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**IMPLEMENTATION OF A BUBBLE MODEL IN *FLAC* AND ITS  
APPLICATION IN DYNAMIC ANALYSIS**

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## ABSTRACT

Methodologies of implementing nonlinear constitutive models of soil in *FLAC* are studied in order to reduce numerical distortion, which has been found to occur in nonlinear dynamic analysis when a nonlinear soil model is implemented using an ‘apparent modulus’ approach. Analyses undertaken using several simple nonlinear soil models indicate that use of ‘plastic correction’ approach can eliminate or minimize the problem. This approach is therefore adopted in the thesis to implement in *FLAC* a bounding surface bubble soil model, *i.e.* the Bubble model. Satisfactory performance of the Bubble model has been obtained in dynamic analysis without using any of the additional mechanical damping given in *FLAC*.

An analytical study on the Bubble model is carried out with *FLAC*. On the basis of the study, the hardening function is modified to better incorporate size ratio effects of the yield surface and is explored to eliminate abrupt transition in stiffness from elastic region to yielding. Pore water pressure is formulated with the assumption that the pore water pressure is generated as a response to the constant volume constraint which prevents the tendency for volume change when plastic volumetric strain takes place. The formulation is added to the Bubble model so that pore water pressure can be generated automatically by the model for fully saturated and undrained soil. *FLAC* analyses indicate that the Bubble model is generally in good agreement with published experimental data.

The parameters and initial conditions associated with the Bubble model are studied with *FLAC* analyses in triaxial stress space to investigate their influence on the model and to investigate their effective ranges. Both large and small strain behaviors of the model are explored in the parametric study.

Finally, the Bubble model is applied in the modeling of vertical vibration of rigid strip foundations. The influence of soil nonlinearity on vertical compliance of rigid foundations is investigated. Some major factors are considered, which include initial stress level in soil, level of excitation and mass ratio of foundation.

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## TABLE OF CONTENTS

ABSTRACT.....	i
ACKNOWLEDGEMENTS .....	ii
LIST OF FIGURES .....	vi
LIST OF TABLES .....	x
1 INTRODUCTION .....	1
1.1 General background .....	1
1.2 Objectives .....	1
1.3 Scope of work .....	2
1.4 Layout of the thesis.....	3
1.5 General remarks .....	4
2 LITERATURE REVIEW .....	6
2.1 Constitutive models of soil .....	6
2.1.1 Critical state soil mechanics and Cam-clay model .....	6
2.1.2 Two - surface model (Mróz et al. 1979) .....	10
2.1.3 Kinematic hardening rule (Hashiguchi 1985).....	16
2.1.4 Recent development of two/three-surface models.....	18
2.2 Nonlinear dynamic analysis with <i>FLAC</i> .....	30
2.3 Vertical vibration of rigid foundations .....	33
2.3.1 Classical work.....	33
2.3.2 Nonlinear numerical modeling of vertical vibration of rigid foundations	38
2.4 Summary .....	43
3 SOME KEY ASPECTS OF PROGRAMMING WITH <i>FLAC</i> .....	46
3.1 Finite elements in <i>FLAC</i> .....	46
3.2 Local and global variables .....	50
3.3 Initialisation .....	50
3.4 <i>FISH</i> state variables .....	51
3.4.1 Stress variables.....	51
3.4.2 Sub-element averaging.....	53
3.4.3 Strain increments .....	57
3.4.4 Pore water pressure .....	57
3.5 Summary .....	62
4 PLASTIC CORRECTION APPROACH .....	63
4.1 ‘Apparent modulus’ approach.....	63
4.2 ‘Plastic correction’ approach .....	66
4.2.1 Explicit and implicit integration .....	66
4.2.2 Procedures of plastic correction.....	68
4.3 Implementation of two nonlinear models .....	71
4.3.1 A hyperbolic model.....	71
4.3.2 A bilinear model .....	77
4.4 Discussion on numerical distortion.....	82
4.5 Summary .....	84
5 THE BUBBLE MODEL AND ITS IMPLEMENTATION IN <i>FLAC</i> .....	85

5.1	Description of the model (Roumania & Wood, 2000) .....	85
5.2	Modifications of the Bubble model .....	92
5.2.1	Plastic modulus function.....	92
5.2.2	Elastic bulk modulus.....	97
5.3	Discussions .....	97
5.4	Pore water pressure .....	105
5.5	Implementation .....	106
Appendix 5.1	Notation.....	108
Appendix 5.2	Expanded equations .....	111
Appendix 5.3	Derivation of stress gradient .....	124
Appendix 5.4	<i>FISH</i> code of the Bubble model.....	127
6	VERIFICATION OF THE BUBBLE MODEL.....	141
6.1	Modelling of non-structured soil .....	141
6.1.1	Example 1 .....	141
6.1.1.1	Drained behaviour.....	143
6.1.1.2	Undrained behaviour.....	147
6.1.2	Example 2 .....	150
6.2	Modelling of structured soil.....	155
6.2.1	Example 1 .....	155
6.2.2	Example 2 .....	161
6.3	Comparison with the modified Cam-clay model .....	165
6.4	Comparison with experiment.....	166
6.5	Dynamic behaviour of the Bubble model .....	170
6.5.1	A single-zone <i>FLAC</i> triaxial model .....	170
6.5.2	A multizone <i>FLAC</i> triaxial model.....	177
6.6	Summary .....	180
7	PARAMETRIC STUDY OF THE BUBBLE MODEL.....	181
7.1	Typical values of parameters .....	181
7.2	Size of bubble .....	184
7.3	Plastic modulus parameters ‘ <i>B</i> and $\psi$ ’ .....	191
7.4	Standard parameters.....	195
7.5	Destructuration parameters .....	201
7.6	Initial conditions .....	204
7.7	<i>G</i> – $\gamma$ curves .....	210
7.7.1	Laboratory <i>G</i> – $\gamma$ curves .....	210
7.7.2	<i>G</i> – $\gamma$ curves of the Bubble model .....	211
7.7.3	Smooth transition in shear modulus.....	215
7.8	Summary .....	217
Appendix 7.1	Data file for modelling triaxial/oedometer tests .....	219
Appendix 7.2	MathCad programme for processing <i>G</i> – $\gamma$ curves .....	224
8	APPLICATION OF THE BUBBLE MODEL .....	225
8.1	Parameters of the Bubble model .....	225
8.2	<i>FLAC</i> model.....	227
8.3	Typical response .....	229
8.4	Vertical compliance of the rigid foundation .....	233
8.4.1	Comparison between nonlinear and elastic models .....	234

8.4.2	Effects of dynamic stress level .....	236
8.4.3	Effects of initial static stress ( <i>i.e.</i> factor of safety) .....	238
8.4.4	Effects of mass ratio.....	240
8.5	Compliance and impedance functions .....	241
8.6	Summary .....	244
	Appendix 8.1 <i>FLAC</i> data files for vertical vibration of rigid foundations .....	245
<b>9</b>	<b>CONCLUSIONS AND DISCUSSIONS .....</b>	<b>248</b>
9.1	Main results and conclusions .....	248
9.1.1	Implementation of soil constitutive models in <i>FLAC</i> .....	248
9.1.2	Numerical distortion in nonlinear dynamic analysis .....	249
9.1.3	The Bubble model.....	249
9.1.4	Parameters of the Bubble model .....	250
9.1.5	Application of the Bubble model .....	252
9.2	Discussion .....	253
9.3	Suggestions for further research .....	256
	<b>LIST OF REFERENCES .....</b>	<b>257</b>

## LIST OF FIGURES

Figure 2.1 State boundary surfaces.....	7
Figure 2.2 Yield locus of Original Cam-clay model.....	8
Figure 2.3 Yield locus of modified Cam-clay model .....	9
Figure 2.4 The two-surface model by Mróz .....	11
Figure 2.5 Illustration of kinematic hardening rule .....	13
Figure 2.6 Two- surface model (Hashiguchi 1985).....	17
Figure 2.7 The original bubble model (Al Tabbaa & Wood, 1989) .....	19
Figure 2.8 Bubble model for structured soil (Wood, 1995).....	23
Figure 2.9 3-SKH model.....	25
Figure 2.10 Two-surface by Kavvadas and Amorosi (2000).....	27
Figure.2.11. Cyclic response of a hyperbolic model in simulating a triaxial dynamic test .....	31
Figure 2.12 Stress-strain response with 2 % local damping. ....	32
Figure 2.13 A lumped parameter vibrating system.....	36
Figure 2.14 Vertical compliance functions for rigid circular foundations (redrawn after Lysmer and Richart, 1966) .....	36
Figure 2.15 Vertical compliance functions for rigid strip foundations (redrawn after Gazetas and Roesset, 1979) .....	38
Figure 2.16 Vertically oscillating circular foundation on elastoplastic half space (redrawn after Borja et al, 1993) .....	39
Figure 2.17 Cyclic displacement amplitude of a rigid foundation (redrawn after Pender, 2000) .....	40
Figure 2.18 Compliance function of a rigid foundation subject to vertical vibration (stress amplitude =120 kPa) .....	41
Figure 2.19 Vertical displacements a rigid strip foundation subject to rocking vibration	41
Figure 2.20 Vertical compliance function (cyclic pressure = 140 kPa).....	42
Figure 2.21 Influence of static factor of safety on displacement amplitude .....	43
Figure 3.1 Sub-elements in a quadrilateral element.....	46
Figure 3.2 Three single-element 2D axisymmetric models .....	47
Figure 3.3 Influence of boundary condition and shape of element on stress.....	48
Figure 3.4 Comparison between FISH state variable and user-defined variable.....	53
Figure 3.5 Comparison between FISH state variable and user-defined variable.....	55
Figure 3.6 Influence of location where FISH stress variables are updated .....	57
Figure 3.7 Effective and total stresses .....	59
Figure 3.8 Effective stress using user defined variable .....	61
Figure 4.1 Illustration of ‘apparent modulus’ approach .....	65
Figure 4.2 Sudden change in apparent modulus .....	66
Figure 4.3 Conceptual illustration in general stress space of plastic correction.....	67
Figure 4.4 Flow chart of ‘plastic correction’ approach.....	69
Figure 4.5 Conceptual illustration of ‘plastic correction’ approach .....	70
Figure 4.6 Flow chart of a hyperbolic model modified for cyclic loading .....	72
Figure 4.7 Dynamic response of the hyperbolic model .....	73

Figure 4.8 Dynamic response of the hyperbolic model without additional damping .....	73
Figure 4.9 Static response of the hyperbolic model.....	74
Figure 4.10 Dynamic response with a timestep of $1.0 \times 10^{-6}$ second .....	74
Figure 4.11 A bilinear model using ‘plastic correction’ approach .....	78
Figure 4.12 Flow chart of implementation of a bilinear model .....	78
Figure 4.13 Dynamic response of a multi-zone bilinear model.....	80
Figure 4.14 Comparison of the bilinear model with Mohr-Coulomb model.....	80
Figure 5.1 Three surfaces of the Bubble model .....	88
Figure 5.2 Initial state of a non-structured soil .....	93
Figure 5.3 Stress-strain behaviour with variable bubble size ( $B = 4$ ).....	93
Figure 5.4 Response of the bubble model to different combinations of $B$ and $R$ .....	94
Figure 5.5 Relationship between $B$ and $R$ .....	94
Figure 5.6 Relationship of $C$ and $R$ .....	96
Figure 5.7 Response of the model with new hardening function and constant $B$ ..... ( $R = 0.001 \sim 1.0$ , $B = 2000$ ).....	96
Figure 5.8 Yield surface.....	98
Figure 5.9 Normal consolidation line and swelling line .....	101
Figure 5.10 Normalised distance of bubble to structure surface .....	102
Figure 5.11 Kinematic hardening of bubble .....	103
Figure 5.12 Flow chart for implementation of the Bubble model .....	107
Figure 6.1 Initial stress conditions of three cases (same structure surface).....	142
Figure 6.2 Drained stress-strain behaviour for Case 2.....	143
Figure 6.3 Location of surfaces at 5% vertical strain .....	144
Figure 6.4 Location of surfaces at 10% vertical strain .....	144
Figure 6.5 Location of surfaces at 20% vertical strain .....	145
Figure 6.6 Drained stress-strain behaviour (same initial structure surface) .....	145
Figure 6.7 Volumetric strain (same initial structure surface) .....	146
Figure 6.8 Drained stress path (same initial structure surface).....	146
Figure 6.9 Undrained stress-strain behaviour (same initial structure surface) .....	148
Figure 6.10 Isotropic hardening of the structure surface .....	148
Figure 6.11 Undrained effective stress path (same initial structure surface).....	149
Figure 6.12 Pore water pressure (same initial structure surface).....	150
Figure 6.13 Initial stress states of three cases (variable initial structure surface).....	151
Figure 6.14 Undrained stress-strain behaviour (variable initial structure surface).....	151
Figure 6.15 Effective stress path (variable initial structure surface) .....	153
Figure 6.16 Pore water pressure (variable structure surface) .....	153
Figure 6.17 Influence of ‘B’ on effective stress path.....	154
Figure 6.18 Initial stress conditions of structured soil.....	155
Figure 6.19 Undrained stress-strain behaviour of structured soil .....	156
Figure 6.20 Destructuration of structured soil .....	157
Figure 6.21 Surfaces at 5% strain .....	158
Figure 6.22 Surfaces at 10% strain .....	158
Figure 6.23 Surfaces at 30% strain .....	159
Figure 6.24 Initial stress conditions of structured soil with anisotropy .....	159
Figure 6.25 Influence of anisotropy on stress-strain behaviour.....	160
Figure 6.26 Surfaces after destructuration at 30% strain ( $\eta_0 = 0.5$ ) .....	161

Figure 6.27 Initial stress conditions for three cases of a structured soil .....	162
Figure 6.28 Undrained stress-strain behaviour of structured soil .....	163
Figure 6.29 Effective stress path of structured soil.....	163
Figure 6.30 Pore water pressure response of the structured soil.....	164
Figure 6.31 Comparison with modified Cam-clay model (Drained).....	165
Figure 6.32 Pressure-void ratio curve for loose sands.....	167
Figure 6.33 Initial stress conditions of three tests.....	167
Figure 6.34 Comparison of Bubble model with published experiment data .....	169
Figure 6.35 A single zone triaxial model.....	170
Figure 6.36 Initial stress conditions .....	171
Figure 6.37 Drained dynamic stress-strain behaviour of Case 1 soil .....	172
Figure 6.38 Dynamic volumetric response of Case 1 soil .....	172
Figure 6.39 Undrained dynamic stress-strain response of Case 1 soil .....	173
Figure 6.40 Dynamic pore water pressure of Case 1 soil .....	173
Figure 6.41 Drained dynamic stress-strain behaviour of Case 2 soil .....	174
Figure 6.42 Dynamic volumetric response of Case 2 soil (40 cycles).....	174
Figure 6.43 Dynamic volumetric response of Case 2 soil (200 cycles).....	175
Figure 6.44 Undrained dynamic stress-strain response of Case 2 soil .....	176
Figure 6.44a Cyclic response of undrained deviator stress of Case 2 soil.....	176
Figure 6.45 Dynamic pore water pressure of Case 2 soil .....	177
Figure 6.46 Initial stress state of the multizone FLAC model.....	178
Figure 6.47 Undrained dynamic stress-strain response of a multizone triaxial model ...	178
Figure 6.47a Cyclic response of undrained deviator stress of a multizone triaxal model .....	179
Figure 6.48 Dynamic pore water pressure (40 cycles) .....	179
Figure 7.1 Initial locations of surfaces associated with reference parameters.....	182
Figure 7.2 Single-element triaxial model.....	184
Figure 7.3 Elastic range vs bubble size (B = 300, drained) .....	185
Figure 7.4 Stress-strain behaviour (B =500, drained).....	186
Figure 7.5 Stress-strain behaviour (B=600, drained).....	186
Figure 7.6 Influence of R on volumetric behaviour (B=600, $\psi=0.5$ , drained) .....	187
Figure 7.7 Volumetric strain vs mean pressure (B = 600).....	187
Figure 7.8 Stress-strain behaviour ( $\psi=1.0$ , B=1500, drained).....	188
Figure 7.9 Stress-strain behaviour ( $\mu = 0.35$ , B =250, drained).....	189
Figure 7.10 Stress-strain behaviour ( $\lambda^* = 0.2$ , B = 350, drained) .....	189
Figure 7.11 Stress-strain behaviour ( $\kappa^* = 0.04$ , B = 280, drained) .....	190
Figure 7.12 Undrained stress-strain behaviour .....	190
Figure 7.13 Pore water pressure.....	191
Figure 7.14 Influence of ‘B’ on stress-strain behaviour ( $\psi=0.5$ , R = 0.2).....	192
Figure 7.15 Influence of parameter B ( $\psi = 0.5, R=0.01$ ).....	192
Figure 7.16 Influence of parameter $\psi$ (B=600) .....	193
Figure 7.17 Relationship between B and $p_{c0}$ ( $\psi = 0.5$ ) .....	194
Figure 7.18 Relationship between B and $p_{c0}$ ( $\psi = 1.0$ ) .....	194
Figure 7.19 Relationship between B and $p_{c0}$ ( $\psi = 1.5$ ) .....	195
Figure 7.20 Influence of Poisson’s ratio on stiffness (undrained) .....	196

Figure 7.21 Influence of Poisson's ratio on stiffness (drained) .....	196
Figure 7.22 Influence of $\lambda^*$ on stiffness and strength (undrained, $\kappa^* = 0.02$ ) .....	197
Figure 7.23 Influence of $\lambda^*$ on stiffness and strength (drained, $\kappa^* = 0.02$ ) .....	197
Figure 7.24 Influence of $\kappa^*$ on stiffness and strength (undrained, $\lambda^* = 0.3$ ) .....	198
Figure 7.25 Influence of $\kappa^*$ on stiffness (drained, $\lambda^* = 0.3$ ) .....	198
Figure 7.26 Dimensionless scaling function, $M_\theta$ ( $M = 1.0$ ) .....	199
Figure 7.27 Influence of parameter 'm' (undrained axisymmetrical compression, $m = 0.7 \sim 1.0$ ) .....	200
Figure 7.28 Influence of parameter 'm' (undrained axisymmetrical extension) .....	200
Figure 7.29 Influence of parameter 'A' ( $r_0 = 2.0$ , undrained) .....	201
Figure 7.30 Influence of parameter 'A' ( $r_0 = 4.0$ , undrained) .....	202
Figure 7.31 Influence of parameter 'k' on destructuration ( $r_0 = 2.0$ , drained) .....	202
Figure 7.32 Size of structure surface, $r_{pc}$ ( $r_0 = 2.0$ , drained) .....	203
Figure 7.33 Influence of parameter 'k' on stress-strain behaviour ( $r_0 = 2.0$ , drained) .....	203
Figure 7.34 Influence of initial stress (undrained, $p_{co} = 200$ kPa) .....	204
Figure 7.35 Influence of initial size of reference surface (undrained, $\sigma_3 = 200$ kPa) .....	205
Figure 7.36 Influence of initial size of structure surface ( $p_{co} = 200$ kPa, $k=8$ , undrained) .....	206
Figure 7.37 Influence of initial anisotropy ( $r_0 = 2.0$ , $p_{co} = 200$ kPa, $k=4$ , undrained) .....	206
Figure 7.38 Calculated bulk modulus during undrained triaxial compression .....	208
Figure 7.39 Influence of $K_{max}$ on static analysis .....	208
Figure 7.40 Dynamic response of a single-zone triaxial model (10 Hz, 20 cycles, vertical strain amplitude = 2%) .....	209
Figure 7.41 A typical laboratory $G - \gamma$ curve for cohesive soils .....	210
Figure 7.42 Influence of $\kappa^*$ on small strain behaviour ( $K_0 = 0$ ) .....	213
Figure 7.43 $G - \gamma$ behaviour when $K_0 \neq 0$ .....	214
Figure 7.44 $G - \gamma$ curves with smooth transition in shear modulus ( $R=0.01$ , $B=1000$ ) .....	216
Figure 8.1 Undrained stress-strain response of soil ( $\sigma_3 = 100$ kPa) .....	226
Figure 8.2 Pressure-settlement curve .....	227
Figure 8.3 FLAC model in dynamic loading stage .....	228
Figure 8.4 Vibration of the soil-foundation model under gravity .....	229
Figure 8.5 Dynamic displacements (24 Hz, 5 kPa stress amplitude, 100 cycles) .....	230
Figure 8.6 Dynamic displacements (24 Hz, 20 kPa stress amplitude, 48 cycles) .....	230
Figure 8.7 Dynamic displacements (24 Hz, 100 kPa stress amplitude, 5 cycles) .....	231
Figure 8.8 Influence of plastic strain ratio on displacement response (24 Hz, 60 kPa stress amplitude, 10 cycles) .....	232
Figure 8.9 Transient vibration (0.5 cycle, 24 Hz, 60 kPa, no additional damping) .....	233
Figure 8.10 Amplitude of displacement (Stress amplitude = 20 kPa) .....	235
Figure 8.11 Normalised amplitude of displacement (Stress amplitude = 20 kPa) .....	236
Figure 8.12 Influence of dynamic stress .....	237
Figure 8.13 Influence of dynamic stress ( $a_0 = 0.5$ ) .....	237
Figure 8.14 Influence of dynamic stress ( $a_0 = 2$ ) .....	237
Figure 8.15 Influence of factor of safety ( $a_0 = 0.5$ , stress amplitude = 20 kPa) .....	239

Figure 8.16 Influence of factor of safety (stress amplitude = 20 kPa).....	239
Figure 8.17 Time history of displacement ( $a_0 = 0.5$ , stress amplitude = 20kPa) .....	240
Figure 8.18 Influence of mass ratio (stress amplitude = 20 kPa, FOS = 3).....	241
Figure 8.19 Comparison of compliance functions of a rigid foundation between FLAC modeling and the semi-analytical solution by Gazetas & Roesset, 1979 .....	242
Figure 8.20 Comparison of impedance functions of a rigid foundation between FLAC modeling and the semi-analytical solution by Gazetas & Roesset, 1979 .....	243
Figure 9.1 Badly deformed elements .....	255

## LIST OF TABLES

Table 5.1 Typical values of parameters $\lambda^*$ and $\kappa^*$ .....	98
Table 7.1 Typical soil parameters for parametric study .....	182
Table 8.1 Soil parameters in modeling foundation vibration .....	225
Table 8.2 Average amplitude of displacement in 10 cycles .....	232