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**Amplitude Resolution by Human and Ideal Observers
for Rayleigh Noise and Other Gaussian Processes**

A thesis submitted to the University of Auckland
in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Michael J. Hautus
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ABSTRACT

The ability of human observers to discriminate aural differences in the amplitude of sinusoids, narrow-band noise, and wide-band noise was measured by the rating method of detection theory. Although each sinusoid (always 1000 Hz) was presented at a fixed amplitude, its amplitude on any trial was drawn from one of two Rayleigh probability distributions that differed in mean amplitude. Similarly, the amplitudes of the narrow-band noises were distributed as the Rayleigh distribution by virtue of the reciprocal relation between their bandwidth (100 Hz centred on 1000 Hz) and duration (10 ms). The amplitudes of the wide-band noises were distributed as chi with 82 degrees of freedom. A detection-theoretic model based on chi-square density functions was fitted to the obtained receiver operating characteristics (ROCs) and psychometric functions. The best-fitting ROCs required, on the average, 4.1 degrees of freedom for the 100-ms sinusoids and 7.3 degrees of freedom for the Rayleigh noise. The best-fitting psychometric function for both the Rayleigh noise and the sinusoids required about one degree of freedom. The results obtained for these two waveforms were not significantly different. The obtained ROCs for the wide-band noise were well fitted by the chi-square model with 82 degrees of freedom and the psychometric functions with 6.1 degrees of freedom. The best-fitting parameters for the wide- and narrow-band waveforms were significantly different. Furthermore, the performance of the observers on the narrow-band waveforms was closer to that of the corresponding ideal observer than was their performance on the wide-band waveforms. For the narrow-band noise and sinusoids, the duration of the waveform affected the variability of the best-fitting degrees of freedom for the obtained ROCs. This variability was greater for waveforms of 10-ms duration (Rayleigh noise and sinusoids) than for sinusoids of 100-ms duration; this greater variability was interpreted as stemming from the difficulty of correctly windowing short-duration waveforms. Sound pressure level had no effect on the amplitude resolution of observers for four different levels of Rayleigh noise and of sinusoids. Therefore Weber's Law, and not the near miss to Weber's Law, holds for these waveforms. The chi-square detection-theoretic model provided a good fit to the data from all experiments, but not as good as predicted by statistical theory. It was concluded that the chi-square model provides a close approximation to the underlying mechanisms involved in the amplitude resolution of Gaussian noise processes.

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