Reactance of hollow, solid, and hemispherical-cap cylindrical posts in rectangular waveguide

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Reactance of Hollow, Solid, and Hemispherical-Cap Cylindrical Posts in Rectangular Waveguide

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Abstract—An accurate experimental technique is employed to determine the reactance of three commonly used symmetrical waveguide discontinuities: cylindrical posts with hollow, solid, and hemispherical-cap ends, and a useful data set is presented. Length adjustment factors are derived for the case of symmetrical excitation that relate the reactance of each post form. When used in conjunction with an existing analysis, it is shown that very accurate results can be predicted for the reactance of solid and hemispherical-cap posts.

Index Terms—Filters, tuning networks, waveguides.

I. INTRODUCTION

ONE OF the most commonly used reactive elements in waveguide matching and filter networks is a cylindrical metallic post that extends into the waveguide, as shown in Fig. 1(a). The post axis is orientated parallel to the dominant \( \text{TE}_{10} \)-mode electric field. The current distribution induced on the post by the incident \( \text{TE}_{10} \) mode will depend on the physical dimensions of the waveguide and the size and form of the post itself; hollow, solid, or hemispherical cap. By changing \( \ell \), the induced current and, hence, the reactive effect of the post, can be adjusted (in practice, this is usually facilitated by using a threaded post that enters through a threaded hole in the waveguide). Notwithstanding its widespread use, there is little published data for the reactance of cylindrical posts other than in [1] where experimental data is given for solid and hemispherical-cap posts. Over the years, these measurements have received many citations.

In the full-height case, when \( \ell = h \), the axial current induced by the incident \( \text{TE}_{10} \) mode is constant along the post surface, significantly reducing the difficulty of the theoretical analysis. Schwinger and Saxon solved this problem using a variational procedure [2] by approximating the circumferential variation of the axial current around the post by two terms of a Fourier series, the results of which appear in [1]. Leviatan et al. [3] also considered the full-height case by representing the post as a number of filaments and solving the boundary conditions using a point-matching technique. This analysis allows the circumferential variation of the induced axial current around the post to be approximated to a high order, making it very accurate for posts up to large radii. Their results were shown to be in good agreement with Schwinger’s theoretical analysis within his stated bounds of accuracy.

In the variable-length case, when \( \ell \neq h \), the axial current induced by the incident \( \text{TE}_{10} \) mode is no longer constant along the post, significantly increasing the complexity of any theoretical analysis. One of the earliest studies was by Lewin [4] who represented the post current by a filament with an assumed sinusoidal distribution. A more accurate theoretical analysis was developed by Williamson [5] for the analytically tractable case of a hollow post. Unlike Lewin, no assumption was made about the form of the induced current, although the effect of the circumferential variation around the post surface was averaged, making the analysis applicable to relatively thin posts only. More recently, an analysis has been reported [6] in which the post is approximated as a prism consisting of a number of strips each carrying an axial current.

In practice, the post would most likely be solid or, for high power applications, have a hemispherical cap. It would be expected that the reactance of a hollow, solid, or hemispherical-cap post of a certain length would only be slightly different, and the variation of the reactance of each form as a function of the physical dimensions and position would be very similar. Indeed, it might be expected that the reactance of one form could be closely represented by that of another of a slightly different length. If this length adjustment was known, then data available for one form could be used to predict the reactance values for others. Moreover, the theoretical treatment in [5] could then be...
used to predict reactance results for solid and hemispherical-cap posts.

In this paper, a simple and accurate experimental procedure for finding the reactance of symmetrical waveguide discontinuities is used to obtain a data set for the reactance of hollow, solid, and hemispherical-cap posts in a rectangular waveguide. The experimental data set is then used to produce length adjustment factors, relating the reactance of solid and hemispherical-cap posts to hollow posts. Using these length adjustments with the factors, relating the reactance of solid and hemispherical-cap posts, was used as the reference point in a waveguide, can be modeled by the T-equivalent circuit shown in Fig. 1(b) as with any symmetrical lossless two-port network. By considering an incident wave at one port and assuming that the other is terminated in a perfect match, it is possible to show that

\[ jx_a = \frac{2\pi}{(1 - \rho)^2 - \tau^2} \]

\[ jx_b = \frac{\rho - \tau + 1}{\rho - \tau - 1} \]

where \( x_a \) and \( x_b \) are normalized reactances, and \( \rho \) and \( \tau \) are the reflection and transmission coefficients, respectively, relative to the central plane of the discontinuity.

An experimental study was conducted to obtain the \( S \)-parameters of five different diameter posts (2\( \ell = 15, 12, 9.6, \) and 3 mm) within a \( C \)-band waveguide (\( d = 47.55 \) mm, \( h = 22.15 \) mm) over the usual operating frequency range of the waveguide (3.95–5.85 GHz) using an Agilent PNA series network analyzer. The posts and waveguide test setup were constructed from brass. Due to practical limitations, hemispherical-cap posts were considered for only the two largest post diameters and hollow posts for only the three largest post diameters. The experimental setup is shown in Fig. 2 and was calibrated to the plane \( A-A \) (at the connection between the network analyzer port hardware and the waveguide test section) by a thru-reflect line (TRL) calibration. The reference plane was subsequently shifted a length \( D \) to the plane \( B-B \) at the central plane of the discontinuity. Since the cylindrical post is symmetrical about \( B-B \), in theory, \( S_{11} = S_{22} \) and \( S_{12} = S_{21} \). In practice, there was a slight difference due to small imperfections in the experimental setup and, thus, the average of \( S_{11} \) and \( S_{22} \), and \( S_{21} \) and \( S_{12} \) was used as the reflection and transmission coefficients, respectively.

**II. EXPERIMENTAL PROCEDURE**

The effect of a symmetrical discontinuity, located at some point in a waveguide, can be modeled by the T-equivalent circuit shown in Fig. 1(b) as with any symmetrical lossless two-port network. By considering an incident wave at one port and assuming that the other is terminated in a perfect match, it is possible to show that

\[ jx_a = \frac{2\pi}{(1 - \rho)^2 - \tau^2} \]

\[ jx_b = \frac{\rho - \tau + 1}{\rho - \tau - 1} \]

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**III. RESULTS**

Tables I–X present data for the normalized reactances \( x_a \) and \( x_b \) versus \( \ell/\hbar \) of each post diameter and form at several frequencies. It can be seen that \( x_a \) is capacitive at most insertion depths, but becomes inductive for large \( \ell/\hbar \). Correspondingly,
the posts resonate at a certain length, the resonant length increasing for increasing post diameters. The reactance \( \tau_h \), for electrically thin posts, varies almost linearly with insertion and is small. However, as the post diameter increases, the variation of \( \tau_h \) with insertion no longer behaves linearly and the reactance becomes significant. Consequently, \( \tau_h \) is often referred to as the “post-thickness reactance,” as in [7].

The hollow post theory in [5] considers the average electric field incident over the post surface, the boundary condition on the post being applied in that sense. This was shown [5] to give accurate results for the average axial surface current, but neglects the circumferential variation around the post (and, thus, \( \tau = 1 + \rho \) and \( \tau_h = 0 \)). However, for the case of symmetrical excitation, the circumferential variation of the current would be somewhat lessened and it might be expected that the theory in [5] would accurately model that situation. For symmetrical excitation, the equivalent circuit of Fig. 1(b) would reduce to a single reactance \( \tau_T \).

### IV. LENGTH ADJUSTMENT FACTORS

Fig. 3 presents a graph of \( \tau_T \) for the three post forms considered in this study. It can be seen that the physical differences of
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anumnities and cross sections. This has permitted a useful data set

length cylindrical posts in a rectangular waveguide. These factors are applicable to

two different situations, each requiring a different挽救on method.

Lee and Mittra [8] and Ting [9] considered the case of a cylin-
drical antenna in free space driven by a coaxial line, and by com-
paring the input admittance (which depends on the near

fields) of cylindrical posts in a rectangular waveguide.

Fig. 3. Experimental results for the variable-length cylindrical post reactance 

\( x_T \) as a function of \( \ell/h \). (a) \( h = 22.15 \, \text{mm}, \alpha = 47.55 \, \text{mm}, \)

\( \alpha /d = 0.5, 2\alpha = 15 \, \text{mm}, \) and \( f = 3.95 \, \text{GHz} \). (b) \( \ell/h = 0.9 \).:

experimental results for solid post. \( \bigcirc \bigcirc \bigcirc \) : experimental results for hemispherical-cap post. \( \bigcirc \bigcirc \bigcirc \) : experimental results for hollow post.

The above results, i.e., (3) and (4), were deduced from the scattered fields (far fields) of cylindrical posts in a rectangular waveguide. It is interesting to note that they are in good agreement with previously published results for a different situation. Lee and Mittra [8] and Ting [9] considered the case of a cylindrical antenna in free space driven by a coaxial line, and by comparing the input admittance (which depends on the near fields) of hollow and solid and hollow and hemispherical-cap antennas, respectively, deduced length adjustment factors of \( \pm 0.13 \alpha \) and \( \pm 0.25 \alpha \).

V. COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES FOR \( x_T \)

Fig. 4. Theoretical and experimental results for the variable-length cylindrical post reactance \( x_T \) as a function of \( \ell/h \). \( h = 22.15 \, \text{mm}, \alpha = 47.55 \, \text{mm}, \)

\( \alpha /d = 0.5, 2\alpha = 15 \, \text{mm}, \) and \( f = 4.90 \, \text{GHz} \) : theoretical results using

length adjustment factors and [5]. \( \bullet \bullet \bullet \) : experimental results for solid post. \( \bigcirc \bigcirc \bigcirc \) : experimental results for hemispherical-cap post. \( \bigcirc \bigcirc \bigcirc \) : experimental results for hollow post.

If the experimentally derived adjustment factors, i.e., (3) and (4), are now applied to the hollow post theory in [5], it should be possible to predict the reactance of solid and hemispherical-cap posts. Theoretical results for the reactance \( x_T \) obtained in this manner are shown in Fig. 4 along with the experimental measurements of the three post forms. The accuracy of the length adjust-
ment factors is evident even when the post insertion is quite large. Furthermore, it is interesting that the results for \( x_T \) for hollow posts are quite accurate even for the electrically thick posts, up to 32% of the waveguide width, that were considered in this study.

VI. CONCLUSION

An accurate means of measuring the reactance of variable-
length cylindrical posts in a rectangular waveguide has been used
that could easily be applied to other waveguide discontinuities and cross sections. This has permitted a useful data set to be obtained. Furthermore, experimentally derived length adjust-
ment factors have been presented that relate the reactance of solid and hemispherical-cap posts to that of a hollow cylindrical post in a rectangular waveguide. These factors are applicable to the case of symmetrical excitation and can be used for relatively thick diameter posts, the largest investigated being over 30% of the waveguide width. When combined with a readily available theory for the reactance of hollow cylindrical posts, theoretical results have been obtained that are in excellent agreement with
the experimental measurements. The knowledge of these factors is of benefit in the practical design of waveguide filters and microwave tuning networks, providing a method for accurately predicting commonly used, but little analyzed reactive elements. Developing a means of analyzing the case of general excitation is a focus of ongoing research by the authors.

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REFERENCES


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