http://researchspace.auckland.ac.nz

ResearchSpace@Auckland

Copyright Statement

The digital copy of this work is protected by the Copyright Act 1994 (New Zealand).

General copyright and disclaimer

Items in ResearchSpace are protected by copyright, with all rights reserved, unless otherwise indicated.

Previously published items are reproduced in accordance with the copyright policy of the publisher.

Sources for information about publishers rights and policies:

Australasian Region - OAKList Database:  http://www.oaklist.qut.edu.au/

International – JISC UK Database http://www.sherpa.ac.uk/index.html

For more information about Copyright at The University of Auckland, including takedown notice.

------------------------------------------------------------

To contact ResearchSpace administrators, please use the Feedback form on our webpage. http://researchspace.auckland.ac.nz/feedback
STRUT-AND-TIE COMPUTER MODELLING OF REINFORCED CONCRETE BRIDGE JOINT SYSTEMS

NICHOLAS H. T. TO and JASON M. INGHAM
Department of Civil and Environmental Engineering,
University of Auckland, New Zealand

SRI SRITHARAN
Department of Civil and Construction Engineering, Iowa State University, USA

Received 7 January 2002
Revised 15 October 2002
Accepted 25 October 2002

Nonlinear inelastic force-displacement response envelopes of large scale bridge tee-joints and multi-column bridge bents were established using strut-and-tie models (STMs) representing the entire structural system. The computer based STMs employed in the current study were formulated based on the force transfer mechanisms in the joint panel region reported in the literature, in conjunction with the formulation procedure for beam and column members that has been previously reported by the authors. Analysis was performed using the nonlinear analytical program Drain-2DX. Obtained analytical results, including response envelopes and sequences of structural failure, were found to correlate satisfactorily with those obtained from the experimental data.

Keywords: Strut-and-tie model; bridge tee-joints; multi-column bridge bent.

1. Introduction

A strut-and-tie model (STM) is a discrete representation of the stress field developed within a reinforced concrete structure that is subjected to external actions. This is a generalised form of the truss modelling technique, which enables rational analysis and design of all reinforced concrete structural types to be performed.

Within a STM, uniaxially stressed struts and ties define the actual compressive and tensile stress fields respectively. The pin connections joining struts and ties together correspond to the biaxially or triaxially stressed nodal zones, where a change in direction of forces is assumed to take place.

The strut-and-tie modelling technique has traditionally been employed in design practice to predict strength and to investigate equilibrium of the applied loads, reactions, and internal forces within a disturbed (D-) region. D-regions are identified as regions where the internal stress-strain distribution within the structural member is highly disturbed and irregular, as a result of discontinuities arising from externally applied actions and/or physical geometries. Due to the highly irregular internal stress-strain distribution, design procedures for these members cannot readily be
reduced to simple mathematical expressions, instead requiring empirical or “rule of thumb” design approaches. Portions of structural members not included in D-regions are referred to as Bernoulli (B-) regions, where the member internal stresses and deformations can be accurately predicted assuming plane section response for flexure and assuming compression field theory [Collins, 1978; Collins and Mitchell, 1980] for shear.

The conventional procedure for designing reinforced concrete structures can be divided into three stages: (a) selecting member dimensions, (b) determining the quantity, placing and anchorage details of reinforcement and ensuring ultimate strength criteria are satisfied and, (c) satisfying member deformation under service loading conditions.

Traditionally, STM was utilised only in the aforementioned second design phase (b) to ensure force equilibrium [ACI, 1997]. Using data acquired from testing large-scale bridge structural systems, the study reported herein expands the application of STM to performance evaluation by predicting nonlinear force-displacement response using properly formulated models.

As STMs can be formulated for both B- and D-region structural members, the expanded application of this modelling technique implies that the three previously described design phases can be accomplished using a single model. This may ultimately result in a consistent standard for the dimensioning and reinforcement detailing of structural systems.

The primary goal of the study reported in this paper was to verify the legitimacy of the STM formulation procedure that was previously established by the authors [To et al., 2001, 2002]. The monotonic performance of large-scale bridge tee-joint and multicolumn bridge bent test units was analysed using STMs and analytical results were then correlated with the experimentally recorded data. It is anticipated that the study reported herein, in conjunction with an ongoing investigation into the use of STM to consider cyclic response, will in due course lead to an integrated STM analysis and design procedure being developed.

2. Reinforced Concrete Bridge Joints

In this section, the design philosophy of reinforced concrete bridge joint systems resembling that shown in Fig. 1 is briefly addressed.

2.1. Experimental program overview

The large-scale bridge test units analysed in this paper were adopted from testing conducted by Sritharan, et al. [1996, 1997, 1999, 2001] at the University of California, San Diego. This testing was sponsored by the California Department of Transportation, and represented part of an extensive experimental research project carried out over the past decade, aiming at improving the seismic design of bridge structures subsequent to widespread damage in the 1989 Loma Prieta and 1994 Northridge earthquakes.
2.2. Design philosophy

The conventional design method for bridge tee- and knee-joint panels requires isolation of the joint shear as an independent force rather than regarding the shear force as an integrated force path for complete force transfer through the neighbouring structural members. The design procedure is primarily governed by empirical rules in conjunction with several other constraints, such as limiting maximum joint shear stress, providing a minimum quantity of confining reinforcement and requiring minimum development length for longitudinal reinforcement. Structural design based upon this method usually results in congested reinforcement details and is often difficult to implement in practice. The deficiency of this design approach was clearly demonstrated by the widespread unexpected damage to bridge structures in recent large earthquakes that occurred in California, indicating that a new design approach for reinforced concrete bridge joints was required.

Following a decade of experimental and analytical study, a rational and less conservative bridge joint design method was proposed by a group of researchers from the University of California, San Diego [Ingham, 1995; Priestley et al., 1996; Srittharan, 1998]. This new rational design method, referred to as the force transfer method, was established by examining the complete force transfer across the joint, in which the joint shear is considered as an integrated part of the joint equilibrium mechanism. Furthermore, the new design approach ensures that the longitudinal column reinforcement is sufficiently anchored into the joint and that damage to the joint region is controlled to a minimum, allowing the overstrength flexural capacity at the critical column section adjacent to the column-joint interface to be developed. Notably, the bridge joint systems analysed here using STMs were detailed according to this new design approach to avoid premature failure.

3. Strut-and-Tie Modelling Example

The strategy adopted in model formulation is briefly addressed in this section. A STM example is illustrated, and the obtained results are discussed.
3.1. Modelling strategy

When formulating a suitable D-region STM, for the reasons of simplicity and consistency, Schlaich, et al. [1987] recommended that a model established based upon force transfer across the member at the cracked serviceability condition could be used to analyse the ultimate strength. Although this approach is not satisfactory, in particular because both internal stress redistribution resulting from progressive concrete damage and significant reinforcement inelastic strain cannot be captured, it is nonetheless acceptable when primarily used to establish appropriate reinforcement detailing in D-regions since it guarantees a conservative solution. Marti [1999] proposed that elastic analysis could be used in conjunction with some modification according to the redistribution of internal forces and moment when determining the compression path in D-regions. Contrarily, other critics [MacGregor, 1988; Sritharan, 1998] argued that a D-region STM that is established based upon the elastic state stress path may be inadequate when the structure is subjected to a combined action. This was confirmed in an experimental research program conducted by Bhide and Collins [1989] in which a series of reinforced concrete panels were subjected to pure shear and combined shear and tension. Results demonstrated that the crack pattern that developed at the serviceability and ultimate limit state load conditions were significantly different when the concrete panel was subjected to a combined action.

It has been widely observed that the stress trajectories occurring in D-regions can be very different at various loading states, which is not usually the case for B-regions. Furthermore, D-regions usually exhibit brittle failure resulting in significant strength degradation, while B-regions often exhibit ductile response and are mostly responsible for nonlinear inelastic behaviour of the entire structural system. Since only one model can be used in a computer analysis, this model should be capable of predicting the ultimate strength and nonlinear inelastic deformation of D- and B-region structural members respectively. Accordingly, the strategy employed in the current study to formulate suitable STMs was to adopt the force transfer mechanism that occurred at the ultimate limit state for the joint panel regions and the mechanism that occurred at the serviceability limit state for the beam and column members.

3.2. Test unit reinforcement details

The tee-joint test unit, IC1, considered in the example was adopted from testing conducted by Sritharan, et al. [1996, 1999]. This unit was one amongst three tested that represented redesigns of the as-built interior joint from the three-column Bent 793 + 57 of the Santa Monica Viaduct in Los Angeles, California. The as-built joint represented a typical joint from the pre-1960 era, with no shear reinforcement, and was found to provide poor seismic performance [MacRae et al., 1994]. The tee-joint unit, IC1, was constructed at half scale and was tested in an inverted position. To
simplify the testing procedure and data interpretation, the soffit and deck slab were not constructed in the test unit.

The overall dimensions and reinforcement details of tee-joint IC1 are illustrated in Fig. 2. The test unit was composed of a circular column and rectangular beam, and was detailed with conventional reinforcement in the joint panel region. Notably, additional cap beam bottom longitudinal rebars (i.e. the top layer of longitudinal reinforcement for an inverted position) and an extra amount of stirrups in the vicinity of the joint region were provided to facilitate effective joint force transfer by mobilising the external joint strut as denoted \( C_{AC} \) in Figs. 3(a) and 3(b). Horizontal joint reinforcement was provided by \#3 \((d_b = 9.5 \text{ mm})\) spiral at 57 mm spacing and 2 sets of 4 legged \#3 hairpin type stirrups were employed as the vertical joint reinforcement. In order to allow the longitudinal column rebars to be effectively anchored into the joint strut, the column rebars were extended close to the top of the cap beam (i.e. the bottom layer of the longitudinal reinforcement for an inverted position), terminating at a distance of 63.5 mm away from the beam edge.

Fig. 3. Force transfer mechanism in tee-joint IC1.
3.3. Modelling technique highlights

A nonlinear analytical computer program, Drain-2DX [Allahabadi and Powell, 1998], was employed to perform all strut-and-tie computer analyses in this study. Analytical results were correlated with the experimentally recorded data to illustrate the effectiveness of STMs in capturing the structural force-displacement response.

As theoretical background relevant to the model formulation procedure has been thoroughly reported [To et al., 2001, 2002], this information is not reconsidered here. Instead, the key modelling features are illustrated in this section using an example of a tee-joint test unit.

3.3.1. Model formulation procedure for B-region

A section force analysis was first performed when formulating the STMs representing the B-regions. Analytical results recorded at the 1st yield state, including neutral axis depth, c, and the force centroids of concrete compression, \( x_c \), and reinforcement tension, \( x_t \), were used in the following steps to determine member area and position. The 1st yield state was defined by the commencement of extreme tension reinforcement yielding or the extreme concrete compression fibre reaching a strain value of 0.002, whichever occurred first.

3.3.1.1. Member area

As the rebars are usually approximately uniformly distributed adjacent to the perimeter of a circular column section, see Fig. 4, but are often concentrated at the compression and tension edges of beam sections, the cross sectional area of reinforcement struts, \( A_{rs} \), and reinforcement ties, \( A_{rt} \), located in the columns and beams are computed separately using Eqs. (1)–(4). Similar to the treatment of rebar members, the cross sectional area of concrete struts, \( A_{cs} \), and concrete ties, \( A_{ct} \), located in the beam and column sections was also computed separately using Eqs. (5)–(7).

The cross-sectional area of longitudinal rebar ties located in a rectangular beam section is:

\[
A_{rs} = A'_{s},
\]

(1)

\[
A_{rt} = A_s,
\]

(2)

where

\( A_s \) is the total area of longitudinal tension reinforcement in the beam;

\( A'_s \) is the total area of longitudinal compression reinforcement in the beam.

The cross-sectional area of longitudinal rebar ties located in a circular column section is:

\[
A_{rs} = 2 \cdot (\pi - \alpha)tr, \]

(3)
\[ A_{rt} = 2\alpha r_o \]  
\[ \alpha = \cos^{-1} \left( \frac{2c - D_c}{D' - d_b - d_{vs}} \right), \]  
and  
\[ t = \frac{A_{st}}{2\pi r_o}, \]  
\[ r_o = \frac{D' - d_b - d_{vs}}{2}, \]  
where  
\[ A_{st} \] is the total area of longitudinal reinforcement;  
\[ d_b \] is the rebar diameter;  
\[ d_{vs} \] is the sectional diameter of transverse rebar;  
\[ D_c \] is the sectional diameter of a circular section;  
\[ D' \] is the concrete core diameter measured from centre to centre of peripheral hoop or spiral;  
\[ r_o \] is the radius of circular concrete core measured from the section centre to the centreline of the longitudinal rebar, see Fig. 4(a);  
\[ t \] is the thickness of imaginary longitudinal steel tube, see Fig. 4(a);  
\[ \alpha \] is the half angle of the flexural tension sector, see Fig. 4(a).  

The cross-sectional area of longitudinal concrete struts is:  
\[ A_{cs} = c \cdot b_w \] for rectangular beam sections;  
\[ A_{cs} = \frac{D_c^2 \cdot (\phi - \sin \phi \cos \phi)}{4} \] for circular column sections;  
and  
\[ \phi = \cos^{-1} \left( \frac{0.5D_c - c}{0.5D_c} \right), \]  
\[ A_{ct} = \frac{A_g - A_{cs}}{2} \] for both types of sectional shapes,  
where  
\[ A_g \] is the gross sectional area;  
\[ b_w \] is the sectional web width;  
\[ c \] is the neutral axis depth recorded at 1st yield state from the section force analysis;  
\[ \phi \] is the half angle of the compression section as defined in Fig. 4(a).  

For the transverse reinforcement tie representing the secondary reinforcement in beams and columns, the effective member area, \( A_e \), was given by:  
\[ A_e = \frac{V_s + V_c + V_p}{f_{vy}}, \]
where

\( V_s \) is the reinforcement contribution to member shear strength;
\( V_c \) is the concrete contribution to member shear strength;
\( V_p \) is the externally applied axial load contribution to shear strength;
\( f_{vy} \) is the measured yield strength of transverse reinforcement.

To compute the shear strength capacity attributed to the transverse rebar \( V_s \), Eqs. (9a) [Priestley et al., 1996] and (9b) were employed when the transverse reinforcement is provided in circular spirals and hoops or rectangular ties respectively.
\[ V_s = \frac{\pi D'}{4s \cdot \tan \theta} f_{ey} \cdot A_{vs}, \quad (9a) \]
\[ V_s = \frac{d}{s \cdot \tan \theta} f_{ey} \cdot A_{vs}, \quad (9b) \]

where

- \( A_{vs} \) is the total area of transverse reinforcement in a layer in the direction of the shear force;
- \( d \) is the member effective depth;
- \( s \) is the pitch distance between transverse reinforcement;
- \( \theta \) is the angle between diagonal struts and member longitudinal axis.

For member shear capacity attributable to concrete tensile strength, \( V_c \), the modified compression field theory (MCFT) [Collins and Mitchell, 1997] recommended by the Canadian Concrete Design Standard [1994] was employed to account for the effect of diagonal concrete strut inclination angle, \( \theta \). \( V_c \) is given by:

\[ V_c = \beta \sqrt{f'_c} A_{ve}, \quad (10) \]

where

- \( A_{ve} \) is the effective shear area, \( A_{ve} = 0.628 A_g \) for circular section and \( A_{ve} = 0.628 A_g \) for rectangular section [Priestley et al., 1996];
- \( f'_c \) is the concrete cylinder strength;
- \( \beta \) is the factor accounting for the shear resistance of cracked concrete, which is computed according to the MCFT [Collins and Mitchell, 1997].

Note that the concrete contribution to shear capacity in the plastic hinge zone is expected to be minimal when significant concrete damage is developed in this region. Hence, for the transverse reinforcement ties located in the potential plastic hinge zone, \( V_c \) is ignored when computing \( A_v \) using Eq. (8).

The contribution to member shear strength from externally applied axial load is considered as an independent component resulting from a diagonal compression strut, which is given by:

\[ V_p = N \cdot \tan \alpha_N, \quad (11) \]

where

- \( N \) is the externally applied compression (+ve) force and tension (−ve) force;
- \( \alpha_N \) is the angle between the column axis and the line joining the positions of concrete strut, \( x_c \), at the top and bottom of the column, see Fig. 4(b).

### 3.3.1.2. Effective member strength

To establish the effective strength of concrete strut, \( f_d \), the maximum concrete flexural compression force, \( C_{c(max)} \), prior to failure was first elevated using section
force analysis. The effective strut capacity was then given as:

\[ f_d = \frac{C_{c(\text{max})}}{A_{cs}}. \]  \hspace{1cm} (12)

The effective tensile strength, \( f_{dt} \), of concrete ties is calculated using Eqs. (13)–(15). Equation (13) was proposed by Chan et al. [1992] assuming cracking to be initiated by the tensile stress in concrete, which is transferred from the embedded reinforcement through bond.

\[ f_{dt} = \bar{\sigma}_{ct} \quad \text{for concrete ties with favourable conditions defined by} \quad \text{Chan et al. [1992]}; \]  \hspace{1cm} (13)

\( \bar{\sigma}_{ct} \) is the average tensile stress in reinforced concrete, which is calculated according to Chan et al. [1992].

Note that the capability of applying Eq. (13) to compute \( f_{dt} \) depends upon several interrelated physical conditions of tie members defined by Chan et al. [1992]. These conditions include member length, steel-concrete area ratio and diameter of embedded rebars. When these interrelated tie member conditions are not satisfactory to provide sufficient tension transfer to induce concrete cracking, no solution is provided by Eq. (13). In such case, Eqs. (14) and (15) were employed to compute \( f_{dt} \) for the concrete ties located in rectangular and circular sections respectively.

\[ f_{dt} = \frac{D_r \cdot b_w \cdot f'_t}{4 \cdot A_{ct}} \quad \text{for concrete ties located in rectangular sectional members}; \]  \hspace{1cm} (14)

\[ f_{dt} = \frac{D_e \cdot f'_t}{6 \cdot A_{ct}} \quad \text{for concrete ties located in circular sectional members}, \]  \hspace{1cm} (15)

where

\( f'_t \) is the plain concrete tensile strength, \( = 0.6 \sqrt{f_c} \) [NZS: 3101(1995)] for concrete subjected to flexural tension;

\( D_r \) is the total sectional depth of rectangular sections.

Again, due to the different longitudinal reinforcement arrangement in columns and beams, the effective strength of reinforcement struts, \( f_{cy} \), and reinforcement ties, \( f_{ty} \), are treated separately.

For beams:

\[ f_{cy} = f_{ty} = f_y. \]  \hspace{1cm} (16)

For circular columns:

\[ f_{cy} = f_y \cdot \frac{\sin \alpha + \cos(\pi - \alpha)}{(\pi - \alpha) \cdot (1 + \cos \alpha)}, \]  \hspace{1cm} (17)

\[ f_{sy} = \phi_f \cdot f_y \cdot \frac{\sin \alpha - \alpha \sin \alpha}{2 \alpha \cos \left( \frac{\pi - \alpha}{2} \right) \sin \left( \frac{\pi - \alpha}{2} \right)}, \]  \hspace{1cm} (18)
where

\[ f_y \] is the measured yield strength of longitudinal reinforcement;
\[ \phi_o = 4/3 \] is an effective overstrength factor only applied for circular columns.

3.3.1.3. Member position

In all cases, the positions of struts, \( x_c \), and ties, \( x_t \), as shown in Fig. 6 are located respectively at the concrete compression force centroid and reinforcement tension force centroid measured at the 1st yield state.

3.3.2. Model formulation procedure for D-region

As the force path developed within a D-region varies significantly for different load combinations and loading states, it is very difficult, if not impossible, to establish rigorous model formulation rules for this structural region. Experience and insight into the actual reinforcement arrangement are always necessary when developing a suitable D-region STM. In this study, the interior joint STMs depicting the force path that occurred at the ultimate limit state were adopted from those previously proposed by Sritharan et al. [1996, 1999, 2000], see Fig. 3.

The cross sectional area of concrete struts located in the D-region was computed graphically and the area of reinforcement tie was calculated as the total area of reinforcement it represented. The measured material strength of rebars was used as the effective strength of reinforcement struts and ties located in the D-region. For the effective strength of concrete struts, values shown in Table 1 as previously reported in the literature, were adopted.

\[ N = 400 \text{ kN} \]

(a) Push (+ve displacement) direction.  
(b) Pull (−ve displacement) direction.

Fig. 5. STMs of tee-joint IC1.
Table 1. D-region effective compressive strength of strut members [Collins and Mitchell, 1980; Rogowsky and MacGregor, 1986; Schlaich et al., 1987; Sritharan, 1998]

<table>
<thead>
<tr>
<th>Effective Strut Strength</th>
<th>Strut Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85f'_c</td>
<td>This value is applicable for struts representing a prism stress distribution, where there is minimal transverse tensile strain induced by neighbouring rebar.</td>
</tr>
<tr>
<td>0.68f'_c</td>
<td>This value can be adopted for struts locating in regions where minor cracking is expected, An example of this application would be the struts residing in a prestressed joint.</td>
</tr>
<tr>
<td>0.51f'_c</td>
<td>This value is appropriate for concrete struts when the neighbouring rebar is not subjected to significant strain hardening (ε_s ≤ 0.01).</td>
</tr>
<tr>
<td>0.34f'_c</td>
<td>This is the maximum permissible stress for concrete struts when there is potential development of significant inelastic strain (ε_s &gt; 0.02) in the neighbouring reinforcement. This value can also be applied to struts modelling bottle-shaped stress zones when no effective confinement is provided.</td>
</tr>
</tbody>
</table>

*For 0.01 < ε_s < 0.02, consider linear interpolation to obtain appropriate permissible stress.

3.4. Strut-and-tie model properties

The formulated STM replicating the force path developed in the tee-joint IC1, when subjected to the push (+ve displacement) and pull (−ve displacement) actions, are depicted in Fig. 5. The STMs resemble the force transfer models shown in Fig. 3, except that the concrete strut CEF located adjacent to the closing corner of the joint was omitted to ensure stability of the computer analyses. Since the force transfer mechanisms within the joint region were similar for both action directions, the pull STM was assumed to mirror the push STM. The calculated properties of model members, as denoted in the push STM shown in Fig. 6, are listed in Table 2. Note

![Fig 6. STM properties of tee-joint IC1.](image-url)
Table 2. STM member properties and stiffness of tee-joint IC1.

<table>
<thead>
<tr>
<th>Member Number</th>
<th>Descriptions</th>
<th>Area (mm$^2$)</th>
<th>Effective Strength (MPa)</th>
<th>Young’s Modulus (GPa)</th>
<th>Strain Hardening Ratio, s.h.r.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Longitudinal reinforcement tie at column</td>
<td>3400</td>
<td>0.82$f_y$</td>
<td>200</td>
<td>2.5%</td>
</tr>
<tr>
<td>2</td>
<td>Longitudinal concrete strut at column</td>
<td>86500</td>
<td>0.66$f_c'$</td>
<td>32.3</td>
<td>0%</td>
</tr>
<tr>
<td>3</td>
<td>Diagonal concrete strut at column</td>
<td>119100</td>
<td>0.85$f_c'$</td>
<td>32.3</td>
<td>0%</td>
</tr>
<tr>
<td>4</td>
<td>Transverse tie at column</td>
<td>1060</td>
<td>1.0$f_y$</td>
<td>200</td>
<td>5%</td>
</tr>
<tr>
<td>5</td>
<td>Longitudinal reinforcement tie at top of beam</td>
<td>2720</td>
<td>1.0$f_y$</td>
<td>200</td>
<td>5%</td>
</tr>
<tr>
<td>6</td>
<td>Diagonal concrete strut at beam</td>
<td>82320</td>
<td>0.85$f_c'$</td>
<td>34.5</td>
<td>0%</td>
</tr>
<tr>
<td>7</td>
<td>Transverse tie at beam</td>
<td>1720</td>
<td>1.0$f_y$</td>
<td>200</td>
<td>5%</td>
</tr>
<tr>
<td>8</td>
<td>Longitudinal reinforcement tie at bottom of beam</td>
<td>2720</td>
<td>1.0$f_y$</td>
<td>200</td>
<td>5%</td>
</tr>
<tr>
<td>9</td>
<td>Longitudinal concrete strut at top of beam</td>
<td>93100</td>
<td>0.34$f_c'$</td>
<td>34.5</td>
<td>0%</td>
</tr>
<tr>
<td>10</td>
<td>Joint concrete strut</td>
<td>109800</td>
<td>0.68$f_c'$</td>
<td>34.5</td>
<td>0%</td>
</tr>
<tr>
<td>11</td>
<td>Column reinforcement tie</td>
<td>2330</td>
<td>1.0$f_y$</td>
<td>200</td>
<td>5%</td>
</tr>
<tr>
<td>12</td>
<td>Column reinforcement tie</td>
<td>1600</td>
<td>1.0$f_y$</td>
<td>200</td>
<td>5%</td>
</tr>
<tr>
<td>13</td>
<td>Joint concrete strut</td>
<td>77300</td>
<td>0.51$f_c'$</td>
<td>34.5</td>
<td>0%</td>
</tr>
<tr>
<td>14</td>
<td>Joint concrete strut</td>
<td>53100</td>
<td>0.51$f_c'$</td>
<td>34.5</td>
<td>0%</td>
</tr>
<tr>
<td>15</td>
<td>Joint concrete strut</td>
<td>86500</td>
<td>0.51$f_c'$</td>
<td>34.5</td>
<td>0%</td>
</tr>
<tr>
<td>16</td>
<td>Joint concrete strut</td>
<td>54900</td>
<td>0.51$f_c'$</td>
<td>34.5</td>
<td>0%</td>
</tr>
</tbody>
</table>

that the member properties for the pull model are not listed, as they are identical to those for the push model. Based on the joint reinforcement detailing and the analysis reported by Sritharan et al. [1999], the joint strut capacity for repeated cyclic action was taken as 0.51$f_c'$, see Table 1.

3.5. Strut-and-tie analytical results

The derived force-displacement response envelopes of tee-joint IC1 for both push (+ve displacement) and pull (−ve displacement) directions are illustrated in Fig. 7, accompanied by the experimentally recorded data. Excellent correlation was obtained for stiffness and strength in the elastic and plastic regimes, confirming the applicability of STMs to conduct structural performance evaluation. Relevant analytical events captured by the STM are illustrated using solid circles on the respective force-displacement response envelopes shown in the same figure.

For the push model, analysis suggested that the yielding of column longitudinal reinforcement first occurred at the column-joint interface and in the joint region, as indicated by events 1–3 in Fig. 5(a). Crushing of the main diagonal joint strut was captured by event 4, indicating that the test unit reached its maximum capacity at
a load level of approximately 400 kN. The pull model predicted very similar analytical events to that captured in the push model. Yielding of column longitudinal reinforcement first occurred at the column-joint interface and in the joint region, see events 1–2 in Fig. 5(b). This was followed by event 3, which indicated the crushing of the joint diagonal concrete strut at a load level of approximately 400 kN.

According to experimental observations [Sritharan et al., 1996, 1999], the performance of tee-joint IC1 was satisfactory until cycled to $\mu = 6.0$, corresponding to a column deflection of 100 mm. Gradual deterioration was observed at this ductility level due to a progressive tension failure of the joint, resulting in concrete spalling in the joint panel region and in the column plastic hinge zone. Accordingly, it is concluded that the captured analytical events agreed satisfactorily with the observed structural response.

4. Reinforced Concrete Bridge Joints

Following the successful completion of the STM example, this section details the strut-and-tie analysis of two large-scale tee-joints and two multicolumn bridge bents using the identical model formulation procedure. Reinforcement details of the respective test units is first addressed, with the STM analytical result subsequently discussed.

4.1. Test units reinforcement details

The two tee-joint test units, IC2 and IC3, considered in this section were comparable to tee-joint IC1, but were detailed with partial and full prestressing, corresponding to $0.1f'_c$ and $0.2f'_c$ respectively.
The first (MCB1) of the two multicolonmn bridge bents analysed in this study was designed with a precast fully prestressed cap beam, while the second unit, MCB2, was designed with a mix of conventional and headed reinforcement, and with mechanical couplers [Sritharan et al., 1997, 2001]. Notably these two multicolonmn bridge bents were also designed according to the force transfer models proposed by Ingham [1995]; Priestley [1993] and Sritharan [1998], resulting in substantially reduced amounts of joint reinforcement and more constructible details. In conjunction with the design concept of force transfer models, reinforcement quantities in the joint region were further decreased when utilising the prestressing strategy employed for the cap beams.

4.1.1. Partially prestressed tee-joint, IC2

As it was anticipated for the partially prestressed unit IC2 that all of the column longitudinal rebars could be directly anchored into the joint diagonal strut of the prestressed joint, no extra stirrups were provided in the cap beam adjacent to the joint region, see Fig. 8(a). The column longitudinal rebars were also extended to the top of the cap beam (i.e. bottom layer of longitudinal reinforcement in the inverted position). Additional bottom beam reinforcement required by the joint design model was also provided in this unit to allow the development of external struts that utilised the reserve capacity of the beam shear reinforcement. The horizontal joint reinforcement was provided by #3 ($d_b = 9.5$ mm) spiral at 121 mm spacing and 2 sets of 4 legged hairpin type reinforcement was utilised as the vertical joint reinforcement, which was identical to that provided in IC1. The employed horizontal and vertical joint reinforcement were 32% and 18% of that required for the maximum horizontal and vertical shear force respectively. Cap beam prestressing was applied using straight bar with zero eccentricity, and the total prestressing force was 1737 kN.

4.1.2. Fully prestressed tee-joint, IC3

Reinforcement details of the fully prestressed unit, IC3, are illustrated in Fig. 8(b). The cap beam of this unit was constructed with precast units to examine the feasibility of precast construction of concrete multicolonmn bents and their associated behaviour when subjected to simulated seismic loading. The column longitudinal reinforcement detail of unit IC1 was duplicated in unit IC3 for strength comparison.

Again, cap beam prestressing was applied using straight bars with zero eccentricity. No continuous longitudinal mild steel reinforcement was provided in the cap beam, however 6 #4 ($d_b = 12.7$ mm) rebars were provided in each precast segment in the top and bottom corner, primarily to support the vertical beam ties during construction. A layer of epoxy with a thickness of 3.2 mm was used between the precast elements to improve the grouting procedure. The joint reinforcement detail of this test unit was identical to that of IC2 except that the joint vertical stirrups were provided as closed ties. Due to buckling of longitudinal rebars in the plastic hinge
zones of unit IC1 and IC2, resulting in the development of pronounced column shear cracks, the volumetric ratio of the column spiral reinforcement in IC3 was increased by 50%. Again, cap beam prestressing was applied using straight bar with zero eccentricity, and the total prestressing force was 3002 kN.

4.1.3. Multicolumn bridge bent, MBC1
The reinforcement details of the precast fully prestressed multicolumn test unit, MCB1, are illustrated in Fig. 9(a). A reinforcement quantity of 4% was provided
to the interior column in order to ensure development of the maximum practicable joint shear, while a longitudinal reinforcement content of 3% was used for the exterior column in order to avoid excessive cap beam seismic moment. Also illustrated in Fig. 9(a) is the reinforcement detail of the pin connection at the column bases. The pin connection was achieved by terminating all the column longitudinal reinforcement just above the footing, reducing the gross area of the column section at the column-footing interface by 50%. Adequate reinforcement was provided at the centre of the column to transfer axial tension force.

Prestressing was applied to the bent cap with zero eccentricity. There was no continuous longitudinal mild steel reinforcement provided in the cap beam, in order to allow construction of the test unit using precast modules, see Fig. 9(a). However, 6 #4 (d_b = 12.7 mm) rebars were provided in each precast segment at the top and bottom, primarily to support the vertical beam ties during construction.

A nominal quantity of horizontal spirals and vertical rectangular ties were provided to both tee and knee-joints. In order to increase the effectiveness of rectangular ties in the out-of-plane direction, two J-hooks were also provided as crossties with each stirrup in the tee-joint. Minimal reinforcement was also provided within the knee-joint. The horizontal spirals of this joint were identical to that of the tee-joint, while only 3 sets of 6 legged #3 (d_b = 9.5 mm) stirrups were placed as vertical joint reinforcement. Moreover, for the column longitudinal rebars to be effectively anchored in the diagonal joint strut, these rebars were extended into the joint as close to the top beam reinforcement as possible.

4.1.4. Multicolumn bridge bent, MCB2

The multicolumn bent unit, MCB2, had the same column reinforcement detail as MCB1, but the cap beam was detailed with a mix of conventional and headed reinforcement, mechanical couplers and zero prestressing, see Fig. 9(b).

To simplify construction of the bent cap, short u-shaped headed reinforcement was fed through the knee-joint and was connected to headed longitudinal beam reinforcement adjacent to the column face using mechanical couplers. In addition, mechanical couplers were also provided in the low bending moment region of the bent cap to reduce the bar diameter of beam side reinforcement over zero lap length. For the reinforcement detail of the tee-joint, 5 sets of 6 legged #3 (d_b = 9.5 mm) stirrups were placed within a 381 mm distance from each face of the interior column, see Fig. 10(a), in order to support an effective joint force-transfer mechanism. Three of the same stirrup set and #3 (d_b = 9.5 mm) stirrups at 57 mm spacing were provided in the joint as the vertical and horizontal reinforcement, as illustrated in Fig. 10(c). Furthermore, the longitudinal reinforcement was extended into the joint as close to the top beam reinforcement as possible.

Since the exterior column had less longitudinal reinforcement content than the interior column, a reduced amount of knee-joint reinforcement was provided. In
order to support an effective joint force-transfer mechanism, 4 sets of 6 legged #3 ($d_b = 9.5$ mm) stirrups were placed within a 381 mm distance adjacent to the exterior column face, see Fig. 10(b). The exterior joint reinforcement detail was identical to that of the interior joint.
Fig. 10. Cap beam reinforcement details of MCB2: (a) adjacent to T-joint; (b) adjacent to knee joint; and (c) joint stirrup detail. (Note: all dimensions in mm and cover to main bar is 25 mm.)

4.2. Strut-and-tie analytical results

All STMs that represent the stress path developed in the joint panel region were formulated according to the force-transfer models established by Sritharan [1998]. As they were primarily constituted to facilitate unit strength examination using hand calculation, some of them were modified slightly to be suitable for computer analyses.

Results obtained from the strut-and-tie analyses were compared with the recorded structural response to determine the effectiveness and the advantage of STM.

4.2.1. Partially prestressed tee-joint, IC2

The ultimate state force transfer models of the partially prestressed unit, IC2, for both action directions are illustrated in Figs. 11(a) and 11(b) and the corresponding STMs are shown in Figs. 12(a) and 12(b). Unlike the force transfer models,
additional concrete struts that run along the length of the cap beam were incorporated in the STMs to represent the concrete mobilised by prestressing. These concrete struts were required to allow a stable model for computer analysis.

For the push STM, due to tension demand to the right of the joint that arose from the externally applied action, additional reinforcement ties were mobilised in the joint and beam. Therefore the push model was slightly different from the pull STM. As the cap beam was partially prestressed, a broader concrete strut was mobilised to transfer shear in the joint region and the development of an effective force-transfer mechanism was less reliant upon joint reinforcement. Therefore, the
joint reinforcement was not expected to sustain significant strain hardening $0.51f'_c$
and was adopted as the effective strength of diagonal joint struts.

When tested, the unit was initially subjected to an impulsive horizontal load of
388 kN in the pull (−ve displacement) action direction, corresponding to a system
ductility of 3. Unfortunately, no test data was recorded during this mishap. The
test unit was then unloaded and the horizontal displacement of the column was
brought back to zero, requiring an actuator force of 158 kN in the push direction.
From this point, the test was started again.

As a result of the erroneous testing procedure, the elastic stiffness in the pull
(−ve displacement) action direction was substantially softer than that predicted
by the STM, see Fig. 13(a). However, as the elastic stiffness was accurately calcu-
lated for the push displacement direction, it was assumed that the pull model had
captured the elastic response satisfactorily. Also suggested by the derived response envelopes in both displacement directions is that the unit strength was accurately predicted.

Relevant analytical events captured by the STMs for both displacement directions are illustrated on the response using solid circles, see Fig. 13(a). Analysis

![Graph showing force-displacement response envelopes correlation for tee-joint test units.]

(a) Partially prestressed tee-joint, IC2.

(b) Fully prestressed tee-joint, IC3.

Fig. 13. Force-displacement response envelopes correlation for tee-joint test units.
suggested that the events captured by both models were identical. Minimal damage with no concrete crushing in the joint panel region was predicted. Yielding of column longitudinal reinforcement first occurred at the column-joint interface and in the joint region, see events 1–3 in Figs. 12(a) and 12(b). Further reinforcement yielding that extending up to the mid-column height was captured by event 4 in both models. Event 5 in both models was concrete crushing in the column at the joint interface. Again, analytical events obtained from both models correlated satisfactorily to experimental observations.

4.2.2. Fully prestressed tee-joint, IC3

The ultimate state force transfer models of the fully prestressed unit, IC3, for both action directions are illustrated in Figs. 11(c) and 11(d) and the corresponding STMs for the same tee-joint test unit are shown in Figs. 12(c) and 12(d). A noteworthy modelling feature is that the column longitudinal reinforcement tie was directly anchored into the joint concrete strut and the diagonal concrete strut that extended from the test unit support. Similar to IC2, the STMs for IC3 for both action directions had additional concrete struts in the cap beam to represent concrete being mobilised by the prestressing and also to provide a stable model when performing the computer analysis. As the force transfer models suggested that similar force paths occurred in the joint for both action directions, the pull STM was assumed to be the mirror image of the push STM. Since the cap beam was fully prestressed, minimal cracking was expected in the joint panel region. Therefore, \(0.68 f_c\) was adopted as the effective strength of joint concrete struts.

The force-displacement response envelopes derived from both of the push and pull models are shown in Fig. 13(b), accompanied by the experimental data. As illustrated, elastic response of the test unit in both displacement directions was accurately captured. However, the analysis slightly underestimated the unit yield strength.

Relevant analytical events captured by both push and pull models are illustrated in Fig. 13(b) using solid circles. Again, the analytical predictions for both models were identical. Yielding of column longitudinal reinforcement first occurred at the column-joint interface, see events 1–2 in Figs. 12(c) and 12(d). This was followed by event 3 in both models, which suggested the crushing of the concrete strut in the compressively stressed joint corner at a high ductility level. Again, satisfactory agreement between the captured analytical events and experimental observations was obtained.

4.2.3. Multicolumn bridge bent, MCB1

The STMs formulated for MCB1, when subjected to externally applied action in both push (+ve displacement) and pull (−ve displacement) directions, are illustrated in Figs. 14(a) and 14(b) respectively. The force transfer models of the joints in MCB1 corresponded to those reported for IC3 in Figs. 11(c) and 11(d) and are
Prestressing = 6875 kN

156 kN + \( P \tan(29.7^\circ) \)

178 kN 178 kN 178 kN

Analytical Events:
1-8, Rebars yield in tension
9-11, Concrete crushes

(a) MCB1 in push (+ve displ.) action direction.

Prestressing = 6875 kN

156 kN + \( P \tan(29.7^\circ) \)

178 kN 178 kN 178 kN

Analytical Events:
1-8 & 10-12, Rebars yield in tension
9. Concrete crushes

(b) MCB1 in pull (−ve displ.) action direction.

Prestressing = 6875 kN

156 kN + \( P \tan(29.7^\circ) \)

178 kN 178 kN 178 kN

Analytical Events:
1-2, 4-5, 7 & 10, Rebars yield in tension
3, 6, 8-9 & 11, Concrete crushes

(c) MCB 2 in push (+ve displ.) action direction.

Fig. 14. The strut-and-tie models of multicolumn bridge bents.
not reconsidered here. Since the force transfer mechanism within the tee- and knee-joint regions were similar for both action directions, the push and pull direction joint models were assumed to be exact reflections.

Also noteworthy is that the rebars connecting the column and footing were assumed to be anchored at a distance of $\ell_{dc}/2$ away from the bar end, where $\ell_{dc}$ is the reinforcement anchorage length calculated using the recommended equations by Priestley et al. [1996] as:

$$\ell_{dc} = \frac{0.3d_b f_y}{f'_c} \text{ (mm, MPa)}.$$  \hfill (19)

As the cap beam was fully prestressed, a large concrete area was mobilised for joint force transfer and minimal cracking was expected. Therefore $0.68f'_c$ was adopted as the effective strength of joint concrete struts.

The derived structural response envelope using both push and pull STMs are illustrated in Fig. 15(a), accompanied by the experimentally recorded data. Excellent correlation of the stiffness and strength between analytical and experimentally data was observed in both displacement directions. Relevant analytical events captured by the push and pull models are illustrated on the respective force-displacement response envelope using solid circles. Both models predicted very similar analytical events. Yielding of longitudinal reinforcement located at the pin connection in both columns first developed, see events 1 and 3 in Figs. 14(a) and 14(b). This was followed by reinforcement yielding in the tee- and knee-joint regions, and at the column-joint interfaces, see events 3–6 for both models. Further reinforcement yielding propagating up the reinforcement at both pin connections was captured by events 7 and 8, and concrete crushing occurred in the knee-joint as suggested by event 9 of both models. For the push model, subsequent analytical events 10 and

Fig. 14. (Continued)
11 predicted concrete crushing in the pin connection. For the pull model, further reinforcement yielding in the pin connections at the exterior column and interior column was captured by events 10–12, see Fig. 14(b). Obtained analytical data were found to correlate satisfactorily with the experimentally observed response.
4.2.4. *Multicolumn bridge bent, MCB*

As the force transfer model of the tee-joint in MCB2, when subjected to push action, was similar to that established for IC1 [shown in Fig. 3(a)], force transfer across the tee-joint was presented in Fig. 11(e) for the pull direction only. Also, illustrated in Figs. 11(f) and 11(g) are the force transfer models of the knee-joint for both action directions.

The formulated STMs for MCB2, when subjected to the externally applied push and pull actions, are illustrated in Figs. 14(c) and 14(d). The STMs of the joint region resembled the corresponding force transfer models, but were slightly different for the two action directions. Additional reinforcement ties in both tee- and knee-joint regions were mobilised in the pull model due to tension demand upon the beam arising from the externally applied action. Again, the reinforcement connecting columns and footings were assumed to be anchored at half the distance given by Eq. (17) away from the bar end. As the beam member of this unit was not prestressed, only narrow concrete struts were mobilised to transfer shear across the joint regions. Furthermore, as the joints were well reinforced, no significant inelastic reinforcement strain was anticipated and concrete damage was expected to occur at the joint surface only. Therefore, $0.51f'_c$ was adopted as the effective strength of joint concrete struts.

The calculated force-displacement response envelopes using the push and pull STMs are illustrated in Fig. 15(b), accompanied by the experimental data. Unit strengths in both push (+ve displacement) and pull (−ve displacement) action directions were captured satisfactorily. Elastic response in the push direction was predicted very accurately but was less precise in the pull direction, particularly after concrete cracking, as indicated by the softened elastic stiffness at a load level of approximately 300 kN.

Relevant analytical events captured by the STMs for both action directions are illustrated on the respective force-displacement response envelope using solid circles, see Fig. 15(b). For the push model, yielding first occurred in the rebars connecting columns and footings, see events 1–2 in Fig. 14(c). This was followed by concrete crushing in the exterior column below the column-joint interface, see event 3, and reinforcement yielding in the cap beam and column in the vicinity of the knee-joint panel region, see events 4–5. Further yielding that propagated up the rebars connecting the columns and footings was predicted by events 7 and 10. Subsequently, significant joint damage due to concrete crushing was suggested by events 6, 8–9 and 11.

For the pull direction mode, yielding first occurred at the rebars in the pin-connections, see events 1–2 in Fig. 14(d). Yielding of column longitudinal reinforcement embedded within both joint regions followed, see events 3–5. Subsequently, concrete crushing in the tee-joint panel region was captured by event 6 and further yielding propagating up the rebars connecting columns and footings was predicted by events 7 and 8. Further analytical data suggested that reinforcement yielding
developed in the beam and column adjacent to the knee-joint panel, see events 9 and 10. Finally, concrete crushing in the knee-joint region was predicted by event 11 at a large displacement ductility level.

The suggested analytical events agreed satisfactorily with the experimental observations, particularly the substantial damage in the joint regions due to significant concrete crushing.

5. Conclusion

The study reported in the current research paper represents an attempt to employ STM for determining the elastic response and strength of reinforced concrete structures.

Analytical results demonstrated that a well formulated STM is capable of assessing the internal force demand, capturing the stiffness in the elastic regime and the strength in the plastic regime. Based upon these findings, it is reasonable to claim that there is considerable potential for combining different phases of the conventional procedure for designing reinforced concrete into a single modelling routine using STMs. The analytical procedure employed in the current study can be used as a blue print to develop practical strut-and-tie design solutions for various reinforced concrete structural types.

Appendix

List of Symbols

\( A_{cs} \) effective area of concrete strut
\( A_{ct} \) effective area of concrete ties
\( A_g \) gross sectional area
\( A_{rs} \) effective area of reinforcement struts
\( A_{rt} \) effective area of reinforcement ties
\( A_s \) total area of tension reinforcement in beams
\( A'_s \) total area of compression reinforcement in beams
\( A_{lt} \) total area of longitudinal reinforcement
\( A_p \) effective area of transverse reinforcement ties
\( A_{ve} \) effective shear area
\( A_{vs} \) total area of transverse reinforcement in a layer in the direction of the shear force
\( b_w \) sectional web width
\( c \) neutral axis depth recorded at 1st yield state from the section force analysis
\( C_{e(max)} \) maximum concrete flexural compression measured prior to section failure
\( d_b \) longitudinal rebar diameter
\( d_{ss} \) sectional diameter of transverse rebar
\( D_c \) total sectional diameter of a circular section
\( D_r \) total sectional depth of a rectangular member section
$D'$  circular concrete core diameter measured from centre to centre of peripheral hoop
$f'_c$  concrete cylinder strength
$f_{cy}$  effective strength of reinforcement struts
$f_d$  effective strength of concrete struts
$f_{dt}$  effective strength of concrete ties
$f'_t$  plain concrete tensile strength
$f_{ty}$  effective strength of reinforcement ties
$f_{vy}$  measured yield strength of transverse reinforcement
$f_y$  measured yield strength of longitudinal reinforcement
$\ell_{dc}$  reinforcement development length
$N$  externally applied axial load
$r_o$  radius of circular concrete core measured from the section centre to the centreline of the longitudinal rebar
$s$  pitch distance of transverse reinforcement
$t$  thickness of imaginary longitudinal steel tube
$V_c$  member shear strength attributed to the concrete tensile capacity
$V_p$  member shear strength attributed to the externally applied action
$V_p$  member shear strength attributed to the transverse reinforcement
$x_c$  longitudinal strut position measured from the compression edge of beam or column section
$x_t$  longitudinal tie position measured from the compression edge of beam or column section
$\alpha$  half angle of the flexural tension sector
$\alpha_N$  angle between the column axis and the line joining the positions of concrete strut at the top and bottom of the column
$\beta$  factor accounting for the shear resistance of cracked concrete
$\phi$  half angle of the compression section
$\phi_o$  effective overstrenth factor only applied for circular columns
$\mu$  structural ductility level
$\sigma_o$  average tensile stress in reinforced concrete
$\theta$  orientated angle of diagonal struts located in beams or columns

**References**


