Free Will and Randomness

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Abstract

It is frequently claimed that randomness conflicts with free will because "[i]f our actions are caused by chance we lack control" and "[r]andomness, the operation of mere chance, clearly excludes control". In this paper we challenge this position. To this aim we propose a simple, two-stage, contextual (not absolute) definition of free will and we show that, relative to this definition, randomness is not incompatible with free will. Crucial for our argument is the fact that there are no random events in nature. Randomness is only a theoretical concept which is defined and produced in deterministic ways: it is not a direct cause of actions.

Our analysis is relative: it does not provide a proof for the existence nor impossibility of free will.

1 Introduction

The fact that randomness is incompatible with freedom is often stated as being intuitive and is postulated by many authors [14]. The following quotations are illustrative.

> One horn of this dilemma is the argument that if an action was caused or necessitated, then it could not have been done freely, and hence the agent is not responsible for it. The other horn is the argument that if the action was not caused, then it is inexplicable and random, and thus it cannot be attributed to the agent, and hence, again, the agent cannot be responsible for it. [34, p. 14].

> On the other hand, if indeterminism is true, then, though things could have happened otherwise, it is not the case that we could have chosen otherwise, since a merely random event is no kind of free choice. That some events occur causelessly, or are not subject to law, or only to probabilistic law, is not sufficient for those events to be free choices. [25, p. 80]

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> Yet if an uncaused action is a random happening, then this no more comports with human value than does determinism. Random acts and caused acts alike seem to leave us not as the valuable originators of action but as an arena, a place where things happen, whether through earlier causes or spontaneously. Clearly if our actions were random, like the time of radioactive decay of uranium 238 emitting an alpha particle, their being thus undetermined would be insufficient to ground human value or provide a basis for responsibility and punishment. [31, p. 91–92]

Our aim is to challenge this view: a form of indeterminism is required for an agent to have free will, but we will argue that using randomness in the decision making does not hinder the agent's freedom.

The special relation between free will and randomness has been noticed and examined from the very beginning of the debate. Randomness too is considered only in an intuitive way, sometimes identified with chance, probability, even indeterminism [14]. Here is a quote similar to the previous ones which uses chance not randomness:

> Either it is an accident that I choose to act as I do or it is not. If it is an accident, then it is merely a matter of chance that I did not choose otherwise; and if it is merely a matter of chance that I did not choose otherwise, it is surely irrational to hold me morally responsible for choosing as I did. [5, p. 285]

Contrary to intuition, randomness is not "naturally or spontaneously produced": it is only a theoretical concept, not a direct cause of actions. There are no random events in nature. Randomness is the object of deep mathematical studies, see [8, 13], and some results are directly relevant for the present discussion: for example, randomness (of any form) is defined and produced in a deterministic way.

The plan of the paper is the following. We first describe a framework necessary for the discussion of free will and we propose a simple, two-stage, contextual (not absolute) definition of free will. We operate with this definition—which we do not claim to be the best nor the right one—through the paper.

In this framework we argue that determinism is incompatible with free will, hence a form of indeterminism[1] is necessary—not sufficient—for free will. Quantum indeterminism illustrates such a form. Next we examine the notions of indeterminism and randomness and their mutual relations: we show that neither of them implies the other. Finally, we provide a detailed analysis showing that randomness is not incompatible with free will.

Our arguments are relative: they do not provide a proof for the existence or impossibility of free will. More work is required in this direction.

2 Agents, Objects, Contexts and Choices

We start by defining four main components of a manifestation of free will: agent, objects, contexts and choices. An *agent* is an (abstract) entity which *can make a choice (decision)* in some

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[1] Aristotle was probably the first major philosopher to argue convincingly for indeterminism.

context. By choice we mean that the agent is able to pick an element from an abstract set of *objects*. The context $\mathfrak{C}$ gives the environment and the relevant constraints for the choice to be made. A context may include the position in space and time where an agent must make a choice, as well as various constraints the objects should satisfy. The object that the agent picks is called the *chosen object* in the context $\mathfrak{C}$.

We illustrate these definitions with some examples. Consider first a girl at a cat store looking to buy a pet. Here the agent can be taken to be the girl, the objects are the pets available for purchase in the shop, and the context is the shop at some time; the girl can choose any pet. On the other hand, an individual animal in the shop can be an agent in the same context: the objects are the foods the animal can eat and a choice consists in selecting one of the available foods. In the same context, the same animal can be an agent for the set of actions $\{$eat, not eat$\}$; here a choice consists in deciding on an action in that set. An abstract object can be an agent or an object, depending on the context.

Consciousness is not required for an object to be an agent. Consider as agent an experimental physicist and as objects the set of experiments she can perform; the context includes the physical set-up for the experiment at a given location and time. For example, a possible choice is to look through a microscope for a particle in an uncertain location. Now consider a particle with an uncertain location and an uncertain velocity in a given context. When you look at it through a microscope and locate it, the particle gives you an answer *here I am*: a given location from the available ones in that context was chosen by the particle. According to Zeilinger [40],

> I call that the two freedoms: first the freedom of the experimenter in choosing the measuring equipment—that depends on my freedom of will; and then the freedom of nature in giving me the answer it pleases. The one freedom conditions the other...

The context is crucially related to the idea of *possibility*. The choices the agent can make are assumed to be *contextually possible*, i.e. possible in the given context. They may or may not be considered rational, morally justified or politically acceptable. The contextual possibility should be distinguished from a wider notion of possibility; for instance, there is a sense in which it is possible for an agent to go scuba diving, but it is not possible in a given context because the agent not close to water. Contextual possibility is also different from Mele's intentional ability. An ability is intentional [29, p. 17–19] if an agent has enough control over its ability to deliberately choose certain outcomes. Intentional ability implies contextual possibility, but the converse does not hold. For example, an agent can intentionally flip a coin, but cannot intentionally get heads.

3 A Definition of Free Will

In the literature on free will (which includes thousands of articles and books [14]) a lot of misunderstanding comes from the fact that—with a few exceptions including [12, 11]—free will is not precisely defined, so different authors operate with different intuitive notions of "free will". Moreover, much of the discussion of free will ends with an "absolute" conclusion: either there is or there is no free will.

In what follows we propose a simple, two-stage, contextual (not absolute) definition of free will, and consistently operate with it. We do not claim that this proposal is *the right definition* nor *better than the current ones*: its goal is to propose a more precise and detailed framework for studying the relation between free will and randomness.

The definition of free will models the idea that an agent has free will in some context if it has the ability to make a decision that is not completely determined by or the result of prior events. More precisely, an agent A *acted freely* in the context $\mathfrak{C}$ with respect to the set of objects $O_{\mathfrak{C}}(A)$ if A could have acted differently to the way it did and A had full control over the outcome of the choice. This means that in the context $\mathfrak{C}$:

(P) The set of objects $O_{\mathfrak{C}}(A)$ available to A contains at least two elements and every choice for A is possible in the given context.

(C) The agent A has full control to choose an object in $O_{\mathfrak{C}}(A)$.

This is a two-stage Valerian definition.[2] It is simple because the Possibility assumption **(P)** guarantees that the act of choice is meaningful and can lead to different outcomes (if $O_{\mathfrak{C}}(A)$ has less than two elements then this is impossible): no other assumption is needed. The set of objects $O_{\mathfrak{C}}(A)$ available to A depends on the context $\mathfrak{C}$ and can vary with $\mathfrak{C}$. The set $O_{\mathfrak{C}}(A)$ may even not be completely known by A[3]; by changing the context $\mathfrak{C}$ some elements may disappear and new elements can be added to $O_{\mathfrak{C}}(A)$, sometimes in indeterministic ways.[4] The main point of **(P)** is that in the context $\mathfrak{C}$, *there are at least two objects in* $O_{\mathfrak{C}}(A)$ *available to A to choose from.* This assumption satisfies a broadly libertarian view of free will, i.e. to be free an agent must have the ability to *do otherwise* from what it did [30] (for a detailed discussion regarding the Possibility assumption see [37] (ch. 3 and 4)).

Contextual availability of every element is $O_{\mathfrak{C}}(A)$ is essential. For example, if A is a human being, then in any context $\mathfrak{C}$ the set $O_{\mathfrak{C}}(A) = \{\text{to be immortal, not to be immortal}\}$ violates the condition of contextual possibility: "to be immortal" is not available to A. In this case $O_{\mathfrak{C}}(A)$ does not contain two elements, hence A cannot have free will in $\mathfrak{C}$. Neuroscience shows evidence for contextually [14].

The Choice assumption **(C)** guarantees that the agent A can fully decide in the context $\mathfrak{C}$ the chosen object in $O_{\mathfrak{C}}(A)$ irrespective of the information available to it; moreover, the agent *has to choose* an object in $O_{\mathfrak{C}}(A)$: not choosing any object is incompatible with this assumption. For example, a human being A has control over $O_{\mathfrak{C}}(A) = \{\text{eat, not eat}\}$ in some contexts $\mathfrak{C}$, but not in all (for example, if A is in hospital treatment). However, a cat has no control over $O_{\mathfrak{C}}(A) = \{$get run over by a bus, not get run over by a bus$\}$ in almost any context.

The proposed definition of free will is contextual, not absolute nor global. It is in agreement with the quantum physicist's notion of free will [22]; it does not assume any forms of rationality, faith, morality or political correctness. More, the definition is compatible with van Inwagen's

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[2] Dennett [12, p. 293] called his model of free will "Valerian" after Paul Valéry who was quoted by saying: "It takes two to invent anything. The one makes up combinations; the other one chooses."

[3] Actually, humans make choices without knowing more than a few elements of $O_{\mathfrak{C}}(A)$.

[4] According to [23], "…one major aspect of biological evolution is the continual change of the pertinent phase space". Some two-stage models use "randomness" for creating alternatives [14]; given that randomness is only a theoretical concept—see the discussion in Section 6—such a model has to explain how "randomness is produced".

classic argument for incompatibilism [38, p. 70] and is supported by most of Dennett's [12] (ch. 15) reasons in favour of his two-stage model.

In what follows "free will" will always refer to the above definition. If the context $\mathfrak{C}$ is understood we may write $O(A)$ instead of $O_{\mathfrak{C}}(A)$.

4 Determinism Conflicts with Freedom

Is free will compatible with determinism? That is, can free will exist in a deterministic universe? The short answer is *no*. However, this is a crucial point which must be carefully considered.

We take determinism to be the view that given the state of the universe at time t, the way things go thereafter is fixed as a matter of natural law [5]. This means that given one context $\mathfrak{C}_0$, all later contexts $\mathfrak{C}_i$ are, in principle, Laplacian, i.e. fully determined. Thus, under this assumption, given an agent A and a context $\mathfrak{C}$ the set of possible objects which A can choose $O_{\mathfrak{C}}(A)$ contains only one element.

A simple argument for this may be set out as follows. Let $O_{\mathfrak{C}}(A)$ denote the set of objects from which A can choose an element given the context $\mathfrak{C}$.

For illustration we consider the case of just two contexts, $\mathfrak{C}_1$ and $\mathfrak{C}_2$, where $\mathfrak{C}_2$ occurs after $\mathfrak{C}_1$. Because of determinism, the contexts $\mathfrak{C}_1$ and $\mathfrak{C}_2$ are pre-determined and distinct. The context $\mathfrak{C}_2$ includes the agent A and its choice made in the context $\mathfrak{C}_1$. Consequently, it would have been impossible for A to make in $\mathfrak{C}_1$ a different choice, so in fact $O_{\mathfrak{C}_1}$ contains a single element; A may have the illusion that $O_{\mathfrak{C}_1}$ contains more than one element, but all the other elements different from its choice in $\mathfrak{C}_1$ are made impossible by $\mathfrak{C}_2$.[5]

This argument above shows that determinism is not compatible with the notion of free will.[6] It also avoids direct reference to laws of nature (the existence of which may be disputed, see [33, 10]).

The point of this argument is essentially to show that if determinism were true, an agent A's choices would be determined by the laws of nature, together with the state of the world before A's existence, neither of which A has any control over. Hence, A would not be able to do otherwise than what was determined, and hence would not have free will.

Any consolidation between A having free will and determinism being true thus fails given our definition because the Possibility assumption **(P)** requires that $O(A)$ contains at least two elements while determinism ensures that it only contains one. So in a deterministic universe A could not have done otherwise to what it did. It should be pointed out however that it is only the Possibility assumption which gives rise to this conflict, for A may happily choose the only element of $O(A)$ (though A would presumably be unaware that there was only one object available for choice).

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[5] This example also shows the importance of the fact that every element in $O_{\mathfrak{C}}$ is possible, i.e. available to be chosen.

[6] Using a different definition of free will in cf. [36] a similar conclusion is reached: "Although the Free Will Theorem can't prove if we have free will, it does have a fundamental consequence: if the Universe is deterministic, and a particles behaviour is always described by a function of the past, then we can't have free will". The free will theorem roughly says that under three specific assumptions "If experimenters have free will, then so do elementary particles", [11].

In arguing that free will is in fact compatible with determinism one would therefore have to reject, or at least reinterpret, the Possibility assumption **(P)**. Different examples of such strategies are famously offered in [5] and [19]; however, these strategies ultimately fail to show that **(P)** is not required for an agent to act freely.

5 Indeterminism and Quantum Indeterminism

We now turn our attention to the relation between indeterminism and free will. Recall that we take determinism to be the view that given the state of a system at some time, all future states are fixed, so given any context, we can, in principle, predict all future contexts.

Broadly, indeterminism occurs when the state of a system at one time does not uniquely fix the state of the system at some future time [16]. Classifying indeterminism[7] is more difficult because simply negating determinism does not give a unique thesis.

In what follows we will discuss indeterminism with reference to quantum mechanics, technically referred to as quantum indeterminism. We stress, however, that we are not arguing for or against the existence of free will on the basis that quantum phenomena occur indeterministically in an intrinsic way. Rather, quantum mechanics provides a theoretical framework in which we find a form of indeterminism which we will argue is required for free will to occur.

First and foremost, *quantum indeterminism is not naturally/spontaneously "produced" in the world*, it is only part of the interpretation of a theoretical model of the quantum world.[8] Quantum indeterminism—which plays a key role in quantum mechanics—was postulated by Born when he proposed that the modulus-squared of the wave function should be interpreted as a probability density [6]. Quantum indeterminism is a fundamental, irreducible form of indeterminism in relation to measurement, a human-made operation. The nature of individual measurement outcomes in quantum mechanics was, for a period, a subject of much debate. Einstein famously dissented, stating his belief that [7, p. 204] "*He does not throw dice*." Over time the conjecture that measurement outcomes are themselves fundamentally indeterministic has been vindicated both theoretically and experimentally, subsequently cementing the view that quantum mechanics is intrinsically indeterministic with respect to measurement [39]. We note that the quantum indeterminism is by no means universally accepted: there are sets of assumptions, "interpretations", leading to a purely deterministic form of quantum mechanics, see [22].

In all discussions about free will, quantum indeterminism is illustrated with reference to Bell's inequalities (see [14]): this approach is not adequate as the form of indeterminism described by these inequalities is purely *statistical*, so it does not reflect an agent's individual behaviour. Fortunately, the stronger Kochen-Specker theorem [22] can be used to prove that maximal quantum indeterminism applies to individual observers [3].

To make our discussion precise, we shall refer to a standard model of quantum mechanics where some assumptions have to be made. Before listing these assumptions we need to define when a physical quantity is *value definite*. Informally, an observable is value definite if its

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[7] A possible classification can be based on the fact that a deterministic function can be i) incomputable—in an infinity of stronger and stronger forms—ii) computable, but not feasibly computable, iii) feasibly computable—again in an infinity of stronger and stronger forms.

[8] This idea will be discussed in detail in the next section.

value is fixed (can be, in principle, known) *before* measurement. To make this intuition precise we refer to the famous paper [18, p. 777] in which Einstein, Podolsky and Rosen define the *physical reality* in terms of certainty and predictability. Based on this largely accepted notion of an element of physical reality, following [3] we identify the notion of an "element of physical reality" with "value definiteness" thus obtaining the following "EPR principle":

> *EPR principle*: If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists a *definite value* prior to observation corresponding to this physical quantity.

The EPR principle renders a definition of value definiteness and physical reality based on the ability to predict. For us the opposite property is important: value indefiniteness corresponds to the absence of physical reality. Indeed, if no unique element of physical reality corresponding to a particular physical quantity exists, this is reflected by the physical quantity being value indefinite. Consequently, if a physical property is value indefinite we cannot predict with certainty the outcome of any experiment measuring this property.

The following three assumptions will be adopted; more technical descriptions and commentaries can be found in [1].

Admissibility Definite values must not contradict the statistical quantum predictions for compatible observables on a single quantum. For example, given a set $\{P_1, \ldots, P_n\}$ of commuting projection observables, if P_1 were to have the definite value 1, all other observables in this set must have the value 0.

Noncontextuality of definite values If a measurement is made of a value definite observable, the outcome obtained (and thus the preexisting physical property) is *noncontextual*. That means it does not depend on other compatible (i.e. simultaneously co-measurable) observables which may be measured alongside the value definite observable.

Eigenstate principle If a quantum system is prepared in the state $|\psi\rangle$, then the projection observable $P_\psi = |\psi\rangle\langle\psi|$ is value definite, as are (by the previous two assumptions) all observables which commute with P_ψ.

With these assumptions one can prove the following strong form of the Kochen-Specker theorem [1, 2] which gives a formal base for the understanding of value indefiniteness in quantum physics:

Theorem. *Let a quantum system be prepared in the state* $|\psi\rangle$ *in dimension* $n \geq 3$ *Hilbert space* $\mathbf{C}^n$*, and let* $|\phi\rangle$ *be any state neither orthogonal nor parallel to* $|\psi\rangle$*, i.e.* $0 < |\langle\psi|\phi\rangle| < 1$. *Then the projection observable* $P_\psi = |\phi\rangle\langle\phi|$ *is* value indefinite.

According to the above theorem, one can precisely identify and measure a value indefinite observable, that is, a quantum state prepared to be orthogonal to the projection observable measured. Using the EPR principle and the above three assumptions, it has been proved in [1, 3] that the individual quantum not only cannot be predicted with certainty by any agent: such an agent can do no better than blindly guessing the outcome of the measurement. In this sense the quantum behaves *maximally unpredictably*. More, the maximal unpredictability itself is almost everywhere [2].

For our purposes this means that given an agent A and a context $\mathfrak{C}$ in which the above theorem is true, it is impossible to determine *in advance* which object in $O_{\mathfrak{C}}(A)$ will be chosen regardless of how much information is available. Up until the moment of choice—which is the moment of measurement—any object can be chosen.

If $O_{\mathfrak{C}}(A)$ contains more than one element—for example, two elements in the context of the above theorem—this implies that various possible outcomes can consistently be brought about given the context and the agent. So the possibility assumption **(P)** is satisfied. The problem now lies with the **(C)** assumption.

Indeterminism is not only compatible with free will, but as we argued in Section 4, a necessary condition for free will: *there is no free will without indeterminism.*

6 Indeterminism and Randomness

The alleged problem with indeterminism is that it seems to leave too much to chance. Although we are able to overcome the problems with the **(P)** assumption, we do so at the cost of introducing randomness into our system of agents and contexts. Does this make agent's control problematic?

Being as old as civilisation itself doesn't make randomness less "mysterious" or less difficult to grasp. The 5th BCE philosopher Leucippus was probably the first to note (see [14, p. 133]) that

> Nothing occurs by chance, but there is a reason and necessity in everything.

Under the influence of mathematician A. de Moivre, Hume [21, p. 56] called chance a mere word:

> . . . there be no such thing as Chance in the world.

Some concepts like chance, number, speed are theoretical concepts with no *direct* counterpart in nature: they are used to model some "reality" , but are not directly observed in the natural world. Randomness is also such a concept: *there are no random events in nature*. So, what is randomness? Where does it appear? As observed in [24],

> . . . randomness . . . is in the interface between our theoretical descriptions and 'reality' as accessed by measurement. Randomness is unpredictability with respect to the intended theory and measurement.

What are the relations between indeterminism and randomness? Probability theory doesn't answer this question. Algorithmic information theory [8, 13] is a mathematical theory which deals with randomness in "its individuality", i.e. not only as a statistical phenomenon: we will use it in what follows.

What are the characteristics usually associated with randomness? Intuitively, randomness is identified with unpredictability (see [15]), lack of correlations (irregularity) and typicality. These characteristics of randomness can be tested by considering examples of "random" events, like coin-tossing. For example, a sequence of coin tosses looks very irregular, and no matter how many times we have tossed the coin, even thousands and thousands of times, predicting the outcome of the next toss seems impossible. We toned down the last sentence by saying that prediction of coin-tossing "seems" impossible. Why? Because, in principle, coin-tossing is as predictable as the motion of the planets, once the initial conditions are given. However, we "believe" that prediction is impossible—and this feeling is confirmed by experiment—because of the peculiar combination of circumstances of coin tossing, more precisely, the sensitive dependence on (some set of) initial conditions coupled with the inability to know these conditions with infinite precision.[9] Coin-tossing randomness is a simple example that shows that determinism is compatible with some forms of randomness.

Stronger forms of randomness exist from a mathematical point of view, but are they "available", can one produce them? The best bet is to consider quantum randomness. In the previous section we have described a quantum experiment in which the outcome is maximally unpredictable. Apparently, having at hand a repeatable maximally unpredictable event is enough to claim that "quantum behaviour under measurement is truly random", in the sense that no correlations exist between successive measurement results. Such a claim would be *false*. Indeed, the notion of "perfect/true/real/genuine randomness" is mathematically vacuous [26, 8]. In particular, *in every sequence of zeros and ones there are correlations, so no such sequence can be truly random. There exist only degrees of randomness with no upper limit.* For example, quantum randomness obtained by measuring a value indefinite observable is much more random than coin-tossing [1, 2].

Is indeterminism necessary for randomness? The answer is negative: coin-tossing or software-generated randomness are (weak) forms of randomness which are produced in computable (hence deterministic) ways. In fact both algorithmic information theory [8, 13] and the practice of generating randomness show that *randomness (of any form) is defined and produced in deterministic ways*. Here are two examples, one mathematical and one physical.

The halting probability of a universal self-delimiting Turing machine U is called the Omega number of U and denoted by Ω_U (see [9] and more in [8, 13]). Given the machine U and a positive integer n we can uniquely calculate the nth bit of the binary expansion of the real number Ω_U: the infinite sequence given by the binary expansion of Ω_U is uniquely determined by U. However, this sequence is "highly random"—technically, Martin-Löf random—because it passes all Martin-Löf tests of randomness. Randomness appears when we "don't know" that the sequence is the Omega number of U. This phenomenon is general because in every sequence there are correlations.

In the same way, in quantum mechanics the Schrödinger equation describes how the quantum state of some physical system changes with time: this evolution is deterministic. Quantum randomness appears when we observe certain individual quanta.

There is a loose analogy between the above two examples. The description of Ω_U by U corresponds to the Schrödinger equation. A first analogy appears when we compare the processes of computing the bits of Ω_U and measuring value indefinite observables in a quantum experiment $\mathfrak{E}$ precisely described in [1]. One can prove that the sequence of bits of Ω_U cannot be algorithmically computed (given U) and, similarly, the sequence of bits obtained by performing ad infinitum the quantum experiment $\mathfrak{E}$ cannot be algorithmically computed (given full information

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[9] Technically, this is expressed by the phenomenon of deterministic chaos [27].

about the experiment [1, 3]). A second, more interesting analogy, appears when we compare the unpredictability of individual bits. An individual bit of Omega is maximally Martin-Löf unpredictable: a bit obtained in the quantum experiment $\mathfrak{E}$ is maximally unpredictable in the sense of [3].[10] These results are provable in both cases.[11]

Is randomness necessary for indeterminism? Again the answer is negative: cellular automata, as well as a plethora of computing machines, work in a non-deterministic way without any use of randomness.

Identifying indeterminism with randomness is misleading and renders problematic any analysis which is based on this assumption (see [14] for a historical review).

7 Is Randomness Incompatible with Freedom?

Being aware that true randomness can be proved to not exist even mathematically, less in nature, we can now examine closely the compatibility between randomness and freedom.

All arguments against the compatibility between free will and randomness based on "pure chance", "pure randomness", "true randomness" are simply unsound because they rest on a vacuous concept. An example is the argument based on Hume's and Schlick's ontological thesis according to which there is nothing intermediate between chance and determinism. In Eddington's words ([17, p. 182]):

> There is no half-way house between random and correlated behavior. Either the behavior is wholly a matter of chance, in which case the precise behavior within the Heisenberg limits of uncertainty depends on chance and not volition. Or it is not wholly a matter of chance, in which case the Heisenberg limits . . . are irrelevant.

Popper [32, p. 227] disagreed:

> Hume's and Schlick's ontological thesis . . . seems to me not only highly dogmatic (not to say doctrinaire) but clearly absurd.

Indeed, as we have seen, there are only degrees of randomness. *Randomness is not a direct cause of actions.*

A more interesting, still unsound, argument for the incompatibility between free will and randomness is the following: randomness exists (in various degrees), so if an agent's actions are caused by randomness, the agent lacks control, so the assumption **(C)** is violated. In more details the argument runs as follows.

1. Assume **(P)**.
2. An agent A has free will with respect to $O(A)$ in the context $\mathfrak{C}$ if the assumption **(C)** is satisfied.
3. So A has ultimate control of which object in $O(A)$ to choose.

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[10] The Schrödinger equation cannot give the exact result of an individual measurement.

[11] The unpredictability/randomness of *every* individual quantum outcome was conjectured/postulated by Born [6].

4. If the object was chosen *randomly*, then no one had ultimate control of which object in $O(A)$ was chosen.
5. Hence, A cannot not have ultimate control on which object in $O(A)$ to choose.
6. Therefore, A has no free will with respect to $O(A)$ in the context $\mathfrak{C}$.

The argument may have some force if one could offer a clear explanation of how the object chosen was *chosen sufficiently randomly* to prevent A having control over its decision. Unfortunately, this explanation is not given, rather it is often claimed that **(P)** is inherently able to provide this, cf. [4, 35], [28, p. 46–51]. As we discussed in the previous section, randomness does not "float around", it is not "imposed on the agent", it is just *produced* and *used* by the agent's decision. To make *random decisions* the agent needs to use a random generator, which is a device producing random bits of a certain quality (but never "truly random bits"); there is no alternative.

According to our proposed definition of free will, the detailed process used by the agent A to choose an object in some context $\mathfrak{C}$ is irrelevant: the only onus in arguing for free will is to show that condition **(C)** is satisfied. For the purposes of making a decision, using a random generator is no different than using the advice of a friend or getting more information from Wikipedia! Indeed, as explained in the previous section, randomness is defined and produced in a deterministic way.

Still, for clarification we will look closely at the process of choosing at *random*. We henceforth assume that **(P)** is satisfied and consider first how a *random generator* may interact with an agent's decision making process, and second how the *quality* of the randomness generated may affect the agent's freedom. With the former in mind, we discuss four possible cases of interactions between the agent and the random generator.

Suppose that in a context $\mathfrak{C}$, $O(A) = \{0,1\}$, A chooses a random generator G with two possible outputs, 0 or 1, and then operates as follows:

(0) A uses G which outputs x, but ignores the output and picks an element of $O(A)$.

(1) A uses G which outputs x and A uses x to pick an element of $O(A)$.

(2) A uses G which outputs x and continues as follows:

  - A uses x to determine whether to use G or not,
  - depending on x, A makes no use of G or uses G to produce another (independent) output y which becomes its decision (in the last case G actually takes the decision on A's behalf).

(3) A uses G which outputs x and its decision is x (G is used to take the decision on A's behalf).

In case (0) it is clear that A's freedom is not hindered by randomness. Still it is worth pointing out that there was a *random element* in his decision process (though it did not impact A's decision). To show that case (1) does not undermine A's freedom, we restate that the information which A uses to make a decision is irrelevant to A's decision being free. A merely *asks G for advice*. In fact cases (0) and (1) are identical if A picks something other than G's output. In either of these cases however, A can consistently choose any element in $O(A)$ and, regardless of what

G outputs, A has the final say on the decision. Hence, neither of these cases will disturb A's freedom.

Cases (2) and (3) are more difficult to analyse. Case (2) is a hybrid between the earlier cases and the substantially more severe case (3). In (2) A operates G to generate a random bit. Depending on what this generated bit was, G either stops (leaving A to make the decision) or generates with G another random element of $O(A)$ and chooses this element on A's behalf. Thus (2) will either reduce to (0) or (3) depending on the result of the first computation. The point is that in cases (2) and (3), once A starts G, which object is chosen may potentially be decided by G rather than A. Is **(C)** fulfilled in these cases? Does the quality of random bits matter (e.g. if they form an incomputable sequence)?

The reason these cases seem to violate the **(C)** assumption is because the agent gives up its final decision, *not* because the agent gives up its final decision to a random process. Asking another agent to make a decision on its behalf is no different than asking a random generator. Notice that whether A retains its freedom in asking another agent B to make the decision for it, is a delicate issue and is, in practice, judged on a case by case basis. This shows that from the point of view of free decisions, B's action is as harmful as G's. Giving up freedom to randomness is as harmful as giving up freedom to any other agent: A retains its freedom when it gives up its decision to B if and only if A retains its freedom when it gives up its decision to G.

Similarly, even if A does not voluntarily give up its decision, another agent B choosing an element of $O(A)$ is no different than some random generator G doing the same. Thus, the fact that indeterminism allows for randomness should not lead us to conclude that free will is impossible any more than the fact that there are other free agents which are capable of choosing on behalf of others.

8 Conclusions

We have proposed a definition of free will which makes specific reference to agents, contexts and objects which is simple, two-stage and contextual (not absolute). Operating with this definition we argued that determinism is incompatible with free will, hence a form of indeterminism is necessary—not sufficient—for free will. Quantum indeterminism illustrates such form of indeterminism. We showed that indeterminism and randomness do not imply each other. Finally, we provided a detailed analysis showing that randomness is not incompatible with free will.

Indeterminism is a necessary condition for freedom. Under indeterminism it is possible for agents to be free *in some contexts*, but under determinism this is never the case.

Our arguments are relative and do not answer the main question, *does free will exist?* The exact meaning of this question is not obvious and, even within the framework of the present paper, the question may be answered differently depending on how it is interpreted. An examination of the precise requirements for the existence of free will—as in the free will theorem [11]—is beyond our scope.

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