehicle due to the initial queue at the start of the analysis time period, in seconds

The average delay due to uniform arrivals is computed with the following equation:
\[ d_1 = \frac{0.5C(1 - \frac{g}{c})^2}{1 - \min(1, x) \cdot \frac{g}{c}} \]  

The incremental delay formulation in HCM 2010 is as follows:

\[ d_2 = 900T \left[ (x - 1) + \sqrt{(x - 1)^2 + \left( (x - 1)^2 + \frac{4x}{cT} \right)} \right] \]

The incremental delay is based on the analysis period \( T \), which is taken in hours. Rest of the variables are as defined previously.

The initial delay covers three scenarios. The first scenario covers the initial delay when the initial queue is clearing during analysis period \( T \). The second situation is dealt when initial queue is decreasing and it measures the initial queue delay during analysis period. The third scenario covers a situation when the initial queue is increasing during time period \( T \).

### 2.11 Queue Process at Signalised Intersections

The queue process is important as the traffic arrivals are generally stochastic in nature and the queue process during the signal process is controlled under two conditions. The first condition is when the arrivals are well below the capacity of that approach. In this condition, the queue is formed during the part of the time when no departures are allowed during the red time. Another significantly important aspect is that the vehicles do not necessarily depart in order of arrival. This random nature of the queue process makes it difficult to focus on individual vehicles and therefore attention is given mainly to the total delay due to this queue process during a signal cycle. The methodologies of different capacity guides, however, provide methods to measure the individual delay by dividing the total delay for a cycle by the expected number of vehicles to arrive in a cycle.

#### 2.11.1 ARR 123 Methodology

Akçelik (1981) was among pioneers to explain queue process at signalised intersection in detail. The methodology of ARR 123 (Akçelik, 1995) establishes based on the full stop rate at signalised intersections for uniform arrivals with adding an overflow component.
The uniform component is based on the assumption that the arrivals are uniform with constant headways. The overflow component caters for the random fluctuations in the arrival flow pattern. The effect due to random fluctuations in arrival flow rate is sensitive to the degree of saturation and becomes negligible for low degree of saturation values. The effect is significant for degree of saturation closer to 1. For over saturated conditions, since the queue length increasing infinitely, the time period of oversaturation becomes significant. The queue process is explained as:

\[
N_o = \frac{cT_f}{4} \left( z + \sqrt{z^2 + \frac{12(x - x_o)}{ctT_f}} \right) \]  

(2.23)

where

- \(N_o\) is the average overflow queue in vehicles,
- \(c\) is the capacity in vehicles per hour,
- \(T_f\) is the flow period,
- \(x\) is \(q/c\) (degree of saturation),
- \(z\) is equal to \(x - 1\), and
- \(x_0\) is the degree of saturation below which the average overflow queue is approximately zero and is given by

\[
x_o = 0.67 + \frac{sg}{600} \]  

(2.24)

### 2.11.2 SIDRA Methodology

The explanation of queue process in SIDRA is based on the principles contained in ARR 123 (Akçelik, 1984). It is widely used in intersection capacity analyses and utilises lane-by-lane analysis to carry out all capacity and performance calculations. SIDRA determines the queue length in two ways. It calculates the back-of-queue and cycle average queue during a particular time period. The back-of-queue is the maximum extension of the queue in terms of the number of vehicles during a typical cycle (SIDRA GUIDE, 2010). This concept is important in determining the how far a queue can go and is frequently used for determining the queue storage ratio (Akçelik, 1996). The cycle average
queue is the average length of the queue during each cycle. This queue incorporates all of the states throughout the cycle time (Akçelik, 2003).

2.11.3 HCM 2000 Methodology

The back-of-queue methodology in the HCM (2000) covers two conditions; the first depends on the arrival patterns of vehicles and the second on the number of vehicles that do not clear the intersection during a given green phase (overflow). The procedure is also capable of handling situations that involve multiple time periods, in which an overflow queue may be carried from one time period to the next.

\[ v_L = v + \frac{Q_b}{T} \]  \hspace{1cm} (2.25)

where

- \( v_L \) is the lane group flow rate including initial queue present (veh/h)
- \( v \) is the arrival flow rate (veh/h)
- \( Q_b \) is the lane group initial queue at the start of the analysis period (veh)
- \( T \) is the length of analysis period (h)

In the previous version of HCM (2000), the average back-of-queue is calculated as;

\[ Q = Q_1 + Q_2 \]  \hspace{1cm} (2.26)

where

- \( Q \) is the back-of-queue size (veh/ln),
- \( Q_1 \) is the first term back-of-queue size in veh/ln \( (Q_1 = N_f) \)
- \( Q_2 \) is the second term back-of-queue size in veh/ln

The first term back-of-queue, determined first by assuming a uniform arrival pattern and then adjusting for the effects of the progression for a given lane group. The first term is calculated using the equation;

\[ Q_1 = P F_2 \frac{v_L C \left(1 - \frac{g}{C}\right)}{1 - \left[min(1.0, X_L) \frac{g}{C}\right]} \]  \hspace{1cm} (2.27)

where
2. Literature Review

\[ Q_1 \] is the first term back of queue size in veh/ln \((Q_1 = N_f)\)

\( PF_2 \) is the adjustment factor for the effects of progression

\( v_L \) is the lane group flow rate per lane

\( X_L \) is the ratio of flow rate to capacity \((v_L/c_L)\) ratio

The second term \( Q_2 \) is an incremental term associated with the randomness of flow and overflow queues that may result because of temporary failures, which can occur even when demand is below capacity.

\[
Q_2 = 0.25c_L T [(X_L - 1) + \sqrt{\frac{8k_B X_L}{c_L T} + \frac{16k_B Q_{bl}}{(c_L T)^2}]]
\]

(2.28)

where

\( PF_2 \) is the adjustment factor for the effects of progression

\( c_L \) is the lane group capacity per lane (veh/h)

\( k_L \) is the second-term incremental factor

\( Q_{bl} \) is the initial queue at start of analysis period (veh)

The above equations are dependent on v/c ratio, and the queue increases as v/c ratio increases.

2.11.4 HCM 2010 Methodology

An improvement in the method is introduced based on the queue accumulation and intersection capacity estimation methods in the new release of the HCM (2010). The new methodology is an extended work of Akçelik (1998) which provides a method to estimate the full stop rate at signalised intersections for uniform arrivals with provision for random fluctuations in arrival rates. The initial work relating to the back-of-queue is extended later for other types of arrivals (Olszewski, 1993, Bonneson et al., 2008). In the new methodology, the back-of-queue length is measured in three terms of queue lengths. The significant change adopted is to eliminate the slow and partially stopped vehicles.

2.12 SCATS System

The Sydney Coordinated Adaptive Traffic System (SCATS) was developed in the
seventies by the Department of Main Roads of New South Wales, Australia (Akcelik, 1997, Akcelik et al., 1998). By 1980, SCATS was operational in over 30 cities around the world.

SCATS is a traffic management system that operates in real time, adjusting signal timings in response to variations in traffic demand and system capacity as they occur. SCATS manages groups of intersections through its unique control philosophy, rather than treating intersections as an isolated entity. It combines theoretical models with previously stored library plans. Signal timings at critical intersections are continuously calculated and implemented by this system based on real traffic demand. These calculations are then transferred and implemented to sub-areas which consist of the critical intersection and its adjacent surrounding signalised intersections (Lowrie, 1990, Wolshon and Taylor, 1999).

The philosophy of the SCATS system was to optimize the signal settings to minimise the vehicle stops and delays incorporating radial road operation. It also provides flexibility to incorporate special functions like maximizing the throughput traffic at certain times (Dinopoulou et al., 2000).

SCATS architecture is developed to incorporate groups of signals on a street network. The core of these signal groups is a critical intersection which makes a subsystem that contains the critical intersection and its neighbouring intersections. These subsystems are controlled by a main regional computer. The system is expandable and more regional computers can be connected to extend the control system management area. These regional computers are controlled by a central computer (Sims and Dobinson, 1980). Regional control computers within its sphere can analyse, modify and implement subsystems dynamically. Figure 2.9 shows the typical arrangement in SCATS system.

The control in SCATS can be divided into two levels (Lowrie, 1982).

**Strategic Level:** This control level calculates the signal timings based on typical traffic conditions. Whereas, cycle time and splits are calculated for a subsystem using the loop detector flow and occupancy data. This control can handle up to 10 intersections. The detectors are normally placed at critical points for this strategic level control.
**Tactical Level:** This control level is for optimisation of individual intersections subject to strategic control allowance. The controller can skip stages if no demand exists, however it cannot skip some stages like main roads by default. The control is operated through loop detectors.

### 2.13 Use of Micro-Simulation at Signalised Intersections

Microscopic simulation models are rapidly gaining acceptance as promising tools to analyse and evaluate applications of ITS and other traffic control and management measures by traffic engineers and transportation professionals. Microscopic traffic simulation models describe the system entities and their interactions at high levels of detail. Their applications in solving complex traffic engineering problems have received approval as well as criticism. These models can track and record the movements of individual vehicles, which help and allow the analyst to test a wide range of roadway configurations and operational conditions that far exceed the limits of typical analytical tools (Akçelik and Besley, 2001, Barcelo and Casas, 2002). Additionally, micro-simulation models include highly sophisticated graphical user interfaces (GUI) that allow
visual displays and the demonstration of traffic operations on a computer screen, which was not possible previously in conventional computational tools. These significant characteristics help to enhance understanding both within the transportation professionals as well as those outside the profession (Boxill, 2007). A better visualization of traffic operations thereby allows the graphical presentation (simulating real traffic behaviour), which better conveys the effects of traffic improvements for a better understanding at public meetings. However, micro-simulation models have been criticised for being time consuming, requiring extensive resources and expense when compared with more traditional analytical tools. Figure 2.10 illustrates a general overview of micro-simulation.

Figure 2.10: Overview of Micro-simulation (Adopted from doctoral dissertation of Ranjitkar (2004))

A limited number of studies have been undertaken using micro-simulation approaches to investigate capacity and queue discharge estimations. A general perception is that micro-simulation under-represents the capacity when compared with field observations (FHWA, 1981, Akçelik, 2008). A study is conducted recently using a hypothetical model of a signalised intersection in Q-Paramics micro-simulation software (Q-Paramics, 2006), produced queue discharge flow rates close to what is suggested in ARR 123 (Andjic and Matthew, 2008). However, the input parameters used in the model were unrealistic. For example, 0.5 seconds was used for reaction time which is much lower than the values observed in field (Ranjitkar et al., 2003). Chaudhry and Ranjitkar (2009) modelled a...
signalised intersection in Auckland, New Zealand (the same intersection modelled in this research) in AIMSUN micro-simulation software (AIMSUN, 2008). The results of the study indicated that the queue discharge flow rates obtained from the micro-simulation software are similar to those taken from loop detectors managed by SCATS.

2.14 Gene Expression Programming

The Gene Expression Programming (GEP) is a type of genetic algorithm that comprised of two components of chromosomes and the expression trees. Chromosomes are strings of genes that can take on some value from a specified finite range and represent a solution to a particular problem. The expression trees consist of the expression of the genetic information encoded in the chromosomes. A process of information decoding from the chromosomes to the expression trees is conducted under a certain set of rules. This code adopts a one-to-one relationship between the symbols of the chromosome and the functions and terminals. GEP is similar in nature and falls in the same category of Genetic Algorithm and Genetic Programing (GP), but the main difference is in the approach in which solutions are represented. The procedure in the selection of the genes is same as Darwinian principle and uses the populations of candidate solutions to a given problem in order to evolve new ones.

In GEP, an individual process is represented by a genotype which is comprised of one or more chromosomes. The chromosome is a linear and of fixed length which can be easily manipulated with genetic operators. Roulette-wheel method is used in GEP for selecting individuals. The GEP allows the fast application of a wide variety of mutation and cross-breeding techniques while guaranteeing that the resulting expression will always be syntactically valid. For this purpose cloning of the best individual with the simple elitism is used to preserve for the next phase. The mutation introduces a random modification into a given chromosome. The process GEP adopts is shown in the Figure 2.11.
Figure 2.11 GEP Algorithm
Studies indicated that the GEP performs better and faster than genetic algorithms. The GEP belongs to the wider class of genetic algorithms (GA) along with Genetic Programming (GP). This class of GA uses populations of individuals, selects them according to fitness and introduces genetic variation using one or more genetic operators (Ferreira, 2001). The main difference is that while GA uses linear strings of fixed length, GP uses nonlinear entities of different sizes and shapes and GEP uses linear strings of fixed length converted to nonlinear entities of different sizes and shapes.

The process GEP adopts during model development is systematically explained in Figure 2.11. Mathematical evolution in the GEP starts with the creation of an initial population of candidate functions. The evolution engine then mutates, breeds and, finally, a mathematical function is evolved as a result of natural selection. The main steps involved in the training and evolution of a GEP are as follows:

1. Create an initial population of viable individuals (chromosomes)
2. Use evolution to attempt to create individuals that fit the data well
3. Use evolution to try to find a simpler, more parsimonious function
4. Use nonlinear regression to find the optimal values of constants

The process of transformation of the parameters into a corresponding form is called encoding. This process of transformation is called the Karva Language which was devised by Ferreira (Ferreira, 2001). The encoding process produces a natural tree structure where internal nodes as operations and leaf nodes as variables or constants.

Genetic algorithms work by evolving sets of individuals called chromosomes. This evolution process is performed on the basis of an initial founder population of individuals that can mutate, breed and be selected for subsequent generations. The initial population creation is done by randomly selecting functions and terminals for the genes.

In terms of performance, GEP is considered as extremely useful and it outshines the existing evolutionary techniques. Some experiments have indicated that it performs better than genetic algorithms (GA). The algorithm in GEP allows the creation of multiple genes to form a sub-expression tree. These multiple genes are the core capability that enables not only the encoding of any conceivable program, but also allows an efficient evolution. Gene
operators then search the solution space and generate valid structures in the form of mathematical expressions. The capability of gene expression is used for the validation of the proposed nonlinear model.
3 METHODOLOGY

This section discusses the methodology adopted in this research. Six intersections were selected to collect the data for this study. A video recording technique was used for recording the traffic flow in peak traffic flow hours. The recorded data was analysed to evaluate the headways. Signal stop line at each approach was taken as a reference line for the analysis and measurements were taken when the front bumper of the car crosses the stop line. Saturation flow rates were calculated by the headway method. Trends were analysed to examine the queue discharge behaviour and a new model is proposed based on the observed behaviours.

3.1 Study Framework

Capacity analysis of a signalised intersection is calculated by determining the saturation flow rate. Determination of saturation flow rate in the field is a cumbersome job and different methods are proposed in literature. Analytical methods, including HCM (2010) and ARR 123 (1981), assume almost identical methodologies in saturation flow rate calculations. HCM (2000) describes direct measurement of prevailing saturation flow rates in Chapter 16, Appendix H. According to HCM (2010), the saturation flow rate represents the maximum rate of flow measured at stop line during the green time. The state of stability is said to be achieved after 10 to 14 s of green time. The direct measurement of saturation flow rate is conducted to cater for the prevailing conditions. The prevailing conditions are those conditions that affect the saturation flow rate due to effect of lane width, traffic composition (heavy vehicles), terrain, and other geometric, control and traffic conditions. In absence of field measurements, a base saturation flow rate is assumed and then saturation flow rate for the prevailing conditions is determined after applying the adjustment factors. The base saturation flow rate is defined as the discharge rate from a standing queue that carries only through passenger cars and is otherwise unaffected by
conditions such as grade, parking, and turning vehicles.

The methodology for determining field saturation flow rate in ARR 123 (Akçelik, 1995) divides the green time into three portions. Saturation flow rate is measured after excluding the first 10 seconds of green time. ARR 123 (Akçelik, 1995) recommends a number of cycles to be observed for greater accuracy in calculation of saturation flow rate. This research adopted existing techniques to measure and verifies the generally assumed sustained saturation flow rate theory. Six intersections were identified for collecting data in Auckland city. The selection of these intersections was made according to a number of factors that are discussed in the subsequent sections in detail.

The methodological framework for this study is presented in Figure 3.1. The research is split into two phases. The first phase covers the practical aspect in which data is collected to study the queue discharge behaviour at signalised intersections. The objective of this stage is to verify the variability of saturation flow rate and determine the tendencies of queue discharge flow rate with respect to queue position and green time. A pilot study was conducted to observe the field trends. Based on the preliminary verification, a detailed methodology is worked out to conduct the extensive data collection from six intersections in Auckland, New Zealand. A criterion was set for selection of the intersections for data collection. The main concern in the selection of the intersections was to highlight those intersections which are not affected by capacity reducing factors.

The data was collected through the video data collection method as well as SCATS data was requested from the Auckland Council. The main factors considered in data collection include: headway, cycle time, saturation flow rate, and delay. The outcome of this phase helped to establish a framework for model development in the second phase. The second phase reviews all the model development methodologies and revisit the cycle time calculation formulation to incorporate results obtained in phase I.
3. Methodology

CAPACITY ANALYSIS

LITERATURE REVIEW

STAGE I DATA COLLECTION

STAGE II MODELLING

QUEUE DISCHARGE BEHAVIOUR

SATURATION FLOW RATE

MICRO-SIMULATION APPROACH

TEST BED / SITE SELECTION

LINEAR MODELS

NON-LINEAR MODELS

MODEL STUDIES

MODEL FORMULATION

COMPARISON

MODEL IMPLEMENTATION

VIDEO DATA

SCATS DATA

HEADWAY

SIGNAL TIME

SATURATION FLOW RATE

DELAY

ANALYSIS

Figure 3.1: Flow Chart of the research work
3. Methodology

3.2 Data Collection

3.2.1 Assumptions

It is a well-known fact that a number of factors affect saturation flow rate. The HCM (2000), and ARR 123 (Akçelik, 1995) described a number of urban conditions and various geometric and traffic configurations that can cause substantial variability in saturation flows. A description of these conditions is highlighted in Table 3.1. Factors used to modify the saturation flow of a given lane or approach generally include the urban environment; local driver behaviour; parking or transit interference; interaction with priority, opposing or adjacent flows; and limited queuing or discharge space. Some methodologies even consider the impact of weather conditions and green interval. While selecting signalised intersection for this particular study, it is particularly crucial that an ideal or close to ideal situation should be established for data collection. Therefore those approaches at signalised intersections are selected that contained baseline geometric conditions and are not influenced by geometric factors that have been reported to be influential in reducing saturation flow rates. Following factors are considered in selection of the intersections;

Geometry

While selection process, the approaches that are crossing perpendicular or closeness to perpendicular, are being selected for data collection. The best alignment at grade intersections is believed to be when the intersecting roads meet at right or nearly right angles (Garber and Hoel, 2010).

Configuration

The lane configuration factor is taken into account when seeking the ideal intersection and priority is given to intersections with at least one exclusive through lane. Balmoral – Dominion Road intersection and Balmoral – Sandringham Road Intersection have exclusive left turn slip lanes.
3. Methodology

Traffic composition

Cars constitute the commonest vehicle type in the Auckland urban area. While selecting an intersection, great care is taken to ensure that those intersections are selected where cars form the maximum percentage of vehicles. This is done in order to study the real field situation with minimum conversions.

Bunching Effects

One of the most important factors in analysis of isolated signalised intersections is the effect of a nearby intersection if it is closely spaced. This effect is very important as it can reduce the saturation flow rate substantially. This factor is also considered and the approaches that are likely to produce the bunching effect are simply crossed out.

Other factors that could affect saturation flow including bus blockage, parking, pedestrian and bicycles activity.

The selection of intersections is intended to minimize factors that can affect the saturation flow rate. The intersections are highlighted with minimum disturbance in terms of bus blockage, parking, pedestrians, bicycle activity and other factors.

Table 3.1: Intersection selection conditions

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Condition or Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lane width</td>
<td>3.6m</td>
</tr>
<tr>
<td>Percent heavy vehicle</td>
<td>0 %</td>
</tr>
<tr>
<td>Approach grade</td>
<td>0 %</td>
</tr>
<tr>
<td>Parking condition</td>
<td>No curb parking</td>
</tr>
<tr>
<td>Local bus stop</td>
<td>No nearside or far side bus stop activity</td>
</tr>
<tr>
<td>Area type</td>
<td>Suburban area, no central business district or residential area</td>
</tr>
<tr>
<td>Permitted movement</td>
<td>At least one exclusive lane for through movement</td>
</tr>
</tbody>
</table>
3. Methodology

3.2.2 Field Surveys

The control for cameras were planned to setup at the green area, and recording were to be done in the morning peak hours from 6:55AM to 9:00AM and in the evening peak hours from 3:55PM to 6:00PM. All the safety procedures of University of Auckland and Auckland Council were adopted during the data collection activity. Permission for conducting data collection activity was acquired from Auckland Council. Recorded data is stored in a hard drive and is analysed later for saturation flow and delay calculations.

3.2.3 Data Matrix

Six intersections selected have four approaches each which make the total number of approaches 24. Table 3.2 below shows the calculated data matrix for proposed data collection on six intersections. Out of these 24 approaches, four approaches were crossed out for data collection due to different reasons. At Balmoral – Dominion Road intersection, the south and north approach have parking, commercial activity and bunching effect due to pedestrian signals respectively. On St. Lukes – New North Road intersection, the south approach has no exclusive through lane, so this approach was crossed out for data collection. The northern approach at Great South Road – South Eastern Highway intersection has a curve which can affect the saturation flow rate. This approach was also excluded from the data collection.

3.2.4 Safety Aspects

Sufficient safety arrangements were made to ensure the safety of the people and equipment during data collection activity. The University of Auckland policy for safety was followed during collection of data. Safety and operational aspects of traffic were carefully monitored. At the mounting and dismounting of camera assemblies at intersections, all measures were adopted to avoid interference with the traffic.
3. Methodology

Table 3.2: Data collection matrix for selected intersections

<table>
<thead>
<tr>
<th>Location</th>
<th>Approach</th>
<th>EAST</th>
<th>West</th>
<th>South</th>
<th>North</th>
<th>Number of Cycles</th>
<th>Data Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balmoral-Dominion Road</td>
<td>Through</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td>60</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shared</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pah - Mt Albert Road</td>
<td>Through</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>480</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shared</td>
<td>1LT</td>
<td>1LT</td>
<td>1LT</td>
<td>1LT</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manukau - Greenlane East Road</td>
<td>Through</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td></td>
<td>Shared</td>
<td>1LT</td>
<td>1LT</td>
<td>1LT</td>
<td>1LT</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balmoral - Sandringham</td>
<td>Through</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>60</td>
<td>840</td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
<td>Shared</td>
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<td>0</td>
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<td>0</td>
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</tr>
<tr>
<td></td>
<td>Total</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St Lukes- New North Road</td>
<td>Through</td>
<td>2</td>
<td>1</td>
<td></td>
<td>2</td>
<td>60</td>
<td>480</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>1</td>
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<tr>
<td></td>
<td>Shared</td>
<td>1LT</td>
<td>1LT</td>
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<td>1LT</td>
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</tr>
<tr>
<td></td>
<td>Total</td>
<td>3</td>
<td>2</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Great South Road - South Eastern Highway</td>
<td>Through</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
<td>60</td>
<td>480</td>
</tr>
<tr>
<td></td>
<td>Right</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Shared</td>
<td>1RT</td>
<td>0</td>
<td>1LT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>Approach</strong></td>
<td><strong>7</strong></td>
<td><strong>7</strong></td>
<td><strong>7</strong></td>
<td><strong>7</strong></td>
<td><strong>120</strong></td>
<td><strong>120</strong></td>
</tr>
</tbody>
</table>

The assembly of the cameras was carefully prepared to allow it to be attached to the street light pole. Velcro strips were used for tying this assembly with the signal light or street light pole. A person remained close to the assembly at all times to ensure safety of the equipment and persons.

3.2.5 Data Collection using SCATS loop detectors

The major portion of data collection was conducted through video camera. In addition to that it was also planned to request the SCATS pulse data for the same intersections from Auckland Council for the same period of time. Auckland council was approached for the
3. Methodology

data collection. However, we were only able to access Balmoral – Dominion Road Intersection data from SCATS system. The data provided was in the form of CSV file which was later imported into spreadsheets for further analysis.

3.2.6 Data Retrieval

Saturation flow rate is the maximum queue discharge rate during the green time. The sample worksheet is prepared for recording the retrieved data from the video. The retrieval of the data was conducted manually by playing the video repeatedly. More than one screen was used in order to play and stop the video for noting down necessary data. Signal stop lines were used as reference lines and the front bumper of a vehicle was taken as the vehicle reference point. An on-screen video timer was used for recording the phase times, start up and lost times. Frame rates are used for determining headway between successive vehicles. The data was recorded for only queued vehicles, while those vehicles included in the queue after the signal phase turns to green was excluded from the calculations.

3.2.7 Data Analysis

The retrieved data was transferred to the excel sheets. This data was analysed for necessary statistical analyses and saturation flow rates were calculated based on HCM (2000) and ARR 123 (Akçelik, 1995) methodologies. Each signal phase was divided into three portions for the calculations of saturation flow rate according to ARR 123 (Akçelik, 1995) methodology. HCM (2000) methodology describes the saturation flow rate as the stabilised rate of flow after the 5th vehicle. Trends were calculated based on these two methodologies. These trends were then compared to verify the findings of the previous studies.

3.3 Selected Intersections for Data Collection

3.3.1 Criteria for Site Selection

Six sites (signalised intersections) were selected around Auckland from which video data for a two hour peak period have been collected. Peak traffic period in Auckland is during
7-9AM and 4-6PM. A selection criterion was developed for selecting the intersections to be studied. Criteria details are listed as:

a) High through traffic volume during peak periods
b) Low percentage of heavy vehicles
c) Flat gradient
d) No bunching of signalised intersections
e) Perpendicular signalised intersection
f) Low pedestrian movement
g) Equal lane widths for all sites.

High through traffic was necessary as a queue needed to have formed prior to the signal changing to the green phase. A low percentage of heavy vehicles were needed so that these vehicles would not give large headways and therefore would not interfere with our data. A flat gradient was required so that vehicle speed would be consistent and would not change. The site should not have signalised intersection close to each other as this would cause queues to be formed while the green phase was running and would not allow for a free flow of vehicles. Sites should not have high level of pedestrians as this would cause signal timings and phases to be governed by pedestrians.

### 3.3.2 Selected Sites

As the purpose of the research was to study the saturation flow rate which is an ideal rate of flow for any intersection, the selection of signalised intersections was made to find ideal or close to ideal intersections in Auckland City. During the selection process, the professionals in Auckland City Council and NZTA were consulted. A list of candidate sites for data collection was prepared and forwarded to Senior Engineer in Auckland City Council for his comments. Following the comments of Auckland City Council, candidate sites were reduced to six intersections. The basic purpose of this exercise was to reduce the external factors that can cause reduction in a saturation flow rate which is based on the capacity of a signalised intersection.

Six signalised intersections located in Auckland City are selected for the study. The reason for selecting Auckland City is the traffic flow volume which is very high during
peak periods. Care has been taken in selection of the candidate intersections that no significant approach grades would be present. High volume is expected during peak traffic hours. Bunching effect caused by nearby intersections is offset by selecting intersections that are not too close to adjacent intersections.

For the data collection, the months of February, March and April were selected. At least two cameras were used to cover the full queue length as well as signal times. Recording was planned for two hours in the evening peak hours from 4PM to 6PM. Simultaneously SCATS data was requested for the same period of time. The geometric conditions of the approaches were also considered for any potential impact on detailed analysis. Intersections closer to ideal conditions were shortlisted and data was collected from the following six intersections;

1. Balmoral – Dominion Road Intersection
2. Pah – Mt Albert Road Intersection
3. Manukau Greenlane East/Balmoral Road Intersection
4. Balmoral – Sandringham Road Intersection
5. St Lukes – New North Road Intersection
6. Great South Road – South Eastern Highway Intersection

Intersections at Manukau – Greenlane East/Balmoral Road and St Lukes – New North Road have lower frequency of buses. Balmoral – Dominion Road Intersection, and Balmoral – Sandringham Road intersection have high frequency of buses in north south direction. However, an exclusive bus lane in the north south direction separates the bus traffic from other traffic.

The location of the intersections is shown in Figure 3.2.

3.3.3 Balmoral – Dominion Road Intersection

Balmoral-Dominion Road intersection is one of the important intersections of Auckland urban road network. This intersection, located in the Central District of Auckland, is the junction of two major urban arterials, Dominion Road and Balmoral Road.
Figure 3.2: Selected sites at Auckland

Dominion Road (approximately 7 kilometres) is one of Auckland’s oldest main roads. It was planned in 1940’s to be used as an expressway. Dominion Road is an arterial route and a key transport corridor in Auckland, stretching between the fringe of the CBD and Mt Roskill. It is also a major passenger transport route, carrying 50,000 bus passengers per week (Auckland City Council, 2009, NZTA, 2007). It is a central-spinal arterial connecting the Central Business District (CBD) to other arterial routes including Balmoral Road. Balmoral Road is a central cross city arterial, connecting the eastern suburbs to the central-western suburbs.

Balmoral Road crosses Dominion Road at 90°. There are four exclusive slip lanes for left turns for all approaches. There are no significant approach grades on this intersection. On the northern approach, a pedestrian signal is in place at about 250m from the intersection. On the southern approach, a pedestrian signal is located at 180m. This approach also has commercial activity. Eastern and western approaches are clear from any disturbance factors so these approaches are selected for data collection.
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3.3.4 Pah – Mt Albert Road Intersection

Mt Albert Road is classified as a regional arterial road. It is a cross-city arterial. Pah – Mt Albert Road intersection is a signalised intersection with two through lanes and one exclusive right turning lane at each approach. Three approaches to this intersection have negligible gradients. The southern approach has some gradient. The nearest hindrance to this intersection is a pedestrian intersection at 380m on the western approach. Hui (1981) collected data on this intersection for saturation flow rate and noted the effects of opposed right turns on this intersection. The layout of the intersection is shown in Figure 3.4.

Figure 3.3: Layout of Balmoral - Dominion Road Intersection
3.3.5 Manukau Greenlane East/Balmoral Road Intersection

Manukau Greenlane East/Balmoral intersection is one of those intersections which face high traffic volumes during peak times. This intersection serves the Greenlane East/Balmoral Road which connects the eastern suburbs to the western suburbs including Point Chevalier and Mt Albert. Manukau Road serves the city traffic to the southern suburbs including Royal Oak and Mt Roskill.

Manukau Road crosses Greenlane Road at an angle. Approach slopes are mild and can be neglected at this intersection. The western approach consists of a large horizontal curve, hence it is excluded from the data collection. The layout of the Manukau Greenlane East/Balmoral Road intersection is shown in Figure 3.5.

Figure 3.4: Layout of Pah – Mt Albert Road Intersection
3. Methodology

3.3.6 Balmoral – Sandringham Road Intersection

Sandringham – Balmoral Road intersection is similar in character to the Dominion – Balmoral Road intersection. Sandringham Road is classified as a district arterial road and it is one of the busiest roads with a lot of recreational and commercial activity all along its length. At one end it serves the Kingsland commercial area and Eden park which is the home of cricket and rugby in Auckland and on the other side it connects the Sandringham shops and Mt Roskill commercial area.

Figure 3.4 shows the layout of Balmoral- Sandringham Road Intersection. This intersection consists of the same characteristics as Balmoral- Dominion Road intersection, except that at the northern approach no exclusive slip lane is provided. Slopes are mild and there is no significant disturbance from parking. On the western approach, the intersection at Kingsway Ave is at 280m which can cause a bunching effect.
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3.3.7 St Lukes – New North Road Intersection

New North Road is a regional arterial road and serves the city centre to the southern suburbs including Avondale, Mt Roskill and New Lynn. St Luke Road is also a Regional Arterial road serving major traffic movements between principal sectors of the region. It connects Balmoral Road to the east of the site with Great North Road to the west of the site.

St Luke’s road crosses New North road at right angles. Three approaches are level, however on the southern approach a mild gradient is observed. A pedestrian signal is at 270m on the southern approach which can cause bunching; therefore this approach was excluded for video recording. The eastern approach is not provided with a slip lane. The layout of the intersection is shown in Figure 3.7.

Figure 3.6: Layout of Balmoral – Sandringham Road Intersection
3. Methodology

3.3.8 Great South Road – South Eastern Highway Intersection

The intersection at South Eastern Highway and Great South Road is located in Penrose and both featured roads are high volume. The layout of the intersection is shown in Figure 3.8. The intersection geometry is flat with no significant slopes, however on the northern approach a curve and another signalised intersection is present at 240m, which makes this a poor approach for data collection. Therefore data is collected from the eastern, western and northern approaches. The intersection is at right angles with no noticeable hindrances geometrically.

Figure 3.7: Layout of St Lukes – New North Road Intersection
3.4 Field Observations

The most commonly used recording system is based on the inductive loop detectors embedded in the roadway. In Auckland, this system is used for getting real time data for operating the SCATS system. Most traffic professionals have a long history of working with inductive loop detectors, which work within a 5-10% error margin. This error margin is acceptable for most traffic operations. However, for intermittent flow conditions, particularly in case of signalised intersections, this system is not very useful. One of the main issues with the inductive loop detectors is that they are not meant to measure queue length, which is important aspect of signalised intersection performance. In normal circumstances the system is embedded in the roadway close to the stop line to produce pulse data for arriving vehicles for phase adjustments.

Video camera recordings, if cameras are placed properly, give a good idea of the traffic flow at the intersection. They can also be used for measuring queue length during

Figure 3.8: Great South Road – South Eastern Highway Intersection
recording time. For the purpose of this study, the video cameras are installed on a rod with a small box at the bottom in which the battery assembly is fixed. The camera rod is planned to be assembled well away from the pole location to minimise any hindrance to vehicle and pedestrian traffic. The rod is attached to the street light pole or signal light pole with special velcro strips. A minimum of four to six strips are used for attachment at approximately 0.5m to 0.75m intervals. These velcro strips supported and tied the assembly with the street or traffic light pole. The details of the assembly are shown in Figure 3.9.

![Figure 3.9: Location of camera fitting assembly](image)

### 3.5 Microscopic and Micro-Analytical Analysis

SIDRA is extensively used in Australia and New Zealand for capacity calculations. This software is based on the methodology in ARR 123 (Akçelik, 1995), however a number of amendments and improvements are introduced in the recent version of SIDRA to incorporate the research in Australia and US. Now a full module is available in the SIDRA software which calculates performance measures based on HCM (2000) methodology.
Microscopic models were also be utilised for analysis of the data with micro-simulation tools in order to observe the capability of these tools to replicate actual driving behaviour at signalised intersections.

### 3.6 Modelling the Observed Queue Discharge Behaviour

Signalised intersections are considered as bottlenecks in urban road networks. Road traffic congestion on intersections is a common observation in peak traffic hours. Signalised intersections are analysed based on analytical techniques. These techniques calculate green times based on a constant saturation flow rate. Field observations confirm that the maximum saturation flow rate does not remain constant and variations can be seen during the span of green time. Thus it is important to incorporate these variations in the design of signal control. Various factors can contribute towards these variations. To understand and incorporate these factors, it is essential to analyse the origins of congestion and quantifying its effects that can help optimise use of existing signalised intersections.

Vehicular traffic is conventionally modelled in either macroscopic or microscopic models. Microscopic models are used to describe the individual traffic behaviour in the network, and these are based on car-following, lane changing and route choice models. These models are based on behavioural characteristics and yield detailed performance measures. The usefulness of these models is limited due to their requirement of large amounts of detailed data and its output in the form of performance measures that are stochastic in nature and computationally expensive to estimate. Thus, applying them in the attempt to optimise a network remains a difficult task.

On the other hand, macroscopic models are flow based models and represent vehicular traffic continuously or discretely. The representation of continuous traffic corresponds to fluid approximations and discrete traffic corresponds to queuing theory. These models produce aggregate performance measures. The signalised intersection is a source of urban congestion and it is therefore very important to optimise its performance. Traditionally, models are formulated that can be embedded within the optimisation framework in order to maximise the performance of signalised intersections. Analytical macroscopic models with sound mathematical properties naturally fit within such an optimisation framework.
3. Methodology

The main challenge in formulating these models lies in choosing the trade-off between efficiency and realism.

### 3.7 Equipment Preparation

Data was collected with a video recording technique using high resolution video cameras. The use of cameras for collecting data at signalised intersection dates back to at least the mid-nineteenth century. Greenshild et al. (1946) were among pioneers who collected data through “time-motion pictures” to record everything that happened at signalized intersections within a field of view. Advances in technology made video recording significantly more capable of precisely recording and retrieving the desired information.

Four wireless cameras were acquired for use in the data collection. An extendable survey level staff rod was used and a special assembly was made to fix the cameras to a pole. Initially it was planned that one staff rod would erect four cameras, covering all approaches and signal timings. A demonstration of the camera assembly is shown in Figure 3.10.

![Wireless cameras installed on staff rod](image)

**Figure 3.10:** Wireless cameras installed on staff rod
The cameras were planned to be elevated to a height of between 3.5 m and 4.5 m to get a better view angle of the traffic at stop lines. The extendable survey staff rod was securely tied with the help of velcro strips on the light poles.

The cameras didn’t require cable connections to transfer data, however, power was required for the cameras. A battery pack was prepared for this purpose and the cameras were attached with special connectors. A 12 volt 12AH SLA battery was used in the power pack. The battery used for this purpose was planned to be installed at the base of the staff rod within a casing for the safety purposes.

During first instance of data collection with four wireless cameras, heavy interruptions of the data transfer were encountered. Therefore, the assembly for data collection was modified. In the new assembly, two cameras were installed rather than four. One camera was used to focus on the stop line, while the other camera was pointed towards the signal light. These two cameras were time coordinated for later use for data retrieval. The placement of the first camera is shown in Figure 3.11.

![Figure 3.11: Front camera pointed at stop line](image)
3.8 Site Set-up

An early arrival to the site ensured enough time to select an appropriate location and set up equipment. The tentative setup locations were marked beforehand based on the Auckland Council GIS Viewer. Early arrival to the site also allowed time to move the set up to a better location, in case the earlier designated location was unsuitable for some reason. Equipment used for the recording of the video data included cameras, the extendable survey staff rod, LAN cables, a network box, a power pack and a computer laptop. The cameras were to be mounted on the extendable staff rod so that the height needed for recording could be reached. The laptop was used to transfer video data from the camera to a hard drive. A third party recording software – Milestone XProtect Essential - was used for this purpose. Video data could only be collected when the weather was fine as none of the equipment was waterproof and wet conditions could influence people’s driving.
4 DATA COLLECTION

This chapter discusses the data collection activities conducted on six signalised intersections in Auckland, New Zealand. At signalised intersections, there are several influencing factors which affect the overall capacity of intersections, including pedestrian activity, approach grade, and geometry. Therefore, the selection of sites was made based on avoiding these capacity reducing factors.

Video recording techniques were used to collect data at these intersections with the help of high resolution cameras. The cameras were installed at vantage points that gave a reasonable view of the stop line, as well as the signal light display. Before conducting the detailed data collection, a pilot study was conducted at a single intersection to verify the hypothesis established based on the literature review. Once the necessary verification was conducted and adequate results were obtained, further data collection activity was performed at the rest of the intersections.

The traffic in urban arterials makes two distinctive peaks during a normal week-day. These peaks form during the morning, when people are moving towards their work places, and during the evening, when people are returning home. To find out the peak hours, data from the SCATS was obtained through the NZTA and the Northcote Traffic Management Centre (ATTOMS) for one of the intersections. The ATTOMS traffic management centre in Northcote manages and monitors the Auckland regional road network. The traffic data was in the form of 15-minute loop detector recordings. Figure 4.1 shows 24 hours’ traffic passing at the Balmoral – Dominion Road intersection. It is evident from Figure 4.1 that there are two distinctive peaks, one from 7AM until 9AM and the other from 4PM until 6PM.
4. Data Collection

Figure 4.1: Traffic flow in 24 hours in Auckland during normal week day

4.1 Pilot Study

A pilot study was conducted at one of the intersection in Auckland to verify the trends in saturation flow rate. The Balmoral Road – Dominion Road intersection was selected for this pilot study. The selection for the pilot study was made based on a number of factors, including accessibility, usability, data availability and simplicity. The close vicinity and convenient location of the intersection made it the first choice. The intersection connects two major urban arterials and therefore carries a heavy traffic volume. There were two main considerations for the selection of the intersection for the pilot study:

- The intersection should be heavily congested during peak periods.
- Pedestrian movement should not be noticeable during peak periods

The data for the Balmoral – Dominion Road intersection was obtained before the actual data collection from SCATS system. The data also contained the phasing information and signal timing screen shots of all approaches. The actual data was collected at the north approach of the intersection and the through and right turning approaches were considered. The intersection has an exclusive slip lane for left turning movements, so this movement was not considered in the final analysis.
4. Data Collection

The data itself was collected on a normal week day with no extraordinary conditions for three hours. The summary of three hours’ traffic count data is shown in Figure 4.2. The three hours’ data revealed that a total of 11539 vehicles entered the intersection. The traffic mainly consisted of passenger cars with a volume of 11324, making up 98.1% of the traffic composition. The share of buses was only 1% and the remainder consisted of is general purpose vehicles (GV). The volume of buses was concentrated on the North-South Corridor at Dominion Road with buses making up 1.6% of the vehicles coming from the Northern approach and 1.7% of vehicles coming from the Southern approach. This North-South corridor is one of the most important bus-based public transportation routes in Auckland. Buses make up less than 0.5% of the total number of vehicles for the remaining intersections. A high percentage of passenger car composition with an ideal geometric layout made this intersection an ideal study site for the calculation of queue discharge flow rates.

4.2 Detailed Data Collection

The collection of data from the six intersections involved the recording of the vehicle headway for each vehicle in the queue for that phase. This was done for two hours during peak period. The headways were calculated by measuring the time between successive vehicles crossing over the datum. The stop line was used as the datum in this case. A vehicle is considered departed when the front axle of the vehicle crosses the stop line and headway measured between successive departures. The departures are shown in the Figure 4.3 below. In this case, headway between two vehicles is calculated as a function of time.

The headways obtained were then used to calculate the saturation flow rate for each vehicle in the queue. The headways for each vehicle in the queue were averaged so as to give an average headway for the position of vehicles in the two hour peak period.

4.3 Data Analysis

Individual headway data is processed from the recorded video data for each vehicle in the queue. Table 4.1 presents the results of the data analysis, which shows some variation in
the drivers’ reaction times, however, the averages were between 1 and 1.2 seconds for the first five intersections, which are located in the close proximity to each other. However, the sixth intersection (the Great South Road – South Eastern Highway intersection) shows a different trend with an average reaction time of 2.05 seconds. The start-up lost times were recorded within the range of 1.18 to 3.08 seconds. The high start-up delay for the St. Lukes Road – New North Road intersection could be due to the downstream approach grade followed by a sharp curve resulting an increase in the start-up lost time.

Figure 4.2: Three hours traffic count at Balmoral - Dominion Road Intersection
Individual headway data obtained from the video data depicted variations in queue discharge rate along the queue length. The observations made from the data collection depicted that the minimum constant discharge headway of a traffic stream is not reached after the fifth vehicle as shown in Figure 4.4. The tendency of queue discharge headway to decrease continues until later queue positions. This decrease in discharge headway results an increase in saturation flow rate towards the back of the queue.

**Figure 4.3:** The departure headway at signalised intersection

**Figure 4.4:** Average headway observations at six intersections at Auckland
Much variation is noted for the stopping position of the first vehicles from the stop line. In calculation of start-up lost time, this distance has a significant influence as the stop line is taken as a reference line for headway calculations.

**Table 4.1: Relationship between queue discharge rate and green time**

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Number of Phases</th>
<th>Reaction Time (Start of green to movement of first vehicle)</th>
<th>Start-up Lost Time</th>
<th>Queue Discharge Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balmoral - Sandringham Rd</td>
<td>58</td>
<td>R, Std Avg Med Std</td>
<td>Equation</td>
<td>R²</td>
</tr>
<tr>
<td>Balmoral - Dominion Rd</td>
<td>59</td>
<td>1.02 0.79 1.51 1.78 1.72 0.0099 t + 0.3496 0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GT South Rd - SE Highway</td>
<td>22</td>
<td>1.20 0.57 1.18 1.03 1.26 0.004 t + 0.4853 0.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manukau - Greelane East Road</td>
<td>58</td>
<td>2.05 0.48 2.64 2.34 1.22 0.0098 t + 0.4113 0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pah - Mt Albert Road</td>
<td>60</td>
<td>1.15 0.45 1.24 0.94 1.12 0.0027 t + 0.4711 0.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St Lukes - New North Road</td>
<td>54</td>
<td>1.18 0.41 1.73 2.14 1.41 0.045 t + 0.4618 0.67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( t \) is the time passed after onset of green

To convert the average headways to the saturation flow rate, the first four or five vehicles are excluded and then Eq. (2.1) is used. An increasing trend can be observed for all of the intersections with a varying slope and R² values ranging from 0.44 to 0.83.

The results presented in this section are in line with the findings of previous researchers - that queue discharge rate increases towards the back-of-queue. A possible explanation to increasing queue discharge behaviour observed in the field would be the fact that the drivers located at the back of the queue are generally more likely to be stopped by change of signal to red than those located at the beginning of the queue. This consideration possibly influences driving behaviour at the back-of-queue that results in drivers maintaining minimal headway from the preceding vehicle.
5 SATURATION FLOW RATE

(This Chapter is a slightly revised version of an article published in “Journal of the Eastern Asia Society for Transportation Studies, Vol. 9, No. 0, pp.1628-1643, 2011”)

Abstract

The basic model of traffic operations at signalised intersections is based on the assumption that when a signal changes to green, the flow across the stop line increases rapidly to saturation flow rate, which remains constant until either the queue is exhausted or the green period ends. In recent years, this assumption has been challenged by some field observations in Taiwan and USA. These observations showed marginal increase in queue discharge rate along the queued vehicles’ position. This chapter investigates queue discharge behaviour at signalised intersections based on video data collected from one of the busiest at-grade intersections. The methods proposed in the HCM (2000) and Australian Road Research Report 123 to estimate saturation flow rates were assessed against field observations. The results from field observations were similar to those observed in Taiwan and USA, whereas the micro-simulation results went against some common perceptions that believed micro-simulation under represents queue discharge rate.

5.1 Introduction

Capacity is the most widely used concept in traffic engineering practice (HCM, 2000, Akçelik, 1981). In the planning, design, and operation of signalised intersections, an estimate of lane capacities is required under prevailing road geometric, traffic and signal control conditions. The U.S. “Highway Capacity Manual (2000)” published by Transportation Research Board and a research report ARR 123 on “Traffic Signals:
Capacity and Timing Analysis” published by the Australian Road Research Board are two important references on this concept (HCM, 2000, Akçelik, 1981). Saturation flow rate (SFR) represents the most important single parameter in the capacity and signal timing analysis of signalised intersection (Akçelik, 1981). It is defined as the steady maximum rate at which queued vehicles can be discharged into an intersection while the traffic signal is green.

The basic model of traffic operations at signalised intersections is based on an assumption that when the traffic signal changes to green, the flow across the stop line increases rapidly to saturation flow rate, which then remains constant until either the queue is exhausted or the green period ends (Akçelik, 1981). The model is shown in Figure 5.1. In recent years, this assumption has been challenged by some field observations conducted in Taiwan and USA (Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004). The average headway between successive vehicles in these studies was measured to have a decreasing trend along the platoon through to the last queued vehicle discharged from the intersection, causing some marginal increase in saturation flow rate. Large variations observed in saturation flow measurements challenged the presumption that the base saturation flow rate of 1,900 passenger cars per hour of green time per lane (as specified in HCM (2000)) is either stable or a constant value.

While most of the analytical models are built on the capacity concept, micro-simulation models do not. Micro-simulation models are built on a different modelling paradigm that is based on car-following and lane-changing behaviour of individual vehicles. Various studies have challenged the validity of micro-simulation models in their ability to replicate queue discharge behaviour observed at signalised intersections (FHWA, 1981, Akçelik, 2008).

This chapter reports on a study that investigates queue discharge characteristics at signalised intersections. The methods proposed in HCM (2000) and ARR 123 to estimate saturation flow rate were verified against field observations taken from a busy signalised intersection in Auckland, New Zealand during peak hours. AIMSUN micro-simulation model is used to analyse the Intersection. AIMSUN is developed in Spain and frequently used for micro-simulation modelling in traffic engineering field (AIMSUN, 2008). The
5. Saturation Flow Rate

A micro-simulation model is built to observe performance of the model in queue discharge behaviour after making a comparison with the observed field data.

![Diagram of saturation flow rate](image)

Figure 5.1: Traditional concept of saturation flow (Source: ARR 123 (Akçelik, 1981))

5.2 Saturation Flow Rate

As discussed earlier, saturation flow rate (SFR) is the most important parameter used in intersection design to determine the intersection capacity and level of service. Clayton (1941) suggested 1500 passenger cars per hour of green time per lane (pcphgpl) for saturation flow rate based on his field observations. The same value was proposed in the first edition of HCM (HCM, 1950), which was later revised in the second edition of HCM published in 1965 and increased to 1,800 pcphgpl (HCM, 1965). In a special report # 209 published by Transportation Research Board in 1997, the value was further increased to 1900 pcphgpl (HCM, 1998). This value has then remained the same for base saturation flow rate in the latest edition of HCM (2000). Saturation flow rate can be estimated from the base saturation flow rate after applying some adjustment factors to better represent
prevailing roadway geometric, traffic and signal control conditions (HCM, 2010). There is no better substitute to accurately collected field data. When there is such data available, saturation flow rate can be directly measured from the average headway measurements between successive vehicles entering the intersection.

Since the research of Clayton (1941) a great deal of research has been conducted on the saturation flow rate, proposing values ranging from 1500 to 2500 pcphgpl (Lin et al., 2004, Teply, 1983). The base saturation flow rate has been assumed to be constant for traffic signalised intersections during the green time. The methodology proposed in HCM (2000) measures saturation flow rate by averaging the saturation headways, which is defined as the average headway after fourth vehicle in the queue till the queue dissipates. The saturation flow rate is then determined by the following equation;

\[ s = \frac{3600}{h_s} \]  

(5.1)

where

\[ s \] is the saturation flow rate in pcphgpl and

\[ h_s \] is the saturation headway in seconds.

The following equation can then be used to calculate the subsequent capacity of the signalised intersection.

\[ c = s \frac{g}{C} \]  

(5.2)

where

\[ c \] is the capacity of approach in passenger cars per hour (pcph),

\[ g \] is the effective green time (s), and

\[ C \] is the cycle length (s).

In recent years, the assumption that the saturation flow rate remains constant after the first three to five cars has passed through the stop line until either the queue is exhausted or the green period ends has been challenged by field observations conducted in Taiwan and USA (Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004). Li and Prevedouros (2002) observed fluctuations in the queue discharge flow rate (QDFR)
for different queue positions in Hawaii. Lin et al. (2004) indicated, based on their study in Taiwan, that the queue discharge flow rate does not rise quickly to maximum level, instead, it rises gradually. This rising trend of maximum discharge flow rate is also observed in another study of three intersections in New York (Lin and Thomas, 2005). Teply (1983) based on his field observations conducted in Canada also noted that saturation flow rate varies with the length of green time.

Long (2007) presented an analytical model to replicate queue discharge behaviour at signalised intersections and verified his model against data collected in USA. The significance of this model is that, unlike others, it did not assume the saturation flow rate to be a constant rate. The gradual increase in saturation flow rates observed in the field data was well replicated by the model however, the complete mathematical expression of the model is not available which makes it difficult to follow. It was mentioned that the analytical solution for the model is difficult to obtain and it must be solved by some root-search techniques.

5.3 Micro-Simulation and Capacity of Signalised Intersections

Microscopic simulation models are rapidly gaining acceptance as a promising tools to analyse and evaluate applications of ITS and other traffic control and management measures by traffic engineers and transportation professionals. Microscopic traffic simulation models describe the system entities and their interactions at high levels of detail (Bleile et al., 1996). Its applications in solving complex traffic engineering problems have received popularity as well as criticism. These models can track and record the movements of individual vehicles, which help and allow the analyst to test a wide range of roadway configurations and operational conditions that far exceed the limits of typical analytical tools (Bayarri et al., 2004). Additionally, micro-simulation models include highly sophisticated graphical user interfaces (GUI) that allow visual displays and the demonstration of traffic operations on a computer screen, which was not possible previously in conventional computational tools. These significant characteristics help to enhance understanding both within the transportation professionals as well as those outside the profession (Manstetten et al., 1998). A better visualization of traffic operations thereby allows the graphical presentation (simulating real traffic behaviour) and a better
understanding in public meetings; the effects of traffic improvements. However, micro-simulation models have been criticised for being time consuming requiring extensive resources and expense when compared with more traditional analytical tools (Lee et al., 2007).

A limited number of studies have been undertaken using micro-simulation approach to investigate capacity and queue discharge estimations. A general perception is that micro-simulation under represents the capacity when compared with field observations (FHWA, 1981, Akçelik, 2008). Andjic and Nigel (2008) using a hypothetical model of a signalised intersection in Q-Paramics micro-simulation software (Q-Paramics, 2006); produced queue discharge flow rates close to what is suggested in ARR 123 (Akçelik, 1995). However the input parameters used in the model were unrealistic. For example, 0.5 second was used for reaction time which is much lower than the values observed in field (Ranjitkar et al., 2003). Chaudhry and Ranjitkar (2009) modelled a signalised intersection in Auckland, New Zealand in AIMSUN micro-simulation software (AIMSUN, 2008). The results produced on queue discharge flow rates were quite comparable to those taken from loop detectors managed by SCATS; without compromising on input parameters values.

5.4 Data Collection

The Balmoral -Dominion Road intersection was selected as a test bed for this research. This is one of the busiest signalised intersections in Auckland, New Zealand. It is a four legged urban intersection with a posted speed limit of 50km/h and roadway lane widths of 3.3 m. The intersection is located on flat terrain with a small proportion of heavy commercial vehicles (less than 3%) and quite low pedestrian movements. The Dominion Road is a major arterial road that connects several suburbs in central Auckland with the CBD. The Balmoral Road is also a major arterial road connecting North-Western Motorway No. 16 with Auckland Hamilton Motorway SH 1. The layout of the intersection is shown in Figure 5.2. Most of the major urban intersections in the Auckland region operate under the Sydney Co-ordinated Adaptive Traffic System (SCATS). SCATS is a traffic management system that operates in real time, adjusting signal timings in response to variations in traffic and system capacity as they occur (Sims, 1979). The maximum intersection cycle time being employed by SCATS is 120 seconds. The composition of the
traffic at this particular intersection mainly consists of cars commuting from home to work place and back.

The field verification data was collected by the use of video recording techniques. The original video data has a resolution of 30 frames per second, which was later reduced to 5 frames per second during the data reduction steps to simplify the calculations with an accuracy of 0.2 seconds. Signal green times, start-up response time (SRT) and headways are calculated from the video data in the evening peak period. Thirty four cycles have been observed for the north and east approach, which carry the heavy tidal traffic load in evening peak hours. The weather at the time of data collection was clear and there were no special events which could cause any disturbance to normal daily traffic conditions. The screenshot of the video data is shown in Figure 5.3. The recorded data contains onscreen timer in a time format of 00:00:00 (hh:mm:ss).

In the evening peak hours, the east and north approaches were observed to have high traffic volume density as the traffic commuting from work places in the central city area to their homes predominates. The data was retrieved frame by frame by playing the video and noting the time of each vehicle crossing the stop line. After noting each vehicle the video was stopped to subsequently transfer the data to spread sheets. Then the video was played again. Each frame corresponds to 0.2 seconds in the video and headways are calculated with reference to the stop line. The start-up response time (SRT) was calculated from the time the signal turns green to the time a vehicle starts moving. Additionally, signal timings were recorded for actual green, red and amber phases. Individual headways from the first vehicle through to the last vehicle were measured. These measured headways were then used for calculations of average headway for each vehicle position in the queue and subsequently saturation headways from the fourth to the last vehicle in a queue is determined.

The methodology in ARR 123 (Akçelik, 1995) recommends saturation flow estimation by dividing the green time into three portions. The first 10 seconds are counted towards lost time and saturation flow rate is measured on the basis of the second portion of green time in which queuing vehicles passing the stop line is counted and divided by the second portion. Saturation flow rates based on the ARR 123 (Akçelik, 1995) methodology was then calculated and compared with the HCM (2000) method.
The micro-simulation approach is a good means of identifying the capacity and saturation flow of intersections. Observed field data was also compared with the micro-simulation approach in order to verify the applicability of the micro-simulation approach. Results of the micro-simulation model were also considered in the particular perspective of a general hypothesis regarding micro-simulation; that it generally under-represents the capacity. The micro-simulation model used for this study was AIMSUN (2008).
5.5 Data Analysis and Results

The summary of three hours of traffic count data is presented in Table 5.1. The three hours data revealed that a total of 11539 vehicles entered the intersection. The traffic mainly consists of passenger cars with a volume of 11324 making a 98.1% of traffic composition. The share of buses is only 1% and the remaining composition is general purpose vehicles (GV). The volume of buses is concentrated on the north-south corridor at Dominion Road with 1.6% buses coming from the north approach and 1.7% buses coming from the south approach. This north-south corridor is one of the most important bus based public transportation routes in Auckland. The percentage of bus volumes on the remaining intersection approaches are less than 0.5%. A high percentage of passenger car composition with an ideal geometric layout makes this intersection an ideal study site for the calculation of queue discharge flow rates.

Figure 5.3: Screen shot from video data taken of Balmoral-Dominion Road Intersection
5.5.1 Headway Characteristics

Time headways data measured from the video data was processed to plot the frequency distribution graph as presented in Figure 5.4. This figure shows that the headways observed in the particular intersection studied is broadly scattered in the range of 1.4 seconds to 2.8 seconds with the maximum proportion of drivers following at 1.6 second headways. The observed mean headway is 1.95 seconds. A chi-square test was performed on the headway distribution. The test results are presented in Figure 5.4. Chi-square test revealed that the headway data neither fit with a log-normal nor normal distribution curve.

Table 5.1: Traffic counts at the intersection (3 hours summary)

<table>
<thead>
<tr>
<th></th>
<th>TO BALMORAL RD EAST</th>
<th>TO DOMINION RD NORTH</th>
<th>TO DOMINION RD NORTH</th>
<th>TO DOMINION RD NORTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>595</td>
<td>593</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>GV</td>
<td>3</td>
<td>379</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cars</td>
<td>528</td>
<td>520</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

For all approaches:
- Total cars: 11324 (98.1%)
- Total GV: 99 (0.9%)
- Total Buses: 117 (1.0%)
5. Saturation Flow Rate

Figure 5.4: Distribution of time headway from field observations

Figure 5.5: Variations in queue discharge flow rate on cycle by cycle bases

5.5.2 Queue Discharge Flow Rate

Figure 5.5 presents the cycle by cycle variation of maximum queue discharge flow rate based on the HCM (2000) and ARR 123 (Akçelik, 1995) methodology. The saturation
flow rate is measured with the HCM (2000) and ARR 123 methodology. The methodology in HCM (2000) describes saturation flow rate is achieved after the 4th vehicle enters the intersection. The ARR 123 (Akçelik, 1995) divides the total green time in three parts and the saturation flow rate is measured after the first 10 seconds. Both methodologies recommend measurements for a number of cycles to determine the saturation flow rate. The average saturation flow rate calculated is 1876 pcphgpl by the HCM (2000) method and 1883 pcphgpl by ARR 123 (Akçelik, 1995) method.

Figure 5.6 presents the average queue discharge rate as a function of queue position in a group of three vehicles. A steady increase in the average discharge rate can be observed along the platoon. The increase from one group to another is statistically significant particularly between groups 4-6 and 7-9 and groups 10-12 and 13-15. The saturation flow rate obtained by the analytical methodologies is close to the groups average discharge rate of 7-9 and 9-12. The results are in the line with similar studies conducted earlier by Lin and Thomas (2005) on three intersections on Long Island, New York.

Figure 5.6: Average queue discharge rate as a function of queue position group
Figure 5.7 presents the observed average headways and queue discharge rate with respect to queue position. A slightly decreasing tendency of headway is observed in this study causing a slightly increasing trend in queue discharge flow rate. The theoretical flow discharge curve becomes horizontal after the 4th vehicle when a constant saturation flow rate is assumed, however the observed field trend shows that towards the end of the green time, the discharge flow rate slightly increases. This increasing trend is not well represented by the theoretical curve.

![Figure 5.7: Field observations of queue discharge rate with respect to queue position](image)

### 5.5.3 Modelling using AIMSUN

In order to use these scenarios in a way that produces the best possible simulation results, it is important to validate these scenarios. The first step in any micro-simulation analysis is to calibrate the input parameters according to local conditions. The parameters used in this research for this vehicle category are presented in Table 5.2.
The micro-simulation model is calibrated by adjusting the saturation flow rate. The saturation flow rate is derived in the micro-simulation model by maintaining a continuous queue and by allowing a warm up simulation time of 20 minutes. The average of the maximum flow rate in the three simulations is noted as 1948 pcphgpl over 20 simulations.

Figure 5.8 denotes that results from micro-simulation show a closer correlation with the reference analytical approaches including HCM (2000) and ARR123 (Akçelik, 1995) methods. Increasing trend observed in the field data towards the end of the queue is not seen in the results from micro-simulation. However, the results obtained do not support the widely known belief in profession that micro-simulation under-represents the capacity.

Table 5.2: Input parameters after calibration

<table>
<thead>
<tr>
<th>Detection Cycle</th>
<th>1.0 seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car-following model</td>
<td>Deceleration estimation (avg. of follower &amp; leader)</td>
</tr>
<tr>
<td>Queuing up speed</td>
<td>1 m/sec</td>
</tr>
<tr>
<td>Queuing leaving speed</td>
<td>4 m/sec</td>
</tr>
<tr>
<td>Speed</td>
<td>50 km/h</td>
</tr>
<tr>
<td>Reaction time</td>
<td>1.0 seconds</td>
</tr>
<tr>
<td>Reaction time at stop</td>
<td>1.35 seconds</td>
</tr>
<tr>
<td>Arrival distribution</td>
<td>Normal distribution</td>
</tr>
<tr>
<td>Acceleration rate Max</td>
<td>2 m/sec2</td>
</tr>
<tr>
<td>Min</td>
<td>1.2 m/sec2</td>
</tr>
<tr>
<td>Normal deceleration rate Max</td>
<td>4.5 m/sec2</td>
</tr>
<tr>
<td>Min</td>
<td>3.5 m/sec2</td>
</tr>
</tbody>
</table>

5.5.4 Queue Discharge Time

Figure 5.9 presents discharge times predicted by micro-simulation model and those taken from field observations. The discharge time is plotted in y-axis while distance to stop-line is on x-axis. The discharge time was computed from time headway measurements averaged for each queue position. It can be observed that micro-simulation model replicates the discharge time quite close to the field observations down to 80 meters from the stop-line, after which it started to diverge slightly from the observed values.
Figure 5.8: Forecasted queue discharge behaviour by AIMSUN

Figure 5.9: Discharge time versus distance to stop-line plot for AIMSUN
5.6 Conclusions

In this chapter, an investigation on the queue discharge behaviour at a signalised intersection is presented. Video data collected in evening peak hours has been analysed and assessment has been carried out to compare theoretical and field values of saturation flow rate and queue discharge flow rate of an urban intersection. This comparative assessment was carried out in three phases. Firstly, a comparison of headway characteristics was carried out. Secondly, after investigating the field queue discharge behaviour it was compared with HCM (2000) and ARR123 (Akçelik, 1995). In the last phase micro-simulation was computed for queue discharge time and was compared with respect to the position of vehicles in the queue.

The collected data depicts broad categories of drivers’ behaviour and it is observed that the value of headway time scatters widely. The study reveals a decreasing trend in headway time for the vehicles towards the end of the queue. This trend is in line with the hypothesis presented in previous studies (Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004, Li and Prevedouros, 2002) and the possible reason of this decreasing trend could be due to behaviour of drivers at the end of queue who try to push forward in order to cross the intersection before change of signal phase.

The study has observed slightly higher value of saturation flow rate for the vehicles at the end of queue. A comparison between average queue discharge rates as a function of queue position in a group of three vehicles has revealed an increase of about 6% queue discharge rate compared to theoretical saturation flow rate at queue positions 16 – 18. These results do not confirm the default constant saturation flow rate concept of HCM (2000) and ARR 123 (Akçelik, 1995). This aspect has been further investigated by using AIMSUN micro-simulation model and the results indicate that they are closely aligned with the analytical approaches.

Queue discharge time analysis shows that field observations are closely aligned with micro-simulation results, for the vehicles in the queue up to a distance of 80m from the stop line. Comparison shows that micro-simulation behaviour is flatter towards the end of queue, which indicates that headway time remains stable towards the end of queue. The observed field value curve deviates away from the micro-simulation trend line for the
vehicles at the end of the queue reflecting an aggressive behaviour of drivers to pass the stop line with lesser headway.

As the study is based on data collected from only one signalised intersection, the findings of the study have limited scope and the findings cannot be generalized. For better understanding of observed trends and verification of results, considerable amount of data is needed.
6 VARIABLE SATURATION FLOW RATE

(This Chapter is a slightly revised version of an article accepted for publication in “Journal of the Asian Transport Studies, 2013”)

Abstract

The traditional method of traffic signal design is based on the assumption of a constant queue discharge rate; also termed saturation flow rate. However, some field studies conducted in the past contradict such an assumption as they report a marginally increasing trend was observed for queue discharge rates towards the back-of-queue. This chapter first verifies the variable nature of the queue discharge rate based on field data collected from six different signalised intersections in Auckland. The observed trends are then modelled using empirical models and assessed by comparing against models developed using the Gene Expression Programming (GEP) algorithm. The proposed model shows compatibility with the existing traffic signal design methodologies for calculating different performance measures. More importantly, it shows prospects to overcome the shortcomings of the traditional method for practical applications.

6.1 Introduction

The convenient assumption of a steady saturation flow rate over a saturated green time makes it considerably easier to calculate the lane group capacity. Several factors are identified in the literature as influencing saturation flow rate; however the basic concept remains the same that when a signal changes to green, the flow across the stop line
increases rapidly to the saturation flow rate, which remains constant until either the queue is exhausted or the green period ends (Akçelik, 1981, HCM, 2010).

A great deal of research conducted on queue discharge behaviour at signalised intersections reported large variations in queue discharge rate (Chaudhry and Ranjitkar, 2013, Chaudhry et al., 2011, Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004, Tarko and Tracz, 2000, Teply, 1983). Figure 6.1 presents the mean headway for queued vehicles at the stop line of signalised intersections as reported in the literature. Most of them reported having observed a decreasing trend in the mean headway and hence an increasing queue discharge rate towards the back-of-queue even after the first four vehicles. The only two exceptions were those reported by Gerlough and Wagner (1967) and Carstens (1971), in which queue discharge rate was either nearly constant or fluctuating slightly up and down.

![Figure 6.1: Departure headways with respect to queue position](image)

The strong evidence of increasing queue discharge rate observed at signalised intersections raises serious doubts regarding the assumption of a steady queue discharge rate termed as saturation flow rate. Several models were proposed in the literature to approximate queue discharge behaviour at signalised intersections (Bonneson, 1992, Long,
2007). However, these models are yet to be implemented in the real world. Such implementation would be challenging as their formulation cannot be solved easily and requires use of the root-search method.

A new model is developed for signalised intersections using Gene Expression Programming (GEP). GEP is a genetic algorithm (GA) based tool; developed by Ferreira (2001). It is quite useful to develop new models based on field observations and also to compare them against alternative formulations. GEP has been used in different fields and proven to be more accurate and stable than genetic programming and linear regression.

The proposed model incorporates the increasing queue discharge rate reported in the literature and also addresses some of the deficiencies of the traditional method of traffic signal design based on a constant saturation flow rate. The model is calibrated and validated against data collected from six signalised intersections in Auckland. The rest of this chapter is organized as follows: a review of related literature is presented in the following section. Section 6.3 provides an overview of GEP and its core functioning. Section 6.4 outlines the details of the data collection from six signalised intersections in Auckland and verifies the presence of an increasing trend for queue discharge rate from the field data observations. In section 6.5, new models are proposed to depict the queue discharge behaviour that was observed in the field. Finally, some concluding remarks are drawn in the last section.

6.2 Traffic Operations at Pre-Timed Signalised Intersection

6.2.1 Theoretical Concept

Figure 6.2 illustrates typical queue discharge behaviour observed at signalised intersections. The queue starts building up at the red signal just before the stop line and dissipates on the green signal. The upper section of the figure represents this phenomenon in terms of the positions of vehicles as a function of time. Webster (1958) proposed an idealized representation of the discharge flow based on an assumption of uniform maximum discharge flow rate which is shown in the lower section of Figure 6.2.
6. Variable Saturation Flow Rate

6.2.2 Variability in Saturation Flow Rate

A number of studies have been previously conducted to quantify the maximum queue discharge rate at signalised intersections. Clayton (1941) stated that the saturation flow rate at signalised intersections typically ranges from 1200 to 1800 pc/h/l. Greenshield et al. (1946) observed a queue discharge rate of 1714 pc/h/l after 6 vehicles passed through the stop line. The 1950 Highway Capacity Manual (HCM) suggested a value of 1500 pc/h/l for saturation flow rate (HCM, 1950). It was later revised to 1800 pc/h/l in the 1965 HCM (1965). In 1997, this value was revised again to 1900 pc/h/l, which remained the same for all subsequent editions of the manual (HCM, 1998, HCM, 2000, HCM, 2010).

![Diagram of traffic flow at a signalised intersection](image)

**Figure 6.2:** Typical traffic flow at a signalised intersection (HCM, 2010)

Teply (1983) conducted a study in Canada and noted that saturation flow rate depends not only on site-specific conditions but also on the duration of the green period and type of
community. Briggs (1977) and Bonneson (1992) proposed some modifications to the traditional method to incorporate the variable nature of the queue discharge rate observed in the field. However, the practical implementation of the variable discharge rate models remained a challenge due to their reliance on several factors that made the calculation of capacity and other performance measures very complicated. Jin et al. (2009) reported their observations, along with the reference to the past literature, showing that departure headway decreases sequentially with queue position.

A series of studies conducted in Taiwan and USA revealed a more consistent increase in queue discharge rate with green time (Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004). Li and Prevedouros (2002) conducted a study in Hawaii that revealed a rather complex relationship between queue discharge rate and queue position. For through and left turning movements, the minimum headway could not be reached until the 9th to 12th vehicle in the queue crossed the stop line. Lin et al. (2004) reported that queue discharge rate often does not conform to the notion of a quick rise to a steady state. They quantified the extent of errors by conducting a statistical analysis on 38 urban lanes in Taiwan. They noted that queue discharge rates increase by an average of 24% for through movement and 16% for protected left turn movement when compared to those determined using the HCM (2000) method. They also noted that there is a 40% chance that the lost time will differ from the correct value by 2 seconds and a 50% chance that the estimated capacity will deviate from the actual capacity by 5%, if the average lost time and saturation flow rate of a group of similar lanes are used as estimates for each lane in the study group. Lin et al., (2007) conducted a similar study at three signalised intersections in Long Island, New York, in which all of the intersections examined exhibited a general trend of gradual compression of headway as green time passed.

6.2.3 Headway Prediction Models

The assumption of a constant saturation flow rate is comparatively easier and less complicated to apply, making it convenient for use in capacity and cycle time calculations, though there were several studies questioning the use of a constant value. Several models were also proposed to cater for the increasing trend in saturation flow rate. Briggs (1977) was the first to propose a deterministic model to incorporate an increasing trend in queue
discharge rate. His model assumes a constant acceleration for queued vehicles and takes the following form;

For \( d \cdot n < d_{i_{\text{max}}}, \)

\[
h_n = T_r + \frac{2. d_i \cdot n}{A} - \frac{2. d_i \cdot (n - 1)}{A}\] (6.1)

Otherwise,

\[
h_n = T_r + \frac{d_i}{V_q}\] (6.2)

where

- \( h_n \) is the headway of the \( n \)th vehicle
- \( n \) is the queue position
- \( V_q \) is the desired speed of queued traffic
- \( d_i \) is the distance between vehicles in a stopped queue
- \( d_{i_{\text{max}}} \) is the distance travelled to reach speed \( V_q \)
- \( T_r \) is driver starting response time
- \( A \) is constant acceleration of queued vehicles

This model has two parts. The first part works when vehicles try to achieve a desired speed, \( V_q \), at the beginning of the green phase. The model accurately replicates the queue discharge behaviour during this state. However, it predicts a constant headway in the second part which represents a state when the vehicles have already achieved the desired speed and are traveling at a constant speed. The model suggests that the minimum headway is achieved after 6 vehicles.

Bonnezon (1992) reported that headway continues to decrease until the eighth or ninth vehicle in the queue crosses the stop line, after which a constant level is attained. He presented his model based on driver reaction time, vehicle speed and acceleration. Unlike Briggs’ (1977) model, Bonnezon’s (1992) model was devised on a non-constant acceleration rate. The headway of \( n^{th} \) vehicle according to this model is:
6. Variable Saturation Flow Rate

\[ h_n = \tau N_1 + T_r + \frac{d_i}{V_{\text{max}}} + \frac{V_{\text{max}} \cdot (1 - e^{-nk}) - V_{\text{sl}(n-1)}}{A_{\text{max}}} \]  

(6.3)

where

\( \tau \) is the additional response time of the first queued vehicle

\( N_1 \) is 1 if \( n=1 \), or 0 if \( n>1 \)

\( T_r \) is the driver response time

\( d_i \) is the distance between vehicles in a stopped queue

\( V_{\text{sl}(n)} \) is the stop line speed of the \( n \)th queued vehicles

\( V_{\text{max}} \) is the maximum speed

\( A_{\text{max}} \) is the maximum acceleration

\( k = \beta / V_{\text{max}} \), where \( \beta \) is an empirical calibration constant

Long (2006) presented a driver behaviour model of queue discharge rate in terms of the discharge times of each vehicle in the queue. His model considers the distances of the vehicles in the queue from the stop line, acceleration, and the average start up lost time, taking the following form:

\[ t d_n = w_n + t a_n = \tau + n.T_r + t_a(sa_n, \alpha, \beta) \]  

(6.4)

where

\( t d_n \) is the average discharge time at the stop line of the \( n \)th vehicle in queue,

\( w_n \) is the start-up waiting time for the \( n \)th vehicle in a queue,

\( t a_n \) is the average acceleration time,

\( \tau \) is the excess start-up time of the lead vehicle in a queue,

\( T_r \) is the uniform or average start-up response time of each driver,

\( sa_n \) is the average distance of acceleration of the \( n^{th} \) vehicle from the start of motion to time \( t a_n \),

\( \alpha \) is the average initial rate of acceleration and

\( \beta \) is the average rate of decrease in acceleration with increasing speed.
Despite its ability to replicate the increase in queue discharge rate, it is difficult to implement this method for practical applications as equation (6.4) cannot be solved easily, requiring application of the root-search method (Long, 2007).

Although there are quite a few models proposed in the literature to approximate variations in queue discharge rate, which were calibrated and validated against field observations data with reasonable accuracy, they could not get much attention for practical applications mainly due to the complexity of the models and the number of assumptions made in those models. It would be wise to develop a model which is relatively simple with a minimum number of variables used to capture the observed queue discharge behaviour. Such a model should also be easy to implement in practice.

6.3 Gene Expression Programming

Gene expression programming (GEP) is a type of genetic algorithm that allows the fast application of a wide variety of mutation and cross-breeding techniques while guaranteeing that the resulting expression will always be syntactically valid. Studies indicated that GEP performs better and faster than genetic algorithms (Qu et al., 2004). The GEP belongs to the wider class of Genetic Algorithms (GA) along with Genetic Programming (GP). This class of GA uses populations of individuals, selects them according to fitness and introduces genetic variation using one or more genetic operators (Ferreira, 2001). The main difference is that GA uses linear strings of fixed length, GP uses nonlinear entities of different sizes and shapes and GEP uses linear strings of fixed length then converted as nonlinear entities of different sizes and shapes.

The process GEP adopts during model development is systematically explained in Figure 6.3. Mathematical evolution in the GEP starts with the creation of an initial population of candidate functions. The evolution engine then mutates, breeds and, finally, a mathematical function is evolved as a result of natural selection. What follows are the main steps involved in the training and evolution of a GEP:

1. Create an initial population of viable individuals (chromosomes)
2. Use evolution to attempt to create individuals that fit the data well
3. Use evolution to try to find a simpler, more parsimonious function
4. Use nonlinear regression to find the optimal values of constants.

The process of transformation of the parameters into a corresponding form is called encoding. This process of transformation is called the Karva Language, which was devised by Ferreira (2006). The encoding process produces a natural tree structure where internal nodes are operations and leaf nodes are variables or constants. An example of the expression tree is shown in Figure 6.4.

Genetic algorithms work by evolving sets of individuals called chromosomes. This evolution process is performed on the basis of an initial founder population of individuals that can mutate, breed, and be selected for subsequent generations. The initial population creation is done by randomly selecting functions and terminals for the genes.

![Flow Chart of Gene Expression Programming](image-url)
In terms of performance, GEP is considered extremely useful, outshining the existing evolutionary techniques, with some experiments indicating that it performs better than genetic algorithms (Ferreira, 2001, Qu et al., 2004, Yu et al., 2010). The algorithm in GEP allows the creation of multiple genes to form sub-expression tree. These multiple genes are the core capability that enable not only the encoding of any conceivable program, but also an efficient evolution. Gene operators then search the solution space and generate valid structures in the form of mathematical expressions. The capability of gene expression is used for the validation of the proposed nonlinear model.

6.4 Field Observations

6.4.1 Data Collection

The data was collected using video recording techniques under normal driving conditions (with no unusual traffic and sunny weather) from six signalised intersections in Auckland, including the St. Lukes Road – New North Road intersection, Balmoral Road–Sandringham Road intersection, Dominion Road – Balmoral Road intersection, Manukau Road – Greenlane East Road intersection, Pah Road – Mount Albert Road intersection, and Great South Road – South Eastern Highway intersection. Figure 6.5 shows locations and layouts of these intersections. Two hours of data was collected from each intersection except one during evening peak periods.

These intersections were selected for this study based on the selection criteria set for choosing intersections minimally affected by capacity reducing factors as suggested in a
study by Le et al. (2000). These criteria are helpful to locate relatively ideal intersections to minimize the need to adjust for prevailing conditions. The selection criteria is composed of three main components including the absence of heavy traffic with at least one exclusive through lane; that the intersection be close to ideal geometric and roadway conditions (with 3.6m lane widths, a level approach grade, minimal or no pedestrian movements, no curb parking, and no bus stop in the vicinity); and a reasonable distance from any adjacent intersections.

6.4.2 Data Analysis

Individual headway data is processed from the recorded video data for each vehicle in the queue. Table 6.1 presents the results of data analysis that shows some variations in the driver’s reaction time from 1 to 1.2 seconds for the first five intersections that are located in the close proximity to each other. While the sixth intersection (the Great South Road – South Eastern Highway intersection) shows a different trend with an average reaction time of 2.05 seconds. The start-up lost time were recorded within the range of 1.18 to 3.08 seconds. The high start-up delay for the St. Lukes Road – New North Road intersection could be due to the downstream approach grade followed by a sharp curve resulting from an increase in the start-up lost time.

A large variation is noted in stopping positions of the first vehicles from the stop line. During the calculation of start-up lost time, this distance has a significant influence as the stop line is taken as a reference line for headway calculations. Figure 6.6 presents the queue discharge rate versus green time plots for each intersection. An increasing trend can be observed for all the intersections with a varying slope and $R^2$ values ranging from 0.44 to 0.83.

The results presented in this section are in line with the findings of previous researchers that the queue discharge rate increases towards the back-of-queue. A possible explanation for the increasing queue discharge behaviour observed in the field could be the fact that the drivers located at the back of the queue are generally more likely to be stopped by a change of signal to red than those located at the beginning of the queue. This consideration possibly influences driving behaviour at the back-of-queue that results in drivers maintaining minimal headway from their preceding vehicle.
6. Variable Saturation Flow Rate

Figure 6.6 shows the average headways in 2 hours of evening peak traffic at six intersections. Saturation flow rate is calculated based on the ARR123 (Akçelik, 1995) and HCM (2000) methods and the average values are shown as a solid line. This figure clearly indicates how much of a difference an increasing trend can produce when compared to the traditional, bell shaped saturation flow rate model.

![Figure 6.5: Selected Sites at Auckland](image)

### Table 6.1 Relationship between queue discharge rate and green time

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Number of Phases</th>
<th>Reaction Time (Start of green to movement of first vehicle)</th>
<th>Start-up Lost Time</th>
<th>Queue Discharge Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$R_t$</td>
<td>Std</td>
<td>Avg</td>
</tr>
<tr>
<td>Balmoral - Sandringham Rd</td>
<td>58</td>
<td>1.02</td>
<td>0.79</td>
<td>1.51</td>
</tr>
<tr>
<td>Balmoral - Dominion Rd</td>
<td>59</td>
<td>1.20</td>
<td>0.57</td>
<td>1.18</td>
</tr>
<tr>
<td>GT South Rd - SE Highway</td>
<td>22</td>
<td>2.05</td>
<td>0.48</td>
<td>2.64</td>
</tr>
<tr>
<td>Manukau - Greelane East Road</td>
<td>58</td>
<td>1.15</td>
<td>0.45</td>
<td>1.24</td>
</tr>
<tr>
<td>Pah - Mt Albert Road</td>
<td>60</td>
<td>1.18</td>
<td>0.41</td>
<td>1.73</td>
</tr>
<tr>
<td>St Lukes - New North Road</td>
<td>54</td>
<td>1.00</td>
<td>0.47</td>
<td>3.08</td>
</tr>
</tbody>
</table>

$t$ is the time passed after onset of green
6. Variable Saturation Flow Rate

6.5 Data Analysis and Results

In the development of a new model, both linear and nonlinear approaches are considered. The main objective is to propose an appropriate model that can not only represent varying discharge flow rate at signalised intersections, but can also be used for determining further performance measures, including capacity and delay, without complicated computations.

Figure 6.6: Plot of queue discharge rate versus green time
6. Variable Saturation Flow Rate

6.5.1 Linear Model

A simple linear model can be proposed based on a simple curve fitting method.

\[ q_s = ag + b \] (6.5)

Here, the values of \( a \) and \( b \) can be computed from field observations. For the six intersections in Auckland, values of 3.23 and 1788 were computed for \( a \) and \( b \) respectively. The simple linear model can be an easier way to reduce the errors caused by a constant saturation flow rate. However, it can create the deception that an infinite increase of queue discharge rate is present, which is not realistic and can lead to erroneous calculations.

A detailed inspection of the traffic flow data after onset of the green signal shows the intricacies of behaviour at signalised intersections. Though the average values produce a visible increasing trend towards the back-of-queue, the cycle by cycle data shows significant variations in discharge flow rate. To evaluate these variations, a sample comparison was conducted to gauge the performance of alternative models for traffic flow. The results of the sample comparison are shown in Table 6.2.

The comparative analysis indicates that the linear model does not provide a good fit for the data. A linear curve fit reports an average \( R^2 \) value of 47.23%. This \( R^2 \) value indicates that the model as it is fitted explains 47.23% of the variability in flow. However, on the other hand, the nonlinear double reciprocal model predicts an average \( R^2 \) value of 83.71%. The best model predicted by the data includes an S-curve of 84.61%, a double reciprocal of 84.35% and a Reciprocal X of 80.77%.

From the sample comparison, a clear indication of nonlinear behaviour for the queue discharge flow rate can be seen. To extend the findings of the sample comparison, a further analysis between linear and nonlinear curve fits is shown in Figure 6.7. Linear curves return low \( R^2 \) values and, on average, a value of 47% can be achieved for these intersections. On the other hand, a nonlinear approach returns a high value for \( R^2 \) and, on average, an \( R^2 \) value of 83% can be achieved, with the highest value being 91.58% for the Pah Road - Mt Albert intersection.
Table 6.2: Comparison of Alternative Models

<table>
<thead>
<tr>
<th>Model</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-curve model</td>
<td>84.61%</td>
</tr>
<tr>
<td>Double reciprocal</td>
<td>84.35%</td>
</tr>
<tr>
<td>Square root-Y reciprocal-X</td>
<td>83.19%</td>
</tr>
<tr>
<td>Reciprocal-X</td>
<td>80.77%</td>
</tr>
<tr>
<td>Squared-Y reciprocal-X</td>
<td>73.59%</td>
</tr>
<tr>
<td>Logarithmic-X</td>
<td>65.15%</td>
</tr>
<tr>
<td>Square root-Y logarithmic-X</td>
<td>64.55%</td>
</tr>
<tr>
<td>Squared-Y logarithmic-X</td>
<td>63.99%</td>
</tr>
<tr>
<td>Multiplicative</td>
<td>63.12%</td>
</tr>
<tr>
<td>Squared-Y square root-X</td>
<td>54.08%</td>
</tr>
<tr>
<td>Square root-X</td>
<td>53.26%</td>
</tr>
<tr>
<td>Double square root</td>
<td>51.86%</td>
</tr>
<tr>
<td>Logarithmic-Y square root-X</td>
<td>49.81%</td>
</tr>
<tr>
<td>Squared-Y</td>
<td>44.18%</td>
</tr>
<tr>
<td>Linear</td>
<td>42.36%</td>
</tr>
<tr>
<td>Square root-Y</td>
<td>40.66%</td>
</tr>
<tr>
<td>Exponential</td>
<td>38.49%</td>
</tr>
<tr>
<td>Reciprocal-Y</td>
<td>33.29%</td>
</tr>
<tr>
<td>Double squared</td>
<td>29.20%</td>
</tr>
<tr>
<td>Squared-X</td>
<td>27.05%</td>
</tr>
<tr>
<td>Square root-Y squared-X</td>
<td>25.49%</td>
</tr>
<tr>
<td>Logarithmic-Y squared-X</td>
<td>23.68%</td>
</tr>
<tr>
<td>Reciprocal-Y squared-X</td>
<td>19.71%</td>
</tr>
</tbody>
</table>

6.5.2 Nonlinear model

As seen from the comparison of alternative models, the nonlinear model seems more appropriate to represent queue discharge rate. However, restricting a model that can incorporate varying trends while not being so complicated that it makes further calculations arduous was a primary objective for the model development. In order to achieve this, the most influential parameters are identified that may impact how discharge rates progress after vehicles are allowed to move at the onset of green time. Based on field observations, the most influential parameters are identified and given as:

\[
q_s = f(g, l_s, h_m, \gamma)
\]  

(6.6)

where

\( q_s \) is the discharge rate after \( t \) seconds of the movement (in veh/ sec),
Figure 6.7 Comparisons of linear models with nonlinear models
$g$ is the time in seconds,

$l_s$ is the initial lost time in seconds,

$h_m$ is the minimum average headway recorded after a number of cycle, and

$\gamma$ is a correction factor.

A training set is selected from the data to establish a model and rest of the data is used as the testing set. The fitness mechanism established to measure how well a model is returning the predicted function as follows;

$$f_i = \sum_{j=1}^{c_i}(M - |C_{(i,j)} - T_j|)$$  \hspace{1cm} (6.7)

where

$M$ is the range of selection,

$C_{(i,j)}$ is the value returned by the individual chromosome $i$ for fitness case $j$ (out of $C_t$ fitness cases) and

$T_j$ is the target value for fitness case $j$.

If $|C_{(i,j)} - T_j|$ is less than or equal to 0.01, then the precision is equal to 0, and $f_i = f_{max} = ctM$. For the fitness purposes, $M$ and $f_{max}$ are set as 100 and 1000 respectively.

The GEP can find the optimum solution by itself then.

An important aspect in model development was to translate speed and queue position into somewhat simple and determinable. The vehicles start moving from stop position and they tend to achieve their desired speed as soon as conditions permit. The speed attained by the stop line depends on various factors, including vehicle characteristics as well as driver behaviour. The posted speed limit can be attained within 20 to 30 seconds with a normal acceleration rate after the onset of green time, however observed data and past literature have indicated that the cases are present where the discharge flow rate continues to increase until 45 to 50 seconds after the onset of green time. The distance to the stop line for a standing vehicle in the queue is also depends on driver behaviour. These parameters are influential but can make further calculations complicated. To simplify the
model, the impact of these parameters is translated into green time and model is developed based on the variation in maximum queue discharge rate during the allocated green time.

Expressions are developed with a restrictive number of mathematical operators in the curve fitting equation. GEP has the inbuilt ability to quickly mutate valid expressions in the form of encoding symbols in genes. Table 6.3 presents the various forms of mathematical expressions used to establish model and the resulting statistical curve fit parameters. About 10 different combinations of mathematical functions are used to establish the most appropriate expression to fit the available data. Addition, division and addition, inverse functions produce optimum expressions with the highest predicted correlations. For the Balmoral-Sandringham Road intersection, addition, multiplication functions also predict a good fit with $R^2$ value of 84.41%. However, addition, inverse functions not only predicts a good fit with an $R^2$ value of 84.21% but also predicts the lowest MAPE value of 5.01 in all functions. GEP also predicts the lowest value of MAPE as 4.03 with n $R^2$ value of 82.88% for the Balmoral-Dominion Road intersection. A relatively weaker correlation is found by the GEP for the Great South Road - South East Highway intersection with an $R^2$ value of 70.16% and a MAPE value of 7.25. A MAPE value of less than 10 represents less error comparatively. For the Manukau-Greenlane East intersection, the best fit curve comes from the addition, division functions with $R^2$ value of 71.15% and a MAPE value of 3.53. However addition, inverse functions also predict a good fit with an $R^2$ value of 80.99% and a MAPE value of 3.62. The Pah Road – Mt Albert Road intersection predicts a strong correlation with an $R^2$ 94.14% and small error with a MAPE value of 2.98 for the addition/inverse functions. The St Lukes-New North Road intersection predicts a relatively low correlation with an $R^2$ value of 62.17% and a MAPE value of 12.06. It can be seen for almost all cases that the addition, inverse mathematical expression returns a comparatively better fit for the available data of six intersections in Auckland.

The most common expression found by GEP is shown below;

$$q_s = \frac{a}{g} + bg + c$$  \hspace{1cm} (6.8)
where \( a, b \) and \( c \) are constants and \( g \) is the green time.

Table 6.3: Model development comparison in GEP

<table>
<thead>
<tr>
<th>Intersections</th>
<th>Balmoral Sandringham Rd</th>
<th>Balmoral Dominion Rd</th>
<th>Great South - South East Highway</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functions</td>
<td>( R^2 )</td>
<td>MAPE</td>
<td>RMSE</td>
</tr>
<tr>
<td>Addition, Division</td>
<td>80</td>
<td>6</td>
<td>138</td>
</tr>
<tr>
<td>Addition, Power</td>
<td>72</td>
<td>7</td>
<td>161</td>
</tr>
<tr>
<td>Addition, Logistic</td>
<td>49</td>
<td>10</td>
<td>217</td>
</tr>
<tr>
<td>Addition, Sqrt</td>
<td>73</td>
<td>7</td>
<td>160</td>
</tr>
<tr>
<td>Addition, Log</td>
<td>76</td>
<td>7</td>
<td>151</td>
</tr>
<tr>
<td>Addition, Subtraction</td>
<td>80</td>
<td>6</td>
<td>138</td>
</tr>
<tr>
<td>Addition, Negation</td>
<td>79</td>
<td>6</td>
<td>139</td>
</tr>
<tr>
<td>Addition, Inverse</td>
<td>84</td>
<td>5</td>
<td>121</td>
</tr>
<tr>
<td>Addition, Multiplication</td>
<td>84</td>
<td>5</td>
<td>121</td>
</tr>
<tr>
<td>Addition, Division, Power</td>
<td>79</td>
<td>6</td>
<td>142</td>
</tr>
</tbody>
</table>

The comparison of the inverse model is shown in Figure 6.8. For all six intersections, the inverse model predicts quite satisfactory results and comparison between actual and predicted values indicates good predictability for the model. Translating the constants into the influential parameters set at the start, the following nonlinear model can capture the scenario observed in field with a relatively sharp transition from a low to high queue discharge rate followed by a marginal increase throughout green time.

\[
q_s = \frac{g - l_s}{1 + h_m(g - \gamma)}
\]  

(6.9)

The analogy of maximum queue discharge rate is not used here, as it is highly variable in the initial stages. Instead a queue discharge rate that is achievable in time \( g \) is used, which is dependent on the initial lost time and a signal management parameter. The initial lost time parameter incorporates the initial reaction time of the drivers after having seen the green light and starting to respond. This initial lost time could vary with vehicle type...
and drivers’ aggression. $h_m$ is the minimum average headway recorded after a number of cycles. Field observation for $h_m$ is a better way to calculate a more accurate discharge flow rate.

### 6.5.3 Nonlinear Model Characteristics

The proposed models have two main characteristics. First, the queue discharge rate varies with as green time passes. Secondly, it increases sharply initially and then tends to increase marginally towards the end of green time. To encapsulate these characteristics, various factors are introduced in the model as discussed in the following paragraphs.

**The difference between $l_s$ and Start-up lost time:** The lost time notation used here is different from the traditional start-up lost time. The traditional model makes another assumption - that the portion other than saturation flow rate is start up lost time. This start-up lost time is measured by adding the additional time consumed by the first few vehicles in a queue. With the decreasing headway situation, this assumption cannot work as studies have indicated that headway cannot be sustained after the first few vehicles and continues to reduce until 7th or 8th vehicles and sometimes even later. $l_s$ denotes the time when the signal turns green to the time when first vehicle starts moving, considering the driver’s reaction and perception time.
b) Balmoral – Dominion Road Intersection

\[ R^2 = 0.70 \]
\[ \text{MAPE} = 7.25 \]
\[ \text{RMSE} = 229.30 \]


c) Great South Road - South East highway Intersection

\[ R^2 = 0.81 \]
\[ \text{MAPE} = 3.62 \]
\[ \text{RMSE} = 96.71 \]


d) Manukau - Greenlane East Road Intersection

\[ R^2 = 0.79 \]
\[ \text{MAPE} = 4.32 \]
\[ \text{RMSE} = 156.89 \]


e) Pah Road - Mt Albert Rd Intersection

\[ R^2 = 0.62 \]
\[ \text{MAPE} = 12.06 \]
\[ \text{RMSE} = 224.50 \]

f) St Lukes – New North Rd Intersection

\[ R^2 = 0.70 \]
\[ \text{MAPE} = 7.25 \]
\[ \text{RMSE} = 229.30 \]

\[ R^2 = 0.81 \]
\[ \text{MAPE} = 3.62 \]
\[ \text{RMSE} = 96.71 \]

\[ R^2 = 0.79 \]
\[ \text{MAPE} = 4.32 \]
\[ \text{RMSE} = 156.89 \]

\[ R^2 = 0.62 \]
\[ \text{MAPE} = 12.06 \]
\[ \text{RMSE} = 224.50 \]

Figure 6.8: Inverse model and actual versus predicted values of flow
6. Variable Saturation Flow Rate

The difference between \( h_m \) and \( h_s \): The \( h_m \) (minimum average headway) used in this model differs from \( h_s \) (average saturation headway) as the former deals with the minimum achievable headway for the intersection while the latter deals with the average headway after first 4 or 5 vehicles. The observed data indicated that the flow rate at signalised intersections grows gradually. For example, at the Balmoral – Dominion Road intersection, the headway after 15 seconds was 2.05 seconds and, after 50 seconds, the headway was 1.9 seconds. The average saturation headway \( (h_s) \) for the intersection was 2 seconds, but \( h_m \) was 1.9 seconds in this case.

![Figure 6.9: GEP evaluation of the proposed model](image-url)
Description of $\gamma$: This factor is related to the general behaviour of the traffic stream at signalised intersections. The traffic behaviour in different parts of the world is different once the vehicles have observed the green signal and are allowed to move. In one scenario, the acceleration pattern of the traffic stream is more aggressive soon after seeing the green signal, as is often the case of the countries with high traffic volume. In other scenario, the traffic stream takes a smooth acceleration pattern. In the first instance, the value of $\gamma$ will be higher than that of the latter scenario. A value between 0.2 and 0.4 gives a good fit for the model of New Zealand conditions.

6.5.4 Performance of Nonlinear model

Figure 6.9 shows the chart for the actual discharge rate against the predicted discharge rate for proposed model. A comparison of the values predicted by the nonlinear model with the actual model is compared. GEP suggested a nonlinear solution that returns the optimum solution for the curve fitting of this data set. Almost all of the intersections showed a reasonable correlation with the nonlinear model. The Pah Road – Mt Albert Road intersection returns the highest $R^2$ value of 0.82. A MAPE value of 1.98 shows an impressive fit with an $R^2$ value of over 72% predicted for the Balmoral – Sandringham Road intersection. The results of model performance based on GEP are shown in Figure 6.9.

Figure 6.10 presents a comparison between the proposed model and other existing models including the traditional model, Bonneson’s (1992) model and Briggs’ (1977) model along with the field measurements. The proposed model gives the best estimate of queue discharge rate when compared with the observed data. The traditional model and Briggs’ (1977) model come to a steady state after initial increase in queue discharge rate. In Briggs’ (1977) model, queue discharge rate increases sharply until the speed reaches its maximum value, after which the trend flattens. In the case of the traditional model, after first few vehicles it flattens for the rest of the green time, whereas Bonneson’s (1992) model gives a similar shape to that of the proposed model, except that it overestimates the queue discharge rate towards the back-of-queue.
This chapter investigates queue discharge behaviour at signalised intersections. The variable nature of queue discharge rate as reported in literature is first verified based on the data collected from six different signalised intersections in Auckland. The observed trend of variations in queue discharge rate is modelled using empirical models, which are then assessed by comparing against models developed using GEP algorithm. The model captures an increasing trend observed for queue discharge rate a reasonable $R^2$ values ranging from 0.62 to 0.84 for calibration and 0.55 to 0.62 for validation. The proposed model is simple yet more accurate compared to the other models assessed in this chapter that model queue discharge behaviour at signalised intersections. The model shows the potential to estimate capacity and other operational measures including signal timing and delay. The implementation of the model on cycle time and delay calculations may improve existing methodologies to determine operational performance measures at signalised intersections.
7 IMPACT ON CAPACITY AND CYCLE TIME CALCULATIONS

Abstract

Capacity of a signalised intersection is calculated based on the assumption that when a signal changes to green, the flow across the stop line increases rapidly to a maximum level called saturation flow rate, which remains constant until either the queue is exhausted or the green period ends. Some studies in recent past indicated that the drivers at the back of a long queue shorten the headways resulting in a marginal increase in queue discharge flow rate along the queued vehicles position. A model has been proposed and presented elsewhere to incorporate the marginally increasing trend at the back-of-queue. This chapter investigates the impact of the new model on capacity and cycle time calculations. The results revealed prospects for this model to improve the existing methodology to estimate capacity and cycle time calculations.

7.1 Introduction

Capacity is among the most concerned parameters in transportation engineering. Capacity analysis is attributed to the methods and procedures used to estimate the traffic carrying capacity of transportation facilities. These procedures and tools are extensively used in facility planning and design and operation (HCM, 2010). For new facility design, these tools can provide a good design platform to determine the suitable size of a facility as well as for predicting future performance of the facility. For existing facility, these procedures are helpful to carry out the operational analysis to assess the functionality at current state (Arnold Jr and McGhee, 1995).
Capacity analysis procedures are normally employed to determine the design of signalised intersection including number of lanes for each approach, cycle time and phasing that is needed to be able to accommodate the traffic volume. On signalised intersections, traffic signals facilitate conflicting traffic movements on same space by splitting the time share. Therefore capacity is measured based on the proportion of the time given to a particular approach and saturation flow rate. Mathematically it can be expressed as:

\[ c = s \frac{g}{C} \]  \hspace{1cm} (7.1)

where \( c \) is capacity of approach in passenger cars per hour (pcph), \( s \) is the saturation flow (pcphgpl), \( g \) = effective green time (s), and \( C \) = cycle length (s).

Saturation flow rate is the basic parameter used in intersection design to determine the intersection capacity and level of service. Clayton (1941) suggested 1500 passenger cars per hour of green time per lane (pcphgpl) for saturation flow rate based on his field observations. The same value was proposed in the first edition of HCM published in 1950 (HCM, 1950), which was later revised in the second edition of HCM published in 1965 and increased to 1,800 pcphgpl (HCM, 1965). In a Special Report # 209 published by Transportation Research Board in 1997, the value was further increased to 1900 pcphgpl (HCM, 1998). This value has then remained the same for base saturation flow rate in the later editions of HCM. Methodologies suggests that the saturation flow rate can be estimated from the base saturation flow rate after applying some adjustment factors to better represent prevailing roadway geometric, traffic and signal control conditions. However the measurement in the field is a better substitute to accurately estimate relevant performance measures.

Since the research of Clayton (1941) a great deal of research has been conducted on the saturation flow rate, proposing values ranging from 1500 to 2500 pcphgpl (Lin et al., 2004, Teply, 1983). The base saturation flow rate has been accepted as a constant for traffic signalised intersections during the green time. In recent years, the assumption that the saturation flow rate remains constant after the first three to five cars has passed through the stop line until either the queue is exhausted or the green period ends has been challenged by field observations conducted in many parts of the world. The studies
indicated that the queue discharge flow rate does not rise quickly to maximum level, instead, it rises gradually. The strong evidence of variations in queue discharge flow rate suggests a further investigation to verify and incorporate such trends in a new model. A study has been conducted at six signalised intersections in Auckland, New Zealand and a new model is proposed that incorporate the marginally increasing trend observed in the field.

\[
q_s = \frac{t - l_s}{1 + h_m(t - \gamma)}
\]  \hspace{1cm} (7.2)

where

- \(q_s\) is the discharge rate in veh/ sec,
- \(t\) is the time in seconds,
- \(l_s\) is the initial lost time in seconds,
- \(R_t\) is the reaction time of the drivers,
- \(h_m\) is the minimum average headway recorded after a number of cycles and
- \(\gamma\) is a correction factor.

Here \(q_s\) is discharge rate after \(t\) seconds of the movement. The analogy of the maximum discharge rate is not used here, as the maximum discharge rate is highly variable at the initial stages, instead a discharge rate achievable in time \(t\) is used. This discharge rate is dependent on the lost time a signal management parameter encounter during the start of the green time. This model also incorporates the initial reaction time of the drivers after seeing the green signal and start responding. This reaction time or response time varies with the type of transmission of vehicles and drivers’ behaviour. In Eq. (7.2), \(h_m\) is the minimum average headway recorded after a number of cycles.

### 7.2 Model Implementation

In this section, the proposed model is evaluated for its possible implementation on signalised intersection for performance measures including capacity and signal timing estimates.
7. Impact on Capacity and Cycle Time Calculations

7.2.1 Impact of the Proposed Model on Capacity Calculations

In a scenario where the queue discharge rate does not achieve a constant high value after few vehicles, the queue discharge rate displays a tendency to steadily increase; Eq. (7.1) can be modified by replacing $S$ with Eq. (7.2) and the new form of the equation will be:

$$c = \frac{g(g - l_s)}{C(1 + h_m(g - \gamma))}$$  \hspace{1cm} (7.3)

Eq. (7.3) gives a capacity calculation that is dependent on the green time allocated to the phase. The model clearly depicts that more green time means that more capacity is added in the system. This impact can be seen in two states as shown in Figure 7.1. In the first state, the vehicles start moving at the onset of the green phase and the capacity predicted by nonlinear model is less than the traditional model. This prediction is justifiable as the traditional model calculates capacity with the maximum discharge rate and does not incorporate the initial transition period in which vehicles are attaining the maximum flow rate. In the second state, the nonlinear model predicts a gradual increase and the difference reaches about 5% after 45 seconds of green time compared with traditional capacity models.

To simplify this equation, assuming a value of initial lost time and the correction term of $\propto$ as zero, the equation will become:

$$c = \frac{g^2}{C(1 + h_m(g))}$$  \hspace{1cm} (7.4)

The comparison of the Eq. (7.4) with the traditional capacity model shows that a 6.9% increase can be gained in the 45 seconds of green time as shown in Figure 7.1b. Figure 7.1a and Figure 7.1b show the increase in capacity, however the difference is in the shape of the percentage gain curve, which is increasing sharply at the start and then continues increasing gradually for the later equation case.
7. Impact on Capacity and Cycle Time Calculations

A comparison between the two figures shows that the difference in percentage increase is mainly in the shape of the curve and overall difference remains between 5% and 7%. The existing capacity formula that accounts for only the green time ratio \( \frac{g}{c} \) does not account for the green time variation that affects the increase in queue discharge rate. For example, a 0.4 green time ratio for the 100 second cycle time will allow 40
7. Impact on Capacity and Cycle Time Calculations

seconds green time for that particular approach. The same ratio with 30 seconds cycle time will allow 12 seconds of green time. The traditional approach does not differentiate between these two cycle times and the capacity of that approach will be 720 pc/h/l. However, the modified model indicated that a 3.9% increase in capacity is possible at 40 seconds green time with a capacity of 748 pc/h/l instead of 720 pc/h/l.

7.2.2 Impact of the Proposed Model on Signal Timing Calculation

The effect of the green time on the queue discharge rate makes it difficult to calculate flow ratios directly as flow ratios will be changed if the green time is changed. For example, the flow ratios that give a green time of 20 seconds and 45 seconds based on a fixed saturation flow rate might not be correct, as the queue discharge rate may be different at these two points. Therefore, instead of introducing a flow ratio concept, a green time demand concept is required that allocates green time based on the demand flow.

The green time demand calculated with this model indicates that relatively more green time is required for queues formed due to low traffic volumes. This is self-explanatory, as for low volumes, the queue may not be long enough to reach the traditional maximum discharge flow rate. The traditional model fails to mention this condition and flow ratios calculated are based on the maximum discharge rate. In high volumes, the green time demand decreases. Figure 7.2 explains this phenomenon in terms of traffic volume and green time demand. The basis of the green time ratio in traditional models is to consider a single value of saturation flow rate regardless of the number of vehicles that the green time ratio is calculated for. However, the proposed model is capable of handling this shortcoming and proposes a demand with respect to the time required to clear the queue in varying flow conditions.

The green time demand for all approaches can help in determining critical movements, the sum of which can be utilized for the cycle time calculations. For a pre-timed signalised intersection, this demand concept does not change, and the only limiting situation is where the maximum signal timing is restrained. In that case, the green times can be adjusted based on priorities and the effects of oversaturation can be measured. The formula for the minimum cycle time can be derived by equating the sum of the green time calculations with the total lost time.
Mathematically it can be described as

\[ C \left( 1 - \sum g_r \right) = L \]  

(7.5)

where \( g_r \) is green time demand (unitless). From this equation, cycle time can be derived as:

\[ C = \frac{L}{1 - \sum g_r} \]  

(7.6)

Multiplying both sides of Eq. (7.6) with \( X_c \) will make this equation similar to Equation 31-68 described in chapter 31 of the HCM (2010). However, with this formulation, the value of \( X_c \) that is arbitrarily assumed is unnecessary and cycle time can be derived directly.

The modified form of Webster cycle time formula, based on the green time demand is:

\[ C = \frac{\alpha L + \beta}{1 - \bar{X} \sum g_r} \]  

(7.7)
where \( X \) is the degree of saturation and \( \alpha \) and \( \beta \) can be used as per Webster (\( \alpha = 1.5 \) and \( \beta = 5 \)) or as per ARR 123 (Akçelik, 1995) (\( \alpha = 1.4 \) and \( \beta = 6 \)).

The integral of the Eq. (7.2) gives the number of vehicles that can pass through the intersection in the time \( t \). Assuming the simplified case, with replacing \( t \) with green time, \( g \):

\[
\int \frac{g}{1 + h_m g} \cdot dg.
\]

Total number of vehicles passing through intersection

\[
= \frac{h_m \cdot g - \log(1 + h_m \cdot g)}{h_m^2}
\]

(7.8)

This integration is useful as it can describe how many vehicles can cross the intersection during given \( g \) time.

To implement cycle time methodology developed in the nonlinear model, a simple comparison has been made for Cycle time-calculations based on the critical flow ratios (Y) calculated by Webster (1958) and the HCM (2000) method as shown in Figure 7.3. With increased capacity, the cycle time calculations indicate lower cycle times where Y is greater than 0.85. Applying the same methodology to the HCM (2000) cycle time calculations, results in even more flat cycle times where Y is greater than 0.85, as shown in Figure 7.4.
7. Impact on Capacity and Cycle Time Calculations

Figure 7.3: Webster cycle time formula

Figure 7.4: HCM cycle time formula
7.2.3 Example

A case used in Webster’s (1958) paper is verified with modified traffic volumes for the methodology described above. A four-legged signalised intersection that allows through movements only has an hourly flow rates of 900 pc/h/l for the north and south approach and 600 pc/h/l for the east and west Approach as shown in Figure 7.5. A saturation flow rate of 1800 pc/h/l gives flow ratios for the north and south approach to be 0.50 and 0.33 for east and west approach.

![Figure 7.5: Signalised intersection example](image)

The traditional approach with a fixed saturation flow rate and the assumption of a two phase signal estimates the critical sum of flow ratios to be 0.83, which gives an optimum cycle time of 102 seconds from the Webster (1958) formula with a total lost time of 8 seconds and a practical cycle time of 108 seconds. Assuming a $X_c$ value of 0.9, the cycle time from the HCM (2000) will be same as the practical cycle time. This result does not depend on the green time allocation on approaches. The model indicated that more allocated green time will increase capacity, hence reduces the overall cycle time. The
results for the practical cycle time calculated based on a variable queue discharge rate as shown in Figure 7.6(a). For an optimum cycle time, the model predicted the same pattern with an increase in queue discharge rate reducing cycle time as shown in Figure 7.6(b).

**Figure 7.6:** Cycle time based on variable queue discharge rate
7. Impact on Capacity and Cycle Time Calculations

7.3 Concluding Remarks

This chapter has investigated the impacts of a nonlinear model proposed in a previous study to cater for variable saturation flow trend. The proposed model is simple yet more accurate than other models assessed in this chapter to model queue discharge behaviour at signalised intersections. The model shows prospect to estimate capacity and other operational measures including signal timing and delay with reasonable accuracy. Traffic signal design using the proposed model shows an increase in the approach capacity by 5% to 7% by influencing the cycle time calculations. The implementation of the model on cycle time and delay calculations shows its compatibility with the existing methodologies to determine operational performance measures at signalised intersections.
8 Delay Estimation at Signalised Intersections with Variable Queue Discharge Rate

(This Chapter is a slightly revised version of an article conditionally accepted for publication in “Journal of the Eastern Asia Society for Transportation Studies, 2013”)

Abstract

The techniques developed for delay estimation in the most traffic signal design guidelines including HCM (2010) and Australian Guide are based on the assumption that queue discharge rate at signalised intersections becomes stable after a few vehicles passes through the stop line, which is termed as saturation flow rate. This assumption has been challenged in recent times as a number of field observation in different parts of the world reported an increasing queue discharge rate observed along the back-of-queue. This chapter proposes an empirical model that is capable of capturing the queue discharge behaviour observed at signalised intersection. The model is implemented to estimate delay and compared with the existing delay models. The results revealed that the proposed model can overcome the deficiencies of the existing models and can estimate delay more accurately.

8.1 Introduction

Delay is an important measure widely used in traffic engineering to evaluate the operational efficiency of signalised intersections. It is one of the main factor on which level of service rating is determined. Beckmann et al. (1956) were among the pioneers who studied and presented delay formulas for signalised intersections based on queuing theory. Later, Webster (1958) extended his work on signalised intersection capacity and delays estimation. He developed expressions to estimate signal timing and delay at a signalised
intersections based on a simplified assumption that when signal turns to green the queue discharge rate across the stop line increases rapidly until it reaches a sustained maximum level termed as saturation flow rate where it remains stable (unchanged) until either the queue is exhausted or the green phase ends. This assumption makes it considerably easy to calculate the lane group capacity, which is equal to a product of saturation flow rate and green to cycle time ratio (g/C).

Several factors were identified in the past influencing the saturation flow rate; however the concept remained the same as proposed by Webster (1958). Figure 8.1 presents this traditional model of traffic signal design at signalised intersections, which is based on the assumption that the saturation flow rate remains constant for a fully saturated intersection for all portions of the green interval except at the beginning and at the end.

The concept of saturation flow rate is the basic parameter in estimation of almost all performance indicators at signalised intersections including delay. Delay is the single most important factor that drivers can perceive and there it drew a lot of attention of past researchers. Webster (1958) was among those who presented a delay formula expressed as follows:

\[
d = \frac{C(1 - \lambda)^2}{2(1 - \lambda x)} + \frac{x^2}{2q(1 - x)} - 0.65 \left( \frac{C}{q^2} \right)^{1/3} x^{(2+5\lambda)}
\]  

Figure 8.1: Traditional saturation flow rate model
where

\[ d \] is the average delay per vehicle in seconds
\[ C \] is the cycle time in seconds
\[ \lambda \] is the portion of the cycle which is effectively green for the phase under consideration i.e. \( g/C \).
\[ x \] is the degree of saturation.
\[ q \] is the arrival rate in vehicles per second

Webster (1958) model considered as a fundamental method for estimating performance measures at signalised intersections. The delay model in Eq. (8.1) comprised of three main elements. The first two terms was derived based on queuing theory where the first term estimates delay assuming the traffic arrivals and departures are completely uniform and the second term assumes a steady state condition and accounts for randomness in arrival rate that leads to overflow queues. The third term was proposed as a correction term based on empirical observations to compensate overestimation of delay when using only the first two terms.

While a great deal of investigations was conducted in the past to quantify the effects of varying arrival flow rate on delay at signalised intersections, the variability in departure flow rate could not get much attention. The assumption of a constant discharge flow rate was one of the reasons for this low attention. A number of studies conducted on queue discharge behaviour at signalised intersection reported to have observed large variations in the saturation flow rate (Chaudhry and Ranjitkar, 2013, Chaudhry et al., 2011, Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004, Tarko and Tracz, 2000, Teply, 1983). Teply (1983) noted that the saturation flow rate depends not only on site-specific conditions but also on the duration of green period and type of community. In a study conducted in Canada, he observed that the maximum queue discharge rate usually drops after about 50s of green interval. A similar study conducted in Hawaii, USA by Li and Prevedouros (2002) reported a rather complex relationship between the saturation flow rate and queue position. It was noted that the minimum
headway was not reached until the 9th to 12th vehicle crossed the stop line in queue for through and left turn movements.

A series of investigations conducted more recently in Taiwan and USA revealed a more consistent increase trend in the saturation flow rate (Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004). Lin et al. (2004) noted that in Taiwan queue discharge often does not confirm to the notion to a quick rise to a steady state. Similar trends were observed at three intersections in Long Island, New York (2005). They all exhibit a general trend of gradual compression of headways as the queue discharge continues. Lin et al. (2007) quantified the extent of errors in the observed data based on statistical analysis conducted on 38 urban lanes in Taiwan. He noted that the discharge rates were increased on average by 24% for straight through movement and 16% for protected left turn movement when compared with HCM (2000) approach. Lin et al., (Lin et al., 2007) also noted that there is a 40% chance that lost time will differ from the assumed value by 2 sec and 50% chance that the estimated capacity will deviate from the actual capacity by 5%, if average HCM (2000) lost time and saturation flow of a group of similar lanes are used as estimates for each lane in the study group.

The strong evidence of variations in queue discharge flow rate reported in the literature above suggests a need for further investigation to verify such trends and then incorporate it in the delay formulation, which might contribute to improve the accuracy of delay estimation. This chapter proposes an empirical model incorporating an increasing queue discharge rate observed at six signalised intersections in Auckland, New Zealand. The model is implemented to estimate delay at signalised intersections and to then compare this with the existing methods of delay estimation. Delay models proposed in literature are reviewed in the following section; followed by results from field observations in Auckland, New Zealand. An empirical model is proposed that incorporate an increasing queue discharge rate. The impact of this model on the delay is investigated in subsequent section. Finally, some concluding remarks are drawn in the last section.

### 8.2 Delay Models

In signalised intersections, delay is the difference between the actual travel time a vehicle experience in traffic control and the travel time in the absence of any traffic signal control.
The calculation of the delay depends on several factors including probabilistic distribution of arrival flow (demand), signal timings and departure flow rate (supply), and the time when the vehicle arrives at the approach. Many of these factors are highly variable, thus making estimation of delay very complicated. The level of service (LOS) criteria for signalised intersections is set on estimation of delay. The LOS is an indicator of operational efficiency of the intersections by which quality of service is determined. The value of LOS is represented by letters A through F, with A being best and F being worst. Delay is used to determine the level of service of a signalised intersection, being the only element that is truly perceived by the drivers.

Delay experienced at traffic signals can be divided into two components: uniform delay and incremental delay. Uniform delay can be estimated using deterministic queuing approach considering a simple case of D/D/1. It is a well-known fact that during the congested period, the arrival flow rate is approaching to a uniform state. In traditional approach, assuming a uniform rate for arrivals and departures makes it a simple case of area calculation to estimate delay. For this purpose, an assumption is made that all vehicles accumulated during phase passes during green time. Solving this case results in formulation of the first term of Webster (1958) equation of delay Eq (8.1). This assumption cannot be implemented on isolated signalised intersections where the flow pattern is randomly distributed. To resolve this issue, a component of random delay equation is introduced that assume Poisson distribution for arrivals (Kendall, 1951). The random delay or overflow delay component includes that portion of delay which occurs due to temporary overflow of queues resulting from the random nature of arrivals. The random delay is an additional term introduced to incorporate delay component above uniform delay. This random delay component is adopted as a second term in the Webster (1958) delay equation.

Theoretically, incorporation of these two terms should represent actual delay, however it was observed that the first two terms produces a higher value of delay. Therefore a third term was introduced which was empirical in nature and it was derived from the simulation of traffic flow and generally refers to correction term (Webster, 1958). Webster formula for delay was later refined to eliminate correction term and a factor is introduced instead to reduce the sum of first and second term by 10% (Courage and Papapanou, 1977).
One of the major drawbacks of the Webster’s (1958) model was its inability to compute delay at saturation level (\(x\approx1\)). Webster’s model performs reasonably well in under-saturated (\(x<1\)) condition. However when the approach frequently faces a condition during which accumulated queue cannot dissipate fully in one cycle, a phenomenon of growing queue is developed, which is termed as overflow delay or incremental delay as referred in HCM (2010). Akcelik (1981) developed a formula to overcome this shortcoming of Webster model for overflow delay component;

\[
d_2 = \frac{cT}{4} \left[ (x - 1) + \sqrt{(x - 1)^2 + \left(\frac{12(x - x_0)}{cT}\right)} \right]
\]  

(8.2)

where \(T\) is analysis period duration (h) and \(c\) is capacity (veh/h) and other variables are as previously defined.

Later, this model in Eq (8.2) was incorporated in HCM (2000) with some modifications. The HCM (2000) model for signalised intersection contains three terms;

\[
d = d_1 PF + d_2 + d_3
\]  

(8.3)

where
\( d \) is average signal delay per vehicle in seconds
\( d_1 \) is average delay per vehicle due to uniform arrivals in seconds
PF is progression adjustment factor
\( d_2 \) is average delay per vehicle due to random arrivals in seconds
\( d_3 \) is average delay per vehicle due to initial queue at start of analysis time period, in seconds

The average delay due to uniform arrivals is computed with the following equation:

\[
d_1 = \frac{0.5C(1 - \frac{g}{c})^2}{1 - \min(1,x) \cdot \frac{g}{c}}
\] (8.4)

The incremental delay formulation in HCM (2010) is as follows:

\[
d_2 = 900T \left[ (x - 1) + \sqrt{(x - 1)^2 + \left( (x - 1)^2 + \frac{4x}{cT} \right)} \right]
\] (8.5)

The strong evidence of variations in queue discharge flow rate reported in the literature suggests a need for further investigation to verify such trends and then incorporate it in the delay formulation, in order to obtain better accuracy in delay estimations.

8.3 Traffic Observations in Auckland

To verify the variability in saturation flow rate, a study was conducted based on data collected from video recording at six intersections in Auckland, New Zealand. A number of site selection criteria parameters were established in order to locate the ideal intersection without applying adjustment factors for prevailing conditions in the light of recommendations made by Le et al. (2000). Three main selection criteria includes presence of heavy traffic with at least one exclusive through lane, the ideal geometric and roadway conditions with at least 3.6m lane width, level approach grade, minimal or no pedestrian movements, no curb parking, no bus stop in the vicinity and acceptable distance from adjacent intersection. The data was collected during peak hours with minimum two hours of data recording on all intersections except one intersection in fair weather conditions. The sites selected include Dominion – Balmoral Road intersection, Balmoral –
Sandringham Road Intersection, Great South Road – South Eastern Highway, St lukes – New North Road, Manukau – Greenlane East, and Pah – Mt Albert Road.

Individual headways are recorded for each vehicle in queue. The data analysis process involved collecting the headways between successive vehicles for two hour time period for each of the six intersections. Initial examination of the headways for each intersection shows some differences in the driver’s reaction time as shown in Table 8.1. Five intersections are located in the close proximity and show the reaction time varying from 1 second to 1.2 seconds. The results of initial examination of Great South Road – South Eastern Highway intersection shows a different trend than the remaining five intersections with average Reaction Time of 2.05 seconds. The start-up lost time observed within the range of 1.18 seconds to 3.08 seconds. The high start-up delay at St Lukes – New North Road is probably due to downstream approach grade with a sharp curve that resulted in increases of start-up lost time.

The possible reasoning of this increasing trend could be the driver’s behaviour towards the end of queue to pass the intersection before signal changes to red. This driver behaviour indicates the need to examine the car-following variables at signalised intersections and further investigations are required which are out of scope for this study. The results are summarized in the Table 8.1. These results give a clear indication that there is a relationship between queue discharge rate and the green time as previously reported in some studies. Based on these results, an empirical model is proposed to predict the varying nature of queue discharge flow rate.

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Number of Phases</th>
<th>Reaction Time (Start of green to movement of first vehicle)</th>
<th>Start-up Lost Time</th>
<th>Queue Discharge Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balmoral - Sandringham Rd</td>
<td>58</td>
<td>1.02 0.79 1.51 1.78 1.72</td>
<td>0.0099 t + 0.3496</td>
<td>0.83</td>
</tr>
<tr>
<td>Balmoral - Dominion Rd</td>
<td>59</td>
<td>1.20 0.57 1.18 1.03 1.26</td>
<td>0.004 t + 0.4853</td>
<td>0.44</td>
</tr>
<tr>
<td>GT South Rd - SE Highway</td>
<td>22</td>
<td>2.05 0.48 2.64 2.34 1.22</td>
<td>0.0098 t + 0.4113</td>
<td>0.54</td>
</tr>
<tr>
<td>Manukau - Greenlane East Road</td>
<td>58</td>
<td>1.15 0.45 1.24 0.94 1.12</td>
<td>0.0027 t + 0.4711</td>
<td>0.57</td>
</tr>
<tr>
<td>Pah - Mt Albert Road</td>
<td>60</td>
<td>1.18 0.41 1.73 2.14 1.41</td>
<td>0.0045 t + 0.4618</td>
<td>0.67</td>
</tr>
<tr>
<td>St Lukes - New North Road</td>
<td>54</td>
<td>1.00 0.47 3.08 3.16 0.96</td>
<td>0.0098 t + 0.3842</td>
<td>0.60</td>
</tr>
</tbody>
</table>

$t$ is the time passed after onset of green
8.4 Model Formulation

Based on the field observations, an empirical model is proposed to accommodate expected increasing queue discharge rate behaviour at signalised intersections. The details of the empirical model have been presented in another paper (Chaudhry and Ranjitkar, 2013). The model proposed is nonlinear in nature and is in the following shape;

\[
q_s = \frac{t - l_s}{1 + h_m(t - \gamma)}
\]

(8.6)

where

- \( q_s \) is the discharge rate in veh/ sec,
- \( t \) is the time in seconds,
- \( l_s \) is the initial lost time in seconds,
- \( h_m \) is the minimum average headway recorded after a number of cycles and
- \( \gamma \) is a correction factor.

Here \( Q \) is discharge rate after \( t \) seconds of the movement. The analogy of the maximum discharge rate is not used here, as the maximum discharge rate is highly variable at the initial stages, instead a discharge rate achievable in time \( t \) is used. This discharge rate is dependent on the lost time a signal management parameter encounter during the start of the green time. This model also incorporates the initial reaction time of the drivers after seeing the green time and start responding. This reaction time and response time varies with the type of transmission of vehicles and drivers’ behaviour. \( h_m \) is the minimum average headway recorded after a number of cycles.

8.4.1 Derivation of Delay under Uniform Arrivals

A D/D/1 case is considered to derive a delay formula for uniform arrivals and varying departure rate as shown in Figure 8.3. In order to coincide with the existing methodologies being used in practice, certain assumptions are necessary to make. The first assumption made in this derivation is pertaining to the signal capacity which is exceeding the arrival flow rate. The second assumption is that there is no initial queue at the start of the green
time. Lastly it is assumed that the queue formed during the red phase dissipates during the green time in the same cycle. These assumptions make it possible to establish a point where arrival line and departure line meet after vehicles are allowed to move during green phase.

The arrival flow rate line gives the total number of vehicle arrival at time t as shown in Figure 8.3. A red phase breaks the flow at signalised intersection for \( r \) seconds, then at the onset of green the flow is continuous with the departure flow rate. The dotted line indicates the traditional discharge flow rate and the broken line indicates the modified model’s discharge flow rate. Under the traditional concept of a uniform arrival and departure rate, the problem of delay can be solved as presented in the first term of the Webster (1958) Eq. (8.1). The derivation of the first term can be seen in any textbook on traffic engineering. When the departure rate varies, a modified form of Eq. (8.6) can be used in D/D/1 queuing analysis, replacing \( g \) with \( t \) and assuming \( \alpha \) and \( l_s \) are equal to zero:

\[
s = \frac{t}{1 + h_{m'}(t)}
\]  

(8.7)

The assumption of more capacity than arrivals means that all the vehicles that come in a cycle are cleared within the same cycle as shown in Figure 8.3. In the absence of constant departures, a new line is formed that represents a nonlinear model as proposed in Eq. (8.7). In Figure 8.3, at the horizontal axis, point o-r is denoting the red time, \( r \), and \( r-g \) and \( r-g' \) represent the time required to completely dissipate the queue, \( t_D \). Multiplying the departure flow rate with time, \( t_D \) gives the departures:

\[
\text{Number of vehicles departing} = \frac{t_D^2}{1 + h_{m'}(t_D)}
\]  

(8.8)

The slope of cumulative arrival line represents the uniform arrival rate approaching at the signalised intersection. The red time stops the traffic flow during which departure flow is zero. During the green time, the slope of cumulative departure line remains zero and equal to \( s \) at the onset of green signal for that approach.

\[
\text{Number of vehicles arriving in time}(r + t_D) = v(r + t_D)
\]  

(8.9)
The intersection of the arrivals and departures can be calculated by equating arrivals and departure terms.

\[ v(r + t_D) = \frac{t_D^2}{1 + h_m(t_D)} \]  

(8.10)

Simplifying above equation by using quadratic equation and ignoring negative value, the time required to dissipate the queue can be calculated as:

\[ t_D = \frac{(v + v_h.r) + \sqrt{(v + v_h.r)^2 + 4vr}}{2(1 - v_h.m)} \]  

(8.11)

Integrating the arrival triangle and deducting the area under the curve of departures gives this expression:

\[ D_t = \frac{v}{2}(t_D + r)^2 - \left[ \frac{t_D(h_m t_D - 2)}{2h_m^2} + \frac{\log(h_m t_D + 1)}{h_m^3} \right] \]  

(8.12)

where

\[ D_t \]  

is the aggregate uniform delay,
\[ v \]  

is the traffic flow,
\[ r \]  

is the length of the red phase and
\[ s \]  

is the saturation flow rate.

The effect of the log term is related to the gain of discharge rate at the onset of green time, which becomes insignificant in this case, so it is neglected.

\[ D_t = \frac{v}{2}(t_D + r)^2 - \frac{t_D(h_m t_D - 2)}{2h_m^2} \]  

(8.13)

Eq. (8.11) and Eq. (8.12) estimate aggregate uniform delay in vehicle – seconds for one signal cycle. To get an estimate of average uniform delay per vehicle, the aggregate is delay is divided by the number of vehicles arriving during the cycle,

\[ \bar{d} = \frac{D_t}{N} \]  

(8.14)
where $\bar{d}$ is the average uniform delay per vehicle and $N$ is the number of arrivals during cycle time $T$ which can be represented by:

$$N = vC$$  \hfill (8.15)

### 8.4.2 Incremental Delay

The component of incremental delay used in the HCM (2010) is also incorporated in the proposed nonlinear model. The model proposed in the HCM (2010) is in a form of a general time-dependent delay model which was conceived in late seventies (Kimber and Hollis, 1979, Robertson, 1979). Empirical evidence indicates that this model predicts reasonable results, though no rigorous theoretical basis for this approach is reported (Dion et al., 2004). A coordinate transformation technique is used to transform the equation that defines a steady-state stochastic delay model asymptotic to the deterministic oversaturation model. Due to the purely empirical nature of this equation, no direct derivation is made and instead same model is used after replacing volume to capacity ratio from modified model.

![Figure 8.3: Signalised intersection queuing with traditional and modified model II](image)

**Figure 8.3:** Signalised intersection queuing with traditional and modified model II
8. Delay Estimation at Signalised Intersections with Variable Queue Discharge Rate

8.5 Model Validation

In order to verify the proposed uniform component of delay, an example case is considered in which an arrival flow rate of 1200 vph is analysed with the traditional model and with the modified model. 30 seconds of red time breaks the traffic flow pattern, and after the onset of the green phase, the traditional model predicted a time of 60 sec to a state of arrival flow pattern. The modified model indicated that after 53.32 seconds, the traffic flow would revert to the arrival flow pattern. This shows a saving of 6.68 seconds of green time \((g'-g)\), approximately 11\%, which can be allocated to next phase as shown in Figure 8.3.

In order to verify the performance of the proposed model, two cases were analysed. In one case, the incremental delay model of the HCM (2010) was used without making any changes. In the second case, the v/c ratio and average capacity, \(c\), were replaced with the proposed nonlinear model in the incremental delay component of the HCM (2010). The first term in each case was replaced by the proposed model. The results indicated that the calculated delay values are fit closely with the delay model of the HCM (2010) as shown in Figure 8.4. In the first case, when only the uniform term is replaced, the results show a lower delay at degree of saturation between 0.85 to 0.95 and then a gradual increase surpassing HCM (2010) delay value in over-saturation state. The curve formed due to the replacement of both terms indicates a further compression where the v/c ratio is 0.8 and 1.0. The curve then gradually surpasses the HCM (2010) curve but remains below the curve of first case.

8.6 Discussion

When delay models were first introduced for signalised intersections, it was observed that the delay predicted by the models were about 5 to 15\% above the actual delay. Webster (1958) realized it first and introduced a correction term. The approximate over-estimation of delay goes as high as 15\% of the actual delay; however an average value remains 10\% of the sum of the uniform and the random delay component (Roess et al., 2010). The first term of traditional delay models is based on a uniform pattern of arrivals and departures. Adoption of new model based on the variable departure flow rate provides some of the
reasoning behind this over-estimation in delay from traditional approach. The results indicated that about 7% decrease in delay is recorded after 45 seconds of green time from the traditional model. A comparison of the 1st term of Webster (1958) model and proposed nonlinear model is shown in Figure 8.5. The lower volume of at a particular approach is indicative of a lower green time. At the lower green time, the nonlinear model predicts a higher delay value which is obvious because of the reason that the delay curve of Webster (1958) model is based on an average value of Saturation flow rate. For lower green times, the queue discharge rate observed is lower and therefore the nonlinear model predicts a higher delay than Webster (1958) model. Later when the queue discharge rate improves, gradually departing curves appear where nonlinear model predicts lower delay than Webster model.

Figure 8.5 indicates another benefit of adopting variable departure flow rate. Incorporating variability in the departure flow rate can help in predicting relatively accurate performance measures during the short green times. Traditional approach cannot make distinction between short cycle times and long cycle times which frequently occur on signalised intersections. Although the impact of short green time is not significant, however vehicles may have to wait for next green cycle due to the reason green times are allocated on the basis of fixed saturation flow rate.

![Graph showing delay estimate comparison between HCM, Webster and nonlinear model](image)

**Figure 8.4:** Delay estimate comparison between HCM, Webster and nonlinear model
8.7 Concluding Remarks

This chapter has implemented an empirical model to estimate delay at signalised intersections. The variations in the departure flow rate can impact the delay calculations. A decrease in delay estimation reduces the need of a correction term which was recommended by Webster (1958) due to an over estimation of delay in traditional delay formula. The reduction in delay in first term indicates that the queue dissipates earlier during long green cycles and saving of the green time gained from this early dissipation can be utilized in other phases. For a single cycle time, the effect of this reduction in delay might not be significant however on a large network where a number of intersections can add up the delays, this overall saving could be significant. The empirical model proposed in this chapter is capable of capturing the queue discharge behaviour observed at signalised intersections in Auckland. The proposed delay model is compared with the existing delay models. The results show that the proposed model can better approximate delay for uniform arrival rate. This investigation confirms the findings of the previous researchers that Webster (1958) model overestimates the delay. The incorporation of variable discharge flow rate in the uniform component of the delay formulation lowered the delay

![Graph showing delay comparison between Webster and nonlinear model](image)

**Figure 8.5**: Delay comparison between Webster and nonlinear model for uniform delay

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estimation by 5 to 6%. This decrease in delay compensate for a significant proportion of delay overestimation by Webster (1958) formulation which is approximately 10%. The results revealed that the proposed model can overcome the deficiencies of the existing models and can estimate delay more accurately.
9 ACCELERATION MODEL TO INCORPORATE VARIABLE QUEUE DISCHARGE HEADWAY AT SIGNALISED INTERSECTIONS

Abstract

Queue discharge headway is an important indicator of the capacity and operational efficiency of a traffic stream at a signalised intersection. In most studies, this factor is simplified and taken as a constant value after first few vehicles in a signal phase. Some researchers showed concerns regarding this simplification as queue discharge headway is a result of a very complex process that involves both driver and vehicle characteristics. A better understanding of this complex process can be obtained by studying these characteristics, which include perception, reaction time, speed and acceleration. This chapter conducts a detailed analysis of individual vehicles’ characteristics with the traffic stream. A data collection activity is conducted at six intersections and observed trends are modelled and verified using Gene Expression Programming (GEP). A nonlinear acceleration model based on the field trends is proposed for signalised intersections. The model indicates reasonable correlation in capturing queue discharge behaviour. More importantly, it shows prospects to overcome the shortcomings of the existing traffic flow models which do not incorporate the variable headway scenario.

9.1 Introduction

Signalised intersections are considered to be bottlenecks in the urban transportation network and are responsible for majority of delays experienced by the drivers. Vehicles are intermittently stopped to give way to conflicting movements and, consequently, a queue is formed on the approach which is stopped. This queue is then allowed to move by
allocating appropriate green time. Traditional traffic theories state that the queue discharge is at a constant maximum level after excluding first few vehicles (Akçelik, 1995, HCM, 2010). Several studies have criticised this statement and have indicated that the flow across the stop line does not reaches to a constant maximum rate after a certain number of initial vehicles (Li and Prevedouros, 2002, Lin et al., 2007, Teply, 1983). Some authors have suggested that this queue discharge rate continues increasing even after the eighth or ninth vehicles (Li and Prevedouros, 2002, Lin et al., 2007, Lin et al., 2004).

Micro-simulation modelling involves the detailed analysis of an individual vehicle’s characteristics within the traffic streams. Over the years, traffic simulation models have been widely accepted as a useful tool in traffic engineering, planning and research. Its applications in solving complex traffic engineering problems have received popularity as well as criticism. The usefulness in predicting various factors such as desired speed, red/yellow traffic light signals and gap acceptance, in addition to analysing roadway traffic conditions, showed a great potential to be explored (Buckholz and Courage, 2008). One of the major components of micro-simulation simulation models consists of a car-following model. This model describes the interaction of a stream of traffic in a lane. A lot of work has been conducted in the past on car-following models in running traffic streams (Ranjitkar et al., 2004, Ranjitkar et al., 2005a, Gagnon et al., 2008). However in stop and go conditions, such as at signalised intersections, the car following models are not producing acceptable results (Chaudhry and Ranjitkar, 2009). While, theoretically, micro-simulation models are considered to be a different focus in terms of input and output parameters, one of the main reasons is that only a limited number of studies are bridging this gap. Due to this shortcoming, micro-simulation models are not normally used in the operational analysis of signalised intersections.

A car-following model represents how vehicles interact in a stream of traffic. The major variables in car-following models are acceleration/deceleration, speed, relative position and perception/reaction time. In general, car-following models are relatively simple to predict traffic flow conditions on a section of a road as the main focus is the behaviour of the individual vehicles for a certain length of road. The methodology of signalised intersections, on the other hand, is mainly focused on the behaviour of the traffic at one point (usually the stop line). The performance of signalised intersections is
judged on how quickly the queue, formed during red time, dissipates which actually depicts the overall capacity of that particular approach. The major variables involved therefore consist of saturation flow rate, v/c ratio and headway (HCM, 2010). Unlike the micro-simulation approach, the variables are usually calculated at the stop line. Several studies indicated that some variables used in car-following models are insignificant in signalised intersection analysis. For example, speed is considered an important parameter in the micro-simulation models, however, it is totally neglected in the capacity and LOS estimations for signalised intersections. This is due to the fact that speed is not very sensitive to flow at saturation level (Tian et al., 2002).

Another issue with the limited use of the micro-simulation concept is that it has a different meaning for its key parameters. For example, the headway terminology is taken to be the measurement of the rate of crossing vehicles at the stop line for the signalised intersection. The same terminology in micro-simulation models are, however, based on establishing a safe distance between successive vehicles; and headway is used to describe the safe distance between two vehicles in terms of time or space as a function of speed. The translation of micro-simulation time-dependent headway into signalised intersection point-dependent headway is not a straight-forward procedure. Most micro-simulation models produce constant headway when they attain a stable condition in speed at the stop line, while the actual field conditions are quite different (Chaudhry et al., 2011).

In traditional signalised intersection methodology, the convenient assumption of a steady saturation flow rate over the saturated green time makes it considerably easier to calculate the lane group capacity. Several vehicular and human behavioural factors are identified in the literature that can influence saturation flow rate (HCM, 2010). However, the basic concept remains the same that when a signal changes to green, the flow across the stop line increases rapidly to the saturation flow rate, which remains constant until either the queue is exhausted or the green period ends (Akçelik, 1981, HCM, 2010).

A great deal of research conducted on queue discharge behaviour at signalised intersections reported large variations in queue discharge rate (Chaudhry and Ranjitkar, 2013, Chaudhry et al., 2011, Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005, Lin et al., 2004, Tarko and Tracz, 2000, Teply, 1983). Several of these studies observed a decreasing trend in the mean headway, and hence an increasing queue
discharge rate towards the back-of-queue even after the first four vehicles (Li and Prevedouros, 2002, Lin et al., 2007, Lin and Thomas, 2005).

This strong evidence for an increasing queue discharge rate observed at signalised intersections raise serious doubts on the assumption of a steady queue discharge rate termed saturation flow rate. Several models were proposed in the literature to approximate queue discharge behaviour at signalised intersections (Bonneson, 1992, Long, 2007). However, these models are yet to be implemented in the real world, which seems difficult as their formulation cannot be easily solved and require the use of the root-search method.

A new car-following model is developed for signalised intersections using Gene Expression Programming (GEP). GEP is a genetic algorithm (GA) based tool, developed by Ferreira (2001). It is quite useful to develop new models based on field observations and also to compare them against alternative formulations. GEP has been used in different fields and proven to be more accurate and stable than genetic programming and linear regression.

The objectives of this chapter are to present and compare a car-following model at signalised intersections that incorporates a marginally decreasing headway trend. Due to the fact that speed and acceleration are mathematically related, this chapter focuses on developing a time-based integrated driver acceleration model without introduction of excessively complicated parameters. This model is then used to derive mathematical relationships for vehicle discharge time, vehicle discharge headway, speed, and saturation flow rate at the stop line.

9.2 Literature Review

Car-following modelling involves the detailed analysis of individual vehicles characteristics within the traffic streams. Over the years, car-following models have been widely accepted as a useful tool in traffic engineering, planning and research. Its applications in newly emerged intelligent transportation systems (ITS) applications including V2V and V2I communications have received widespread interest (Sinha et al., 2013). While a great deal of work has been conducted on vehicle to vehicle interactions on a continuous section of the road, there are not enough literature currently available for the
use of car-following in stop and go situation such as signalised intersections. Most of the previous research mainly focused on driving behaviour along the straight sections without interruptions and the effects of control strategy is either ignored or lack a detailed analysis (Ranjitkar et al., 2005b).

In car-following models, the lead vehicle plays a significant role in establishing traffic flow condition for followers. The traditional car-following models are based on the concept that driver accelerates or decelerates in response to a stimulus or several stimuli from surroundings. The theories of car-following models are generally built to depict a running traffic stream where acceleration and deceleration happen as conditions change. Reuschel (1950) and Pipes (1953) were among the earliest researchers who studied the running state of vehicles in a queue. The research group at General Motors made a significant contributions to the car-following theory (Chandler et al., 1958, Herman et al., 1959). Chandler et al. (1958) proposed the response of a driver as a function of sensitivity and stimuli

\[ \text{Response} = f(\text{sensitivity, stimuli}) \]  \hspace{1cm} (9.1)

The model considered a single stimulus factor of relative speed to trigger response in the form of acceleration.

\[ a_n(t + T) = \lambda \Delta v_n(t) \]  \hspace{1cm} (9.2)

where \( \lambda \) is the sensitivity factor and \( \Delta v_n \) is the difference between the speed of the leader and follower vehicles. Sensitivity is defined as

\[ \lambda = \alpha \frac{v_n(t + T)}{\Delta x_n(t)} \]  \hspace{1cm} (9.3)

Eq (9.3) describes a simple linear relationship between acceleration and relative speed. This model eventually became the basis of more detailed generalized general motors model after a series of investigations (Gazis et al., 1961).

\[ a_n(t + T) = \alpha_0 \left[ v_{n+1}(t + T) \right]^m \frac{v_n(t) - v_{n+1}(t)}{[x_n(t) - x_{n+1}(t)]^l} \]  \hspace{1cm} (9.4)

where \( m \) and \( l \) are the scaling constants to describe the speed and spacing term.
At signalised intersections, however, the phenomenon of queue discharge is mainly dependent on acceleration behaviour only. Briggs (1977) was the first to propose a deterministic model to incorporate an increasing trend in queue discharge rate. His model assumes a constant acceleration for queued vehicles and takes the following form:

For $d.n < d_{i\text{max}}$,

$$h_n = T_r + \frac{2.d_i.n}{A} - \frac{2.d_i.(n-1)}{A}$$

(9.5)

Otherwise,

$$h_n = T_r + \frac{d_i}{V_q}$$

(9.6)

where $h_n$ is the headway of the $n$th vehicle, $n$ is the queue position, $V_q$ is the desired speed of the queued traffic, $d_i$ is the distance between the vehicles in a stopped queue, $d_{i\text{max}}$ is the distance travelled to reach the desired speed ($V_q$), $T_r$ is the driver starting response time, and $A$ is the constant acceleration of queued vehicles.

Briggs’ (1977) model has two parts. The first part works when vehicles try to achieve a desired speed $V_q$ at the beginning of green phase. The model replicates the queue discharge behaviour during this state well. However, it predicts a constant headway in the second part, which represents a state when vehicles have already achieved the desired speed and are traveling at a constant speed. The model suggests that the minimum headway is achieved after 6 vehicles.

Bonneson (1992) reported that the headway continues to decrease until eighth or ninth vehicle in the queue crosses the stop line, after which a constant level is attained. He presented his model based on driver reaction time, vehicle speed and acceleration. Unlike Briggs’ (1977) model, Bonneson’s (1992) model was devised based on a non-constant acceleration rate. The headway of the $n^{th}$ vehicle according to this model is:

$$h_n = \tau N_1 + T_r + \frac{d_i}{V_{max}} + \frac{V_{max} \cdot (1 - e^{-n.k}) - V_{sl(n-1)}}{A_{max}}$$

(9.7)
where $\tau$ is the additional response time of the first queued vehicle, $N_1$ is 1 if $n=1$, or 0 if $n>1$, $d_i$ is the distance between vehicles in a stopped queue, $V_{sl(n)}$ is the stop line speed of the $n$th queued vehicles, $V_{max}$ is the maximum speed, $A_{max}$ is the maximum acceleration and $k = \beta/V_{max}$, where $\beta$ is an empirical calibration constant.

Long (2006) presented a driver behaviour model of queue discharge rate in terms of the discharge times of each vehicle in the queue. His model considers the distances of the vehicles in the queue to the stop line, acceleration, and the average start up lost time, taking the following form:

$$td_n = w_n + ta_n = \tau + n.T_r + t_a(sa_n, \alpha, \beta) \quad (9.8)$$

where

- $td_n$ is the average discharge time at the stop line of the $n$th vehicle in queue,
- $w_n$ is the start-up waiting time for the $n$th vehicle in a queue,
- $ta_n$ is the average acceleration time,
- $\tau$ is the excess start-up time of the lead vehicle in a queue,
- $T_r$ is the uniform or average start-up response time of each driver,
- $sa_n$ is the average distance of acceleration of the $n$th vehicle from the start of motion to time $ta_n$,
- $\alpha$ is the average initial rate of acceleration and
- $\beta$ is the average rate of decrease in acceleration with increasing speed.

Despite its ability to replicate the increase in queue discharge rate, it is difficult to implement this method for practical applications as Eq. (9.8) cannot be easily solved and requires application of the root-search method (Long, 2007).

The acceleration behaviour is usually considered to be constant. Evans and Rothery (1983) conducted some experiments on saturation flow and presented their observations as shown in Figure 9.1. They showed that vehicles start moving from stationary position with the maximum acceleration rate. Gradually this acceleration decreases as vehicles reaches closer to the desired speed.
9.3 Gene Expression Programming

Gene expression programming (GEP) is a type of genetic algorithm that allows fast application of a wide variety of mutation and cross-breeding techniques while guaranteeing that the resulting expression will always be syntactically valid. Studies indicated that GEP perform better and faster than Genetic algorithms (Qu et al., 2004). The GEP belongs to the wider class of Genetic Algorithms (GA) along with genetic programming (GP). This class of GA uses populations of individuals, selects them according to fitness, and introduces genetic variation using one or more genetic operators (Ferreira, 2001). The main difference is that GA uses linear strings of fixed length, GP uses nonlinear entities of different sizes and shapes and GEP uses linear strings of fixed length then converted as nonlinear entities of different sizes and shapes.

Mathematical evolution in the GEP starts with creation of an initial population of candidate functions. Then the evolution engine mutates breeds and finally mathematical function is evolved as a result of natural selection.

In terms of performance, GEP is considered as extremely handy and it outshines the existing evolutionary techniques and some experiments indicated that it performs better.
than genetic algorithms (Ferreira, 2001, Qu et al., 2004, Yu et al., 2010). The algorithm in GEP allows the creation of multiple genes to form sub-expression tree. These multiple genes are the core capability that enable to not only encode any conceivable program but also allows an efficient evolution. Gene operators then search the solution space and generate valid structures in the form of mathematical expressions.

### 9.4 Driver Acceleration Model

Akçelik and Biggs (1987) indicated that queued drivers initially adopt a high acceleration and then decrease this acceleration as travel time increases. The high initial acceleration is observed as drivers want to achieve the desired speed and as they reach near to their desired speed, they gradually reduce acceleration. The general form of the model with respect to time \( t \) is given by;

\[
a(t) = r a_m \theta^n (1 - \theta^m)^2 (n > 0, m > -0.5n)
\]  

where \( a(t) \) is the acceleration rate at time \( t \), \( a_m \) is the maximum acceleration, \( \theta \) is the time ratio \( (t/t_a) \), \( t_a \) is the acceleration time, \( m \) and \( n \) are parameters to be determined, and \( r \) is a parameter which depends on the values of \( m \) and \( n \).

The model proposed was relatively simple as the input parameters do not require speed at the stop line. This factor is also highlighted by other research that the speed parameter becomes insignificant when traffic streams are operating at capacity, as in the case of signalised intersections (Evans and Rothery, 1983).

An earlier study conducted on driver acceleration characteristics indicated that acceleration decreased linearly with increasing speed (Buhr et al., 1969).

\[
a = A_{max} \left[ 1 - \frac{V}{V_{max}} \right]
\]  

where \( a \) is the instantaneous acceleration, \( V \) is the velocity of vehicle, \( A_{max} \) is the maximum acceleration and \( V_{max} \) is the maximum speed corresponding to zero acceleration.
9. Acceleration Model to incorporate Variable Queue Discharge Headway at Signalised Intersection

This model was used by Bonneson (1992) to develop expressions for speed and acceleration distances. He used the exponential approach to depict speed profiles at signalised intersections. The proposed model accurately represents the decrease in queue discharge headway until eighth or ninth vehicle in the queue at stop line after which a constant level is predicted.

When signal turns to green, the vehicle in \(n^{th}\) position does not move until the vehicle in front of it moves. The movement of early vehicles depicts how vehicles behind will follow. The first vehicle in the queue does not have any constraint and the driver can select the acceleration pattern as he wishes. After the preceding vehicle begins to move, the \(n^{th}\) vehicle starts moving.

Numerous attempts have been made to describe the acceleration behaviour including constant acceleration model and, linearly decreasing acceleration model polynomial acceleration models. The simplest model developed was based on a constant acceleration model. The linearly decreasing constant model returns reasonable results, however the actual scenario in the field depicts a nonlinear decrease in acceleration rate as vehicles start moving after onset of the green period. The vehicles start at a high acceleration rate and the acceleration curve becomes flatter as the vehicles reach the desired speed. The effect of car engine power could impact the acceleration rate, however studies indicate that there is not much difference in the acceleration rate due to engine capacity. Vehicles usually start moving at a normal acceleration rate, which is much less than what vehicles can produce due to engine power. This normal acceleration rate is a totally site specific phenomenon.

To incorporate the nonlinear acceleration model, RTK Japanese car-following experiment data is used (Suzuki et al., 2005). Nonlinear models seem more appropriate to represent the queue discharge rate. However, restricting a model that can incorporate varying trends as well as not being too complicated as to make further calculations arduous was a primary objective for the model development. In order to achieve this, the most influential parameters that may impact how acceleration rate progresses after vehicles are allowed to move at the onset of green time are identified. For car-following models, speed, acceleration, and gap are important parameters. At a signalised intersection, stationary vehicles correspond to zero speed. Similarly, when vehicles start moving, the way in which acceleration rate is changing can be used to calculate speed, as acceleration and
speed are related terms. Studies indicate that, at capacity, the speed parameter becomes insignificant at signalised intersections. Therefore the speed parameter is not used in model formulation.

The stop position of each vehicle in the queue is an important parameter for signalised intersections. Table 9.1 below shows the average gap between stopped vehicles. As the data was collected on a test track, no reference stop line was present and an arbitrary stop line imagined. In the literature, studies that collect headways or discharge times at the stop line rarely collect data on the distances of the front wheels of queued vehicles from the stop line when they were stopped.

<table>
<thead>
<tr>
<th>Vehicle Position</th>
<th>Average Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between 1st and 2nd Vehicle</td>
<td>5.57</td>
</tr>
<tr>
<td>Between 2nd and 3rd Vehicle</td>
<td>6.36</td>
</tr>
<tr>
<td>Between 3rd and 4th Vehicle</td>
<td>8.0</td>
</tr>
<tr>
<td>Between 4th and 5th Vehicle</td>
<td>7.42</td>
</tr>
<tr>
<td>Between 5th and 6th Vehicle</td>
<td>7.45</td>
</tr>
<tr>
<td>Between 6th and 7th Vehicle</td>
<td>7.98</td>
</tr>
<tr>
<td>Between 7th and 8th Vehicle</td>
<td>8.5</td>
</tr>
<tr>
<td>Between 8th and 9th Vehicle</td>
<td>6.08</td>
</tr>
<tr>
<td>Between 9th and 10th Vehicle</td>
<td>6.5</td>
</tr>
<tr>
<td><strong>Aggregate Average</strong></td>
<td><strong>7.1</strong></td>
</tr>
</tbody>
</table>

A quick review of the data shows that vehicles stop at almost equal distance. A simplification can be made assuming equal distances between stopped vehicles, and most influential factor that can affect instantaneous acceleration:

\[ a = f(n, t) \]  

(9.11)

where \( a \) is the instantaneous acceleration and \( n \) is the position of the vehicle in the queue. A training set is selected from the data to establish a model and rest of the data is used as the testing set. The fitness mechanism established to measure how well a model is returning the predicted function as follows:
where $M$ is the range of selection, $C(i,j)$ is the value returned by the individual chromosome $i$ for fitness case $j$ (out of $C_t$ fitness cases) and $T_j$ is the target value for fitness case $j$. If $|C(i,j) - T_j|$ is less than or equal to 0.01, then the precision is equal to 0, and $f_i = f_{max} = C_t M$. For the fitness purposes, $M$ is used as 100 and $f_{max}$ is used as 1000. The GEP can find the optimum solution by itself then.

Expressions are developed while restricting the number of mathematical operators in the curve fitting equation. GEP has the inbuilt ability to quickly mutate valid expressions in the form encoding symbols in genes. Table 9.2 presents the various forms of mathematical expressions used to establish the model and the resulting statistical curve fit parameters. About 10 different combinations of mathematical functions are used to establish the most appropriate expression to fit the available data. The addition, division and addition, and inverse functions produce optimum expressions with the highest predicted correlations. The addition, multiplication functions which produce a linear model return the lowest statistical parameters with $R^2$ value of 74%, MAPE value of 41, and RMSE and MAE value of 0.2. The addition, division and addition, and inverse functions produce the most optimal values with $R^2$ values above 95% and MAPE values above 26. RMSE and MAE values are also reasonable. The SQRT, LOG, subtraction and negation functions produced no results with the selected set of conditions. It can be seen for almost all cases that the addition, inverse mathematical expression returns a relatively better fit for the available data.

An important aspect in model development was to translate speed $V$ and queue position $n$, in something simple and determinable. The vehicles start moving from stop position and tend to achieve their desired speed as soon as conditions permit. The speed attained at the stop line depends on various factors, including vehicle characteristics as well as driver behaviour. The posted speed limit can be attained within 20 to 30 seconds with the normal acceleration rate after the onset of green time, however, observed data and previous literature indicates that cases exist where the discharge flow rate continue to increase until between 45 and 50 seconds. Table 9.2 shows a comparison between different
models. A closer look into the models shows that a time based model can be deduced by translating vehicle position into distance and applying the law of motion. Graphical representation of the different models predicted by GEP is shown in Figure 9.2. The distance to the stop line of a standing vehicle in the queue depends on driver behaviour. An equal distance can simplify the model, and the model is developed based on the instantaneous acceleration at time, \( t \). This assumption is made to make the model in line with a study conducted previously for variable saturation flow rate (Chaudhry and Ranjitkar, 2013).

### Table 9.2: Model development comparison in GEP

<table>
<thead>
<tr>
<th>Functions</th>
<th>Mathematical Expression</th>
<th>Statistical Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition, Division</td>
<td>( a = \frac{3.85}{n} - \frac{2.23}{n^2} + 0.08 )</td>
<td>95 26.2 0.1 0.4</td>
</tr>
<tr>
<td>Addition, Power</td>
<td>( a = \text{pow}(0.89,n) + 0.12 + \text{pow}(0.66,n) )</td>
<td>94 28.1 0.1 0.7</td>
</tr>
<tr>
<td>Addition, Inverse, square</td>
<td>( a = \left( \frac{91.47}{(n+0.03)} \right) + \left( \frac{29677.09}{(n+435.75)} \right) + (2082.95(n+112.20)) + (789.92(n+112.11)) + 51.76 + \left( \frac{-88.66}{(0.02/n)+n} \right) + \left( \frac{-3795.76}{(n+437.62)} \right) )</td>
<td>96 27.2 0.1 0.1</td>
</tr>
<tr>
<td>Addition, SQRT</td>
<td>No Result produced by GEP under set of conditions selected</td>
<td></td>
</tr>
<tr>
<td>Addition, LOG</td>
<td>No Result produced by GEP under set of conditions selected</td>
<td></td>
</tr>
<tr>
<td>Addition, Subtraction</td>
<td>No Result produced by GEP under set of conditions selected</td>
<td></td>
</tr>
<tr>
<td>Addition, Negation</td>
<td>No Result produced by GEP under set of conditions selected</td>
<td></td>
</tr>
<tr>
<td>Addition, Inverse</td>
<td>( a = \left( \frac{0.6}{((1.48/n)-2.48)+n} \right) + (-0.1) + \left( \frac{(-0.96)}{((n-2.32)+1.32/n)} \right) + (7.51/(((-1.03)/n)+n-(-2.49))) )</td>
<td>96 27.6 0.1 0.6</td>
</tr>
<tr>
<td>Addition, Multiplication</td>
<td>( a = (-(0.06)*n)+1.2 )</td>
<td>74 41.0 0.2 0.2</td>
</tr>
<tr>
<td>Addition, Division, Power</td>
<td>( a = \left( 1.1371282/n \right) + ((-0.0290559)*n) + 0.6984866 )</td>
<td>94 30.5 0.1 0.1</td>
</tr>
</tbody>
</table>
A simplified model based on the observations on can be represented as:

\[ a(t) = \frac{A_{\text{max}}}{1 + \frac{1}{2} h(t - T)} \]  \hspace{1cm} (9.13)

where \( a \) is the instantaneous acceleration, \( A_{\text{max}} \) is the maximum acceleration and \( t \) is the time elapsed after signal turns green.

At the stop line, assuming that there are no capacity reduction conditions downstream, first vehicle accelerates and reaches its desired speed as he wishes or as the vehicle performance allows. The following vehicles’ accelerations depend mostly on how the lead vehicle accelerates.  

The distance to the stop line

\[ S_n = n \cdot d + S_1 \]  \hspace{1cm} (9.14)

where \( S_n \) is the distance from the stop line, \( n \) is the position of the vehicle in queue, \( d \) is the average spacing between the vehicles in queue and \( S_1 \) is the distance of the first vehicle from the stop line.

\[ t_n = \frac{S_n + \sqrt{S_n(8A_{\text{max}} + S_n)}}{2A_{\text{max}}} \]  \hspace{1cm} (9.15)
where \( t_n \) is the discharge time from the \( n^{th} \) vehicle in queue and \( A_{\text{max}} \) is the maximum acceleration.

Translating the time of each vehicle to reach at the stop line from there stationary position in the queue to the traditional headway at the stop line showed a decreasing trend as shown in Figure 9.3. It can be seen from this figure that the headway gradually decreases and resultant saturation flow rate increases with respect to the time (green time in case of signalised intersection).

![Figure 9.3: Proposed model outcome for headway and saturation flow rate](image)

**Figure 9.3**: Proposed model outcome for headway and saturation flow rate

### 9.5 Model Validation

Figure 9.4 shows the actual versus predicted values of acceleration for the proposed model. GEP suggested a nonlinear solution that returns the optimum solution for the curve fitting of this data set. The validation results indicated that a 96% correlation is present for the proposed model with actual values.
Figure 9.4: Actual versus predicted value of acceleration

Figure 9.5 presents a comparison between the proposed model and actual readings of acceleration. The proposed model gives a good estimate of the decrease in acceleration when compared with the observed data.

Figure 9.5: Performance of acceleration with respect to queue position
9. Concluding Remarks

This chapter proposed a new acceleration model for signalised intersections. The variable nature of the acceleration behaviour of the vehicles in the queue is first verified using the RTK GPS dataset and then observed trends are assessed using the GEP algorithm. The proposed model is simple and captures the car-following behaviour observed at signalised intersections. A comparison with the saturation produced by the proposed model depicts an increase of 5.5% at 30 seconds of green time from the flow rate at 10 seconds. This shows a marginal increase of saturation flow rate. The implementation of this model can bridge the gap between existing car-following models and the methodologies used for signalised intersections.
10 CONCLUSIONS AND RECOMMENDATIONS

10.1 Summary

Saturation flow rate is a key parameter in traffic signal design on the basis of which most of the performance measures including capacity and delay are estimated. The main objective of this research was to investigate queue discharge behaviour at signalized intersections and to verify increasing queue discharge rate reported in the literature. With an improved understanding of queue discharge behaviour at signalized intersections gained from empirical study of field observations, a nonlinear model is devised to approximate the queue discharge behaviour more realistically. To implement the proposed queue discharge model in practice, the respective signal timing and uniform delay formulations are also derived. Finally, a microscopic car-following model is proposed to replicate the observed queue discharge behaviour at signalised intersections. The proposed method is verified against the data collected from signalized intersections in Auckland. The key findings of this research are summarised in the following paragraphs; categorised into the following subheadings: verification of saturation flow rate, development for queue discharge models, model implementation and development of car-following model for signalized intersections. Finally some recommendations on future research directions are made in the last section.

10.2 Verification of Saturation Flow Rate

A pilot study was conducted to verify some of the observations reported in the literature contradicting with the assumption of a constant saturation flow rate. The signalised intersection selected for the pilot study was chosen based on its proximity to ideal road geometry and traffic conditions.
10. Conclusions and Recommendations

Observations and Interpretations

Following are the main observations and interpretations that can be drawn from this pilot study:

- The traffic was mainly composed of passenger cars which constituted 98.1% of the total traffic with only 1% buses and the remaining is general purpose vehicles (GV), making it an ideal choice for this study as there were no capacity reducing factors.
- There was a slight decreasing tendency observed for queue discharge headway towards the back of queue, resulting in a slightly increasing trend for queue discharge rate.
- Around 6% increase in queue discharge rate was observed, for queued vehicles positioned between 16 to 18, when compared with the constant saturation flow rate computed using the HCM method.
- These results are in line with what is reported in the literature that is an increasing queue discharge rate, which contradicting with the notion of a constant saturation flow rate concepts adopted in the traditional capacity guides.

10.3 Development of Queue Discharge Models

The process involved in queue discharge flow is comprised of complex phenomena that depend on many behavioural and vehicular factors. A detailed investigation has been conducted to investigate the queue discharge behaviour at signalised intersections. Six signalised intersections were identified for data collection using video cameras. The results obtained confirmed the findings of the pilot study outcomes and showed an increasing queue discharge rate.

Linear and nonlinear models are proposed to approximate queue discharge behaviour observed at signalised intersections. However nonlinear model performed better than linear model to replicate the queue discharge behaviour. The Gene Expression Programming (GEP) tool was used to develop and compare a number of different linear and nonlinear models based on their goodness of fit values. The results revealed that the inverse function model produces the highest correlation with the field data.
Observations and Interpretations

Following observations are made from the development of queue discharge model to approximate the field observations at signalised intersections:

- The nonlinear queue discharge model shows that the maximum discharge rate is sensitive to the green time allocated to the phase. In general, a lower green time not only adds to the delays due to the acceleration and deceleration required to stop and move, but also the queue discharge rate may not reach its maximum value.

- At the Barmoral – Dominion Road intersection, the discharge headway after 15 seconds of green time was recorded as 2.05 seconds which corresponds to a queue discharge flow rate of 1756 pcphgpl which further decreased to a value of 1.9 seconds after 45 seconds of green time that corresponds to a queue discharge rate of 1895 pcphgpl.

- Similar observations were made at other intersections as well. After 45 seconds of green time, queue discharge headway decreased by 5% to 9% when compared with the discharge headway after 15 seconds of green time. The latter corresponds to fifth or sixth vehicle in the queue.

- The observed trends of variations in the queue discharge rate at signalised intersections are modelled using empirical models. A comparison between different models indicated that the inverse function model produces better correlations with the field observations.

- The $R^2$ values computed for the inverse function model ranges from 62% and 84% for different intersections, where the Balmoral – Sandringham Road intersection witnessed the highest $R^2$ values of 84.41% and the St Lukes-New North Road intersection witnessed the lowest of 62.17%. MAPE values ranged from 3 to 12. The highest MAPE value of 12.06 is witnessed for the St-Lukes – New North Road intersection while the lowest value of 2.98 is witnessed for the Pah Road – Mt Albert Road intersection.
10.4 Model Implementation

10.4.1 Capacity and Signal Timing Formulations

In general, the capacity of a signalised intersection approach is dependent on the proportion of the green time allocated to the phase. However, capacity is calculated based on a constant saturation flow rate. A nonlinear model is proposed to approximate the variable queue discharge rate observed in the field and implemented for the capacity and signal timing calculations. However the calculations were somewhat cumbersome using the proposed model, which is then simplified assuming initial lost time and correction term equals to zero.

While implementing the proposed model, two approaches were adopted. In one approach, the proposed model is used with the initial lost time and correction term. The results indicated that the capacity at the beginning of the green period is reduced compared to traditional capacity model. This is well justified, as the fixed nature of the traditional capacity model does not incorporate the lower queue discharge rate. On the other hand, the proposed model is capable of incorporating the initial phase as well, and therefore it can be applied to the approaches with low traffic volumes. In the second approach, taking out the value of initial lost time $l_\ell$ and correction term $\propto$ correction term to make the model simpler to be used in signal and cycle time calculations. The simplified model produced a reduced capacity, however, it helps in using the model in further calculations.

Observations and Interpretations

Following observations are made on the implementation of the proposed model to update the capacity and signal timing formulations:

- The proposed model improved the capacity by 6.9% with 45 seconds of green time when compared with the traditional model.
- Using simplified version of the proposed model that is after removing initial lost time $l_\ell$, and correction term $\propto$, the capacity is increased by 5% with 45 seconds of green time compared with the traditional model.
- A concept of green time demand is introduced to incorporate the variable queue discharge rate. The green time demand is calculated based on the flow ratio.
When compared with the traditional method, the proposed method, generally yields relatively higher green time for low traffic demand conditions as discharge headway is relatively higher towards the beginning of the queue.

10.4.2 Delay Formulation

The traditional uniform delay model is modified to implement the proposed variable queue discharge model instead of a constant saturation flow rate used in the traditional model. The modified uniform delay model yields relatively lower delay values when compared with the traditional model indicating an early dissipation of queues due to increased capacity. This addresses one of the discrepancies that the traditional uniform delay model has as it overestimates the uniform delay. An empirical correction term was introduced then in the traditional delay model to compensate for this discrepancy.

Observations and Interpretations

Following observations are made on the implementation of the proposed model to update uniform delay formulation:

- The implementation of the variable queue discharge model to derive uniform delay formulation partially addressed overestimation problem of the traditional delay model that overestimates the delay by approximately 10% when compared with the field observations.
- The delay estimates using the new formulation is 5 to 6% lower than those produced by the traditional model.

10.5 Development of Car-following Model for Signalised Intersections

A car-following model is proposed to incorporate variable queue discharge headway. It is observed that the drivers accelerate more aggressively when they start to move after seeing the green signal. The acceleration decreases as they are closer to the allowable speed limit. The proposed model yields a strong correlation with the observed data.
10. Conclusions and Recommendations

Observations and Interpretations

Following observations are made on the development of car-following model for signalised intersection:

- The vehicles start moving at a high acceleration rate and then the acceleration curve becomes flatter as it reaches a desirable speed.
- A car-following (acceleration) model is developed by restricting the variables to minimum. Speed is generally an important factor in car-following models. However, it is observed that, at the capacity, the speed becomes insignificant parameter for signalised intersections.
- The proposed model yields 5.5% increase in flow rate at 30 seconds of green time when compared with the flow rate at 10 seconds.

10.6 Recommendations for Future Research

In light of the favourable research findings, it is demonstrated that a non-linear model for queue discharge rate provides a viable tool for the evaluation of variable queue discharge rate at signalised intersections. The non-linear models proposed in this dissertation are ready for field testing for traffic signal design and delay estimations. The research presented in this dissertation is a first step in establishing and validating non-linear model. The proposed model has shown potential to integrate with existing signalised intersection evaluation methodologies. However, this model is proposed based on the data which was predominantly consisted of cars and light vehicles and hence it is needed to test this model in varying conditions. Queue discharge rate is sensitive to several influencing factors. Localised driving conditions are most influential among these influencing factors. It is therefore required to calibrate and validate the proposed model before implementing in the field. Calibration and validation of the model is also required to implement the proposed model to address the more challenging network and traffic configuration in the field.

Some examples for further research include the following;

a) Quantifying the impact of downstream frictional element such as parking, and bus stops. For example, what are the impacts of moving a bus stop from near side to far side on the queue discharge rate?
b) Validating the queue discharge rate under mixed vehicle composition across the traffic lanes. The research findings in this dissertation is based on the traffic flow which is largely consists of passenger cars. The impact of other vehicles in variable saturation flow rate is yet to validate.

c) The delay model is proposed based on uniform arrivals and only first term is changed. It is recommended that the additional regression equations be generated relating to 2nd term of delay equation. Comparison should be made with the field observations.

Further validation of the models proposed in this dissertation would require intensive and systematic field data collection and implementation of the proposed model on actual traffic signals.
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